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SINR Analysis of OFDM and f-OFDM for Machine Type Communications

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***Abstract*—Machine type communications (MTC) have been growing significantly in recent years and this tendency is foreseen to be kept in the near future playing an increasingly important role in the industry. The signals used for MTC will coexist with the current and next generation cellular systems. Therefore it is of interest to study how they can perform jointly and the viability of coexistence of signals from both systems. We focus in one of the new waveforms being discussed for 5G, namely on filtered-OFDM (f-OFDM), along with traditional OFDM. The interference is analysed for both types of signals and the expression of the SINR is found allowing us to compare the behavior of OFDM and f-OFDM in these circumstances. Some simulations are shown to validate the theoretical analysis and explore some foreseen MTC scenarios.**

I. Introduction

Machine Type Communications (MTC) and the Internet of Things (IoT) are based on a very large network of interconnected physical objects. These devices will coexist with the cellular broadband communication systems, either Fourth Generation (4G) or the future Fifth Generation (5G). Therefore, it is crucial to analyse how to efficiently integrate MTC within existing mobile communication systems and also in the context of new waveforms that are being discussed for 5G.

MTC have some very specific technical requirements that may conflict with classical cellular systems. These devices need to extend their battery life as much as possible, so their energy consumption must be very reduced. Most sensors send sporadically short data frames, hence the transmission is asynchronous, the packet size is small and the data rate is reduced. The number of foreseen machine subscribers is considerably higher than the human subscribers, thus some low complexity but effective medium access control is needed to manage an unprecedented number of connections. Additionally, we can take advantage of the fact that MTC do not require high data rates and consequently bandwidth, placing them in any available slice of fragmented spectrum, achieving a more efficient use of the already crowded spectral resources.

Currently, different organizations are proposing potential waveform candidates for MTC. In particular, the Third Generation Partnership Project (3GPP) is proposing several options and the most popular one is based on Orthogonal Frequency Division Multiplexing (OFDM) and Single Carrier - Frequency Division Multiple Access (SC-FDMA) [1], as used today in Long Term Evolution (LTE), but with several modifications to make them more suitable for the requirements of MTC.

On the other hand, mobile subscribers are demanding an enormous increase of the data rates and the capacity is to be increased up to 1,000 times within the next 5 years [2]. Several new waveforms are already proposed for 5G: Filter Bank Multi Carrier (FBMC) [3], Generalised Frequency Division Multiplexing (GFDM) [4], Universal Filtered Multi-Carrier (UFMC) [5], filtered OFDM (f-OFDM) [6], etc. The main objective of all these modulations is to reduce the out-of-band emissions and take advantage of those gap bands allowing narrowband communication to be carried out in the resources left unused by bandwidth-hungry services. Among these waveforms, traditional OFDM has the worst performance in terms of out-of-band interference. FBMC has the best behaviour but it has an additional complexity due to filtering computations. In this work we focus on f-OFDM [6] which has a very good trade-off between the complexity and interference.

In order to make an efficient use of the spectrum as explained above, it is important to analyse the behaviour of f-OFDM as compared to pure OFDM when two or more waveforms are placed in contiguous spectral resources. It is well known that orthogonal modulations are very sensitive to the inter-carrier interference (ICI) which can destroy their orthogonality and make it impossible to recover the transmitted information for certain Signal to Noise Ratios (SNR). This issue has been extensively researched for OFDM [7] [8], analysing how the ICI due to a frequency mismatch of the oscillators or a time-varying channel affects a given OFDM signal. However the interference caused to others in the context of spectrum sharing for MTC has not been analysed, to the best of our knowledge. In this paper we study the effect of the ICI created from one signal to another and we compare the performance of OFDM and f-OFDM.

The remainder of the paper is organized as follows. Section II provides an explanation of the waveforms under study. In section III we analyse the ICI interference for OFDM and f-OFDM. Section IV presents some numerical and simulation results to validate the theoretical analysis and provide some understanding of the best configurations for MTC. Finally, in section V some conclusions will be pointed out.

