Essays in Dynamic Public Finance

Onursal Bağırgan

In partial fulfilment of the requirements for the degree of

Doctor in Economics

Universidad Carlos III de Madrid

Advisor:
William Fuchs

January 2022

To my parents, İmran and Yaşar,
for raising me to live my dreams
by giving up most of theirs.

Acknowledgements

I want to thank William Fuchs for his tremendous help as my main advisor. He was always available and ready to help whether he was in Madrid or abroad. He always tried to lead me to the right path while respecting my freedom to do my research at my own pace and style. His handling approach helped me define and improve myself throughout my Ph.D. studies.

I want to thank all the Department of Economics faculty members at Universidad Carlos III de Madrid. I had fruitful conversations with many of them and felt productive in positive and helpful environments. Some faculty members need to be mentioned here specifically. Andrés Erosa was my advisor at the beginning of my studies, and I also took a few courses from him. I was a naive economist wanna-be when I first met him, and he is one of the prominent influencers that shaped my view of economics. Another professor with a significant impact on my economic thinking was Matthias Kredler. I enjoyed and learned a lot during his reading group meetings. My understanding of economics developed considerably with every conversation I had with him, even small conversations while commuting to Madrid from Getafe. I would like to thank the professors who helped me at different stages of my research; Aditya Kuvalekar, Evi Pappa, Juan José Dolado, Johannes Schneider, Emircan Yurdagül and Marco Celentani. I am grateful that they have dedicated their time and effort to help improve my work.

I participated in different reading groups while working on my thesis. I want to thank especially the participants of the Macro Reading Group participants. They have been my colleagues to have intellectual conversations with and my friends to take a drink after a working day. I also want to thank the Micro Reading Group participants for surviving with me in the harsh, but educational, environment of the reading group and the participants of the infamous Marxian Economics Reading Group to broaden my understanding of economic thinking.

I made many friends while studying in Madrid. I would like to thank everyone in my cohort for being so nice and welcoming. I wish all of them to have a happy life with successful careers. I will always remember Fabrizio Colella and Richard Jaimes for suffering with me during our first year. Ismael Gálvez Iniesta was great flatmate and the hard worker that demonstrates the true researcher mentality. I also want to thank Ruben Veiga, Javier Rodriguez, Michele Rosenberg, Federico Curci, Federico Masera, Salvatore LoBello, Michelangelo Rossi, Jose Carbo, Tomas Martinez, and Beatriz Gonzalez Lopez for being great friends and colleagues.

There were a few from the younger cohorts with whom I feel connected. I want to thank Florencia Airaudo, Agustina Martinez, Henry Redondo, Ivan Acosta, and especially Bilal Sali, the best flatmate ever, for sharing great memories with me. I always had a lot of fun when I was with them. They enabled me to be cheerful and happy all the time which made my time in Madrid unforgettable.

I had many friends from the previous stages of my life and a few of them were always with me throughout this journey. Although we mainly communicated via texting, I would like to thank Zahid Çağlıöz, Ali Yılmaz, Hüseyin Gürkan, Murat İplikçi and Oğuz Çetin for their support and their true friendship.

My last thanks should go to my sister and my parents. They always supported me no matter the circumstances. I hope to have them around me for a long and happy lifetime.

Abstract

An upcoming trend in public finance is applying economic theory to understand how private agents' incentives, expectations, and perceptions affect the efficiency and sustainability of government policies. This doctoral dissertation presents novel studies that fall into this category. The first three chapters examine many aspects of tax amnesties from this perspective, while the last chapter studies intergenerational risk-sharing under limited enforcement.

The first chapter introduces a selection of stylized facts about tax amnesties by examining the recent state-level tax amnesty experiences in the US. We provide observational evidence on the heterogeneity of tax amnesty implementations among US states. The heterogeneity is also persistent throughout the last four decades. We show that a few states declaring amnesties very rarely in the 80s and 90s start to implement amnesties much more frequently in recent decades. We also observe that the tax amnesty declarations of US states cluster around the US recession periods.

The second chapter introduces a theoretical framework to investigate the strategic interaction between a government and taxpayers in an economy. Our model predicts four factors that make a tax amnesty more likely in an economy: high personal income; high tax rates; low political cost for declaring an amnesty; and low audit rates. We examine US state-level data to test this prediction and show that the states with high personal income and high tax rates are using tax amnesties more frequently. We also show that the self-fulfilling characteristic of tax amnesties may lead to sub-optimal outcomes under lack of commitment.

In the third chapter, we present a theoretical framework to explain the recurring nature of tax amnesties. We construct a model with the strategic interaction of a government and a mass of taxpayers. The government and the taxpayers interact repeatedly, and each interaction can result in a tax amnesty. We show that an amnesty can cause another amnesty in the near future by altering taxpayers' beliefs about unobservable government characteristics. Therefore, an economy may get into a sequence of successive tax amnesties through a reputational channel referred to as an "expectation trap." The expectation trap mechanism can explain the series of tax amnesties in some US states that rarely experienced tax amnesties in the past. Extending our baseline model shows that a recession may cause a tax

amnesty, which can trigger a sequence of further amnesties that can spread into the years after recovery from the recession.

In the fourth chapter, we study the sustainable intergenerational insurance schemes under lack of enforcement. The welfare improving insurance policies may not be implementable under limited enforcement since any agent can opt-out of the insurance scheme if it does not benefit her. We develop a 2-period overlapping generations model to study the impact of different government policies. A standard tax-and-transfer scheme cannot provide perfect risk-sharing. To improve intergenerational risk sharing, we first introduce money into the system. We show that introducing money into the baseline model may improve risk-sharing. We also investigate the possibility of providing further risk-sharing by using monetary policy. We show that a monetary policy rule improves welfare under a numerical example.

Contents

List of Tables

List of Figures

Chapter 1

An Introduction to the Study of Tax Amnesties

This introductory chapter lays the foundations for the detailed examination of tax amnesties throughout the following two chapters. We present some stylized facts about the tax amnesties using US state-level tax amnesty data from 1982 to 2018. The following two chapters provide theoretical and empirical analyses that examine the key aspects of tax amnesties and provide explanations to the stylized facts. Here, we also briefly explain our theoretical analyses' key ingredients and results and their relation with the existing tax amnesty literature. Chapter 4 investigates a distinct topic, so we prefer to leave its introduction to that chapter.

1.1 Some Stylized Facts on Tax Amnesties

Tax amnesties raise important revenues and uncover taxable assets. For instance, the 2003 Illinois tax amnesty program collected USD 532 million, approximately 2.2 percent of the total tax revenue in 2003. Italy's 2009 tax amnesty uncovered USD 80 billion in previously undeclared assets. This amount is approximately five percent of gross domestic product.[1]

Governments also frequently employ tax amnesty programs. US states implemented 130 tax amnesties between 1982 and 2018.[2] Italy, Ireland, and others implemented multiple tax amnesties during the same period. Emerging economies use tax amnesties quite often,

[1] Italy tax amnesty yields record 80bn Euros *(Financial Times, 2009)*

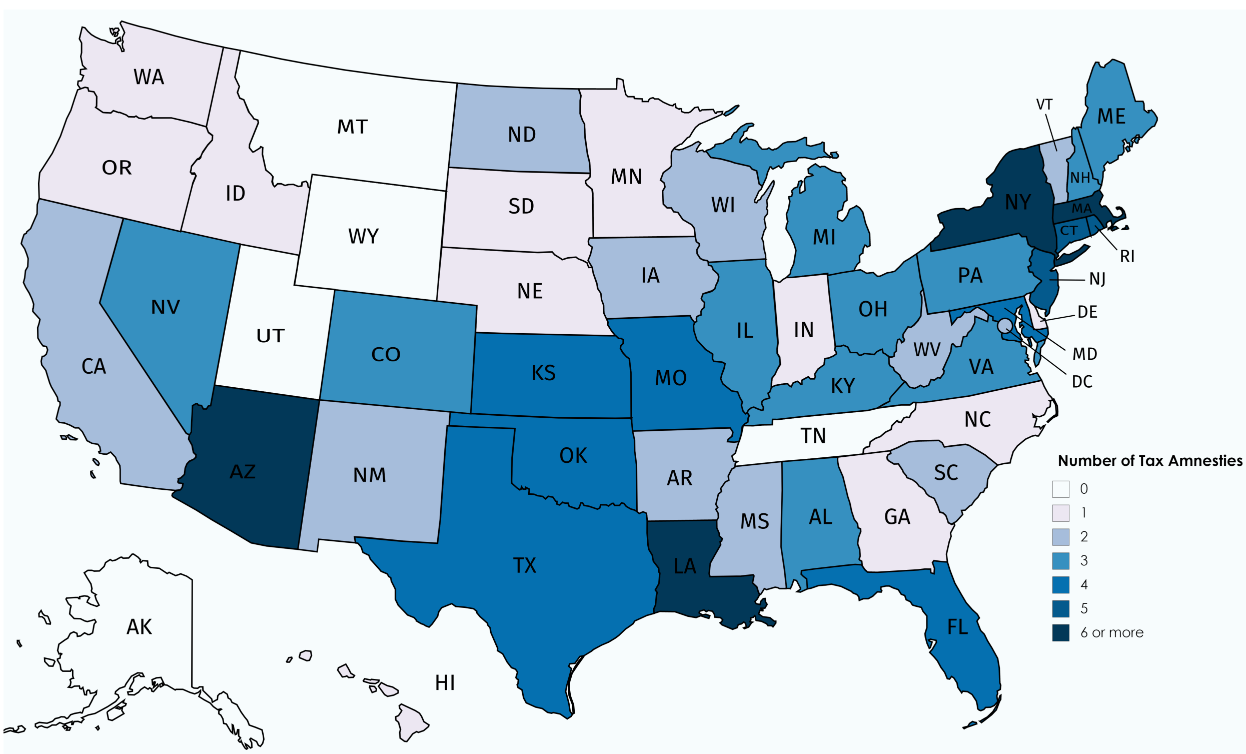
[2] Note that this figure, and all the statistics about US state-level tax amnesties, are from the data provided online by Federation of Tax Administrators (FTA). Alternatively, there is a dataset that is collected by Mikesell and Ross (2012). It includes a few more amnesties than the FTA data, but it is shorter. Throughout this paper, we will use the FTA dataset. However, all qualitative results hold with a combined dataset of FTA and Mikesell and Ross (2012) as well.

as well. Argentina had seven tax amnesties during 1995-2004, while Turkey implemented 19 tax amnesties during 1981-2016.

We focus on US state-level tax amnesties between the years 1982-2018. We have two main reasons to do so. First, we need to have a set of economies that are similar in dimensions unobservable in data, such as economic culture, political system, while exhibiting enough variations in tax amnesty implementations. Although we recognize that the US states differ in many key aspects, such as their tax systems, the difference between independent countries would be much more. Second, the availability of data strengthens our results. We use a dataset publicly available on the Federation of Tax Administrators website. The dataset includes all tax amnesty implementations since 1982 for all the US states.

From 1982 to 2018, US state governments enacted 130 tax amnesties. The states differed in their frequency of implementing tax amnesties. While five states did not implement any tax amnesties, 13 states implemented at least four tax amnesties during the same period. We visualize the heterogeneity in tax amnesty implementations in Figure 1.1.

Figure 1.1: Number of tax amnesties by US states during 1982-2018



The heterogeneity is also persistent through time. Seventeen states implemented at least one amnesty in three of the last four decades. Among them, four states employed at least one in each of the last four decades, for a total of 24. This suggests that for a significant number of states, taxpayers have been presented with opportunities to pay their previously evaded taxes at least once per decade. Further evidence on persistence reveals that it is hard to find states that stopped implementing tax amnesties after the first one. Twenty-nine states implemented a tax amnesty during the 1980s, the first decade in our sample. Almost all, 26, employed a second amnesty. The majority already employed multiple amnesties since 1990. These 29 states have averaged 2.28 tax amnesties since 1990, while the rest averaged only 1.57 amnesties. These figures suggest that the states that tended to implement tax amnesties the 1980s were also more likely to employ them in the subsequent decades.

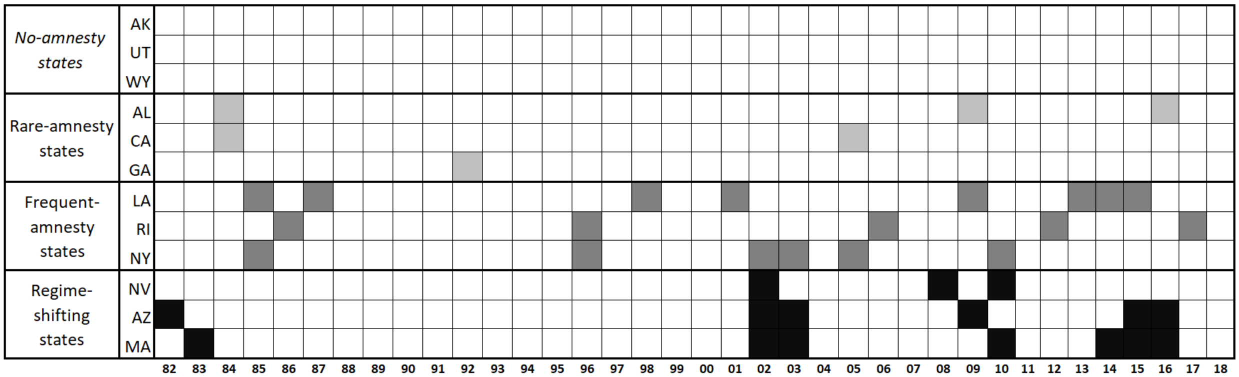
A simple accounting of tax amnesties suggests that some states implement amnesties much more frequently than others. Their residents probably start to see tax amnesties as a natural part of fiscal administration since they are accustomed to see recurring amnesties over multiple decades. We categorize the states into four different groups in terms of their tax amnesty frequency. *No-amnesty states* consists of states with no amnesties. *Rare-amnesty states* consists of states with infrequent amnesties: at least one but at most three amnesties. *Frequent-amnesty states* consists of states with at least four amnesties. Lastly, a fourth group consists of states that rarely implemented tax amnesties, but began to use them much more frequently. We can argue that these states switched their regime in terms of tax amnesties and have behaved like *Frequent-amnesty states* lately. We call this group *Regime-shifting states*. This group is interesting, since there is no reverse regime shift in our database. There are no states that stopped implementing frequent amnesties. We provide three examples for each of these groups in Figure 1.2. An extended figure with all the states can be found in Appendix A.

Lastly, the data suggest that tax amnesties are counter-cyclical. We see clustering around post-recession years. Tax amnesties are clustered around years 2002 and 2009 especially, See Figure 1.3. Also, we observe that some regime shifts are triggered by amnesties that can be associated with US recessions.

1.2 A Brief Discussion of Our Theory

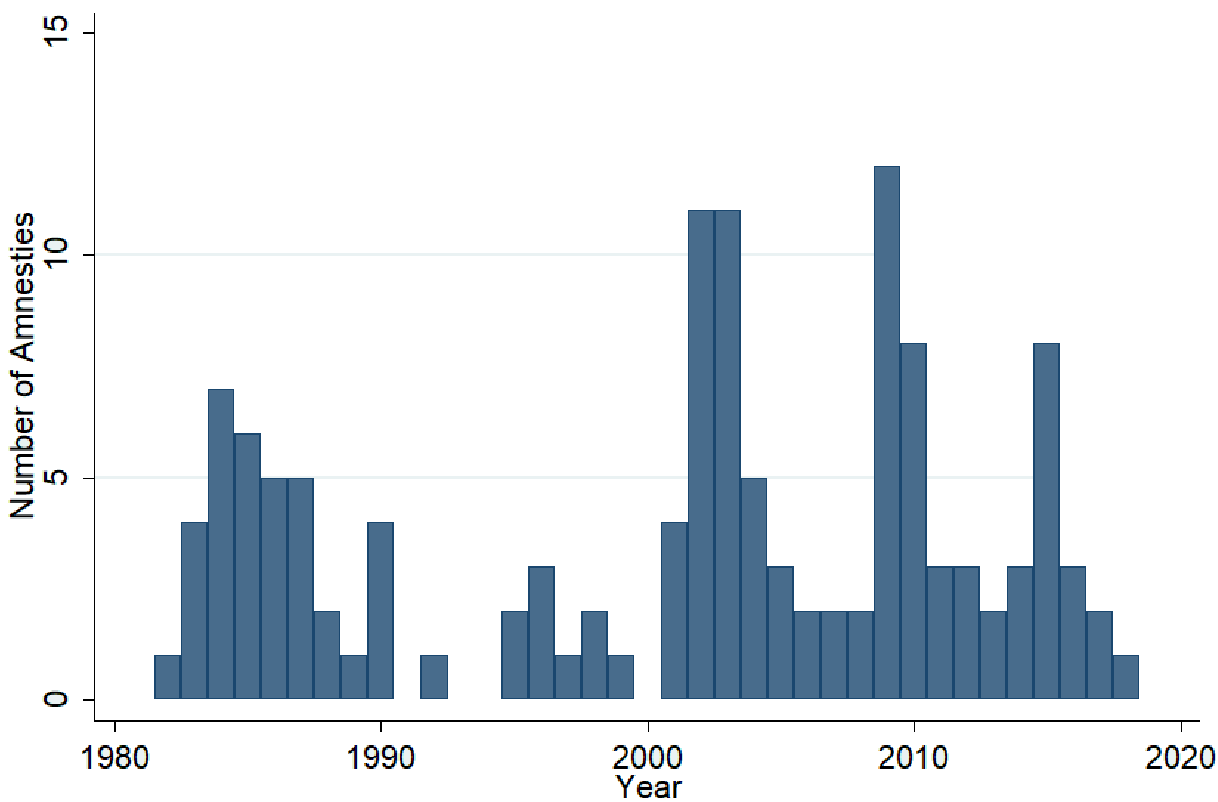
In Chapter 2, we introduce a 1-period model of tax amnesties. We model the tax amnesties as the strategic interaction of a government and a mass of taxpayers. First, taxpayers draw an income privately, make an income declaration, and pay taxes based on their declared income. Since the income is private, they can evade taxes. The government can perfectly estimate total evasion, but identifying individual evasions would require further efforts. The government can audit individuals imperfectly, and a taxpayer who is identified as an evader

Figure 1.2: Amnesty frequencies of some selected states by groups during 1982-2018



An empty box corresponding to state i year j means that there is no amnesty in state i in year j. A colored box means that there is one amnesty in that state in that year. Different shades of black filling are used for different state categories. An extended version of this figure with all the states can be found in Appendix C. There are five *No-amnesty states*, 30 *Rare-amnesty states*, six *Frequent-amnesty states* and nine *Regime-shifting states*.

Figure 1.3: Frequency of state-level tax amnesties in US over the years



must pay a penalty. Between initial tax declaration and auditing, the government may also implement a tax amnesty, which gives taxpayers a second chance to declare their income truthfully. Incomes declared under the amnesty program are subject to a special amnesty period tax rate that the government chooses at the time of the amnesty declaration. The government implements an amnesty if the potential revenue benefit from declaring an amnesty is higher than its political cost.

Our model suggests an explanation for the observed heterogeneity in tax amnesty implementations. The tax amnesties are the results of the optimal decisions of the government and taxpayers. The combination of specific economic parameters, some of which we can easily be observed in data, directly affect the aggregate outcome of the taxpayer's behaviors. The aggregate behavior of taxpayers then gives the attractiveness of a tax amnesty from the government's perspective. More precisely, our model shows that tax amnesties can be supported in the set of equilibria if the state has high taxpayer income, high tax rates, low audit rates, and low political cost of declaring a tax amnesty.

Tax amnesties also have a self-fulfilling nature in our setup. Since government's revenue maximization problem is common knowledge, taxpayers can understand that the government would decrease income tax rate during an amnesty. If taxpayers expect an amnesty, many taxpayers hide their income with the expectation of declaring them later with a lower tax rate. When there is high amount of hidden income, the benefit of a potential tax amnesty is also high. Then, it is more likely for the government to declare an amnesty.

The first variable of interest that affects the occurrence of an amnesty is the income level of taxpayers. The model suggests that the more income individuals get, the more likely an amnesty becomes attractive for the government. The decision to evade or not is orthogonal to the income of taxpayers in our model. Therefore, the ratio of taxpayers who hide income from the government does not depend on the taxpayers' income level. With a constant ratio of individuals evading taxes by not declaring their income to the government, the total hidden income from the government is higher with a higher level of individual income. Then, the total value of evaded taxes is also higher. The extra revenue generated by the government through declaring an amnesty is higher if the total value of evaded taxes is higher. Hence, declaring an income is more attractive for a government when the individual income levels are higher.

The second variable that determines the occurrence probability of a tax amnesty is the income tax rate. The higher the tax rates, the higher the benefit of an amnesty for the government at an equilibrium. The level of income tax rates affects amnesty occurrence via two distinct channels. The first channel is the evasion behavior of taxpayers. Higher tax rates create higher incentives to hide income from the government. Therefore, the aggregate hidden income of taxpayers is positively correlated to the income tax rates. A tax amnesty is more attractive for the government if more hidden income can potentially be uncovered with an amnesty. The second channel is the direct effect on the amount of evaded taxes. Our model only studies income taxes. It implies that the total tax evasion is simply the total hidden income multiplied by the income tax rate. With higher tax rates, the same amount of hidden income corresponds to a higher amount of tax evasion. Even if we have taken the aggregate hidden income exogenous, we would still affect tax rates on total evaded taxes. Since the revenue benefit of a potential tax amnesty is higher when there is more tax evasion, the government has more incentives to declare an amnesty when income taxes are higher.

The third variable of interest for us is the probability of a successful audit by the government. The level of the probability of a successful audit affects the occurrence of a tax amnesty in two different ways, similar to tax rates. However, the effect of higher successful audit probability has the inverse effect of higher income tax rates. Firstly, a higher successful audit probability decreases the government's dependency on tax amnesties to uncover hidden income. High enough audit success probability makes tax amnesties redundant in any equilibrium. Secondly, from the taxpayers' perspective, a higher probability of audit success means a higher probability of facing punishment. This creates a disincentive to evade for the taxpayers in the first place.

Lastly, the cost of an amnesty for the government affects the occurrence of an amnesty. The government decides whether to declare an amnesty or not. The government does this by making a cost-benefit comparison. The other determinants of tax amnesties affect the benefit of a tax amnesty. Keeping them constant, a higher cost of tax amnesty would lead to a lower net benefit of a tax amnesty. If the cost is high enough, the net benefit would be negative and would make the amnesty unattractive for the government.

To test the predictions of our model, we use state-level personal income data and state-level tax burden data to construct a sufficient statistic suggested by our model. We provide evidence that the states with higher personal income and tax burden tend to have more tax amnesties in the last four decades, as our model predicted.

First, we order the US states from the highest to lowest based on the value of the sufficient statistics. Our model suggests that a tax amnesty is more likely in the states with higher values for the sufficient statistics. We show that the states in the highest quartile declares at least one amnesty in our database. On the other hand, four of the five states without any amnesty is in the lowest quartile. Also, the states in the lowest quartile average 2.08 amnesties for the period 1982-2018. The average number of amnesties per state in the highest quartile is 3.67.

Second, we run regressions to provide further evidence on the predictive power of our model. Our estimations suggest that the number of tax amnesties are significantly higher on states with higher values for our sufficient statistics. More precisely, the states with higher values of personal income per capita and domestic tax burden are the states with higher number of tax amnesties in our database. We run both an OLS estimation as well as a negative binomial estimation to check the robustness of our results.

Chapter 3 extends the model introduced in Chapter 2 by adding the time dimension. In Chapter 3, a government and a mass of taxpayers interact repeatedly. The government may be behavioral — i.e, one that never declares an amnesty; or optimizing — i.e, one that would declare an amnesty if it were revenue-maximizing. The government type may change after every period according to a Markov process.

Taxpayers do not observe the government type but form rational beliefs given the information they observe. In each period, the taxpayers form their beliefs on the probability of tax amnesties, and the 1-period game introduced in Chapter 2 is played out. At the end of the period, taxpayers observe the outcome of the period and update their beliefs on the government type, which would impact their expectation of tax amnesties next period.

Our main finding is that the recurring nature of tax amnesties can be explained through a mechanism called an "*expectation trap*". The more an amnesty is expected, the more evasion occurs, and so the government has more incentive to declare an amnesty. If the government declares an amnesty, it reveals its type as optimizing. Then, the government type is most likely to be optimizing in the next period. This leads taxpayers to expect an amnesty in the next period, as well. Through this channel, a tax amnesty may trigger a sequence of further amnesties. We show that the expectation trap mechanism exists in all the equilibria for a wide range of parameters. We also conduct further tests on the robustness of expectation traps in our modeling assumptions by studying small variations of our model.

The expectation traps, which imply that a tax amnesty may trigger a sequence of further amnesties, can explain the regime shifts. A state with no amnesty in its history might start implementing them frequently, if the first amnesty it declares puts the state in an expectation trap. It is possible that the *Regime-shifting states* got caught in expectation traps in recent years. We can also argue that *Frequent-amnesty states* were caught in this trap due to an earlier amnesty in the 1980s.

Our model abstracts from recessions. In our model, the first amnesty, which can trigger further amnesties through an expectation trap, arises endogenously as a result of reputation dynamics. However, we later enrich our model of repeated interactions to capture the effects of recessions. We show that a one-period recession may result in a tax amnesty. Then, through the expectation trap mechanism, this single amnesty may trigger a sequence of further amnesties. In this way, a temporary, one-period shock to the economy may have consequences that persist for many periods.

1.3 Literature Review

Some studies provide evidence on the conditions leading to expectation traps in tax amnesty environments. Ross and Buckwalter (2013), for example, use differences in eligibility periods of tax amnesty programs to estimate the effects of tax amnesty expectations. They estimate that about 12.9% to 16.5% of state tax amnesty revenues in the US until 2011 can be attributed to strategic delinquency. Taxpayers evade taxes when they expect a tax amnesty in the near future, and they later participate in the amnesty to benefit from the amnesty discounts. Using US state-level data, Bayer et al. (2015) claim that tax amnesties are self-fulfilling. They show that one additional amnesty in other states in the previous year raises a state's amnesty probability by 0.007%. They interpret this as a possible change in taxpayers' expectations of an amnesty due to seeing amnesties in other states. Alm et al. (1990) conduct laboratory experiments about tax amnesties and conclude that in a repeated tax evasion setup, tax compliance drops after the first unexpected tax amnesty. Langenmayr (2017) uses a synthetic analysis to estimate the effect of the 2009 US off-shore voluntary disclosure program on tax evasion. The estimates show that US tax evasion increased in 2010 and 2011, although the voluntary disclosure program had ended in 2009.

The closest model in the tax amnesty literature to our work is that of Bayer et al. (2015) who construct a game between a government and infinitely many taxpayers to show that tax amnesties have self-fulfilling characteristics. This is, indeed, one of the results of our stage game analysis. Our work differs from theirs in two main aspects. First, their paper focuses on a single-period model while our main mechanism is delivered through repeated interactions. Second, tax amnesties are fully endogenous in our setup. In their work, although the government optimally chooses whether to declare an amnesty, the discount on tax rates during the amnesty period is exogenous. Our model allows the government to choose the optimal amnesty tax rate.

Studies on tax amnesties focus mainly on their effects. Malik and Schwab (1991), Andreoni (1991), Garz and Pagels (2018) and many others investigate the potential benefits of tax amnesties, such as an increase in long-run revenue and welfare improvements through consumption smoothing. A key difference between the tax amnesty literature and our model is that the prior literature conducts no dynamic analysis of tax amnesties: the analyses are generally set in a static environment. In the works of Stella (1991) and Macho-Stadler et al. (1999), tax amnesties are one-time events in a dynamic tax evasion environment.

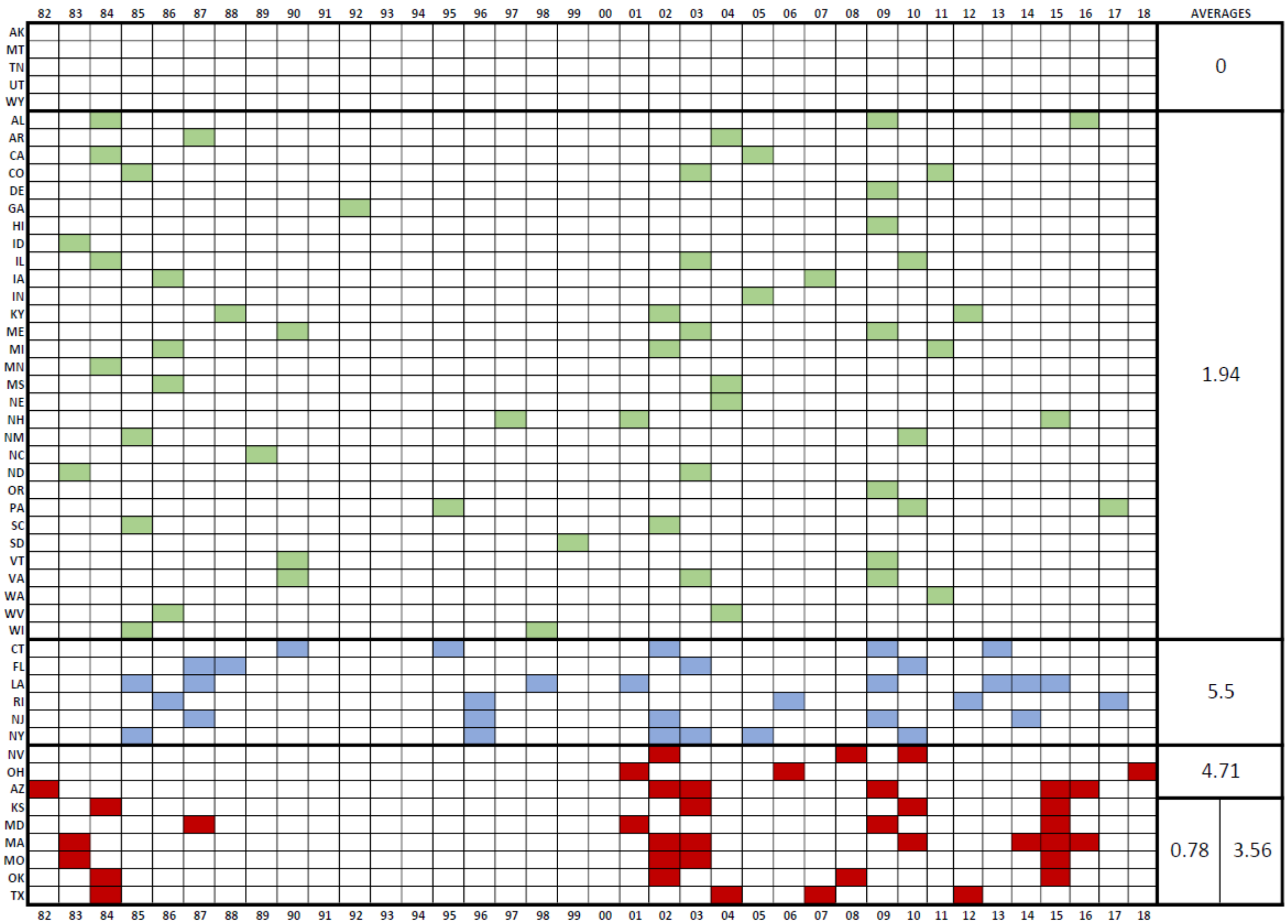
This paper relates to the literature on government policies with reputation. The seminal papers by Kreps and Wilson (1982) and Milgrom and Roberts (1982) introduce reputation in repeated games. Following these, Barro and Gordon (1983) model a repeated policy game in which consumers form an expectation of inflation, and the government sets an inflation rate. They show that a high inflation period may lead consumers to expect high inflation in the future. Since inflationary expectations are self-fulfilling, the model explains the serial high inflationary years in US. Their work opened the way for a sequence of policy analyses with reputation, such as Barro (1986), Persson et al. (1987), Chari and Kehoe (1990), Stokey (1991), Albanesi et al. (2003), Sleet (2001), Alvarez et al. (2002) and many others. A few of these studies point out expectation traps as an explanation of the persistence of undesirable outcomes. Chari et al. (1998), Albanesi et al. (2003) and Armenter (2008) are examples of monetary policy papers that explain high inflation periods as a result of expectation traps. In a recent paper, Amador and Phelan (2021) study sovereign defaults through government reputation. Their result suggests that the expectation trap can explain the serial defaulting.

We contribute to both the tax amnesty literature and the government reputation literature. We contribute to the tax amnesty literature by constructing the first dynamic model of tax amnesties. We show that the dynamic aspect is important since the lack of commitment may lead to sub-optimal outcomes for a government in a static setup. Since we know that reputation can be a commitment device in repeated games, a dynamic model is essential to analyze tax amnesties. Our second contribution is to the government reputation literature. Here, we point out the dynamic inconsistency in the tax amnesty environment. Therefore, the impact of tax amnesty relies on government reputation. The existing government reputation literature tends to work on continuous variables such as inflation. However, tax amnesties are binary events. Thus, in our analysis, this can explain the robustness of the expectation trap to change in parameter values and modeling assumptions.

1.4 Appendix

Appendix A. Tables and Figures

Figure 1.4: Tax amnesties by state governments throughout years



Chapter 2

Theory and Evidence on the Causes of Tax Amnesties

Chapter 1 is structured as follows. In Section 2.1, we introduce our 1-period theoretical model. In Section 2.2, we solve the model. Section 2.3 shows and discusses our empirical results. Then, the last section concludes our analysis.

2.1 Model

Environment

Consider a stage game where a government faces a continuum of taxpayers. The government collects income taxes from taxpayers. Taxpayers can evade taxes. If they choose to do so, they face a possibility of prosecution for tax evasion and a fine. Between the evasion and prosecution, they might have an opportunity to come clean if a tax amnesty is declared by the government. The timing of the game is as follows.

1. Each taxpayer i forms her belief on the probability of seeing an amnesty, which we denote as $\phi_i \in [0, 1]$.

2. Each taxpayer i draw an income y_i and a preference shock ϵ_i, which are private information. The income y_i can be either 0 or w with equal probabilities.[1] The preference shock is distributed uniformly, i.e. $\epsilon_i \sim U[0, 1]$.[2] Both distributions are common

[1]we can assume equal probabilities without loss of generality. All results hold as long as the distribution is binary.

[2]Uniform distribution assumption is not necessary. It makes our equations simpler and analysis easy to follow. However, we acknowledge that our analysis may not hold for non-continuous distributions since it may break the uniqueness of the optima for the government's amnesty tax rate decision problem.

knowledge.

3. Taxpayers declare their income y_i^d and pay τy_i^d amount of taxes.

4. The government plays a mixed strategy by selecting the probability of an amnesty, $x \in [0,1]$. In case of declaring an amnesty, the government needs to pay a fixed cost C_A.[3]

5. If there is an amnesty, G_O sets a special tax rate for the amnesty program which we denote by a.

6. Taxpayers decide how much to declare during the amnesty, y_i^a and pay ay_i^a.

7. Independent of whether there was an amnesty, the agents who still hide income at the end of the game can get caught with probability p, and give all their income to the government in that case. The probability p is exogenous.[4]

Our approach here is to make all amnesty decisions endogenous while keeping other parts of the model as simple as possible. We have a continuum of taxpayers, which means that with the given income distribution half of the population will draw 0. Their optimal choice is trivial, to declare zero income. However, the other half will draw w and their problem is our subject of interest. We normalize the measure of taxpayers who draw w to one. The assumption of uniform distribution of income is without loss of generality. As long as we have a binary distribution, all our assumptions hold. However, binary income distribution is important. In an environment with more than two possible income levels, taxpayers' income declaration becomes a signaling game among different income types. This would add a layer of complication to our model.

Note that ϵ captures the idiosyncratic differences among taxpayers. It can be interpreted as moral cost of being an evader or psychological cost of being punished as an outlaw. The distribution of ϵ should be continuous to ensure a unique solution to the government's amnesty tax rate decision. For the ease of computation, we assume it to be uniform. The lower and upper bounds of the distribution are important. For the purpose of our model, we want to have the individuals who draw the highest value of ϵ to be a truthful taxpayer independent of the amnesty probability, while the lowest ϵ individual should be a tax evader even if the probability of amnesty is zero. Otherwise, the importance of amnesty expectations may not be visible in our results.

[3]This can be interpreted as the political cost of declaring an amnesty.

[4]We can think the environment is such that investigation is costless for the government, so everyone who declared zero income is investigated at the end of the game. When government investigates a tax evader, it is able to find evidence of evasion with probability p.

