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A CAPACITATED FACILITY LOCATION MODEL WITH BIDIRECTIONAL FLOWS

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Abstract. Supply chains with returned products are receiving increasing attention in the operations management community. The present paper studies a capacitated facility location model with bidirectional flows and marginal value of time for returned products. The distribution system consists of a single supplier that provides one new product to a set of distribution centers (DCs), which then ship to the final retailers. While at the retailers' site, products can be shipped back to the supplier for reprocessing. Each DC is capacitated and handles stocks of new and/or returned product. The model is a nonlinear mixed-integer program that optimizes DC location and allocation between retailers and DCs. We show that it can be converted to a conic quadratic program, which can be efficiently solved. Some valid inequalities are added to the program to improve computational efficiency. We conclude by reporting numerical experiments that reveal some interesting properties of the model.

Keywords. capacitated facility location model; conic quadratic programming; valid inequalities; closed-loop supply chain

1. Introduction

In the increasingly competitive global manufacturing environment, the success of a corporation depends on its ability to favorably manage its supply chains. A supply chain includes all the components necessary to design, fabricate, distribute, sell, support, use, and recycle (or dispose of) a product. Competitive and regulatory pressures present new challenges in supply chain management. Consequently, green supply chains, reverse supply chains, closed-loop supply chains, and sustainable supply chains are getting more attention. In particular, supply chain managers are interested in economically handling returned products by reusing them to obtain numerous financial benefits (Blackburn et al. [8]).

Companies face time and cost trade-offs in the implementation of integrated supply chains. Time-varying prices of returned products, especially for time-sensitive and short life-cycle products, complicate the problem. For example, consumer electronics products such as PCs can lose its value at rates of 1% per week (Guide and Van Wassenhove [17]). In response, it would be preferable to shorten the flow time of returned products in a reverse supply chain. This strategy results in more profits from the salvage value of returned products by reentering them into the market as quickly as possible. Meanwhile, batch processing of products to benefit from

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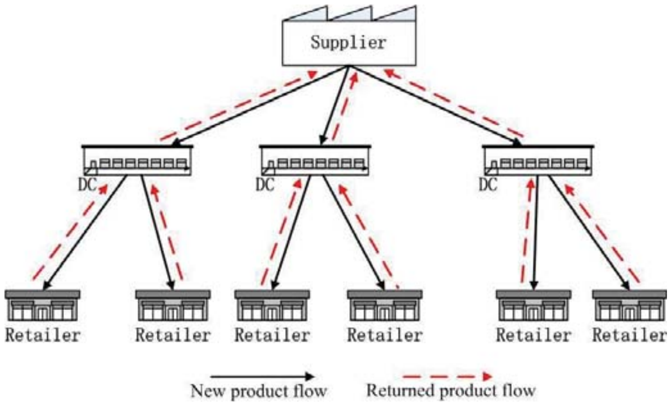

FIGURE 1. The structure of the three-tiered supply network.

economies of scale is also a recommended strategy but requires a slower flow time supply chain. Thus, the time versus cost trade-off enters into planning.

We consider a three-tiered supply network: one supplier, some distribution centers (DCs) with capacity limitation, and retailers. The product can simultaneously flow in two directions. The forward direction is the flow of the retailer's order of the product from the DC. In turn, the DCs get replenished from the supplier based on the specified inventory policy. The reverse direction is the flow of returned products from the retailers to the corresponding DCs and then back to the supplier to be reprocessed. Note that the DCs can hold stocks of both new and returned products. Figure 1 illustrates the structure of the three-tiered supply network.

The main contributions of this paper can be summarized as follows:

(1) It is the first model in closed-loop supply chain design to jointly consider capacitated DCs, stochastic demands of new and returned products, risk pooling to buffer random demands, savings from co-locating of forward and reverse flows in the same DC, and value loss related to inventory and transportation times.
(2) It employs a novel and powerful solution technique, the conic integer programming approach, that is convenient for the model presented and many others with similar nonlinear optimization forms.
(3) The convex hull of the feasible solutions is explored to add valid inequalities that improve computational efficiency.
(4) From our computational studies we obtain interesting managerial insights. For example, we show that the more time-sensitive the returned product is the least costly is to retrieve salvage value. We also show the effects of value loss related to inventory and transportation times, where a smaller or larger optimal number of opened DCs is recommended depending on whether the dominant factor is time spent in inventory or in transportation.

The remainder of the paper is organized as follows. In section 2, we present a literature review on the integrated forward/reverse network design and the capacitated facility location problem. Section 3 develops a nonlinear mixed-integer programming formulation of the supply chain. In section 4, the model is converted into a conic quadratic mixed-integer program to be solved efficiently. Subsequently, some valid inequalities are developed to improve the computational efficiency of the branch and cut algorithm and the quality of the solution. Next, in section 5 we explore the behavior of the supply chain under this optimization strategy through computational experiments on real data. In the last section, we conclude and discuss future research avenues.

2. Literature review

The integrated forward/reverse supply chain network design is an emerging research topic. The interested reader can refer to the works of Guide and Van Wassenhove [17] and Akçalı et al. [3] for comprehensive reviews on closed-loop supply chains. Below we briefly review some of the most relevant papers.

Sahyouni et al. [34] developed three generic uncapacitated integrated closed-loop supply chain design models that minimize fixed locating and transportation costs, while Lu and Bostel [24] presented a two-level uncapacitated location problem with three types of facilities that minimize fixed setup costs and transportation costs. Ko and Evans [20] proposed a genetic algorithm-based heuristic to solve a multi-period, two-echelon, multi-commodity, capacitated facility location model. Üster et al. [42], Easwaran and Üster [12], and Easwaran and Üster [13] studied multi-product closed-loop supply chain network design problems. The objective was to locate collection centers and finite-capacity manufacturing facilities while coordinating the forward and reverse flows in the network so as to minimize the processing, transportation, and fixed location costs. Pishvaee et al. [30] developed a bi-objective mixed integer program to minimize the total costs and maximize the responsiveness of an integrated forward/reverse logistics network. Pishvaee et al. [31] proposed a closed-loop supply chain network design model in a robust optimization framework. Our work employs conic programming (Atamtürk et al. [4]) as the solution approach for our closed-loop supply chain network design model. Further, a significant difference between our model and previous work is the integration of capacitated facility location and inventory decisions in which facilities can accept forward and/or returned products with the possibility of pooling forward inventory for different retailer sites.

We have seen models in the literature that integrate location decisions with other types of decisions in supply chains, such as transportation decisions, robustness and reliability considerateness. Daskin [10], Langevin and Riopel [21], and Melo et al. [25] provide good surveys on this subject. We are particularly interested in models that combine location with inventory decisions. Shen et al. [39] and Daskin et al. [11] studied the impact of inventory costs on location decisions in a stochastic demand environment. Their model incorporates nonlinear working inventory costs and nonlinear safety-stock inventory costs. Shen et al. [39] applied a column generation technique whereas Daskin et al. [11] used Lagrangian relaxation to solve this joint location-inventory model. During the last decade, scholars have studied different versions of this problem by extending it to capacitated warehouses (Ozsen et al. [29]), customer service considerations (Shen and Daskin [40]), multi-commodities (Shen [36]), supply uncertainty (Qi and Shen [32]), profit maximization (Shen [37]), disruptions (Qi et al. [33]), etc. The present paper can be considered a novel extension of the integrated supply chain design problem since it is the first to integrate reverse flows into a joint location-inventory model in which warehouses are assumed to be capacitated. For a summary of publications that study integrated supply chain design problems we refer to Shen [38].

Capacity restrictions in the facility location problem are a natural extension of the original problem and play a critical role. The capacitated facility location problem (CFLP) and its variants are well studied in the literature. For a review, please refer to the book of Mirchandani and Francis [27]. We note that most solution algorithms for capacitated facility location problems are adaptations of algorithms for uncapacitated problems. Therefore, heuristics such as Lagrangian relaxation-based algorithms Holmberg et al. [18], Daskin et al. [11], Langevin and Riopel [21], Sahyouni et al. [34], Lu and Bostel [24], Ozsen et al. [29], Liu et al. [23], Benders decomposition-based solution approaches Üster et al. [42], Easwaran and Üster [12, 13] and meta-heuristics such as genetic algorithm Ko and Evans [20], tabu search Easwaran and Üster [12], memetic algorithm Pishvaee et al. [30] are used extensively.

Both uncapacitated and capacitated supply chain design solution algorithms share the same problem: the subproblems generated are still intractable or only near-optimal solutions can be obtained. Therefore, some scholars have dedicated to the study of the cutting plane method, which explores the polyhedral convex hull of the feasible solutions and constructs valid inequalities to combine with the branch and bound or branch and cut algorithm. Normally, it can dramatically improve the efficiency of these algorithms. The cutting plane method for solving uncapacitated facility location problems has been studied since Cornuejols et al. [9]. At the end of 1980s, it was extended to solve the capacitated facility location problem (Leung and Magnanti [22]).

Valid inequalities have been used in the literature to improve the efficiency and quality of the models' solutions. Some of the most used valid inequalities are: clique inequalities (Leung and Magnanti [22]), odd cycle inequalities (Leung and Magnanti [22], Klose [19]), submodular inequalities (Aardal et al. [2], Klose [19]), (k, I, S, I) inequalities (Aardal et al. [2]), flow cover inequalities (Aardal et al. [2], Aardal [1], Klose [19]), knapsack cover inequalities, effective capacity inequalities, combinatorial inequalities (Aardal et al. [2], Aardal [1]), single depot inequalities (Aardal [1]), lifted cover inequalities (Klose [19]), and extended polymatroid inequalities (Atamtürk et al. [4]). Some of them are facets for the capacitated facility location problems. Some other valid inequalities are incorporated into Lagrangian-relaxation based methods to tighten the feasible region for capacitated facility location problems (Klose [19], Miranda and Garrido [26]). The present article adds extended polymatroid inequalities to tighten the feasible region of a conic quadratic mixed-integer program.