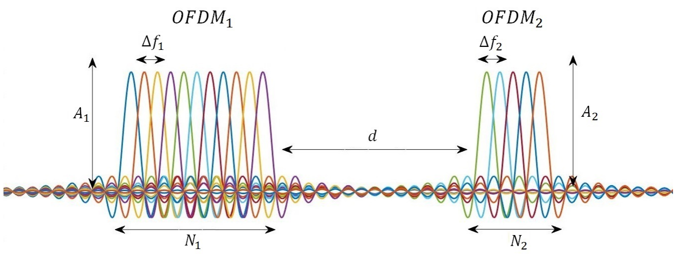


Fig. 1. OFDM interference parameters.

II. System Model

We consider two signals that are transmitted in contiguous spectral resources. The waveforms are based on OFDM, either the traditional approach or a filtered one f-OFDM. The details about the waveforms are given in the following sub-sections.

A. OFDM

The modulated signal is obtained in blocks of N_i samples following a general multi-carrier scheme which has the following expression

$$s_i[m] = A_i \sum_{k=0}^{N_i-1} X_i[k] e^{j\frac{2\pi mk}{N_i}} w_{N_i}[m], \quad (1)$$

where A_i is an amplitude scaling of the signal, $X_i[k]$ are symbols of a complex constellation and the subindex i is used to distinguish different OFDM signals that will coexist. The exponential factor corresponds to a frequency shift for each of the sub-carriers, and $w_{N_i}[m]$ is the prototype filter. In the particular case of OFDM, this filter is a rectangular pulse which corresponds to a discrete $sinc$ in the frequency domain

$$w_{N_i}[m] = \left\{ \begin{array}{ll} 1 & |m| \leq N_i \\ 0 & |m| > N_i \end{array} \right\} \xrightarrow{\mathscr{F}} \frac{\sin(w\left(N_i + \frac{1}{2}\right))}{\sin\left(\frac{w}{2}\right)}, \quad (2)$$

where N_i is the pulse width in samples that equals the OFDM symbol duration.

Fig. 1 shows two OFDM signals, where the spectrum of each of them is made of shifted $sincs$ over the frequency. In general, sub-carriers do not interfere to each other thanks to their orthogonality, where it can be shown that

$$\langle e^{j\frac{2\pi km}{N_i}} w_{N_i}[m], e^{j\frac{2\pi lm}{N_i}} w_{N_i}[m] \rangle = \delta[k-l]. \quad (3)$$

In this equation $< a, b >$ denotes the inner product of a and b. For standard OFDM, (1) can be simplified removing the prototype filter $w_{N_i}[m]$ because it is a rectangular pulse, and making use of the Inverse Fast Fourier Transform (IFFT) as

$$s_i[m] = A_i \sum_{k=0}^{N_i-1} X_i[k] e^{j\frac{2\pi km}{N_i}} = A_i \cdot IFFT\left\{\mathbf{X_i}\right\}, \quad (4)$$

where $\mathbf{X_i}$ is a vector containing the set of N_i complex information symbols to be transmitted. Before sending each block of N_i samples, it is necessary to add a cyclic prefix (CP) to protect it against inter-symbol interference (ISI) caused by the multipath channel. Once the CP is added, the expression of the signal to be transmitted is

$$s_{cp,i}[m] = \left\{ \begin{array}{ll} s_i[m] & m = 0 \ldots N-1 \\ s_i[m+N_i] & m = -M_i \ldots -1 \end{array} \right\}, \quad (5)$$

where M_i is the length of the CP.

At the OFDM receiver, we only need to remove the CP and perform an FFT to each block to demodulate the entire signal. Thanks to the orthogonality we can process each sub-carrier as one of a set of N_i independent sub-channels.

B. Filtered-OFDM

f-OFDM is one of the proposed waveforms for the evolution towards 5G [6]. Its main feature is its simplicity and its similarity to the well-known OFDM makes it appealing to the mobile operators. The main idea is filtering the OFDM signal that we have described before. Thanks to this filter, the out-of-band emissions will be reduced making adjacent gap bands available and easier to use for other purposes.