We assume that the taxpayers are rational. This assumption disciplines our model. The expectations of taxpayers on the probability of an amnesty, which are formed at the beginning of the game, are rational. Hence, we will only focus on the equilibria with rational expectations which implies $\phi_i = \phi$. To simplify the notation, we drop the taxpayer subscript and use only ϕ on taxpayers' belief.

All taxpayers have the same risk-neutral utility function. The payoff of a taxpayer i who draw income y and preference ϵ_i is

$$\begin{aligned} u_i(y_i^d, y_i^a) = y - &\tau y_i^d \\ &- \phi_t[ay_i^a + \underbrace{p(y - y_i^d - y_i^a) + \epsilon_i(y - y_i^d - y_i^a)}_{\text{Cost of hiding income after amnesty}}] \\ &- (1 - \phi_t)\underbrace{[p(y - y_i^d) + \epsilon_i(y - y_i^d)]}_{\text{Cost of being an evader}}. \end{aligned}$$

The payoff is simply the income minus expected cost. First subtracted term is the tax payment associated with the initial income declaration. Then, with probability ϕ the individual believes that there will be an amnesty. If there is an amnesty, she can participate in the amnesty by declaring an income and pay taxes based on the special amnesty tax rate. If she still hides income after an amnesty program, she needs to suffer from the psychological cost. Also, the government catches her with probability p and confiscates all the evaded income. With probability $1 - \phi$, there won't be an amnesty and if she didn't declare truthfully initially, she can get caught and lose all her income. She also suffers a loss of utility from the psychological cost of being an evader.

The government tries to maximize the income from taxes and evasion penalties. The revenue of the government can be divided into three categories, revenues from initial declarations, revenues from the amnesty program and revenues from auditing.

$$\textit{Revenues with amnesty: } A(\phi, a) = \int \tau y_i^d di + \int a y_i^a di + \int p(w - y_i^d - y_i^a) di \tag{2.1}$$

$$\textit{Revenues without amnesty: } R(\phi) = \int \tau y_i^d di + \int p(w - y_i^d) di \tag{2.2}$$

Notice that the revenue benefit of an amnesty may not be equal to the revenues collected during the amnesty program. By declaring an amnesty, government attracts some individuals to the program that it would catch through auditing process with some probability. The difference between the values A and R is the revenue increase associated to an amnesty program.

Assumption 1 ensures the existence of an interior equilibrium where there are positive masses of honest taxpayers, tax evaders and, in case of an amnesty, amnesty participants.

Assumption 1. $p < \tau < p + \frac{1}{2}$

In the next section, we will analyze the optimal decisions of taxpayers and the government.

2.2 Solving the Model

Consider the theoretical framework described above, which is essentially a sequential game played by the taxpayers and the government. We will focus on the sequentially rational equilibria with rational expectations. We will introduce the problems of agents in the following order: Tax evaders' tax amnesty participation problem; the government's amnesty tax rate decision if it declares one; and taxpayers' initial income declaration problem. Lastly, we will discuss the government's amnesty declaration decision.

Tax Evaders

Notice that any partial evasion cannot be optimal. To see this, remember the publicly known distribution which taxpayers are drawing income from only has two income levels. For individuals who draw 0, the unique optimal decision is to declare their true income. Therefore, any individual who declares an income different than zero would reveal that she drew w. Then, the evaders in this economy are the ones who draw w as income, but declare that they have zero income. From now on, I will mention the set of individuals with income $y_i = w$ and declared income $y_i^d = 0$ as tax evaders. Realize that if there is no tax amnesty, there is no decision to make for tax evaders. In that case, the expected payoff of a tax evader would be

$$w - (p + \epsilon_i)w$$

Let us focus on the case where there is a tax amnesty. Then, the problem of tax evader i in case of an amnesty is

$$\max_{y_i^a \in [0,w]} w - a y_i^a - p(w - y_i^a) - \epsilon_i (w - y_i^a).$$

Notice that declaring anything other than 0 would reveal that the person has an income of w. Then, the maximization problem of a tax evader is trivial to solve. There are only two potentially optimum declaration $\{0, w\}$. Then, for an evader, the optimal income declaration during the amnesty period is

$$y_i^{a*} = \begin{cases} w & a \leq p + \epsilon_i \\ 0 & a > p + \epsilon_i \end{cases}.$$

Tax evaders declare their true income during an amnesty if the cost they pay is weakly lower than the expected cost of not declaring true income. Given this optimal decision function we can go one-step backward and solve the government's optimal decision for the amnesty tax rate a.

Government's revenue maximization during an amnesty

Note that we first need to find what is the optimal amnesty tax rate, a, if the government decides to declare an amnesty. Before we start to solve the government's problem, we can simplify it by organizing our thoughts on how the agents split up into tax evaders and taxpayers in this economy. We only focus on individuals who draw income w. Therefore, the income of taxpayers as well as their characteristics are totally identical, except their private preference parameter ϵ. It is the deciding factor on splitting the population into subgroups in terms of their tax behavior. At the equilibrium, there is an $\bar{\epsilon}$ such that all the individuals with $\epsilon_i < \bar{\epsilon}$ declare $y_i^d = 0$ while all the individuals with $\epsilon_i \geq \bar{\epsilon}$ declare their true income $y_i^d = w$.

Notice that $\bar{\epsilon}$ is an equilibrium object, which arises before the government's amnesty decision. The government doesn't observe any individual's income or preference parameter. However, it knows the distributions that taxpayers draw these values, so it can perfectly predict $\bar{\epsilon}$ after seeing the total taxes collected. With this observation, we can write the government's problem given the threshold of truthful income declaration $\bar{\epsilon}$ as

$$\max_a \int_0^{\bar{\epsilon}} \mathbb{I}_{\{y_i^{a*}(a)=w\}} awd\epsilon + \int_0^{\bar{\epsilon}} \mathbb{I}_{\{y_i^{a*}(a)=0\}} pwd\epsilon$$

Since we know the optimal amnesty participation strategy of tax evaders, $y_i^{a*}(a)$ for all i, we can re-write government's problem as

$$\max_a \int_0^{\bar{\epsilon}} \mathbb{I}_{\{\epsilon \geq a-p\}} awd\epsilon + \int_0^{\bar{\epsilon}} \mathbb{I}_{\{\epsilon \geq a-p\}} pwd\epsilon.$$

Then, the optimal amnesty tax rate is

$$a^*(\bar{\epsilon}) = \frac{\bar{\epsilon}}{2} + p. \tag{2.3}$$

Taxpayers

Given the taxpayers' belief on the probability of an amnesty and the optimal decision functions we derived from the later stages, taxpayer i's problem at the beginning of the game becomes:

$$\max_{y_i^d \in [0,w]} y_i - \tau y_i^d - \phi a^*(\bar{\epsilon}) y_i^{a*}(\bar{\epsilon}, y_i^d) - (p + \epsilon_i)(y_i - y_i^d - y_i^{a*}(\bar{\epsilon}, y_i^d))$$

As we discussed, the threshold $\bar{\epsilon}$ determines the optimal tax evasion decision of a taxpayer at the beginning of the game.

$$y_i^{d*} = \begin{cases} w & \epsilon_i \geq \bar{\epsilon} \\ 0 & \epsilon_i < \bar{\epsilon} \end{cases}$$

We should determine the cutoff value $\bar{\epsilon}$ and the initial belief on the probability of an amnesty ϕ to fully characterize an equilibrium in this setup. We start by finding the the cutoff value of $\bar{\epsilon}$ and incorporate that information into the set of optimal behaviors derived until now.

To find the cutoff value $\bar{\epsilon}$, let us focus on the taxpayer who is indifferent between declaring the true income and evading. Such a taxpayer exists because of Assumption 1. Because of the linear nature of our model, she is unique. The value of declaring truthfully should be equal to the value of declaring zero income for her. In other words, the following condition should hold with her private value of ϵ_i.

$$w - w\tau = w - \phi\left(\frac{\bar{\epsilon}}{2} + p\right) w + (1 - \phi)(p + \epsilon_i) w.$$

Notice that her private ϵ_i should be the equilibrium $\bar{\epsilon}$, which would give us the threshold

$$\tau = \phi\left(\frac{\bar{\epsilon}}{2} + p\right) + (1 - \phi)(p + \bar{\epsilon}) \implies \bar{\epsilon}^* = \frac{2(\tau - p)}{2 - \phi}. \tag{2.4}$$

By plugging in the value of $\bar{\epsilon}$, we can give a characterization of a taxpayer's initial income declaration, the government's amnesty tax rate decision and a tax evader's amnesty participation decision as functions of the initial taxpayer belief.

$$y_i^{d*} = \begin{cases} w & \epsilon_i \geq \dfrac{2(\tau - p)}{2 - \phi} \\ 0 & \epsilon_i < \dfrac{2(\tau - p)}{2 - \phi} \end{cases}, \tag{2.5}$$

$$a^* = \frac{\tau + p(1-\phi)}{(2-\phi)}, \tag{2.6}$$

$$y_i^{a*} = \begin{cases} w & \epsilon_i \geq \dfrac{\tau - p}{2-\phi} \\ 0 & \epsilon_i < \dfrac{\tau - p}{2-\phi} \end{cases}. \tag{2.7}$$

Government's amnesty declaration decision

Since they all observe the same information, every taxpayer has the same initial belief. We can then derive the aggregate values of government revenues, given the optimal decisions presented in equations 2.5-2.7.

Proposition 1. If taxpayers maximize their utility given an initial belief of an amnesty ϕ,

$$\text{Revenue with amnesty, } A(\phi) = w\tau - w(\tau - p)^2 \left(\frac{3-2\phi}{(2-\phi)^2}\right), \quad \text{is decreasing in } \phi.$$

$$\text{Revenue without amnesty, } R(\phi) = w\tau - w(\tau - p)^2 \frac{2}{2-\phi}, \quad \text{is decreasing in } \phi.$$

$$\text{Benefit of an amnesty, } B(\phi) = A(\phi) - R(\phi) = \frac{w(\tau-p)^2}{(2-\phi)^2}, \quad \text{is increasing in } \phi.$$

Proof. See Appendix A. □

Proposition 1 gives us some important characteristics. Note that focusing on the amnesty revenues would be misleading. A government which does not declare an amnesty will collect more revenues in tax evasion prosecutions at the end of the period. The appropriate measure for a benefit of an amnesty would be the difference between total government revenues with and without declaring amnesty. Since we know the optimal amnesty tax rate the government would choose in case of an amnesty, we can compute these revenues. The most important implication of Proposition 1 is that the more taxpayers expect an amnesty, the more benefit the amnesty brings to the government's revenues. Intuitively, if taxpayers believe that a tax amnesty is likely, more taxpayers hide their income by expecting to pay lower taxes by participating in the amnesty. Higher aggregate evasion leads to a higher benefit of tax amnesty programs. This makes the amnesty programs more attractive from the government's perspective.

Given the above results, the government declares an amnesty if

$$B(\phi) \geq C_A. \tag{2.8}$$

The benefit of an amnesty for the government budget is a function of the taxpayers' beliefs. The probability they assign to an amnesty effects the benefit of an amnesty for the government.

Equilibrium

We only need an initial belief on the probability of a tax amnesty that is also consistent with the government's optimal decision to fully characterize an equilibrium. Since the government's decision is given by the benefit of an amnesty, studying the relation between the belief on the probability of an amnesty and the benefit of an manesty is key to equilibrium characterization. Proposition 1 already establishes it. Then, we can examine the government's optimal decision on declaring an amnesty under different initial taxpayer belief on the probability of an amnesty.

Depending on the parameters, there are three different cases, which give three different set of equilibria.

$\boldsymbol{B(1) < C_A}$**:** Cost of amnesty is so high that a tax amnesty is not optimal for the government. Even if all taxpayers believe that the government will implement an amnesty with certainty, the benefit of amnesty is still not high enough to surpass the fixed cost of a tax amnesty. In this case, the only rational expectations equilibrium is that taxpayers do not expect an amnesty at all, and the government does not declare an amnesty:

$$(\phi^*, x^*) = (0, 0).$$

$\boldsymbol{B(0) > C_A}$**:** Even if all taxpayers believe that the government will not implement an amnesty, and therefore evade very little, the extra revenue generated by a tax amnesty is still greater than the fixed cost of it. In this case, there is a unique rational expectations equilibrium. Taxpayers set their expectations such that the government declares an amnesty with certainty:

$$(\phi^*, x^*) = (1, 1).$$

$\boldsymbol{B(0) \leq C_A \leq B(1)}$**:** In this case, we have multiple self-fulfilling equilibria. The multiple equilibria arise from the fact that the potential benefit of a tax amnesty increases with the total evaded taxes. Also, taxpayers' initial belief on the probability of a tax amnesty effects total evasion. Therefore, we may have several initial beliefs which end up rationalized by the government's decision.

$$(\phi_1^*, x_1^*) = (0,0), \quad (\phi_2^*, x_2^*) = (1,1), \quad (\phi_3^*, x_3^*) = \left(\frac{2\sqrt{2C_A} - 2(\tau - p)}{\sqrt{2C_A}}, \frac{2\sqrt{2C_A} - 2(\tau - p)}{\sqrt{2C_A}} \right)$$

This is the parameter region that gives the most interesting results. If taxpayers expect to have an amnesty, they evade more which, in turn, makes an amnesty more beneficial for the government. In such a case, the government's optimal decision would be to declare an amnesty and it is consistent with taxpayers' initial beliefs. On the other hand, if taxpayers do not expect an amnesty at all, the fraction of agents who evades is small, and that leads to a small potential benefit of an amnesty for the government. In this case, the government's optimal decision might be to *not declare* a tax amnesty, which is again consistent with the initial beliefs. This result is in line with Bayer et al. (2015), which claimed tax amnesties have a self-fulfilling characteristic.

There is also a third possibility in this parameter region. Taxpayers may divide into tax evaders and honest taxpayers in such proportions that the government can become indifferent between declaring an amnesty or not. This division of taxpayers is possible when taxpayers' expectations set a particular probability to the occurrence of a tax amnesty. Since government is indifferent in this scenario, it can optimally choose this exact probability as its mixed strategy. This should constitute an equilibrium where amnesty probability is between zero and one.

Since we know the exact functional form of the benefit function of an amnesty, we can derive some intuition out of these results. Depending on the values that the set of parameters get, we are in three different region that results in three distinct set of equilibria. Let us analyze how the changes in parameter values would lead us move among the three regions we discussed above that give three distinct set of equilibria.

The benefit function $B(\phi)$, ceteris paribus, is increasing in w. Given the values of other parameters, the unique equilibrium would be the one with an amnesty for high enough w. A decrease in w would eventually lead to the region where there are multiple equilibria. A further decrease in the value of w would fall the model into the region where the unique equilibrium is with zero probability of amnesty. The effect of w is not continuous, in the sense that a small change in w may not effect the probability of an amnesty. However, the economy still jumps from an equilibrium with amnesty to a multiple equilibria region, and then to a region without an amnesty with high enough changes in w. This exercise suggests that the economies with higher values for w are more likely to experience tax amnesties. We can get the intuition behind this result from the magnitude of the evaded taxes. For a constant tax rate, the value of evaded taxes is higher with the same amount of delinquent taxpayers. Keeping the cost of amnesty constant, the government would have more incentives to declare an amnesty with higher values of total evaded taxes since it would mean that it can extract more revenue from an amnesty. This result will be one of the hypotheses we are going to test in our empirical analysis.

Assumption 1 gives us $\tau > p$. As long as we are in the parameter region that satisfies this assumption, the effect of τ on the set of equilibria is similar to the effect of w. The model suggests that the economies with higher τ are more likely to experience tax amnesties. Intuitively, for a given amount of hidden income, the value of evaded taxes is higher if the tax rate is higher. Then, this leads to an increase in the potential income of an amnesty and therefore an increase in the benefit of an amnesty. Moreover, notice that the cutoff value for the psychological cost of being an outlaw, $\bar{\epsilon}$ is higher with a higher tax rate. Then, the increase in tax rates increases the benefit of an amnesty further since it increases the ratio of evaders in the population. By accounting both effects, a government has more incentives to declare an amnesty if the tax rate is higher in the economy.

The probability of a successful audit at the end of the game, p, has the exact opposite effect of the tax rate. An increase in p makes an amnesty less attractive for the government via two different channel. First, a higher probability of successful auditing would mean that the government can collect more income through catching tax evaders. It certainly decreases the need for an amnesty. Second, the higher probability of being caught discourages the taxpayers from tax evasion in the first place. The less total evasion, the less amnesty revenue, and therefore less benefit from an amnesty.

Lastly, given the values of w, τ, and p; the set of equilibria also depends on the value of C_A. The higher the cost of amnesty, the less incentives there is to declare one for the government. Note that a change in C_A does not directly effect the tax evasion decision of the taxpayers. It may only change the equilibria via increasing the stakes for the government to declare an amnesty.

Before testing the combination of these effects by using a data from US state-level tax amnesties, we comment on some secondary yet economically meaningful results derived from our model. The next subsection is dedicated to these results.

Further Results

From the government's perspective, not all the equilibria are equally desirable. The Proposition 2 establishes this result.

Proposition 2. Assume $C_A \geq 1$. If a commitment technology exists, committing to not declaring an amnesty is optimal for the government.

Proof. See Appendix A. □

Remember that the government's total revenue with amnesty, $A(.)$, is decreasing in the taxpayers' expectation of an amnesty. If the government would be given an opportunity to commit to not declaring an amnesty, it would certainly exercise that option. However, without a proper commitment technology, not every government's promise is credible. For a government with low amnesty cost, although the initial commitment of not having an amnesty is optimal, declaring an amnesty might be optimal after taxpayers decide their income declarations. This points out a time inconsistency problem. It is also a motivation for a study on the relation between the government reputation and tax amnesties, since reputation can behave as a commitment device in repeated interactions. Chapter 2 of thesis will explore that dimension.

The economic role of a tax amnesty for the government in our model resembles certain pricing behavior of the firms that arise under certain assumptions. The following lemma establishes this resemblance.

Proposition 3. For a given value of ϕ, the tax amnesty is a price discrimination tool for the government.

Proof. Given Assumption 1, $0 < \bar{\epsilon}^* < 1$ and $p + \bar{\epsilon}^* > a^* > p$. Then, $\tau \geq a > p$ for any $\phi \in [0, 1]$, and the first inequality holds strictly if $\phi \neq 1$. □

Proposition 3 shows that government always wants to implement a tax amnesty. Intuitively, it is a result of better price discrimination. One can create parallels between our model and a model where a monopolist selling a good to costumers with different valuations of the good. Since taxpayers' preferences are heterogeneous on their willingness to evade, the price that they are willing to pay varies. Without an amnesty, government divides the taxpayers into two groups and charges two different prices, i.e. tw and pw. By implementing an amnesty, the government is able to divide the taxpayers into three groups in terms of their willingness to pay. It provides an intertemporal price discrimination as Cassone and Marchese (2000) suggested[5].

2.3 Empirical Analysis

The model suggests that the ratio $\frac{w(\tau-p)^2}{C_A}$ has a particular importance on observing tax amnesties in an economy. The set of equilibria moves from no amnesty with certainty

[5]Cassone and Marchese (2000) finds a similar result with a slightly different framework. The most important deviation from our framework is that they take tax amnesty as certainty.

to observing an amnesty with certainty while the ratio gets bigger. If the ratio is low enough, there won't be any amnesty. Similarly, if the ratio is high enough, the set of equilibria includes some equilibria with positive amnesty probability and perhaps amnesty with certainty. Intuitively, an economy is less likely to experience a tax amnesty if the benefit of a tax amnesty is relatively small when normalized with its potential cost. Briefly, the model suggests a state is more likely to experience a tax amnesty if the ratio $\frac{w(\tau-p)^2}{C_A}$ is bigger.

To test this, we can create a proxy for the non-linear function of parameters. Unfortunately, reliable data on the political cost of amnesties and the probability of detecting evaders are not available, at least not on the state level. Therefore, we constructed a proxy which includes per capita personal income and domestic tax burden of residents in a state.

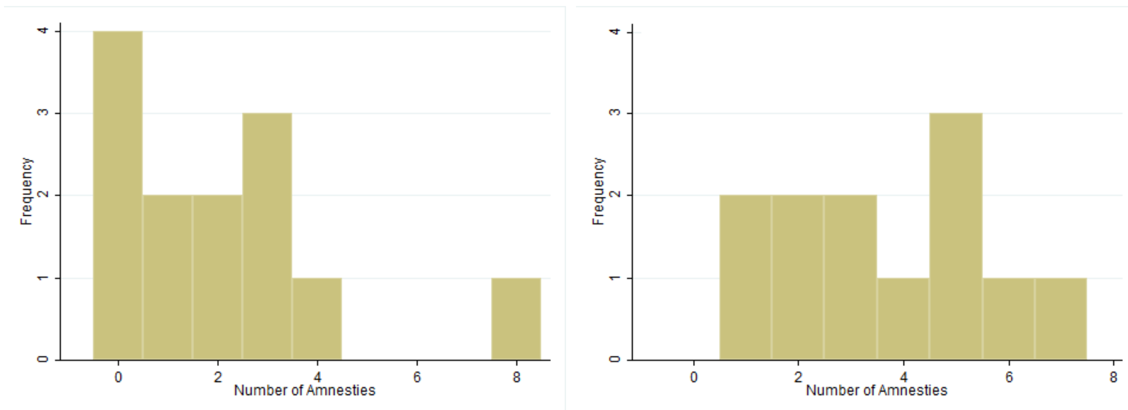
$$\frac{w(\tau-p)^2}{C_A} \approx w\tau^2 \approx (Personal\ Income\ Per\ Capita) \times (Domestic\ Tax\ Burden)^2 \quad (2.9)$$

When we match the data with our theoretical variables, we use personal income per capita in a US state as the data for the taxpayer income w. For the our variable t, we prefer to use tax burden instead of any tax rates. Tax burden represents the total taxes paid by a state's residents as a ratio of that state's net national product. There are many tax buckets that flows taxes from residents to the state governments, such as income taxes, corporate taxes, sales taxes. The tax rates of different tax types differ a lot from one state to another. In one state the most important tax might be the income tax while in the other it might be the sales tax. Therefore, selecting a particular tax rate data would create weighting problems. Tax burden is an attempt to aggregate these different tax buckets and we choose to use it in our analysis.

We order fifty US states from highest to lowest in terms of the proxy we created. Our theory predicts that amnesties are more likely in states which are high in this order. We compare the average number of amnesties in the highest quartile and the lowest quartile. As it is shown in Figure 2.1, it is less likely to see amnesties in the lowest quartile states. In the lowest quartile, there are several states which didn't declare any amnesty at all and the average number of amnesties per state is 2.08. In comparison, the states in the highest quartile declared at least one amnesty while averaging 3.67 amnesties.

Louisiana is an outlier in this database and requires an explanation. It is in the lowest quartile while being the state with the highest number of amnesties (see the state with eight amnesties in the lowest quartile in Figure 2.1). Our theory predicts that there are at least two other factors in an economy which affects the likelihood of an amnesty that we leave out in this analysis because of data availability. Political cost of an amnesty or audit efficiency

Figure 2.1: Histogram of the number of state amnesties in the highest and lowest quartiles



(a) Lowest Quartile **(b)** Highest Quartile

The lowest quartile represents the 12 states with the lowest values of (*Personal Income per Capita*) × (*Tax Burden*)^2. The highest quartile represents the 12 states with the highest values of the same statistic. Four of the five zero amnesty states are in the lowest quartile, while there is no zero amnesty state in the highest quartile. Lowest quartile states averaged 2.08 tax amnesties in our sample, while highest quartile states averaged 3.67.

might be very low in Louisiana and that might lead this state to implement amnesties more frequently. In fact, Louisiana its inefficiency in their general tax administration. It gets persistently low grades in Council of State Taxation reports on state tax administrations. Although their grading methodology does not include audit efficiency, it might point to an inefficiency in tax enforcement as well.

We made an OLS estimation by regressing *Number of Amnesties* on our proxy wt^2. We control for several other factors. We divide the states into three categories in terms of their political party tendencies. We computed the number of years a state was governed by a Republican Party affiliated governor. If this number is greater than 24, approximately two-third of the years, *Republican Dummy* takes the value one for that state. Republican Party is known for its low-tax, pro-business mentality. A state which is generally governed by Republican party might have lower tax rates, and therefore may have less tax amnesties according to our model. Alternatively, a governor of a state with a majority of Republican voters may face a small political cost when she declares an amnesty. If the number of years with a republican governor is between 15 and 24 for a state, we label that state as *Swing-state*, to represent the fact that it is generally a battleground for both parties. In a state which both political parties have similar chances to win elections, the government's might discount the future more, and therefore they may be more inclined to use amnesties to increase current revenues rather than maintaining government reputation.

We also control for a state's debt-to-GDP ratio. This variable clearly creates an endogeneity problem. A state with high debt-to-GDP ratio may need fast revenue collections into its budgets more frequently. Therefore, they might be more inclined to declare an amnesty. On the other hand, a tax amnesty raises extra revenues and may decrease the government's debt. One way to tackle with this issue might have been using the state debt-to-gdp ratios at the first year of our database. However, US Census database, which is the only database with state government finances, provides historical data only as back as 1992. Therefore, we used state debt in 1992 as our control variable.

One unplausible feature of our estimation is that the number of observations is limited. Since our analysis focuses on US states, we cannot deal with this problem under our model. Another concern might be that our dependent variable is count data. It cannot take a value less than zero. To provide a robustness check, we run a regression with negative binomial estimation, which is more suitable for a count data estimation.

The results of our linear estimation is supporting the hypothesis of our model. The states which have higher values of income and tax rates also have higher number of amnesties in our database, even after controlling for some other potential characteristics. Our results suggest that state debt is negatively correlated with number of amnesties, but as we discussed earlier this variable suffers from an obvious endogeneity problem. Hence, it is not meaningful to interpret its coefficient. Lastly, the political party leanings of states seem to be unimportant in their tax amnesty implementations. Since we are working in a count data environemnt, we also provide a robustness check by making a negative binomial estimation. Qualitative interpretation of the results is unchanged.

Apart from the interaction variable, income of a state also has a direct effect on the number of tax amnesties. Average income in a state might be correlated with many other characteristics and that may lead to an increase in amnesty numbers in that state. One possible channel can be through ability to hide income. We can argue that in a state with higher average personal income, individuals also have more means to hire more tax experts and to invest in tax evasion schemes. This might lead to a decrease in tax probability of getting caught. Our model suggest that when this probability decreases, the potential benefit of an amnesty increases. Therefore, higher average income might increase likelihood of a tax amnesty in a state independent of the tax rates. We also made a separate OLS estimation in which we omit the individual control variables of personal income and domestic tax burden. The coefficient of our sufficient statistics is still positive and significant.

Table 2.1: Estimations to test the hypothesis of the 1-period model

VARIABLES	OLS Number of Amnesties	Negative Binomial Number of Amnesties	OLS 2 Number of Amnesties
$PersonalIncome \times (DTB)^2$	0.00136**	0.000403**	0.00121***
	(0.000622)	(0.000201)	(0.000213)
$PersonalIncome$	0.0490***	0.0198***	
	(0.0175)	(0.00680)	
$(DTB)^2$	-0.0289	-0.00793	
	(0.0251)	(0.00850)	
State Debt-to-GDP	-0.134**	-0.0660**	-0.0851
	(0.0591)	(0.0265)	(-0.1901)
Republican Dummy	-0.318	-0.133	0.144
	(0.622)	(0.272)	(-0.8282)
$Swing - state\ Dummy$	0.908	0.355	1.380
	(0.626)	(0.231)	(0.2576)
Observations	50	50	50
R-squared	0.773		0.752

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

DTB: Domestic Tax Burden. All variables, except dummies, are in averages on years between 1982-2018. Personal Income is in per capita terms and in thousands. Domestic Tax Burden is in percent terms, i.e. if tax burden is 10 percent for state i, then Domestic Tax Burden for state i is equal to 10. State Debt-to-GDP ratio represents the state debt in 1992. Republican Dummy is 1, if the state has been governed by a Republican for more than 23 years during 1982-2018. Swing-state Dummy takes the value 1, if the has been governed by a Republican less than 24 years, but more than 14 years. Dependent variable is Number of Amnesties. It represents the number of amnesties a state implemented during the period 1982-2018.

2.4 Concluding Remarks

We build a model of tax amnesties that provides hypotheses on the occurrence of tax amnesties in an economy. It shows that an economy with higher personal income, higher tax rates, lower audit success, and lower cost of declaring amnesty is more likely to experience an amnesty. We then test our models hypotheses with an empirical analysis that provide some evidence on the validity of our model's predictions.

Another important feature arises from our model is the role of commitment. A government would like to give signals, to commit if possible, not to implement any amnesty in the future. However, after private agents make their decisions on tax evasions, it might be optimal for the government to declare an amnesty to raise extra revenues. We can assume that such commitment technology does not exist. This aspect creates a room for the study of the government reputation in a dynamic tax amnesty environment since we know that reputation can play the role of commitment in repeated interactions.

Our model can be interpreted as a model of sales taxes as well, if we interpret w as the income from selling a product and t as the sales tax rate. However, some tax types such as capital income tax and corporate income tax have different dynamics that we cannot capture without allowing savings. To check the validity of our hypotheses and to discover new economic variables relevant to tax amnesty decisions we need to study theoretical setups that can allow for other types of tax amnesties.

There are possible paths that future research can follow. Considering the increasing popularity of tax amnesties on uncovering assets in tax havens, a study focusing on this type of amnesties may provide valuable insights. The tax amnesty dynamics in a game of hiding assets might be significantly different. Another important step might be to study the effect of tax amnesties on wealth inequality.

2.5 Appendix

Appendix A. Proofs

A1. Proof of Proposition 1

Proof. 1. Revenues with amnesty: Notice that the government's total revenue can be written as

$$
\begin{aligned}
A =& R_T + R_A + R_P \\
=& \int_{\bar{\epsilon}^*}^{1} \tau w d\epsilon + \int_{\frac{\bar{\epsilon}^*}{2}}^{\bar{\epsilon}^*} a^* w d\epsilon + \int_0^{\frac{\bar{\epsilon}^*}{2}} p w d\epsilon \\
=& (1-\bar{\epsilon}^*) w\tau + \left(\bar{\epsilon}^* - \frac{\bar{\epsilon}^*}{2}\right) w \left(\frac{\bar{\epsilon}}{2} + p\right) + \frac{\bar{\epsilon}^*}{2} wp \\
=& (1-\bar{\epsilon}^*) w\tau + \frac{\bar{\epsilon}^{*2}}{4} w + \bar{\epsilon}^* wp \\
=& \left(1 - \frac{2(\tau - p)}{2-\phi}\right) wt + \frac{\left(\frac{2(\tau-p)}{2-\phi}\right)^2}{4} w + \frac{2(\tau-p)}{2-\phi} wp \\
=& w\tau - w(\tau - p)\left(\frac{2(\tau-p)}{2-\phi}\right) + w\frac{4(\tau-p)^2}{(2-\phi)^2} \\
=& w\tau - w(\tau-p)^2 \left(\frac{3-2\phi}{(2-\phi)^2}\right).
\end{aligned}
$$

2. Revenues without amnesty:

$$
\begin{aligned}
R =& R_T + R_P \\
=& \int_{\bar{\epsilon}^*}^{1} \tau w d\epsilon + \int_0^{\bar{\epsilon}^*} p w d\epsilon \\
=& (1-\bar{\epsilon}^*) w\tau + \bar{\epsilon}^* wp \\
=& \left(1 - \frac{2(\tau-p)}{2-\phi}\right) w\tau + \frac{2(\tau-p)}{2-\phi} wp \\
=& w\tau - w(\tau-p)^2 \frac{2}{2-\phi}.
\end{aligned}
$$

3. Benefit of amnesty:

$$
\begin{aligned}
B(\phi) =& A(\phi) - R(\phi) \\
=& \frac{w(\tau-p)^2}{(2-\phi)^2}.
\end{aligned}
$$

Lastly, it is easy to see that $B(.)$ is increasing and $R(.)$ is decreasing on the interval $\phi \in [0,1]$.

Let us also check $A(.)$.

$$\begin{aligned}\frac{\partial A}{\partial \phi} &= -\, w(\tau - p)^2 \frac{(-2)(2-\phi)^2 + 2(2-\phi)(3-2\phi)}{(2-\phi)^4} \\ &= -\, 2w(\tau - p)^2 \frac{2 - 3\phi + \phi^2}{(2-\phi)^4} \\ &= -\, 2w(\tau - p)^2 \frac{(\phi - 1)(\phi - 2)}{(2-\phi)^4} \leq 0, \text{ when } \quad \phi \in [0,1].\end{aligned}$$

□

A2. Proof of Proposition 2

Proof. We need to show that the outcome $(\phi^*, x^*) = (0, 0)$ is weakly better than any equilibrium outcome without commitment for the government. First, if $B(1) < C_A$ the unique equilibrium is the same with or without commitment.

Second, assume $B(0) > C_A$. Then, without commitment, government gets $A(1) - C_A$. With commitment, government gets $R(0)$. We know $R(0) \geq A(1) - C_A$.

Lastly, assume $B(0) \leq C_A \leq B(1)$. Again from $R(0) \geq A(1) - C_A$ we know that $(\phi_1^*, x_1^*) = (0, 0)$ is better equilibrium than $(\phi_2^*, x_2^*) = (1, 1)$ for government revenues. Now, let's focus on the mixed strategy equilibrium. Note that in this equilibrium, government needs to be indifferent between declaring an amnesty or not. Therefore,

$$A(\phi_3^*) - C_A = R(\phi_3^*).$$

The expected payoff of government under the mixed strategy equilibrium is

$$\phi_3^*[A(\phi_3^*) - C_A] + (1 - \phi_3^*)R(\phi_3^*) = R(\phi_3^*) < R(0).$$

Hence, government would like to commit in order to secure $(\phi^*, x^*) = (0, 0)$ equilibrium. □

Chapter 3

Repeated Tax Amnesties

The rest of the paper is structured as follows. In Section 3.1, we introduce our infinite horizon framework. In Section 3.2, we derive results from the stage game which exhibits similar characteristics to the 1-period model of Chapter 2. Section 3.3 makes a Markov-perfect analysis of the infinite horizon model. Section 3.4 discusses the equilibrium results. Section 3.6 introduces an extension of our baseline model to introduce the notion of recessions.

3.1 Theoretical Framework

Consider the following infinite horizon game. Time is discrete. In each period, a government which can be of type *no-amnesty* or *opportunistic*, denoted by G_N and G_O respectively, faces a continuum of short-lived taxpayers. Taxpayers can hide their income from the government. In that case, the government can catch the evaders with a certain probability and implement a monetary punishment. Between the evasion and government audit, the government can provide a second opportunity to declare hidden incomes of taxpayers.

The government is a long-run player. Its type is not observable by taxpayers. The *no-amnesty* type is a behavioral type that never implements tax amnesties. On the other hand, the *opportunistic* type may use amnesties when they are optimal. The government type can change at the end of a period with the following publicly known Markov transition matrix.[1]

	G_N	G_O
G_N	π_N	$1-\pi_N$
G_O	$1-\pi_O$	π_O

Since the government type is private, taxpayers will use their best prediction given the information they have. Taxpayers' optimal behavior is a function of their beliefs on the probability

[1] With the introduction of such a reputation mechanism, our model is similar to Ball (1995) and Phelan (2005), which are building on seminal papers such as Kreps and Wilson (1982), Milgrom and Roberts (1982), Barro and Gordon (1983) and many others.

that the government is *opportunistic*. Let ρ_t represents the probability that the government is the *opportunistic* type from the point of view of taxpayers at the beginning of period t. Notice that ρ_t represents the reputation of government in terms of its willingness to offer a tax amnesty.

The government type G_N does not have an actual decision since it is a behavioral type. Therefore, our interest will be on the game between taxpayers and the *opportunistic* type. The timing of a period is as follows:

1. Given the reputation ρ_t, each taxpayer i forms her belief on the probability of seeing an amnesty in this period, which we denote as $\phi_{i,t} \in [0, \rho_t]$.

2. Each taxpayer i draw an income $y_{i,t}$ and a preference shock $\epsilon_{i.t}$, which are private information. The income $y_{i,t}$ can be either 0 or w with equal probabilities. The preference shock is distributed uniformly, i.e. $\epsilon_{i.t} \sim U[0, 1]$. Both distributions are public information.

3. Taxpayers declare an income to the government $y^d_{i.t}$ and pay taxes with an income tax rate τ.

4. G_O decides whether to declare an amnesty, $x_t \in [0, 1]$. If the government declares and amnesty, it pays a fixed cost C_A.

