

Essays in Macroeconomics

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A dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in

Economics

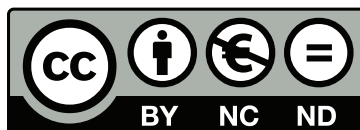
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Other Research Merits

Chapter 2 has been awarded with the ‘III Premio Nada es Gratis a Job Market Papers en Economía’ by the economics spanish blog Nada es Gratis. A blog post about this Chapter was published on January 27, 2021, under the title ‘La caída del dinamismo empresarial: Una historia de manzanas y naranjas’.

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Summary

During the last three decades, there have been many fundamental changes regarding the evolution of most advanced economies, particularly the United States. Some remarkable trends characterizing this evolution are the decline in business dynamism, the increase in average profits and average markups of firms, the rise in market concentration, the increase of robotization and automation, the decline in the labor share, the decline in the entry rate of new firms, and the stagnation of productivity growth. The combination of these facts has raised concerns, among others, about increasing inequality, declining entrepreneurship due to the crowding-out of new entrants, and declining consumer purchasing power. Although identifying the reasons underlying these facts is crucial from a policy-making perspective, there is little agreement on these macroeconomic trends' underlying cause(s). This dissertation consists of two chapters that attempt to shed light on this discussion.

In the first chapter, *'Inside the decline of the labor share: Bringing the tales together'*, I analyze the decline in the United States labor income share during the last two decades. This decline has been widely documented in the economic literature and is also contemporaneous to a strong structural change process from manufacturing to services. In other words, during the last two decades, the U.S. has continued its transformation into a more service-oriented economy. In particular, I document the following trends regarding both phenomena: i) the evolution of the labor share of income is widely heterogeneous across industries, with a much more substantial decline in manufacturing than in services; ii) the average wage, employment, and value added – the main components of the labor share – exhibit a different pattern between manufacturing and services industries; iii) there is a strong (relative) reallocation of labor and capital from manufacturing to services industries; and iv) substantial capital deepening has taken place, increasing of the capital-labor ratio of the economy together with the capital-output ratio of the economy.

I analyze both phenomena through the lens of a multi-sector model in which the main mechanisms that have been proposed to explain the decline in the labor share – biased

technological change and market power – also affect the process of structural change between sectors. Moreover, I show that structural change is relevant for the overall labor share as it affects the weight of each industry’s labor share. I use the model to infer the rate of technological change and market power – in the form of exogenous markups – that match the U.S. data. The model can explain up to 87.4% of the decline in the labor share of the U.S. economy. Moreover, I find that the increase in markups in the manufacturing and services industries is the main reason underlying the decline. The remaining part is accounted for by technological change, which after 2008 becomes more important in explaining the decline in the labor share, especially in services industries. Although market power also affects the pace of structural change from manufacturing to services, I show that technological change is its fundamental driver.

In the second chapter, *‘Provider-driven complementarity and firm dynamics’*, I concentrate on the significant decline in business dynamism experienced in the U.S. since the mid-1980s. This decline is (non-exhaustively) characterized by the following trends: i) the entry rate of new firms has declined; ii) market concentration, measured by the share of sales accruing to the biggest firms, has increased; iii) expenditure on R&D activities, measured both as a fraction of total cost or total sales, has increased; and iv) the growth rate of the economy has slowed down.

I offer a new explanation based on the assumption of provider-driven complementarity, which makes seemingly independent products become complements when provided by a single firm. Provider-driven complementarity boils down to the idea that during the process of product innovation – the introduction of new and improved products to the market – firms can embed differential characteristics to their products. These characteristics are such that, absent quality differences across products, consuming several goods from a single provider is preferable to purchasing each good from a different firm. Based on this idea, I propose a framework that explains increasing R&D expenditures and concentration yet decreasing entry rates and economic growth.

Theoretically, I develop a quality ladder growth model where provider-driven complementarity is crucial in determining firms’ incentives to challenge incumbents in their established markets. Specifically, I assume that the complementarity effect increases in the number of products supplied by each firm. Moreover, consumers buy each good from the firm that delivers the highest quality, adjusted by provider-driven complementarity, relative to its market price. I show that provider-driven complementarity generates an endogenous barrier to entry in new markets. Consequently, despite firms use innovation to increase profits, it also affects firms’ industrial organization and can ultimately deter

firm entry.

I use the theory of provider-driven complementarity to perform a quantitative exercise in which I reduce the size of the average quality jump obtained after any successful innovation. The exercise is motivated by the recent literature on ideas becoming harder to find and can also be thought of as innovations becoming less radical over time. I show that such decline induces a growth slowdown. Additionally, I find that the entry rate declines, and incumbents become bigger and spend more resources on R&D, even as the economy's overall growth rate declines. This contrasts with the predictions of a standard quality ladder model without provider-driven complementarities, which implies the reverse.

Chapter 1

Inside the decline of the labor share: Bringing the tales together

1.1 Introduction

The labor share – the part of national income allocated to labor compensation – is a key variable in economics, crucial for analyzing technological change or inequality. It comes as no surprise that the influential papers of [Elsby, Hobijn and Sahin \(2013\)](#), [Karabarbounis and Neiman \(2014\)](#), and [Koh, Santaeuilàlia-Llopis and Zheng \(2020\)](#), among others, documenting a decline in the labor share in the U.S. during the last decades, have received a great deal of attention.¹ A declining labor share poses many relevant macroeconomic and policy-making concerns. For example, it can indicate lower average labor compensation growth relative to labor productivity, leading to increasing inequality and declining consumer purchasing power. As a consequence, it is crucial to understand the reasons underlying the decline.² However, the literature is far from a consensus when it comes to isolating the drivers of the decline, despite the effort made trying to unravel its cause.

In this paper, I use industry-level data from the [U.S. Bureau of Economic Analysis \(2017\)](#) to examine the evolution of the U.S. labor share and its main components from 1998 to 2016.³ At the same time, I also examine the contemporaneous structural change process, characterized by the relative reallocation of production inputs (namely employment and capital stock) from manufacturing to services industries, and the increasing contribution to value added generation of the latter. This analysis establishes the follow-

¹The labor share has also declined in some EU countries, as shown in [Kostarakos \(2020\)](#).

²For example, for the design of optimal fiscal policy, as shown in [Atesagaoglu and Yazici \(2020\)](#).

³Although previous research has established that the decline of the labor share may have already started in the early 1980s, at least half of the decline has taken place since the late 1990s.

ing facts:

1. The evolution of the labor share has been substantially heterogeneous across industries:
 - (a) The (linear) trend of the labor share shows a steeper decline in manufacturing than in services industries. In manufacturing, the labor share decline is roughly five times higher than in services.
 - (b) The evolution of compensation of employees – the main component of the labor share, formed by wages, employment, and value added – exhibits a different pattern between manufacturing and services industries. In manufacturing, the average wage has grown faster than that of value added, while employment has declined during the last two decades. In contrast, employment in services has increased, while the growth rate of value added has been higher than that of the average wage.
2. There has been a strong (relative) reallocation of labor and capital from manufacturing to services industries. The reallocation of labor has been more intense than that of capital. At the same time, the share of value added generated by services industries has increased.
3. Substantial capital deepening has taken place during the last two decades:
 - (a) Consequently, the capital-labor ratio of the U.S. economy measured in real 2009\$ per hour worked has increased from 69.5 in 1998 to 91.4 in 2016.⁴ In manufacturing, the capital-labor ratio strikingly increased from 69.2 in 1998 to 137.5 in 2016.
 - (b) The economy's capital-output ratio also increased from 1.55 in 1998 to 1.61 in 2016, especially during the Great Recession. Despite starting at similar levels in 1998, the capital-output ratios of manufacturing and services have diverged during the last two decades.

Previous literature has highlighted two reasons that underlie the decline in the labor share: technological change (e.g. in the form of automation)⁵ and changes in market power (reflected in the increasing profits and concentration of firms)⁶. However, as the

⁴capital excludes residential assets and publicly-owned capital.

⁵Some examples are Acemoglu and Restrepo (2019), Hémous and Olsen (2021) or Martínez (2019).

⁶See, among others, Autor, Dorn, Katz, Patterson and Van Reenen (2020), Barkai (2020) or De Loecker and Eeckhout (2020).

labor share of an economy can be computed by appropriately weighting industry-level labor shares, the previous analysis highlights an additional source of variation for the labor share. Provided that the labor shares of manufacturing and services are different, an increase in the relative importance of services in the production of total value added will be reflected in the economy's labor share. Therefore, the strong process of structural change from manufacturing to services has an obvious effect on the evolution of the labor share.

In this paper, I use structural change data to help to quantify the contribution of technological change and market power in explaining the decline of the labor share and the process of structural change itself. To do this, I develop a two-sector model of structural change – an extended version of the [Álvarez-Cuadrado, Long and Poschke \(2017, 2018\)](#) framework – which stresses the role of the supply side on the process of structural change. Specifically, the model allows for industry-specific technological change and production technology differences across industries.

I incorporate heterogeneous competition levels across sectors to enrich this framework, which I model as the ability to charge (exogenous) industry-specific time-varying markups. Consequently, industry-specific technological change, production technology differences, and heterogeneous markups across industries can explain structural change and the evolution of the labor share at the industry and aggregate level. While the effect of markups in explaining the decline of the labor share is well known and has been already studied in [Karabarbounis and Neiman \(2014\)](#) or [Barkai \(2020\)](#), the novel part of the model is that markups can also play a role in explaining structural change. The idea is simple: if one industry can charge higher markups than others, it will affect the relative price between industries. Therefore, higher prices driven by higher markups in one industry will affect the quantity demanded of output from other industries. The model is intentionally reduced to the main elements that are quantitatively relevant. In this sense, the model restricts to intra-temporal decisions and abstracts from the inter-temporal dimension. In other words, I consider a static allocative equilibrium model repeatedly: given an endowment of capital stock and labor, the solution of the model reduces to determining the equilibrium allocations of capital and labor across sectors, abstracting from capital accumulation or labor supply decisions.

I then use the model to perform a quantitative analysis where I analyze the joint evolution of the U.S. labor share (both at the aggregate and industry-level) and the process of structural change during the last two decades. The exercise is conducted as follows. First, I obtain the time series of the stock of capital and labor from the data

and feed them period-by-period into the model as an endowment. I assume that the deep parameters governing consumption and production are fixed. In contrast, technological change and market power parameters can vary over time. Second, by repeatedly using the first-order and market clearing conditions that characterize the static equilibrium, I retrieve the evolution of technological change and market power that match quantitatively the evolution of both industry labor shares and the process of structural change between sectors. The model can explain up to 87.4% of the decline in the labor share of the economy. Moreover, I find that the increase in markups in the manufacturing and services industries is the main reason underlying the decline. Specifically, market power accounts for 64.1% of the decline, being more relevant in manufacturing. The remaining part of the decline is produced by capital-biased technological change,⁷ which after 2008 becomes more important in explaining the decline in the labor share, especially in services industries. Finally, although market power affects the pace of structural change from manufacturing to services, technological change is its fundamental driver.

I conduct a series of robustness exercises that reinforce the findings of the baseline quantitative exercise. On the one hand, I show that the qualitative nature of the explanations for the decline in the labor share and the structural change process does not change when using different estimates for the elasticity of substitution between capital and labor in the production function of manufacturing and services goods. These are crucial parameters of the model and play a fundamental role in determining the evolution of each industry's capital-labor ratio and its labor share. Most of the literature that estimates these elasticities finds values below one, i.e., capital and labor are slight complements in the production process. I rely on this literature in the baseline exercise and fix values smaller than one for these elasticities. To provide additional validation on the baseline exercise results, I conduct a series of robustness exercises where I consider alternative estimates for these elasticities. I find that the importance of the evolution of markups in explaining the decline of the labor share is preserved under a wide range of estimates for this elasticity.

In the Appendix, I conduct a series of additional exercises where I consider alternative assumptions on the joint evolution of technological change and market power. In particular, following the narrative on the relevance of technological change, I focus on explanations that stress its crucial role while considering different assumptions on both the level and the evolution of markups over time. I show that technological change can

⁷That is, the evolution of technology is such that it increases the relative productivity of capital with respect to labor.

go a long way in explaining the declining trend in the labor share across sectors. However, this comes at several costs. Without market power, it is not possible to match the declining trend of the labor share in services, and the levels of the labor shares across sectors are similar, opposite to what is observed in the data. If I introduce market power but assume that markups are homogeneous across industries and constant over time, the fit of the model improves. However, the level of the labor shares is still far from what I observe in the data, and the declining trend of both industry labor shares is slightly more robust than in the data. Finally, if markups are assumed to be heterogeneous but constant over time, the model can generate a difference in the average level of the labor shares as observed in the data. Still, it produces a stronger declining trend of the labor share in services. This reinforces the baseline quantitative analysis results. I conclude that markups must be heterogeneous across industries and increase over time to match the data.⁸

Ultimately, the results of this paper are aligned with the stream of the literature on the labor share decline that explains it as a consequence of a reduction in the levels of competition, which favors that firms increase their markups over marginal costs. Besides, other phenomena as automation or the rise of artificial intelligence (which may be understood as a type of biased technological change) also have an important effect on the labor share, albeit smaller. However, these phenomena are essential to understand the process of structural change from manufacturing to services.⁹

Literature review. This paper contributes to two strands of the economic literature. First, to the literature on the labor share decline. Several explanations that try to understand the reasons underlying its decline have been put forward. In this sense, the decline is increasingly related to the continuously increasing capabilities of computers, artificial intelligence, and robots, which raise concerns about the risks of marginalization of human labor soon. Although these concerns are nowadays receiving much attention, they are far from being new. Technological change has always allowed substituting human labor for that performed by machines (Herrendorf, Herrington and Valentinyi, 2015). More than half a century ago, Keynes (1931) and Leontief (1952) argued that eventually, economies worldwide would experience a significant decrease in human labor, which would be replaced by the introduction of automatized processes and machines. While their claims did

⁸Moreover, if markups do not change over time, the process of technological change is very strong and highly volatile, especially compared to the baseline quantitative analysis.

⁹Following a different approach, Dixon and Lim (2020) find similar results regarding the decline in the labor share in contemporaneous work.

not seem to come true at that time, technology is also being introduced in more service-oriented sectors nowadays, which raises the question of whether this time is different from previous times (Autor, 2015). There is a stream of the literature directly related to these concerns, the automation literature, that has gained much interest during the last few years (see, e.g. Hémous and Olsen, 2021, Acemoglu and Restrepo, 2019, 2020, Martínez, 2019). These papers develop models with automation, horizontal innovation, endogenous technologies, and the creation of new tasks where labor has a comparative advantage and study the dynamic properties of these environments. In this line of research, Frey and Osborne (2017) and Arntz, Gregory and Zierahn (2016) try to quantify the risks of automation and computerization by estimating which activities are more likely to be automated. A similar explanation, although without considering automation explicitly, is given by Álvarez-Cuadrado et al. (2017), who argue that in a multi-sector model growth in which there exist differences in the elasticity of substitution between capital and labor across different sectors, economic growth can lead to structural change and the decline of the overall labor share.

The decline in the labor share is also tied up to the literature that addresses the labor markets' polarization and the increase in inequality, where some of the most relevant contributions are (Autor, Levy and Murnane, 2003) and (Autor, Katz and Kearney, 2003). Furthermore, it is also related to the literature on capital accumulation and its macroeconomic consequences (see Piketty and Goldhammer, 2014). In this sense, if in some industries the decrease of the labor share is mostly due to the destruction of employment, then the reallocation of workers that lose their jobs will be, in many cases, to occupations with the lower need of specific abilities, and as a consequence, with lower wages. Moreover, if the decline in the labor share is also due to stagnant wages, this will also contribute to the increase in inequality.

A newer stream of this literature argues that the decline of the labor share may be due to a reduction in competition, allowing firms to increase their profits. Recently, Autor et al. (2020) and Barkai (2020) have put forward a new explanation relating the decline of the labor share with the concentration of market share in the so-called superstar firms. These firms are favored by technology advances and become less labor-intensive over time. Moreover, Barkai (2020) also tries to quantify the macroeconomic importance of these causes and documents that the decline in the labor share is further accompanied by a decline in the capital share. His results show that these two events can only jointly happen if the share of profits increases. Besides, De Loecker, Eeckhout and Unger (2020a), De Loecker and Eeckhout (2020) use firm-level data to document an increase in markups,

which is consistent with the decrease in labor share, decrease in capital share, and also with other secular trends as the decrease in low skill wages, decrease in the labor force, or the slowdown in aggregate output. [Bergholt, Furlanetto and Faccioli \(2019\)](#) use a structural vector autoregressive model and show that an increase in firms' markups can explain a significant part of the decline in the labor share. This paper contributes to this literature by bringing both technological and competition explanations together, developing a model where both can potentially explain the decline of the labor share.

Secondly, this paper also contributes to the literature on structural change, a strand in which exists a vast literature, combining both demand-side and supply-side explanations. In their seminal paper, [Ngai and Pissarides \(2007\)](#) show that economic growth occurs at different rates across sectors of an economy and that differences in total factor productivity across sectors can generate shifts in employment, generating structural change. Another notable contribution is that of [Acemoglu and Guerrieri \(2008\)](#), who provide an additional channel to account for structural change. In particular, using a two-sector model, they show that the joint combination of capital deepening with differences in factor proportions across sectors leads to unbalanced growth across industries, as the capital-labor ratio raises output more in industries with higher capital intensity. More recently, [Álvarez-Cuadrado et al. \(2017\)](#) further extend the supply-side explanations, showing that cross-sectoral differences in the substitutability of capital and labor can also have implications for structural change. If sectors have different possibilities of substituting capital for labor in the production process, then as a factor becomes more abundant (or relatively cheaper), the industry with higher substitutability will employ more units of this factor, again leading to structural change. I contribute to this literature by showing that if sectors also exhibit different levels of competition, the relative allocation of capital and labor across industries will also be affected and, consequently, the final production of manufacturing and services goods. In other words, the level of competition can affect the pace of structural change.

This paper is not the first to use industry-level data to analyze the evolution of the labor share.¹⁰ However, to the best of my knowledge, it is the first paper that decomposes each industry's labor share into its main components (wage, employment, and value added) and documents their heterogeneous evolution at the industry-level. However, this paper's main contribution is the analysis of the combined effect of technological change and the level of competition on both the labor share and the process of structural change.

¹⁰See, e.g., ([Diez-Catalan, 2017](#)), ([Kehrig and Vincent, 2017](#)) or ([Álvarez-Cuadrado et al., 2018](#))

Layout. The rest of this paper is organized as follows. In Section 1.2, I explain the methodology followed in computing the labor share, and I analyze the evolution of the main components of the labor share and the process of structural change. In Section 1.3, I set out the model. In Section 1.4, I conduct the quantitative analysis. Section 1.5, concludes. The Appendix includes additional evidence on the decline of the labor share, additional results of the model, and further validation for the quantitative analysis results.

1.2 Labor share and structural change in the U.S.

1.2.1 Labor share

In this paper, I use data from the online public interactive database published by the [U.S. Bureau of Economic Analysis \(2017\)](#). In particular, I have gathered aggregate data for the U.S. from 1947 to 2016 and industry-level data for 67 industries from 1998 to 2016.¹¹ Several methodologies can be used to compute the labor share, among them [Valentinyi and Herrendorf \(2008\)](#) that use Input-Output tables, [Elsby et al. \(2013\)](#) that use the compensation of employees and estimate the compensation of the self-employed workers, [Karabarbounis and Neiman \(2014\)](#) that focus on computing the labor share of the corporate sector, avoiding many of the problems related with the imputation of the wages of self-employed workers, and [Koh et al. \(2020\)](#) that measure ambiguous income, i.e., the share of income that can not be unambiguously assigned either to capital or labor and distribute proportionally to the unambiguous retribution of capital and labor.¹²

This paper follows the latter methodology, based ultimately on [Cooley and Prescott \(1995\)](#). Following this approach, I show that the evolution of the trend of the labor share for the overall economy turns out to be qualitatively similar and quantitatively very close to the evolution of the trend of the naive labor share (namely, compensation of employees over value added), which is much simpler to compute. One of the main drawbacks of using industry-level data is the lack of consistency in the industry definitions, which precludes from obtaining long time series without altering the original data.¹³ Therefore, in this

¹¹The exhaustive list of industries can be found in Appendix 1.6.1.

¹²It is essential to remark that recent work by [Atkinson \(2020\)](#) casts doubts about the BEA's ability correctly measure firm's investments.

¹³While industry-level data under the Standard Industrial Classification (SIC) classification is available at least since 1947, the migration to the North American Industry Classification System (NAICS), introduced a break in the time-series data. As [Yuskavage \(2007\)](#) points out, NAICS provides a more consistent classification of establishments into industries based on the similarity of their production processes, rather than considering similarities in the produced output. However, this change in the industry

paper, I compute the naive labor share for each industry rather than relying on Koh et al. (2020) to compute the labor share at the industry level. This is a similar approach as the one followed by Diez-Catalan (2017) or Kehrig and Vincent (2017). The overall labor share of the economy can then be obtained by appropriately weighing and aggregating each industry's naive labor share. The most obvious advantage of using industry-level data is that it allows characterizing the evolution of each industry's labor share, allowing to identify which industries contribute more to the decline in the labor share.

I further investigate the decline in the labor share by decomposing the naive labor share into its components: the average wage, the number of full-time equivalent employees, and the value added generated by each industry. A detailed analysis of the first two shows that their evolution is heterogeneous between manufacturing and services industries. During this period, I find that manufacturing industries are generally characterized by exhibiting negative employment growth rates, while services industries show positive growth rates. Additionally, value added in services industries grows faster than the average wage, while the opposite happens in manufacturing industries. Combining these two findings suggests that the reallocation of workers from manufacturing to services industries, or structural change, together with the different evolution of wages and value added across industries, may explain the decline of the overall economy's labor share. Consequently, it is key to understand which mechanisms drive this heterogeneous evolution at the industry level.

