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Financial Frictions, Occupational Choice and Economic Inequality*

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Abstract

The Lucas (1978) model is extended to incorporate heterogeneity in working ability and a time allocation decision by entrepreneurs (work versus manage). Financial frictions distort not only the average skill of entrepreneurs but also the average skill of workers. The model economy accounts for half of the association between entrepreneurship and external finance to GDP in the data, whereas a standard span of control model explains only about one tenth. The variation in entrepreneurship is mostly due to the variation in self-employed entrepreneurs rather than in employers. Moreover, financial frictions have larger effects on output per worker, TFP, and inequality.

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1 Introduction

A theory of entrepreneurship that distinguishes between entrepreneurs that are self-employed and those that are employers is built in order to study the impact of financial frictions on occupational sorting, output, and inequality. The importance of modelling this distinction is motivated by cross-country evidence on the negative association between entrepreneurship and the ratio of external finance to GDP across countries (see Figure 1). While the fraction of employers tends to be slightly increasing with financial development, the fraction of self-employed households is strongly negatively associated with external finance to GDP. As a result, the evidence suggests that the variation in self-employed entrepreneurs is crucial for understanding the negative relationship between entrepreneurship and financial development in the cross-country data.

Evidence from Brazil on household earnings inequality between and within occupations points that the mean earnings of entrepreneurs is not higher than that of workers.[1] Moreover, Figure 2 shows that there is substantial earnings heterogeneity within all occupations (workers, employers, and self-employed) so that there are households at the top and bottom of the income distribution in all of these occupations. The standard span of control model with heterogeneity in one skill cannot account for these observations.

The standard Lucas (1978) framework is extended by assuming that agents are heterogeneous in managerial and working skills, as in Jovanovic (1994), and by modeling a time allocation decision in which entrepreneurs divide their time between managing their busi-

[1] In Brazil, mean earnings are lower for self-employed entrepreneurs than for workers, and are higher for employers than for workers. These facts apply to other countries as well. See Hamilton (2000), Hurst and Pugsley (2011), Poschke (2013), Vissing-Jorgensen and Moskowitz (2002) for papers documenting facts on entrepreneurial returns in the US.

nesses and working. Households choose how much to consume, save and whether to be a worker or an entrepreneur (self-employed or an employer). Entrepreneurs decide how much time to devote to the supply of labor and/or managerial inputs and how much (outside) labor to hire. Financial frictions are modeled with a collateral constraint limiting the amount of capital entrepreneurs can use, as in Buera and Shin (2013) and Moll (2014).

The theory implies that some entrepreneurs will rely only on their own labor and managerial inputs and become self-employed, while other entrepreneurs will decide to hire labor services in the market and become employers. By modeling heterogeneity in the form of two skills the theory can distinguish between a comparative advantage in entrepreneurship (a high ratio of managerial to working skills) and an absolute advantage (a high value of both skills). Heterogeneity in absolute advantage implies that both at the top and bottom of the income distribution there are entrepreneurs and workers, which is needed for the theory to be consistent with household evidence on earnings inequality between and within occupations.

The model economy is calibrated to Brazilian household and aggregate data. The skill distribution and its evolution over time is chosen to match moments on occupational choices, occupational transitions, and earnings inequality between and within occupations. To benchmark the results, we also calibrate a version of the model economy in which households only differ in their managerial skills and entrepreneurs do not face a time allocation problem, as in the Lucas (1978) model.

One novel implication of the theory in this paper is that financial frictions distort not only the productivity of entrepreneurs but also the productivity of workers. In the baseline economy, as financial frictions are reduced the changes in the equilibrium sorting of workers are such that the productivity of employers increases and that of workers decreases. The first effect increases labor demand and the second one decreases labor supply in *effective units* of labor. This asymmetric effect is associated to a large increase in the number (quantity) of workers in order to equate labor supply and labor demand, which works through a large reduction in the fraction of self-employed households. As a result, the elimination of financial frictions causes the rate of entrepreneurship to decrease much more in the baseline economy (from 32% to 12%) than in the standard model economy (from 10.5% to 7%).

The quantitative performance of the theory is assessed by computing, for the two calibrated model economies, equilibrium allocations that differ on financial frictions. We find that the baseline model accounts for half of the association in the cross-country data between entrepreneurship and external finance to GDP while the standard model explains only about one tenth. The baseline model also explains the fact that the fraction of employers shows a mildly positive association with the external finance to GDP ratio in the cross-country data and that variation in entrepreneurship is mostly due to the variation in self-employment rates. In the baseline economy, a reduction in financial frictions shifts households from self-employment into paid work, thereby increasing the share of employee compensation in national income. This prediction of the theory is consistent with the cross-country data. Quantitatively, the baseline economy accounts for about 30% of the empirical association between the ratio of employee compensation to GDP and the ratio of external finance to GDP. The standard model cannot account for this evidence because it abstracts from self-employment.

The output gain of eliminating financial frictions in the baseline economy is much larger than the one in the standard model economy (58% versus 47%). Crucial to this result is that the decrease in the fraction of entrepreneurs is more than 5 times bigger in the baseline economy than in the standard model economy (0.20 versus 0.035), implying that the reallocation of capital and labor towards the most productive entrepreneurs is more important in the baseline economy than in the standard model. The larger reallocation of capital towards a smaller number of entrepreneurs implies a larger credit expansion in the baseline economy, leading to a higher increase in output.

Given that the baseline economy provides a good account of household inequality in Brazil, the impact of financial frictions on household inequality is evaluated. The elimination of financial frictions in the baseline model economy leads to a substantial reduction in the inequality across households, decreases the persistence of income over time and the correlation of earnings and assets. Interestingly, the most important effect of eliminating financial frictions on inequality is not on the reduction of inequality *per se* but on the sources that drive inequality. The elimination of financial frictions allow talented households to obtain the full reward to their skills increasing the proportion of the income variance explained by skills.

The paper relates to a large literature in macroeconomics investigating the consequences of financial frictions on aggregate output, entrepreneurship, and productivity.[2] Relative to this literature, we show that self-employment and heterogeneity in working skills are important for understanding the strong negative association between entrepreneurship and financial development in the cross-country data as well as the output and productivity gains of removing financial frictions.

Jovanovic (1994) extends the Lucas model with two dimensional skill heterogeneity to

[2] See, for instance, Erosa (2001), Erosa and Hidalgo-Cabrillana (2008), Jeong and Townsend (2007), Buera and Shin (2011), Buera et al. (2011), Greenwood et al. (2013), Midrigan and Xu (2014) and Moll (2014). Buera et al. (2015) provide a recent and comprehensive survey of this literature. Also, see Quadrini (2009) for a survey of papers on entrepreneurship.

show that entrepreneurs are not necessarily drawn from the top of the ability distribution. Our paper extends the analysis in Jovanovic by considering a dynamic model with financial frictions and a time allocation problem of entrepreneurs. D'Erasmo and Boedo (2012), Hurst and Pugsley (2011), Poschke (2013), Yurdagul (2017) develop theories of entrepreneurship with small businesses.[3] None of these papers account for own-account workers.

2 The model

Consider a closed economy that is on steady state and is populated by a large number (normalized to 1) of infinitely-lived households. Households are heterogeneous in wealth and in working (z_w) and managerial skills (z_m). They face uninsurable idiosyncratic shocks to their skills and maximize expected discounted utility over stochastic consumption sequences.

2.1 Production technology

Following Lucas (1978), output is produced with a constant returns to scale production technology in managerial, labor, and capital inputs. Entrepreneurs can only use their managerial input because there is no market for managers. The supply of the managerial input is equal to the product of the households' managerial ability (z_m) and the time devoted to managing a business (t_m). The output produced by a household supplying $m = z_m t_m$ units

[3] Gollin (2008) emphasizes the dominant presence of small businesses in developing economies. Hurst and Pugsley (2011) and Yurdagul (2017) study, respectively, the role of non-pecuniary benefits of entrepreneurship and a preference for flexible work hours in explaining the low entrepreneurial returns. Poschke (2013) emphasizes the option value of entrepreneurship.

of managerial input and using k units of capital and n efficiency units of labor is:

$$Y(m,k,n) = m^{\gamma} k^{\eta} n^{\theta}, \text{ where } \gamma + \eta + \theta = 1. \quad (1)$$

The time allocation decision of entrepreneurs ($t_m \in [0,1]$) is modeled to introduce self-employment (own-account workers) in the Lucas (1978) framework. When $0 < t_m < 1$ entrepreneurs supply *both* managerial and labor inputs to their own businesses. Specifically, the labor input supplied by entrepreneurs to their business is equal to the product of their working ability (z_w) and the time devoted to non-managerial activities ($1 - t_m$). The total labor input used by an entrepreneur is the sum of the labor supplied by the entrepreneur ($(1-t_m)z_w$) and the labor hired in the market (n_d) from workers outside the family, $n = n_d + (1-t_m)z_w$ where z_w is the working ability of the household. All entrepreneurs choose $t_m > 0$ because the managerial input is needed for production. Entrepreneurs can be partitioned into two subgroups depending on whether they hire outside labor or not. The first subgroup is given by the employers, who are those entrepreneurs hiring labor outside the family ($n_d > 0$). Entrepreneurs that hire outside labor are assumed to incur a fixed per period operating cost of c_e.[4] The second subgroup are the self-employed, comprising entrepreneurs that only use their own household labor input ($n = (1-t_m)z_w$ and $n_d = 0$). Workers are those households who use all their time as workers ($t_m = 0$), obtaining labor earnings wz_w.

[4]The fixed cost is introduced so that employers demand a non-trivial amount of labor (an amount bounded away from zero), thereby making the distinction between self-employed and employer meaningful.

2.2 Stochastic process on skills

Household's skills evolve stochastically over time and there are no insurance markets to insure skill risk. The logarithm of skills of household i is governed by

$$ln(z_{wit}) = \alpha_{wi} + u_{wit}, \tag{2}$$

$$ln(z_{mit}) = \alpha_{mi} + u_{mit} + \zeta_{it}, \tag{3}$$

where z_{wit} (z_{mit}) denotes the working (managerial) skills at date t, α_{wi} and α_{mi} represent household fixed effects on working and managerial productivities, ζ_{it} is a transitory shock to managerial ability, and u_{wit} and u_{mit} denote persistent shocks to skills. The latter shocks follow a first order auto-regressive process

$$u_{jit} = \rho_j u_{jit-1} + \epsilon_{jit}, \text{ for } j = w, m, \tag{4}$$

with $\epsilon_t = (\epsilon_{wt}, \epsilon_{mt})$ jointly drawn from a bivariate normal distribution with correlation coefficient $corr(\epsilon_{wt}, \epsilon_{mt}) = \rho_{\epsilon_{w,m}}$.

The transitory shock ζ_{it} on managerial ability is drawn every period from a normal distribution with mean zero and variance σ_ζ^2. We assume that each period there is a (small) probability that households draw new skills from the invariant distribution of abilities. In particular, with probability p_α the household draws a new pair of fixed effects $(\alpha_w^{'}, \alpha_m^{'})$ from a fixed bivariate distribution $F_{(\alpha_w, \alpha_m)}$.