5. If there is an amnesty, G_O sets a special income tax rate for the amnesty program which is denoted by a_t.

6. Taxpayers decide how much to declare during the amnesty, $y^a_{i.t}$ and pay taxes based on the amnesty tax rate, $a_t y^a_{i.t}$.

7. At the end of the period, the government can catch the the agents who still hide income with probability p, and confiscate all their income. The audit success probability p is exogenous.

8. Taxpayers update their beliefs on government type at the end of the period.

Our approach here is to add government reputation to the model introduced in Chapter 2. We already have a 1-period model in Chapter 2 that captures the essence of a tax amnesty game between taxpayers and the government. For the purpose of this paper, we use that model as a building stone for our infinitely repeated game. To avoid repetition in the thesis, we will go over the main ingredients of the model. However, we will skip some details that already overlap with the 1-period model of the previous chapter.

We assume that the taxpayers are rational. All taxpayers have the same risk-neutral utility function. The payoff of a taxpayer i who draw income y and preference ϵ_i in period t is

$$\begin{aligned} u_i(y^d_{i,t}, y^a_{i,t}) = y - &\tau y^d_{i,t} \\ &- \phi_t[a y^a_{i,t} + \underbrace{p(y - y^d_{i,t} - y^a_{i,t}) + \epsilon_i(y - y^d_{i,t} - y^a_{i,t})}_{\text{Cost of hiding income after amnesty}}] \\ &- (1-\phi_t)\underbrace{[p(y - y^d_{i,t}) + \epsilon_i(y - y^d_{i,t})]}_{\text{Cost of being an evader}}. \end{aligned}$$

The payoff is simply the income minus expected cost. If the taxpayer decides to declare any income at the beginning of the period, it has to pay income tax with the fixed rate t. The taxpayer beliefs the probability of seeing an amnesty is ϕ_t . If there is an amnesty, she can participate in the amnesty by declaring an income and pay taxes with the special amnesty tax rate a. If she still hides income after an amnesty, she suffers a psychological cost of being an outlaw denoted with $\epsilon_{i.t}$. Also, the government catches her with probability p_t and confiscates all the evaded income. The taxpayers beliefs that with probability $1 - \pi_t$, there won't be an amnesty. The cost of hiding income in that case is again the psychological cost plus the expected fine that can be paid to the government.

Government type G_O tries to maximize the total income from taxes, from evasion penalties, and in case of declaring one, from tax amnesty. For convenience, we assume that the *opportunistic* government only maximizes revenues of its own type. Therefore, when it is discounting the future, it takes the probability of survival into account. The period revenue of the government can be divided into three categories, revenues from initial declarations, revenues from the amnesty program and revenues from auditing.

$$\textit{Revenues with amnesty: } A(\phi_t, a_t) = \int \tau y^d_{i.t} di + \int a_t y^a_{i.t} di + \int p(w - y^d_{i.t} - y^a_{i.t}) di \tag{3.1}$$

$$\textit{Revenues without amnesty: } R(\phi_t) = \int \tau y^d_{i.t} di + \int p(w - y^d_{i.t}) di \tag{3.2}$$

Assumption 1, introduced in Chapter 2, continues to ensure the existence of an interior equilibrium where there are positive masses of honest taxpayers, tax evaders and, in case of an amnesty, amnesty participants. We first analyze the economic environment as a simple, one-period model. Then, we add the time dimension to study the dynamic aspect of tax amnesties. For this purpose, our theoretical analysis will be divided into two sections: stage game analysis and Markov-perfect analysis.

3.2 Stage Game Analysis

Consider the stage game played by taxpayers and government which corresponds to one period of our theoretical framework. It is useful to compare the stage game we have here with the model we analyzed in Chapter 2. The only difference in this paper is the addition of reputation. For a 1-period game, the reputation is a parameter that is publicly known by all the players.

The reputation of the government restricts the set of beliefs that taxpayers can rationally form at the beginning of the game. More precisely, without introducing a reputation mechanism, the taxpayers have the freedom to assign any value on the interval $[0, 1]$ to the probability of seeing an amnesty. However, the theoretical framework we introduced in this paper restricts the interval of expectations that taxpayers can form at the beginning of a period. If the probability of the government being *opportunistic* is $\rho \in [0, 1]$, rational taxpayers would assign a probability on the interval $[0, \rho]$ to the probability of an amnesty. [2]

The optimal decisions other than the governments' amnesty declaration decision can be written as a function of the taxpayers' initial belief about the amnesty probability. Then, we can write the government's revenue as the sum of the initial tax collection, the revenue generated from audits, and the revenue from the amnesty program if the government decides to implement it.

Proposition 4. If taxpayers maximize their utility given an initial belief of an amnesty ϕ,

$$\text{Current revenue with amnesty, } A(\phi) = w\tau - w(\tau - p)^2 \left(\frac{3 - 2\phi}{(2 - \phi)^2} \right), \quad \text{is decreasing in } \phi.$$
$$\text{Current revenue without amnesty, } R(\phi) = w\tau - w(\tau - p)^2 \frac{2}{2 - \phi}, \quad \text{is decreasing in } \phi.$$
$$\text{Benefit of an amnesty, } B(\phi) = A(\phi) - R(\phi) = \frac{w(\tau - p)^2}{(2 - \phi)^2}, \quad \text{is increasing in } \phi.$$

Proof. See Appendix A. □

The government type G_O declares an amnesty if

$$B(\phi) \geq C_A. \tag{3.3}$$

The initial reputation of the government, ρ, pins down the set of equilibria. There are three regions with three different set of equilibria.

$\boldsymbol{B(\rho) < C_A}$**:** If this condition holds, then independent of the initial belief taxpayers form, the benefit of an amnesty is always lower than the cost of it for the government. The rational expectations dictate a unique equilibrium such that taxpayers do not expect an amnesty at all, and the *opportunistic* government does not declare an amnesty:

$$(\phi^*, x^*) = (0, 0).$$

[2]This is the only difference that government reputation brings to a 1-period model. To avoid repetition, we will jump into the equilibria of the stage game, since the optimization problems are identical to Chapter 2. Nonetheless, we dedicate Appendix D to the detailed analysis of the stage game.

$B(0) > C_A$: If this condition holds, then there is always enough revenue benefit to overpass the cost of an amnesty. In this case, the *opportunistic* government declares an amnesty with certainty at the unique equilibrium.

$$(\phi^*, x^*) = (\rho, 1).$$

$B(0) \leq C_A \leq B(\rho)$: In this case, we have multiple self-fulfilling equilibria.

$$(\phi_1^*, x_1^*) = (0, 0), \quad (\phi_2^*, x_2^*) = (\rho, 1), \quad (\phi_3^*, x_3^*) = \left(\frac{2\sqrt{2C_A} - 2(\tau - p)}{\sqrt{2C_A}}, \frac{2\sqrt{2C_A} - 2(\tau - p)}{\rho\sqrt{2C_A}} \right)$$

If taxpayers expect an amnesty, they evade more. The high evasion leads to an amnesty since it increases the revenue benefit of an amnesty. If taxpayers do not expect any amnesty, then less taxpayers hide their income. This makes the benefit of an amnesty lower than the fixed cost. Finally, there is also a middle scenario where taxpayers' expectation of an amnesty results in a particular amount of evasion that makes the *opportunistic* government to be indifferent between declaring a tax amnesty or not. This case creates a mixed-strategy equilibrium.

From the government's perspective, not all the equilibria is equally desirable. The Proposition 5 establishes this result.

Proposition 5. Assume $C_A \geq A(\rho) - R(0)$. If a commitment technology exists, committing to not declaring an amnesty is optimal for the government.

Proof. See Appendix A. □

The government's total revenue with amnesty, $A(.)$, is decreasing in the taxpayers' expectation of an amnesty, so the result given in Proposition 5 holds only for high enough ρ. Therefore, if a government with a bad reputation would have a commitment technology, it would use it to decrease evasion.

The set of equilibria, equilibrium revenue of the government and the optimality of commitment depend on the reputation ρ. For lower values of ρ, which can be interpreted as a high reputation, the set of equilibria moves to the case of *no amnesty*. Note that the government revenue at the *no amnesty* equilibrium is equal to the revenue of government under commitment. Even when the outcome of the game is with a tax amnesty, the government revenue is higher for lower values of ρ, since $A(.)$ is a decreasing function. Although we can establish the importance of reputation with a stage game analysis, it is only a parameter for this setup. However, reputation should be an endogenous object which evolves with the observable government actions. Therefore, a dynamic analysis is necessary to fully understand the role of reputation on tax amnesty occurrence.

3.3 Markov Perfect Analysis

In this section, we discuss the dynamic model. We restrict our attention to Markov Perfect equilibria. We first define the equilibrium. Then, we narrow our focus by imposing some restrictions on parameters to eliminate some trivial equilibria. Then, our equilibrium analysis is divided into two subsections. Firstly, we introduce a Markov-perfect equilibrium, Its purpose is to explain and visualize the expectation trap mechanism in a simple way. It also shows the existence of Markov-perfect equilibria in our setup. Secondly, we show that the expectation trap mechanism exists in every Markov perfect equilibria under the restrictions we impose on parameters.

In the dynamic setup, an the *opportunistic* government's objective is to maximize the sum of discounted revenues. The following is the *opportunistic* government's problem.

$$V(\rho) = \max_{x\in[0,1]} \quad x\left[A(\phi) - C_A + \beta V(\pi_O)\right] \tag{3.4}$$

$$+ (1-x)\left[R(\phi) + \beta V(\rho')\right] \tag{3.5}$$

where $A(\phi)$ and $R(\phi)$ represent government revenues with and without amnesties, respectively, when taxpayers expect an amnesty with probability ϕ. If the *opportunistic* government declares an amnesty in a given period, it reveals its type. Then, the probability that the government is *opportunistic* is π_O in the next period. Belief update ρ' in case of no amnesty is given by a law of motion of beliefs, $\Gamma(\rho)$.

The government's action in a period effects the beliefs of taxpayers in the future periods. It might be optimal for a government to manipulate beliefs of taxpayers to increase future revenues. This gives the government another tool to maximize its revenues, which was lacking in the stage game. Note that we didn't put the optimal amnesty tax rate as a control variable for the government's problem. The reason is that once the government decides to declare an amnesty, the amnesty tax rate is immediately given by equation 3.30. The following is the recursive equilibrium definition of the repeated game.

Definition 1. Given ρ, *MPE* is a pair of $\{\phi^*(\rho), x^*(\rho)\}$ and a law of motion $\Gamma(\rho)$ such that

- Given $\phi^*(\rho)$, taxpayers' evasion and amnesty participation decisions are optimal.
- $x^*(\rho)$ solves the *opportunistic* government's problem, given in equation 3.5.
- Initial beliefs are consistent with government's decision; i.e. $\phi^*(\rho) = \rho x^*(\rho)$.
- At the end of the period t, beliefs are updated rationally, i.e.

$$\rho' = \Gamma(\rho) = (\pi_O)\left(\frac{\rho(1-\frac{\phi}{\rho})}{(1-\rho)+\rho(1-\frac{\phi}{\rho})}\right) + (1-\pi_N)\left(\frac{(1-\rho)}{(1-\rho)+\rho(1-\frac{\phi}{\rho})}\right)$$

The law of motion of the beliefs is given by Bayesian updating. Given the taxpayers' belief and the belief updating function, government's optimal decision is

$$x^*(\rho) = \begin{cases} 1 & B(\phi^*(\rho)) > \beta(V(\rho') - V(\pi_O)) + C_A \\ \\ [0,1] & B(\phi^*(\rho)) = \beta(V(\rho') - V(\pi_O)) + C_A \\ \\ 0 & B(\phi^*(\rho)) < \beta(V(\rho') - V(\pi_O)) + C_A \end{cases} . \tag{3.6}$$

There are multiple equilibria in the stage game. It is natural that we have many equilibria in the repeated game as well, even when focusing on Markov-Perfect equilibria. We will restrict our attention to the region of parameter space which satisfies the following condition.

$$A(1 - \pi_N) - R(0) > C_A. \tag{3.7}$$

There are some Markov-Perfect equilibria where reputation does not play an important role. Condition 3.7 makes sure that reputational cost is the deciding factor in a tax amnesty decision of the government in our focus of reputation set $[1 - \pi_N, \pi_O]$. Notice that when π_N converges to 1, condition 3.7 converges to $B(0) > C_A$. It states that declaring tax amnesty is always beneficial if there is no reputational cost. From now on, we will assume that condition 3.7 holds.

A Markov-Perfect Equilibrium with The Expectation Trap

A Markov Perfect Equilibrium is a pair of state-dependent best response functions which defines strategy pairs $(\phi^*(\rho), x^*(\rho))$ for each $\rho \in [1 - \pi_N, \pi_O]$. In order to characterize an equilibrium, we need to understand what are the optimal strategies for each player in every reputation level ρ. In this subsection, we will introduce an equilibrium in pure strategies which features the main mechanism we try to emphasize in this paper.

In a pure strategy equilibrium, in any given state ρ, the pair of actions which constitutes an equilibrium should satisfy one of the following:

- $(\phi^*(\rho), x^*(\rho)) = (0, 0)$ (We will call this *no amnesty*),
- $(\phi^*(\rho), x^*(\rho)) = (\rho, 1)$ (We will call this *declaring amnesty*).

The former set of strategies specifies that taxpayers do not expect an amnesty at all and the *opportunistic* government does not declare an amnesty. The latter set of strategies specifies that taxpayers expect that an opportunistic government would for sure declare an amnesty and the *opportunistic* government indeed declares it with certainty.

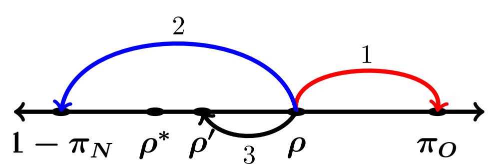
Before showing an equilibrium, we will give a set of results on Bayesian updating which will help us to understand the dynamics of a proposed pure strategy equilibrium. As is shown in Figure 3.1, in a given reputation level ρ:

1. If an amnesty occurs, the opportunistic government reveals its type. Therefore, next period's reputation will be $\rho' = \pi_O$.

2. If the taxpayers expect an amnesty with probability ρ, but do not see an amnesty in that period, then they will think that government must be the *no-amnesty* one. Therefore, the next period's reputation will be $\rho' = 1 - \pi_N$.

3. If the taxpayers do not expect an amnesty and they do not see an amnesty in that period, they will update their beliefs with Bayes Rule: $\rho' = \Gamma(\rho, 0) = \rho\pi_O + (1-\rho)(1-\pi_N)$.

Figure 3.1: Three possible belief update under pure strategies.



The belief update under different scenarios. The point ρ^* is the fixed point that the belief converges with the infinite iterations of belief update 3. The exact form of it is given in Lemma 1.

Lemma 1. The Bayesian belief update has the following characteristics:

1. There exists a unique fixed point, $\rho^* = \Gamma(\rho^*, 0)$, which is $\rho^* = \dfrac{1-\pi_N}{(1-\pi_O)+(1-\pi_N)}$.

2. $\forall \rho < \rho^*$, $\rho^* > \Gamma(\rho, 0) > \rho$.

3. $\forall \rho > \rho^*$, $\rho^* < \Gamma(\rho, 0) < \rho$.

Proof. See Appendix A. □

Lemma 1 suggests that there is a fixed point towards which the reputation converges to when taxpayers do not expect an amnesty and there is no amnesty realization. For the rest of the analysis, we will assume that $\rho^* \in (1-\pi_N, \pi_O)$. Since π_N and π_O are very close to 1, the assumption is reasonable. Now, we can introduce a pure strategy equilibrium.

Proposition 6. Suppose the following conditions hold:

$$B(0) - C_A \leq \beta[A(\rho^*) - A(\pi_O)] \tag{3.8}$$

$$B(\rho^*) - C_A > \frac{\beta}{1-\beta}[R(0) - A(\pi_O) + C_A] \tag{3.9}$$

Then, there exists an $\underline{R} \in (1 - \pi_N, \rho^*)$ such that the following set of Markov strategies constitutes an equilibrium:

$$(\phi^*(\rho), x^*(\rho)) = (0, 0), \quad \forall \rho < \underline{R}$$
$$(\phi^*(\rho), x^*(\rho)) = (\rho, 1), \quad \forall \rho \geq \underline{R}.$$

Proof. See Appendix A. □

Proposition 6 introduces an equilibrium. The Markov strategies which constitute this equilibrium are depicted in Figure 3.2. In this particular equilibrium, the taxpayers do not expect an amnesty if the probability that the government type being *opportunistic* is below a certain threshold $\underline{R}$. Low probability of facing an *opportunistic type* can be interpreted as high reputation. When reputation is high, even if the taxpayers would believe that an opportunistic government declares an amnesty, the overall probability of seeing an amnesty is still small. Therefore, the taxpayers would evade little and there would be small potential benefit from an amnesty. Also, remember that the government reputation jumps to its lowest level when the government declares an amnesty. The government would perceive that burning reputation is more costly when reputation is high. Hence, there would not be enough incentives for an *opportunistic* government to declare an amnesty. Then, the only rational belief is to assign zero probability to tax amnesty occurrence. Consider a period where the probability of facing an *opportunistic* government is higher than the threshold $\underline{R}$. In this case, the equilibrium outcome delivers an amnesty if the government type is *opportunistic*. If taxpayers believe that an *opportunistic* government would declare an amnesty with certainty, the overall probability of facing an amnesty is high. This leads to high tax evasion by taxpayers. The high tax evasion creates enough incentives to an *opportunistic* government to implement an amnesty.

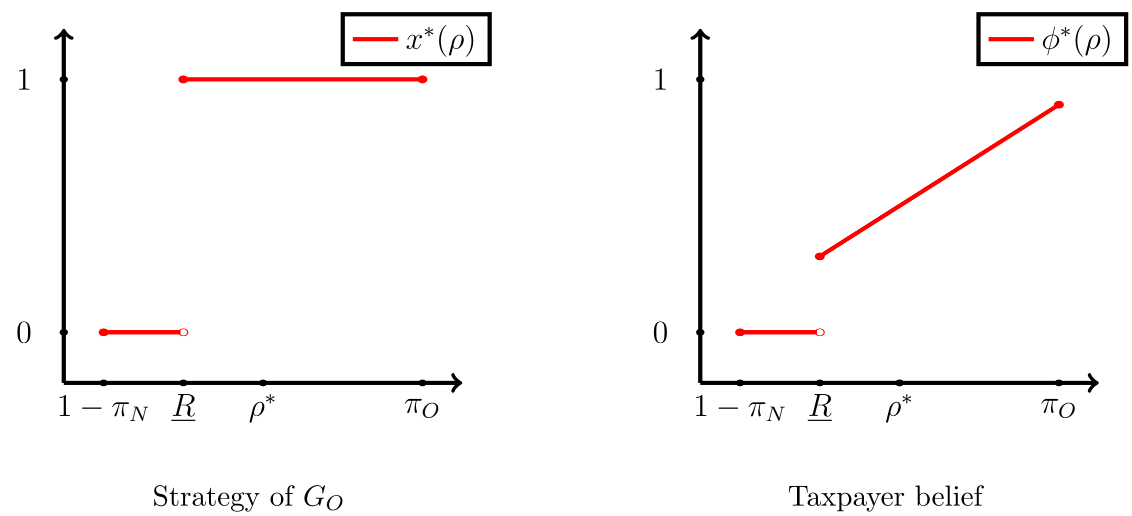
The equations 3.8 and 3.9 are not necessary conditions for the existence of an equilibrium.[3] However, they are key to some of the dynamics that this equilibrium delivers. The condition 3.9 makes sure that ρ^* is close enough to π_O, so the switching threshold $\underline{R}$ can be in the interval $(1 - \pi_N, \rho^*)$. One of the plausible features of the equilibrium is that we can observe an amnesty even after some consecutive periods without an amnesty. It provides dynamics which allow for a switch from sequence of no amnesty outcomes to a sequence of amnesty outcomes. We need $\underline{R}$ to be in the interval $(1 - \pi_N, \rho^*)$ to have these dynamics. We also need that the government is patient enough to not implement an amnesty when it knows that the reputation will pass into the interval $[\underline{R}, \rho^*]$ in the next period. The condition 3.8 makes sure that if taxpayers do not expect an amnesty and therefore evade little, the government is patient enough to wait even if the reputation will enter the region $[\underline{R}, \rho^*]$ in the next period.[4]

[3] We show existence of an equilibrium with a less restrictive parameter conditions in Appendix B.

[4] One might wonder whether the parameter set which satisfies both conditions is non-empty. The following numerical example satisfies the conditions 3.8 and 3.9 as well as other assumptions we make up to this point:

Consider a game that starts with an initial value $\rho_0 < \underline{R}$. In the first period, the taxpayers do not expect an amnesty and the government does not implement an amnesty. This period does not provide new information on the government type. However, the government reputation will be updated for the next period since the public knows that government type might change through Markov transition. Consider that the game is continued to be played out like this. Every period is another iteration of the belief update with Markov transition probabilities. Lemma 1 and the fact that $\underline{R} < \rho^*$ imply that each iteration will increase the probability that the government is *opportunistic*. They also imply that after a finite number of periods, the government reputation diminishes so much that the probability of being the opportunistic type will be greater than $\underline{R}$. Then, the taxpayers expect an opportunistic government to declare an amnesty and evade high enough that it will be optimal for an *opportunistic* government to declare an amnesty. In such a period, if the government is *opportunistic*, it declares an amnesty and reveals its type. Then, the probability of government type being *opportunistic* becomes π_O in the next period. It means that the government reputation is again in the region where taxpayers expect an amnesty from an *opportunistic* government. Since the probability that the government is *opportunistic* also reaches its highest level in this period, the probability of an amnesty reaches its maximum possible value. If the government type persists, the outcome of the period is again occurrence of an amnesty. In fact, as long as the government type persists, the game will continue to provide tax amnesties in every period.

Figure 3.2: Strategies of The Markov Perfect Equilibrium



The strategy function of the *opportunistic* government and belief function of taxpayers in the Markov-perfect equilibrium introduced in Proposition 6. Note that the taxpayers are outside observers. From their point of view, the probability of amnesty reaches its maximum at the worst reputation level, i.e. π_O.

This equilibrium can explain the main observations we made on the tax amnesties. The

$\beta = 1/2, w = 9, \tau = 2/3, p = 1/3, C_A = 1/8, \pi_O = 0.99, \pi_N = 0.99$.

probability of amnesty reaches its highest value at the worst reputation. The reputation jumps to its worst level after a realization of tax amnesty. Intuitively, a government which uses amnesties as a fiscal tool might be seen as soft in its willingness to enforce tax law. Such an interpretation by taxpayers would lead them to expect another amnesty in the future. It leads to higher tax evasion and results in an increase in the probability of another tax amnesty. In this way, one amnesty triggers a sequence of amnesties through a reputation channel. We call this reputation mechanism an *expectation trap*. Moreover, the equilibrium delivers shifts from a sequence of *no amnesty* outcomes to a sequence of *declaring amnesty* outcomes. A game which starts with any reputation level will eventually enter a zone where an *opportunistic* government would declare amnesty with certainty. It brings an explanation on the regime shifts we pointed out in Figure 1.2.

The Expectation Trap in All Markov Perfect Equilibria

In this subsection, we show that for an outside observer, the probability of amnesty reaches its maximum in the period after an amnesty realization. This result holds under any initial reputation and any Markov-perfect equilibrium. Hence, the expectation trap mechanism exists in any equilibrium. We only need to assume condition 3.7 for this result. Therefore, this subsection shows the strength and robustness of the expectation traps in a tax amnesty environment.

Note that in any MPE, the reputation jumps to π_O after a period with amnesty. Therefore, the relevant continuation game after an amnesty should start with an initial reputation π_O. For convenience, we denote the probability of an amnesty in period t of the relevant continuation game as ϕ_t, i.e. $\phi_0 = \phi(\pi_O)$. Let us start with a simple but crucial lemma.

Lemma 2. Consider the relevant continuation game between taxpayers and the *opportunistic government* after a tax amnesty realization, i.e. $\rho_0 = \pi_O$. In any MPE, for all sequences $\{\rho_k\}_{k=0}^{\infty}$ with positive probability, there exists a period $t \in \mathbb{Z}_{\geq 0}$ such that $\phi_t > 0$.

Proof. See Appendix A. □

Lemma 2 states that in the relevant continuation game after an amnesty, an *opportunistic* government would put a positive probability on declaring amnesty after a finite period of time. Intuitively, an opposite scenario would lead to an inconsistency between taxpayers' belief and government strategies. Since we assumed condition 3.7 holds, the benefit of amnesty is higher than the fixed cost under any taxpayer belief. The only reason an *opportunistic* government may not declare an amnesty in any period would be the reputational cost induced in continuation values. However, the reputational cost disappears if taxpayers believe that there won't be any amnesty in the relevant continuation game after an amnesty. This should lead to a tax amnesty implementation of an *opportunistic* government, even in the relevant continuation game. Since taxpayers form rational beliefs, they should assign positive probability of amnesty at some point in the relevant continuation game.

In a game in which players foresee that there will be a positive probability of a tax amnesty in finite period of time, they form beliefs and choose strategies in earlier periods based on this information. By exploiting this idea, Lemma 3 introduces a result for the first period after a tax amnesty realization.

Lemma 3. Consider the relevant continuation game between taxpayers and the *opportunistic government* after a tax amnesty realization, i.e. $\rho_0 = \pi_O$. In any MPE, $\phi_0 > 0$.

Proof. See Appendix A. □

Lemma 3 shows that there is a positive probability of amnesty in the first period after an amnesty realization. We already established that there will be a period with positive probability of amnesty occurrence in the relevant continuation game, say period t. Higher probability of amnesty means higher evasion and therefore lower total revenue for the government. A forward-looking *opportunistic* government foresees this outcome. In period $t-1$, an *opportunistic* government essentially decides whether to wait one more period to declare an amnesty. It foresees that the next period will include high enough evasion to incentivize an amnesty. However, high evasion delivers low period revenues in total. The next period will include high amount of evasion and then the government will reveal its type by declaring amnesty anyway. Therefore, the government does not have incentives to wait and improve its reputation. If the probability of amnesty is zero in the period after an amnesty, the government is tempted to declare an amnesty in period $t-1$. With the same logic, a forward-looking government would also put a positive probability of amnesty in period $t-2$ and in period $t-3$ and so on. With enough iterations of this argument, we can see that an *opportunistic* government always chooses a positive probability of amnesty in the first period after an amnesty.

Tax amnesty probability is positive in the first period of the relevant continuation game. However, if this probability is very small or increasing after the first period, it is still difficult to argue that a MPE features an expectation trap. To eliminate this possibility, we introduce two results in Lemma 4 and Lemma 5.

Lemma 4. In any MPE, $\phi(\pi_O) > 1 - \pi_N \geq \phi(1 - \pi_N)$.

Proof. See Appendix A. □

The second inequality is obvious. The first inequality is a direct result of condition 3.7. Consider the period right after an amnesty realization. If taxpayers would believed that the probability of an amnesty were smaller than $1-\pi_N$, there would not be enough punishment to deter an amnesty declaration. Then, an *opportunistic* government would strictly prefer to declare it. Therefore, there cannot be a Markov-perfect equilibrium which assigns a probability of amnesty less than $1-\pi_N$ at the reputation π_O.

Lemma 5. Consider the relevant continuation game between taxpayers and the *opportunistic government* after a tax amnesty realization, i.e. $\rho_0 = \pi_O$. In any MPE, for all sequences $\{\rho_t\}_{t=0}^{\infty}$ with positive probability, $\phi_0 > \phi(\rho_t)$, $\forall \rho_t \in [1-\pi_N, \pi_O)$, $\forall t \geq 0$.

Proof. See Appendix A. □

Lemma 5 shows the high reputational cost associated with an amnesty declaration. The highest probability of amnesty is reached in the first period of the relevant continuation game after an amnesty realization. The idea behind Lemma 5 is the same as the idea behind Theorem 1. Since Theorem 1 delivers our main result, we will discuss the incentives and the decision processes in detail after Theorem 1. Now, we can introduce our main result.

Theorem 1. Consider a game with any initial reputation $\rho_0 \in [1-\pi_N, \pi_O]$. Take any Markov-perfect equilibrium $\{\phi^*(.), x^*(.)\}$. Then, for all sequences $\{\rho_t\}_{t=0}^{\infty}$ with positive probability

$$\phi^*(\pi_O) > \phi^*(\rho_t), \quad \forall \rho_t \neq \pi_O, \ \forall t \geq 1.$$

Proof. See Appendix A. □

Theorem 1, the main result of the paper, shows that the amnesty probability is highest right after an amnesty. It implies that every Markov-perfect equilibrium features the expectation trap mechanism. Specifically, an amnesty decision of the government in every Markov-perfect equilibrium may potentially lead to another in the next period by means of causing taxpayers to assign the highest probability to the subsequent amnesty.

The intuition behind Theorem 1 is based on a contradiction. Assume that there exists a Markov-strategy profile which constitutes an equilibrium but does not feature expectation trap, i.e., there exists a period t at which the probability of amnesty is higher than $\phi^*(\pi_O)$ but there is no amnesty in period $t-1$. If there is no amnesty in a period, either i) the government type is *no-amnesty* or ii) the *opportunistic* government does not declare amnesty. If the government type was *no-amnesty* in the previous period, the probability of amnesty in this period would be $\phi(1-\pi_O)$. However, Lemma 4 shows that $\phi(1-\pi_O)$ is lower than $\phi^*(\pi_O)$, so we eliminate i). We next explain why ii) contradicts with the assumption of equilibrium, i.e., the *opportunistic* government prefers declaring amnesty. In case of declaring amnesty, the government collects extra revenue and the next period probability of amnesty is $\phi^*(\pi_O)$. In the other case, the government collects less revenue, and the next period probability of amnesty is higher than $\phi^*(\pi_O)$ — by the contradiction assumption. Note that higher the amnesty probability lower the total government revenue. Therefore, an *opportunistic* government is better off declaring amnesty in period t-1, and this contradicts with the assumption of equilibrium.

Note that this result requires only the condition 3.7, which is the necessary condition to eliminate the trivial equilibrium where we do not see any amnesty under any reputation level.

Therefore, it consolidates the robustness and strength of the expectation trap mechanism in our environment.

3.4 Analysis of Results

The main result of the Markov-perfect analysis is the existence and robustness of expectation traps. The expectation trap mechanism helps to understand the main observations we derived from the tax amnesty data. As we have shown in Figure 1.2, some states start to implement multiple tax amnesties per decade after a long period without any amnesty at all. This sudden regime shift can be a result of the weakness that government signals when it declares a tax amnesty. A tax amnesty is, along with a revenue increasing fiscal tool, a statement of weakness in tax enforcement. A government which declares an amnesty also signals that it is not able or willing to enforce tax laws properly.[5]. The bad reputation comes with this signal is the root of the recurring tax amnesties we see in some economies.

The bad reputation cycle which we call expectation trap can be analyzed through a series of implications triggered by the weakness signal generated by a tax amnesty declaration. If taxpayers interpret an amnesty as a weakness signal on the government side, they would update their belief about future tax amnesties. Their increased belief on a future tax amnesty creates an incentive to tax evasion. An increase in the total tax evasion provides higher incentives for government to implement another tax amnesty. This vicious cycle can explain both the heterogeneity and regime shifts in tax amnesty implementations. A visual depiction of the expectation trap mechanism can be seen in Figure 3.3.

3.5 Special Cases

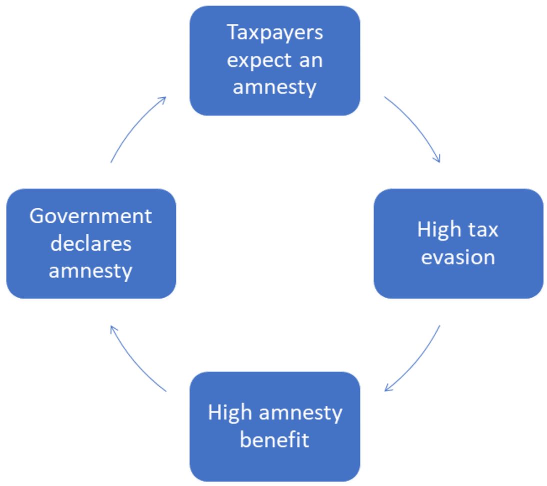
It is beneficial to study small variations of the baseline model in order to test the robustness of our results an also its relation to some of the more common reputation models in the literature. In the baseline model, we set multiple private government types, which are not permanent but persistent. Government type may switch with a Markov transition matrix. We should test whether some of these assumptions are necessary for the existence of expectation traps in the equilibria.

Private Permanent Type

Consider the special case of the baseline model where $\pi_N = \pi_O = 1$. Government type is still unobservable to the taxpayers but it is permanent. Notice that this brings our model closer to some well-known works on reputation such as Kreps and Wilson (1982), Milgrom and Roberts (1982) etc. In this case, declaring a tax amnesty would reveal the type forever as *opportunistic.*

[5] For a result which is in line with this intuition see Ross and Buckwalter (2013).

Figure 3.3: The mechanism of an expectation trap



The optimization of an *opportunistic* government requires comparing the one-period net benefit of a tax amnesty and present discounted value of future revenue loss associated with revealing its type.

Since we assumed that condition 3.7 holds, the stage game with $\rho = 1$ has a unique equilibrium: taxpayers expect an amnesty with certainty and government implements an amnesty. If the type is revealed as *opportunistic*, the game will repeat the unique stage game outcome forever. Then, the continuation value of declaring an amnesty would be

$$A(1) - C_A + \beta[A(1) - C_A] + \beta^2[A(1) - C_A] + \ldots = \frac{A(1) - C_A}{1 - \beta}.$$

Consider the case where if the type is not revealed yet, taxpayers believe there will not be any amnesty. In the first period, the net benefit of declaring amnesty is

$$B(0) - C_A.$$

The present discounted value of the revenue loss associated to revealing the type is

$$\beta\frac{R(0) - A(1) + C_A}{1 - \beta} = \beta\frac{C_A}{1 - \beta}.$$

Therefore, we can support an equilibrium where *opportunistic* government mimics *no-amnesty* type if

$$B(0) \leq \frac{C_A}{1 - \beta}. \tag{3.10}$$

Intuitively, if government is patient enough, it is optimal to pretend being *no-amnesty* type. If the condition 3.10 does not hold, then it is optimal to implement an amnesty in the first period. Then, the game would produce tax amnesties in every period forever. This result is not surprising given the many similar works on reputation.

The expectation trap still exists in this variation of our model. However, dynamics arising from it is slightly different than the baseline model. Declaring amnesty moves the game to an absorbing state of permanent amnesty outcomes. Considering that it repeats the worst stage game equilibrium forever, the expectation trap is a more powerful mechanism in this version.

Public Permanent Type

In addition to the assumption of $\pi_N = \pi_O = 1$, assume that government type is *opportunistic* and it is publicly known. We can think as after all histories, it is common knowledge that the type is *opportunistic*. Notice that this version of the model resembles Barro and Gordon (1983). In this case, there is no more a payoff relevant variable. Thus, Markov-perfect analysis is not suitable. However, we can still construct equilibria by allowing history-dependence in beliefs and strategies. Consider the following particular belief update of taxpayers:

$$\phi_t = \begin{cases} \phi_{t-1} & \text{if there was no amnesty in the past} \\ 1 & \text{if there was an amnesty in the past} \end{cases} \tag{3.11}$$

Assuming this particular belief update, we can construct a rational expectations equilibrium which features expectation trap.

Proposition 7. Assume that the belief update is as given in 3.11. Under the condition

$$(1-\beta)B(0) < C_A < B(1),$$

there exists a real number $\phi^* \in (0,1)$ such that;

$$\phi_0 = \phi^*, \quad x_t^* = \begin{cases} \phi^* & \text{if there was no amnesty in the past} \\ 1 & \text{if there was an amnesty in the past} \end{cases}$$

is an equilibrium.

Proof. See Appendix A. □

Starting from the first period, the probability of a tax amnesty realization is positive, but there is still a possibility of not declaring an amnesty. If a tax amnesty does not occur, the game moves to a subgame which is identical to the game starts in first period. If a tax amnesty occurs, the game moves to a subgame where we see a tax amnesty in every period forever. This result is not surprising considering the similarity between this particular variation of our model and the model analyzed in Barro and Gordon (1983). A tax amnesty starts an infinite sequence of tax amnesties in this particular equilibrium. This is a direct result of the particular belief update we assumed. It punishes the *cheating* action, in our case declaring amnesty, forever. However, it may not be the case in some other equilibria. It possible to construct similar equilibria where the punishment of *cheating* is for some finite period. In such a case, an amnesty triggers a sequence of amnesties, but its effect would not be permanent. Another possibility would be to make the length of punishment random. An equilibrium with such belief update might produce very similar outcomes with the switching strategy equilibrium we introduced in our Markov-perfect analysis.