3. Problem formulation

There are three types of distribution centers in the network: forward (new products), reverse (returned products), and joint DCs (both new and returned products). We determine the DC locations among potential sites and the assignment of retailers to the DCs. The objective is to minimize the fixed charges of locating the distribution centers, working inventory costs, transportation costs, and the value loss of returned products.

To benefit from the risk pooling strategy, inventories are not kept at the retailers' sites but at the DCs. The DCs can fill retailer demand and can store returned products temporarily. An approximation to the (Q, R) model with Type-I service (Ozsen et al. [29]) is used for managing the stock of new products. The inventory policy followed by the return products is an approximation of the EOQ since not always the EOQ formula will provide the optimal quantity (the system is capacitated).

To exploit economies of scale in transportation costs, returned products will be shipped back to the supplier for reprocessing after a predetermined quantity at the DCs is reached. At the same time, getting returned products back to the market quickly will bring more profit. Blackburn et al. [8] investigated reverse supply chains for commercial returns (in particular, products returned by customers for any reason within 90 days of sale). In a real example, for \$1000 worth of product returns nearly half the product value ($> 45\%$) is lost in the return process by waiting for the product to be reprocessed. Indeed, a returned consumer product could wait in excess of 3.5 months before it is sent to disposition in a real case. Thus, we analyze the trade-off between efficiency and responsive costs when designing a forward/reverse supply chain network.

Before proposing the model, some important assumptions are followed. First of all, customer demands are Poisson distributed. Thus, variances of daily demand and returns are identical to the means of daily demand (μ_i^F) and returns (μ_i^R), respectively, for each retailer i. Further, demands at the retailers are uncorrelated over time and across retailers. Demand of returned products is independent from new product's demand. The model also assumes that there is sufficient transportation capacity but controls capacity at each DC. Tables 1, 2, and 3 define the variables and parameters.

TABLE 1. Sets.

I	Set of retailers indexed by i
J	Set of candidate DC sites indexed by j

TABLE 2. Decision variables per each DC j.

X_j^F	1, if candidate location j is selected as a forward DC, and 0 otherwise
X_j^R	1, if candidate location j is selected as a reverse DC, and 0 otherwise
X_j^C	1, if candidate location j is selected as a joint DC, and 0 otherwise
Y_{ij}^F	1, if demand of new products of retailer $i \in I$ is served by DC j, and 0 otherwise
Y_{ij}^R	1, if returned products of retailer $i \in I$ is collected by DC j, and 0 otherwise
$\mathbf{Y_j^R}$	$= (Y_{1j}^R, ..., Y_{Ij}^R)^T$
Q_j^F, Q_j^R	Shipment quantity of new and returned products at DC j

Table 3. Inputs.

C_j	Capacity of DC j, $j \in J$
μ_i^F, μ_i^R	Mean (daily) volumes of new and returned products at retailer i
f_j^F, f_j^R	Fixed (yearly) costs of locating a DC for forward/reverse flow at DC j
F_j^F, F_j^R	Fixed costs of placing an order of new and returned products at DC j
g_j^F, g_j^R	Fixed transportation costs between supplier and DC j for new and returned products
a_j^F, a_j^R	Cost per unit to ship between DC j and the supplier for new and returned products
d_{ij}	Cost per unit to ship between DC j and retailer i in forward/reverse flows
S_j^C	Fixed location cost savings at joint DC j
$WI_j^F(\cdot)$	The total annual cost of working inventory at forward DC j
$WI_j^R(\cdot)$	The total annual cost of working inventory at reverse DC j
β	Weight factor associated with the transportation cost in forward/reverse flows
θ	Weight factor associated with the inventory cost in forward/reverse flows
W	Weight factor associated with loss in value of returned products
γ	Returned products' (daily) marginal value of time
α	Desired percentage of retailers orders satisfied
z_α	Standard normal deviate such that $P(z \le z_\alpha) = \alpha$
h	Inventory holding cost per unit of products per year for each DC
L_j	Lead time in days at a DC j
χ	Number of days in a year

In summary, model $(\mathcal{P})$ is:

$$
\begin{aligned}
\min_{X,Y} Z \quad &= \quad \sum_{j\in J}\left\{ f_j^F X_j^F + \sum_{i\in I}\beta\chi d_{ij}\mu_i^F Y_{ij}^F + \theta h z_\alpha \sqrt{L_j \sum_{i\in I}\mu_i^F Y_{ij}^F} + WI_j^F(D_j^F, Q_j^F)\right\} \\
&+ \quad \sum_{j\in J}\left\{ f_j^R X_j^R + \sum_{i\in I}\beta\chi d_{ij}\mu_i^R Y_{ij}^R + WI_j^R(D_j^R, Q_j^R)\right\} - \sum_{j\in J} S_j^C X_j^C \\
&+ \quad W \sum_{j\in J} R(\mathbf{Y_j^R}, Q_j^R), && (1) \\
s.t. \quad & \sum_{j\in J} Y_{ij}^F = 1, \sum_{j\in J} Y_{ij}^R = 1, \ \forall i \in I, && (2) \\
& Y_{ij}^F \le X_j^F, Y_{ij}^R \le X_j^R, \ \forall i \in I, \forall j \in J, && (3) \\
& X_j^C \le X_j^F, X_j^C \le X_j^R, \ \forall j \in J, && (4) \\
& Q_j^F + z_\alpha \sqrt{L_j \sum_{i\in I}\mu_i^F Y_{ij}^F} + L_j \sum_{i\in I}\mu_i^F Y_{ij}^F + Q_j^R \le C_j, \ \forall j \in J, && (5) \\
& Q_j^F, Q_j^R \ge 0, \ \forall j \in J, && (6) \\
& X_j^F, X_j^R, X_j^C \in \{0,1\}, \ \forall j \in J, && (7) \\
& Y_{ij}^F, Y_{ij}^R \in \{0,1\}, \ \forall i \in I, \forall j \in J. && (8)
\end{aligned}
$$

where,

$$WI_j^F(D_j^F, Q_j^F) = \begin{cases} F_j^F \dfrac{D_j^F}{Q_j^F} + \beta(g_j^F + a_j^F Q_j^F)\dfrac{D_j^F}{Q_j^F} + \dfrac{\theta h}{2} Q_j^F, \ \forall j \in J, & Q_j^F > 0, \\ 0, & Q_j^F = 0. \end{cases} \tag{9}$$

$$WI_j^R(D_j^R, Q_j^R) = \begin{cases} F_j^R \dfrac{D_j^R}{Q_j^R} + \beta(g_j^R + a_j^R Q_j^R)\dfrac{D_j^R}{Q_j^R} + \dfrac{\theta h}{2} Q_j^R, \ \forall j \in J, & Q_j^R > 0, \\ 0, & Q_j^R = 0. \end{cases} \tag{10}$$

$$D_j^F = \chi \sum_{i \in I} \mu_i^F Y_{ij}^F, D_j^R = \chi \sum_{i \in I} \mu_i^R Y_{ij}^R, \ \forall j \in J.$$

The objective function (1) consists of four parts: cost of forward flows, cost of reverse flows, savings from co-location of forward and reverse DCs, and the time value of returned products.

The first part sums the costs of handling new products including the fixed charge of locating forward DCs, the DC-to-retailer shipping costs, the safety stock costs to ensure customer satisfaction, and the working inventory cost. The working inventory cost of new products is formulated as equation (9) which is the sum of the fixed costs for handling orders, the DC-to-supplier shipping costs, and the average order holding costs per year. The detailed explanation of equation (9) can be referred to Shen et al. [39].

The second part of the objective contains the costs of the reverse flows. Except for the safety stock costs, it has the same cost components as the first part of the objective. The working inventory cost of returned products is represented as equation (10). We suppose that the return rates of used products are constant among different retailers and are integrated in the definition of D_j^R. Geyer et al. [15] provide a discussion of calculating return rates in practical settings.

The third part of the objective represents the fixed cost savings created by the co-location of forward and reverse DCs at the same site. Note that normally the cost saving must be less than the minimum of the fixed charges of forward and reverse DCs. We, therefore, assume that $S_j^C \leq \min\{f_j^F, f_j^R\}$ (Sahyouni et al. [34]).

The fourth part concerns the time value of returned products. $R(\mathbf{Y_j^R}, Q_j^R)$ is the total average value loss of returned product per year and it is related to returned product's marginal value of time. Derivation of the formula of $R(\mathbf{Y_j^R}, Q_j^R)$ is given at the end of this section.

Constraints (2) ensure that each retailer is served by exactly one DC. Constraints (3) state that a retailer can only be assigned to an open DCs. Constraints (4) stipulate that if a DC is assigned to serve both forward and reverse flows, i.e. a joint DC, then it acts as not only a forward DC but a reverse DC as well. Consequently, cost savings occur. Note that forward (reverse) DCs refer to stand-alone forward (reverse) DCs and forward (reverse) facilities at joint DCs throughout the rest of this paper. Constraints (5) are the capacity restrictions of each DC j (further described in the next paragraph.) Constraints (6) are nonnegative constraints. Constraints (7) and (8) are standard integrality constraints.

