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Modelling Electricity Swaps with Stochastic Forward Premium Models

Iván Blanco[a], Juan Ignacio Peña[b] and Rosa Rodríguez[c]

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Abstract

We present a new model for pricing electricity swaps. Two general factors affect all contracts but unique risk factors affect each contract. General factors are average swap prices and deterministic trend-seasonal components, and unique factors are forward premiums. Innovations follow MNIG distributions. We estimate the model with data from the European Energy Exchange. The model outperforms four competitors, both in in-sample valuation and in out-of-sample forecasting, and in fitting the term structure of volatilities by market segments. Competitor models are (i) diffusion spot prices, (ii) jump-diffusion spot prices with time dependent volatility, (iii) HJM-based and (iv) Lévy multifactor model with NIG distributions. Value-at-Risk measures based on normality strongly underestimate tail risk whereas our model gives estimates that are more accurate.

Keywords: Electricity swaps; Stochastic forward premium; Multivariate Normal Inverse Gaussian distribution; Lévy processes

JEL Codes: C51; G13; L94; Q40

[a] Universidad Carlos III de Madrid, Department of Business Administration, c/ Madrid 126, 28903 Getafe (Madrid, Spain). ivan.blanco@uc3m.es;[b] *Corresponding author. Universidad Carlos III de Madrid, Department of Business Administration, c/ Madrid 126, 28903 Getafe (Madrid, Spain). ypenya@eco.uc3m.es*; [c] Universidad Carlos III de Madrid, Department of Business Administration, c/ Madrid 126, 28903 Getafe (Madrid, Spain). rosa.rodriguez@uc3m.es.

Executive Summary

We present a new model for pricing electricity swaps. We posit that swap electricity prices result from at least three driving forces. First, a stochastic factor acting as an anchor of the overall level of the forward curve, this factor represents the average "consensus" price for the contracts within a given maturity slot (yearly, quarterly, monthly). Second, a factor reflecting deterministic trend-seasonal components, because we assume that market anticipates weather-related variations in demand. Third, a factor accounting for (mean-reverting) stochastic deviations from the previous two factors. These deviations depend on time to maturity and length of delivery period. By using a MNIG distribution, our model embodies realistic probabilities of occurrence of extreme prices.

We test the model using EEX data for the German market. The model outperforms four competitors, both in in-sample valuation and in out-of-sample forecasting, and in fitting the term structure of volatilities by market segments. Competitor models are based on (i) diffusion spot prices, (ii) jump-diffusion spot prices with time dependent volatility, (iii) HJM approach and (iv) Lévy multifactor model based on the NIG distribution. The model presents noticeable ability in capturing extreme tail risk, as suggested by results of VaR analysis. A practical implication is that capital charges to traders using EEX electricity contracts, based on risk-adjusted capital and on the normality assumption, are too conservative. We suggest increasing them also that evaluators of the performance of traders should adjust their recommendations accordingly.

1. Introduction

A growing number of electricity market participants use derivatives contracts as a way of trading synthetic electricity generation plants[1]. Therefore, electricity derivatives are becoming progressively an important part of the global energy commodities market. By far, the most liquid derivatives contracts in the electricity markets are forwards, futures and swaps[2]. Thus, developing proper valuation models of these electricity contracts is an important step for computing risk measures and for the pricing of other derivatives (e.g. options).

Initial pricing models for electricity forwards and swaps relied on the classic cost-of-carry pricing model (i.e. spot-price-based approach). This approach has been reasonably successful in the case of many storable commodities; see Schwartz (1997). In this vein, Hilliard and Reis (1998), Schwartz and Smith (2000), Lucia and Schwartz (2002), Cartea and Figueroa (2005) and Casassus and Collin-Dufresne (2005), among others, presented extensions of the basic model to electricity forwards. However, this approach faces some challenges. First, the non-storability of electricity creates risks that cannot be hedged, leading to an incomplete market setting. Consequently, the risk-neutral measure cannot be determined uniquely. Second, model-based swap prices are not necessarily consistent with market prices. For these reasons, we propose a pricing approach based on modelling

[1] For instance in Europe the EFET (www.efet.org) a group of more than 100 energy trading companies from 27 European countries promotes energy trading throughout Europe and provides templates of many standardized energy derivatives contracts.

[2] In most electricity markets, forward and futures contracts guarantee delivery of the electricity over a period of time (e.g. monthly or yearly contracts) rather than at a fixed future time. As Benth and Koekebakker (2008) argue the nature of these contracts are very similar to a swap exchanging a fixed price for floating (spot) electricity price during the defined period. In fact, swap contracts are integrals of traditional forward contracts, with fixed delivery time. Therefore, in this paper, a swap is a futures contract with delivery over a given period.

the swap curve directly, allowing for a very general distribution for the sources of uncertainty (innovations) influencing state variables. Our model embodies three factors accounting for (a) average swap price within each market segment, (b) deterministic trend-seasonal factor and, (c) stochastic changes in the shape of the swap curve for each contract. We name this model Stochastic Forward Premium model (SFP henceforth). SFP model relies on a particular case of the Multivariate Generalized Hyperbolic (MGH) distributions, namely the Multivariate Normal Inverse Gaussian (MNIG) distributions, which allow stochastic dynamics in terms of correlated NIG Lévy processes. We test SFP model using data from the German Power Market (European Energy Exchange, EEX) which is Europe's largest power market in terms of consumption[3] and perform in sample and out-of-sample valuation exercises against four competing models.

This study extends current literature in several ways. First, Kiesel, Schindlmayr and Börger (2008) suggest a two-factor model for electricity swaps calibrated to at-the-money options traded in the EEX market. However, liquidity of these options varies substantially across contracts and over time. Thus, we work directly with swap contracts, because, at least some of them are liquid all the time. Second, in contrast with Benth and Koekebakker (2008) who allow for just one Brownian motion as the driver of the dynamics of the swap curve, we argue that correlated Lévy process for each market segment are useful to explain the dynamic behaviour of swap prices, allowing for a more realistic representation. Third, Borovkova and Geman (2006a, b), propose a two-factor

[3] According to Quarterly Reports on European Electricity Markets (European Commission, 2016) the largest electricity consumer in the EU is Germany. Regarding market liquidity, as measured by churn ratios, data suggest that Germany is the most liquid electricity market in Europe in recent years. See the Quarterly Reports on European Electricity Markets. Regarding efficiency, Growitsch and Nepal (2009) present evidence supporting the efficiency of German wholesale electricity spot market and empirical evidence in Peña and Rodriguez (2016) suggests that German electricity derivatives markets are time-zero efficient.

model, in which the first factor is the average forward price and the second factor is analogous to the stochastic convenience yield. This concept is problematic in the case of electricity because it is not storable. Furthermore, they use monthly dummies to deal with seasonality and assume normal distributions for the innovations, both choices being at odds with empirical evidence. Fourth, Fleten and Lemming (2003), Keppo, Audet, Heiskanen and Vehviläinen (2004), Koekebakker and Ollmar (2005) and Bjerksund, Rasmussen, and Stensland (2010) build a continuum of instantaneous-delivery forward contract by smoothing market prices, or by combining market prices with forecasts generated by bottom-up models. However, we argue that working directly with liquid swap prices is more transparent, instead of using ad hoc numerical procedures to extract smooth curves from quoted prices[4]. In the empirical application in this paper, liquid contracts are those six contracts closer to maturity for the monthly, quarterly and yearly delivery periods.

Summing up, our contributions are as follows. First, we propose a new SFP model based on three factors: (1) average swap prices within each market segment, (2) deterministic trend-seasonal factors, and (3) contract-specific mean-reverting stochastic deviations from average prices. Innovations of factors one and three follow MNIG processes. In doing so, SFP model effectively captures realistic, time-varying characteristics in swap prices, overcoming limitations in standard models of swap curves that cannot account for asymmetries and fat tails. Second, empirical results during the period from 2004 to 2013 provide strong evidence supporting SFP model in comparison with diffusion spot price models and HJM-based model, largely because SFP presents

[4] Advocates of the smoothing algorithms posit that futures prices with fixed time to maturity can be extracted each day from the smoothed curve. The main criticism is however that the prices are not "true" market prices but interpolations and these smoothed prices may distort the empirical analysis. We prefer to concentrate on actual market prices and include the effect of the changing time to maturity in the SFP component

better in sample fit. Third, SFP model outperforms Cartea and Figueroa (2005) jump-diffusion model and Di Poto and Fanone (2012) Lévy multifactor model in an out-of-sample pricing exercise in the period 2013-2015 and in fitting the term structure of volatilities by market segments. Fourth, we show that models who do not account for the impact of non-normality are not able to replicate market prices, and, in particular, are unable of taking into account tail risk. This result is important because Gonzalez-Pedraz, Moreno and Peña (2014) present evidence suggesting that tail risk measures for energy portfolios based on standard assumptions (e.g. normality) underestimate actual tail risk, especially for short positions and short time horizons. Another important practical implication of VaR analysis is that capital charges to traders using EEX electricity swap contracts (based on risk adjusted capital under the normality assumption) should be adjusted (most likely upwards) and that evaluators of traders' performance should adjust their recommendations accordingly.

The rest of paper is as follows. In Section 2, we revise related literature and present SFP model. After we describe data in Section 3, we report results of empirical analysis in Section 4. Section 5 compares SPF model against four competing alternatives. Section 6 presents Value-at-Risk results. Section 7 concludes.

2. The Model

In this section, after presenting a literature review, we outline basic characteristics of SFP model and present some guidelines for its practical implementation. As a general reference for MNIG distributions, we suggest McNeil, Frey, and Embrechts (2005).

2.1. Literature review

Models for pricing and hedging financial derivatives on electricity prices belong to three broad categories: (i) based on fundamental equilibrium (ii) based on spot electricity prices and other key variables such as interest rates and (iii) based on forward price processes. Models in the first category focus on supply and demand relationships to obtain power prices as a solution of an optimization problem. This optimization problem embodies information on market prices and trading activity. This approach allows computation of forward prices, using the condition that they provide equilibrium in demand for forward contracts; see Bessembinder and Lemmon (2002). In a similar vein, Supatgiat, Zhang and Birge, (2001) show that market-clearing prices are determined by solving a Nash equilibrium problem for the bidding strategies of market agents. Within this class, another group of models is the hybrid one including a combination of parameterized full production cost models (Eydeland and Wolyniek, 2002) and econometric models (Karakatsani and Bunn, 2008). However, traditional frameworks find difficult to incorporate information about the future (e.g. addition of new generation facilities) because the evolution of all relevant pricing information (the information filtration) is usually determined by past and current realizations of the (spot) price process of the electricity. Benth and Meyer-Brandis (2009) propose the enlargement of the information filtration as a mean for meeting this challenge. Current efforts to enlarge the information filtration focus on including information about forecasts of relevant state variables such as fuel prices, load and available capacity. In this vein, Füss, Mahringer and Prokopczuk (2015) report that, in terms of improvement in pricing of forward contracts, the usefulness of enlarging the filtration with short-term forecasts of state variables is especially remarkable in periods during which electricity prices are highly sensitive to those state variables. Although useful for a wide range of applications, models in this category face challenges when it comes to capture appropriately price dynamics, which is what market participants need in order to develop effective hedging and risk management strategies.

The second category is based on specifying stochastic processes for spot prices and possibly for a limited set of other state variables, calibrating their parameters using market data. Then, one may resort to closed formulas or numerical approximations, in order to price contingent claims. Examples of this approach are Schwartz (1997), Hilliard and Reis (1998), Schwartz and Smith (2000), and Casassus and Collin-Dufresne (2005), among others. This approach has been successfully applied to some energy commodities, particularly crude oil, but its adequacy is less clear in the case of electricity markets because of the very specific features present in electricity spot prices. These characteristics are strong seasonality, mean reversion, jumps, stochastic volatility and regime switching (see Escribano, Peña and Villaplana, 2011) among others) and are caused by the difficulties of storing electricity efficiently; see also Lucia and Schwartz (2002), Geman and Roncoroni (2006) and Cartea and Figueroa (2005). Besides that, this approach has some disadvantages because endogenously generated swap prices are not necessarily consistent with observable market prices. Quinn, Reitzes and Scumacher (2005) argue that electricity swap prices are a function of market expectations of demand and cost conditions during the actual delivery period, and these expectations are not necessarily influenced by current market behaviour (i.e. spot prices), and they present evidence supporting their claim in the PJM market. Furthermore, historical correlation between electricity spot prices and the nearby swap prices is not particularly stable. Benth, Šaltyte-Benth and Koekebakker (2008) find that contracts located in the very short end of the swap curve are the only ones with sizeable correlations with the spot price at Nord

Pool.

The third category is based on modelling directly the term structure of electricity swap prices. Within this category, a first line of research relies on Heath, Jarrow and Morton (1992) (HJM) which focus on the dynamics of the swap curve as a whole. Examples of this approach are Cortazar and Schwartz (1994), Amin, Ng, and Pirrong (1995), Miltersen and Schwartz (1998), Clewlow and Strickland (1999), Koekebakker and Ollmar (2005), Miltersen (2003), Keppo, Audet, Heiskanen, and Vehviläinen (2004) and Trolle and Schwartz (2009), among others. In all these cases, market price curves are inputs into the derivative pricing model and therefore derivatives prices thus generated should be consistent with observable market prices. A second line of research focuses on modelling a given function of observed swap prices and then analysing stochastic deviations from this function by means of additional state variables. An example of this approach is Borovkova and Geman (2006a), who propose a two-factor model, in which the first factor is average forward price and the second factor is analogous to the stochastic convenience yield. However, a potential problem affecting all models within this category is their assumption of Gaussian distributions for the innovations[5]. As suggested by Frestad, Benth and Koekebakker (2010) in the case of electricity swaps, this assumption is unlikely to be appropriate, because innovations of electricity swap prices are strongly non-normal. Besides that, factor structure of swap curves in electricity markets is probably much more complex than in other energy markets. For instance, Koekebakker and Ollmar (2005) report that, in order to explain more than 98% of the variation in the sample covariance matrix in the Nord Pool, they need more than ten factors. Interestingly,

[5] Exceptions are Andresen, Koekebakker and Westgaard (2010) who present a discrete random-field model based on the multivariate NIG distribution, and Di Poto, and Fanone, (2012) who apply a Lévy multifactor market model by using Independent Component Analysis.

factors explaining a large proportion of the variation in the long end of the curve seem to have very low explanatory power in the short end of the curve. This fact suggests that, besides a few general factors affecting the whole curve, some parts of the curve are exposed to unique risk factors that other parts of the curve are not exposed to. Frestad (2008) find strong support of common and unique factors in the Nordic electricity market. In most papers, innovation processes driving state variables are Gaussian. In this paper, we rely on a more general process for innovations. There is growing evidence suggesting that the Multivariate Normal Inverse Gaussian (MNIG) distribution, which was first used for modelling speculative returns in Barndorff-Nielsen (1997), fits heavy-tailed and skewed financial data well and is, at the same time, analytically tractable, see, for instance, Rydberg (1999), Barndorff-Nielsen and Prause (2001), Forsberg and Bollerslev (2002), and Karlis (2002) among others. Therefore, the MNIG distribution is especially suitable for modelling financial prices, and in particular for the term structure individual contract dynamics (Benth, et al, 2008) as well as their joint evolution (Andresen, Koekebakker and Westgaard (2010)). These are the reasons why we posit that innovations follow MING distributions in SFP model.

2.2. The SFP model

Assume T<∞ and let (Ω,F, Q) be a complete filtered probability space, with an increasing and right-continuous filtration $\{F_t\}_{t\in[0,T]}$ where, as usually, F_o contains all sets of probability zero in *F*. We assume that the market trades swap contracts with different delivery periods and a bond that yields a constant risk free rate *r* >0, so futures and forward prices are equal. Consider the price $F_i(t,T)$ of a swap contract with expiry date $T = 1, \ldots, N$, which is also the start of the delivery period, time to maturity $\tau = (T - t)$,and delivery length period given by the subscript $i = 1, \ldots, I$ (e.g. *i*=1 (M) for monthly contracts, i=2 (Q) for quarterly contracts and i=3(Y) for yearly contracts)[6]. These contracts are settled against the daily average spot price during the delivery period and, in agreement with market practice, we call them electricity swaps. Hence, $F_{\boldsymbol{i}}(t,\boldsymbol{T})$ denotes a price vector of a completely observable swap curve at time t, where vector $\boldsymbol{T}$ indicates starting delivery dates which are available at trading date t, and vector subscript $\boldsymbol{i}$ denotes the underlying asset (delivery period) over which is defined each curve's swap contract. Notice that in a general term structure model, we consider the evolution of a continuum of swap contracts for all possible expirations. Since only subsets of them trade in the market, we develop a version of the model that specifies the dynamics only for tradable contracts that are liquid enough. Therefore, our model follows the spirit of the market model; see Brace, Gatarek, and Musiela (1997), which was originally developed for interest rates. Borovkova and Geman (2006b) propose a similar model to ours, but they consider only two sources of uncertainty, driven by uncorrelated Gaussian innovations. We instead allow for multiple sources of uncertainty, and these sources of uncertainty follow correlated MNIG distributions. In doing so, SPF model captures correlations among sources of uncertainty, as well as salient stylized facts in the distribution of electricity prices, such as fat tails. We model swap prices for any maturity T and delivery length period i as follows,

$$F_i(t,T) = \overline{F}_i(t)e^{(s_i(t)+\gamma_i(t,T))} \tag{1}$$

where the first component in the right-hand side is the average level of swap prices within each market segment i . We define average price for a given market segment as a weighted

[6] For example, the swap price $F_M\ (t,1)$ is the price, at time t, of the contract that matures at the end of the current month (e.g. January) and provides delivery of electricity at a fixed price during the next month (e.g. February). This contract is the M+1 or M1 monthly contract in market parlance. Similarly, $F_Q\ (t,2)$ is the price of the Q2 quarterly contract, and $F_Y\ (t,3)$ is the price of the Y3 yearly contract.

average of available swap contracts corresponding to this segment. Weights depend on the present value of each contract included in the average (Benth, Benth, and Koekebakker, 2008). Consider a discount function *w(j)*, with components $w(j) = (1+r)^{-j/h}$ where *h* takes value 12, 4, and 1 for monthly, quarterly and yearly contracts respectively. We define a weight function *g(t,j)* as $g(t,j) = \frac{w(j)}{\sum_j w(j)}$, indexed by set *j*={1,2,…,*N*}. This weight function integrates to one. Therefore, we calculate average price $\bar{F}_i(t)$ for segment *i,* based on N contracts as follows

$$\bar{F}_i(t) = \sqrt[N]{\prod_{j=1}^{N} g(t,j) F_i(t,j)} \qquad (2)$$

In the empirical application, we set N=6 for all market segments. Thus, set *j* is defined as *j* ={1,2,3,4,5,6} and the elements of the weight function are *g(t,j)* = *w(j)/[w(1)+w(2)+w(3)+w(4)+w(5)+w(6)]*. For low values of *r* and *j,* the weight function can be approximated by $g(t,j) = |j|^{-1}$ where |*j*| is the cardinality of the set *j*[7]. We use this approximation in what follows.[8]

Average swap prices do not contain seasonal factors. Following Benth, Kiesel, and Nazarova (2012), we estimate deterministic trend and seasonal factors $s_i(t)$ by means of a deterministic periodical function:

$$s_i(t) = \alpha_{i,1} + \alpha_{i,2}\frac{t}{250} + \alpha_{i,3}\cos\left(\alpha_{i,4} + 2\pi\frac{t}{250}\right) + \alpha_{i,5}\cos\left(\alpha_{i,6} + 4\pi\frac{t}{250}\right)$$

The quantity $\gamma_i(t,T)$ is the stochastic forward premium[9] (SFP) for delivery period *i* and

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[7] We thank an anonymous referee for clarifying this point.