2.3 Capital markets and occupational choice

The financial intermediation industry is assumed to be competitive. Intermediaries take deposits from households and pay the interest rate r. They rent capital to entrepreneurs at a rate $r + \delta$. Enforcement problems limit the capital rented to entrepreneurs. Following Buera and Shin (2013) and Moll (2014), among others, the capital rented by entrepreneurs is limited by a collateral constraint $k \leq \lambda a$, where a denotes household assets. The parameter $\lambda \geq 1$ determines the degree of financial frictions, with $\lambda = \infty$ indicating perfect financial markets and $\lambda = 1$ an economy with no credit.

Given steady state factor prices (w, r), the profit made by an entrepreneur in state (a, z_w, z_m) is given by the function

$$\pi(a, z_m, z_w) \equiv \max_{k,n,0 \leq n_d, 0 \leq t_m \leq 1} \{m^\gamma k^\eta n^\theta - wn_d - rk - \delta k - c_e \mathrm{I}_{n_d > 0}\} \tag{5}$$

$$\text{s.t. } k \leq \lambda a, \quad m = t_m z_m, \quad n = (1 - t_m) z_w + n^d,$$

The household problem is given by

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} E \left\{ \sum_{t=0}^{\infty} \beta^t \ln(c_t) \right\} \tag{6}$$

$$\text{s.t. } c_t + a_{t+1} = \max\{wz_w, \pi(a, z_m, z_w)\} + (1 + r)a_t, \ a_0 \text{ given},$$

where the max above represents the occupational choice decision.

3 Time Allocation and Occupational Maps

We now study, in partial equilibrium (e.g. for a fixed wage rate and interest rate), how the theory can give rise to three active occupational choices: worker, self-employed, and employer. When capital markets are perfect, occupational choices are entirely determined by the ability ratio $\frac{z_w}{z_m}$. Individuals with a high $\frac{z_w}{z_m}$ ratio have a comparative advantage at working and choose to become workers, individuals with a low $\frac{z_w}{z_m}$ ratio have a comparative advantage at entrepreneurship and choose to become employers, and those with intermediate skill ratios prefer to be self-employed. The discussion below also characterizes how tight borrowing constraints distort occupational choices and the rates of return to skills (see Appendix A for the analytical derivations).

Consider the determinants of self-employment income. Self-employed individuals choose how much time to allocate to managerial versus working activities and how much capital to use in production. The marginal product of their time is equated across its two uses (managerial and working time) and satisfies:

$$MPT_{se} = r_{mw}(\mu)\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}, \quad \text{where } r_{mw}(\mu) = \left[\frac{\gamma^{\gamma}\eta^{\eta}\theta^{\theta}}{(r+\delta+\mu)^{\eta}}\right]^{\frac{1}{1-\eta}} \tag{7}$$

is the rate of return to the composite skill input $\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}$ and μ is the Lagrange multiplier associated with the borrowing constraint.[5] The marginal product of the self-employment time is the product of the skill composite $\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}$ and the rate of return r_{mw}. Note that

[5] Although the tightness of the borrowing constraint μ is a function of asset holdings a and skills (z_m, z_w), we do not explicitly reflect this in our notation (e.g. write $\mu(a, z_m, z_w)$) to simplify the exposition.

the return to the skill composite decreases with μ. The income of a self-employed individual with assets a is given by

$$y_{se} = r_{mw}(\mu) \left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}} + \mu k + ra, \tag{8}$$

where $k = \lambda a$ and μ vary across the self-employed with different characteristics (z_m, z_w, a).

Now consider the decisions of employers. The marginal product of employers' time (MPT_e) can be expressed as the product of managerial skills z_m and the rate of return $r_m(\mu)$ on the employer's managerial skill:

$$\begin{aligned} &MPT_e = z_m r_m(\mu) \geq z_w\, w \text{ with strict inequality if } t_m = 1, \text{ where} \\ &r_m(\mu) = \gamma \left[\left(\frac{\eta}{(r+\delta+\mu)} \right)^{\eta} \left(\frac{\theta}{w} \right)^{\theta} \right]^{\frac{1}{1-(\eta+\theta)}} \end{aligned} \tag{9}$$

Note that $r_m(\mu)$ is the rate of return to the managerial input z_m and μ is the Lagrange multiplier associated to the borrowing constraint.[6] The income of an employer with ability (z_m, z_w) and assets a is given by

$$y_e = z_m r_m(\mu) + \mu k + ra - c_e, \tag{10}$$

where $k = \lambda a$. Note that borrowing constraints (μ) generate heterogeneity in rates of return to skills among employers. The results below characterize the occupational choice decisions in an economy with perfect enforcement.

[6] In our calibrated economy 90% of employers choose $t_m = 1$, and the difference in aggregate managerial input $(t_m * z_m)$ and aggregate managerial ability (z_m) of employers is 1%.

Proposition 1: Perfect Enforcement. *Assume that* $\lambda = \infty$. *Let* $R_1^* \equiv \left(\frac{r_{mw}^*}{w}\right)^{\frac{\theta+\gamma}{\gamma}}$ *and* $R_2^* \equiv \left(\frac{r_m^*}{r_{mw}^*}\right)^{\frac{\theta+\gamma}{\theta}}$, *where* r_{mw}^* *and* r_m^* *are the rates of return to the skill composite* $\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}$ *and the managerial skill defined in* (7) *and* (9) *when* $\mu = 0$. *i) The rates of return on skills do not vary across individuals* ($r_m = r_m^*, r_{mw} = r_{mw}^*$) *so that income inequality is all due to heterogeneity in skills and assets. ii) If there is no fixed cost of operation* ($c_e = 0$)*, then the optimal occupational choice is the one that maximizes the marginal product of time and is only determined by the skill ratio* ($\frac{z_w}{z_m}$) *as follows: Individuals with an ability ratio* $\frac{z_w}{z_m} > R_1^*$ *work for a wage, individuals with* $R_1^* > \frac{z_w}{z_m} > R_2^*$ *are self-employed, and those with* $\frac{z_w}{z_m} < R_2^*$ *are employers. iii) If the fixed cost of operation is positive* ($c_e > 0$)*, the decision to be an employer depends on the skill ratio* ($\frac{z_w}{z_m}$) *and on the absolute level of managerial ability* (z_m)*. Individuals prefer to become employers relative to self-employment when* $\frac{z_w}{z_m} < R_2^*(1 - \frac{c_e}{z_m r_{mw}^*})$.

When there is perfect enforcement (and the fixed cost of operation is zero), occupational choices can be characterized in terms of two skill ratios R_1^* and R_2^*. Individuals with comparative advantage at working (an ability ratio $\frac{z_w}{z_m} > R_1^*$) work for a wage, individuals with $R_1^* > \frac{z_w}{z_m} > R_2^*$ are self-employed, and those with comparative advantage at managing ($\frac{z_w}{z_m} < R_2^*$) are employers. A positive fixed cost of operation ($c_e > 0$), implies that employers require a minimum scale in order to operate a profitable business so that the decision to be an employer depends both on the skill ratio $\frac{z_w}{z_m}$ and on the level of managerial ability z_m. Proposition 2 characterizes occupational choice decisions when borrowing constraints bind (λ finite).

Proposition 2: Imperfect Capital Markets *Assume that* $\lambda < \infty$ *and consider individuals for which the borrowing constraint binds* ($\mu > 0$)*. i) The rates of return on* *skills (r_m, r_{mw}) vary across individuals and decrease with μ. Skill returns positively covary with asset holdings. Income inequality is due to heterogeneity in skills, assets, and rates of return. ii) Occupational choices depend on the skill ratio, assets, and the absolute level of skills (the last two affect the rates of return to skills because they determine the tightness of the borrowing constraint).*

Capital market imperfections distort returns to skills and, thus, occupational choices. A tight borrowing constraint depresses the rate of return to the managerial ability of employers and the return to the composite skill input supplied by self-employed individuals. It also increases the rate of return to capital faced by entrepreneurs. For fixed skills (z_m, z_w), an increase in assets makes the borrowing constraint less tight (μ), increasing the rates of return to skills and depressing the shadow cost of capital. As a result, asset holdings matter importantly for occupational choice decisions in the presence of financial frictions. Panel a in Figure 3 shows how occupational choices vary with skills in our calibrated model economy for an individual with assets equal to the median in the economy. Panel b draws the occupational map when perfect enforcement is introduced in the calibrated model economy. A comparison of the occupational maps, reveals that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices.

4 Calibration

The baseline model economy is calibrated to assess the quantitative implications of the theory. To benchmark the results, a version of the model economy in which entrepreneurs use all their time to manage their businesses and households are not heterogeneous in working skills is also calibrated. This version of the model economy is labelled as the “standard model economy”.

To simplify the calibration, some parameters are set (externally) using estimates from other studies in the literature. In particular, for the two calibrated model economies, the parameters of the production function are set to standard values in the literature: $\gamma = 0.198, \eta = 0.3256, \theta = 0.4764$ (see Guner et al. (2008)). The model period is set to a year and the annual depreciation rate of capital is set to $\delta = 0.06$. The rest of the parameters are assigned by solving the model economy and minimizing a loss function. The loss function consists of the sum of square deviations between key moments in the model economy and in the Brazilian data.

The household level data used to calibrate the model comes mainly from the Monthly Labor Survey (PME for its name in Portuguese, *Pesquisa Mensal de Emprego*) for the periods 2003-2010. The PME has a sampling structure similar to the CPS in the US: selected households are interviewed for four months, stay out of the sample for the next eight and then they are interviewed for four months again before leaving the sample. This structure allows us to exploit the short panel dimension to compute transitions of earnings and occupations. The calibration also targets consumption data from the Household Expenditure Survey (POF for its name in Portuguese *Pesquisa de Orçamento Familiar*) for the period 2008/2009.[7]

[7]See Appendix C for further details about the datasets.

4.1 Calibration of the baseline economy

The managerial fixed effects (α_m) are assumed to be drawn from a standard Pareto distribution with tail parameter η_m. The location of the Pareto distribution is shifted with a parameter μ_m that controls the mean value of managerial fixed effects. The Pareto distribution is discretized and the first grid point is set to the one that accumulates probability mass $p_{\underline{m}}$. The fixed effects on working ability are assumed to be drawn from a Normal distribution with variance σ^2_{wf} and a mean value that depends on the realization of the managerial fixed effect according to a parameter $\rho_{\alpha_{mw}}$ that controls the correlation of fixed effects on working and managerial skills.[8]

In total 13 parameters are calibrated internally by minimizing the distance between moments in the model and 15 moments in the data. While the moments targeted will all be jointly determined by all of these parameters, in presenting the calibration procedure it is useful to associate to each target a specific parameter.

