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Who Quits Next? Firm Growth in Growing Economies*

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Abstract

We document novel facts about the relationship between aggregate growth and firm dynamics using a large set of countries. We argue that firm employment patterns are not necessarily informative about cross-country differences in aggregate growth because they are induced by changes in the productivity of a firm relative to others. In contrast, aggregate growth is linked to average firm-level productivity growth and firm age. We formalize this intuition through a tractable model of endogenous aggregate growth and firm dynamics where firms realize positive returns to investment with some probability. We find that cross-country disparities in this probability can account for two-thirds of the variation in aggregate growth.

Keywords: firm dynamics; productivity; selection; economic growth; cross-country data.

JEL codes: D21, D22, E23, O4.

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I Introduction

An extensive literature relates differences in the level of development across countries to differences in firm employment and productivity distributions (among them, Hsieh and Klenow, 2009; Bartelsman et al., 2009; Moscoso Boedo and Mukoyama, 2012; Poschke, 2017; Bento and Restuccia, 2017). However, not much emphasis has been made on linking firms' productivity and employment growth profiles to aggregate growth across countries. In this paper, we depart from the previous literature towards a more dynamic analysis and study the link between aggregate growth and growth at the firm level.

We start by documenting new facts relating the industry dynamics to economic growth. We use the standardized dataset of the World Bank Enterprise Surveys (ES), which provides harmonized plant-level data for more than 91 countries for years 2003 to 2014 in service and manufacturing sectors. We document that countries with rapid growth have (i) a larger share of firms undergoing labor productivity growth, but (ii) no significant differences in the share of firms increasing employment. In addition, (iii) the fraction of firms increasing employment, among the firms increasing productivity exhibits no correlation with aggregate growth. Finally, faster growing economies have (iv) younger firms on average. We show that our results are robust to controlling for the countries' level of income and sectoral composition.

In light of the canonical firm dynamics model, Hopenhayn (1992), the fact that firm-level productivity growth and aggregate growth are positively correlated is not surprising. Nevertheless, we highlight our first fact because the fraction of firms with positive productivity growth did not necessarily have to correlate with the cross-country variation in aggregate growth. The intensive margin of productivity growth may also matter and within-country heterogeneity along this margin can potentially shape aggregate growth. The correlation with aggregate growth that we get by counting firms with productivity growth is useful for us to model the probability of success (realizing positive returns to investments in productivity) as a fundamental of the economy. First, it simplifies the analysis to think of this probability as a parameter common within an economy. Second, as will be apparent from our quantitative exercise, it is straightforward to identify this probability from the data.

We also find it informative to observe no correlation between aggregate growth and the fraction of firms with positive employment growth, facts (ii) to (iii). For simplicity, suppose that all firms that increased productivity did it by the same rate $\hat{\phi} > 0$. If the average productivity increases at a rate smaller than $\hat{\phi} > 0$, firms that increase productivity also increase employment (because their productivity is growing relative to average productivity). This would in turn imply that the fraction of firms with employment growth should be positively correlated with aggregate growth, which is counterfactual.

In this paper, we argue that firms in different countries face differences in the likelihood of receiving returns to a given investment ("probability of success" hereafter). Firms of an economy with a fruitful business environment achieve frequent productivity growth (higher probability of success), which leads to higher aggregate growth (Fact (i)). On the other hand, firms' employment growth/decline arises from changes in their position in the productivity distribution relative to other firms (rank reversals). Higher probability of success does not necessarily affect rank reversals if it is common to all firms in an economy. Accordingly, faster growing economies can have a similar fraction of firms increasing employment as in other economies (Facts (ii) and (iii)). Finally, as average productivity growth is higher in faster growing economies, so is firm-level competition for scarce resources and firm selection (exit and entry). These mechanisms lead to higher firm churning, and younger firms (Fact (iv)).

To formalize these links, we build a novel and tractable general equilibrium model with endogenous growth and firm dynamics. We split the firms space into two. In particular, firms can be of the "innovative" type, or they can operate with a constant productivity technology. At the beginning of each period, innovative firms decide whether or not to invest in technology improvements and how much to produce in the current period. Innovative firms can also decide to liquidate. If so, their technology is taken over by a "non-innovative" firm. Non-innovative firms decide whether to stay in the market, produce and pay operating costs, or exit. This market structure allows us to characterize the growth rate of firm and aggregate productivity analytically along a balanced growth path (BGP). The tractability stems from the homogeneity in the value of the innovative firms. Two key assumptions warrant this feature. First, innovation costs are proportional to firms' revenues. Second, we introduce a novel exit-entry mechanism from the innovative to the non-innovative sector which implies that innovative firms' investment decisions are independent of the firm size and age, albeit the exit option is non-trivial. We assume that an entering non-innovative firm buys the technology from an exiting innovative firm and makes a take-it-or-leave-it offer. In equilibrium, the optimal offer is such that the innovative firm is compensated as if it were to operate the innovative technology for infinitely many periods.

This model allows us to quantify the role of the probability of success and return uncertainty in generating aggregate growth. This is key to quantitatively characterize the link between features of the firm dynamics and aggregate growth. We identify cross-country differences in this probability of success through the fraction of firms with positive productivity growth episodes. In addition, we calibrate the exogenous component of the exit rate and the cost of productivity investment to match a synthetic economy whose aggregate growth and average firm age coincides with the average in our sample. The model produces 67 percent of the variation in aggregate output observed in the data. The model is also consistent with a negative correlation between average firm age and aggregate growth, and it explains 94 percent of the cross-country variation in average firm age.

In our benchmark economy, firms' endogenous productivity growth is constant along the balanced growth path. If the firm is successful, it grows proportionally to average productivity and its employment does not change (because its productivity relative to the average did not change). If instead the firm is unsuccessful, its employment shrinks. This prediction is overturned once additional shocks on the size of the returns are introduced. This allows the model to generate the observed fraction of firms that experience employment growth in the data for each country. However, we show that matching the employment growth profiles does not improve the model's fit to the aggregate growth rates across countries. This result suggests that employment growth patterns are not necessarily informative about aggregate growth.

We view the main contributions of this paper as two-fold. First, to the best of our knowledge, this is the first paper to empirically document the cross-country relationship between characteristics of the firm dynamics and aggregate growth for a large number of countries. Second, the paper presents a parsimonious model of endogenous growth and firm heterogeneity with the minimal structure needed to generate the aforementioned facts.[1] That is, a stochastic process for productivity that entails the random arrival of an endogenous innovation level (to endogenize selection), and the random arrival of shifts in the innovation level (to match employment dynamics).

Due to the homogeneity built into our model, income levels are not pinned down. However, one of the mechanisms that may induce disparities in growth rates across economies is their position in the development spectrum, i.e. transition dynamics. Aggregate growth rates and firm dynamics may also vary in response to large economic shocks such as trade reforms (Eslava et al., 2004), financial crises (Buera et al., 2015), etc. To address these issues while maintaining the model tractability, we run robustness exercises in which we compute residual variation in growth rates across countries after controlling for income; volatility in growth rates, and time trends. These alternative exercises do not change our main findings: variation in the probability of success explains the bulk of the differences in aggregate growth rates, whereas variation in firm employment dynamics has a negligible role.

Related literature

This paper directly relates to two strands of literature. First, we take part in the literature that documents differences in industry dynamics across countries and their implications for aggregate growth. Asturias et al. (2017) focuses on the differential impact of entry and exit patterns in accounting for aggregate productivity dynamics along the transition path for fast and slow growing economies. Moreover, McGowan et al. (2017) uses a sample of rich

[1] The model can be readily extended to allow for additional heterogeneity in investment across firms and the use of other factors of production.

economies to show that a higher market share of zombie firms (defined as old firms that have persistent problems meeting their interest payments) can significantly dampen the aggregate productivity and output growth, through a congestion effect on the remaining firms in the market. These results are consistent with our finding of a higher share of young firms in economies that grow relatively fast. Moreover, it relates to our finding that the variation in the fraction of firms with positive productivity growth can account for aggregate growth performances of different economies. Criscuolo et al. (2014) also uses a set of rich economies to show that the recent episodes of aggregate downturns are accompanied with a decline in start-up rates. Decker et al. (2014) shows a similar pattern for the US.[2] Our results from a much broader set of countries are consistent with this finding, in that economies with lower aggregate growth rates exhibit lower firm churning, therefore older firms.