Aggregate level

Readers familiar with the evolution of the labor share at the aggregate level, and in particular with the work of Koh et al. (2020), may jump directly to the next subsection. Following Koh et al. (2020), the labor share at time t , LS_t , is obtained as

$$LS_t = 1 - \frac{DEP_t + UCI_t + ACI_t}{Y_t},$$

where DEP_t is depreciation, UCI_t is unambiguous capital income, ACI_t is ambiguous capital income and Y_t is a measure of value added generated in period t . The variable UCI_t is defined as

$$UCI_t = RI_t + CP_t + NT_t + CSGE_t,$$

where RI_t is rental income, CP_t are corporate profits, NT_t is net interest and $CSGE_t$ are current surplus government enterprises. Moreover, the variable ACI_t is obtained as

$$ACI_t = \theta_t AI_t,$$

classification hinders the availability of long time series.

where, on the one hand, AI_t is ambiguous income and can be computed as

$$AI_t = PI_t + TPS_t + BCTP_t + SD_t,$$

where PI_t is proprietors' income, TPS_t are taxes on production net of subsidies, $BCTP_t$ are business current transfers payments and SD_t is statistical discrepancy. On the other hand, θ_t is the share of ambiguous income assigned to capital, which is obtained as

$$\theta_t = \frac{UCI_t}{UI_t}$$

where

$$UI_t = UCI_t + DEP_t + CE_t,$$

is unambiguous income and CE_t is the compensation of employees in period t .

Following this methodology, Figure 1.1 shows the labor share at the aggregate level for the United States from 1987 to 2016 and the share of ambiguous income.¹⁴ Besides, Figure 1.1 also shows the naive labor share of the overall U.S. economy, an alternative measure of the labor share which is constructed as follows. First, for any industry i , define its naive labor share at time t , NLS_{it} , as

$$NLS_{it} = \frac{CE_{it}}{Y_{it}}. \quad (1.1)$$

Then, the naive labor share at the aggregate level (or overall naive labor share of the economy) at time t can be obtained as

$$NLS_t = \frac{\sum_i CE_{it}}{\sum_i Y_{it}} = \frac{\sum_i CE_{it}}{Y_t} = \sum_i \frac{Y_{it}}{Y_t} \frac{CE_{it}}{Y_{it}} = \sum_i \eta_{it} NLS_{it}, \quad (1.2)$$

which implies that the overall naive labor share of the economy is a weighted average of the naive labor shares of each industry, with weights given by $\eta_{it} = \frac{Y_{it}}{Y_t}$, the relative size of each industry in value added generation. In the data all these variables are measured in nominal terms.¹⁵ As measure of value added I use the nominal gross value added of each industry, which in the data is obtained as the sum of expenditures on labor or CE_{it} , gross operating surplus GOS_{it} and taxes on production and imports minus subsidies $TPIS_{it}$, i.e.

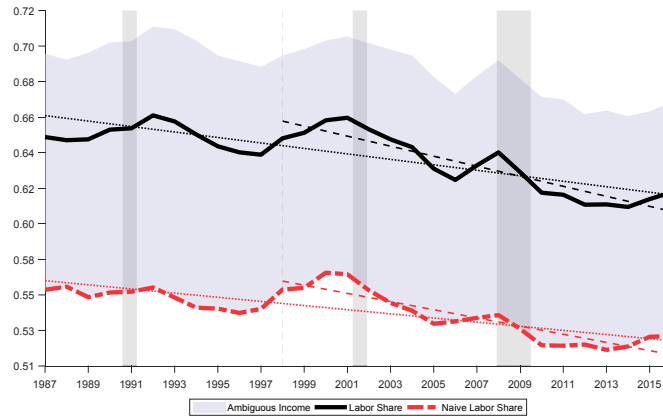
$$Y_{it} = CE_{it} + GOS_{it} + TPIS_{it}.$$

¹⁴Extended results from 1947 to 2016 can be found in Appendix 1.6.2.

¹⁵Therefore, without loss of generality, in the proposed notation $Y_{it} = P_{it}^Y Y_{it}^R$, where Y_{it}^R is value added measured in goods and P_{it}^Y is its price.

The allocation between capital and labor of $TPIS_{it}$ is unclear (see Mućk, McAdam and Growiec (2018)). While in previous research these taxes are usually not allocated, in this paper I allocate them to capital.¹⁶ By doing so, the resulting naive labor share practically coincides with the labor share of Koh et al. (2020) if the share of ambiguous income is totally assigned to capital, i.e., when $\theta_t = \theta_{it} = 1$, for all i and t .¹⁷

Figure 1.1: U.S. labor income share, 1987 - 2016



Note: Vertical shaded areas represent recession periods.

Throughout this paper, I approximate the time-series of the labor share with a linear time trend to compute the labor share variation over a period. In general, let x_t , $t = 1, 2, \dots$ be a time series, and define as \tilde{x}_t the fitted value of x_t from the linear trend of the time series at time t . I define the variation, or fitted change, of the time series between t and $t + 1$, $\Delta x_{t,t+1}$ is as

$$\Delta x_{t,t+1} = \tilde{x}_{t+1} - \tilde{x}_t,$$

and its fitted percentage change is obtained as

$$\Delta \% x_{t,t+1} = \frac{\tilde{x}_{t+1} - \tilde{x}_t}{\tilde{x}_t} = \frac{\Delta x_{t,t+1}}{\tilde{x}_t}.$$

The fitted labor share computed under Koh et al. (2020) methodology was 0.6597 in 1987 and 0.6131 in 2016. Therefore its fitted change was -0.0466 (a percentage fitted

¹⁶If not allocated, then (1.1) would be

$$NLS_{it} = \frac{CE_{it}}{Y_{it} - TPIS_{it}}.$$

However, as the share of $TPIS_{it}$ is relatively constant during the period analyzed both at the aggregate and industry-level, allocating taxes only changes quantitatively the results.

¹⁷There is, though, a small difference, as Koh et al. (2020) use the gross national product as measure of value added, while I use the aggregate nominal gross value added.

change of -7.06%). Simultaneously, the fitted naive labor share was 0.5627 in 1987 and 0.5275 in 2016 and thus declined in 0.0353 labor share points during the same period (a 6.27% fitted percentage decline). Between 1998 and 2016, the labor share declines from 0.6564 to 0.6030 (an 8.08% reduction), and the naive labor share falls from 0.5625 to 0.5193 (a 7.68% decline). Together with Figure 1.1, these results allow characterizing two aspects of the evolution of the U.S. labor share during the last three decades. First, both labor share measures show a fast decline during this period, which has intensified during the period from 1998 to 2016, as the trend of the labor share during this period is steeper than during the period from 1987 to 2016. Second, the naive labor share can qualitatively match the decline of the labor share computed under Koh et al. (2020) methodology. Besides, the decline is quantitatively very close. In this sense, Figure 1.1 exhibits that the naive labor share delivers an excellent approximation of the movements of the labor share during this period. These findings suggest that the secular decline of the labor share, which according to previous research starts in the early 1980s, is far from being over and has intensified during the last decades. In Figure 1.1 I also highlight in the gray shaded areas the periods of economic recession. From 1998 to 2016, the periods of recession seem to be contemporary to periods of substantial declines in the labor share, particularly during the Great Recession.

Industry level

The analysis of the labor share at the industry level is slightly more complicated due to data limitations. While industry-level data under the Standard Industrial Classification (SIC) classification is available at least since 1947, the migration to the North American Industry Classification System (NAICS) introduced a break in the time-series data. To circumvent this, I focus on the period from 1998 to 2016, using industry-level data under the NAICS classification. Even in this short and recent period, the industry-level data is not as detailed as the aggregate data, which hinders the labor share's computation following Koh et al. (2020) methodology. Consequently, given the previous findings, I rely on the naive labor share to analyze each industry's labor share. I will refer simply to the labor share hereafter.¹⁸ Moreover, to obtain further information on the movements of the naive labor share that allow to analyze it in more detail, I decompose the compensation

¹⁸In *Labor share* I conduct a similar analysis for 8 industries of the U.S. and show that the naive labor share can deliver a good approximation for the evolution of the 'true' labor share for the period I consider. This is in line with the results of Mućk et al. (2018).

of employees (the numerator of (1.2)) for each industry i at time t into

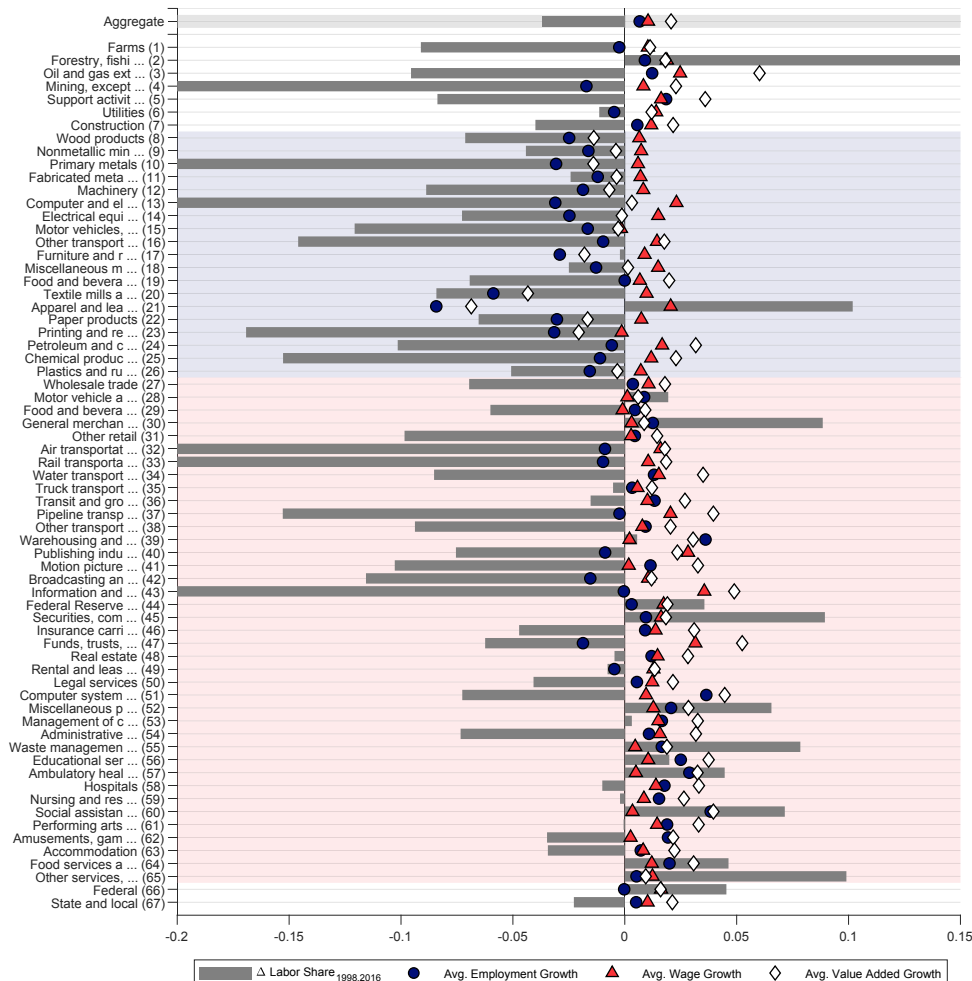
$$CE_{it} = WS_{it} + SUPL_{it} = W_{it}L_{it} + ECGSI_{it} + ECEPIF_{it},$$

where WS_{it} are wage and salary accruals and disbursements. $SUPL_{it}$ are supplements to wages and salaries. According to the BEA methodology, the term WS_{it} comprehends the monetary remuneration of employees, including the compensation of corporate officers; commissions, tips, and bonuses; voluntary employee contributions to certain deferred compensation plans and receipts in kind that represent income. It can be further decomposed into the product of the average wage of each industry W_{it} and the number of full-time equivalent employees within that industry L_{it} . Moreover, the term $SUPL_{it}$ consists of employer contributions for government social insurance $ECGSI_{it}$ and employer contributions for employee pension and insurance funds $ECEPIF_{it}$.

Figure 1.2 shows the fitted change of the naive labor share for each industry between 1998 and 2016.¹⁹ Moreover, I also show the average growth rate of the number of full-time equivalent employees, the average growth rate of real wages, and the average growth rate of real gross value added in each industry during this period. To clarify this figure's interpretation, manufacturing industries (industries ranging from indexes 8 to 26, both included) are those in the blue shaded area. Private services industries (industries ranging from indexes 27 to 65, both included) are those in the red shaded area.

Not surprisingly, the aggregate labor share decline mentioned above has been accompanied by a decline in many industries' naive labor share. In this sense, the decline in the overall economy's labor share is further characterized by the remarkable heterogeneity in the evolution of the naive labor share trend at the industry level. This heterogeneity is present between manufacturing and services industries and within these two aggregations of industries. In particular, out of the 67 industries, only 16 show a positive trend during this period. Focusing on manufacturing industries, only apparel and leather products (industry 21) delivers a positive trend. In contrast, the remaining industries show a negative trend, with the highest declines taking place in computer and electronic products (industry 13), primary metals (industry 10), and printing and related support activities (industry 23). Within services industries, the highest declines have happened in rail transportation (industry 33) and air transportation (industry 32). It has also declined in industries as relevant as wholesale trade (industry 27) or administrative and support

¹⁹Motor vehicles, bodies and trailers, and parts (industry 15); information and data processing services (industry 43) and securities, commodity contracts, and investments (industry 45) show a naive labor share bigger than 1 during some periods. Moreover, those values are, in general, very far from the remaining observations.

Figure 1.2: Naive labor income share fitted change and average growth rates of employment, wages

Note: Light blue shaded area comprehends manufacturing industries (industries 8 to 26). Light red shaded area comprehends private services industries (industries 27 to 65); Industries 1 to 7 are agricultural activities, mining, utilities, and construction; Industries 66 and 67 are government industries. The exhaustive list of industry index identifiers is available in Appendix 1.6.1. The labor share fitted change of some industries are off-scale to simplify the exposition of the results. Their specific values are: -0.2756 (4, mining except oil and gas), -0.2095 (10, primary metals), -0.2311 (13, computer and electronic products), -0.2678 (32, air transportation), -0.2822 (33, rail transportation), -0.5475 (43, information and data processing services).

services (industry 54). Among the industries outside manufacturing and private services, the most substantial decline has happened in mining, except oil and gas (industry 4) and the strongest increase in forestry, fishing, and related activities (industry 2).

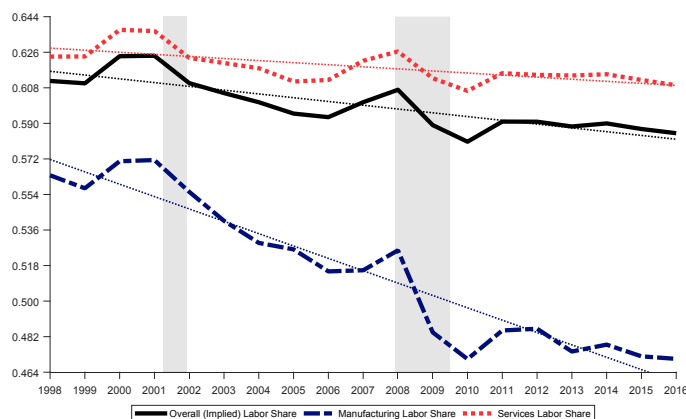
While these facts are important in themselves, the implications at the aggregate level

of these strong declines may be minimal if these industries' weight in terms of value added generation is small. In this sense, the industry with the highest weight in 2016 is real estate (industry 48); however, given the characteristically low naive labor share of this industry, even though the decline in the trend of the naive labor share is very small, declining by 0.0046 labor share points, it implies a 9.34% decline. Other industries with high weights are state and local government (industry 67) with a small fitted change of 0.0228 labor share points, a 2.56% decline; wholesale trade with a modest decline of 0.0695 percentage points which implies a 13.10% decline; miscellaneous professional, scientific, and technological services (industry 52) with an increase of 0.0655 percentage points, or a 10.26% increase; and construction (industry 7), with a decline of 0.0399, a 6.10% decline. Therefore, in 4 out of these 5 industries, which account for roughly 35% of value added generation in 2016, the labor share is declining.

The heterogeneity in the decline of the naive labor share trend is a consequence of the underlying heterogeneity in the growth rates of employment, wages, and value added across industries. Figure 1.2 shows that the average growth rate of employment has been either negative or very close to zero in all manufacturing industries. Interestingly, pooling all manufacturing observations from 1998 to 2016, I find that the correlation between the growth rate of employment and the decline in the trend of the naive labor share is very close to zero, which implies that industries with higher declines in the labor share are not necessarily those in which more employment is destroyed. Moreover, except in four industries, the average growth rate of wages in manufacturing industries has been higher than that of value added. These two facts' joint effect causes the generalized and robust decline of the manufacturing industries' naive labor share. On the contrary, in services industries, I find the totally opposite situation. With a few exceptions, employment growth has been either positive or close to zero. Besides, except in two industries, the average growth rate of wages in services industries has been lower than that of value added. The joint combination of these two effects is the cause underlying the decline naive labor share trend in many services industries. Nevertheless, out of the sixteen industries that show a positive trend for their labor share, thirteen are private services industries.

Naturally, the heterogeneity between and within manufacturing and services industries implies a different evolution of their labor share, shown in Figure 1.3.²⁰ In particular, the decline in manufacturing is much steeper than the one in services: the fitted labor share in manufacturing was 0.5716 in 1998 and 0.4590 in 2016, and thus declined in 0.1126 labor share points during this period (a 19.7% fitted percentage decline), while in services

²⁰Given the specific evolution of the real estate industry labor share, I exclude it from services.

Figure 1.3: Naive labor share: manufacturing and services industries, 1998 - 2016

Note: Services exclude Real Estate. The overall implied naive labor share is obtained by using (1.2) after excluding agricultural activities, mining, utilities, construction, government and real estate industries.

was 0.6280 in 1998 and 0.6091 in 2016, and thus declined in 0.0189 labor share points during the same period (a 3.01% fitted percentage decline). Given the relative weights of these two industries, the implied labor share (computed as in (1.2)) declines in 0.0344 labor share points (a 4.48% fitted percentage decline) from 1998 to 2016.²¹ In fact, one can conclude that manufacturing and services industries exhibit a different evolution of the labor share, clearly motivated by the heterogeneity on the evolution of their main components, namely, employment, wages, and value added.

1.2.2 Structural change

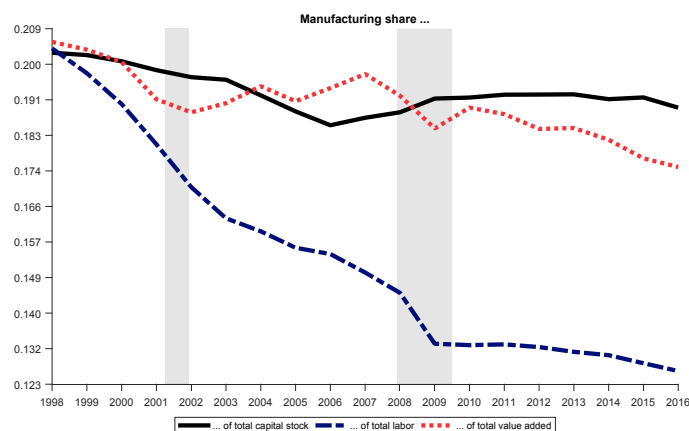
The enormous differences in the growth rates of employment and value added between manufacturing and services industries suggest that contemporaneously to the decline in the labor share, the last two decades have also witnessed an intense structural change period. To examine this, I gather additional data on (non-residential) capital stock and employment from the [U.S. Bureau of Economic Analysis \(2017\)](#).²²

Figure 1.4 depicts three series that allow evaluating the process of structural change. The solid (black) line represents the share of capital stock in manufacturing with respect to the economy's total capital stock, where the total is the sum of capital in manufacturing

²¹Note that for the purposes of this paper, restricting the analysis to manufacturing and services (and, as a consequence, omitting agricultural activities, mining, utilities, construction, government, and real estate) only changes the results quantitatively.

²²In Appendix 1.6.1 I show how I derive the capital stock and discuss the computation of the capital-labor ratio.

Figure 1.4: Manufacturing share of total capital stock, total labor and total value added, 1998 - 2016

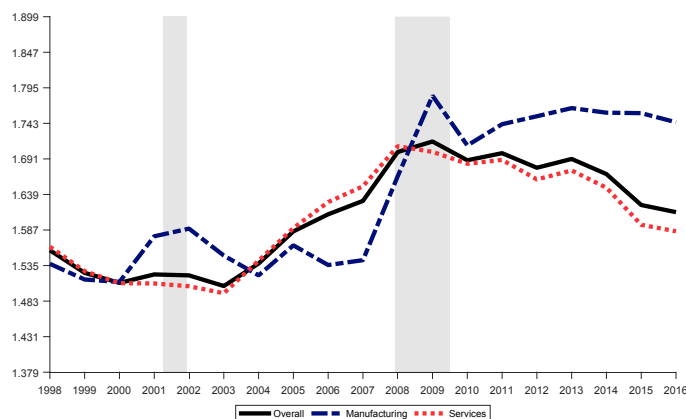


Note: Total excludes agricultural activities, mining, utilities, construction, government and real estate industries.

and services industries.²³ This relative share has declined slightly over the last two decades, even though the level of the capital stock in manufacturing has increased. The dashed (blue) line represents the share of labor in manufacturing with respect to total labor. The evolution is striking, with a very steep decline implied by the high destruction of employment in manufacturing, as opposed to services, where it has grown over the last two decades. The joint evolution of these two shares shows that there has been an intense relative reallocation of inputs from manufacturing to services. Finally, the dotted (red) line shows the share of value added produced by manufacturing industries. The picture is the same from an output perspective, as the contribution of manufacturing to value added has declined during the last two decades.

