Ganuza, J.-J., & Penalva, J. (2019). Information disclosure in optimal auctions. *International Journal of Industrial Organization*, 63, pp. 460–479.

DOI: 10.1016/j.ijindorg.2018.11.004

Information disclosure in optimal auctions[☆]

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JEL classification:
C72
D44
D82
D83

Keywords:
Optimal auction
Private values
Information disclosure and reserve price

A B S T R A C T

This paper analyzes a situation in which the seller controls the accuracy of what potential buyers learn about their valuation of a good to be sold. This setting is related to many real situations such as home sales, antique auctions, and digital platforms such as Google and Facebook selling online advertising slots. Two important questions arise: what is the optimal selling mechanism, and what is the optimal disclosure policy of the seller. Under the assumption of private values, a simple auction with a reserve price is the optimal mechanism. What we show is that the amount of (costly) information provided increases with the number of potential bidders when using the optimal mechanism and is greater than when the object is always sold. Because information changes the distribution of a bidder's expected valuations, the optimal reserve price also changes, so that the number of bidders (indirectly)

[☆] We are grateful for comments by four anonymous referees as well as Roberto Burguet, Preston McAfee, Marco Ottaviani and participants at JEI (Barcelona), CSEF (Capri) and SAE (Barcelona), as well as seminars at Bocconi University and Universitat Pompeu Fabra. The usual disclaimers apply.

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[1] Juan-José Ganuza gratefully acknowledges the support of the Barcelona GSE Research, the government of Catalonia, and the Agencia Estatal de Investigación through project ECO2017-89240-P (AEI/FEDER, UE).

[2] Jose Penalva acknowledges the support of the Ministerio de Economía, Industria y Competitividad through project ECO 2012–36559 and 2016/00118/001 (MINECO/FEDER, UE), and the Comunidad de Madrid S2015/HUM-3353 (EARLYFIN-CM).

1. Introduction

We analyze a situation in which the seller of an object controls the accuracy with which N risk-neutral bidders learn their private valuations.[3] The object is sold using an optimal mechanism, a standard auction with reserve price in this setting. A well-known result in auction theory is that the optimal reserve price depends on the distribution of bidder valuations but not on the number of bidders. In our framework, where the auctioneer chooses how much information to disclose, competition between bidders (captured by the number of bidders, N) affects the incentives of the auctioneer to disclose information, which in turn affects the distribution of bidder valuations, and hence, the reserve price. What we find is that a greater number of bidders increases the optimal amount of information disclosed, leading to a more restrictive reserve price.

Many situations exist in which the auctioneer can affect bidder information to some extent. Take the sale of goods on the Internet through auctions (e.g., eBay), for example. In most of these auctions, sellers have most of the information about the goods for sale and decide how much information to reveal through electronic images and text descriptions. Similarly, when selling a house, real estate agents control the information disclosed to potential buyers, and when selling antiques at auction, sellers also manage how much detail they provide. Governments soliciting bids for a public project or a company selling a subsidiary have plenty of information about the goods and control how much will reach bidders.

In online advertising, platforms such as Google and Yahoo! sell advertising slots on websites. Each of these advertising slots is sold to potential advertisers, usually via an automatic auction-type mechanism.[4] The platform selling the advertising slots has access to a great deal of information from previous interactions about the person viewing the page (obtained directly from the person or extracted from correlating information on observed browsing patterns). However, the potential advertisers are the ones who know the use and benefit of this information (private values). In this interaction, the platform

[3] For a review of real world examples where this is the case, such as in the Internet advertising business, see Hummel and McAfee (2015) or Ganuza and Penalva (2010).

[4] In this paper, we analyze a single unit auction setting. Such a setting is useful to analyze problems like the auction of a slot for a banner to be displayed in a webpage or streaming video. Thus, our analysis helps understand a subset of auction mechanisms used in the online advertising industry, though not all. We do not address other types of auctions, such as those used to analyze auctions for the position of search results which are used by search engines.

must determine how much information to make accessible to potential buyers of the advertising slot and how to organize the selling mechanism.[5]

The current paper looks at the interaction between access to information the seller gives to potential buyers and the selling mechanism, in particular the reserve price used in the auction. The reserve price can be significant for seller revenue, as shown in Ostrovsky and Schwarz (2016). Their paper analyzes a large-scale field experiment on reserve prices in "sponsored search" auctions conducted by Yahoo! to sell advertisements. In particular, the authors show that when reserve prices are set according to the theory, revenues increase substantially relative to a control group with fixed and suboptimal reserve prices.

The current paper contributes to two branches of the literature. Jullien and Mariotti (2006) and Cai et al. (2007) show that in an affiliated value setting, the auctioneer uses the reserve price to signal the valuation. In that setting, the number of bidders affects the seller's incentive to signal through the reserve price. In our private value setting, the seller's valuation is not relevant for bidder decision problems, and the reserve price has no informational content. We also contribute to the literature that analyzes the auctioneer's incentives to disclose information in private value settings where the assumption is that the object is always sold (see, for example, Board, 2009; Ganuza, 2004; Ganuza and Penalva, 2010; and Hummel and McAfee, 2015[6]).[7] We show that the auctioneer provides more information when using an optimal mechanism, an auction with reserve price. Providing information to bidders has the positive effect of increasing the efficiency of the allocation (and bidders' willingness to pay). It also, however, has a negative effect, increasing bidders' informational rents. As the reserve price reduces bidder informational rents, the auctioneer's incentive to disclose more information increases.[8] As the reserve price reduces bidders' informational rents, it increases the incentives of the auctioneer to disclose more information.

This result sheds light on the targeting problem in the online advertising industry. Platforms that provide highly accurate information about consumers may end up with little competition in the auction. The result would be large market power for firms whose products are a good match with the preferences of those particular consumers. For this reason, platforms may prefer to increase competition by being less precise about consumer characteristics. We show that applying the optimal mechanism alleviates the trade-off

[5] De Corniere and De Nijs (2016) study the auctioneer's information revelation problem in the setting of platforms selling advertising slots on pages people visit. Their analysis focuses on how the auctioneer's optimal decisions affect prices and competition in downstream markets.

[6] Hummel and McAfee (2015) also consider the provision of information in a setting with reserve prices. The key difference with the current paper is that the auctioneer's information provision is costless and they only consider all-or-nothing information structures.

[7] All these papers, as well as the current one, assume that the auctioneer chooses from a given family of indexed information structures. Bergemann and Pesendorfer (2007) studied the joint problem of costlessly designing bidders' private information and the corresponding optimal mechanism. While they consider optimal mechanisms, in their setting it is very difficult to study the effects of competition on information provision as the resulting (optimal) information structures are not ordered in terms of informativeness.