Starting from the time domain signal $s_{cp,i}[m]$ we are going to filter it before transmission

$$x_i[m] = b[m] * s_{cp,i}[m] = \sum_{n=-\infty}^{\infty} b[n-m] s_i[n], \quad (6)$$

where $*$ denotes a convolution operation and $b[m]$ represents the filter coefficients. Then the signal at the receiver after going through the propagation channel is

$$y_i[m] = h_i[m] * x_i[m] + n_i[m], \quad (7)$$

where $h_i[m]$ are the multipath channel coefficients and $n_i[m]$ is a zero-mean complex Gaussian random variable with variance σ_i^2 accounting for Additive White Gaussian Noise (AWGN).

The filter $b[m]$ is designed to reduce the out-of-band emission. We have chosen a Finite Impulse Response (FIR) filter to be applied to the signal in the digital domain before transmission. The choice is motivated by the fact that it has a linear phase and it is always stable.

The FIR filter will be designed following the "Windowing Method" [9]. In principle, the ideal filter is a rectangular pulse in the frequency domain which corresponds to a $sinc$ in the time domain

$$b_0[m] = \frac{\sin(Wm)}{\pi m} \xrightarrow{\mathscr{F}} B_0[jw] = \left\{ \begin{array}{ll} 1 & 0 \leq |w| \leq W \\ 0 & W < |w| \leq \pi \end{array} \right\}. \quad (8)$$

However, since in the time domain we cannot convolve our signal with an infinite length filter, we need to truncate it multiplying by a rectangular pulse $p[m]$, giving

$$b[m] = p[m] \cdot b_0[m]. \quad (9)$$

With this approach we do not have an ideal rectangular pulse any more in the frequency domain, as it can be seen in Fig. 2, and the filter order has to be increased to make it as steep as needed.

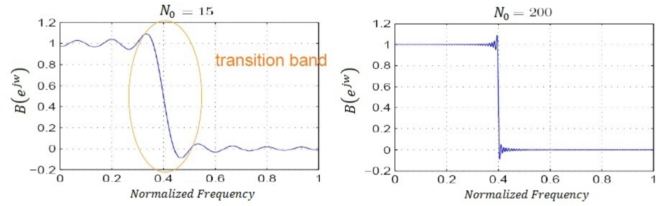


Fig. 2. Filter frequency response.

In this regard we must be extremely careful, because incrementing the filter order N_0 implies also an increment of its length N_l, which is given by

$$N_0 = N_l - 1. \tag{10}$$

The increment of this length will impact the OFDM signal, because it inserts long tails at the beginning and ending of each OFDM symbol. These tails are equivalent to introducing an additional ISI, which must be cancelled using a long enough CP. Hence, there is a trade-off between achieving very narrow filters and the increase of the ISI.

Finally, it is important to mention that, depending on the filter design, we may have a high peak-to-peak ripple which will introduce an additional distortion to the signal. This can be avoided again at the expense of not having the filter as narrow as wanted but this implies an increment of the number of required guard band sub-carriers to separate the two coexisting signals enough, as we will see later.

III. Interference Analysis

As we mentioned before, the orthogonality of OFDM is exploited to carry different complex symbols in each sub-carrier. The loss of orthogonality causes interference among the sub-carriers known as ICI. In a general scenario this distortion is caused by the Doppler effect in time-varying channels and the non-ideality of oscillators. In our particular case of two OFDM signals collocated in contiguous spectrum, even in the absence of these impairments, we have ICI that is caused by the fact that both OFDM signals are not synchronized and there is a mismatch of their frequency spacing.

According to our system model, we have two OFDM signals that are transmitting information symbols denoted as $X_1[k]$ and $X_2[k]$ based on (1), with their corresponding parameters shown in Fig. 1. Hence at the receiver side we will receive two signals according to (7) and denoted as $y_1[n]$ and $y_2[n]$.

