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Towards Low Latency and Resource-Efficient FPGA Implementations of the MUSIC Algorithm for Direction of Arrival Estimation

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***Abstract*—The estimation of the Direction of Arrival (DoA) is one of the most critical parameters for target recognition, identification and classification. MUltiple SIgnal Classification (MUSIC) is a powerful technique for DoA estimation. The algorithm requires complex mathematical operations like the computation of the covariance matrix for the input signals, eigenvalue decomposition and signal peak search. All these signal processing operations make real-time and resource-efficient implementation of the MUSIC algorithm on Field Programmable Gate Arrays (FPGAs) a challenge.**

In this paper, a novel design approach is proposed for the FPGA-implementation of the MUSIC algorithm. This approach enables a significant reduction in both FPGA resources and latency. In more detail, the proposed design enables the estimation of DoA in real-time scenarios in 2μsec with 30% to 50% fewer resources as compared to existing techniques.

***Index Terms*—Direction of arrival, MUSIC, Array Signal Processing, FPGA.**

I. Introduction

Direction of Arrival (DoA) estimation is a major component in many electronic warfare for tactical and strategic applications. Active surveillance systems like Radars transmit a pulse and receive its reflected echo to locate a target. Instead, self-protection systems are passive schemes that intercept a target without transmitting any signal. The basic functionality of such systems is to provide an early warning of possible threats by intercepting the radiated emission, separating it from the noise and measuring the signal parameters to gather strategic and tactical electronic intelligence. Self-protection systems also need to differentiate between friendly and hostile emissions based on the received signal characteristics. The received signal has many associated parameters like frequency, duration of the transmission, pulse repetition frequency and DoA. Among all these, DoA is the only parameter that cannot be changed on a pulse to pulse basis [1]. So the accuracy of DoA estimation is of extreme importance for the classification and identification of the emitter.

Accuracy for DoA estimation is a major concern which depends upon several factors. These factors include the array structure, receiver structure, computing platform, and the algorithms used. DoA estimation is also a function of the antenna pattern. It is also dependent upon the resolution of the digitized data, the synchronization between data acquisition of the different antenna chains, the computation response time and the implemented algorithm. The ever-increasing density of the Electromagnetic (EM) environment further adds to the challenges of accurate measurement of DoA.

Many algorithms have been devised that offer different levels of complexity, output accuracy, resolution, and closely spaced target distinction. These algorithms include mono pulse amplitude comparison methods, phase comparison methods and subspace methods [2]. With the advent of array signal processing [3], [4] the spatial estimation of the DoA is becoming more common due to its inherent characteristics [5]. The subspace or the spatial estimation methods produce reliable and stable results even with signals having a low Signal to Noise Ratio (SNR) and that is an ever-growing demand for DoA estimation in the saturated EM environment. The MUSIC algorithm is a subspace method that offers high accuracy and closely spaced target distinction at the cost of high complexity [6].

The MUSIC algorithm has three main building blocks i.e. covariance matrix computation, Eigenvalue Decomposition (EVD) and peak search. All stages of the MUSIC algorithm offer different levels of complexity, EVD being the most expensive in terms of time and resources. This complexity makes the MUSIC algorithm less attractive for real-time implementation despite its high accuracy and closely spaced target distinction.

The design approach proposed in this paper modifies the data flow of the MUSIC algorithm by the addition of Virtual Array Reduction (VAR) and Normalization blocks. The VAR block reduces the dimensionality of the input space by intelligent sensor selection while the normalization enables EVD simplification leading to resource and latency savings.

In more detail, the main contributions presented in the rest of the paper are summarized in the following points:

- Introducing the VAR (Virtual Array Reduction) technique that reduces the number of antenna signals that need to be processed reducing the the processing requirement to almost one fourth along with improving the FPGA resource utilization.
- Normalization of the covariance matrix. The algorithm is devised to provide reliable outputs in a dense EM environment. Stable output at SNR as low as 4.37dB requires a larger number of bits for each element of the covariance matrix resulting in high FPGA resource utilization. This requirement grows exponentially with each

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subsequent stage. The normalization of covariance matrix reduces the resources needed.

- Reducing the processing time by limiting the array dimension for spectral peak search. Peak search is restricted within 3dB intercept points of the maximum receiving antenna.
- The concurrent processing of amplitude and phase of a signal requires considerable resources in the FPGA. In this paper MUSIC algorithm is implemented by considering only amplitude (real part) of the signal. This follows the works in references [7]-[9] who devised different transformations to convert the complex data set to a real- valued data set in order to avoid complex value computation in the MUSIC algorithm. Similarly, [10] and [11] also considered only the amplitude of signal in MUSIC computation by separating the data set into real and imaginary parts and then using real parts for DoA estimation in order to reduce the computational complexity of the algorithm. All the designs proposed in [7] - [11] acquire complex data and perform some computation to obtain a real-valued data set at certain stage of the algorithm. Contrary to these approaches our proposed design does not require any complex data at any level. Our algorithm only acquires the real-valued signal as the input to the MUSIC algorithm.

The proposed modifications by employing simple mathematical techniques provide significant savings in resource utilization and processing latency without compromising the accuracy of the DoA estimation. This is due to the fact that these techniques not only simplify the overall design but also reduce complex operations that require costly IPs like CORDIC and limit the RMS error to 0.49°. This meets the requirement of most real-time systems as for such systems an RMS error less than 1°is required. The rest of the paper is organized as follows: Section II discusses the basics of the MUSIC algorithm, section III gives a brief overview of related works, the proposed design is presented in section IV. Evaluation results are presented in section V and the conclusion with future work in section VI.

II. Multi Signal Classification algorithm (MUSIC)

As discussed in Section I, the accuracy of DoA estimation is of utmost importance in many applications like radars, sonar, medical imaging, and many others. Various algorithms have been devised to address this problem. In 1979 Schmidt [12], proposed the MUSIC algorithm for determining the DoA in the spatial domain. It provided the basis for DoA estimation using array signal processing. Estimation of the angle through the spatial spectrum is a specialized signal estimation technique of array signal processing that utilizes sensor arrays to extract space signal parameters. The entire signal space is divided into three parts; the incident signal space, array receiver, and parameter estimation [13]. The MUSIC algorithm is implemented in the parameter estimation part. The MUSIC algorithm has three stages; computing the covariance matrix of the acquired data, characteristic decomposition of the covariance matrix in signal subspace and the corresponding noise subspace that is orthogonal to it, and finally the spectral peak search using these orthogonal subspaces and hence the DoA. Each stage is described briefly in the following subsections.

A. Signal Acquisition

The MUSIC algorithm requires the complex signals of the antenna array to compute the DoA. It assumes that the signals are received by planar waves from a far-field source and that all the receiving channels have the same characteristics. The elements of the array are uncorrelated with variance σ^2, zero mean and Gaussian noise η_m(t).

B. Covariance Matrix

Although all the chains process the received data from each antenna independently, the received signals have certain relation with each other that can be exploited to find the DoA. The covariance matrix of the input data is computed to develop a linear relationship between signals received by all receiver chains that is then used to estimate the DoA.