Ozsen et al. [29] point out that the capacity of a DC must withstand the worse-case scenario because the amount of space the warehouse needs is proportional to peak inventory. In particular, this happens when there is no demand of new products and no shipment of returned products during the replenishment lead time.

Thus, the capacity constraints can be formulated as:

$$Q_j^F + z_\alpha \sqrt{L_j \sum_{i \in I'} \mu_i^F} + L_j \sum_{i \in I'} \mu_i^F + Q_j^R \leq C_j, \ \forall j \in J,$$

where I' is set of retailers served by DC j. The first and fourth terms represent the order quantity of new products and returned products, respectively. The second term is the safety stock under the assumption of Normal demands and it covers stock-outs that occur with a probability of α or less. Note that z_α is a standard Normal deviate such that $P(z \leq z_\alpha) = \alpha$. The third term is the average demand during lead times. We could tighten the right hand side of this capacity constraint by multiplying C_j by $(X_j^F + X_j^R - X_j^C)$ for the cases in which $X_j^C \geq X_j^F + X_j^R - 1$. This alternative formulation does not provide significantly better results for our experiments.

Derivation of the average value loss of returned product

We start the derivation by defining the average value loss of returned product associated with inventory times at DC j to build lot Q_j^R per year for a linear decay rate ($R_{inv}(Q_j^R)$). An exponential decay rate could also be used but, as shown in Appendix A, a linear decay rate is a good enough approximation.

Based on Blackburn et al. [8], the daily marginal value of time (MVT), denoted by γ, can be represented by the slopes of the lines in Figure 2. The greater γ is the more sensitive the price of returned products is to time. The average value loss of returned product associated with inventory times at DC j, denoted by $R_{inv}(Q_j^R)$, can be defined in a similar way as we calculate inventory holding cost of returned products in a lot sizing problem. Since, on average, there will be $\chi Q_j^R/2$ returned product of inventory on hand per year, the average value loss of returned products per year is defined as:

$$R_{inv}(Q_j^R) = \frac{\gamma V \chi}{2} Q_j^R, \tag{11}$$

where V is the initial price of returned products.

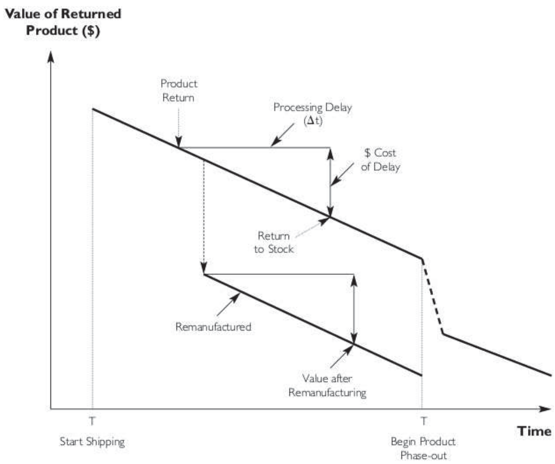

FIGURE 2. Time value of product returns [8]

To completely define the average values loss of returned product we need to add the value loss related to transportation times of returned products to the average value loss function associated to building up Q_j at DC j. For a linear decay rate, we define the average amount of returned product in transportation per year associated to DC j as $\sum_{i\in I}(\frac{d_{ij}+a_j^R}{k})\chi\mu_i^R Y_{ij}^R$, where k is the daily transportation cost per unit returned product and so $\frac{d_{ij}+a_j^R}{k}$ represents the time spent by a returned product in transportation from retailer i to DC j and from DC j back to the supplier. Thus, we define the total average value loss per year as

$$R(\mathbf{Y_j^R}, Q_j^R) = R_{inv}(Q_j^R) + R_{tr}(\mathbf{Y_j^R}) = \gamma V\left(\frac{\chi Q_j^R}{2} + \sum_{i\in I}(\frac{d_{ij}+a_j^R}{k})\chi\mu_i^R Y_{ij}^R\right). \quad (12)$$

4. Model properties and reformulation

The problem is modeled as a nonlinear mixed-integer program, which, in general, its optimal solutions are very hard to find in a reasonable amount of time. However, we note that our model could be identified as a novel version of the family of joint location-inventory models first time introduced by Shen et al. [39]. From Atamtürk et al. [4] we can define an equivalent conic quadratic mixed-integer program that will be directly solved via commercial optimization packages. Further, Atamtürk et al. [4] suggest that some cuts can be beneficial valid inequalities for models of the mentioned family. In the current work, we show how the polymatroid cuts are beneficial for our specific model.

Definition 1. A conic quadratic mixed-integer program (CQMIP) is an optimization problem of the form:

$$\begin{aligned} \min \quad & c'x \\ \text{s.t.} \quad & \| A_i x + b_i \|_2 \leq \ d_i' x + e_i, \ i = 1, \ldots, p, \end{aligned}$$

where $x \in \mathbb{Z}^n \times \mathbb{R}^m$, $c \in \mathbb{R}^{(n+m)}$, $A_i \in \mathbb{R}^{n_i \times (n+m)}$, $b_i \in \mathbb{R}^{n_i}$, $d_i \in \mathbb{R}^{(n+m)}$, $e_i \in \mathbb{R}$, $\|\cdot\|_2$ is the Euclidean norm, and all parameters are rational.

The following proposition provides an equivalent CQMIP formulation of problem $(\mathcal{P})$.

Proposition 1. *Problem $(\mathcal{P})$ is equivalent to the following (CQMIP):*

$$
\begin{aligned}
\min_{X,Y} Z^S &= \sum_{j\in J}\left\{f_j^F X_j^F + \theta h z_\alpha \omega_j + \sum_{i\in I}\beta(d_{ij}+a_j^F)\chi\mu_i^F Y_{ij}^F + \frac{\theta h}{2}u_j\right\} \\
&+ \sum_{j\in J}\left\{f_j^R X_j^R + \sum_{i\in I}\bar{\beta}(d_{ij}+a_j^R)\chi\mu_i^R Y_{ij}^R + \frac{W\gamma V\chi+\theta h}{2}v_j\right\} - \sum_{j\in J} S_j^C X_j^C, \\
s.t. \quad & \sum_{j\in J} Y_{ij}^F = 1, \sum_{j\in J} Y_{ij}^R = 1, \ \forall i\in I, && (13)\\
& Y_{ij}^F \le X_j^F, Y_{ij}^R \le X_j^R, \ \forall i\in I, \forall j\in J, && (14)\\
& X_j^C \le X_j^F, X_j^C \le X_j^R, \ \forall j\in J, && (15)\\
& \omega_j^2 \ge L_j \sum_{i\in I}\mu_i^F (Y_{ij}^F)^2, \ \forall j\in J, && (16)\\
& \frac{1}{2}(u_j+Q_j^F)^2 \ge H_j^F\chi\sum_{i\in I}\mu_i^F (Y_{ij}^F)^2 + \frac{3}{2}(Q_j^F)^2 + \frac{1}{2}u_j^2, \ \forall j\in J, && (17)\\
& \frac{1}{2}(v_j+Q_j^R)^2 \ge H_j^R\chi\sum_{i\in I}\mu_i^R (Y_{ij}^R)^2 + \frac{3}{2}(Q_j^R)^2 + \frac{1}{2}v_j^2, \ \forall j\in J, && (18)\\
& Q_j^F + z_\alpha\omega_j + L_j\sum_{i\in I}\mu_i^F Y_{ij}^F + Q_j^R \le C_j, \ \forall j\in J, && (19)\\
& \omega_j, u_j, v_j, Q_j^F, Q_j^R \ge 0, \ \forall j\in J, && (20)\\
& X_j^F, X_j^R, X_j^C, Y_{ij}^F, Y_{ij}^R \in \{0,1\}, \ \forall i\in I, \forall j\in J, && (21)
\end{aligned}
$$

where $\bar{\beta} = \beta + W\gamma V/k$, $H_j^F = \dfrac{2(F_j^F+\beta g_j^F)}{\theta h}$ *and* $H_j^R = \dfrac{2(F_j^R+\beta g_j^R)}{W\gamma V\chi+\theta h}$.

Proof. A conic transformation is employed to linearize the objective of problem $(\mathcal{P})$ in order to convert it into a CQMIP model. First, three sets of auxiliary variables ω_j, u_j and v_j are introduced, which satisfy the following inequalities:

$$
\begin{aligned}
\omega_j &\ge \sqrt{L_j\sum_{i\in I}\mu_i^F Y_{ij}^F}, && (22)\\
\frac{\theta h}{2}u_j &\ge (F_j^F+\beta g_j^F)\frac{D_j^F}{Q_j^F} + \frac{\theta h}{2}Q_j^F, && (23)\\
\left(\frac{W\gamma V\chi}{2}+\frac{\theta h}{2}\right)v_j &\ge (F_j^R+\beta g_j^R)\frac{D_j^R}{Q_j^R} + \left(\frac{W\gamma V\chi}{2}+\frac{\theta h}{2}\right)Q_j^R. && (24)
\end{aligned}
$$

Recall $Y_{ij}^2 = Y_{ij}$ if Y_{ij} is a binary variable, so we transform the above inequalities as follows:

$$\begin{aligned}
\omega_j^2 &\geq L_j \sum_{i\in I} \mu_i^F (Y_{ij}^F)^2, \\
\frac{1}{2}(u_j + Q_j^F)^2 &\geq \frac{2(F_j^F + \beta g_j^F)}{\theta h} \chi \sum_{i\in I} \mu_i^F (Y_{ij}^F)^2 + \frac{3}{2}(Q_j^F)^2 + \frac{1}{2}u_j^2, \\
\frac{1}{2}(v_j + Q_j^R)^2 &\geq \frac{2(F_j^R + \beta g_j^R)}{W\gamma V\chi + \theta h} \chi \sum_{i\in I} \mu_i^R (Y_{ij}^R)^2 + \frac{3}{2}(Q_j^R)^2 + \frac{1}{2}v_j^2.
\end{aligned}$$

Then, the objective of problem $(\mathcal{P})$ is reformulated as:

$$\begin{aligned}
&\sum_{j\in J} \left\{ f_j^F X_j^F + \theta h z_\alpha \omega_j + \sum_{i\in I} \beta (d_{ij} + a_j^F) \chi \mu_i^F Y_{ij}^F + \frac{\theta h}{2} u_j \right\} \\
+ &\sum_{j\in J} \left\{ f_j^R X_j^R + \sum_{i\in I} \bar{\beta} (d_{ij} + a_j^R) \chi \mu_i^R Y_{ij}^R + \frac{W\gamma V\chi + \theta h}{2} v_j \right\} - \sum_{j\in J} S_j^C X_j^C.
\end{aligned}$$

The set of capacity constraints (5) is linearized by substituting the nonlinear term by $z_\alpha \omega_j$ obtaining the set of constraints (19).