The calibration targets the following moments in the Brazilian data: First, the mean fixed effect on managerial ability (μ_m) and the fixed cost of employers (c_e) to match the occupational choice structure with 68% of workers, 24% of self-employed, and 8% of employers (2 independent targets). Second, the variance of shocks to working ability ($\sigma^2_{\epsilon_w}$) and the Pareto tail on fixed effects on managerial ability (η_m), to match the variance of log-earnings of workers and entrepreneurs with values of 0.76 and 1.06 (2 targets). Third, the variance and serial correlation of innovation to managerial skills ($\sigma^2_{\epsilon_m}, \rho_m$), the correlation between innovations to working and managerial skills ($\rho_{\epsilon_{m,w}}$) and the transitory shock to manage-

[8]Appendix B presents a detailed description of how the discretization of the skill distribution as well as a discussion of the calibration strategy. It also reports on the performance of the model economy on non-targeted dimensions.

rial ability (σ_ζ^2) to match the yearly transition rates across occupations (worker to worker, worker to employer, self-employed to self-employed, self-employed to employer, employer to self-employed, employer to employer) (6 independent targets). Fourth, the correlation of fixed effects on working and managerial skills ($\rho_{\alpha_{m,w}}$) and the probability of the lowest fixed effect on managerial skill ($p_{\underline{m}}$) to match the difference in mean log-earnings between employers and workers (0.66), between employers and self-employed (0.94), and between workers and entrepreneurs (0.04) (2 independent targets). Fifth, the probability of drawing new fixed effects (p_α) to match the variance of log consumption of 0.80 (1 target). Sixth, the parameter on the collateral constraint (λ) to match the credit to (anual) GDP ratio of 42% (1 target).[9] Seventh, the discount factor (β) to match the capital to output ratio of 2.1 (1 target).[10]

4.2 Calibration of standard model economy

Following Buera and Shin (2011), the shocks to managerial skill are specified to follow an AR(1) process. The calibration involves choosing four parameters to match four moments in the data: (i) the serial correlation of managerial shocks to match the persistence of entrepreneurship between two consecutive years of 87%; (ii) the variance of innovations to managerial shocks to match the variance of log entrepreneurial income of 1.06; (iii-iv) λ and β to match the target for the credit to GDP ratio (42%) and the capital to income ratio in the Brazilian data (2.1).

[9] See The World Bank (2017).

[10] The capital to output ratio was computed following the same methodology as in Greenwood et al. (2013) using the Penn World Tables 7.0 (see Heston et al. (2011)), and corresponds to the average for the year 2004-2010.

4.3 Calibration results

The parameter values and the calibration results for the baseline economy are reported in Panel A of Tables 1 and 2. Panel B in these tables reports similar statistics for the standard model economy. While the two calibrated model economies match the calibration targets well, the baseline economy provides a much better account of the occupational structure, income differences *between* and *within* occupations, and household inequality in Brazil.

The baseline model economy matches perfectly the occupational structure in Brazil: It matches the overall entrepreneurship rate in Brazil of 32% and the composition of entrepreneurs between self-employed household (24%) and employers (8%). These observations stand in contrast to the standard model economy, which implies an entrepreneurship rate less than a third the one in the data (10%) and cannot account for self-employment. The baseline model can match the fraction of self-employed households in the data with modest fixed cost of operation by employers. The fixed cost of operation faced by employers in the baseline economy represents 0.9% of GDP.[11]

The baseline model quantitatively accounts for the substantial heterogeneity in earnings *within* and *between* occupations. The variance of log-earnings among workers and entrepreneurs are 0.76 and 1.07 in the baseline economy (versus 0.76 and 1.06 in the data). The theory is also consistent with observed earnings inequality between occupations. Mean earnings are higher for employers than for workers, and are the lowest for self-employed households. The difference in mean log-earnings between employers and workers is 0.66

[11] Alternatively, the fixed cost represents 0.7% of the mean output of employers or 18% of the mean wage in the economy. Buera et al. (2011) calibrate a fixed cost of operation for firms in the manufacturing sector that is about 3 times the average wage. In Midrigan and Xu (2014) the fixed costs of operation represent 4.6% of GDP, which is about 5 times larger than the one we obtain.

(0.66 in the data) and between employers and self-employed is 0.93 (0.94 in the data). Self-employed households make on average 28% less earnings than workers (29% in the data). To match these facts on earnings differences across occupations, the theory requires a positive but moderate correlation of managerial and working skills (the correlation of fixed effects is 0.145).

By design, the standard model economy cannot account for the substantial earnings heterogeneity across workers in the data because it assumes away heterogeneity in working ability. Moreover, because it cannot distinguish between comparative advantage and absolute advantage at entrepreneurship, it largely overstates earnings differences between occupations: The model implies a difference in mean log-earnings between entrepreneurs and workers of 1.35 versus the value of -0.04 in the data. On the other hand, the model generates too little inequality across households since the variance of log consumption is 0.32 relative to 0.80 in the data.

The baseline economy is consistent with key data patterns on occupational transitions, though the match is not perfect. First, the model predicts that being a worker is quite persistent: 94% of workers in the model economy and in the data are workers one year later. Second, both in the baseline economy and in the data, entrepreneurs are less likely to remain in their occupations than workers. The model matches pretty closely the persistence of self-employment in the data (75% versus 77%) but overstates the persistence of employers (76% vs 68%). Third, the baseline model economy matches the fact that households are much more likely to transit into employer from self-employment than from being a paid worker. The flow of self-employed households into employers is 8% in the data and 7% in the model. The annual transition rate from worker into employer is quite low both in the model and in the data (1% in the data and negligible in the model). Fourth, the model is consistent with the fact that employers are much more likely to switch to self-employment than paid work.

Summing up, the calibrated model economy with heterogeneity in two skills provides a reasonable account of occupational choices, income inequality within and across occupations, occupation transitions, and household inequality in Brazil. In Appendix B, we evaluate the model economy along many non-targeted dimensions.[12]

5 Financial frictions, entrepreneurship, and the cross-country data

The quantitative performance of the theory is assessed by computing, for the two calibrated model economies, the equilibrium allocations for economies that differ in the parameter λ determining the strength of financial frictions.[13]

Figure 4a plots the relationship between the entrepreneurship rate and the ratio of external finance to GDP in the model-simulated data and in the cross-country data. The short-dashed line corresponds to the data generated from the baseline economy, the long-dashed line from the standard model economy, and the solid line to the actual cross-country data. Relative to the data, the rate of entrepreneurship predicted by the standard model economy is too low and varies too little with the level of financial development. The range

[12] In particular, the calibrated model economy is consistent with (i) the fact that within group (occupation) inequality explains 96% of the variation of earnings in the data; (ii) non-targeted moments of the earnings distribution; (iii) salient features regarding mean earnings gains/losses for workers/employers/self-employed households switching to other occupations versus those that do not switch; (iv) measures of consumption inequality as well as differences in mean consumption across households in different occupations.

[13] Equilibrium is computed for 19 economies with λ varying in the interval (1,2500). The grid for λ is not equally spaced since the grid size is smaller when λ is close to 1. Equilibrium statistics do not change for values of $\lambda > 2500$. The economy with $\lambda = 2500$ is interpreted as an economy with perfect credit markets.

of variation of the entrepreneurship rate in this model goes from 7% to 11.5%. The baseline model economy accounts much better for the relationship between entrepreneurship and financial development in the cross-country data. The range of variation of entrepreneurship predicted by this model economy goes from 12% to 33% (which is about 5 times the one predicted by the standard model). To gain some quantitative sense on the performance of the model economies relative to the data, the rate of entrepreneurship is regressed on external finance to GDP in the cross-country and model simulated data. We find that the regression coefficient of external finance is -0.11 in the baseline economy, -0.025 in the standard model, and -0.23 in the data. The baseline economy accounts for half of the variation in the data which is quite impressive giving that only one factor (financial frictions) is varying across economies. The standard model on the other hand explains only about one tenth of the response of entrepreneurship to changes in external finance to GDP.

In the baseline economy the fractions of employers and self-employed households vary in opposite directions with financial development. While the rate of self-employed households is strongly decreasing with the level of financial development, the fraction of employers shows a mildly positive relationship with external finance to GDP ratio (see Figures 4b and 4c).[14] Both of these implications are validated by the data: first, the regression coefficient of the fraction of employers on external finance to GDP is 0.01 both in the model and in the data. Second, the regression coefficient of the fraction of self-employed households on credit to GDP ratio is -0.12 in the model and -0.24 in the data.

In the baseline economy, a reduction in financial frictions shifts households from self-

[14] The model economy was calibrated to Brazil. Since the fraction of employers in Brazil is well above the implied by the regression line in the cross-country data, the regression line for the model is above the one in the data.

employment into paid work, thereby increasing the share of employee compensation in national income. To test this prediction of the theory, Figure 5 graphs the relationship between the ratio of employee compensation to GDP and the ratio of external finance to GDP in the model-simulated data and in the cross-country data. The baseline model economy is consistent with the fact that these two variables are positively associated in the cross-country data. The standard model cannot account for this evidence because it abstracts from self-employed households. Quantitatively, the baseline economy accounts for about 30% of the empirical association between the ratio of employee compensation to GDP and the ratio of external finance to GDP.

Consider the effects of financial frictions in output per worker and TFP.[15] The coefficient of the regression of output per worker on credit to GDP is 0.23 for the baseline model, 0.24 for the standard model, and 0.34 for the data. In a similar exercise Buera et al. (2011) find a regression coefficient of 0.22 in their model economy. Hence, consistently with their analysis, we conclude that the model accounts for about two thirds of the cross-country relationship between output per worker and financial development. The regression coefficient of TFP on the external finance to GDP ratio is 0.12 for both the baseline and the simple model, and 0.26 for the cross-country data. Hence, the two calibrated model economies account for about 50% of the cross-country association between TFP and financial development. Nonetheless, the next section of the paper shows that modeling heterogeneity in two skills amplifies substantially the estimates of the gains in output per worker and TFP of removing financial frictions in Brazil. It also amplifies the effect of financial frictions on household

[15] TFP is measured as in standard growth accounting exercises $TFP = \frac{Y}{K^{\frac{\eta}{\eta+\theta}}}$, where Y denotes aggregate output (net of fixed costs) and K aggregate capital.

inequality.

6 The impact of removing financial frictions in Brazil

This section studies in detail the impact on entrepreneurship, resource allocation and production efficiency of removing financial frictions in Brazil. This is done by comparing some key model statistics in the baseline economy with those of an identical economy with perfect credit markets.

6.1 Output, credit, and entrepreneurship

The elimination of financial frictions leads to a much larger increase in output in the baseline economy than in the standard model economy (58% versus 47%, as shown in columns a and b in Table 3). In understanding this result, it is useful to decompose the impact of financial frictions in output in two terms: how much output changes in response to an increase in the credit to GDP ratio and how much the credit to GDP ratio changes in response to a decrease in financial frictions. The first effect was estimated in the previous section of the paper where it was found that the two calibrated model economies imply similar regression coefficients on credit to GDP on output. However, the second effect is much stronger in the baseline model economy: The elimination of financial frictions leads to an increase in the credit to GDP ratio of 1.53 in the baseline economy and of 1.28 in the standard model economy, explaining why the increase in aggregate output is larger in the former economy.

