Dixit, D., et al. On the ASER Performance of UAV-Based Communication Systems for QAM Schemes. In: *IEEE Communications Letters, 25(6), June 2021, Pp. 1835-1838*

DOI: https://doi.org/10.1109/LCOMM.2021.3058212

On the ASER Performance of UAV-Based Communication Systems for QAM Schemes

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***Abstract*—In this paper, we derive an average symbol error rate (ASER) expression of rectangular quadrature amplitude modulation (RQAM) scheme for unmanned aerial vehicle-enabled communication systems operating over double-shadowing and double-scattering composite fading channel. A moment generating function for the receiver output signal-to-noise ratio is obtained to analyze the ASER expression of non-coherent modulation schemes. An asymptotic expression of ASER for RQAM scheme is also derived to examine diversity order of the considered system. Further, the impact of composite fading parameters and path loss on system performances is highlighted. Finally, we validate all the theoretical results through Monte Carlo simulations.**

***Index Terms*—Unmanned aerial vehicles, Double scattering, Composite fading, Quadrature amplitude modulation.**

I. Introduction

In wireless communications, the use of unmanned aerial vehicles (UAVs) is attracting huge research attention in both academia and industry due to its ability to improve both the coverage and capacity of terrestrial cellular networks, and its various applications. Due to high mobility and low operational cost, UAV-assisted wireless communications can provide flexible and cost-effective seamless communication services within an observed area [1], [2]. Further, UAV can be used for on-demand wireless communications in the desired area, for crowd monitoring and environment surveillance features, etc. However, UAV-enabled communication needs to address various challenges regarding physical layer communications such are: UAV selection strategy, placement/trajectory planning, channel modeling, control and navigation, and interference management.

Further, to achieve high data rates, quadrature amplitude modulation (QAM) is a preferred modulation scheme in 802.11 Wi-Fi standards, LTE-Advanced standards, and beyond due to its high spectral and power efficiency [3]. In literature, rectangular QAM (RQAM) is considered as a versatile modulation scheme since it includes several coherent modulation schemes namely binary phase-shift keying (BPSK), square QAM (SQAM), binary frequency-shift keying (BFSK), and quadrature phase-shift keying (QPSK), etc. [4]. RQAM can be implemented in asymmetric subscriber loop and telephone-line modems, high speed mobile communication systems, microwave communications, etc. Therefore, in this paper, we consider UAV-assisted wireless networks using RQAM to support higher data rates.

To access viability of UAV-assisted communication systems, various performance metrics have been evaluated in literature [5]-[10]. Outage performance of UAV-based communications is studied in [5]. A multi-hop UAV based system is analyzed in the presence of both multipath and shadowing effects in [6] in terms of the error rate and the channel capacity performance metrics. Further, outage probability of hybrid-duplex UAV-assisted communication system is analyzed over Rician-fading scenario in [7] and low-altitude aerial relaying environment in [8]. Outage performance of UAV based communication is analyzed by considering user mobility and fading in [9]. Recently, authors in [10] have investigated thoroughly the UAV-based communications considering channel modeling and UAV selection schemes. Furthermore, the average symbol error rate (ASER) performance of several QAM schemes, operating under different fading environments, for various wireless communication scenarios have been analyzed in the literature such as [4], [11], [12], etc. However, a UAV-assisted communication system using generic modulation scheme RQAM is not considered for analysis yet.

Contributions: In this paper, novel closed-form expression of ASER for general order RQAM scheme is derived by using cumulative distribution function (CDF)-based approach over double-shadowing (DS) and double-scattering (DSc) composite fading channel. Further, a moment generating function (MGF) expression of the receiver output signal-to-noise ratio (SNR) is evaluated to analyze the ASER for non-coherent modulation schemes such as non-coherent BFSK (NCBFSK) and differential BPSK (DBPSK). Furthermore, an asymptotic ASER expression for RQAM scheme is also investigated to examine the diversity order of the considered system. In this paper, we consider UAV-assisted networks which are suggested as a promising technology for beyond fifth-generation (5G) networks, to support the requirements for massive connectivity, ultra reliability, and increased throughput. Hence, the derived results in this paper can be useful to provide insights for beyond 5G wireless networks.