The rest of constraints of problem $(\mathcal{P})$ remain untransformed since they are linear. □

Extremal extended polymatroid inequalities:

Utilizing submodularity, the conic quadratic constraints (16) $\sim$ (18) lead to a class of valid inequalities that can improve the performance of the solution algorithm. Before presenting the results, some definitions are introduced. To simplify the notation, we drop the superscripts F and R in this subsection.

Definition 2. A set function $f : 2^N \to R$ is submodular if $f(S) + f(T) \geq f(S \cup T) + f(S \cap T)$ for all $S, T \in N$.

Definition 3. (Schrijver [35]) The polyhedron associated with the submodular function f on N:

$$EP_f := \left\{ \pi \in R^N \mid \pi(S) \leq f(S) for\ each\ S \subseteq N \right\}$$

is called the extended polymatroid associated with f if $f(\phi) = 0$ where $\pi(S) = \sum_{i\in S} \pi_i$.

Definition 4. (Atamtürk and Narayanan [5]) The inequalities associated with the extended polymatroid of f, $\pi x \leq w,\ \pi \in EP_f$ are called extended polymatroid inequalities. When the inequalities are defined by the extreme points of the extended polymatroid EP_f, they are called extremal extended polymatroid inequalities of Q_f.

Proposition 2. *Let $\mathcal{Q}_f$ denote the lower convex envelope of the sets of solutions which satisfy constraints (16), i.e.*

$$\mathcal{Q}_f = conv \left\{ (Y_j, \omega_j) \in \{0,1\}^{|I|} \times R : \omega_j \geq f(S) = \sqrt{L_j \sum_{i\in S} \mu_i}\ \forall S \subseteq I \right\}.$$

Then, the inequality $\sum_{i\in I} \pi_i Y_{ij} \leq \omega_j$ is valid for $\mathcal{Q}_f$,

where $\pi_i = \sqrt{L_j \sum_{i\in S(i)} \mu_i} - \sqrt{L_j \sum_{i\in S(i-1)} \mu_i} \in EP_f, S = \{i \mid Y_{ij} = 1\}$, *and* $S(i) = \{(1),(2),\cdots,(i)\}$, $1 \le i \le |I|$ *for some permutation.*
This valid inequality is an extremal extended polymatroid inequality.

Proof. See Appendix B. □

A similar result can be derived for the set of constraints (17) and (18).

Proposition 3. *Let* $\mathcal{Q}_u$ *denote the lower convex envelope of the sets of solutions which satisfy constraints (17) and (18), i.e.,*

$$\mathcal{Q}_u = conv\left\{(Y_j, u_j, Q_j) \in \{0,1\}^{|I|} \times R^2 : \frac{1}{2}(u_j+Q_j)^2 \ge H_j\chi\sum_{i\in I}\mu_i(Y_{ij})^2 + \frac{3}{2}(Q_j)^2 + \frac{1}{2}u_j^2\right\},$$

Then, $\sum_{i\in I}\pi_i Y_{ij} \le u_j + Q_j$ *is a valid inequality for* $\mathcal{Q}_u$ *where* $\pi_i = \sqrt{8H_j\chi\sum_{i\in S(i)}\mu_i} - \sqrt{8H_j\chi\sum_{i\in S(i-1)}\mu_i}$, $S = \{i \mid Y_{ij} = 1\}$, *and* $S(i) = \{(1),(2),\cdots,(i)\}$,$1 \le i \le |I|$ *for some permutation. This valid inequality is an extremal extended polymatroid inequality of* $\mathcal{Q}_{\overline{u}} = conv\left\{(Y_j, \overline{u}_j) \in \{0,1\}^{|I|} \times R : \frac{1}{2}\overline{u}_j^2 \ge 4H_j\chi\sum_{i\in I}\mu_i(Y_{ij})^2\right\}$.

Proof. See Appendix B. □

To find these valid inequalities, we introduce the concept of separation problem.

Definition 5. The separation problem associated with a combinatorial optimization problem is the problem: Given $x^* \in R^n$, is $x^* \in conv(X)$? If not, find an inequality $\pi x \le \pi_0$ satisfied by all points in X, but violated by the point x^*.

The separation problem for the extremal extended polymatroid inequality can be computed by a greedy algorithm described in Edmonds [14] and Atamtürk and Narayanan [5].

The greedy algorithm will find valid extremal extended polymatroid inequalities of the types described in Propositions 2 and 3 and we will add them to our formulation to speed up the solution process.

5. Computational experiments

In this section, we perform computational experiments to test the model and check how the addition of valid inequalities can speed up the computation. We start by varying the inventory and transportation weights (Table 5) without adding valid inequalities. The subsequent analysis studies the effect of the valid inequalities over a range of different DC capacity values (Tables 6, 7, and 8). The second subsection is devoted exclusively to evaluating the impact of the DC capacities (Table 9). We continue by studying the impact of marginal value of time of returned products (Table 11, 12 and 13), and the trade-off between inventory and value loss of returned products (Figures 3, 4, 5, 6, and 7).

All experiments are based on two data sets from the 1990 U.S. Census described in Daskin [10]: a 49-city data set and an 88-city data set. The first data set reports the demand of each of the lower 48 U.S. state capitals plus Washington D.C. The second data set adds to the first data set the 50 largest U.S. cities eliminating duplicates. Multiple papers on location-inventory models with stochastic demand have based their experiments on the following modifications of these data sets (Shen et al. [39], Daskin et al. [11], Atamtürk et al. [4], etc.). For the first data set, the mean demand of new products is obtained by dividing the first group of demand data by 100 and the fixed forward facility location costs are obtained by dividing the facility location costs by 100. For the second data set, the mean demand is obtained by dividing the first group of demand data by 1000 and the fixed forward facility location costs are obtained as in the first data set. The mean quantity of returned products is calculated by multiplying the return rate with the second group demand data from each data set. The return rate is identical among all retailers. The fixed reverse facility location costs are equal to the fixed forward facility location costs. Each retailer location is also a candidate DC location. The cost savings of joint DCs are set to $0.2 \min(f_j^F, f_j^R)$ in all experiments. The capacities of all DCs are equal to each other for the same experiment. The parameter values and descriptions of this model are listed in Table 4.

We directly employ these data in all the experiments except for Tables 6, 7, and 8 that are build to show computational performance. In these tables we report the average of ten random instances per row. In turn, each instance is generated by adding noise to some of the main parameters defined above. In particular, we multiply the values of the mean demand, standard deviation, and fixed costs by $(1+\epsilon)$ where ϵ is drawn from a uniform [-0.1, 0.1].

Table 4. The parameters of the model.

Parameter	**Value**	**Description**
h	1	Inventory holding cost per unit of products per year for each DC
α	97.5%	Service level
z_α	1.96	Standard normal deviate such that $P(z \leq z_\alpha) = \alpha$
F_j^F, F_j^R	10	Fixed order costs
g_j^F, g_j^R	10	Fixed transportation costs between the DCs and supplier
a_j^F, a_j^R	5	Per unit shipment costs between the DCs and supplier
L_j	1	Lead time in days
χ	1	Number of days worked in a year
W	1	Weight factor associated with loss in value of returned products
γ	10%	Marginal value of time of returned products
Return rate	40%	
V	1	Initial price of returned products
k	100	Daily transportation cost per unit returned product

The computational experiments are conducted on HP 380 G7 server running the CentOS5.4 operating system. We used the MIQCP solver of CPLEX 12.1, which solves CQMIP relaxations at the nodes of the branch-and-bound tree, with CPLEX heuristics turned off.