C. Eigen Value Decomposition

The main idea behind the MUSIC algorithm is the characteristic decomposition of the covariance matrix into signal and noise subspaces. This takes the signal from the time domain into the subspace domain. The eigenvalue decomposition that involves eigen values and eigen vectors that are used to find the point where the maxima of covariance of input signal lies. Many methods exist for finding the eigenvalues of the incident signal covariance matrix like the Exact Jacobi method, the Jacobi method [14],[15], and the algebraic method [16]. Some other techniques for fast computation of SVD (Singular Value Decomposition) and EVD have also been proposed in [17]. Among all these techniques [16] is found to be the most efficient and has less computational requirements. This method implements the following mathematical model. If a, b, c are the variance of received data and d, e, f are the covariance of received data with respect to each other. The covariance matrix is represented as:

$$M = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix} \tag{1}$$

$$p = det(M - I\lambda) \tag{2}$$

To find the eigenvalues p, the equation (2) is set equal to zero. The characteristic equation can be written in simplified form as:

$$\lambda^3 + k\lambda^2 + l\lambda + m = 0 \tag{3}$$

where

$$k = -(a + b + c) \tag{4}$$

$$l = ab + bc + ac - f^2 - e^2 - d^2 \tag{5}$$

$$m = af^2 - abc - 2fde + be^2 + cd^2 \tag{6}$$

The implementation of the solution for (3) is non-trivial when implemented in FPGAs. Simplifying the equation by substituting $\lambda = x - \frac{k}{3}$. The resulting equation is:

$$x^3 + px + q = 0 \tag{7}$$

where

$$p = -\frac{1}{3}k^2 + l \tag{8}$$

$$q = m + \frac{2}{27}k^3 - \frac{1}{3}lk \tag{9}$$

As the covariance matrix is symmetric and positive definite we can substitute:

$$x = \sqrt{\frac{-4p}{3}}y \tag{10}$$

This produces

$$4y^3 - 3y = \frac{3q}{p\sqrt{\frac{-4p}{3}}} \tag{11}$$

From trigonometric relations:

$$cos3\alpha = 4cos3\alpha - 3cos\alpha \tag{12}$$

and substituting $y = cos\alpha$, we have:

$$cos3\alpha = \frac{3q}{\sqrt{\frac{-4p}{3}}} \tag{13}$$

Thus the eigenvalue decomposition produces the three eigenvalues. The eigenvalues are expressed as:

$$\lambda 1 = x1 - \frac{k}{3} = \beta cos\alpha - \frac{k}{3} \tag{14}$$

$$\lambda 2 = x2 - \frac{k}{3} = \beta cos(\alpha - 2\frac{\pi}{3}) - \frac{k}{3} \tag{15}$$

$$\lambda 3 = x3 - \frac{k}{3} = \beta cos(\alpha + 2\frac{\pi}{3}) - \frac{k}{3} \tag{16}$$

where

$$\beta = \sqrt{\frac{-4p}{3}} \tag{17}$$

$$\alpha = \frac{1}{3}(\frac{\pi}{2} - arcsin\frac{3q}{p\beta}) \tag{18}$$

D. Spectral Peak Search

To find the spectral peak, the eigen vectors of the incident signal covariance matrix are steered over the antenna response. The point where the noise subspace finds its minima is the point of maxima for the signal subspace. The DoA is estimated from the noise subspace. The spectrum function is defined as:

$$P_m(\theta) = \frac{1}{a^H(\theta)E_n E_n^H a(\theta)} \tag{19}$$

The denominator in equation (19) shows the inner product of the signal vector $a(\theta)$, noise matrix E_n, their Hermitian matrices $a^H(\theta)$ and E_n^H. When $a(\theta)$ is orthogonal with each column of E_n, the value of this denominator is zero, but because of the existence of the noise, it actually has some minimum value. At the point of minima for denominator $P_m(\theta)$ has its peak. By this formula, making θ change will estimate the arrival angle by finding the peak. The direction where the spectrum function has the maximum value corresponds the estimated DoA.

The effectiveness of the MUSIC algorithm depends on the signals being uncorrelated. The more the signals are uncorrelated, the better the performance of the MUSIC algorithm is. It can distinguish between two closely spaced targets through spatial spectrum analysis for the estimation of DoA. Optimal implementation of any algorithm for a practical application imposes multiple constraints. These may include processing platform and environment characteristics. For example, in any passive direction-finding system, the signal received from the noisy environment has considerable variations. Then this signal is digitized by the Analog to Digital Converters (ADC) for further processing. The accuracy of the calculated data can be increased by a high resolution ADC. However, there is always a trade-off to be made between the number of bits of the ADC and the required FPGA resources. It is desirable to use a small number of bits as possible to manage the FPGA resources but not compromising the overall performance of the algorithm. Similarly, the operational SNR of the system also contributes towards the resource utilization. To make the system stable for lower SNR values, more FPGA resources are required. The MUSIC algorithm is based on estimating the expectation of the input signal. The expectation provides better accuracy with a large number of samples (ideally approaching infinity) from as many sensors as possible. However, for a real-time implementation a compromise has to be made between the algorithm complexity and output accuracy. Further implementing the phase discrimination hardware to acquire amplitude and phase of signal is costly and challenging. Its processing requires more complex algorithms and significant resources on the FPGA. Utilizing only the real part of the signal not only simplifies the receiving end but also reduces the FPGA resources to almost half.

III. Related Work

MUSIC is a subspace algorithm for estimating DoA. There is plenty of research available on the implementation of the MUSIC algorithm on FPGA. The algorithm has been studied extensively for different array structures, input data type/length, accuracy, SNR and number of snapshots [18] - [22]. Variants of the MUSIC algorithm like root-MUSIC algorithm [23], improved MUSIC algorithm [24] or modified MUSIC algorithm [25] have been proposed to reduce the algorithm complexity. The authors in [5] propose a robust technique for DoA estimation based on the MUSIC algorithm. The proposed methods offer a high degree of accuracy but they require considerably high time for DoA estimation. The advent of modern radars impose strict timing constraints on processing algorithms. To work in a real-time system the algorithm should be able to process millions of pulses per second. However the method presented in [5] produces the result in 1 ms, thus can process only 1000 pulses per second which does not suffice the requirements of a real-time system. Another modification of the MUSIC algorithm, HFMA is proposed in [26]. The HFMA algorithm bypasses the eigenvalue decomposition which is the most expensive block of MUSIC algorithm implementation. It also uses conjugate symmetry of covariance matrix and iterative data storage to reduce data exchange time. Though [26] estimates the angle in 25.5μ sec which is significantly less than 1ms as presented in [5], it still can only process at maximum of 40000 pulses per second which again does not meet the real-time DoA estimation requirements. All the proposed modifications in HFMA utilize complex mathematical models that require complex IPs (like CORDIC) for implementation resulting in high resource utilization. Likewise, high speed parallel optimizations for implementation of the MUSIC algorithm are presented in [27]. Though the scheme proposes a MUSIC algorithm without eigenvector decomposition of covariance matrix still it requires 354μ sec to estimate DoA that is impractical for any real-time system.