In this scenario, the ICI value depends on the following parameters:

- Normalized distance d is the integer number of sub-carriers between the two OFDM signals (see Fig. 1). It is defined as
$$d = \left\lfloor \frac{D}{\Delta f_1} \right\rfloor \tag{11}$$
where $\lfloor a \rfloor$ stands for the smallest integer lower than a, D is the spectrum distance of the two OFDM signals measured in Hz, and Δf_1 is the sub-carrier spacing of the first OFDM signal that is taken as reference.
- Normalized carrier frequency offset (CFO) ϵ caused by the mismatch of the frequency spacing of the two OFDM signals. It is defined as
$$\epsilon = \mathrm{mod}\left(D, \Delta f_1\right), \tag{12}$$
where $mod(a, b)$ is the modulo operation which retrieves the remainder of dividing a/b.
- Number of sub-carriers N_1 and N_2 of each of the OFDM signals.
- Amplitude ratio in the frequency domain α which is defined as
$$\alpha = \frac{A_1}{A_2}, \tag{13}$$
where A_1 and A_2 are the amplitude scaling of each of the OFDM signals.

If we look at the first OFDM signal, once received, removed the CP, and performed the FFT, we have

$$R_1[k] = A_1 H_1[k] X_1[k] + I_2[k] + N_1[k], \tag{14}$$

where $H_1[k]$ and $N_1[k]$ are the Fourier transform of $h_1[m]$ and $n_1[m]$, the channel response and noise of the first OFDM signal (7) respectively. Additionally, the term $I_2[k]$ is the total amount of ICI caused by the second OFDM signal $y_2[n]$.

The whole interference term at sub-carrier k can be expressed as

$$I_2[k] = A_2 \sum_{n=0}^{N_2-1} H_2[n] X_2[n] S_2[n-k], \tag{15}$$

where $S_2[n-k]$ denotes the interference caused by sub-carrier n of the second OFDM in sub-carrier k of the first one and is obtained from (2) with

$$w = \frac{2\pi l}{N}, \tag{16}$$

where $l = n-k$ and $N = N_1+d+N_2$ is the total bandwidth of these two OFDM measured in number of sub-carriers . Hence, we have

$$S_2[l] = \frac{\sin[\pi(l+\epsilon)]}{N \sin[\frac{\pi(l+\epsilon)}{N}]} \exp\left[\frac{j\pi(l+\epsilon)(N-1)}{N}\right], \tag{17}$$

where ϵ is the normalized CFO shift due to the ICI as defined in equation (12). In the absence of ICI, the value of ϵ is zero, so the sine of multiples of π will be zero too, cancelling this term.

In order to characterize the impact of ICI on our system, we will analyse the signal-to-noise-plus-interference ratio (SINR) due to this effect. Without interference we define the SNR per sub-carrier as

$$SNR_i[k] = \frac{E\left[\left|A_i H_i[k] X_i[k]\right|^2\right]}{\sigma_i^2}. \tag{18}$$

In the presence of ICI, considering the first OFDM signal, its SINR per sub-carrier is

$$SINR_1[k] = \frac{E\left[\left|A_1 H_1[k] X_1[k]\right|^2\right]}{\sigma_1^2 + E\left[\left|I_2[k]\right|^2\right]}. \tag{19}$$

We need to simplify (19) developing the interference term

$$E\left[|I_2[k]|^2\right] = E\left[\left|A_2 \sum_{n=0}^{N_2-1} H_2[n]X_2[n]S_2[n-k]\right|^2\right] = \\ = E\left[\left(\sum_{n=0}^{N_2-1} H_2[n]X_2[n]S_2[n-k]\right) \left(\sum_{n=0}^{N_2-1} H_2[n]X_2[n]S_2[n-k]\right)^H\right], \quad (20)$$

where H denotes the Hermitian and the scaling amplitude A_2 is a deterministic value. This can be further simplified as

$$E\left[|I_2[k]|^2\right] = |A_2|^2 \sum_n \sum_m S_2[n-k]S_2[m-k] \\ E\left[H_2[n]X_2[n]H_2[m]X_2[m]\right], \quad (21)$$

which is accurate for $(\pi\epsilon)^2 < 3$ according to [7].

