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# Thwarting Selfish Behavior in 802.11 WLANs

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***Abstract*—The 802.11e standard enables user configuration of several MAC parameters, making WLANs vulnerable to users that selfishly configure these parameters to gain throughput. In this paper we propose a novel distributed algorithm to thwart such selfish behavior. The key idea of the algorithm is for stations to react, upon detecting a misbehavior, by using a more aggressive configuration that penalizes the misbehaving station. We show that the proposed algorithm guarantees global stability while providing good response times. By conducting an analysis of the effectiveness of the algorithm against selfish behaviors, we also show that a misbehaving station cannot obtain any gain by deviating from the algorithm. Simulation results confirm that the proposed algorithm optimizes throughput performance while discouraging selfish behavior. We also present an experimental prototype of the proposed algorithm demonstrating that it can be implementated on commodity hardware.**

## I. Introduction

The mechanisms defined in 802.11e, which have been incorporated into the 802.11 standard since 2007, expose a number of configurable parameters that can be modified by a simple command. This gives users direct control of the contention parameters used by their wireless adapter and allows them to modify the behavior of the wireless interface. Users can therefore easily configure the 802.11e parameters of their wireless device with aggressive values that increase their share of the medium at the expense of the other users.[1] Such *selfish* behavior can lead to severe unfairness in the allocation of network capacity amongst users sharing the same WLAN.

In the literature the approaches proposed to address this selfishness problem can be classified as being either centralized [2]–[5] or distributed [6]–[8]. In this paper, we propose a novel *distributed* approach to address the selfishness problem. The advantage of distributed approaches is that they do not rely on a central authority and thus can be used both in infrastructure and ad-hoc modes, in contrast to centralized approaches which can only be used in infrastructure mode.

Previous analysis of the selfishness problem in a WLAN [8] has shown that, if misbehaving stations are not penalized, the WLAN naturally tends to either great unfairness or network collapse. Following this result, in this paper we focus on the design of a penalizing mechanism in which any station that misbehaves will be punished by the other stations and thus will have no incentive to misbehave. A key challenge when designing such a penalizing scheme is to carefully adjust the punishment inflicted on a misbehaving station. If, on the one hand, the punishment is not sufficiently severe, a station could benefit from misbehaving. However, on the other hand, an overreaction could trigger the punishment of other stations leading to an endless loop of punishments. Our design makes use of Lyapunov stability theory to address this challenge. The main contributions of our paper are as follows:

- We propose a novel distributed algorithm that penalizes misbehaving stations by making use of a more aggressive configuration of the 802.11e parameters upon detecting misbehavior.
- We conduct a stability analysis of the algorithm to show that when all stations implement our algorithm, the WLAN convergences to the optimal point of operation.
- We conduct an analysis of the effectiveness of the algorithm against selfish behavior that shows that a station cannot increase its throughput by deviating from the algorithm.
- We extensively evaluate the performance of the proposed algorithm via simulation under a wide variety of conditions that confirm its good properties.
- We show the feasibility of implementing the algorithm by deploying a prototype and evaluating it in a small experimental testbed.

The rest of the paper is structured as follows. In Section II we discuss the selfishness problem in 802.11. Section III presents the algorithm proposed. The algorithm is evaluated analytically in Section IV: we first analyze its performance when all stations implement the algorithm and then study the case when stations may deviate from the algorithm. The performance of the algorithm is exhaustively evaluated via simulation in Section V and its feasibility of implementation is validated in Section VI by means of a prototype. Finally, Section VII closes the paper with some concluding remarks.

## II. Selfishness in 802.11

In this section, we briefly summarize the EDCA mechanism of 802.11e, identify the selfishness problem and briefly review related work.

### A. 802.11e EDCA

The 802.11e EDCA mechanism works as follows. When a station has a new frame to transmit, it senses the channel. If the channel remains idle for a period of time equal to the $AIFS$ parameter, the station transmits. Otherwise, if the channel is detected busy, the station monitors the channel until it is measured idle for an $AIFS$ time, and then executes a backoff process.

When the backoff process starts, the station computes a random number uniformly distributed in the range $(0, CW-1)$, and initializes its backoff time counter with this value. $CW$ is called the contention window and for the first transmission

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[1] Indeed, in [1] it has been shown that a station can obtain very substantial gains in throughput by misbehaving.

attempt the minimum value ($CW_{min}$) is used. In case of a collision $CW$ is doubled, up to a maximum value $CW_{max}$.

As long as the channel is sensed idle, the backoff time counter is decremented once every time slot of duration $T_e$, and "frozen" when a transmission is detected on the channel. When the backoff time counter reaches zero, the station accesses the channel in the next time slot. Upon accessing the channel, a station can transmit several consecutive frames for a duration given by the $TXOP$ parameter.

A collision occurs when two or more stations start transmitting simultaneously. An acknowledgment (Ack) frame is used to notify the transmitting station that the frame has been successfully received. If the Ack is not received within a given timeout, the station reschedules the transmission by doubling $CW$, up to a maximum value $CW_{max}$, and reenters the backoff process. If the number of failed attempts reaches a predetermined retry limit, the frame is discarded. Once the backoff process is completed, $CW$ is set again to $CW_{min}$.

It can be seen from the above description that the behavior of a station depends on a number of parameters, namely $CW_{min}$, $CW_{max}$, $AIFS$ and $TXOP$. As these are (according to the 802.11 standard) configurable parameters whose setting can be modified by means of simple commands, a user can easily configure these parameters selfishly to gain extra throughput (i.e., to increase the individual throughput received by the user). We refer to this as the *problem of selfishness in 802.11*.

### *B. Related work*

The above problem of selfishness in 802.11 has been modeled using game theory. In these models, the players are the wireless stations which configure their 802.11e parameters to obtain as much throughput as possible. The simplest models are those based on static games, in which each player makes a single move and all moves are made simultaneously. The modeling of the selfishness problem in 802.11 based on static games [6], [7] leads to the following two families of Nash equilibria: in the first family, there is one player that receives a non-null throughput while the rest of the players receive a null throughput, and in the second family, all players receive a null throughput (the latter is known as the *tragedy of the commons*: the selfish behavior of each player leads to a tremendous misuse of the public good).

Both of the above families of solutions are highly undesirable, as they lead either to extreme unfairness or network collapse. One alternative to avoid these undesirable solutions is to allow users to make additional moves (i.e., change their 802.11e configuration) during the game. This has been modeled by making use of the theory of *repeated games* [9]. With repeated games, time is divided into stages and at each stage a player can take a new decision based on the observed behavior of the other players in the previous stages. Therefore, if a station is detected misbehaving, the other stations can *punish* this station and thus discourage such behavior.

Making use of repeated games, [6] and [7] have proposed two approaches to address the problem of selfishness in 802.11. The approach proposed by [6] is based on *selective jamming*: if a station detects that another station is misbehaving, thereafter it listens to its transmitted packets and switches to transmission mode, *jamming* enough bits so that the packets cannot be properly recovered at the receiver. While the use of jamming punishes misbehaving stations, it has the major drawback of relying on functionality not available in current wireless devices. Indeed, due to the accurate timing required, the implementation of such a jamming mechanism would need to be performed at the hardware level and entails substantial complexity.

The approach proposed by [7] does not suffer from the above drawback but addresses only two types of misbehaving stations: the so-called selfish stations, with $CW = 2$, and the so-called greedy stations, with $CW = 1$. While the scheme proposed is effective when dealing with these two particular configurations, other $CW$ configurations that may greatly benefit a misbehaving station are neither detected nor punished by this mechanism, as we show in the simulation results of Section V-E. Additionally, the algorithm of [7] is based on heuristics that do not guarantee quick convergence, and indeed we show in a further simulation result in Section V-G that this approach may suffer from convergence issues.

In this paper, we propose a novel approach that, in contrast to the previous two approaches, relies exclusively on functionality readily available in current wireless devices and is effective against any selfish configuration of the 802.11e parameters. Additionally, by relying on Lyapunov stability techniques, our approach is guaranteed to quickly converge to the desired point of operation.

In [2]–[5] the issue of selfishness in 802.11 WLANs is also addressed. However, in contrast to our distributed algorithm, these papers propose a centralized approach and therefore can only be applied to a WLAN operating in infrastructure mode. Additionally, many of these approaches only address the detection of misbehaving stations while our approach not only detects but also punishes such stations.

A number of other work addresses selfishness in wireless networks from a game theoretic point of view [10]–[13]. Besides focusing on a different MAC protocol, these studies differ from ours in that they consider some kind of transmission cost or pricing mechanism that plays a key role in the resulting equilibria. In contrast to these approaches, we achieve the desired equilibrium by means of a penalizing mechanism.

Perhaps the most closely related to this paper is our previous work of [14], which uses a similar technique to counteract selfish behavior. However, both the scope of the work and the algorithm design are substantially different. Indeed, while here we address the problem of selfishness in 802.11, [14] deals with a completely different random access protocol, namely distributed opportunistic scheduling (DOS), and hence focuses on a different problem.[2] Furthermore, [14] relies on local linearized analysis, while here we use Lyapunov theory for the global design and analysis of the algorithm. As a consequence, the algorithm proposed in this paper provides much stronger guarantees on stability and convergence than that of [14].

[2]With DOS, upon gaining access to the channel a station measures the radio conditions and only transmits when the channel quality is above a certain threshold; hence, this mechanism is fundamentally different from 802.11.

## III. PAS ALGORITHM

In this section, we present our algorithm to address the problem of selfishness in 802.11, which we call the *selfishness-Proof Adaptive Stable* (PAS) algorithm. In the following, we first present the objectives pursued and then describe the algorithm design to achieve these objectives.

### A. Algorithm objectives and scope

The central objective of the PAS algorithm is to drive the configuration of the 802.11e EDCA parameters to target values that maximize the overall WLAN performance. To achieve this objective, PAS enforces that a misbehaving station cannot benefit from using a different configuration, which provides stations with an incentive to adopt the target configuration.

Following the arguments given in [6], [15], [16], in this paper we aim at the following setting of the four EDCA parameters, which maximizes the throughput performance of the WLAN (hereafter we refer to this setting as the *target configuration* or *optimal configuration*):

- The $AIFS$ parameter is set to its minimum value ($AIFS = DIFS$).
- The $TXOP$ parameter is set such that a fixed payload is transmitted upon accessing the channel. For simplicity, in the rest of the paper we assume that this payload corresponds to one packet (i.e., $TXOP = 1$ packet); however, the analysis would be identical for any other payload value.[3]
- The maximum backoff stage $m$ is set equal to 0.[4] This yields the same value for $CW_{min}$ and $CW_{max}$ (i.e., $CW_{max} = CW_{min}$); in the following, we refer to this value simply as $CW$.
- The $CW$ parameter is set equal to the value that, given the above setting for the other parameters, maximizes the throughput of the WLAN when all stations are saturated. Hereafter, we refer to this value as $CW_{opt}$.

With the above, the objective of the PAS algorithm can be reformulated as being to achieve the following two goals: ($i$) when all stations implement PAS (i.e., they are well-behaved), the system should converge to the target configuration, namely $CW_i = CW_{opt}$ $\forall i$ (where $CW_i$ is the $CW$ used by station $i$); and ($ii$) if a station misbehaves (by using a $CW_i$ different from $CW_{opt}$ and/or a different setting of the other parameters), this station should not obtain a throughput benefit from such misbehavior.

In the following, we address the design of the PAS algorithm. Like the previous work in [6], [7], in the design of the algorithm we assume that all stations are saturated (i.e., always have a packet ready for transmission), they are within transmission range of each other (i.e., no hidden nodes) and use the same modulation-coding scheme. In the simulations section, we show that the proposed algorithm can be extended to effectively prevent selfish behavior with non-saturated stations. While the design assumes no hidden terminals, the algorithm also works for hidden terminals as long as the RTS/CTS mechanism is used.[5] Furthermore, in the case of different modulation-coding schemes, the algorithm can be applied to enforce the target configuration proposed in [17].

