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Endogenous Hours and the Wealth of Entrepreneurs*

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Abstract

US entrepreneurs typically work long hours in their firms and these hours form a large part of the firms' labor input. This paper studies the role of endogenous owner hours in shaping the wealth distribution among entrepreneurs. We introduce owners' endogenous labor supply into a model of entrepreneurial choice and financial frictions. The model fits well the levels and the dispersion of wealth among entrepreneurs. Long owner hours incentivize poor, highly productive individuals to be owners and help the most productive owners to accumulate large quantities of wealth. On net, owners working long hours decreases the median owner wealth and increase wealth dispersion among owners. Differently, the ability to work sufficiently short hours incentivizes owners to run low productivity firms with high wealth to income ratios. Finally, alternative calibrations ignoring the endogenous labor supply of owners lead to owners that are much richer than in the data and overstate the effect of financial frictions in the economy.

Keywords: Entrepreneurship, wealth accumulation, labor supply, firm dynamics

1. Introduction

There is a large variation in wealth levels of US entrepreneurs, with the top 25th percentile owner holding 3 times the average worker wealth and the bottom 25th percentile owner holding one-fifth of the average worker wealth. The entrepreneurship literature typically attributes a part of this variation to the interaction of financial frictions (typically in the form of collateral constraints for factor costs) and firm-level heterogeneity in the history of productivity shocks. In these models, such as Quadrini (2000) and Cagetti and De Nardi (2006), entrepreneurship arises from good business ideas of their owners. Owners have to finance part of their firms' operating costs with their wealth, thus, providing

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an incentive for owners to accumulate wealth.[1] Productivity heterogeneity leads to heterogeneity in desired firm sizes and, thus, in the required wealth to finance operations. Moreover, firms' histories, i.e. the tenure in entrepreneurship and the accumulated productivity shocks, add to the variation in wealth as the longer a firm is already operating and the more profitable it has been the more wealth the owner can accumulate to overcome the financial frictions.

A relatively less explored pattern in entrepreneurship is a high level and a significant dispersion of the labor supply of owners into their own firm. On average, owners work more than 40 hours per week, largely contributing to the total labor input in most firms. Owner hours are the only labor input for 75 percent of the firms in the Survey of Business Owners (SBO), and, even within employers, the mean ratio of owner hours to hired labor is 57 percent. Moreover, owner hours exhibit high dispersion, with an interquartile ratio of 1.7 for the SBO, and 1.8 in the Survey of Income and Program Participation (SIPP).[2]

This paper studies the role of endogenous owner hours in (i) shaping the wealth distribution among entrepreneurs, (ii) explaining the wealth of entrepreneurs relative to workers, and (iii) inferring financial frictions and their impact on the entrepreneurial sector and the owners' wealth distribution. We use an entrepreneurship model with financial frictions and endogenous owner hours to show that owners working sufficiently long hours in their firms makes it attractive to operate firms with little wealth and, at the same time, allows owners to accumulate large quantities of wealth. Accordingly, it expands the left and the right tails of the wealth distribution within entrepreneurs. Moreover, the ability to work sufficiently short hours generates a higher number of low income, yet moderately wealthy, owners. In regards to our second objective, we show that capturing owner hours implies on average less severe financial frictions and more modest wealth levels of owners relative to a model without endogenous owner hours.

In our model, individuals differ in their productivity as entrepreneurs, productivity as workers, their wealth, and disutility of working. They can be either workers or entrepreneurs, and entrepreneurs face financial constraints in the form of a collateral constraint. Workers supply a fixed number of hours. In contrast, entrepreneurs supply their hours flexibly in their firm and also rent capital and hire labor to produce output. Hired labor is not a perfect substitute for owner hours, reflecting the need for supervision and the unique knowledge of an owner in her own firm.

We identify the substitutability between owner hours and hired labor using the average owner hours in intermediate and large firms relative to smaller ones. We calibrate the remaining parameters to match further moments of owner hours, firm size, and labor market outcomes in the US. The model replicates well the non-targeted features of the relationship between owner hours and firm performance. First, the model generates a hump-shaped relationship between firm size and owner hours. The calibrated degree of substitutability between owner hours and hired labor is low enough to make owner hours increase with firm size for smaller firms but high enough to generate reduced owner hours in larger firms, where the marginal disutility of working is too high compared to the diminished effect of their hours on consumption. Second, the model generates a positive correlation of owner hours with contemporaneous and future firm performance. Since financial frictions dampen current firm performance and future growth, long owner hours lead to higher firm output today and faster accumulation of wealth leading to high output in the future. These long owner hours are particularly

[1] Similar mechanisms are used in papers focusing on developing countries, such as Buera and Shin (2013), Midrigan and Xu (2014), and Allub and Erosa (2014).

[2] See the Appendix for descriptions of the data. To assure that entrepreneurs working little in their business do not work long hours as an employee, we consult to the SIPP, which has information on the number of hours spent in one's own business and as an employee. We find that on average, business owners work less than 3 percent of their hours outside of their business (and earn less than 3 percent of total income from outside of their business). Even owners who spend less than 20 hours per week in their business, work on average only 25 percent of their total hours outside of this business.

relevant for highly productive young firms, and the model matches the growth rates observed in the data early in firms' life-cycles.

Importantly, we also establish that the model replicates closely the wealth features of entrepreneurs observed in the data. It matches not only the corresponding dispersion but also the fact that while most entrepreneurs have substantially more wealth than the median worker, the bottom 25% of the entrepreneurial wealth distribution holds almost no wealth. Moreover, despite the median owner income being lower than median worker income, the median owner wealth is higher than the median worker wealth.

We then use our model to study the role of endogenous owner hours in shaping these wealth patterns. In doing so, we distinguish between owners' ability to supply sufficiently long hours and their ability to supply sufficiently short hours. The ability of owners to work significant hours in their firm increases the profitability of businesses. It allows highly productive owners, particularly those with a low disutility of working, to accumulate the wealth to get closer to their optimal size; hence, it expands the right tail of the wealth distribution. Moreover, it makes it feasible to operate firms with little wealth, hence, expanding also the left tail. In fact, a counterfactual experiment where owners cannot work more than 10 hours a week generates a 20 percent decrease in the 95th percentile level of wealth and makes the 25th percentile more than double. Consequently, the interquartile ratio decreases to half in this counterfactual. Put differently, our results show that owners' ability to exert their hours into the firm makes the two tails of the distribution thicker and increases the wealth dispersion.

Next, we study the role of the ability to work sufficiently short hours, namely shorter than salaried workers, for the owners' wealth distribution. In line with the above, the existence of poor, productive owners and also that of very rich owners is related to upward, not downward, flexibility. In contrast, owners that value the ability to work short hours are the unproductive ones, who are over-represented in intermediate wealth ranges. Many of these owners operate their firm because they have more wealth than the typical worker and, therefore, higher consumption and lower optimal hours. This generates owners with lower income, yet, higher wealth than the typical worker. Hence, a counterfactual imposing a lower bound of 40 hours on entrepreneurs' weekly hours reduces the number of owners with modestly high wealth levels. This pushes apart the two tails of the wealth distribution and increases dispersion measures such as the interquartile ratio that rises from 8.1 in our benchmark to 9.5 in this counterfactual. That is, owners' ability to work sufficiently short hours decreases the wealth dispersion.

This paper also shows how the endogenous labor supply of entrepreneurs as a modeling device changes the inferred strength of financial frictions and the resulting wealth levels among business owners. We recalibrate two versions of our model, one that rules out owners' labor supply in their firm altogether, and one that highlights the importance of flexibility by fixing the labor supply of owners at 40 hours per week. As discussed above, owners' ability to supply significant hours to their firm contributes largely to the value of operating a firm and tilts the distribution towards low wealth firms. Therefore, all else equal, ignoring owners' labor supply decreases the number of operating firms, particularly low wealth ones. As a consequence, such a model's recalibration infers a higher average entrepreneurial ability of individuals to rationalize the number of active entrepreneurs. In turn, the model without owner hours infers that financial frictions are more severe to rationalize that the average firm size is quite small, despite entrepreneurs being highly productive. Consequently, the share of financially constrained firms increases to 67% in the model without owner hours up from 24% in the baseline model. Higher average productivity coupled with more stringent financial frictions implies stronger incentives to accumulate wealth. Hence, owners' wealth levels are too high at all points of the distribution in this alternative model. To be specific, the average owner wealth relative to average worker wealth, which is in line with the data in our model, increases by a factor of 15. The dispersion of wealth among entrepreneurs is similar to the baseline model, however, as we argue above, endogenous owner hours coupled with relatively weak financial frictions explain the data better than strong financial frictions

because the model is consistent with the substantial number of low wealth owners. In line with the higher fraction of firms being financially constrained, this model also differs largely in the predicted effects of relaxing financial constraints. If we double the borrowing ability of firms for the operating costs, this alternative model predicts the output in the entrepreneurial sector to almost triple, whereas the predicted increase in our benchmark is 50 percent.

Turning to the recalibrated model with fixed owner hours, we find that this model, similar to our benchmark, captures that most owners have little income. It fails to rationalize, however, that many of these low-income owners are relatively wealthy because it misses selection into entrepreneurship based on wealth. Moreover, the underlying impact of financial frictions on this model are stronger, with 32 percent of firms being financially constrained instead of the 24 percent in our model. This is due to two reasons. First, the upward flexibility of owner hours provides owners with a cushion against financial frictions. Second, similar to the model without owner hours, this recalibrated model implies larger optimal firm sizes and more severe financial constraints. Accordingly, doubling the borrowing ability of entrepreneurs generates a positive impact on the entrepreneurial output that is 25 percent larger than the one in benchmark model.[3]

The labor supply of entrepreneurs to their businesses so far has not received much attention from the macroeconomics literature. Two recent exceptions are Yurdagul (2017) and Allub and Erosa (2014). The former introduces owner hours as shifters of Hicks-neutral productivity to study the value of hours flexibility to entrepreneurs. Meanwhile, Allub and Erosa (2014) study the self-selection of entrepreneurs into employer and non-employer firms where entrepreneurs supply a fixed amount of hours to their firm. The difference in our paper is that first, we allow for endogenous variation in entrepreneurs' labor supply so that we can capture its interaction with financial frictions, propensities to work, and firms' performance. Second, our focus is on the wealth distribution of entrepreneurs whereas these papers' objectives are directly about income differences and flexibility in labor supply (the first paper), or firm dynamics (the second paper).

There is also a strand of literature related to our paper, where the owner may supply managerial input into her firm. Bhattacharya et al. (2013) study the effect of endogenous managerial skill accumulation on firms' life-cycle profiles. In Lee (2019), managerial ability enters the production function in a similar way, and this ability depends on whether the manager is also the owner of the firm or not. In contrast to this line of research, we restrict attention to owners' endogenous labor supply when it comes to modeling their input to the firm and study the effects of this input on entrepreneurs' wealth outcomes.[4]

The paper is organized as follows. Section 2 describes the model. Section 3 explains the calibration and Section 4 discusses the model's predictions for relevant empirical features relating firm performance to owner hours. Section 5 studies the role of owner hours in wealth accumulation of business owners. Section 6 concludes.

2. Model

We study an infinite horizon closed economy with a continuum of individuals. Time is discrete, and the discount factor is β. Individuals derive utility from consumption and leisure, over which they have

[3] Our results draw a connection to Domeij and Floden (2006), who argue that not taking into account the financial frictions biases downward the estimates of the labor supply elasticity of workers. Instead, our focus here is in the reverse direction, in that we find that not taking into account the labor supply adjustments of business owners lead to the overstating of financial frictions.

[4] Some recent papers study the role of management in shaping firm dynamics across countries. See, for instance, Bloom et al. (2016) for an overall review of the role of management practices; and Akcigit et al. (2016) specifically for the role of the ease of delegation to hired managers.

separable preferences. The objective of each individual i is to maximize her life-time utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_{it}) - v_{it} \frac{h_{it}^{1+\phi}}{1+\phi} \right\},$$

where c_{it} is consumption, h_{it} are the number of working hours, ϕ determines the labor supply elasticity, and v_{it} governs the disutility of work. At the beginning of a period, an individual is characterized by her asset holdings, a_{it}, entrepreneurial ability, z_{it}, labor efficiency, x_{it}, and disutility of working, v_{it}. The latter three follow independent AR(1) processes:

$$\begin{aligned} \log z_{it} &= (1-\pi_z)\log z_0 + \pi_z \log z_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_\epsilon^2) \\ \log x_{it} &= \pi_x \log x_{it-1} + \xi_{it}, \quad \xi_{it} \sim N(0, \sigma_\xi^2) \\ \log v_{it} &= (1-\pi_v)\log v_0 + \pi_v \log v_{it-1} + u_{it}, \quad u_{it} \sim N(0, \sigma_v^2). \end{aligned}$$

Each period, an individual is either an entrepreneur (e) or a worker (w). We fix the labor supply of workers at $\bar{h} = 40$ weekly hours.[5] Workers earn an hourly wage of ω per efficiency units of labor.