A stark implication of the relatively small decline in the share of capital stock employed in manufacturing industries compared to the substantial reduction in employment share is that the capital-labor ratio in manufacturing industries has almost doubled during this period. The capital-labor ratio has also increased in services industries, but this increase is much smaller. Figure 1.5 shows the evolution of the aggregate capital-output ratio (solid black line), which has increased from 1998 to 2016, especially during the Great Recession. Note that the capital-output ratios in manufacturing (dashed blue line) and services (dotted red line) are very similar and, despite some fluctuations, remain relatively close until the Great Recession. From that point onwards, the capital-output ratios of manufacturing and services have diverged, staying relatively constant at a higher level in

²³In other words, total excludes the agricultural, mining, utilities, construction, government and real estate industries.

Figure 1.5: Capital-output ratio, 1998 - 2016

manufacturing and seemingly converging back to in pre-recession level in services.

1.3 Model

The previous Section documents that the last two decades have witnessed a heterogeneous decline in the labor share of manufacturing and services industries, which has been contemporary to an intense period of structural change from manufacturing to services industries. Many explanations have been proposed to explain both phenomena; however, there is no consensus on the key determinants that explain the decline in the labor share. In this Section, I study how the two main mechanisms that have recently been put forward to explain it – biased technological change and changes in market power – can simultaneously have relevant effects on the observed process of structural change in the U.S. economy.

Following the previous Section results, I restrict my attention to a two-sector model, where the production side of the economy consists of manufacturing and services industries. In particular, I build a model based on [Álvarez-Cuadrado et al. \(2017, 2018\)](#), where I allow for imperfect competition in both industries, given by the (exogenous) ability to fix a price equal to a markup over marginal cost. Similar to their model, differences in the evolution of the labor share and structural change across sectors may come from differences in technological change, capital-labor ratios, and differences in the elasticity of substitution between capital and labor or their intensity in the production function. Moreover, I show that the (heterogeneous) evolution of market power across industries can shape the declining labor shares, which is a straightforward and well-known result, and the pace of structural change between sectors.

The baseline model is intentionally reduced to the quantitatively relevant elements. I restrict my attention to intra-temporal decisions considering a static model repeatedly, and I do not model the inter-temporal dimension. The model's equilibrium reduces to determining the allocations of capital and labor across sectors in an endowment economy without capital accumulation or labor supply decisions.

1.3.1 Environment

Time is discrete.²⁴ At each particular moment in time t , the economy is formed by a final good and two types of intermediate goods that are produced by aggregating a continuum of differentiated varieties in the unit interval $i \in [0, 1]$. In what follows, I describe each agent in detail.

Final good

There is a unique final good in the economy, Y_t , produced by a single price-taker firm that combines a manufacturing good M_t , and a services good S_t , with the technology

$$Y_t = \left[\gamma M_t^{\frac{\theta-1}{\theta}} + (1-\gamma) S_t^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where $\gamma \in [0, 1]$ and $\theta \geq 0$ is the elasticity of substitution between manufacturing and services goods. Each of these goods $J \in \{M, S\}$ is produced by aggregating a continuum of differentiated intermediate inputs $i \in [0, 1]$, with the technologies

$$J_t = \left(\int_0^1 j_t(i)^{\frac{\varepsilon_{j,t}-1}{\varepsilon_{j,t}}} di \right)^{\frac{\varepsilon_{j,t}}{\varepsilon_{j,t}-1}}, \quad (1.3)$$

where $j_t(i)$, $j \in \{m, s\}$ is the quantity of intermediate variety i used to produce good J and $\varepsilon_{j,t}$ is the time-varying elasticity of substitution across different varieties.

Assuming a perfectly competitive environment, the problem of this firm can be broken into two steps. First, given $p_{j,t}(i)$, the price of intermediate variety i of type j , the firm optimally chooses $m_t(i)$ and $s_t(i)$, $\forall i$, to minimize the cost of producing some given quantities S_t and M_t . Second, given the implied prices $P_{M,t}$ and $P_{S,t}$ for manufacturing and services goods, the firm optimally balances M_t and S_t to minimize the cost of producing a given quantity Y_t . The first step yields the conditional input demand

$$j_t(i) = \left(\frac{p_{j,t}(i)}{P_{J,t}} \right)^{-\varepsilon_{j,t}} J_t, \quad \forall i, \quad J = \{M, S\}, \quad j = \{m, s\}, \quad (1.4)$$

²⁴Abstracting from the inter-temporal dimension simplifies the definition of time, which could either be finite or infinite.

where $P_{J,t}$ is the ideal price index.²⁵ The second step yields the relative demand

$$MRTS_{M,S} = \frac{\gamma}{1-\gamma} \left(\frac{S_t}{M_t} \right)^{\frac{1}{\theta}} = \frac{P_{M,t}}{P_{S,t}}. \quad (1.5)$$

Normalizing to unity the price of the final good we obtain the price index

$$1 = P_{Y,t} = \left[\gamma^\theta P_{M,t}^{1-\theta} + (1-\gamma)^\theta P_{S,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

Intermediate varieties

There exist two types of intermediate goods that are used to produce manufacturing and services goods. Within these two types, there exist a continuum of differentiated varieties in the unit interval $i \in [0, 1]$, i.e., each manufacturing (services) intermediate producer supplies an intermediate variety $m_t(i)$ ($s_t(i)$) which is used as an input in the production of the manufacturing (services) good.

Each variety i of intermediate type j , where $j = m, s$ (henceforth, a pair $\{i, j\}$), is produced using a constant returns to scale technology $y_{j,t}(i) = F_j[k_{j,t}(i), l_{j,t}(i)]$. This technology combines capital, $k_{j,t}(i)$, rented at a rate R_t , with labor, $l_{j,t}(i)$, hired at a wage W_t . Each producer of an intermediate variety takes input prices and aggregate demand as given. Intermediate varieties are produced under monopolistic competition. I assume that the technology for producing a pair $\{i, j\}$ is given by the constant elasticity of substitution production function

$$y_{j,t}(i) = A_{j,t} \left[\alpha_j [\tilde{B}_{j,t} k_{j,t}(i)]^{\frac{\sigma_j-1}{\sigma_j}} + (1-\alpha_j) [l_{j,t}(i)]^{\frac{\sigma_j-1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j-1}}, \quad \forall i \quad j \in \{m, s\},$$

where σ_j is the elasticity of substitution between capital and labor, α_j is the distributional parameter, $A_{j,t}$ is neutral technological change, and $\tilde{B}_{j,t}$ is capital-augmenting technological change. The profit-maximization problem of the intermediate producer of pair $\{i, j\}$ is characterized by the equations

$$p_{j,t}(i) F_{j,t}^k(i) = \frac{\varepsilon_{j,t}}{\varepsilon_{j,t} - 1} R_t = \mu_{j,t} R_t, \quad (1.6)$$

$$p_{j,t}(i) F_{j,t}^l(i) = \frac{\varepsilon_{j,t}}{\varepsilon_{j,t} - 1} W_t = \mu_{j,t} W_t, \quad (1.7)$$

²⁵As is standard with Dixit-Stiglitz aggregators, the price $P_{J,t}$ is given by

$$P_{J,t} = \left(\int_0^1 p_{j,t}(i)^{1-\varepsilon_{j,t}} di \right)^{\frac{1}{1-\varepsilon_{j,t}}}.$$

where $p_{j,t}(i)$ is the price of the variety pair $\{i, j\}$, $F_{j,t}^k(i)$ ($F_{j,t}^l(i)$) is the marginal product of capital (labor), and

$$\mu_{j,t} = \frac{\varepsilon_{j,t}}{\varepsilon_{j,t} - 1}, \quad (1.8)$$

is a sector-specific markup over factor prices. Profits earned by the variety producer of pair $\{i, j\}$ represent a constant share of output sold, given by

$$\pi_{j,t}(i) = \frac{1}{\varepsilon_{j,t}} p_{j,t}(i) y_{j,t}(i).$$

1.3.2 Static Allocation

I assume that an exogenous supply of capital K_t and labor L_t flow into the economy each period, and that there is no capital accumulation. This assumption allows characterizing a period-by-period static allocation as in Acemoglu and Guerrieri (2008) and Álvarez-Cuadrado et al. (2017).²⁶ In this sense, the concept of equilibrium for the purpose of this paper reduces to finding output prices P_m, P_s , input prices R_t, W_t , outputs M_t, S_t , and allocations of inputs across sectors K_m, L_m, K_s, L_s that clear the labor and capital markets (with exogenously given K_t and L_t supplies). As the environment across varieties in the production of intermediates is symmetric, it follows that the static equilibrium satisfies $m_t(i) = y_{m,t}(i) = M_t$, $s_t(i) = y_{s,t}(i) = S_t$, $p_{m,t}(i) = P_{m,t}$, $p_{s,t}(i) = P_{s,t}$, $k_{m,t}(i) = K_{m,t}$, and $k_{s,t}(i) = K_{s,t}$, for all i and t . Define

$$k_t = \frac{K_t}{L_t}, \quad \kappa_t = \frac{K_{m,t}}{K_t}, \quad \lambda_t = \frac{L_{m,t}}{L_t}, \quad (1.9)$$

which are, respectively, the capital-labor ratio of the economy and the shares of capital and labor allocated to the production of the manufacturing good. The static equilibrium can then be summarized in the system of equations²⁷

$$\frac{1 - \alpha_s}{1 - \alpha_m} \frac{\alpha_m}{\alpha_s} \frac{\tilde{B}_{m,t}^{\frac{\sigma_m - 1}{\sigma_m}}}{\tilde{B}_{s,t}^{\frac{\sigma_s - 1}{\sigma_s}}} k_t^{\frac{1}{\sigma_s} - \frac{1}{\sigma_m}} \left(\frac{1 - \kappa_t}{1 - \lambda_t} \right)^{\frac{1}{\sigma_s}} \left(\frac{\lambda_t}{\kappa_t} \right)^{\frac{1}{\sigma_m}} = 1, \quad (1.10)$$

$$\frac{1 - \gamma}{\gamma} \frac{\mu_{m,t}}{\mu_{s,t}} \frac{1 - \alpha_s}{1 - \alpha_m} \left(\frac{A_{s,t}}{A_{m,t}} \right)^{\frac{\theta - 1}{\theta}} \frac{g_{m,t}(\kappa_t, \lambda_t, k_t)^{\frac{1}{\theta} - \frac{1}{\sigma_m}}}{g_{s,t}(\kappa_t, \lambda_t, k_t)^{\frac{1}{\theta} - \frac{1}{\sigma_s}}} k_t^{\frac{1}{\sigma_s} - \frac{1}{\sigma_m}} \frac{\lambda^{\frac{1}{\sigma_m}}}{(1 - \lambda_t)^{\frac{1}{\sigma_s}}} = 1, \quad (1.11)$$

²⁶The analysis could be extended by including a typical representative household. The equilibrium could then be broken into two parts: first the static model shown in this Section and second a dynamic problem of capital accumulation.

²⁷See Appendix 1.6.3 for a step-by-step derivation of (1.10) and (1.11).

where

$$g_{m,t}(\kappa_t, \lambda_t, k_t) = \left[\alpha_m \left(\tilde{B}_{m,t} \kappa \right)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) \left(\frac{\lambda}{k_t} \right)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}},$$

$$g_{s,t}(\kappa_t, \lambda_t, k_t) = \left[\alpha_s \left(\tilde{B}_{s,t} (1 - \kappa_t) \right)^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) \left(\frac{1 - \lambda_t}{k_t} \right)^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}}.$$

The static equilibrium reduces to finding the (unique) solution of (1.10) and (1.11) in the (κ, λ) space given k_t and a set of values for the parameters.²⁸ The static equilibrium shares the same comparative static properties concerning capital deepening and changes in technological change parameters shown in [Álvarez-Cuadrado et al. \(2017\)](#). As a consequence, I focus on the novel properties of this static model. Once κ_t and λ_t are determined, it is trivial to obtain the remaining components of the equilibrium and characterize any moment of interest from the model. Consider first the labor share. Given that both intermediate firms exhibit constant returns to scale, from the profit maximizing condition of intermediate producers follows

$$J_t = \frac{\mu_{j,t}}{P_{j,t}} (R_t K_{j,t} + W_t L_{j,t}),$$

which allows defining the sectoral shares of labor $s_{J,t}^L$, capital $s_{J,t}^K$, and profits $s_{J,t}^\Pi$ as

$$s_{J,t}^L = \frac{W_t L_{j,t}}{P_{j,t} J_t} = \frac{W_t L_{j,t}}{\mu_{j,t} (R_t K_{j,t} + W_t L_{j,t})},$$

$$s_{J,t}^K = \frac{R_t K_{j,t}}{P_{j,t} J_t} = \frac{R_t K_{j,t}}{\mu_{j,t} (R_t K_{j,t} + W_t L_{j,t})},$$

$$s_{J,t}^\Pi = 1 - s_{J,t}^K - s_{J,t}^L = 1 - \frac{1}{\mu_{j,t}},$$

with $j \in \{m, s\}$ and $J \in \{M, S\}$.²⁹ These shares can also be expressed as

$$s_{J,t}^L = \frac{W_t L_{j,t}}{P_{j,t} J_t} = \frac{1}{\mu_{j,t}} \epsilon_{J,t}^L, \quad (1.12)$$

$$s_{J,t}^K = \frac{R_t K_{j,t}}{P_{j,t} J_t} = \frac{1}{\mu_{j,t}} \epsilon_{J,t}^K, \quad (1.13)$$

²⁸See [Álvarez-Cuadrado et al. \(2017\)](#) for a detailed discussion on the uniqueness of the solution.

²⁹From (1.8), $s_{J,t}^\Pi$ can also be expressed as

$$s_{J,t}^\Pi = 1 - \frac{\epsilon_{j,t} - 1}{\epsilon_{j,t}} = \frac{1}{\epsilon_{j,t}},$$

which shows that profits are simply the inverse of the elasticity of substitution across varieties in each sector.

where

$$\begin{aligned}\epsilon_{J,t}^L &\equiv (1 - \alpha_j) \left(\frac{A_{j,t} L_{j,t}}{J_t} \right)^{\frac{\sigma_j - 1}{\sigma_j}}, \\ \epsilon_{J,t}^K &\equiv \alpha_j \left(\frac{A_{j,t} \tilde{B}_{j,t} K_{j,t}}{J_t} \right)^{\frac{\sigma_j - 1}{\sigma_j}},\end{aligned}$$

are the sectoral elasticities of output with respect to labor, $\epsilon_{J,t}^L$, or capital, $\epsilon_{J,t}^K$, at time t . Note that both markups and technology parameters characterize the level of the labor share in each sector.

Now define the relative share of labor income as

$$\frac{s_{J,t}^L}{s_{J,t}^K} = \frac{W_t L_{j,t}}{R_t K_{j,t}} = \frac{1 - \alpha_j}{\alpha_j} (\tilde{B}_{j,t} k_{j,t})^{\frac{1 - \sigma_j}{\sigma_j}}. \quad (1.14)$$

It follows from this expression that both capital-biased technological change and the capital-labor ratio of a sector will affect its relative share of labor income. In particular, if $\sigma_j < 1$ and there is capital deepening, i.e., $k_{j,t}$ increases over time (the relevant case for the quantitative analysis performed in the next section), a decline in the relative share of labor income can only occur if there is a sufficiently strong decline in $\tilde{B}_{j,t}$. Intuitively, if there is more capital, its relative price will decline, and firms will hire more capital. If capital and labor are gross substitutes, then the demand for labor will also increase, introducing an upward pressure on its relative price. As the former effect dominates the latter, the combination of both effects leads to an increase in the relative share of labor income. However, as $\sigma_j < 1$, a decline in $\tilde{B}_{j,t}$ generates a reduction in both the rental rate and the wage rate, being relatively stronger in the latter. If the decline in $\tilde{B}_{j,t}$ is sufficiently strong, it can completely offset the effect of capital deepening and generate a decline in the relative share of labor income.

Note that the evolution of markups does not directly affect the relative income share in a sector. However, it can be through an indirect channel by altering the allocation of inputs across sectors (i.e., through structural change). The latter result is established in the following proposition.

Proposition 1 (Competition displacement effect). *Let $\eta \equiv \mu_{m,t}/\mu_{s,t}$ be the relative markup between manufacturing and services. Then, a decline in η shifts up (1.11) in (κ, λ) space.*

Proof. As the relative markup η only appears in the equilibrium equation (1.11), we just need to show how (1.11) is affected when markups change asymmetrically across sectors.

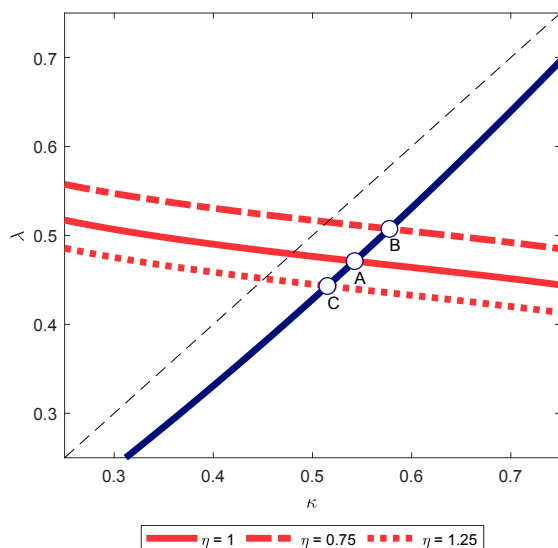
Consider a given (constant) parametrization of the deep parameters of the model. The proof is divided into two parts. Suppose first that κ and k are fixed. If η increases (i.e., markups increase relatively more in manufacturing than in services), then (1.11) will hold with strict inequality. As the equilibrium adjustment can only be made through λ and (1.11) is increasing in λ , for each value of κ , less labor will be demanded, and therefore λ must decline. In other words, if the relative markup η increases, the equilibrium resource allocation of capital and labor to manufacturing declines. Following the same reasoning, if η declines λ must increase. Now suppose that λ is fixed instead of κ . As the equilibrium adjustment can only be made through κ and (1.11) is increasing in κ , for each value of λ , less capital will be demanded, and therefore κ must decline. ■

This result is intuitive. Suppose first a one-sector economy with a representative firm. From the firm's perspective, for a given demand of its output, an exogenous increase in markups allows setting a higher price. To the extent that this exogenous change does not affect its relative demand of capital and labor, the increase in markups is neutral for the capital and labor shares, which follows from (1.14).³⁰

However, in a two-sector model, the competition displacement effect implies that changes in markups affect the allocation of inputs across sectors and, as a consequence, can also affect the sectoral capital-labor ratios. To see this, suppose that ε_m increases so that the intermediate inputs are more substitutable in the production function of manufacturing goods. This implies a decline in μ_m , as the ability to set up higher markups over marginal cost declines. Thus, the relative markup also declines. Consequently, given that any intermediate firm's cost function does not change, P_m declines, and the relative price across sectors also declines. This finally implies that the production of the final good will become more manufacturing intensive. Thus both κ and λ will increase or, in other words, (1.11) shifts up in κ, λ space. Therefore, changes in markups (or in the level of competition) can affect the allocation of resources across sectors, generating structural change. This is precisely the competition displacement effect.

To illustrate the effect of markups, figure 1.6 shows the results for a numerical example. The (solid blue) upward sloping line depicts (1.10), and the (red solid, dashed, and dotted) downward sloping lines depict (1.11) for different values of the relative markup η . The intersection between both lines yields the shares of total capital and labor employed in the production of manufacturing goods. Additionally, the thin dashed black line represents the 45° line, thus the fact that (1.10) lies below it implies that manufacturing is relatively more capital intensive than services.

³⁰In other words, the effect of an increase in markups in the capital and labor shares is symmetric.

Figure 1.6: Competition displacement effect


Parameter values: $\gamma = \theta = 0.5$, $\sigma_m = 0.8$, $\sigma_s = 0.6$, $\alpha_m = 0.45$, $\alpha_s = 0.35$, $\tilde{B}_m = \tilde{B}_s = A_m = A_s = k = 1$.

Consider the initial equilibrium represented by allocation A in which both sectors have the same level of markups (i.e., $\eta = 1$). Now suppose that markups decline in manufacturing so that the relative markup declines. By proposition 1, (1.11) shifts up in (κ, λ) space, and the new allocative equilibrium becomes B . In this new equilibrium, more manufacturing goods are demanded in the final good production, which trivially implies that more inputs must be used in manufacturing, thus K_m and L_m increase.³¹ Given that the shrinking sector is more labor-intensive, the parallel shifting of (1.11) implies that the capital-labor ratio declines in both sectors, even though the aggregate capital labor-ratio remains constant. As a by-product, the relative share of labor income given by (1.14) also declines in both sectors. The results are the opposite if, starting from equilibrium A , the relative markup increases, attaining the allocative equilibrium C .

In general, the key implication of proposition 1 is that the evolution of markups may generate structural change both in terms of capital and labor. Consequently, as long as sectors are not symmetric, the relative share of labor income, given by (1.14), will vary with markups.

Ultimately, as the quantitative analysis's main purpose is quantifying the decline in the labor share, an expression for the aggregate labor share of the economy must be

³¹As $K = k = 1$ by assumption, it follows that $\kappa = K_m$ and $\lambda = L_m$. Then $k_m = \kappa/\lambda$ and $k_s = (1 - \kappa)/(1 - \lambda)$.

established. To that end, define the share of value added generated by sector $J \in \{M, S\}$ over total valued added as

$$\psi_{J,t} = \frac{P_{J,t}J_t}{Y_t}, \quad \forall t.$$

This allows deriving the overall economy shares of labor, s_t^L ,³² capital, s_t^K and profits, s_t^Π , as weighted sums of the sectoral shares, i.e. for $Z \in \{L, K, \Pi\}$

$$s_t^Z = \psi_{M,t}s_{M,t}^Z + \psi_{S,t}s_{S,t}^Z = \sum_J \psi_{J,t}s_{J,t}^Z.$$

Naturally, changes in the income shares across sectors will affect the overall shares of income with an effect proportional to each sector's value added share.