[8] The approximation implies that weights w(j) are all equal to one and therefore the weight function g(t,j) takes value equal to 0.1666 in all cases. As a comparison , assuming r = 2.2543% (which was the average 3-month EURIBOR during our sample period 2004-2012) , the exact values of the weight function are g(t,1)=0.1674 , g(t,2)=0.1671 ,g(t,3) = 0.1668, g(t,4) = 0.1665, g(t,5) = 0.1661, and g(t,6)=0.1659.

[9] We could write SFP as $\gamma_i(t, T-t)(T-t)$ as in Borovka and Geman (2006b) to point out that the effect of time to maturity *(T-t)* enters into swap prices via SPF. However, for the sake of clarity, we choose our more compact notation, but we stress the fact that the time-to-maturity effect is included in the SFP

expiry date *T*. By construction, SPF factors are zero on average, and we obtain them as

$$\gamma_i(t,T) = \ lnF_i(t,T) - ln\overline{F_i}(t) - \ s_i(t) \tag{3}$$

Next, we specify stochastic dynamics for state variables in terms of Multivariate Normal Inverse Gaussian (MNIG), see Bandorff-Nielsen (1997), Øigård, et al. (2005) and Lévy processes. Notice that the dimension of the system is d = I+ I×N. Let ***L(t)*** be a d-dimensional vector of MNIG Lévy processes. Thus, this vector has stationary and independent increments. In other words, the distribution of ***L(t) – L(s)****, t > s ≥ 0* , is only dependent on *t-s* and not on *t* and *s* separately, such that its increments *d**L(t)**=**L(t+dt)** – **L(t)*** = ***X*** are[10], standardized (i.e. zero-mean, unit variance) and MNIG distributed with probability density function $MNIG_d(\boldsymbol{X};\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$

$$f(\boldsymbol{X}) = \frac{\delta}{2^{(d-1)/2}}\left[\frac{\alpha}{\pi q(\boldsymbol{x})}\right]^{(d+1)/2} K_{\frac{d+1}{2}}[\alpha q(\boldsymbol{x})] \times e^{p(\boldsymbol{x})} \tag{4}$$

Where scalar α is a measure of tail heaviness; large values of α implies light tails, while smaller values of α implies heavier tails, vector $\boldsymbol{\beta}$ contains asymmetry parameters, scalar δ is a scale parameter, vector $\boldsymbol{\mu}$ contains location parameters and matrix $\boldsymbol{\Sigma}$ contains covariances.

$$q(\boldsymbol{x}) = \sqrt{\delta^2 + (\boldsymbol{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})} \ \ , \quad p(\boldsymbol{x}) = \delta\sqrt{\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}} + \boldsymbol{\beta}'(\boldsymbol{x}-\boldsymbol{\mu})$$

and $K_{\frac{d+1}{2}}[x]$ is the modified Bessel function of the second kind with index *(d+1)/2,* and the parameters have the following characteristics, $\delta > 0$, $\alpha^2 > \boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}, \boldsymbol{\beta} \in \mathbb{R}^{\boldsymbol{d}}, \boldsymbol{\mu} \in$

component.

[10] In the special case where the increments ***L(t)-L(s)*** are normally distributed with zero mean and covariance matrix Π, we have a standard multivariate Brownian motion.

$\mathbb{R}^{\boldsymbol{d}}, \boldsymbol{\Sigma} = \{\sigma_{ij}\} \in \mathbb{R}^{\boldsymbol{d}\times\boldsymbol{d}}$ and we require[11] $\boldsymbol{\Sigma}$ to be positive definite and $|\boldsymbol{\Sigma}| = 1$. Mean vector of $\boldsymbol{X}$ is

$$E[\boldsymbol{X}] = \boldsymbol{\mu} + (\delta\boldsymbol{\Sigma\beta}/\sqrt{\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta}})$$

and covariance matrix $V[\boldsymbol{X}] = \{v_{i,j}\}\ i = 1, \ldots, d\,; j = 1, \ldots, d$ is defined as

$$V[\boldsymbol{X}] = [\frac{\delta}{\sqrt{\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta}}}][\boldsymbol{\Sigma} + (\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta})^{-1}\boldsymbol{\Sigma\beta\beta}'\boldsymbol{\Sigma}'] \tag{5}$$

When skewness parameter is zero ($\boldsymbol{\beta}$ = 0), mean vector coincides with $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ solely determines the correlation structure. For asymmetric MNIG distributions, correlation structure depends on all parameters, excepting $\boldsymbol{\mu}$.

Marginal distributions of MNIG are univariate NIG distributions (Lillestøl, 2000). Denoting the parameters of the marginal distributions of i th component of $\boldsymbol{X}$ as X_i ,and using obvious notation, it is easy to see that $\mu_i = \boldsymbol{\mu}_i$, $\delta_i = \delta(\sigma_{ii})^{1/2}$, $\beta_i = \sigma_{ii}^{-1}\sum_{k=1}^{d}\sigma_{ik}\boldsymbol{\beta}_k$ and $\alpha_i = \sqrt{\sigma_{ii}^{-1}(\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta}) + \beta_i^2}$. Scale-free indicators of asymmetry χ_i and kurtosis ξ_i , $\{(\chi_i, \xi_i) \in R^2; |\chi_i| < \xi_i < 1)$ are respectively defined as (Rydberg, 1997):

$$\chi_i = \frac{\xi_i\,\beta_i}{\alpha_i}\ ;\ \xi_i = \frac{1}{\sqrt{(1+\delta_i\gamma_i^2\,)}}$$

[11] The distributions MNIG$_d$($\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma}$) and MNIG$_d$($\alpha/\kappa, \kappa\boldsymbol{\beta}, \kappa\delta, \kappa\boldsymbol{\mu}, \kappa\boldsymbol{\Sigma}$) are identical for any $\kappa > 0$ Therefore, an identifying problem occurs when we start to fit the parameters of a MNIG distribution to data. This problem is solved by introducing a suitable constraint, for instance requiring that the determinant of the dispersion matrix $\boldsymbol{\Sigma}$ is equal to one.

where $\gamma_i = \sqrt{\alpha_i^2 \quad - \beta_i^2}$. If scale-free parameters are close to zero (i.e. $(\chi_i , \xi_i) \approx (0,0)$) marginal NIG distributions are close to being normal. On the other hand, when scale-free parameters ξ_i are very close to one, marginal NIG distributions are very similar to heavy-tailed Cauchy distributions.

We define dynamics of $ln\overline{F}_i(t)$ and $\gamma_i(t,T)$ under the market probability measure by the stochastic differential equations:

$$dln\overline{F}_i(t) = \kappa_i\big(\zeta_i - \ ln\overline{F}_i(t)\big)dt + \theta_{\overline{F,\iota}}dL_{\overline{F}_\iota}(t)\ ;\ i = 1, \dots, I \tag{6}$$

$$d\gamma_i(t,T) = -\varpi_{i,T}\gamma_i(t,T)\ dt + \theta_{\gamma_i(T)}dL_{\gamma_i(T)}(t)\ ;\ i = 1, \dots, I\ \ T = 1, \dots., N \tag{7}$$

SFP factors are subject to their own sources of uncertainty, given by standardized MNIG Lévy processes, $dL_{\overline{F}_\iota}(t)$, and $dL_{\gamma_i(T)}$ which are assumed to be correlated. We can substitute (6) and (7) into (1) and derive the dynamics of the swap log-prices under the market probability measure as follows:

$$\begin{aligned} dln\big(F_i(t,T)\big) = \big[\kappa_i\big(\zeta_i - \ ln\overline{F}_i(t)\big) - \varpi_{i,T}\gamma_i(t,T) + ds_i(K)\ \big]dt + \theta_{\overline{F,\iota}}dL_{\overline{F}_\iota}(t) \\ + \ \theta_{\gamma_i(T)}dL_{\gamma_i(T)}(t) \\ i = 1, \dots, I\ \ T = 1, \dots., N \end{aligned}$$

Therefore, $lnF_i(t,T)$ is obtained by integrating the above differential equation with initial condition[12]

[12] If X_1 and X_2 are independent NIG random variables with common parameters α ,β but having different

$$lnF_i(0,T) = ln\overline{F_i}(0) + s_i(t) + \gamma_i(0,T) \quad i = 1, \dots, I \;\; T = 1, \dots., N$$

The term structure of swap prices variances is:

$$\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,\iota}}\theta_{\gamma_i(T)}v_{\overline{F_\iota},\gamma_i(T)}\,;\;\; i = 1, \dots, I\,; T = 1, \dots, N \qquad (8)$$

where $\eta_i^2 = {\theta_{\overline{F,\iota}}}^2 \times v_{\overline{F_\iota,F_\iota}}$ and $\tau_{i,T}^2 = {\theta_{\gamma_i(T)}}^2 \times v_{\gamma_i(T),\gamma_i(T)}$ are the variances of the corresponding average factor and of the stochastic discount factor respectively and $v_{\overline{F_\iota},\gamma_i(T)}$ is the covariance between the average factor and the stochastic discount factor, all of them elements of (5).

As we are interested in pricing other derivatives, we now consider how to price these derivatives in the risk-neutral world. First, notice that we specify the dynamics (1), (6) and (7) under the market (real-world) probability measure *P*; therefore, we must select a risk-neutral probability measure *Q*. A common choice (see Benth, Šaltyte-Benth and. Koekebakker , 2008) is Esscher transform which generalizes Girsanov transform to Lévy processes and guarantees that ***L(t)*** process is still a NIG Lévy process under *Q*. This Esscher transform implies that, under the risk-neutral measure *Q*, the transformed $\boldsymbol{L}^Q(t)$ vector is MNIG-distributed with density function $MNIG_d(\boldsymbol{X};\alpha, \boldsymbol{\beta}+\boldsymbol{\eta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$, or in other words, Esscher transform only changes the asymmetry of the process. Vector $\boldsymbol{\eta}$ measures the price of jump risk, that is, the price that market players charge for assuming the risk of not being able to hedge. A positive price leads to a more right-skewed distribution.

scale and location parameters δ_1,μ_1, and δ_2, μ_2, then $X_1+X_2 = X$ is NIG with parameters $(x;\alpha,\beta\; \delta_1+\delta_2, \mu_1+\mu_2)$.

Given an estimate[13] of the vector $\boldsymbol{\eta}$, and the transformed process under Q, we can apply standard techniques to price other derivatives such as options.

2.3. Implementation

Consider a dataset of n swap curves $F_i(t,\boldsymbol{T})$ $t = 1, \dots n$, where vector $\boldsymbol{T}$ indicates delivery dates available at trading day t, and subscript $\boldsymbol{i}$ denotes the underlying asset (delivery period, e.g. during next month). We present the implementation as a step-by-step algorithm.

Step 1: Take natural logarithm of market prices.

Step 2: Estimate average price factor for each segment i as the simple average of the number of contracts available ($T(i)$) in each segment i.

$$ln\overline{F}_i(t) = \frac{1}{T(i)} \sum_{T=1}^{T(i)} lnF_i(t,T) \tag{9}$$

Step 3: Estimate deterministic trend and seasonal factors $s_i(t)$ by means of the following OLS regression:

$$lnF_i(t,T) - ln\overline{F}_i(t) = s_i(t) + \varepsilon_i(t,T) \tag{10}$$

where $s_i(t)$ is

$$s_i(t) = \alpha_{i,1} + \alpha_{i,2}\frac{t}{250} + \alpha_{i,3}\cos\left(\alpha_{i,4} + 2\pi\frac{t}{250}\right) + \alpha_{i,5}\cos\left(\alpha_{i,6} + 4\pi\frac{t}{250}\right)$$

In addition, calculate standard errors of the estimates using estimators robust to auto correlated and heteroskedastic residuals[14].

Step 4: Estimate SFP factors $\gamma_i(t,T)$ by means of residuals of (10)

$$\widehat{\gamma}_i(t,T) = lnF_i(t,T) - ln\overline{F}_i(t) - \widehat{s}_i(t) = \hat{\varepsilon}_i(t,T) \tag{11}$$

[13] Estimation methods in the case d=1 are proposed in Benth et al. (2008) and in Frestad, Benth and Koekebakker (2010)..

[14] If there are large spikes in prices it is advisable to bound the data by the 70% quantile of the empirical distribution, see Geman and Roncoroni (2006).

Step 5: Estimate mean reversion, volatility and covariance parameters in discrete-time versions of Equations (6) and (7) by means of a system of seemingly unrelated equations (SURE, Zellner 1962). In doing so, we assume that error terms may have cross-equation contemporaneous covariance. The system[15] takes the form[16]

$$\begin{pmatrix} \nabla ln\overline{F_i}(t) \\ \nabla\widehat{\gamma_i}(t,T) \end{pmatrix} = \begin{pmatrix} \kappa_i(\zeta_i - ln\overline{F_i}(t-1)\ ) \\ -\varpi_{i,T}\widehat{\gamma}_i(t-1,T) \end{pmatrix} + \begin{pmatrix} \theta_{\overline{F_i}}\varepsilon_{\overline{F_i}}(t) \\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t) \end{pmatrix} \tag{12}$$

Include suitable autoregressive terms in (12) to take into account residual autocorrelation. Usually, including one or two lags is enough in order to obtain residuals free of autocorrelation.

Step 6: Define residuals from Equation (12) as

$$\boldsymbol{Y} = \begin{pmatrix} \varepsilon_{\overline{F_i}}(t) \\ \varepsilon_{\gamma_i(T)}(t) \end{pmatrix} \ t = 1, \dots, n \tag{13}$$

where $\boldsymbol{Y}$ (dimensions d×n) has a mean vector of zero and covariance matrix $\Omega = \{\omega_{i,j}\}$.

We require the covariance matrix of MNIG process $\boldsymbol{\Sigma}$ to be positive definite and $|\boldsymbol{\Sigma}| = 1$. Thus, we apply a convenient normalization suggested in Urzua (1997). Let Γ denote the orthogonal matrix whose columns are the standardized eigenvectors of Ω, and Λ denote the diagonal matrix of the eigenvalues of Ω . Define $\Omega^{-1/2}$ as the inverse of the square root decomposition of Ω ; or, in other words, that

$$\Omega^{-1/2} = \Gamma\Lambda^{-1/2}\Gamma' \tag{14}$$

---

[15] For instance, in our sample from EEX, we consider three market segments and six contract for each market segment. Therefore, system (12) contains 21 equations (3+6×3) and we estimate its parameters using feasible generalized least squares (FGLS, Greene 2012).

[16] $\nabla x(t) = x(t) - x(t-1)$

Then, random variable $\boldsymbol{X}$ defined as

$$\boldsymbol{X} = \Omega^{-1/2}\boldsymbol{Y} \tag{15}$$

has zero mean, and an identity matrix as its covariance matrix. This variable $\boldsymbol{X}$, contains standardized (zero mean and unit variance) and orthogonal residuals. These residuals are free of autocorrelation.

Step 7: Use $\boldsymbol{X}$ as an estimation of vector $d\boldsymbol{L}(t)$, which follows a MNIG distribution with probability density function $\mathrm{MNIG}_d(\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$. Obtain point estimates of parameters of MNIG distributions by using the EM algorithm developed by Øigård, Hanssen, Hansen, and Godtliebsen (2005), which is an extension of Karlis (2002)[17] . Parameters of marginal univariate NIG distributions can be obtained using this algorithm as well.

Step 8: Compute bootstrapped confidence intervals for MNIG parameters based on 2000 replications. It is advisable to use stationary bootstrap, see Politis and Romano (1994) because of its consistency in accommodating a large class of heterogeneous weakly dependent time series with data that arise as functions of mixing processes, such as ARCH and GARCH processes (Paparoditis and Politis, 2009)[18].