Second, this paper relates to the theoretical literature that links firm innovation decisions, aggregate productivity and output dynamics. Our model differs from the quality ladder framework (Grossman and Helpman, 1993) in that the size of the productivity improvement is endogenous and the frequency of upgrades is exogenous. For the study of multiproduct firms, where innovation relates to the introduction of new goods to the firm portfolio, such an assumption is natural. In our economy however, we think of productivity in broad terms to include managerial practices, the quality of the capital used in production, technological updating. A sensible assumption is that the quality choice of such margins is an endogenous decision to the firm, and the return of those investments is only randomly realized. More importantly, unlike the knife-edge endogenous growth case presented in Atkeson and Burstein

[2] Decker et al. (2014) also serves as a survey of the literature studying the reasons and implications of this decline in entry for the US.

(2010) where growth is induced exclusively by product innovation (entry), growth in our economy is induced by firms' choices on the step size of the innovation, and entry affects this endogenous choice through general equilibrium effects. In Klette and Kortum (2004), growth can be accounted for by either incumbents or entrants. Higher entry generates lower average age in equilibrium, but whether aggregate growth increases or not depends on whether the contribution of entrants to growth exceeds the losses in the rate of innovation by incumbents.

The rest of the paper is organized as follows. Section II documents our empirical facts. Section III presents our model and Section IV gives its quantitative assessment. Section V concludes.

II Evidence

In this section, we document the motivating facts of our study. Our main data source is the standardized dataset of the World Bank Enterprise Surveys (ES), which provides harmonized plant-level data across countries in service and manufacturing sectors targeting formal establishments with at least 5 employees. We use information for a total of 138 country-year samples from 91 countries between years 2003 to 2014.[3] Our data includes establishments operating in manufacturing and service sectors. We postpone a detailed description of the dataset to the Appendix.

We start by documenting the relationship between the fraction of establishments ("firms" will be used interchangeably) that report positive productivity growth in the last two years

[3]For a technical overview of data quality, see Garcia-Santana and Ramos (2015) and Bento and Restuccia (2017). Garcia-Santana and Ramos (2015) show that the measure of aggregate employment implied by the ES data correlates strongly with the employment measure in the Penn World Table 7.0. Bento and Restuccia (2017) show that for the case of India, their estimate of the elasticity of aggregate productivity to changes in firm-level distortions using the ES, is close to the estimate by Hsieh and Klenow (2014) using census data.

and the aggregate growth rate. To obtain current and past productivity levels for each establishment, we use information on their current and past sales (y, adjusted by country-specific inflation) and number of employees (l). In particular, consistently with the model we will introduce in the next section, we assume a production function of the form $y = z^{1-\theta} l^{\theta}$. This implies that establishment-level productivity can be obtained as: $z = \left(\frac{y}{l^{\theta}}\right)^{\frac{1}{1-\theta}}$.

The productivity parameter z captures all factors of production that can be accumulated, including the technology level of an establishment as well as capital goods. Moreover, because we use sales data as a measure of output to compute productivity, the latter also includes shifts in profitability at the establishment level. We use a labor share of $\theta = 0.66$.[4]

Our measure of the aggregate output growth rate is the average growth rate of GDP per capita in the 20 years prior to each survey year as reported in the World Development Indicators (WDI). We choose a 20 years horizon to be consistent with the time span in which an average firm in our sample operates (around 16 years), and to be less subject to short term movements in aggregate growth.[5]

Panel (a) of Figure 1 shows the relationship between the fraction of establishments with positive productivity growth within a two-year time span and average annual GDP per capita growth. The correlation coefficient is 0.18 in spite of the large variation across countries.

It is not surprising that a measure of firm success correlates positively with a measure of country-level success, such as aggregate income growth. However, not all measures of firm success correlate positively with aggregate growth. For instance, Panel (b) of Figure 1 shows that the fraction of establishments with employment growth is not positively correlated

[4] In Section IV we estimate the average labor share in our sample to run robustness checks. The stylized facts presented in this section are robust to using this estimated labor share.

[5] We get similar results when using different windows, such as a 10-years horizon, or only the 2000s.

with aggregate growth. Notice that this is true in spite of having more establishments increasing productivity in faster growing economies. In fact, Panel (c) of Figure 1 shows that the fraction of establishments that increase employment, among those that increase their productivity, is not significantly different across slow and fast growing economies.

As our dataset is built with repeated cross-sections, it lacks information about entry-exit behavior of establishments. However, by looking at the average age of establishments, we can infer establishment churning. In particular, in economies with strong selection (high exit and entry rates) we expect to have younger establishments on average.[6] The last panel of Figure 1 plots the average establishment age against the growth rate of GDP per capita in our sample. It shows that faster growing economies have younger establishments on average (with a correlation between aggregate growth and average plant age of -0.46). This fact is consistent with the findings in Asturias et al. (2017) when using longitudinal data to account for firm churning during episodes of fast growth.

A few caveats are in order regarding our empirical approach. First, data limitations prevent us from incorporating capital accumulation into our analysis. In particular, the ES data does not provide information on past capital of a plant (neither owned nor rented). Hence, constructing the residual productivity growth net of capital accumulation is not possible. In the Appendix, we discuss the implications of using the growth rates of aggregate capital to construct measures of plant-level productivity growth and aggregate output growth net of capital accumulation. We argue that our quantitative results are robust to this alternative accounting.

[6] Our measure of establishment's age corresponds to the difference between the survey year, and the reported year in which the establishment began operations.

