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# Automatic Portmanteau Tests with Applications to Market Risk Management

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**Abstract.** This article reviews some recent advances in testing for serial correlation, provides Stata code for implementation and illustrates its application to market risk forecast evaluation. The classical and widely used Portamenteau tests and their data-driven versions are the focus of this article. These tests are simple to implement for two reasons: first, the researcher does not need to specify the order of the autocorrelation tested, since the test automatically chooses this number; second, its asymptotic null distribution is chi-square with one degree of freedom, so there is no need of using a bootstrap procedure to estimate the critical values. We illustrate the wide applicability of the methodology with applications to forecast evaluation for market risk measures, such as Value-at-Risk and Expected Shortfall.

**Keywords:** dbptest, rtau, Autocorrelation, consistency, power, Akaike's AIC, Schwarz's BIC, Market Risk

## 1. Introduction

Testing for serial correlation has held a central role in time series analysis since its inception (see the early contributions by Yule (1926) and Quenouille (1947)). Despite the many proposals and variations since the seminal contribution of Box and Pierce (1970), still the so-called Portmanteau tests are the most widely used. In its simplest form, the employed statistic is just the sample size times the sum of the first $p$ squared sample autocorrelations, which is compared with critical values from a chi-square distribution with $p$ degrees of freedom (with a correction if the test is applied to residuals). The basic Box-Pierce statistic has been slightly modified to improve its finite sample performance, see Davies et al. (1977), Ljung and Box (1978), Davies and Newbold (1979) or Li and McLeod (1981). The properties of the classical Box-Pierce tests have been extensively studied in the literature; see e.g. the monograph by Li (2004) for a review of this literature. Much of the theoretical literature on Box-Pierce tests was developed under the independence assumption, and hence is generally invalid when applied to dependent data (the asymptotic size of the test is different from the nominal level); see Newbold (1980) or more recently Francq et al. (2005) for valid tests. This limitation of classical Box-Pierce tests is by now well understood. This paper focuses on another limitation of Classical Box-Pierce tests: the selection of the employed number of autocorrelations is arbitrary. We review the contribution of Escanciano and Lobato (2009), who proposed a data-driven Portmanteau statistic where the number of correlations is not fixed but selected automatically from the data. In this paper, we give a synthesis of this methodology, introduce new general assumptions for its validity, review new applications in risk management and provide Stata code for its implementation.

## 2. Automatic Portmanteau Tests: A Synthesis

Given a strictly stationary process $\{Y_t\}_{t\in Z}$ with $E[Y_t^2] < \infty$ and $\mu = E[Y_t]$, define the autocovariance of order $j$ as

$$\gamma_j = Cov(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)], \qquad \text{for all } j \geq 0,$$

and the $j-th$ order autocorrelation as $\rho_j = \gamma_j/\gamma_0$. We aim to test the null hypothesis

$$H_0 : \rho_j = 0, \qquad \text{for all } j \geq 1,$$

against the fixed alternative hypotheses

$$H_1^K : \rho_j \neq 0, \qquad \text{for some } 1 \leq j \leq K, \tag{1}$$

and some $K \geq 1$.

Suppose we observe data $\{Y_t\}_{t=1}^n$. Then, $\gamma_j$ can be consistently estimated by the sample autocovariance

$$\widehat{\gamma}_j = \frac{1}{(n-j)} \sum_{t=1+j}^{n} (Y_t - \overline{Y})(Y_{t-j} - \overline{Y}), \quad j = 0, ..., n-1,$$

where $\overline{Y}$ is the sample mean, and also introduce $\widehat{\rho}_j = \widehat{\gamma}_j / \widehat{\gamma}_0$ to denote the $j-th$ order sample autocorrelation.