Step 9: In order to compute the term structure of swap prices given by (8) we set $\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,i}}\theta_{\gamma_i(T)}\omega_{\overline{F_i},\gamma_i(T)}$ where $\eta_i^2 = \theta_{\overline{F,i}}{}^2 \times \omega_{\overline{F_i,F_i}}$ and $\tau_{i,T}^2 = \theta_{\gamma_i(T)}{}^2 \times \omega_{\gamma_i(T),\gamma_i(T)}$ are the variances of the corresponding average factor and of the

[17] An alternative method is the Multi-cycle Expectation Conditional Maximization (MCECM) algorithm developed in McNeil, Frey, and Embrechts (2005).

[18] Consider a series that is heteroskedastic. If the subsample values of the pertinent statistic have distributions that converge to the limit in a uniform way, then subsampling remains consistent; see Politis et al. (1997). For instance, in the case of OLS regression with dependent errors subsampling works properly without assuming a stationary or homoscedastic error structure (Politis et al. 1997, Theorem 3.4).

stochastic discount factor respectively and $\omega_{\bar{F}_i,\gamma_i(T)}$ measures correlation between two factors, all of them obtained from (12).

3. Data

Germany is Europe's largest power market in terms of consumption and therefore we choose this market as the source of the data[19]. Our data set consists of daily data from June 1, 2004 until December 31, 2012, on settlement prices for the following available baseload[20] swap contracts traded in EEX: Yearly, Quarterly and Monthly (Phelix-Base/Month/Quarter/Year-Futures). The company operating EEX market (EEX AG) has provided the data.[21] We choose the six most liquid contracts within each market segment, that usually are the closest to maturity ones. Within each market segment, these six contracts represent the 99% (100%), 97% (99%) and 100% (100%) of the total trading volume (open interest) in the case of monthly, quarterly and yearly contracts, respectively. We discuss liquidity of each contact in more detail in Section 3.2. We define a continuous series as a perpetually linked series of swap settlement prices. For example, M1 starts at the nearest contract month, which forms the first values of the continuous series, until the contract reaches its expiry date, or until the first business day of the actual contract month. At this point, we take the next trading contract month. [22] For all series, we compute returns as the first difference of log prices. [23]

[19] See also footnote 3. .

[20] Specifically, we use EEX Power Derivatives Phelix-Base Month, Quarter and Year Future (Exchange codes F1BM, F1BQ, F1BY). As an illustration of this kind of contracts, the 1MW baseload Jan13 contract is a monthly swap contract that gives the holder the obligation to buy 1MWh of energy for each hour of January 2013, paying the futures price in Euros/MWh. The seller provides the buyer the amount of energy of 1MW × 24h × 31. The settlement is financial.

[21] The futures market at the EEX started trading financial futures on base and peak block contracts in the spring of 2001. In 2004 option trading on these contracts was introduced, and since 2005 futures with physical settlement were introduced. Other basic facts on the EEX futures market can be found in for instance in European Energy Exchange (2005), see also Viehmann (2011).

[22] The continuous series M1, Q1 and Y1 match the series from Datastream: EBMCS00, EBQCS00 and EBYCS00.

[23] In the original prices series, a level change appears on the day when one contract expires and a new one

3.1. Summary Statistics

Figure 1 contains graph of all swap price series. Monthly series seem to be more volatile, followed by quarterly series, being yearly series more stable.

[INSERT FIGURE 1 HERE]

Table 1 summarizes statistics for all series in levels. Data for individual series is in Panel A and averages of each market segment are in Panel B. Looking at averages of each market segment (monthly, quarterly and yearly), average price tends to increase with maturity and volatility tends to decrease with maturity, as expected. Volatility is usually higher for the closest-to-maturity contract (Samuelson effect); confirming the well-known fact that short-dated swaps tend to be more volatile than long-dated swaps. Within each market segment, average prices follow similar patterns, but volatility presents a behaviour that is more complex. In the case of monthly contracts, volatility follows an inverted u-shape. However, in the case of quarterly contracts, it has a u-shape and in the case of yearly contracts, it increases with time to maturity. In returns series (not shown), volatility follows expected patterns, decreasing with time to maturity. Volatility ranges (in annualized terms) from 33% in the case of M1 to 13% in the case of Y6. All series present positive asymmetry and kurtosis coefficients higher than 3. JB tests suggest that normal distributions are unlikely to be appropriate for these series.

[INSERT TABLE 1 HERE]

is included. This "rolling" effect sometimes generates jumps in the returns series. Therefore, to avoid this artificial effect in the returns series, we apply intervention analysis (see Box and Tiao, 1975) in each day when there is a "rolling" effect. Notice that those jumps are not caused by market behavior, but they are simply a technical problem caused by the definition of the continuous series.

4. Empirical Results

In this section, we present estimates of the components of swap price as defined in Equation (1), that is, average swap price, seasonal component and stochastic forward premium.

4.1. Average swap prices

We compute average swap prices by using Equation (2). Figure 2 contains graphs. Table 2 and summary statistics are in Table 2.

[INSERT FIGURE 2 HERE]

[INSERT TABLE 2 HERE]

We may observe that average prices increase with maturity and volatility decreases with maturity. Average returns are close to zero. Volatility is higher in the monthly segment and lower in the yearly segment, and all series present positive asymmetry and high kurtosis. It is interesting to note that correlations (both in levels and in first differences) are lower than one in all cases. Correlation is lowest between monthly and yearly average prices both in levels (77%) and in first differences (68%). This fact supports models based on specific average components for each market segment, as we posit in this paper.

4.2. Seasonal Components

As an illustration of seasonal components, described in Equation (10), Figures 3A and 3B present the elements of OLS regression $lnF_i(t,T) - ln\overline{F_i}(t) = s_i(t) + \varepsilon_i(t,T)$ where Actual (in red) is the dependent variable $lnF_i(t,T) - ln\overline{F_i}(t)$, Fitted (in green) is the seasonal factor $\hat{s}_i(t) = \hat{\alpha}_{i,1} + \hat{\alpha}_{i,2}\frac{t}{250} + \hat{\alpha}_{i,3}\cos\left(\hat{\alpha}_{i,4} + 2\pi\frac{t}{250}\right) + \hat{\alpha}_{i,5}\cos\left(\hat{\alpha}_{i,6} + \right.$

$4\pi \frac{t}{250}\Big)$ and Residual (in blue) is the SPF factor defined as $\widehat{\gamma_i}(t,T) = lnF_i(t,T) - ln\overline{F_i}(t) - \widehat{s_i}(t) = \hat{\varepsilon}_i(t,T)$. Figure 3A is based on M1 prices and Figure 3B on Q1 prices. In both cases, sample period ranges from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.

[INSERT FIGURE 3A]

[INSERT FIGURE 3B]

In the case of M1 contract, the fitted line suggests a seasonal behaviour of swap prices over the year. As expected, seasonal component is above average for contracts expiring in fall and winter, and it is below average for contracts expiring in spring and summer. Explanatory power of seasonal components for monthly contracts ranges from 60% (M6) to 32% (M3). We may observe similar behaviour in Figures 3A and 3B suggesting the presence of a fall-winter, spring-summer cycle also in the case of Q1 contract. Explanatory power of seasonal factors tends to be similar for quarterly contracts and ranges from 68% (Q3) to 39% (Q1). A statistical significance test (not shown) reveals that most seasonal components are significantly different from zero at 95% confidence level. We obtain similar results for other monthly and quarterly contracts.

4.3. Stochastic forward premium factors

Using Equation[24] (11), we compute estimated SFPs. Summary statistics are in Table 3, in levels $\widehat{\gamma_i}(t,T)$ in Panel A and in first differences $\widehat{\nabla\gamma_i}(t,T)$ in Panel B. All $\widehat{\gamma_i}(t,T)$ series present zero means, as expected, and volatilities generally fall with maturity. Furthermore,

[24]We adjust all SPF series to take into account the rolling effect. See footnote 14.

asymmetry coefficients range from -0.7 to 1.0 and kurtosis ranges from 1.8 to 4.6. Jarque-Bera test rejects the null hypothesis of normality in all cases. First order autocorrelation coefficients are close to one (not shown). In the case of series $\nabla\widehat{\gamma_i}(t,T)$, they present means that are essentially zero and volatilities tend to decrease with time to maturity. Skewness ranges from -0.4 to 0.7 and kurtosis ranges from 5 to 22. Again, Jarque-Bera test rejects the null hypothesis of normality in all cases. First-order autocorrelation coefficients (not shown) are usually below 0.1, suggesting a slow mean-reverting behaviour. In summary, evidence suggests that SFP series (in levels and in first differences) do not follow normal distributions.

[INSERT TABLE 3 HERE]

Table 4 presents correlations between returns of average series and returns of SFPs series. Correlation coefficients between monthly SFP factors and the monthly average factor range from -0.35 to 0.46, and are statistically significant in five out of six cases. Correlations among monthly SPF factors range from -0.60 to 0.43, and are significant in thirteen out of fifteen cases. In the cases of quarterly (yearly) SFP factors correlation coefficients between quarterly (yearly) SFP factors and the quarterly (yearly) average factor range from -0.35 to 0.50 (-0.10 to 0.69) , and are statistically significant in three (six) out of six (six) cases. Correlations among quarterly (yearly) SPF factors range from -0.50 to 0.13 (-0.68 to 0.58), and they are significant in ten (thirteen) out of fifteen (fifteen) cases. Correlations are significant in 61% of cases (22 over 36), between monthly and quarterly SFP factors, in 22% of cases between monthly and yearly SFP factors and in 11% of cases between quarterly and yearly SFP factors.

Overall, evidence suggests significant correlations between many factors (average and SFPs) and tends to support assumptions in our theoretical model, because it allows for correlations both within and across market sectors and factors, thus permitting a realistic representation of market prices.

[INSERT TABLE 4 HERE]

4.4. Relative weight of components

An important practical question is the proportion of the total variation explained by each component for different market segments. To study this issue we run a regression, where explanatory variables are components, included sequentially. In the monthly segment, on average, average swap price explains around 75% of total variation, SFP component explains an additional 15% and seasonal components explain the remaining 10% of total variation, and differences across maturities are not very marked. The same situation appears in the quarterly segment, where average swap price explains around 80%, SFP component explains an additional 5% and seasonal components explain the remaining 15% of total variation. In the case of the yearly segment, and on average, average swap price explains around 85%, and SFP components explain the remaining 15% of total variation[25].

One implication of these results is that SPF (specific) components in each swap contract represents a non-negligible source of risk. This risk is specific of each contract and cannot

[25] In the yearly segment some more noticeable differences across maturities appear, because the average swap price explains around 93%, 85% and 78% for the one-year, two year and three year maturities respectively, and the SFP component explains the additional 7%, 15% and 22%.

be hedged using other contracts. For instance, if one trader wants to hedge a position in a yearly contract using monthly contracts, risk associated to SPF factors (contract-specific risk) may be more than 20% of total risk. This idiosyncratic risk cannot be hedged using monthly contracts. If a trader uses monthly contracts to hedge yearly contracts, he assumes additional idiosyncratic risks for each monthly contract used. As a consequence of this, a trader wishing to hedge a given electricity swap contract in the EEX market using other (shorter maturity) electricity swap contracts may assume significant idiosyncratic (contract-specific) and basis (maturity-specific) risks.

4.5. Model Estimation

Table 5 presents estimation results of model (12) for average swap prices and stochastic forward premium factors. Panel A contains estimations of mean reversion κ_ι, long term mean ζ_ι and volatility $\theta_{\bar{F}_\iota}$ parameters for average series factors, and also estimates of parameters corresponding to their marginal NIG distributions using standardized (0,1) residuals. Panel B contains estimates of parameters of MNIG distribution, based on autocorrelation-free, standardized and orthogonalized residuals from Equations (12) and (15). Panels C and D contain estimations of similar parameters for SFPs within each market segment (monthly, quarterly and yearly). Mean reversion parameters are significant in almost all cases, ranging from 0.0017 to 0.0022 for average series and from 0.0010 to 0.0070 for SFP. Residual volatility varies considerably, ranging from 0.08 to 0.013 for average series and from 0.003 to 0.008 for SFP factors. Volatility tends to be decreasing with time to maturity in all cases, as expected.

Regarding estimated NIG parameters for marginal distributions, scale-free indicators of asymmetry χ are not significant, but indicators of kurtosis ξ are highly significant and range from 0.589 to 0.968. Likelihood ratio tests[26] applied to standardized residuals strongly support NIG distributions as better alternatives than standard normal distributions for all individual distributions. It is also interesting to note that SFP components corresponding to each swap contract, share some common characteristics with other SFPs within each market segment (symmetry, degree of mean reversion) but also present specific features (different volatility levels and tail heaviness).

A test of multivariate Normality (Urzua, 1997) of standardized residuals obtained from (15) (not show) clearly reject the null of normality. Estimated parameters of MNIG distribution shown in Panel C give a similar message than in the case of marginal distributions. Asymmetry parameters are not significant, but we may observe significant values in the cases of parameters measuring tail heaviness. In summary, the distribution of standardized residuals is largely consistent with a symmetric MNIG distribution with heavy tails.

[INSERT TABLE 5 HERE]

Table 6 contains correlations among the ordinary residuals from Equation (12) and in boldface; we highlight correlations used in the computations of the volatility term structure.

[INSERT TABLE 6 HERE]

[26] Strictly speaking, the likelihood ratio test can be applied only for nested models. In our case, the standard Normal distribution N(0,1) is nested within a general NIG(χ , ξ) distribution, because a NIG(0,0) distribution follows the same distribution as the N(0,1). *LR = -2*LOGLK(N(0,1)) + *LOGLN(NIG(*χ *,* ξ*)).*

4.6. In-sample Goodness-of-Fit Tests

As a formal test of the extent to which the NIG distribution is successful in representing the (one-day) innovations in the marginal distributions of average swap prices and SFPs, we implement tests of fit based on the empirical distribution function (EDF). These statistics measure the discrepancy between the EDF and a given theoretical distribution (e.g. Normal or NIG). We calculate two statistics: Cramer-von Mises and Kolmogorov-Smirnov. [27] We calculate the parameters of the NIG distribution by maximum likelihood. Next, we face the problem of how to evaluate these test statistics, given that the true parameter values of the (NIG) distribution are unknown. To solve this problem, and following Capasso, Alessi, Barigozzi and Fagiolo (2009), we generate 5000 Monte Carlo simulations of i.i.d. NIG random numbers for each market segment. In doing so, we obtain approximate distributions of the EDF test statistics. We briefly summarize results. The null hypothesis is that marginal distributions follow NIG distributions. The number of violations of the null hypothesis at the 5% significance level is always lower than the critical value suggesting that, in all cases, one-day returns follow NIG distributions.

4.7. Volatility term structure

In Table 7 we present a volatility term structure computed by using Equations (8) and (12) with calibrated parameters in Table 5 and Table 6.

[INSERT TABLE 7]

As may be seen in Table 7 the model is able to reproduce the overall volatility

[27] For details on EDF statistics see D'Agostino and Stephens (1986).

structure with high degree of precision, irrespective of the market segment. Average absolute and relative errors are lower than 0.5%. Average contributions of each element in (8) to total variance explained by the model are as follows: 87.5% corresponds to the variance of average factors, 18.5% corresponds to the variance of stochastic discount factors, and -6.1% corresponds to covariance terms between average factors and stochastic discount factors. However, the range of variation is substantial being in the first case from 41% to 111%, in the second case from 9% to 37% and in the third case from -49% to 31%. The implication of these results is that the order of magnitude of determinants of the variance for each contract should be analysed carefully because, although in some cases impact of average factors dominate, in other cases the impact of covariance components can be determinant.

5. Comparison against alternative models

In this section we compare the performance of the SFP (baseline) model introduced in section 2 against four competing models, (i) one-factor spot price model, (ii) HJM-based model (iii) Cartea and Figueroa (2005) jump-diffusion model and (iv) Di Poto and Fanone (2012) Lévy multifactor market model based on Independent Component Analysis. Details are in Appendix A, B, C and D respectively.

5.1. One-factor spot price Model

We present a summary estimation of this model and pricing exercises with computations of Root Mean Squared Errors (RMSE) defined as RMSE = $\sqrt{\frac{1}{T}\sum_{t=1}^{t=T}(price_{market}(t) - price_{model}(t))^2}$ in Appendix A. As an illustration, we present here results for contracts M1, Q1 and Y1 (which are liquid contracts within each market segment) during 2010 and compare them against market prices. Results suggest that this model is not able to capture basic features of swap market prices. In particular, theoretical prices are much more volatile than market prices, as Figure A2 suggests. The reason is that model-based forward prices are simple functions of spot prices, which are always much more volatile than market prices. Correlations between theoretical prices and market prices are (on average) 0.53, 0.41 and 0.09, for monthly, quarterly and yearly prices respectively. Pricing errors are not independent and model-based variances do not appropriately reflect market volatility. Besides that, residuals from the model are strongly non-normal, contradicting basic assumptions underlying this model. In summary, this one-factor model does not capture basic characteristics of market prices and therefore is unlikely to be useful for pricing or hedging purposes.

5.2. HJM-based Model

Estimation results of this model are in Appendix B. Eigenvalues resulting from eigenvector decomposition tell us the importance of each eigenvector and hence the number of factors that we should include. The first eigenvector is the most important, explaining 87%, 89% and 94% of total variation in the evolution of the swap curve for monthly, quarterly and yearly contracts respectively.

Table B2 presents, in Panel B, least squares estimates of parameters from the volatility functions obtained when applying PCA (Principal Component Analysis) using equation $\sigma_{1i}(t, \boldsymbol{T_i}) = e^{-k_i(\boldsymbol{T_i}-t)}\sigma_{1i}$ for the full sample period 2004-2012. Panel A reports in-sample Root Mean Squared pricing Errors (RMSEs). We compute daily errors based on fitted swap prices; results are in Panel B. We compute the volatility function implied by the HJM model and by the SFP model and compare results against market prices. In the case of HJM model, RMSEs are 6.05%, 19.32%, and 11.96% for monthly, quarterly and yearly contracts respectively. By contrast, the RMSEs for those contracts are 0.10%, 0.12% and 0.30% respectively in the case of the SFP model. The degree of fit of SFP model is substantially higher than HJM model. It is fair to say that, although HJM model seems to fit the volatility term structure to some extent, it is unable to recover the skewness and kurtosis observed in the empirical distributions. By contrast, SFP model not only fits market's volatility term structure better, but also it is able to take into account skewness and kurtosis.