To overcome the high computational and processing cost of eigenvalue value decomposition for DoA estimation using the MUSIC algorithm, an efficient approach for EVD is presented in [28]. The proposed method exploits the symbolic consistency of tangent function to avoid the calculation of rotation angle. The efficiency of the method proposed in [28] is compared for different number of sensors. The processing time decreases while reducing the array size. An array of 4 elements requires nearly 380μ sec for DoA estimation. However, it offers a slightly higher accuracy of 0.3°while consuming considerably high resources. A LU(Lower-Upper) factorization based technique is presented in [29]. The implemented technique performs LU factorization of the covariance matrix of the input signal before performing EVD. LU factorization performed on 4 element ULA(Uniform Linear Array) leads to reduce the utilized FPGA resources, but still it does not satisfy the real-time requirement of pulse density. Among all presented algorithms in [29] LU-L(Lower-Upper factorization - Lower Triangle Matrix) is the most computationally efficient. It requires 3.95μ sec at 40 MHz for DoA estimation that enables to process 0.254 MPPS. Another efficient FPGA-based hardware implementation for DoA estimation using MUSIC algorithm is detailed in [30]. The paper proposes two approaches for DoA estimation using Cholesky and LDL decomposition techniques to avoid the high computational cost of EVD. Both techniques give reasonably good results for the consumed FPGA resources and computation time. The techniques are able to process maximum 0.34 MPPS for LDL for 16 bits data size. The techniques presented in [29], [30] produce comparably good results at low SNR.
Therefore, real-time implementations of algorithms with a fewer number of FPGA resources are of interest to fit all the algorithms in a single FPGA. However, the implementation of the MUSIC algorithm on FPGAs with fewer resources and an acceptable real-time response is a challenge. In this paper, a real-time implementation of the MUSIC algorithm with fewer resources, better performance and improved output accuracy is proposed and evaluated. To reduce complexity, only the signal strength is taken as input. The complexity of the MUSIC algorithm is further reduced by selecting the antennas with maximum signal and discarding all others. To reduce the size of the covariance matrix VAR is introduced in our proposed design. Although the reduction of the antenna array has never been used in tandem with the MUSIC algorithm, it has been widely used in monopulse amplitude comparison techniques as mentioned in [31] - [34].

IV. Implementation of the Proposed Optimized MUSIC Algorithm on Xilinx FPGAs

In our approach, the implementation of the MUSIC algorithm is modified by introducing two additional blocks: Virtual Array Reduction block and Normalization block at appropriate stages, as shown in Fig. 1.The proposed algorithm is implemented for a six antenna UCA (Uniform Circular Array), but the same approach is applicable to larger antenna arrays with a slight modification in the Virtual Array Reduction block. Each stage of the MUSIC algorithm is modeled carefully to achieve the real-time implementation requirements. The normalization of the covariance matrix reduces the required FPGA resources and

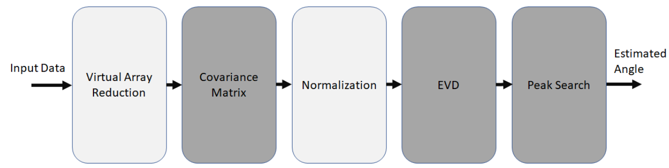

Fig. 1. Block diagram of the proposed Optimized MUSIC Algorithm.

simplifies the eigenvalue decomposition model. The algorithm runs at 100MHz and estimates the DoA in 200 cycles only. The eigenvalues computed from the covariance matrix of the input signals are utilized to compute the eigen vectors. Eigenvalue decomposition is a complex operation for implementation on a hardware platform. As we are considering that the array will receive a single signal at a given time t, the highest eigenvalue will correspond to the signal subspace while the other two will correspond to the noise subspace. The eigen vectors corresponding to the noise subspace are steered through the antenna gain pattern to search the peak for DoA estimation [35]. The resource and time optimization is achieved by normalizing the covariance matrix and limiting the peak search in an interval of 60°. The algorithm is designed to perform well with low SNR for the intercepted signal.

The additional blocks shown in Fig. 1 not only satisfactorily manage the hardware resources but also significantly reduce the processing time. These blocks also enabled the optimization of the Algebraic method for EVD presented in [16]. This led to two design approaches for the implementation of the MUSIC algorithm:

- Design Approach I - Optimized MUSIC Algorithm
- Design Approach II - Optimized MUSIC Algorithm with modified EVD block

The design is implemented in fixed point arithmetic.The design is required to work for dynamic range of 84dB. According to [36], 14 bit digital data will suffice the requirement. So, inputs from the sensors are taken as unsigned binary with fixed point precision of 14 bits and 0 binary points (UFix14_0). Signed binary numbers are also used in the design depending upon the data. Data carries the nomenclature as UFixa_b or Fixa_b, where $UFix$ represents unsigned data, Fix represents signed data, a and b represent fixed point precision of bits and binary points respectively.

A. Optimized MUSIC Algorithm

1) Virtual Array Reduction

The number of array elements along with some other parameters define the resolution and accuracy of the estimated DoA. Increasing the number of array elements increases the algorithm complexity [37], FPGA resource requirements, and processing time. Signal processing techniques and DoA estimation in particular, are driven by matrix operations [29]. The matrix size and the required operations determine the complexity of the implemented algorithms. This is managed by adding a Virtual Array Reduction block as presented in Fig. 2. The Virtual Array Reduction block receives the data from all antennas and compares the signal strength received by each channel for maximum signal localization. As the algorithm is only based on the amplitude of the received signal, the Virtual Array

Reduction block compares the input of the antennas in pairs. The comparison operation is performed in three stages in order to reach the antenna that is receiving the maximum. The index generator computes the index of the maximum signal receiving antenna. Based on which the left and right antennas are selected. The final stage of the VAR block ensures to select the maximum signal strength with amplitude received by two neighbouring antennas from the antenna array. This virtually reduces the array elements from six to three. As the VAR block selects between the input signals, the word length of the output is the same as that of the input i.e UFix14_0.

Apparently, reducing the number of antennas may result in some inaccuracies. It is worth mentioning that all the DoA algorithms (either MUSIC or any other algorithm) work on the basic assumption that the direction from where a signal with maximum strength is received is taken as the Direction of Arrival. Particularly in a UCA, if a system receives a signal from a certain direction the antenna with maximum signal reception along with two neighboring ones will receive the largest signals and rest will receive either weaker, no or reflected signals, that do not have much contribution to the computation. Thus excluding those antennas from computation should not introduce any relevant error.