The remaining paper is organized as follows: The system and channel model are given in Section II. The expressions

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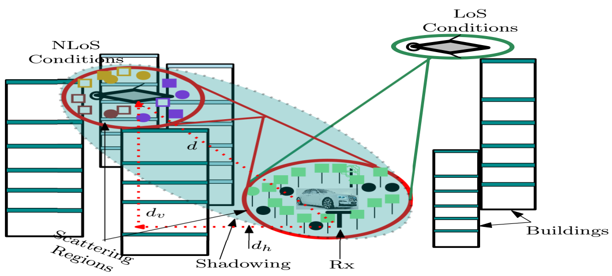


Fig. 1. System model.

for exact ASER and asymptotic ASER are obtained in Section III. In Section IV numerical results are given and discussed. Finally, conclusions are described in Section V.

II. SYSTEM AND CHANNEL MODEL

We choose a UAV-assisted communication as illustrated in Fig. 1, where a transmitter (Tx) node is placed in a UAV, and a receiver (Rx) node is placed at the ground. Further, it is also considered that the Tx node and/or the Rx node and/or significant scatterers around them are in motion. On the other hand, the scatterers near the Tx node and the Rx node are separated by a large distance. Thus, the received signal is subjected to DSc due to keyhole propagation [10]. Shadowing fading exists between the Tx node and the Rx node due to the presence of obstacles such as buildings. Furthermore, we consider DS scenario (due to non-light-of-sight (NLoS) conditions) which means shadowing exists in both local scattering regions at the Tx node and the Rx node. Assume the Tx node is transmitting a unit energy symbol x_t. At the Rx node, the base-band signal received can be expressed by $y_r = \sqrt{P_t\, d^{-\eta}} h\, x_t + w_r$, where P_t (in Watt) denotes the transmission power of the Tx node, h is the composite fading channel coefficient of the link, η is the path loss exponent, d_v (in meter), d_h (in meter), and $d = \sqrt{d_v^2 + d_h^2}$ (in meter) are the vertical distance and the horizontal distance, and the propagation distance between the Tx node at the UAV and the Rx node at the ground, respectively, and w_r is additive white Gaussian noise with zero mean and N_0 variance at the Rx node. The instantaneous output SNR at the Rx node is given by $\gamma = P_t\, d^{-\eta}|h|^2/N_0$. For the performance analysis, we need a CDF expression $F_\gamma(\gamma)$ for output SNR γ over DS and DSc scenario which is given by [10, (12)]

$$F_\gamma(\gamma) = \mathbb{S}_0 G_{3,3}^{2,3}\left(\frac{m_1 m_2}{\bar{\gamma}}\gamma \middle| \begin{array}{c} 1-\alpha_2, 1-\alpha_1, 1 \\ m_1, m_2, 0 \end{array}\right), \tag{1}$$

where $\bar{\gamma} = P_t\, d^{-\eta}\mathbf{E}[|h|^2]/N_0$ denotes average SNR, $\mathbf{E}[\cdot]$ is a statistical averaging operator, $G_{p,q}^{m,n}(\cdot)$ represents the Meijer's G-function, m_1 and m_2 are multi-path fading parameters, α_1 and α_2 are shadowing parameters, and $\mathbb{S}_0 = \frac{1}{\Gamma(m_1)\Gamma(m_2)\Gamma(\alpha_1)\Gamma(\alpha_2)}$. Furthermore, by using the CDF-based approach and with the help of [13, (7.813.1)], the MGF expression can be obtained as

$$\begin{aligned}\mathcal{M}_\gamma(s) &= \int_0^\infty s\, \exp(-s\gamma) F_\gamma(\gamma) d\gamma \\ &= \mathbb{S}_0 G_{4,3}^{2,4}\left(\frac{m_1 m_2}{s\bar{\gamma}} \middle| \begin{array}{c} 0, 1-\alpha_2, 1-\alpha_1, 1 \\ m_1, m_2, 0 \end{array}\right).\end{aligned} \tag{2}$$