5.1. **Computational performance of the algorithm.** In this section, we confirm the validity of the model and the efficiency of the algorithm. In Table 5, we report the results of numerical experiments carried out over different values of θ and β (the second and third columns, respectively). Note that the quantity of returned products is less than the demand of new products as defined in both data sets. The

TABLE 5. Performance of the model without adding valid inequalities, time limits=3600s

Num. cities	θ	β	**Total cost**	**DCs**^{F}	**DCs**^{R}	**DCs**^{C}	**CPU time (s)**	**GAP(%)**	**Nodes**
49	0.10	0.0010	122663.26	10	2	2	371.99	0.0099	27138
49	0.10	0.0020	161920.12	10	2	2	11.33	0.0094	1930
49	0.10	0.0030	192657.53	13	3	3	1.82	0.0039	46
49	0.10	0.0040	216988.24	15	3	3	1.24	0.0036	40
49	0.10	0.0050	239955.55	16	3	3	1.20	0.0000	33
49	0.20	0.0020	163068.68	10	2	2	26.23	0.0097	4366
49	0.50	0.0050	244652.01	15	3	3	1.51	0.0075	40
49	1.00	0.0050	249012.60	15	3	3	1.12	0.0032	28
49	2.00	0.0050	256340.58	15	3	3	0.99	0.0047	35
49	5.00	0.0050	274761.25	15	3	3	1.20	0.0091	81
88	0.10	0.0010	24545.54	10	9	9	9.98	0.0000	121
88	0.10	0.0020	33869.69	12	10	10	8.38	0.0054	148
88	0.10	0.0030	41693.04	15	11	11	9.35	0.0058	146
88	0.10	0.0040	47569.63	22	11	11	3.96	0.0087	45
88	0.10	0.0050	52248.90	23	15	14	3.05	0.0024	21
88	0.20	0.0020	34542.68	11	10	10	9.24	0.0098	296
88	0.50	0.0050	55268.35	22	14	14	6.99	0.0099	321
88	1.00	0.0050	57923.66	22	13	13	18.16	0.0100	1500
88*	2.00	0.0050	62222.82	21	13	13	136.71	0.0100	11338
88*	5.00	0.0050	72004.29	17	12	12	3601.80	0.0792	126865

*: For related computational experiments, see Table 6 and 7.

capacities of the DCs are set to 31000 and 7700 for the 49-city and 88-city data sets, respectively. They are 1.05 times the maximum daily demand of new products. Total costs are listed in the fourth column. DC usage is shown in the next three columns, by displaying the number of forward DCs, reverse DCs and joint DCs, respectively (labeled DCs^F, DCs^R and DCs^C). The algorithm's performance is measured in terms of CPU time, the gap between the upper and lower bounds, and the number of nodes searched.

From Table 5, we observe that:

- Total costs increase when weight factors (θ or β) increase.
- More forward/reverse DCs are opened if unit transportation cost is expensive (larger β). In contrast, some forward/reverse DCs are closed due to the fact that holding cost becomes expensive (larger θ).
- In most cases, joint DCs are preferred due to cost savings but we can find some cases in which the numbers of reverse DCs and joint DCs are different due to capacity restrictions.
- Computational times have an increasing trend when we increase the value of θ while they decrease when β increases.

Finally, we present Tables 6, 7, and 8 to confirm the computational benefits of the extremal extended polymatroid inequalities. In these tables, "C" reports the capacities of the DCs, "CPU time" reports the average running time, and "Nodes" the average number of nodes in branch and bound tree. The last three columns report the number of valid inequalities added to the root node of the corresponding branch and bound tree. “W”, “U” and “V” are the average number of extremal extended polymatroid inequalities generated based on constraints (17), (18), and (16), respectively. Note that the valid inequalities contribute to finding the optimal solution in less time. For these experiments, the optimality gap is set to CPLEX’s default optimal gap at 0.01%. Even in instances where the optimal solutions cannot be found in the time limit specified, adding the valid inequalities improves the quality of the solutions by providing a solution closer to the optimal (smaller % GAP).

Table 6. The comparison between instances with and without the valid equalities, 88-city, W=1, $\theta = 2$, $\beta = 0.005$, time limits=3600s.

	CPLEX			CPLEX+CUTS					
C	**CPU time (s)**	**Nodes**	**GAP (%)**	**CPU time (s)**	**Nodes**	**GAP (%)**	**W**	**U**	**V**
10000	123.98	8548	0.0100	50.81	2912	0.0100	43	13	2
9000	130.12	9431	0.0100	43.99	2549	0.0100	31	10	0
8000	758.97	53770	0.0100	73.70	5048	0.0100	36	10	0
7900	624.83	48511	0.0100	90.38	6910	0.0100	26	9	0
7800	378.02	30033	0.0100	173.04	13994	0.0100	34	10	0
7700	152.83	11338	0.0100	53.07	2713	0.0100	32	12	1

Table 7. The comparison between the instances with and without the valid equalities, 88-city, W=1, $\theta = 5$, $\beta = 0.005$, time limits=3600s.

	CPLEX			CPLEX+CUTS					
C	**CPU time (s)**	**Nodes**	**GAP (%)**	**CPU time (s)**	**Nodes**	**GAP (%)**	**W**	**U**	**V**
10000	3602.39	123846	0.2067	1417.73	58577	0.0100	61	28	8
9000	3602.02	123682	0.1269	369.69	13229	0.0100	56	15	8
8000	3602.19	118435	0.2454	780.19	27418	0.0100	52	16	4
7900	–	3	–	1381.11	59816	0.0100	44	14	4
7800	–	6	–	785.52	28430	0.0100	53	21	5
7700	3601.80	118520	0.0855	667.01	30314	0.0100	80	33	13

– means do not find any feasible solution within the time limits

5.2. **The impact of DC capacity.** In this section, we study the effect of DC capacity on both the number of open DCs and the operations at the DCs, as illustrated in Table 9. From this table, we see that if we tighten the capacities the number of forward DCs increases and more DCs are utilized at capacity.

The impact of DC capacity on order sizes at joint DCs is summarized in the following property:

Table 8. The comparison between the instances with and without the valid equalities, 88-city, W=1, $\theta = 10$, $\beta = 0.005$, time limits=3600s.

	CPLEX			CPLEX+CUTS					
C	**CPU time (s)**	**Nodes**	**GAP (%)**	**CPU time (s)**	**Nodes**	**GAP (%)**	**W**	**U**	**V**
10000	3601.61	42141	0.8330	3601.33	55875	0.1584	107	53	25
9000	3601.54	41320	0.9515	2898.96	47697	0.0100	108	56	37
8000	3601.44	51072	0.9232	3601.51	42392	0.1007	108	61	25
7900	3601.51	47565	0.9614	3601.22	34153	0.1365	118	52	33
7800	3601.63	44235	0.8668	3601.56	52347	0.1216	121	62	37
7700	–	4	–	3601.51	48237	0.0543	109	67	33

– means do not find any feasible solution within the time limits

Property 1. *Given a joint DC j,*

(a) If capacity at DC j is binding, the optimal order quantities of new products (Q_j^{F*}) and returned products (Q_j^{R*}) are either

- *strictly less than the corresponding EOQ quantities;*
- *or, equal to the corresponding EOQ quantities.*

(b) If capacity at DC j is not binding, the optimal order quantities are the corresponding EOQ quantities:

$$Q_j^{F*} = Q_{j\_EOQ}^F = \sqrt{\frac{2(F_j^F + \beta g_j^F)D_j^F}{\theta h}},$$

$$Q_j^{R*} = Q_{j\_EOQ}^R = \sqrt{\frac{2(F_j^R + \beta g_j^R)D_j^R}{\theta h + W\gamma V\chi}}.$$

Proof. See Appendix B for proof. □

5.3. **The impact of returned products' marginal value of time.** Returned products' marginal value of time is associated with the degree of time sensitivity of the product's price. If we consider it along with DC capacity it leads to distinct characterizations of DC location decisions. Table 11 shows the impacts of γ with $k = 500$. To emphasize the impacts of returned products' inventory, we also report some results with different return rates and $k = \infty$, which simulates the cases when transportation times are ignored (Tables 12 and 13). The marginal value of time, γ, in the experiments is set to 1%, 10%, 30%, 50%, 70%, and 90%. The columns labeled "Cost^F" and "Cost^R" list the costs associated with the forward and reverse flows, respectively. Table 10 lists the parameters of the experiments. From the experiments, we find that γ has different impact on the results with and without the considerations of loss values associated with transportation times. We find that:

(1) Fewer reverse DCs are needed for highly time-sensitive returned products (higher γ) when transportation times are neglected. Intuitively, storage time of time-sensitive returned products has been reduced in order to retrieve more salvage value of them. This implies more shipments with smaller quantity so the storage space needed decreases. On the contrary, more reverse DCs are build for higher γ if transportation times are part of the

TABLE 9. The impact of the capacity of the DCs ($\theta = 0.1$ and $\beta = 0.001$).

Num.	**Capacity**	**Total cost**	**DCs**^{F}	**DCs**^{R}	**DCs**^{C}
1	50000	23594	9 (0)	9 (0)	9 (0)
2	20000	23594	9 (0)	9 (0)	9 (0)
3	15000	23594	9 (0)	9 (0)	9 (0)
4	14000	23618	9 (1)	9 (1)	9 (1)
5	13000	23794	9 (1)	9 (1)	9 (1)
6	12000	23980	10 (1)	9 (1)	9 (1)
7	11000	24038	10 (1)	9 (1)	9 (1)
8	10000	24144	10 (1)	9 (1)	9 (1)
9	9000	24221	10 (2)	9 (1)	9 (1)
10	8000	24327	10 (2)	9 (1)	9 (1)
11	7900	24367	10 (2)	9 (1)	9 (1)
12	7800	24427	10 (2)	9 (1)	9 (1)
13	7700	24546	10 (2)	9 (1)	9 (1)

The numbers in brackets in the last three columns indicate how many of the DCs opened have binding capacities.

value loss function. This is because having more DCs reduces transportation times.

(2) Returned products impact the forward DCs in number and location. While more forward DCs are constructed in the case of non-negligible transportation times (Table 11) if the product is more time-sensitive, fewer forward DCs are constructed in the case of negligible transportation times and highly time-sensitive returned products (Table 13). Even if the number of forward DCs is identical, the locations of some forward DCs are different. For instance, in Table 12 a DC is opened in Atlanta as a forward DC in experiment 2 while Charlotte is constructed in experiment 3 and Atlanta is closed. Similar results are also observed in reverse DCs. However, the number of the stand-alone forward DCs increases for highly time-sensitive returned products (the difference between the sixth column and the eighth column) in Tables 12 and 13 to offset for the reduction of forward product capacity created by the drop of the number of joint DCs. Similarly, the number of stand-alone forward DCs decreases for higher γ in Table 11 to offset for the increment of capacity created by the increase in the number of joint DCs.