Define the upper envelope over choosing salaried work or entrepreneurship as:

$$W(z, x, v, a) = \max \left\{ V^e(z, x, v, a), V^w(z, x, v, a) \right\},$$

where V^e and V^w are the values of being an entrepreneur or a worker. Workers can choose each period to become an entrepreneur. Then, the value as a worker is given by:

$$\begin{aligned} V^w(z, x, v, a) &= \max_{a' \geq 0} \left\{ \log(c) - v \frac{\bar{h}^{1+\phi}}{1+\phi} + \beta E_{z', x', v' | z, x, v} \left[W(z', x', v', a') \right] \right\} \\ \text{s.t.} \qquad & c = \omega x \bar{h} + (1+r)a - a' \end{aligned}$$

An entrepreneur's production depends on whether she employs outside labor or operates her firm as a non-employer. An employer entrepreneur's production function is given by

$$Y_{it} = z_{it} \left[K_{it}^{\alpha} \left[h_{it}^{\rho} + L_{it}^{\rho} \right]^{\frac{1-\alpha}{\rho}} \right]^{\eta},$$

where h_{it} is her own labor supply, L_{it} is the hours of hired efficient labor units, and K_{it} is the quantity of rented capital. The parameter ρ captures in a reduced form that an owners' hours are not perfectly substitutable by hired labor possibly resulting from the need for supervision or unique knowledge of the owner in her own firm. The parameter $\eta < 1$ gives the span-of-control.

For non-employer entrepreneurs, we assume a production function that is similar to the one for the employers:

$$Y_{it} = z_{it} \kappa \left[K_{it}^{\alpha} h_{it}^{\lambda} \right]^{\eta}, \tag{1}$$

where we maintain the parameter guiding the returns to capital and the overall span-of-control as in the employers' production function. We do allow the average productivity and the concavity of the labor input to be different.

[5] We discuss the reasoning behind some of the modeling choices, including the fixed hours for salaried workers, at the end of this section.

An entrepreneur decides on her future wealth, a', her own labor supply, h, how much outside labor, L, to hire, and how much capital, K, to rent. She has the option to become a worker when a job offer as a salaried worker arrives, which happens with probability $(1-\chi_E)$. Then the value of entrepreneurship is:

$$
\begin{aligned}
V^e(z,x,v,a) \quad = \quad & \max_{a',K,h,L}\left\{ \log(c) - v\frac{h^{1+\phi}}{1+\phi} + \beta E_{z',x',v'|z,x,v}\Big[(1-\chi_E)W(z',x',v',a') + \chi_E\, V^e(z',x',v',a')\Big]\right\} \\
\text{s.t.} \qquad & c = \max\{\Pi_E, \Pi_N\} + (1+r)a - a' \\
& \Pi_E = z\left[K^\alpha\left[h^\rho + L^\rho\right]^{\frac{1-\alpha}{\rho}}\right]^\eta - wL - (r+\delta)K \\
& \Pi_N = z\kappa\left[K^\alpha h^\lambda\right]^\eta - (r+\delta)K \\
& wL + (r+\delta)K \leq \theta a \\
& h \in \{0, h^2, .., h^N\},\ L \geq 0,\ a' \geq 0,
\end{aligned}
$$

where we assume the firm faces a collateral constraint that requires it to prefund its operational expenses by using a fraction of owners' wealth, θ, and we restrict the choices of owner hours to be in a discrete grid with N points. We denote the decision to be an entrepreneur with $E(z,x,v,a)$, the labor supply decision in case of being an entrepreneur with $h(z,x,v,a)$, capital rental and labor hiring decisions of entrepreneurs by $K(z,x,v,a), L(z,x,v,a)$, and saving decisions by $A(z,x,v,a,o)$ which also depends on the current occupation, o.

Finally, following Quadrini (2000) and Cagetti and De Nardi (2006), there is a non-entrepreneurial sector represented by a production function exhibiting constant returns to scale that hires any resources not used in the entrepreneurial sector. The factor shares are the same as those in the production function of employer entrepreneurs:

$$Y_n = K_n^\alpha L_n^{1-\alpha}.$$

Stationary General Equilibrium. A SGE for this economy is a set of policy functions by individuals $\{E(z,x,v,a), h(z,x,v,a), K(z,x,v,a), L(z,x,v,a), A(z,x,v,a,o)\}$, prices $\{r,\omega\}$, factor demand from the non-entrepreneurial sector $\{K_n, L_n\}$, and a stationary distribution $\Phi(z,x,v,a,o)$ for entrepreneurial productivity (z), worker ability (x), value of leisure (v), wealth (a), the state of owning a firm $o \in \{e,w\}$ such that:

- Given the prices, policy functions solve the individuals' problem.
- Given the prices, K_n, and L_n solve the non-entrepreneurial firms' problem.
- Total demand for labor equals the total supply of efficiency units of labor by workers.
- Total demand for capital equals the total wealth.
- Total output of entrepreneurs and the non-entrepreneurial sector is equal to total consumption and the depreciated capital.
- The distribution Φ is consistent with individuals' policy functions.

Discussion of model features

Here, we briefly discuss the reasons lying behind some of the modeling choices and discuss their implications for firm dynamics and wealth heterogeneity. In Appendix E, we go further in this discussion by showing the sensitivity of our quantitative results to these features.

Separate production function for non-employers. In the data, there is a high rate of non-employer business owners, and these owners typically work short hours, for instance shorter than the hours of small employers. We introduce a separate function for non-employers to be able to explain the low labor supply of non-employer entrepreneurs relative to small employers while replicating the observed number of non-employer businesses. To begin with, we allow for a potentially different weight of owner hours in non-employer firms (λ) than the weight of labor in employer firms ($1-\alpha$) to capture this. As will be apparent from our calibration results, the relatively low weight of owner hours for the non-employer firms makes this option attractive for individuals with low desired hours. To generate the prevalence of non-employer entrepreneurs, we allow for a productivity shifter, κ, to pick up any residual peculiarities of non-employer businesses. We will later show that our model implies realistic output differences between employers and non-employers and income differences between entrepreneurs and workers. These will serve as a check for our modeling choices on non-employer businesses.

Probability of not having salaried work options for entrepreneurs. The stochastic probability of being able to return to salaried work, $1-\chi_E$, works as a reduced form way to capture the long-term commitment owners make when founding a firm. In its absence, marginal firms would enter and exit the market at a much higher frequency than observed in the data. In fact, we will calibrate this probability by targeting the number of 5-year old firms relative to new start-ups.[6] Having a commitment to entrepreneurship changes the productivity distribution of firms in two ways. First, firms enter with relatively higher productivity, because they know that they cannot exit at will. Second, some firms may have had a poor productivity history, yet, were not able to exit. Thus, the model also features more low productivity firms which help in generating the left tail in the income and productivity distribution of entrepreneurs.

Disutility of working shocks. In the data, there is considerable heterogeneity in the labor supply of owners, which is potentially relevant for individuals' self-selecting into entrepreneurship and the firm-level outcomes conditional on entry. Some of this variation is due to the endogenous responses of owners to their wealth and productivity levels. In addition, some of the variation in owner hours can be due to shocks to their attitudes towards work, which will affect their selection into entrepreneurship and the observed wealth levels conditional on entry. Without the heterogeneity in the disutility of working, our model generates some variation in owner hours but falls short in matching the observed dispersion. This suggests that the exogenous component in this variation is quantitatively important and capturing this will allow us to understand the interplay between owners' labor supply and their wealth distribution, which is the focus of this paper.

Fixed labor supply of salaried workers. This assumption is based on the literature in labor economics suggesting that working hours in salaried work are inflexible. (See Dickens and Lundberg (1993), Stewart and Swaffield (1997), Boheim and Taylor (2004), Aaronson and French (2004), among others.) In addition, Yurdagul (2017) shows that the inflexibility in the working hours in salaried work can account for some of the observed cases of entrepreneurship. Accordingly, the fixed hours assumption joint with the heterogeneity in the disutility of working provides incentives for relatively unproductive firms, who value the flexibility in their hours, to operate in the market, often as non-employers. A high disutility of working as the primary reason to operate a firm, thus, captures some of the non-pecuniary motives that Hurst and Pugsley (2011) find are important to understand entrepreneurship. Different from alternative rationales for firms to operate as non-employers, such as fixed costs of hiring, non-pecuniary motives imply that financial frictions are relatively unimportant for non-employers because the productivity of these firms may be far away from the productivity of an employer firm.

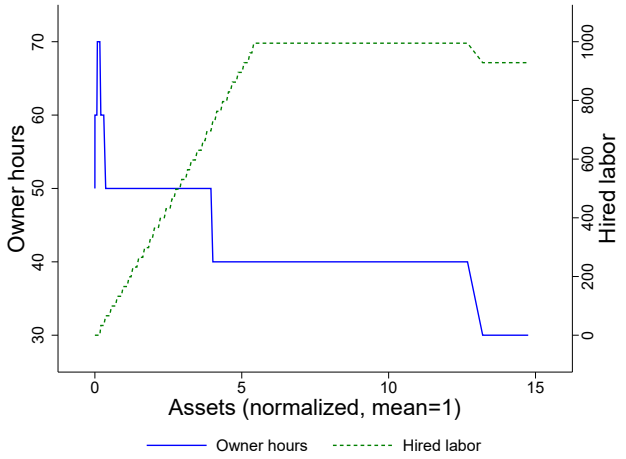

Figure 1: Policies on owner hours and hired labor across wealth

Note: The figure shows the policy functions, conditional on being an entrepreneur for the 98th percentile productivity and median disutility of working in the underlying ergodic distribution. In these plots we use the parameterizations that we highlight in the next section. Hours are given in weekly units. Owner hours in each task are restricted to be on a discrete grid, as detailed in Appendix D.

Discussion on wealth, financial frictions, and owner hours

The model does not allow for a complete analytical characterization, and we solve it numerically given the first order conditions derived in Appendix C.[7] Nevertheless, it is useful to discuss the implications of the model for decisions on owner hours and hired labor before moving on to the calibration section.

Figure 1 plots the optimal choices of an entrepreneur for her own labor supply and hired labor into the firm for a particular level of productivity and disutility of work. The solution uses the model calibration described in the next section, which entails a substitutability parameter, ρ, set at 0.5. On the x-axis, we plot the level of wealth (relative to the average) to highlight the role of financial frictions in these policies.

For low levels of wealth, the firm does not hire workers due to the financial constraint. Accordingly, the owner shoulders all the tasks in the firm. As the wealth level increases, the firm can afford to hire external labor. As hired labor and owner hours are partial complements, the owner increases initially her labor supply. Nevertheless, with yet higher levels of wealth, owners gradually decrease their labor supply into the firm. For any combination of states, there is a threshold wealth level that allows the firm to operate at the optimal level; hence, the hired labor stops increasing for wealth levels higher than this threshold.[8] Importantly, the owners' optimal labor supply is still significant at 40 hours per week even at the point at which the firm operates at the optimal size hiring slightly below 25 full-time worker equivalent hours. Owners still provide significant hours into their firm at high wealth levels because owner hours and hired labor are only partial substitutes. With a linear labor aggregation ($\rho = 1$), high wealth owners would find it optimal to work almost zero hours because their consumption level is high and, thus, their optimal labor supply is low.

[6] Alternative ways of avoiding this would be a certain cost of entering or exiting.

[7] Appendix D gives the details of our numerical strategy for solving the model.

[8] For high enough wealth, the hired labor actually starts going down since wealth effects pull owners' labor supply further down and owners' hours and hired labor are complements.

3. Calibration

We calibrate the model taking a period as a year. We set the parameter governing the labor supply elasticity, ϕ, at 2 corresponding to an intermediate degree commonly estimated in the literature. We follow Cagetti and De Nardi (2006) in setting the capital share, α, to 0.33, the depreciation rate, δ, to 0.06, the span of control, η, to 0.88, and the autocorrelation of worker productivity, π_x, to 0.95.