5.3. Cartea and Figueroa (2005) Jump-diffusion model

Estimation results of this model are in Appendix C. We compare this model against SFP model by looking at out-of-sample prediction errors for eighteen swap prices in the period from January 1, 2013 to December 30, 2015. Sample size is 756 observations. At any time in this period, we predict the swap price with maturity T in segment i at time t+h using the conditional expectation $E[F_i(t+h,T) \mid F_t]$, which is the best prediction in the mean-square sense. We calculate its value using the parameter values calibrated from the eight-year in-sample period. For the prediction interval h, we use 1 day, 5 days and 20 days, representing daily, weekly and monthly horizon. Let $e_j = [E[F_i(t_j+h,T) \mid F_t] - F_i(t_j+h,T)]$. We measure the overall prediction error by the Root Mean Squared Error (RMSE) , $RMSE = \sqrt{\sum_{j=1}^{N} e_j^2 / N}$. For h=1,5,20 days , we have N = 755, 750, 735, respectively. We check the statistical significance of the difference in the prediction error among alternative models by using the Diebold-Mariano (DM) test (see Diebold and Mariano, 1995). We consider the squared error loss function. The null hypothesis is that model 1 and 2 have the same prediction accuracy. If the test takes a negative (positive) value this means that model 1 (2) is more accurate than model 2 (1).

[INSERT TABLE 8]

As may be seen in Table 8, for all contracts and prediction horizons SFP outperforms CF in terms of forecasting accuracy. The difference in prediction error is statistically significant in all cases.

5.4. Di Poto and Fanone (2012) Lévy multifactor market

Estimation results of this model are in Appendix D. We compare this model against the SFP model and against CF model by looking at out-of-sample prediction errors for eighteen swap prices in the period from January 1, 2013 to December 30, 2015. As was the case in the previous section, we check the statistical significance of the difference in the prediction error among alternative models by using the Diebold-Mariano (DM) test with squared error loss function. The null hypothesis is that model 1 and 2 have the same prediction accuracy. If the test takes a negative (positive) value this means that model 1 (2) is more accurate than model 2 (1).

[INSERT TABLE 9]

Table 9 contains DM test for the comparison between SFP and DF. In 52% of cases both models present similar forecasting accuracy, in 33% of cases SFP is more accurate than DF whereas in the remaining 15% DF is better than SFP.

[INSERT TABLE 10]

Table 10 contains DM test for the comparison between DF and CF. In all cases and for all prediction horizons, DF outperforms CF in terms of forecasting accuracy. The difference in prediction error is statistically significant in all cases. As a further comparison, in Table 11 we present root mean squared errors for fitting the term structure of volatilities by market segments using HJM, SFP and DF. In all market segments, SFP provides best fit, followed by DF whereas HJM presents highest errors.

[INSERT TABLE 11]

6. Value at Risk

In order to compare performance of alternative models in comparison with SFP, we compute Value-at-Risk (VaR) at different probability levels over a one-day horizon. Details of the procedure are in Appendix E. We present results in Table 12. It may be seen that the VaR_Q^{Normal} , calculated under the assumption of normality tends to understate risk, this understatement being strong for high confidence levels (99.5% and 99.99%) suggesting that tail risk is underestimated. On the other hand, VaR_Q^{NIG} tends to mildly overestimate risk at relatively low significance levels but it is able to properly account for extreme tail risk. It is worth noting that the overestimation of risk provided by NIG distributions are proportionally much lower than the underestimation of risk produced by the normal distribution.

One important practical implication of our results is as follows. We know that VaR models allow users to control risk and decide how to allocate limited resources.

Financial intermediaries impose a capital charge to traders based on risk-adjusted capital. This creates a natural incentive for traders to take a position only when they have strong views on markets. If they have no views, they should abstain from trading. Our results suggest that the risk-adjusted capital for traders using EEX swap electricity contracts should be increased in comparison with the standard practice based on the normality assumption. Traders should also rationally adjust positions as risk changes (in the face of an increasingly volatile environment a sensible response is to scale down positions). Furthermore and given that VaR is also a performance evaluation tool, the evaluators of the performance of the traders should adjust their measures accordingly.

[INSERT TABLE 12]

7. Conclusions

Modelling the dynamics of electricity swap prices is a challenging task for academics as well as for market participants, such as generators, traders, and speculators. The main reason is that risk measures and option prices depend crucially on the distributions driving state variables. From a structural point of view, we posit that swap electricity prices result from at least three driving forces. First, a stochastic factor acting as an anchor of the overall level of the curve, this factor representing average “consensus” price for contracts within a given maturity slot (yearly, quarterly, monthly). Second, a factor reflecting deterministic trend-seasonal components because of market anticipation of weather-related variations in demand. Third, a contract-specific factor accounting for (mean-reverting) stochastic deviations from above factors, these deviations depend on time to maturity and length of delivery period.

Besides that, innovations driving stochastic factors are non-normal, a critical fact that should be taken properly into account. In particular, failure for account for asymmetries and fat tails leads to theoretical prices that are not compatible with market prices. Our SFP model takes into account all these features. By using a MNIG distribution, our model embodies realistic probabilities of occurrence of extreme prices. We test the model using EEX data for the German market in the period 2004-2015. SFP outperforms four competitors, both in in-sample valuation and in out-of-sample forecasting, and in fitting the term structure of volatilities by market segments. Competitor models are based on (i) diffusion spot prices, (ii) jump-diffusion spot prices with time dependent volatility, (iii) HJM approach and (iv) Lévy multifactor model based on the NIG distribution. Another remarkable point is the ability our model has in capturing extreme tail risk, as suggested by results of VaR analysis. A practical implication of our results is that capital charges to traders using EEX electricity contracts, based on risk-adjusted capital and on normality assumptions, are likely to be too low. We suggest increasing them. In addition, evaluators of traders' performance should adjust their recommendations accordingly.

Looking forward, an application of our model to other electricity markets offers an interesting topic for further investigation, given the local behaviour of many electricity markets. The application of our model to pricing of options, structured derivative contracts, hedging strategies, portfolio diversification, and risk management purposes represent other natural directions for further research.

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Table 1: Summary Statistics

This table reports summary statistics for daily swap prices over the full sample period from 6/1/2004 to 12/31/2012. Sample size is 2179 observations. JB is the Jarque-Bera normality test. Probability is the p-value of this test under the null of normality.

	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6
Panel A: Daily Swap Prices																		
Mean	48.74	49.92	50.60	50.99	51.45	51.50	50.44	51.47	51.46	51.74	52.47	52.76	52.00	52.61	53.52	55.23	56.69	57.54
Median	47.88	48.85	49.20	49.17	49.13	49.66	48.75	49.60	49.98	51.79	52.41	51.84	51.94	53.70	54.59	55.30	56.15	57.30
Maximum	98.41	96.76	98.23	101.94	101.00	102.75	97.50	100.93	94.95	84.75	94.07	98.33	90.15	89.00	89.67	90.30	96.30	96.80
Minimum	26.50	26.45	28.25	29.05	30.35	30.55	28.69	31.17	31.05	30.50	30.55	31.70	33.12	33.70	34.40	36.69	37.51	38.29
Std. Dev.	12.82	12.95	13.15	13.01	12.86	12.48	12.76	12.25	10.89	10.33	11.30	11.54	9.90	9.58	9.73	10.04	11.09	11.13
Skewness	0.80	0.79	1.04	1.23	1.25	1.19	0.98	1.19	0.56	0.36	0.72	0.81	0.64	0.30	0.27	0.25	0.58	0.55
Kurtosis	3.76	3.68	4.29	5.06	5.37	5.37	4.25	5.28	3.43	3.21	4.22	4.65	4.72	4.48	4.33	3.76	4.18	4.07
JB	287.2	269.4	541.4	939.8	1072.9	1024.5	488.0	985.3	132.3	51.67	324.9	485.2	415.6	232.4	187.5	73.78	249	215.5
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel B: Average Series M, Q and Y																		
	M	Q	Y															
Mean	50.53	51.73	54.60															
Median	48.98	50.73	54.83															
Maximum	99.85	95.09	92.04															
Minimum	28.53	30.61	35.62															
Std. Dev.	12.88	11.51	10.24															

Table 2: Average Swap Price Statistics

This table reports summary statistics for average swap prices computed as the geometric average of daily swap prices within each market segment. Sample period is from 6/1/2004 to 12/31/2012.Sample size is 2179 observations. JB is the Jarque-Bera normality test. Probability is the p-value of this test under the null of normality.

	M	Q	Y
	Panel A: $\bar{F}$		
Mean	50.330	51.470	54.520
Median	48.630	50.520	55.190
Maximum	93.630	92.470	91.620
Minimum	30.410	32.210	35.780
Std. Dev.	12.010	10.260	10.010
Skewness	1.090	0.870	0.450
Kurtosis	4.840	5.020	4.520
JB	742	644	283
Probability	0.000	0.000	0.000
Correlation	$ln\bar{F}_M$	$ln\bar{F}_Q$	$ln\bar{F}_Y$
$ln\bar{F}_M$	1.00		
$ln\bar{F}_Q$	0.94	1.00	
$ln\bar{F}_Y$	0.77	0.91	1.00
	Panel B: $\nabla ln\bar{F}$		
Mean	0.000	0.000	0.000
Median	0.000	0.000	0.000
Maximum	0.130	0.080	0.070
Minimum	-0.080	-0.070	-0.060
Std. Dev.	0.013	0.011	0.008
Skewness	0.330	0.090	0.330
Kurtosis	9.810	9.610	12.610
JB	4251	3973	8426
Probability	0.000	0.000	0.000
Correlation	$\nabla ln\bar{F}_M$	$\nabla ln\bar{F}_Q$	$\nabla ln\bar{F}_Y$
$\nabla ln\bar{F}_M$	1.00		
$\nabla ln\bar{F}_Q$	0.84	1.00	
$\nabla ln\bar{F}_Y$	0.68	0.89	1.00

Table 3. Summary Statistics Stochastic Forward Premium (SFPs)

This table reports summary statistics for SFPs, in levels $\gamma_i(T)$ (Panel A) and in first differences $\nabla\hat{\gamma}_i(t,T)$ (Panel B) for the full sample period from 6/1/2004 to 12/31/2012. Sample size is 2179 observations. JB is the Jarque-Bera normality test. Boldface indicates rejection of the null hypothesis of normality.

	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6
Panel A: SFPs Levels $\gamma_i(T)$																		
Mean	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	0.007	0.003	0.004	0.001	-0.001	0.000	0.016	0.004	0.000	0.001	-0.001	-0.007	0.026	0.009	0.001	-0.011	-0.012	-0.014
Maximum	0.171	0.173	0.156	0.191	0.202	0.284	0.215	0.130	0.214	0.311	0.206	0.189	0.142	0.053	0.040	0.079	0.085	0.077
Minimum	-0.241	-0.208	-0.186	-0.158	-0.197	-0.228	-0.305	-0.163	-0.269	-0.182	-0.125	-0.175	-0.187	-0.069	-0.041	-0.044	-0.062	-0.067
Std. Dev.	0.072	0.059	0.053	0.053	0.058	0.071	0.088	0.053	0.060	0.063	0.054	0.063	0.062	0.028	0.012	0.027	0.032	0.035
Skewness	-0.574	-0.439	-0.360	0.124	0.394	0.341	-0.779	-0.437	-0.176	0.492	0.324	0.262	-0.806	-0.380	-0.183	1.073	0.581	0.346
Kurtosis	3.614	3.638	3.519	3.354	4.222	4.572	3.516	3.260	4.652	4.668	3.000	2.873	2.859	2.037	3.423	3.274	2.288	1.821
JB	**154.04**	**106.9**	**71.43**	**16.97**	**191.89**	**266.69**	**244.67**	**75.56**	**259.09**	**340.62**	**38.09**	**26.35**	**238.00**	**136.5**	**28.36**	**425.32**	**168.8**	**169.7**
Panel B: First Differences of SFPs $\nabla\hat{\gamma}_i(t,T)$																		
Mean	-0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	0.072	0.037	0.033	0.043	0.028	0.042	0.062	0.025	0.028	0.028	0.026	0.031	0.022	0.022	0.018	0.017	0.020	0.047
Minimum	-0.077	-0.067	-0.023	-0.034	-0.035	-0.041	-0.045	-0.025	-0.022	-0.024	-0.032	-0.037	-0.040	-0.023	-0.015	-0.016	-0.022	-0.032
Std. Dev.	0.011	0.006	0.005	0.006	0.007	0.008	0.007	0.004	0.004	0.005	0.005	0.005	0.005	0.003	0.003	0.003	0.003	0.004
Skewness	-0.099	-0.120	0.225	0.070	-0.195	0.107	0.258	-0.219	0.360	0.343	0.211	-0.019	-0.484	-0.130	0.558	0.158	-0.050	0.745
Kurtosis	9.826	11.41	5.558	6.478	5.068	5.782	10.302	5.986	5.585	5.415	6.364	5.706	7.944	8.518	9.409	6.747	10.36	22.06
JB	**4231.7**	**6434.**	**612.16**	**1099.8**	**401.99**	**706.67**	**4863.5**	**826.72**	**653.29**	**571.71**	**1042.7**	**664.49**	**2303.7**	**2769.**	**3840.**	**1283.4**	**4926**	**33197**

Table 4. Correlations Average factors and SPFs

This table presents the correlations between returns of average factors $\nabla ln\bar{F}_i(t)$ and returns of SPF factors $\nabla\hat{\gamma}_i(t,T)$ for each segment. Sample size is 2178 observations. Boldface indicates statistical significance at 5% level.

	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6	$\nabla ln\bar{F}_M$	$\nabla ln\bar{F}_Q$	$\nabla ln\bar{F}_Y$
M1	1.00																				
M2	**0.43**	1.00																			
M3	**-0.28**	-0.06	1.00																		
M4	**-0.58**	**-0.43**	**0.10**	1.00																	
M5	**-0.60**	**-0.56**	-0.10	**0.23**	1.00																
M6	**-0.59**	**-0.56**	**-0.19**	**0.15**	**0.32**	1.00															
Q1	**0.49**	**0.45**	**0.14**	**-0.29**	**-0.45**	**-0.53**	1.00														
Q2	-0.05	-0.13	**-0.19**	-0.01	0.14	**0.18**	**0.13**	1.00													
Q3	**-0.14**	**-0.13**	0.01	0.04	**0.10**	**0.18**	**-0.35**	-0.05	1.00												
Q4	**-0.10**	-0.07	0.02	0.06	0.05	**0.09**	**-0.42**	**-0.38**	0.07	1.00											
Q5	**-0.19**	-0.08	0.07	**0.15**	**0.12**	0.07	**-0.35**	**-0.44**	**-0.25**	0.06	1.00										
Q6	**-0.25**	**-0.27**	-0.12	**0.18**	**0.26**	**0.29**	**-0.50**	**-0.22**	**-0.13**	-0.09	0.07	1.00									
Y1	**0.18**	**0.15**	0.07	**-0.09**	**-0.19**	**-0.18**	**0.14**	-0.01	0.02	0.05	-0.03	**-0.22**	1.00								
Y2	-0.01	0.04	0.02	0.02	-0.03	-0.02	-0.08	-0.06	0.04	0.04	**0.11**	-0.01	**0.57**	1.00							
Y3	-0.08	-0.06	-0.05	0.03	**0.11**	0.08	-0.11	0.00	0.01	0.00	0.03	**0.11**	**-0.14**	**0.20**	1.00						
Y4	-0.08	-0.08	-0.04	0.05	0.08	**0.09**	-0.07	0.01	-0.01	0.00	-0.01	0.10	**-0.49**	**-0.39**	0.05	1.00					
Y5	-0.05	-0.06	-0.03	0.00	0.06	**0.09**	0.01	0.05	-0.03	-0.06	-0.04	0.06	**-0.66**	**-0.70**	**-0.32**	**0.13**	1.00				
Y6	-0.06	-0.05	-0.01	0.04	0.07	0.04	0.01	0.00	-0.04	-0.05	-0.03	0.08	**-0.61**	**-0.68**	**-0.39**	-0.02	**0.58**	1.00			
$\nabla ln\bar{F}_M$	**0.46**	**0.35**	-0.07	**-0.31**	**-0.33**	**-0.35**	**0.50**	**0.21**	**-0.10**	**-0.18**	**-0.29**	**-0.35**	**0.60**	**0.30**	**-0.13**	**-0.34**	**-0.35**	**-0.30**	1.00		
$\nabla ln\bar{F}_Q$	**0.46**	**0.35**	-0.07	**-0.31**	**-0.33**	**-0.35**	**0.50**	**0.21**	-0.10	-0.18	-0.29	**-0.35**	**0.60**	**0.30**	**-0.13**	**-0.34**	**-0.35**	**-0.30**	**0.99**	1.00	
$\nabla ln\bar{F}_Y$	0.07	0.07	-0.02	-0.04	-0.03	-0.08	-0.03	-0.01	0.02	0.08	0.03	-0.07	**0.40**	**0.29**	**-0.10**	**-0.30**	**-0.26**	**-0.18**	**0.69**	**0.69**	1.00

Table 5. Estimation of SURE and MNIG Models   OLD OLD

This table presents estimates of model (12) $\begin{pmatrix} \nabla ln\bar{F}_i(t) \\ \nabla\hat{\gamma}_i(t,T) \end{pmatrix} = \begin{pmatrix} \kappa_i(\zeta_i - ln\bar{F}_i(t-1)) \\ -\varpi_{i,T}\hat{\gamma}_i(t-1,T) \end{pmatrix} + \begin{pmatrix} \theta_{\bar{F}_i}\varepsilon_{\bar{F}_i}(t) \\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t) \end{pmatrix}$ *i=1(M),2(Q),3(Y)   T=1,…,6   t=1,…,n*

SURE system contains 21 equations (3+6×3) and is estimated using the feasible generalized least squares (FGLS) method. To take into account residual autocorrelation, a first order AR(1) term is included in all equations. In the second step we use standardized residuals from Equation (12) $\varepsilon_{\bar{F}_i}$ and $\varepsilon_{\gamma_i(T)}$ using the Urzua (1997) method based on Equation (15) as estimations of the vector ***dL****(t)*, which is MNIG. The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations. Standard errors for the parameters in the first step are heteroscedasticity- and autocorrelation consistent. Standard errors for the NIG parameters are based on generating 2000 bootstrapped samples. * denotes significance at 5% level and ** at 1% level. Likelihood ratio test is computed as follows LR = -2*LOGLK(N(0,1)) + 2*LOGLN(NIG(α,β,δ,μ)). For each equation and marginal distribution, Panel A contains results for average series factors and the estimation of their marginal NIG distributions using standardized (0,1) residuals. Panel B contains the estimation of the parameters of MNIG distribution, using standardized and orthogonalized residuals from Equations (12) and (15). Panels C and D contain the same information for SFPs within each market segment (monthly, quarterly and yearly).