(3) Reverse flows impact forward flows' decisions not only on facility location but also on inventory management. In Table 13, it is interesting to note that the forward flow costs slightly diminish for highly time-sensitive returned products. This shows an opposite trend with the total costs and reverse flow costs.

Figure 3 aims to describe the trade-off between working inventory costs and value loss of returned products associated to inventory times. As shown, the working inventory cost decreases while the loss in value increases in the range of $(0, Q_j^{R'}]$. From the proof of property 2 (Appendix A), Q_j^{R*} refers to the optimal shipment quantity of returned products when considering the loss in value. Once taking the

TABLE 10. The parameter values in the experiments shown in Tables 11, 12 and 13.

Parameter	**Value**
W	10
θ	0.1
β	0.005
Capacity of DCs	7700

TABLE 11. Impact of returned products' marginal time value (return rate = 60%, $k = 500$).

Num.	γ **(%)**	**Total cost**	**Cost**^{F}	**Cost**^{R}	**DCs**^{F}	**DCs**^{R}	**DCs**^{C}
1	1	55143	32033	25124	23	20	19
2	10	60212	32300	26970	23	23	23
3	30	68062	32298	28304	24	24	24
4	50	74858	32471	31280	24	28	24
5	70	80148	32595	34790	25	33	25
6	90	84663	32676	36367	25	35	25

TABLE 12. Impact of returned products' marginal time value (return rate = 60%, $k = \infty$).

Num.	γ **(%)**	**Total cost**	**Cost**^{F}	**Cost**^{R}	**DCs**^{F}	**DCs**^{R}	**DCs**^{C}
1	1	54755	32033	25124	23	20	19
2	10	56489	32033	25521	23	17	16
3	30	58439	32033	26360	23	16	15
4	50	59751	32033	26940	23	13	12
5	70	60745	32033	27438	23	13	12
6	90	61605	32033	27863	23	13	12

TABLE 13. Impact of returned products' marginal time value (return rate = 100%, $k = \infty$).

Num.	γ **(%)**	**Total cost**	**Cost**^{F}	**Cost**^{R}	**DCs**^{F}	**DCs**^{R}	**DCs**^{C}
1	1	60957	32412	31708	24	24	24
2	10	63351	32387	32639	24	24	24
3	30	66230	32376	33983	24	24	24
4	50	68176	32370	34876	23	21	21
5	70	69722	32349	35616	23	21	21
6	90	71054	32330	36320	23	20	20

loss in value of returned produces into account, Q_j^{R*} must be less than $Q_j^{R'}$. This leads to different decisions due to different priorities/preferences of the decision-makers. It is therefore interesting to find the corresponding non-inferior solutions.

We vary the weight factor W, and plot the trade-off curves corresponding to working inventory cost and value loss of returned products (Figures 4 and 5). Table 14 summarizes the parameters used in these experiments.

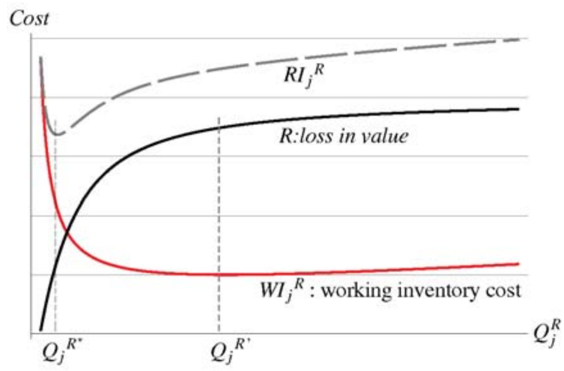

FIGURE 3. Schematic diagram of the value loss in inventory and the working inventory cost of returned products, where $Q_j^{R'}$ denotes the optimal shipment quantity of returned products without the consideration of the loss in value.

As shown in Figure 4, we find that more DCs are constructed if we try to reduce the loss in value of returned products (or, in other words, we try to retrieve more salvage value from returned products). While, Figure 5 shows the opposite results if we ignore the transportation times between the retailers, the DCs, and the supplier. As already observed in Tables 11 and 12, this implies that time in transportation and time in inventory of returned products have opposed influences on supply chain design decisions.

TABLE 14. The parameter values in the experiments shown in Figures 5 and 7.

Parameter	Value
γ	50%
Return rate	80%
θ	0.1
β	0.005

Figures 6 and 7 report the trade-off curves between working inventory cost and loss in value of returned products with different γ (i.e. returned products with different marginal values of time). According to the figures, given a fixed change of loss in value, the change of working inventory cost of time-sensitive returned products (higher γ) is smaller than that of time-insensitive ones (lower γ). Therefore, and confirming our intuition, it makes more sense to salvage the value of time-sensitive returned products. We also note that the working inventory costs in Figure 6 are larger than those in Figure 7 (similarly, in Figures 4 and 5). This is due to the influence of transportation times in the value loss function of the model behind Figure 6. Since there is a major influence of transportation times, more DCs are opened, and this implies that smaller amounts of returned product will be shipped to each opened DC (Q_j^R). Figure 3 shows that smaller Q_j^R leads to larger working inventory costs.

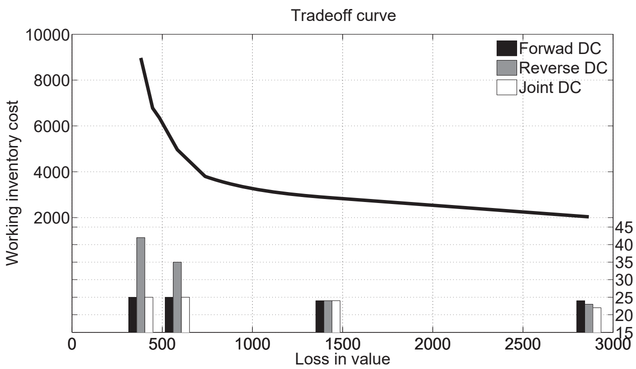

FIGURE 4. Trade-off curve between working inventory cost and value loss of returned products (DC capacity=7700, $k = 500$).

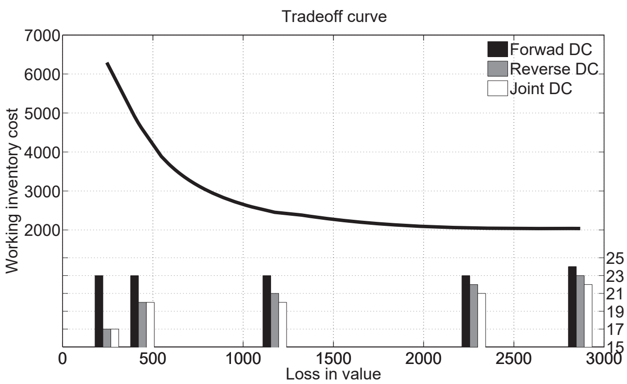

FIGURE 5. Trade-off curve between working inventory cost and value loss of returned products (DC capacity=7700, $k = \infty$).

6. Conclusions and future research

This paper studies the capacitated facility location problem with bidirectional flows, which is starting to receive much attention in the literature. This model minimizes the fixed location costs, the working inventory, and the transportation costs. Moreover, we consider the loss in value of returned products when making location decisions. We transform the model into a conic quadratic mixed integer program. The model can be solved efficiently in most cases by using CPLEX. Some valid inequalities are added to improve the efficiency of the branch and cut algorithm and the quality of the solutions.

We perform an extensive computational study and observe the following interesting results:

(1) Extremal extended polymatroid inequalities are computationally beneficial for the formulation presented.
(2) DC capacity has impact on facility location decisions and inventory operations. The optimal order quantities of new/returned products at a joint

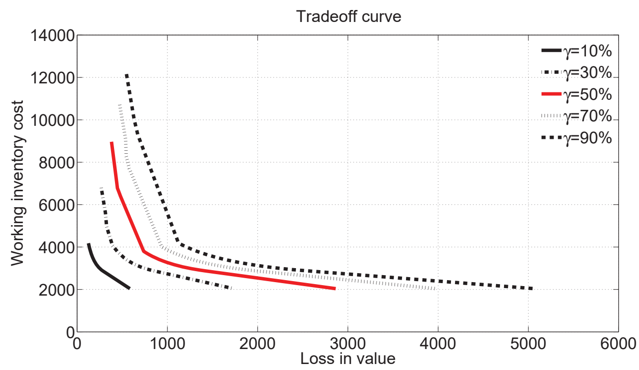

FIGURE 6. Trade-off curves for returned products with different γ, capacity=7700, $k = 500$. Note that for small γ the corresponding curve's domain is smaller than that of larger values of γ since the loss in value is defined as $\frac{\chi\gamma}{2}Q_j^R$ and Q_j^R is in the range $(0, EOQ\ value]$.

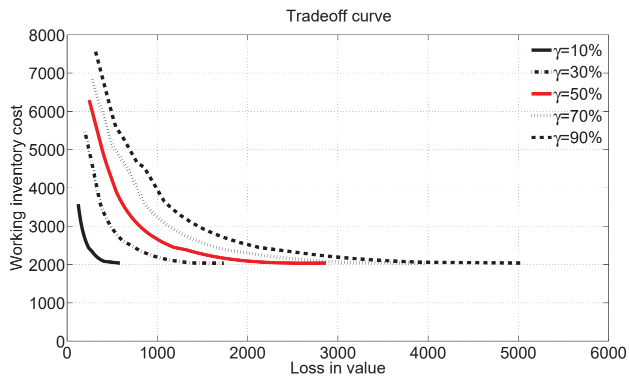

FIGURE 7. Trade-off curves for returned products with different γ, capacity=7700, $k = \infty$.