1.4 Quantitative analysis

In this Section, I use the theory to perform a quantitative analysis that ultimately intends to quantify the contribution of technological change and market power to the decline of the labor share and the process of structural change documented in Section 1.2.

I start by describing the calibration procedure that yields technological change and markups processes across sectors, henceforth, the unobserved exogenous processes. According to the theoretical framework, I obtain a sequence of static allocations that deliver time-series for the labor share of manufacturing, the labor share of services, and a process of structural change that replicate those observed in the data.

Before discussing this analysis's main findings, it is important to highlight the intuition behind the exercise. According to the theory, if we were only interested in explaining the evolution of the labor share, technological change and market power could individually deliver a decline aligned with that observed in the data. However, there is no guarantee that the resulting process of structural change is consistent with that experienced in the U.S.³³ Therefore, structural change offers an additional source of identification that allows characterizing the joint evolution of technological change and market power. In other words, the observed process of structural change informs the evolution of technological change and market power across sectors while being consistent with the observed decline in the labor share. Consequently, considering the labor share and the process of structural

³²Note that this statistic is the model counterpart of (1.2).

³³In Appendix 1.6.4, I show that the implications of technological change and market power for structural change are inconsistent with the path observed in the data when the only interest is matching the evolution of the labor share.

change jointly is crucial to derive meaningful conclusions regarding the reasons underlying the decline in the labor share.

The quantitative analysis results show that to match the data: i) markups must increase over time, and ii) technological change must be capital biased.³⁴ The model can explain 87.4% of the decline in the aggregate labor share of the economy between 1998 and 2016. Moreover, this exercise allows quantifying the contribution of technological change and markups for the decline in labor share and the process of structural change. I find the increase in markups is the main driver underlying the decline in the labor share. Technological change is also relevant for the decline, especially for services since 2008, and is the main reason that explains the structural change process from manufacturing to services. To stress the baseline experiment results, in Appendix 1.6.5 I conduct a series of additional experiments where I consider alternative assumptions regarding the level and evolution of market power over time. I conclude that without industry-specific time-varying markups, it is not possible to fully reconcile the decline in the labor share and the transformation of the U.S. into a more service-oriented economy during the last two decades.

1.4.1 Calibration

Here I present the calibration of the quantitative experiment. The model has 12 structural parameters that need to be calibrated, which I partition into two sets.

Parameters fixed without solving the model

I start by fixing some parameters of the model exogenously by relying on previous literature. First, regarding the final good production, following [Buera and Kaboski \(2009\)](#) I fix the elasticity of substitution between manufacturing and services goods, θ , to 0.5. This implies that manufacturing and services goods are gross complements in the production of the numeraire of the economy. Moreover, I fix the relative weight of manufacturing and services, γ , to its average value over the period 1998 to 2016.

For the sectoral production functions, I fix the elasticities of substitution between capital and labor $\sigma_m = 0.8$ and $\sigma_s = 0.75$, the estimates of [Herrendorf et al. \(2015\)](#). Although there exists a vast literature that tries to identify these elasticities, there is no consensus on whether capital and labor are indeed gross substitutes or gross complements in production (see, among others, [Karabarbounis and Neiman \(2014\)](#), [Herrendorf et al. \(2015\)](#),

³⁴In other words, technology must evolve in such a way that the relative productivity of capital increases and, therefore, its relative demand.

Alonso-Carrera, Freire-Serén and Raurich (2017) or Wemy (2021).³⁵ Fixing values below unitary elasticity implies that capital and labor are gross substitutes in the production of manufacturing and services goods, being more easily substitutable in the production of the former. Additionally, given that using (away from unitary) constant elasticity of substitution production functions necessarily involves dealing with their normalization, fixing the elasticities of substitution outside the model alleviates this problem in my quantitative analysis.³⁶ Finally, following previous literature, I fix $1 - \alpha_m$ and $1 - \alpha_s$ to the geometric average of the labor share in each sector. Table 1.1 lists the values assigned to these parameters.

Table 1.1: Parameters fixed outside the model

Parameter	Value	Definition	Source
γ	0.19	Average manufacturing/services	Data
θ	0.5	Subst. elasticity M and S	Buera and Kaboski (2009)
σ_m	0.8	Subst. elasticity K_m and L_m	Herrendorf et al. (2015)
σ_s	0.75	Subst. elasticity K_s and L_s	Herrendorf et al. (2015)
α_m	0.49	Manufacturing average labor share	Data
α_s	0.39	Services average labor share	Data

Targeted moments

The next step consists of using moments from the data to infer the unobserved exogenous processes' evolution: technological change and market power. The parameters governing both processes are calibrated to exactly replicate the moments of interest. Technically, the experiment is conducted as follows:

1. For a given t guess a set of parameters $\{\tilde{B}_{m,t}, \tilde{B}_{s,t}, A_{m,t}, A_{s,t}, \varepsilon_{m,t}, \varepsilon_{s,t}\}$.
2. Take K_t and L_t (and thus k_t , the capital-labor ratio of the economy in period t) as given, and solve the static model characterized in Sec. 1.3 by Equations (1.10) and (1.11). The solution yields the equilibrium allocation of capital and labor across sectors.
3. Compute the following moments in the model:

³⁵In Appendix 1.6.6 I show that the results are robust for a wide range of estimates of these elasticities.

³⁶See Klump, McAdam and Willman (2007b), León-Ledesma, McAdam and William (2010), Klump, McAdam and Willman (2012), Temple (2012) among others for a detailed analysis on the normalization of CES production functions.

- i. Manufacturing labor share, $s_{M,t}^L$,
- ii. Services labor share, $s_{S,t}^L$,
- iii. Capital-output ratio of the economy, K_t/Y_t ,
- iv. Capital-labor ratio in manufacturing, $k_{m,t}$,
- v. Manufacturing share of total capital, κ_t
- vi. Manufacturing share of total labor, λ_t .

Denote this set of moments as $\hat{g}(\Theta)$, where Θ is the full set of parameters (both exogenously fixed and calibrated) needed to solve the model.

4. Compute the distance between the moments generated in the model $\hat{g}(\Theta)$ and their counterpart in the data, g . If not close, go back to step 1.

To implement this procedure, I develop an algorithm that performs a random search in the parameter space and employs a minimum-distance criterion function that compares empirical moments from the data to their model-implied counterparts. Technically, it minimizes the weighted sum of squared relative deviations between the moments generated within the model, $\hat{g}(\Theta)$ and those computed in the data, g . The calibration procedure is performed jointly. Hence all the moments are interdependently affected by changes in all the parameters

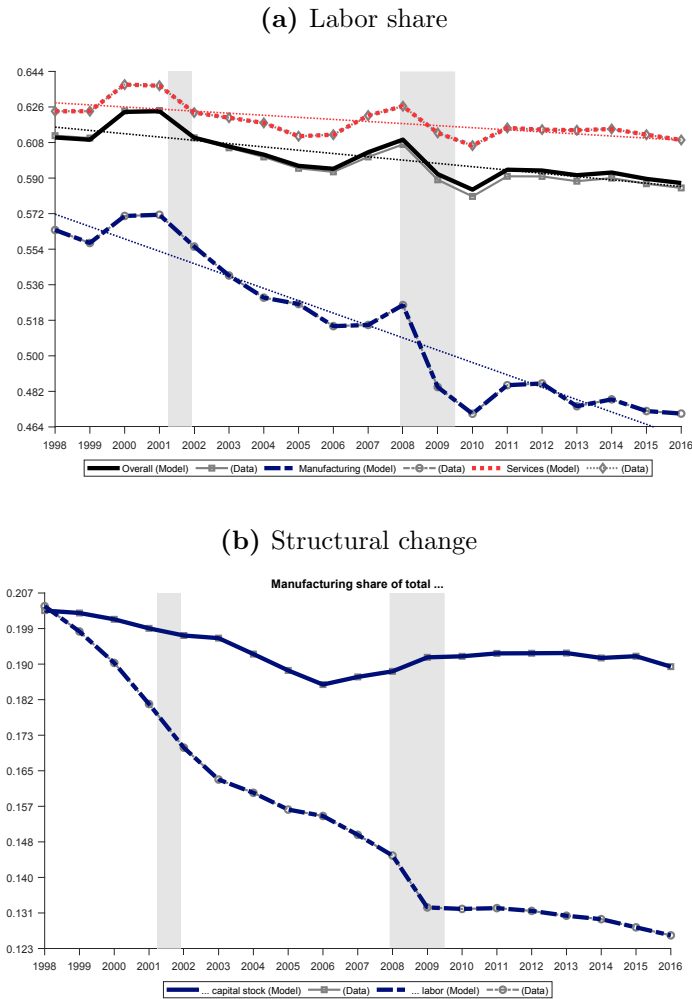
1.4.2 Results

In what follows, I discuss the fit of the model and the estimates obtained for technological change and markups and quantify their contribution to the decline in the labor share. To do that, I first show the baseline experiment results where both technological change and markups vary over time and jointly interact. As a consequence, both shape the evolution of the labor share, generating at the same time structural change. Second, I decompose the effects of technological change and market power by comparing the results of the baseline economy with two counterfactual (non-recalibrated) economies: i) an economy that shows what would have been the evolution of the U.S. economy had markups evolved as in the baseline experiment but without technological change; ii) an economy that shows what would have been the evolution of the U.S. economy had technological change evolved as in the baseline experiment but without a change in markups.

Fit of the model, parameter values and discussion

Figure 1.7 shows the fit of the industry level shares of the baseline economy and the allocation of capital and labor across industries. By construction, it perfectly replicates their evolution along the period considered.

Figure 1.7: Fit of the model, 1998 - 2016



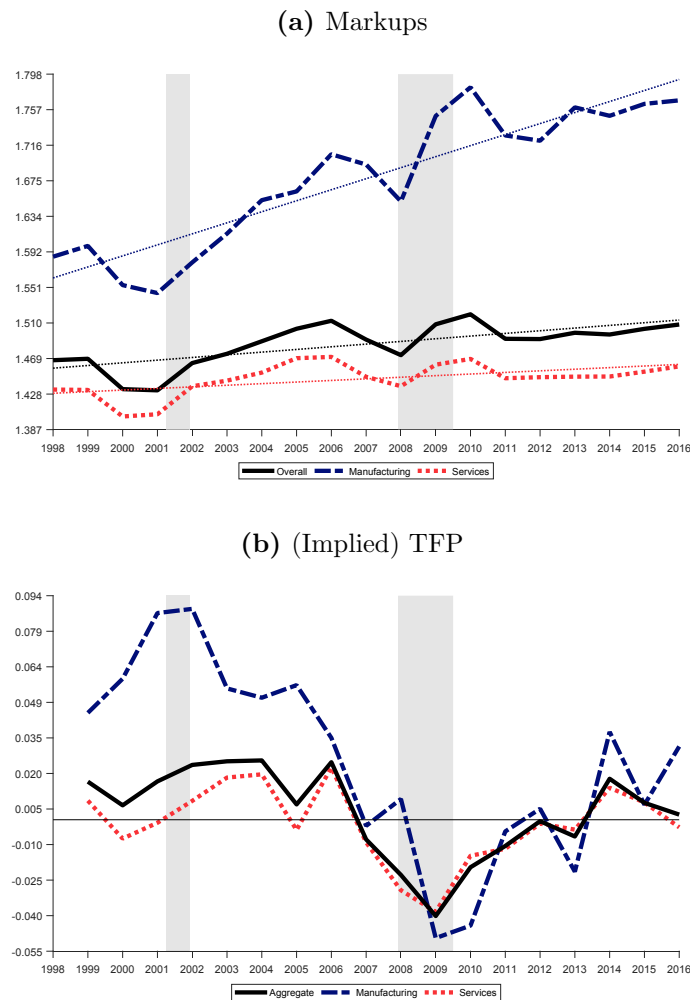
Concentrating on the labor share, figure 1.7a reflects how the calibrated economy in the baseline experiment matches both the observed ups-and-downs and the declining trend in the industry-level labor shares. This decline is much steeper in the labor share of manufacturing industries (represented by the dashed blue line) than in services (represented by the dotted red line). Although the overall labor share is not targeted – is obtained by weighting the industry labor shares – it is remarkably close to its data counterpart.³⁷

³⁷The smaller decline is a consequence of the resulting higher weight in the value added of manufacturing

Regarding structural change moments, figure 1.7b exhibits that the baseline economy is also consistent with the strong relative reallocation of employment from manufacturing to services industries (the dashed blue line) and the relatively smaller fluctuations of the share of capital stock in manufacturing with respect to the total capital stock of the U.S. economy (the solid blue line).

Figure 1.8 exhibits the calibrated evolution of markups at the industry and overall (implied) level (figure 1.8a) and the TFP growth rates implied by the calibrated evolution of the technological change parameters (figure 1.8b).³⁸ Both mechanisms interact in equilibrium to characterize the phenomena of interest.

Figure 1.8: Calibrated exogenous processes: markups and (implied) TFP



relative to services obtained in the calibrated model.

³⁸TFP is computed by using Kmenta approximation. See Kmenta (1967) and Klump, McAdam and Willman (2007a)

The steep increase in markups in manufacturing (the dashed blue line) contrasts with the flatter evolution in services in the benchmark exercise. Both jointly imply an overall increase in the economy-wide markup of 0.05 during the last two decades. Simultaneously, industry-level biased technological change implies a positive average TFP growth in manufacturing, depicted as the dashed blue line, and almost zero TFP growth in services. Besides, TFP growth in both sectors is declining, being much more noticeable in manufacturing. This result has the essence of TFP growth slowdown pointed out in [Duernecker, Herrendorf and Valentinyi \(2019\)](#). Besides, it is aligned with the commonly accepted view that ongoing low productivity growth is related to competition's weaknesses, reflected in the calibration through markups.

The calibrated series for technological change parameters involve negative growth rates of capital augmenting technological change (\tilde{B}) in both sectors and positive growth rates of neutral technological change (A). Through the lens of the model, (1.12) and (1.13) imply that the evolution of markups produces a decline in both labor and capital shares. Moreover, according to (1.14), the evolution of technological change implies a decline in the relative share of labor income in both sectors, even though the capital-labor ratio increases. Consequently, the decline in the labor share is driven by a parallel increase in the pure profit share. In contrast, the capital share remains flat during this period.

In the baseline experiment, markups turn out to be highly negatively correlated with the labor share time series. This finding is not new and is a feature of the data already pointed out by [De Loecker and Eeckhout \(2020\)](#). Moreover, the quantitative evolution of markups is aligned with the results of [Hall \(2018\)](#) and, to a lesser extent, to those of [De Loecker et al. \(2020a\)](#).

Driving processes

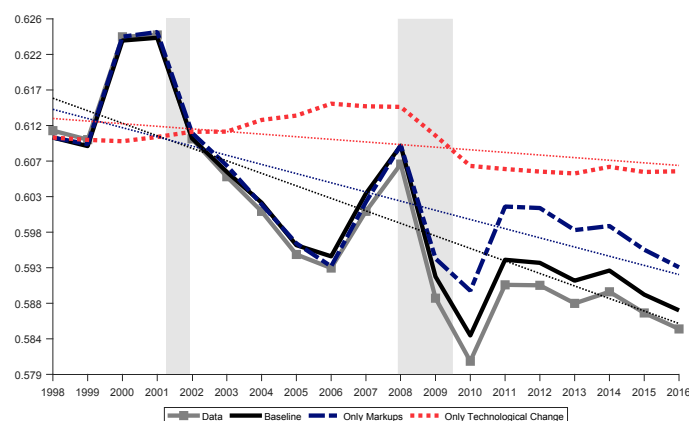
To quantify the contribution of technological change and market power to the decline of the (aggregate) labor share and the process of structural change, I compare the results of the baseline economy with those of two non-recalibrated counterfactual economies:

1. There is no technological change in the first one, and markups vary as in the baseline experiment. In other words, this economy represents the evolution of the economy had markups evolved as in the baseline economy, but if technological change had not happened.
2. There are no market power changes in the second one, and technological change evolves as in the baseline experiment. In turn, this economy represents the evolution

of the economy had technological change evolved as in the baseline economy but if there had not been changes in market power.

Figure 1.9 shows the evolution of the labor share from 1998 to 2016. The baseline exercise is able to explain 87.4% of the decline observed in the data. The main reason underlying this decline is the evolution of market power, as if technological change had not taken place, market power would individually account for 64.1% of the decline as depicted by the dashed blue line. Instead, if there are no changes in market power, technological change would individually account for 18.2% of the decline as depicted by the dotted red line.³⁹ As a consequence, the increase in markups is the main reason that underlies the decline of the labor share since the late 1990s.

Figure 1.9: Labor share, baseline and counterfactual experiments, 1998 - 2016



Focusing on the industry-level labor shares, Figure 1.9 depicts the resulting labor shares in manufacturing and services in the baseline and counterfactual economies. If the only exogenous change (other than capital deepening) had been markups, the labor shares in manufacturing and services would have declined, accounting for 61.5% and 38.4%, respectively. In the opposite situation, i.e., if technological change had taken place, but markups had remained constant at its 1998 level, the labor shares in manufacturing and services would have also declined, accounting for 38.7% and 23.9% of the decline, respectively.⁴⁰

³⁹To stress the relevance of these results, in Appendix 1.6.5 I conduct a series of experiments where I recalibrate alternative economies with different assumptions regarding the joint evolution of technological change and market power. In particular, I show that in an economy without markups, technological change can be pushed to explain up to 62.9% of the observed decline in the data. However, this comes at the cost of not being able to reconcile the observed evolution of the industry-level labor shares.

⁴⁰Note that the volatility of both labor shares is lower in absence of changes in markups, which implies

Table 1.2: Decomposition of the effects of market power and technological change, 1998 - 2016

	s^L	s_M^L	s_S^L	λ	κ
<i>1998 - 2016</i>					
Both	-3.01 (87.4%)	-11.26	-1.89	-7.69	-1.03
Market Power	-2.21 (64.1%)	-6.93 (61.5%)	-0.72 (38.4%)	-0.96 (12.4%)	-0.59 (56.7%)
Technological Change	-0.62 (18.2%)	-4.35 (38.7%)	-0.45 (23.9%)	-7.03 (91.5%)	-0.14 (13.1%)
<i>Only Technological Change</i>	-2.16 (62.9%)	-12.8 (114%)	-0.31 (16.5%)	-7.58 (98.6%)	-1.11 (107%)
<i>1998 - 2008</i>					
Market Power	-1.76 (87.5%)	-4.04 (69.7%)	-1.06 (91.1%)	-0.47 (7.9%)	-0.14 (7.8%)
Technological Change	0.55 (-27.4%)	-1.42 (24.5%)	0.54 (-46.4%)	-5.61 (94.2%)	-1.43 (77.5%)
<i>2008 - 2016</i>					
Market Power	-0.63 (60.1%)	-1.98 (58.9%)	-0.19 (26.9%)	-0.27 (21.3%)	-0.29 (-477%)
Technological Change	-0.66 (63.9%)	-1.64 (48.8%)	-0.58 (78.6%)	-1.11 (86.39%)	0.33 (544%)

Note: 'Both' represents the baseline economy. 'Market Power' and 'Technological Change' values are obtained by allowing technological change and fixing markups at its 1998 level, or fixing technological change and changing markups. 'Only technological Change' shows the results of an alternative economy without markups. More details can be found in Appendix 1.6.5.