### B. Computation of $CW_{opt}$

We use the model of [18] to compute the throughput $r_i$ of station $i$ as a function of the transmission probabilities $\hat{\tau}_i$,

$$r_i(\hat{\boldsymbol{\tau}}) = \frac{l}{T_s(\hat{\boldsymbol{\tau}})}\hat{\tau}_i \prod_{j\neq i}(1-\hat{\tau}_j) = \frac{\hat{\tau}_i}{1-\hat{\tau}_i}\frac{l}{T_s(\hat{\boldsymbol{\tau}})}\prod_{j=1}^{n}(1-\hat{\tau}_j) \quad (1)$$

where $\hat{\boldsymbol{\tau}} = [\hat{\tau}_1, \cdots, \hat{\tau}_n]$, $n$ is the number of active stations in the WLAN, $l$ is the packet payload in bits, $T_s(\hat{\boldsymbol{\tau}}) = T_t + (T_e - T_t)\prod_j(1-\hat{\tau}_j)$ is the average duration of a slot in seconds, $T_t$ the duration of a transmission and $T_e$ the duration of an empty time slot.

By [19, Lemma 1], the rate region boundary is the set of throughput vectors such that $\sum_{i=1}^{n} T_{air,i}(\hat{\boldsymbol{\tau}}) = 1$ where $T_{air,i}(\hat{\boldsymbol{\tau}}) = \hat{\tau}_i \frac{T_t}{T_s(\hat{\boldsymbol{\tau}})}$ is the fraction of airtime (including both successful and colliding transmissions) used by station $i$. When all stations use the same transmission probability, it follows immediately that the value $\tau_{opt}$ maximising throughput is the unique solution to

$$\frac{1-n\tau_{opt}}{(1-\tau_{opt})^n} = 1 - \frac{T_e}{T_t} \quad (2)$$

Once we have $\tau_{opt}$ then $CW_{opt} = \frac{2}{\tau_{opt}} - 1$. When $\frac{T_e}{T_t}$ is small, a good approximation is $CW_{opt} = n\sqrt{\frac{2T_t}{T_e}} - 1$.

The following fundamental property will also prove useful:[6]

**Theorem 1.** *Consider the set of points $C = \{\hat{\boldsymbol{\tau}} : \hat{\boldsymbol{\tau}} \in [\tau_m, \tau_M]^n\}$, $0 \leq \tau_m \leq \tau_M \leq 1$. For any $\hat{\boldsymbol{\tau}} \in C$ the following inequality holds:*

$$D(\hat{\boldsymbol{\tau}}) := nr_{opt} - \sum_j r_j(\hat{\boldsymbol{\tau}}) \leq n\rho\Delta \quad (3)$$

*where $\Delta = \max(\tau_{opt} - \tau_m, \tau_M - \tau_{opt})$, $\rho = \frac{r_{opt}}{\tau_{opt}(1-\tau_{opt})}$ and $r_{opt}$ is the maximum achievable throughput of a station when $\hat{\tau}_i = \hat{\tau}_j$ $\forall i, j$.*

This theorem is a continuity result that can be used to bound the difference $D(\hat{\boldsymbol{\tau}})$ between the optimum and actual WLAN sum-rate throughput.

### C. Algorithm description

Following the 802.11 standard, which updates the configuration of the 802.11e parameters upon receiving a beacon frame, PAS implements an *adaptive algorithm* in which each station updates its $CW_i$ at every beacon interval, while keeping the configuration of the other parameters fixed to the values provided in Section III-A; hereafter we refer to each beacon

[3] As explained in [15], the setting of this parameter responds to a trade-off between throughput and delay. Indeed, by increasing $TXOP$ throughput is improved at the expense of increasing delay. Based on this, [15] suggests to set this parameter to the largest possible value that gives an acceptable delay.

[4] The maximum backoff stage is defined as the number of times that the $CW$ is doubled until reaching $CW_{max}$ (i.e., $CW_{max} = 2^m CW_{min}$).

[5] The reason for this is that, as in [6], [7], in order to detect selfish behaviors we need to know the throughput received by the other stations.

[6] The proof of all theorems is provided in the Appendix.

interval as a *stage* of the algorithm. The central idea behind PAS is that, when a station is detected as misbehaving, the other stations reduce their $CW_i$ in subsequent stages to prevent this station from benefiting from misbehaving.

A key challenge in PAS is to carefully adjust the reaction against a misbehaving station. Indeed, as mentioned in the introduction, if the reaction is not severe enough such a station may benefit from its misbehavior, but if the reaction is too severe the system may become unstable by entering an endless loop where all stations indefinitely reduce their $CW_i$ to punish each other. In order to address this challenge we use techniques from Lyapunov theory [20] in the design of PAS to guarantee that the $CW_i$ of all stations converges to $CW_{opt}$ and avoid that the system enters into a spiral of increasing punishments that lead to throughput collapse.

The iterative PAS update of the $CW_i$ can be modeled as a discrete time dynamical system whose state is given by $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_n]$, where $\tau_i$ is related to the probability with which station $i$ transmits in a slot time. That is:

$$\boldsymbol{\tau}(t+1) = f(\boldsymbol{\tau}(t)) \tag{4}$$

where $f : [0,1]^n \rightarrow [0,1]^n$ is a non-linear function that models the system dynamics. The main design challenge is to determine the function $f$. To this end, we adopt a standard feedback approach [21] and update $\tau_i$ at each stage as:

$$\tau_i(t+1) = \tau_i(t) + \gamma g_i(t), \; i = 1, \cdots, n \tag{5}$$

where $\gamma > 0$ is a scalar parameter and $g_i : [0,1]^n \rightarrow [0,1]$ .

In order to allow for large values of $\gamma$, which reduces the convergence time of the algorithm,[7] we impose that the probability of transmitting in a slot time does not fall below $\tau_{opt}/2$ (where $\tau_{opt}$ is the optimal $\tau_i$ value, given later). Similarly, if $\tau_i(t)$ exceeds 1, we transmit with probability 1. Thus,

$$\hat{\tau}_i(t) = \min(1, \max(\tau_i(t), \tau_{opt}/2)) \tag{6}$$

where $\hat{\tau}_i(t)$ is the probability that the station $i$ transmits in a slot time, after imposing the above constraint. Given $\hat{\tau}_i(t)$, the $CW_i$ parameter of the station at stage $t$ is $CW_i(t) = 2/\hat{\tau}_i(t) - 1$.

We next address the design of function $g_i$ in (5). Our requirements when designing $g_i$ are twofold: $(i)$ misbehaving stations should not be able to obtain extra throughput from the WLAN by following a different strategy from PAS, and $(ii)$ as long as there are no misbehaving stations that deviate from PAS, the $\tau_i$ of all stations should converge to the optimal value $\tau_{opt}$. To meet the above requirements, we select $g_i$ as follows

$$g_i(t) = \sum_{j \neq i} (r_j(t) - r_i(t)) - F_i(t) \tag{7}$$

where $F_i(t)$ is a function that we design below, and $r_j(t)$ is the throughput received by station $j$ over the beacon interval corresponding to stage $t$. Similarly to [6], [7], to obtain the $r_j(t)$ values we rely on the broadcast nature of the wireless medium, which provides WLAN stations with the ability to measure the throughput received by the other stations.

Observe that, according to (7), $g_i(t)$ consists of the following two components, each of which fulfills one of the requirements identified before:

- The first component, $\sum_{j \neq i} r_j(t) - r_i(t)$, serves to punish misbehaving stations: if a station $i$ receives less throughput than the other stations, this component will be positive and hence station $i$ will increase its transmission probability $\tau_i$ to punish the other stations.
- The second component, $F_i(t)$, drives the system to the target configuration in the absence of misbehaving stations.

Regarding $F_i(t)$, to drive the $\tau_i$ to the target value $\tau_{opt}$ we require $F_i(t)$ to be positive when $\tau_i > \tau_{opt}$, and negative otherwise. Furthermore, $F_i(t)$ should not allow misbehaving stations to obtain a throughput gain over well-behaved stations. To gain insight, we consider a scenario with one misbehaving station in steady-state operation, which implies that the misbehaving station uses a static configuration. (In the analysis of Section IV-B we show that PAS is also effective against selfish strategies that change the configuration over time.) In steady-state the LHS and RHS of update (5) must be equal for the well-behaved stations using PAS, i.e., $g_i(t^\infty) = 0 \; \forall i \neq s$, where $s$ is the misbehaving station and the $\infty$ superscript indicates values when the system is in steady state, and so

$$F_i(t^\infty) = \sum_{j \neq i} (r_j(t^\infty) - r_i(t^\infty)) = r_s(t^\infty) - r(t^\infty) \tag{8}$$

where $r(t^\infty)$ is the throughput of a well-behaved station (which, by symmetry, is the same for all such stations in steady-state). We require that the throughput of the misbehaving station does not exceed the target throughput, $r_s(t^\infty) \leq r_{opt}$. That is,

$$\begin{aligned} nr_s(t^\infty) &= r_s(t^\infty) + (n-1)r(t^\infty) + (n-1)(r_s(t^\infty) - r(t^\infty)) \\ &= \sum_{j=1}^{n} r_j(t^\infty) + (n-1)F_i(t^\infty) \leq nr_{opt} \end{aligned}$$

which is satisfied when

$$F_i(t) \leq \frac{1}{n-1} D(t) \tag{9}$$

where $D(t) = nr_{opt} - \sum_j r_j(t)$.

The intuition here is that when a station misbehaves, it receives more throughput than the well-behaved stations. This, however, moves the point of operation away from the optimal one, reducing the overall efficiency in terms of the aggregate throughput. The bound (9) ensures that the additional throughput received by the misbehaving station does not outweigh the throughput it loses due to the overall loss of aggregate throughput. This guarantees that in steady-state the misbehaving station does not receive more throughput and hence does not benefit from its misbehavior.

Following the above requirements, we select $F_i(\boldsymbol{\tau})$ as:

$$F_i(t) = \begin{cases} D(t)/[2(n-1)], & \tau_i > \tau_{opt} \;\&\; D(t) \geq 0 \\ -D(t)/[2(n-1)], & \tau_i \leq \tau_{opt} \;\&\; D(t) \geq 0 \\ D(t)/(n-1), & D(t) < 0 \end{cases} \tag{10}$$

[7]The fact that imposing a lower bound on $\tau_i$ allows for larger $\gamma$ values can be seen from the proof of Theorem 2.

The above choice meets the design requirements set above for the function $F_i(t)$: $(i)$ it satisfies (9), preventing misbehaving stations from obtaining any gain; and $(ii)$ for well-behaved stations it fulfills $F_i(t) > 0$ for $\tau_i > \tau_{opt}$ and $F_i(t) < 0$ for $\tau_i < \tau_{opt}$, which ensures convergence to optimal operation.[8] Note that, as long as $D(t) < 0$, we set $F_i(t)$ equal to one half of the upper bound provided by (9). This choice has been made to ensure that, when misbehaving, a station is punished and sees its throughput reduced (this would not happen if our choice of $F_i$ was equal to the upper bound).

To compute update (5) with these choices of $g_i$ and $F_i$, each station only needs to measure at the end of every stage the throughput it has received during this stage as well as the throughput that each of the other stations has received.

## IV. Algorithm Analysis

In this section we study analytically the performance of the system. First, we prove that when all the stations are well-behaved and implement the PAS algorithm, the WLAN converges to the optimal configuration (Section IV-A). Then, we show that a misbehaving station does not have any incentive to deviate from the PAS algorithm (Section IV-B).

### A. Stability Analysis

We show that when all stations implement PAS, the WLAN is driven to the optimal configuration, i.e., $\tau_i = \tau_{opt}\ \forall i$.