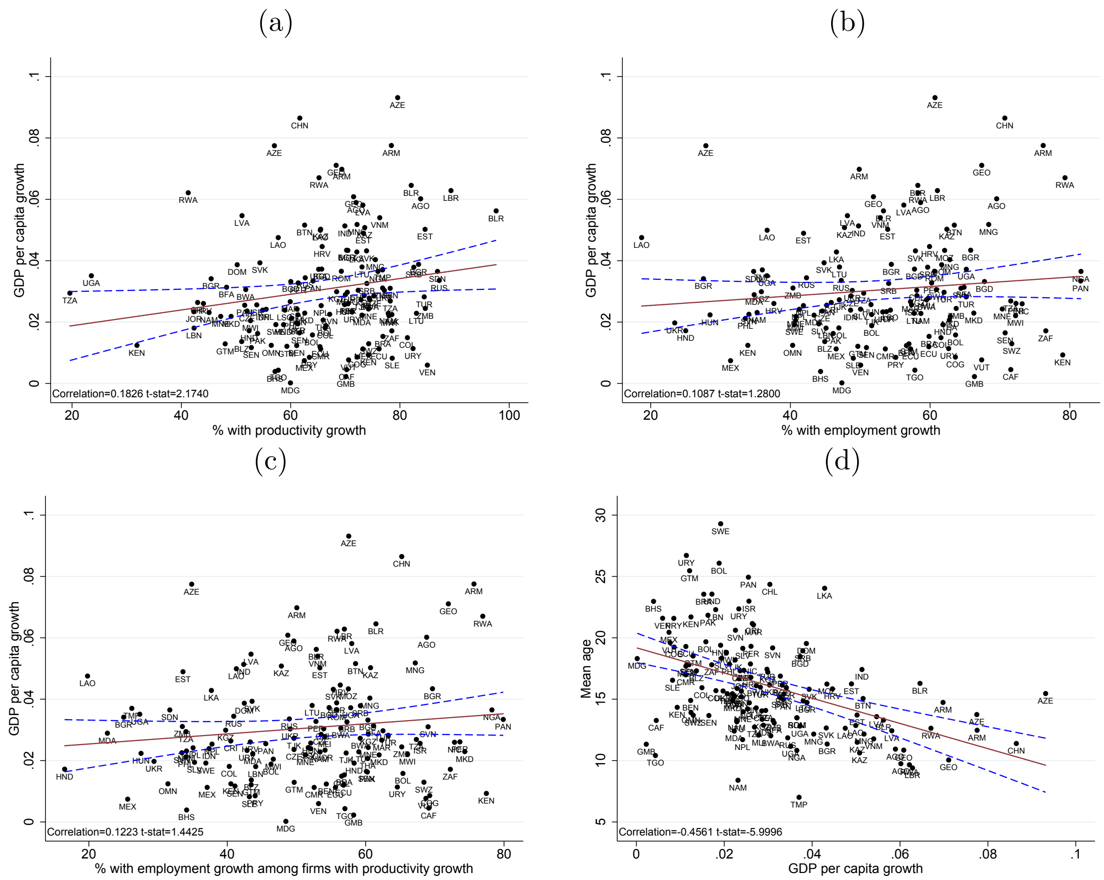

Figure 1: Features of the variation in GDP per capita growth

Note: Panels (a) GDP per capita growth vs. fraction of establishments with productivity growth; (b) GDP per capita growth vs. fraction of establishments with employment growth; (c) GDP per capita growth vs. fraction of establishments with employment growth among the establishments with productivity growth; (d) average establishment age vs. GDP per capita growth.

Second, our benchmark analysis does not control for cross-country differences in the sectoral composition of firms. In the Appendix, we replicate Figure 1 using the detailed information on plants' sector given in the ES.[7] For plant-level (productivity and employment) growth, we use growth rates relative to that of the median plant in the same sector in our cross-country data. For aggregate growth, we use the growth rates conditional on the fraction of firms from each sector in our ES sample. The correlations implied by this exercise are also similar to those reported above.

To further assess the reliability of our data, we generate moments of firm dynamics and

[7]There is a total of 37 subsectors that an establishment can operate in.

aggregate income that can be compared to others previously used in the literature. There is a long standing debate as to the relationship between employment size distributions and income per capita (Alfaro et al. (2009); Hsieh and Olken (2014)). In our sample, the correlation between GDP per capita and average establishment size (log employment) is positive, 0.25, and significant at 5%. This result differs from Alfaro et al. (2009) who find a significant negative correlation.[8] But our finding is consistent with Poschke (2017) who finds: a) a positive correlation between average establishment size and income per capita and b) a positive (but lower) correlation in the dispersion of establishment's employment with GDP per capita (in our dataset this correlation is 0.15 but not statistically significant). Our results are also in line with the positive correlation documented in Bento and Restuccia (2017) who find an elasticity of manufacturing establishment size with respect to GDP per capita of 0.29. As in Bartelsman et al. (2009), we find that the correlation between the share of small establishments and GDP per capita is negative and significant, -0.49 at 1%. Overall, these results are encouraging in terms of the quality and predictions of our benchmark dataset.

III A model of endogenous growth and firm selection

Next, we build a tractable model of endogenous firm productivity growth. In doing so, we rely on our empirical finding that the fraction of firms with positive productivity growth is positively correlated with the aggregate growth. Hence, we treat firms' probability of realizing returns to investments in productivity (probability of success) as a country-specific fundamental.

[8]These authors trim their sample to firms that have at least 20 employees, hence by construction, the smaller firms in each country are not accounted for. In our sample, the average establishment in the poorest economies hires between 7 and 20 employees.

In the economy, there is a continuum of heterogeneous firms. Each firm operates a technology that uses labor l to produce a homogeneous good y, following the production function introduced in Section II, i.e. $y = z^{1-\theta} l^{\theta}$.

There are two types of firms: "Innovative" (I) and "Non-innovative" (N). The first type can change their productivity via investment, and they are the main drivers of growth. Whenever a firm with productivity z undertakes an investment $\phi \geq 1$ in period t, the productivity in period $t+1$ is $z' = \phi z$ with probability q and $z' = z$ otherwise. The probability of success in investment (q) is the same for all firms operating in the economy. Investment is costly and depends on $C(\phi, z, w) = c \frac{(\phi^{\tau} - 1) z}{w^{\frac{\theta}{1-\theta}}}$, where w denotes the wages and τ is a parameter shaping the convexity of the cost function. "Non-innovative" firms have a productivity level constant in time. Firms' objective is to maximize the present discounted value of life-time profits discounting the future at the risk-free interest rate $(R - 1)$. In this paper, we focus on partial equilibrium and we take R as an exogenous parameter of the model.

There is an inelastic supply of labor equal to 1. An innovative firm has an overhead cost of labor f_I, and a non-innovative firm has an overhead cost of $f_N < f_I$. At any point in time, a non-innovative firm can decide to exit the market at no cost. An innovative firm has also an option of liquidation. In particular, an innovative firm, upon observing whether or not the previous period's investment was successful, can sell its technology to an entrepreneur in the non-innovative sector. Entry of non-innovative firms occurs through the purchase of a technology (z) from an innovative firm. The transaction takes place at a price $P(z, w)$, which is determined through a take-it-or-leave-it offer from the entrant. Finally, after production and investment take place, firms are subject to an exogenous exit shock which arrives at rate

δ each period. Innovative firms exiting the market are replaced with new firms which draw their productivity from the incumbent distribution of innovative firms. Entry occurs until the expected value of an entrant equals the entry cost.

Next, we solve for the static allocation of labor given the distribution of productivity in the market. Then, we characterize the dynamic decisions of the firm, i.e. entry, exit and technology investment.

III.A Firms' problem (static)

The static problem of a firm is:

$$\Pi(z, w) = \max_{l} z^{1-\theta} l^{\theta} - wl,$$

where θ is the share of labor in production.

Using the optimal labor demand from this problem, we can characterize equilibrium wages. Denote by M_I and M_N the measure of innovative and non-innovative firms in the market, by $M \equiv M_I + M_N$, the total measure of firms, and by $\alpha \equiv \frac{M_I}{M}$ the fraction of innovative ones in the market. Notice that the total labor supply that can be used for productive purposes is equal to $1 - (\alpha f_I + (1 - \alpha) f_N) M$. Equilibrium in the labor market dictates that the total labor demand should equal the total labor supply for productive purposes. Equilibrium wages satisfy:

$$w = \frac{\theta}{(1 - (\alpha f_I + (1 - \alpha) f_N) M)^{1-\theta}} \left(\int z dv^I(z) + \int z dv^N(z) \right)^{1-\theta}, \tag{1}$$

where $v^I(v^N)$ is the equilibrium distribution of productivity for innovative (non-innovative) projects. By definition, $M_j = \int dv^j(z)$ for $j = I, N$.

Using the equilibrium cost of labor we can show that profits, labor demand and firm output are proportional to firm productivity and labor costs, $\frac{z}{w^{\frac{\theta}{1-\theta}}}$.[9]

III.B Firms' problem (dynamic)

Non-Innovative firms. Let $V_N(z, w)$ be the value of a firm operating in the non-innovative sector, when the cost of labor is w and its productivity is z. The value of a firm operating in sector N satisfies:

$$V_N(z, w) = (1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}} - f_N w + \frac{1-\delta}{R} \max\{V_N(z, w'), 0\}.$$

Because the productivity of the firm is fixed z, but labor cost rises as the economy grows $w' > w$, if profits are negative in a given period, they remain negative thereafter. Hence, the value of the firm is negative if and only if the flow in that period is negative:

$$V_N(z, w) \leq 0 \Leftrightarrow (1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}} - f_N w \leq 0. \tag{2}$$

Notice that the only decision for this firm is whether to exit.