DC are the EOQ quantities, if DC capacity is non-binding, and below the EOQ level if capacity is binding.

(3) Marginal value of time of returned products impacts the location and inventory decisions not only of reverse facilities but also of forward facilities. If the transportation times of returned products are negligible, fewer DCs are constructed for highly time-sensitive returned products. However, the reverse effect occurs when transportation times become longer.

(4) In order to retrieve more salvage value from returned products, it is necessary to tolerate higher working inventory costs of returned products. However, for a fixed change of loss in value the respective working inventory cost increment will be smaller for time-sensitive products than for time-insensitive products. In addition, retrieving more salvage value might result in a smaller or larger optimal number of opened DCs, depending on

whether inventory or transportation times of returned products have a major influence. Thus, it is helpful to consider carefully the salvage value of time-sensitive returned products when making location decisions.

We suggest some avenues of further research. First, this model can naturally be extended to incorporate multiple products (Shen [36]). Second, it is interesting to explore the impact of pricing on the design of the integrated network with returned products (Shen [37]). Finally, other decisions, such as the vehicle routing decisions for collecting returned products (e.g., Berger et al. [6] and Shen and Qi [41]), can be integrated into the capacitated facility location problem.

7. APPENDIX

7.1. **Appendix A. The linear decay rate is a good enough approximation to define $R_{inv}(Q_j^R)$**

We can define the average value loss of returned product associated with inventory times ($R_{inv}(Q_j^R)$) to tightly approximate the well-studied case of returned products with exponential price decay function. In this case

$$Price(t) = Ve^{-\gamma t},$$

where t denotes the time the unit is held at DC. This case is extensively used in the literature to investigate the loss in value of returned products (Guide et al. [16], Blackburn and Scudder [7]).

The salvage value of returned products (with batch size equal to Q_j^R) can be found by integrating over $[0, Q_j^R]$, $\int_0^{Q_j^R} Price\left(\frac{Q_j^R - q}{Q_j^R}\frac{\chi Q_j^R}{D_j^R}\right) dq$. Then, the loss in value per batch equals $VQ_j^R - \int_0^{Q_j^R} Price\left(\frac{Q_j^R - q}{Q_j^R}\frac{\chi Q_j^R}{D_j^R}\right) dq$. Subsequently, $R_{inv}(Q_j^R)$ can be determined multiplying by the number of shipments to the supplier per year, $\frac{D_j^R}{Q_j^R}$,

$$\begin{aligned} R_{inv}\left(Q_j^R\right) &= \frac{D_j^R}{Q_j^R}\left\{VQ_j^R - \int_0^{Q_j^R} Price\left(\frac{Q_j^R - q}{Q_j^R}\frac{\chi Q_j^R}{D_j^R}\right) dq\right\} \\ &= VD_j^R - \frac{V(D_j^R)^2}{\chi\gamma Q_j^R}\left(1 - e^{-\frac{\chi\gamma Q_j^R}{D_j^R}}\right). \end{aligned} \tag{25}$$

It is intractable to find the optimal solution of the model due to the complexity of the resulting $R_{inv}(Q_j^R)$. Therefore, we replace $e^{\frac{-\chi\gamma Q_j^R}{D_j^R}}$ with its second-order Taylor-series expansion

$$e^{-\frac{\chi\gamma Q_j^R}{D_j^R}} \approx 1 - \frac{\chi\gamma Q_j^R}{D_j^R} + \frac{\chi^2\gamma^2(Q_j^R)^2}{2(D_j^R)^2}.$$

Then, equation (25) can be approximated as follows:

$$R_{inv}(Q_j^R) \approx \hat{R}_{inv}(Q_j^R) = \frac{V\chi\gamma}{2} Q_j^R. \tag{26}$$

Let us define $RI_j^R(Q_j^R) = WI_j^R(D_j^R, Q_j^R) + W \cdot R_{inv}(Q_j^R)$, which represents the sum of working inventory costs and value loss of returned products associated with inventory. We next present the following property regarding this expression that is studied experimentally in section 5.3.

Property 2. *$RI_j^R(Q_j^R)$ is unimodal in Q_j^R.*

Proof. See Appendix B. □

From the approximation in (26) we have that

$$RI_j^R(Q_j^R) \approx \hat{RI}_j^R(Q_j^R) = WI_j^R(D_j^R, Q_j^R) + W \cdot \hat{R}_{inv}(Q_j^R).$$

In order to examine the accuracy of the approximation, we define

$$ERR(Q_j^R) \quad = \quad RI_j^R(Q_j^R) - \hat{RI}_j^R(Q_j^R) = W\left(R_{inv}(Q_j^R) - \hat{R}_{inv}(Q_j^R)\right),$$

which measures the error between RI_j^R and its approximation, $\hat{RI}_j^R$.

Property 3. *$ERR(Q_j^R)$ is a concave function of Q_j^R.*

Proof. See Appendix B. □

Owing to Property 3, the quantities of returned products with respect to the maximal error, denoted by $Q_{j\_max}^R$, can be determined by using search algorithms such as golden section method or bisection method. Since the optimal shipment quantity of returned products with capacity constraints must be less than that without consideration of capacity constraints, we also calculate the later one, denoted by Q_j^{R*}. Due to Property 2, search algorithms are employed as well. Then, $Q_j^R = min\left(Q_{j\_max}^R, Q_j^{R*}\right)$ is used to examine the accuracy of the approximation.

We perform 2,000,000 numerical experiments with the parameters drawn uniformly from the range given in Table 15. The values of $Q_{j\_max}^R$ and Q_j^{R*} are found by bisection method. The results are summarized in Table 16 and Table 17, in which we normalize the error by RI_j^R, $err = \dfrac{|ERR|}{RI_j^R}$, and by R_j^R, $err_2 = \dfrac{|ERR|}{R_j^R}$, respectively.

The results show that $\hat{RI}_j^R$ is a quite tight approximation of RI_j^R.

Note that equation (26) has the same form as equation (11). Therefore, we can adopt equation (11) to approximate the loss in value of returned products associated with inventory times.

7.2. **Appendix B: Proofs.**

Proof of PROPERTY 1

Table 15. Parameter intervals for numerical experiments.

Parameter	Interval	Parameter	Interval
F_j^R	[5, 15]	V	[1, 10]
g_j^R	[5, 15]	χ	[1,10]
a_j^R	[1, 10]	γ	[1,10]
h	[1, 5]	θ	[0.01,1]
W	[0.1, 100]	β	[0.001,0.1]
D_j^R	[1, 1000000]		

Table 16. Statistical results for numerical experiments of err.

err (%)	**number of experiments**	**percentage**
<0.01	291,687	29.2%
<0.02	513,487	51.3%
<0.05	854,207	85.4%
<0.1	951,443	95.1%
<0.2	1,000,000	100%
average error=0.031%, maximal error=0.126%, minimal error=0.0054%		

Table 17. Statistical results for numerical experiments of err_2.

err (%)	**number of experiments**	**percentage**
<0.01	235,898	23.6%
<0.02	331,440	33.1%
<0.05	712,973	71.3%
<0.1	904,566	90.5%
<0.2	952,496	95.2%
<0.5	984,736	98.5%
<1.0	1,000,000	100%
average error=0.090%, maximal error=0.993%, minimal error=0%		

Proof. The shipment quantities of new and returned products can be obtained exogenously by solving the following program:

$$W_j(D_j^F, D_j^R) = \begin{cases} Min \quad F_j^F \dfrac{D_j^F}{Q_j^F} + \beta\left(g_j^F + a_j^F Q_j^F\right)\dfrac{D_j^F}{Q_j^F} + \theta\dfrac{hQ_j^F}{2} + \theta h z_\alpha \sqrt{L_j \dfrac{D_j^F}{\chi}} \\ \qquad +F_j^R \dfrac{D_j^R}{Q_j^R} + \beta\left(g_j^R + a_j^R Q_j^R\right)\dfrac{D_j^R}{Q_j^R} + \dfrac{\theta h + W\gamma V\chi}{2} Q_j^R, \\ s.t. \quad Q_j^F + z_\alpha\sqrt{L_j\dfrac{D_j^F}{\chi}} + L_j\dfrac{D_j^F}{\chi} + Q_j^R \le C_j, \\ \qquad Q_j^F, Q_j^R \ge 0. \end{cases}$$

Because $W_j(D_j^F, D_j^R)$ is convex, we apply KKT conditions and obtain the equations:

$$\begin{cases} -\dfrac{(F_j^F+\beta g_j^F)D_j^F}{(Q_j^F)^2}+\dfrac{h\theta}{2}+\lambda_j=0, \\ -\dfrac{(F_j^R+\beta g_j^R)D_j^R}{(Q_j^R)^2}+\dfrac{h\theta+W\gamma V\chi}{2}+\lambda_j=0, \\ \lambda\left(Q_j^F+L_j\dfrac{D_j^F}{\chi}+z_\alpha\sqrt{L_j\dfrac{D_j^F}{\chi}}+Q_j^R-C_j\right)=0, \\ Q_j^F, Q_j^R, \lambda \geq 0, \end{cases}$$

where λ_j is a nonnegative Lagrangian multiplier. If the capacity constraint is strictly satisfied, then λ_j=0 and the shipment quantities of new and returned products can be determine by the economic order quantities of them, i.e.,

$$Q_j^F = \sqrt{\frac{2(F_j^F+\beta g_j^F)D_j^F}{h\theta}},$$

$$Q_j^R = \sqrt{\frac{2(F_j^R+\beta g_j^R)D_j^R}{h\theta+W\gamma V\chi}}.$$

If the capacity constraint is binding, then $\lambda_j \geq 0$ and the shipment quantities of new and returned products are:

$$Q_j^F = \sqrt{\frac{2(F_j^F+\beta g_j^F)D_j^F}{h\theta+2\lambda_j}},$$

$$Q_j^R = \sqrt{\frac{2(F_j^R+\beta g_j^R)D_j^R}{h\theta+W\gamma V\chi+2\lambda_j}}.$$