3.6 An Extension with Stochastic Cost

We have established that one tax amnesty may trigger a sequence of further amnesties through an expectation trap and it might explain the regime shift we see in some economies. This shift occurs endogenously in our model. Considering that tax amnesties cluster around years after US recessions (see Figure 1.3), one may argue that most of these shifts are triggered by a recession.

A natural way to introduce recessions in our model is to allow stochastic shocks to the fixed cost of declaring amnesty, C_A. Particularly, consider the following modification to the baseline model. Assume that the cost of declaring an amnesty is stochastic and a new cost is drawn at the beginning of each period. The cost can either be C_L with probability p_L or it can be C_A with probability $1 - p_L$. The stochastic process is i.i.d. and the draw is publicly observable. The cost level C_A is the standard fixed cost in the baseline model whereas C_L is a very small cost. We can interpret drawing C_L as facing a recession. Governments generally look for extra sources of revenue during recessions. They may enter a deficit crisis and in need of some urgent cash injection into the state budget. Moreover, since increasing government spending helps boost the economy, a tax amnesty might be an important tool to get out of a recession. Therefore, during a recession period, a tax amnesty might not seem as costly to the government. We may even argue that the government can gain an indirect political benefit in amnesty implementations. It is natural to think that we may even allow C_L to be negative.

We acknowledge that the individual income levels also drop during a recession. We abstract from the changes in income levels to present our main point in a simpler way. A temporary drop in income levels during a recession does not change our results as long as C_L is small enough.

We are interested in equilibria where recessions trigger a sequence of tax amnesties. Proposition 8 sets up such an equilibrium.

Proposition 8. Assume the following restrictions on parameters.

$$B(0) - C_A < \frac{\beta(1-p_L)}{1-\beta(1-p_L)}[R(0) - A(\pi_O) + C_A] \tag{3.12}$$

$$B(\pi_O) - C_A > \frac{\beta}{1-\beta}[R(0) - A(\pi_O) + C_A] \tag{3.13}$$

$$B(1-\pi_N) - C_L > \frac{\beta}{1-\beta}[R(0) - A(\pi_O) + C_A] \tag{3.14}$$

If $p_L \in (0,1)$ is small enough, then the following is an equilibrium:

$$\{\phi(\rho, C_L), x(\rho, C_L)\} = \{\rho, 1\}$$
$$\{\phi(\pi_O, C_A), x(\pi_O, C_A)\} = \{\pi_O, 1\},$$
$$\{\phi(\rho, C_A), x(\rho, C_A)\} = \{0, 0\}, \quad \forall \rho \in [1-\pi_N, \pi_O).$$

Proof. See Appendix A. □

In this equilibrium, an economy with an initial reputation different than π_O never experiences an amnesty until a recession. When recession hits, political cost becomes very small and reputation cost alone cannot convince a *opportunistic* government to mimic the behavioral type. However, the declaration of a tax amnesty in a period still reveals the government type. Thus, it provides a jump to the worst reputation. Although it is very likely to get out of the recession in the next period, the probability of a tax amnesty realization is still close to certainty. When a recession period results with an amnesty, the expectation trap mechanism provides a persistence in amnesty implementations. The game produces repeated tax amnesties until a government type change occurs. The dynamics of this equilibrium is particularly interesting since a temporary shock to the economy leaves a long-lasting shift in tax amnesty occurrence probability.

3.7 Concluding Remarks

Data suggest that tax amnesties tend to be repetitive. The frequency of tax amnesties is quite heterogeneous among different states and this heterogeneity looks persistent. Some states start to implement tax amnesties very frequently while they were using them rarely in previous decades. We construct a model to show that these characteristics can be explained via a reputational mechanism. Declaring an amnesty can increase the taxpayers' expectations for another one in the near future. An increase in expectations can make an amnesty more valuable for the government because it leads to an increase in tax evasion. Our model also makes predictions about which characteristics make economies more vulnerable against such traps. In this sense, it provides some valuable insights for policymakers.

Some shortcomings of our model should be noted. The taxpayers are short-lived in our setup. This is a simplification to maintain tractability. However, this comes with some costs. It eliminates the possibility of forward-looking taxpayer decisions on evasion by taking into account savings and multiple tax prosecution periods. Our model can be interpreted as a model of sales taxes as well. However, some tax types such as capital income tax and corporate income tax have different dynamics that we cannot capture without allowing savings.

Note that our model can be used as a baseline to study other aspects of tax amnesties. One such aspect is the effect of tax amnesties on individuals' perception of tax evasion. One can argue that seeing repetitive tax amnesties may decrease the psychological cost of evading taxes for private agents in the economy. Such a topic can be studied by making the distribution of ϵ as a state variable depends on past realizations of tax amnesties.

There are possible paths that future research can follow. An important step might be to study the effect of tax amnesties on wealth inequality. Studying the correlation between the ability to evade and the wealth of an individual would be a natural extension starting from our work. This would again require a model with savings and long-lived taxpayers. Building such a model itself would be a great step for the tax amnesty studies.

3.8 Appendix

Appendix A. Proofs

A1. Proof of Proposition 4

Proof. 1. Revenues with amnesty: Notice that the government's total revenue can be written as

$$\begin{aligned}
A =& R_T + R_A + R_P \\
=& \int_{\bar{\epsilon}^*}^{1} \tau w d\epsilon + \int_{\frac{\bar{\epsilon}^*}{2}}^{\bar{\epsilon}^*} a^* w d\epsilon + \int_{0}^{\frac{\bar{\epsilon}^*}{2}} p w d\epsilon \\
=& (1-\bar{\epsilon}^*) w\tau + \left(\bar{\epsilon}^* - \frac{\bar{\epsilon}^*}{2}\right) w \left(\frac{\bar{\epsilon}}{2} + p\right) + \frac{\bar{\epsilon}^*}{2} wp \\
=& (1-\bar{\epsilon}^*) w\tau + \frac{\bar{\epsilon}^{*2}}{4} w + \bar{\epsilon}^* wp \\
=& \left(1 - \frac{2(\tau-p)}{2-\phi}\right) wt + \frac{\left(\frac{2(\tau-p)}{2-\phi}\right)^2}{4} w + \frac{2(\tau-p)}{2-\phi} wp \\
=& w\tau - w(\tau-p)\left(\frac{2(\tau-p)}{2-\phi}\right) + w\frac{4(\tau-p)^2}{(2-\phi)^2} \\
=& w\tau - w(\tau-p)^2\left(\frac{3-2\phi}{(2-\phi)^2}\right).
\end{aligned}$$

2. Revenues without amnesty:

$$\begin{aligned}
R =& R_T + R_P \\
=& \int_{\bar{\epsilon}^*}^{1} \tau w d\epsilon + \int_{0}^{\bar{\epsilon}^*} p w d\epsilon \\
=& (1-\bar{\epsilon}^*) w\tau + \bar{\epsilon}^* wp \\
=& \left(1 - \frac{2(\tau-p)}{2-\phi}\right) w\tau + \frac{2(\tau-p)}{2-\phi} wp \\
=& w\tau - w(\tau-p)^2 \frac{2}{2-\phi}.
\end{aligned}$$

3. Benefit of amnesty:

$$\begin{aligned}
B(\phi) =& A(\phi) - R(\phi) \\
=& \frac{w(\tau-p)^2}{(2-\phi)^2}.
\end{aligned}$$

Lastly, it is easy to see that $B(.)$ is increasing and $R(.)$ is decreasing on the interval $\phi \in [0, 1]$.

Let us also check $A(.)$.

$$\begin{aligned}\frac{\partial A}{\partial \phi} &= -w(\tau-p)^2\frac{(-2)(2-\phi)^2+2(2-\phi)(3-2\phi)}{(2-\phi)^4} \\ &= -2w(\tau-p)^2\frac{2-3\phi+\phi^2}{(2-\phi)^4} \\ &= -2w(\tau-p)^2\frac{(\phi-1)(\phi-2)}{(2-\phi)^4} \leq 0, \text{ when } \quad \phi\in[0,1].\end{aligned}$$

□

A2. Proof of Proposition 5

Proof. We need to show that the outcome $(\phi^*, x^*) = (0, 0)$ is weakly better than any equilibrium outcome without commitment for the government. First, if $B(\rho) < C_A$ the unique equilibrium is the same with or without commitment.

Second, assume $B(0) > C_A$. Then, without commitment, government gets $A(\rho) - C_A$. With commitment, government gets $R(0)$. We know $R(0) \geq A(\rho) - C_A$.

Lastly, assume $B(0) \leq C_A \leq B(\rho)$. Again from $R(0) \geq A(\rho) - C_A$ we know that $(\phi_1^*, x_1^*) = (0, 0)$ is better equilibrium than $(\phi_2^*, x_2^*) = (\rho, 1)$ for government revenues. Now, let's focus on the mixed strategy equilibrium. Note that in this equilibrium, government needs to be indifferent between declaring an amnesty or not. Therefore,

$$A(\phi_3^*) - C_A = R(\phi_3^*).$$

The expected payoff of government under the mixed strategy equilibrium is

$$\phi_3^*[A(\phi_3^*) - C_A] + (1-\phi_3^*)R(\phi_3^*) = R(\phi_3^*) < R(0).$$

Hence, government would like to commit in order to secure $(\phi^*, x^*) = (0, 0)$ equilibrium. □

A3. Proof of Lemma 1

Proof. 1. $\rho^* = \Gamma(\rho^*, 0) \implies \rho^* = \rho^*\pi_O + (1-\rho^*)(1-\pi_N) \implies \rho^* = \dfrac{1-\pi_N}{(1-\pi_O)+(1-\pi_N)}$.

2. Take arbitrary ρ, ρ' such that $\rho < \rho^*$ and $\rho' = \Gamma(\rho, 0)$. Then,

$$\begin{aligned}
&\rho' - \rho > 0 \\
\iff &\rho\pi_O + (1-\rho)(1-\pi_N) - \rho > 0 \\
\iff &1 - \pi_N > \rho((1-\pi_O) + (1-\pi_N)) \\
\iff &\frac{1-\pi_N}{(1-\pi_O)+(1-\pi_N)} > \rho \\
\iff &\rho^* > \rho.
\end{aligned}$$

Therefore, $\rho' > \rho$. Now, realize that $\Gamma(\rho, 0)$ is increasing in ρ. Then,

$$\rho^* > \rho \implies \Gamma(\rho^*, 0) > \Gamma(\rho, 0) \implies \rho^* > \rho'.$$

3. Symmetric arguments with the previous point proves it trivially.

□

A4. Proof of Proposition 6

It is clear that the beliefs are consistent with the government strategies. To show that government strategy function is optimal, first define the sequence S and function $W(.)$ as described before.

$$\begin{aligned}
S =& (r_0, r_1, r_2, r_3, \ldots) \text{ where} \\
r_0 =& 1 - \pi_N \text{ and } r_{t+1} = \Gamma(r_t, 0), \ \forall t \in \mathbb{Z}_{\geq 0}
\end{aligned}$$

$$W(T) = (1 + \beta + \beta^2 + \ldots.. + \beta^{T-1})R(0) + \beta^T(A(r_T) - C_A) + \beta^{T+1}(V(\pi_O))$$

First, let us show that $W(.)$ is an increasing function. For any period $T \geq 0$,

$$\begin{aligned}
&W(T+1) - W(T) \geq 0 \\
\iff &\beta^T(R(0) - A(r_T) + C_A) + \beta^{T+1}(A(r_{T+1}) - A(\pi_O)) \geq 0 \\
\iff &\beta(A(r_{T+1}) - A(\pi_O)) \geq A(r_T) - R(0) - C_A
\end{aligned}$$

Remember that function $A(.)$ is decreasing and proposition 1 makes sure that $0 < r_T < r_{T+1} < \rho^*$. Then, the last inequality holds, since $A(0) > A(r_T)$ and $A(r_{T+1}) > A(\rho^*)$. Now, define the set

$$D = \{(R, T) \mid r_T \geq R \text{ and } B(R) - C_A = \beta[W(T) - V(\pi_O)]\}.$$

Notice that the set is non-empty by the condition 3.9. To see that realize $r_T < \rho^*$ and $W(0) \geq W(T), \forall T$. Now take

$$(\underline{R}, \underline{T}) = \arg\min_{(R,T)\in D} R$$

First, realize that $r_{\underline{T}-1} < \underline{R}$. To see that, assume it is not. Then, $r_{\underline{T}-1} \geq \underline{R}$ which means

$$B(r_{\underline{T}-1}) - C_A \geq \beta[W(\underline{T}) - V(\pi_O)] > \beta[W(\underline{T}-1) - V(\pi_O)]$$

where the second inequality comes from the fact that $W(.)$ is increasing. Define L such that

$$B(L) - C_A = \beta[W(\underline{T}-1) - V(\pi_O)].$$

Clearly, $r_{\underline{T}-1} \geq L$ which makes $(L, \underline{T}-1) \in D$. Since $W(.)$ and $B(.)$ are increasing functions, $L < \underline{R}$, which is a contradiction.

Second, realize that by construction of the set D, $r_{\underline{T}} \geq \underline{R}$. Now, note that the proposed Markov strategy profile suggests $V(1-\pi_N) = W(\underline{T})$. We also know

$$B(\underline{R}) - C_A = \beta[W(\underline{T}) - V(\pi_O) = \beta[V(1-\pi_N) - V(\pi_O)].$$

Then, it is easy to see that for any $\rho \geq \underline{R}$,

$$B(\rho) - C_A \geq \beta[V(1-\pi_N) - V(\pi_O)].$$

Now, let's take an arbitrary $\rho_0 < \underline{R}$. Remember that $\underline{R} < \rho^*$. So, in the game that starts at ρ_0, reputation must pass to the other side of $\underline{R}$ in finite number of periods, which will end up being a *declaring amnesty* equilibrium. Then, there must exist a positive integer k such that $\rho_{k-1} < \underline{R} \leq \rho_k$ where the set $\Sigma = \{\rho_0, \rho_1, \rho_2, \ldots, \rho_{k-1}, \rho_k\}$ represents the set of first k states the game will follow if it starts at ρ_0, i.e.

$$\rho_m = \Gamma(\rho_{m-1}, 0), \quad \forall m \leq k.$$

Note that it makes

$$(\phi^*(\rho_m), x^*(\rho_m)) = (0,0), \text{ if } m < k$$
$$(\phi^*(\rho_k), x^*(\rho_k)) = (\rho_k, 1)$$

For this to constitute an equilibrium, the following set of inequalities must be satisfied.

$$\begin{aligned}
&B(0) - C_A \leq \beta(V(\rho_1) - V(\pi_O)) \\
&B(0) - C_A \leq \beta(V(\rho_2) - V(\pi_O)) \\
&B(0) - C_A \leq \beta(V(\rho_3) - V(\pi_O)) \\
&\ldots \\
&B(0) - C_A \leq \beta(V(\rho_{k-1}) - V(\pi_O)) \\
&B(0) - C_A \leq \beta(V(\rho_k) - V(\pi_O)) \\
&B(\rho_k) - C_A \geq \beta(V(1-\pi_N) - V(\pi_O)).
\end{aligned}$$

where

$$\begin{aligned}
V(\rho_k) =& A(\rho_k) - C_A + \beta V(\pi_O) \\
V(\rho_m) =& R(0) + \beta V(\rho_{m+1}), \quad \forall m < k \\
V(1-\pi_N) =& W(\underline{T}) \\
V(\pi_O) =& A(\pi_O) - C_A + \beta V(\pi_O).
\end{aligned}$$

First, focus on the last inequality:

$$\rho_k \geq \underline{R} \implies B(\rho_k) - C_A \geq B(\underline{R}) - C_A = \beta(W(\underline{T}) - V(\pi_O)) = \beta(V(1-\pi_N) - V(\pi_O)).$$

Second, realize that

$$V(\rho_k) - V(\pi_O) = A(\rho_k) - A(\pi_O).$$

Then,

$$B(0) - C_A \leq \beta(V(\rho_k) - V(\pi_O)) = \beta(A(\rho_k) - A(\pi_O))$$

which holds, since the condition 3.8 holds. Lastly, realize that $V(\rho_m) \leq V(\rho_{m-1})$ for any $m < k$. To see that,

$$\begin{aligned}
V(\rho_{m-1}) =& (1 + \beta + \beta^2 + \ldots + \beta^{k-m-2})R(0) + \beta^{k-m-1}(A(\rho_k) - C_A) + \beta^{k-m}V(\pi_O) \\
V(\rho_m) =& (1 + \beta + \beta^2 + \ldots + \beta^{k-m-1})R(0) + \beta^{k-m}(A(\rho_k) - C_A) + \beta^{k-m+1}V(\pi_O) \\
\implies V(\rho_{m-1}) - V(\rho_m) =& \beta^{k-m-1}[A(\rho_k) - C_A - R(0)] + \beta^{k-m}[A(\pi_O) - A(\rho_k)] \\
\text{(A is decreasing)} \quad \leq& \beta^{k-m-1}[A(0) - C_A - R(0)] + \beta^{k-m}[A(\pi_O) - A(\rho^*)] \\
\leq& \beta^{k-m-1}[B(0) - C_A] + \beta^{k-m}[A(\pi_O) - A(\rho^*)] \\
\text{(From condition 3.8)} \quad \leq& 0
\end{aligned}$$

This result implies that

$$B(0) - C_A \leq \beta(V(\rho_m) - V(\pi_O)) \quad \forall m < k.$$

Since ρ_0 was arbitrary, the proposed Markov strategy profile is an equilibrium.

A5. Proof of Lemma 2

Proof. Suppose otherwise. The only possibility is that players play *no amnesty* in every period in the relevant continuation game. Then, there must be a MPE such that in the relevant continuation game, taxpayers do not expect an amnesty in any period and opportunistic government plays $x^* = 0$. Denote the sequence of reputation levels the relevant continuation game follows under such an equilibrium as $(\pi_O, \rho_1, \rho_2, \rho_3, \ldots)$, where $\rho_1 = \Gamma(\pi_O, 0)$, $\rho_2 = \Gamma(\rho_1, 0)$, $\rho_3 = \Gamma(\rho_2, 0)$, and so on. Now, consider a one-shot deviation of playing $\hat{x}(\pi_O) = 1$ from opportunistic government. It should not be a profitable deviation, so the following inequality must hold.

$$A(0) - C_A + \beta V(\pi_O) \leq R(0) + \beta V(\rho_1)$$

Notice that if government does not deviate then the continuation value will be $V(\rho_1)$. Since the equilibrium will continue as *no amnesty* forever, the value should be

$$V(\rho_1) = R(0) + \beta R(0) + \beta^2 R(0) + \beta^3 R(0) + \ldots = \frac{R(0)}{1-\beta}$$

The equilibrium suggests that the game will continue as *no amnesty* forever, if reputation jumps to π_O. Therefore,

$$V(\pi_O) = R(0) + \beta R(0) + \beta^2 R(0) + \beta^3 R(0) + \ldots = \frac{R(0)}{1-\beta}$$

Then, we can re-write our inequality as

$$A(0) - C_A + \leq R(0) \iff B(0) - C_A \leq 0$$

which is a contradiction with condition 3.7. □

A6. Proof of Lemma 3

Proof. Assume that it is not, i.e. $\phi_0 = 0$. We have already proven before that an infinite sequence of zero amnesty probabilities cannot be an equilibrium. Then, there must be a finite number of periods, say $\underline{k}$, such that the *opportunistic* government puts a positive probability on declaring amnesty for the first time. Then, we would have the following set of equality with value functions.

$$\begin{aligned}
V(\pi_O) =& R(0) + \beta V(\rho_1) \\
V(\rho_1) =& R(0) + \beta V(\rho_2) \\
V(\rho_2) =& R(0) + \beta V(\rho_3) \\
& \dots \\
V(\rho_{\underline{k}-1}) =& R(0) + \beta V(\rho_{\underline{k}}) \\
V(\rho_{\underline{k}}) =& A(\phi_{\underline{k}}) - C_A + \beta V(\pi_O).
\end{aligned}$$

where $\phi_{\underline{k}} > 0$. With the help of these set of equations we can find the value of $V(\rho_{\underline{k}}) - V(\pi_O)$.

$$\begin{aligned}
V(\rho_{\underline{k}}) - V(\rho_{\underline{k}-1}) =& A(\phi_{\underline{k}}) - C_A - R(0) + \beta[V(\pi_O) - V(\rho_{\underline{k}})] \\
V(\rho_{\underline{k}-1}) - V(\rho_{\underline{k}-2}) =& \beta[V(\rho_{\underline{k}}) - V(\rho_{\underline{k}-1})] \\
V(\rho_{\underline{k}-2}) - V(\rho_{\underline{k}-3}) =& \beta[V(\rho_{\underline{k}-1}) - V(\rho_{\underline{k}-2})] \\
& \dots \\
V(\rho_2) - V(\rho_1) =& \beta[V(\rho_3) - V(\rho_2)] \\
V(\rho_1) - V(\pi_O) =& \beta[V(\rho_2) - V(\rho_1)]
\end{aligned}$$

By summing up all the equations we have

$$\begin{aligned}
V(\rho_{\underline{k}}) - V(\pi_O) =& A(\phi_{\underline{k}}) - C_A - R(0) + \beta[V(\pi_O) - V(\rho_1)] \\
=& A(\phi_{\underline{k}}) - C_A - R(0) - \beta[V(\rho_1) - V(\pi_O)] \\
=& A(\phi_{\underline{k}}) - C_A - R(0) - \beta[\beta^{\underline{k}-1}(V(\rho_{\underline{k}}) - V(\rho_{\underline{k}-1}))] \\
=& A(\phi_{\underline{k}}) - C_A - R(0) - \beta^{\underline{k}}[A(\rho_{\underline{k}}) - C_A - R(0) + \beta(V(\pi_O) - V(\rho_{\underline{k}}))] \\
=& (1 - \beta^{\underline{k}})[A(\phi_{\underline{k}}) - C_A - R(0)] - \beta^{\underline{k}+1}(V(\pi_O) - V(\rho_{\underline{k}})) \\
\implies V(\rho_{\underline{k}}) - V(\pi_O) =& \frac{1 - \beta^{\underline{k}}}{1 - \beta^{\underline{k}+1}}[A(\phi_{\underline{k}}) - C_A - R(0)]
\end{aligned}$$

Now, consider the period $\underline{k} - 1$. The following inequality must hold in that period.

$$B(0) - C_A \leq \beta[V(\rho_{\underline{k}}) - V(\pi_O)] = \beta\frac{1 - \beta^{\underline{k}}}{1 - \beta^{\underline{k}+1}}[A(\phi_{\underline{k}}) - C_A - R(0)]$$

Remember that the function A is decreasing. Therefore,

$$\begin{aligned}
B(0) - C_A \leq& \beta\frac{1 - \beta^{\underline{k}}}{1 - \beta^{\underline{k}+1}}[A(\phi_{\underline{k}}) - C_A - R(0)] \\
&< A(\phi_{\underline{k}}) - C_A - R(0) < A(0) - C_A - R(0) = B(0) - C_A
\end{aligned}$$

which is a contradiction. Hence, $\phi_0 > 0$. □

A7. Proof of Lemma 4

Proof. Suppose that it is not true. Then, there exists a MPE such that $\phi(\pi_O) \leq \phi(1-\pi_N)$. Denote $\rho_0 = \pi_O$, ρ_1 as the reputation of period 1 if there is no amnesty in period 0, ρ_2 as the reputation of period 2 if there is no amnesty in period 1 and so on... First, we will show that $\phi(\rho_t) \neq \rho_t$ for any t under our supposition. Since we assumed π_O to be close to 1 and $\phi(\pi_O) \leq \phi(1-\pi_N)$, $\phi(\rho_0) < \rho_0$. Now, to get a contradiction, assume that st. $\phi(\rho_1) = \rho_1$. Then, the following equation must hold.

$$V(\rho_1) = A(\rho_1) - C_A + \beta V(\pi_O).$$

Lemma 3 already shows that $\pi_O > 0$. Then, the following equation must hold, as well.

$$V(\pi_O) = A(\pi_O) - C_A + \beta V(\pi_O). \tag{3.15}$$

Combining these two, we have

$$V(\rho_1) - V(\pi_O) = A(\rho_1) - A(\pi(\pi_O)) < 0,$$

where the last inequality comes from the fact that A is decreasing. Now, the supposition of $\phi(\pi_O) \leq \phi(1-\pi_N)$ gives us

$$V(\pi_O) = A(\phi(\pi_O)) - C_A + \beta V(\pi_O) = R(\phi(\pi_O)) + \beta V(\rho_1).$$

By rearranging terms we reach

$$V(\rho_1) - V(\pi_O) = \frac{B(0) - C_A}{\beta} > 0,$$

which is a contradiction. Therefore, $\phi(\rho_1) > \rho_1$. Now, to get a contradiction, assume that there exists a $t \geq 2$ such that $\phi(\rho_t) = \rho_t$ and $\phi(\rho_k) < \rho_k$ for all $k < t$. Then, notice that the following equations must hold.

$$V(\rho_t) = A(\rho_t) - C_A + \beta V(\pi_O), \tag{3.16}$$

$$V(\rho_{t-1}) = R(\phi(\rho_{t-1})) + \beta V(\rho_t) \geq A(\rho_{t-1}) - C_A + \beta V(\pi_O). \tag{3.17}$$

The equations 3.15 and 3.16 together implies

$$V(\rho_t) - V(\pi_O) = A(\rho_t) - A(\pi_O) < 0,$$

where the last inequality comes from the fact that A is a decreasing function. The equation 3.17 implies that

$$V(\rho_t) - V(\pi_O) \geq \frac{B(\phi(\rho_{t-1})) - C_A}{\beta} > 0,$$

where the inequality comes from the condition 3.7. This is a contradiction, so $\phi(\rho_t) < \rho_t$. From mathematical induction it shows that $\phi(\rho_i) < \rho_i$ for any $i \geq 0$.

Second, we will reach a contradiction when $\phi(\rho_i) < \rho_i$ for any $i \geq 0$, to disprove our initial supposition. Note that $\phi(\rho_i) < \rho_i$ implies that the *opportunistic* government puts positive probability on *not declaring amnesty* in period i. Then, the following set of conditions needs to be satisfied.

$$\begin{aligned}
V(\pi_O) =& A(\phi(\pi_O)) - C_A + \beta V(\pi_O) \leq R(\phi(\pi_O)) + \beta V(\rho_1) \\
V(\rho_1) \leq& R(\phi(\rho_1)) + \beta V(\rho_2) \\
V(\rho_2) \leq& R(\phi(\rho_2)) + \beta V(\rho_3) \\
V(\rho_3) \leq& R(\phi(\rho_3)) + \beta V(\rho_4) \\
& \dots
\end{aligned}$$

Then, we can write the following.

$$A(\phi(\pi_O)) - C_A + \beta V(\pi_O) \leq \frac{R(0)}{1-\beta} \implies A(1-\pi_N) - R(0) \leq C_A$$

which is a contradiction with condition 3.7. □

A8. Proof of Lemma 5

Proof. First, we have just shown that $\phi_0 > 0$. Second, if $\phi_0 = \pi_O$, the proof follows from the fact that $\phi_i \leq \rho_i < \pi_O$, $\forall i \in \mathbb{Z}_{\geq 1}$. Therefore, we only need to prove the case where $\phi_0 \in (0, \pi_O)$.

Suppose, to get a contradiction, the claim is not true. So, there exists $t > 0$ such that $\phi_0 \leq \phi_t$. Then, value functions should satisfy the following set of conditions.

$$\begin{aligned}
V(\pi_O) =& A(\phi_0) - C_A + \beta V(\pi_O) \\
V(\rho_t) =& A(\phi_t) - C_A + \beta V(\pi_O). \\
\implies& V(\rho_t) - V(\pi_O) = A(\phi_t) - A(\phi_0) \leq 0.
\end{aligned}$$

where the last inequality comes from the fact that A is decreasing. Consider the period $t-1$. To be able reach reputation ρ_t, government should put a positive probability on not declaring an amnesty in its strategy at period $t-1$. Then, we can simply write the following.

$$\begin{aligned} V(\rho_{t-1}) =& R(\phi_{t-1}) + \beta V(\rho_t) \geq A(\phi_{t-1}) - C_A + \beta V(\pi_O) \\ &\implies \beta[V(\rho_t) - V(\pi_O)] \geq B(\phi_{t-1}) - C_A \geq B(0) - C_A > 0 \end{aligned}$$

which is a contradiction. Hence, $\phi_0 > \phi_i, \forall \in \mathbb{Z}_{\geq 1}$. □

A9. Proof of Theorem 1

Proof. Take an arbitrary MPE $\{\phi(.), x(,)\}$ and an arbitrary initial reputation ρ_0. Denote reputation in period i as ρ_i. Suppose, to get a contradiction, there exists a $t > 1$ such that $\phi(\rho_t) > \phi(\pi_O)$ and $\rho_t \neq \pi_O$. It means that there was no amnesty in period $t-1$. Then, there are two possibilities; either the *opportunistic* government did not choose to declare an amnesty or an although an *opportunistic* government would have declared an amnesty, the government type was *no-amnesty*. Let us first focus on the first possibility. We already know that $\phi(\pi_O) > 0$. Then,

$$\begin{aligned} V(\pi_O) =& A(\phi(\pi_O)) - C_A + \beta V(\pi_O) \\ V(\rho_t) =& A(\phi(\rho_t)) - C_A + \beta V(\pi_O) \\ &\implies V(\rho_t) - V(\pi_O) = A(\phi(\rho_t)) - A(\phi(\pi_O)) < 0 \end{aligned}$$

where last inequality comes from the fact that $A(.)$ is a decreasing function and π_O is the highest possible value for the state variable. Now, consider period $t-1$. We must have $\phi(\rho_{t-1}) < 1$ since $\rho_t \neq \pi_O$. Then, it must be

$$\begin{aligned} R(\phi(\rho_{t-1})) + \beta V(\rho_t) \geq A(\phi(\rho_{t-1})) - C_A + \beta V(\pi_O) \\ \implies \beta[V(\rho_t) - V(\pi_O)] \geq B(\phi(\rho_{t-1})) - C_A \geq B(0) - C_A > 0 \end{aligned}$$

where the last inequality comes from condition 3.7. Notice that this is a contradiction. Now, assume that in period $t-1$, an *opportunistic* government would have liked to declare an amnesty with probability one, but the government type was *no-amnesty* and therefore we didn't see any amnesty. Then, $\phi_t = 1 - \pi_N$. Lemma 4 already implies that $\phi(\pi_O) > \phi(1 - \pi_N)$.

Hence, $\phi(\pi_O) > \phi(\rho_i), \quad \forall \rho_i \neq \pi_O, \quad \forall i \geq 1$. □

Proof of Proposition 7

Proof. It is clear that the taxpayer beliefs are consistent with the government strategies. Therefore, we will only show the the optimality of government strategy. Consider any period m. The belief update requires that if there has been any amnesty in the past, the belief is $\phi_m = 1$. In such a case, the outcome of period m does not effect the successor subgame, since belief will stay as $\phi_k = 1$

$\forall k > m$ independent of government's actions. Assumption 7 requires that government's strategy is to declare an amnesty with certainty.

Now, consider an arbitrary period t where there has not been any amnesty in the past. Then, the belief is $\phi_t = \phi^*$. Then, government's payoff if follows the suggested strategy x_t^* is

$$\frac{R(\phi^*)}{1-\beta}. \tag{3.18}$$

If government deviates and declares an amnesty

$$A(\phi^*) - C_A + \beta\frac{A(1) - C_A}{1-\beta}. \tag{3.19}$$

Notice that suggested government strategy must be a proper mixed strategy, since $\phi^* \in (0,1)$. For suggested strategy and beliefs to constitute an equilibrium, government must be indifferent between implementing an amnesty and not implementing one. In other words, the payoff from suggested strategy must be equal to the deviation payoff:

$$\frac{R(\phi^*)}{1-\beta} = A(\phi^*) - C_A + \beta\frac{A(1) - C_A}{1-\beta} \tag{3.20}$$

We know that $A(\phi^*) - R(\phi^*) = B(\phi^*)$. Then,

$$\begin{aligned}
B(\phi^*) =& \beta[A(\phi^*) - A(1)] + C_A \\
\frac{w(\tau-p)^2}{(2-\phi^*)^2} =& \beta\left[w\tau - w(\tau-p)^2\left(\frac{3-2\phi^*}{(2-\phi^*)^2}\right) - w\tau + w(\tau-p)^2\right] + C_A \\
\frac{w(\tau-p)^2}{(2-\phi^*)^2} =& \beta\frac{w(\tau-p)^2}{(2-\phi^*)^2}\left[(1-\phi^*)^2\right] + C_A \\
\frac{w(\tau-p)^2}{(2-\phi^*)^2} =& \beta\frac{w(\tau-p)^2}{(2-\phi^*)^2}\left[(1-\phi^*)^2\right] + C_A \\
\frac{1-\beta(1-\phi^*)^2}{(2-\phi^*)^2} =& \frac{C_A}{w(\tau-p)^2}.
\end{aligned}$$

Note that this is a necessary and sufficient condition for the equilibrium. Therefore, we only need to show that such a real number exists. Define

$$f(\phi) = \frac{1-\beta(1-\phi)^2}{(2-\phi)^2}$$

Moreover, note the following:

$$f(0) = \frac{1-\beta}{4} \text{ and } f(1) = 1.$$

Now, realize that we can re-write the assumption 7 in the following way as well.

$$\frac{1-\beta}{4} < \frac{C_A}{w(\tau-p)^2} < 1 \iff f(0) < \frac{C_A}{w(\tau-p)^2} < f(1)$$

Note that in the domain $[0,1]$, the function $f(.)$ is continuous in ϕ. Then, there must be a $\phi^* \in (0,1)$ such that

$$f(\phi^*) = \frac{C_A}{w(\tau-p)^2}.$$

□

A10. Proof of Proposition 8

Proof. It is clear that the taxpayer beliefs are consistent with the government strategies. Therefore, it is enough to show that government strategies are optimal given the taxpayer beliefs. First, take the government strategies when C_L is drawn as given. Now, consider any reputation level $\rho_0 \neq \pi_O$ and define $\rho_{i+1} = \Gamma(\rho_i, 0)$. Note that as long as there is no amnesty realization, the reputation will always be different than π_O. Then, government's value function if it follows the suggested strategy is

$$\begin{aligned}
V(\rho_0) &= p_L[A(\rho_0) - C_L + \beta V(\pi_O))] + (1-p_L)[R(0) + \beta V(\rho_1)] \\
V(\rho_1) &= p_L[A(\rho_1) - C_L + \beta V(\pi_O))] + (1-p_L)[R(0) + \beta V(\rho_2)] \\
V(\rho_2) &= p_L[A(\rho_2) - C_L + \beta V(\pi_O))] + (1-p_L)[R(0) + \beta V(\rho_3)] \\
&\dots \\
V(\rho_t) &= p_L[A(\rho_t) - C_L + \beta V(\pi_O))] + (1-p_L)[R(0) + \beta V(\rho_{t+1})] \\
&\dots \\
\implies V(\rho_t) &= \sum_{k=0}^{\infty}((1-p_L)\beta)^k[p_L(A(\rho_{t+k}) - C_L + \beta V(\pi_O)) + (1-p_L)R(0)]
\end{aligned}$$

Similarly, government's value function in reputation level π_O if it follows the suggested strategy is

$$\begin{aligned}
V(\pi_O) &= p_L[A(\pi_O) - C_L + \beta V(\pi_O))] + (1-p_L)[A(\pi_O) - C_A + \beta V(\pi_O)] \\
V(\pi_O) &= \frac{A(\pi_O) + p_L C_L + (1-p_L)C_A}{1-\beta}
\end{aligned}$$

or, equivalently

$$V(\pi_O) = \sum_{k=0}^{\infty}((1-p_L)\beta)^k[p_L(A(\pi_O) - C_L + \beta V(\pi_O)) + (1-p_L)(A(\pi_O) - C_A)]$$

Now, realize that the given strategy profile constitutes an equilibrium if the following conditions hold.

$$A(0) - C_A + \beta V(\pi_O) \leq R(0) + \beta V(\rho_t), \quad \forall \rho_t \in [1 - \pi_N, \pi_O) \tag{3.21}$$

$$A(\pi_O) - C_A + \beta V(\pi_O) \geq R(\pi_O) + \beta V(1 - \pi_N) \tag{3.22}$$

We can re-write the inequality 3.21 as

$$\begin{aligned} & A(0) - C_A + \beta V(\pi_O) \leq R(0) + \beta V(\rho_t) \\ \iff & B(0) - C_A \leq \beta[V(\rho_t) - V(\pi_O)]. \end{aligned}$$

Now, realize that

$$\begin{aligned} V(\rho_t) - V(\pi_O) &= \sum_{k=0}^{\infty}((1-p_L)\beta)^k[p_L(A(\rho_{t+k}) - A(\pi_O)) + (1-p_L)(R(0) - A(\pi_O) + C_A)] \\ &> \sum_{k=0}^{\infty}((1-p_L)\beta)^k[(1-p_L)(R(0) - A(\pi_O) + C_A)] \\ &= \frac{(1-p_L)}{1-(1-p_L)\beta}(R(0) - A(\pi_O) + C_A). \end{aligned}$$

Then condition 3.12 implies that

$$B(0) - C_A \leq \frac{\beta(1-p_L)}{1-(1-p_L)\beta}(R(0) - A(\pi_O) + C_A) < \beta[V(\rho_t) - V(\pi_O)].$$

Consider the inequality 3.13. We can re-write it as

$$\begin{aligned} & A(\pi_O) - C_A + \beta V(\pi_O) \leq R(\pi_O) + \beta V(1 - \pi_N) \\ \iff & B(\pi_O) - C_A \leq \beta[V(1 - \pi_N) - V(\pi_O)]. \end{aligned}$$

$$V(1-\pi_N) - V(\pi_O) = \sum_{k=0}^{\infty}((1-p_L)\beta)^k[p_L(A(\rho_k) - A(\pi_O)) + (1-p_L)(R(0) - A(\pi_O) + C_A)]$$

where $\rho_0 = 1 - \pi_N$. Notice that

$$\lim_{p_L \to 0}[V(1-\pi_N) - V(\pi_O)] = \frac{1}{1-\beta}(R(0) - A(\pi_O) + C_A).$$

Then, condition 3.13 makes sure that there exists a $p_L > 0$ small enough such that inequality 3.22 holds.