We calibrate the remaining parameters to match moments from the US data during the 2000s. No single US data set exists that has detailed panel information on firms' input factors, individual wealth, and workers' income. Therefore, we combine two data sets. The SBO has the advantage of a large representative sample of US firms with detailed information on their employment and output. The SIPP, differently from the SBO, has information from both salaried workers and business owners, and it has a panel structure. However, the number of observations on owners is much smaller than in the SBO and the employment information gives the firm size over only three categories. Hence, we use the SIPP to obtain targets that require panel data or those that involve information from the salaried workers. For the rest of the moments, we use the SBO. Appendix A provides a more detailed description of the data samples.

We set the level parameter of entrepreneurial productivity, z_0, to match the 9 percent entrepreneurship rate in the SIPP, where we define an entrepreneur to be an individual who reports having a business with fewer than 100 employees and whose business income is above her salaried income. We use the persistence of entrepreneurial productivity shocks and the probability to be able to enter salaried work to match aspects of the age structure of firms. More persistent entrepreneurial productivity, i.e. higher π_E, implies a higher average firm age, and we use the parameter to target an average firm age of 10.7 years. A lower probability to be able to become a salaried worker, $1 - \chi_E$, implies lower exit rates from entrepreneurship not only through the direct effects on the opportunity to do so but also through discouraging the marginal entrepreneurs from starting a firm. The latter is especially relevant for reducing the exit rates of young firms. Accordingly, we use this parameter to match the survival rate of 44% after 5 years.

Regarding the size distribution of firms, we calibrate the degree of financial frictions, θ, to match the average hours of labor hired by employer firms.[9] With the dispersion of productivity shocks, σ_z, we target the level of employment at the right tail of employers, namely the 95th percentile employment level. In the data, 75 percent of all entrepreneurs are non-employers. Without further differences across the production functions of employers and non-employers, our calibration would imply too few entrepreneurs compared to the data. Accordingly, we set the relative productivity of non-employers, κ, targeting the 75 percent observed in the data.

We target several moments of owner hours. We set the autocorrelation parameter, π_v, and the dispersion of the shocks to the disutility of working, σ_v, to match the autocorrelation and the standard deviation of owner hours in the SIPP. We use the mean of the log disutility of working, v_0, to match that employers work 49.7 hours per week. In the employer entrepreneur production function, the substitutability between own and hired labor, ρ, affects negatively the relationship between firm size and owner hours. Accordingly, we target with ρ the average owner hours for firms with at least 5 employees. Finally, we use the weight of owner hours in non-employer production, λ, to match their average hours worked of 41.8 hours. Our calibration implies that non-employers are relatively productive, possibly resulting from avoiding agency problems, however, the size of their operation is difficult to scale up, i.e., λ is relatively small.

Finally, our calibration targets the labor income dispersion among workers and the wealth to income

[9] In the model, mean labor efficency is one. In the data, we assume that hired labor in each firm has mean efficency of one.

Parameter	Value	Basis
		Panel A: Literature
ϕ	2	Inverse of labor supply elasticity
α	0.33	Cagetti and De Nardi (2006)
δ	0.06	''
η	0.88	''
π_x	0.95	''
		Panel B: Calibrated
z_0	1.08	Fraction of owners = 0.09
π_z	0.99	Average firm age = 10.7
χ_E	0.65	Number of 5-year-old, rel. to 1-year-old firms= 0.44
σ_z	0.14	$95th$ percentile labor of employers = 1190
θ	0.24	Mean size employers = 242
κ	1.47	Fraction of non-employers = 0.75
σ_v	0.90	Std. owner hours = 0.63
π_v	0.47	Persistence owner hours = 0.6
v_0	0.35	Mean owner hours, employers = 49.7
ρ	0.5	Mean owner hours, $\geq$ 5 employees = 50.4
λ	0.61	Mean owner hours, non-employers = 41.8
σ_x	0.29	Gini coefficient labor income = 0.41
β	0.98	Wealth to income ratio = 6.7

Table 1: Parameters

Note: The table gives the model parameterization. Appendix A provides detailed information on our sample construction, variable definitions, and definitions of entreprenuership. Appendix B gives the model counterparts of the targeted moments.

ratio in the economy. In particular, the dispersion to shocks of worker productivity, σ_x, targets the Gini-coefficient of labor income in the SIPP. The discount factor, β, is calibrated to match the ratio of total wealth to total income (labor income and firm profits) in the overall economy, which we estimate to be at 6.7 in the SIPP.[10]

We show in Appendix B that the model matches the targeted moments well. In the next section, we compare the model implications in non-targeted moments to those in the data.

4. Model fit in non-targeted features on owner hours and firm performance

In this section, we compare non-targeted model moments to the data. Given our focus on the endogenous wealth distribution, we pay special attention to the cross-sectional distributions in firms' output and its link to the distributions of firm sizes and owner hours. We supplement this analysis with moments from the firms' age distribution to better understand the role of financial frictions in shaping these distributions.

Though we only target the 95th percentile of employment within employers, Table 2 shows that the model matches the thickness of the right tail in general, with a 95th to 90th percentile ratio of 1.9 (2.4 in the data) and a 90th to 75th percentile ratio of 2.8 (2.0 in the data). The model overpredicts the differences between the left and the right tail within employers because it does not allow indivisible labor demand and, hence, features firms with very little but positive hired labor. This results in a 90th to 50th percentile ratio of 6.8 in the model (4.7 in the data).

The thick right tail in the size distribution translates into a thick right tail in the output distribution that is similar to the data. Table 2 documents that the 90th to 50th percentile ratio in the model is 5.2 and the 90th to 75th percentile ratio is 2.5. Both of these are close to their data counterparts, which are at 5.7 and 2.4, respectively. The model undershoots the 75th to 25th percentile output

[10]This implies a value of the discount factor, β relatively high, at 0.98. Using a value of 0.96 does not change the main implications of our model, except that the ratio of total wealth to total income ratio in the economy would be 5.9.

Moment	Data	Model	Moment	Data	Model	Moment	Data	Model
Hired labor			*Output*			*Owner hours*		
P95/P90	2.4	1.9	P90/P50	5.7	5.2	P90/P10	4.0	3.0
P90/P75	2.0	2.8	P90/P75	2.4	2.5	P90/P75	1.2	1.3
P90/P50	4.7	6.8	P75/P25	5.5	3.7	P75/P25	1.8	1.8

Table 2: Distribution of firms and owner hours

Note: The data source for hired labor and output is the SBO for employer firms with fewer than 100 employees. The data for owner hours is the SIPP for all firms with fewer than 100 employees. PX/PY denotes the percentile ratios X to Y.

ratio, with 3.7 whereas this is 5.5 in the data. Our model features a different production function for non-employer firms than for employers that we discipline by observed labor supply choices of owners and the prevalence of non-employers. Reassuring, the model implies an average output for employer firms as relative to the average non-employer firm of 17.9, close to the 13.2 in the data. For medians, this ratio is 8.9 in the model and 11.7 in the data.

The high share of non-employers and these firms having relatively low output also allows the model to replicate that the median entrepreneur earns less than the median worker. The ratio of median owner to median worker income of 0.76 in the model and 0.51 in the data. In our model, the key for this is the multi-dimensional heterogeneity together with the flexibility in owner hours. Individuals with either a high disutility of working, low labor productivity, or high wealth have an incentive to start a firm, even with a poor business idea, and work few hours. Looking at means instead of medians, the model generates a ratio of average entrepreneurial income relative to average worker income of 1.3, similar to the 1.2 that we get from the data.

The calibration, targeting the standard deviation in owner hours, generates a 90-10 percentile ratio of 3.0, and a 75-25 percentile ratio of 1.8, while these ratios are 4.0 and 1.8 in the data, respectively. Different from the size and output distributions, the right tail of the owner hours distribution is not very thick neither in the model nor in the data: The 90th percentile works only 30 percent more than the owners working at the 75th percentile, whereas in the data the difference is 20 percent.

We now study the model implications for the relationship between owner hours and firm size. Our calibration targets related to this are the average hours of non-employers, the average of employers, and the average among employers with at least 5 employees. Accordingly, looking at how the hours change across narrower size groups is a test for the model performance. Figure 2a shows that in the data, the owner hours increase from a level of 42 hours per week for the non-employers, to a level of around 51 hours for the firms of intermediate size. This pattern slightly reverses for firms that are even larger, with about 49 hours per week for employers of 50 to 99 workers. Figure 2c shows that the model replicates the observed hump-shaped pattern of owner hours across firm sizes. Average owner hours are longer in firms with 5-19 employees than at firms with 1-4 employees because of the imperfect substitutability of owner hours and because larger firms are more productive and, thus, face on average stronger financial constraints that the owner wishes to overcome. Owners at the very large firms are on average very wealthy and have high consumption levels and, therefore, wish to consume more leisure. Moreover, hired labor is a partial substitute for owner hours. These two mechanisms lead to lower hours at the top of the size distribution.

Consequently, the model also matches the behavior of owner hours as a share of total hours in a firm. In the SBO, owner hours are about 80 percent of the hired hours in firms employing 1 to 4 workers. Meanwhile, in larger firms, the owner hours relative to hired hours decline in a monotone way, though, even at firms with 10 to 19 employees, owner hours are as large as 10 percent of the total hired labor. At larger firms, this weight converges to zero due to the increase in the total hired labor. In the model, the variation of this ratio across size groups is quantitatively close to the pattern in the data.

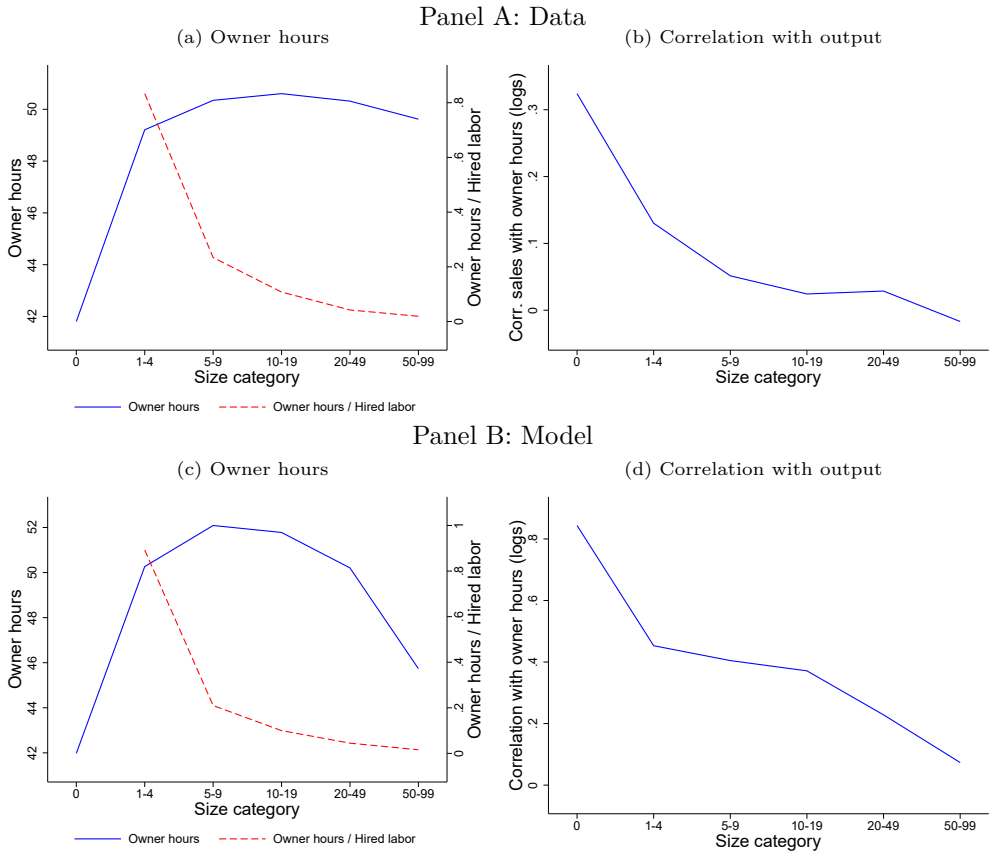

Figure 2: Owner hours, labor, and output

Note: In constructing the figures, we pool all the firms both for the data and for the model. Figures 2a and 2c give the average owner hours and owner hours-to-hired labor ratio in each size category, in the data and in the model. Hours are given in weekly units. Profits in the model is the output net of all costs of externally hired or rented factors. We provide details on data variable definitions in Appendix A.