Innovative firms. An innovative firm has the same static revenues as a non-innovative firm, but is different in two important aspects. First, it can engage in risky investment

[9] Profits are $\Pi(z, w) = (1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}}$, labor demand is $l(z, w) = \theta^{\frac{1}{1-\theta}} \frac{z}{w^{\frac{1}{1-\theta}}}$, and firm output is $y(z, w) = \theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}}$.

in productivity, which results in successful innovation with probability q. Second, after realizing returns from previous period's productivity investment, it can sell its technology to an entrant into the non-innovative sector. We can write the value of an innovative firm as:

$$\begin{aligned} V_I(z,w) &= \max_{\phi\geq 1}(1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} - c\frac{(\phi^\tau-1)z}{w^{\frac{\theta}{1-\theta}}} - f_I w \\ &+ \frac{1-\delta}{R}[q\max\{V_I(\phi z,w'),P(z,w')\} + (1-q)\max\{V_I(z,w'),P(z,w')\}]. \end{aligned}$$

We assume that if a firm sells the technology in a period it does not realize any improvements in technology. Also, we disregard the option of exiting the market without liquidation with a conjecture that the liquidation price is always positive.

If the firm decides to liquidate, it meets one potential entrant that will use its technology z to operate a non–innovative firm. We assume that the entrant makes a take-it-or-leave-it offer $P(z,w)$ to buy the firm. Its pay-off is $V_N(z,w)-P(z,w)$. The innovative firm can meet at most one entrant per period and it accepts the offer if and only if $P(z,w)\geq V_I(z,w)$. In case of indifference, we assume that the offer is accepted. Hence, a weakly dominant strategy for an entrant non-innovative firm is to offer:

$$P(z,w) = \min\{V_I(z,w),V_N(z,w)\}.$$

Without loss of generality we assume that this is the strategy of the entrant firm. In other words, the transaction occurs when its surplus is non-negative, and the continuation value of an innovative firm in equilibrium is $\max\{V_I(z,w'),P(z,w')\} = V_I(z,w')$.[10] Thus, an

[10] If $V_I(z,w)\leq V_N(z,w)$, the offer is accepted for any $P(z,w)\geq V_I(z,w)$. Hence the optimal strategy is to offer $P(z,w)=V_I(z,w)$. If $V_I(z,w)>V_N(z,w)$, any $P(z,w)>V_N(z,w)$ gives at most zero payoff, and any $P(z,w)\leq V_N(z,w)$ gives zero payoff. So there is no strategy that gives a strictly higher payoff

innovative firm makes its investment decisions as if it was infinitely-lived:

$$V_I(z,w) = \max_{\phi \geq 1} (1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}} - c\frac{(\phi^\tau - 1)z}{w^{\frac{\theta}{1-\theta}}} - f_I w + \frac{1-\delta}{R}[qV_I(\phi z, w') + (1-q)V_I(z,w')].$$

This feature allows us to compute the equilibrium investment of the firms quasi-analytically.[11] When we describe the solution of the model, we show that given overhead labor costs, any innovative firm finds it optimal to sell its technology to an entrant non-innovative firm in finitely many periods.

Finally, we assume that entry into the innovative sector is frictionless. Entrants draw their productivity from the incumbent distribution and enter up to the point in which the expected value of an innovative firm is zero.[12]

III.C Balanced Growth Path

Definition 1 *A balanced growth path (BGP) is a sequence of aggregate output, a measure of aggregate productivity and wages that grow at a constant rate. In addition, there exists a constant measure of firms in the market, and an invariant distribution of firm productivity around a mean which grows at a constant rate.*

than $P(z,w) = \min\{V_I(z,w), V_N(z,w)\}$ for any state of the world. With this pricing scheme, we can rewrite the value of innovative firms using $\max\{V_I(\phi z, w'), P(z,w')\} = V_I(\phi z, w')$ and $\max\{V_I(z,w'), P(z,w')\} = V_I(z,w')$.

[11] Notice that this would not be the case if an innovative firm was maximizing between $V_I(z,w)$ and $V_N(z,w)$, because then the firms close to the exit threshold would take into account the continuation value if they were to exit the market. With our pricing assumptions such a consideration does not arise.

[12] Because firms draw productivity from the incumbent distribution they can be immediately large. The model can be extended to allow for a shifted distribution of entrants relative to that of incumbents. If combined with additional uncertainty on firm returns, it can accommodate fast growth for small (younger on average) firms and slower for large (older on average) firms.

Guess: *There exists a BGP along which output and wages grow at the same rate. i.e. there exists* $\gamma \geq 1$ *such that:*

$$[Y_t, w_t] = [Y, w]\gamma^t. \tag{3}$$

Under this guess, we solve for the firms' optimal policies and show the existence of an invariant distribution and describe it. Once we solve for the equilibrium distribution and allocation of firms across innovative and non-innovative projects, we compute the equilibrium growth rate for aggregate output and wages. We then verify that both are constant to confirm the existence of the BGP.

III.C.1 Firms

Define the function that describes the number of periods left in the market for a non-innovative firm (including the current one) as:

$$T(z, w) \equiv \min\{t : (1-\theta)\theta^{\frac{\theta}{1-\theta}} z \leq f_N w^{\frac{1}{1-\theta}} \gamma^{\frac{t}{1-\theta}}\}.$$

To ease notation we will denote $T \equiv T(z, w)$.

Proposition 1 *Along the BGP:*

(a) the value of a non-innovative firm satisfies: $V_N(z, w) = B_{NT} \frac{z}{w^{\frac{\theta}{1-\theta}}} - D_{NT} w$,

where for any $T \geq 0$*:*

$$B_{NT} = (1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{1 - \left(\frac{1-\delta}{R\gamma^{\frac{\theta}{1-\theta}}}\right)^T}{1 - \frac{1-\delta}{R\gamma^{\frac{\theta}{1-\theta}}}}, \quad D_{NT} = f_N \frac{1 - \left(\frac{1-\delta}{R}\gamma\right)^T}{1 - \frac{1-\delta}{R}\gamma}$$

(b) the value of an innovative firm satisfies: $V_I(z,w) = B_I \frac{z}{w^{\frac{\theta}{1-\theta}}} - D_I w$*, where*

$$B_I = (1-\theta)\theta^{\frac{\theta}{1-\theta}} - c(1-\tau)\left[\frac{\frac{1-\delta}{R} q B_I}{c\tau\gamma^{\frac{\theta}{1-\theta}}}\right]^{\frac{\tau}{\tau-1}} + c + \frac{\frac{1-\delta}{R}(1-q)B_I}{\gamma^{\frac{\theta}{1-\theta}}}, \qquad D_I = \frac{f_I}{1-\frac{1-\delta}{R}\gamma}.$$

Proof. See the Appendix for the proofs of all propositions.

Hence, the value of the innovative firm is monotonically decreasing in labor costs. Changes in the growth rate of the economy and adoption costs affect the equilibrium value of the firm through the constant B_I. This last condition provides the first equation relating the aggregate growth rate (γ), the probability of success (q), and the optimal firm productivity growth (ϕ). Given the equilibrium value function of the innovative firm we can show that the optimal investment strategy is $\phi = \left[\frac{\frac{1-\delta}{R} q B_I}{c\tau\gamma^{\frac{\theta}{1-\theta}}}\right]^{\frac{1}{\tau-1}}$. Additional parametric restrictions assure positive levels of optimal investment, $\phi \geq 1$.

Assumption 1: $c\tau < 1$.

Assumption 2: $c\tau(1-\frac{1-\delta}{R}(1-q)) < \frac{1-\delta}{R}q((c\tau-1) + ((1-\theta)\theta^{\frac{\theta}{1-\theta}} + c))$.[13]

Given the optimal value of the firms, we can describe the optimal transfer of firms from the I-sector to the N-sector.

Proposition 2 *There exists a unique productivity level,* $\hat{z} > 0$*, that makes an innovative*

[13] Using the euler equation and the relationship between firm investment and aggregate growth, we can show that in equilibrium

$$c\tau(1-\frac{1-\delta}{R}(1-q)) = (c\tau-1)\frac{1-\delta}{R}q\phi^{1-\theta} + \frac{1-\delta}{R}q\phi^{1-\theta-\tau}((1-\theta)\theta^{\frac{\theta}{1-\theta}} + c).$$

The right hand side (RHS) is monotonically decreasing in ϕ for ($c\tau < 1$). Hence, if at $\phi = 1$, RHS > LHS the solution should be larger than 1.

firm indifferent between liquidation or not, and it is given by:

$$\hat{z} = \frac{D_I - D_{NT}}{B_I - B_{NT}} w^{\frac{1}{1-\theta}}. \tag{4}$$

Above this threshold, an innovative firm continues to operate. Moreover, every innovative firm is sold to an entrant non-innovative firm in finitely many periods.