[8] In an alternative setting, Eső and Szentes (2007) shows that if the auctioneer can commit to the informativeness of the signals provided and charge bidders for it, then the auctioneer can extract all the informational rents ex-ante, and the tradeoff between efficiency and bidders' rents vanishes.

between improving the consumer-advertiser match (and advertiser willingness to pay) and competition among advertisers. Thus, the platform is willing to provide a more accurate description of consumer preferences. In addition, we show that for consumers looking for products in niche markets, where less competition for their attention is expected, the optimal choice is to provide less information and use a less restrictive reserve price.

This paper is structured as follows: In the next section, we present the model and known results in the setting where reserve prices are not used. Section 3 solves the model when the seller can set the optimal reserve price, and describes the main results. Section 4 considers alternative signal structures, and Section 5 is the conclusion.

2. The model

An auctioneer wishes to sell a single object to one of N risk-neutral bidders using a standard auction, such as a second-price sealed-bid auction.[9] The auctioneer's valuation of the object is 0. Bidder valuations, v_i, $i \in \{1, 2, \dots, N\}$, are identically and independently distributed with cumulative distribution $F(v)$, support on [0,1], a strict positive and differentiable density, $f(v)$, on [0,1] and mean v_m. $F(v)$ satisfies the monotone hazard rate assumption, which implies that the virtual valuation function, $J(v) = v - \frac{1-F(v)}{f(v)}$, is increasing in v.

Bidders do not know their valuations. Before the auction, the auctioneer discloses information that generates a private signal for each bidder, X_i. The auctioneer chooses how much information to reveal by determining the precision of bidders' signals, denoted by δ, which is public. For the auctioneer obtaining and transmitting information is costly. In particular, creating N signals with precision δ has a cost $C(\delta)$, where $C'(\delta) > 0$, $C''(\delta) > 0$, and $C'(0) = 0$. We assume that the information release is symmetric and that all private signals have the same precision.[10]

Finally, after updating their expected valuations using their private signals, bidders submit offers to the auctioneer, and the auction takes place.

In summary, the sequence is as follows:

1. The auctioneer, knowing the number of bidders, N, decides how much information to disclose to the market by choosing δ at a cost $C(\delta)$.
2. Given δ, each bidder receives a realization x_i of the private random signal X_i. Bidders update their valuations of the object using δ and x_i.
3. The auction takes place and the object is awarded.

We solve the model by backward induction.

[9] The Revenue Equivalence Theorem applies in our setting. For concreteness, it may be useful to think of the auctioneer as using a second-price sealed-bid auction although all our results hold for any standard auction.

[10] In the working paper version of the current document (available upon request) we consider the case where the seller can discriminate among bidders regarding the information disclosed. In a setting with costless information disclosure, we find that it is better to keep one bidder uninformed than to use a reserve price. Furthermore, with a sufficiently large number of bidders it is optimal to release all information to all bidders.

2.1. Updating bidder valuations

We focus on a particular family of information structures ordered in terms of informativeness by δ. In particular, following (Lewis and Sappington, 1994), we concentrate on truth-or-noise signals (the realization of the signal is either the underlying value or only noise). This information structure has the advantages of a conditional expectation that is linear in the signal realization, and a conditional probability of being truth or noise that does not depend on the particular realization of the signal.[11]

For given δ, agent i receives a realization, x_i, of the signal, X_i, where this signal is known to be equal to the bidder's true private valuation, $X_i = v_i$, with probability δ, while with the complementary probability, X_i is pure noise with distribution $F(\cdot)$. Thus, independently of signal accuracy, the marginal distribution of the signal will be the same as the distribution of valuations, i.e. $\Pr\{X_i \leq x_i\} = F(x_i)$. With this signal, the expected valuation of the object for a bidder i who receives a signal X_i when precision is δ, $v_i^E(x_i, \delta)$ is equal to:

$$
\begin{aligned}
v_i^E(x_i, \delta) &= E[v_i | X_i = x_i, \delta] \\
&= x_i \delta + (1 - \delta) v_m
\end{aligned}
\tag{1}
$$

The auctioneer, by choosing δ, determines the distribution of the expected valuations.[12] However, because the auctioneer does not observe signal realizations, the setup is still one of standard private values.

Each bidder's expected valuation $v_i^E(x_i, \delta)$ is a function of the bidder's privately observed signal realization x_i. Because the private information of the bidder is the realization of the signal, we can think of x_i as the bidder's type, and write the virtual valuation of the bidder as follows:

$$
v^E(x, \delta) - \left(\frac{\partial v^E(x, \delta)}{\partial x} \right) \frac{1 - F(x)}{f(x)}
$$

which simplifies to

$$
\begin{aligned}
J_\delta(x) &= x\delta + (1 - \delta) v_m - \delta \frac{1 - F(x)}{f(x)} \\
&= \delta J(x) + (1 - \delta) v_m.
\end{aligned}
$$

This new virtual valuation function $J_\delta(x)$ is a convex combination of (a) the standard virtual valuation over the distribution of types (signals x), $J(x)$, which is the virtual valu-

[11] The truth-or-noise information structure appears in Banerjee (1992) and it has been used extensively in the literature of information release and mechanism design with endogenous information structures (Bergemann and Valimaki, 2006; Johnson and Myatt, 2006; Ganuza and Penalva, 2010; Shi, 2012; De Corniere and De Nijs, 2016; Wang, 2017; Hagiu and Wright, 2018 among others). See also Ottaviani and Sø rensen (2006) for a more comprehensive discussion of linear information structures.

[12] In particular, it can be shown that a greater δ makes the distribution of expected valuations more spread out in the sense of the dispersive order (which is stronger than the convex order). Then, the signals are ordered in terms of informativeness in the sense of supermodular precision as defined in Ganuza and Penalva (2010).

ation with full information, and (b) the mean v_m (the virtual valuation that would result if no information was revealed at all). Note that $J_\delta(x)$ trivially preserves the monotonicity of the virtual valuation function $J(x)$ which allows us to focus our analysis without loss of generality on standard auction mechanisms such as the second-price sealed-bid auction.

2.2. Information disclosure when the object is always sold

Let the expected revenue of the auctioneer when the object is always sold be denoted by $\pi(\delta,\ N)$ where π is defined by the following equation:

$$\pi(\delta, N) = \int_0^1 \left[y\,\delta + (1-\delta)v_m - \delta \frac{1-F(y)}{f(y)} \right] m(y,N)dy - C(\delta),$$

where $m(y,\ N)$ is the density function of $X_{N:\ N}$ which is the maximum of N independent draws from $F(x)$. Then, the auctioneer's optimal information disclosure decision is obtained as the solution to the following problem:

$$\delta^* \in \text{argmax}_\delta\ \pi(\delta, \text{N}) \tag{2}$$

From existing results (see Board, 2009; Ganuza and Penalva, 2010) we obtain the following proposition.