The Box-Pierce $Q_p$ statistic (cf. Box and Pierce (1970)) is just

$$Q_p = n \sum_{j=1}^{p} \widehat{\rho}_j^2,$$

which is commonly implemented via the Ljung and Box (1978) modification

$$LB_p = n(n+2) \sum_{j=1}^{p} (n-j)^{-1} \widehat{\rho}_j^2. \tag{2}$$

When $\{Y_t\}_{t=1}^n$ are independent and identically distributed ($iid$), both $Q_p$ and $LB_p$ converge to a chi-square distribution with $p$ degrees of freedom, in short $\chi_p^2$. When $\{Y_t\}_{t=1}^n$ are serially dependent, like for example when $Y_t$ is a residual from a fitted model, the asymptotic distribution of $Q_p$ or $LB_p$ is generally different from $\chi_p^2$ and depends on the data generating process in a complicated way, see for instance Francq et al. (2005) and Delgado and Velasco (2011).

In this section we provide a synthesis of the automatic Portmanteau test methodology suggested in Escanciano and Lobato (2009), thereby extending the methodology to other situations. The main ingredients of the methodology are: (1) the following asymptotic results for individual autocorrelations, for $j = 1, ..., d$, where $d$ is a fixed upper bound,

$$\sqrt{n} \left( \widehat{\rho}_j - \rho_j \right) \xrightarrow{D} N(0, \tau_j), \tag{3}$$

for a positive asymptotic variance $\tau_j > 0$, with[1]

$$\widehat{\tau}_j \xrightarrow{P} \tau_j; \tag{4}$$

and (2) a data-driven construction of $p$ given below. For $iid$ observations $\tau_j = 1$, and trivially we can take $\widehat{\tau}_j = 1$, but in other more general settings with weak dependence

1. In this paper we use $\xrightarrow{D}$ and $\xrightarrow{P}$ to denote convergence in distribution and in probability, respectively.

or estimation effects we will have an unknown $\tau_j \neq 1$ that needs to be estimated. Our definitions of Portmanteau tests allow for general cases. Define

$$Q_p^* = n \sum_{j=1}^{p} \widetilde{\rho}_j^2,$$

where $\widetilde{\rho}_j = \widehat{\rho}_j / \sqrt{\widehat{\tau}_j}$ is called a *Generalized Autocorrelation* here. Then, the Automatic Portmanteau test is given by

$$AQ = Q_{\widetilde{p}}^* \tag{5}$$

where

$$\widetilde{p} = \min\{p : 1 \leq p \leq d; L_p \geq L_h, h = 1, 2..., d\},$$

with

$$L_p = Q_p^* - \pi(p, n, q),$$

$\pi(p, n, q)$ is a penalty term that takes the form

$$\pi(p, n, q) = \begin{cases} p \log n, & \text{if} \quad \max_{1 \leq j \leq d} \sqrt{n} \left|\widetilde{\rho}_j\right| \leq \sqrt{q \log n}, \\ 2p, & \text{if} \quad \max_{1 \leq j \leq d} \sqrt{n} \left|\widetilde{\rho}_j\right| > \sqrt{q \log n}, \end{cases} \tag{6}$$

and $q = 2.4$. The penalty term in (6) has been proposed by Inglot and Ledwina (2006a) for testing the goodness of fit for a distribution. The value of $q = 2.4$ is motivated from extensive simulation evidence in Inglot and Ledwina (2006b) and Escanciano and Lobato (2009). The value of $q = 0$ corresponds to the Akaike Information Criterion (AIC), see Akaike (1974). The value of $q = \infty$ corresponds to the Bayesian Information Criterion (BIC), see Schwarz et al. (1978). In the context of testing for serial correlation, it is known that AIC is good in detecting non-zero correlations at long lags, at the cost of leading to size distortions. In contrast, BIC controls the size accurately and is good for detecting non-zero correlations at short lags. As shown empirically in Figures 1 and 2 in Escanciano and Lobato (2009), the choice of $q = 2.4$ provides a "switching effect" in which one combines the advantages of the two selection rules involved (AIC and BIC). Thus, we recommend $q = 2.4$ in applications. The upper bound $d$ does not affect the asymptotic null distribution of the test, although it may have an impact on power if it is chosen too small. The finite sample performance of the automatic tests is not sensitive to the choice of $d$ for moderate and large values of this parameter, as shown in Table 5 of Escanciano and Lobato (2009) and Table 6 of Escanciano et al. (2013). Extensive simulation experience suggests that the choice of $d$ equals to the closest integer around $\sqrt{n}$ performs well in practice.