III. ASER PERFORMANCE

The ASER expression for several modulation schemes with the aid of the CDF expression for the system output SNR γ can be derived as [4, eq. (5)]

$$P_s(e) = -\int_0^\infty P_s^{'}(e|\gamma) F_\gamma(\gamma) d\gamma, \tag{3}$$

where $P_s^{'}(e|\gamma)$ denotes the first derivative of the conditional symbol error rate (SER) $P_s(e|\gamma)$ for AWGN channels w.r.t. γ.

A. Coherent Generalized RQAM Scheme

The desired conditional SER for $M_1 \times M_2$-ary RQAM scheme is given by [4]

$$P_s(e|\gamma) = 2[pQ(a\sqrt{\gamma}) + qQ(b\sqrt{\gamma}) - 2pqQ(a\sqrt{\gamma})Q(b\sqrt{\gamma})], \tag{4}$$

where $M = M_1 \times M_2$, $p = 1 - \frac{1}{M_1}$, $q = 1 - \frac{1}{M_2}$, $a = \sqrt{\frac{6}{(M_1^2-1)+(M_2^2-1)\beta^2}}$, $b = \beta a$, wherein M_1 and M_2 are the number of in-phase and quadrature-phase constellation points, respectively. $\beta = d_2/d_1$, where d_1 and d_2 represent in-phase and quadrature phase decision distances, respectively. The first derivative of $P_s(e|\gamma)$ w.r.t. γ given in (5), can be obtained by using the identity $Q(x\sqrt{z}) = 0.5\,\text{erfc}\left(\frac{x\sqrt{z}}{\sqrt{2}}\right) = \frac{0.5}{\sqrt{\pi}} G_{1,2}^{2,0}\left(\frac{x^2 z}{2} \middle| \begin{array}{c} 1 \\ 0, 0.5 \end{array}\right)$ [14, (06.27.26.0006.01)]. Further, substituting (1) and (5) into (3), and solving required integration using identity $\exp(-x) = G_{0,1}^{1,0}\left(x \middle| \begin{array}{c} - \\ 0 \end{array}\right)$ [14, (01.03.26.0004.01)], and with the aid of [13, (7.813.1), (9.301)] and [15], a new closed-form ASER expression of RQAM scheme can be obtained as given in (6), where $H_{p_1,q_1:p_2,q_2:p_3,q_3}^{0,n_1:m_2,n_2:m_3,p_3}(x, y|\cdot)$ denotes bi-variate Fox-H function [15, (2.57)]. From the ASER expression of RQAM scheme, one can also obtain ASER expression for SQAM as a special case by substituting $M_1 = M_2 = \sqrt{M}$ and $\beta = 1$. Moreover, it is interesting to mention that by putting $M_1 = 2$, $M_2 = 1$, $p = 0.5$, $q = 0$, $a = \sqrt{2}$, and $\beta = 0$ into (6), the ASER for BPSK can also be obtained which is equal to [10, (15)].

B. Non-Coherent Modulations

The ASER expression for any modulation schemes using probability density function (PDF)-based method can also be evaluated as [16]

$$P_s(e) = \int_0^\infty P_s(e|\gamma) f_\gamma(\gamma) d\gamma, \tag{7}$$

where $f_\gamma(\gamma)$ represents PDF of γ. The conditional SER for binary non-coherent modulation schemes, $P_s(e|\gamma)$ is given in [16] as

$$P_s(e|\gamma) = 0.5\exp(-\mathcal{B}\gamma), \tag{8}$$

where $\mathcal{B}$ is a modulation dependent constant. For instance, such modulation scheme includes NCBFSK for $\mathcal{B} = 0.5$ and DBPSK for $\mathcal{B} = 1$. By substituting (8) into (7), and manipulating the integration, we can obtain the ASER expression for non-coherent modulation schemes in terms of MGF as