Now, take the government strategy when C_A is drawn as given. At any reputation $\rho \in$

$[1-\pi_N, \pi_O]$, declaring an amnesty for government is optimal if

$$A(\rho) - C_L + \beta V(\pi_O) \geq R(\rho) + \beta V(1-\pi_N) \iff B(\rho) - C_L \geq \beta[V(1-\pi_N) - V(\pi_O)]$$

We already know the limit properties of the difference in value functions. We also know that function $B(.)$ is increasing. Hence, condition 3.14 implies the above inequality. □

Appendix B. Some Further Results

In this section, we will derive some further results by focusing on pure-strategy Markov-perfect equilibria. Since we know how the game is played out after an amnesty, we can compute the continuation value of declaring amnesty easily. Given this continuation value, we can find some further results. For our analysis on this section, we will make one more assumption on the parameters.

$$B(0) - C_A < \beta[A(1-\pi_N) - A(\pi_O)]. \tag{3.23}$$

Lemma 6. Consider any Markov Perfect Equilibria in pure strategies. In the subgame which starts with reputation $1-\pi_N$, the equilibrium outcome is *no amnesty*.

Proof. Suppose not. Then, there should a MPE in pure strategies such that $(\phi^*(1-\pi_N), x^*(1-\pi_N)) = (1-\pi_N, 1)$. Now, consider a one-shot deviation of $\hat{x}(1-\pi_N) = 0$ for opportunistic government. It cannot be a profitable deviation, so

$$A(1-\pi_N) - C_A + \beta V(\pi_O) \geq R(1-\pi_N) + \beta V(1-\pi_N)$$

Now, remember that we know it must be $(\phi^*(\pi_O), x^*(\pi_O)) = (\pi_O, 1)$. Then, we can compute the continuation values as follows.

$$\begin{aligned} V(\pi_O) =& A(\pi_O) - C_A + \beta V(\pi_O) \\ V(1-\pi_N) =& A(1-\pi_N) - C_A + \beta V(\pi_O) \end{aligned}$$

Now, we can substitute them into the inequality above.

$$\begin{aligned} A(1-\pi_N) - C_A + \beta[A(\pi_O) - C_A + \beta V(\pi_O)] \geq& R(1-\pi_N) + \beta[A(1-\pi_N) - C_A + \beta V(\pi_O)] \\ A(1-\pi_N) - C_A + \beta[A(\pi_O)] \geq& R(1-\pi_N) + \beta[A(1-\pi_N)] \\ B(1-\pi_N) - C_A \geq& \beta[A(1-\pi_N) - A(\pi_O)] \end{aligned}$$

which contradicts with the condition 3.23. □

With Lemma 6, we established that any MPE in pure strategies should have *no amnesty* in the best reputation. With Proposition 3 we have already established that any MPE in pure strategies should have *declaring amnesty* in the worst reputation. It is difficult to characterize the whole set of Markov-Perfect equilibria. However, we can set what are the best and worst equilibria for the government. The following definition will provide us a way to compare different equilibria.

Definition 2. Consider two arbitrary MPE in pure strategies, e_1 and e_2. We say "e_1 is a better equilibrium for government than e_2" if the set of reputation levels with *no amnesty* outcome under e_2 is a subset of the set of reputation levels with *no amnesty* outcome under e_1:

$$\forall \rho \in [1-\pi_N, \pi_O], \quad (\phi^*_{e_2}(\rho), x^*_{e_2}(\rho)) = (0,0) \implies (\phi^*_{e_1}(\rho), x^*_{e_1}(\rho)) = (0,0)$$

Proposition 9. Assume that the following condition holds:

$$\frac{B(\pi_O) - C_A}{R(0) - A(\pi_O) + B(\pi_O)} \geq \beta \tag{3.24}$$

Then, the best MPE in pure strategies is

$$(\phi^*(\pi_O), x^*(\pi_O)) = (\pi_O, 1), \quad (\phi^*(\rho), x^*(\rho)) = (0,0) \quad \forall \rho \in [1-\pi_N, \pi_O)$$

Proof. Let us first show that it is an equilibrium. First, given that the taxpayer belief is $\phi^*(\pi_O) = \pi_O$, $x^*(\pi_O) = 1$ is a best-response if

$$B(\pi_O) - C_A \geq \beta[V(1-\pi_N) - V(\pi_O)]$$

Given the equilibrium, we can compute the continuation values as follows.

$$V(\pi_O) = A(\pi_O) - C_A + \beta(A(\pi_O) - C_A) + \beta^2(A(\pi_O) - C_A) + \ldots = \frac{A(\pi_O) - C_A}{1-\beta}$$
$$V(1-\pi_N) = R(0) + \beta(R(0)) + \beta^2(R(0)) + \ldots = \frac{R(0)}{1-\beta}$$

We can incorporate them into our inequality.

$$B(\pi_O) - C_A \geq \beta\left[\frac{R(0)}{1-\beta} - \frac{A(\pi_O) - C_A}{1-\beta}\right]$$

which holds if the condition 3.24 holds. Second, for any other reputation $\rho \in [1-\pi_N, \pi_O)$, $x^*(\rho) = 0$ is optimal if

$$B(0) - C_A \leq \beta[V(\Gamma(\rho, 0)) - V(\pi_O)]$$

Given the equilibrium, and given the fact that $\rho^* \in (1-\pi_N, \pi_O)$, we can pin down

$$V(\Gamma(\rho, 0)) = \frac{R(0)}{1-\beta}$$

Then, we can re-write the inequality as

$$\begin{aligned}
B(0) - C_A \leq & \beta \left[\frac{R(0)}{1-\beta} - \frac{A(\pi_O) - C_A}{1-\beta} \right] \\
(1-\beta)(B(0) - C_A) \leq & \beta \left[R(0) - A(\pi_O) - C_A \right] \\
B(0) - C_A - \beta B(0) + \beta C_A \leq & \beta R(0) - \beta A(\pi_O) - \beta C_A \\
B(0) - C_A \leq & \beta R(0) - \beta A(\pi_O) + \beta B(0) \\
B(0) - C_A \leq & \beta A(0) - \beta A(\pi_O)
\end{aligned}$$

which holds since the condition 3.23 holds. It is trivial to show that beliefs are consistent, so it is an equilibrium. Then, Lemma 3 implies that this equilibrium must be the best MPE in pure strategies. □

Now, let us try to construct the worst equilibrium. In a given reputation level ρ, $(\phi^*(\rho), x^*(\rho)) = (\rho, 1)$ can be an equilibrium if the following inequality holds.

$$B(\rho) - C_A \geq \beta [V(1 - \pi_N) - V(\pi_O)] \tag{3.25}$$

From proposition 3 we know that the above inequality should hold when $\rho = \pi_O$. Also, from lemma 6 we know that it cannot hold when $\rho = \pi_N$. Since $B(.)$ is a strictly increasing function, there should be a point $R \in (1 - \pi_N, \pi_O]$ such that

$$B(R) - C_A = \beta [V(1 - \pi_N) - V(\pi_O)] \tag{3.26}$$

We know the continuation value $V(\pi_O)$ from Proposition 3. The continuation value $V(1 - \pi_N)$ is ambiguous and depends on equilibrium payoffs. Lemma 6 implies that the period starts with state $1 - \pi_N$ should be played as *no amnesty*. However, we don't know the rest of the periods. The game may continue with no amnesty forever. If the game is played as *no amnesty* forever, then the continuation value is easy to compute, i.e. discounted infinite some of $R(0)$. Then, the threshold R would be

$$B(R) - C_A = \beta \left[\frac{R(0)}{1-\beta} - \frac{A(\pi_O) - C_A}{1-\beta} \right].$$

Alternatively, at some point we may see an amnesty and jump to the state π_O. To give a general form for that case, let us define a sequence

$$\begin{aligned}
S = & (r_0, r_1, r_2, r_3, \ldots) \text{ where} \\
r_0 = & 1 - \pi_N \text{ and } r_{t+1} = \Gamma(r_t, 0), \ \forall t \in \mathbb{Z}_{\geq 0}
\end{aligned}$$

We can generalize the continuation value $V(1-\pi_N)$ as follows:

$$V(1-\pi_N) = (1+\beta+\beta^2+.....+\beta^{T-1})R(0) + \beta^T(A(r_T)-C_A) + \beta^{T+1}(V(\pi_O))$$

where T is the period that government declares amnesty and with a little abuse of notation $T=\infty$ represents the case where the game is played as *no amnesty* forever. Realize that for an equilibrium to exists with a $T \in \mathbb{Z}_{\geq 0}$, the inequality 3.25 should hold when $\rho = r_T$, so $r_T > R$. Before going further, we will introduce a useful result.

Proposition 10. Assume that the condition 3.8 holds. Define the sequence S and function $W(.)$ as described before.

$$S = (r_0, r_1, r_2, r_3, \ldots) \text{ where}$$
$$r_0 = 1-\pi_N \text{ and } r_{t+1} = \Gamma(r_t, 0), \; \forall t \in \mathbb{Z}_{\geq 0}$$

$$W(T) = (1+\beta+\beta^2+.....+\beta^{T-1})R(0) + \beta^T(A(r_T)-C_A) + \beta^{T+1}(V(\pi_O))$$

Consider the set

$$D = \{(R,T) \mid r_T \geq R \text{ and } B(R) - C_A = \beta[W(T) - V(\pi_O)]\}$$

The following is the full characterization of the worst MPE in pure strategies:

1. If D is an empty set, then the worst MPE in pure strategies is

$$\forall \rho \geq \underline{R}, \quad (\phi^*(\rho), x^*(\rho)) = (\rho, 1)$$
$$\forall \rho < \underline{R}, \quad (\phi^*(\rho), x^*(\rho)) = (0, 0)$$
$$\text{where } \underline{R} \text{ is st. } B(\underline{R}) - C_A = \beta\left[\frac{R(0) - A(\pi_O) + C_A}{1-\beta}\right]$$

2. If D is non-empty, define

$$(\underline{R}, \underline{T}) = \arg\min_{(R,T)\in D} R.$$

Then, the worst MPE in pure strategies is

$$\forall \rho \geq \underline{R}, \quad (\phi^*(\rho), x^*(\rho)) = (\rho, 1)$$
$$\forall \rho < \underline{R}, \quad (\phi^*(\rho), x^*(\rho)) = (0, 0)$$

Proof. 1. Assume that D is empty. Consider the Markov strategy profile (ϕ^*, x^*) such that

$$\forall \rho \geq \underline{R}, \quad (\phi^*(\rho), x^*(\rho)) = (\rho, 1)$$
$$\forall \rho < \underline{R}, \quad (\phi^*(\rho), x^*(\rho)) = (0, 0)$$
$$\text{where } \underline{R} \text{ is st. } B(\underline{R}) - C_A = \beta \left[\frac{R(0) - A(\pi_O) + C_A}{1-\beta} \right].$$

First, notice that $\underline{R} \geq \rho^*$. To see that, assume it is not. Then, $\underline{R} < \rho^*$. Now, since $\lim_{T\to\infty} r_T = \rho^*$, there exist a big enough integer K such that $r_K > \underline{R}$. Define L such that

$$B(L) - C_A = \beta[W(K) - V(\pi_O)]$$

Since $W(.)$ is increasing and $\lim_{T\to\infty} W(T) = \frac{R(0)}{1-\beta}$, $W(K) < \frac{R(0)}{1-\beta}$. Then, $L < \underline{R}$. But, then $r_K > L$ and the pair $(L, K) \in D$, which is a contradiction. Therefore, $\underline{R} \geq \rho^*$.

Second, notice that the proposed Markov strategy profile implies

$$V(1 - \pi_N) = \frac{R(0)}{1-\beta}.$$

We can show that the proposed profile is an equilibrium. The beliefs ϕ^* are consistent with x^* for any reputation. We just need to show that x^* is a best-response, given taxpayers' behavior is based on the belief ϕ^*. Take any reputation level $\rho \geq \underline{R}$. Since B is increasing,

$$B(\rho) - C_A \geq \beta \left[\frac{R(0) - A(\pi_O) + C_A}{1-\beta} \right] = \beta[V(1 - \pi_N) - V(\pi_O)].$$

Therefore, $x^*(\rho) = 1$ is a best-response.

Now, take any reputation level $\rho < \underline{R}$ and consider the following inequality

$$B(0) - C_A \leq \beta \left[V(\rho') - V(\pi_O) \right]$$

where $\rho' = \Gamma(\rho, 0)$. Proposition 1 implies that $\rho' < \underline{R}^*$ and $\Gamma(\rho', 0) = \rho'' < \underline{R}$ and $\Gamma(\rho'', 0) = \rho''' < \underline{R}$ and so on. The proposed equilibrium suggests that this will a infinite sequence of *no amnesty* plays. Therefore,

$$V(\rho') = \frac{R(0)}{1-\beta}.$$

Then, our inequality becomes

$$B(0) - C_A \leq \beta \left[\frac{R(0) - A(\pi_O) + C_A}{1-\beta} \right]$$

Since we know that $\underline{R} \geq \rho^* > 0$ and the function $B(.)$ is increasing, the above inequality must hold. Hence, the proposed Markov strategies are indeed an equilibrium.

To see that it is the worst equilibrium, suppose that it is not. Then, there must be another equilibrium $\hat{\phi}, \hat{x}$ in which

$$\forall \rho \geq \underline{R}, \quad (\hat{\phi}, \hat{x}) = (\rho, 1)$$
$$\exists \hat{\rho} < \underline{R}, \quad (\hat{\phi}, \hat{x}) = (\hat{\rho}, 1)$$

Then, it must be

$$B(\hat{\rho}) - C_A \geq \beta[V(1-\pi_N) - V(\pi_O)].$$

If $V(1-\pi_N) = \dfrac{R(0)}{1-\beta}$, then above inequality cannot hold by definition of $\underline{R}$. So, it must be the case that $V(1-\pi_N) = W(\hat{T})$ for some positive integer $\hat{T}$. It means that the following inequality holds,

$$B(r_{\hat{T}}) - C_A \geq \beta[W(\hat{T}) - V(\pi_O)].$$

Now define

$$B(\hat{R}) - C_A = \beta[W(\hat{T}) - V(\pi_O)].$$

Since $B(.)$ is increasing, $r_{\hat{T}} \geq \hat{R}$. But then, $(\hat{R}, \hat{T}) \in D$, which is a contradiction. Hence, the proposed equilibrium must be the worst one.

2. Now, D is non-empty and

$$(\underline{R}, \underline{T}) = \arg \min_{(R,T) \in D} R$$

First, realize that $r_{\underline{T}-1} < \underline{R}$. To see that, assume it is not. Then, $r_{\underline{T}-1} \geq \underline{R}$ which means

$$B(r_{\underline{T}-1}) - C_A \geq \beta[W(\underline{T}) - V(\pi_O)] > \beta[W(\underline{T}-1) - V(\pi_O)]$$

where the second inequality comes from the fact that $W(.)$ is increasing. Define L such that

$$B(L) - C_A = \beta[W(\underline{T}-1) - V(\pi_O)].$$

Clearly, $r_{\underline{T}-1} \geq L$ which makes $(L, \underline{T}-1) \in D$. However, since $W(.)$ and $B(.)$ are increasing functions, $L < \underline{R}$, which is a contradiction.

By definition, $r_{\underline{T}} \geq \underline{R}$. The proposed Markov strategy profile suggests $V(1-\pi_N) = W(\underline{T})$. We also know

$$B(\underline{R}) - C_A = \beta[W(\underline{T}) - V(\pi_O) = \beta[V(1-\pi_N) - V(\pi_O)].$$

Then, it is easy to see that for any $\rho \geq \underline{R}$,

$$B(\rho) - C_A \geq \beta[V(1-\pi_N) - V(\pi_O)].$$

Now, let's take an arbitrary $\rho_0 < \underline{R}$. By definition, $\underline{R} < \rho^*$. So, the game starts at ρ_0, must pass to the other side of $\underline{R}$ in finite number of periods, which will end up being a *declaring amnesty* equilibrium. Then, there must exist a positive integer k such that $\rho_{k-1} < \underline{R} \leq \rho_k$ where the set $\Sigma = \{\rho_0, \rho_1, \rho_2, \ldots, \rho_{k-1}, \rho_k\}$ represents the set of first k states the game will follow if it starts at ρ_0, i.e.

$$\rho_m = \Gamma(\rho_{m-1}, 0), \quad \forall m \leq k.$$

Note that it makes

$$\begin{aligned}
&(\phi^*(\rho_m), x^*(\rho_m)) = (0,0), \text{ if } m < k \\
&(\phi^*(\rho_k), x^*(\rho_k)) = (\rho_k, 1)
\end{aligned}$$

For this to constitute an equilibrium, the following set of inequalities must be satisfied.

$$\begin{aligned}
&B(0) - C_A \leq \beta(V(\rho_1) - V(\pi_O)) \\
&B(0) - C_A \leq \beta(V(\rho_2) - V(\pi_O)) \\
&B(0) - C_A \leq \beta(V(\rho_3) - V(\pi_O)) \\
&\ldots \\
&B(0) - C_A \leq \beta(V(\rho_{k-1}) - V(\pi_O)) \\
&B(0) - C_A \leq \beta(V(\rho_k) - V(\pi_O)) \\
&B(\rho_k) - C_A \geq \beta(V(1-\pi_N) - V(\pi_O)).
\end{aligned}$$

where

$$\begin{aligned}
V(\rho_k) =& A(\rho_k) - C_A + \beta V(\pi_O) \\
V(\rho_m) =& R(0) + \beta V(\rho_{m+1}), \quad \forall m < k \\
V(1-\pi_N) =& W(\underline{T}) \\
V(\pi_O) =& A(\pi_O) - C_A + \beta V(\pi_O).
\end{aligned}$$

First, focus on the last inequality:

$$\rho_k \geq \underline{R} \implies B(\rho_k) - C_A \geq B(\underline{R}) - C_A = \beta(W(\underline{T}) - V(\pi_O)) = \beta(V(1-\pi_N) - V(\pi_O)).$$

Second, realize that

$$V(\rho_k) - V(\pi_O) = A(\rho_k) - A(\pi_O).$$

Then,

$$B(0) - C_A \leq \beta(V(\rho_k) - V(\pi_O)) = \beta(A(\rho_k) - A(\pi_O))$$

which holds, since condition 3.8 holds. Lastly, realize that $V(\rho_m) \leq V(\rho_{m-1})$ for any $m < k$. To see that,

$$\begin{aligned}
V(\rho_{m-1}) =& (1 + \beta + \beta^2 + \ldots + \beta^{k-m-2})R(0) + \beta^{k-m-1}(A(\rho_k) - C_A) + \beta^{k-m}V(\pi_O) \\
V(\rho_m) =& (1 + \beta + \beta^2 + \ldots + \beta^{k-m-1})R(0) + \beta^{k-m}(A(\rho_k) - C_A) + \beta^{k-m+1}V(\pi_O) \\
\implies V(\rho_{m-1}) - V(\rho_m) =& \beta^{k-m-1}[A(\rho_k) - C_A - R(0)] + \beta^{k-m}[A(\pi_O) - A(\rho_k)] \\
\text{(A is decreasing)} \quad & \leq \beta^{k-m-1}[A(0) - C_A - R(0)] + \beta^{k-m}[A(\pi_O) - A(\rho^*)] \\
& \leq \beta^{k-m-1}[B(0) - C_A] + \beta^{k-m}[A(\pi_O) - A(\rho^*)] \\
\text{(From condition 3.8)} \quad & \leq 0
\end{aligned}$$

This result implies that

$$B(0) - C_A \leq \beta(V(\rho_m) - V(\pi_O)) \quad \forall m < k.$$

Since ρ_0 was arbitrary, the proposed Markov strategy profile is an equilibrium.

To see that it is the worst equilibrium, suppose that it is not. Then, there must be another equilibrium $\hat{\phi}, \hat{x}$ in which

$$\begin{aligned}
\forall \rho \geq \underline{R}, \quad (\hat{\phi}, \hat{x}) = (\rho, 1) \\
\exists \hat{\rho} < \underline{R}, \quad (\hat{\phi}, \hat{x}) = (\hat{\rho}, 1)
\end{aligned}$$

Then, it must be

$$B(\hat{\rho}) - C_A \geq \beta[V(1 - \pi_N) - V(\pi_O)].$$

If $V(1 - \pi_N) = W(\underline{T})$, then above inequality cannot hold. Remember that $W(.)$ is increasing. So, it must be the case that $V(1 - \pi_N) = W(\hat{T})$ for some positive integer $\hat{T} < \underline{T}$. Also, the following inequality must hold:

$$B(r_{\hat{T}}) - C_A \geq \beta[W(\hat{T}) - V(\pi_O)].$$

Define

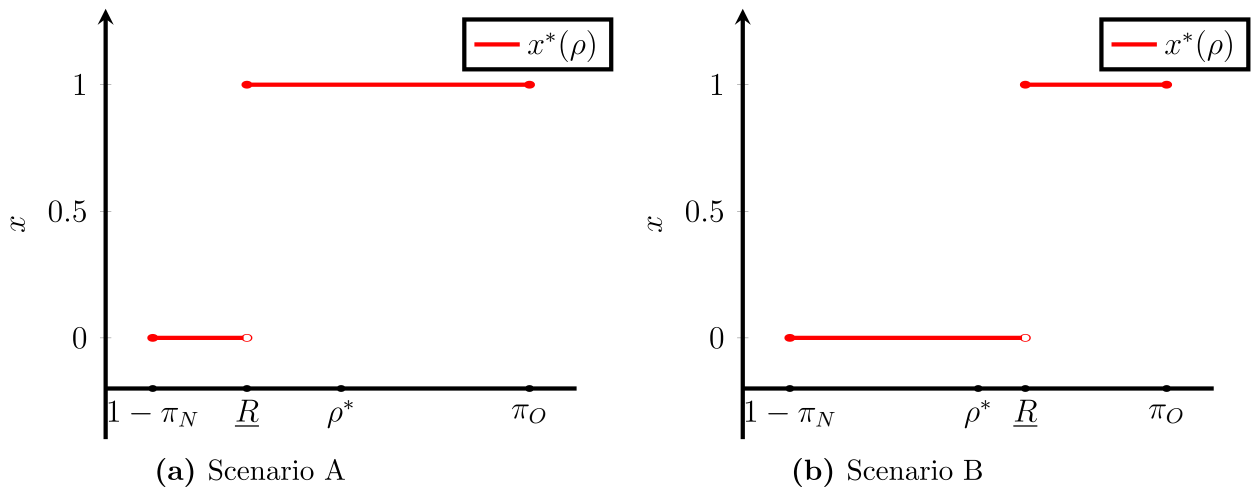
$$B(\hat{R}) - C_A = \beta[W(\hat{T}) - V(\pi_O)].$$

Since $W(.)$ is increasing, $\hat{R} < \underline{R}$. Since $B(.)$ is increasing, $r_{\hat{T}} \geq \hat{R}$. But then, $(\hat{R}, \hat{T}) \in D$, which is a contradiction. Hence, the proposed equilibrium must be the worst one.

□

Shortly, Proposition 10 says that the worst equilibrium should look like one of the following:

Figure 3.4: Two scenarios for the worst MPE in pure strategies



Notice that whether we are in scenario A or scenario B depends on further conditions parameters of Markov transition matrix, π_O and π_N, which pin down the value of ρ^*. This is an interesting result from two reasons. First, in both scenarios our model can give an explanation for the heterogeneity among different economies in terms of tax amnesty frequency. Just a small difference in initial reputation or a temporary difference in government type might make two otherwise identical countries see very different paths of tax amnesty implementations. Second, scenario B also explains how an economy may enter a sequence of tax amnesties after a period of no-amnesty. To see this, take an initial reputation on the interval $[1-\pi_N, \underline{R})$. Since the equilibrium is *no amnesty* on this interval, the game will start with a no amnesty period. The game will continue without an amnesty until it passes to the interval $[\underline{R}, \rho^*)$, and it will happen in a finite period of time since the reputation will converge to ρ^* with a sequence of *no amnesty* periods.

Appendix C. Detailed Solution to the Stage Game

Our single period model is essentially a sequential game. We will focus on the sequentially rational equilibria with rational expectations. To simplify notation, we will drop the time subscript for this section. Also, we will take the reputation ρ as a given parameter in this section and assume that $\rho \in (0, 1)$. We will introduce the problems of agents in the following order: Tax evaders' tax amnesty participation problem; the *opportunistic* government's amnesty tax rate decision if it declares one; and taxpayers' initial income declaration problem. Lastly, we will discuss the government's amnesty declaration decision.

Tax Evaders- Notice that any partial evasion cannot be optimal. To see this, remember the publicly known distribution which taxpayers are drawing income from only has two income levels.

For individuals who draw 0, the unique optimal decision is to declare their true income. Therefore, any individual who declares an income different than zero would reveal that she drew w. Then, the evaders in this economy are the ones who draw w as income, but declare that they have zero income. From now on, I will mention the set of individuals with income $y_i = w$ and declared income $y_i^d = 0$ as tax evaders. Realize that if there is no tax amnesty, there is no decision to make for tax evaders. In that case, the expected payoff of a tax evader would be

$$w - (p + \epsilon_i)w$$

Let us focus on the case where there is a tax amnesty. Then, the problem of tax evader i in case of an amnesty is

$$\max_{y_i^a \in [0,w]} w - ay_i^a - p(w - y_i^a) - \epsilon_i(w - y_i^a).$$

Notice that declaring anything other than 0 would reveal that the person has an income of w. Then, the maximization problem of a tax evader is trivial to solve. There are only two potentially optimum declaration $\{0, w\}$. Then, for an evader, the optimal income declaration during the amnesty period is

$$y_i^{a*} = \begin{cases} w & a \leq p + \epsilon_i \\ 0 & a > p + \epsilon_i \end{cases}.$$

Tax evaders declare their true income during an amnesty if the cost they pay is weakly lower than the expected cost of not declaring true income. Given this optimal decision function we can go one-step backward and solve the government's optimal decision for the amnesty tax rate a.

Government- Note that we first need to find what is the optimal amnesty tax rate, a, if the government decides to declare an amnesty. Before we start to solve the government's problem, we can simplify it by organizing our thoughts on how the agents split up into tax evaders and taxpayers in this economy. We only focus on individuals who draw income w. Therefore, the income of taxpayers as well as their characteristics are totally identical, except their private preference parameter ϵ. It is the deciding factor on splitting the population into subgroups in terms of their tax behavior. At the equilibrium, there is an $\bar{\epsilon}$ such that all the individuals with $\epsilon_i < \bar{\epsilon}$ declare $y_i^d = 0$ while all the individuals with $\epsilon_i \geq \bar{\epsilon}$ declare their true income $y_i^d = w$.

Notice that $\bar{\epsilon}$ is an equilibrium object, which arises before the government's amnesty decision. The government doesn't observe any individual's income or preference parameter. However, it knows the distributions that taxpayers draw these values, so it can perfectly predict $\bar{\epsilon}$ after seeing the total taxes collected. With this observation, we can write the government's problem given the threshold of truthful income declaration $\bar{\epsilon}$ as

$$\max_a \int_0^{\bar{\epsilon}} \mathbb{I}_{\{y_i^{a*}(a)=w\}} aw d\epsilon + \int_0^{\bar{\epsilon}} \mathbb{I}_{\{y_i^{a*}(a)=0\}} pw d\epsilon$$

Since we know the optimal amnesty participation strategy of tax evaders, $y_i^{a*}(a)$ for all i, we can re-write government's problem as

$$\max_a \int_0^{\bar{\epsilon}} \mathbb{I}_{\{\epsilon \geq a-p\}} a w d\epsilon + \int_0^{\bar{\epsilon}} \mathbb{I}_{\{\epsilon \geq a-p\}} p w d\epsilon.$$

Then, the optimal amnesty tax rate is

$$a^*(\bar{\epsilon}) = \frac{\bar{\epsilon}}{2} + p. \tag{3.27}$$

Taxpayers- Given the taxpayers' belief on the probability of an amnesty and the optimal decision functions we derived from the later stages, taxpayer i's problem at the beginning of the game becomes:

$$\max_{y_i^d \in [0,w]} y_i - \tau y_i^d - \phi a^*(\bar{\epsilon}) y_i^{a*}(\bar{\epsilon}, y_i^d) - (p + \epsilon_i)(y_i - y_i^d - y_i^{a*}(\bar{\epsilon}, y_i^d))$$

As we discussed, the threshold $\bar{\epsilon}$ determines the optimal tax evasion decision of a taxpayer at the beginning of the game.

$$y_i^{d*} = \begin{cases} w & \epsilon_i \geq \bar{\epsilon} \\ 0 & \epsilon_i < \bar{\epsilon} \end{cases}$$

To find the cutoff value $\bar{\epsilon}$, let us focus on the taxpayer who is indifferent between declaring the true income and evading. Such a taxpayer exists because of Assumption 1. Because of the linear nature of our model, she is unique. The value of declaring truthfully should be equal to the value of declaring zero income for her. In other words, the following condition should hold with her private value of ϵ_i.

$$w - w\tau = w - \phi \left(\frac{\bar{\epsilon}}{2} + p\right) w + (1 - \phi)(p + \epsilon_i) w.$$

Notice that her private ϵ_i should be the equilibrium $\bar{\epsilon}$, which would give us the threshold

$$\tau = \phi \left(\frac{\bar{\epsilon}}{2} + p\right) + (1 - \phi)(p + \bar{\epsilon}) \implies \bar{\epsilon}^* = \frac{2(\tau - p)}{2 - \phi}. \tag{3.28}$$

By plugging in the value of $\bar{\epsilon}$, we can give a characterization of a taxpayer's initial income declaration, the government's amnesty tax rate decision and a tax evader's amnesty participation decision as functions of the initial taxpayer belief.

$$y_i^{d*} = \begin{cases} w & \epsilon_i \geq \frac{2(\tau - p)}{2 - \phi} \\ 0 & \epsilon_i < \frac{2(\tau - p)}{2 - \phi} \end{cases}, \tag{3.29}$$

$$a^* = \frac{\tau + p(1-\phi)}{(2-\phi)}, \tag{3.30}$$

$$y_i^{a*} = \begin{cases} w & \epsilon_i \geq \dfrac{\tau - p}{2-\phi} \\ 0 & \epsilon_i < \dfrac{\tau - p}{2-\phi} \end{cases}. \tag{3.31}$$

The government's amnesty declaration- Since they all observe the same information, every taxpayer has the same initial belief. We can then derive the aggregate values of government revenues, given the optimal decisions presented in equations 3.29-3.31.

Chapter 4

Sustainable Intergenerational Insurance with Money

4.1 Introduction

Some macroeconomic shocks disproportionately affect certain age groups. The Great Recession in 2008 and the economic crisis caused by the coronavirus pandemic are two prominent examples of such shocks. Glover et al. (2020b) show that the Great Recession of 2008 had a disproportionately greater negative impact on the older generations while Glover et al. (2020a) show that the sectors young generations work more are the ones that are hit harder by the Covid-19 pandemic.

Governments can aim to provide insurance schemes that offset the intergenerational risks on income distributions. The sustainability of such policies is debated under limited enforcement. The policies would involve intergenerational transfers, and some generations may not want to make them if it does not benefit them in the future. Then, the study of sustainable intergenerational insurance is crucial to understanding the feasibility of intergenerational risk-sharing under limited enforcement.

This paper studies intergenerational insurance by incorporating monetary policy into a simple macroeconomic framework with intergenerational income risk. We build a 2-period overlapping generations model with a stochastic endowment process. The total endowment is constant, but the stochastic process distributes it to the young and the old agents. There is no intra-generational heterogeneity in income. The endowed consumption goods are perishable. No investment technology can transfer goods to future periods. The government's objective is to maximize social welfare. Welfare is defined as a weighted sum of the expected utilities of all agents where earlier generations have higher weights - the government discounts future generations. The government can introduce risk-sharing mechanisms that transfer wealth among agents. There is limited enforcement, i.e., an individual can opt out of the government plan in any given period. The government also controls money creation and can use monetary policy. Individuals cannot block any monetary policy decisions, but they can choose the amount of money they hold.

The lack of enforcement prevents the government from providing perfect risk-sharing. If the agent does not benefit from participation, there is no way to convince an agent to participate in the risk-sharing mechanism. If an agent is asked to pay a part of her income to the government, the government needs to promise a high enough future payoff to get a part of her current period income. However, such promises are not always feasible. The government cannot implement any welfare-improving transfers in some histories since it cannot convince agents to participate in the scheme. Our main result is that the monetary policy can improve welfare even in those states. Individuals have incentives to hold money for old age. Since individuals hold money, the government can use inflation as a tool for wealth transfers.

The government faces two main limitations as a result of its lack of enforcement. First, there is no way to get anything from an old agent. Since the old agent knows that she will be dead in the next period, any promise of future transfer is meaningless to her. Then, she would not participate in any plan that requires her to give up a part of her endowment. Second, a young agent would give a part of her endowment only if promised a future payoff to make the participation incentive compatible. Recent work by Lancia et al. (2020) shows that intergenerational transfers from the young to the old is infeasible after a certain amount of consecutive periods with the same transfers. It implies that transferring wealth from the young agents to the old agents every period is unsustainable. Hence, perfect risk-sharing is not possible under limited enforcement.

The introduction of money enhances the government's capabilities of providing intergenerational risk-sharing. The government can use simple monetary policies that indirectly transfer wealth from a generation to another. We analyze the effects of money on intergenerational risk-sharing in three steps. We first give money to the initial old and let it circulate in the economy without any other intervention. Second, we allow the government also to make surprise interventions to the economy. Third, we study a scenario in which the government declares a monetary policy rule at the beginning of time and credibly commits to it.

Money acts as a state-contingent asset and improves insurance in our setup. Money is given to the initial old and circulates with intergenerational trade in a monetary equilibrium. In each period, the young generations give a certain amount of their endowments to buy money from the old agents, and they sell their money to the newly born generation in the next period. In this way, money enables a saving mechanism for individuals. However, this is not the only role of the money in our setup. The price of money is driven by the young agents' demand for money. If the young agents' endowment is high, their demand for money is high, and the price of money in terms of consumption good is high. In other words, the price of money is higher when the old agents' endowment is lower. It means that when the old generation is unlucky from the endowment process, they get a higher income from selling the paper money. Hence, money is a state-contingent asset that insures the agents against income shocks.