The Appendix shows that for this mechanism to be optimal, the surplus of the transaction has to increase in unsuccessful innovations. Hence, unlucky firms eventually would like to liquidate. It is also necessary that firms that are very productive relative to labor costs (high z relative to w) have a negative surplus from the transaction (the firm does not liquidate) and that relatively unproductive firms the surplus have a positive surplus (the firm liquidates). If this holds, there is a finite point in time at which an innovative firm is traded, and the population of both types of firms is non-degenerate.

III.C.2 Distribution dynamics

The equilibrium values of each type of firms are monotonic in productivity z. Hence, exit decisions take the form of thresholds in productivity, above which the firm remains active and below which the firm exits. In Proposition 2, we defined the threshold $\hat{z}$ such that an innovative firm sells its technology. Using condition (2) for the survival of a non-innovative firm, we define also the lower bound of productivity for the non-innovative firms operating in the market as $\tilde{z}$, i.e.

$$\tilde{z} = \frac{f_N}{(1-\theta)\theta^{\frac{\theta}{1-\theta}}} w^{\frac{1}{1-\theta}}. \tag{5}$$

Given that the growth rate of wages along the BGP is γ, equations (4) and (5) imply that the thresholds $\hat{z}$ and $\tilde{z}$ grow at a constant rate μ (i.e. $\hat{z}' = \hat{z}\mu$, $\tilde{z}' = \tilde{z}\mu$) that is tied to the growth rate of aggregate output, $\mu \equiv \gamma^{\frac{1}{1-\theta}}$. Define the number of periods that a non-innovative firm with productivity $\hat{z}$ has left in the market as $\hat{t} \equiv T(\hat{z}, w)$. Since the growth rates of the two productivity thresholds are constant and equal, $\hat{t}$ is constant and it satisfies: $\hat{z} = \tilde{z}\mu^{\hat{t}}$. This feature proves useful to show the properties of the productivity distribution.

Proposition 3 *If the initial distribution of productivity in the innovative sector is Pareto with shape parameter λ, and entrants draw productivity from the incumbent distribution*

1. *the growth rate of the threshold levels (μ) is the same as the investment rate (ϕ).*
2. *the equilibrium distribution of productivity across innovative firms is also Pareto with shape parameter λ.*

This proposition shows that conditional on having entrants draw their productivity from a Pareto distribution, there is an invariant endogenous productivity distribution of firms in the market. This implies that the growth rate of productivity at the firm level is proportional to aggregate growth: $\phi = \gamma^{\frac{1}{1-\theta}}$. The reason is that average productivity under a Pareto distribution is proportional to the productivity of the lower threshold of the distribution, and innovative firms choose the same rate of innovation, irrespective of their level of productivity.

It remains to be shown that the relative population of innovative and non-innovative projects in the market is constant along the BGP.

Proposition 4 *The share of innovative firms in the market (α) is constant along the BGP and solves*

$$(1-\alpha) = \alpha(1-q)[1-\mu^{-\lambda}]\frac{1-\delta}{\delta}[1-(1-\delta)^{\hat{t}}].$$

Moreover, the total measure of firms (M) is also a constant determined by equation (1), which describes equilibrium wages.

Two remarks are in order regarding the model implications. First, since the firm value is homogeneous in the ratio of firm productivity to labor costs, $\frac{z}{w^{\frac{1}{1-\theta}}}$, only the productivity of the marginal firm relative to labor costs is determined, i.e. economies can be scaled up and down by shifting the productivity of the marginal firm and wages. Second, the expected value of an innovative firm in the market is determined by the equilibrium threshold for the marginal innovative firm and labor costs.

Verifying the growth rate of output. Total output in the economy satisfies:

$$Y = \int \theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}} dv^I(z) + \int \theta^{\frac{\theta}{1-\theta}} \frac{z}{w^{\frac{\theta}{1-\theta}}} dv^N(z).$$

Using the equilibrium condition for wages given by Equation (1), this implies that:

$$Y = (1-(\alpha f_I + (1-\alpha) f_N)M)\frac{w}{\theta}.$$

From Proposition 3, we know that M and α are constant along the BGP. Hence, Y grows at the same rate (γ) as wages. This verifies the guess given in condition (3).

Age distribution. Here, we describe the age distribution of firms, which is tightly linked to firm selection and aggregate growth in our model. In the non-innovative sector, the probability of surviving for only t years is:

$$\Phi^N(t) = \begin{cases} (1-\delta)^t \delta, & \text{if } t < \hat{t}, \\ (1-\delta)^t, & \text{if } t = \hat{t}. \end{cases}$$

Thus, the mean age of N-firms is: $\sum_{t=0}^{\hat{t}} t\Phi^N(t) = \frac{1-\delta}{\delta}\left[1-(1-\delta)^{\hat{t}}\right]$, which is decreasing in the time to exit the market. Non-innovative firms' productivity is constant and the average productivity overall grows at a constant rate, making these firms less profitable with time. Selection gives a downward sloped age profile for non-innovative firms.

Next, let the number of innovative firms operating for t years be N_t:

$$N_t = x(1-\delta)^{t-1}\left[1 - F(\hat{z}\mu^{t-1}) + \sum_{j=1}^{t-1}\binom{t-1}{j} q^j (1-q)^{t-1-j} \overbrace{\left(F(\hat{z}\mu^{t-j}) - F(\hat{z}\mu^{t-j-1})\right)}^{\text{Firms needing } j \text{ innovations}}\right],$$

where x is the measure of entrants into the innovative sector.[14] The age distribution can then be characterized by a probability density function, $\Phi^I(t) = \frac{N_t}{\sum_{j=1}^{\infty} N_j}$, and average age for I-firms is $\sum_{t=0}^{\infty} t\Phi^I(t)$. The average age of innovative firms is non-monotonic in aggregate growth. For a given growth rate of exit thresholds (μ) the ratio of the measure of firms that are t years old to those that are $t-1$ years old, N_t/N_{t-1}, increases with the probability of success q. Higher probability of success increases firm investment and aggregate growth, inducing stronger selection. If selection is not too strong, average age increases with the probability of success. Otherwise, the increase in the growth rate of exit thresholds (μ) makes the ratio N_t/N_{t-1} fall for any age t. Hence, average age falls.

[14] The relevant measure of entry for this statistic is that of t periods ago, x_t. However, the measure of firms is constant along the BGP, which gives $x_t = x = \delta + (1-\delta)(1-q)(1-\phi^{-\lambda})$.

IV Quantitative exploration

This section has three parts. In the first one, we identify the probability of success in the data (q) and discuss its role for aggregate outcomes. In the second part, we present a full calibration of our model economy in which we are able to isolate the contribution of differences in the probability of success to the cross-country disparity in aggregate growth rates. We then extend our benchmark economy to allow for return uncertainty and assess its contribution to explaining differences in growth rates across countries. Finally, we run various robustness checks for the quantitative performance of our benchmark model.

IV.A Probability of success

Identification. A key component of the theory that we present in this paper is the probability with which firms receive returns on productivity investment: the probability of success (q). Identifying such a parameter in the data is problematic. A firm may not improve its productivity because it did not invest in productivity improvement previously, or because it did invest, and it was unsuccessful. In addition, firms that were unsuccessful may exit the market, inducing selection among the observed firms.