The key to this result is that the decrease in the fraction of entrepreneurs is more than 5 times bigger in the baseline economy than in the standard economy (0.20 versus 0.035).

The larger reduction in the number of entrepreneurs in the baseline economy is associated with a larger increase in the average productivity of entrepreneurs. While the mean skill of entrepreneurs increases by about 200% in the baseline economy, it increases by about 50% in the standard economy. The reallocation of capital and labor towards the most productive entrepreneurs is thus stronger in the baseline economy which, in turn, accounts for the larger credit expansion in this economy.

6.2 Resource allocation and TFP

Using the production function of the model economy, the efficiency of production is measured as $TFP_{ideal} = Y/K^{\eta}N^{\theta}$, where Y denotes the aggregate output (net of fixed costs) and (K, N) the aggregate capital and labor input. The gains in production efficiency are much larger in the baseline economy than in the standard model economy (26% versus 21%) despite the fact that the two model economies imply the same regression coefficient of external finance relative to GDP on TFP_{ideal} (0.15). Again, this finding is explained by the larger credit expansion in the baseline economy. We also measure TFP as in standard growth accounting exercises: $TFP_{data} = Y/K^{\frac{\eta}{\theta+\eta}}$. The changes in measured TFP are smaller than the ones in TFP_{ideal} for both economies (23% in the baseline economy versus 19% in the standard economy). Hence, standard TFP measurement underestimates the true production efficiency gains by 3 percentage points in our baseline economy and by 2 percentage points in the standard economy.

The baseline economy has novel implications on how financial frictions affect output and production efficiency by employers and self-employed entrepreneurs. In the baseline economy, the elimination of financial frictions changes dramatically the contribution to aggregate output from employers and self-employed entrepreneurs. While the aggregate output of employers increases by 80 percent, the aggregate output of the self-employed decreases by 87 percent. As a result, the share of aggregate output produced by self-employed households decreases from 13 percent in the baseline economy to 1 percent in the economy with no financial frictions.

The elimination of financial frictions leads to large production efficiency gains by employers (28%) and large losses by self-employed households (-14.3%). Standard growth accounting measures of TFP underestimate the gains by employers and the losses by self-employed entrepreneurs (14% relative to 28% for employers and -11% relative to -14.3% for self-employed). These differences are large because there are large changes in the allocation of capital and employment across the two types of entrepreneurs. The large increase in the capital and employment allocated to employers reduces their measured TFP gains relative to their true productivity gains and the opposite occurs for the self-employed.

6.3 Financial frictions and inequality

The elimination of financial frictions leads to a reduction of the Gini coefficient in assets from 0.91 to 0.84, decreases the Gini coefficient of consumption from 0.60 to 056, reduces the auto-correlation of earnings from 0.89 to 0.80, and decreases the correlation of earnings and assets from 0.71 to 0.38. These effects are present in the standard model economy but to a lesser extent. Interestingly, financial frictions have a large effect on the *sources* that drive inequality.[16]

Table 4 decomposes the variance of income that is due to the variance of capital income, returns to skills, and their covariance. Financial frictions diminish the inequality in skill returns by limiting the rents that talented individuals obtain from their skills. Since entrepreneurs with high managerial skills need to operate at a large scale to obtain all the potential rents from their skills, borrowing constraints diminish the rate of return attained by these individuals (see Proposition 2). The elimination of financial frictions allows talented individuals to obtain the full reward to their skills.

The variation in skill returns explains 20% of the income variance in the baseline economy whereas it accounts for 64% of the income variance in the economy with no financial frictions (see Table 4). Capital income (directly) accounts for 31% of the income variance in the baseline economy, which is about twice the 12% figured in the PCM economy.

6.4 Sensitivity analysis

The sensitivity of the results to changes in some key parameters and modeling choices is now briefly examined. In a first experiment, the variance of working skills in the baseline economy is set to zero. This experiment allows to isolate how modeling the entrepreneurial time allocation decision matters for the quantitative impact of financial frictions vis a vis the effect of heterogeneity in working skills. The baseline economy assumes that employers pay a fixed cost of operation. In a second experiment, it is examined how the results vary

[16] In the standard model, the elimination of financial frictions diminishes the Gini coefficient of assets from 0.93 to 0.89, the Gini of consumption from 0.386 to 0.364, and the correlation of earnings and assets from 0.71 to 0.41. To measure the sources of inequality, returns to skills are computed as the total rents on working and managerial skills: wz_w for workers, $r_m z_m$ for employers, and $r_{mw} z_{mw}$ for self-employed, as defined in Section 3.

when this cost is set to zero or when it is expressed in terms of the wage rate (so that an increase in the equilibrium wage rate rises the cost of operation). The results are presented in columns c to e in Table 3.[17]

When financial frictions are eliminated in the economy with no heterogeneity in working skills, the decrease in the fraction of self-employed workers is 0.09 which is about a half the one in the baseline economy (0.20). Hence, modeling the entrepreneurial time allocation decision and heterogeneity in working ability are both important for the quantitative response of entrepreneurship to financial frictions.

The large credit expansion (associated to the large reduction of the entrepreneurship rate) explains why aggregate output and TFP increase more with the removal of financial frictions in the baseline economy than in the standard model economy. Table 3 offers some interesting clues for understanding this result. Financial frictions affect labor demand and labor supply through changes in the skill distribution of workers and entrepreneurs. The elimination of financial frictions increases the average managerial quality of entrepreneurs augmenting aggregate labor demand. This effect is much weaker in the standard model economy than in the other economies (the average skill of entrepreneurs rises by about 50% in the standard economy and by more than 180% in the other economies). The economies with heterogeneity in working skills feature an additional effect: The removal of financial frictions leads to a decrease in the average ability of workers of more than 8%, reducing the effective aggregate labor supply. Hence, a large change in the fraction (quantity) of workers is needed to equate labor supply and labor demand in these economies.

[17]Following a suggestion form an anonymous referee, B.4 shows how these model economies fit the calibration targets of the baseline model economy

The calibrated fixed cost is not important for the output gains of removing financial frictions: The output gain is 55% in the economy with no fixed costs ($c_e = 0$), 57% when the fixed cost rises with the wage rate, and 58% in the baseline economy (columns a, d, e in Table 3). While c_e has a small effect on how financial frictions affect the entrepreneurship rate, it matters for the effect of financial frictions on the *composition* of entrepreneurs. The fraction of employers does not change with the elimination of financial frictions in the baseline economy. However, when $c_e = 0$ the decrease in entrepreneurship is explained by a substantial reduction in both the fractions of self-employed (12%) and employers (10%), which is at odds with the cross-country data.

7 Conclusions

The Lucas (1978) model is extended to distinguish between entrepreneurs that are self-employed from those that are employers. The proposed theory assumes that households are heterogeneous in managerial and working skills and entrepreneurs decide how much time to devote to the supply of labor and/or managerial input. While the baseline model accounts for half of the association in the data between entrepreneurship and external finance to GDP, the standard model explains only about one tenth of the cross-country variation in entrepreneurship. The baseline model is also consistent with the fact that the fraction of employers shows a mildly positive association with the external finance to GDP ratio and that the cross-country variation in entrepreneurship is mostly explained by the variation in self-employment rates. Moreover, the output gain of eliminating financial frictions in the baseline economy is much larger than in the standard model (58% versus 47%). The larger reallocation of capital towards a smaller number of entrepreneurs in the baseline economy implies a larger credit expansion and output gain.

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Panel A.	Baseline Model Economy	
$p_{\underline{m}}$	Prob. first fixed effect on manag. skill	0.229
η	Pareto tail managerial skill	5.40
μ_m	Location managerial skill distrib.	-2.976
$\rho_{\alpha_{m,w}}$	Correlation working and managerial fixed effects	0.145
$\sigma^2_{\epsilon_w}$	Variance innovation working skill	0.073
$\sigma^2_{\epsilon_m}$	Variance innovation managerial skill	1.145
ρ_m	Autocorrelation managerial skill	0.788
$\rho_{\epsilon_{m,w}}$	Correlation innovations managerial and working skills	0.335
σ^2_ζ	Variance transitory shock managerial skill	0.212
β	Discount factor	0.886
λ	Collateral constraint	1.27
c_e	Fixed cost of employers	0.08
p_α	Probability of new skills draw	0.041
Panel B.	**Standard Model Economy**	
σ^2_m	Variance innovation managerial skill	1.25
ρ_m	Autocorrelation managerial skill	0.929
β	Discount factor	0.898
λ	Collateral constraint	1.29

Table 1: Calibrated Parameters. *Note:* The *baseline model economy* (Panel A) refers to the economy with heterogeneity in both entrepreneurial and working ability, while in the *standard model economy* (Panel B) there is heterogeneity only in the entrepreneurial ability while workers are homogeneous.

Panel A: Baseline Model Economy		
	Data	Model
K / Y	2.11	2.04
Credit / Y	0.42	0.43
Income inequality: Within occupations		
Var(y_w)	0.76	0.76
Var(y_{ent})	1.06	1.07
Income inequality: Between occupations		
Avg y_{EMP}-Avg y_W	0.66	0.66
Avg y_{SE}-Avg y_W	-0.29	-0.28
Avg y_{EMP}-Avg y_{SE}	0.94	0.93
Occupational Structure		
Workers	68%	68%
Self-Employed	24%	24%
Employers	8%	8%
Occupational Transitions		
W to W	94%	94%
W to SE	5%	6%
SE to SE	77%	75%
SE to E	8%	8%
E to SE	22%	23%
E to E	68%	76%
Consumption inequality		
Var(log(c))	0.80	0.73

Panel B; Standard Model Economy		
	Data	Model
K / Y	2.11	2.0
Credit / Y	0.42	0.42
Persistence Entrep.	0.87	0.88
Var(y_{ent})	1.06	1.06

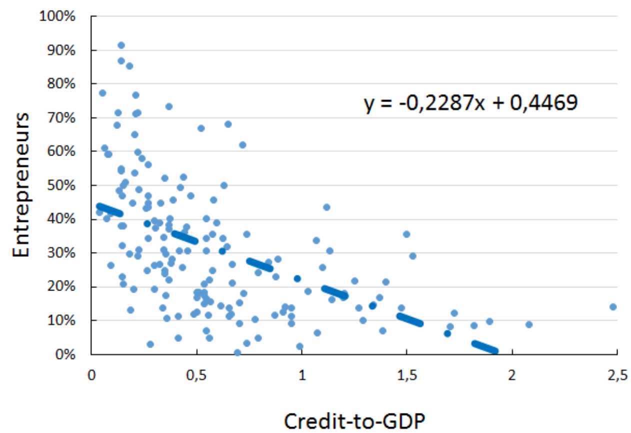
Table 2: Calibration Results. *Note:* In the table W stands for Workers, SE for Self-Employed and E for Employers. Variances and differences of earnings between occupations are computed using the natural logarithm of earnings. Occupational transitions are computed comparing the same month of two consecutive years, keeping only households that were employed in both years.