Money provides insurance for income shocks, but risk-sharing is still imperfect. The government can improve risk-sharing further when we allow for surprise monetary interventions. We show that it is now possible for the government to transfer wealth from the old generation to the young by simply printing money and distributing it to the young generations. It creates inflation by decreasing the young agents' demand for money. Therefore, they buy the same amount of paper money from the old agents by giving them fewer goods in the new monetary equilibrium. We also show that news about a potential intervention in the next period affects the income distribution of the current period. If the young agents of the current period receive news that the government will distribute newly printed money to the young agents in the next period, they may change their demand for the money in the current period. We construct a numerical example that the news of an intervention next period increases the money demand in the current period. The young agents of the current period are willing to pay more consumption goods to buy the old agents' paper money. Then, the old agents consume more in the new monetary equilibrium. The news of the monetary intervention next period transfers wealth from the young agents to the old agents in the current period.

Allowing the government to make surprise monetary interventions enables policies that can transfer wealth between generations in both ways. However, the improvements provided by the government policies may disappear - they may even become harmful - when they are anticipated. We study the effect of anticipated government interventions by studying a particular monetary policy rule. The government declares a state-dependent policy rule at the beginning of time and credibly commits to it. The policy rule dictates no intervention when the young generation is lucky, but the government has to distribute some newly printed money to the young agents when the old generation is lucky. We show that this policy might be desirable by constructing a numerical example that the introduction of such a policy improves welfare.

Committing to the monetary policy rule affects the economy in different ways. It has a direct impact on the periods the government intervenes. The policy rule dictates money creation if the state is o. This newly created money is distributed to the young agents of that period. This directly benefits the young agents of that period while decreasing the wealth of the old agents. The second impact of the monetary policy rule is through affecting the expectations. All agents know the money they invest in today may partially lose its value with a monetary intervention next period. If the state of the next period is o, the government will create inflation that will lower the return of the savings on money. They incorporate this information on their money demand decision today. Therefore, the outcome of today is affected by the policy rule regardless of the state of the economy. Lastly, the agents of today also know that the young agents of tomorrow will face similar prospects for their futures. Then, the agents of today should also understand that the monetary policy rule impacts the money demand tomorrow for both states. It further affects the money demand today.

Since there are many forces that may affect the agents' behaviors in different ways, the net impact of all the forces on the money demand of a young generation depends on which force is dominant. We can only analyze the aggregate effect of all forces by comparing the outcomes with and without the monetary policy rule. Our variable of interest is the money demand. The change in money demand with the implementation of the policy rule gives us the distributional effect of the policy. If money demand is higher (lower) in a certain state of the economy, the young agents give more (less) consumption goods to the old agents in that state.

Although committing to a monetary policy certainly affects the total welfare, the direction and magnitude of the impact depend on the magnitude of the monetary intervention. The particular monetary policy we study only increases the money supply if the old generation is lucky in that period. Our numerical analyses suggest that there seems to be an optimal increase in the money supply. For small increases in money supply, the total welfare improves. The welfare improvement comes from the impact of the monetary policy on the unluckiest generations. The monetary policy improves the lifetime utility of an agent who is young in-state o and old in-state y. However, since the monetary policy is only a redistribution tool, some agents experience the negative impact of the policy on their lifetime utilities. On the other hand, big increases in money supply disrupt the utility of other agents so much that the total welfare may decrease with the introduction of the monetary policy. Hence, the committed magnitude of the monetary intervention is crucial.

There is already an established literature on risk-sharing in overlapping generations models. The literature treated the public policies as the main tool to provide intergenerational risk-sharing. The seminal works by Enders and Lapan (2021) and Shiller (1999) are examples of such scholarly contributions. These papers and many others in the literature assume perfect enforcement. A planner can easily take income from an individual and transfer it to another one. Our paper distinguishes itself by assuming limited enforcement. In our setup, anyone can opt-out of any policies.

The closest paper to our work is Lancia et al. (2020). They show that the perfect intergenerational risk-sharing is not sustainable with lack of enforcement. They study the performance of a range of policies, such as history-dependent policies, state-dependent policies, etc. Their setup does not have money and, therefore, monetary policy. They only allow the government to use simple transfer schemes without any enforcement. We, in fact, build on their framework by adding money and monetary policy. Our contribution is that the limited risk-sharing provided by the policies without enforcement can be improved by using money and monetary policies.

A branch of the social security literature studies the political economy aspect of it. The papers like Cooley and Soares (1999), Gonzalez-Eiras and Niepelt (2005), Boldrin and Rustichini (2000) work on intergenerational transfers with limited enforcement. However, the political authority can enforce the policies that are able to pass a certain voting procedure. This branch of the literature works on particular setups where policies can be implemented if a majority of the public accepts them. Our setup is more restrictive in this sense since the government cannot enforce a policy on an individual in public if that individual does not want it.

Our paper is also related to the role of money and monetary policy on risk-sharing and distribution. Many influential studies show the role of money on imperfect economic and contractual environments such as Samuelson (2021), Kocherlakota (1998), Aiyagari and Williamson (2000) etc. The imperfect economic environments give money a purpose and, therefore, value in these setups. Our paper is similar to them conceptually since the money is valuable in our setup thanks to its role in saving and insurance. Scheinkman and Weiss (1986), İmrohoroğlu (1992), Reed and Waller (2006) and many others study the effect of monetary policy on income distribution. Our paper distinguishes itself by focusing on a state-dependent monetary policy that can be perfectly anticipated.

The inflation decreases the value of the accumulated wealth of the old generations in our model. This is a key mechanism and it may rely on the fact that money is the only saving mechanism in our model. However, the households usually have other instruments such as bonds, real estates, stocks, etc. The empirical evidence seems to suggest that inflation has a similar intergenerational distributive impact even when considering households' other asset holdings. Doepke and Schneider (2006) analyzes the US household data on nominal asset holdings. They show that the main losers from a moderate inflationary period in US are the old households since they are the main bondholders. On the other hand, the young middle-class households gain the most since they usually have fixed-rate mortgage debt. Bielecki et al. (2021) builds a life cycle model and calibrates it by using Euro area household data. They show that a typical monetary easing transfers wealth from old generations to young generations.

The rest of the paper is structured as follows. Section 4.2 builds the baseline framework. Section 4.3 adds money to the baseline model and studies its effects. Section 4.4 studies the role of both surprise and anticipated monetary policies. Section 4.5 provides some concluding remarks.

4.2 Theoretical Framework

In this section, we introduce a baseline 2-period overlapping generations model with distributional risk. This is a simplified version of Lancia et al. (2020) framework that keeps the core of the setup unchanged. It is nice to see the limitations of the baseline economy to understand the importance of money and monetary policy in this setup.

Consider a 2-period overlapping generations endowment economy. There is no production. Every agent from each generation receives the single good of the economy as endowment in both periods of their lives. There is no technology for saving. The total endowment in the economy is constant, denoted with E. The distribution of total endowment depends on a stochastic process. The economy can be in two states, $Z = \{y, o\}$, which are i.i.d. The state of the economy determines the distribution of income to generations. There is no intra-generational risk meaning that different agents from the same generation receives exactly the same endowment. Let us also assume that the young (old) generation receives higher endowments than the old (young) in state y (state o). Agents receive utility only from consumption. The utility function u is strictly increasing and strictly concave; i.e. $u' > 0$ and $u'' < 0$. The agents discount the future with a discount factor

$\beta \in (0,1)$. We assume perfect information and constant population throughout this paper.

Let us write the maximization problem of a young agent in period t,

$$\begin{aligned}
\max_{c_t^y(z_t), c_t^o(z_{t+1})} \quad & u(c_t^y(z_t)) + \beta u(c_t^o(z_{t+1})) \\
s.to \quad & c_t^y(z_t) \leq e^y(z_t), \\
& c_t^o(z_{t+1}) \leq e^o(z_{t+1}), \\
& c_t^y(z_t) \geq 0, c_t^o(z_{t+1}) \geq 0
\end{aligned}$$

where e_t^y and e_t^o represents the endowment of a young agent in period t and the endowment of an old agent in period t. Note that the endowments depend only on the current state of the economy by construction. On the other hand, we could have entertained the possibility of history-dependent consumption patterns. Since the consumption levels will always be state dependent at an equilibrium throughout this paper, we abstain from doing so to simplify the notation from the beginning.

Let us also write the maximization problem of an old agent in period t,

$$\begin{aligned}
\max_{c_t^o(z_t)} \quad & u(c_t^o(z_t)) \\
s.to \quad & c_t^o(z_t) \leq e^o(z_t), \\
& c_t^o(z_t) \geq 0.
\end{aligned}$$

It is well-established that trade among agents usually suffers from the necessity of double coincidence in single good overlapping generations models, especially when there is no intra-generetional heterogeneity and there is no other asset to trade with. The best thing agents can do is to consume what they have, so the unique equilibrium is autarky.

Suppose that there is a government that has an objective to maximize the welfare in the economy. We define government's welfare function as the expected value of a weighted sum of utilities of all possible agents that will be born in this economy. More precisely, the government's welfare function at the beginning can be written as

$$\frac{\beta}{\delta} \sum_{z_0} u(c^0(z_0)) + \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \delta^t \left(u(c^y(z^t)) + \beta \sum_{z_{t+1}} \pi(z_{t+1}) u(c^o(z^t, z_{t+1})) \right) \right].$$

where δ is government's discount factor. For simplicity, we assume throughout this paper that the government's discount factor is equal to the agents' discount factor, $\delta = \beta$. It also provides the most interesting setup of intergenerational risk. This assumption makes sure that the government would like to transfer wealth from the young generation to the old generation in a period with state y, and it is vice versa in a period with state o.

Autarky is clearly suboptimal for most parametrization of this model, since there is intergenerational risk. Whether we can achieve the first-best depends on the assumptions on the capabilities of the government. Let us start the analysis by assuming full enforcement, i.e. the government can enforce any redistributional policy on the agents. Under full enforcement, only the balanced budget requirement constrains the set of feasible policies. The government can simply collect taxes from any agent and make transfers to any agent as long as transfers are equal taxes in total. The government can simply tax lucky generations and make transfers to unlucky generations to provide perfect risk-sharing to the agents. Hence, the first best is attainable.

The problem becomes more complicated if we assume limited enforcement. Suppose that the government cannot enforce agents to pay taxes. In such a setup, the government can only implement a policy if agents voluntarily participate it. Then, any agent which is worse off under a particular policy can opt out of the system. Therefore, limited enforcement greatly reduces the set of feasible policies. Lancia et al. (2020) shows that the government cannot achieve perfect risk-sharing under limited enforcement.

There are two underlying mechanisms that prevent perfect risk sharing under limited enforcement. First, the government can only tax a young agent by promising a considerable transfer when she is old. An agent would only give up some of her income if she will get a positive transfer in the future that keeps her lifetime utility at least at the level of her outside option, which is the autarky consumption. Second, an old agent would opt out from any policy that requires her to give up a part of her endowment. As a result, the government cannot transfer wealth from an old generation to a young generation in any time. Transferring wealth from the young generation to the old generation is also infeasible after a finite number of consecutive periods that the government transfers wealth from young to old. In each iteration, the government has to promise a bigger transfer to the young generations to keep them at the system, and after finite iterations the required level of promised future transfers become infeasible. The rest of the paper investigates how the money and monetary policy may overcome, at least partially, the limitations arising from the limited enforcement.

4.3 Introducing Money

Suppose that the government prints some amount of money at the beginning of time and distributes it to the initial old. We will denote the amount of money an initial old agent gets with m. There is no other intervention to the economy. Since our focus is the effect of money on the risk-sharing, we will focus on the monetary equilibria. However, just introducing money is not enough to make sure that money is valued and traded at an equilibrium. Assumption 2 makes sure that there exists a monetary equilibrium.

Definition 3. An equilibrium is a *monetary equilibrium* if money has a positive value in all periods and histories.

Assumption 2. Utility function, stochastic process and endowment levels are such that for every state z_t in every period t,

$$u'(e^y(z_t)) < \beta \sum_{z_{t+1} \in Z} \pi(z_{t+1}) u'(e^o(z_{t+1}))$$

The assumption says that the expected utility of the next period must be low enough for young agents so that saving a positive amount of their income is optimal at a zero interest rate. To have a monetary equilibrium in a deterministic 2-period overlapping generations model, saving on money must be attractive enough for young agents so that they would hold money if they expect it be demanded by also the next generations. Therefore, the young agents must have incentives to save for the future in every period. Assumption 2 is the stochastic counterpart of this argument. It ensures that saving is attractive enough in every possible state in every period. Therefore, a young agent would demand money in every history if she expects the money to be demanded in every possible state of the next period.

This assumption is satisfied when the endowment of unlucky old agents, $e^o(y)$, is sufficiently low compared to the endowment of unlucky young, $e^y(o)$. Since the total endowment is constant in every state, it also means that the endowment of lucky young agents, $e^y(y)$ must be the higher than the endowment of lucky old agents, $e^o(o)$. It implies that the endowment levels are higher on average for young agents. We expect an economy to have such a characteristic without a pension system.

Let us examine the maximization problem of a young agent at period t.

$$\begin{aligned} \max_{c_t^y(z_t), \{c_t^o(z_{t+1})\}_{z_{t+1} \in Z}, m_t(z_t)} \quad & u(c_t^y(z_t)) + \beta u(c_t^o(z_{t+1})) \\ s.to \quad & c_t^y(z_t) + p_t(z_t) m_t(z_t) \leq e^y(z_t), \\ & c_t^o(z_{t+1}) \leq e^o(z_{t+1}) + p_{t+1}(z_{t+1}) m_t(z_t),, \quad \forall z_{t+1} \in Z \\ & c_t^y(z_t) \geq 0, c_t^o(z_{t+1}) \geq 0, m_t(z_t) \geq 0. \end{aligned}$$

where $p_t(z_t)$ represents the price of money in terms of consumption goods at period t at state z_t. The young agents take the price of money as given and decides how much money they should buy. They can now save by buying money when they are young and selling it when they are old.

Maximization problem of an old agent in period t is

$$\begin{aligned} \max_{c_t^o(z_t)} \quad & u(c_t^o(z_t)) \\ s.to \quad & c_t^o(z_t) \leq e^o(z_t) + p_t(z_t) m_{t-1}(z_{t-1}) \\ & c_t^o(z_t) \geq 0. \end{aligned}$$

Note that the population and the amount of money in the economy are constant. It implies that every old agent will hold exactly m amount of money at the beginning of a period and every young agent will buy exactly m amount of money in every period; i.e. $m_t(z_t) = m, \forall z_t \in Z, \forall t$.

There is not a unique equilibrium in this new setup. We know at least one more equilibrium which is autarky. A standard approach in the analyses of overlapping generations model with money is to focus on the symmetric monetary equilibria. This assumption corresponds to stationary consumption patterns in deterministic setups. In our model, it should corresponds to state dependent consumption patterns, i.e. if $z_t = z_k$ for some periods t and k, then $c_t^i(z_t) = c_k^i(z_k), \forall i$.

The price of money should evolve with the demand for money. The demand for money depends on the saving decisions of young agents. All young agents are identical within a generation, so their demand for money should be the same. The young generations are heterogeneous only in their endowment. Then, we can conclude that the price of money is not history dependent, it is only state dependent, $p_t(z) = p_k(z), \forall z \in \{o, y\}$. From now on, we will drop the time subscript from the prices.

Proposition 11. Money is a state-contingent asset and it improves risk-sharing.

Proof. See Appendix A. □

It is interesting to see that money is not just a saving mechanism for the old age. It is also a state-contingent asset that offset some of the risk induced by the endowment process. To see this, let us focus on the total income of an old agent from an arbitrary period t,

$$e_t^o(z_t) + p_t(z_t)m. \tag{4.1}$$

Realize that the price of money is driven by the demand for money by the young agents. Their incentive for consumption smoothing implies that the more endowment they have, the more demand they should have for money. Then, $p(y) > p(o)$. Notice that the higher the endowment of the young agents, the lower the endowment of the old agents. When an old agent gets low income from the endowment process, her money will have higher value. The situation is similar from the perspective of a young agent as well. Remember that a young agent buys m units of money at the equilibrium independent of the state of the economy. However, the more income she gets from the endowment process, the higher the equilibrium price of money is . Hence, the money is a state-contingent asset that improves risk-sharing.

Although money provides some insurance, it does not provide perfect risk-sharing. Lemma 7 highlights this result.

Lemma 7. Money can only provide limited insurance.

Proof. See Appendix A. □

Notice that the economy still lacks a saving technology for the consumption good. There is no way to transfer goods from one period to another. Therefore, the feasible allocations for a planner, in this case the government, is very limited. The perfect risk-sharing can only be achieved by equalizing the consumption of agents who live in that period.[1] Then, Lemma 7 shows that there is still a room for improvement.

It is well-established that the monetary policy may have some redistributional consequences. In the next section we will allow government to use monetary policy to provide further risk sharing.

4.4 Monetary Policy

The analysis of monetary policy is two-fold. First, we investigate the effects of one-time monetary interventions. For this, we introduce an unanticipated monetary shock and analyze the effect of it on the consumption allocations at the monetary equilibrium. We also analyze the effect of a one-time news shock about a future monetary intervention. The examination of these one-time shocks hints us the characteristics of a monetary policy rule that can potentially improve welfare. We see that a one-time monetary intervention can transfer wealth from the old generation to young while a news shock about a future one-time intervention can transfer wealth from young to old. Second, we allow the government to declare a monetary policy rule at the beginning of time and we assume that the government credibly commits to it. Lastly, we discuss the results from the both parts of our monetary policy analysis.

4.4.1 One-Time Shocks

One-time monetary intervention

Consider the baseline model with money where initial old holds m amount of money. Suppose that in period t, the government prints μm amount of new money and distributes it to the young generation of period t equally. We assume that this was unanticipated by agents in previous periods and the agents do not anticipate such an intervention in the future as well[2]. Essentially, this is a one-time surprise intervention.

The consumption levels of the agents who are not alive at period t are unaffected by this intervention. Lemma 8 establishes this result.

Lemma 8. One-time monetary intervention only affects the consumption levels of the generations t and $t-1$.

[1] Here we implicitly use the assumption that $\beta = \delta$. In other words, the government discounts future generations exactly at the amount that an agent discounts future utility. Then, the utility an old agent gets tomorrow and the utility a young agent gets tomorrow have the same weight in government's objective.

[2] Assuming that the government can credibly commit to not intervene again in the future periods would be a way to do it.

Proof. See Appendix A. □

The intuition of Lemma 8 is pretty straightforward. The agents of previous periods did not anticipate the monetary intervention. Therefore, they did not change their decisions. The agents of future generations are also unaffected since everyone knows that the money supply will remain constant in the future.

The one-time monetary intervention we described in this subsection affects only the money demand of the young agents at period t. In other words, it affects the young agents' willingness to give up their endowed consumption goods to buy money from the old agents. Lemma 9 shows how the consumption levels of the young and old agents of period t are changed.

Lemma 9. One-time monetary intervention increases the consumption of the young agents and decreases the consumption of old agents at period t.

Proof. See Appendix A. □

The driving force of the result highlighted at Lemma 9 is the change in money demand. Since now the young agents already have a certain amount of money from the government, they have less willingness to give up consumption goods for the money the old agents hold. It implies that the price of money is lower at the equilibrium. Then, the young generation buys the same amount of money by giving less consumption goods to the old generation. Hence, the one-time monetary intervention makes the generation t better off while making the generation $t-1$ worse off.

Lemma 9 shows that distributing newly printed money to young generation can be a way to transfer wealth from an old generation to a young generation. However, our analysis here suffers from two fundamental problems. First, we assume that the intervention is unanticipated. Second, we assume that the intervention does not change the expectations of agents for the future periods. We tackle the first problem in the following subsection.

News about the one-time intervention

Consider that the information about a one-time monetary intervention that will happen in period $t+1$ becomes public in period t. Such news may alter the consumption allocation in period t. The young agents in period t want to buy money, but now they also anticipate that the government will decrease the value of their investment on money next period by printing new money. This may change their willingness to invest.

There is already some amount of intergenerational transfer from the young agents to the old agents in every period at a monetary equilibrium. The young agents voluntarily transfer consumption goods to the old agents to buy money.Let us define the saving of the young agents at period t as $s_t(z_t) = p_t(z_t)m_t(z_t)$.

This transformation helps us to see the immediate effect of potential policies. Focusing on the change of $s_t(z_t)$ should give us an immediate idea about how our policy affects the intergenerational distribution. A policy that increases $s_t(z_t)$ is actually increasing $c^o(z_t)$ while decreasing $c^y(z_t)$. Lemma 10 shows the change of the equilibrium $s_t z_t$ with the news shock we described.

Lemma 10. Suppose in period t the government declares that *there will be a one-time intervention next period, if the state of the next period is realized as o.* The one-time intervention is as described in Lemma 9. The effect of the news on period t is ambiguous.

Proof. See Appendix A. □

Lemma 10 shows that it is possible to transfer wealth from the young agents to the old agents of an arbitrary period with a news shock about the future. However, this effect depends on the parametrization. The reason is that the news shock creates two forces that pull the saving rate of the young agents to opposite directions. One of them is the substitution effect. The monetary intervention decreases the value of the money. If the young agents receive the news of an intervention next period, they understand that it will decrease the return of money that they buy this period. Therefore, they are less willing to pay for the money. Second force is the wealth effect. Since the value of the money will be less with the intervention, the future expected income of the young agents decreases. Their incentive for saving might be increased to provide better insurance for the future income risk.

4.4.2 Monetary Policy with Commitment

Our analysis up to now hints us the channels that a monetary intervention can affect the consumption allocation at the equilibrium. The government can transfer wealth from the young agents to old agents, or from the old agents to young agents by using surprise intervention and news shocks about them. However, the effects of the monetary interventions may change if they are anticipated by the agents. We would expect rational agents to anticipate such a move from the government if there is no commitment. For this reason, we work on a monetary policy with commitment in this subsection.

Suppose that the government commits credibly to the following monetary policy rule at the beginning of time.

$$\begin{aligned}&\text{For any period t,}\\&\text{if } z_{t+1} = o, \text{print } \mu m_t \text{ amount of money and distribute it to young agents}\\&\text{If } z_{t+1} = y,\ \text{do not intervene.}\end{aligned}$$

This is a promising candidate for a welfare improving monetary policy. Remember that the government would like to transfer wealth from the old agents to the young agents if the state is o. We know that distributing newly printed money to the young agents may be a way to do it. We also know that the probability of a monetary intervention in the next period might affect the saving decision of the agents in the current period. Therefore, although the government does not do anything in a period with state y, the agents' savings may still be affected by government's commitment to the described monetary policy. The young agents still expect that there is a probability of intervention in the next period. This might work like the news shock we described in the previous subsection.

Study of such a policy rule with commitment is more complicated than the study of one-time surprise interventions. The young agents in period t knows that if the state is o in period $t+1$, the government will intervene the economy. They make their saving decision based on this knowledge. Similarly, the young agents of period $t+1$ will also take the possibility of government intervention in period $t+2$ into account. The crucial part is that the young agents in period t knows this fact as well. They should take into account the fact that the decisions of the agents in period $t+1$ will be affected by the government's commitment to the policy rule. Hence, the state dependent structure of the policy rule makes its analysis complicated.

We assume that the government declares the policy rule at the beginning of period $t = 0$ and credibly commits to it. We take μ as a positive constant for now, but we will discuss the possibility of making it a choice variable for the government. We focus on the state-dependent equilibria as the baseline model. Since our policy rule is also state-dependent in this analysis, we will have two types of young agents; the one that is born in state y and the one that is born in state o. To solve for an equilibrium, we should solve the Euler equations of both types of agents simultaneously.

$$u'(e^y(y) - p_t(y)m_t) = \beta \left[\pi_o \frac{p_{t+1}(o)}{p_t(y)} u'(e^o(o) + p_{t+1}(o)m_t) + \pi_y \frac{p_{t+1}(y)}{p_t(y)} u'(e^o(y) + p_{t+1}(y)m_t)\right]$$
$$u'(e^y(o) - p_t(o)m_t) = \beta \left[\pi_o \frac{p_{t+1}(o)}{p_t(o)} u'(e^o(o) + p_{t+1}(o)m_t) + \pi_y \frac{p_{t+1}(y)}{p_t(o)} u'(e^o(y) + p_{t+1}(y)m_t)\right]$$

The price of money should change with every monetary intervention. However, saving of the young agents should be state-dependent only. We can re-write the Euler equations in the following form to simplify our analysis.

$$u'(e^y(y) - s(y)) = \beta \left[\pi_o \frac{s(o)}{s(y)} u'(e^o(o) + s(o)) + \pi_y u'(e^o(y) + s(y))\right] \quad (4.2)$$
$$u'(e^y(o) - s(o)) = \beta \left[\pi_o \frac{1}{1+\mu} u'(e^o(o) + s(o)) + \pi_y \frac{s(y)}{s(o)(1+\mu)} u'(e^o(y) + s(y))\right] \quad (4.3)$$
$$\text{where } s(z_t) = p_t(z_t)m_t = e^y(z_t) - c^y(z_t). \quad (4.4)$$

Note that solving the equations 4.2 and 4.3 together should give us the equilibrium characterization. The equilibrium is the same with the baseline economy if $\mu = 0$. Our focus is to study whether we can improve the risk-sharing by selecting $\mu > 0$.

Remember that the government wants to transfer wealth from the young agents to old agents in state y, and from the old agents to the young agents in state o. Since the intergenerational transfers are only possible through saving on money, the government wants to increase the equilibrium level of s_y and decrease the equilibrium level of s_o. If increasing μ gives such a result, that would improve risk-sharing. On the other hand, if increasing μ has the opposite effect for all $\mu \geq 0$, then the optimal choice for the government would be $\mu = 0$.

Without making any further assumptions, it is impossible to solve for closed-form solutions. Our results with the current assumptions is very limited. Lemma 11 summarizes them.

Lemma 11. Take an economy with $\mu \geq 0$. An increase in μ cannot have the following two effects at the same time:

- $s^*(o)$ increases
- $s^*(y)$ decreases.

Proof. See Appndeix A. □

Lemma 11 rules out the worst case scenario, but the effect of an increase in μ is ambiguous. We need further assumptions to advance our analysis. From now on, we will assume that the utility function is logarithmic, i.e. $u(.) = \ln(.)$. Then, we can characterize an equilibrium.

Proposition 12. Assume that $\pi_y \beta e^y(y) > e^o(y)$ and the utility is logarithmic. For a small enough $\mu > 0$, the equilibrium can be characterized with the following set of equations:

$$s^*(y) = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \tag{4.5}$$

$$s^*(o) = \frac{s^*(y)e^y(o)}{(1+\mu)e^o(y) - \mu s(y)}, \tag{4.6}$$

$$c^{y*}(y) = e^y(y) - s^*(y), \tag{4.7}$$

$$c^{o*}(y) = e^o(y) + s^*(y), \tag{4.8}$$

$$c^{y*}(o) = e^y(o) - s^*(o), \tag{4.9}$$

$$c^{o*}(o) = e^o(o) + s^*(o). \tag{4.10}$$

where

$$\begin{aligned}
a =& \mu e^o(o) - e^y(o) - \beta\pi_o e^y(o) + \beta\pi_y \mu e^o(o) - \beta\pi_y e^y(o), \\
b =& \mu e^o(o)e^o(y) - e^o(y)e^y(o) - e^o(o)e^y(y)(1+\mu) + \beta\pi_o e^y(o)e^y(y) - \beta\pi_y \mu e^o(o)e^y(y) \\
&+ \beta\pi_y e^y(o)e^y(y) - \beta\pi_o e^y(o)e^o(y) - \beta\pi_y e^o(o)e^y(y)(1+\mu), \\
c =& \beta\pi_o e^y(o)e^o(y)e^y(y) + \pi_y \beta e^o(o)e^y(y)^2(1+\mu) - e^o(y)e^o(o)e^y(y)(1+\mu).
\end{aligned}$$

Proof. See Appendix A. □

Although proposition 12 can characterize the equilibrium, it is not sufficient for our analysis. We aim to study the effect a change in monetary policy. However, we can only argue that the effect is ambiguous with our current knowledge.

For the rest of this section, we will continue with a particular numerical example to show that a monetary intervention can improve welfare even when it is anticipated. We use the parametrization given on Table 4.1. We also keep the assumption that the utility function is logarithmic, i.e. $u(.) = \ln(.)$.

Parameter	π_o	π_y	$e^y(y)$	$e^o(o)$	$e^y(o)$	$e^o(y)$	β
Value	0.5	0.5	0.99	0.51	0.49	0.01	0.9

Table 4.1: Numerical values assigned to the parameters.

We assume that the probability of the states are the same. It simplifies the welfare analysis by ensuring that the different history nodes of the same periods have equal weights in the government's objective function. Notice that the income distribution is more unequal in state y. Together with the discount factor, it ensures that the set of numerical values satisfy Assumption 2.

Given the numerical assumptions we make, we can find the unique symmetric monetary equilibrium for each value of μ. We can show that when $\mu = 0$, an infinitesimal increase in μ decreases s_o and s_y. Lemma 12 formalizes this result.

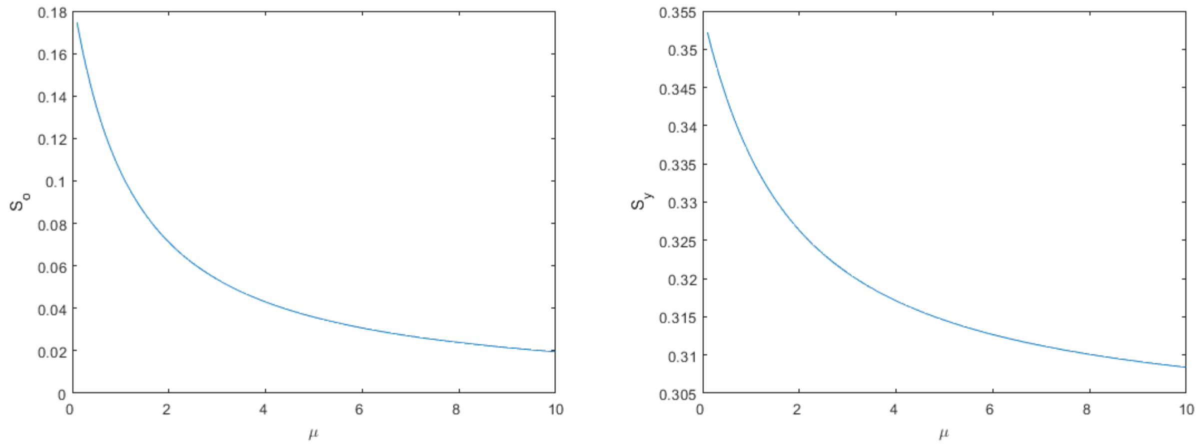
Lemma 12. Given the parametrization in Table 4.1 and assuming logarithmic utility,

$$\frac{\partial s_o}{\partial \mu}\Big|_{\mu=0} < 0, \quad \frac{\partial s_y}{\partial \mu}\Big|_{\mu=0} < 0. \tag{4.11}$$

Proof. See Appendix A. □

Lemma 12 shows a mixed picture in terms of risk-sharing. Selecting a monetary policy rule with small enough μ improves inequality in state o by decreasing the value of money. It implies that the young agents can buy money by giving up less consumption goods. It is a step towards better risk-sharing since it benefits the young agents in a state where they are unlucky. However, it also has the same effect at state y, which is undesirable since it benefits lucky young agents in that case. Figure 4.1 shows that the effect of μ on the saving levels is monotone in our numerical example.

Our numerical assumptions provides us more tools to analyze the change in welfare for monetary policy rules with different μ. We can numerically find the equilibrium for each value of μ. The ideal scenario for the government would be to decrease the saving of the young agents when the state is o, but increase the saving of the young agents when the state is y. Then, we cannot make a definite claim about the effect of the monetary policy on the government's objective. The proposition 13 establishes the result for the government's objective.



Saving of the young agents when the state is o Saving of the young agents when the state is y

Figure 4.1: Saving levels for different values of μ

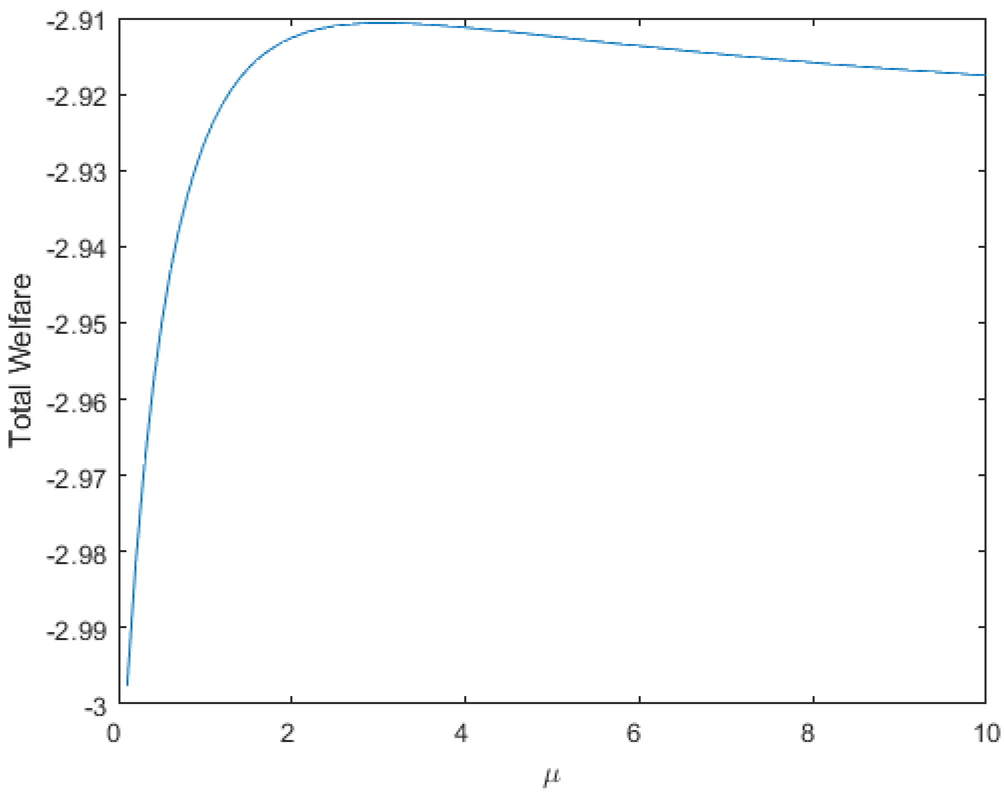


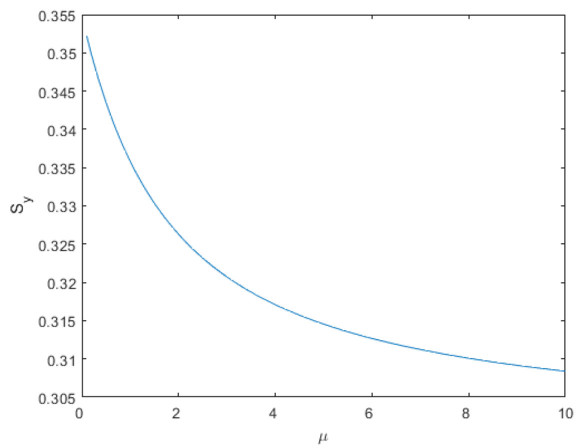
Figure 4.2: Equilibrium value of the welfare function for different values of μ

Proposition 13. There is a parametrization, e.g. Table 4.1, such that the government's commitment to a monetary policy rule is welfare improving.

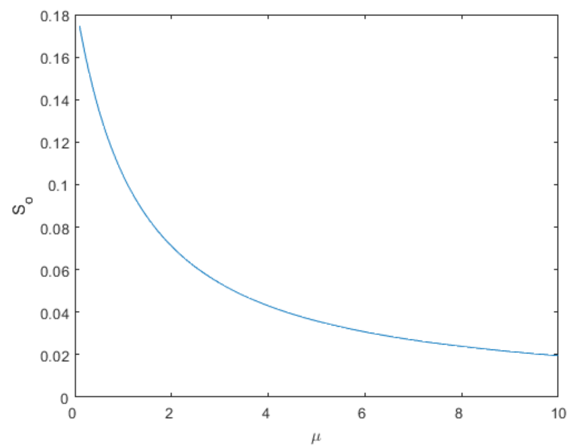
Proof. See Appendix A. □

The numerical analysis shows that selecting $\mu > 0$ improves welfare. Therefore, an active monetary policy where the government intervenes the economy with increasing money supply in state o is welfare improving. However, there is a limit for the magnitude of the intervention. After a point, the stronger monetary intervention provides suboptimal results.[3]

The effect of an increase in μ on total welfare is not monotone. To see this, we need to analyze the effect of μ on different types of agents in the economy. We can start with the initial old. Remember that the saving levels represent the amount of consumption goods the young agents give up to buy money. We already established that the equilibrium saving levels decrease with an increase in μ. Hence, the initial old should suffer from an increase in μ although the magnitude of the effect might depend on the state of the initial period. Figure 4.3 shows the utility of initial old for different value of μ.