$$P_s(e) = 0.5\,\mathcal{M}_\gamma(\mathcal{B}). \tag{9}$$

$$P_s^{'}(e|\gamma) = -\frac{ap}{\sqrt{2\pi}}\gamma^{-0.5}\exp\left(-0.5a^2\gamma\right) - \frac{bq}{\sqrt{2\pi}}\gamma^{-0.5}\exp\left(-0.5b^2\gamma\right) + \frac{apq}{\sqrt{2\pi}}\gamma^{-0.5}\exp\Big(-0.5a^2\gamma\Big) \times G_{1,2}^{2,0}\Big(0.5b^2\gamma\Big|\begin{array}{c}1\\0,0.5\end{array}\Big) + \frac{bpq}{\sqrt{2\pi}}\gamma^{-0.5}\exp\Big(-0.5b^2\gamma\Big)G_{1,2}^{2,0}\Big(0.5a^2\gamma\Big|\begin{array}{c}1\\0,0.5\end{array}\Big). \quad (5)$$

$$P_s^{RQAM}(e) = \frac{\mathbb{S}_0 p}{\sqrt{\pi}}G_{4,3}^{2,4}\Big(\frac{m_1m_2}{0.5a^2\bar{\gamma}}\Big|\begin{array}{c}0.5,1-\alpha_2,1-\alpha_1,1\\m_1,m_2,0\end{array}\Big) + \frac{\mathbb{S}_0 q}{\sqrt{\pi}}G_{4,3}^{2,4}\Big(\frac{m_1m_2}{0.5b^2\bar{\gamma}}\Big|\begin{array}{c}0.5,1-\alpha_2,1-\alpha_1,1\\m_1,m_2,0\end{array}\Big) - \frac{\mathbb{S}_0 pq}{\pi}H_{1,0:1,2:3,3}^{0,1:2,0:2,3}\left(\frac{b^2}{a^2},\frac{m_1m_2}{0.5a^2\bar{\gamma}}\middle|\begin{array}{c}0.5,1,1\\-\end{array}\middle|\begin{array}{c}(1,1)\\(0,1),\Big(0.5,1\Big)\end{array}\middle|\begin{array}{c}(1-\alpha_2,1),(1-\alpha_1,1),(1,1)\\(m_1,1),(m_2,1),(0,1)\end{array}\right) - \frac{\mathbb{S}_0 pq}{\pi}H_{1,0:1,2:3,3}^{0,1:2,0:2,3}\left(\frac{a^2}{b^2},\frac{m_1m_2}{0.5b^2\bar{\gamma}}\middle|\begin{array}{c}0.5,1,1\\-\end{array}\middle|\begin{array}{c}(1,1)\\(0,1),\Big(0.5,1\Big)\end{array}\middle|\begin{array}{c}(1-\alpha_2,1),(1-\alpha_1,1),(1,1)\\(m_1,1),(m_2,1),(0,1)\end{array}\right). \quad (6)$$

$$G_{3,3}^{2,3}\Big(\frac{m_1m_2}{\bar{\gamma}}\gamma\Big|\begin{array}{c}\triangle_1\\\triangle_2\end{array}\Big) \approx \sum_{k=1}^{2}\Big(\frac{m_1m_2}{\bar{\gamma}}\gamma\Big)^{\triangle_{2,k}}\frac{\prod_{l=1,l\neq k}^{2}\Gamma(\triangle_{2,k}-\triangle_{2,l})\prod_{l=1}^{3}\Gamma(1-\triangle_{1,l}+\triangle_{2,k})}{\prod_{l=3}^{3}\Gamma(1-\triangle_{2,l}+\triangle_{2,k})}, \quad (10)$$