Formally, a point $\boldsymbol{\tau}_e \in [0,1]^n$ is said to be a *globally asymptotically stable equilibrium* of the system (4) if $(i)$ $\forall \epsilon > 0\ \exists \delta > 0$ such that $\|\boldsymbol{\tau}(0) - \boldsymbol{\tau}_e\| < \delta \Rightarrow \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_e\| < \epsilon\ \forall t$; and $(ii)$ $\lim_{t\to\infty} \boldsymbol{\tau}(t) = \boldsymbol{\tau}_e\ \forall \boldsymbol{\tau}(0) \in [0,1]^n$. These conditions ensure that the system converges to $\boldsymbol{\tau}_e$ independently of its initial state and that the equilibrium point is unique. We have the following result:

**Theorem 2** (Global stability)**.** *The target configuration $\boldsymbol{\tau}_{opt} = [\tau_{opt} \cdots \tau_{opt}]^T$ is a globally asymptotically stable equilibrium point under update (5) provided $n \geq 2$ and*

$$\gamma < \gamma_{max} := \left( \frac{nl}{T_m} \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} \right)^{-1}$$

*where $T_m := T_t + (T_e - T_t)\left(1 - \frac{\tau_{opt}}{2}\right)^n$.*

The proof of Theorem 2 makes use of Lyapunov's direct method [20]. Namely, a point $\boldsymbol{\tau}_e$ is the globally asymptotically stable equilibrium of the system if there exists a continuous radially unbounded function $V : [0,1]^n \to [0,1]$ such that $V(\boldsymbol{\tau} - \boldsymbol{\tau}_e) > 0\ \forall \boldsymbol{\tau} \neq \boldsymbol{\tau}_e$, $V(\boldsymbol{\tau}_e) = 0$ and

$$V(\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_e) < V(\boldsymbol{\tau}(t) - \boldsymbol{\tau}_e) \tag{11}$$

To apply this result we must find a Lyapunov function $V$ with these properties. Selecting $V(\boldsymbol{\tau} - \boldsymbol{\tau}_e) = \|\boldsymbol{\tau} - \boldsymbol{\tau}_e\|_\infty$ as a candidate Lyapunov function, with the equilibrium point $\boldsymbol{\tau}_e = \boldsymbol{\tau}_{opt} = [\tau_{opt} \cdots \tau_{opt}]^T$, the proof of Theorem 2 establishes that

$$\|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty < \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty \tag{12}$$

That is, Lyapunov equation (11) is satisfied by this choice of $V$ and so $\boldsymbol{\tau}_{opt}$ is a globally asymptotically stable equilibrium.

It remains to select the value of the parameter $\gamma$. This involves a tradeoff: the smaller $\gamma$, the slower the rate of convergence is; however, if $\gamma$ is set too large the system risks instability (as shown by Theorem 2). Following the same rationale as the *Ziegler-Nichols* rules [22], which have been proposed to address a similar tradeoff in the context of classical control theory, we recommend setting $\gamma$ to half of the value at which the system turns unstable, i.e., $\gamma = \gamma_{max}/2$.

The analysis conducted so far assumes that stations have perfect knowledge of the $r_j$ values. However, since these values are measured by the stations, measurement errors may introduce disturbances into the system and make its trajectory differ from that of the undisturbed system. In this case, Theorem 2 of [23] (along with Corollary 1) can be used to guarantee that, as long as the disturbances are bounded, the trajectory of the disturbed system approaches a ball around the equilibrium of the undisturbed system.[9] The following sufficient conditions are given in [23]: $(i)$ there exist two class $\mathcal{K}$ functions $\alpha_1$ and $\alpha_2$ such that $\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|)$ for $|x| \leq r$, $(ii)$ there exists a class $\mathcal{K}$ function $\alpha_3$ such that $V(x(t+1)) - V(x(t)) \leq -\alpha_3(|x(t)|)$ for $|x| \leq r$, $(iii)$ there exists a Lyapunov function for the undisturbed system which is uniformly continuous in $\boldsymbol{\tau}$, and $(iv)$ the update function (5) is uniformly continuous in the disturbance. Condition $(i)$ is satisfied for $\alpha_1 = \alpha_2 = \|\boldsymbol{\tau} - \boldsymbol{\tau}_{opt}\|_\infty$. Condition $(ii)$ is guaranteed by Theorem 3 below for $\|\boldsymbol{\tau} - \boldsymbol{\tau}_{opt}\|_\infty \leq \tau_{opt}/2$. Condition $(iii)$ is satisfied by the Lyapunov function we have selected. Finally, condition $(iv)$ holds since, for a given $\tau_i$, $g_i(t)$ is uniformly continuous in the $r_i$'s and so in the disturbances of the $r_i$ measurements. Thus, PAS does not only converge when measurements are error-free, but it is also robust to measurement errors.

**Theorem 3.** *There exists a $\mathcal{K}$ function $f(x)$, $x \in [0, \tau_{opt}/2]$, that satisfies $\|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty - \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty \leq -f(\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty)$.*

### B. *Effectiveness against selfish behavior*

In the previous section we have seen that, when all stations implement PAS, the system converges to the target configuration, i.e., all stations have $\tau_i = \tau_{opt}$ and receive a throughput equal to $r_{opt}$. In this section we conduct an analysis to show that a station cannot obtain a throughput greater than $r_{opt}$ by following a different strategy from PAS. In what follows, we say that a station is *well-behaved* when it implements PAS to configure its 802.11e parameters, while we say that it is *misbehaving* when it follows a different strategy from PAS to configure its parameters with the aim of obtaining greater throughput.

The analysis conducted in this section assumes that users are *rational* and want to maximize their own throughput. Like other previous analyses on selfishness in 802.11 [6],

[8] The only exception to this is when $D(t) < 0$: in this case (10) gives $F_i(t) < 0$ independent of the value of $\tau_i$. However, as we show later in Section IV-A, this does not affect the convergence of the algorithm to the desired point of operation.

[9] Furthermore, [23] also shows that the radius of this ball is proportional the supremum norm of the disturbance, which guarantees that as disturbances fade, so does their effect.

[7], we consider the throughput obtained by a station over an infinite interval, which is a common assumption when users do not know when they will leave the system. Under these assumptions, the following theorem shows the effectiveness of PAS against a misbehaving station. Note that the theorem does not impose any restriction on the strategy followed by the misbehaving station, which may play with all four of the 802.11e parameters, changing their settings over time.

**Theorem 4.** *Consider a misbehaving station that uses a configuration that can vary over time. If all the other stations implement the PAS algorithm, the throughput received by this station will be no larger than* $r_{opt}$.

A consequence of the above theorem is that, if all other stations run PAS, the best alternative for a given station is to run PAS as well, since it cannot benefit from following a different strategy. Therefore a station does not have any incentive to deviate from PAS, establishing the effectiveness of the PAS algorithm against selfish behavior.[10]

## V. Performance Evaluation

In this section we thoroughly evaluate PAS by conducting an extensive set of simulations to show that $(i)$ a misbehaving station cannot benefit from following a different strategy from PAS, and $(ii)$ when all stations are well-behaved, PAS provides optimal performance, is stable and reacts quickly to changes. For the simulations, we have implemented our algorithm in OMNET++. The physical layer parameters of IEEE 802.11g and a fixed payload size of 1500 bytes have been used in all of the experiments. We set the beacon interval to 100 ms, a typical value used in 802.11 WLANs (this is the interval over which the PAS algorithm measures the throughput of the stations and updates the $CW_i$ configuration).

In the simulations of Sections V-A to V-H, we focus on the $CW$ parameter: we assume that all stations (both well-behaved and misbehaving) use a fixed configuration of their $AIFS$, $TXOP$ and $m$ parameters equal to the optimal configuration and play only with the $CW$ parameter. Then, in the simulations of Section V-I we study all four parameters and show that misbehaving stations cannot obtain any benefit from any configuration of these parameters. Unless otherwise stated, we assume that all stations are sending traffic under saturated conditions and measurements are error-free. For all simulation results, 95% confidence intervals are below 0.5%.

### A. Impact of selfish behavior

In order to gain insight into the impact of the configuration used by a misbehaving station, we evaluate the resulting throughput distribution when the misbehaving station uses a fixed $CW$ value and all other stations implement PAS. Figure 1 shows the throughput obtained by the misbehaving station and by a well-behaved station as a function of the $CW$ value used by the misbehaving station, $CW_s$, when there are $n = 10$ stations in the WLAN. We observe from this figure

[10]From a game theoretic perspective, this corresponds to symmetric Nash equilibrium: when all other stations run PAS, a station cannot gain any extra utility (in this case, throughput) by following a different strategy.

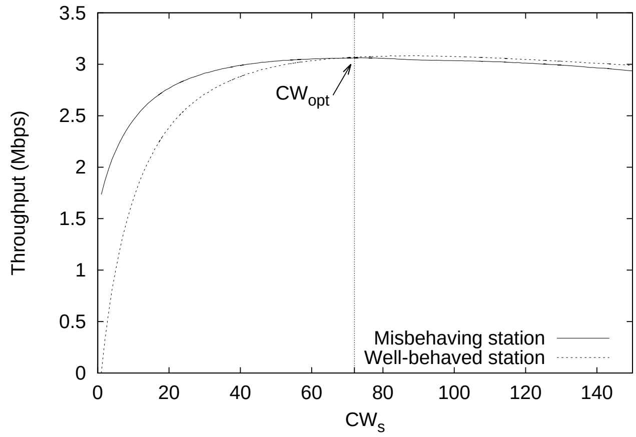


Fig. 1. Throughput of a misbehaving and a well-behaved station as a function of the $CW_i$ of the misbehaving station.

that there are some $CW_s$ values for which the misbehaving station obtains a larger throughput than a well-behaved station, and others for which a well-behaved station obtains a larger throughput. However, if the misbehaving station chooses the $CW_s$ that maximizes its throughput, then it does not receive more throughput than a well-behaved station, and hence does not have any gain over a well-behaved station as a result of its selfish behavior. We further observe that, as a consequence of our design choice for $F_i$, when a station misbehaves (by setting $CW_s < CW_{opt}$) it is punished, seeing its throughput reduced to as much as half of the optimal throughput for the most aggressive $CW_s$ configuration.

### B. Protection against selfish behavior

According to the analysis conducted in Section IV-B, a station cannot obtain more throughput with a selfish strategy than by running PAS. To validate this result, we evaluate the throughput obtained by a misbehaving station with the following strategies. In the first strategy (*static*), the misbehaving station uses the fixed configuration of $CW_i$ that provides the largest throughput, obtained from performing an exhaustive search over all possible configurations (as in Figure 1). In the second strategy (*adaptive 1*), the misbehaving station periodically tries $CW_i = 2$ to gain throughput and when it realizes that its throughput is below $r_{opt}$, it assumes that it has been detected as misbehaving and switches back to $CW_i = CW_{opt}$. The third strategy (*adaptive 2*) is similar to the previous one but instead of switching back to $CW_{opt}$, the station increases its $CW_i$ by 5. In the last strategy (*adaptive 3*), the misbehaving station decreases its $CW_i$ by 5 as long as its throughput is larger than in the previous stage and increases it by 5 otherwise. Figure 2 compares the throughput obtained with each of these strategies against that obtained with PAS for different $n$ values. We observe that, when all other stations run PAS, a given station maximizes its payoff by running PAS as well, as it obtains a larger throughput with PAS than when using any of the other strategies.

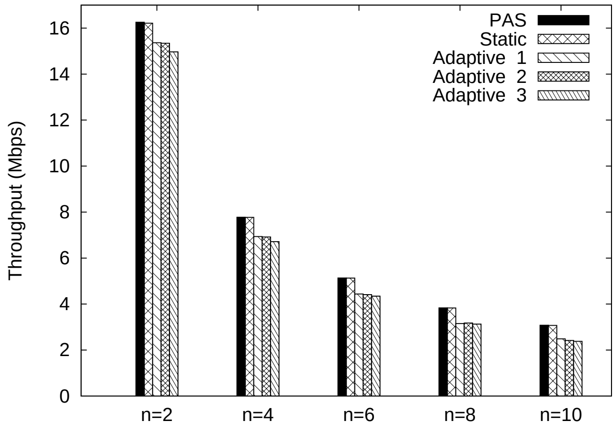


Fig. 2. Throughput of a station when using different strategies.

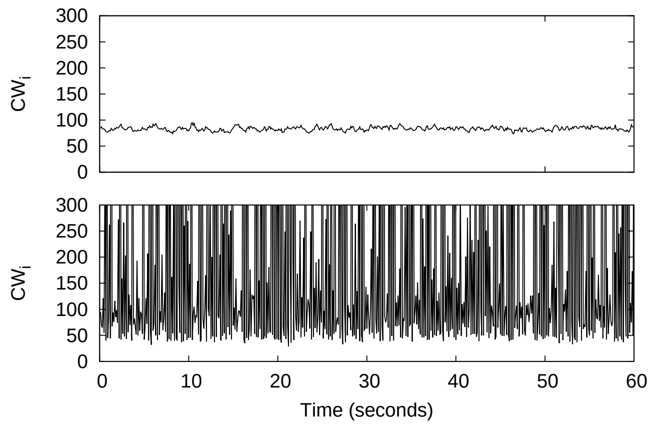


Fig. 3. System stability for different $\gamma$ settings.