In our benchmark model, all innovative firms try to innovate and they succeed with probability q. High productivity, large firms choose to remain in the market regardless of their success in innovating. Endogenous selection in the model only occurs at the bottom of the productivity and size distribution (in our model, relative productivity and firm employment map one-to-one). Hence, at the top of the employment size distribution, the fraction of firms that grow in productivity should be approximately equal to q.[15]

Accordingly, we compute the fraction of firms in the top decile of the employment distribution that realize productivity growth over a two-year time span.[16] This yields a country-year specific probability of success (q). The average q in our sample is 0.441, and the standard deviation is 0.115.[17]

Importantly, our estimates for q are stable for a large group of countries for which we have two samples. In total 46 out of 91 countries in our sample have two ES samples instead of one. Within this group, the median absolute difference per year between the estimated q across two samples is 0.03 and the 25th percentile is 0.02.

The role of q for aggregate outcomes. In order to better highlight the mechanisms at play in our model, we start by discussing how variation in the probability of success, as inferred from the data, affects equilibrium outcomes. In particular, we focus on the change in aggregate output and on various statistics describing the operating firms.[18]

The results are illustrated in Figure 2. First, Panel (a) shows a strong positive relationship between the probability of success and the implied aggregate growth rate of the economy. Changes in the probability of success affect equilibrium aggregate growth through two channels: a direct one, through more frequent episodes of firm productivity growth; and

[15]Firms at the top of the productivity distribution may exit the market exogenously at rate δ. However, since these shocks are independent of the size and the innovative status of the firm, they do not invalidate our identification strategy.

[16]As emphasized at the beginning, in the data we observe establishments behavior rather than firms behavior. Establishments report past sales and employment for two fiscal years before. Hence, we adjust the q measure from the data by $q = 1 - (1 - q^{\text{data}})^{1/2}$. Using top 50, 20 or 5 percent of the employment distribution of firms does not vary our results except small changes in the levels of the measured q.

[17] This sample corresponds to 126 country-year observations for which we solved the model in the next section. The full sample presented in Section 2 includes countries with no observations for previous employment. The full sample for which we can identify q consists of 138 country-year pairs. There are 12 country-year observations for which we cannot solve the model given the calibrated costs (common across countries) and the observed probability of success.

[18]To do this, we use benchmark model parameters – to be described in Section IV.B.

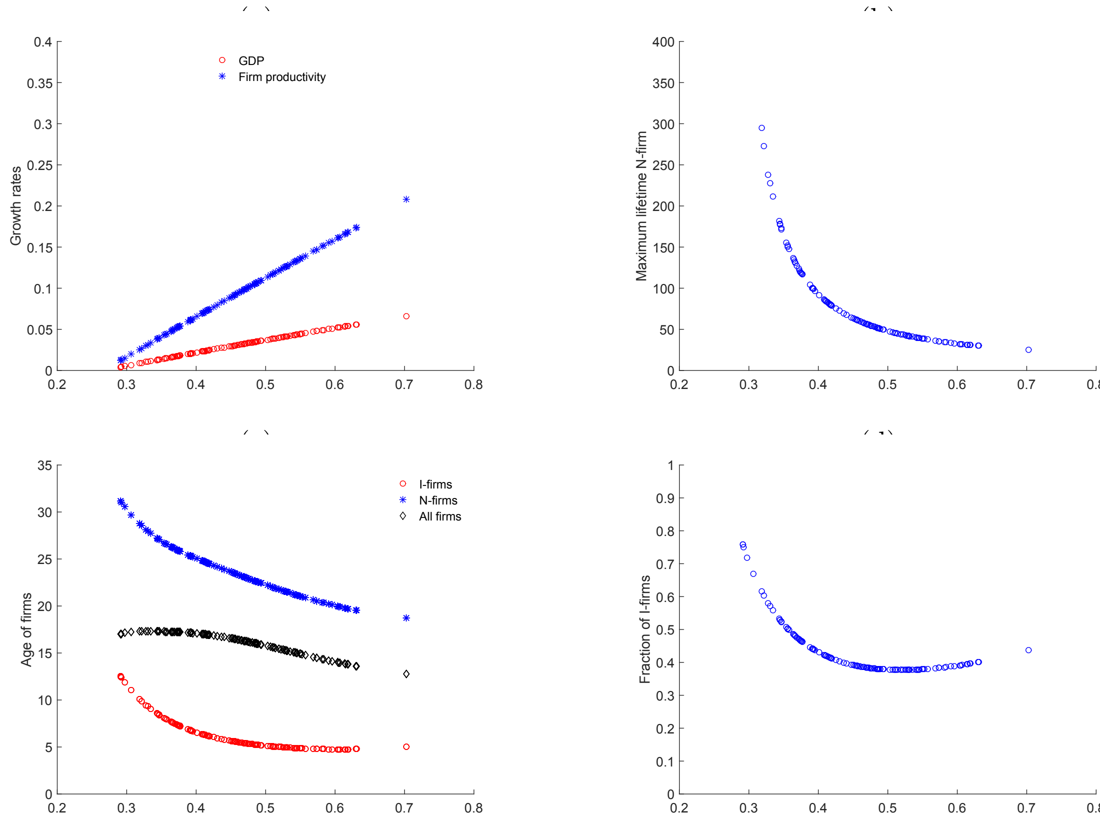

Figure 2: Variation in the probability of success

Note: Panels from the top-left to bottom-right: Impact of changes in the probability of success on (a) aggregate growth and productivity growth of successful firms; (b) life-time of a non-innovative firm in the market, conditional on not receiving an exogenous exit shock; (c) average age of innovative and non-innovative firms; (d) share of innovative firms in the market.

an indirect one, through stronger incentives for firm-level productivity investment (ϕ). A first-order impact of faster growth at the firm and the aggregate level is stronger selection, which is apparent in the decrease of the expected lifetime of non-innovative firms in the market (Panel (b)). As the life-time of non-innovative firms gets shorter, the average age among these firms goes down. This selection effect is also present for innovative firms. However, there is also an opposing positive effect on average age through higher probability of success. The latter effect counterbalances the negative selection effect especially for higher levels of q, where the average age of innovative firms remains stable around 5 years (Panel (c)). Nevertheless, average firm age decreases monotonically with q. Finally, the share of innovative firms in the market varies non-monotonically with the probability of success. This feature stems from a stronger selection effect for lower levels of q, and from a stronger increase in the probability of success for the top portion (Panel (d)).

Possible determinants of probability of success. In this paper, we treat the probability of success as exogenous. Nevertheless, in the Appendix, we discuss the possible factors that might be shaping this probability across countries. Through various reduced form estimations, we show that better investor protection and better quality of infrastructure, including the absence of electricity outages and an educated labor force, can account for the variation in probability of success across countries.

IV.B Calibration

Here, we bring the model to the data. To isolate the role of the probability of success for cross-country differences in aggregate growth, we set common parameters across countries and allow only for differences in the probability of success, as estimated in Section IV.A.

The model parameters can be split into two groups. A collection of them is pinned down outside the model following the literature. A second group is jointly calibrated to match moments of a synthetic economy with growth rate and average firm (establishment) age corresponding to the average across the economies in our sample (See Table 1).

The labor share in our production function (θ) is set to 0.66. In the Appendix, we run robustness checks by estimating this parameter directly from our data. In each exercise we recalibrate adoption costs, depreciation rates and operating costs to hit the same targets as in our benchmark economy. These alternative calibrations produce very similar results to the ones we document in this section.

We estimate the Pareto tail (λ) for the employment distribution for each country-year sample, following Axtell (2001). We use a common parameter across countries corresponding to the average in the sample, $\lambda = 2$.[19] We set the real interest rate (r) to 5.8%, which is the median real lending rate for our sample countries as reported by the WDI.[20]

We assume that the cost of innovative activity is a quadratic function of the attempted jump in productivity, hence $\tau = 2$. In the Appendix, we show that our results are robust to alternative values of this parameter.