Without financial frictions, the total labor input would be at its optimal level given the entrepreneurial productivity. Owner hours would help increase firm profits by saving payroll expenses, but this would not alter the firm output. The only link between firm output and owner hours would be a negative relationship that arises due to the concavity in the marginal utility of consumption. In particular, owners of firms with high output would have high consumption and, thus, a low propensity to exert effort to substitute hired labor and to increase firm profits. Yet, Figure 2b shows that owner hours (in logs) are positively correlated with output in the data. This correlation is particularly strong at small firms (around 0.30) and falls towards zero at firms with 50 to 99 employees. The model replicates this qualitative feature, as it allows for financial frictions. Some firms do not operate at their optimal size and owners' labor supply helps overcome this friction, thereby, increasing firm output. Figure 2d shows that this correlation is about 0.8 for firms with no employees and decreases with size. The mechanism through which the model generates this decreasing pattern is that with more workers, the role of owner hours in increasing output is more limited due to diminishing returns to labor.

A rapid average firm growth during the start-up years is usually seen as another indicator for firms overcoming financial frictions. In our model, two particular features contribute to the observed firm growth. First, individuals with a high disutility of working find it optimal to found firms mostly because it allows them to work short hours, and they usually operate as non-employers. Our calibration implies that the disutility of work is less persistent than productivity and, hence, these firms exit the market relatively quickly, leading to average firm size growth from the extensive margin. Second, long

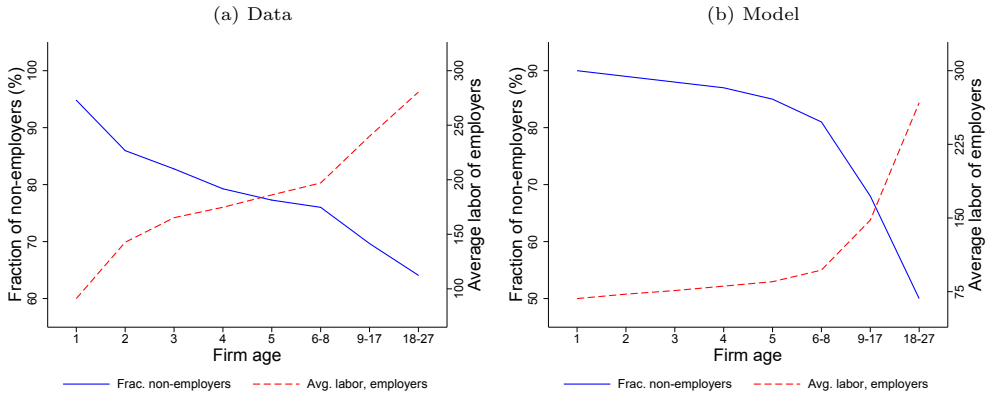

Figure 3: Employment patterns over age

Note: The figure plots the averages for each age group in the data and in the model. We provide details on data variable definitions in Appendix A.

owner hours allow the most productive firms to overcome financial frictions quickly, leading to rapid intensive margin growth. In our calibration, we do not target any of these growth patterns from the data. Figure 3a shows that the fraction of non-employers among owners drop from 90 percent to 50 percent from the 1-year-old firms to firms of age 18 to 27 in the model. This is similar to the patterns in the data, with the corresponding decline from slightly above 90 percent to about 60 percent. The average employment of employers also shows a quantitatively similar pattern in the model to the data, with the average hours hired by start-ups being around 75 hours which goes up to 270 by age 18-27. The corresponding change in the data is from 100 hours to 270.

Previously, we have shown a positive correlation between owner hours and contemporaneous firm output. Entrepreneurial effort can also be positively correlated with future firm performance. The SBO does not allow us to track firms over time. Therefore, we employ the Kauffman Firm Survey (KFS) that follows a single cohort of entering firms over time. The drawback of this is that the average firm size is considerably higher in the KFS than in the SBO raising questions about the comparability of firms. Nevertheless, we still find it worthwhile to study how the differences in owner hours in the initial phases of the life-cycle can be related to the differences in firms' performance later on. Life-cycles of firms whose owners work very long and short hours initially can look different for two reasons. First, owners that work long hours in the first years of the business will work more in the following years to the extent that the factors that make them work long hours (low disutility of working, high firm productivity) in the beginning are persistent. Figure 4a shows that in the data, owners that work particularly long (short) hours at the beginning of the operations typically work long (short) hours in the years to come, though we observe some mean reversion. The same pattern arises in our model, as Figure 4d illustrates. Second, the initial effort of owners itself helps them build the capital they need to increase the output levels in the future. In fact, figures 4b and 4e illustrate that the differences in the output of the two groups of firms remain at around 2 log points in the data and 1 log points in the model. Similar patterns are also visible for employment in figures 4c and 4f. In the data, the differences in the hired labor between the two groups are initially around 50 hours and after eight years, this difference increases to around 150 hours. In the model, the corresponding difference starts at around 10 hours and becomes as large as 20 hours after eight years. Large persistent differences in firm performance that are related to firm observables during the first year of operation also link to recent evidence by Pugsley et al. (2017) who show that most post-entry firm growth is predictable by founding year observables.

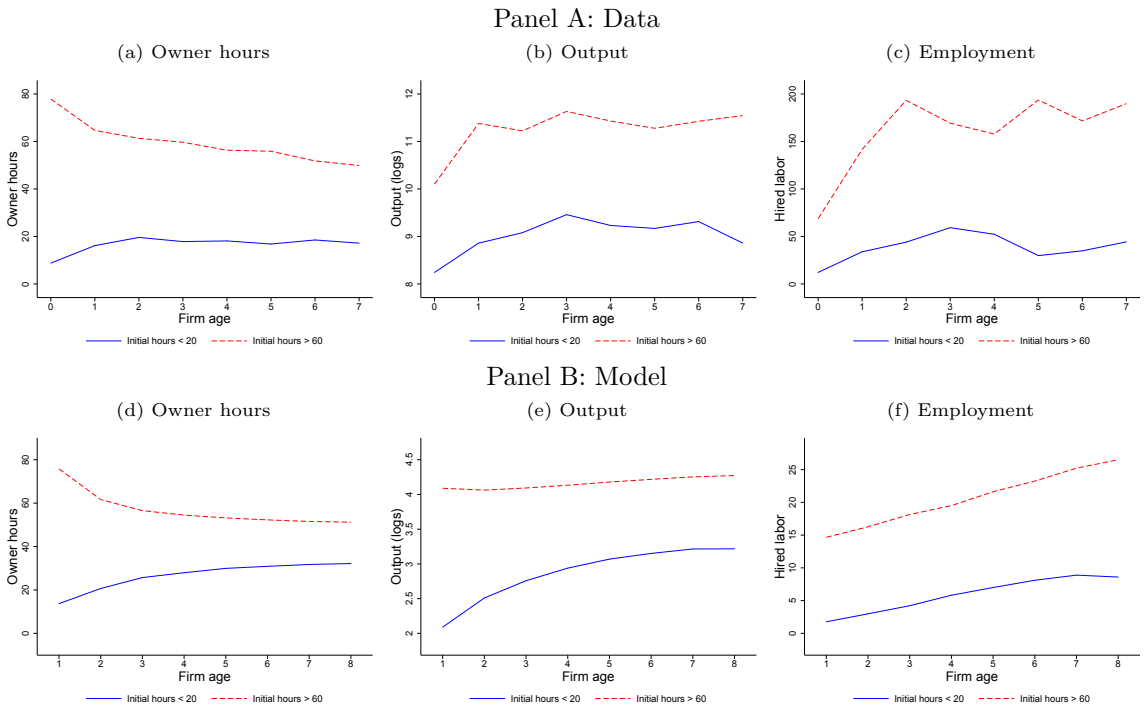

Figure 4: Initial hours and the life-cycle

Note: The figures in Panel A use two subsamples of the population of firms that survive for eight years in the Kauffman Firm Survey. The figures in Panel B use two subsamples of the population of firms from our model simulation surviving the first eight years in business. For each panel, one group consists of the firms whose owners work fewer than 20 hours per week, and the other by those whose owners work more than 60 hours per week, during the first year of operations. Then they plot the averages for these firms in each period.

5. Entrepreneurs' labor supply and wealth

In the previous section, we show that the model matches well the cross-sectional distributions of firm size and output and the relationship between firm size and owners' labor supply. Moreover, the model is also consistent with the income differences between non-employers and employers and the differences between owners and workers. In this section, we show that these features translate into a wealth distribution that is similar to the data and highlight that owners' endogenous labor supply is key for this.

5.1. Model fit in wealth patterns

Our first finding is that the model replicates closely the levels of wealth held by the entire spectrum of entrepreneurs, from rich to poor. Importantly, it does so without targeting any features of wealth except the ratio of the total wealth relative to the total income in the overall economy. This is highlighted in Table 3 where we show the distribution of owners' wealth, normalized by the average worker wealth, in the data and the model. In particular, the average entrepreneurial wealth in the model is 3.4 times that of the average worker, which is close to the corresponding ratio of 2.6 in the model. Behind this average lies a significant heterogeneity, with the top 5th percentile of the entrepreneurial wealth distribution holding about 16 and 10 times the average worker wealth in the model and the data; and the bottom 25th percentile holding 30 and 20 percent of the average worker wealth in the model and the data, respectively.

The right panel of Table 3 highlights the implication of our model for the wealth dispersion within entrepreneurs. The standard deviation of the log-wealth is 1.6 in the model, similar to 1.7 in the data.

Moment	Data	Model	Moment	Data	Model
Levels (rel. to avg. worker)			*Dispersion*		
Average	2.6	3.4	log (sd)	1.7	1.6
P95	10.1	15.9	P90/P75	2.1	3.2
P90	6.4	6.7	P90/P50	6.1	8.0
P75	3.0	2.1	P75/P50	2.9	2.5
P50	1.1	0.8	P75/P25	13.8	8.1
P25	0.2	0.3	P50/P25	4.8	3.3

Table 3: Levels and dispersion of owners' wealth, data and model
Note: Data refers to net wealth from the SIPP. The left panel gives the average and different percentiles of entrepreneurs' wealth relative to average worker wealth. The right panel gives the dispersion measures within entrepreneurs' wealth. PX denotes the level of wealth percentile X.

The ratios of 90th to 75th, 90th to 50th, and 75th to 50th and 50th to 25th are roughly in line with the data. The caveat is that the 75th to 25th percentile ratio is somewhat lower in the model (8.1) than in the data (13.8).

Another test for the model is the allocation of entrepreneurs into different wealth groups in the economy-wide wealth distribution. In the data, there is considerable variation across the wealth groups that entrepreneurs fall into, which is in line with the coexistence of high and low wealth levels among entrepreneurs shown in Table 3. The model matches this well. In particular, 17 percent of owners are in the top 5 percent of the overall wealth distribution in the economy, close to the 15 percent observed in the data. 20 percent of the owners are in the top decile compared to 25 percent in the data. That said, the model undershoots the fraction of owners in the top quartile, with a fraction of 32 percent compared to the 54 percent in the data. Importantly, many entrepreneurs find themselves in the bottom of the wealth distribution both in the model and in the data. 43 percent of the owners are in the bottom half of the distribution, 12 percent are in the bottom quartile. In the data, these are at 30 percent and 14 percent. Nevertheless, both in the data and in the model, the median owner has more wealth than the median worker despite, as shown above, the median owner having a lower income than the median worker. Finally, the model is also consistent with owners' wealth and income being only mildly positively correlated implying that some wealthy owners generate only little firm income. The correlation between log profits and log wealth is 0.24 in the model and 0.32 in the data.

In this paper, we focus on wealth patterns among entrepreneurs and not among workers or in the overall economy. The model gives a decent fit to some measures of overall dispersion such as the standard deviation of log-wealth which is 2.0 in the model and 1.9 in the data, and the dispersion in the upper end, such as the P90/75 ratio which is 1.6 in the model and 2.0 in the data. The model generates too few very low wealth individuals compared to the data, which implies missing the dispersion on the lower end, such as in the P50/P25 ratio that is 4.8 in the model and 13.7 in the data.