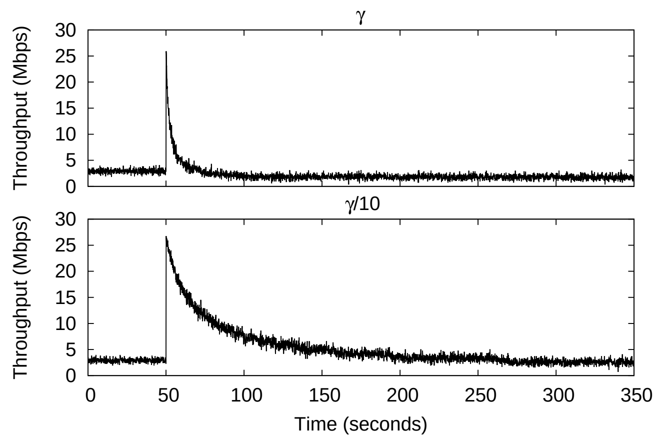


Fig. 4. Speed of reaction for different $\gamma$ settings.

### C. Throughput performance

The PAS algorithm has been designed with the goal of optimizing throughput performance when all stations run PAS. To verify this goal, we evaluate the throughput performance as a function of the number of stations $n$. As a benchmark against which to compare throughput performance, we consider a WLAN in which the $CW_i$ of all stations is statically set to the optimal value $CW_{opt}$. We observe from the results (not shown here for space reasons) that throughput performance when all stations run PAS follows very closely the optimal configuration (differences are always below 0.5%). Based on this, we conclude that the proposed algorithm is effective in providing optimal throughput performance.

### D. Stability and speed of reaction

To validate that the PAS algorithm guarantees stable behavior, we analyze the evolution over time of the parameter $CW_i$ for our $\gamma$ setting and a configuration of this parameter 10 times larger, in a WLAN with 10 stations. We observe from Figure 3 that with the proposed configuration (label "$\gamma$"), the $CW_i$ only presents minor deviations around its stable point of operation, while if a larger setting is used (label "$\gamma * 10$"), the $CW_i$ exhibits strongly unstable behavior with large oscillations.

To investigate the speed with which the system reacts against a misbehaving station, we consider a WLAN with 10 stations where initially all stations run PAS and then, after 50 seconds, one station changes its $CW_i$ to 2. Figure 4 shows the evolution of the throughput of the misbehaving station over time. We observe from the figure that with our setting (label "$\gamma$"), the system reacts quickly, and in less than a few tens of seconds the misbehaving station does no longer benefit from its misbehavior. In contrast, for a setting of this parameter 10 times smaller (label "$\gamma/10$"), the reaction is very slow and even 2 minutes afterwards, the misbehaving station is still receiving more than 1 Mbps extra throughput.

Since with a larger setting of $\gamma$ the system suffers from instability while with a smaller one it reacts too slowly, we conclude that the proposed setting provides a good tradeoff between stability and speed of reaction.

### E. Comparison against other approaches

In order to illustrate the advantages of PAS over other approaches, we compare the performance of PAS against CRISP [7] and the standard DCF configuration when there is a misbehaving station in the WLAN. In particular, we consider a WLAN with a misbehaving station that uses the $CW_i$ value that maximizes its throughput and show the throughput received by the misbehaving station and by a well-behaved station.[11] The results, depicted in Figure 5, show that PAS outperforms very substantially CRISP and DCF. Since CRISP has been designed to punish only severely misbehaving users with $CW_i = 2$ or $CW_i = 1$, a misbehaving station with a slightly larger $CW_i$ goes undetected and can gain very significant throughput, leaving well-behaved stations with low throughputs as shown in the figure. With the standard DCF configuration, a misbehaving station maximizes its gain with $CW_i = 1$, which yields zero throughput for the well-behaved stations. The results also show that both with DCF and with CRISP, a station has a strong incentive to misbehave as it

[11] To obtain the $CW_i$ that maximizes the misbehaving station's throughput, we evaluated all possible $CW_i$ values and choose that which provides the largest throughput to the misbehaving station.

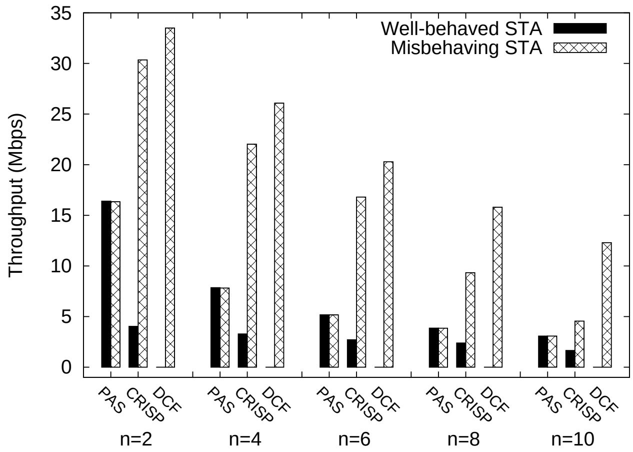


Fig. 5. Comparison against CRISP and DCF.

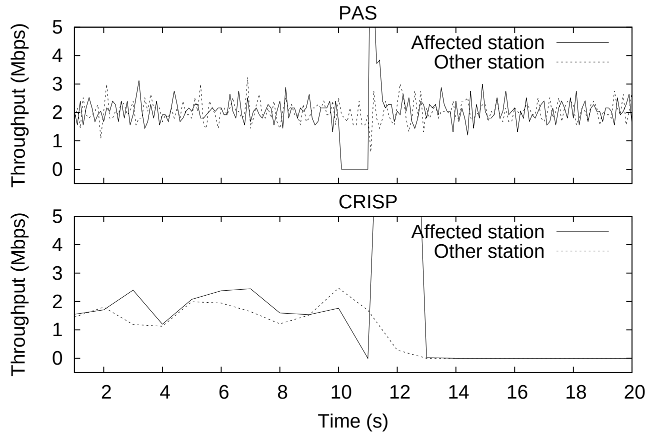


Fig. 6. Robustness to perturbations of PAS vs. CRISP.

obtains substantial gains (if it was well-behaved, it would receive the throughput given by PAS).

### F. Robustness to measurement errors

By the robustness analysis in Section IV-A, provided the measurement noise is not too large the operation of the system with measurement errors should remain close to that when error-free. To validate this result, we consider a scenario with 10 well-behaved stations where the probability that a station running PAS does not correctly decode an ongoing transmission in the channel (so inducing a measurement error) is 10%.[12] The total throughput in the network in this case is equal to 30.79 Mbps, the same throughput that we obtain for the error-free case. To further validate the effectiveness of the algorithm with measurement errors, we revisit the above scenario with one of the stations misbehaving by using contention window of $CW_{opt}/2$; in this case the misbehaving station receives a throughput of 2.58 Mbps, while it would receive a throughput of 3.08 Mbps if it ran PAS. These results confirm the algorithm's robustness to measurement errors; indeed, even with a measurement error rate as high as 10%, PAS is effective in driving the system to optimal operation as well as in preventing selfish behaviors.

### G. Robustness to perturbations

In addition to robustness against measurement errors, PAS has also been designed to achieve robustness against perturbations. Indeed, by Theorem 2, PAS is guaranteed to converge to the desired point of operation independent of the initial state. Therefore, no matter the state to which the system is brought by a perturbation, it will always tend to recover. In order to demonstrate this feature, we consider the following experiment. In a WLAN with 15 stations, all running PAS, we introduce a burst of errors that affect one of the stations for one second. Figure 6 shows the evolution of the throughput of the affected station and one of the other stations over time. The figure also shows the behavior provided by CRISP under the same conditions.

We observe from the figure that PAS quickly converges to the desired point of operation after the perturbation. Indeed, immediately after the perturbation the station that suffered the burst of errors believes that the other stations are misbehaving (as they have received a larger throughput) and so uses a smaller $CW_i$ for a period of time, resulting in a larger throughput for this station. However, after a short transient all stations return to using the optimal $CW_{opt}$. In contrast to the above, CRISP does not show a robust behavior. With CRISP, the affected station selects $CW_i = 2$ after the burst to punish the others. These react by decreasing their $CW_i$ to 2 and eventually to 1, and from this point on the stations keep punishing each other which brings the total throughput in the WLAN practically to 0. The WLAN remains in this state for the rest of the simulation run, which is 300 seconds long (only the first 20 seconds are shown in the figure).

### H. Non-saturated stations

So far we have assumed that all stations are saturated, which is the most relevant case for selfishness and the only one considered in [6], [7]. However, PAS can be easily extended to support non-saturated stations as follows: $(i)$ to avoid reacting to other stations receiving more throughput, a non-saturated station does not use the PAS algorithm to compute its configuration; $(ii)$ a saturated station only includes in the sum of (7) those stations that are receiving more throughput, thus excluding the non-saturated stations; and $(iii)$ to compute $CW_{opt}$, we take into account the sending rate of the non-saturated stations (following, e.g., [16]). To show the performance of PAS with non-saturated stations, we consider the a WLAN with 10 stations, half of them saturated and the other half sending at a rate equal to half of the saturation throughput. In this scenario, a station that misbehaves by using the $CW_i$ value that maximizes its gain obtains a throughput of 4.51 Mbps, while it would obtain 4.52 Mbps if it ran PAS. This confirms the effectiveness of the algorithm in thwarting selfish behaviors in the presence of non-saturated stations.

[12] To evaluate the impact of measurement errors, we consider that a station only suffers from errors when decoding frames addressed to other stations, i.e., when measuring their throughput, and never when receiving a frame addressed to itself.

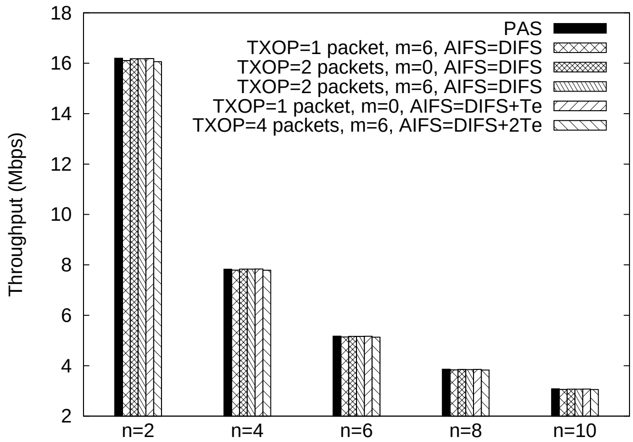


Fig. 7. Protection of PAS against selfish strategies that may use any of the 802.11e parameters.

### *I. Other 802.11e parameters*

In the experiments so far, we have only considered situations where misbehaving stations adjust the $CW_i$ parameter. However, according to the 802.11e standard, there are a number of additional parameters a user can play with, namely the backoff stage $m$, the arbitration interframe space $AIFS$ and the transmission opportunity duration $TXOP$. In order to show that a misbehaving user cannot benefit from adjusting any of these parameters, we have conducted a number of experiments in which the parameters differ from the target values given in Section III-A. For each of the settings considered for these parameters, the misbehaving station uses the $CW_{min}$ that maximizes its throughput.

The results of the above experiment are given in Figure 7 for different $n$ values. We observe that the misbehaving station never obtains any throughput gain by deviating from PAS independent of the parameters it adjusts. We conclude that PAS is effective not only against selfish adjustment of the $CW_i$ parameter but also against all of the other configurable parameters of the 802.11e standard. This is in line with Theorem 4, according to which a station cannot benefit from following a strategy different from PAS. This result is particularly relevant since previous approaches [6], [7] focus only on the $CW_i$ parameter and are not evaluated against any of the other 802.11e parameters.

## VI. Experimental Prototype

One of the advantages of PAS is that it relies on functionality readily available in standard devices and therefore can be implemented with current off-the-shelf hardware. In order to show this, we have implemented the PAS algorithm on Linux-based laptops. Our implementation is based on Linux kernel 2.6.24 laptops equipped with Atheros AR5212 cards operating in 802.11a mode and employing the MadWifi v0.9.4 driver. The PAS algorithm runs as a user-space application. In order to collect information about other stations' throughput, PAS uses a virtual device configured in promiscuous mode and monitors all frames that belong to the same BSS. With this information, it computes the $CW$ configuration by executing the algorithm described in Section III and updates the computed $CW_{min}$ and $CW_{max}$ parameters in the driver every beacon interval by means of a private `IOCTL` call.