	(a) Baseline	(b) Standard	(c) $\sigma_w^2 = 0$	(d) $c_e = 0$	(e) $c_e * w$
Change in fraction of					
Workers	0.20	0.035	0.07	0.22	0.20
Entrepreneurs	-0.20	-0.035	-0.07	-0.22	-0.20
Self-Employed	-0.20	0	-0.09	-0.12	-0.19
Employers	0.0	-0.035	0.03	-0.10	-0.01
Credit to GDP	1.53	1.28	1.29	1.52	1.53
% Change					
Output	58%	47%	54%	55%	57%
Avg skill workers	-8%	0%	0%	-14%	-8%
Avg manag skill entrep.	201%	52%	180%	217%	193%
TFP_{data}	23%	19%	26%	21%	22%
TFP_{ideal}	26%	21%	31%	24%	26%

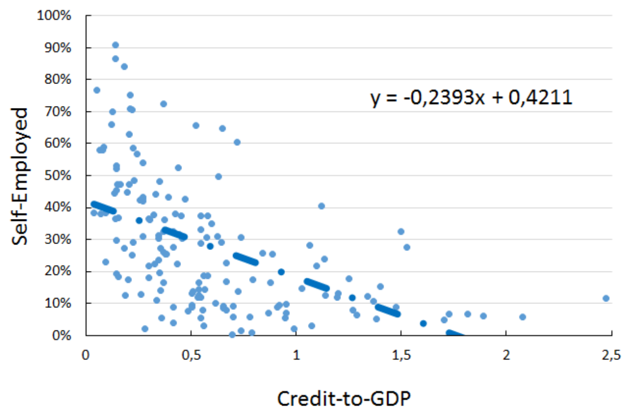
Table 3: Aggregate Effects of Removing Financial Frictions in Brazil. *Note:* This table reports the effect of removing financial frictions in different economies: i) Baseline economy, ii) Standard economy, iii) Baseline economy with variance of working skills set to zero, iv) Baseline economy no fixed costs of operation by employers, iv) Baseline economy with fixed costs in terms of labor input.

	Baseline Model		Standard Model	
	Baseline	PCM	Baseline	PCM
Capital income	31%	12%	31%	17%
Skill returns	20%	64%	20%	57%
Covariance term	49%	24%	48%	25%

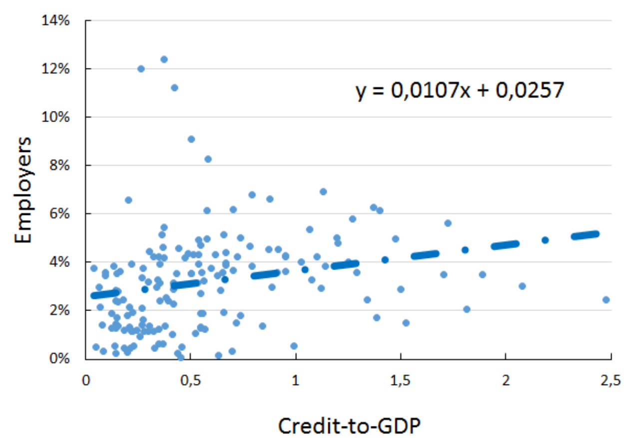
Table 4: Variance Decomposition of Income. *Note:* Total income can be decomposed in capital income and skill returns. Then $VAR(y) = VAR(\text{skill returns}) + VAR(\text{capital income}) + 2 * COVAR(\text{skill returns, capital income})$. PCM refers to the economy with perfect capital markets.



(a) Entrepreneurs

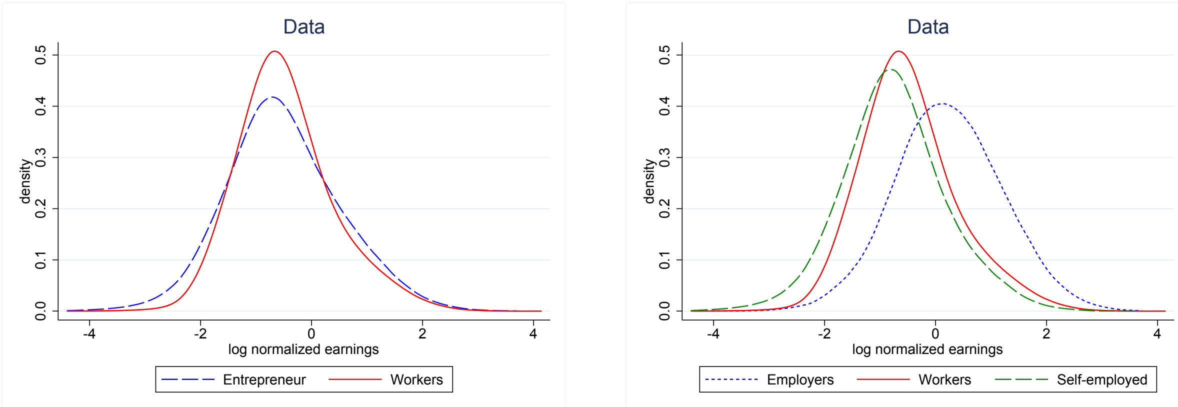


(b) Self-Employed



(c) Employers

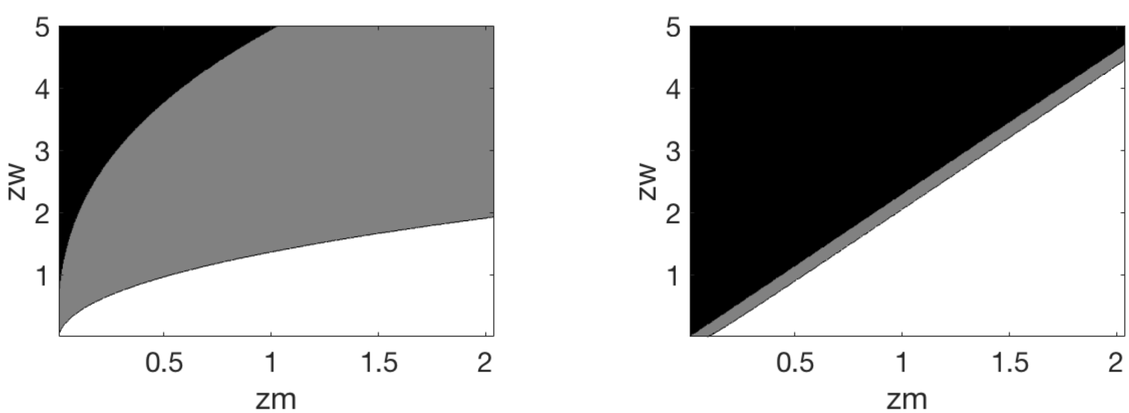
Figure 1: Occupational Structure and Credit across Countries. *Note:* Entrepreneurs are defined as those own-account workers where the remuneration depends on the goods and services produced. Self-employed are entrepreneurs who do not hire labor, while employers are those entrepreneurs who hire at least one person. Credit-to-GDP is taken from the World Bank development indicators database and is defined as domestic credit to the private sector. The dots indicate countries for which we have data on occupation and credit, while the dashed-line is the linear regression line of occupation on credit. (This is also true for the rest of figures.)



(a) Two Occupations

(b) Three Ocupations

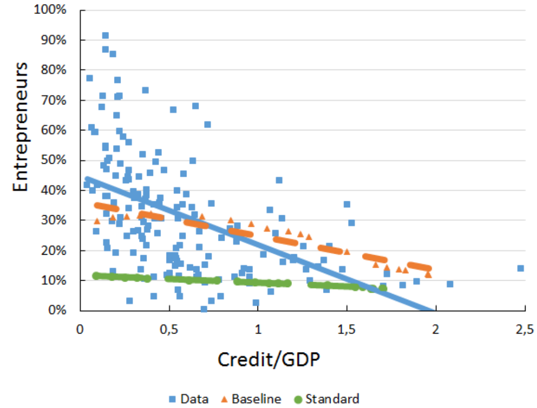
Figure 2: Earnings Distribution in Brazil. *Note:* The figure plots the normalized log-earnings for each occupation. Earnings are effectively perceived earnings by the individual in the month from all the works done. The normalization is done by dividing the earnings of each household by average earning of the economy.



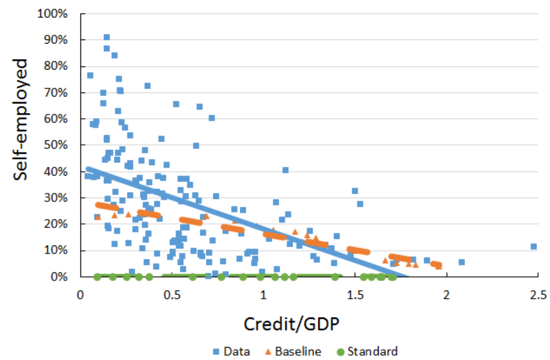
(a) Imperfect Capital Markets

(b) Perfect Capital Markets

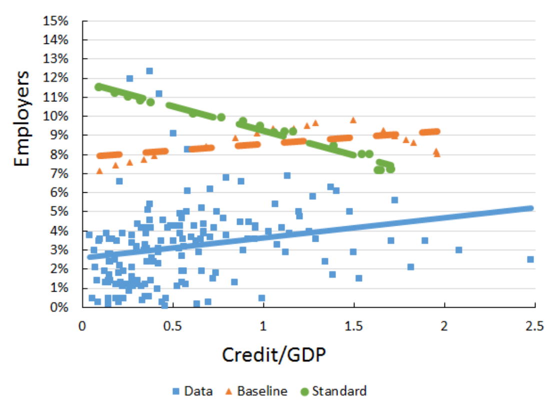
Figure 3: Occupational Map. *Note:* The graphs present the occupational choice for each combination of entrepreneurial and working ability for an individual with the median asset in the baseline economy and for the economy with perfect capital markets (in this case the occupational choices do not depend on the asset level). To do the graph both abilities were uniformly distributed, and so the sizes of the areas do not represent the actual distribution of skills.



(a) Entrepreneurs



(b) Self-Employed



(c) Employers

Figure 4: Occupational Structure and Credit across Countries. *Note:* In the graphs the baseline model corresponds to the economy with heterogeneity in both managerial and working ability, while the standard model corresponds to an economy with heterogeneity only in the managerial ability.