Proposition 1. *There exists N_0 such that for $N \leq N_0$ it is not optimal to reveal any information, while for $N > N_0$ it is optimal to reveal some amount of information, $\delta^*(N) > 0$. The optimal amount of information (when the object is always sold) δ^* is increasing with the number of bidders, N.*[13]

In a setting where the auctioneer chooses whether or not to costlessly reveal a given level of information, Hummel and McAfee (2015) show that N_0 is 3 or 4. Board (2009) does not impose the monotone hazard rate assumption. Using an example based on the power distribution $F(x) = x^\beta$ Board proves that N_0 may be unbounded even with costless information disclosure.[14]

To illustrate Proposition 1 consider a special case with a uniform bidder valuation distribution on [0,1], and a cost of obtaining and transmitting information that is quadratic,

[13] As δ^* may not be a singleton, this statement as well as further comparative statics results below, should be interpreted in the sense of Veinott's strong set order: $\delta^*_{N+1} \geq \delta^*_N$ iff $\forall \delta \in \delta^*_{N+1}, \delta' \in \delta^*_N$, $\max\{\delta, \delta'\} \in \delta^*_{N+1}$ and $\min\{\delta, \delta'\} \in \delta^*_N$.

[14] Another reason for optimally withholding information is the possibility of entry. Vagstad (2007) analyzes whether the auctioneer should provide information to bidders before they decide about paying an entry cost for participating in a second-price auction without a reserve price. Vagstad analyzes two extreme information structures: (i) no information, bidders take the entry decision without knowing their valuations (Levin and Smith (1994)), and (ii) full information, bidders learn perfectly their valuation before deciding whether to pay the entry cost (Samuelson, 1985). Vagstad shows that early provision of information leads bidders to self-select into the auction but also increases the informational rents of bidders in the auction. The total effect of information is ambiguous in terms of the number of entrants, welfare and revenues.

i.e. $C(\delta) = \theta\delta^2$. For a given δ, the distribution of expected valuations, v^E is uniform over the interval, $[\frac{(1-\delta)}{2}, \frac{(1+\delta)}{2}]$. The optimal amount of information is $\delta^* = \frac{1}{2\theta}(\frac{N-1}{N+1} - \frac{1}{2})$ if N is larger than 3; otherwise it is 0. Note that the optimal amount of information, $\delta^*(N)$, is strictly increasing for $N > 3$.

3. Information disclosure in optimal auctions

We now study the same model but allow the auctioneer to use an optimal mechanism to sell the object. After the release of information, we are still in a standard private value setting, so the optimal mechanism can be implemented with a simple second-price sealed-bid auction with a reserve price. Then, the characterization of the optimal mechanism reduces to identifying the optimal reserve price in the auction. The time sequence is as follows:

1. The auctioneer, knowing the number of bidders, N, decides how much information to disclose to the market by choosing δ at a cost $C(\delta)$.
2. Given δ, each bidder receives a realization x_i of the private random signal X_i. Bidders update their valuations of the object using δ and x_i.
3. The auctioneer selects and announces a reserve price.
4. The auction (with the optimal reserve price) takes place and the object is awarded.

First, we analyze how the optimal reserve price depends on the information provided. Then we analyze the optimal information provision decision.

3.1. The optimal reserve price

We take as given the level of information provided by the seller, δ. Because the new virtual valuation $J_\delta(x)$ is monotone, we can derive the optimal reserve price through the type x^{RP} that makes this new virtual valuation function equal to zero (the opportunity cost of the seller in our setting), *i.e*, $J_\delta(x^{RP}) = 0$:

$$x^{RP}\delta + (1-\delta)v_m - \delta\frac{1-F(x^{RP})}{f(x^{RP})} = 0. \qquad (3)$$

It can also happen that $J_\delta(0) > 0$ and that the new virtual valuation is positive for all values of x^{RP}. Then the reserve price will be not binding because the seller does not want to exclude any type from bidding. The next Lemma characterizes the optimal reserve price, and when the solution is interior.

Lemma 1. *The optimal reserve price is $v^{RP} = x^{RP}\delta + (1-\delta)v_m$ where*

$$x^{RP} = \begin{cases} J^{-1}\left(-\frac{(1-\delta)}{\delta}v_m\right) & \text{if} J(0) < -\frac{(1-\delta)}{\delta}v_m \\ 0 & \text{Otherwise} \end{cases}$$

Giving this characterization, we can proceed to study the relationship between a given level of precision, δ, and the optimal reserve price.

Proposition 2. *There exists a level of information* $\overline{\delta} = \frac{v_m}{v_m - J(0)} \in [0,1]$ *such that: (i) if the auctioneer sets* $\delta \leq \overline{\delta}$*, the optimal selling strategy is to use a non-binding reserve price,* $F(x^{RP}) = 0$ *and (ii) if the auctioneer sets* $\delta > \overline{\delta}$*,* $x^{RP} = J^{-1}(-\frac{(1-\delta)}{\delta} v_m)$ *and* x^{RP} *is increasing in* δ*. (iii) The probability that an individual bidder has an expected valuation lower than the optimal reserve price,* $F(x^{RP})$ *is increasing in* δ*.*

With no information, all bidders have the same expected valuation (v_m) so there are no informational rents and the (non-binding) reserve price is equal to v_m. Part (i) of Proposition 2 says that using a non-binding reserve price continues to be optimal if the auctioneer provides only limited information.

The intuition behind parts (ii) and (iii) of Proposition 2 is based on the following two facts: (a) the informational rents are increasing in the amount of information provided, and; (b) the reserve price is a seller's tool to reduce informational rents. Putting these two facts together, the greater the amount of information provided by the seller, the larger the informational rents, which in turn leads to larger seller incentives to reduce such rents by increasing the reserve price relative to the distribution of expected valuations.

3.2. Information disclosure with an optimal reserve price

Having determined the auctioneer's optimal selling strategy we now analyze the auctioneer's information disclosure problem. Let the auctioneer's expected profit for disclosing information δ while using the optimal reserve price be denoted by $\hat{\pi}(\delta, N)$, where

$$\hat{\pi}(\delta, N) = \int_{x^{RP}(\delta)}^{1} \left[y\,\delta + (1-\delta)v_m - \delta \frac{1-F(y)}{f(y)} \right] m(y,N)dy - C(\delta),$$

Then, the optimal choice of precision when using an optimal mechanism, $\widehat{\delta}^*$, is

$$\widehat{\delta}^* \in \text{argmax}_\delta\ \hat{\pi}(\delta, \text{N}). \tag{4}$$

We now look at the effect of greater competition on the information disclosed, $\widehat{\delta}^*$.

Proposition 3.