Moment	Data	Model
Top 5%	0.15	0.17
Top 10%	0.25	0.20
Top 25%	0.54	0.32
Bottom 50%	0.30	0.43
Bottom 25%	0.14	0.12

Table 4: Allocation of owners in wealth groups, data and model
Note: The table gives the fraction of entrepreneurs that fall into a given category in the economy-wide wealth distribution. Data refers to net wealth from the SIPP.

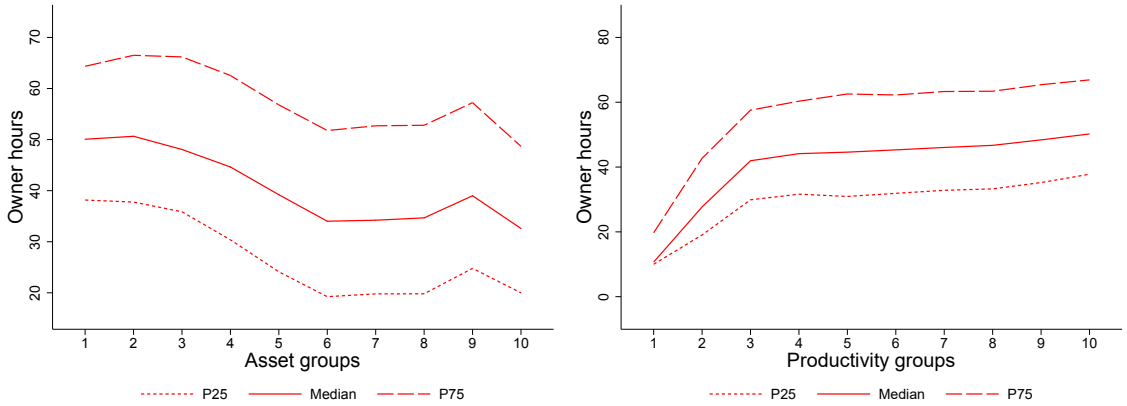

Figure 5: Owner hours across productivity and wealth groups

Note: The figures show the 25th, 50th, and 75th percentile of owner hours within wealth and entrepreneurial productivity groups in the model. The groups correspond to the wealth (a) and entrepreneurial productivity (z) deciles in the economy.

5.2. Role of endogenous owner hours for the wealth distribution

This section studies the role that endogenous owners hours play in shaping these wealth patterns. Before quantifying this role, we build the intuition by showing that owners' labor supply choices are systematically related to their wealth and productivity. Moreover, we illustrate that owners' wealth is not only affected by the history of their productivity but also by the history of their hours worked.

To start building intuition for the model mechanisms, Figure 5 shows the variation in average owner hours across owners' productivity and wealth. It documents that owner hours are particularly long with relatively low levels of wealth and with high levels of productivity. Owners can use their own labor as an input, without accruing factor costs, to increase the output of their firm. This is especially relevant for entrepreneurs who need to overcome the financial frictions, i.e., those with low wealth and those with high productivity. In contrast, firms with low productivity require little labor and, hence, little of the owners' labor input. There are two peculiar factors in our model that contribute to the number of low productive firms and also reduce further the optimal hours of these firms. In particular, higher wealth and higher disutility of working shocks decrease the desired hours and, due to the inflexible hours in salaried work, motivate founding firms even with low productivity.

To highlight the way that the variations in owner hours lead to a causal relationship between owners' labor supply and wealth accumulation, Figure 6 plots the averages of entrepreneurial productivity, disutility of working, owner hours, and hired labor during the first eight years of operation for "rich" and "poor" owners. We define an owner as "rich" ("poor") when she reaches (drops to) the top (bottom) wealth decile of all owners at some point within the current entrepreneurship spell. We also compare these profiles with the corresponding patterns for all firms that remain active for at least eight years. Since the richer (poorer) owners employ more (fewer) workers, as highlighted in Figure 6a, this exercise also illustrates what kind of histories lead to large and small firms.

Our results show that owners that ultimately become rich work particularly long hours (compared to the average owner) during the start-up period of their firm. Their hours increase during these years with age because more wealth leads to more hired labor which complements owner hours. Figure 6c shows that, not surprisingly, owners that enter the top wealth decile within the current spell of entrepreneurship have on average a high and increasing profile of productivity. Moreover, Figure 6b documents that the eventually rich owners also have a persistently low profile of the disutility of working. Since these labor supply shocks are independent of the other states of the owners, this suggests a causal link from the high owner hours to the high wealth levels of these owners. These

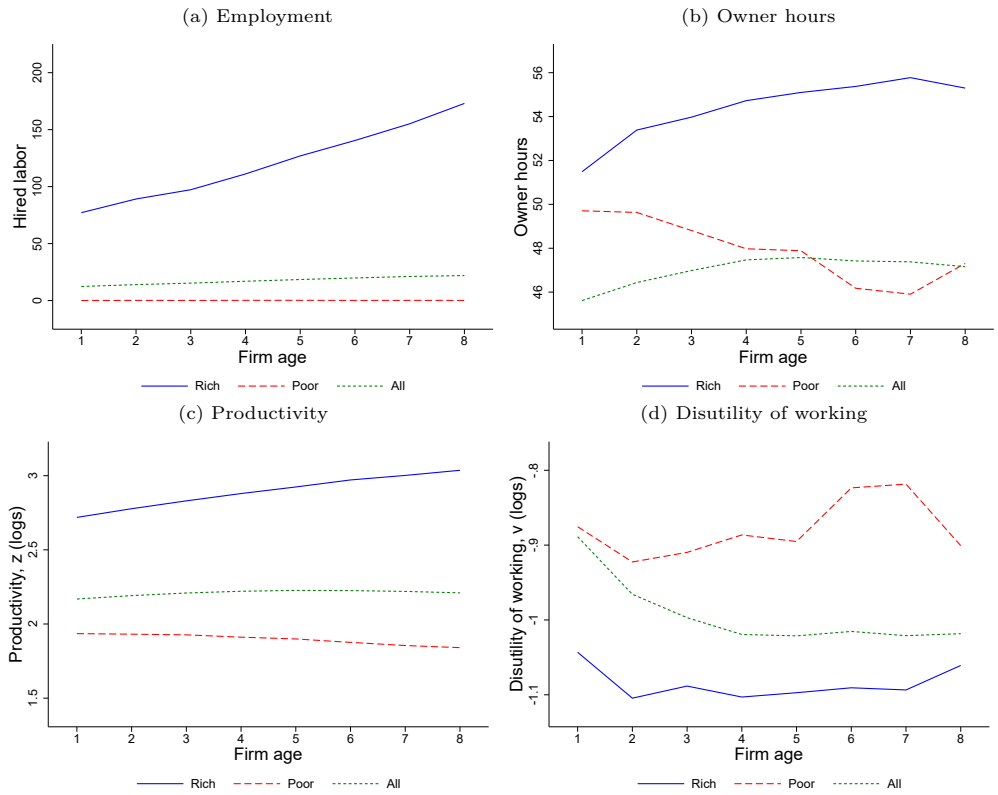

Figure 6: Histories of rich and poor at age 8

Note: The figure shows the histories of average entrepreneurial productivity, disutility of working, owner hours, and hired labor for firms of age 8. For the "Rich" group, we consider the maximum wealth level ever attained after age 7, and identify those for which this level is within the top 10% of the cross-sectional wealth distribution of owners and is larger than the starting wealth of the firm. For the "Poor" group, we consider the minimum wealth level ever attained after age 7, and identify those for which this level is below the bottom 10% of the cross-sectional wealth distribution of owners.

owners' profiles contrast sharply with the early life-cycle of an average owner, and more so with the profiles of the owners who eventually drop into the bottom decile. The typical owner and those owners that remain poor have lower productivity and a higher disutility of working when starting their firm than these eventually rich owners. The poor owners, differently from the average, have continuously high disutility of working shocks paired with low and decreasing productivity. Accordingly, they supply hours that are low and decreasing, leading to bottom decile wealth levels.

In what follows, we quantify the role of entrepreneurial labor supply for the wealth distribution among owners. We split the analysis into two. First, we show the role of owners' ability to work significant hours in the firm. We then contrast this with the role of owners' ability to work short hours.

5.2.1. Role of sufficiently long owner hours

Our strategy here is to run a counterfactual exercise by imposing an upper bound of 10 hours for owners' labor supply. An alternative would be to set the owner hours to zero, but this implies almost no entrepreneurship in our model. Accordingly, we choose a milder experiment to have the effects more visible with a non-degenerate population of owners.

As a first illustration of the changes in the owners' wealth distribution, Figure 7 shows the number of owners in this counterfactual (denoted by $h \leq 10$) whose wealth would fall into a given wealth decile from the baseline. The first finding is that throughout the entire spectrum of wealth, fewer owners operate than in the baseline because the value of being an entrepreneur relative to being a worker

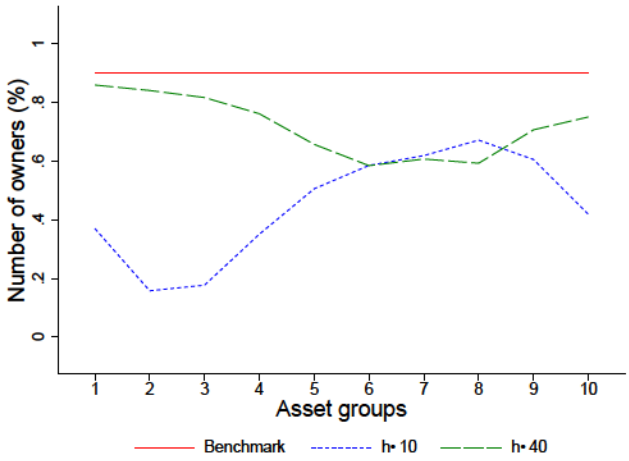

Figure 7: Selection on the wealth dimension

Note: The figure gives the number of entrepreneurs in each asset group, relative to the baseline. The threshold wealth levels for each group are the same across economies, and correspond to the thresholds for wealth deciles in the benchmark. The number of owners in each group is normalized by 0.009, which is the number of owners in each group in the baseline. The total number of owners in each economy are (in the order they appear in the legend) 0.09, 0.04 and 0.07.

decreases. Overall, the fraction of entrepreneurs in the economy decline from 9 percent to 4 percent.

The second finding is that the reduction in the number of owners varies systematically across wealth groups. In particular, Figure 7 shows larger reductions in the number of owners at lower levels of wealth. The number of owners that belong to the second-lowest wealth decile decreases by about 80 percent relative to the benchmark, and that for the third-highest wealth group declines only by 30 percent.

This asymmetry is in line with the high owner hours at lower wealth levels shown in Figure 5 and poor, constrained owners using these hours to overcome financial frictions. Overcoming these frictions is the most relevant for intermediate and highly productive owners. Figure 8 shows that the share of intermediate productive owners declines the most when restricting hours. The share of highly productive owners declines little because their value in entrepreneurship is higher than that of salaried work even with the reduced hours. The share of the least productive owners is almost unchanged because their optimal hours are low. In our baseline model, many of these low productive owners operate their firm because they are relatively wealthy and their optimal hours are low, thus, leading to the small decline in the number of owners at intermediate to high wealth levels.

There are two exceptions to the decreasing relationship between wealth and the declining share of owners resulting from reduced hours. First, Figure 7 shows a relatively large drop in the richest owner group. Given that the most productive owners are still active, this reduction, then, is due to the role of the long hours in allowing the most productive owners to accumulate the high wealth levels, as we have shown in Figure 6. The second exception is at the lower end, as the decline in the number of entrepreneurs from the lowest wealth group is milder than in the two higher wealth categories. In our benchmark, the lowest wealth category not only over-represents the lowest productivity owners but also includes a disproportionally high number of the highest disutility of working owners, which is consistent with the role of high disutility in creating the poorest owners (Figure 6). Accordingly, the number of owners in the lowest wealth category declines rather mildly when we restrict the owners' hours to low levels.