Figure 1.10: Industry labor shares, baseline and counterfactual experiments, 1998 - 2016

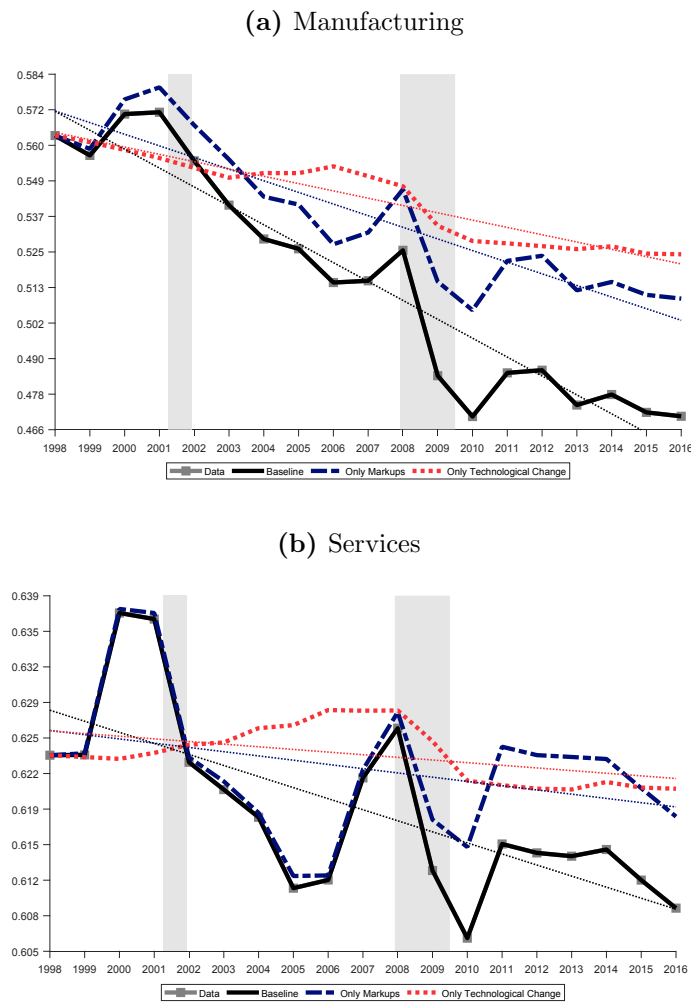
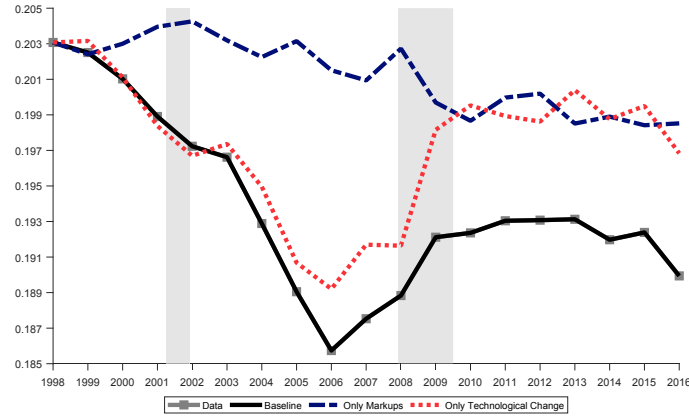
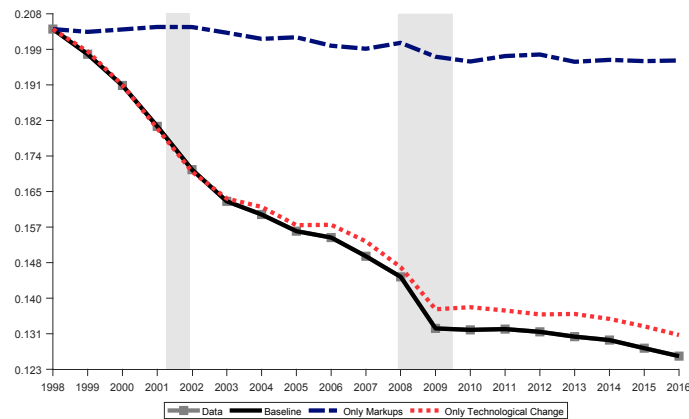


Figure 1.11 depicts the time-series of the shares of total capital and labor employed in manufacturing industries. Following the previous discussion and the theoretical predictions laid out in Section 1.3, market power generates structural change, but its effect is smaller than that of technological change. According to the dashed blue lines, if technology did not evolve as in the baseline economy, structural change between manufacturing and services in terms of employment would have been almost negligible during the last two decades. In other words, markups alone cannot generate the strong structural change observed in the data. On the other hand, the evolution of technology can account for almost all employment reallocation across sectors. This is related to the automation lit-

that the underlying process of technological change in the baseline economy is also not that volatile. A detailed discussion regarding the volatility of these series can be found in Appendix 1.6.5.

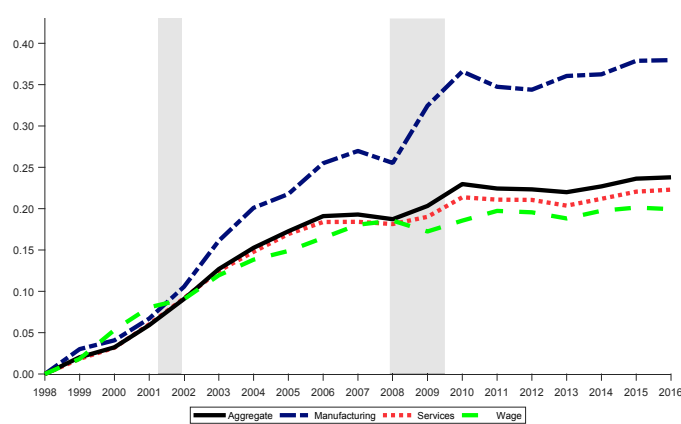
Figure 1.11: Structural change, baseline and counterfactual experiments, 1998 - 2016**(a)** Share of capital in manufacturing out of total capital stock**(b)** Share of labor in manufacturing out of total labor

erature, where labor can be easily replaced in manufacturing, further supported by the substantial capital deepening observed in the economy.

Considering the entire period from 1998 to 2016 hides substantial heterogeneity in the relevance of technological change and market power for the moments of interest. Table 1.2 provides additional results of the baseline economy by considering two sub-periods: 1998 to 2008 and 2008 to 2016. The evolution of market power is key to explain the decline in the labor share from 1998 to 2008. Specifically, it accounts for 69.7% of the decline in manufacturing and 91.1% of the decline in services. This yields an 87.4% decline at the aggregate level. Technological change can explain up to 24.5% of the decline in manufacturing; however, it would have implied an increase in the labor share in services. As a consequence, in the absence of market power, the estimated technological

process would have implied an increase in the aggregate labor share. Market power is still the primary reason underlying the decline of the labor share in manufacturing from 2008 to 2016, accounting for 58.9% of the decline, while technological change accounts for 48.8%. Interestingly, in the services industries, market power is less relevant during this period, accounting for 26.9% of the decline, while technological change becomes much more relevant, accounting for 78.6% of the decline. Overall, technological change is slightly more relevant during this period.

Figure 1.12: Cumulative growth rate of labor productivity and wage, baseline economy, 1998 - 2016



The implications of the baseline economy non-targeted dimensions are well aligned with the data. The growth rates of value added, employment and wages across sectors exhibit the same pattern as in the data: the average wage growth is higher than that of value added in manufacturing and smaller than that of services. Besides, the average employment growth rate is negative in manufacturing and positive in services.

Additionally, Figure 1.12 depicts the cumulative growth rate of labor productivity and wages. The joint combination of technological change and market power implies an incomplete pass-through of labor productivity gains to wages, a well-known documented fact in previous literature. Through the lens of the model, this gap has increased steadily since 1998, being particularly noticeable and persistent in manufacturing. Since the start of the Great Recession, the gap is also noticeable in services.

1.5 Conclusion

The decline in the majority of advanced economies' labor share is possibly one of the most troubling facts of modern macroeconomics. The commonly accepted view is that this decline started in the early 1980s and has intensified during the last two decades.

This paper uses U.S. industry-level data from 1998 to 2016 and shows that not only is the decline heterogeneous and pervasive across industries, but it is also contemporaneous to an intense structural change process from manufacturing to services. A simple decomposition of the (naive) labor share into its main components shows that manufacturing and services industries experience a different evolution. The former are generally characterized by exhibiting negative employment growth rates, while on the contrary, services industries exhibit positive growth rates. Besides, value added in services industries grows at a higher rate than that of wages, while the opposite happens in manufacturing industries.

Despite the numerous contributions to the literature on the decline in the labor share, there is still no consensus on the causes underlying this decline. Intuitively, the two leading explanations for the decline of the labor share, namely technological change (e.g., in the form of automation) and changes in the level of competition (e.g., rise in the concentration of firms), can also affect the contemporaneous pace of structural change. With this idea in mind, and to understand the industry-level evolution of the labor share and the process of structural change from manufacturing to services, I develop an extension of the model proposed by [Álvarez-Cuadrado et al. \(2017, 2018\)](#). This is a multi-sector model with several supply-side mechanisms that allow for structural change and changes in factor shares, which I augment with heterogeneity in the level of markups across industries. Therefore, in the proposed model, industry-specific technological change, production technology differences across industries, and heterogeneous markups play a role in explaining both changes in the labor share and in structural change.

The model is then taken to the data to perform a quantitative analysis. The baseline experiment results exhibit that both time-varying markups and biased technological change are necessary to explain the observed declines in the labor share and the reallocation of labor and capital from manufacturing to services. Through the lens of the model, markups are the main driver of the decline in manufacturing and services industries' labor share. Technological change has also contributed to the decline, especially since 2008, and is the fundamental reason underlying structural change from manufacturing to services. I also show that if markups are not allowed to vary over time, technological change can go a long way in explaining the decline of the labor share, but it cannot replicate the levels of the labor share across sectors while being consistent with the process of structural change.

Consequently, this paper's results are aligned with the stream of the literature that relates the decline of the labor share with a decline in competition, which then favors an increase in markups. Other phenomena as automation and the rise of AI (which could be understood as a type of capital augmenting technological change) also play a role in the

decline in the labor share, especially after 2008, and are the reason underlying the process of structural change from manufacturing to services during the last two decades.

1.6 Appendix

1.6.1 Data

Industries

The industries are defined according to the 2007 North American Industry Classification System (NAICS). The relationship between the chosen level of detail (BEA summary) and the 2007 North American Industry Classification System (NAICS) code structure can be found in further detail on the BEA methodology website.

Others: Agricultural activities, Mining, Utilities and Construction

1. Farms (NAICS 111)
2. Forestry, fishing, and related activities (NAICS 113[4,5])
3. Oil and gas extraction (NAICS 211)
4. Mining, except oil and gas (NAICS 212)
5. Support activities for mining (NAICS 213)
6. Utilities (NAICS 221)
7. Construction (NAICS 23)

Manufacturing

8. Wood products (NAICS 321)
9. Nonmetallic mineral products (NAICS 327)
10. Primary metals (NAICS 331)
11. Fabricated metal products (NAICS 332)
12. Machinery (NAICS 333)
13. Computer and electronic products (NAICS 334)
14. Electrical equipment, appliances, and components (NAICS 335)
15. Motor vehicles, bodies and trailers, and parts (NAICS 3361[2,3])
16. Other transportation equipment (NAICS 3364[5,6,9])
17. Furniture and related products (NAICS 337)
18. Miscellaneous manufacturing (NAICS 339)

19. Food and beverage and tobacco products (NAICS 311)
20. Textile mills and textile product mills (NAICS 313)
21. Apparel and leather and allied products (NAICS 315[6])
22. Paper products (NAICS 322)
23. Printing and related support activities (NAICS 323)
24. Petroleum and coal products (NAICS 324)
25. Chemical products (NAICS 325)
26. Plastics and rubber products (NAICS 326)

Private Services

27. Wholesale trade (includes durable goods and nondurable goods) (NAICS 42)
28. Motor vehicle and parts dealers (NAICS 441)
29. Food and beverage stores (NAICS 445)
30. General merchandise stores (NAICS 452)
31. Other retail (NAICS 4424, 4468, 451, 4534)
32. Air transportation (NAICS 481)
33. Rail transportation (NAICS 482)
34. Water transportation (NAICS 483)
35. Truck transportation (NAICS 484)
36. Transit and ground passenger transportation (NAICS 485)
37. Pipeline transportation (NAICS 486)
38. Other transportation and support activities (NAICS 487[8])
39. Warehousing and storage (NAICS 493)
40. Publishing industries (includes software) (NAICS 511)
41. Motion picture and sound recording industries (NAICS 512)
42. Broadcasting and telecommunications (NAICS 515)
43. Information and data processing services (NAICS 518[9])
44. Federal Reserve banks, credit intermediation, and related activities (NAICS 521[2])

45. Securities, commodity contracts, and investments (NAICS 523)
46. Insurance carriers and related activities (NAICS 524)
47. Funds, trusts, and other financial vehicles (NAICS 525)
48. Real estate (NAICS 531)
49. Rental and leasing services and lessors of intangible assets (NAICS 532[3])
50. Legal services (NAICS 5411)
51. Computer systems design and related services (NAICS 5415)
52. Miscellaneous professional, scientific, and technical services (NAICS 5412[2,3,4,6,7,8,9])
53. Management of companies and enterprises (NAICS 55)
54. Administrative and support services (NAICS 561)
55. Waste management and remediation services (NAICS 562)
56. Educational services (NAICS 61)
57. Ambulatory health care services (NAICS 621)
58. Hospitals (NAICS 622)
59. Nursing and residential care facilities (NAICS 623)
60. Social assistance (NAICS 624)
61. Performing arts, spectator sports, museums, and related activities (NAICS 711[2])
62. Amusements, gambling, and recreation industries (NAICS 713)
63. Accommodation (NAICS 721)
64. Food services and drinking places (NAICS 722)
65. Other services, except government (NAICS 81)

Government

66. Federal government, includes general government and government enterprises (NAICS n/a)
67. State and local government, includes general government and government enterprises (NAICS n/a)

Capital stock and employment

One of the main data statistics needed to take the model to the data in a realistic way is the capital-labor ratio. Given that, by definition, capital and labor measure intrinsically different goods, the capital-ratio is not a unit-free measure, and thus it is key to define the measurement units of both the numerator and the denominator properly. In this paper, the capital to labor ratio is measured in real 2009 dollars per hour. In this Appendix, I carefully explain how to obtain the appropriate measure for capital and labor (or employment).

Capital Stock The capital stock is computed as the current-cost net stock of private fixed assets by adding Equipment (BEA, table 3.1.E), Structures (BEA, table 3.1.S), and IPP (BEA, table 3.1.I). These tables include residential assets, but given the model used in the paper, the relevant measure of capital shall not include these types of assets.⁴¹ However, the BEA does not provide a sufficiently detailed disaggregation of private non-residential fixed assets. Thus, I need to construct that series from the available data.

To do so, I also collect data on the current-cost net stock of private nonresidential fixed assets by industry group and legal form of organization (BEA, table 4.1). The BEA definition of nonfarm nonmanufacturing includes services industries and other industries as utilities or construction. Therefore, to compute the level of nonresidential fixed assets for services industries, I assume that the share of services of private fixed assets including residential fixed assets (but omitting real estate) in nonfarm nonmanufacturing is the same as in private nonresidential fixed assets.⁴² Finally, chain-type quantity indexes for the net stock of private fixed assets are used to obtain the measure of real capital stock.

Labor (Employment) To construct hours worked in each industry, I combine use data on hours worked by full-time and part-time employees by industry (BEA, tables 6.9C and 6.9D). The within industry classification (which type of firms are included in which industry) changes slightly in 2000. To solve this, I smooth out the existing discrepancy across tables by assuming that the employment share between 1998 and 2000 across industries is constant at its 2000 level, allowing aggregate employment to vary as in the data.

⁴¹Including (or excluding) residential private fixed assets only makes an enormous difference for the real estate industry. In any case, as it is explained in Section 1.2, I omit the real estate industry.

⁴²The difference between the aggregate value of both series after omitting real estate is indeed very small.

The data available aggregates hours worked in finance and insurance, real estate, rental, and leasing (FIRERL), thus for consistency reasons, I need to exclude those hours worked in real estate industries. To do this, I also gather data on the number of full-time equivalent employees by industry (BEA, table 6.5D), which allows identifying separately the number of employees in each of the sub-industries of FIRERL. A crucial step is assuming that hours distribute evenly among all these sub-industries. Under this assumption, the value of hours in manufacturing and services excluding real estate can finally be pinned down.

1.6.2 Labor share

Aggregate level

This section extends the analysis of the labor share made in Figure 1.1 in Section 1.2. In particular, Figure 1.13 shows the labor share obtained for the U.S. at the aggregate level⁴³. This figure shows the same results as in (Koh et al., 2020), with the now widely accepted fact that the labor share has decreased since 1947. Specifically, in 1947 the labor share was 0.678 while in 2015, the labor share was 0.620, which implies a decline of roughly five labor share points. While a linear fit from 1947 to 2016 exhibits a clear decreasing trend, it seems that a more appropriate fit should consider at least two different trends for the periods before and after 1980. In doing so, I show the labor share was almost constant before 1980, while it is decreasing ever after.

Figure 1.13: U.S. labor share, 1947 - 2016.

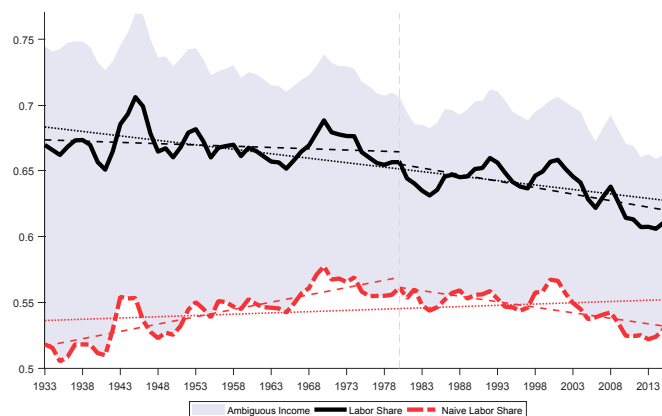


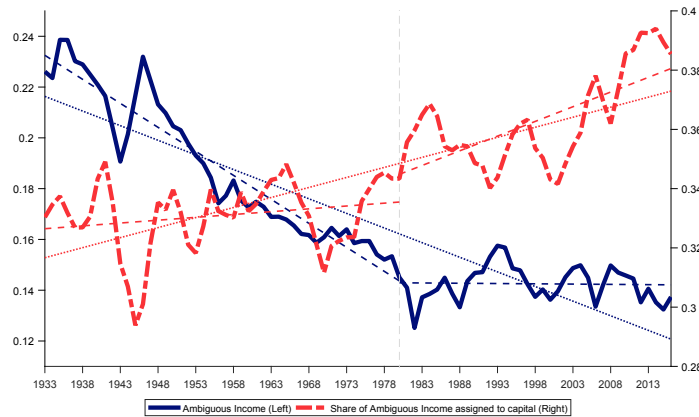
Figure 1.13 also represents the evolution of the naive labor share, which seems not to

⁴³I will refer to this value of the labor share as aggregate LS. However, this must not be confused with the value of labor share that would result in aggregating the industries that we consider.

be a good approximation in terms of the labor share trend over the entire period. As Mućk et al. (2018) suggest, it shows a clear hump-shaped pattern, which is not present in the labor share. However, focusing on the period starting in 1980, the trend of both measures is remarkably similar. In other words, while the labor share can only be adequately measured by following the proposed methodology, at the aggregate level, since the 1980's an approximation to its trend can be obtained by just computing the naive labor share. Finally, Figure 1.13 also shows the evolution of the AI share of income (the grey shaded area), which has also shrunk since 1947, when its level was 0.223, and remains roughly stable since the 1980s at around 0.145, i.e., 14.5 labor share points.

As it has become clear from the industry-level data analysis of Section 1.2, precise computation of the ambiguous income (AI) component is crucial for the precise computation of the labor share. Both the evolution of the AI share and the parameter θ , which measures the part of ambiguous income that accrues to capital, are shown in Figure 1.14. Contrary to what happened with the AI share, the value of θ has been steadily increasing since the 1960s, especially since the 1980s. Therefore, over the last decades, at the same time as the compensation of employees has been declining, the share of ambiguous income that is assigned to labor has also been declining, due to the increase of θ and the decrease of AI.

Figure 1.14: U.S. AI share (left axis) and θ (right axis), 1933 - 2015.



Industry level

In this section, I complete the analysis of the labor share at the industry level. The lack of data mentioned above prevents following the methodology presented in Section 1.2 to compute the labor share at the industry level. Therefore, I modify the methodology by redefining the following variables:

1. Unambiguous Capital Income (UCI) = Corporate Profits + Net Interest.
4. Ambiguous Income (AI) = Gross Operating Surplus (GOS) - Corporate Profits - Net Interest - DEP + Taxes on Production - Subsidies.

This methodology's main difference is that AI is now computed as a residual instead of arising as a definition from the data available. Besides, as a measure of aggregate output, I use the gross domestic product (GDP), which is still the sum of total unambiguous income and ambiguous income, i.e., $Y = \text{UCI} + \text{DEP} + \text{CE} + \text{AI} \equiv \text{GDP}$.

In the following figures, I show the results at the industry level for eight industries of the U.S. Each figure shows, for each industry, its labor share under this modified methodology, its naive labor share, its AI share, and, as a benchmark, the aggregate labor share of the economy. Note that the labor share is only computed for the period 1998 to 2015, the unique period when it is possible to obtain detailed data under the NAICS classification. From 1987 to 1997, the best that one can do is computing the naive labor share, as the BEA only provides a series of converted data for compensation employees CE from SIC to NAICS for this period. Unfortunately, obtaining longer series is not possible, as the aggregation of industries is not consistent before 1987.

Figures 1.15 and 1.16 focus on the manufacturing sector, where the data available allows considering the manufacturing industry of durable goods and the manufacturing industry of nondurable goods independently. The labor share in manufacturing has declined much faster than the aggregate labor share. These results are in line with the results of [Álvarez-Cuadrado et al. \(2018\)](#), though in contrast to them, this computation shows that in 1998 the manufacturing sector's labor share was lower than the aggregate labor share of the whole economy.

Figure 1.15: U.S. Manufacturing (durable goods) industry labor share, 1987 - 2015.

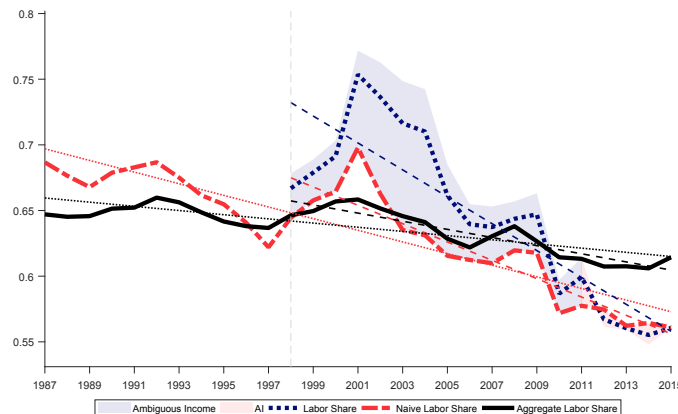
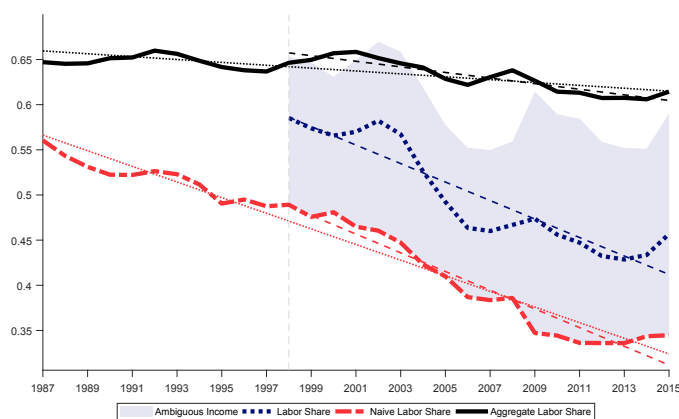


Figure 1.16: U.S. Manufacturing (nondurable goods) industry labor share, 1987 - 2015.