(i) *There exists* $\widehat{N}_0$ *such that for* $N \leq \widehat{N}_0$ *it is not optimal to reveal any information, while for* $N > \widehat{N}_0$ *it is optimal to reveal some amount of information,* $\widehat{\delta}^*(N) > 0$*. The optimal amount of information* $\widehat{\delta}^*$ *is non-decreasing in the number of bidders, N.*

(ii) *When the auctioneer uses an optimal reserve price, the auctioneer discloses some information with less competition than when the object is always sold,* $\widehat{N}_0 \leq N_0$*.*

Part (i) extends the insights of Proposition 1 to optimal mechanisms. The intuition is the same: Competition between bidders (N) reduces informational rents and increases the incentives of the auctioneer to provide information. Despite the similarity in the results, the proof of part (i) of Proposition 3 is much more involved as the auctioneer's objective function with an optimal mechanism, $\hat{\pi}(\delta, N)$, does not satisfy the single-crossing condition globally. This implies that we cannot appeal to the usual comparative statics results. Part (ii) follows directly from the fact that for all N, it is more profitable to give information when using a reserve price than when not using one. Then, if the auctioneer wants to give some information when not using a reserve price, the auctioneer will necessarily also want to give information when using one. This result is in line with that of Hummel and McAfee (2015). In a setting of all-or-nothing costless information disclosure, they show that the minimum number of bidders required for the optimality of full information provision is lower with a reserve price than without one.

Propositions 2 and 3 jointly imply that a higher number of bidders leads the auctioneer to optimally set a more restrictive reserve price, in the sense that for any bidder, the probability of having a valuation higher than the reserve price is lower.

Corollary 4. *The probability that any individual bidder's valuation is below the optimal reserve price is weakly increasing in N.*

This relationship between the number of bidders and the reserve price is indirect. It arises through the effect of competition on the auctioneer's incentives to provide information. By changing the amount of information disclosed the auctioneer changes the distribution of bidder valuations and hence the optimal reserve price. By Proposition 3, more competition leads to more information disclosure. By Proposition 2, more information disclosure generates more informational rents and consequently, greater incentives to use a more restrictive reserve price.

Finally, the next proposition compares $\widehat{\delta}^*$ with the optimal one chosen when the object is always sold, δ^*.

Proposition 5. *When the auctioneer uses an optimal reserve price, the auctioneer provides more information than when the object is always sold, $\widehat{\delta}^* \geq \delta^*$.*

Because the reserve price reduces bidder rents, having a reserve price weakens the trade-off between efficiency and bidder rents, increasing the auctioneer's incentives to provide more information.

3.3. Example

We continue with the previous example: uniformly distributed valuations and quadratic costs. The virtual valuation function of the prior distribution is, $J(x) = 2x - 1$ and $\overline{\delta} = \frac{v_m}{v_m - J(0)} = \frac{1}{3}$. Therefore, (i) if $\delta < \frac{1}{3}$, the optimal selling strategy is to use a non-binding reserve price. (ii) If $\delta > \frac{1}{3}$, $J(x^{RP}) = -\frac{(1-\delta)}{2\delta} \Longrightarrow x^{RP} = \frac{3}{4} - \frac{1}{4\delta}$, so that only bidders with signals greater than $\frac{3}{4} - \frac{1}{4\delta}$ will participate in the auction. Then, x^{RP}, the optimal reserve price in terms of the signal, is increasing in the amount of information. The optimal reserve price in terms of expected valuations is $v^{RP} = x^{RP}\delta + (1-\delta)\frac{1}{2} = \frac{1+\delta}{4}$. Note that in this example, v^{RP} is increasing in the amount of information. However, this increase may not necessarily be the case because $\frac{\partial v^{RP}}{\partial \delta} = (x^{RP} - v_m) + \frac{\partial x^{RP}}{\partial \delta}$. If $x^{RP} < v_m$, the first term is negative and the formal reserve price v^{RP} may decrease in δ. The optimal amount of information is implicitly defined by the following expression:

$$\int_{\frac{3}{4}-\frac{1}{4\widehat{\delta}^*}}^{1} \left(2y - \frac{3}{2}\right) N y^{N-1}\, dy = 2\theta\widehat{\delta}^*,$$

where we are using $m(y, N) = Ny^{N-1}$. Numerical computations show that $\widehat{\delta}^* > \delta^*$ is increasing in N. This effect then makes x^{RP} (and in this case also v^{RP}) increase in N. For example, when $\theta = \frac{1}{6}$.

N	4	5	6	7	8	9	10
δ^*	0.3000	0.5000	0.6429	0.7500	0.8333	0.9000	0.9545
$\widehat{\delta}^*$	0.3000	0.5034	0.6490	0.7552	0.8369	0.9023	0.9560
x^{RP}	0	0.2534	0.3648	0.4190	0.4513	0.4729	0.4885
v^{RP}	0	0.3758	0.4123	0.4388	0.4592	0.4756	0.4890

4. General information structures

In the main model, we have focused on the seller's problem where the information provided comes from a special family of information structures, and we found that disclosing more information (induced by higher bidder competition) leads to a more restrictive optimal reserve price. In this section we want to identify conditions for this core result to be true for a larger set of information structures.

In the general case, the seller chooses to give bidders access to one of two possible arbitrary signals X_δ and $X_{\delta'}$. Let $E_\delta[V|x]$ denote the conditional expectation of V given $\{X_\delta = x\}$. This expectation defines the random variable $E[V|X_\delta]$, with distribution $G(v^E, \delta) = \Pr\{x : E_\delta[V|x] \leq v^E\}$, and marginal distribution $G_\delta(x)$. Similarly, define $E[V|X_{\delta'}]$, with distributions $G(v^E, \delta')$ and $G_{\delta'}(x)$ in the same way. We assume that $G(v^E, \delta)$ and $G(v^E, \delta')$ have the monotone likelihood ratio property.

We are interested in the relationship between the information disclosed and the optimal reserve price. Once the auctioneer discloses information and the distribution of bidders' posterior conditional expectations is fixed, the choice of the optimal reserve price coincides with the seller's optimal take-it-or-leave-it offer to a single potential buyer. We thus focus on the latter problem and how it relates to the information ordering between X_δ and $X_{\delta'}$. The expected profit received by a seller who makes a take-it-or-leave-it offer, v^{RP}, to a potential buyer whose valuation is randomly drawn from the distribution

$G(v^E, \delta)$ is given by:

$$U(v^{RP}, \delta) = \big(1 - G(v^{RP}, \delta)\big)v^{RP}.$$

Let $v^{RP*}(\delta)$ denote the optimal solution to this problem. Our question is whether there is an informational ordering that will imply that $v^{RP*}(\delta)$ is monotone in δ. Or alternatively, whether there is an information ordering that ensures that $U(v^{RP}, \delta)$ satisfies the single-crossing condition in v and δ.