Note that $S_2[n-k]$ is a deterministic value according to (17) and $H_2[n]$ and $X_2[n]$ are independent processes. Hence, the expression can be reduced as follows

$$E\left[|I_2[k]|^2\right] = |A_2|^2 \sum_n |S_2[n-k]|^2 E\left[H_2^2[n]\right] E\left[X_2^2[n]\right]. \quad (22)$$

Additionally, we assume that the mean gain of the channel and mean power of the input constellation symbols are normalized to one, $E\left[|H_i[k]^2|\right] = 1$, $E\left[|X_i[k]^2|\right] = 1$. Hence (19) can be expressed as

$$SINR_1[k] = \frac{|A_1|^2}{\sigma_1^2 + |A_2|^2 \sum_{n=0}^{N_2-1} |S_2[n-k]|^2}. \quad (23)$$

where the numerator of (19) has been developed in the same way as the interference term.

The above analysis is valid for unfiltered OFDM signals. In order to consider f-OFDM, (15) is modified as

$$I_2[k] = \sum_{n=0}^{N_2-1} B[n]A_2H_2[n]X_2[n]S_2[n-k], \quad (24)$$

where $B[n]$ is the frequency response of the transmission filter $b[m]$ in (6). So (23) is modified as

$$SINR_1[k] = \frac{|A_1|^2}{\sigma_1^2 + |A_2|^2 \sum_{n=0}^{N_2-1} |B[n]S_2[n-k]|^2}. \quad (25)$$

Therefore, the mean value of all sub-carriers is

$$\overline{SINR_1} = \frac{1}{N_1} \sum_{k=0}^{N_1-1} SINR_1[k]. \quad (26)$$

Finally, we can find a closed-form bound on the interference terms by approximating (17) using a trigonometric rule

$$x\csc(x) < \frac{\pi}{2}, -\frac{\pi}{2} < x < \frac{\pi}{2}, \quad (27)$$

checking first the bounds of x

$$-\frac{\pi}{2} < \frac{\pi(l+\epsilon)}{N} < \frac{\pi}{2} \rightarrow -\frac{N}{2} < (l+\epsilon) < \frac{N}{2}. \quad (28)$$

This condition is satisfied because N is the total bandwidth, so the distance l can never be larger than it and ϵ is upper-bounded by 1 according to (12).

Applying (27) to (17), the resulting expression of interference term is

$$\sum_{n=0}^{N_2} |S_2[n-k]|^2 \leq \sum_{n=0}^{N_2} \left|\frac{\sin[\pi((n-k)+\epsilon)]}{2((n-k)+\epsilon)}\right|^2, \quad (29)$$

which has a closed-form expression

$$\sum_{n=0}^{N_2} |S_2[n-k]|^2 \leq \\ \frac{1}{4}\sin^2(\pi(k-\epsilon))\left(\psi^{(1)}(\epsilon-k) - \psi^{(1)}(N_2+\epsilon+1-k)\right), \quad (30)$$

where $\psi^{(n)}(x)$ is defined as

$$\psi^{(n)}(x) = \frac{d^{n+1}}{dx^{n+1}} \ln(\Gamma(x)), \quad (31)$$

and, in turn, $\Gamma(x)$ is defined as

$$\Gamma(x) = (x-1)!. \quad (32)$$

IV. Numerical Results

In this section we will show some simulation results to verify the theoretical analysis. Then we will also examine some proposed scenarios by the 3GPP for MTC.

A comparative of different proposed waveforms for 5G in terms of out-of-band emission is shown in Fig. 3. For f-OFDM the performance of two filters is shown, namely Hanning and Chebysev. We can see that the out-of-band emissions of f-OFDM depend directly on the filter frequency response. In addition, its performance is much better than OFDM.