Note that both of them are less than the respective economic order quantities if $\lambda_j > 0$ or are equal to the economic order quantities if $\lambda_j = 0$. □

Proof of Proposition 2

Proof. $f(S) = \sqrt{L_j\mu(S)}$, where $\mu(S) = \sum_{i\in S}\mu_i$, is a submodular function due to its concavity based on the following result studied in Nemhauser and Wolsey [28] and Shen et al. [39]:

A set function $f: 2^N \to R$ defined by $f(S) = g(a(S))$, where $g(\cdot)$ is concave and $a(S)$ is the sum of the components of $a \in R_+^N$ on $S \subseteq N$, is submodular.

Hence, $\pi_i = \sqrt{L_j\mu(S_{(i)})} - \sqrt{L_j\mu(S_{(i-1)})}$ is an extreme point of the extended polymatriod EP_f based on Edmonds [14]. That is, $\pi_i \in EP_f$. Therefore, $\pi(S) \leq f(S) \leq \omega_j$, which completes the proof. □

Proof of Proposition 3

Proof. Let $\overline{u}_j = u_j + Q_j$, from constraints (17) and (23) we obtain the following relaxed form of constraints (17)

$$\begin{aligned} \overline{u}_j^2 &\geq \frac{4(F_j+\beta g_j)}{\theta h}D_j + 3(Q_j)^2 + u_j^2 \\ &\geq \frac{4(F_j+\beta g_j)}{\theta h}D_j + 3(Q_j)^2 + \left(\frac{2(F_j+\beta g_j)}{\theta h}\frac{D_j}{Q_j}+Q_j\right)^2. \end{aligned} \tag{27}$$

Taking the derivative of the right-hand side of the above inequality with respect to Q_j, we obtain

$$6Q_j + 2\left(1 - \frac{2D_j(F_j+\beta g_j)}{h\theta(Q_j)^2}\right)\left(Q_j + \frac{2D_j(F_j+\beta g_j)}{h\theta Q_j}\right) = 0.$$

Solving for Q_j, we obtain $Q_j = \sqrt{\frac{(F_j+\beta g_j)D_j}{h\theta}}$. Substituting this into the inequality (27) we obtain the following relaxed constraint

$$\begin{aligned}
\overline{u}_j^2 &\geq \frac{4(F_j+\beta g_j)}{\theta h}D_j + 3(Q_j)^2 + u_j^2 \\
&\geq \frac{4(F_j+\beta g_j)}{\theta h}D_j + 3(Q_j)^2 + \left(\frac{2(F_j+\beta g_j)}{\theta h}\frac{D_j}{Q_j} + Q_j\right)^2 \\
&\geq \frac{4(F_j+\beta g_j)}{\theta h}D_j + \frac{12(F_j+\beta g_j)}{\theta h}D_j \\
&= \frac{16(F_j+\beta g_j)}{\theta h}D_j.
\end{aligned}$$

According to Proposition 2, we can get a valid inequality (that is also an extremal extended polymatroid inequality), $\sum_{i\in I}\pi_i Y_{ij} \leq u_j + Q_j$, for the lower convex envelope of the relaxed constraint, that is $\mathcal{Q}_{\overline{u}} = conv\left\{(Y_j,\overline{u}_j)\in\{0,1\}^{|I|}\times R : \frac{1}{2}\overline{u}_j^2 \geq 4H_j\chi\sum_{i\in I}\mu_i(Y_{ij})^2\right\}$, where $\pi_i = \sqrt{8H_j\chi\sum_{i\in S(i)}\mu_i} - \sqrt{8H_j\chi\sum_{i\in S(i-1)}\mu_i}$.

Note that the suggested inequality is also valid for $\mathcal{Q}_u$ of constraints (17), where $\mathcal{Q}_u = conv\left\{(Y_j,u_j,Q_j)\in\{0,1\}^{|I|}\times R^2 : \frac{1}{2}(u_j+Q_j)^2 \geq H_j\chi\sum_{i\in I}\mu_i(Y_{ij})^2 + \frac{3}{2}(Q_j)^2 + \frac{1}{2}u_j^2\right\}$.

The same proof can be derived for constraints (18) and (24).

□

Proof of PROPERTY 2

Proof. RI_j^R is the sum of a concave function ($R_{inv}(Q_j^R)$) and a convex function ($WI_j^R(D_j^R, Q_j^R)$). The second-order derivative of RI_j^R with respect to Q_j^R is:

$$\begin{aligned}
\frac{\partial^2 RI_j^R}{\partial(Q_j^R)^2} &= \frac{1}{\chi\gamma(Q_j^R)^3}\Big\{-2D_j^R\left[D_j^R VW - (F_j^R+\beta g_j^R)\chi\gamma\right] \\
&\quad + e^{-\frac{\chi\gamma}{D_j^R}Q_j^R} VW\left[2(D_j^R)^2 + 2D_j^R Q_j^R\chi\gamma + (Q_j^R)^2\chi^2\gamma^2\right]\Big\}. \quad (28)
\end{aligned}$$

We cannot determine whether it is positive or not. As such, RI_j^R is neither convex nor concave.

Let Q_j^{R*} denotes Q_j^R such that

$$\begin{aligned}\frac{\partial RI_j^R}{\partial Q_j^R} &= \frac{1}{2\chi\gamma(Q_j^R)^2}\left\{-2e^{-\frac{\gamma\chi}{D_j^R}Q_j^R} D_j^R VW(D_j^R + Q_j^R\chi\gamma)\right.\\ &\quad \left. + \left[2(D_j^R)^2 VW - 2D_j^R(F_j^R + \beta g_j^R)\chi\gamma + h(Q_j^R)^2\chi\gamma\theta\right]\right\} = 0.\end{aligned}$$

Substituting Q_j^{R*} into equation (28), we find

$$\begin{aligned}\frac{\partial^2 RI_j^R}{\partial (Q_j^R)^2}\Big|_{Q_j^R = Q_j^{R*}} &= \frac{1}{\chi\gamma(Q_j^{R*})^3}\left\{h(Q_j^{R*})^2\chi\gamma\theta - 2e^{-\frac{\gamma\chi}{D_j^R}Q_j^{R*}} D_j^R VW(D_j^R + Q_j^{R*}\chi\gamma)\right.\\ &\quad \left. + e^{-\frac{\chi\gamma}{D_j^R}Q_j^{R*}} VW\left[2(D_j^R)^2 + 2D_j^R Q_j^{R*}\chi\gamma + (Q_j^{R*})^2\chi^2\gamma^2\right]\right\}\\ &= \frac{1}{\chi\gamma(Q_j^{R*})^3}\left\{h(Q_j^{R*})^2\chi\gamma\theta + e^{-\frac{\chi\gamma}{D_j^R}Q_j^{R*}} VW(Q_j^{R*})^2\chi^2\gamma^2\right\} \geq 0.\end{aligned}$$

It shows that $RI_j^R(Q_j^R)$ is unimodal in Q_j^R and Q_j^{R*} is global minimum. □

Proof of PROPERTY 3

Proof. Taking the first- and second-order derivative of $R_{inv}(Q_j^R)$ with respect to Q_j^R, we can show that $R_{inv}(Q_j^R)$ is an increasing and concave function of Q_j^R.

$$\frac{dR_{inv}(Q_j^R)}{dQ_j^R} = \frac{V(D_j^R)^2 e^{-\frac{Q_j^R\gamma\chi}{D_j^R}}}{(Q_j^R)^2\gamma\chi}\left(e^{\frac{Q_j^R\gamma\chi}{D_j^R}} - 1 - \frac{Q_j^R\gamma\chi}{D_j^R}\right) > 0.$$

$$\begin{aligned}\frac{d^2 R_{inv}(Q_j^R)}{d(Q_j^R)^2} &= \frac{Ve^{-\frac{\gamma\chi Q_j^R}{D_j^R}}}{(Q_j^R)^3\gamma\chi}\left[-2(D_j^R)^2\left(-1 + e^{\frac{\gamma\chi Q_j^R}{D_j^R}}\right) + 2D_j^R Q_j^R\gamma\chi + (Q_j^R)^2\gamma^2\chi^2\right]\\ &= \frac{2(D_j^R)^2 Ve^{-\frac{\gamma\chi Q_j^R}{D_j^R}}}{(Q_j^R)^3\gamma\chi}\left(-e^{\frac{\gamma\chi Q_j^R}{D_j^R}} + 1 + \frac{Q_j^R\gamma\chi}{D_j^R} + \frac{(Q_j^R)^2\gamma^2\chi^2}{2(D_j^R)^2}\right) < 0\end{aligned}$$

and $\hat{R}_{inv}(Q_j^R)$ is linear. Therefore, $ERR(Q_j^R)$ is a concave function of Q_j^R. □

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