Answering this question seems best approached by rewriting the problem in terms of the quantiles, $\pi = G(v^{RP}, \delta)-$ the probability that the offer is rejected. The probability, π, can also be interpreted as the realization of a signal, $\Pi_\delta = G_\delta(X_\delta)$, where this new signal is informationally equivalent to the original one, as it is only a relabeling obtained by applying a monotone transformation (the probability integral transformation). We can define $W_\delta(\pi) = \mathbb{E}[V|X_\delta = G_\delta^{-1}(\pi)]$ as the expected valuation of the buyer who receives signal π. Given the monotonicity of posterior conditional expectations, $W_\delta(\pi)$, in π, we refer to the reserve price as π^{RP} or $v^{RP} = W_\delta(\pi^{RP})$ interchangeably.

We can now rewrite the seller's objective function which is to maximize V, where

$$V(\pi, \delta) = (1 - \pi)G(\pi, \delta)^{-1} = (1 - \pi)W_\delta(\pi).$$

Let $\pi^{RP*}(\delta)$ denote the optimal solution to this problem, which, by the monotonicity of $W_\delta(\pi)$, identifies a solution $v^{RP*} = G^{-1}(\pi^{RP*}, \delta)$ of the original optimal reserve price problem. Having reframed the problem in this way, a natural approach to order information structures is to consider how more information affects the sensitivity of the expected valuation function $W_\delta(\pi)$ to realizations of the normalized signal, π. Ganuza and Penalva (2010) also follow this approach: more information makes conditional expectations more sensitive to the realization of the signal and thus generates greater dispersion of the distribution of posterior conditional expectations. Ganuza and Penalva (2010) provide three nested definitions of informativeness (supermodular precision, SC precision, and integral precision) based on different notions of dispersion that can be used to describe when the random variable $W_\delta(\Pi_\delta)$ is more "spread out" than $W_{\delta'}(\Pi_{\delta'})$. Of these, the strongest is supermodular precision, which is useful in determining a sufficient condition for the reserve price to be increasing in informativeness.

Definition 6. X_δ is more supermodular precise than $X_{\delta'}$ if for all q, $p \in (0, 1)$, $q > p$

$$W_\delta(q) - W_\delta(p) \geq W_{\delta'}(q) - W_{\delta'}(p).$$

To establish our sufficient condition, we define the point, $\overline{\pi}$, at which the posterior conditional expectation functions for δ and δ' cross. If $W_\delta(q) - W_{\delta'}(q)$ is continuous, $\overline{\pi}$ is defined as the solution to $W_\delta(\overline{\pi}) - W_{\delta'}(\overline{\pi}) = 0$.[15]

[15] More formally, $\bar{\pi}$ is defined as a crossing point of $W_\delta(\pi) - W_{\delta'}(\pi)$. A crossing point for this function is a point c such that for $\pi \leq c$ $W_\delta(\pi) - W_{\delta'}(\pi) \leq 0$ and for $\pi \geq c$ $W_\delta(\pi) - W_{\delta'}(\pi) \geq 0$. If X_δ is more

Proposition 7. *Let X_δ be more supermodular precise than $X_{\delta'}$, $\pi^{RP*}(\delta) \leq \overline{\pi}$, and $\pi^{RP*}(\delta') \leq \overline{\pi}$. Then, $\pi^{RP*}(\delta) \geq \pi^{RP*}(\delta')$.*

Therefore, Proposition 7 provides a sufficient condition for greater informativeness to translate to a more restrictive optimal reserve price. This sufficient condition relies on supermodular precision which is a demanding informativeness criterion.[16] However, it is satisfied by many commonly used information structures.

5. Entry fees

In the main text, we focus on optimal mechanisms that take the form of a standard auction with a reserve price. An alternative way to implement the optimal mechanism is to use an entry fee. We first consider the case in which bidders, after learning their valuations, decide whether or not to pay the entry fee the auctioneer sets and then bid in an auction without a reserve price. As we discuss below, we can implement the optimal mechanism using entry fees. If the auctioneer sets the entry fee optimally, the outcome is equivalent to that obtained with the optimal reserve price in the standard setting (see, for example, Krishna, 2009), and all our results hold.

For fixed δ, let $M(y, N-1)$ be the distribution function of the maximum of $N-1$ independent draws from $F(x)$. The optimal reserve price $v^{RP} = x^{RP}\delta + (1-\delta)v_m$ is the expected valuation of the marginal type x^{RP}. We know that when the reserve price is set at v^{RP}, only bidders with type x^{RP} and higher participate in the auction. Suppose the entry fee is set in this way with this effect. In that case, apart from the entry fee, the expected profit of the marginal type when the bidder participates in a second-price auction without a reserve price is $v^{RP}M(x^{RP}, N-1)$, the probability that the bidder is the only person in the auction and gets the object for free. If the auctioneer sets the entry fee, e^{RP}, equal to this expected profit, then the marginal type (x^{RP}) is exactly indifferent between entering the auction and not entering. Then, the sets of bidders who participate in the auction with an entry fee equal to e^{RP} and no reserve price, and who participate in an auction with no entry fee and an optimal reserve price are the same.

Because the assumptions of the revenue equivalence theorem are satisfied in our setting, the outcome of both auction environments will be the same. Hence, the results of the continuation game after setting δ are the same with an optimal entry fee with no reserve price or an optimal reserve price with no entry fee. Our results regarding incentives to provide information therefore hold equally in both settings.

---

supermodular precise than $X_{\delta'}$ then the existence of such a crossing point is obtained directly from the definition, where $\overline{\pi}$ is a point that satisfies the following condition: for all $\pi \in [0, \overline{\pi})$, $W_\delta(\pi) - W_{\delta'}(\pi) \leq 0$, and for all $\pi \in (\overline{\pi}, 1]$, $W_\delta(\pi) - W_{\delta'}(\pi) \geq 0$. The existence of $\overline{\pi}$ follows from iterated expectations and the monotonicity of $W_\delta(\pi)$ in π.

[16] Although this is a very demanding order, there are commonly used set of signals that are ordered in terms of supermodular precision. For example, linear location experiments such as the true-noise information structures used in this paper or the normal location experiments. See Ganuza and Penalva (2010).

Consider now that the auctioneer can charge an entry fee before providing information in a context where the auctioneer can commit to the quality of the information to be disclosed. The possibility of this commitment to signal quality allows the auctioneer to extract ex-ante all future expected informational rents the provided information generates. The auctioneer then can extract all the surplus and has incentives to provide the "efficient" level of information and not use a reserve price. Eső and Szentes (2007) follow this approach.