The labor share level is very different in the durable and nondurable goods industries, although both constitute the manufacturing industry. Both industries show a decreasing labor share, as the manufacturing industry, and much faster than the aggregate labor share. The labor share in the durable goods industry in 1998 is 0.667, while in the nondurable goods industry, the labor share is 0.585, eight labor share points lower. In 2015 the values were 0.560 and 0.457, respectively, which implies that the gap has widened in 3 labor share points. Omitting the period 2011 - 2015, the differences between both industries in the AI share are noticeable.⁴⁴ In particular, in the durable goods industry, it is decreasing very fast, while in the nondurable goods industry, I find that the AI share has increased since 1998, passing from 0.164 to 0.245. In the manufacturing industry and the nondurable goods industry, the naive labor share can match the labor share trend.

The following four figures cover services industries. In particular, Figures 1.17 and 1.18 show the wholesale trade industry and the retail trade industry, respectively. In 1998, both industries had a higher labor share than the aggregate labor share of the economy, which remains true during all the timeframe analyzed. In 1998, the wholesale trade industry labor share was 0.723, and the retail trade labor share was 0.768, while

⁴⁴Notice that during this period, the labor share in the durable goods industry is smaller than the naive labor share. This happens because the AI share is also negative for this period, a drawback of the developed methodology for the industry level. Recall that the AI share is computed as a residual, which implies that in this case, the sum of gross operating surplus plus taxes (net of subsidies) on production is smaller than the sum of corporate profits, net interest, and depreciation. According to the proposed methodology, when the AI share is negative, and for a given value of θ , when computing the labor share we effectively add a negative value to the CE, which results in a lower labor share than the naive labor share. In this case, we must stick with the naive labor share in the durable goods industry for the period 2011 - 2015. Without more detailed data, it is not possible to pin down the source of the negativeness of the AI share of income.

in 2015 were 0.675 and 0.666, respectively. Consequently, the trend of the labor share in the former is flatter than in the latter. Besides, the wholesale trade industry trend has followed the aggregate labor share trend closely, while the retail trade industry trend has been much steeper.

Figure 1.17: U.S. Wholesale trade industry labor share, 1987 - 2015.

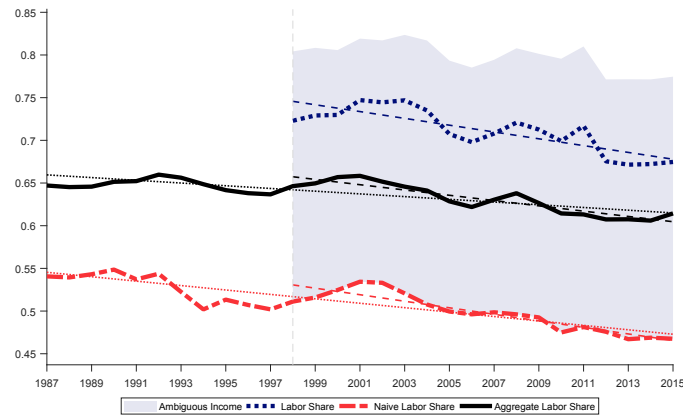
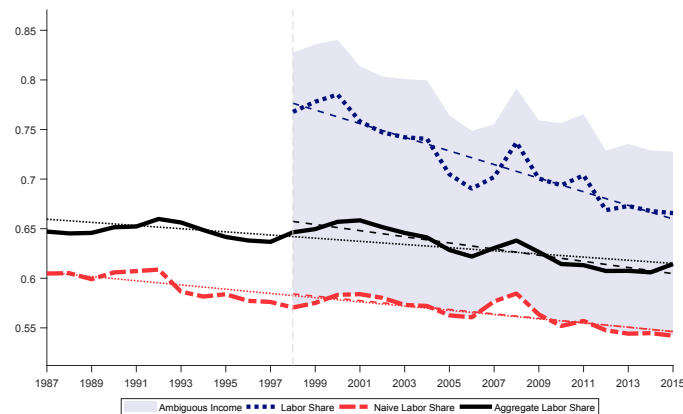


Figure 1.18: U.S. Retail trade industry labor share, 1987 - 2015.



Both wholesale and retail trade industries have a more significant AI share than the aggregate labor share, an expected result due to its computation. However, while in the wholesale trade industry, this share remains roughly constant during the period at around 0.300, in the retail trade industry, it has diminished from 0.255 to 0.186, a decrease of 7 labor share points. The difference between both industries in 2015 is higher than 11 labor share points. Finally, we can point out that the naive labor share delivers a good approximation of the trend of the labor share in the wholesale trade industry, but this is not the case for the retail trade industry.

Figure 1.19: U.S. Transportation and warehousing industry labor share, 1987 - 2015.

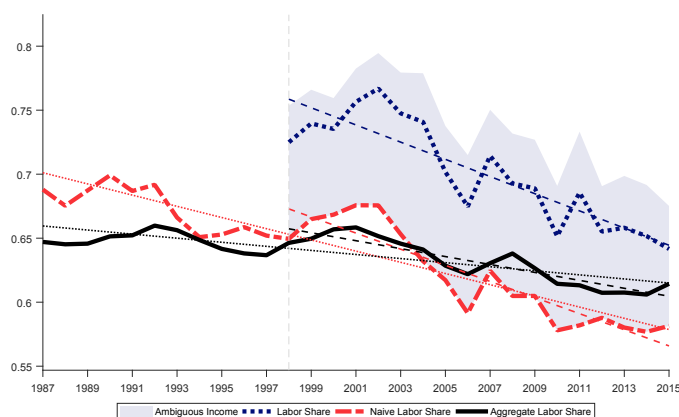


Figure 1.20: U.S. Information industry labor share, 1987 - 2015.

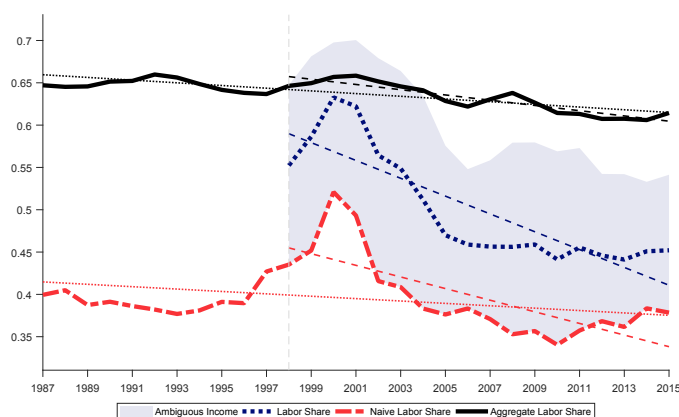


Figure 1.19 shows the results for the transportation and warehousing industry. As the previous services industries analyzed, the transportation industry exhibits a higher labor share than the aggregate labor share all along the considered period. However, its decline is considerably faster, from 0.725 in 1997 to 0.642, almost eight labor share points lower. The AI share remains roughly constant in this industry, averaging 0.117, and the naive labor share delivers a good approximation of the trend of the labor share.

The results of the Information industry are shown in Figure 1.20. This industry presents a different behavior than the previous ones, increasing in the late 1990s, peaking in the early 2000s, undoubtedly motivated by the boom of the dot-com firms, and decreasing ever after. In any case, considering the labor share pre-peak value in 1998, which was 0.552, the labor share has declined to 0.452, exactly ten labor share points, though it peaked in 2000, being 0.633. The AI share has slightly decreased, and it has averaged 0.206, again higher than the aggregate share of AI. The naive labor share again delivers

a good approximation of the labor share, though the trend is somewhat flatter.

1.6.3 Model

The static equilibrium equations are

$$\mu_{m,t}R_t = P_{m,t}\alpha_m(A_{m,t}\tilde{B}_t^m)^{\frac{\sigma_m-1}{\sigma_m}}\left(\frac{M_t}{K_{m,t}}\right)^{\frac{1}{\sigma_m}}, \quad (1.15)$$

$$\mu_{m,t}W_t = P_{m,t}(1-\alpha_m)A_{m,t}^{\frac{\sigma_m-1}{\sigma_m}}\left(\frac{M_t}{L_{m,t}}\right)^{\frac{1}{\sigma_m}}, \quad (1.16)$$

$$\mu_{s,t}R_t = P_{s,t}\alpha_s(A_{s,t}\tilde{B}_{s,t})^{\frac{\sigma_s-1}{\sigma_s}}\left(\frac{S_t}{K_{s,t}}\right)^{\frac{1}{\sigma_s}}, \quad (1.17)$$

$$\mu_{s,t}W_t = P_{s,t}(1-\alpha_s)A_{s,t}^{\frac{\sigma_s-1}{\sigma_s}}\left(\frac{S_t}{L_{s,t}}\right)^{\frac{1}{\sigma_s}}, \quad (1.18)$$

$$\frac{P_{m,t}}{P_{s,t}} = \frac{\gamma}{1-\gamma}\left(\frac{S_t}{M_t}\right)^{\frac{1}{\theta}}, \quad (1.19)$$

$$1 = [\gamma^\theta P_{m,t}^{1-\theta} + (1-\gamma)^\theta P_{s,t}^{1-\theta}]^{\frac{1}{1-\theta}},$$

$$M_t = A_{m,t}\left[\alpha_m(\tilde{B}_{m,t}K_{m,t})^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m)L_{m,t}^{\frac{\sigma_m-1}{\sigma_m}}\right]^{\frac{\sigma_m}{\sigma_m-1}}, \quad (1.20)$$

$$S_t = A_{s,t}\left[\alpha_s(\tilde{B}_{s,t}K_{s,t})^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s)L_{s,t}^{\frac{\sigma_s-1}{\sigma_s}}\right]^{\frac{\sigma_s}{\sigma_s-1}}, \quad (1.21)$$

together with the market clearing conditions

$$K_t = K_{m,t} + K_{s,t},$$

$$L_t = L_{m,t} + L_{s,t}.$$

Given a value for k_t , the equilibrium allocation of the static model is characterized by the solution of a system of two equations in two unknowns, κ_t and λ_t , where k_t , κ_t and λ_t are defined as in (1.9). To obtain these two equations proceed as follows. On the one hand, arbitrage ensures that in equilibrium the interest rate will be equated across industries, which from (1.15) and (1.17) implies

$$R_t = \frac{1}{\mu_{m,t}}P_{m,t}\alpha_m(A_{m,t}\tilde{B}_{m,t})^{\frac{\sigma_m-1}{\sigma_m}}\left(\frac{M_t}{K_{m,t}}\right)^{\frac{1}{\sigma_m}} = \frac{1}{\mu_{s,t}}P_{s,t}\alpha_s(A_{s,t}\tilde{B}_{s,t})^{\frac{\sigma_s-1}{\sigma_s}}\left(\frac{S_t}{K_{s,t}}\right)^{\frac{1}{\sigma_s}},$$

or, equivalently,

$$\frac{\mu_{s,t}}{\mu_{m,t}}\frac{P_{m,t}}{P_{s,t}}\frac{\alpha_m}{\alpha_s}[A_{m,t}\tilde{B}_{m,t}]^{\frac{\sigma_m-1}{\sigma_m}}\left(\frac{M_t}{K_{m,t}}\right)^{\frac{1}{\sigma_m}}[A_{s,t}\tilde{B}_{s,t}]^{-\frac{\sigma_s-1}{\sigma_s}}\left(\frac{S_t}{K_{s,t}}\right)^{-\frac{1}{\sigma_s}} = 1. \quad (1.22)$$

The same is true for the equilibrium wage, thus following the same procedure with (1.16) and (1.18), substituting in the previous equation and rewriting yields

$$\frac{1 - \alpha_s}{1 - \alpha_m} \frac{\alpha_m}{\alpha_s} \frac{\tilde{B}_{m,t}^{\frac{\sigma_m-1}{\sigma_m}}}{\tilde{B}_{s,t}^{\frac{\sigma_s-1}{\sigma_s}}} \left(\frac{K_{s,t}}{L_{s,t}} \right)^{\frac{1}{\sigma_s}} \left(\frac{L_{m,t}}{K_{m,t}} \right)^{\frac{1}{\sigma_m}} = 1. \quad (1.23)$$

By using (1.9), (1.23) can be expressed as

$$\frac{1 - \alpha_s}{1 - \alpha_m} \frac{\alpha_m}{\alpha_s} \frac{\tilde{B}_{m,t}^{\frac{\sigma_m-1}{\sigma_m}}}{\tilde{B}_{s,t}^{\frac{\sigma_s-1}{\sigma_s}}} k_t^{\frac{1}{\sigma_s} - \frac{1}{\sigma_m}} \left(\frac{1 - \kappa_t}{1 - \lambda_t} \right)^{\frac{1}{\sigma_s}} \left(\frac{\lambda_t}{\kappa_t} \right)^{\frac{1}{\sigma_m}} = 1, \quad (1.24)$$

which determines the first equilibrium equation.

On the other hand, the equilibrium prices given by (1.16) and (1.18) can be substituted in (1.19) to obtain

$$\frac{\mu_{m,t} W_t \frac{1}{1 - \alpha_m} A_{m,t}^{-\frac{\sigma_m-1}{\sigma_m}} \left(\frac{M_t}{L_{m,t}} \right)^{-\frac{1}{\sigma_m}}}{\mu_{s,t} W_t \frac{1}{1 - \alpha_s} A_{s,t}^{-\frac{\sigma_s-1}{\sigma_s}} \left(\frac{S_t}{L_{s,t}} \right)^{-\frac{1}{\sigma_s}}} = \frac{\gamma}{1 - \gamma} \left(\frac{S_t}{M_t} \right)^{\frac{1}{\theta}}, \quad (1.25)$$

which can be rewritten as

$$\frac{\mu_{m,t}}{\mu_{s,t}} \frac{1 - \alpha_s}{1 - \alpha_m} \frac{A_{s,t}^{\frac{\sigma_s-1}{\sigma_s}}}{A_{m,t}^{\frac{\sigma_m-1}{\sigma_m}}} \frac{M_t^{\frac{1}{\theta} - \frac{1}{\sigma_m}}}{S_t^{\frac{1}{\theta} - \frac{1}{\sigma_s}}} \frac{L_{m,t}^{\frac{1}{\sigma_m}}}{L_{s,t}^{\frac{1}{\sigma_s}}} \frac{1 - \gamma}{\gamma} = 1. \quad (1.26)$$

Now, rewrite (1.20) and (1.21) as

$$M_t = K_t A_{m,t} \left[\alpha_m \left(\tilde{B}_{m,t} \kappa \right)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) \left(\frac{\lambda}{k_t} \right)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} \equiv K_t A_{m,t} g_{m,t}(\kappa_t, \lambda_t, k_t),$$

$$S_t = K_t A_{s,t} \left[\alpha_s \left(\tilde{B}_{s,t} (1 - \kappa_t) \right)^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) \left(\frac{1 - \lambda_t}{k_t} \right)^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}} \equiv K_t A_{s,t} g_{s,t}(\kappa_t, \lambda_t, k_t).$$

Then, using (1.9) and after some algebra (1.26) can be expressed as

$$\frac{1 - \gamma}{\gamma} \frac{\mu_{m,t}}{\mu_{s,t}} \frac{1 - \alpha_s}{1 - \alpha_m} \left(\frac{A_{s,t}}{A_{m,t}} \right)^{\frac{\theta-1}{\theta}} \frac{g_{m,t}(\kappa_t, \lambda_t, k_t)^{\frac{1}{\theta} - \frac{1}{\sigma_m}}}{g_{s,t}(\kappa_t, \lambda_t, k_t)^{\frac{1}{\theta} - \frac{1}{\sigma_s}}} k_t^{\frac{1}{\sigma_s} - \frac{1}{\sigma_m}} \frac{\lambda^{\frac{1}{\sigma_m}}}{(1 - \lambda_t)^{\frac{1}{\sigma_s}}} = 1, \quad (1.27)$$

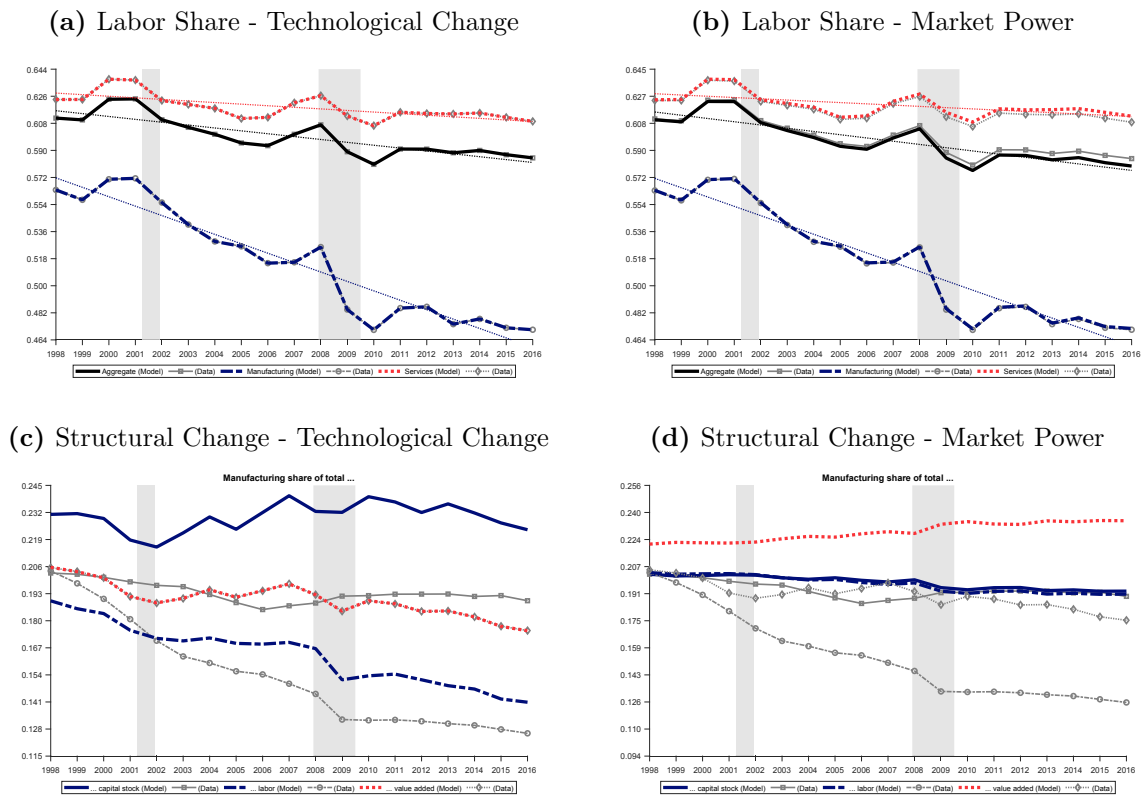
which determines the second equilibrium equation.

1.6.4 Labor share and structural change: individual effects of technological change and market power

This Section shows the resulting paths for the labor share and structural change that are obtained when targeting the aggregate and industry-level labor shares and individually estimating technological change and market power.

As prescribed by the theory, Figure 1.21 reveals that technological change or market power can individually replicate the observed declined in manufacturing and services' labor shares. However, this comes at the cost of obtaining capital and labor allocations across industries that are far from those observed in the data.

Figure 1.21: Technological change or market power, 1998 - 2016.



1.6.5 Alternative experiments

In the baseline experiment, both technological change and the level of markups are heterogeneous across industries and are allowed to vary over time. The results show that to match the data: i) markups must be heterogeneous across industries and increase over

time, and ii) technological change must be capital biased.⁴⁵

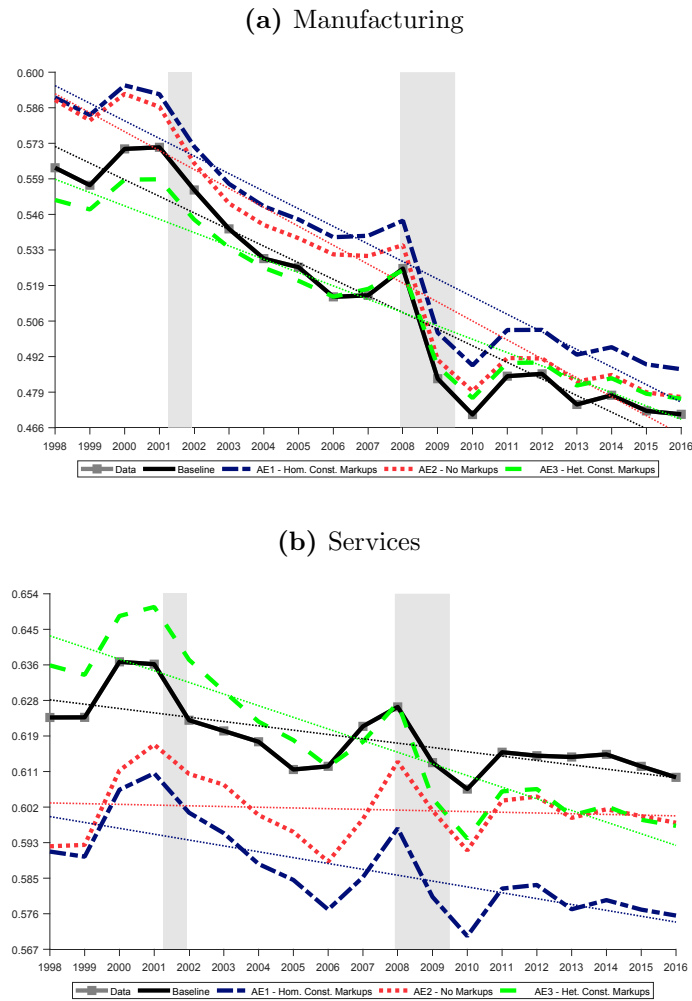
To stress the baseline experiment results, I conduct three additional experiments where I consider alternative assumptions of both the level and the evolution of markups over time. Many different explanations for the decline of the labor share have been put forward, particularly those stressing the crucial (and sometimes, solely) role of technological change. In this Section, I perform a series of experiments with alternative specifications on the joint evolution of technology and markups, stressing the former's explanatory power.