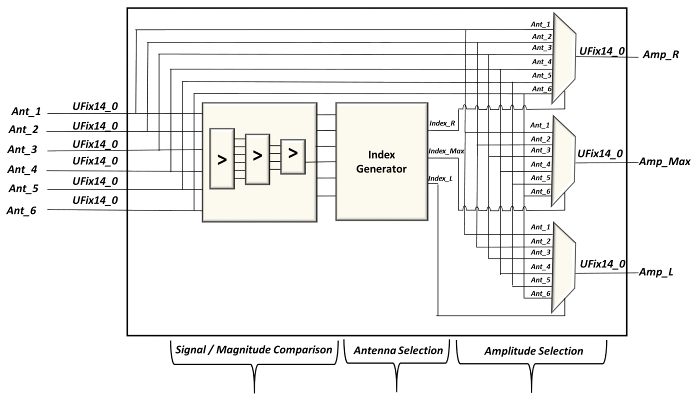

Fig. 2. Virtual Array Reduction.

2) Covariance Matrix

In a passive surveillance system, all the signal receiving channels are closely matched but independent of each other. When a signal is received by any antenna array, different elements of the array receive different signal strength and phase for the same signal. The covariance block calculates the variance and covariance of the selected channels. Variance calculates the deviation of received data from its mean. It is given by the following equation:

$$V = \frac{1}{N-1} \sum_{i=1}^{N} \mid A_i - \mu \mid^2 \tag{20}$$

where

$$\mu = \frac{1}{N} \sum_{i=1}^{N} A_i \tag{21}$$

Covariance computes the linear association of the received data from the selected channels with each other. It is given by the following formula:

$$Cov = \frac{1}{N-1} \sum_{i=1}^{N} (A_i - \mu_A)(B_i - \mu_B) \tag{22}$$

If we consider all the six antennas of the UCA, the covariance matrix will be having 21 distinct matrix elements. Selecting the antennas using the Virtual Array Reduction block lowers the processing requirement of covariance matrix calculation. In this case only 6 distinct matrix elements are required. So reducing the number of parallel running modules almost to one fourth. The implementation of the covariance block is shown in Fig. 3.

All the calculation is performed using three snapshots for each signal, which is the minimum requirement for correctly characterizing a virtual array of 3 antennas. Increasing the snapshots to any number will still result in 3x3 covariance matrix. Addition of more snapshots does not affect the linear relationship between the signals intercepted by the three antennas. However, it surely increases the data acquisition time, calculation of mean and covariance matrix. The mean μ of the snapshots received by each antenna is calculated as shown in equation (21). The value of mean is used to calculate variance and covariance of the received signal using equation (20) and equation (22) respectively.

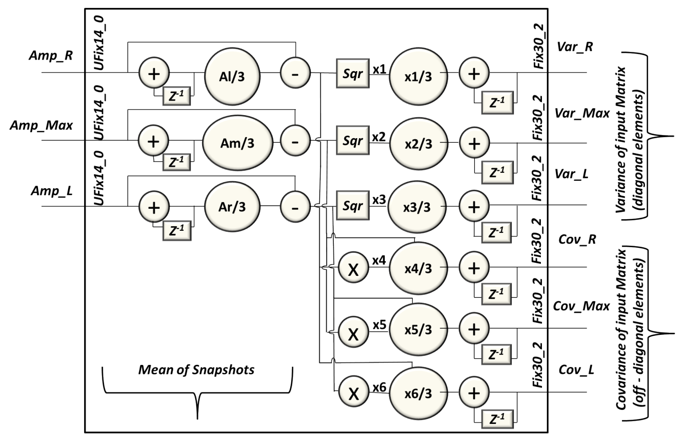

Fig. 3. Covariance Matrix Calculation.

Equation (20) and (22) show that calculation of Variance and Covariance involve Multiplication of input data followed by addition and subsequently division. So for an input of UFix14_0, the word length of Fix30_2 would suffice for accurate computation.

3) Normalization of the Covariance Matrix

The algorithm should perform reliably for an input signal with an SNR as low as 4.37dB (that is sufficiently low to characterize a highly noisy environment). The requirement of such a low SNR exponentially increases the required number of bits for the covariance matrix. To deal with increase in the number of bits and to keep the FPGA resource requirements within limits it is proposed that data should be normalized at the appropriate stage. In particular, once the covariance matrix is computed, it is normalized for further calculation as shown in Fig. 4.The normalization scales down the values of variance and covariance and keeps the linear relationship between the received signals unaltered. That is why normalization offers optimization without compromising the accuracy at output. The normalization enables us to reduce the number of bits for each variable from 30 to 14 and a further reduction in subsequent stages. The output of the normalization block varies between 0-1.Thus we set the output word length of normalization block to Fix14_12 to represent the number accurately.

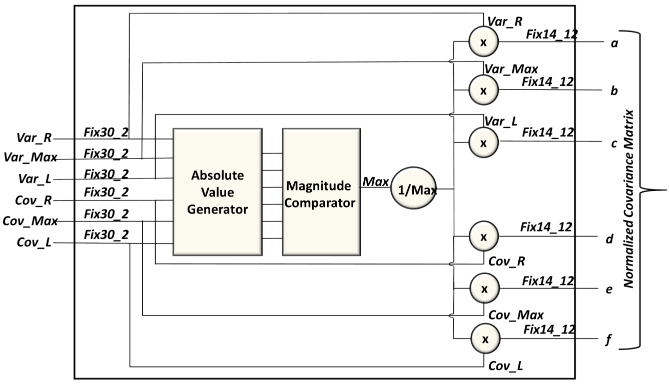

Fig. 4. Normalization of Covariance Matrix.

4) Eigenvalue Decomposition (EVD)

As the number of antennas is limited to three; the maximum receiving antenna and two neighboring antennas on either side, the size of the covariance matrix and subsequently the size of the normalized covariance matrix are limited to 3x3. It is worth mentioning that the covariance matrix for any set of data is always symmetric and positive definite. Eigenvalue decomposition of the covariance matrix is performed. It is the most significant and critical block for DoA computation when using the MUSIC algorithm. In this paper, the algebraic method [16] is followed with a few modifications, for determining the eigenvalues of the normalized covariance matrix. The mathematical model presented in (equations (4) to (18))is used to perform eigenvalue decomposition. Fig. 5 shows the implementation of the EVD for Design Approach I. These include an Arithmetic Logic Unit (ALU) for performing basic mathematical operations followed by arcsin of angle α. These values are used to calculate the eigenvalues and eigen vectors.

The ALU performs the basic arithmetic operations to calculate k, l, m, p and q as per equations (4) to (9). These values are used to caculate β. Arcsin unit calculates angle α utilizing CORDIC and CORDIC SINCOS IPs.

We assume that one signal is incident at a time. So the largest eigenvalue corresponds to the signal subspace, whereas the rest of the eigenvalues correspond to the noise subspace. The row reduction technique is used for calculating eigen vectors.