The third ($h \leq 10$) column in Table 5 summarizes these changes in terms of moments of the wealth distribution. Resulting from the particularly large reduction in low wealth entrepreneurs, the left tail

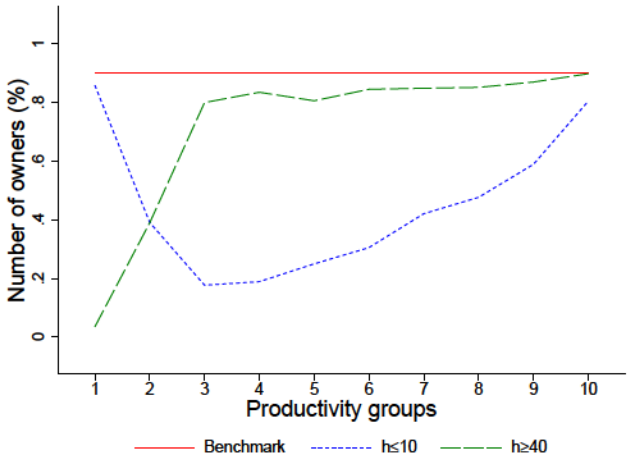

Figure 8: Selection on the productivity dimension

Note: The figure gives the number of entrepreneurs in each group of entrepreneurial productivity, relative to the baseline. The threshold productivity levels for each group are the same across economies, and correspond to the thresholds for productivity deciles in the benchmark. The number of owners in each group is normalized by 0.009, which is the number of owners in each group in the baseline. The total number of owners in each economy are (in the order they appear in the legend) 0.09, 0.04 and 0.07.

of the owners' wealth distribution shrinks substantially. For instance, the bottom 25th percentile of the distribution increases from 0.3 in the benchmark (relative to benchmark median) to 0.7 with the tight upper bound on hours. Similar effects are visible for the median, which increases by 50 percent, and for the 75th percentile, which increases from 2.5 to 2.9. However, the role of long hours for the richest owners manifests itself with a shrinking of also the right tail. The top 5 percentile level decreases from 18.9 to 15.4, and the top 10 percentile decreases from 8 to 7.4.

In turn, the shrinking of the two tails reduces the wealth dispersion among owners. In particular, the measures of overall dispersion, such as the standard deviation, the P90/P50 ratio, the P75/P50 ratio, and the P75/P25 ratio all decrease. Similarly, measures of tail dispersion, such as the P90/P75 ratio and the P50/P25 ratio decrease from 3.2 and 3.3 in the benchmark to 2.5 and 2.1 in the counterfactual with a cap at 10 hours.

To sum up, the poor and highly productive owners choose long hours to accumulate wealth and overcome financial frictions. The ability to work significantly long hours incentivizes poor individuals to be owners and, at the same time, allows higher wealth levels at the very top. Accordingly, supplying significant hours extends the two tails of the wealth distribution and increases the dispersion within entrepreneurs. The expansion on the left tail of the distribution dominates that in the right tail reducing the observed median wealth among owners.

5.2.2. Role of sufficiently short hours

We now show that the flexibility to work short hours is also important for the number and composition of active owners and the wealth inequality among them. For this, we impose a lower bound on owners' labor supply at 40 hours per week. This is the exogenous level we set for workers and roughly the average labor supply of non-employers. Naturally, using a higher threshold, such as the mean of employers, makes the implications of the counterfactual stronger.

The line labeled "$h \geq 40$" in Figure 7 shows that, similar to the previous experiment, the number of firms that are active at any point of the wealth distribution declines, implying an overall drop in the entrepreneurship rate from 9 percent to 7 percent. Intuitively, this experiment roughly leads to opposite effects of the previous one that reduces owner hours in terms of the selection across wealth

Moment	Benchmark	Counterfactual	
		$h \leq 10$	$h \geq 40$
	Percentile (bm med.=1)		
P95	18.9	15.4	19.6
P90	8.0	7.4	8.5
P75	2.5	2.9	2.4
P50	1.0	1.5	0.8
P25	0.31	0.70	0.25
	Dispersion owners		
log (sd)	1.6	1.5	1.7
P90/P75	3.2	2.5	3.5
P90/P50	8.0	5.0	10.5
P75/P50	2.5	2.0	3.0
P75/P25	8.1	4.2	9.5
P50/P25	3.3	2.1	3.2

Table 5: Wealth of entrepreneurs in counterfactuals

Note: The top panel gives the percentile levels of wealth relative to the benchmark median. The bottom panel shows the log-standard deviation and percentile ratios among entrepreneurs in the benchmark and in the two counterfactuals.

categories. In particular, the largest decreases in the number of entrepreneurs occur at intermediate levels of wealth. The mechanisms are in line with the first experiment. Because long owner hours help poor, yet, productive owners operate, forcing long hours has almost no impact on the number of poor owners. In contrast, the experiment leads to a reduction in the number of owners in more intermediate wealth categories and, as Figure 8 illustrates, in the lowest productivity levels. Regarding the richest group of owners, as shown in Figure 6, working long hours in the start-up phase is needed to reach the top of the wealth distribution. This mechanism limits the negative effect of prohibiting short hours on the number of entrepreneurs at the very top wealth levels, making the number of owners from the intermediate wealth levels decrease the most.

Table 5 shows the resulting wealth distribution among owners following the shrinkage in the middle of the wealth distribution. The levels of the bottom part of the distribution, such as the bottom 25th percentile and the median decrease (relative to the benchmark) from 0.31 and 1 to 0.25 and 0.8. In line with the reversal of the pattern of selection at the higher end of the wealth distribution (Figure 7), the levels at the top, such as the top 5th and 10th percentiles, increase from 18.9 and 8.0 to 19.6 and 8.5. In other words, there is an expansion in the two tails of the distribution. In terms of the overall dispersion, the log standard deviation increases from 1.6 to 1.7, and the interquartile ratio increases from 8.1 to 9.5. Regarding the upper end of the distribution, the P90/P75 and the P90/P50 ratio both increase, with the most pronounced change being the latter from 8.0 to 10.5.

In short, our results show that the flexibility of owners to work short hours is valuable, as the entrepreneurship rate declines by around 20 percent when owners cannot work less than 40 hours. Working short hours is particularly valuable for modestly wealthy individuals with low entrepreneurial productivity. Hence, this flexibility shrinks the left and the right tails of the wealth distribution, restricting the dispersion within.

5.3. Implications of a model without endogenous owner hours

The previous subsection studies the role of endogenous owner hours in the context (and parametrization) of our benchmark model. In this subsection, we first show that a quantitative model of entrepreneurship ignoring endogenous owner hours performs worse in matching the wealth distribution among owners. Second, we document that such a model overstates the effects of financial frictions on entrepreneurial activity.

5.3.1. Parametrization ignoring endogenous owner hours

To highlight the importance of owner hours, we compare our baseline model to an alternative parametrization shutting down owner hours altogether. Hence, in this model, no firms operate as non-employers. Variants of this model are the workhorse framework to understand entrepreneurial choice in the literature. To separately emphasize the importance of flexibility in owner hours, we consider a second model that fixes owner hours at 40 hours per week. In this model, firms may still operate as non-employers.

We calibrate these models following the same calibration strategy as in our benchmark to the extent possible. We display the full recalibration in Appendix B. To make the entrepreneurial choice as comparable as possible to the baseline model, we retain the preferences for leisure, and we keep the same mean log disutility of working, v_0. That is, in the two models, the choice between founding a firm or remaining a worker is on average as in the baseline model when the owner chooses $h = 0$ and $h = 40$, respectively.[11] In the model with fixed owner hours, we keep the calibrated dispersion in the disutility of working and the same elasticity of substitution between owner hours and outside labor as in the baseline model. In the model without owner hours, we set the dispersion of the disutility of working to zero and assume production requires only capital and hired labor, i.e., $\rho = 1$.

Two features in which the parametrizations of the alternative models strongly differ from those of the benchmark are firms' average productivity and the strength of the financial frictions. In line with our findings in the counterfactuals of Section 5.2, without recalibration, the models feature a large drop in the share of entrepreneurs.[12] In either case, the recalibrations push the average productivity parameter z_0 upwards to increase the profitability of the firms relative to worker income. The recalibrations imply that the average productivity of operating firms increases by 430 and 8 percent in the model without owner hours and the model with fixed owner hours relative to the baseline. Resulting from the higher productivity, financial frictions, captured by the inverse of θ, need to be larger than the benchmark to explain the small size of employers in the data. In fact, in the model without owner hours, we do not decrease the borrowing ability, θ, further than the one-half of the benchmark to maintain certain comparability with the benchmark.[13]

5.3.2. Positive implications of the alternative models for the wealth distribution

There are two main determinants of the changes in observed wealth levels going from the benchmark calibration to the model without owner hours. First, as Section 5.2.1 shows, forcing owner hours to low levels reduces the number of poor owners and, thus, increases average owners' wealth in our benchmark parameterization. The second determinant is the recalibration of the parameters on entrepreneurial productivity and borrowing ability. In particular, the increase in average productivity increases the observed wealth levels because it increases the optimal firm size and the ability to accumulate wealth. Moreover, the decrease in the borrowing ability, on the one hand, increase owners' incentives to hold wealth. On the other hand, it reduces firms' profits and, hence, their ability to accumulate wealth. In total, we find that the model without owner hours features substantially higher wealth levels among owners. To be specific, the average entrepreneur's wealth level in this calibration is about 52 times that of the average worker, while this ratio is 2.6 in the data and 3.4 in the benchmark model.

Not only is average owner wealth counterfactually large in general, but the model also fails to rationalize any owners in the bottom quarter of the economy's wealth distribution in particular. Table 7 shows that the fraction among owners that are in the bottom quarter of the economy-wide wealth

[11] The results become even stronger when we abstract from the disutility of working in the $h = 0$ model because in that case being an entrepreneur would be relatively less attractive than being a worker.

[12] Since the model without owner hours cannot generate non-employers, we target the 2.3 percent employer rate in the data. In the model with fixed hours, we again target an entrepreneur share of 9 percent.

[13] θ decreases from 0.24 to 0.12 in the model without owner hours and to 0.18 in the model with inflexible hours. The magnitudes of the changes in model implications from the benchmark to the alternative without owner hours would be much larger had we let θ decrease further than the one-half of the benchmark value.

Moment	Data	Benchmark	$h = 0$ model	$h = 40$ model
		Levels (rel. to avg. worker)		
Average	2.6	3.4	52.0	3.1
P95	10.1	15.9	267.0	13.6
P90	6.4	6.7	126.4	5.3
P75	3.0	2.1	33.1	1.6
P50	1.1	0.8	6.0	0.5
P25	0.2	0.2	2.1	0.2
		Dispersion		
log (sd)	1.7	1.6	1.9	1.7
P90/P75	2.1	3.2	3.8	3.3
P90/P50	6.1	8.0	21.1	10.9
P75/P25	13.8	8.1	15.7	9.7
P75/P50	2.9	2.5	5.5	3.3
P50/P25	4.8	3.3	2.8	3.0

Table 6: Wealth of entrepreneurs in alternative models

Note: The top panel gives the average and different percentile levels of wealth within entrepreneurs relative to the average worker in each model. The bottom panel gives the log-standard deviation and percentile ratios of wealth within entrepreneurs.

distribution is zero in the alternative without owner hours. In our benchmark, this fraction is 12 percent, which is close to the data. In line with this, this model allocates too many entrepreneurs to the highest wealth portions in the economy-wide distribution. For instance, 71 percent of owners in this model enter into the top decile of the overall wealth distribution, which in the benchmark is about 20 percent.

In terms of the wealth dispersion, the results from the alternative model without owner hours show that the increase in the wealth levels at the top dominates the increase at the bottom, due to the stronger financial frictions. Accordingly, the overall dispersion measures and those for the upper tail increase. For instance, the interquartile ratio increases from 8.1 to 15.7, the P90/P75 ratio increases from 3.2 to 3.8, and the P90/P50 increases from 8.0 to 21.1.

Next, we turn to the model where we fix owner hours at 40 per week. In this model, owners with high wealth still find it attractive to operate highly productive firms, however, high wealth levels do no longer lead individuals to start low productive firms and work few hours. Instead, in the alternative model, individuals found low productive firms only when their productivity as a worker is even lower than their entrepreneurial productivity. Selection into entrepreneurship upon wealth allows our benchmark model to be consistent with median owner income being lower than the median worker income (0.76), yet, the median owner wealth being larger than the median worker wealth (1.4). Without selection upon wealth, in the alternative with fixed owner hours, the wealth to income ratio of low productive owners, those who need little wealth to finance their operations, is almost the same as for workers. Therefore, though the model features an income ratio of the median owner to median worker similar to the baseline (0.74), it fails to generate a higher median owner wealth (0.8). As a consequence, the model also features too many owners in the bottom half of the wealth distribution (54%).