In order to validate our implementation, we deployed a small testbed consisting of three laptops, two of them sending traffic to the third one. For the traffic generation, nodes ran the `iperf` tool using 1470 byte UDP packets. The sending rate at each station was set to 20 Mbps, ensuring that they always had a packet ready for transmission. We evaluated the following strategies: $(i)$ both stations employing the PAS algorithm to compute their configuration, $(ii)$ both stations using a fixed $CW$ configuration (for a wide range of $CW$ values), and $(iii)$ one station executing PAS and the other one using a fixed $CW$ configuration. The results of these experiments (summarized in Table I) confirm the good properties of PAS: $(i)$ when all stations are well-behaved, PAS outperforms any static $CW_i$ configuration; and $(ii)$ when one station runs PAS, the other station is better off running PAS than any other configuration.

## VII. Conclusions

With 802.11e, users may selfishly configure the parameters used by their station so as to increase their share of throughput at the expense of other users. In order to prevent such undesirable behavior, in this paper we design a novel adaptive algorithm called PAS (*selfishness-Proof Adaptive Stable*). With the PAS algorithm, upon detecting misbehavior, users react by using a more aggressive configuration of the parameters that serves to punish the misbehaving station.

A critical aspect in the design of such an adaptive algorithm is to carefully adjust the reaction against a misbehaving station to avoid the system becoming unstable. By conducting a Lyapunov stability analysis of the PAS algorithm, we show that, when all of the stations in the WLAN run PAS, the system is globally stable and converges to the desired configuration. Furthermore, by conducting an analysis of the effectiveness against selfish behavior, we show that a misbehaving station cannot benefit by following a different strategy from PAS (either with a fixed or a variable configuration). These results are confirmed by means of simulations as well as experiments.

While the focus of this paper has been on 802.11 MAC protocol, the main ideas of the paper can be generalized to other MAC protocols as long as we can compute the loss of efficiency when operating with non-optimal configurations.

## Acknowledgements

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TABLE I
EXPERIMENTAL RESULTS

| Strategy station 1 ($CW_1$) | PAS | 2 | 4 | 8 | 16 | 32 | 64 | 128 | PAS | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Strategy station 2 ($CW_2$) | PAS | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 2 | 4 | 8 | 16 | 32 | 64 | 128 |
| Throughput station 1 ($Mbps$) | 15.92 | 13.17 | 14.00 | 15.41 | 15.70 | 14.97 | 11.94 | 9.85 | 11.96 | 13.94 | 15.11 | 15.61 | 14.75 | 13.31 | 12.21 |
| Throughput station 2 ($Mbps$) | 15.94 | 13.06 | 14.01 | 15.42 | 15.66 | 14.93 | 11.86 | 9.86 | 11.96 | 14.13 | 15.06 | 15.65 | 14.62 | 12.94 | 8.10 |


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## APPENDIX

### A. Proof of Theorem 1

We proceed by establishing two useful Lemmas, and then present the proof of Theorem 1.

**Lemma 1.** *Consider the set of points* $C(p_e) = \{\hat{\boldsymbol{\tau}} : \hat{\boldsymbol{\tau}} \in [\tau_m, \tau_M]^n, \prod_{j=1}^n (1-\hat{\tau}_j) = p_e\}$, $0 \le \tau_m < \tau_M \le 1$. *Over set* $C(p_e)$*, the vector* $\hat{\boldsymbol{\tau}}$ *minimising* $\sum_{i=1}^n r_i(\hat{\boldsymbol{\tau}})$ *has all elements equal, i.e.* $\hat{\tau}_i = \hat{\tau}_j$*,* $i, j \in \{1, 2, \cdots, n\}$*.*

*Proof:* By (1), $\sum_{i=1}^n r_i = \frac{l}{T_t} \frac{1}{\prod_{j=1}^n (1+x_j)+a} \sum_{i=1}^n x_i$ where $x_i = \hat{\tau}_i/(1-\hat{\tau}_i)$ and $a = (T_e - T_t)/T_t$. Minimising $\sum_{i=1}^n r_i$ over set $C(p_e)$ then corresponds to the following optimisation

$$\min_{x_i, i=1,2,\cdots,n} \sum_{i=1}^n x_i \quad s.t. \prod_{j=1}^n (1+x_j) = \frac{1}{p_e}, x_m \le x_i \le x_M \forall i$$

which we can rewritten equivalently as

$$\min_{z_i, i=1,2,\cdots,n} \sum_{i=1}^n e^{z_i - 1} \quad s.t. \sum_{j=1}^n z_i = \log \frac{1}{p_e}, z_m \le z_i \le z_M \forall i$$

where $z_i = \log(1+x_i)$, $z_m = \log(1+x_m)$, $z_M = \log(1+x_M)$. It is enough to show that any optimum $\mathbf{z}^*$ satisfies $z_i^* = z_j^*$, $i, j \in \{1, 2, \cdots, n\}$.

The objective is convex, the equality constraint is linear and the inequality constraints convex, hence this is a convex optimisation. Since $\tau_m < \tau_M$ the Slater condition is satisfied and so strong duality holds. The Lagrangrian $L$ is

$$\sum_{i=1}^n e^{z_i - 1} - \lambda(\sum_{j=1}^n z_i - \log \frac{1}{p_e}) + \sum_{i=1}^n \underline{\theta}_i (z_m - z_i) + \sum_{i=1}^n \bar{\theta}_i (z_i - z_M)$$

and the main KKT conditions are

$$\left. \frac{\partial L}{\partial z_i} \right|_{z_i = z_i^*} = e^{z_i^* - 1} - \lambda - \underline{\theta}_i + \bar{\theta}_i = 0, \; i = 1, 2, \cdots, n \quad (13)$$

which must be satisfied by any optimal point $\mathbf{z}^*$. When $\underline{\theta}_i = 0 = \bar{\theta}_i$, $i = 1, 2, \cdots, n$ it follows from the KKT conditions that the minimum occurs when $z_i^* = z_j^*$, $i, j \in \{1, 2, \cdots, n\}$.

When $\underline{\theta}_i > 0$ for some $i$ (and so by complementary slackness $z_i^* = z_m$), we would like to show that we must have $\underline{\theta}_i > 0$ for all $i = 1, 2, \cdots, n$ (and so $z_i^* = z_m$ for all $i$). Firstly, when $\log(1/p_e) = n z_m$, since $z_i^* \ge z_m$ it follows immediately that $z_i^* = z_m$ for all $i$. Otherwise, we proceed by contradiction. Suppose that $n z_m < \log(1/p_e) \le n z_M$ and $z_i^* = z_m$ for some $i$. The elements of $\mathbf{z}^*$ are therefore not all the same value. Consider the point $y_i = \log(1/p_e)/n$, $i = 1, 2, \cdots, n$. This point satisfies the constraints

$$\sum_{i=1}^n y_i = \log \frac{1}{p_e}, \; z_m \le y_i = \log(1/p_e)/n \le z_M \quad (14)$$

and so is feasible. By the strict convexity of the exponential we have $n e^{\frac{1}{n} \sum_{i=1}^n z_i^*} = n e^{\log(1/pe)/n} < \sum_{i=1}^n e^{z_i^*}$ (with strict inequality since the elements of $\mathbf{z}^*$ are, by assumption, not all the same value). Observing that $\sum_{i=1}^n e^{y_i} = n e^{\log(1/p_e)/n}$, it follows immediately that $\sum_{i=1}^n e^{y_i - 1} < \sum_{i=1}^n e^{z_1^* - 1}$ yielding a contradiction. That is, we must either have $z_i^* = z_m$ for all $i$ or $z_i^* \ne z_m$ for all $i$. Since in the case we are analyzing we have $z_i^* = z_m$ for some $i$, this implies $z_i^* = z_m$ for all $i$. By an almost identical argument, it also follows that when $\underline{\theta}_i < 0$ for some $i$, we must have $z_i^* = z_M$ for all $i$. ■

**Lemma 2.** *Consider the set of points $C = \{\hat{\boldsymbol{\tau}} : \hat{\boldsymbol{\tau}} \in [\tau_m, \tau_M]^n\}$, $0 \le \tau_m \le \tau_M \le 1$. Over set C, the vector $\hat{\boldsymbol{\tau}}$ minimising $\sum_{i=1}^n r_i(\hat{\boldsymbol{\tau}})$ satisfies either $\hat{\tau}_i = \tau_m$ for all i or $\hat{\tau}_i = \tau_M$ for all i.*

*Proof:* If $\tau_m = \tau_M$ the result follows trivially. Suppose therefore $\tau_m < \tau_M$. By Lemma 1, minimising $\sum_{i=1}^n r_i$ s.t. $\tau_m \le \hat{\tau}_i \le \tau_M$, $i = 1, 2, \cdots, n$ is equivalent to finding a $\tau^*$ solving

$$\min_{\tau_m \le \hat{\tau} \le \tau_M} \frac{n\hat{\tau}}{1-\hat{\tau}} \frac{(1-\hat{\tau})^n}{T_t + (T_e - T_t)(1-\hat{\tau})^n} l \tag{15}$$

and setting $\hat{\tau}_i = \tau^*$, $i = 1, 2, \cdots, n$. Taking logs and letting $x = \frac{\hat{\tau}}{1-\hat{\tau}}$, $\tilde{x} = \log x$ this optimisation can be rewritten as $\min_{\tilde{x}_m \le \tilde{x} \le \tilde{x}_M} \tilde{r}(\tilde{x})$ with $\tilde{r}(\tilde{x}) = \tilde{x} - \log((1 + e^{\tilde{x}})^n + a) + \log \frac{l}{T_t}$, $a = (T_e - T_t)/T_t$, $\tilde{x}_m = \log \frac{\tau_m}{1-\tau_m}$, $\tilde{x}_M = \log \frac{\tau_M}{1-\tau_M}$. Importantly, the objective function $\tilde{r}(\cdot)$ is concave in $\tilde{x}$, since $(i)$ the first term is linear; $(ii)$ expanding the $(1+e^{\tilde{x}})^n$ term, it can be verified that the second term is convex [24]; and $(iii)$ the third term is constant. Hence, for any $\tilde{x} = \alpha\tilde{x}_m + (1-\alpha)\tilde{x}_M$, $0 \le \alpha \le 1$ lying in the interval $[\tilde{x}_m, \tilde{x}_M]$ we have $\tilde{r}(\tilde{x}) \ge \alpha\tilde{r}(\tilde{x}_m) + (1-\alpha)\tilde{r}(\tilde{x}_M)$. It follows immediately that the minimum of $\tilde{r}(\tilde{x})$ over interval $[\tilde{x}_m, \tilde{x}_M]$ must be located at one of the boundary points. ■

*Proof of Theorem 1:* By Lemma 2, $r_{opt} - \frac{1}{n}\sum_j r_j$ is minimized either when $\hat{\tau}_i = \tau_m\ \forall i$ or $\hat{\tau}_i = \tau_M\ \forall i$. For $\hat{\tau}_i = \tau_m\ \forall i$, we have

$$\frac{1}{n}\sum_j r_j = \frac{\tau_m(1-\tau_m)^{n-1} l}{T_{s,m}} \ge \frac{\tau_m(1-\tau_{opt})^{n-1} l}{T_{opt}} \tag{16}$$

where $T_{s,m}$ and $T_{opt}$ are the values of $T_s$ when $\tau_i = \tau_m\ \forall i$ and $\tau_i = \tau_{opt}\ \forall i$, respectively. From the above,

$$r_{opt} - \frac{1}{n}\sum_j r_j \le (\tau_{opt} - \tau_m)\frac{(1-\tau_{opt})^{n-1} l}{T_{opt}} \le \Delta\frac{r_{opt}}{\tau_{opt}} \le \Delta\frac{r_{opt}}{\tau_{opt}(1-\tau_{opt})}$$

from which we have that (3) holds for this case.