The row reduced echelon form of covariance matrices for both eigen values are used for calculating the eigen vectors as shown in Fig. 6. If the normalized covariance matrix is given by equation (1). Equation (2) is modified as follows to obtain the row reduced echelon form for eigenvalues corresponding to the noise subspace. The row reduced echelon form of matrix obtained will be:

$$\begin{pmatrix} 1 & X & Y \\ 0 & 1 & Z \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} EV_1 \\ EV_2 \\ EV_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{23}$$

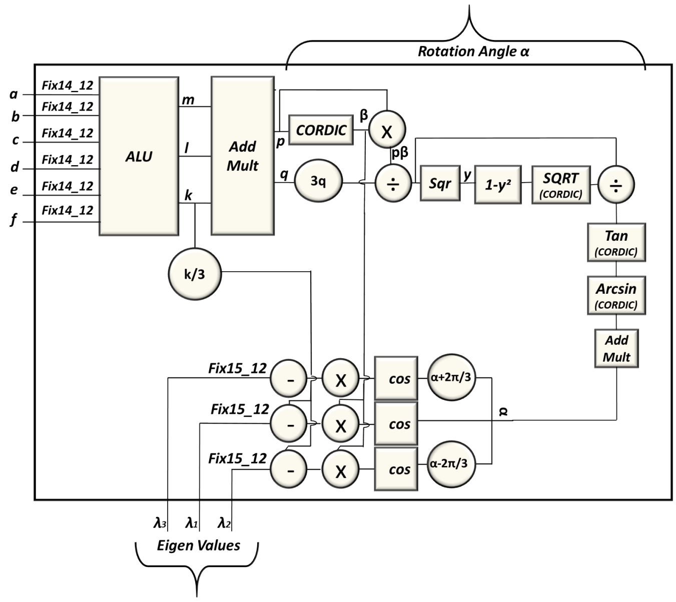

Fig. 5. Eigenvalue Decomposition.

From equation (23) the eigen vectors obtained will be:

$$EV_1 = XZ - Y \tag{24}$$

$$EV_2 = -Z \tag{25}$$

$$EV_3 = 1 \tag{26}$$

where

$$X = \frac{d}{(a-\lambda)} \tag{27}$$

$$Y = \frac{e}{(a-\lambda)} \tag{28}$$

$$Z = \frac{f - \frac{de}{(a-\lambda)}}{(b-\lambda) - \frac{d^2}{(a-\lambda)}} \tag{29}$$

This is the most expensive block of the entire design as its implementation is based on Divide Generator, CORDIC and CORDIC SINCOS IPs. All of these blocks require a large amount of resources and more clock cycles for operation. Therefore, efforts were made to optimize the algorithm in the eigenvalue decomposition block to reduce the number of clock cycles and optimize the FPGA resources.

5) Spectrum Peak Search

The MUSIC algorithm searches for the maxima in the signal subspace. In fact, many techniques for finding spectral peak search in DoA estimation using the MUSIC algorithm have been proposed [38] - [40]. Here, to optimally use the hardware resources and reduce the number of cycles, the minima of noise subspace is used to find the angle of arrival in the given sector. The peak search algorithm finds the angle within the 3 dB intercept point of antenna pattern in the sector. As the UCA assumed in this paper has six equally spaced antennas, the sectors formed by the six equally spaced antennae spans

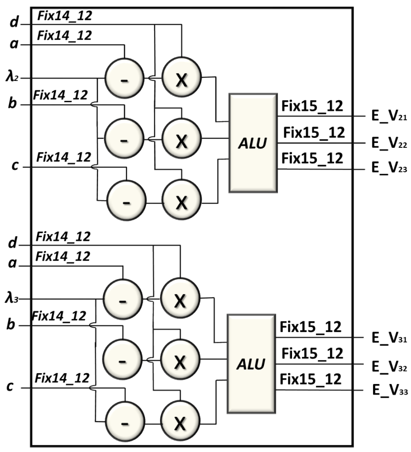

Fig. 6. Eigen Vector Computation.

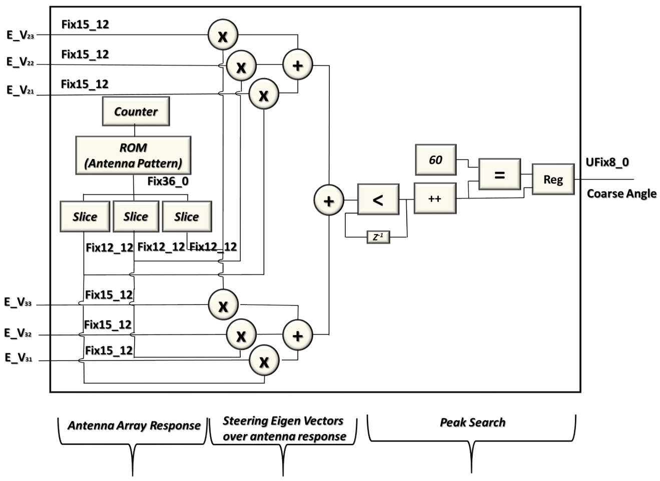

Fig. 7. Spectral Peak Search.

almost 60°. It is known that the gain pattern of the maximum receiving antenna decreases on either side of its main lobe. The antenna gain pattern is an important variable of the peak search algorithm and if we go beyond the 3dB intercept point it will not only increase computation but can also cause false triggering. Therefore, the peak search algorithm iteratively runs for sixty angles with a step size of 1°in the sector of maximum receiving antenna as shown in Fig. 7.

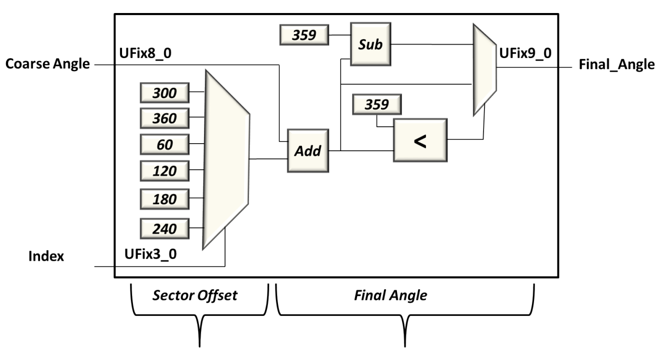

Fig. 8. Final Angle Estimation.

The final angle of arrival is calculated with respect to the index of the maximum receiving antenna Fig. 8. The proposed algorithm produces a result with desired accuracy (that is RMS error of estimated angle is less than 1°). As the DoA has maximum value of 359 it is represented in UFix9_0.

B. Optimized MUSIC Algorithm with modified EVD block

The implementation of the MUSIC algorithm on FPGAs has two main issues. The first one is the logarithmic increase in the number of bits with every computational stage that adds up to the resource utilization of the FPGA. This is properly addressed in design presented in the previous subsection as the logarithmic increase of data is reduced by introducing normalization of the covariance matrix. The second challenge is the use of costly IPs like Divide Generators, CORDIC, and SINCOS in terms of resources and time.