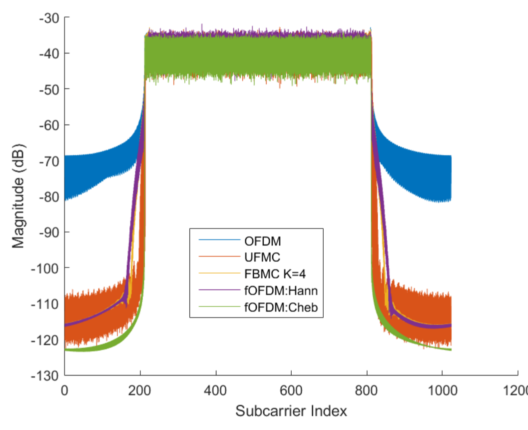
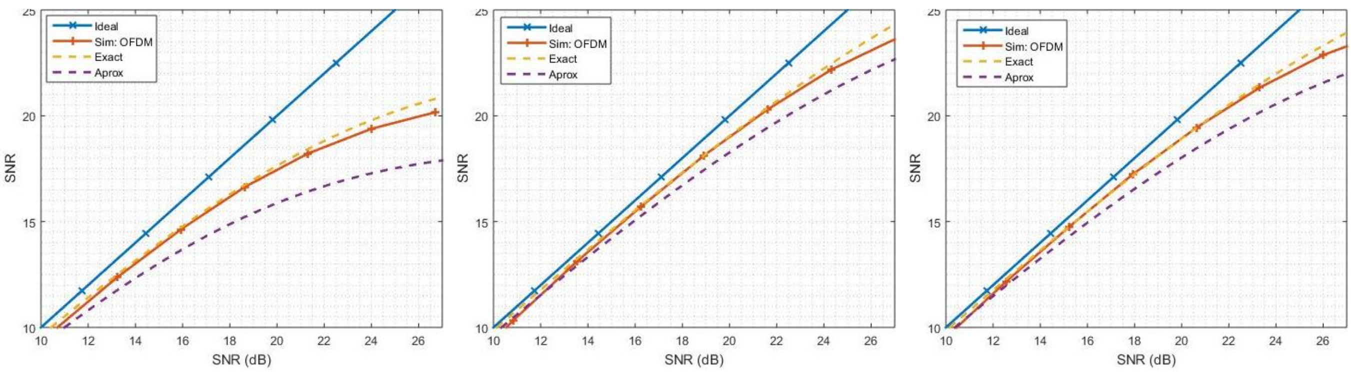


Fig. 3. Comparative of the spectra of the modulation schemes.

We consider now the scenario based on two OFDM signals introduced in section II. Table I shows the values of the different parameters where one of the coexisting signals is assumed to belong to a mobile broadband system, similar to LTE with a bandwidth of 10 MHz but using f-OFDM instead of OFDM. The other one is a narrowband MTC using a



(a) OFDM (b) f-OFDM Hanning (c) f-OFDM Chebyshev

Fig. 4. SINR of MTC with $\epsilon = 0.2$

TABLE I
SYSTEM PARAMETERS

Purpose	Evolved LTE	MTC
Waveform	f-OFDM	OFDM
FFT Points N_i	1024	16
Used subcarriers $N_{sc,i}$	600	12
Center frequecency	0	309
CP Length $N_{cp,i}$	160 in 1st symbol	4 in 1st symbol
	144 in other symbol	2 in other symbol
LTE Parameters	Δf = 15KHz	T_slot = 0.5ms
Filter Length	72 samples	None

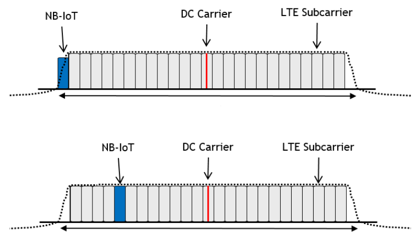


Fig. 5. MTC operation modes. Top one is guard-band and botton one is in-band.

traditional OFDM according to [1]. Note that we have enlarged the CP to avoid the additional ISI caused by the f-OFDM.

To evaluate the performance of the different alternatives, we will use as a reference the SNR due only to AWGN and we will evaluate the SINR that includes also the interference generated by the neighbour OFDM signal. Because the OFDM signals have some unused sub-carriers and the CP contains samples that are not used, we define the mean effective SNR of the i-th OFDM signal as

$$\overline{SNR_i}(dB) = 10\log\left(\frac{1}{N_{sc,i}}\sum_{k=0}^{N_{sc,i}-1} SNR_i[k]\right) + + 10\log\left(\frac{N_i}{N_{cs,i}}\right) + 10\log\left(\frac{N_i}{N_i + N_{CP,i}}\right). \quad (33)$$

A. Theoretical results validation

Fig. 4 shows the SINR of the OFDM and f-OFDM signals in the described scenario. We can see that OFDM suffers the highest interference, and the performance of f-OFDM has an improvement compared to OFDM. Furthermore, the SINR is always between the lower-bound and the upper-bound.