Theorem 1: *Under the null hypothesis, (3) and (4),* $AQ \xrightarrow{D} \chi^2_1$.

This Theorem justifies the rejection region

$$AQ > \chi^2_{1,1-\alpha},$$

where $\chi^2_{1,1-\alpha}$ is the $(1-\alpha)-$quantile of the $\chi^2_1$. The following Theorem shows the consistency of the test.

Theorem 2: *Assume* $\widehat{\rho}_j \xrightarrow{P} \rho_j$ *for* $j = 1, ..., d$*, and let (4) hold. Then, the test based on AQ is consistent against* $H_1^K$, for $K \leq d$.

Note that joint convergence of the vector of autocorrelations is not needed, in contrast to much of the literature. Thus, the methodology of this paper does not require estimation of large dimensional asymptotic variances.
The proofs of both theorems follow from straightforward modification of those in Escanciano and Lobato (2009), and hence they are omitted.

Remark 1. The methodology can be applied to any setting where (3) and (4) can be established. This includes raw data or residuals from any model. There is an extensive literature proving conditions such as (3) and (4) under different assumptions, see examples below.

Remark 2. The reason for the $\chi^2_1$ limiting distribution of the Automatic Portmanteau test is that under the null hypothesis $\lim_{n\to\infty} P(\widetilde{p} = 1) = 1$. Heuristically, under the null $Q_p^*$ is small, and $\pi(p, n, q)$ increases in $p$, so the optimal choice selected is the lowest dimensionality $p = 1$ with high probability.

## 3. Applications To Risk Management

We illustrate the general applicability of the methodology with new applications in risk management. There is a very extensive literature on the quantification of market risk for derivative pricing, for portfolio choice and for risk management purposes. In particular, this literature has long been interested in the evaluation of market risk forecasts, the so-called backtests, see Jorion (2006) and Christoffersen (2009) for comprehensive reviews. A leading market risk measure has been the Value at Risk ($VaR$), and more recently

Expected Shortfall ($ES$). $VaR$ summarizes the worst loss over a target horizon that will not be exceeded at a given level of confidence called coverage level. $ES$ is the expected value of losses beyond a given level of confidence.[2] We review popular backtests for $VaR$ and $ES$ and derive automatic versions using the general methodology above.

Let $R_t$ denote the revenue of a bank at time $t$, and let $\Omega_{t-1}$ denote the risk manager's information at time $t-1$, which contains lagged values of $R_t$ and possibly lagged values of other variables, say $X_t$. That is, $\Omega_{t-1} = \{X_{t-1}, X_{t-2}, ...; R_{t-1}, R_{t-2}, ...\}$. Let $G(\cdot, \Omega_{t-1})$ denote the conditional cumulative distribution function (*cdf*) of $R_t$ given $\Omega_{t-1}$, i.e. $G(\cdot, \Omega_{t-1}) = \Pr\left(R_t \leq \cdot | \Omega_{t-1}\right)$. Assume $G(\cdot, \Omega_{t-1})$ is continuous. Let $\alpha \in [0,1]$ denote the coverage level. The $\alpha$-level $VaR$ is defined as the quantity $VaR_t(\alpha)$ such that

$$\Pr\left(R_t \leq -VaR_t(\alpha) | \Omega_{t-1}\right) = \alpha. \tag{7}$$

That is, the $-VaR_t(\alpha)$ is the $\alpha - th$ percentile of the conditional distribution $G$,

$$VaR_t(\alpha) = -G^{-1}(\alpha, \Omega_{t-1}) = -\inf\left\{y : G(y, \Omega_{t-1}) \geq \alpha\right\}.$$