The analysis of different stages of the algorithm in the previous subsection revealed that when the covariance matrix is normalized, for any set of inputs, the values of the covariance matrix varies between 0 and 1, thus producing l and m to very small decimal values almost equal to zero. This is because when the parameters a, b, c, d, e, f are normalized to be in the 0 to 1 range and represented in Q2.12 (aka Fix14_12) format (though Q1.13 format is enough), any product would also be a value in this range and it would get smaller whether it is a squaring operation of a variable or a product term consisting of two or more variables. Except for the equation of k which is a simple addition of three parameters a, b and c, the equations for l and m contain many terms which contribute very little, so if neglected do not degrade the system output. The squaring terms can be neglected, and the same logic applies to for example the product term abc which will be orders of magnitude smaller than the original terms. As such small values have no contribution towards the calculation of eigenvalues so the equations can be modified for eigenvalue decomposition.

Keeping this in mind, equations (8) to (18) are modified for implementation of a further optimized design referred as Design Approach II. The modified mathematical model is as follows:

$$p = -\frac{1}{3}k^2 \tag{30}$$

$$q = \frac{2}{27}k^3 \tag{31}$$

Substituting the value of equation (30) and (31) the value of α is modified as:

$$\alpha = \frac{1}{3}(\frac{\pi}{2} - arcsin(-1)) \tag{32}$$

Which shows that α is now a constant. It also shows that many operations like multiplication, division, arcsin calculation are no longer required and this enables to implement the algorithm with fewer FPGA resources and number of clock cycles. It also eliminates the computation of COS using CORDIC SINCOS IP, replacing it by multiplication with a constant. Thus the modified equations for eigenvalue computation become:

$$\lambda 1 = x1 - \frac{k}{3} = \beta * CONST - \frac{k}{3} \tag{33}$$

$$\lambda 2 = x2 - \frac{k}{3} = \beta cos(CONST + 2\frac{\pi}{3}) - \frac{k}{3} \tag{34}$$

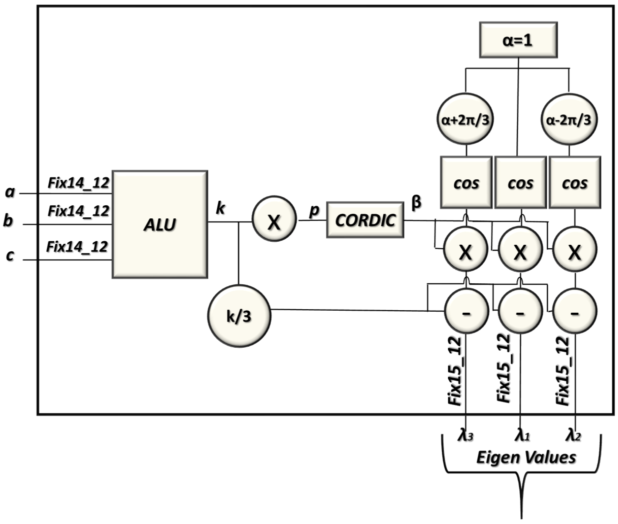

Fig. 9. Optimized Eigenvalue Decomposition.

$$\lambda 3 = x3 - \frac{k}{3} = \beta cos(CONST - 2\frac{\pi}{3}) - \frac{k}{3} \quad (35)$$

The implementation of the EVD optimizaton algorithm is illustrated in Fig. 9. All the changes and modifications are in the eigenvalue decomposition block. The comparison of Fig. 5 and Fig. 9 shows the differences in the implementation. It can be inferred from the figures that the simplified mathematical model devised after the normalization of the covariance matrix not only reduced the number of signals but also eliminated a significant number of sub-blocks for the EVD. The results obtained are better than for Design Approach I. It also improves the latency and FPGA resource utilization as will be seen in the evaluation results presented in the following section.

V. Evaluation of the Proposed Designs

Both variants of the MUSIC algorithm (Design Approach I and Design Approach II) have been implemented for FPGA using the Xilinx System Generator Tool [41] and released as open-source [42]. The input from each sensor is taken as an unsigned 14 bit that provides a 84dB dynamic range to the system. The algorithm is implemented in fixed-point. The antenna pattern is stored in 12 bits because increasing the bit resolution for antenna patterns did not produce any difference in the results. The algorithm was tested in software and subsequently mapped to an FPGA using Zynq (XC7Z020-CLG484-1) as the target hardware.

A. *Comparison of the Two Design Approaches*

Both designs are developed such that, the operating frequency is 100MHz, output accuracy is comparable, acceptable range of input amplitude is the same and both techniques should be able to generate correct results for SNRs in the range of 4.37dB to 30dB. Keeping these constraints the same for both approaches, the designs are compared for FPGA resource utilization, number of clock cycles, and accuracy.

1) *Test Setup*

A Simulink test setup was designed to generate the test signal in a sector that covers the FOV (Field of View) for the maximum receiving antenna. The input signal is generated from the 3dB intercept point of the maximum receiving antenna. The span between the two intercept points depends upon the placement of antennas. For a UCA of six antennae, each antenna is placed 60°apart. So the FOV of each antenna is approximately equal to 60°. The snapshots are varied by adding noise in order to characterize the algorithm on wider range of SNR. The Simulink test setup used is illustrated in Fig. 10. The MATLAB function block provides the flexibility to change the FOV, SNR and the input amplitude. Pulse generators provide the necessary controls. 1D serializer along with the FIFO provide rate transition of samples for hardware testing.

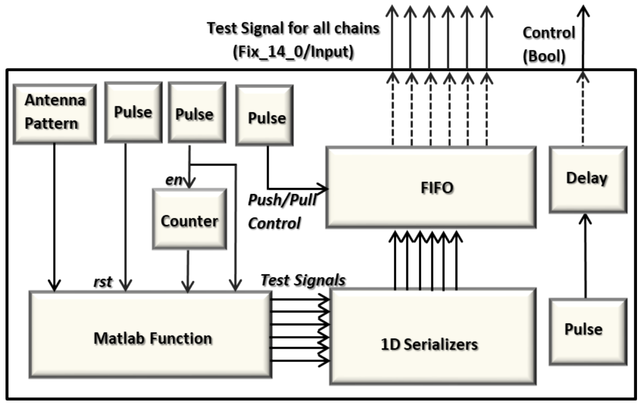

Fig. 10. Simulink Test Setup.

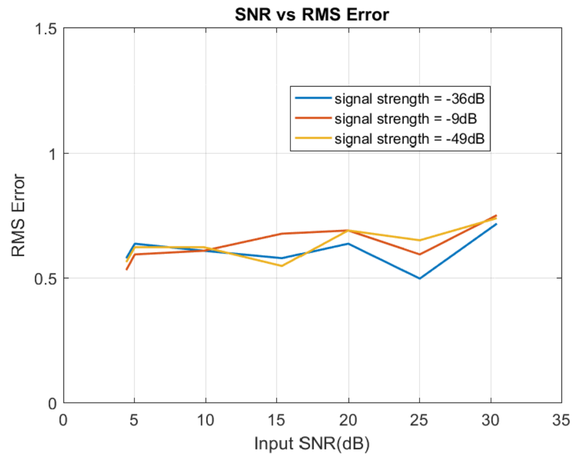

Fig. 11. SNR vs RMS Error on different input signal strength.