Utility of the initial old when $z_0 = y$.



Utility of the initial old when $z_0 = o$.

Figure 4.3: Equilibrium utility levels of the initial old for different values of μ

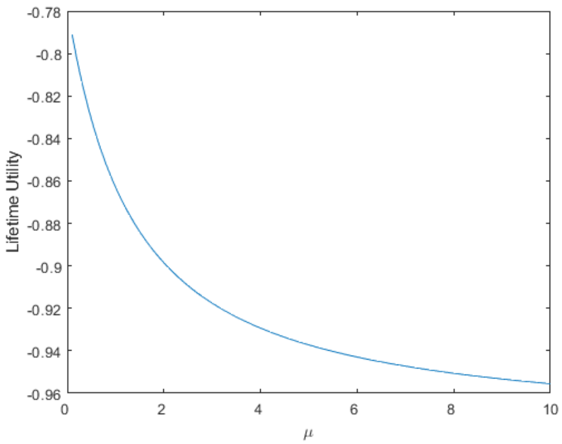
We have four different types of agents in this economy exluding the initial olds[4];

1. lucky when young and lucky when old,
2. lucky when young and unlucky when old,
3. unlucky when young and lucky when old,
4. unlucky when young and unlucky when old.

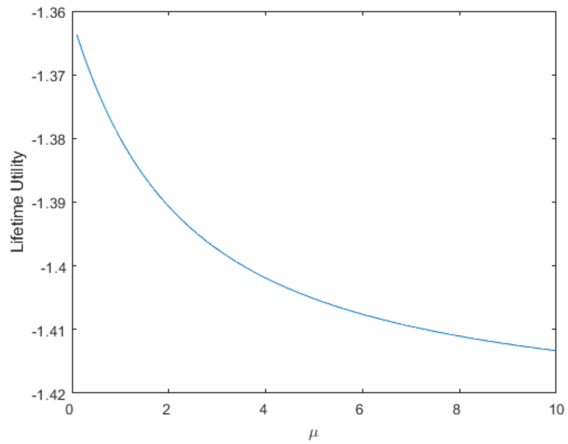
[3]We also check the validity of assumptions. We see that the assumptions hold for each value of μ on the grid.

[4]We do not include the initial olds into this part of the analysis since the effect of the monetary policy is independent of the numerical assumptions. Since the saving of young agents decreases monotonically in both states, the initial old suffers from an increase in μ.

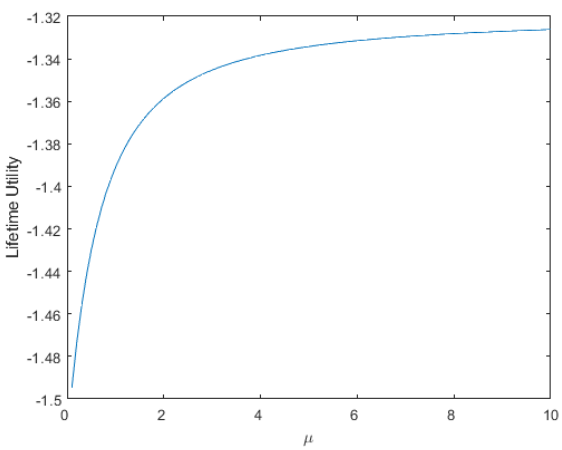
where being lucky in a period refers to the realization of the state that gives more endowment to the generation of the agent. For example, an agent who experiences state y when young and state o when old is under the first category, i.e. lucky when young and lucky when old. Let us examine how the lifetime utility of these agents change with different values of μ.



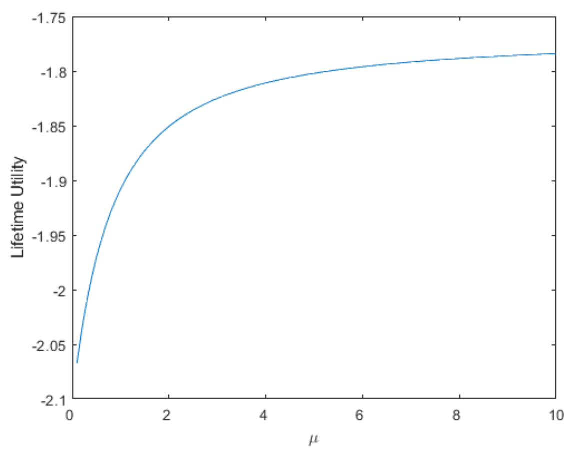
Lucky when young and lucky when old



Lucky when young and unlucky when old



Unlucky when young and lucky when old



Unlucky when young and unlucky when old

Figure 4.4: Lifetime utilities of four types of agents with different values of μ

Figure 4.4 highlights two important effects of the monetary policy rule. First, an increase in μ increases the lifetime utility of the unluckiest agents while decreases the lifetime utility of the luckiest agents. This is encouraging for improving welfare. Second, the results are mixed for the agents in between. The agents who are lucky only when they are old benefits from the stronger monetary policy while the agents who are lucky only when they are young suffers from it. Notice that the positive effect of the increase in μ on the agents who are lucky only when old flattens rapidly, while the negative effect of it on the lifetime utility of the agents who are lucky only when young remains strong in our numerical analysis. This unbalanced effect together with the decrease in the utility of initial olds explains how increasing μ becomes suboptimal after a certain point. It shows that there is an optimal μ that maximizes the government's objective. We discuss how it evolves with changes in numerical assumptions in Appendix B.

4.5 Concluding Remarks

We study a 2-period overlapping generations model with stochastic endowments. It has imperfect risk-sharing due to lack of enforcement. Two underlying limitations of the model brings imperfect risk-sharing; the inability to transfer wealth from old to young and the inability of making consecutive transfers from young to old a la Lancia et al. (2020). We show that introducing money into this setup improves risk-sharing by providing an asset for young agents to save that has a state-contingent return at the equilibrium. We also show that surprise monetary interventions may improve risk-sharing further. Lastly, we study a numerical example where the monetary policy improves risk-sharing further even when it is declared by the government at the beginning of time. These results should attract attention of the policymakers since it shows the effectiveness of the monetary policy on intergenerational redistribution, when the fiscal policy is constrained with limited enforcement.

Our analysis on the sustainable intergenerational insurance and monetary policy has limitations on certain aspects that can be investigated in future research. Although we show the effectiveness of monetary policy on improving intergenerational risk-sharing, we should also consider the side effects of a risk-sharing focused monetary policy rule. We should investigate the other possible consequences of using monetary policy to provide intergenerational insurance before making definitive conclusions about the benefits of such a policy. Also, it might be possible to further improve risk-sharing by combining the monetary policy and fiscal policy together. It would be an insightful analysis although it would make the decision making of the government more intricate.

4.6 Appendix

Appendix A. Proofs

Proof of Proposition 11

Proof. The key is to show $p(y) > p(o)$. To prove by contradiction, assume it is not. Realize that we have two types of agents and one Euler equation for each type. When we combine the two Euler Equations we have

$$p(o)u'(e^y(o) - p(o)m) = p(y)u'(e^y(y) - p(y)m) \tag{4.12}$$

First, suppose $p(y) = p(o)$. Then, $e^y(y) = e^y(o)$; a contradiction. Second, suppose that $p(y) < p(o)$. Then,

$$\frac{u'(e^y(y) - p(y)m)}{u'(e^y(o) - p(o)m)} > 1 \implies e^y(o) - p(o)m > e^y(y) - p(y)m \implies e^y(y) < e^y(o), \tag{4.13}$$

also a contradiction. Hence, $p(y) > p(o)$. It implies that the money is a state-contingent asset. Now, notice that the income of an old agent in an arbitrary period t is

$$e^o(z_t) + p(z_t)m \tag{4.14}$$

Since $p(y) > p(o)$ and $e^o(y) < e^o(o)$, the state that the income from the endowment process is high (low) is the state that the price of money is low (high). Therefore, money is an insurance for the endowment shocks. □

Proof of Lemma 7

Proof. We can derive the following equation from the Euler Equations for the monetary equilibrium,

$$p(o)u'(e^y(o) - p(o)m) = p(y)u'(e^y(y) - p(y)m). \tag{4.15}$$

We know from the Proposition 11 that $p(y) > p(o)$. For above equation to hold, we need $e^y(o) - p(o)m < e^y(y) - p(y)m$. It implies $c^y(o) < c^y(y)$. Since the total endowment is constant for all states, we have $c^o(o) > c^o(y)$ in the monetary equilibrium. In other words, the old agents still enjoy higher consumption levels in state o. Hence, the money only provides limited insurance in the monetary equilibrium. □

Proof of Lemma 8

Proof. Remember that we assumed the government intervention only occurs at period t and the government can convince agents that there will not be any more interventions in the future. The optimal decisions of a young agent at a period k in state z_k is given by the following Euler equation.

$$u'(e^y(y) - p_k(z_k)m_k) = \beta \left[\pi_o \frac{p_{k+1}(o)}{p_k(z_k)} u'[e^o(o) + p_{k+1}(o)m_k] \right. \\ \left. + \pi_y \frac{p_{k+1}(y)}{p_k(z_k)} u'[e^o(y) + p_{k+1}(y)m_k] \right]$$

We can define sacing of the young agents as $s_k(z_k) = p_k(z_k)m_k$. Now, let us re-write the Euler equation by substituting saving in.

$$u'(e^y(y) - s_k(z_k)) = \beta \left[\pi_o \frac{s_{k+1}(o)m_k}{s_k(z_k)m_k(1+\mu_{k+1}(o))} u'[e^o(o) + p_{k+1}(o)(m_k)] \right. \\ \left. + \pi_y \frac{p_{k+1}(y)}{p_k(z_k)} u'[e^o(y) + p_{k+1}(y)(m_k)] \right]$$

□

Proof of Lemma 9

Proof. To prove this, we will analyze the economy with the defined one-time intervention and the counterfactual economy without any government intervention. In the former case, there is a t and z_t such that $\mu_t(z_t) > 0$ and $\mu_i(z_i) = 0 \ \forall i \neq t, \quad \forall z_i$ while the latter refers to the case that $\mu_k(z_k) = 0$ $\forall k, z_k$.

Consider the one time intervention at an arbitrary period t with state z_t. Let us re-write the Euler equation of a young agent of period t with state z_t by incorporating law of motion of money,

$$u'(e^y(z_t) - s_t(z_t)) = \beta \left[\pi_o \frac{s_{t+1}(o)}{s_t(y)} \frac{1}{1+\mu_t(z_t)} u'(e^o(o) + s_{t+1}(o)) + \pi_y \frac{s_{t+1}}{s_t(y)} \frac{1}{1+\mu_t(z_t)} u'(e^o(y) + s_{t+1}(y))\right] \tag{4.16}$$

Note that $\mu > 0$ implies that there is monetary intervention in period t, while $\mu = 0$ implies no intervention.

We already assumed that the intervention is limited to one period, $\mu_t > 0$ and $\mu_k = 0 \ \forall k \neq t$. As a result of this assumption, the intervention does not effect any real variables at any period. To see this, realize that the amount of money can be normalized in every period without any impact on the real variables. What matters for the agents' decision making is the expectation on the growth rate of money in the future since it determines the return on money. Then, an intervention in period t cannot effect $s_{t+1}(z_{t+1}) \ \forall z_{t+1} \in Z$. Therefore, a monetary intervention, a change in μ_t, can only effect the saving of the young agent at period t; i.e. $s_t(z_t)$.

The Euler equation at period t in state z_t is a necessary condition for any monetary equilibrium. Let $s_t^*(z_t)$ denote the equilibrium level of the saving of the young agent at period t in state

z_t when $\mu_t(z_t) = 0$. Let us define function F as

$$F(\mu, s) = u'(e^y(z_t) - s) - \beta \left[\pi_o \frac{s_{t+1}(o)}{s} \frac{1}{1+\mu} u'(e^o(o) + s_{t+1}(o)) + \pi_y \frac{s_{t+1}}{s} \frac{1}{1+\mu} u'(e^o(y) + s_{t+1}(y)) \right]$$

Notice that there exists an equilibrium for every value of μ given our assumptions. Also, since the Euler equation holds in any equilibrium. Then, the following should also hold

$$\frac{\partial F(\mu, s_t^*(z_t))}{\partial \mu} = 0. \tag{4.17}$$

Notice that $s_t^*(z_t)$ is a decision variable for the young agent at period t and the decision of the agent depends on the value of μ. Therefore, it may change with a change in μ as well. Then, we have

$$\begin{aligned} \frac{\partial F(\mu, s_t^*(z_t))}{\partial \mu} =& - u''(e^y(z_t) - s_t^*(z_t)) \frac{\partial s_t^*(z_t)}{\partial \mu} \\ &+ \beta \pi_o \frac{s_{t+1}(o)}{(s_t^*(z_t))^2} \frac{\partial s_t^*(z_t)}{\partial \mu} \frac{1}{1+\mu} u'(e^o(o) + s_{t+1}(o)) \\ &+ \beta \pi_y \frac{s_{t+1}}{(s_t^*(z_t))^2} \frac{\partial s_t^*(z_t)}{\partial \mu} \frac{1}{1+\mu} u'(e^o(y) + s_{t+1}(y)) \\ &+ \beta \pi_o \frac{s_{t+1}(o)}{s_t^*(z_t)} \frac{1}{(1+\mu)^2} u'(e^o(o) + s_{t+1}(o)) \\ &+ \beta \pi_y \frac{s_{t+1}}{s_t^*(z_t)} \frac{1}{(1+\mu)^2} u'(e^o(y) + s_{t+1}(y)). \end{aligned}$$

Realize that

$$\begin{aligned} \frac{\partial s_t^*(z_t)}{\partial \mu} > 0 \implies \frac{\partial F(\mu, s_t^*(z_t))}{\partial \mu} > 0 \\ \frac{\partial s_t^*(z_t)}{\partial \mu} = 0 \implies \frac{\partial F(\mu, s_t^*(z_t))}{\partial \mu} = 0 \end{aligned}$$

Then, it is clear that $\frac{\partial s_t^*(z_t)}{\partial \mu} < 0$. Then, an increase in μ decreases the saving of the young agents in that period. Hence, the consumption of the young agents increase and consumption of the old agents decrease in period t. As we already established, no other period is affected by the one-time intervention. □

Proof of Lemma 10

Proof. Define $s_t(z_t) = p_t(z_t)m_t(z_t)$. Let us write the Euler equation of an agent who borns at period t.

$$u'(e^y(z_t)-s_t(z_t)) = \beta\left[\pi_o\frac{s_{t+1}(o)}{s_t(z_t)}\frac{m}{m_{t+1}(o)}u'(e^o(o)+s_{t+1}(o)) + \pi_y\frac{s_{t+1}(y)}{s_t(z_t)}\frac{m}{m_{t+1}(s)}u'(e^o(y)+s_{t+1}(y))\right]$$

The government declares that it will print money and distribute to the young in period t, if $z_t = o$. Then,

$$m_{t+1}(o) = m(1+\mu), \quad m_{t+1}(s) = m.$$

Let us re-write the Euler equation,

$$u'(e^y(z_t)-s_t(z_t)) = \beta\left[\pi_o\frac{s_{t+1}(o)}{s_t(z_t)}\frac{1}{1+\mu}u'(e^o(o)+s_{t+1}(o)) + \pi_y\frac{s_{t+1}(y)}{s_t(z_t)}u'(e^o(y)+s_{t+1}(y))\right] \quad (4.18)$$

There is no government intervention in the future if $z_{t+1} = y$. Therefore, $s_{t+1}(y)$ should be unchanged. From Lemma 9 we know that $\frac{\partial s_{t+1}(o)}{\partial \mu} < 0$. Then it follows that $\frac{\partial s_t(z_t)}{\partial \mu} \neq 0$, since in that case the Euler equation cannot hold.

The sign of $\frac{\partial s_t(z_t)}{\partial \mu}$ is ambiguous. To see this, consider the numerical example described in Table 1. □

Proof of Lemma 11

Proof. The following equation can be derived by combining the equations 4.2 and 4.3,

$$(1+\mu)s_o u'(e^y(o)-s(o)) = s_y u'(e^y(y)-s(y)) \quad (4.19)$$

We can define

$$f(\mu) = (1+\mu) \text{ and } g(s_y, s_o) = \frac{s_y u'(e^y(y)-s(y))}{s_o u'(e^y(o)-s(o))} \quad (4.20)$$

and from equation 4.19 we know that $f(\mu^*) = g(s_y^*, s_o^*)$ for a monetary equilibrium $\{\mu^*, s_y^*, s_o^*\}$. Clearly,

$$\frac{\partial f}{\partial \mu} > 0.$$

Also since $u'' < 0$,

$$\frac{\partial g}{\partial s_y} > 0 \text{ and } \frac{\partial g}{\partial s_o} > 0.$$

If μ^* increases, we cannot have ($s^*(o)$ increases) and ($s^*(y)$ decreases) at the same time. □

Proof of Proposition 12

Proof. Utility function is $u(.) = \ln(.)$. When we solve the Euler equations simultaneously by embedding this information, we have

$$as_y^2 + bs_y + c = 0$$

where

$$\begin{aligned}
a =& \mu e^o(o) - e^y(o) - \beta\pi_o e^y(o) + \beta\pi_y \mu e^o(o) - \beta\pi_y e^y(o),\\
b =& \mu e^o(o)e^o(y) - e^o(y)e^y(o) - e^o(o)e^y(y)(1+\mu) + \beta\pi_o e^y(o)e^y(y) - \beta\pi_y\mu e^o(o)e^y(y)\\
&+ \beta\pi_y e^y(o)e^y(y) - \beta\pi_o e^y(o)e^o(y) - \beta\pi_y e^o(o)e^y(y)(1+\mu),\\
c =& \beta\pi_o e^y(o)e^o(y)e^y(y) + \pi_y\beta e^o(o)e^y(y)^2(1+\mu) - e^o(y)e^o(o)e^y(y)(1+\mu).
\end{aligned}$$

The roots of this equation are

$$s(y)_- = \frac{-b-\sqrt{b^2-4ac}}{2a}, \text{ and } s(y)_+ = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

We also know that the feasible region for the savings is st. $0 \leq s(y) \leq e^y(y)$. Also, for a small enough $\mu > 0$, we have

$$a < 0, \quad b < 0, \text{ and if we also assume } \pi_y\beta e^y(y) > e^o(y) \quad c > 0.$$

Then, we only have one feasible root, which is $s(y)_-$. From the pair of Euler equations we have, we can also derive the following equation:

$$(1+\mu)s(o)u'(e^y(o) - s(o)) = s(y)u'(e^y(y) - s(y)).$$

Taking the utility functions as logarithmic, we can solve for $s(o)$:

$$s^*(o) = \frac{s^*(y)e^y(o)}{(1+\mu)e^o(y) - \mu s(y)}.$$

The rest of the results that characterize the equilibrium are a direct result of the budget constraints of the agents. □

Proof of Lemma 12

Proof. With the assumption of logarithmic utility, previous proposition already gives us the closed form solutions of the saving levels at the equilibrium:

$$
\begin{aligned}
s^*(y) =& \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \\
s^*(o) =& \frac{s^*(y)e^y(o)}{(1+\mu)e^o(y) - \mu s(y)}.
\end{aligned}
$$

where

$$
\begin{aligned}
a =& \mu e^o(o) - e^y(o) - \beta\pi_o e^y(o) + \beta\pi_y \mu e^o(o) - \beta\pi_y e^y(o), \\
b =& \mu e^o(o)e^o(y) - e^o(y)e^y(o) - e^o(o)e^y(y)(1+\mu) + \beta\pi_o e^y(o)e^y(y) - \beta\pi_y \mu e^o(o)e^y(y) \\
& + \beta\pi_y e^y(o)e^y(y) - \beta\pi_o e^y(o)e^o(y) - \beta\pi_y e^o(o)e^y(y)(1+\mu), \\
c =& \beta\pi_o e^y(o)e^o(y)e^y(y) + \pi_y\beta e^o(o)e^y(y)^2(1+\mu) - e^o(y)e^o(o)e^y(y)(1+\mu).
\end{aligned}
$$

The derivatives of these functions with respect to μ have ambiguous signs when evaluated at $\mu = 0$. However, by introducing the numerical values given on the table we have

$$
\frac{\partial s_o}{\partial \mu}\Big|_{\mu=0} \approxeq -0.1249, \text{ and } \frac{\partial s_y}{\partial \mu}\Big|_{\mu=0} \approxeq -0.0255.
$$

□

Proof of Proposition 13

Proof. Since the probability of different states are equal and the utility is logarithmic, the total welfare function can be reduced to

$$
W = \ln(c^y(y)) + \ln(c^y(o)) + \ln(c^o(y)) + \ln(c^o(o)).
$$

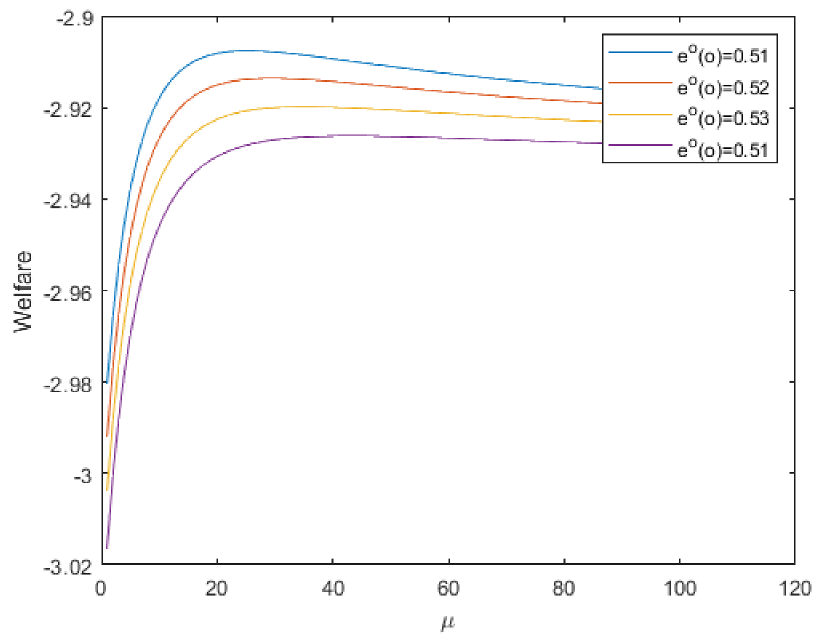
Given the numerical values in Table 4.1, we have

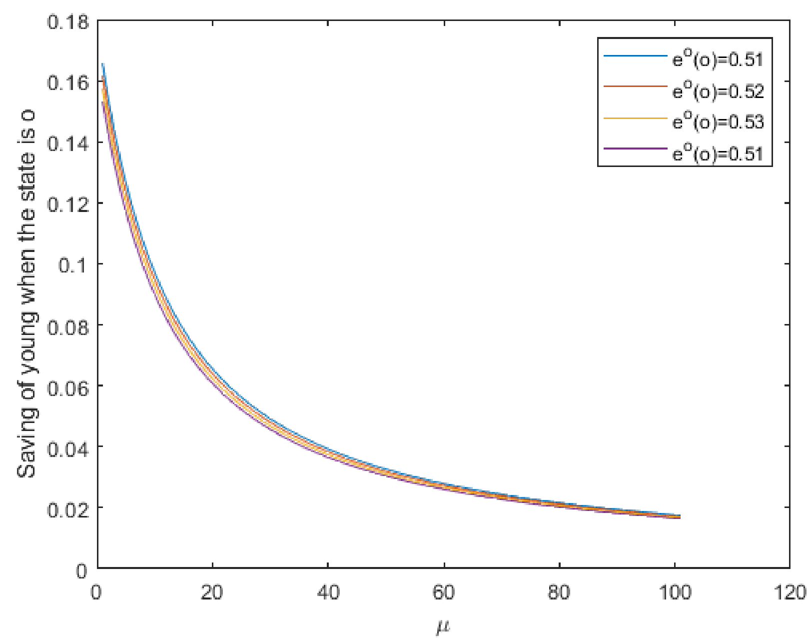
$$
\frac{\partial W}{\partial \mu}\Big|_{\mu=0} \approxeq 0.1828.
$$

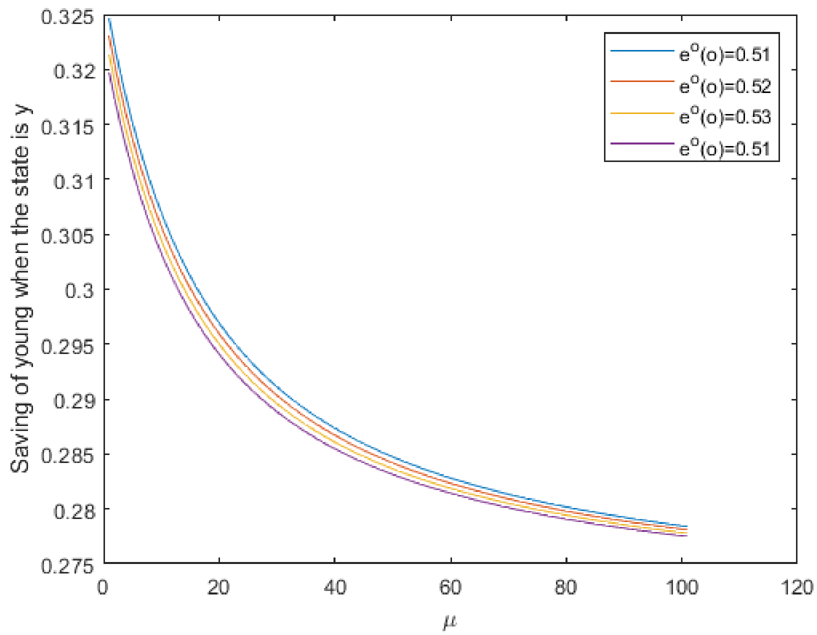
□

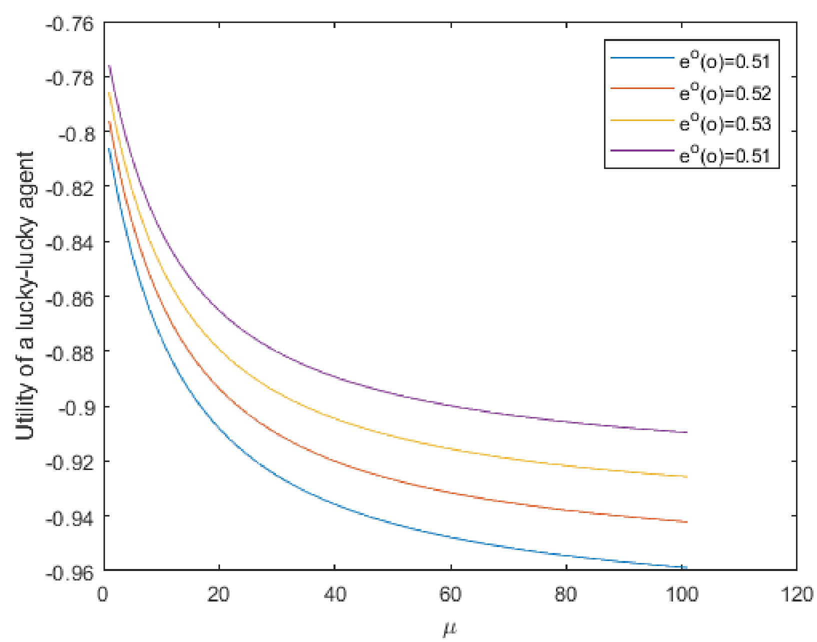
Appendix B. Further Numerical Analysis

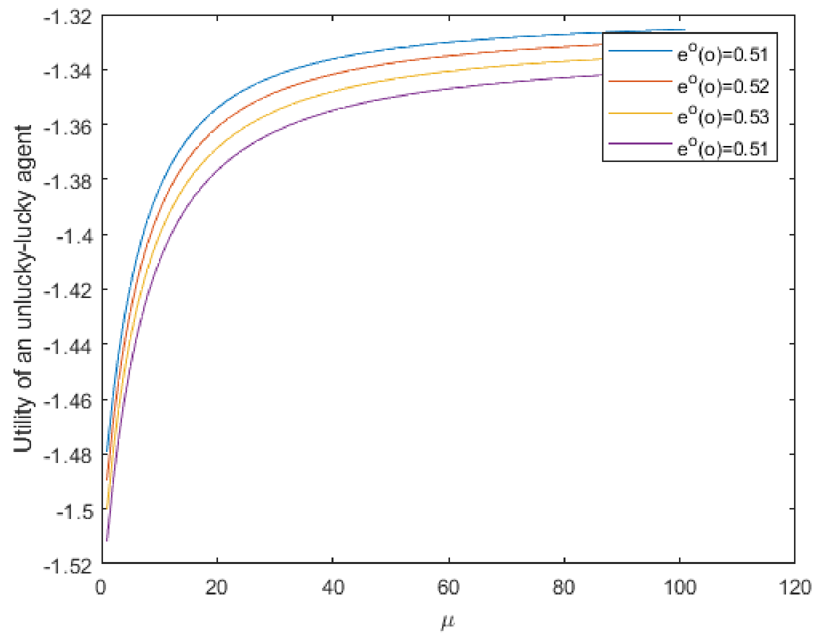
In this subsection, we will study numerically the equilibrium levels of variables and welfare as functions of the magnitude of monetary interventions, μ, with different parametrization. Let us first analyze changes in equilibrium values with different parameter values of $e^o(o)$. Then, there are analyses with changes in $e^y(y)$, and lastly with changes in β.



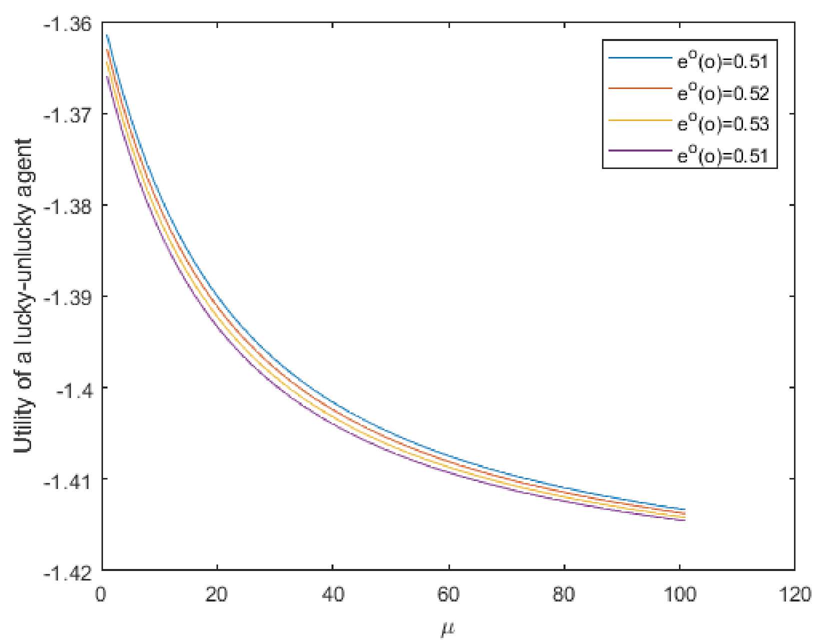



Saving of young when the state is y
0.325
0.32
0.315
0.31
0.305
0.3
0.295
0.29
0.285
0.28
0.275
0
20
40
60
80
100
120
μ
$e^{o}(o)$=0.51
$e^{o}(o)$=0.52
$e^{o}(o)$=0.53
$e^{o}(o)$=0.51

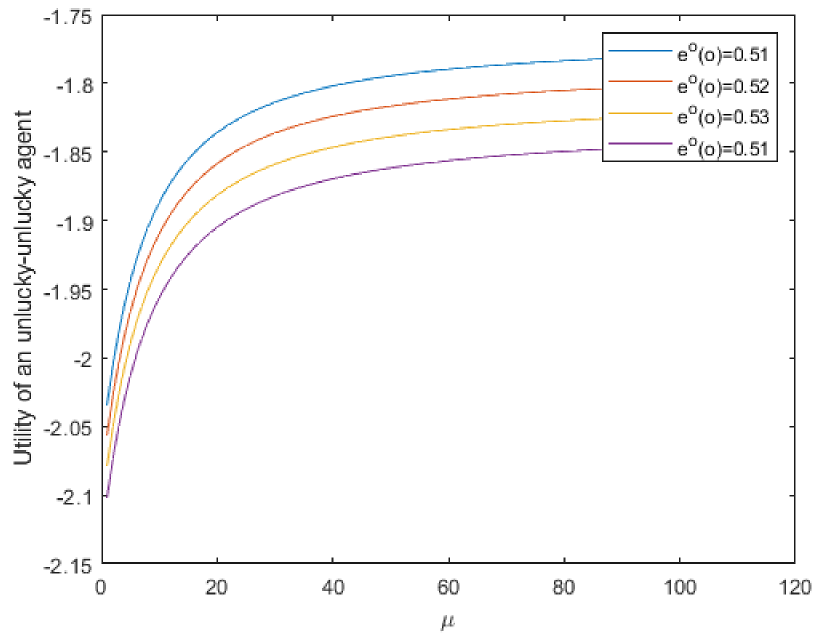

Utility of a lucky-lucky agent
-0.76
-0.78
-0.8
-0.82
-0.84
-0.86
-0.88
-0.9
-0.92
-0.94
-0.96
0
20
40
60
80
100
120
μ
$e^{o}(o)$=0.51
$e^{o}(o)$=0.52
$e^{o}(o)$=0.53
$e^{o}(o)$=0.51

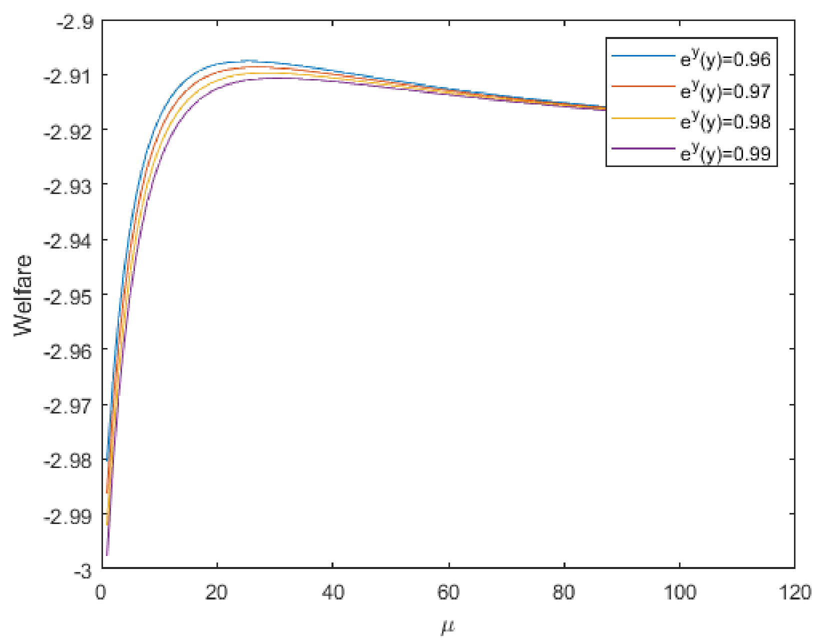


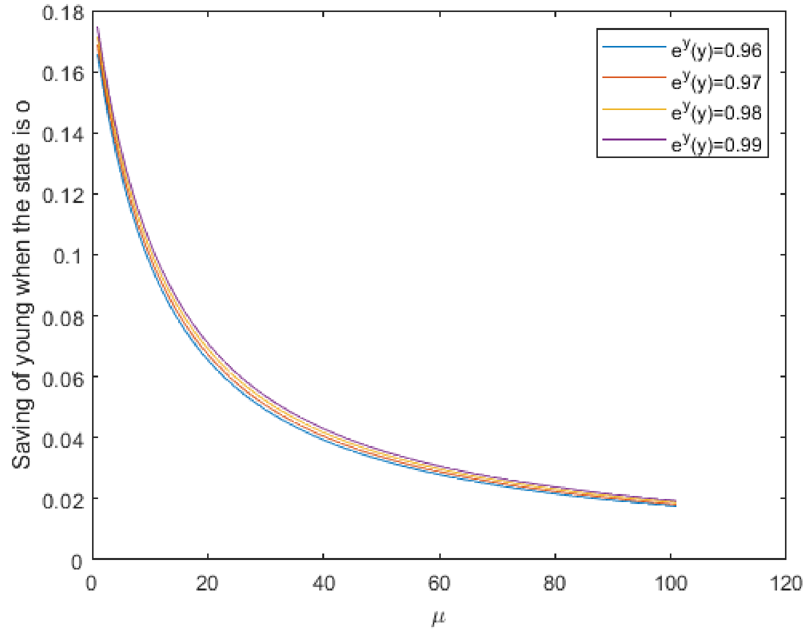
-1.32
-1.34
-1.36
-1.38
-1.4
-1.42
-1.44
-1.46
-1.48
-1.5
-1.52
0
20
40
60
80
100
120
μ
Utility of an unlucky-lucky agent
e^{o}(o)=0.51
e^{o}(o)=0.52
e^{o}(o)=0.53
e^{o}(o)=0.51