	M	Q	Y
	Panel A: Estimation $\nabla ln\bar{F}_i(t)$ and Marginal NIG		
κ_i	0.0017**	0.0022**	0.0019**
ζ_i	3.726**	3.889**	3.998**
$\theta_{\bar{F}_i}$	0.013	0.011	0.008
α	0.711**	0.686**	0.556**
β	0.036	0.022	0.045
δ	0.708**	0.685**	0.550**
μ	-0.036	-0.022	-0.045
χ	0.043	0.028	0.075
ξ	0.858**	0.869**	0.924**
LR	41119**	37137**	29287**
	Panel B: Parameters of MNIG residuals		
α	0.855**	0.855**	0.855**
β	0.011	0.001	0.025
δ	0.848**	0.848**	0.848**
μ	0.012	0.002	0.001

Table 5. Estimation of SURE and MNIG Models (cont.)

This table presents estimates of model (12) $\begin{pmatrix}\nabla ln\bar{F}_i(t)\\ \nabla\hat{\gamma}_i(t,T)\end{pmatrix} = \begin{pmatrix}\kappa_i(\zeta_i - ln\bar{F}_i(t-1))\\ -\varpi_{i,T}\hat{\gamma}_i(t-1,T)\end{pmatrix} + \begin{pmatrix}\theta_{\bar{F}_i}\varepsilon_{\bar{F}_i}(t)\\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t)\end{pmatrix}$ *i=1(M),2(Q),3(Y)* *T=1,…,6* *t=1,…,n*

SURE system contains 21 equations (3+6×3) and is estimated using the feasible generalized least squares (FGLS) method. To take into account residual autocorrelation, a first order AR(1) term is included in all equations. In the second step we use standardized residuals from Equation (12) $\varepsilon_{\bar{F}_i}$ and $\varepsilon_{\gamma_i(T)}$ using the Urzua (1997) method based on Equation (15) as estimations of the vector ***dL(t)***, which is MNIG. The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations. Standard errors for the parameters in the first step are heteroscedasticity- and autocorrelation consistent. Standard errors for the NIG parameters are based on generating 2000 bootstrapped samples. * denotes significance at 5% level and ** at 1% level. Likelihood ratio test is computed as follows LR = -2*LOGLK(N(0,1)) + 2*LOGLN(NIG(α,β,δ,μ)). For each equation and marginal distribution, Panel A contains results for average series factors and the estimation of their marginal NIG distributions using standardized (0,1) residuals. Panel B contains the estimation of the parameters of MNIG distribution, using standardized and orthogonalized residuals from Equations (12) and (15). Panels C and D contain the same information for SFPs within each market segment (monthly, quarterly and yearly).

	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6
Panel C: Estimation $\nabla\hat{\gamma}_i(t,T)$ and Marginal NIG																		
$-\varpi_{i,T}$	-0.006**	-0.007**	-0.004**	0.006**	-0.006**	-0.005**	0.0006*	-0.001*	-0.003**	0.000	-0.0006*	-0.001*	-0.0008*	-0.003**	-0.021**	-0.003**	-0.0017*	-0.004**
$\theta_{\gamma_i(T)}$	0.011	0.006	0.005	0.006	0.007	0.008	0.007	0.004	0.004	0.005	0.005	0.005	0.005	0.003	0.003	0.003	0.003	0.004
α	0.659**	0.590**	1.114**	0.929**	1.240**	1.037**	0.644**	0.970**	0.867**	1.035**	0.961**	1.054*	0.838**	0.766**	0.708**	0.906**	0.655**	0.409**
β	-0.024	-0.020	0.096	0.021	-0.102	0.036	0.024	-0.067	0.089	0.102	0.065	-0.003	-0.112	-0.029	0.084	0.072	-0.016	0.040
δ	0.6578**	0.589**	1.102**	0.929**	1.227**	1.035**	0.642**	0.963**	0.853**	1.020**	0.955**	1.054**	0.816**	0.765**	0.693**	0.898**	0.654**	0.403**
μ	0.024	0.020	-0.095	-0.021	0.101	-0.036	-0.024	0.067	-0.088	-0.101	-0.064	0.003	0.110	0.029	-0.083	-0.072	0.016	-0.040
χ	-0.032	-0.030	0.056	0.016	-0.048	0.024	0.033	-0.050	0.080	0.068	0.049	-0.002	-0.107	-0.032	0.102	0.060	-0.022	0.095
ξ	0.881**	0.910**	0.650**	0.744**	0.589**	0.687**	0.888**	0.724**	0.782**	0.692**	0.729**	0.678**	0.799**	0.830**	0.862**	0.759**	0.883**	0.968**
LR	31519**	37140**	23287**	14753**	17695**	18847**	29519**	41137**	38767**	16113**	19655**	14567**	37779**	48127**	35541**	14432**	16755**	14517**
Panel D: Parameters of MNIG residuals																		
α	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**	0.855**
β	0.011	0.001	0.025	-0.025	-0.008	0.045	-0.013	0.009	0.029	0.032	0.033	0.039	-0.048	0.005	0.019	0.034	0.016	0.011
δ	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**	0.848**
μ	0.012	0.002	0.001	0.0345	0.008	0.005	0.011	0.01	0.0243	-0.034	-0.016	-0.048	-0.071	0.031	0.047	0.007	0.078	0.053

Table 6. Correlations of the residuals of model (12)

This table presents the correlations of the residuals from (12) $\begin{pmatrix}\nabla ln\bar{F}_i(t)\\ \nabla\hat{\gamma}_i(t,T)\end{pmatrix} = \begin{pmatrix}\kappa_i(\zeta_i - ln\bar{F}_i(t-1))\\ -\varpi_{i,T}\hat{\gamma}_i(t-1,T)\end{pmatrix} + \begin{pmatrix}\theta_{\bar{F}_i}\varepsilon_{\bar{F}_i}(t)\\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t)\end{pmatrix}$ *i=1(M),2(Q),3(Y)* *T=1,…,6* *t=1,…,n* . The sample size is 2178 observations. Boldface indicates the correlations needed in order to compute the term structure of swap prices given by (8).

	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6	$\nabla ln\bar{F}_M$	$\nabla ln\bar{F}_Q$	$\nabla ln\bar{F}_Y$
M1	1.00																				
M2	0.42	1.00																			
M3	-0.30	-0.09	1.00																		
M4	-0.57	-0.42	0.10	1.00																	
M5	-0.60	-0.52	-0.05	0.23	1.00																
M6	-0.60	-0.54	-0.14	0.16	0.30	1.00															
Q1	0.51	0.44	0.11	-0.31	-0.44	-0.53	1.00														
Q2	0.00	-0.04	-0.14	-0.05	0.06	0.10	0.19	1.00													
Q3	-0.21	-0.19	0.00	0.10	0.16	0.24	-0.37	0.02	1.00												
Q4	-0.19	-0.15	0.01	0.12	0.12	0.18	-0.49	-0.35	0.10	1.00											
Q5	-0.22	-0.14	0.01	0.15	0.18	0.15	-0.47	-0.46	-0.18	0.12	1.00										
Q6	-0.24	-0.21	-0.07	0.18	0.21	0.24	-0.52	-0.41	-0.11	0.01	0.16	1.00									
Y1	0.18	0.15	0.08	-0.10	-0.19	-0.20	0.16	0.00	0.01	0.04	-0.03	-0.25	1.00								
Y2	-0.01	0.02	0.02	0.01	-0.01	-0.01	-0.09	-0.06	0.05	0.05	0.11	0.00	0.56	1.00							
Y3	-0.08	-0.06	-0.05	0.03	0.11	0.08	-0.11	-0.01	0.01	0.00	0.04	0.12	-0.15	0.21	1.00						
Y4	-0.08	-0.08	-0.04	0.05	0.08	0.10	-0.07	0.00	0.00	0.00	0.00	0.11	-0.51	-0.40	0.07	1.00					
Y5	-0.05	-0.06	-0.03	0.02	0.05	0.09	0.00	0.05	-0.02	-0.06	-0.05	0.06	-0.67	-0.71	-0.32	0.16	1.00				
Y6	-0.06	-0.05	-0.02	0.05	0.07	0.04	0.00	0.01	-0.03	-0.05	-0.03	0.09	-0.61	-0.68	-0.39	0.00	0.59	1.00			
$\nabla ln\bar{F}_M$	**0.47**	**0.37**	**-0.06**	**-0.32**	**-0.35**	**-0.38**	0.53	0.23	-0.14	-0.25	-0.29	-0.38	0.60	0.31	-0.14	-0.35	-0.35	-0.31	1.00		
$\nabla ln\bar{F}_Q$	0.17	0.15	0.02	-0.11	-0.14	-0.18	**0.18**	**0.11**	**-0.01**	**-0.07**	**-0.08**	**-0.22**	0.69	0.45	-0.13	-0.44	-0.46	-0.38	0.84	1.00	
$\nabla ln\bar{F}_Y$	0.06	0.07	-0.02	-0.04	-0.02	-0.08	-0.02	0.01	0.02	0.04	0.04	-0.06	**0.40**	**0.30**	**-0.11**	**-0.31**	**-0.27**	**-0.19**	0.69	0.89	1.00

Table 7: Volatility Term Structure

This table presents the market volatility of swap returns and estimated volatility using the term structure of swap prices variances. The term structure of swap prices variances is given by $\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,\iota}}\theta_{\gamma_i(T)}v_{\overline{F_\iota},\gamma_i(T)}$; $i = 1,\dots,I$; $T = 1,\dots,N$ where $\eta_i^2 = \theta_{\overline{F,\iota}}{}^2 \times v_{\overline{F_\iota,F_\iota}}$ and $\tau_{i,T}^2 = \theta_{\gamma_i(T)}{}^2 \times v_{\gamma_i(T),\gamma_i(T)}$ We set $\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,\iota}}\theta_{\gamma_i(T)}\omega_{\overline{F_\iota},\gamma_i(T)}$ where $\eta_i^2 = \theta_{\overline{F,\iota}}{}^2 \times \omega_{\overline{F_\iota,F_\iota}}$ and $\tau_{i,T}^2 = \theta_{\gamma_i(T)}{}^2 \times \omega_{\gamma_i(T),\gamma_i(T)}$ are the variances of the corresponding average factor and of the stochastic discount factor respectively and $\omega_{\overline{F_\iota},\gamma_i(T)}$ is the covariance between the two factors, all of them obtained from (12). The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations

	Market	Model	Relative Error	Absolute Error
M1	0.3264	0.3269	-0.15%	0.15%
M2	0.2613	0.2621	-0.31%	0.31%
M3	0.2191	0.2192	-0.05%	0.05%
M4	0.2002	0.2003	-0.05%	0.05%
M5	0.1999	0.1991	0.40%	0.40%
M6	0.1997	0.1988	0.45%	0.45%
Q1	0.2245	0.2248	-0.13%	0.13%
Q2	0.1925	0.1928	-0.16%	0.16%
Q3	0.1831	0.1831	0.00%	0.00%
Q4	0.1815	0.1813	0.11%	0.11%
Q5	0.1843	0.1841	0.11%	0.11%
Q6	0.1759	0.1753	0.34%	0.34%
Y1	0.1738	0.1739	-0.06%	0.06%
Y2	0.1506	0.1511	-0.33%	0.33%
Y3	0.1311	0.1312	-0.08%	0.08%
Y4	0.1237	0.1231	0.49%	0.49%
Y5	0.1264	0.1259	0.40%	0.40%
Y6	0.1336	0.1331	0.37%	0.37%
Average			0.08%	0.22%

Table 8: Diebold-Mariano Test: Stochastic Forward Premium vs Cartea-Figueroa

This table presents test values for DM test. Forecasting period is from 1/1/2013 to 12/30/2015. Sample size is 756 observations. We consider the squared error loss function. The null hypothesis is that model SFP and CF have the same prediction accuracy. If the test takes a negative (positive) value this means that model SFP (CF) is more accurate than model CF (SFP). Boldface means statistical significance at 1% level.

	1-day	5-days	20-days
M1	**-33.71**	**-11.61**	**-5.62**
M2	**-36.13**	**-11.89**	**-5.74**
M3	**-41.78**	**-13.98**	**-6.92**
M4	**-47.04**	**-15.69**	**-7.74**
M5	**-37.85**	**-12.68**	**-6.31**
M6	**-30.01**	**-9.93**	**-4.71**
Q1	**-46.83**	**-15.59**	**-7.55**
Q2	**-32.16**	**-31.63**	**-29.46**
Q3	**-29.23**	**-28.71**	**-26.21**
Q4	**-42.01**	**-41.83**	**-40.38**
Q5	**-37.03**	**-36.29**	**-34.14**
Q6	**-29.82**	**-29.49**	**-28.01**
Y1	**-37.94**	**-37.87**	**-37.21**
Y2	**-40.99**	**-40.96**	**-40.47**
Y3	**-47.56**	**-47.53**	**-47.12**
Y4	**-49.18**	**-49.16**	**-48.78**
Y5	**-41.95**	**-41.92**	**-41.42**
Y6	**-39.25**	**-39.22**	**-38.67**

Table 9: Diebold-Mariano Test: Stochastic Forward Premium vs Di Poto-Fanone

This table presents test values for DM test. Forecasting period is from 1/1/2013 to 12/30/2015. Sample size is 756 observations. We consider the squared error loss function. The null hypothesis is that model SFP and DF have the same prediction accuracy. If the test takes a negative (positive) value this means that model SFP (DF) is more accurate than model DF (SFP). Boldface means statistical significance at 1% level.

	1-day	5-days	20-days
M1	**2.59**	2.03	1.98
M2	0.98	**3.25**	**2.59**
M3	**-2.56**	-1.21	-1.38
M4	**-2.87**	**-4.96**	-1.89
M5	**-3.69**	**-4.25**	-0.68
M6	-1.89	-1.69	-0.26
Q1	-1.98	2.06	-0.44
Q2	**-2.59**	**-4.36**	-0.98
Q3	**-3.22**	**-6.25**	-1.36
Q4	**-2.98**	**-7.36**	**-2.97**
Q5	-1.98	-1.74	-0.86
Q6	**-4.56**	**2.36**	1.98
Y1	2.02	1.98	**2.98**
Y2	1.26	**2.96**	0.59
Y3	-1.06	**-2.69**	0.46
Y4	-1.96	**-3.12**	**-4.21**
Y5	**-2.78**	2.04	0.71
Y6	**-3.14**	**3.69**	**2.69**

Table 10: Diebold-Mariano Test: Di Poto-Fanone vs Cartea-Figueroa

This table presents test values for DM test. Forecasting period is from 1/1/2013 to 12/30/2015. Sample size is 756 observations. We consider the squared error loss function. The null hypothesis is that model DF and CF have the same prediction accuracy. If the test takes a negative (positive) value this means that model DF (CF) is more accurate than model CF (DF). Boldface means statistical significance at 1% level.

	1-day	5-days	20-days
M1	**-24.39**	**-8.16**	**-3.89**
M2	**-21.03**	**-7.33**	**-8.95**
M3	**-20.98**	**-9.40**	**-9.25**
M4	**-31.24**	**-11.12**	**-5.17**
M5	**-21.09**	**-7.52**	**-4.53**
M6	**-24.38**	**-6.36**	**-3.78**
Q1	**-29.04**	**-9.99**	**-5.89**
Q2	**-30.76**	**-24.61**	**-18.29**
Q3	**-28.26**	**-18.94**	**-17.17**
Q4	**-27.16**	**-27.36**	**-30.18**
Q5	**-23.08**	**-22.60**	**-35.36**
Q6	**-18.90**	**-18.16**	**-18.18**
Y1	**-25.24**	**-25.37**	**-27.22**
Y2	**-34.15**	**-30.05**	**-31.99**
Y3	**-39.47**	**-28.10**	**-35.93**
Y4	**-34.87**	**-26.37**	**-38.08**
Y5	**-37.92**	**-30.07**	**-37.96**
Y6	**-29.06**	**-24.28**	**-35.15**

Table 11: Volatility Term Structure by Segments

This table presents root mean squared errors for fitting the term structure of volatilities by market segments using HJM, SFP and DF. The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations.