The two proposed schemes, i.e Design Approach I and Design Approach II were validated on ZedBoard Zynq Evaluation kit using Hardware Co-simulation. The design was ported to Vivado [41],[43]. The post implementation behavioral simulation was performed via test benches. The results obtained are similar to Hardware Co-simulation. The developed algorithms are tested for different input signals and varying SNR. From Fig. 11 it can be seen that the the RMS error of the estimated angle remains less than 1°for all cases.

2) *Comparison of Resources and Timing*

The resource utilization for both implementations (Design Approach I and Design Approach II) are summarized in Table I. It also presents the comparison of resources used by both approaches which shows that Design Approach II requires

TABLE I
COMPARISON OF TWO DESIGN APPROACHES

Resource Type	Design Approach I				Design Approach II				Savings (%)
	VAR + Cov	EVD	Peak Search + Final Angle	Total Resources	VAR + Cov block	EVD	Peak Search + Final Angle	Total Resources	
LUTs	2460	6304	133	8897	2149	4804	133	7086	20.3%
FF	3578	6483	212	10273	3009	4762	212	7983	22.3%
BRAM	0	0	0.5	0.5	0	0	0.5	0.5	0%
DSPs	24	26	8	58	18	10	8	36	37.9%
Clock(MHz)				100				100	0%
Latency (sec)				2.72 μ				2.0 μ	26.4%

TABLE II
TIMING CONSTRAINT SUMMARY FOR DESIGN FREQUENCY 100MHZ (NS)

Design	WNS	WHS	WPWS	Fmax (1/(10ns - WNS))
Approach I	0.551	0.023	4.02	105 MHz
Approach II	0.270	0.035	4.02	102.7 MHz

WSN - Worst Negative Slack, WHS - Worst Hold Slack,
WPWS - Worst Pulse Width Slack

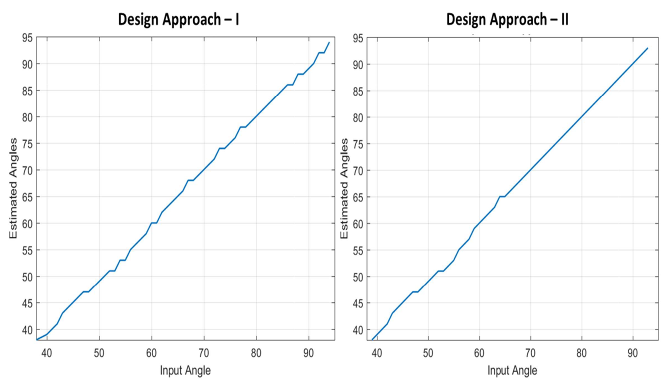

Fig. 12. Input Angle vs Estimated Angle for DA-I and DA-II .

almost 20% fewer resources as compared to Design Approach I. Likewise Design Approach I utilizes 272 cycles, whereas Design Approach II utilizes 200 cycles. Thus almost 72 cycles (almost 26%) are saved while implementing Design Approach II along with FPGA resource optimization. The processing time is about 2μsec, which is an adequate response time for any real-time system. Timing constraints for both designs are summarized in Table II.It can be seen that DA-I can be clocked at a maximum frequency of 105 MHz while DA-II can be operated at most at 102.7 MHz. A detailed analysis of the critical path in DA-I revealed that it is determined by the divider generator IP in the ARCSIN block passing through the FPGA carry-chains. As the chain is connected with dedicated silicon wires for fast propagation, mere application of optimization techniques like pipe-lining cannot improve speed, instead optimization of individual block IPs, for example, better Divider Generator can reduce the critical path.

3) Comparison of RMS Error

Both techniques mentioned above estimate the DoA with a high accuracy. The RMS error for both techniques is less than 1°for a wide range of SNR as shown in Fig. 11. The Design Approach I gives an accuracy of about 0.74°as shown in Fig. 12 that shows that the angle of arrival is calculated accurately through the entire FOV of the maximum receiving antenna. Ideally the estimated angle should vary linearly with the input angle, but due to some errors, there is a slight variation in the estimated DoA.

When EVD block is optimized, it eliminates a few modules from the EVD block. This elimination improves the RMS error in Design Approach II. This is because the quantization in the fixed point implementation introduces some errors at each module and processing step. Therefore, when some of the modules are eliminated the impact those quantization errors is reduced. As a result, the Design Approach II has an RMS error of as low as 0.49°. It is obvious from Fig. 12 that for Design Approach I the estimated angle accurately follows the input angle with exceptions at some points. For Design Approach II from Fig. 12 it can be seen that the estimated angle more linearly follows the change in the input angle.

4) Comparison of Power Consumption

Both design approaches are also compared for power consumption. The power consumption of the proposed designs was estimated with Vivado design suite with default settings. Design Approach I utilizes 496 mW and Design Approach II consumes 392 mW which are acceptable for practical systems. This shows that optimizations implemented in Design approach II also improved the design from a power utilization perspective.

B. Comparison with other Designs

The algorithm presented in this paper implements the MUSIC algorithm in 1D. However the same algorithm can be implemented for 2D arrays. If two concurrent modules of the same MUSIC algorithm are implemented, one for azimuth and the other for elevation, it will estimate the DoA in 2D. It can be approximated that for estimation of 2D angle of arrival the resources mentioned in Table I will double. It may also

TABLE III
COMPARISON WITH OTHER TECHNIQUES IN TERMS OF SPECIFICATIONS

Resources	This Paper Design Approach I/II	Reference [5]	Reference [26]	Reference [28]	Reference [29]LU_L / LU_U	Reference [30] LDL / CHOL
RMS error	0.49°	±0.1°	1°or 0.1°	0.29°	0.022°- 0.027°	0.25°
Array size	6	8	8 (ULA)	4	4 (ULA)	8 (ULA)
SNR(dB)	4.37 - 30	15	–	10	5–30	0-25
Device	Zynq XC7Z020	Virtex-6 LX130T	Virtex-6	Virtex-7 XC7VX690T	Virtex-5	Virtex-5
Signal Source	1	3	1	1	1/2	1/2
Snapshots	3	2048	128	1000	500	500

TABLE IV
COMPARISON WITH OTHER TECHNIQUES IN TERMS OF RESOURCES, CLOCK FREQUENCY, AND LATENCY NORMALIZED W.R.T DESIGN APPROACH II