C. Asymptotic ASER

In this section, we derive the asymptotic expression of ASER for RQAM scheme in the high SNR regime ($\bar{\gamma} \to \infty$). For the high SNR approximation, the Meijer-G function in (1) can be approximately written as (10) [14, (07.34.06.0006.01)], where $\triangle_1 = (1-\alpha_2, 1-\alpha_1, 1)$ and $\triangle_2 = (m_1, m_2, 0)$. Further, putting (10) into (1) and then substituting (1) into (3) along with (5), and solving the required integration with the help of [13, (3.381.4), (7.813.1)], the simplified asymptotic ASER for RQAM can be expressed as

$$P_s^{\infty}(e) \underset{\bar{\gamma}\to\infty}{\approx} \mathbb{S}_0 \sum_{k=1}^{2}\Big(\frac{m_1m_2}{\bar{\gamma}}\Big)^{\triangle_{2,k}}\mathcal{R}_k. \quad (11)$$

where $\mathcal{R}_k$ is given in (12) and $\triangle_{2,1} = m_1, \triangle_{2,2} = m_2$. From (11), the diversity order of the considered system is equal to $\min(m_1, m_2)$. Further, we can observe that only fading parameter shows the impact on the system's diversity, and variation in shadowing parameter does not affect the diversity order.

IV. Numerical and Simulation Results

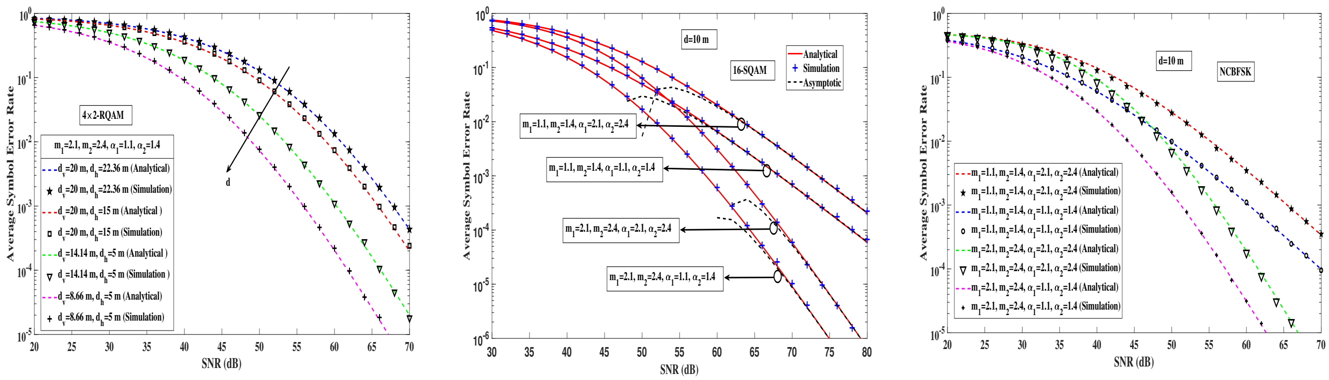
In this section, we present ASER plots with the theoretical and simulation results for several values of system parameters. For theoretical results, the bi-variate Fox-H function is numerically computed using its Mellin-Barnes representation with a suitable finite contour [17]. In the ASER plots, the SNR denotes the normalized average SNR without path loss and a path loss exponent η is chosen to be 2.5 (typically from 2 to 6 for a propagation scenario) [2]. Also, we choose $m_2 = m_1 + 0.3$ and $\alpha_2 = \alpha_1 + 0.3$.

In Fig. 2(a), a comparison between theoretical and simulation curves of ASER for 4×2-RQAM scheme against SNR over DS and DSc channels is demonstrated, considering several values of d_v and d_h for low-altitude UAV case. From the figure, we observe that the theoretical and the simulated results overlap. It can also be noticed that the ASER performance improves as d_v and d_h decrease. For example, to maintain an ASER of 10^{-3}, the SNR gain of 1.95 dB (approx.) is observed by decreasing d_h from 22.36 m to 15 m when d_v is fixed at 20 m. However, the SNR gain of 4.4 dB (approx.) is observed by decreasing d_v from 14.14 m to 8.66 m, when d_h is fixed at 5 m.