In our model, operating costs are closely related to the employment of the smaller firms observed in the market. We calibrate overhead costs in the non-innovative sector (f_N) so that the smallest firm in the market holds one employee (i.e. $l = 1$ for a firm with productivity $\tilde{z}$), which yields $f_N = 4$. To calibrate the operating cost in the innovative sector (f_I) we use estimates of the relative overhead cost in formal and informal firms in Brazil by Ulyssea (2018). In our model, non-innovative firms are smaller, less productive, and less profitable than innovative ones. These characteristics are prevalent among informal firms in developing countries (Perry et al., 2007; La Porta and Schleifer, 2008). Ulyssea (2018) estimates the overhead cost in the innovative sector to be 2.0325 times as large as the one in the non-innovative sector, which implies a calibrated cost of $f_I = 8.13$.[21] We calibrate the cost of investment (c) and the exogenous exit rate (δ) to match the average growth rate in GDP per capita and the average age of firms operating in the market for the countries in our sample.[22]

[19] The Appendix describes in detail how the tail parameters are computed.

[20] This rate corresponds to the average lending rate in a country adjusted for inflation as measured by the GDP deflator.

[21] Alternatively, we could have calibrated the operating cost in the innovative sector to match the level of the implied entry cost in each market (as in Barseghyan and DiCecio, 2011) given the optimal measure of firms along the BGP. This additional source of heterogeneity across economies would make it harder to isolate the role of the probability of success for differences in aggregate growth rates.

[22] In the Appendix, we discuss the implications of having a separate innovation cost level calibrated for each country-year observation.

Table 1: Calibration

Parameter	Value	Basis / Target
Investment cost curvature, τ	2	quadratic
Production function shares, θ	0.66	labor share
Interest rate, R	1.058	median in WDI
Overhead cost, N-firms f_N	4	marginal firm with 1 employee
Overhead cost, I-firms f_I	8.13	relative overhead cost 2.03
Pareto tail, λ	2.00	average in ES sample
Investment cost (level), c	0.1765	average growth 3.09%
Exogenous exit rate, δ	5.09%	average firm (establishment) age 16.09 years

Note: The marginal firm in the N-sector corresponds to the firm with lowest productivity at each point in time. The relative overhead cost is the ratio of costs in the formal and informal sector as reported in Ulyssea (2018).

IV.C Probability of success and aggregate growth disparities

Our first objective is to study the extent to which cross-country variation in the probability of success explains the variation in aggregate growth rates. To do it, we use our calibrated economy to predict equilibrium growth rates when the only source of variation across countries is the probability of success. This is depicted in Figure 3. The correlation between the model predicted growth rates and the data is 0.23. To measure the amount of variation in growth rates accounted for by the model, we construct a similar measure to an R^2 in linear regressions. Our measure is the sum of squared errors adjusted by the square of the values observed in the data: $S^2 = 1 - \frac{(\gamma-\gamma_{model})'(\gamma-\gamma_{model})}{\gamma'\gamma}$. This goodness of fit measure is introduced in Asker et al. (2014). The model explains 67% of the variation in growth rates observed in the data. Meanwhile, our model produces a standard deviation (interquartile range) in growth rates of 1.8% (2.17) whereas in the data it is 1.4% (2.07).

Next, we turn our attention to the variation in firm age across countries. As in the data, the model predicts that countries with higher growth rates have on average younger establishments (Figure 1). This negative correlation depends on the share of innovative firms in the market and the identified values for the probability of success. The correlation between

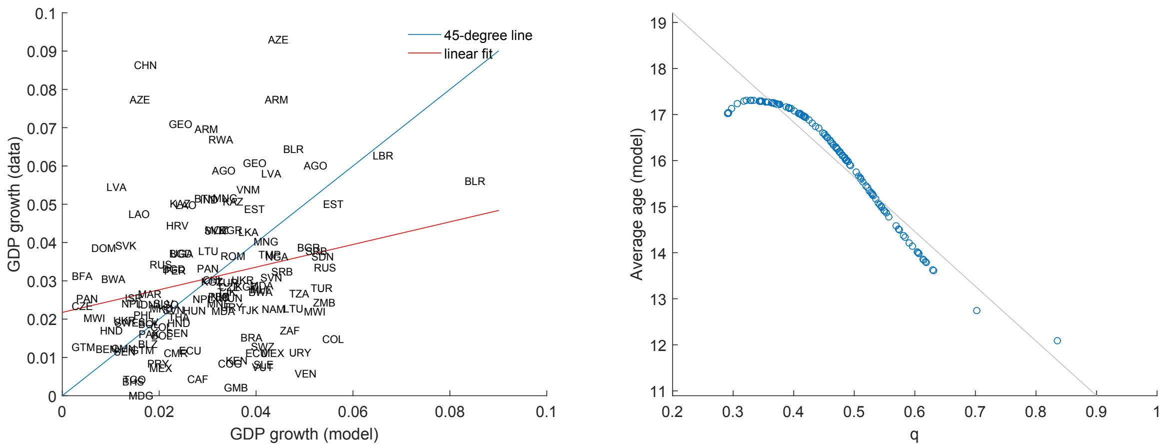

Figure 3: Aggregate growth and average firm age, model fit

Note: The left panel gives the predicted growth rates versus those observed in the data. The right panel plots predicted average age against the one observed in the data. The unique source of variation across countries is the probability of success q.

the model-implied mean age and the data is 0.22 (Figure 3). Using the uncentered measure of fit, we find that the model explains 94.3% of the variation in average age. These findings suggest that aggregate growth rates and average firm age may be tied together through the probability of success.

The lack of employment growth. In the baseline economy, productivity improvement upon success is the same across firms. Hence, firms cannot experience employment growth. Formally, we can describe the change in employment for an innovative firm that remains in the market even if unsuccessful as

$$\frac{l'}{l} - 1 = \begin{cases} \frac{\phi}{\mu} - 1, & \text{with probability } q; \\ \frac{1}{\mu} - 1, & \text{with probability } 1 - q. \end{cases}$$

Since the growth rate of the marginal innovative firm (μ) is equal to firms' investment level (ϕ) in equilibrium, the best outcome for an innovative firm is to stay at the same ranking in the productivity ladder. In the next section, we extend our benchmark economy by introducing

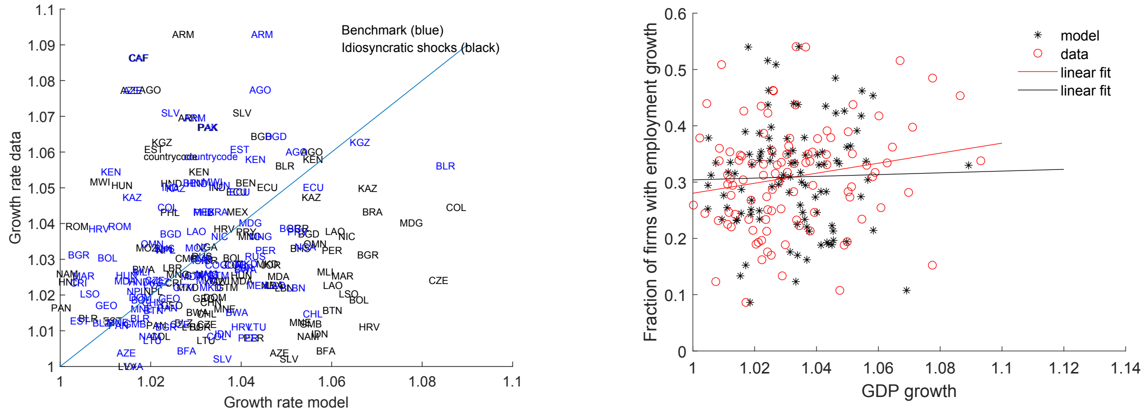

Figure 4: Alternative models, growth rates

Note: The left panel plots the growth rates in the data against the predictions of two alternative models. The "baseline" model corresponds to an economy with no return uncertainty (blue). The "idiosyncratic shocks" economy is one with return uncertainty (black). Return uncertainty is calibrated to match the fraction of firms with employment growth in each country-year and the average employment growth of the average economy in the sample. The right panel plots the relationship between aggregate growth and the share of firms with employment growth in the model and in the data.

idiosyncratic productivity shocks disciplined by the degree of employment growth observed in the data. We show that this additional heterogeneity does not improve the model's ability of explaining the observed aggregate growth rates.