B. Performance of proposed MTC scenarios

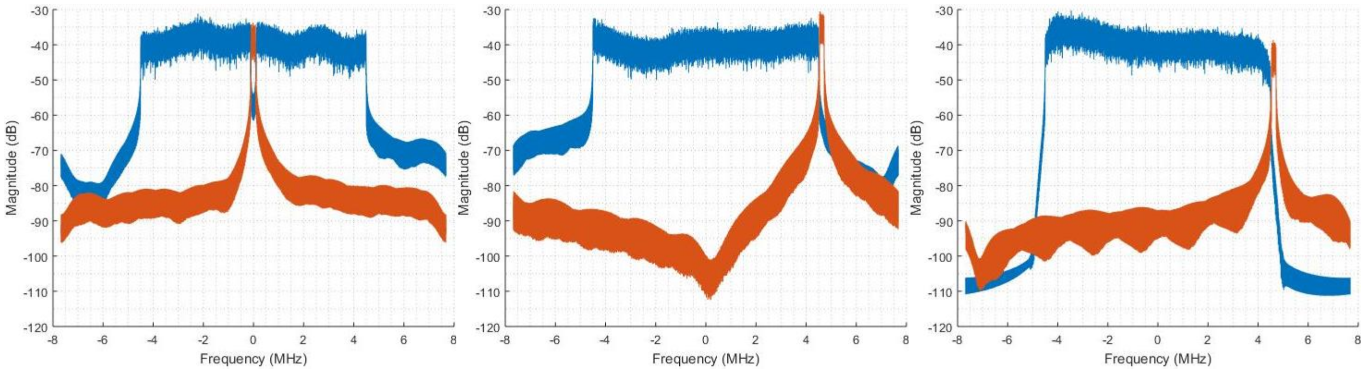
Two scenarios are proposed by 3GPP [1] for MTC, see Fig.5.

Fig. 6 shows the frequency response of two OFDM signals reproducing both scenarios. In Fig. 6a we can see that some LTE sub-carriers are switched off allowing the MTC transmission in the same band. Comparing Fig. 6b and 6c we can see the interference reduction for MTC when it is using f-OFDM for the broadband signal.

In Fig. 7a, we can see the bit error rate (BER) of MTC when the evolved LTE signal is polluted by ICI. The performance of the in-band scenario is the worst because the signal is surrounded by interference. For the guard-band case we can see that filtering the broadband signal improves the performance. In addition, we can see that both filters produce approximately the same results; this is because both of them have the same steepness in the guard-band area (see Fig. 3).

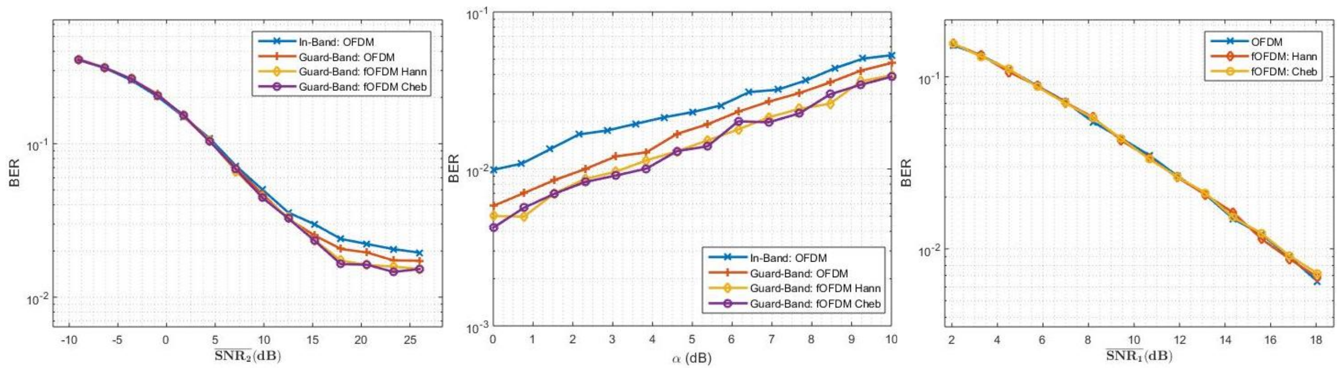
We have also analysed the BER for different amplitude ratios between the two signals, amplifying the broadband signal by a factor α as described in (13). In Fig. 7b we can see that the higher signal magnitude the stronger interference will appear in the MTC band. The same conclusions hold regarding the fact that the guard-band operation outperforms the in-band one, and filtering helps to reduce the interference improving 3dB.