5.3.3. Effects of financial frictions in models with and without owner hours

In our benchmark model, financial frictions have less pronounced effects on the entrepreneurial sector due to two main reasons. First, endogenous owner hours are a tool to mitigate the effects of financial frictions. Second, ignoring this labor input as a model feature pushes the calibrated productivity levels up and the calibrated borrowing ability down as we highlighted in the parameterization of the alternative models. For instance, only 24 percent of owners in our benchmark model operate their firms below the optimal size. The corresponding fractions are 67 and 32 percent in the alternative

Moment	Data	Model	$h = 0$ model	$h = 40$ model
Top 5%	0.15	0.17	0.64	0.15
Top 10%	0.25	0.20	0.71	0.17
Top 25%	0.54	0.32	0.83	0.26
Bottom 50%	0.30	0.43	0.02	0.54
Bottom 25%	0.14	0.12	0.00	0.20

Table 7: Allocation of owners in wealth groups, data and model
Note: The table gives the fraction of entrepreneurs that fall into a given category in the economy-wide wealth distribution. Data refers to net wealth from the SIPP.

calibrations with no owner hours and with fixed owner hours.

We quantify the severity of financial frictions in our benchmark, in terms of its effects on entrepreneurial aggregates, and compare it with the alternative models by doubling the parameter θ, hence relaxing the financial frictions. We focus on steady state comparisons for each model in reaction to this experiment.

The implications of this exercise are in line with the lower fraction of constrained owners before the experiment in our benchmark compared to the alternatives. Table 8 shows that the aggregate moments for the entrepreneurial sector change in a milder fashion in our benchmark than in the alternative models. In particular, doubling θ leads to a 49.5 percent increase in total entrepreneurial output in the baseline, which is a much smaller change than the 190 percent increase in the model without owner hours and the 62 percent increase in the model with fixed hours. The corresponding comparison for the employment within the entrepreneurial sector is similar, with an 82 percent increase in the benchmark, and a 160 and a 96 percent increase in the two alternative models, respectively.

Next, we study the predicted change in total wealth. There are two opposing effects on the total wealth of entrepreneurs when we relax the frictions. There is a positive effect, because, the profits of the firms and, hence, the ability of their owners to accumulate wealth increases.[14] There is a negative effect because the wealth level needed to reach the optimal size, given a productivity level, decreases. For the firms that are still far from their optimal size, the positive effect dominates. For the firms that are at their optimal size, or close to it, the negative effect dominates because frictions form an important reason why owners accumulate wealth in these models. We find that in all the three models the positive effect surpasses the negative one as reflected in the higher total entrepreneurial wealth in all the models. However, the increase in total wealth in the benchmark is smaller than in the alternatives with a 15.7 percent increase compared to increases of 69.7 and 22.6 in the alternative models.

Apart from total wealth, also wealth inequality reacts differently to an improvement in borrowing requirements in the three models. As discussed above, relatively many owners with low entrepreneurial ability operate with relatively high wealth in the baseline model compared to the model with fixed owner hours. These owners are not borrowing constrained and, thus, do not increase their wealth holdings as a response to better borrowing conditions. As a consequence, the interquartile ratio of wealth increases relatively little in the baseline model (0.9) compared to the model with fixed owner hours (3.5). The share of constrained owners that are relatively wealthy is yet higher in the model without owner hours leading to an increase in the interquartile wealth ratio of 10.6.

[14] There is a further positive effect stemming from entrepreneurship becoming more attractive and more individuals operating a firm.

	Benchmark	$h = 0$ model	$h = 40$ model
	Share of constrained owners (%)		
Before experiment	23.6	66.9	32.1
After experiment	13.6	56.5	21.5
	Change (rel. to corresponding baseline)		
Output (%)	49.5	189.6	61.6
Hired labor (%)	82.0	159.8	95.8
Wealth (%)	15.7	69.7	22.6
Wealth, sd (logs)	0.14	0.13	0.19
Wealth, P75/P25 ratio	0.9	10.6	3.5

Table 8: Impact of relaxing the financial constraints (doubling θ)

Note: The table shows the consequences of doubling the parameter θ in the benchmark calibration and in the alternative models without owner hours and fixed owner hours described in Section 5.3.1. The top panel gives the fraction of entrepreneurs that are financially constrained in the alternative calibrations, before and after the experiment. The bottom panel gives the changes relative to the baseline of each calibration. For the output, hired labor and wealth, we give the percentage change in the totals among entrepreneurs. In the last two rows, we show the absolute change in the log standard deviation and in the interquartile ratio.

6. Conclusion

This paper studies owners' labor supply as a factor in shaping the levels and heterogeneity of entrepreneurs' wealth and the strength of financial frictions. The model features endogenous owner hours in an entrepreneurship model and matches well the empirical patterns of owner hours and firm performance. Importantly, the wealth levels and within-owners wealth dispersion also fair well with the data.

We find that endogenous owner hours affect the wealth distribution of business owners, first, because they increase the firm profitability for poor owners and, hence, increase the number of poor owners. Moreover, it helps the owners accumulate the wealth needed to reach their optimal size. Ultimately, owner hours increase the wealth dispersion within entrepreneurs and decreases the median level of owners' wealth.

We show that endogenous owner hours is an important modeling device to explain the number of business owners with low levels of wealth and low productivity. In addition, ignoring owner hours in the production function leads to overstating the importance of financial frictions in the economy. In fact, we find that only 24% of owners are financially constrained, and most owners operate either as non-employers or at a very small size without any desire to grow. We show that as a consequence, improvements in financial markets lead to only small changes in owners' wealth.

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Appendix

A. Data details

In our analysis we use three data sources.

Survey of Business Owners (SBO). SBO data comes from the Public Use Microdata Sample (PUMS) file for 2007, provided by the US Census. We exclude firms with multiple owners in our analysis. The sample also excludes owners of businesses larger than or equal to 100 employees, and those who report that the business is not their primary source of income. We weight observations with their tabular weights (*tabwgt*). To avoid disclosure, SBO categorizes the information on receipts and employment. We take the median integer in the range of a category as the corresponding continuous value of the variable.[15]

For output, we use the total receipts (*receipts_noisy*). For hired labor hours, we multiply the number of workers (*employment_noisy*) by 35, as we do not observe part time and full-time workers separately. Owner hours are given within 6 categories. We replace the category "None", with zero hours; category "Fewer than 20" as 10 hours; category "20 to 39" as 30 hours; "40 hours" as is; "41 to 59 hours" as 50 hours; and "60 and above" as 65 hours.

Survey of Income and Program Participation (SIPP). We use the 1996, 2001, 2004, and 2008 SIPP panels and deflate all data using the CPI. The SIPP is a representative survey of US non-institutionalized households with a panel dimension of up to 4 years. Every 4 months, called a wave, the Census conducts an interview with all adult members of participating households asking them about their work and household characteristics during the preceding 4 months. In order to account for the seam-bias effect generated by the recollection period, we aggregate the monthly information to the wave period. To maximize the number of available information, we use the wave data for cross-sectional statistics. For the data using the panel dimension, we aggregate further to the yearly frequency of the model.

We keep individuals between ages 22 and 65 that are not part of the armed forces. A worker is counted to be an entrepreneur whenever they report having a business, the business income is above their income from being an employee, business income is at least $50 per quarter, and the firm size is at most 100 employees. Individuals may report to have up to two businesses and we aggregate all business income. Business equity is top-coded and we impute top-coded values using a Pareto distribution fitted to the upper tail of non-top-coded values.

Kauffman Firm Survey (KFS). This data is provided by the Kauffman Foundation. Our sample excludes firms with multiple owners. We weight the firms with their initial sampling weights. We use the private version of the KFS accessed through the NORC Data Enclave.[16]

Owner hours is the hours worked by the owner for the business on average per week ($g1b1\_hours\_owner\_01\_X$). To construct the total hired labor per week, we sum the number of full time employees ($c6\_num\_ft\_employees\_X$) multiplied by 40 and the number of part-time employees ($c7\_num\_pt\_employees\_X$) multiplied by 20. For yearly profits and the payroll, we use profits ($f24\_profit\_amt\_X$) and the amount paid to wages, salaries and benefits in the wave of reference ($f18a\_wage\_exp\_Y\_amt\_X$).

We construct our measure of output using the profits, payroll and the total value of capital owned by the business. Since we do not observe the costs of capital rental, we obtain an opportunity cost of capital by multiplying different types of assets by the sum of their depreciation rates and the risk-free interest rate. The depreciation rates for each item comes from the Bureau of Economic Analysis

[15] See http://www.census.gov/econ/sbo/ for complete data description.

[16] For the definitions of the raw variables, see http://www1.kauffman.org/kfs/KFSWiki/Data-Dictionary.aspx.

estimates for year 2004.[17] The risk-free interest rate is set at 5 percent following our calibration of the model. In specific, we construct the cost of capital by adding the value of business-owned land and buildings ($f28e\_asset\_landbuild\_X$) by $0.05 + 0.02$, value of equipment ($f28d\_asset\_equip\_X$) by $0.05 + 0.06$ and value of vehicles ($f28f\_asset\_veh\_X$) by $0.05 + 0.12$.[18] We compute output by adding profits, payroll costs and the costs of capital.[19]

B. Calibration information

Param.	**Moment**	**Data**	Benchmark		$h = 0$ model		$h = 40$ model	
			Parameter	Value	Parameter	Value	Parameter	Value
z_0	Fraction of owners	0.09	1.08	0.09	1.13	0.02	1.18	0.09
π_z	Avg. firm age	10.70	0.99	10.73	0.95	14.19	0.99	10.58
σ_z	$95th$ pct labor, employers	1190	0.14	1029	0.40	1164	0.13	1043
χ_E	No. of 5-y / 1-y-old firms	0.44	0.65	0.45	0.51	0.44	0.41	0.44
θ	Mean size, employers	242	0.24	245	0.12	349	0.18	247
κ	Fraction of non-employers	0.75	1.47	0.75	-	0	1.46	0.75
σ_v	Std. hours, owners	0.63	0.90	0.63	-	0	0.90	0
π_v	Persistence hours	0.60	0.47	0.62	-	-	0.47	-
v_0	Mean hours, employers	49.70	0.35	50.14	0.35	0	0.35	40
ρ	Mean hours, emp. size > 4	50.4	0.50	50.3	1	0	0.50	40
λ	Mean hours, non-employers	41.90	0.61	41.92	-	0	0.61	40
σ_x	Gini coefficient labor income	0.41	0.29	0.42	0.28	0.42	0.29	0.42
β	Wealth-to-income ratio	6.7	0.98	6.99	0.97	6.97	0.98	7.04

Table A1: Parameters in alternative models

Note: The first part of the table shows the calibrated parameters of the benchmark model, the data targets they correspond to, and the model's implications for these corresponding moments. The second part ("$h = 0$ model") shows the parameterization in the alternative model without owner hours that we study in Section 5.3. The final part ("$h = 40$ model") does the same for the model with owner hours fixed at 40 hours per week.

In Section 3, we omitted the documentation of the benchmark model's fit to the targeted moments. The first columns in Table A1 provide this comparison.

In our Section 5.3, we compare the implications of our benchmark calibration with those of calibrations omitting endogenous owner hours. Consider first the model with fixed owner hours, $h = 40$. In that case, relative to the baseline calibration, the model cannot match by design the average hours of employer and non-employer owners, the average hours at different employer sizes, and the dispersion of owner hours. Therefore, we fix the parameters associated with these moments (ν_0, λ, σ_v, ρ) to those from the baseline calibration. We calibrate the remaining parameters following the same calibration strategy as in the baseline calibration. The last columns in Table A1 summarizes the resulting parameter values in this alternative calibration, and the values for the corresponding statistics we target in the benchmark.

Turning to the model without owner hours, we assume production requires only capital and hired labor, i.e., $\rho = 1$. To make the entrepreneurial choice as comparable as possible to the baseline model, we retain the disutility of working as a worker. That is, in the two models, the choice between founding a firm or remaining a worker is as in the baseline model when the owner chooses $h = 0$. Given that no owners operate as non-employers in this model, we target the average age of employers in the data that is 14.5 years. Similarly, we target an entrepreneurship rate of 2.3%; the share of employers in society. However, even with this more conservative calibration target, the model is not able to match the share of owners together with average firm size, even with very low values for the collateral

[17] See https://www.bea.gov/national/FA2004/Tablecandtext.pdf.

[18] The implied cost shares of capital are very much in line with the usual estimates of 0.33.