We illustrate this idea in our setting as follows. Consider the following sequence of events: the auctioneer announces a level of information disclosure, δ, and sets a fee equal to e^I. Bidders pay e^I sequentially and receive a private random signal with accuracy δ. Then, they participate in a second-price auction without a reserve price. Consider that only n of the potential N bidders decide to pay the entry fee. Apart from e^I, the expected profit for the winner of the second-price auction without a reserve price is $v_1^E(x_1, \delta) - v_2^E(x_2, \delta) = (x_1(n) - x_2(n))\delta$, where $x_1(n)$ and $x_2(n)$ are respectively the expected values of the first and second order statistics of the distribution of the signal, F. By setting an entry fee $e^{I*}(n)$ equal to $\frac{(x_1(n) - x_2(n))\delta}{n}$, the auctioneer can extract all the bidder surplus of the n participants.[17] Because bidders are homogeneous before paying the entry fee and receiving their private information, they have ex-ante the same probability of winning and would be indifferent between not participating or paying e^{I*} and participating in the auction.[18] Note that e^{I*} is decreasing in n (more competition/entry leads to lower profits, and lower entry fees naturally will encourage more competition/entry) and increasing in δ (more information leads to higher bidder rents).

By setting $e^{I*}(n)$ the expected ex-ante profits of the auctioneer are

$$\pi^I(\delta, n) = n\frac{(x_1(n) - x_2(n))\delta}{n} + v_2^E(x_2(n), \delta) - C(\delta).$$

The first term is the revenue from entry fees, the second is the expected price in the auction, and the third term captures the cost of providing information. The expected profits expression simplifies to

$$\pi^I(\delta, n) = v_1^E(x_1(n), \delta) - C(\delta).$$

Note that the auctioneer can extract all bidder rents and that the profits coincide with total surplus. This outcome has two implications: First, because profits/total surplus are increasing with the number of firms participating in the auction, restricting entry offers no gains. Then, e^I should be set so as to encourage all N potential bidders to participate, i.e., at $e^{I*} = e^{I*}(N) = \frac{(x_1(N) - x_2(N))\delta}{N}$. Second, the auctioneer's optimal information disclosure decision is the efficient level of information that maximizes total

[17] By setting $e^I = e^{I*}(4)$ then exactly four bidders will pay the fee, get a signal, and participate in the auction.

[18] For simplicity we have assumed sequential entry so that the first $n-1$ bidders will strictly prefer to enter and the nth will be indifferent.

surplus:

$$\delta^{E*} \in \operatorname{argmax}_\delta \; v_1^E(x_1(N), \delta) - C(\delta). \tag{5}$$

In our opening numerical example, with uniformly distributed valuations and quadratic costs, the efficient amount of information is $\delta^{E*} = \frac{1}{2\theta}(\frac{N}{N+1} - \frac{1}{2})$, and the auctioneer wants to provide more information than in the optimal cases we considered previously. Increasing information does not increase profits (they are extracted through entry fees), so the auctioneer is willing to provide information even when the number of sellers is only 2.

6. Conclusions

Using a highly tractable model of information disclosure, we have found that when the seller chooses how much (costly) information to provide and uses an optimal selling mechanism, the seller provides more information than when the object is always sold. The reserve price is no longer independent of the number of bidders. In particular, with more bidders, the auctioneer will optimally increase the amount of information provided and impose a more restrictive reserve price. We also find that the minimum number of bidders needed before disclosing information is profitable is lower when an optimal auction mechanism is used. This paper also extends existing results establishing that more competition leads the auctioneer to provide more information to the market in settings involving optimal mechanisms. The model is standard in all dimensions but not in the choice of the set of signals available to the auctioneer. We consider intermediate levels of information disclosure whereas most of the literature focuses on all-or-nothing decisions, but we constrain our analysis to the particular class of linear information structures. Our strategy of proof uses the linearity of our chosen information structures and cannot be applied directly to a fully general set of signals. We have developed an extension of our model to general information structures and characterized a sufficient condition for greater informativeness to lead to a more restrictive optimal reserve price.

Appendix A

Proof of Lemma 1 and Proposition 2. As we discuss in the main text, the optimal reserve price can be characterized by x^{RP}, where x^{RP} is obtained from the expression:

$$J_\delta(x^{RP}) = 0$$

$$x^{RP}\delta + (1-\delta)v_m - \delta\frac{1 - F(x^{RP})}{f(x^{RP})} = 0.$$

which, gives us the following solution:

$$x^{RP} - \frac{1 - F(x^{RP})}{f(x^{RP})} = -\frac{(1-\delta)}{\delta} v_m$$
$$i.e. \quad J\big(x^{RP}\big) = -\frac{(1-\delta)}{\delta} v_m$$

As $J(x)$ is monotonone, it is possible to invert it, then $x^{RP} = J^{-1}\Big(-\frac{(1-\delta)}{\delta} v_m\Big)$. The reserve price, v^{RP}, is just the expected valuation given x^{RP}:

$$v^{RP} = x^{RP}\delta + (1-\delta)v_m,$$

As $J(x)$ is an increasing function of x and $-\frac{(1-\delta)}{\delta} v_m$ is increasing in δ, a larger δ implies a larger x^{RP} (Proposition 2 (iii)). Notice that x^{RP} will not be well defined when $J(0) = -\frac{1}{f(0)} > -\frac{(1-\delta)}{\delta} v_m$, since then $x - \frac{1-F(x)}{f(x)}$ is always larger than $-\frac{(1-\delta)}{\delta} v_m$. In that case, $x^{RP} = 0$, will be optimal and the auctioneer does not restrict the participation by any type. Summarizing, we have:

$$x^{RP} = \begin{cases} J^{-1}(-\frac{(1-\delta)}{\delta} v_m) & \text{if} J(0) < -\frac{(1-\delta)}{\delta} v_m \\ 0 & \text{Otherwise} \end{cases}$$

Finally, we can rewrite the condition $J(0) > -\frac{(1-\delta)}{\delta} v_m$ as a condition on δ to define $\bar{\delta}$ and prove parts (i) and (ii) of Proposition 2:

$$J(0) \lesseqgtr -\frac{(1-\delta)}{\delta} v_m \iff \delta \lesseqgtr \bar{\delta}, \quad \text{where} \quad \bar{\delta} = \frac{\mathrm{v_m}}{\mathrm{v_m + J(0)}}.$$

□

Proof of Proposition 3.

(i) We first establish that providing some information is optimal if N is sufficiently large. Consider the limit as $N \to \infty$ of $\widehat{\pi}(\delta, N)$. As N goes to infinity, the distribution $m(x, N)$ converges to a mass point at $x = 1$ so that $\lim_{N\to\infty} \widehat{\pi}(\delta, N) = \delta + (1 - \delta)v_m - C(\delta)$. As $C'(0) = 0$, and $v_m < 1$, the optimal $\widehat{\delta}^*$ will be strictly positive. Let $\widehat{N}_0$ be the smallest N such that it is optimal to provide information. Then, for $N > \widehat{N}_0$ it is also optimal to provide information as $\widehat{\pi}(\delta, N)$ is increasing in N for all δ, and hence providing no information cannot be optimal: $\widehat{\pi}(\widehat{\delta}^*, \widehat{N}_0) > \widehat{\pi}(0, N) = v_m$ $\Rightarrow \widehat{\pi}(\widehat{\delta}^*, N) > v_m$ for $N \geq \widehat{N}_0$.