We next address the case $\hat{\tau}_i = \tau_M\ \forall i$. If $\tau_M > 2\tau_{opt}$, it is easy to see that (3) holds, as in this case the denominator of (3) is larger than $\tau_{opt}$ and the numerator is smaller than $r_{opt}$. To prove that (3) also holds for $\tau_M \le 2\tau_{opt}$, we proceed as follows. Let $r(\tau)$ be the throughput of a station as a function of $\tau$ when $\hat{\tau}_i = \tau$ for all $i$. Then, by expressing $r(\tau)$ as the integral of its derivative,

$$r_{opt} - r(\tau_M) = \int_{\tau_M}^{\tau_{opt}} \frac{dr(\tau)}{d\tau} d\tau \tag{17}$$

From $r(\tau) = l\tau(1-\tau)^{n-1}/T_s$,

$$\frac{dr(\tau)}{d\tau} = \frac{l(1-\tau)^{n-2}(T_s - n\tau T_t)}{T_s^2}$$

We next show that the above derivative is negative in the interval $\tau \in [\tau_{opt}, \tau_M]$. The sign of the derivative depends on that of the term $T_s - n\tau T_t$. Since the throughput is maximized at $\tau_{opt}$ and $n > 1$, the derivative at $\tau = \tau_{opt}$ is 0 (when the number of stations $n > 1$ the optimum attempt probability must lie in the interior of $[0, 1]^n$), and so $T_s - n\tau T_t$=0. The derivative of $T_s - n\tau T_t$ is $n(1-\tau)^{n-1}(T_t - T_e) - nT_t$, which is negative for $\tau \in [0, 1]$. Thus, $T_s - n\tau T_t$ equals 0 at $\tau = \tau_{opt}$ and decreases afterwards, which implies that $T_s - n\tau T_t < 0$ for $\tau > \tau_{opt}$. With this, (17) can be rewritten as

$$r_{opt} - r(\tau_M) = -\int_{\tau_M}^{\tau_{opt}} \left|\frac{dr(\tau)}{d\tau}\right| d\tau \tag{18}$$

which can be bounded as follows:

$$r_{opt} - r(\tau_M) \le -\int_{\tau_M}^{\tau_{opt}} \left|\frac{dr(\tau)}{d\tau}\right|_{\max} d\tau = \left|\frac{dr(\tau)}{d\tau}\right|_{\max} (\tau_M - \tau_{opt}) \tag{19}$$

where $|dr(\tau)/d\tau|_{\max}$ is an upper bound for the absolute value the derivative in the interval $\tau \in [\tau_{opt}, \tau_M]$

To find $|dr(\tau)/d\tau|_{\max}$, we proceed as follows. Given that $\tau \in (\tau_{opt}, \tau_M]$ and $\tau_M \le \min(2\tau_{opt}, 1)$, we want to evaluate $dr(\tau)/d\tau$ at $\tau = K\tau_{opt}$ for $1 < K \le \min(2, 1/\tau_{opt})$, which yields

$$\frac{\partial r(\tau)}{\partial \tau} = \frac{l(1-K\tau_{opt})^{n-2}(T_K - nK\tau_{opt}T_t)}{T_K^2} \tag{20}$$

where $T_K$ is the value of $T_s$ for $\tau_i = K\tau_{opt}\ \forall i$. Note that, for $K > 1$, we have $T_K > T_{opt}$ and $T_K - nK\tau_{opt}T_t < 0$ (the latter holds since we have earlier shown that the term $T_s - n\tau T_t$ is negative for $\tau > \tau_{opt}$). With this, the absolute value of $dr(\tau)/d\tau$ can be bounded by

$$\left|\frac{dr(\tau)}{d\tau}\right| \le \frac{l(1-\tau_{opt})^{n-2}(nK\tau_{opt}T_t - T_{opt})}{T_{opt}^2} \tag{21}$$

Before, we have shown that the term $T_s - n\tau T_t$ is equal to 0 at $\tau = \tau_{opt}$, i.e., $T_{opt} - n\tau_{opt}T_t = 0$. Adding this term to $nK\tau_{opt}T_t - T_{opt}$ gives $(K-1)n\tau_{opt}T_t$. Furthermore, since $T_{opt} = n\tau_{opt}T_t$, this can be expressed as $(K-1)T_{opt}$. Combining this with the above equation yields:

$$\left|\frac{dr(\tau)}{d\tau}\right| \le \frac{l(1-\tau_{opt})^{n-2}(K-1)}{T_{opt}} \le \frac{l(1-\tau_{opt})^{n-2}}{T_{opt}} \tag{22}$$

Finally, combining the above bound on the maximum value of the derivative with (19) leads to:

$$r_{opt} - \frac{1}{n}\sum_j r_j \le \frac{l(1-\tau_{opt})^{n-2}}{T_{opt}}(\tau_M - \tau_{opt}) \le \frac{r_{opt}}{\tau_{opt}(1-\tau_{opt})}\Delta$$

from which (3) also holds for this case. ■

### B. Proof of Theorem 2

Once again, we proceed by establishing a number of intermediate Lemmas, and then present the proof of Theorem 2.

**Lemma 3.**

$$(i)\ \sum_{j\ne i}(r_j(\hat{\boldsymbol{\tau}}) - r_i(\hat{\boldsymbol{\tau}})) \le \frac{(n-1)\,l}{T_m}(\hat{\tau}_M - \hat{\tau}_i)\left(1 - \frac{\tau_{opt}}{2}\right)^{n-2}$$

$$(ii)\ \sum_{j\ne i}(r_j(\hat{\boldsymbol{\tau}}) - r_i(\hat{\boldsymbol{\tau}})) \ge \frac{(n-1)\,l}{T_m}(\hat{\tau}_m - \hat{\tau}_i)\left(1 - \frac{\tau_{opt}}{2}\right)^{n-2}$$

*with $n \ge 2$, $\hat{\tau}_M = \max_{i\in\{1,\cdots,n\}} \hat{\tau}_i$, $\hat{\tau}_m = \min_{i\in\{1,\cdots,n\}} \hat{\tau}_i$, $\frac{\tau_{opt}}{2} \le \hat{\tau}_k \le 1$.*

*Proof:* $(i)$ Let $r_M$ be the throughput of station $M$. Since $r_i \le r_M$ we have $\sum_{j\neq i}(r_j - r_i) \le \sum_{j\neq i}(r_M - r_i) = (n-1)(r_M - r_i)$. Substituting from (1) and rearranging we have

$$(n-1)(r_M - r_i) = \frac{(n-1)\,l}{T_s}(\hat{\tau}_M - \hat{\tau}_i)\prod_{k\neq i,M}(1-\hat{\tau}_k)$$
$$\overset{(a)}{\le} \frac{(n-1)\,l}{T_m}(\hat{\tau}_M - \hat{\tau}_i)\left(1-\frac{\tau_{opt}}{2}\right)^{n-2}$$

where $(a)$ follows from the fact that $\frac{\tau_{opt}}{2} \le \hat{\tau}_k \le 1$ and $T_s \ge T_m$ (the latter holds since $\hat{\tau}_j \ge \frac{\tau_{opt}}{2}$).

$(ii)$ Since $r_i \ge r_m$ we have $\sum_{j\neq i}(r_j - r_i) \ge (n-1)(r_m - r_i)$. The second part of the result now follows using an identical argument to $(i)$. ■

**Lemma 4.** *(i) $\hat{\tau}_m - \hat{\tau}_i \ge \tau_m - \tau_i$ and (ii) $\hat{\tau}_M - \hat{\tau}_i \le \tau_M - \tau_i$, where $\tau_m = \min_{i\in\{1,\cdots,n\}}\tau_i$, $\tau_M = \max_{i\in\{1,\cdots,n\}}\tau_i$, $\hat{\tau}_j = \max\{\frac{\tau_{opt}}{2}, \tau_j\}$.*

*Proof:* (i) When $\tau_i \ge \frac{\tau_{opt}}{2}$ then $\hat{\tau}_i = \tau_i$. Since $\hat{\tau}_m \ge \tau_m$ it follows that $\hat{\tau}_m - \hat{\tau}_i \ge \tau_m - \tau_i$. When $\tau_i < \frac{\tau_{opt}}{2}$, then $\hat{\tau}_m = \hat{\tau}_i = \frac{\tau_{opt}}{2}$, and hence $\hat{\tau}_m - \hat{\tau}_i = 0$, while $\tau_m - \tau_i \le 0$. (ii) When $\tau_i \ge \frac{\tau_{opt}}{2}$ then $\hat{\tau}_i = \tau_i$, $\hat{\tau}_M = \tau_M$ and $\hat{\tau}_M - \hat{\tau}_i = \tau_M - \tau_i$. When $\tau_i < \frac{\tau_{opt}}{2}$ we have two cases: (a) if $\tau_M < \frac{\tau_{opt}}{2}$ then $\hat{\tau}_M - \hat{\tau}_i = 0 \le \tau_M - \tau_i$; (b) if $\tau_M \ge \frac{\tau_{opt}}{2}$ then $\hat{\tau}_M - \hat{\tau}_i = \tau_M - \hat{\tau}_i \le \tau_M - \tau_i$ since $\tau_i \le \hat{\tau}_i$. ■

**Lemma 5.** *When $\gamma < \gamma_{max} = \left(\frac{nl}{T_m}\left(1-\frac{\tau_{opt}}{2}\right)^{n-2}\right)^{-1}$, $D(\hat{\boldsymbol{\tau}}) < 0$ and $n \ge 2$ then under update (5),*

$$\tau_i(t+1) < \tau_M(t+1) \text{ if } \tau_i(t) < \tau_M(t) \tag{23}$$

*where $M := \arg\max_i \tau_i(t)$.*

*Proof:* It is sufficient to show that

$$\tau_i + \gamma\left(\sum_{j\neq i}(r_j - r_i) - F_i\right) < \tau_M + \gamma\left(\sum_{j'\neq M}(r'_j - r_M) - F_M\right)$$

where $F_M$ is the value of $F_i$ for station $M$ (in the equation we have dropped the $t$ arguments from all quantities to streamline notation). Since $F_i = F_M$, $i = 1,\cdots,n$ when $D < 0$ this simplifies to $\gamma n(r_M - r_i) < \tau_M - \tau_i$. Substituting from (1) we obtain

$$\gamma\frac{nl}{T_s}(\hat{\tau}_M - \hat{\tau}_i)\prod_{j\neq i,M}(1-\hat{\tau}_j) < \tau_M - \tau_i \tag{24}$$

By Lemma 4, $\hat{\tau}_M - \hat{\tau}_i \le \tau_M - \tau_i$ and a sufficient condition for (24) is $\gamma\frac{nl}{T_s}\prod_{j\neq i,M}(1-\hat{\tau}_j) < 1$. Since $\hat{\tau}_j \ge \frac{\tau_{opt}}{2}$, this holds when $\gamma < \gamma_{max}$. ■

**Lemma 6.** *Under the conditions of Lemma 5, $\tau_M(t+1) \le \tau_M(t)$, with equality only when $\tau_j = \tau_{opt}$, $j = 1,\cdots,n$.*

*Proof:* It is sufficient to show that

$$\tau_M + \gamma\left(\sum_i r_i - nr_M - \frac{nr_{opt} - \sum_i r_i}{n-1}\right) \le \tau_M \tag{25}$$

with equality only when $\tau_j = \tau_{opt}$, $j = 1,\cdots,n$. When $\tau_j = \tau_{opt}$, equality holds. Assume now that $\tau_j \ne \tau_{opt}$ for some $j$. Since $\gamma > 0$, the above condition is satisfied when

$$r_M + \sum_{i\neq M}(r_i - r_M) - r_{opt} < 0 \tag{26}$$

If $\hat{\tau}_M = 1$, then since $n > 1$ and $r_i = 0$, $i = 1,\cdots,n$, (26) is satisfied. Suppose therefore $\hat{\tau}_M < 1$ and define function $G = r_M + \sum_{j\neq M}(r_j - r_M) - r_{opt}$. The partial derivative of $G$ with respect to $\hat{\tau}_i$ is given by $\frac{\partial G}{\partial\hat{\tau}_i} = \frac{\partial r_i}{\partial\hat{\tau}_i} + \sum_{j\neq i,M}\frac{\partial(r_j - r_M)}{\partial\hat{\tau}_i}$. It can be verified that $\partial r_i/\partial\hat{\tau}_i > 0$ (since $\hat{\tau}_j \le \hat{\tau}_M < 1$). Also,

$$\frac{\partial(r_j - r_M)}{\partial\hat{\tau}_i} = -\frac{l}{T_s^2}\left(\frac{\hat{\tau}_j}{1-\hat{\tau}_j} - \frac{\hat{\tau}_M}{1-\hat{\tau}_M}\right)$$
$$\times\left(\prod_{k\neq i}(1-\hat{\tau}_k)T_s + \prod_k(1-\hat{\tau}_k)\frac{\partial T_s}{\partial\hat{\tau}_i}\right) \ge 0$$

since $\partial T_s/\partial\hat{\tau}_i > 0$ and $\frac{\tau}{1-\tau}$ is monotonically increasing in $\tau$. Hence $\partial G/\partial\hat{\tau}_i > 0$, which implies that $G$ takes a maximum for the largest possible value of $\hat{\tau}_i$, for all $i \ne M$. Since $\hat{\tau}_i \le \hat{\tau}_M$, this means that $G$ is maximized when $\hat{\tau}_i = \hat{\tau}_M$ for all $i$. In this case, (26) becomes $r_M(\hat{\boldsymbol{\tau}}_M) - r_{opt} < 0$, where $\hat{\boldsymbol{\tau}}_M = \{\hat{\tau}_M,\ldots,\hat{\tau}_M\}$. Since $r_{opt}$ is the maximum throughput when all stations use the same transmission attempt probability, $r_M(\hat{\boldsymbol{\tau}}_M) - r_{opt} = 0$ only if $\hat{\tau}_M = \tau_{opt}$. But by assumption $\tau_M \ne \tau_{opt}$ and so we must have $r_M(\hat{\boldsymbol{\tau}}_M) - r_{opt} < 0$. ■

*Proof of Theorem 2:* To establish global asymptotically stability we show that $\|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty < \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty$ unless $\boldsymbol{\tau}(t) = \boldsymbol{\tau}_{opt}$. By definition, $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \max(|\tau_M(t) - \tau_{opt}|, |\tau_m(t) - \tau_{opt}|)$, where $\tau_M$ and $\tau_m$ are the maximum and minimum values of the elements of vector $\boldsymbol{\tau}$ respectively. We proceed in a case-by-case fashion.