[19] Unfortunately, we do not observe the costs of intermediary goods.

constrained. We opt to fix the collateral constrained to 1/2 of our baseline calibration and allow firms to be larger than observed on average in the data. Calibrating even more severe financial frictions to match the same moments as in our baseline calibration would amplify the results described in Section 5.3. The columns under "$h = 0$ model" in Table A1 shows the parameter values for this model.

C. Web appendix: Solving entrepreneurs' problem

Here we focus on the first order conditions of an entrepreneurs' problem, taking as given the productivity level z, the disutility of work v, wealth a, and also her choice on asset holdings a' and own labor supply, h.

Case 1: Employers. We start with the solution in case of employing workers. For this, it is convenient to define the following term for aggregate labor:

$$X \equiv \left[\lambda h^{\rho} + (1-\lambda)L^{\rho}\right]^{\frac{1}{\rho}}.$$

The first order conditions, given the corresponding variable is positive, are:

$$(K) \qquad \eta\alpha\frac{Y}{K} = (r+\delta)(1+C)$$
$$(L) \qquad \eta(1-\alpha)(1-\lambda)\frac{Y}{X}X^{1-\rho}L^{\rho-1} = \omega(1+C)$$

Then, if financial constraints do not bind:

$$Y = \left[z\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}X^{(1-\alpha)\eta}\right]^{\frac{1}{1-\alpha\eta}}$$

This equation gives Y as a function of X. The second equation to solve these two unknowns is:

$$X = \left[\lambda h^{\rho} + (1-\lambda)\left(\frac{(1-\lambda)(1-\alpha)\eta Y X^{-\rho}}{\omega}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\rho}}.$$

Putting two together, we get:

$$X^{\rho} = \lambda h^{\rho} + (1-\lambda)\left(\frac{(1-\lambda)(1-\alpha)\eta\left[z\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}\right]^{\frac{1}{1-\alpha\eta}}X^{\frac{(1-\alpha)\eta}{1-\alpha\eta}-\rho}}{\omega}\right)^{\frac{\rho}{1-\rho}}$$

Which is one equation for one unknown X. Once we solve this, we get Y from the first equation. Then, we recover L and K from:

$$L = \left[\frac{(1-\lambda)(1-\alpha)\eta Y X^{-\rho}}{\omega}\right]^{\frac{1}{1-\rho}}, \quad K = \frac{\alpha\eta Y}{r+\delta}.$$

If the financial constraints bind we use the following non-linear equation to find L, and then K:

$$L = \frac{\theta a}{\omega\left[1+\frac{\alpha}{1-\alpha}\frac{1}{1-\lambda}\left(\lambda\left(\frac{h}{L}\right)^{\rho}+1-\lambda\right)\right]}, \quad K = \frac{\alpha}{1-\alpha}\frac{1}{1-\lambda}\frac{\omega}{r+\delta}L\left[\lambda\left(\frac{h}{L}\right)^{\rho}+1-\lambda\right]$$

Case 2: Non-employers. The FOC for capital rental is:

$$\alpha\eta\frac{Y}{K} = (r+\delta)(1+C)$$

If the financial constraints do not bind:

$$Y = \left[z\kappa\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}h^{\lambda\eta}\right]^{\frac{1}{1-\alpha\eta}}, \quad \Pi = Y(1-\alpha\eta), \quad K = \alpha\eta\frac{Y}{r+\delta}$$

If the financial constraints bind, then:

$$Y = z\kappa\left(\frac{\theta a}{r+\delta}\right)^{\alpha\eta}h^{\lambda\eta}, \quad \Pi = Y - \theta a, \quad K = \frac{\theta a}{r+\delta}.$$

Iterating the prices using the non-entrepreneurs' problem. We compute the aggregate supply levels not absorbed by the entrepreneurial sector of each factor, $K_j^{s,n}$, $L_j^{s,n}$ and use the first order conditions from the non-entrepreneurial sector to compute an update for equilibrium prices:

$$\frac{K}{L} = \frac{\alpha}{1-\alpha}\frac{\omega}{r+\delta}, \quad \alpha^{\alpha}(1-\alpha)^{1-\alpha} = (r+\delta)^{\alpha}\omega^{1-\alpha}$$

Using these, we update the prices as follows:

$$r_{j+1} = \epsilon r_j + (1-\epsilon)\left\{\frac{\alpha}{1-\alpha}\omega_j\frac{L_j^{s,n}}{K_j^{s,n}} - \delta\right\}, \quad \omega_{j+1} = \left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r_{j+1}+\delta)^{\alpha}}\right]^{\frac{1}{1-\alpha}}$$

where ϵ is a relaxation parameter to make the updates smoother.

D. Web appendix: Computational details

We discretize the individual state into 245 assets, 21 entrepreneurial productivity levels, 5 worker productivity levels, and 6 disutility levels of work. We allow for off grid asset choices with a total of 2197 possible choices. Regarding entrepreneurial productivity, given a share of only 9% of entrepreneurs in the economy, we are particularly interested in its right tail. Thus, we choose a uneven grid with 67% of the grid points lying in the top 10% of the ergodic productivity distribution. For the discrete hours choices, we allow for 10 grid points.

The algorithm starts by guessing an interest rate and the wage. Given prices, we solve the value functions at the grid points. We use a Monte-Carlo simulation of 10,000 individuals to compute the stationary distribution of the economy. Given the stationary distribution, we compute total asset supply, total labor supply, and the capital and labor demand of entrepreneurs. Given these quantities, we compute the aggregate supply levels not absorbed by the entrepreneurial sector of each factor, K_n^s and L_n^s and use the first order conditions from the non-entrepreneurial sector to compute an update for equilibrium prices.

E. Web appendix: Sensitivity analysis

We dedicate this appendix section to show the sensitivity of our model to a few changes in the modeling and calibration strategy. In particular, we first consider changes to the model features discussed in Section 2: the separate production function for non-employers, the option to return to salaried work, and the heterogeneity in the disutility of work. In addition, we show the sensitivity of our results to having a higher substitutability between owner hours and hired labor. We modify the calibration strategy one by one for each of these, and recalibrate each time the model to match (abstracting from the modified parameter) the same moments as the baseline model.

Our benchmark model allows for different production functions for employers and non-employers. In Section 3, we show that the calibration implies stronger concavity for the labor input in the case of non-employers relative to employers. The column labeled $\lambda = 1-\alpha$ in Table A2 shows the result when we restrict these to be the same, i.e., $\lambda = 1-\alpha$. The first panel shows that the model implies a similar dispersion of owner wealth and a similar mean owner wealth as the baseline. Similarly, the second panel shows that owners are still over-represented at the top of the economy-wide wealth distribution, yet, a sizable fraction of owners are in the bottom half of the distribution. The third panel shows that the financial frictions are weaker in this alternative calibration, particularly that the number of constrained owners is smaller. Relative to the baseline, being a non-employer becomes relatively more valuable for individuals with long hours, i.e., the selection between employers and non-employers becomes weaker. In fact, the average weekly hours of non-employers rise from 42 in the baseline to 46. With more of the productive owners staying non-employers, the collateral constrained needs to become weaker to match the average firm size among employers. The last panel shows that resulting from the less severe financial frictions, the impact of changing the collateral constrained is even weaker than in the baseline model.

Next, we focus on the probability of owners to have an option to become salaried workers, $1-\chi_E$. In our model, the parameter χ_E captures the commitment to entrepreneurship, and we calibrate this parameter to match the share of firms surviving until year 5 relative to those of new startups. This implies a χ_E of 0.65 in our baseline calibration. To test the sensibility of our model to this calibration target, we decrease this parameter to 0.5, i.e. increase the probability of opportunity to become a salaried worker from 0.35 to 0.50. This leads to a drop in the firm survival rate to age 5 from 0.44 to 0.35. The column labeled $\chi_E = 0.5$ shows that this alternative calibration provides a dispersion of owner wealth similar to the baseline and an average level that is, though higher than in the baseline, well below the level implied by the model without owner hours. The second panel shows that the shares of owners in the top 5% and bottom 50% of the economy's wealth distribution are almost unchanged. Having fewer young firms, with a constant (calibrated) average firm age, implies that the typical owner must be further away from the participation threshold than in the baseline model. As a consequence, the second panel shows that financial frictions are somewhat stronger than in the baseline model, and the third panel shows that the effects of changing these frictions become stronger. Nevertheless, most owners are still unconstrained in this calibration and the effects of relaxing financial frictions are substantially weaker than in the model without owner hours.

For our benchmark model, we calibrate the variance parameter of the disutility of working shocks, σ_v targeting the standard deviation of owner hours. Here, we show the implications of reducing this parameter to a two-thirds of the benchmark calibrated value. As a consequence, the standard deviation of owner hours declines from 0.63 to 0.47. The calibration then implies a slight increased level, and an almost unchanged standard deviation of owners' wealth. The share of owners in the top 5% of the economy's wealth distribution rises to 22%, and the share of owners in the bottom 50% declines slightly relative to the baseline model. With fewer owners choosing entrepreneurship because of a high disutility of work, high productivity becomes more predominant as a motive for entrepreneurship. That is, the model becomes more similar to the model without owner hours where average firm productivity is higher, wealth still is highly dispersed, yet, around a counterfactually

	Benchmark	$\lambda = 1 - \alpha$	$\chi_E = 0.5$	$\frac{2}{3}\sigma_v$	$\rho = 0.7$
			Owner wealth		
Average	3.4	2.7	4.0	4.6	4.2
Sd (log)	1.6	1.6	1.6	1.7	1.6
P75/P25 ratio	8.1	8.4	8.2	9.6	8.8
			Share in wealth distribution (%)		
Top 5%	0.17	0.14	0.18	0.22	0.19
Bottom 50%	0.43	0.47	0.42	0.42	0.42
			Share of constrained owners (%)		
Before experiment	23.6	16.8	0.31	33.6	33.3
After experiment	13.6	9.39	0.19	21.3	22.8
			Change (rel. to corresponding baseline)		
Output (%)	49.5	36.5	60.5	63.2	84.3
Hired labor (%)	82.0	61.0	98.8	100.0	122.2
Wealth (%)	15.7	0.2	25.7	31.8	39.4
Wealth, P75/P25 ratio	0.9	1.0	1.9	2.9	2.1

Table A2: Sensitivity Analysis

Note: The table shows the main results of our sensitivity checks: (i) setting weight of owner hours in the non-employer production function to that of the employer function ($\lambda = 1-\alpha$), (ii) increasing the probability of having a salaried work option for entrepreneurs from 0.5 to 0.65 ($\chi_E = 0.65$), (iii) decreasing the volatility of disutility of working by one-third ($\frac{2}{3}\sigma_v$), and (iv) increasing the substitutability parameter between own hours and hired labor from 0.5 to 0.7 ($\rho = 0.7$). The first panel gives the average owner wealth relative to average worker wealth, and two measures of dispersion without entrepreneurs' wealth. The second panel gives the fraction of owners that are in the top 5% and bottom 50% of the economy-wide wealth distribution. The last two panels repeats the experiment that doubles owners' borrowing ability, θ, for each recalibration.

high level of owner wealth. Accordingly, the share of constrained owners increases to 33.6 percent, and the effects of relaxing financial frictions are larger than in the baseline model. Nevertheless, the impact of relaxing financial frictions remains much smaller than in the model without owner hours.

Finally, we study how does it affect our conclusions to increase the parameter guiding the substitutability between owner hours and outside labor, ρ, from 0.5 to 0.7. This change naturally makes the owner hours less important for firm dynamics. Because it becomes easier to substitute owner hours, owners at intermediate and large firms work fewer hours. The average hours of owners with more than 5 employees declines from 50 to 44 hours. The column labeled $\rho = 0.7$ shows that the recalibrated model matches again the standard deviation of owner wealth and implies a somewhat higher mean owner wealth than in the baseline. Also, the shares of owners in the top 5% and bottom 50% of the economy's wealth distribution are almost unchanged. At the same time, making it easier to substitute owner hours implies that they have less of a role to play in mitigating financial frictions. Hence, the model implies that more owners are constrained than in the baseline model; the share increases to 33.3 percent. Consequently, the impact of relaxing financial frictions on the entrepreneurial sector becomes stronger. For example the change in output in the baseline model when relaxing the collateral constrained by one half is 49.5 percent and increases to 84.3 percent in the model with $\rho = 0.7$. Though the effect is substantially larger than in the baseline model, it is still only half of the effect in the model without owner hours.