Define the $\alpha$-violation or hit at time $t$ as

$$h_t(\alpha) = 1(R_t \leq -VaR_t(\alpha)),$$

where $1(\cdot)$ denotes the indicator function. That is, the violation takes the value one if the loss at time $t$ is larger than or equal to $VaR_t(\alpha)$, and it is zero otherwise. An implication of (7) is that violations are Bernoulli variables with mean $\alpha$, and moreover, centered violations are a martingale difference sequence ($mds$) for each $\alpha \in [0,1]$, i.e.

$$E\left[h_t(\alpha) - \alpha | \Omega_{t-1}\right] = 0 \text{ for each } \alpha \in [0,1].$$

This restriction has been the basis for the extensive literature on backtesting $VaR$. Two of its main implications, the zero mean property of the hit sequence $\{h_t(\alpha) - \alpha\}_{t=1}^{\infty}$ and its uncorrelation led to the unconditional and conditional backtests of Kupiec (1995) and Christoffersen (1998), respectively, which are the most widely used backtests. More recently, Berkowitz et al. (2011) have proposed the Box-Pierce-type test for $VaR$

$$C_{VaR}(p) = n\sum_{j=1}^{p} \widehat{\rho}_j^2,$$

with $\widehat{\rho}_j = \widehat{\gamma}_j / \widehat{\gamma}_0$ and $\widehat{\gamma}_j = 1/(n-j)\sum_{t=1+j}^{n}(\widehat{h}_t(\alpha) - \alpha)(\widehat{h}_{t-j}(\alpha) - \alpha)$, and where $\{\widehat{h}_t(\alpha) = R_t \leq -\widehat{VaR}_t(\alpha)\}_{t=1}^{n}$, for an estimator of the $VaR$, $\widehat{VaR}_t(\alpha)$. An automatic

2. Other names for ES are Conditional VaR, Average VaR, tail VaR or expected tail loss.

version of the test statistic in Berkowitz et al. (2011) can be computed following the algorithm above with $\tau_j = 1$. This test is only valid when there is no estimation effects. If $T$ is the in-sample size for estimation and $n$ is the out-of-sample size used for forecast evaluation, the precise condition for no estimation effects in backtesting VaR and ES is that both $T \to \infty$ and $n \to \infty$ at a rate such that $n/T \to 0$ (i.e. the in-sample size is much larger than the out-of-sample size). More generally, Escanciano and Olmo (2010) provided primitive conditions for the convergences (3) and (4) to hold in a general setting where there is estimating effects from estimating $VaR$. When estimation effects are present $\tau_j$ no longer equals 1, but Escanciano and Olmo (2010) provide suitable estimators $\widehat{\tau}_j$ satisfying (4). Let $AC_{VaR}$ denote the Automatic Portmanteau version of $C_{VaR}(p)$.

More recently there has been a move in the banking sector towards $ES$ as a suitable measure of market risk that is able to capture "tail risk" (the risk coming from very big losses). $ES$ is defined as the conditional expected loss given that the loss is larger than $VaR_t(\alpha)$, that is,

$$ES_t(\alpha) = E\left[-R_t|\Omega_{t-1}, -R_t > VaR_t(\alpha)\right]. \tag{8}$$

Definition of a conditional probability and a change of variables yield a useful representation of $ES_t(\alpha)$ in terms of $VaR_t(\alpha)$,

$$ES_t(\alpha) = \frac{1}{\alpha}\int_0^\alpha VaR_t(u)du. \tag{9}$$

Unlike $VaR_t(\alpha)$, which only contains information on one quantile level $\alpha$, $ES_t(\alpha)$ contains information from the whole left tail, by integrating all $VaR$s from 0 to $\alpha$. In analogy with (9), we define the cumulative violation process,

$$H_t(\alpha) = \frac{1}{\alpha}\int_0^\alpha h_t(u)du.$$

Since $h_t(u)$ has mean $u$, by Fubini's Theorem $H_t(\alpha)$ has mean $1/\alpha\int_0^\alpha udu = \alpha/2$. Moreover, again by Fubini's Theorem, the $mds$ property of the class $\{h_t(\alpha) - \alpha : \alpha \in [0,1]\}_{t=1}^\infty$ is preserved by integration, which means that $\{H_t(\alpha) - \alpha/2\}_{t=1}^\infty$ is also a $mds$. For computational purposes, it is convenient to define $u_t = G(R_t, \Omega_{t-1})$. Using that