To assess the performance of these different explanations against the baseline experiment, I recalibrate the model under these new specifications and compare the results with those from the baseline experiment. The specific alternative specifications that I consider are: AE1: technological change is allowed to vary over time while markups are homogeneous across sectors and are fixed at a value $\mu_{m,t} = \mu_{s,t} = 1.25, \forall t$; AE2: technological change is allowed to vary over time while there are no markups in the economy, that is $\varepsilon_{j,t} \rightarrow \infty$ so that $\mu_{j,t} \rightarrow 1, j = \{m, s\}, \forall t$, and thus intermediate producers behave as price takers (this is, in spirit, an application to a different dataset of the mechanism proposed by [Álvarez-Cuadrado et al. \(2018\)](#).); AE3: technological change is allowed to vary over time while markups are heterogeneous across sectors, but are constant at the average level calibrated for the baseline economy.

The results of the alternative experiments exhibit that, in the absence of variation over time in the level of markups, technological change can go a very far way in explaining the declining trend in the labor share across sectors. Regardless of the assumption on markups, the calibrated exogenous processes for technological change under experiments AE1, AE2, and AE3 can generate substantial declines in the labor share of manufacturing, in the labor share of services, or both. Given that markups are constant in these three alternative settings, according to (1.14), the predicted labor share declines are accompanied by a parallel increase in the capital share.

Figure 1.22 shows the labor shares' time series for manufacturing and services industries, comparing the baseline experiment with the alternative specifications. Noticeably, if markups are homogeneous across sectors (either positive, depicted as the dashed (blue) line, or zero, depicted as the dotted (red) line), the model fails to match the levels of the labor shares while being consistent with the process of structural change. Technological change can then replicate the declining trend of the labor share but at the cost of delivering a smaller average labor share in services and a higher average labor share in

⁴⁵In other words, technology must evolve in such a way that the relative productivity of capital increases and, therefore, its relative demand.

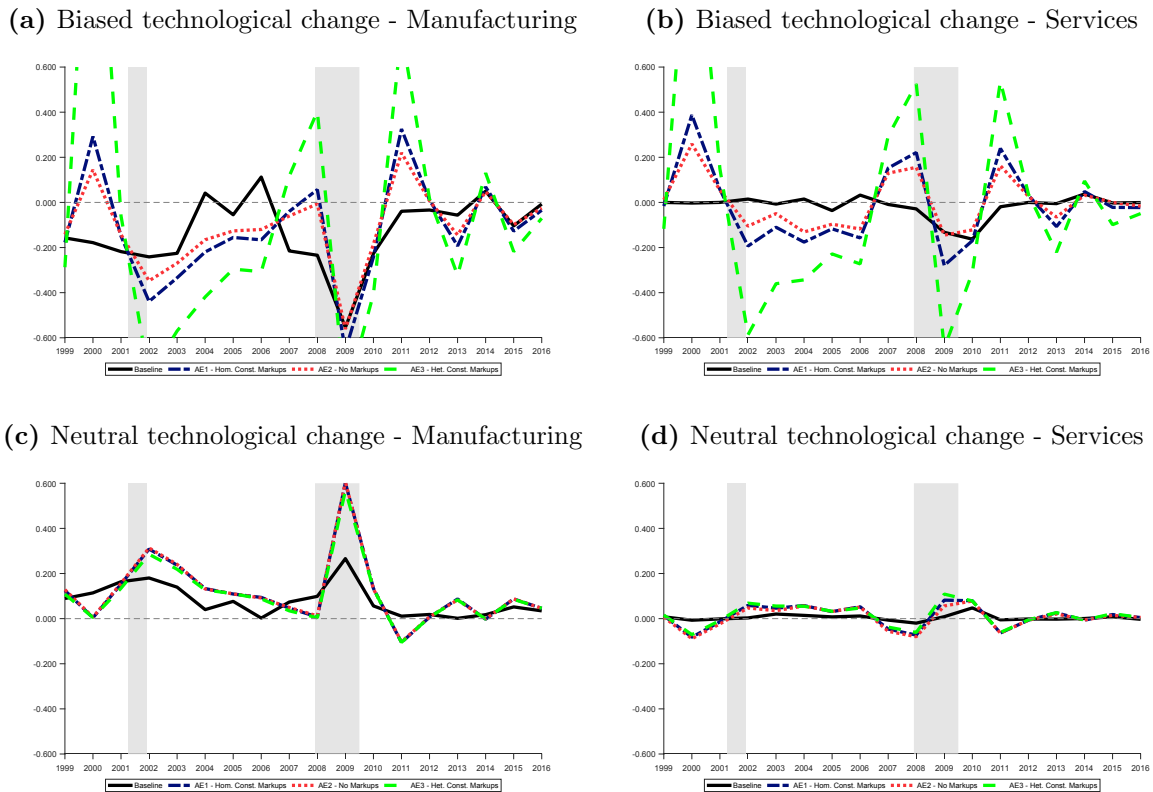
Figure 1.22: Labor share, baseline and alternative experiments, 1998 - 2016.

manufacturing. In other words, so long as markups are homogeneous, the model cannot generate a big gap between both labor shares, like the one measured in the data.

Allowing for heterogeneity in markups and recalibrating the model delivers a new technological change process that yields an evolution of the labor shares of manufacturing and services much closer to the results obtained with the baseline economy (or those observed in reality). However, this comes at the cost of a slight under-prediction of the decline in the trend of the labor share of manufacturing and a high over-prediction of the decline in the trend of the labor share of services. The fitting of structural change moments is remarkably good no matter what is the assumption regarding markups considered.

The joint analysis of all the results discussed so far suggests a strong link between the decline in the labor share and the moments pertaining to structural change. In particular,

Figure 1.23: Technological change parameters, baseline and alternative experiments, 1998 - 2016.



if markups are not heterogeneous and are not allowed to vary over time, technological change alone cannot replicate the levels of the labor share across sectors while being consistent with structural change. If the only interest lies in the labor share’s evolution, the model can be pushed even further by targeting only labor share moments. In that case, the model can perfectly reproduce the level and the trend of the labor share in this period, but that comes at the cost of being inconsistent with the observed structural change process.

All experiments exhibit similar qualitative results concerning the evolution of the capital-output ratio across sectors, the wage-value added growth gap, and employment growth. All specifications over-predict structural change in terms of value added, resulting from the substantial decline in the relative price between manufacturing and services goods.

The evolution of technology is qualitatively similar across the different experiments. However, the volatility of the series of technological change parameters is widely heterogeneous across different specifications. Figure 1.23 shows that the smallest volatility of these series is attained in the baseline economy, while it is remarkably higher in the

Figure 1.24: Share of capital in manufacturing out total capital stock, baseline and alternative experiments, 1998 - 2016.

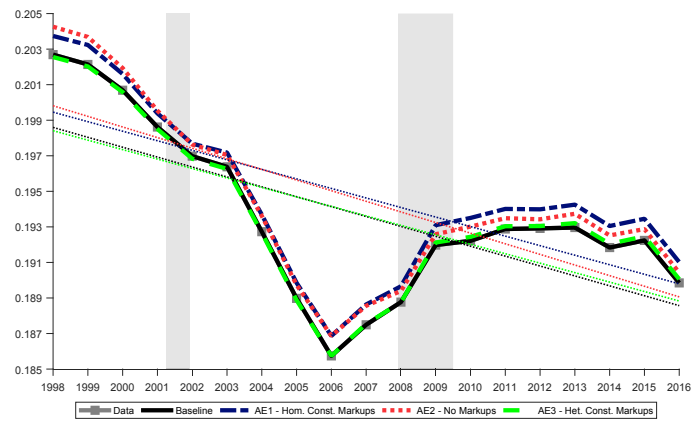


Figure 1.25: Share of labor in manufacturing out of total labor, baseline and alternative experiments, 1998 - 2016.

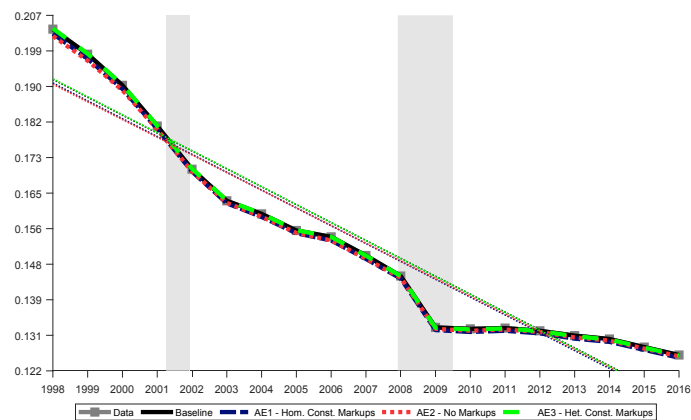
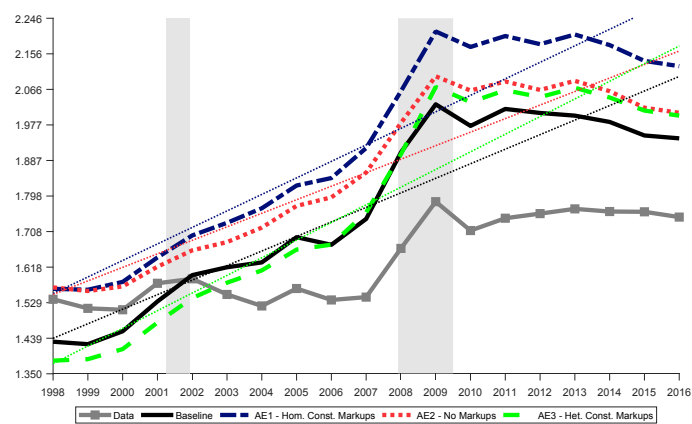


Figure 1.26: Manufacturing capital-output ratio, baseline and alternative experiments, 1998 - 2016.



counterfactual economies where markups are kept fixed. Intuitively, fixing markups reduces the degrees of freedom to match the data, and therefore the calibrated processes of technological change need to be more volatile to match the evolution of the moments of interest. Compared to the remaining experiments, experiment (Exp6), where markups are heterogeneous and constant, is the experiment in which higher volatility is needed.

Figures 1.24 - 1.26 show the evolution of the share of capital in manufacturing, the share of labor in manufacturing, and the capital-output ratio in manufacturing. Although there are no significant differences across the different specifications for the shares of capital in manufacturing and services, the exercises where markups are homogeneous – either zero or positive but constant over time – fail in delivering the correct level of capital allocated to manufacturing. As there are no market power differences across sectors, the equilibrium allocation of capital in manufacturing is more significant than in the other experiments. Consequently, the resulting capital-output ratio in manufacturing is also higher than its counterpart in the data.

Consequently, all the results point towards the need for heterogeneity in both technological change and markups across sectors to explain the observed declines in the labor share and the strong relative reallocation of labor from manufacturing to services. Through the lens of the model, markups are the main driver of the decline in the labor share in manufacturing and services industries, while technological change is the main driver of structural change.

1.6.6 Robustness

The elasticities of substitution between capital and labor in manufacturing and services are crucial parameters of the model. These elasticities play a fundamental role in determining the evolution of the capital–labor ratio of each industry and, ultimately, its labor income share. Given the wide spectrum of available estimates in the literature, I repeat the baseline exercise considering different values for these elasticities. In particular, I consider the following alternative estimates:

1. ‘High σ ’, where I take the estimate $\sigma = 1.25$ from [Karabarbounis and Neiman \(2014\)](#) and assume is the same for both industries so that $\sigma_m = \sigma_s = 1.25$. This implies that capital and labor are slightly substitutes in the production of manufacturing and services goods,
2. ‘Low σ ’, where I take the estimate $\sigma = 0.406$ from [Chirinko and Mallick \(2017\)](#) and assume is the same for both industries so that $\sigma_m = \sigma_s = 0.406$. This exercise

reinforces the complementarity of capital and labor relative to the baseline exercise.

Table 1.3 summarizes each channel's contribution to the decline in the labor share, both at the aggregate and industry level. I find that the baseline exercise's main result is preserved under different specifications of the capital-labor elasticity. In other words, the increase in markups is the main reason underlying the decline in the overall labor share of the U.S. As Figure 1.27 depicts, the level of markups recovered declines as the elasticity of substitution increases. However, the upward trend is remarkably consistent across specifications.

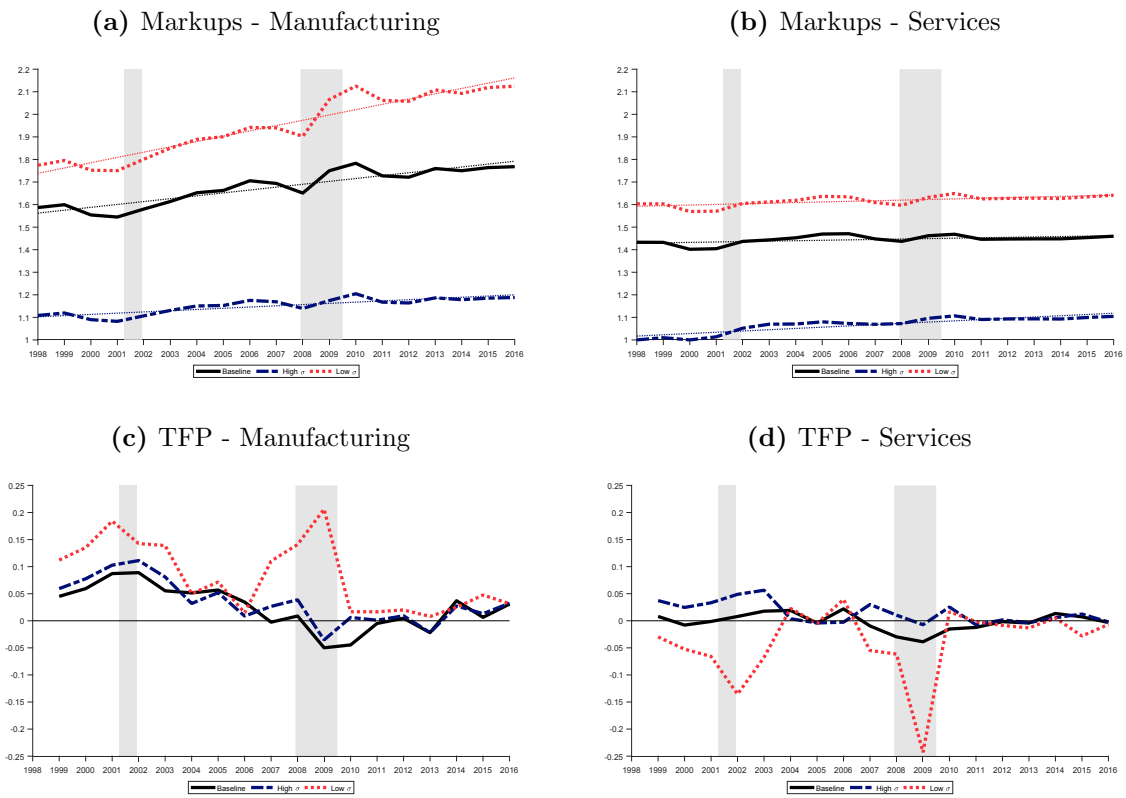
Table 1.3: Robustness: Contribution to the decline in the labor share: markups vs. technological change

	s^L	s_M^L	s_S^L
<i>Baseline</i>			
Both	-3.01	-11.26	-1.89
Market Power	-2.21	-6.93	-0.72
Technological Change	-0.62	-4.35	-0.45
<i>High σ</i>			
Both	-2.99	-11.26	-1.87
Market Power	-6.94	-6.06	-7.19
Technological Change	2.89	-7.07	4.29
<i>Low σ</i>			
Both	-3.00	-11.26	-1.89
Market Power	-4.19	-11.26	-1.89
Technological Change	0.43	0.00	0.00

Note: Non-recalibrated decomposition. 'Both' includes technological change and market power. 'Market Power' and 'Technological Change' values are obtained by allowing technological change and fixing markups at its 1998 level, or fixing technological change and changing markups.

Finally, Figure 1.27 shows the evolution of markups and implied TFP from the evolution of technological change.

Figure 1.27: Evolution of markups and industry total factor productivity (TFP) for different specifications of the elasticity of substitution between capital and labor, 1998 - 2016.



Chapter 2

Provider-driven complementarity and firm dynamics

2.1 Introduction

The recent debate on increasing firm concentration and profits highlights some troubling facts regarding declining business dynamism and competition in the United States. This literature documents that nowadays markets are more concentrated and less competitive than they were decades ago.¹ Adding to this literature, this paper reviews a series of trends on declining business dynamism using data from Compustat, Business Dynamics Statistics, and World Development Indicators. These trends are as follows:

1. The entry rate of new firms has declined.
2. Market concentration, measured by the share of sales accruing to the biggest firms, has increased.
3. Expenditure on R&D activities, measured both as a fraction of total cost or total sales, has increased.
4. Productivity growth has slowed down.

Taken together, while the entry rate of new firms has been declining, the so-called ‘superstar’ firms have become bigger and more profitable.² This in turn has raised concerns

¹See, among others, [Autor et al. \(2020\)](#), or [Akcigit and Ates \(2019a\)](#) for the first feature, and ([Duernecker et al., 2019](#)), ([De Loecker, Eeckhout and Unger, 2020b](#)), ([Shambaugh, Nunn, Breitwieser and Liu, 2018](#)) or ([Feijoo-Moreira, 2020](#)) for the second.

²In line with [Autor et al. \(2020\)](#), the term ‘superstar’ refers to the most productive firms in an industry.

regarding dominant firms crowding out new entrants and reducing entrepreneurship. At the same time, economic growth has been sluggish during the last decades,³ though R&D efforts have increased substantially.

In this paper, I propose a theoretical framework that explains increasing R&D expenditures and concentration yet decreasing entry rates and economic growth. The key novelty of the model is introducing provider-driven complementarities into an otherwise standard quality ladder model. Provider-driven complementarity makes initially independent products become complements when provided by a single firm. It boils down to the idea that during the process of product innovation – the introduction of new and improved products to the market – firms can incorporate differential characteristics to their products. These firm-specific characteristics, which can be associated with the brand, software, or product design, are such that, absent quality differences across products, consuming several goods from a single provider is preferable to purchasing each good from a different firm.

Theoretically, I build on the [Akcigit and Kerr \(2018\)](#) model of endogenous growth through R&D. The economy is formed by a representative household and an endogenous measure of firms. Each firm owns a product portfolio that supplies monopolistically to the market. All firms have access to the same production technology, and product quality grows on a ladder through stochastic quality arrivals arising from investment in R&D. Firms' R&D is of two types: internal (improve the quality of a product within its portfolio) and external (improve the quality of a product outside its portfolio). I model provider-driven complementarity as a demand shifter embedded in the production process, increasing in the number of (different) products supplied by the same firm.⁴ Therefore, upon entering the economy, any firm is ex-ante able to generate the same complementarity level. In other words, there exist only two sources of heterogeneity across firms that evolve endogenously as a result of innovation: the number of products in their portfolio and the quality of each product.

In a standard quality-ladder model, successful R&D improving the quality of a variety enables the innovator to price-out a lower-quality incumbent. However, when firms generate provider-driven complementarity, consumers do not necessarily switch to the state-of-the-art highest quality product. Instead, they may remain attached to the lower-quality incumbent if the provider-driven complementarity derived from this firm is sufficiently large. In equilibrium, each market is supplied by its market leader: the firm able to offer the highest quality, adjusted by provider-driven complementarity, relative to its market

³See ([Duernecker et al., 2019](#)), or ([Gordon, 2018](#)) for some examples on growth slowdown.

⁴This is equivalent to assuming that product complementarities are inherent to consumers' preferences and independent of the production technology of firms.

price. This bears an important effect on R&D decisions: when there exists provider-driven complementarity, the size of the quality improvement that an innovator requires to become a market leader depends not only on the size of its product portfolio but also on the size of the product portfolio of the incumbent. In particular, the probability of obtaining an innovation that allows replacing an incumbent – labeled a successful innovation – is a function of the product portfolio size. Specifically, firms with large portfolios are ex-ante more likely to obtain successful innovations than smaller ones. Put differently; smaller firms need to obtain larger quality innovations than bigger firms to offset the provider-driven complementarity effect of a given incumbent. Therefore, firms conduct R&D for two reasons: i) it allows increasing their market share by selling higher quality goods and/or capturing more markets, and ii) it increases the provider-driven complementarity effect as firms increase their product portfolio. As a result, firms' R&D decisions affect the industrial organization of firms and can ultimately deter firm entry, because provider-driven complementarity generates an endogenous barrier to entry in new markets. In other words, a key novelty of the provider-driven complementarity framework is that the equilibrium distribution of products across firms affects firms' R&D decisions, which in turn affect aggregate variables.

I use the theory of provider-driven complementarity to perform a quantitative exercise in which I reduce the size of the average quality jump stemming from any successful innovation. This exercise is motivated by the recent literature on ideas becoming harder to find, in the spirit of [Bloom, Jones, Van Reenen and Webb \(2020\)](#), and can also be thought of as innovations becoming less radical over time.⁵ The reduction in the average innovation step size mechanically generates a slowdown in the economy's growth rate. Most importantly, it introduces rich dynamics in the R&D decisions of firms when they generate provider-driven complementarity. By targeting the decline in the U.S. growth rate, I show that there is less entry while incumbents become bigger and spend more resources on R&D, even as the economy's overall growth rate declines. This contrasts with the predictions of a standard quality ladder model without provider-driven complementarities, which implies the reverse.