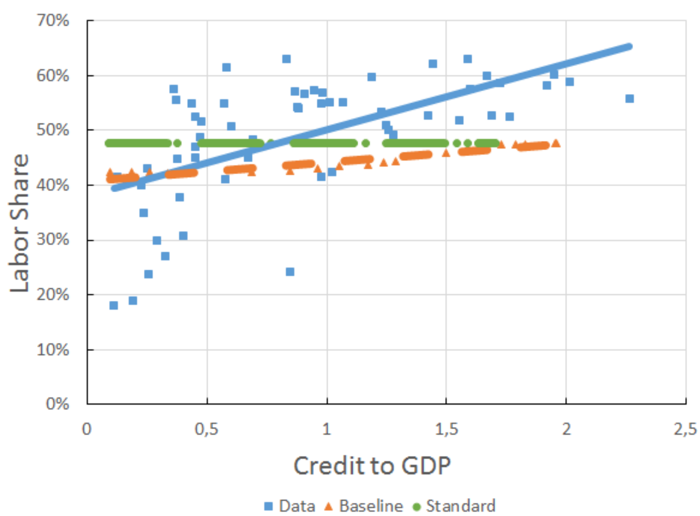


Figure 5: Employee compensation and external finance across countries *Note:* Data for employee compensation is taken from the International Labor Organization, and defined as total remuneration, in cash of in kind, payable by an enterprise to an employee in return for work.

A Proofs of Propositions

Before proceeding to the proofs of Proposition 1 and 2, it is convenient to characterize the production plan of self-employed and employers.

Self-employed: Production Plan. The optimal production plan of self-employed individuals maximizes

$$\pi_{se} = (z_m t_m)^\gamma k^\eta (z_w(1-t_m))^\theta - (r+\delta)k + (1+r)a + \mu_k(\lambda a - k) \tag{A-1}$$

where $\gamma + \eta + \theta = 1$. The FOC imply:

$$\{t_m\} \qquad z_m^\gamma k^\eta z_w^\theta [\gamma t_m^{\gamma-1}(1-t_m)^\theta - t_m^\gamma \theta (1-t_m)^{\theta-1}] = 0 \Rightarrow t_m^* = \frac{\gamma}{\gamma+\theta} \tag{A-2}$$

$$\{k\} \qquad (z_m t_m)^\gamma \eta k^{\eta-1} (z_w(1-t_m))^\theta - r - \delta - \mu_k = 0 \Rightarrow k = \left[\frac{(z_m t_m)^\gamma \eta (z_w(1-t_m))^\theta}{r+\delta+\mu_k}\right]^{\frac{1}{1-\eta}}. \tag{A-3}$$

Note that the first FOC equates the marginal product of entrepreneurial time at managing and working. Combining the FOC implies that the marginal product of entrepreneurial time satisfies:

$$\begin{aligned} MPT_{se} &= \gamma z_m^\gamma (t_m^*)^{\gamma-1} k^{*\eta} (z_w(1-t_m^*))^\theta \\ &= r_{mw} \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}, \\ &\text{where } r_{mw}(\mu) = \gamma \eta^{\frac{\eta}{1-\eta}} \left(\frac{\theta}{\gamma}\right)^{\frac{\theta}{\gamma+\theta}} \left(\frac{1}{r+\delta+\mu}\right)^{\frac{\eta}{1-\eta}}. \end{aligned} \tag{A-4}$$

Income of self-employed individuals can then be written as

$$y_{se} = MPt_m\, t_m + MPt_w\, t_w + MPK\, k + ra - k(r+\delta), \tag{A-5}$$

$$y_{se} = r_{mw}(\mu) \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}} + \mu k + ra. \tag{A-6}$$

Employers: Production Plan. The optimal production plan of employers solves

$$\pi(z_m, z_w, a) \;=\; \max_{t_m, t_w, n_d, k} (z_m t_m)^\gamma k^\eta (n_d + z_w t_w)^\theta - w n_d - (r+\delta)k + (1+r)a - c_e \tag{A-7}$$

$$k \le \lambda a, t_m + t_w = 1, t_w \ge 0. \tag{A-8}$$

The non-negativity constraint on t_w ensures that managerial time cannot be bigger than 1. Associate the multiplier μ_k to the borrowing constraint, μ_t to the time constraint, and μ_{tw} to the non-negative constraint on the working time. The FOC of the problem imply

$$MPK = (z_m t_m)^\gamma \eta k^{\eta-1} (n_d + z_w t_w)^\theta = r + \delta + \mu_k, \tag{A-9}$$

$$MPn_d = (z_m t_m)^\gamma k^\eta \theta (n_d + z_w t_w)^{\theta-1} = w, \tag{A-10}$$

$$MPt_m = z_m \gamma (z_m t_m)^{\gamma-1} k^\eta (n_d + z_w t_w)^\theta = \mu_t, \tag{A-11}$$

$$MPt_w = (z_m t_m)^\gamma \theta k^\eta (n_d + z_w t_w))^{\theta-1} z_w = \mu_t - \mu_{tw}, \tag{A-12}$$

where the parameters have been assumed to be such that it is optimal to hire outside labor ($n_d > 0$). Combining the FOC yields:

$$w z_w = MPt_w \le MPt_m, \text{with equality only if } t_w > 0. \tag{A-13}$$

The analysis is divided into three steps.

Step 1: It is first shown that if the borrowing constraint does not bind ($\mu_k = 0$), then the entrepreneur allocates all his time to managerial tasks ($t_w = 0, t_m = 1$). Assume that $\mu_k = 0$ and let $n \equiv n_d + z_w(1-t_m)$. Furthermore, to find a contradiction assume that $t_w > 0$. Then, $\mu_{tw} = 0$ implies $MPt_m = MPt_w$ so that

$$z_m \gamma n = t_m z_m \theta z_w \Rightarrow t_m = \frac{\gamma n}{\theta z_w}. \tag{A-14}$$

Combining the FOC for MPK and MPn_d, gives

$$(z_m t_m)^\gamma \left(\frac{w \, \eta \, n}{(r + \delta) \, \theta} \right)^\eta \theta n^{\theta - 1} = w. \tag{A-15}$$

Combining (A-14)-(A-15) yields

$$n^{\gamma+\theta+\eta-1} \left(\frac{z_m \gamma}{\theta z_w} \right)^\gamma \left(\frac{w\eta}{\theta(r+\delta)} \right)^\eta \theta = w, \tag{A-16}$$

which is false in general given that $\gamma+\theta+\eta-1=0$. In sum, if the borrowing constraint does not bind, then an employer optimally chooses to devote all his time to managerial tasks.

Step 2: Assume that the borrowing constraint binds ($k = \lambda a$). Now it is shown that there exists a threshold level of assets $a^*(z_m, z_w)$ such that the optimal production plan features $t_w > 0$ if $a < a^*(z_m, z_w)$ and $t_w = 0$ if $a > a^*(z_m, z_w)$. Thus, if the borrowing constraint is not too tight, employers allocate all their time to managerial activities. Below conditions are found for which $t_m < 1$ (or, equivalently, $t_w > 0$). Note that $t_m < 1$ only if $\mu_{tw} = 0$. In this case, the marginal product of entrepreneurial time is equated across the two uses of time. From the FOC it can be obtained that

$$MPt_w = MPt_m \Rightarrow n = \frac{\theta z_w t_m}{\gamma}. \tag{A-17}$$

Plugging n into the FOC with respect to labor demand and solving for t_m gives an expression for the optimal fraction of time dedicated to managerial tasks:

$$t_m = \left[\frac{\theta z_m^\gamma (\lambda a)^\eta}{w} \left(\frac{\gamma}{z_w \theta} \right)^{1-\theta} \right]^{\frac{1}{1\ \gamma\ \theta}}. \tag{A-18}$$

Note that $t_m < 1$ iff

$$a < a^*(z_m, z_w) \equiv \frac{1}{\lambda} \left[\frac{w}{\theta z_m^\gamma} \left(\frac{z_w \theta}{\gamma} \right)^{1-\theta} \right]^{\frac{1}{\eta}}. \tag{A-19}$$

Since $\overline{k}(a, z_m, z_w)$ is increasing in a, the inverse of this function can be used to define a threshold level of assets $a^*(z_m, z_w)$ such that $t_m < 1$ if and only if assets are below this threshold. Otherwise, $t_m = 1$.

Step 3: Compute the marginal product of employers' time. From Step 1 and 2, when assets are below $a^*(z_m, z_w)$ we have $MPt_m = MPt_w = wz_w$. On the other hand, when assets are above $a^*(z_m, z_w)$, $t_m = 1$ and $MPt_m > MPt_w$. To obtain an expression for MPt_m note that the FOC with respect to capital and outside labor imply:

$$k = \frac{w\eta}{(r+\delta+\mu_k)\theta} n_d \tag{A-20}$$

$$n_d = \left(\frac{\theta z_m^\gamma}{w} \left[\frac{w\eta}{(r+\delta+\mu_k)\theta} \right]^\eta \right)^{\frac{1}{1\ (\eta+\theta)}} \tag{A-21}$$

Plugging k and n_d into $MPt_m = \gamma z_m^\gamma k^\eta n_d^\theta$ gives

$$MPt_m = z_m \gamma \left[\left(\frac{\eta}{(r+\delta+\mu)} \right)^\eta \left(\frac{\theta}{w} \right)^\theta \right]^{\frac{1}{1\ (\eta+\theta)}} = z_m r_m(\mu)$$

$$\text{where } r_m(\mu) = \gamma \left[\left(\frac{\eta}{(r+\delta+\mu)} \right)^\eta \left(\frac{\theta}{w} \right)^\theta \right]^{\frac{1}{1\ (\eta+\theta)}}. \tag{A-22}$$

To study occupational choice decisions, the analysis first considers the decision between working for a wage vs. becoming self-employed.