$h_t(u) = 1(R_t \leq -VaR_t(u)) = 1(u_t \leq u)$, we obtain

$$\begin{aligned} H_t(\alpha) &= \frac{1}{\alpha}\int_0^{\alpha} 1(u_t \leq u)du \\ &= \frac{1}{\alpha}(\alpha - u_t)1(u_t \leq \alpha). \end{aligned} \tag{10}$$

Like violations, cumulative violations are distribution-free, since $\{u_t\}_{t=1}^{\infty}$ comprises a sample of *iid* $U[0,1]$ variables (see Rosenblatt (1952)). Cumulative violations have been recently introduced in Du and Escanciano (2017). The variables $\{u_t\}_{t=1}^{\infty}$ necessary to construct $\{H_t(\alpha)\}_{t=1}^{\infty}$ are generally unknown, since the distribution of the data $G$ is unknown. In practice, researchers and risk managers specify a parametric conditional distribution $G(\cdot, \Omega_{t-1}, \theta_0)$, where $\theta_0$ is some unknown parameter in $\Theta \subset \mathbb{R}^p$, and proceed to estimate $\theta_0$ before producing $VaR/ES$ forecasts. Popular choices for distributions $G(\cdot, \Omega_{t-1}, \theta_0)$ are those derived from location-scale models with Student's $t$ distributions, but other choices can be certainly entertained in our setting. With the parametric model at hand, we can define the "generalized errors"

$$u_t(\theta_0) = G(R_t, \Omega_{t-1}, \theta_0)$$

and the associated cumulative violations

$$H_t(\alpha, \theta_0) = \frac{1}{\alpha}(\alpha - u_t(\theta_0))1(u_t(\theta_0) \leq \alpha).$$

Very much like for $VaR$s, the arguments above provide a theoretical justification for backtesting $ES$ by checking whether $\{H_t(\alpha, \theta_0) - \alpha/2\}_{t=1}^{\infty}$ have zero mean (unconditional $ES$ backtest) and whether $\{H_t(\alpha, \theta_0) - \alpha/2\}_{t=1}^{\infty}$ are uncorrelated (conditional $ES$ backtest).

Let $\widehat{\theta}$ be an estimator of $\theta_0$ and construct residuals

$$\widehat{u}_t = G(R_t, \Omega_{t-1}, \widehat{\theta}),$$

and estimated cumulative violations

$$\widehat{H}_t(\alpha) = \frac{1}{\alpha}(\alpha - \widehat{u}_t)1(\widehat{u}_t \leq \alpha).$$

Then, we obtain

$$\widehat{\gamma}_j = \frac{1}{n-j}\sum_{t=1+j}^{n} (\widehat{H}_t(\alpha) - \alpha/2)(\widehat{H}_{t-j}(\alpha) - \alpha/2) \text{ and } \widehat{\rho}_j = \frac{\widehat{\gamma}_j}{\widehat{\gamma}_0}.$$

Du and Escanciano (2017) construct the Box-Pierce test statistic

$$C_{ES}(p) = n \sum_{j=1}^{p} \widehat{\rho}_j^2, \tag{11}$$

and derive its asymptotic null distribution. In particular, they establish conditions for (3) and (4) to hold and provide expressions for the corresponding $\widehat{\tau}_j$. Let $AC_{ES}$ denote the Automatic Portmanteau version of $C_{ES}(p)$.

Compared to the existing backtests, these automatic backtests select $p$ from the data, and only require estimation of marginal asymptotic variances of marginal correlations to obtain known limiting distributions.