In Fig. 7c we can see the BER of the broadband OFDM signal. As it can be expected the performance of OFDM and f-OFDM are almost the same in this case since the very narrowband MTC signal does not interfere as much as this broadband signal.



(a) In-Band OFDM (b) Guard-Band OFDM (c) Guard-Band f-OFDM

Fig. 6. Frequency response of in-band and guard-band operation with $\epsilon = 0.2$ in LTE and multi-path channel effects.



(a) MTC: in-band vs guard-band. (b) MTC: in-band vs guard-band for $\overline{SNR_2}$ = 22dB. (c) Envolved LTE in guard-band scenario.

Fig. 7. Performance comparative with interference of $\epsilon = 0.2$.

V. Conclusions

In this paper we have analysed the performance of OFDM and f-OFDM when they are sharing spectrum bands with application to MTC. Filtering the OFDM makes it easier to place narrowband signals in small frequency spaces left by LTE-like broadband signals. We have provided analytical expressions to evaluate the SINR of OFDM and f-OFDM in the presence of interference from contiguous signals that are not synchronized. Numerical results verify the accuracy of the analysis. We have explored potential scenarios of MTC, where the guard-band operation is shown to be more suitable than the in-band one regarding resilience to interference. The use of f-OFDM improves the results reducing undesirable distortion.

The use of f-OFDM has several advantages. Its reduced complexity and similarity to the well-known OFDM together with the fact that it enables an efficient use of spectral gaps make it one of the candidates for the evolution of mobile communications air interface.

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References

[1] 3GPP, "Cellular system support for ultra low complexity and low throughput internet of things,," 3GPP and GSM/EDGE Radio Access Group, Tech. Rep. 45.820, Dec. 2012.

[2] Ericsson. (2016) Ericsson mobility report. [Online]. Available: http://www.ericsson.com/res/docs/2016/mobility-report/

[3] B. Farhang-Boroujeny, "OFDM Versus Filter Bank Multicarrier," *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 92–112, May 2011.

[4] G. Fettweis, M. Krondorf, and S. Bittner, "GFDM - Generalized Frequency Division Multiplexing," in *Vehicular Technology Conference, 2009. VTC Spring 2009. IEEE 69th*, April 2009, pp. 1–4.

[5] F. Schaich and T. Wild, "Waveform contenders for 5G: OFDM vs. FBMC vs. UFMC," in *Communications, Control and Signal Processing (ISCCSP), 2014 6th International Symposium on*, May 2014, pp. 457–460.

[6] X. Zhang, M. Jia, L. Chen, J. Ma, and J. Qiu, "Filtered-OFDM - Enabler for Flexible Waveform in the 5th Generation Cellular Networks," in *2015 IEEE Global Communications Conference (GLOBECOM)*, Dec 2014, pp. 1–6.

[7] T. Pollet, M. V. Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 191–193, Feb 1995.

[8] J. Lee, H.-L. Lou, D. Toumpakaris, and J. M. Cioffi, "Effect of carrier frequency offset on ofdm systems for multipath fading channels," in *Global Telecommunications Conference, 2004. GLOBECOM '04. IEEE*, vol. 6, Nov 2004, pp. 3721–3725 Vol.6.

[9] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications*. PRENTICE-HALL, 1996.