(ii) We show the monotonicity of $\widehat{\delta}^*(N)$ by proving that $\hat{\pi}(\delta, N)$ satisfies a single crossing condition. Let $\hat{\pi}(\delta, N)$ denote the auctioneer's expected profit for disclosing information δ while using the optimal reserve price:

$$\hat{\pi}(\delta, N) = \int_{x^{RP}(\delta)}^{1} \left[y\,\delta + (1-\delta)v_m - \delta \frac{1-F(y)}{f(y)} \right] m(y, N)\, dy - C(\delta).$$

To reduce the length of the expressions, let

$$\begin{aligned}\Phi(x,\delta) &= \left[x\delta + (1-\delta)v_m - \delta\frac{1-F(x)}{f(x)}\right] \\ &= \delta J(x) + (1-\delta)v_m,\end{aligned}$$

so we can write the auctioneer's profits more concisely as:

$$\hat{\pi}(\delta, N) = \int_{x^{RP}(\delta)}^{1} \Phi(y,\delta)m(y,N)dy - C(\delta).$$

Properties of $\Phi(x,\,\delta)$: Φ is increasing in x and has increasing differences. To see this, let $\delta > \delta'$, $\Phi(x,\delta) - \Phi(x,\delta') = (\delta-\delta')(J(x)-v_m)$ so that increasing differences follows from the monotonicity of $J(x)$.

Using this, together with the fact that the reserve price is increasing with δ (Proposition 2), we analyze the incremental return of information over profits at optimal reserve prices:

$$\begin{aligned}\hat{\pi}(\delta,N) - \hat{\pi}(\delta',N) &= \int_{x^{RP}(\delta)}^{1} \Phi(y,\delta)m(y,N)dy - C(\delta) \\ &- \int_{x^{RP}(\delta')}^{1} \Phi(y,\delta')m(y,N)dy + C(\delta'), \\ &= \int_{x^{RP}(\delta')}^{1} \big(\mathbf{1}_{\{y\geq x^{RP}(\delta)\}}\Phi(y,\delta) - \Phi(y,\delta')\big)m(y,N)dy - (C(\delta)-C(\delta'))\end{aligned}$$

Let $K(x) = \mathbf{1}_{\{x\geq x^{RP}(\delta)\}}\Phi(x,\delta) - \Phi(x,\delta')$ where $\mathbf{1}_{\{x\geq x^{RP}(\delta)\}}$ is the indicator function for the set $\{x \geq x^{RP}(\delta)\}$, i.e. $\mathbf{1}_{\{x\geq x^{RP}(\delta)\}} = 0$ for $x < x^{RP}(\delta)$ and $\mathbf{1}_{\{x\geq x^{RP}(\delta)\}} = 1$ for $x \geq x^{RP}(\delta)$.

$$\begin{aligned}\hat{\pi}(\delta,N) - \hat{\pi}(\delta',N) &= \int_{x^{RP}(\delta')}^{1} K(y)m(y,N)dy - (C(\delta)-C(\delta')) \\ &= \int_{x^{RP}(\delta')}^{1} \left(K(y) - \frac{(C(\delta)-C(\delta'))}{1-M(x^{RP}(\delta'),N)}\right)m(y,N)dy\end{aligned}$$

It is important to notice that $K(x)$ is single-crossing. To see this, notice that $K(x)$ is negative and decreasing in the interval $(x^{RP}(\delta'),\, x^{RP}(\delta))$ since $\Phi(x,\,\delta')$ is positive and increasing and $\mathbf{1}_{\{x\geq x^{RP}(\delta)\}} = 0$. For $x \geq x^{RP}(\delta)$, $K(x) = \Phi(x,\delta) - \Phi(x,\delta') = (\delta-\delta')(J(x)-v_m)$ which is negative at $x = x^{RP}(\delta)$ and increasing on $[x^{RP}(\delta),\, 1]$. Thus, it is single-crossing. Furthermore, $K(x)$ is eventually positive, as $J(1) > v_m$. As $K(x)$ is increasing, let $\overline{x}$ be the point such that for $x < \overline{x}$, $K(\overline{x}) < 0$, and for $x \geq \overline{x}$, $K(x) \geq 0$. Similarly, define $\widetilde{x}$ such that for $x < \widetilde{x}$, $K(x) - \frac{(C(\delta)-C(\delta'))}{1-M(x^{RP}(\delta'),N)} < 0$, and for $x \geq \widetilde{x}$, $K(x) - \frac{(C(\delta)-C(\delta'))}{1-M(x^{RP}(\delta'),N)} \geq 0$. If $K(1) - \frac{(C(\delta)-C(\delta'))}{1-M(x^{RP}(\delta'),N)} < 0$, then let $\widetilde{x} = 1$.

We define $l(x) = \frac{m(x,N+1)}{m(x,N)}$. This ratio is increasing as F^N is dominated by F^{N+1} in the likelihood-ratio order.

Then

$$
\begin{aligned}
& \hat{\pi}(\delta, N+1) - \hat{\pi}(\delta', N+1) \\
= & \int_{x^{RP}(\delta')}^{1} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) m(y, N+1) dy \\
= & \int_{x^{RP}(\delta')}^{1} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) l(y) m(y, N) dy \\
= & \int_{x^{RP}(\delta')}^{\widetilde{x}} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) l(y) m(y, N) dy \\
& + \int_{\widetilde{x}}^{1} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) l(y) m(y, N) dy \\
\geq & \; l(\widetilde{x}) \int_{x^{RP}(\delta')}^{\widetilde{x}} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) m(y, N) dy \\
& + l(\widetilde{x}) \int_{\widetilde{x}}^{1} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) m(y, N) dy \\
= & \; l(\widetilde{x}) \int_{x^{RP}(\delta')}^{1} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)}) m(y, N) dy \\
\geq & \; l(\widetilde{x}) \int_{x^{RP}(\delta')}^{1} (K(y) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N)}) m(y, N) dy \\
= & \; l(\widetilde{x}) (\hat{\pi}(\delta, N) - \hat{\pi}(\delta', N)),
\end{aligned}
$$

where the first inequality holds since $(K(x) - \frac{(C(\delta) - C(\delta'))}{1 - M(x^{RP}(\delta'), N+1)})$ is single crossing and the second inequality from stochastic dominance: $1 - M(x^{RP}(\delta'), N+1) > 1 - M(x^{RP}(\delta'), N)$.