# 4. Stata Implementation

We introduce the `dbptest` command to implement the automatic portmanteau test (5). Notice that $\tau_j = 1$ for *iid* observations, as well as backtesting $VaR$ and $ES$ without estimation effects.

We also provide a Stata command `rtau` to estimate $\tau_j$ for more general cases, including martingale difference sequence as in Escanciano and Lobato (2009), as well as backtests for $VaR$ and $ES$ with estimation effects as in Escanciano and Olmo (2010) and Du and Escanciano (2017), respectively.

## 4.1 Syntax

### Automatic Q Test

`dbptest` *varname* [ *if* ] [ *in* ] [ , `mu(#)` `q(#)` `tauvector(`*matname*`)` `nlags(#)` ]

### Estimating $\tau_j$

`rtau` *varname* [ *if* ] [ *in* ] , `nlags(#)` `seriestype(`*type*`)` [ `cl(#)` `nobs(#)` ]

## 4.2 Options

### Automatic Q Test

`mu(#)` specifies the mean of the variable tested. The default is the variable's sample mean.

`q(#)` is some fixed positive number to control the switching effect between AIC and BIC. The default value is 2.4.

`tauvector(`*matname*`)` specifies a column vector containing variances of the autocorrelations. The default is a vector of 1's.

`nlags(#)` specifies the maximum number of lags of autocorrelations. The default is the closest integer around $\sqrt{n}$, where $n$ is the number of observations. If it is larger than the dimension of `tauvector`, it will be replaced by the dimension of `tauvector`.

#### Estimating $\tau_j$

`nlags(#)` specifies the number of lags of autocorrelations.

`seriestype(`*type*`)` specifies one of the following three types: *mds*, *var* and *es*.

`seriestype(`*mds*`)` specifies *varname* to be a martingale difference sequence as in Escanciano and Lobato (2009).

`seriestype(`*var*`)` corresponds to backtesting $VaR$. *varname* assumes an AR(1)-GARCH(1,1) model with student-$t$ innovations, when deriving the estimation effects.

`seriestype(`*es*`)` corresponds to backtesting $ES$. *varname* assumes an AR(1)-GARCH(1,1) model with student-$t$ innovations, when deriving the estimation effects.

`cl(#)` specifies the coverage level of $VaR$ and $ES$. The default is 0.05.

`nobs(#)` specifies the in-sample size when backtesting $VaR$ and $ES$.

### 4.3 Remarks

One needs to `tsset` the data before using `dbptest` and `rtau`.

#### Automatic Q Test

`dbptest` implements a data-driven Box-Pierce test for serial correlations. The test automatically chooses the order of autocorrelations. The command reports not only the usual outputs of Box-Pierce test as `wntestq`, i.e., the Q statistics and the corresponding P-value, but also the automatic number of lags chosen.

#### Estimating $\tau_j$

`rtau` estimates the asymptotic variances of autocorrelations when necessary. This includes

(1) martingale difference sequence data;

(2) backtesting $ES$ and $VaR$ with estimation effects.

Only when *var* or *es* is specified, `cl(#)` and `nobs(#)` are required.

## 4.4 Saved results

### Automatic Q Test

Scalars

| | | | |
|---|---|---|---|
| `r(stat)` | Q statistic | `r(p)` | probability value |
| `r(lag)` | the number of lag(s) | | |

### Estimating $\tau_j$

Matrix

| | |
|---|---|
| `e(tau)` | variances of autocorrelations |

## 4.5 Example

To illustrate the usage of the two commands, we consider the DAX Index return data from January 1, 1997 to June 30, 2009 as in Du and Escanciano (2017). The in-sample period is from January 1, 1997, to June 30, 2007. The out-of-sample period is from July 1, 2007, to June 30, 2009, which is the financial crisis period.

We use the in-sample data to estimate an AR(1)-GARCH(1,1) model with student's $t$ innovations. After getting the estimates for $u_t$, $h_t(\alpha)$, $H_t(\alpha)$ using the out-of-sample data, we implement the conditional backtests for $VaR$ and $ES$ using the new `dbptest` command.