IV.D Uncertainty on returns conditional on success

To allow for shifts in firm productivity rankings, we augment the model with idiosyncratic productivity shocks. Hence, firm level productivity depends on an endogenous component (ϕ) and an exogenous component (ε). Productivity levels follow,

$$z' = \begin{cases} \phi\varepsilon z, & \text{with probability } q; \\ z, & \text{with probability } 1-q. \end{cases}$$

We assume these shocks are i.i.d. with uniform distribution, $\varepsilon \backsim U\left(\underline{\varepsilon}, \overline{\varepsilon}\right)$. To identify them in the data, we use information on the fraction of firms with employment growth and average firm employment growth. Our main exercise assumes that the support of the distribution of shocks is symmetric around some average value $E(\epsilon)$. We set this average value to match the average firm employment growth in a synthetic economy corresponding to the average employment growth in our sample of countries.[23] We use the average employment growth among firms in the top decile of the employment distribution.[24] The average employment growth between two periods in the model is

$$G \equiv q\int_{\underline{z}}^{\infty}\frac{\phi E(\epsilon)z}{\mu z}dF(z|z>\underline{z})+(1-q)\int_{\underline{z}}^{\infty}\frac{z}{\mu z}dF(z|z>\underline{z})=q\frac{\phi E(\epsilon)}{\mu}+(1-q)\frac{1}{\mu}.$$

The average firm-level employment growth over multiple periods is constant: the corresponding average growth measure over t periods is given by G^t. In the data, average firm employment growth for our synthetic economy is 2.56% per year, which yields and expectation of the shocks equal to 1.03. The growth rate of the economy depends only on the expectation of the shocks and the probability of success. Hence, in this extended version of the model, all variation in growth rates will be accounted for by variation in the probability of success, q.[25]

The model predicts that employment growth occurs whenever $\frac{l'}{l}>1$ or $\frac{\phi\varepsilon}{\mu}>1$. The latter occurs with probability $\Pr(\varepsilon>\frac{\mu}{\phi})$. Hence, the share of firms that experience employment growth is $\frac{q}{\overline{\varepsilon}-\underline{\varepsilon}}(\frac{\phi}{\mu}\overline{\varepsilon}-1)$. We use the above expression and the symmetry assumption on the support of the shocks to calibrate a country-specific distribution of shocks, targeting the share of firms experiencing employment growth. By construction however, targeting these

[23] Alternatively, we can calibrate country-specific mean shocks by targeting average firm (establishment) employment growth in each country.

[24] Using the top decile of firms (establishments) is consistent with our identification strategy for the probability of success across economies. When we get the mean growth, we average across age groups within an economy weighted by the number of firms (establishments) in each age category.

[25] When we calibrate the expectation of the shocks to a country-specific target (i.e. average firm employment growth), some of the variation in growth rates is generated by disparities in this expectation. However, the variation in observed growth rates that is accounted for does not improve relative to our benchmark.

shares has no implications for the variation in growth rates observed in the data.

We recompute predicted growth rates allowing for the probability of success and the size of idiosyncratic shocks to vary across countries. We recalibrate the investment cost parameter (c) to match the average growth rate observed in the data (c increases from 0.1765 in the benchmark to 0.192 in the economy with shocks).[26]

Our main finding is that matching average employment growth in our sample of countries and the share of firms with employment growth in each country does not improve the ability of model to generate the cross-country variation in the aggregate growth rates. Figure 4 shows how the implied growth rates change from the benchmark model to this extension. Our measure of goodness of fit, S^2, is 69% while in the benchmark economy it corresponds to 65% for a comparable sample of countries.[27,28] We can also compare alternative measures of variation in growth rates. The standard deviation (interquartile range) of growth rates in the data is 1.96% (2.37), the benchmark predicts 1.08 (2.09), and the extended economy predicts 1.53 (1.99).

IV.E Robustness

In order to assess the robustness of our numerical results, we perform alternative calibration exercises. Here we briefly describe the motivation, strategy and the results of these. We leave the technical details to Appendix.

Our analysis focuses on an equilibrium in which the aggregate growth rates are constant in time. Furthermore, given the homogeneity built into the model, the levels of income are

[26] If investment costs are not recalibrated, the economy that matches employment growth rates produces an average growth rate in the sample of 3.23% (versus 3.09% in the benchmark).

[27] We are able to solve this calibrated economy for 103 country-year observations.

[28] When we allow for cross-country variation in average employment growth, and hence, in the expectation of the shocks, the model predicts 43% of the variation in growth rates.

not pinned down. While these features dramatically facilitate the solution of the model, the identification of its parameters, and the illustration of its main mechanisms, they leave out important dynamics that might be at play. For example, where an economy stands in the spectrum of development may shape its aggregate growth rate due to transitional dynamics.

To answer this concern, we perform two type of exercises. First, we run a calibration using a measure of residual growth rates after conditioning on the level of income and income group of each economy. Hence, we obtain a data counterpart to the growth rates predicted by our model, in which the levels of income are, by construction, arbitrary. We find that this alternative calibration strategy gives very similar results to our benchmark exercise.

Second, we consider a subsample of countries for which the annual growth rate of GDP per capita over the past 20 years have been relatively stable. In one exercise, we exclude country-year observations with a standard deviation of growth rate of income per capita in the top decile of our sample. In a separate exercise, we regress observed growth rates on a time trend and a constant for each country. Among the cases with a time trend significantly different from zero at 5 percent level, we exclude economies that have a time trend in the top decile. With these two alternative ways of cleaning the sample, we arrive to similar conclusions in terms of the relative role of the variation in the probability of success for aggregate growth disparities. When focusing on the sample of countries with no time trends, the correlation of the model-implied aggregate growth with the one observed in the data increases relative to our benchmark economy (from 0.23 to 0.33). The corresponding correlation for the average firm age also increases (from 0.19 to 0.26). Hence, we view our benchmark results as conservative.

We also build robustness exercises using alternative values for the parameters calibrated outside the model. In particular, the labor share (θ) and the curvature of the cost function (τ). First, we estimate the labor share assuming constant returns to scale from the ratio of labor costs to sales (this yields a value of 0.79). Alternatively, we estimate its value from the product between an estimate of the span of control and the labor share, under the assumption of decreasing returns to scale (this yields a value of 0.572). Our benchmark findings in terms of the variation in growth rates and the average age of firms across countries are robust to these alternative values of θ. With respect to the parameter shaping the curvature of the cost function, empirical estimates are unfortunately unavailable. Therefore, we experiment with specifications closer to a linear cost function, $\tau = 1.2$ and specifications with higher curvature, $\tau = 5$. Our conclusions in terms of the cross-country variation in growth rates and average firm age accounted for by the model are robust to these alternative calibrations.

When we extend any of these exercises to match average employment growth in the data, we consistently find a negligible role for cross-country disparities in firm employment growth patterns in explaining differences in aggregate growth.

V Conclusion

We build a general equilibrium theory that links firm-level uncertainty on returns to investment with the industry dynamics (employment and age distribution of firms) and aggregate growth. For a large set of countries, we document this relationship using plant-level and aggregate data, and highlight key facts that characterize the growth process. We use the model to identify mechanisms through which idiosyncratic uncertainty shapes the industry dynamics and aggregate growth.

We show that cross-country differences in the success probability for firms can account for two-thirds of the observed variation in aggregate growth. We also show that a key statistic related to aggregate growth is the age distribution of firms. The fraction of firms with positive employment growth holds information about the success of some firms relative to others in the economy but not necessarily about the aggregate growth.

Before concluding, we reemphasize that there are other determinants of economic growth that we do not study in this paper. Perhaps the most importantly, we abstract from transition dynamics and instead focus on balanced growth. This is done for tractability, but we expect the mechanisms uncovered in the paper to remain relevant along a transition path.

In addition, we do not model cross-country differences in innovation technologies, or costs. Instead, we focus only on the probability of success, in order to illustrate better its role in the cross-country variation in aggregate growth. It might be important to complement our analysis with an accounting exercise that shows the relative role of all these factors in shaping aggregate growth. We leave this extension for future work.

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