In Fig. 2(b), theoretical and simulated curves of ASER for 16-SQAM scheme are plotted together along with asymptotic curves against SNR over DS and DSc channels, considering several values of composite fading parameters and a fixed $d = 10$ m. The asymptotic curves for all the investigated cases are also aligned with theoretical and simulated curves in high SNR regime which further validate the accuracy of the theoretical analysis. It can also be noted that the ASER performance improves when fading parameter m_1 increases and/or shadowing parameter α_1 decreases, however the impact of m_1 is more than α_1. For example, to maintain an ASER of 10^{-3}, the SNR gains of 9.5 dB (approx.) and 10.5 dB (approx.) are observed by increasing m_1 from 1.1 to 2.1 when α_1 is fixed at 1.1 and 2.1, respectively, however, the SNR gains of 5.4 dB (approx.) and 4.5 dB (approx.) are observed by decreasing α_1 from 2.1 to 1.1 when m_1 is fixed at 1.1 and 2.1, respectively. Using adaptive modulation to enhance the spectral efficiency for wireless communications has been suggested [16]. One can notice that in the low SNR region and at the high-altitude, the lower-order QAM scheme can be preferred over the higher-order QAM scheme. In the high SNR region and at the low-altitude, the higher-order QAM scheme can be preferred over the lower-order QAM scheme.

Fig. 2(c) shows the ASER curves of NCBFSK scheme for a fixed $d = 10$ m. Similar to Fig. 2(b), one can observe that the ASER performance improves when fading parameter m_1 increases and/or shadowing parameter α_1 decreases. For exam-

$$\mathcal{R}_k = \frac{ap}{\sqrt{2\pi}}\Gamma\left(\triangle_{2,k}+0.5\right)\left(0.5a^2\right)^{-(\triangle_{2,k}+0.5)} + \frac{bq}{\sqrt{2\pi}}\Gamma\left(\triangle_{2,k}+0.5\right)\left(0.5b^2\right)^{-(\triangle_{2,k}+0.5)} - \frac{apq}{\sqrt{2\pi}}\left(0.5a^2\right)^{-(\triangle_{2,k}+0.5)}$$
$$\times G_{2,2}^{2,1}\left(\frac{b^2}{a^2}\middle|\begin{matrix}-(\Delta_{2,k}-0.5),1\\0,0.5\end{matrix}\right) - \frac{bpq}{\sqrt{2\pi}}\left(0.5b^2\right)^{-(\triangle_{2,k}+0.5)} G_{2,2}^{2,1}\left(\frac{a^2}{b^2}\middle|\begin{matrix}-(\Delta_{2,k}-0.5),1\\0,0.5\end{matrix}\right). \quad (12)$$



(a) (b) (c)

Fig. 2. (a): ASER performance for 4×2-RQAM, (b): ASER performance for 16-SQAM, (c): ASER performance for NCBFSK.

ple, to maintain an ASER of 10^{-3} for NCBFSK modulation scheme, the SNR gains of 8.75 dB (approx.) and 9.75 dB (approx.) are observed by increasing m_1 from 1.1 to 2.1 when α_1 is fixed at 1.1 and 2.1, respectively, however, the SNR gains of 5 dB (approx.) and 4.75 dB (approx.) are observed by decreasing α_1 from 2.1 to 1.1 when m_1 is fixed at 1.1 and 2.1, respectively.

V. Conclusion

In this paper, new closed-form ASER expressions for RQAM, NCBFSK, and DBPSK for a UAV-assisted communication over DS and DSc composite fading channel have been derived. Moreover, asymptotic ASER expression for RQAM scheme has been derived to study diversity order. The impact of fading parameters and path loss on ASER performance has also been highlighted. Using similar approach one can also analyze the performance of UAV-based communications over single-shadowing channel for QAM schemes.

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