### Without Estimation Effects

Here we carry out the automatic portmanteau test (5) without considering the estimation effects, i.e. $\tau_j = 1$.

```
. dbptest H, mu(0.05)

Automatic Portmanteau test for serial correlation
----------------------------------------------------------

Variable: H
----------------------------------------------------------

Portmanteau (Q) statistic                =    2.8417
Prob > chi2(1)                           =    0.0918
```

```
The number of lag(s) (from 1 to 23)       =        1
---------------------------------------------------------
```

The displayed results are for cumulative violations at 10% risk level, i.e. $H_t(0.1)$. Under the correct model specification, we have $E[H_t(\alpha)] = \alpha/2$, so we set `mu` to be 0.05. We get an $AQ$ statistic of 2.8417 and a P-value of 0.0918. Hence, the $ES$ model is rejected at 10% significance level. It also reports the number of lag(s) chosen, which is 1 in this case.

Likewise, we carry out the conditional backtest for $VaR$ using $h_t(\alpha)$. Following the rule-of-thumb that the coverage level for $ES$ is twice (or approximately twice) that of $VaR$, We examine the autocorrelations of $h_t(0.05)$.

```
. dbptest h,mu(0.05)

Automatic Portmanteau test for serial correlation
---------------------------------------------------------
Variable: h
---------------------------------------------------------
Portmanteau (Q) statistic                 =   0.7972
Prob > chi2(1)                            =   0.3719
The number of lag(s) (from 1 to 23)       =        1
---------------------------------------------------------
```

We now get an $AQ$ statistic of 0.7972 and a P-value of 0.3719, we fail to reject the $VaR$ model.

### With Estimation Effects

To take the estimation effects into account, we use the command `rtau` to estimate $\tau_j$ first before we run the `dbptest` command.

```
. rtau lret,  nlags(15) seriestype(es) cl(0.1) nobs(2658)

---------------------------------------------
Asymptotic Variances of Autocorrelations
---------------------------------------------
    Order      Tau for ES
      1         1.0027636
      2         1.0192228
      3         1.0192343
      4         1.004399
      5         1.0030891
      6         1.0021455
```

```
      7         1.0137747
      8         1.0016341
      9         1.0094143
      10        1.0012676
      11        1.0011319
      12        1.0080588
      13        1.0077699
      14        1.0033674
      15        1.0017961
---------------------------------------------

. mat Tau_ES = e(tau)

. dbptest H, mu(0.05) tauvector(Tau_ES)

Automatic Portmanteau test for serial correlation
---------------------------------------------------------
Variable: H
---------------------------------------------------------
Portmanteau (Q) statistic                  =    2.8338
Prob > chi2(1)                             =    0.0923
The number of lag(s) (from 1 to 15)        =         1
---------------------------------------------------------

. rtau lret,  nlags(15) seriestype(var) cl(0.05) nobs(2658)

---------------------------------------------
Asymptotic Variances of Autocorrelations
---------------------------------------------
    Order      Tau for VaR
      1         1.01014
      2         1.0029985
      3         1.0023986
      4         1.0023737
      5         1.0027832
      6         1.0021056
      7         1.001556
      8         1.0014201
      9         1.0011457
      10        1.0009844
      11        1.001431
      12        1.0013224
      13        1.0013889
      14        1.0009824
      15        1.0011676
---------------------------------------------
```

```
. mat Tau_VaR = e(tau)

. dbptest h, mu(0.05) tauvector(Tau_VaR)

Automatic Portmanteau test for serial correlation
──────────────────────────────────────────────────────────
Variable: h
──────────────────────────────────────────────────────────
Portmanteau (Q) statistic                  =    0.7892
Prob > chi2(1)                             =    0.3743
The number of lag(s) (from 1 to 15)        =         1
──────────────────────────────────────────────────────────
```

Notice that the in-sample size here is 2658. The $AQ$ test statistics for $ES$ and $VaR$ here are slightly lower than those without estimation effects. The tests conclusions remain the same, although the P-values are slightly bigger than before.

■

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