	Yearly Contracts	**Quarterly Contracts**	**Monthly Contracts**
HJM	6.05%	19.32%	11.96%
SFP	0.10%	0.12%	0.30%
DF	1.27%	4.09%	1.07%

Table 12: VaR Wald Test

This table reports the results of Wald test for daily swap prices. The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations. **, or * indicates that the coefficient estimate is significantly different from the null hypotheses at the 1%, or 5 level, respectively. VaR is computed as $VaR(i,T)_Q^{Normal} = k(\sigma_{i,T}\sqrt{\Delta t})$ in the Normal case and as $VaR_Q^{NIG}[F_i(t,T)] = \sqrt{VaR_Q[\overline{F}_i(t)]^2 + VaR_Q[\gamma_i(t,T)]^2 + 2Cov(\overline{F}_i(t),\ \gamma_i(t,T))}$ in the NIG case. Under the assumption that the VaR under consideration is accurate, the z-statistic $z = \frac{\sqrt{T}(\frac{N}{T}-c)}{\sqrt{c(1-c)}}$ has an approximate standard normal distribution. A positive (negative) z statistic indicates that the model underestimates (overestimates) risk.

	N(1%)	N(0.5%)	N(0.01%)	NIG(1%)	NIG(0.5%)	NIG(0.01%)
M1	2.84**	3.07**	14.53**	-1.24	-1.79	1.68
M2	3.27**	4.59**	8.10**	-2.32**	-2.39**	1.68
M3	2.84**	5.50**	10.24**	-3.18**	-3.30**	-0.47
M4	2.63**	3.98**	14.53**	-3.39**	-3.30**	-0.47
M5	3.70**	4.89**	16.67**	-3.18**	-3.30**	-0.47
M6	4.35**	6.43**	18.81**	-3.18**	-3.30**	-0.47
Q1	3.70**	6.10**	10.24**	-3.18**	-2.70**	-0.47
Q2	4.56**	6.41**	18.81**	-4.04**	-3.30**	-0.47
Q3	3.70**	6.41**	16.67**	-2.96**	-2.39**	-0.47
Q4	2.41**	3.67**	18.81**	-2.96**	-3.00**	-0.47
Q5	4.35**	6.41**	23.10**	-3.83**	-2.70**	-0.47
Q6	5.21**	7.93**	14.53**	-3.83**	-2.70**	1.68
Y1	4.35**	6.71**	16.67**	-0.17	-1.49	1.68
Y2	4.13**	6.71**	10.24**	-1.46	-2.39**	1.68
Y3	3.70**	6.10**	10.24**	-1.68	-2.70**	1.68
Y4	2.63**	3.67**	10.24**	-3.18**	-2.70**	1.68
Y5	2.63**	5.50**	20.96**	-1.89	-2.39**	-0.47
Y6	3.70**	6.10**	25.24**	-1.68	-2.09*	-0.47

Figure 1: Swap Price Series

The figure presents swap price series in Euros/MWh. Data is for the full sample period from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.

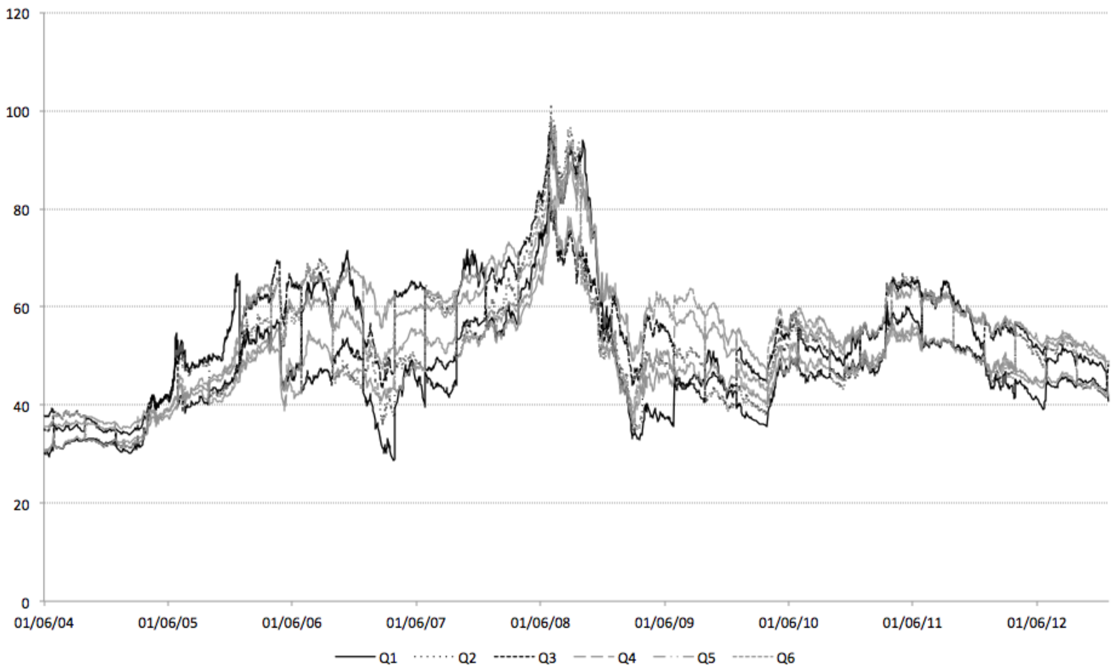


Figure 2: Average Swap Prices

The figure presents average price series in Euros/MWh. Data is for the full sample period from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.

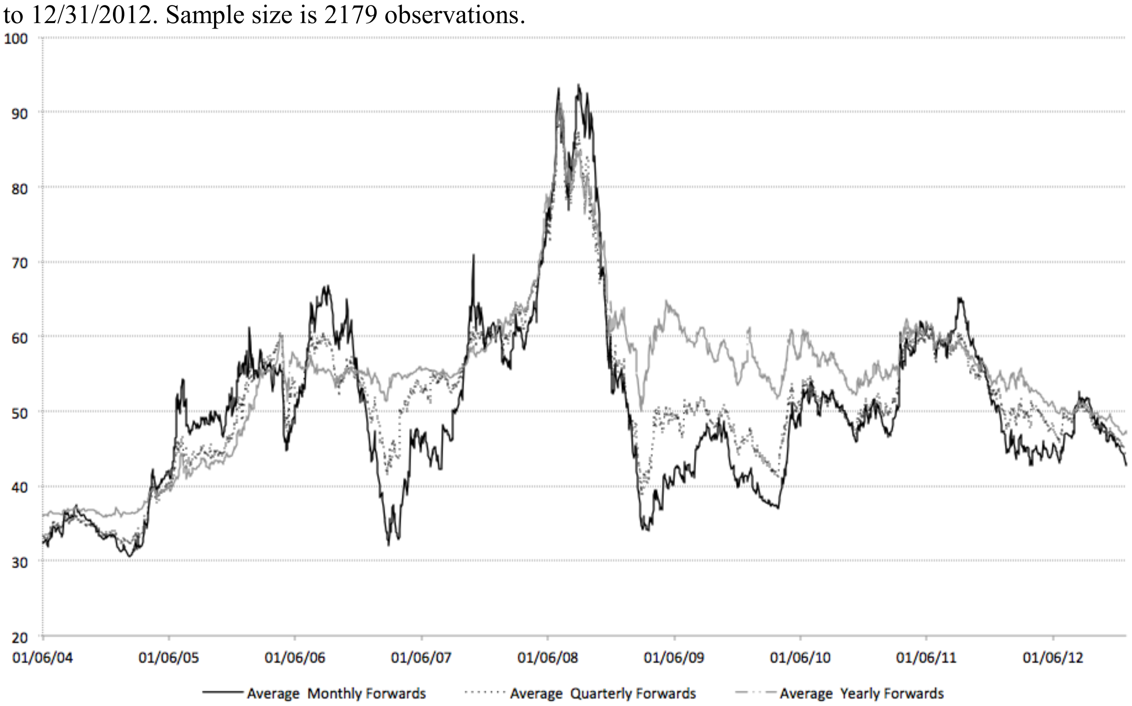


Figure 3A: Seasonal Components M1 Contract

The figure presents the elements of OLS regression $lnF_i(t,T) - ln\bar{F}_i(t) = s_i(t) + \varepsilon_i(t,T)$ where Actual (in red) is the dependent variable $lnF_i(t,T) - ln\bar{F}_i(t)$, Fitted (in green) is the seasonal factor $\hat{s}_i(t) = \hat{\alpha}_{i,1} + \hat{\alpha}_{i,2}\frac{t}{250} + \hat{\alpha}_{i,3}\cos\left(\hat{\alpha}_{i,4} + 2\pi\frac{t}{250}\right) + \hat{\alpha}_{i,5}\cos\left(\hat{\alpha}_{i,6} + 4\pi\frac{t}{250}\right)$ and Residual is the SPF factor defined as $\hat{\gamma}_i(t,T) = lnF_i(t,T) - ln\bar{F}_i(t) - \hat{s}_i(t) = \hat{\varepsilon}_i(t,T)$. Data is M1 prices for the full sample period from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.

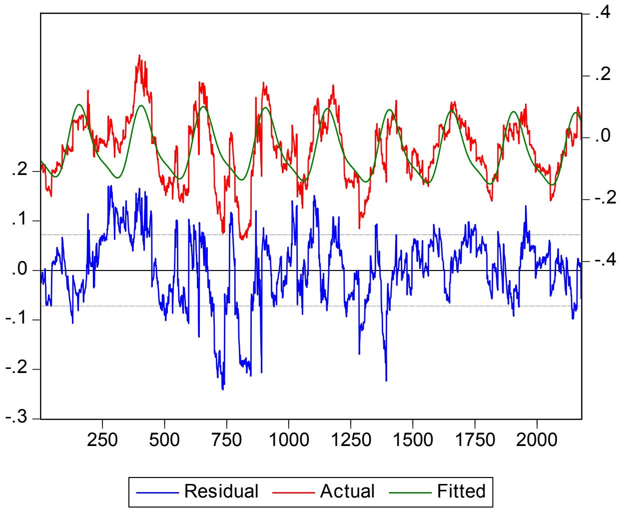
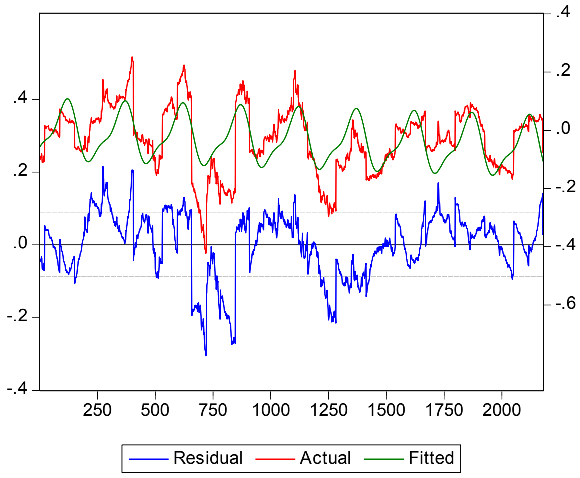


Figure 3B: Seasonal Components Q1 Contract

The figure presents the elements of OLS regression $lnF_i(t,T) - ln\bar{F}_i(t) = s_i(t) + \varepsilon_i(t,T)$ where Actual (in red) is the dependent variable $lnF_i(t,T) - ln\bar{F}_i(t)$, Fitted (in green) is the seasonal factor $\hat{s}_i(t) = \hat{\alpha}_{i,1} + \hat{\alpha}_{i,2}\frac{t}{250} + \hat{\alpha}_{i,3}\cos\left(\hat{\alpha}_{i,4} + 2\pi\frac{t}{250}\right) + \hat{\alpha}_{i,5}\cos\left(\hat{\alpha}_{i,6} + 4\pi\frac{t}{250}\right)$ and Residual is the SPF factor defined as $\hat{\gamma}_i(t,T) = lnF_i(t,T) - ln\bar{F}_i(t) - \hat{s}_i(t) = \hat{\varepsilon}_i(t,T)$. Data is Q1 prices for the full sample period from 6/1/2004 to 12/31/2012. Sample size is 2179 observations.



Appendix A: Spot Prices - One factor Model

We summarize Lucia and Schwartz (2002). Spot electricity prices P_t are characterized as

$$P_t = s(t) + X_t \qquad \text{(A.1)}$$

where $s(t)$ is a deterministic function[28], and X_t, is a mean-reverting stochastic process with constant volatility σ and, under the natural probability measure P follow:

$$dX_t = -kX_t dt + \sigma dZ_t^P \qquad \text{(A.2)}$$

It can be shown (see Cartea and Figueroa, 2005) that under the risk-neutral probability measure Q it follows:

$$dX_t = k(\alpha^* - X_t)dt + \sigma dZ_t^Q \qquad \text{(A.3)}$$

where dZ_t^Q are increments of standard independent Brownian motions Z_t^* the mean reversion parameters are k and X(0)=x_0, where the drift terms are

$$\alpha^* \equiv \frac{-\lambda\sigma}{k} \qquad \text{(A.4)}$$

In this section, we assume the Market Prices of Risk (MPR) of the electricity, which are λ respectively, to be constant over time. Under the risk-neutral measure the spot price P_t follows

$$P_t = s(t) + X_0 e^{-kt} + \alpha^*(1 - e^{-kt}) + \sigma \int_0^t e^{k(s-t)} dZ^Q \qquad \text{(A.5)}$$

The distribution of P_t is Normal with mean given by :

$$E_0^Q(P_t) = s(t) + X_0 e^{-kt} + \alpha^Q(1 - e^{-kt}) \qquad \text{(A.6)}$$

[28] Variables *f(t)* and *fF(t)* include constant terms, deterministic seasonal components as well as other deterministic factors such as calendar effects.

The value of any derivative security must be the expected value, under the risk-neutral measure, of its payoffs discounted to the valuation date at the risk-free rate. Assuming a constant risk-free rate *r*, the value at time zero of a forward contract on the spot price maturing at time *T* must be

$$V_0^T(P_T) = e^{-rT}E_0^*[P_T - F_0(P_0, T)] \tag{A.7}$$

where $F_0(P_0,T)$ is the forward price set at time zero and *T* is the time to maturity. Since the value of a forward contract must be zero when it is first entered into, we obtain a closed form expression for computing forward prices with maturity *T* as follows

$$F_0(P_{0,}T) = E_0^*(P_T) = s(T) + (P_0 - f(0))e^{-kT} + \alpha^*(1 - e^{-kT}) \tag{A.8}$$

The variance of the forward prices are given by

$$Var_0^T(P_T) = \frac{\sigma^2}{2k}(1 - e^{-2kT}) \tag{A.9}$$

These results are for forward contracts providing electricity in a single point in time (T). Given that the swap contract provides delivery of electricity during a period (e.g. during 31 days in January), we use (A.8) to generate prices during the full delivery period (e.g. we generate thirty-one forward prices in the cases of monthly contracts maturing in January and so on), and we take the average. This average is the estimated swap price provided by this spot price model.

Table A1: Estimation of One Factor Model

This table reports the results of regressions (16) and (17). The dependent variable is the average daily EEX spot price (EEX - Phelix Base Hr.01-24 E/Mwh). Our database spans from February 2, 2009 to December 31 2012. Explanatory variables include day of the week dummies as well as the NEG variable that is a dummy variable taking into account negative electricity prices. It is equal to 1 if the price is negative (4 Oct 2009, 26 Dec 2009, 25 Dec 2012 y 26 Dec 2012) and it is zero otherwise. We estimate the coefficients by means of a regression robust to heteroscedasticity, and serial autocorrelation. The results presented correspond to the estimated coefficient, standard errors and t-statistics. The symbol * and ** denotes that the variable is significant at 5% and 1%, respectively.

	Coeff.	s.e.	t-stat
NEG	-79.86**	12.91	-6.18
@WEEKDAY=1	46.25**	0.77	60.09
@WEEKDAY=2	48.06**	0.64	75.30
@WEEKDAY=3	48.14**	0.67	71.49
@WEEKDAY=4	47.48**	0.66	71.76
@WEEKDAY=5	46.22**	0.71	65.28
@WEEKDAY=6	39.84**	0.60	65.99
@WEEKDAY=7	33.60**	0.68	49.21
$\mathbf{1-\hat{k}}$	0.77**	0.03	29.70
$\hat{\boldsymbol{k}}$	0.23**	0.025	8.79
$\hat{\boldsymbol{\sigma}}$	5.86		
$\boldsymbol{R^2}$	58.7		

Figure A1: Parameter Stability

The figure depicts the recursive estimation of the parameters (mean reversion k and volatility σ) for the Spot Series 7 days EEX - Phelix Base Hr.01-24 E/Mwh. We use a 365-day rolling window. There are 1066 estimates. The last window is: 2/01/12-31/12/12. Average values are $k = 0.23$ and $\sigma = 5.86$.

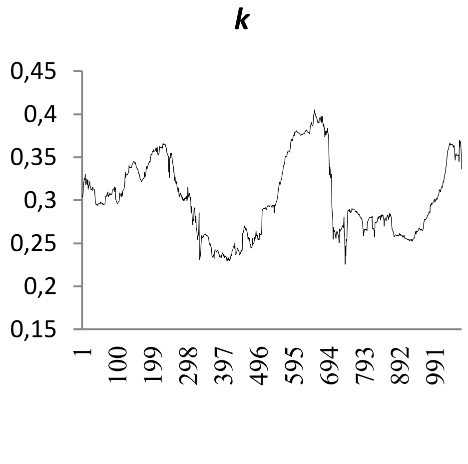
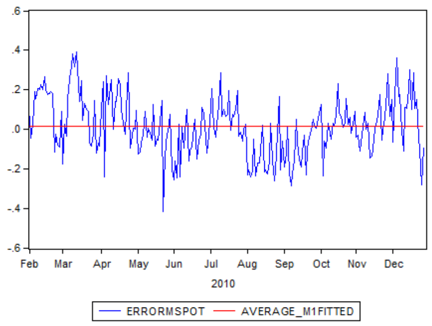


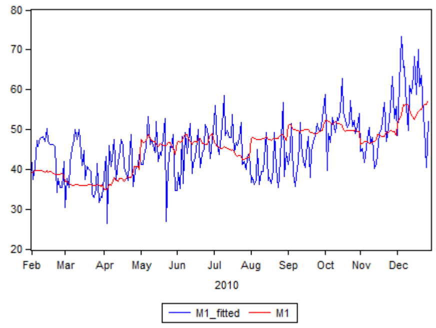
Figure A2: Swap Prices, Fitted Values and Errors in the Spot Model

Using the parameters estimated in Table B1 we compute forward prices we obtain a closed form expression for computing forward prices with maturity T using Equation (A.8) as follows $F_0(P_0,T) = \widehat{s(T)} + (P_0 - f(0))e^{-\hat{k}T} + \hat{\alpha}^*(1 - e^{-\hat{k}T})$. We then generate swap theoretical prices (average of forward prices during the delivery period) using the previous equation for all contracts (denoted M1_fitted, Q1_fitted and Y1_fitted). As an illustration we present the results for M+1, Q+1 and Y+1 denoted M1, Q1 and Y1 (which are the most liquid contracts within each market segment) and compare them against market prices during 2010. The results for the other contracts and periods are available on request.