Worker vs. Self-employment. An individual with ability (z_m, z_w) prefers to be self-employed rather than work for a wage if and only if

$$z_w w + ra < \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\theta+\gamma}} r_{mw}(\mu) + \mu k + ra, \tag{A-23}$$

which holds when the skill ratio satisfies

$$\frac{z_w}{z_m} < \left[\frac{r_{mw}(\mu) + \mu k / \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\theta+\gamma}}}{w}\right]^{\frac{\theta+\gamma}{\gamma}}. \tag{A-24}$$

Self-employment vs. Employer. An individual with ability (z_m, z_w) and assets a prefers being an employer rather than self-employment if and only if

$$\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\theta+\gamma}} r_{mw}(\mu_{se}) + \mu_{se} k_{se} + ra < z_m r_m(\mu_e) + \mu_e k_e + ra, \tag{A-25}$$

where μ_e and μ_{se} are the Lagrange multipliers associated with the borrowing constraints when the individual is an employer or is self-employed, respectively, and k_e and k_{se} are the capital used in production in these occupations. This inequality holds when the ability ratio is such that

$$\frac{z_w}{z_m} < \left[\frac{r_m(\mu_e)}{r_{mw}(\mu_{se})} + \frac{(\mu_e k_e - c_e)}{z_m r_{mw}(\mu_{se})} - \frac{(\mu_{se} k_{se})}{z_m r_{mw}(\mu_{se})}\right]^{\frac{\theta+\gamma}{\theta}}. \tag{A-26}$$

Proof of Proposition 1 (Economy with perfect enforcement). When $\lambda = \infty$ the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$).
Part 1. Setting $\mu = 0$ in (A-4) and (A-22), it follows that the rates of return to skills (r_{mw} and r_m) do not vary across individuals. Denote these returns by r^*_{mw} and r^*_m. Since individuals face the same skill prices, income inequality is all due to heterogeneity in skills and asset holdings. This proves part 1 of the proposition.
Part 2. Setting $\mu = 0$ in (A-24) implies that the individual prefers to be self-employed rather than work for a wage if and only if

$$\frac{z_w}{z_m} < \left(\frac{r^*_{mw}}{w}\right)^{\frac{\theta+\gamma}{\gamma}} \equiv R^*_1. \tag{A-27}$$

Setting $c_e = 0$ and $\mu = 0$ in (A-26) implies that the individual prefers to be an employer rather than a self-employed if and only if

$$\frac{z_w}{z_m} < \left(\frac{r^*_m}{r^*_{mw}}\right)^{\frac{\theta+\gamma}{\theta}} \equiv R^*_2. \tag{A-28}$$

This establishes the result in part 2.
Part 3. Now consider $c_e > 0$. Setting $\mu = 0$ in (A-26) implies that the individual prefers to be an employer rather than a self-employed if and only if

$$\frac{z_w}{z_m} < R^*_2 \left(1 - \frac{c_e}{z^*_m r^*_{mw}}\right)^{\frac{\theta+\gamma}{\theta}}. \tag{A-29}$$

The decision between being a worker or a self-employed is not affected by the fixed cost of operation c_e. This establishes the result in part 3.

Proof of Proposition 2 (Economy with imperfect enforcement). Consider an individual with a binding borrowing constraint (otherwise, occupational choice decisions and returns to skills are characterized as in Proposition 1). From (A-4) and (A-22), it follows that rates of return to skills (r_{wm} and r_m) decrease with the tightness of the borrowing constraint (e.g. returns decrease with μ). Since rates of return to skills vary across individuals, income inequality is due to heterogeneity in skills, assets, and rates of return. From (A-24) and (A-26), it follows that occupational choices now depend on the skill ratio, asset holdings, and the absolute level of skills (the last two matter because they affect the value of μ).

B Calibration and performance of model along non-targeted dimensions.

This appendix explains in more detail the calibration procedure, provides some intuition about the identification of parameters, and evaluates the performance of the calibrated model economies along non-targeted dimensions.

B.1 Calibration

The fixed effects on working ability are assumed to be distributed according to

$$\alpha_w = \epsilon_{wf} + \rho_{\alpha_{mw}} \frac{(\alpha_m - E(\alpha_m))}{std(\alpha_m)} \sigma_{\alpha_w} \text{ where } \epsilon_{wf} \sim N\left(0, \sigma^2_{wf}\right), \tag{B-1}$$

where $\rho_{\alpha_{mw}}$ controls the correlation of fixed effects on working and managerial skills, σ^2_{wf} affects the variance of fixed effects on working ability, and $std(\alpha_m)$ denotes the standard deviation of managerial fixed effects.[18] Note that fixed effects on working and managerial skills are positively correlated when $\rho_{\alpha_{mw}} > 0$. The distribution of fixed effects on working skills satisfy

$$E(\alpha_w) = 0 \tag{B-2}$$

$$E(\alpha_w|\alpha_m) = \rho_{\alpha_{mw}} \frac{(\alpha_m - E(\alpha_m))}{std(\alpha_m)} \sigma_{\alpha_w} \tag{B-3}$$

$$VAR(\alpha_w) = \frac{\sigma^2_{wf}}{1 - (\rho_{\alpha_{mw}})^2} \tag{B-4}$$

The coefficient $\rho_{\alpha_{mw}}$ controls how deviations of managerial skills from its mean value affect the (conditional) mean value of working skills.[19] Hence, two additional parameters need to be calibrated $\left(\rho_{\alpha_{m,w}}, \sigma^2_{wf}\right)$.

Next, the calibration of the persistent shocks to working and managerial abilities is discussed. These shocks have been assumed to follow a first order autoregressive bi-variate process. The autocorrelation of managerial-ability shocks (ρ_m) and working-ability shocks (ρ_w), and the variance-covariance matrix of innovations to the skill process need to be pinned down (three additional parameters: σ_{ϵ_w}, σ_{ϵ_m} and $\rho_{\epsilon_{m,w}}$). To minimize the parameters to be calibrated, Brazilian household data on wages is directly us to determine some of the parameters driving the shocks to the skill distribution of workers' abilities. Following the calibration strategy of Storesletten et al. (2005), we note two observations on the Brazilian household data: first, the variance of log wages grows linearly with age suggesting that the autocorrelation of wage shocks is high. Thus, the autocorrelation of working-ability shocks is set to $\rho_w = 0.96$. Second, the variance of log wages in Brazil at age 20 is equal to 0.42. The variance of fixed effects on working ability is thus set to 0.42. Using (B-4) yields

$$\sigma^2_{wf} = (1 - (\rho_{\alpha_{m,w}})^2) \times 0.42 \tag{B-5}$$

Conditional on the distribution of managerial fixed effects, we only need to set the correlation between fixed effects on working and managerial skills ($\rho_{\alpha_{m,w}}$) to determine the distribution of fixed effects on working ability (see equation (B-1)). Finally, the probability that a household draws new shocks from the invariant distribution of abilities (p_α) and the variance (σ^2_ζ) of transitory shock on managerial ability need to be specified.[20]

[18] The $std(\alpha_m)$ is determined in the calibration of managerial fixed effects.

[19] To fix ideas, consider the case $\rho_{\alpha_{mw}} = 1$. When the fixed effect on managerial skills is above the mean value of managerial skills by about 0.2 standard deviations of managerial skills, the fixed effect on working skill is expected to be above the mean value of working skills by 0.2 standard deviations of workings skills.

[20] Recall that the transitory shocks to managerial ability were assumed to be drawn from a normal distribution with mean zero and variance σ^2_ζ.

The following two-stage procedure allows for important efficiency gains in the estimation of parameters. In a first stage, the procedure searches for the 13 parameters listed in Panel A of Table 1 and the two equilibrium prices (wage rate and interest rate). This is done by adding 2 additional moments in the loss function: (i) the labor market clearing condition and (ii) the credit markets clearing condition. In a second stage, given the 13 parameters obtained in the first stage, the procedure searches for the two equilibrium prices that clear the factor markets. The joint search for model parameters and equilibrium prices allows for important efficiency gains.[21] The loss function is minimized using a simplex routine. The calibration allows for 18 pair of fixed effects for managerial-working ability (the fixed effect on managerial ability can take 6 values, and for each value of managerial ability we allow for 3 different values on fixed effects on working ability, giving a total of 18 pairs of fixed effects on working and managerial skills). Three transitory shocks are set on managerial fixed effects. A Tauchen-Hussey routine is used to discretize the bivariate process on working and managerial skills with 49 pairs of shocks. As a result, the model economy has $18 \times 49 \times 3$ skill types.

B.2 Discussion on the calibration of the skill distribution.

The discussion in this subsection focuses on the most important effects that the parameters determining the bivariate skill distribution have on the calibration targets.

Modeling heterogeneity in two skills allows the baseline model economy to distinguish between comparative advantage in entrepreneurship (a high skill ratio $\frac{z_m}{z_w}$) and absolute advantage (a high value of skills). This distinction is crucial for building a quantitative theory that can account for the occupational structure, transition rates across occupations, and earnings inequality between and within occupations in Brazil. To be concrete, assume for now that there are no credit constraints $\lambda = \infty$. Then, as shown in Section 2, the occupational distribution is determined by the skill ratio $\frac{z_m}{z_w}$. Households with a high ratio of managerial to working skills become employers, households with an intermediate ratio become self-employed, and households with a low ratio become workers. Now, notice that the variation in two skills allows for heterogeneity in earnings *within* occupations since the levels of z_w and z_m could vary across households with a common skill ratio. Moreover, since occupations are determined by the skill ratio but earnings by the absolute level of skills the theory allows for the possibility that some employers make lower earnings than some workers and viceversa. This is not possible in the standard model with heterogeneity in one skill. Since this model cannot distinguish between comparative and absolute advantage at occupations, all entrepreneurs make more earnings than workers and there is no variation in earnings across workers.

The skill distribution determines the occupational structure and the variation in earnings between and within occupations. For a fixed correlation of skills, the higher the variance of skills the more inequality in earnings within occupations. It follows that the variance of skills matter importantly for inequality *within* occupations. The correlation of skills in the population plays a crucial role in determining earnings inequality *between* occupations. The reason is that when skills are positively correlated, households with comparative advantage at managing (high ratio $\frac{z_m}{z_w}$) also have absolute advantage at managing and working (both abilities tend to be high). As a result, the difference in mean earnings between entrepreneurs and workers is high. When the correlation of skills is negative, the opposite happens and the difference in mean earnings between entrepreneurs and workers is negative. In sum, the cross-sectional correlation and variation of skills are crucial for inequality within and between occupations.

In the presence of financial frictions ($\lambda < \infty$), the variation of skills over time also plays a role in determining occupational choices and earnings inequality. Since wealthy individuals have a comparative advantage at entrepreneurship, occupational choices and earnings inequality are determined by the joint distribution of wealth and skills. This distribution, in turn, is determined by the stochastic process on skills. The calibration needs to pin down the importance of fixed effects versus transitory shocks in the variation of skills and the serial correlation and variance of innovations to skills.

[21] Intuitively, relative to the standard procedure of finding first the equilibrium prices before computing equilibrium moments, our procedure of computing moments "out of equilibrium" has the advantage that it allows us to obtain information on how parameters affect model statistics while searching for equilibrium prices (w, r).

To tease apart the variation in skills due to fixed effects versus transitory shocks, the calibration uses information on cross-sectional inequality (consumption and earnings inequality) as well as information on dynamic moments (yearly occupational transition rates). Note that consumption inequality in Brazil is quite large (about five times the one present across US households) and a model with transitory shocks alone can't generate this level of inequality in consumption. Intuitively, heterogeneity in fixed effects is needed to match the large amount of consumption inequality across households in Brazil while keeping the model consistent with the evidence on occupational transitions. On the other hand, the data reveals that a substantial fraction of households changes occupations between two consecutive years (6% of workers, 23% of self-employed, and 32% of employers choose a different occupation the next year). A model with heterogeneity in fixed effects alone cannot account for the observed transitions into and out of entrepreneurship. To match these facts, the calibration requires large transitory shocks to managerial skills (the persistence of managerial shocks has to be well below one). Moreover, the calibration requires a purely transitory shock (an iid shock) to managerial skills in order to match the transition rates from self-employed to employer (8%) and from employer to self-employed (22%).