Resources	This Paper Approach I	Reference [5]	Reference [26]	Reference [28]	Reference [29]LU_L	Reference [29]LU_U	Reference [30](LDL)	Reference [30](CHOL)
LUTs	1.26x	3.20x	3.39x	4.77x	1.65x	1.62x	1.54x	1.55x
FF	1.29x	–	1.89x	1.90x	1.05x	1.05x	1.08x	1.09x
BRAM	1.00x	60.00x	512.00x	–	10.00x	10.00x	10.00x	10.00x
DSPs	1.61x	3.35x	1.33x	–	3.68x	3.33x	3.23x	3.20x
Clock(MHz)	1.00x	1.00x	1.48x	1.45x	0.63x	0.65x	0.59x	0.63x
Latency	1.36x	500.00x	12.75x	190.00x	1.26x	1.20x	1.48x	1.54x

add some more resources to synchronize the two dimensional angle of arrival estimations. A 2D angle of arrival estimation is also presented in [5]. The approach presented in the paper [5] uses a large number of resources. Comparing the results of this paper and [5], it is inferred that covariance block for both approaches utilize same resources even though the approach presented here includes the Normalization block. EVD and Peak Search block use considerably less LUTs and BRAM but more DSPs as compared to the approach presented in [5]. The comparison of resource utilization of two approaches shows that for 2D estimation the of angle of arrival using MUSIC algorithm, the algorithm presented in this paper is efficient as compared to the technique presented earlier in terms of resources and number of consumed clock cycles. Similarly an efficient approach HFMA is also proposed in [26]. Comparing the algorithm presented in this paper with HFMA described in [26] again concludes that HFMA is an expensive approach w.r.t resource utilization. Likewise, on-off rotation method presented in [28] requires high FPGA resources and computational time with an incoming signal of 10dB SNR. The methods presented in [30] and [29] suggest to apply some sort of matrix operation like LU Decomposition (lower and upper triangle) and LDL/ Cholesky decomposition of covariance matrix before performing the eigenvalue decomposition. These operations help to reduce the complexity of EVD and the computational time but still require considerably high FPGA resources. Instead, the methods presented in this paper require only relatively simple operations like Antenna array reduction and normalization to be performed on the covariance matrix that considerably reduce computational time along with required resources of FPGA.

1) *Choice of word lengths for fixed point implementation*

Our choice of word lengths is determined by the off the shelf available ADC that fits the application requirements in terms of sampling frequency and resolution along with the required dynamic range of the system. Therefore, it followed a systematic approach for selection of word lengths from the ADC output to the final angle determination block keeping in mind the computational demands of each block. It is worth mentioning that our architecture utilizes variable word lengths from inputs to outputs compared to literature work which seems to use a fixed quantization. Our DA-I and DA-II are significantly better in terms of slice registers, slice LUTs, BRAMs and DSP48 device utilization when compared to Figure 14 in Reference [29] and Table 2 in Reference [30]. Interestingly, for 12/6 quantization (which means total word length is 12 bits in which 6 bits are used for integer) our DA-II is 3.33x, 2.45x, 2.31x better than LDL in Reference [30] for slice registers, slice LUTs and DSP48 resources. This is due to the fact that we have used optimized variable quantization across the design blocks.

2) *Comparison in terms of specifications*

The comparison of proposed algorithm with [5], [26], [28], [29] and [30] is presented in Table III. In this table designs are compared in terms of their accuracy, array architecture, SNR, simultaneous source handling and number of snapshots. Though other techniques mentioned in this table offer better accuracy, the accuracy offered by the algorithm presented in this paper still satisfies the real-time requirements [44]. SNR is one of most critical design constraints for any application. The ever increasing dense EM environment makes it a very significant design parameter for DoA estimation.This paper presents an algorithm that can operate in a wide range of SNR. The papers presented in [29] and [30] show a slightly better range in terms of signal handling in a noisy environment but the SNR range for design presented in this paper is comparable with them. Signal sources refer to the simultaneous signal handling of any algorithm. The algorithm presented in [5] is capable of handling 3 simultaneous signals but on the other hand it requires 1msec to give a reliable output. The processing time is almost 500 times the processing time of algorithms presented in this paper. Snapshots are the number of the samples for any given signals. Number of snapshots should be carefully monitored to avoid unnecessary computation but not compromising the overall accuracy.

As the design goal is to find the DoA for radar pulses as narrow as 50 ns [45], the optimum number of samples that can be acquired at 100 MHz is 3 (avoiding under/overshoot at the rise/fall time of the signal). Reducing the number of snapshots is not a new research direction in fact the authors in [46] have shown the implementation of their algorithm with a single snapshot. To have more samples per pulse, higher sampling frequencies of the order of GHz should be used that would increase the overall system complexity and cost and may not even be feasible for large values. Therefore, the algorithms presented in this paper compute the DoA with only 3 snapshots (signal values captured at different times/ number of samples) with comparable output accuracy and wide range of SNR. This enables to compute the DoA for signals with duration in order of nanoseconds.

3) *Comparison in terms of Resources, Latency and Power Consumption*

The comparison of normalized resources w.r.t the Design Approach II presented in this paper along with the computation time is summarized in Table IV which shows that the algorithms based on complex mathematical models [5], [26], [28] require much more resources and computation time.However, less complex designs, for example, the DoA estimation techniques mentioned in [29] and [30] still require 50% to 60% more resources and processing time.

Analysing the resource usage in FPGA designs is not so simple. For example, when comparing our design with that in [29] (the LU_U one that is the one with fewer resources), our design approach II requires 36 DSP blocks while LU_U in [29] requires 120 DSP blocks (so 3.33x times). The target device for our implementation is the XC7Z020-CLG484-1 that has 160 DSP blocks. So [29] will use 75% of the DSP blocks leaving very few blocks for the rest of the subsystems. Instead, our approach would leave most of the DSP blocks for other functions. In this case, LUTs or FFs are not so critical as the devices has more than 46K and 92K respectively and both designs used less than 30% of them. In fact, looking closer at Table III, all the alternative designs considered have either much larger latency so not being suitable for real-time or incur a significant (at least 3x) overhead in DSP blocks. Our proposed architecture enables a reduction in both latency and resource usage. In fact, the simplifications that we introduce allow us to eliminate many of the complex operations that use DSP blocks which is a critical resource when implementing signal processing systems on FPGAs. Similarly, the architectures proposed in this paper are also power efficient. The power consumption of the design in [26] utilizes 238.27mW for its proposed accelerator only that is developed in ASIC for TSMC 40nm CMOS technology.

VI. Conclusions and Future Work

In this paper a novel design approach was presented for an efficient implementation of the MUSIC algorithm for DoA estimation in terms of resource utilization and processing time for Xilinx FPGAs. The main aim of the design is to develop an algorithm that can respond in real-time. Special attention is paid to the constraints imposed by dense and noisy EM environments. The algorithm developed successfully works with SNRs as low as 4.37dB that represents a highly noisy EM environment. The results show that the optimized technique not only reduces the FPGA resource and latency requirements but also improves the output accuracy. This may be due to the fact that the elimination of certain stages reduces the cumulative quantization error, contributed by each stage. In summary, the designed algorithm provides a hardware efficient method for DoA estimation in a dense EM environment with real-time response time, considering all variants like SNR and intercepted power.

The algorithm proposed in this paper can be further optimized for resources and latency in future work. The most expensive IP is the Divide Generator. Any mathematical model that can reduce or eliminate the Divide Generator will significantly reduce the resource requirements. Moreover, the same Divide Generator needs significant pipelining adding to the number of clock cycles required to obtain the output. The same modification may serve both purposes. Finally, the proposed designs can be also used to optimize ASIC implementation. Evaluating their benefits for a modern ASIC technology is also an interesting topic for future work.

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