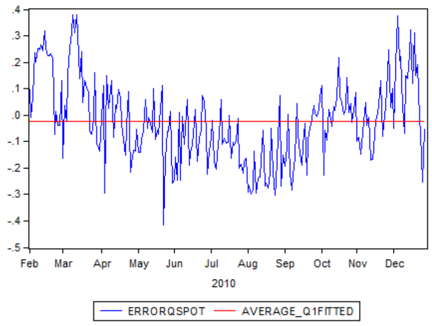
Error M1



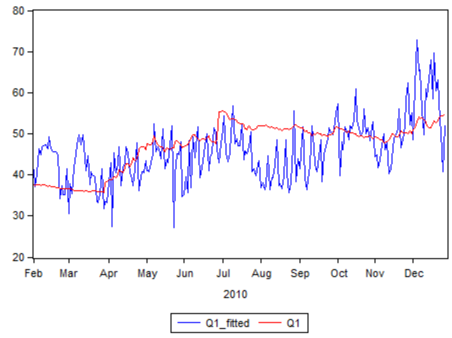
M1 Fitted and M1



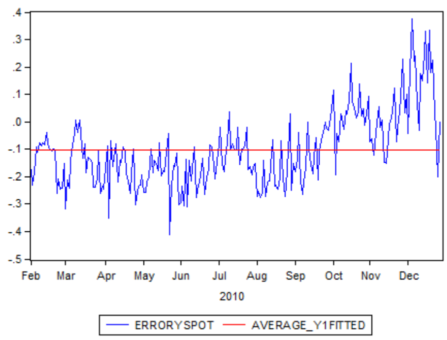
Error Q1



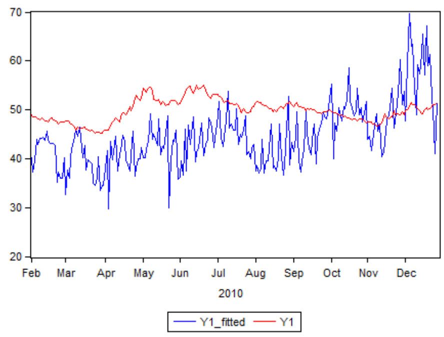
Q1 Fitted and Q1



Error Y1



Y1 Fitted and Y1



Appendix B: HJM Model

We tested several alternative specifications for the HJM model; all of them are available on request. We present the best performing specification, based on using only three contracts with highest liquidity within each segment. Under this specification, the model gives best results in terms of fitting the volatility term structure. [29]. This model is a HJM-based multi-factor stochastic process for electricity swap prices under the real-world probability measure,

$$\frac{dF_i(t,\boldsymbol{T})}{F_i(t,\boldsymbol{T})} = \sum_{k=1}^{N} \alpha_{ki}(t,\boldsymbol{T}) + \sigma_{ki}(t,\boldsymbol{T})dW_t^{ki}, \tag{B.1}$$

Given that we work with liquid contracts within each market segment, we propose specific parameterizations for the volatility functions in (B.1) as follows

$$\frac{dF_i(t,\boldsymbol{T})}{F_i(t,\boldsymbol{T})} = \alpha_i + \sigma_{1i}(t,\boldsymbol{T_i})dW_t^{1i} \tag{B.2}$$

$$\sigma_{1i}(t,\boldsymbol{T_i}) = e^{-k_i(\boldsymbol{T}_i - t)}\sigma_{1i} \tag{B.3}$$

where dW_t^{1i}, are independent Brownian motions for all delivery periods, and $\sigma_{1i}(t,\boldsymbol{T_i})$ are volatility functions. We decide to use the parameterization (B.2)-(B.3) because one factor tends to explain more than 80% of total variation and we look for simple and robust parametrizations. Therefore, we apply a parsimonious representation, that is, one factor. Regarding specific functions to be used, Equation (B.2-B.3) was chosen because of its analytical tractability and at the same time its ability in reflecting the well-known fact that short dated forward returns are more volatile than long dated forwards. To calibrate this model we proceed as follows. We compute returns for contracts available in each case. Yearly contracts (Y1 to Y3), quarterly contracts (Q1 to Q3) and monthly contracts (M1 to M3). Volatility functions are recovered by eigenvector decomposition of the covariance matrix. This decomposition yields a set of independent factors driving the evolution of the variables underlying covariance matrix Σ. We decompose Σ into *n* (n=3) eigenvectors $\boldsymbol{v_i}$ (size 3x1) and associated eigenvalues λ_i such that Σ = RVR' where columns of R are eigenvectors and the principal diagonal in V contains eigenvalues (other

[29] Additional results for other models are available on request.

elements in V are zero). We only consider one eigenvalue. The first volatility function is computed by fitting Equation (B.1) to data $\boldsymbol{v_1}\sqrt{\lambda_1}$.

Statistics of returns series are in Table B1. Average return are not statistically different from zero suggesting overall null drift for the swaps, and therefore we set α_i = 0, $\forall i$. Estimated standard deviation is annualized by the number of trading days (250) and varies from 13% (Y3) to 32% (M1). Volatility is usually higher for the closest to maturity contracts (Samuelson effect), confirming the well-known fact that short dated forward returns tend to be more volatile than long dated forwards. Figure B1 shows (sample period from 2004 to 2012) the term structure of the volatility for each market segment. The distribution of returns presents some skewness and deviates significantly from the normal distribution, as high kurtosis figures on Table B1 suggests.

Eigenvalues resulting from the eigenvector decomposition tell us the importance of each eigenvector and hence the number of factors that we should include in our model. Thus, the first eigenvector is the most important, explaining 87%, 89% and 94% of the total variation in the evolution of the swap curve for the monthly, quarterly and yearly contracts respectively, supporting the reasonableness of assumptions (B.2 –B.3).

Figure B2 shows the first principal component function recovered from the above procedure for each contract type. This first principal component acts to shift forward prices and tilts curves. The most important factor (COMP1) is positive for all maturities, but decreasing with maturity. This implies that a positive shock to the system causes all prices to shift up but by decreasing amounts, depending on the maturity. The longer the maturity, the smaller the increase in prices is. Table B2 presents the parameter estimates from the volatility function obtained in the Principal Component Analysis using equations for the entire sample 2004-2012.

Table B2 presents in Panel B the LS estimates of parameters from volatility functions obtained in PCA using equation $\sigma_{1i}(t,\boldsymbol{T_i}) = e^{-k_i(\boldsymbol{T_i}-t)}\sigma_{1i}$ for the full sample period 2004-2012. Panel A reports the in-sample root mean squared pricing errors (RMSEs). We compute daily errors based on fitted swap prices based on estimated parameters in Panel B. We compute the volatility function implied by the HJM model and by the SFP model and compare results against market prices. In the case of the HJM model, root-mean squared errors (RMSEs) are 6.05%, 19.32%,and 11.96% for monthly, quarterly and yearly contracts respectively. By contrast, RMSEs for those contracts are 0.10%, 0.12% and 0.30% respectively in the case of SFP model. The degree of fit of SFP is substantially higher than HJM's. In the case of the first volatility function, parameter σ_{1i} represents the overall volatility of the forward curve whilst parameter k_i tells us how fast the forward volatility curve decreases with increasing maturity. Parameter σ_{1i} captures the annualized volatility averaged over all contracts of a given class. From Table B1 it is easy to see that average volatility for annual, quarterly and monthly returns is 17%, 19% and 32% respectively. These are very close to estimated parameters σ_{1i} in Table B1, which are 19%, 19% and 38% respectively. The reason of the proximity lies in the low values of the decay factor. Estimated values of this parameter k (0.15, 0.03 and 0.20) suggest a fairly slow decrease in volatility as time to maturity increases. Monthly prices present higher overall volatility and faster decrease in volatility with maturity, followed by quarterly and yearly prices. However, monthly prices present slower volatility attenuation than yearly prices. The degree of fit of the equation is high in the case of yearly prices (99%), followed by monthly (99%) and quarterly prices (83%). To check parameter stability we repeat the calibration exercise using different subsamples 2006-2010 and 2010-2012. A comparison of parameters is in Figure B3, which suggests stability of estimates of parameter. Overall, results suggest that parameters are reasonably stable over time.

It seems fair to say that, although the model seems to fit the volatility term structure to some extent, it is unable to recover the skewness and kurtosis observed in the empirical distributions. By contrast, the SFP model not only fits better the market's volatility term structure, but also is able to take into account skewness and kurtosis.

Table B1: Descriptive Statistics of Returns

The table shows descriptive statistics of returns (1-day changes in the natural logs of swap prices) and the sample covariance matrix of these returns. We study three contracts for each market segment (yearly, quarterly and monthly), contracts M1 to M3, Q1 to Q3 and Y1 to Y3 from 6/1/2004 to 12/31/2012. The "Std. Dev." column reports the standard deviation of the series in annual terms. The nine series are corrected of the rolling effect by means of intervention analysis. p-val is the p-value for the test of zero mean.

	M1	M2	M3	Q1	Q2	Q3	Y1	Y2	Y3
Mean	-0.001	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	0.163	0.149	0.126	0.109	0.099	0.099	0.088	0.070	0.073
Minimum	-0.146	-0.163	-0.239	-0.062	-0.161	-0.074	-0.071	-0.063	-0.064
Std. Dev.	0.329	0.269	0.239	0.193	0.191	0.182	0.174	0.150	0.131
Skewness	0.175	-0.160	-1.837	0.326	-0.905	0.136	0.008	0.163	0.516
Kurtosis	9.999	14.152	37.444	9.801	24.640	10.586	9.424	10.023	14.248
p-val	0.080	0.133	0.195	0.511	0.985	0.984	0.962	0.595	0.998

Table B2: Parameters Estimation and RMSE

The table presents in Panel A the LS estimates of the parameters from the volatility functions obtained in PCA using equation $\sigma_{1i}(t, \boldsymbol{T_i}) = e^{-k_i(T_i-t)}\sigma_{1i}$ for the full sample period 2004-2012. t- statistics are presented in parenthesis. Panel A reports in-sample root mean squared pricing errors for HJM and SFP models. In the case of HJM, the errors are computed daily based on the fitted prices of swap based on estimated parameters in Panel B.

	Yearly Contracts	Quarterly Contracts	Monthly Contracts
Panel B: RMSEs			
HJM	6.05%	19.32%	11.96%
SFP	0.10%	0.12%	0.30%
Panel A: Volatility Function			
σ_1	0.012	0.012	0.024
	(37.34)	(34.75)	(67.90)
K	-0.154	-0.030	-0.201
	(11.32)	(2.23)	(26.09)
R^2	0.992	0.834	0.998

Figure B1: Volatility Functions

The Figure shows the first principal component functions recovered from the above procedure for each contract type (M1, M2, M3; Q1, Q2, Q3 and Y1,Y2, Y3). Sample period 2004-2012

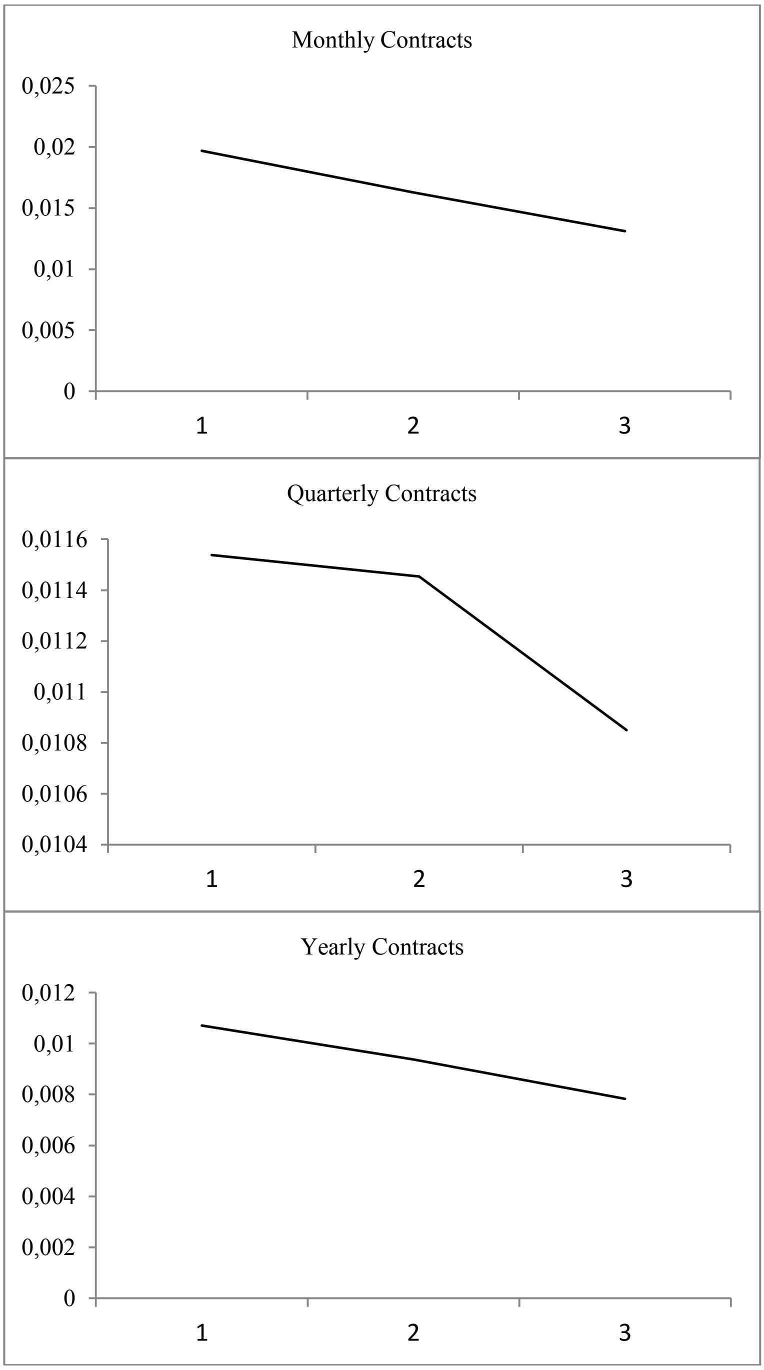
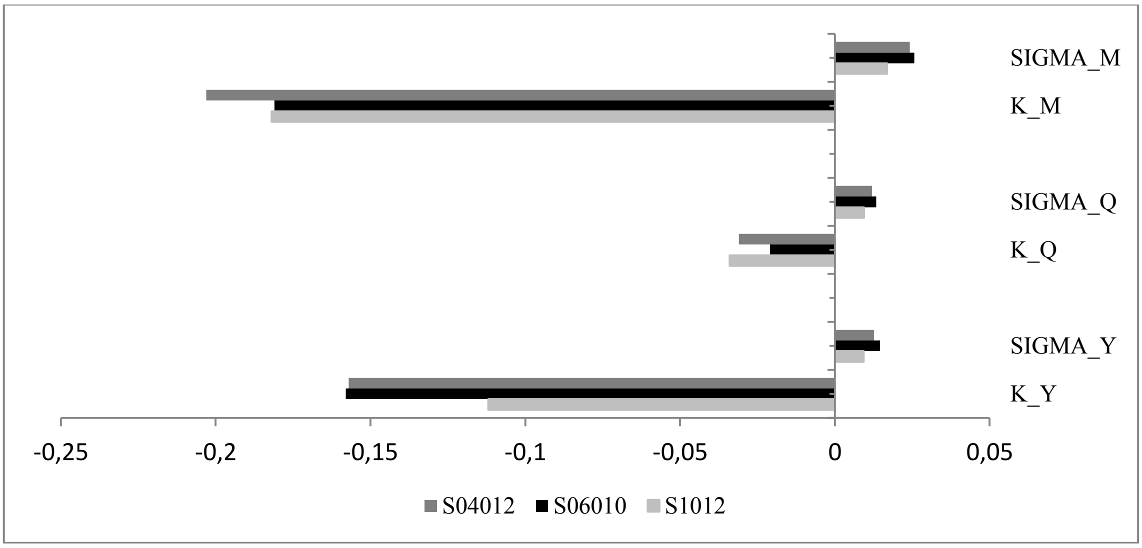


Figure B2: Stability of the parameters by subsamples

To check parameter stability we have repeated the calibration exercise using different subsamples 2006-2010 and 2010-2012. The plot presents a comparison of the parameters.