**Case 1**: $\tau_M(t) > \tau_{opt}$, $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_M(t) - \tau_{opt}$. For $\|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty < \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty$ we require

$$|\tau_i(t+1) - \tau_{opt}| < \tau_M(t) - \tau_{opt}, \qquad i = 1,\cdots,n \tag{27}$$

Substituting from (5) and (7), (27) is satisfied provided:

$$\tau_i + \gamma\left(\sum_{j\neq i}(r_j - r_i) - F_i\right) - \tau_{opt} < \tau_M - \tau_{opt} \tag{28}$$

$$\tau_i + \gamma\left(\sum_{j\neq i}(r_j - r_i) - F_i\right) - \tau_{opt} > \tau_{opt} - \tau_M \tag{29}$$

where the dependency on $t$ has been omitted to simplify notation.

**Case 1a** ($F_i = -D/[2(n-1)]$**:** $\tau_i \le \tau_{opt}$**,** $nr_{opt} \ge \sum_j r_j$). Using Lemma 3 plus Theorem 1 with $\Delta = \tau_M - \tau_{opt}$, (28) is satisfied provided

$$\gamma\left(\frac{(n-1)\,l}{T_m}(\hat{\tau}_M - \hat{\tau}_i)\left(1-\frac{\tau_{opt}}{2}\right)^{n-2} + \frac{n\rho}{2(n-1)}(\tau_M - \tau_{opt})\right) < \tau_M - \tau_i \tag{30}$$

By Lemma 4, $\hat{\tau}_M - \hat{\tau}_i \le \tau_M - \tau_i$. Also, by assumption $\tau_i \le \tau_{opt}$ and so $\tau_M - \tau_i > 0$. It follows that (30) (and so (28)) is satisfied provided

$$\gamma < \left(\frac{(n-1)\,l}{T_m}\left(1-\frac{\tau_{opt}}{2}\right)^{n-2} + \frac{n\rho}{2(n-1)}\right)^{-1} \tag{31}$$

Given that $\rho = l(1-\tau_{opt})^{n-2}/T_{opt} < l(1-\tau_{opt}/2)^{n-2}/T_m$, the above is satisfied provided

$$\gamma < \left(\frac{nl}{T_m}\left(1-\frac{\tau_{opt}}{2}\right)^{n-2}\right)^{-1} \tag{32}$$

Since $-F_i = \frac{D}{2(n-1)} \geq 0$, (29) is satisfied provided

$$\gamma \sum_{j \neq i} (r_j - r_i) > 2\tau_{opt} - \tau_M - \tau_i \quad (33)$$

By assumption $|\tau_M - \tau_{opt}| \geq |\tau_m - \tau_{opt}|$ and so $\tau_m \geq 2\tau_{opt} - \tau_M$. Also, by assumption $\tau_i \leq \tau_{opt} < \tau_M$ and so $\tau_m < \tau_M$, $r_m < r_M$. If $\tau_i = 2\tau_{opt} - \tau_M$ then $r_i = r_m$ and (33) holds provided $\gamma > 0$ (the RHS equals 0 while the LHS is lower bounded by $\gamma(r_M - r_m) > 0$). Otherwise, we have $\tau_i > 2\tau_{opt} - \tau_M$. By Lemma 3, (33) is satisfied provided

$$\gamma \frac{(n-1)\, l}{T_m} (\hat{\tau}_m - \hat{\tau}_i) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} > 2\tau_{opt} - \tau_M - \tau_i$$

By Lemma 4, $\hat{\tau}_m - \hat{\tau}_i \geq \tau_m - \tau_i$. And $\tau_m - \tau_i \geq 2\tau_{opt} - \tau_M - \tau_i$. Hence, (33) is satisfied provided

$$\gamma < \left( \frac{(n-1)\, l}{T_m} \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} \right)^{-1} \quad (34)$$

**Case 1b** ($F_i = D/[2(n-1)]$**:** $\tau_i > \tau_{opt}$**,** $nr_{opt} \geq \sum_j r_j$). From $nr_{opt} \geq \sum_j r_j$, it holds that $F_i = \frac{D}{2(n-1)} = \frac{n}{2(n-1)} \left( r_{opt} - \frac{1}{n} \sum_j r_j \right) \geq 0$. If $\tau_i = \tau_M$ then either $(i)$ $\tau_j = \tau_i$ for all $j$, in which case $F_i > 0$ and $\sum_{j \neq i} (r_j - r_i) = 0$, or $(ii)$ $\tau_j < \tau_i$ for some $j$, in which case $\sum_{j \neq i} (r_j - r_i) < 0$ and (as mentioned above) $F_i \geq 0$. In both cases, (28) is satisfied. Otherwise, we have $\tau_i < \tau_M$. Since $F_i \geq 0$, (28) is satisfied provided $\gamma \sum_{j \neq i} (r_j - r_i) < \tau_M - \tau_i$. By Lemma 3, this holds provided

$$\gamma \frac{(n-1)\, l}{T_m} (\hat{\tau}_M - \hat{\tau}_i) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} < \tau_M - \tau_i \quad (35)$$

By assumption, $\tau_i > \tau_{opt}$ and so $\hat{\tau}_i = \tau_i$, $\hat{\tau}_M = \tau_M$. Also, $\tau_M - \tau_i > 0$. Hence, (35) holds when $\gamma$ satisfies (34).

Using Lemma 3 plus Theorem 1 with $\Delta = \tau_M - \tau_{opt}$, (29) is satisfied provided

$$\gamma \left( \frac{(n-1)\, l}{T_m} (\hat{\tau}_i - \hat{\tau}_m) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} + \frac{n\rho}{2(n-1)} (\tau_M - \tau_{opt}) \right) < \tau_i - (2\tau_{opt} - \tau_M) \quad (36)$$

By assumption, $\tau_m \geq 2\tau_{opt} - \tau_M$ and so by Lemma 4, $\hat{\tau}_i - \hat{\tau}_m \leq \tau_i - (2\tau_{opt} - \tau_M)$. Also, by assumption $\tau_i > \tau_{opt}$ and so $\tau_i - (2\tau_{opt} - \tau_M) \geq \tau_M - \tau_{opt}$. It then follows that (36) (and so (29)) is satisfied when $\gamma$ satisfies (31).

**Case 1c** ($F_i = D/(n-1)$**:** $nr_{opt} < \sum_j r_j$). By Lemmas 5 and 6, $\tau_i(t+1) < \tau_M(t+1) < \tau_M(t)$ and so (28) is satisfied (observe that the LHS of (28) is $\tau_i(t+1) - \tau_{opt}$). Since $F_i \leq 0$, (29) is satisfied provided $\gamma \sum_{j \neq i} (r_j - r_i) > 2\tau_{opt} - \tau_M - \tau_i$. By Lemma 3, this holds provided

$$\gamma \frac{(n-1)\, l}{T_m} (\hat{\tau}_i - \hat{\tau}_m) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} < \tau_i - (2\tau_{opt} - \tau_M)$$

By assumption, $\tau_m \geq 2\tau_{opt} - \tau_M$ and by Lemma 4, $\hat{\tau}_i - \hat{\tau}_m \leq \tau_i - (2\tau_{opt} - \tau_M)$. Hence, the above holds (and so (29) is satisfied) when $\gamma$ satisfies (34).

**Case 2**: $\tau_M(t) < \tau_{opt}$ or $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty \neq \tau_M(t) - \tau_{opt}$. In this case, it necessarily holds that $\tau_m(t) < \tau_{opt}$ and $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_{opt} - \tau_m(t)$. For $\|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty < \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty$ we require

$$\tau_i + \gamma \left( \sum_{j \neq i} (r_j - r_i) - F_i \right) - \tau_{opt} < \tau_{opt} - \tau_m \quad (37)$$

$$\tau_i + \gamma \left( \sum_{j \neq i} (r_j - r_i) - F_i \right) - \tau_{opt} > \tau_m - \tau_{opt} \quad (38)$$

**Case 2a** ($F_i = -D/[2(n-1)]$**:** $\tau_i \leq \tau_{opt}$**,** $nr_{opt} \geq \sum_j r_j$). By Lemma 3 and Theorem 1 with $\Delta = \tau_{opt} - \tau_m$ condition (37) is satisfied provided

$$\gamma \left( \frac{(n-1)\, l}{T_m} (\hat{\tau}_M - \hat{\tau}_i) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} + \frac{n\rho}{2(n-1)} (\tau_{opt} - \tau_m) \right) < 2\tau_{opt} - \tau_m - \tau_i \quad (39)$$

Since $\tau_i \leq \tau_{opt}$ then $2\tau_{opt} - \tau_m - \tau_i \geq \tau_{opt} - \tau_m$. By Lemma 4, $\hat{\tau}_M - \hat{\tau}_i \leq \tau_M - \tau_i$. When $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty \neq \tau_M(t) - \tau_{opt}$, then $|\tau_M - \tau_{opt}| < |\tau_m - \tau_{opt}|$ and so $\tau_m < 2\tau_{opt} - \tau_M$ i.e. $\tau_M < 2\tau_{opt} - \tau_m$. Hence, $\tau_M - \tau_i < 2\tau_{opt} - \tau_m - \tau_i$. When $\tau_M < \tau_{opt}$ then $\tau_M - \tau_i \leq \tau_{opt} - \tau_i = 2\tau_{opt} - \tau_m - \tau_i - (\tau_{opt} - \tau_m) \leq 2\tau_{opt} - \tau_m - \tau_i$ where the last inequality follows from the fact that $\tau_m \leq \tau_i \leq \tau_{opt}$. It follows that (39) holds when $\gamma$ satisfies (31).

If $\tau_i = \tau_m$ then (38) is satisfied since $r_j - r_m \geq 0$ and $F_i > 0$ (unless $\tau_i = \tau_{opt}$ $\forall i$). Otherwise, if $\tau_i > \tau_m$ then since $F_i \leq 0$, (38) is satisfied provided $\gamma \sum_{j \neq i} (r_j - r_i) > \tau_m - \tau_i$. By Lemmas 3 and 4, this holds when $\gamma$ satisfies (34).