B.3 Performance of the baseline economy along non-targeted dimensions

The calibrated model economy is over-identified since we targeted 15 moments using 13 parameters. Below, a discussion follows on how the baseline economy matches some additional moments on the distribution of earnings across occupations and earnings changes upon occupational switches that were not directly targeted.

The calibration of the baseline economy targeted the variance of earnings among workers and among entrepreneurs but did not target the overall level of earnings inequality. The Gini coefficient in the baseline model economy is 0.56, which is close to the 0.53 value in the data (see Table B-1). The standard model, not surprisingly, predicts too little earnings inequality: The Gini coefficient of earnings is 0.22 in this economy. The range of variation of earnings in the baseline model is not far, though somewhat smaller, from that in the data. The earnings ratio between households at the 90th percentile and households at the 10th percentile is about 10.4 in the data. This ratio is about 8.9 in the baseline model economy and 1 in the standard model. The earnings ratio between households in the 75th percentile and at the 25th percentile is 3.1 in the data and 3.5 in the baseline model economy. Overall, the baseline economy accounts for the large amount of earnings inequality in the Brazilian data. Relative to the data, it implies less inequality between the upper and bottom tails of the earnings distribution and more inequality at the middle of the distribution.

In assessing the success of our theory in modeling the sorting of heterogeneous households across occupations, we now evaluate how the calibrated model economy accounts for the variation of earnings within and across occupations. The calibration of the baseline economy targeted the difference between average earnings across the three occupations. The calibration did not target the variance of earnings among the self-employed and among employers. The baseline economy is consistent with the (non-targeted) fact that earnings inequality is higher among employers than self-employed: the variance of log-earnings in Brazil is higher among employers than self-employed households with values of 0.87 and 0.934. In the model economy, these statistics are 0.86 and 1.05.

Figure B-1 plots the distribution of earnings across the three occupation in the model and in the data.[22] Note that the theory is consistent with the fact that the low income employers make less earnings than the median self-employed individual. Moreover, as in the data, the distribution of self-employment earnings is shifted to the left, relative to that of workers, and the distribution of worker's earnings is shifted to the left, relative to that of employers.

Table B-1 also reports the decomposition of earnings inequality in terms of within group (occupations) versus between group inequality in the data and in the model economies. The baseline economy matches the fact that most of earnings inequality is explained by within group inequality (96%). Hence, between group inequality only accounts for a small share of inequality both in the baseline model and in the data. The standard model economy, however, overstates by a factor of 6 the importance of between group inequality

[22]The plots show estimated Epanechnikov kernel densities to the Brazilian and model data using the STATA command kdensity.

Figure B-1: Distribution of Earnings-Data vs Model II. *Note:* The plots present the distribution of normalized earnings for three occupations: workers, self-employed and employers, in the data (panel at the top) and in the model (panel at the bottom). The distribution are estimated Epanechnikiov kernel densities, using band width of 0.2.

(26% of the overall variation in earnings relative to the 4% in the data).

The Brazilian monthly labor survey (PME) has a short panel dimension. In the spirit of Hamilton (2000), for each occupation we compute mean earnings gains for occupational switchers and non switchers in the model and in the data to test the "dynamic" implications of the theory. For instance, the set of worker in the Brazilian data is partitioned in three groups: (i) those that are still workers the next year, (ii) those who switch to self-employment; (iii) and those workers who become employers the next year. For each of the three groups of workers, we compute mean earnings growth between the two consecutive years.[23] The data shows that workers that become employers next year tend to exhibit much stronger growth in earnings than those who did not change occupation. The mean earnings growth in Brazil for workers becoming employers is 57% relative to 1% for those not switching occupation. In the model these statistics are 82% and 1%. Consistently with the data, the model predicts that workers switching to self-employment have negative earnings growth, though the model understates these earnings losses (-6% in the model vs -34% in the data).

Repeating this exercise for employers, the Brazilian data reveals that the employers that switch to workers have important earnings losses (-37%) and that these earning losses are much bigger for the employers switching to self-employment (-65%). Employers that do not switch exhibit earnings gains of 16%. The theory is consistent with these observations. In the baseline economy, on average, employers that do not switch face positive earnings growth (8%) and those who become self-employed or become paid workers have important earning losses. Earnings losses are much larger for those employers who become self-employed rather than workers (-35% versus -98%). Regarding the income transitions for self-employed, both in the model and in the data self-employed individuals switching to employers have important gains in their mean earnings (74% in the model and 45% in the data).

While we do not have data on assets holdings at the household level, we do have data on household consumption. The differences in mean consumption across different occupation groups are viewed as estimates of permanent income differences across households in different occupations. The baseline model economy is consistent with the fact that difference in mean log-consumption between entrepreneurs and workers is

[23]Earnings are normalized by mean earnings of the economy.

small (less than 0.01 log points). Moreover, the difference in mean log consumption between workers and self-employed is large (-0.17 in the model and -0.12 in the data). The same applies to the difference in mean log consumption between employers and workers (0.56 in the model and 0.98 in the data). The standard model cannot account for these facts.

	Data	Baseline Model
Income Transitions		
W to W	0.01	0.01
W to SE	-0.34	-0.06
W to E	0.57	0.83
SE to W	0.00	0.10
SE to SE	-0.03	-0.02
SE to E	0.45	0.74
E to W	-0.37	-0.35
E to SE	-0.65	-0.98
E to E	0.16	0.08
Income Inequality within Entrepreneurs		
Var(y_{SE})	0.869	0.859
Var(y_{EMP})	0.934	1.058
Inequality Meassures - Labor Income		
Gini	0.53	0.56
Earnings ratio $p90/p10$	10.7	8.9
Earnings ratio $p75/p25$	3.2	3.5
Between group inequality	4.0%	4.3%
Within group inequality	96.0%	95.7%

Note: In the table W stands for Workers, SE for Self-Employed and E for Employers. Since in the data we do not have assets we use labor income (skill returns) in the model to compare it with the data.

Table B-1: Calibration Results- Nontargeted Moments

B.4 Performance of the economies in the sensitivity analysis.

The baseline economy was fitted to match the moments in Table 2. The economies in the sensitivity analysis in section 6.4 provide interesting information on which moments of the data are not fitted when one dimension/parameter of the model is shut down. Table B-2 reports the calibrated moments across these model economies. The Standard model economy predicts an entrepreneurship rate of 11% which is far below the target of 32%. This finding should not be surprising since the Standard Model economy does not account for SE entrepreneurs. Moreover, it implies counterfactual large differences in log average income between employers and workers (1.35 versus 0.66 in the data). The model economy with no heterogeneity in working skills ($\sigma_w^2 = 0$) is inconsistent with data on many dimensions. First, it does not match key facts on inequality between occupations. Indeed, the difference in mean log-income between employers and workers is 1.16 (relative to 0.66 in the data) and the mean earnings of self-employed households is higher than that of workers (0.21 log points versus -0.29 log points in the data). Second, it predicts a fraction of entrepreneurs that it is too low relative to the data (0.13 versus 0.32). Third, the variance of log consumption is quite small relative to the data (0.07 versus 0.80). The economy with no fixed costs of operation $c_e = 0$ misses several important dimensions. While it matches the entrepreneurship rate in the data, it over-predicts the fraction of employers (0.21 versus 0.08) and under-predicts the fraction of self-employed (0.12 versus 0.24). It also grossly understates the mean log-income difference between employers and workers (0.10 versus 0.66) and between employers and self-employed (0.34 versus 0.94). Finally, it implies a too high persistence of being an employer (0.86 versus 0.68).

	Data	Model	Standard	$\sigma_w^2 = 0$	$c_e = 0$
K / Y	2.11	2.04	2.0	2.06	2.02
Credit / Y	0.42	0.43	0.42	0.43	0.428
Income inequality: Within occupations					
Var(y_w)	0.76	0.76	0	0	0.736
Var(y_{ent})	1.06	1.07	1.06	0.36	0.963
Income inequality: Between occupations					
Avg y_{EMP}-Avg y_W	0.66	0.66	1.35	1.16	0.10
Avg y_{SE}-Avg y_W	-0.29	-0.28	1.35	0.21	-0.23
Avg y_{EMP}-Avg y_{SE}	0.94	0.93	0	0.95	0.34
Occupational Structure					
Workers	68%	68%	89%	86%	66%
Self-Employed	24%	24%	n.a.	10%	12%
Employers	8%	8%	11%	3%	21%
Occupational Transitions					
W to W	94%	94%	98.5%	98%	94%
W to SE	5%	6%	n.a.	2%	5%
SE to SE	77%	75%	n.a.	74%	50.8%
SE to E	8%	8%	n.a.	0	11.9%
E to SE	22%	23%	n.a.	24%	12.5%
E to E	68%	76%	88%	76%	86.1%
Consumption inequality					
Var(log(c))	0.80	0.73	0.32	0.07	0.70

Table B-2: Sensitivity Analysis. *Note:* In the table W stands for Workers, SE for Self-Employed and E for Employers. Variances and differences of earnings between occupations are computed using the natural logarithm of earnings. Occupational transitions are computed comparing the same month of two consecutive years, keeping only households that were employed in both years.

C Brazilian data

We use two datasets to characterize the distribution of income across occupations, occupational transitions, and other statistics of the Brazilian economy reported in the paper: the Monthly Labor Survey (PME for its name in Portuguese, *Pesquisa Mensal de Emprego*) and the Household Expenditure Survey (POF for its name in Portuguese, *Pesquisa de Orçamento Familiar*).

We use the 2008/2009 wave of the POF, that is a national representative household survey, and contains information on occupations, income and consumption of families. We use this dataset to construct the statistics of consumption. In our sample we keep only households where the head is male, employed and is older than twenty and younger than sixty years old. In order to make the consumption comparable among households we divide it by the number of adults equivalents in the house. The final dataset includes 47,173 households. Unfortunately, the POF is a cross section survey which does not allow us to compute transitions of earnings and occupations for individuals. Therefore we use data from the PME to compute those transitions.

We use the waves of the PME for the years 2003 until 2010. This is a monthly household labor survey covering the metropolitan areas of six Brazilian regions: Rio de Janeiro, São Paulo, Porto Alegre, Belo Horizonte, Recife and Salvador. The PME has a structure similar to the CPS in the U.S. Each individual is followed for four months, left out of the sample the next eight months and interviewed again the following 4 months. We take the first and fifth interview of each individual for the years 2003 until 2010. In this way we keep two observation of each individual, which corresponds to the same month of consecutive years. As in the case of the POF, our sample includes households where the head is male and aged twenty to sixty years old. This survey contains information on individuals' occupation and earnings. The earnings of the household are the sum of the earnings of all members. In order to make the earnings comparable we deflect them with the corresponding month Consumer Price Index (CPI) and divide them by the number of adults equivalents in the house. In addition, we only keep individuals who are employed in both periods of the survey. The final dataset includes 131,056 households with information on earnings. To do the transition matrix on occupations we consider only the head of household data. The variable of earnings is a constructed variable, which includes the earnings effectively perceived by the individual in the month from all the works done.