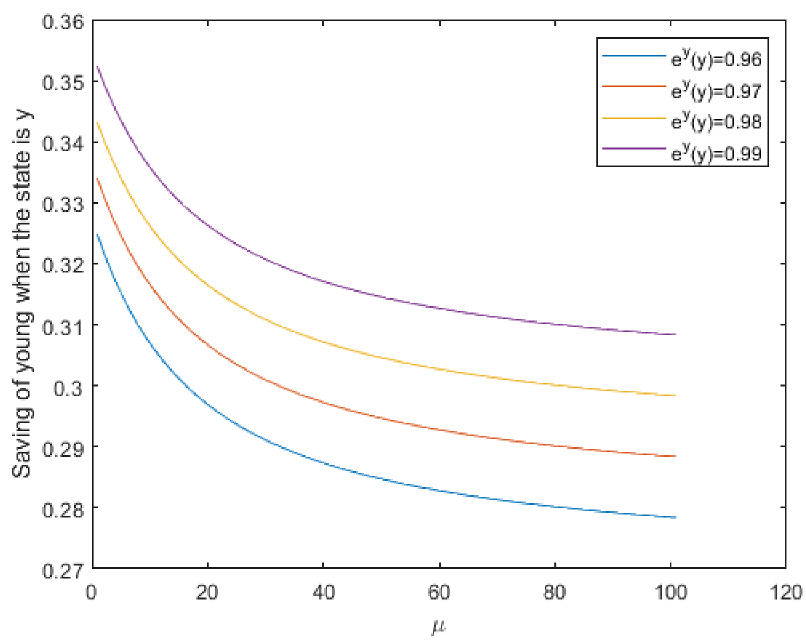


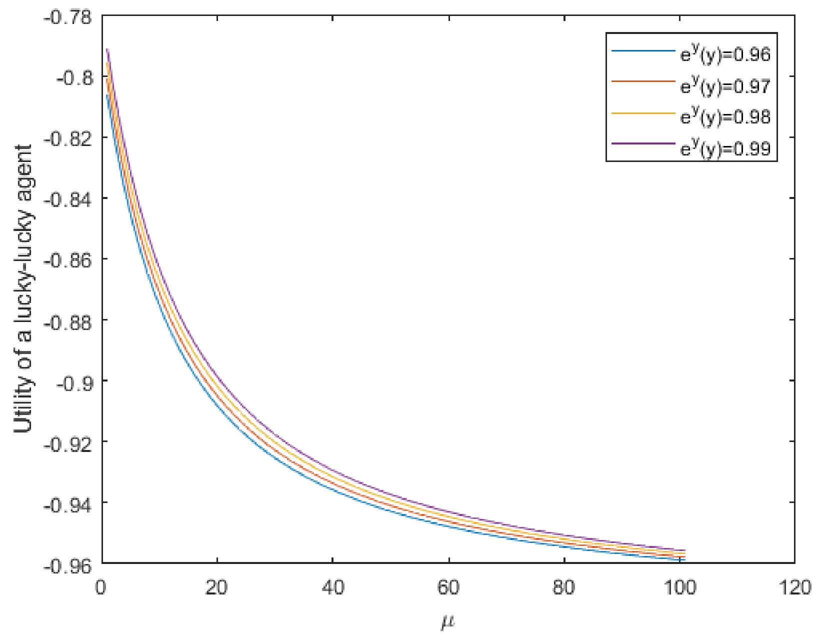
-1.36
-1.37
-1.38
-1.39
-1.4
-1.41
-1.42
0
20
40
60
80
100
120
μ
Utility of a lucky-unlucky agent
e^{o}(o)=0.51
e^{o}(o)=0.52
e^{o}(o)=0.53
e^{o}(o)=0.51

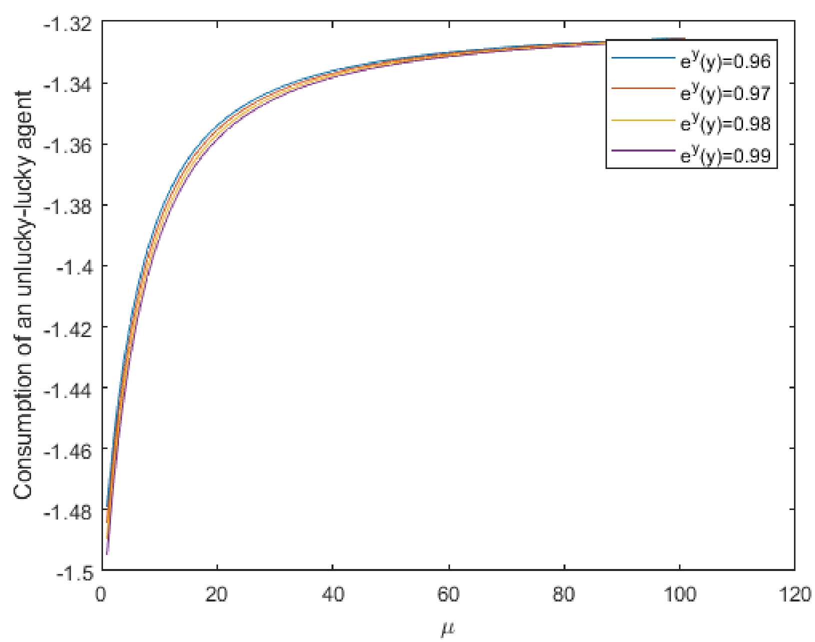

Utility of an unlucky-unlucky agent
μ
e^o(o)=0.51
e^o(o)=0.52
e^o(o)=0.53
e^o(o)=0.51

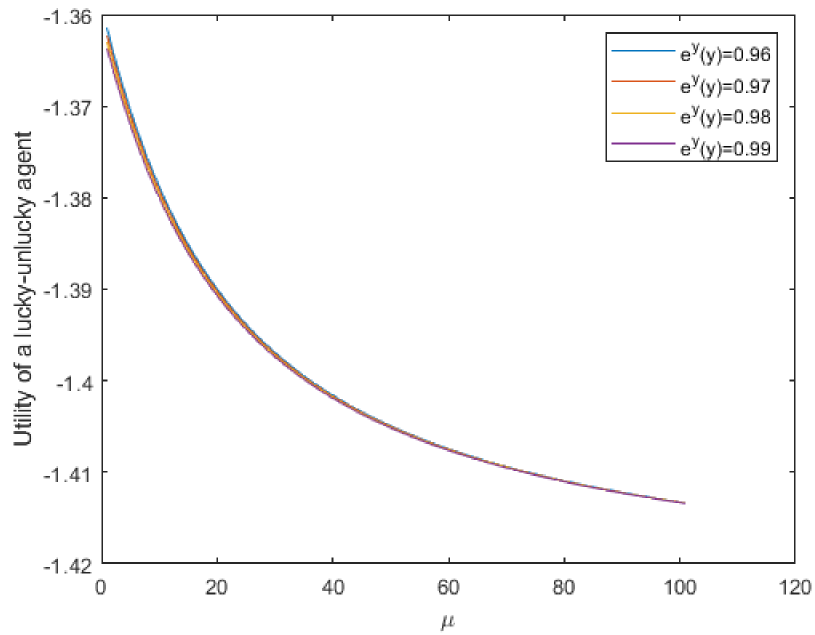

Welfare
μ
e^y(y)=0.96
e^y(y)=0.97
e^y(y)=0.98
e^y(y)=0.99


Saving of young when the state is o
μ
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100
120
$e^y(y)$=0.96
$e^y(y)$=0.97
$e^y(y)$=0.98
$e^y(y)$=0.99

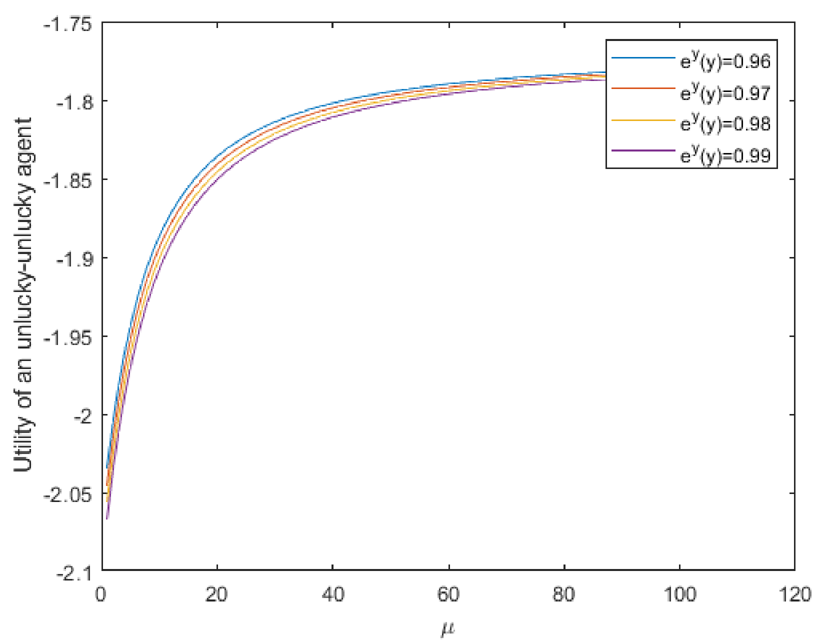

Saving of young when the state is y
μ
0.36
0.35
0.34
0.33
0.32
0.31
0.3
0.29
0.28
0.27
0
20
40
60
80
100
120
$e^y(y)$=0.96
$e^y(y)$=0.97
$e^y(y)$=0.98
$e^y(y)$=0.99


-0.78
-0.8
-0.82
-0.84
-0.86
-0.88
-0.9
-0.92
-0.94
-0.96
0
20
40
60
80
100
120
Utility of a lucky-lucky agent
μ
e^y(y)=0.96
e^y(y)=0.97
e^y(y)=0.98
e^y(y)=0.99

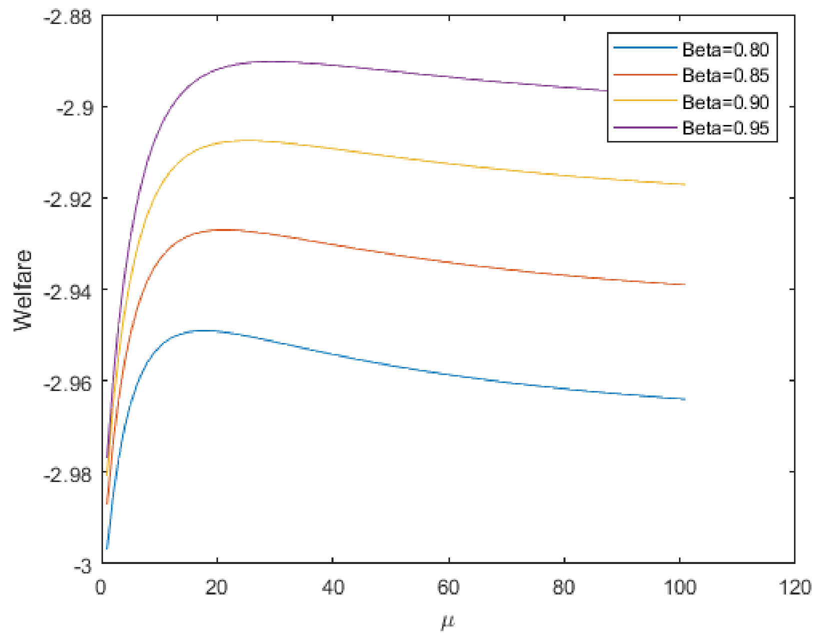

-1.32
-1.34
-1.36
-1.38
-1.4
-1.42
-1.44
-1.46
-1.48
-1.5
0
20
40
60
80
100
120
Consumption of an unlucky-lucky agent
μ
e^y(y)=0.96
e^y(y)=0.97
e^y(y)=0.98
e^y(y)=0.99

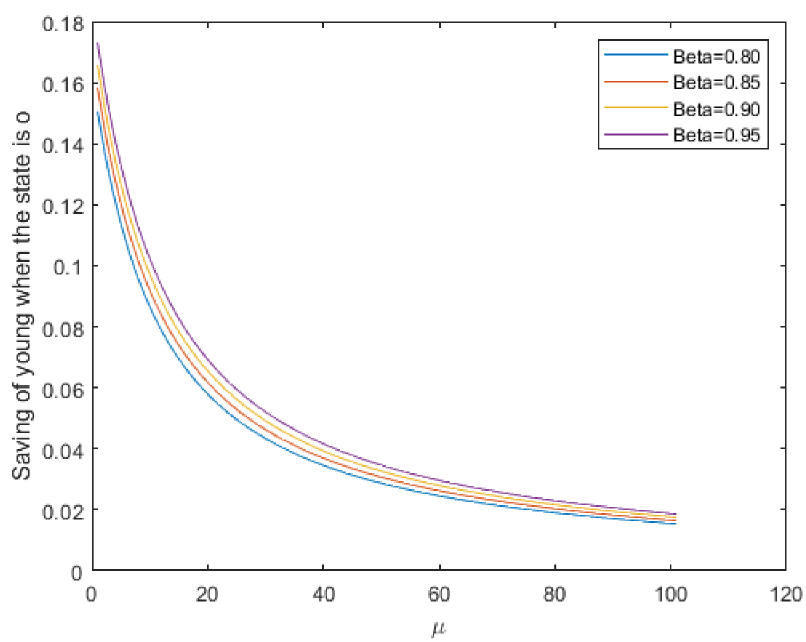


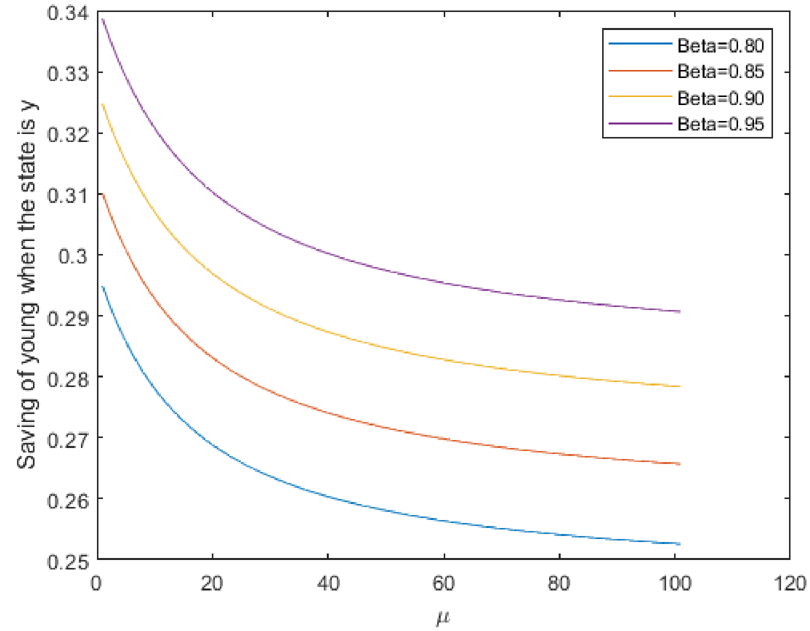
Utility of a lucky-unlucky agent
μ
e^y(y)=0.96
e^y(y)=0.97
e^y(y)=0.98
e^y(y)=0.99

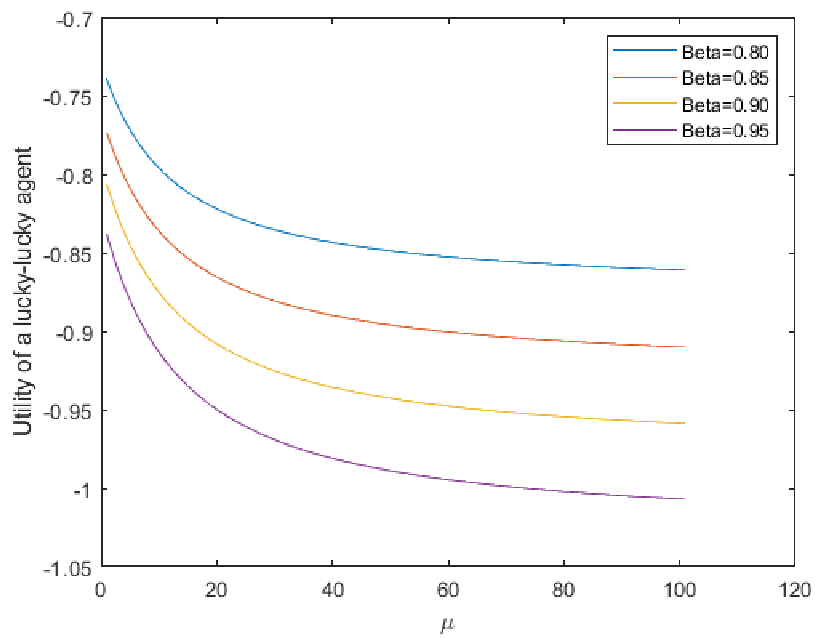


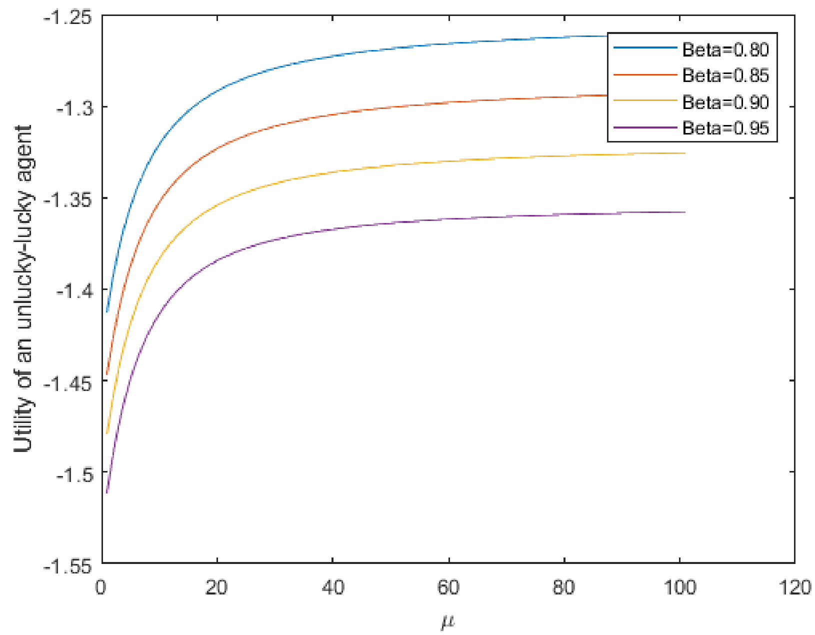
Utility of an unlucky-unlucky agent
μ
e^y(y)=0.96
e^y(y)=0.97
e^y(y)=0.98
e^y(y)=0.99


Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95
Welfare
μ

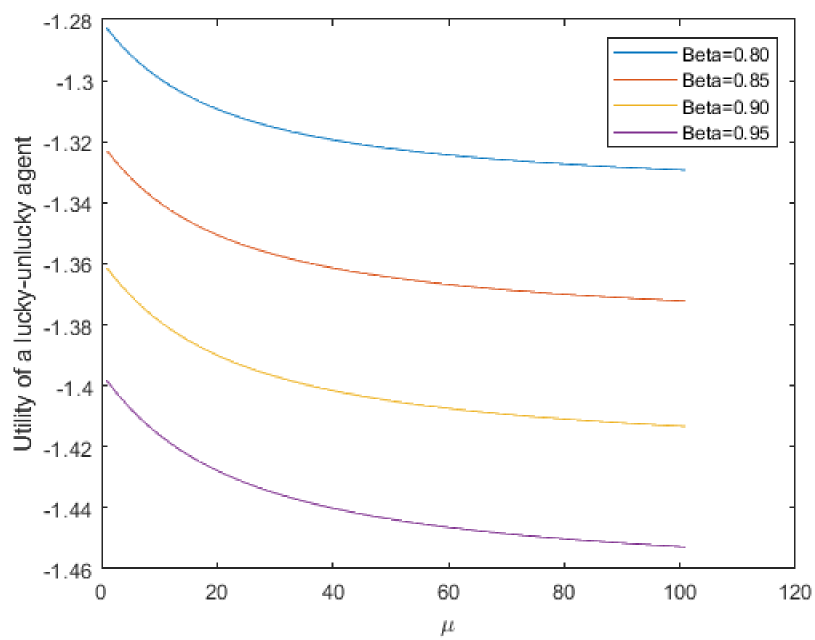

Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95
Saving of young when the state is o
μ


Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95
Saving of young when the state is y
0.34
0.33
0.32
0.31
0.3
0.29
0.28
0.27
0.26
0.25
0
20
40
60
80
100
120
μ

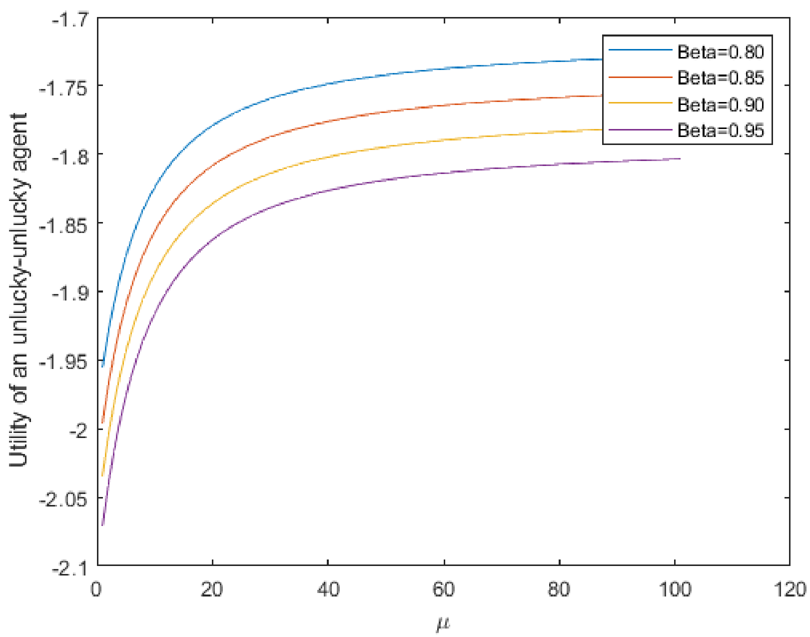

Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95
Utility of a lucky-lucky agent
-0.7
-0.75
-0.8
-0.85
-0.9
-0.95
-1
-1.05
0
20
40
60
80
100
120
μ



Utility of an unlucky-lucky agent
μ
Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95



Utility of a lucky-unlucky agent
μ
Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95



Utility of an unlucky-unlucky agent
-1.7
-1.75
-1.8
-1.85
-1.9
-1.95
-2
-2.05
-2.1
0
20
40
60
80
100
120
μ
Beta=0.80
Beta=0.85
Beta=0.90
Beta=0.95

References

Aiyagari, S., Williamson, S.D., 2000. Money and dynamic credit arrangements with private information. Journal of Economic Theory 91, 248–279.

Aiyagari, S.R., Peled, D., 1991. Optimal Capital Income Taxation and Long Run Debt with Borrowing Constraints .

Albanesi, S., Chari, V.V., Christiano, L.J., 2003. Expectation traps and monetary policy. The Review of Economic Studies 70, 715–741.

Alm, J., Beck, W., 1990. Tax amnesties and tax revenues. Public Finance Quarterly 18, 433–453.

Alm, J., Martinez-Vazquez, J., Wallace, S., 2009. Do tax amnesties work? the revenue effects of tax amnesties during the transition in the russian federation. Economic Analysis and Policy 39, 235 – 253.

Alm, J., McKee, M., Beck, W., 1990. Amazing grace: Tax amnesties and compliance. National Tax Journal 43, 23–37.

Alvarez, F., Kehoe, P.J., Neumeyer, P., 2002. The time consistency of fiscal and monetary policies. Working Paper 616. Federal Reserve Bank of Minneapolis Research Department.

Amador, M., Phelan, C., 2021. Reputation and sovereign default. Econometrica 89, 1979–2010.

Andreoni, J., 1991. The desirability of a permanent tax amnesty. Journal of Public Economics 45, 143 – 159.

Armenter, R., 2008. A general theory (and some evidence) of expectation traps in monetary policy. Journal of Money, Credit and Banking 40, 867–895.

Azariadis, C., Galasso, V., 2002. Fiscal constitutions. Journal of Economic Theory 103, 255–281.

Ball, L., 1995. Time-consistent policy and persistent changes in inflation. Journal of Monetary Economics 36, 329 – 350.

Ball, L., Mankiw, N., 2007. Intergenerational risk sharing in the spirit of arrow, debreu, and rawls, with applications to social security design. Journal of Political Economy 115, 523–547.

Barro, R., 1986. Reputation in a model of monetary policy with incomplete information. Journal of Monetary Economics 17, 3 – 20.

Barro, R., Gordon, D.B., 1983. Rules, discretion and reputation in a model of monetary policy. Journal of Monetary Economics 12, 101 – 121.

Bayer, R.C., Oberhofer, H., Winner, H., 2015. The occurrence of tax amnesties: Theory and evidence. Journal of Public Economics 125, 70 – 82.

Berentsen, A., Camera, G., Waller, C., 2005. The distribution of money balances and the nonneutrality of money. International Economic Review 46, 465–487.

Bielecki, M., Brzoza-Brzezina, M., Kolasa, M., 2021. Intergenerational redistributive effects of monetary policy. Journal of the European Economic Association , jvab032.

Bloise, G., Calciano, F.L., 2008. A characterization of inefficiency in stochastic overlapping generations economies. Journal of Economic Theory 143, 442–468.

Boldrin, M., Rustichini, A., 2000. Political equilibria with social security. Review of Economic Dynamics 3, 41–78.

Borgne, E.L., 2006. Economic and political determinants of tax amnesties in the u.s. states. National Tax Journal, Proceedings of the 98th Annual Conference on Taxation , 443–50.

Borgne, E.L., Baer, K., 2008. Tax Amnesties : Theory, Trends, and Some Alternatives. International Monetary Fund.

Buckwalter, N.D., Sharp, N.Y., Wilde, J.H., Wood, D.A., 2014. Are state tax amnesty programs associated with financial reporting irregularities? Public Finance Review 42, 774–799.

Calvo, G.A., 1978. On the time consistency of optimal policy in a monetary economy. Econometrica: Journal of the Econometric Society , 1411–1428.

Chari, V., Christiano, L.J., Eichenbaum, M., 1998. Expectation traps and discretion. Journal of Economic Theory 81, 462 – 492.

Chari, V.V., Kehoe, P.J., 1990. Sustainable plans. Journal of Political Economy 98, 783–802.

Chari, V.V., Kehoe, P.J., Prescott, E.C., 1988. Time consistency and policy. Staff Report 115. Federal Reserve Bank of Minneapolis.

Chattopadhyay, S., Gottardi, P., 1999. Stochastic olg models, market structure, and optimality. Journal of Economic Theory 89, 21–67.

Christian, C., Gupta, S., Young, J., 2002. Evidence on subsequent filing from the state of Michigan's income tax amnesty. National Tax Journal 55, 703–21.

Cooley, T.F., Soares, J., 1999. A positive theory of social security based on reputation. Journal of Political Economy 107, 135–160.

Das-Gupta, A., Mookherjee, D., 1995. Tax amnesties in india: An empirical evaluation. Boston university, Institute for economic development , 212–36.

Das-Gupta, A., Mookherjee, D., 1996. Tax Amnesties as Asset-Laundering Devices. Journal of Law, Economics, and Organization 12, 408–431.

Demange, G., 2002. On optimality in intergenerational risk sharing. Economic Theory 20, 1–27.

Demange, G., Laroque, G., 2021. Social security and demographic shocks. Econometrica 67, 527–542.

Diamond, P.A., 1977. A framework for social security analysis. Journal of Public Economics 8, 275–298.

Dinmore, G., 2009. Italy tax amnesty yields record 80bn Euros. https://www.ft.com/content/35dfa00a-efd9-11de-833d-00144feab49a.

Doepke, M., Schneider, M., 2006. Inflation and the redistribution of nominal wealth. Journal of Political Economy 114, 1069–1097. URL: https://doi.org/10.1086/508379, doi:10.1086/508379.

Domínguez, B., 2007. On the time-consistency of optimal capital taxes. Journal of Monetary Economics 54, 686–705.

Dubin, J., Graetz, M., Wilde, L., 1992. State income tax amnesties: Causes. Quarterly Journal of Economics 107, 1057–70.

Enders, W., Lapan, H.E., 2021. Social security taxation and intergenerational risk sharing. International Economic Review 23, 647–658.

Feng, Z., 2013. Tackling indeterminacy in overlapping generations models. Mathematical Methods of Operations Research 77, 445–457.

Fisher, R., Goddeeris, J.H., Young, J.C., 1989. Participation in tax amnesties: The individual income tax. National Tax Journal 42, 15–27.

Franzoni, L.A., 2000. Amnesties, settlements and optimal tax enforcement. Economica 67, 153–176.

Garz, M., Pagels, V., 2018. Cautionary tales: Celebrities, the news media, and participation in tax amnesties. Journal of Economic Behavior & Organization 155, 288 – 300.

Glover, A., Heathcote, J., Krueger, D., Ríos-Rull, J.V., 2020a. Health versus Wealth: On the Distributional Effects of Controlling a Pandemic. Working Papers 2020-038. Human Capital and Economic Opportunity Working Group.

Glover, A., Heathcote, J., Krueger, D., Ríos-Rull, J.V., 2020b. Intergenerational redistribution in the great recession. Journal of Political Economy 128, 3730–3778.

Gonzalez-Eiras, M., Niepelt, D., 2005. Sustaining Social Security. CESifo Working Paper Series 1494. CESifo.

Gordon, R.H., Varian, H.R., 1988. Intergenerational risk sharing. Journal of Public Economics 37, 185–202.

Graetz, M., Wilde, L., 1993. The decision by strategic nonfilers to participate in income tax amnesties. International Review of Law and Economics 13, 271–283.

Green, E.J., Zhou, R., 2005. Money as a mechanism in a bewley economy. International Economic Review 46, 351–371.

Klein, P., 2010. Time Consistency of Monetary and Fiscal Policy. Palgrave Macmillan UK, London. pp. 370–375.

Klein, P., Krusell, P., Ríos-Rull, J.V., 2008. Time-consistent public policy. Review of Economic Studies 75, 789–808.

Klein, P., Rull, J.V.R., 2003. Time-consistent optimal fiscal policy. International Economic Review 44, 1217–1245.

Kocherlakota, N.R., 1998. Money is memory. Journal of Economic Theory 81, 232–251.

Kreps, D.M., Wilson, R., 1982. Reputation and imperfect information. Journal of Economic Theory 27, 253 – 279.

Kydland, F.E., Prescott, E.C., 1977. Rules rather than discretion: The inconsistency of optimal plans. Journal of Political Economy 85, 473–491.

Labadie, P., 2021. Comparative dynamics and risk premia in an overlapping generations model. The Review of Economic Studies 53, 139–152.

Lancia, F., Russo, A., Worrall, T.S., 2020. Optimal sustainable intergenerational insurance. CEPR Discussion Paper No. DP15540 .

Langenmayr, D., 2017. Voluntary disclosure of evaded taxes — increasing revenue, or increasing incentives to evade? Journal of Public Economics 151, 110 – 125.

López-Laborda, J., Rodrigo, F., 2003. Tax amnesties and income tax compliance: The case of spain. Fiscal Studies 24, 73–96.

Lucas Jr, R.E., Stokey, N.L., 1983. Optimal fiscal and monetary policy in an economy without capital. Journal of monetary Economics 12, 55–93.

Luitel, H., Tosun, M., 2010. An Examination of the Relation between State Fiscal Health and Amnesty Enactment. Working Papers 10-009. University of Nevada, Reno, Department of Economics.

Luitel, H.S., Sobel, R.S., 2007. The revenue impact of repeated tax amnesties. Public Budgeting & Finance 27, 19–38.

Macho-Stadler, I., Olivella, P., Pérez-Castrillo, D., 1999. Tax amnesties in a dynamic model of tax evasion. Journal of Public Economic Theory 1, 439–463.

Magill, M., Quinzii, M., 2021. Indeterminacy of equilibrium in stochastic olg models. Economic Theory 21, 435–454.

Malik, A.S., Schwab, R.M., 1991. The economics of tax amnesties. Journal of Public Economics 46, 29 – 49.

Manuelli, R., 1990. Existence and optimality of currency equilibrium in stochastic overlapping generations models: The pure endowment case. Journal of Economic Theory 51, 268–294.

Maskin, E., Tirole, J., 2001. Markov perfect equilibrium: I. observable actions. Journal of Economic Theory 100, 191–219.

McCallum, B.T., 1983. The role of overlapping-generations models in monetary economics. Carnegie-Rochester Conference Series on Public Policy 18, 9–44.

Mikesell, J., Ross, J., 2012. Fast money? the contribution of state tax amnesties to public revenue systems. National Tax Journal 65, 529–562.

Milgrom, P., Roberts, J., 1982. Predation, reputation, and entry deterrence. Journal of Economic Theory 27, 280 – 312.

İmrohoroğlu, A., 1992. The welfare cost of inflation under imperfect insurance. Journal of Economic Dynamics and Control 16, 79–91.

İmrohoroğlu, A., Prescott, E.C., 1991. Seigniorage as a tax: A quantitative evaluation. Journal of Money, Credit and Banking 23, 462–475.

Ohtaki, E., 2015. A note on the existence and uniqueness of stationary monetary equilibrium in a stochastic olg model. Macroeconomic Dynamics 19, 701–707.

Parle, W.M., Hirlinger, M.W., 1986. Evaluating the use of tax amnesty by state governments. Public Administration Review 46, 246–255.

Persson, M., Persson, T., Svensson, L.E.O., 1987. Time consistency of fiscal and monetary policy. Econometrica 55, 1419–1431.

Phelan, C., 2006. Public trust and government betrayal. Journal of Economic Theory 130, 27 – 43.

Phelan, C., Stacchetti, E., 2001. Sequential equilibria in a ramsey tax model. Econometrica 69, 1491–1518.

Rangel, A., 2021. Forward and backward intergenerational goods: Why is social security good for the environment? The American Economic Review 93, 813–834.

Reed, R.R., Waller, C.J., 2006. Money and risk sharing. Journal of Money, Credit and Banking 38, 1599–1618.

Reinhart, C.M., Rogoff, K.S., Savastano, M.A., 2003. Debt Intolerance. Brookings Papers on Economic Activity 34, 1–74.

Ross, J.M., Buckwalter, N.D., 2013. Strategic tax planning for state tax amnesties: Evidence from eligibility period restrictions. Public Finance Review 41, 275–301.

Samuelson, P.A., 2021. An exact consumption-loan model of interest with or without the social contrivance of money. Journal of Political Economy 66, 467–482.

Scheinkman, J.A., Weiss, L., 1986. Borrowing constraints and aggregate economic activity. Econometrica 54, 23–45.

Shiller, R.J., 1999. Social security and institutions for intergenerational, intragenerational, and international risk-sharing. Carnegie-Rochester Conference Series on Public Policy 50, 165–204.

Sleet, C., 2001. On credible monetary policy and private government information. Journal of Economic Theory 99, 338 – 376.

Sleet, C., 2004. Optimal taxation with private government information. The Review of Economic Studies 71, 1217–1239.

Song, Z., Storesletten, K., Wang, Y., Zilibotti, F., 2015. Sharing high growth across generations: Pensions and demographic transition in china. American Economic Journal: Macroeconomics 7, 1–39.

Spear, S.E., Srivastava, S., Woodford, M., 1990. Indeterminacy of stationary equilibrium in stochastic overlapping generations models. Journal of Economic Theory 50, 265–284.

Stella, P., 1991. An economic analysis of tax amnesties. Journal of Public Economics 46, 383 – 400.

Stokey, N.L., 1991. Credible public policy. Journal of Economic Dynamics and Control 15, 627 – 656.

Torgler, B., Schaltegger, C., 2005. Tax amnesty and political participation. Public Finance Review 33, 403–31.

Villalba, M.A.S., 2017. On the effects of repeated tax amnesties. Journal of Economics and Political Economy 4, 285–301.