	S04012	S06010	S1012
SIGMA_M	0.0242	0.0257	0.0169
K_M	-0.203	-0.181	-0.182
SIGMA_Q	0.012	0.0134	0.0094
K_Q	-0.031	-0.021	-0.034
SIGMA_Y	0.0126	0.0146	0.0093
K_Y	-0.157	-0.158	-0.112

Appendix C: Cartea and Figueroa (2005) Model

CF denote the electricity spot price at time 0≤t≤T by P(t), and assume that it takes the form P(t) = $e^{s(t)}$X(t) where s(t) is a deterministic function modelling trend an seasonal effects, and X(t) is a stochastic process modelling the random fluctuations around this trend. We choose the following trend model (Benth et al., 2012)

$$s(t) = \alpha_1 + \alpha_2 \frac{t}{250} + \alpha_3 \cos\left(\alpha_4 + 2\pi \frac{t}{250}\right) + \alpha_5 \cos\left(\alpha_6 + 4\pi \frac{t}{250}\right) \quad (C.1)$$

DF specify the X(t) process for the jump-diffusion model as follows

$$dlnX(t) = -\alpha lnX(t)dt + \sigma(t)dW(t) + lnJdQ(t) \quad (C.2)$$

where $\alpha > 0$ is the speed of mean-reversion, W is a Brownian motion, $\sigma(t) > 0$ is a time-dependent volatility, J is a proportional random jump size and $dQ(t)$ is a Poisson process of daily intensity (arrival rate) l with

$$dQ(t) = \begin{Bmatrix} 1 \; with\; probability \;\; ldt \\ 0 \; with \; probability \; (1-l)dt \end{Bmatrix} \quad (C.3)$$

We follow standard practice and assume that the jump size distribution is lnJ~N (μ_J ;σ_J) and E[J] = 1. We estimate parameters using the method in Cartea and Figueroa (2005) with σ(t) estimated by rolling historical volatility. Estimated parameters are shown in Table C.1

Table C1: CF Estimated parameters

This table contains estimates of CF model. Sample period is from 6/1/2004 to 12/31/2012.Sample size is 2179 observations. * denotes significance at 5% level and ** at 1% level

Parameter	Estimate
α	0.1347**
Average σ(t)	0.1618**
μ_j	-0.1971
σ_j	0.6952**
l	0.0188**

Notice that the estimate of daily intensity l implies an average number of seven jumps per year, which is consistent with the empirical evidence. Next, we compute forward prices using Equation 20 in Cartea and Figueroa (2005)

$$F(t,T) = G(T)\left(\frac{S(t)}{G(t)}\right)^{e^{-\alpha(T-t)}} \exp\left[\int_t^T \left[\frac{1}{2}\sigma^2(s)e^{-2\alpha(T-s)} - \lambda\sigma(s)e^{-\alpha(T-s)}\right] ds + \int_t^T \xi\left(\alpha,\sigma_J^2\right) l ds - l(T-t)\right] \quad (C.4)$$

$$\xi\left(\alpha,\sigma_J^2\right) = \exp\left[-\frac{\sigma_J^2}{2}e^{-\alpha(T-s)} + \frac{\sigma_J^2}{2}e^{-2\alpha(T-s)}\right]$$

Notice that in C.4 an additional parameter λ, Market Price of Risk (MPR) is included. By using theoretical forward prices and comparing with average market prices, we extract the implied lambda (MPR). In Table C2 we show values of lambda by contract.

Table C2: CF Market Price of Risk

This table contains estimates of the Market Price of Risk for CF model using Equation C.4. Sample period is from 6/1/2004 to 12/31/2012.Sample size is 2179 observations.

	Average	λ (MPR)
SPOT	50.82	
M1	48.74	0.21
M2	49.92	0.17
M3	50.61	0.16
M4	50.99	0.14
M5	51.45	0.13
M6	51.51	0.11
Q1	50.44	0.14
Q2	51.47	0.11
Q3	51.46	0.05
Q4	51.74	0.01
Q5	52.47	-0.05
Q6	52.76	-0.11
Y1	52.01	0.02
Y2	52.61	-0.17
Y3	53.52	-0.41
Y4	55.23	-0.69
Y5	56.69	-1.02
Y6	57.54	-1.39
Average	52	-0.15

Notice that sign and magnitude of MPR varies across forward maturities depending on hedging pressure from producers and consumers (Benth, Cartea and Kiesel 2008). We associate situations where MPR < 0 with markets where the consumers' desire to cover their positions 'outweighs that of the producers. Consumers are averse to higher electricity prices and willing to pay a risk premium to avoid such higher prices. We observe this situation in the case of contracts Y2-Y6. Conversely, situations where MPR>0 result when the producers' desire of hedge their positions outweighs that of the consumers. We observe this situation in the case of contracts M1 to Q3. Positive (Negative) MPR is usual in positive (negative) beta equity markets and implies that forward prices are (upward) downward-biased estimators of future spot prices.

Appendix D: Di Poto and Fanone (2012) Model

In order to provide comparison against a more recent market model competitor, we develop an adapted version of the model proposed by Di Poto and Fanone (2012), DF from now on. Specifically, with deviate from DF in several aspects. First, we do not apply any smoothing algorithm to our data. This choice make results directly comparable to the model proposed in this paper. Second, we use the loading factor as a direct volatility proxy instead of fixing a polynomial parametrization as in DF. Although this deviation does not make a significant difference, it eliminates the possible fitting error, thus improving DF model performance. Finally, we make use of the first four independent component instead of the first three components of DF. Again, this adaptation should have a positive effect on DF model performance.

D.1. The model

Assume T<∞ and let (Ω, *F, P*) be a complete filtered probability space, with an increasing and right-continuous filtration $\{F_t\}_{t\in[0,T]}$ where, as usually, F_0 contains all sets of probability zero in *F.* The model assumes a simple lognormal market representation given by the following stochastic differential equation:

$$d\ln F_c\,(t) = \delta_c(t)dt + \sum_{k=1}^{n} \Sigma_{\mathrm{c,k}}(\mathrm{t})\mathrm{dL_k}(\mathrm{t}) \quad (D.1)$$

Where, $F_c(t) = F_c(t, \tau_s^c, \tau_e^c)$ is the price at time t for an electricity future with delivery period $[\tau_s^c, \tau_e^c]$. We assume δ_c and $\Sigma_{c,k}$ to be sufficiently regular functions such that the swap dynamic $\ln F_c$ is square integrable, and dL_k, $k = 1, \dots, n$ are independent Lévy processes.

In addition, the following functional form is assumed to model the mean reverting process, $\delta_c(t)$:

$$\delta_c(t) = \frac{ds_c(t)}{dt} + \alpha_c(s_c(t) - \ln F_c(t)) \quad (D.2)$$

Last expression allows us to model a typical market behavior, that is, mean reversion in the direction of seasonality. Finally, a Normal Inverse Gaussian (NIG) distribution is assumed for the independent increments, $dL_k(t)$.

D.2 Model estimation

We begin the estimation with the seasonal component. We model seasonality by using the following parametric periodical function:

$$s_c(t) = \beta_0 + \beta_1 t + \beta_2 \cos\left(\frac{2\pi(t - \beta_3)}{250}\right) \quad (D.3)$$

We estimate parameters in (D.3) with a least square approach for each futures contract. After the seasonal estimation we remove it by subtracting, $s_c(t)$, for the log-price. Once seasonality is removed, in line with DF, we deal with the autoregressive component of order one, AR(1). As in DF, we do not find evidence of a long memory effect. Estimated mean-reversion parameters, $\widehat{\alpha_c}$ are close to zero and non-significant and we report results in Table D1.

Table D1: Estimated mean reversion parameters

Estimated parameters of the AR(1) process for future contracts with different maturities. Sample period is from 6/1/2004 to 12/31/2012.Sample size is 2179 observations. * denotes significance at 5% level and ** at 1% level.

Parameter	Yearly	Quarterly	Monthly
α_1	0,0059	0,0062	0,0093
α_2	0,0039	0,0071	0,0042
α_3	0,0022	0,0096	0,0031
α_4	0,0026	0,0064	0,0038
α_5	0,0038	0,0088	0,0049
α_6	0,0096	0,0081	0,0120

Last, we apply the Independent Component Analysis (ICA) algorithm to residuals $\boldsymbol{x} = d\ln F_c(t) - \delta_c(t)dt$, to decompose them in a mixing matrix $\boldsymbol{A} = \Sigma_{\mathrm{c,k}}$ and a source $\mathbf{s}$. Independent Components (ICs) are obtained by applying the *FastICA* algorithm to the residuals. Table D2 shows model performance by comparing model-generated volatility $\Sigma_{\mathrm{c,k}}$ term structure against actual market volatility.

Table D2: Volatility Term Structure

This table compares actual market volatility of swap returns and estimated volatility using the term structure of swap prices variances. Sample period is from 6/1/2004 to 12/31/2012.Sample size is 2179 observations.

	Market	**DF Model**	**Relative Error**	**Absolute Error**
M1	0.3264	0.3125	4.26%	4.26%
M2	0.2613	0.2526	3.34%	3.34%
M3	0.2191	0.213	2.78%	2.78%
M4	0.2002	0.1963	1.97%	1.97%
M5	0.1999	0.1873	6.30%	6.30%
M6	0.1997	0.1855	7.09%	7.09%
Q1	0.2245	0.1751	22.01%	22.01%
Q2	0.1925	0.1634	15.09%	15.09%
Q3	0.1831	0.1632	10.89%	10.89%
Q4	0.1815	0.1542	15.04%	15.04%
Q5	0.1843	0.1326	28.06%	28.06%
Q6	0.1759	0.1217	30.79%	30.79%
Y1	0.1738	0.1861	-7.09%	7.09%
Y2	0.1506	0.1445	4.03%	4.03%
Y3	0.1311	0.1249	4.70%	4.70%
Y4	0.1237	0.1138	8.01%	8.01%
Y5	0.1264	0.1111	12.07%	12.07%
Y6	0.1336	0.1134	15.09%	15.09%
Average			**10.25%**	**11.03%**

Appendix E: Value at Risk

In order to compare the performance of alternative models, we compute Value-at-Risk (VaR) at different probability levels over a one-day horizon. Given a portfolio *P*, a time *T* and a probability level *Q*, a loss $L*$ is selected, at which exists a probability *Q* that effective losses *L*, are at most $L*$ in period *T*. The loss $L*$ is portfolio´s VaR. Formally,

$$Prob[L^* \geq L] = Q \tag{E.1}$$

and therefore VaR_Q is a quintile of asset´s returns probability density function, which defines the maximum expected loss with confidence level *Q*. In the following, and to be consistent with the empirical evidence in our sample, we assume that the expected one-day swap return is zero. A comparison of the VaR for standardized returns and for different probability levels, *Q* is shown in Figure E1, for the Normal distribution and for the NIG distribution with different kurtosis parameter values. In the case of relatively low significance levels (90% and 95%), the values of VaR_Q^{Normal} tend to be higher (in absolute terms) than those of VaR_Q^{NIG}, so the latter measure will probably underestimate risk. However, in the case of high significance levels (99% and beyond) there is a very substantial difference between the two measures, because the VaR_Q^{Normal} strongly underestimates the risk in comparison with VaR_Q^{NIG}. The difference between the two measures, for a given *Q*, is higher; the closer to the unity is the kurtosis parameter ξ.

For the computation of the 1-day VaR for each swap contract, we proceed as follows. We assume that the innovations in the spot model and in the HJM model are normal. However, and given the limited success of the spot price model in our sample, we employ errors from the HJM model for the VaR calculations. Therefore, we compute $VaR(i,T)_Q^{Normal}$ as follows

$$VaR(i,T)_Q^{Normal} = k(\sigma_{i,T}\sqrt{\Delta t}) \tag{E.2}$$

where the factor *k* (critical values) depends on *Q* as presented in Figure E1 and $\sigma_{i,T}$ is the volatility of the innovations of the forward prices generated by means of the HJM model.

To compute the VaR in the case of the SFP model, we consider that each swap can be thought as a portfolio containing two stochastic factors and therefore its VaR should be computed using the standard VaR formula for a portfolio (Jorion, 2001). Using Equation (E.1), we define the VaR for the swap in market segment *i* and maturity *T* as follows

$$VaR_Q^{NIG}[F_i(t,T)] = \sqrt{VaR_Q[\overline{F_i}(t)]^2 + VaR_Q[\gamma_i(t,T)]^2 + 2Cov(\overline{F_i}(t),\ \gamma_i(t,T))}$$

(E.3)

We compute the cumulative distributions functions for the NIG processes driving $\overline{F_i}(t)$ and $\gamma_i(t,T)$ by means of numerical simulation. We compute the VaR for each component as follows

$$VaR_Q^{NIG}[\overline{F_i}(t)] = k_{\xi,\chi}(\theta_{\overline{F,i}}\sqrt{\Delta t}) \quad \text{(E.4)}$$

$$VaR_Q^{NIG}[\gamma_i(t,T)] = k_{\xi,\chi}\left(\theta_{\gamma_i(T)}\sqrt{\Delta t}\right) \quad \text{(E.5)}$$

where factor $k_{\xi,\chi}$ depends on Q and on skewness and kurtosis. The volatilities $\theta_{\overline{F,i}}$, $\theta_{\gamma_i(T)}$ are the residual standard errors obtained from Equation (12) and reported in Table 5. We obtain covariance matrix $\Omega = \{\omega_{i,j}\}$ (see Table 6) computed as follows

$$Cov\left(\overline{F_i}(t),\gamma_i(t,T)\right) = \theta_{\overline{F,i}} \times \theta_{\gamma_i(T)} \times \omega_{\overline{F_i},\gamma_i(T)}$$

Next, we compare the Failure Ratios (*FR*) of alternative models. We define *FR* as

$$FR = \frac{Realized\ proportion\ of\ VaR\ failures}{Expected\ proportion\ of\ VaR\ failures} = \frac{N/T}{c}$$

where 1-*c* is the confidence level, *T* is number of time periods (e.g. *T*=100 days) and a Failure appears when the realized loss (negative return) is larger than the VaR forecast. If the model producing the VaR forecasts is right in assessing the risk, we expect that *FR*≈1. On the other hand if the model tends to underestimate, (overestimate) risk, then *FR* > 1 (*FR* < 1). To test the statistical difference from one of the estimated FR, we use a variation of Kupiec (1995) test, suggested by Campbell (2007). Under the assumption that the VaR under consideration is accurate, the z-statistic has an approximate standard normal distribution and has a known exact finite sample distribution. The z statistic is actually the Wald variant of the likelihood ratio statistic proposed by Kupiec (1995). One potential advantage of the Wald test over the likelihood ratio test is that it is well-defined in the case that no VaR violations occur. On the other hand, Kupiec's test is not defined in this case. Moreover, the possibility that no violations occur in a relatively short period, is not trivial. The z-statistic is

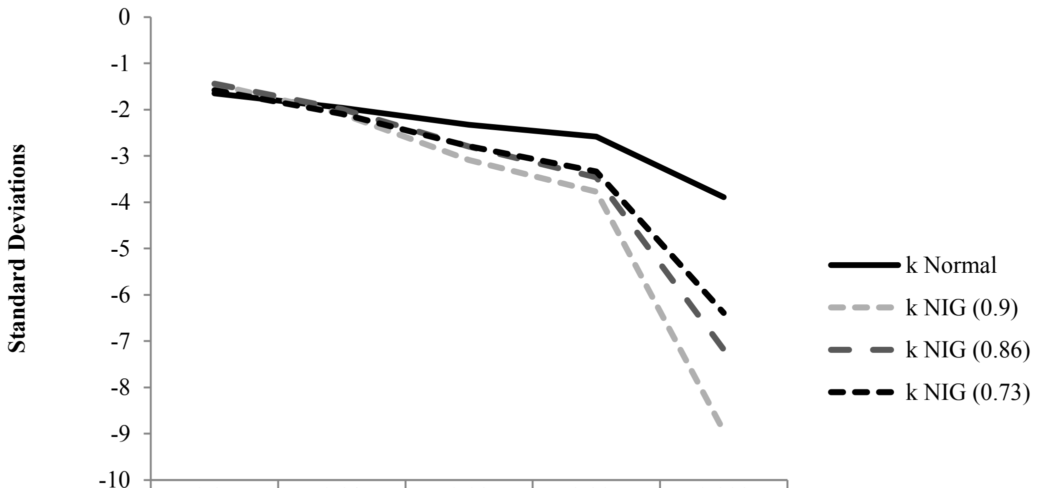
$$z = \frac{\sqrt{T}(\frac{N}{T} - c)}{\sqrt{c(1-c)}} \quad \text{(E.6)}$$

A positive (negative) z statistic indicates that the model tends underestimate (overestimate) risk. We present the results in Table 8. It may be seen that VaR_Q^{Normal} , calculated under the assumption of normality tends to understate risk, and this understatement is very strong for high confidence levels (99.5% and 99.99%) suggesting that tail risk is severely underestimated. On the other hand VaR_Q^{NIG} tends to mildly overestimate risk at relatively low significance levels but it is able to properly account for extreme tail risk. It is worth noting that the overestimation of risk provided by the NIG distribution is proportionally much lower than the underestimation of risk produced by the normal distribution.

One important practical implication of our results is as follows. We know that VaR models allow users to control risk and decide how to allocate limited resources. Financial intermediaries impose a capital charge to traders based on risk-adjusted capital. This creates a natural incentive for traders to take a position only when they have strong views on markets. If they have no views, they should abstain from trading. Our results suggest that the risk adjusted capital for traders using EEX swap electricity contracts should be adjusted upwards in comparison with the standard practice based on the normality assumption. Traders should also rationally adjust positions as risk changes (in the face of an increasingly volatile environment a sensible response is to scale down positions). Furthermore and given that VaR is also a performance evaluation tool, the evaluators of the performance of the traders should adjust their measures accordingly.

Figure E1: Critical values *k*

The figure shows critical values *k* for Value-at-Risk computations in Equations (29), (31) and (32) for a number of Levels of Significance (Q): Standardized Normal N(0,1) and NIG distributions



	0,95	0,98	0,99	0,995	0,999
k Normal	-1,65	-1,96	-2,33	-2,58	-3,89
k NIG (0.9)	-1,45	-2,09	-3,09	-3,77	-8,94
k NIG (0.86)	-1,44	-1,99	-2,8	-3,46	-7,18
k NIG (0.73)	-1,58	-2,09	-2,79	-3,34	-6,39