**Case 2b** ($F_i = D/[2(n-1)]$**:** $\tau_i > \tau_{opt}$**,** $nr_{opt} \geq \sum_j r_j$). Note that $\tau_M \geq \tau_i > \tau_{opt}$. Hence, to be in case 2 we must have $|\tau_M - \tau_{opt}| < |\tau_m - \tau_{opt}|$ and so $\tau_m < 2\tau_{opt} - \tau_M$ i.e. $\tau_M < 2\tau_{opt} - \tau_m$. If $\tau_i = \tau_M$ then (37) is satisfied since the LHS non-negative while the RHS is positive. Otherwise, if $\tau_i < \tau_M$ then since $F_i \geq 0$, (37) is satisfied provided $\gamma \sum_{j \neq i} (r_j - r_i) < 2\tau_{opt} - \tau_m - \tau_i$. By Lemma 3 this holds when

$$\frac{(n-1)\, l}{T_m} (\hat{\tau}_M - \hat{\tau}_i) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} < 2\tau_{opt} - \tau_m - \tau_i \quad (40)$$

As already noted, $\tau_M < 2\tau_{opt} - \tau_m$ and $\tau_M \geq \tau_i > \tau_{opt}$. Hence, $\hat{\tau}_M - \hat{\tau}_i = \tau_M - \tau_i < 2\tau_{opt} - \tau_m - \tau_i$. It follows that (40) is satisfied when $\gamma$ satisfies (34).

Using Lemma 3 and Theorem 1 with $\Delta = \tau_{opt} - \tau_m$, condition (38) is satisfied provided

$$\gamma \frac{(n-1)\, l}{T_m} (\hat{\tau}_m - \hat{\tau}_i) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} + \gamma \frac{n\rho}{2(n-1)} (\tau_m - \tau_{opt}) > \tau_m - \tau_i$$

Since $\tau_i > \tau_{opt}$, $\hat{\tau}_m - \hat{\tau}_i \geq \tau_m - \tau_i$. Also, $\tau_m - \tau_{opt} \geq \tau_m - \tau_i$. It follows that the above holds when $\gamma$ satisfies (31).

**Case 2c** ($F_i = D/(n-1)$**:** $nr_{opt} < \sum_j r_j$). Observe that the LHS of (37) is $\tau_i(t+1) - \tau_{opt}$. By Lemmas 5 and 6, $\tau_i(t+1) < \tau_M(t+1) < \tau_M(t)$. By assumption, $\tau_M(t) - \tau_{opt} < \tau_{opt} - \tau_m$. Therefore, (37) is satisfied.

If $\tau_i = \tau_m$ then (38) is satisfied. Otherwise, if $\tau_i > \tau_m$ then since $F_i \leq 0$ condition (38) is satisfied provided $\gamma \sum_{j \neq i} (r_j - r_i) > \tau_m - \tau_i$. By Lemma 3 this holds provided $\gamma \frac{(n-1)l}{T_m} (\hat{\tau}_i - \hat{\tau}_m) \left(1 - \frac{\tau_{opt}}{2}\right)^{n-2} < \tau_i - \tau_m$. By Lemma 4 this holds when $\gamma$ satisfies (34).

Finally, we take for $\gamma_{max}$ the smallest of the bounds given by (32) and (34). ∎

### C. Sketch of the proof of Theorem 3

We proceed on a case-by-base basis. For each case we show that, either the case is not possible or there is an upper bound on $f$ that belongs to $\mathcal{K}$. Thus, by taking $f$ equal to the minimum of all upper bounds, the theorem is proved. This holds for $\boldsymbol{\tau} \in [\tau_{opt}/2, 1]^n$ and thus $\|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty \le \tau_{opt}/2$.

Let $i$ be such that $|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty = |\tau_i(t+1) - \tau_{opt}|$. Let $\tau_i$, $\tau_m$ and $\tau_M$ denote $\tau_i(t)$, $\tau_m(t)$ and $\tau_M(t)$, respectively.

**Case 1:** ($\tau_M > \tau_i > \tau_{opt}$ & $\tau_i(t+1) > \tau_{opt}$ or $\tau_m < \tau_i < \tau_{opt}$ & $\tau_i(t+1) < \tau_{opt}$). It can be seen that if $\tau_j(t) < \tau_i(t) < \tau_{opt}$, then $\tau_j(t+1) < \tau_i(t+1)$. Similarly, if $\tau_j(t) > \tau_i(t) > \tau_{opt}$, then $\tau_j(t+1) > \tau_i(t+1)$. Thus, this case is not possible.

**Case 2:** ($\tau_i < \tau_{opt}$ & $\tau_i(t+1) > \tau_{opt}$ or $\tau_i > \tau_{opt}$ & $\tau_i(t+1) < \tau_{opt}$). From the proof of Theorem 2 it can be seen that $|\tau_{opt} - \tau_i(t+1)| - \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty \le -(1-\gamma/\gamma_{max})\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty$.

From the above two cases, in the following we only need to look at $\tau_i = \tau_m$ or $\tau_M$, and show that $|\tau_i(t+1) - \tau_i|$ is an increasing function of $|\boldsymbol{\tau}(t+1) - \boldsymbol{\tau}_{opt}\|_\infty$.

**Case 3:** ($D(t) \ge 0$)

*Case 3a:* ($\tau_i = \tau_M$ & $\tau_i(t+1) > \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_M - \tau_{opt}$). In this case $\tau_M - \tau_M(t+1)$ is proportional to $G = -nr_M - (2n-1)\sum_{i \ne M}(r_i - r_M) + nr_{opt}$. Following a similar reasoning to the proof of Lemma 6 it can be seen that $G$ is minimized for $\boldsymbol{\tau} = \boldsymbol{\tau_M}$, where $\boldsymbol{\tau_M} = \{\tau_M, \ldots, \tau_M\}$. From this, $\tau_M - \tau_M(t+1) \ge \frac{\gamma}{n-1}(r_{opt} - r_M(\boldsymbol{\tau_M}))$. Note that this is an increasing function of $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty$ given that $\tau_M = \tau_{opt} + \|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty$.

*Case 3b:* ($\tau_i = \tau_M$ & $\tau_i(t+1) > \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_{opt} - \tau_m$). In this case $\tau_M - \tau_M(t+1) = \frac{\gamma}{2(n-1)}(nr_{opt} - nr_M + (2n-1)\sum_{i \ne M}(r_i - r_m) \ge \frac{\gamma}{2(n-1)}(nr_{opt} - nr_M + n(r_M - r_m)) \ge \frac{\gamma}{2(n-1)}(nr_{opt} - nr_m) \ge \frac{\gamma n}{2(n-1)}(r_{opt} - r_m(\boldsymbol{\tau_m}))$.

*Case 3c:* ($\tau_i = \tau_m$ & $\tau_i(t+1) < \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_{opt} - \tau_m$). In this case $\tau_m(t+1) - \tau_m = \gamma(\sum_j r_j - nr_m + (nr_{opt} - \sum_j r_j)/(2(n-1))) \ge \gamma\frac{n}{2(n-1)}(r_{opt} - r_m) \ge \gamma\frac{n}{2(n-1)}(r_{opt} - r_m(\boldsymbol{\tau_m}))$.

*Case 3d:* ($\tau_i = \tau_m$ & $\tau_i(t+1) < \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_M - \tau_{opt}$). It can be seen that an upper bound for $r_m$ in this case is obtained by setting $\tau_j = \tau_{opt}$ $\forall j \ne M$ and $T_s = T_{opt}$. Thus we have $\tau_m(t+1) - \tau_m \ge \frac{\gamma n l}{2(n-1)T_{opt}}\tau_{opt}(1-\tau_{opt})^{n-1}(\tau_M - \tau_{opt})$.

**Case 4:** ($D(t) < 0$)

*Case 4a:* ($\tau_i = \tau_M$ & $\tau_i(t+1) > \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_M - \tau_{opt}$). From the proof of Lemma 6 we have that $\tau_M - \tau_M(t+1) \ge \frac{\gamma}{n-1}(r_{opt} - r_M(\boldsymbol{\tau_M}))$.

*Case 4b:* ($\tau_i = \tau_M$ & $\tau_i(t+1) > \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_{opt} - \tau_m$). Following a similar reasoning to Lemma 6, it can be seen that if we set $\boldsymbol{\tau}*$ such that $\tau_j = \tau_M$ $\forall j \ne m$, we have an lower bound on $\tau_M - \tau_M(t+1)$. Thus, $\tau_M - \tau_M(t+1) \ge \frac{n\gamma}{n-1}(r_{opt} - r_m(\boldsymbol{\tau}^*)) \ge \frac{n\gamma}{n-1}(r_{opt} - r_m(\boldsymbol{\tau_m}))$.

*Case 4c:* ($\tau_i = \tau_m$ & $\tau_i(t+1) < \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_{opt} - \tau_m$). In this case $\tau_m(t+1) - \tau_m \ge \gamma(r_M - r_m) \ge \gamma(r_{opt} - r_m(\boldsymbol{\tau_m}))$.

*Case 4d:* ($\tau_i = \tau_m$ & $\tau_i(t+1) < \tau_{opt}$ & $\|\boldsymbol{\tau}(t) - \boldsymbol{\tau}_{opt}\|_\infty = \tau_M - \tau_{opt}$). Following a similar reasoning to case 3d, we have $\tau_m(t+1) - \tau_m \ge \gamma(r_M - r_m) \ge \gamma(r_{opt} - r_m) \ge \frac{\gamma l}{T_{opt}}\tau_{opt}(1-\tau_{opt})^{n-1}(\tau_M - \tau_{opt})$. ■

### D. Proof of Theorem 4

The PAS algorithm computes $\tau_i$ at a given stage $t'$ according to the following expression:

$$\tau_i(t') = \tau_i^{initial} + \gamma\sum_{t=0}^{t'}\left(\sum_{j \ne i}(r_j(t) - r_i(t)) - F_i(t)\right) \quad (41)$$

If $\tau_i$ exceeds 1 at any stage, then it decreases in the next stages until it goes below 1. Indeed, for $\tau_i > 1$ we have $\hat{\tau}_i = 1$, which leads to $r_j = 0$ for $j \ne i$ and $F_i > -r_i$, and thus from the above expression $\tau_i$ decreases. This implies that $\tau_i$ can never exceed $\tau_{max} \doteq 1 + \delta$, where $\delta$ is the maximum distance that $\tau_i$ can cover in one stage (which is bounded). Taking this into account, (41) yields

$$\sum_t\left(\sum_{j \ne i}(r_j(t) - r_i(t)) - F_i(t)\right) \le S_{max} \quad (42)$$

where $S_{max} \doteq (\tau_{max} - \tau_i^{initial})/\gamma$.

When there is a misbehaving station that changes its configuration over time and receives a throughput $r_s(t)$ while the rest of the stations are well-behaved, using the same configuration and obtaining the same throughput $r(t)$, the above can be expressed as

$$\sum_t r_s(t) \le \sum_t (r(t) + F_i(t)) + S_{max} \quad (43)$$

If we now consider the throughput of the misbehaving station over an interval $T$, the average throughput over this interval can be computed as $r_s = (1/T)\sum_t r_s(t)T_{beacon}$, where $T_{beacon}$ is the duration of a beacon interval. Thus,

$$r_s \le \frac{1}{T}\sum_t (r(t) + F_i(t))\,T_{beacon} + S_{max}\frac{T_{beacon}}{T} \quad (44)$$

Since we considering a very large interval $T \to \infty$, the term $S_{max}T_{beacon}/T$ tends to 0, which yields

$$r_s \le \frac{1}{T}\sum_t (r(t) + F_i(t))\,T_{beacon} \quad (45)$$

Let us consider now a given stage $t$. From (10) we have $F_i(t) \le \frac{1}{n-1}(nr_{opt} - r_s(t) - (n-1)r(t))$, which yields

$$(n-1)r(t) + r_s(t) + (n-1)F_i(t) \le nr_{opt} \quad (46)$$

Since the above equation is satisfied for all $t$,

$$\sum_t (n-1)r(t) + r_s(t) + (n-1)F_i(t) \le \sum_t nr_{opt} \quad (47)$$

Furthermore, from (45),

$$(n-1)\sum_t r_s(t) \le (n-1)\sum_t (r(t) + F_i(t)) \quad (48)$$

Adding the above two equations yields $n\sum_t r_s(t) \le n\sum_t r_{opt}$, from which

$$r_s = \frac{1}{T}\sum_t r_s(t)T_{beacon} \le \frac{1}{T}\sum_t r_{opt}T_{beacon} = r_{opt} \quad (49)$$

which proves the theorem. Since the right hand side of the above equation is precisely the throughput that the misbehaving station would get if it always ran PAS, this shows that the misbehaving station cannot benefit from using a different strategy no matter how it changes its configuration over time. As the proof does not make any assumption on the configuration of the misbehaving station, this holds for any configuration of all the 802.11e parameters. ■