Then, $\hat{\pi}(\delta, N+1) - \hat{\pi}(\delta', N+1) \geq l(\widetilde{x})(\hat{\pi}(\delta, N) - \hat{\pi}(\delta', N))$, which implies that $\hat{\pi}(\delta, N)$ satisfies the single-crossing condition, namely:

$$
\begin{aligned}
\forall \delta > \delta', \quad & \hat{\pi}(\delta, N) - \hat{\pi}(\delta', N) \geq 0 \Rightarrow \hat{\pi}(\delta, N+1) - \hat{\pi}(\delta', N+1) \geq 0 \\
& \text{and} \hat{\pi}(\delta, \mathrm{N}) - \hat{\pi}(\delta', \mathrm{N}) > 0 \Rightarrow \hat{\pi}(\delta, \mathrm{N}+1) - \hat{\pi}(\delta', \mathrm{N}+1) > 0.
\end{aligned}
$$

We can then apply the results of Milgrom and Shannon (1994) to conclude that $\widehat{\delta}^*(N+1) \geq \widehat{\delta}^*(N)$.

(ii) Follows from $\widehat{\pi}(0, N) = \pi(0, N) = v_m$ and $\widehat{\pi}(\widehat{\delta}^*, N) \geq \pi(\delta^*, N)$. Then,

$$
\pi(\delta^*, N_0) > v_m \Rightarrow \widehat{\pi}(\widehat{\delta}^*, N_0) > v_m.
$$

Furthermore, there may exist N such that

$$\pi(\delta^*, N) < v_m < \widehat{\pi}(\widehat{\delta}^*, N),$$

in which case $\widehat{N}_0 \leq N_0$. $\square$

Proof of Collorary 4. Inmediate from the result (i) in Proposition 3 and results (ii) and (iii) in Proposition 2. $\square$

Proof of Proposition 5. To prove that $\widehat{\delta}^*(N) \geq \delta^*(N)$, it is enough to show that $\widehat{\delta}^*(N) < \delta^*(N)$ is not possible. In order to do so, we first state state that if $\delta > \delta'$ then, the following "increasing differences" condition holds:

$$\hat{\pi}(\delta, N) - \hat{\pi}(\delta', N) > \pi(\delta, N) - \pi(\delta', N).$$

In words, that the impact of increasing information over profits is higher when we use an optimal reserve price. This is equivalent to

$$\hat{\pi}(\delta, N) - \pi(\delta, N) > \hat{\pi}(\delta', N) - \pi(\delta', N).$$

Using the definition of π and $\hat{\pi}$ this is equivalent to

$$\begin{aligned} & -\int_0^{x^{RP}(\delta)} \left[y\,\delta + (1-\delta)v_m - \delta\frac{1-F(y)}{f(y)} \right] m(y, N)dy \\ > & -\int_0^{x^{RP}(\delta')} \left[y\delta' + (1-\delta')v_m - \delta'\frac{1-F(y)}{f(y)} \right] m(y, N)dy \end{aligned}$$

From Proposition 2 we know that $x^{RP}(\delta) > x^{RP}(\delta')$ so we can rewrite the inequality as:

$$\begin{aligned} & -\int_{x^{RP}(\delta')}^{x^{RP}(\delta)} \left[y\delta + (1-\delta)v_m - \delta\frac{1-F(y)}{f(y)} \right] m(y, N)dy \\ > & \ (\delta - \delta')\int_0^{x^{RP}(\delta')} \left[y - v_m - \frac{1-F(y)}{f(y)} \right] m(y, N)dy \end{aligned}$$

This inequality is satisfied since the LHS is positive (given the definition of $x^{RP}(\delta)$, $x\delta + (1-\delta)v_m - \delta\frac{1-F(x)}{f(x)} < 0$ for all $x < x^{RP}(\delta)$) and the RHS is negative (since $(\delta - \delta')$ is positive, $x - v_m - \frac{1-F(x)}{f(x)}$ is increasing, and it is negative at $x^{RP}(\delta')$). This is because

$$\begin{aligned} x^{RP}(\delta')\delta' + (1-\delta')v_m - \delta'\frac{1-F(x^{RP}(\delta'))}{f(x^{RP}(\delta'))} &= 0 \\ x^{RP}(\delta') - v_m - \frac{1-F(x^{RP}(\delta'))}{f(x^{RP}(\delta'))} &= -\frac{v_m}{\delta'} < 0 \end{aligned}$$

Hence, $\hat{\pi}(\delta, N) - \hat{\pi}(\delta', N) > \pi(\delta, N) - \pi(\delta', N)$. Consider the contrary to the statement of the proposition that, $\delta^*(N) = \delta > \delta' = \widehat{\delta^*}(N)$. This would imply that $\pi(\delta, N) - \pi(\delta', N) > 0$ and $\hat{\pi}(\delta, N) - \hat{\pi}(\delta', N) < 0$ which contradicts the "increasing differences" condition stated above.

Let X_δ be more *supermodular precise* than $X_{\delta'}$, $\pi^{RP*}(\delta) \leq \underline{\pi}$, and $\pi^{RP*}(\delta') \leq \underline{\pi}$. Then, $\pi^{RP*}(\delta) \geq \pi^{RP*}(\delta')$. □

Proof of Proposition 7. As we show in the main text, the optimal reserve price is the solution of the following auctioneer's decision problem

$$\pi^{RP*}(\delta) \in \arg\max \Psi(\pi, \delta) = (1 - \pi) W_\delta(\pi).$$

A sufficient condition for the monotonicity of $\pi^{RP*}(\delta)$ would be that $\Psi(\pi,\ \delta)$ has increasing differences for $\pi' < \pi < \underline{\pi}$. This is equivalent to show that, if $\pi > \pi'$:

$$\begin{aligned}(1-\pi)W_\delta(\pi) - (1-\pi')W_\delta(\pi') &\geq (1-\pi)W_{\delta'}(\pi) - (1-\pi')W_{\delta'}(\pi') \\ &(1-\pi)(W_\delta(\pi) - W_\delta(\pi')) - (\pi - \pi')W_\delta(\pi') \\ &\geq (1-\pi)(W_{\delta'}(\pi) - W_{\delta'}(\pi')) - (\pi - \pi')W_{\delta'}(\pi')\end{aligned}$$

This condition is satisfied since

$$(1-\pi)(W_\delta(\pi) - W_\delta(\pi')) \geq (1-\pi)(W_{\delta'}(\pi) - W_{\delta'}(\pi'))$$

and

$$(\pi - \pi')W_\delta(\pi') \leq (\pi - \pi')W_{\delta'}(\pi')$$

Where the first inequality comes from the definition of supermodular precision and the second one from the fact that as $\pi' < \pi < \overline{\pi}$, then $W_\delta(\pi') \leq W_{\delta'}(\pi')$. □

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