The interaction between innovative step size and provider-driven complementarity in determining which firm supplies each product in equilibrium is key to the previous results. When the average innovative step size declines, the probability of obtaining a successful innovation also does. Moreover, this has a direct implication on the rate of creative

⁵The observation of slowing technical change goes back to the mid-1960s and 1970s, as does the idea of 'exhaustion of inventive opportunities', see [Griliches \(1994\)](#) for a review.

destruction. As it turns out, this rate – which is a decreasing function of a firm’s number of products – declines as innovators find it more difficult to come up with successful innovations. Therefore, all else equal, small firms – and mainly potential entrants – find it more difficult to become market leaders. In particular, more quality innovations that would be successful in the absence of provider-driven complementarity do not find their way into the markets. The decline in the probability of obtaining a successful innovation can be broken down into two components. The first one is mechanical: reducing the average step size innovation makes firms less likely to obtain successful innovations. The second component is the change in the distribution of firms that affects firms’ incentives to conduct R&D. As a result, firms’ industrial organization matters for equilibrium outcomes if firms generate provider-driven complementarity, a novel and crucial feature of this framework.

Additionally, the reduction in the step size of innovation affects incumbents and potential entrants asymmetrically in the provider-driven complementarity framework. This results from the interaction between two forces: the *market effect* and the *quality effect*. The *market effect* captures the increase in the value of the discounted stream of profits associated with being a market leader, which enhances the incentives to conduct R&D. In equilibrium, the interest rate and the rates of creative destruction decline, and so does the effective discount rate of firms profits. The *quality effect* captures the decline in the productivity of investing in quality. When the step size of innovation declines, firms find it more difficult to come up with successful innovations and obtain smaller quality improvements if successful. Both effects decrease the incentives to conduct R&D. I show that when the step size of innovation declines, incumbents conduct less internal R&D (the *quality effect* dominates the *market effect*) and more external R&D (the *market effect* dominates the *quality effect*).

As a consequence, this leads to an overall increase in the R&D expenditure of incumbent firms. However, potential entrants – that only conduct external R&D and do not generate complementarity upon entry – conduct less R&D due to the decline in the probability of obtaining a successful innovation (the *quality effect* dominates the *market effect*). This decline drives down the entrants’ innovation rate, which reduces the entry rate of new firms. The joint effect of the decline in entry and the increase in external R&D innovation rates of incumbents is a reduction in the number of active incumbents in equilibrium. Accordingly, the equilibrium firm size distribution shifts to the right as a substantial share of firms become bigger. In turn, this yields an increase in the concentration of sales.

Literature review. This paper relates to two strands of the economic literature. First and foremost, to the recent and growing literature on declining business dynamism starting with (Decker, Haltiwanger, Jarmin and Miranda, 2016). There are many contributions to this literature that offer various explanations for increased market concentration and declining business dynamism. Aghion, Bergeaud, Boppart, Klenow and Li (2019) investigate whether falling firm-level costs of spanning multiple markets due to accelerating IT advances can explain the trends observed in the data. The authors show that as the cost of spanning into multiple markets declines, the most efficient firms expand into new markets while less efficient firms find it more difficult to enter profitably, innovating less. Moreover, while a temporary surge of growth occurs in the short run, in the long run, innovation and productivity growth decline as both high and low productivity firms' incentives to conduct R&D are hampered because incumbent firms tend to be high productivity firms. Akcigit and Ates (2019b) analyze a series of margins that shape competition dynamics and ultimately show that a decline in the intensity of knowledge diffusion from frontier firms to laggards can explain rising market concentration and a slowdown in business dynamism. Liu, Mian and Sufi (2020) find that low interest rates can explain rising concentration and declining dynamism by encouraging investment for industry leaders relative to its followers. Cavenaile, Celik and Tian (2020) find that the increase in markups is mainly driven by a decrease in competition from small firms, but among large firms. The authors show that markups have actually contributed to productivity growth, and estimate that the increase in the cost of innovation is the fundamental reason underlying the productivity slowdown. Peters and Walsh (2019), Hopenhayn, Neira and Singhania (2018), Engbom (2020), Bornstein (2018), and Röhe and Stähler (2020) provide different explanations that relate aging and declining population growth with declining entry rates of new firms and increased concentration. Closest to this paper are De Ridder (2020) and Olmstead-Rumsey (2020). On the one hand, De Ridder (2020) shows that the trends in productivity growth, R&D expenditure, and business dynamism can be explained by an increase in firms' use of intangible inputs. On the other hand, Olmstead-Rumsey (2020) shows that average patent quality has fallen over the same period and can explain the aggregate productivity growth slowdown and market concentration increase. I contribute to this literature by offering an additional explanation based on the role of provider-driven complementarity, a simple mechanism that shifts demand as a function of firm size. Contrary to these two papers, the model I propose does not rely on ex-ante firm heterogeneity. However, the model can successfully explain many empirical findings on business dynamism – e.g., the joint observation of increasing R&D

effort and decreasing economic growth – through the asymmetries that provider-driven complementarity generates between small and big firms.

Second, this paper is tightly related to the literature on (quality ladder) endogenous growth models. The main contributions to this literature are the seminal papers of [Romer \(1990\)](#), [Grossman and Helpman \(1991\)](#), [Aghion and Howitt \(1992\)](#), and the Schumpeterian growth models developed in [Klette and Kortum \(2004\)](#), [Lentz and Mortensen \(2008\)](#), and [Acemoglu and Cao \(2015\)](#) where a single firm operates each product line as a monopolist. My model builds upon the recent and influential [Akcigit and Kerr \(2018\)](#) framework. Although these models usually feature an endogenous distribution of products across firms, this distribution is a crucial equilibrium element in my theory of provider-driven complementarity. Specifically, firms' industrial organization is a determinant not only of individual firm decisions, but also for aggregate outcomes. A key novelty of the provider-driven complementarity framework is that the probability of obtaining a successful innovation that allows entering a market, and the rate of creative destruction experienced by an incumbent, crucially depend on the number of products that each firm supplies to the market. Ultimately, this implies that the distribution of products across firms is not a residual equilibrium outcome like in most growth models, but a key element that affects firms' optimal R&D decisions. Although this substantially complicates the model's solution, the modeling choice of provider-driven complementarity remains tractable and allows obtaining analytical solutions.

Layout. The rest of this paper is organized as follows. In [Section 2.2](#), I briefly review some recent key empirical findings on firm dynamics in the U.S. In [Section 2.3](#), I develop a novel theory of provider-driven complementarity. In [Section 2.4](#), I carry out my main quantitative experiment and discuss the relevance of the asymmetries generated by provider-driven complementarity for the results. I additionally provide a sensitivity analysis of the results. In [Section 2.5](#), I conclude.

2.2 Business dynamism, concentration, R&D expenditure, and provider-driven complementarity

In this Section, I briefly discuss the empirical trends that motivate this paper. The analysis is based on several facts that have already been documented in the literature. The empirical trends I show use data from Compustat, Business Dynamics Statistics, and

World Development Indicators.⁶ I also provide suggestive evidence of the relevance of provider-driven complementarity in shaping consumer demand.

Fact # 1: Declining entry rate. I start by reviewing a well-known and widely documented fact about declining business dynamism in the United States. Figure 2.1 shows the firm entry rate using data from the BDS. Since 1985, the firm entry has contracted in around 1/3, declining by 5 p.p. As Decker et al. (2016) and Akcigit and Ates (2019a) show, the same trend holds for establishments. This decline in the entry rate is also related to the long-run decline in the startup rate Karahan, Pugsley and Şahin (2019).

Figure 2.1: Firm entry rate

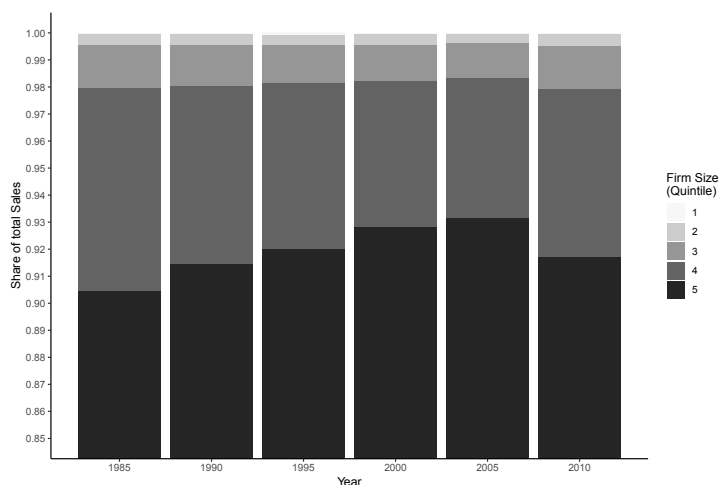


Source: Author's calculations using Business Dynamics Statistics data.

Fact # 2: Increased firm concentration. An additional well documented fact is the contemporaneous increase in the concentration of businesses (see, among others, (Akcigit and Ates, 2019b), (De Loecker et al., 2020b), (Grullon, Larkin and Michaely, 2018), (Van Reenen, 2018), (Aghion et al., 2019), or (Helpman and Niswonger, 2020)). Figure 2.2 shows the five-year average share of sales of each quintile of firms in Compustat. From 1985 to 2010, the share of sales held by the 20% biggest firms in the U.S. economy increased from roughly 90% of the total sales between 1985 and 1990 to 93% between 2005 and 2019. From 2010 to 2015, this share has declined slightly in the aftermath of the Great Depression.

Fact # 3: Increased R&D expenditure. The endogenous growth literature stresses innovation – typically the result of successful R&D activities – as a key driver of aggregate

⁶Specific details about the data can be found in Appendix 2.6.1.

Figure 2.2: Cumulative share of sales by firm quintile

Source: Author's calculations using Compustat data.

growth. Figure 2.3 depicts the total expenditure in R&D as a share of total cost⁷ or total sales for firms in Compustat. During the last decades, R&D expenditure has roughly doubled relative to firms' total cost and total sales.⁸

Figure 2.3: R&D expenditure as a fraction of Total Sales and Total Cost

Source: Author's calculations using Compustat data

⁷In the data, the total cost of firms is Operating Expenditure (OPEX), the sum of Cost of Goods Sold (COGS), and Selling, General, and Administrative Expenses (SGA).

⁸The increase in R&D can also be observed by using aggregate data from the OECD. For example, the share of Gross domestic expenditure on R&D has also increased during the last decades. The interested reader can find additional details in Appendix 2.6.1.

Fact # 4: Growth slowdown. Through the lens of the standard growth literature, the apparent increase in R&D effort would lead to stronger aggregate growth. However, the last decades have been characterized by stagnant or even declining growth rates.⁹ Figure 2.4 shows the slowdown of GDPpc growth. While in the mid-1980s, GDPpc growth averaged 2.5%, this rate has declined to 1.5% in the mid-2010s.

Figure 2.4: U.S. Real GDPpc Growth



Source: World Development Indicators.

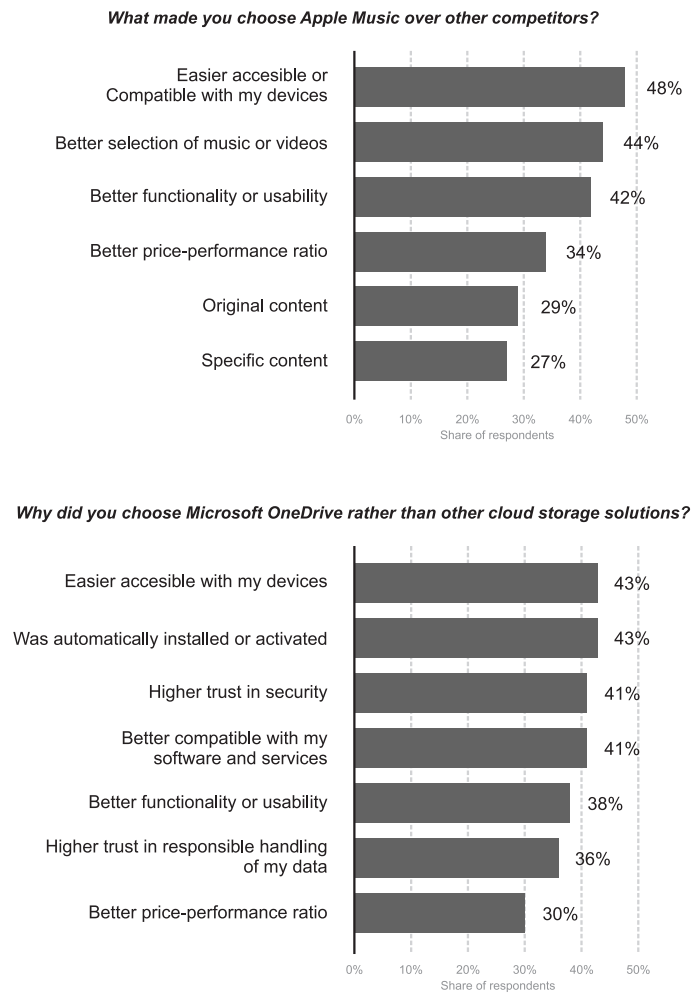
Provider-driven complementarity. Figure 2.5, shows the responses to two questions regarding the consumption decisions of two well-known internet-based services: Apple Music and Microsoft OneDrive.¹⁰ Both questions try to address what is the most relevant aspect shaping consumers’ consumption decisions. As it turns out, both services are designed in a way that are easier to access/are better compatible (read-as more complementary) with other goods or services provided by the same firm. More strikingly, this is a more important factor in shaping demand than the price-performance (read-as price-to-quality) ratio.

Based on this suggestive evidence, in this paper, I model the fact that consumers value more products when the same firm offers them. This novel mechanism, labeled provider-driven complementarity, is defined as firms’ ability to transform seemingly independent products into complements when provided by a single firm. Ultimately, provider-driven complementarity is a firm-specific characteristic that is incorporated into its products during the process of product creation and innovation. The next Section develops a

⁹See, for example, Duernecker et al. (2019), Gordon (2018) or Gordon and Sayed (2018).

¹⁰To the best of my knowledge, extensive consumer data that allows matching consumers’ demand with the different products supplied by a firm is not readily available

Figure 2.5: Consumption decisions for internet-based services in the U.S.



Note: Survey conducted in the United States from October 26 to November 5, 2018. Respondents aged 18-69. Source: Statista Global Consumer Survey

theoretical framework that introduces provider-driver complementarity into an otherwise standard quality ladder model.

2.3 A theory of provider-driven complementarity

In this Section, I describe an endogenous growth model where firms can generate provider-driven complementarities. The model builds upon the quality ladder growth model introduced by [Akcigit and Kerr \(2018\)](#). To clarify and facilitate the exposition and analysis of the effects of provider-driven complementarity, I characterize firms' decisions and equi-

librium outcomes in two steps. First, I consider a simplified version of the model where the quality jump obtained after a successful innovation can always offset the effects of provider-driven complementarity, i.e., the latter does not play a role in determining the market leader of each variety. This simplified version shows some key insights into how provider-driven complementarity affects the optimal decisions of firms. I then consider a generalized version of the model where quality and provider-driven complementarity interact and jointly determine each variety's market leader.

2.3.1 Environment

Time is continuous and infinite. The economy comprises a representative household, a final good producer, and an endogenous measure of (multiproduct) firms that produce a continuum of intermediate goods indexed by $j \in [0, 1]$. In what follows, I describe each agent in detail.

Household

There is a representative household with preferences represented by the logarithmic utility function

$$U_t = \int_0^\infty e^{-\rho t} \ln C_t dt,$$

where C_t denotes consumption of the final good of the economy. At any instant, the household is endowed with one unit of labor that supplies inelastically and receives as counterpart the wage rate w_t . The household is also the owner of all firms in the economy, and thus its wealth A_t is simply the aggregate value of all these firms. Denoting by r_t the continuous rate of return on wealth, one can write the household flow budget constraint as

$$\dot{A}_t = w_t + r_t A_t - C_t.$$

The solution to the maximization problem of the household yields the common Euler equation

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{2.1}$$

Final goods

There is a unique all-purpose final good that can be used both for consumption and R&D. This final good is produced by aggregating a continuum of intermediate varieties $j \in [0, 1]$. At any instant, there is an endogenously determined set \mathcal{F} (of measure $F > 0$) of active

incumbent intermediate producers indexed by $i \in \mathcal{F}$, who operate in a monopolistically competitive product market. For each variety $j \in [0, 1]$ there exist different vintages characterized by q_{ijt} , the quality of variety j that firm i produces at time t . The quantity of variety j (produced by firm i) demanded in the final good production is denoted by k_{ijt} . The final good production function is given by

$$Y_t = \frac{L_t^\beta}{1 - \beta} \left[\int_0^1 \left(\sum_{i \in \mathcal{F}} \left[(m_{it} q_{ijt})^{\frac{1}{\varepsilon-1}} k_{ijt} \right]^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1} \frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon(1-\beta)}{\varepsilon-1}}, \quad (2.2)$$

where $0 < \beta < 1$, $\varepsilon \geq 0$ is the elasticity of substitution across different varieties, and $\theta \geq 0$ is the elasticity of substitution across different vintages of a given variety. The only non-standard feature in (2.2) is $m_{it} \geq 1$ which denotes the provider-driven complementarity effect derived by demanding goods produced by firm i . Before discussing this novel argument in detail, I impose two parametric restrictions regarding the parameters driving the elasticities of substitution between varieties and different vintages within a variety, θ and ε respectively. First, I assume that the different vintages of a given variety are perfect substitutes once adjusted by quality and provider-driven complementarity, i.e., $\theta = \infty$. Second, for tractability I impose $\varepsilon = \beta^{-1}$.

The idea behind provider-driven complementarity is intuitive and straightforward. Provider-driven complementarity is a mechanism that turns goods into ‘complements’ when acquired from the same firm. As an example, consider that two firms were able to produce the same good j at the same quality level q_{jt} . In that case, according to (2.2) in the absence of price differences between both firms, it would be optimal to demand good j from the firm i that gives the highest complementarity. I assume that each firm’s level of provider-driven complementarity is only a function of the number of products that it supplies to the market. Formally

$$m_{it} = \gamma^{1 - \frac{1}{n_{it}}}, \quad (2.3)$$

where $\gamma \geq 1$ is constant across firms and n_{it} denotes the number of varieties currently provided by firm i . Coming back to the previous example, in the absence of quality differences, (2.2) will demand good j from the firm with higher n among all firms able to produce j at some given quality q_{jt} . A direct implication of this functional form is that firms’ industrial organization, i.e., the equilibrium distribution of the number of goods across firms, matters for the economy’s aggregate production.

I assume that intermediate firms compete à la Bertrand. This would naturally lead to the standard limit pricing framework where the firm that can offer the highest quality, adjusted by provider-driven complementarity, relative to its market price – read as the

most productive firm – is limited by the second most productive firm.¹¹ However, to keep the model simple and avoid the case of limit pricing, I follow [Akcigit and Kerr \(2018\)](#) and [Garcia-Maza, Hsieh and Klenow \(2019\)](#) and adopt the following stage-game assumption.¹²

Assumption 1 (Monopoly pricing). *Intermediate firms enter a two-stage price-bidding game when setting prices. In the first stage, every firm pays a fixed fee $\epsilon > 0$ to be able to post a price. In the second stage, prices are revealed.*

Assumption 1 ensures that in the Nash equilibrium of this game, only the firm that can offer the highest provider-adjusted quality to price ratio

$$\frac{m_{it}q_{ijt}}{p_{ijt}} \quad (2.4)$$

per unit of expenditure in variety j pays the fee and enters the second stage, as any other firm can never recover the fee in the second stage. As that firm is the only firm bidding a price, each variety is produced by precisely one firm, which will always operate as a monopoly. I henceforth refer to such a firm as the market leader of variety j . Consequently, the size of each firm n_{it} ultimately denotes the number of markets in which firm i is the market leader. For simplicity, as in equilibrium, only one firm is active in each market, I re-define each firm's provider-driven complementarity effect by m_{jt} where its dependence on j refers to the market leader of that variety.

Taking the intermediate goods' prices as given, the representative final good producer chooses the quantity of each intermediate good k_{jt} , $j \in [0, 1]$ to maximize its profits. Normalizing $P_t^Y = 1$, $\forall t$, dropping time subscripts (to ease notation), and acknowledging that there is only one firm supplying each variety, the problem reads as

$$\begin{aligned} \max_{k_j, L} \quad & Y - \int_0^1 p_j k_j dj - wL \\ \text{s.t.} \quad & \frac{L^\beta}{1 - \beta} \int_0^1 [m_j q_j]^\beta k_j^{1-\beta}, \end{aligned}$$

which yields the well-known inverse demand function

$$p_j = L^\beta [m_j q_j]^\beta k_j^{-\beta}, \quad (2.5)$$

for any variety j .

¹¹I henceforth refer to quality, adjusted by provider-driven complementarity, relative to its market price as provider-adjusted quality to price ratio

¹²In Appendix 2.6.3 I discuss the implications of allowing for limit-pricing in a similar way as in ([Peters, 2018](#)).

Intermediate producers

Intermediate producers are characterized by

- the set $\mathcal{J}_i = \{j : j \text{ is owned by firm } i\}$, with cardinality $n \in \mathbb{Z}_+$, the # of varieties in which i is the outstanding market leader,
- the multiset $\mathbf{q}_i = \{q_j : j \in \mathcal{J}_i\} \in \mathbb{R}_+^n$, which collects the quality level of each variety $j \in \mathcal{J}_i$.

Each intermediate variety is produced with the production technology $k_j = \bar{q}l_j$, where

$$\bar{q} = \int_0^1 q_j \, dj$$

denotes the average quality level of the economy. Under Assumption 1, in equilibrium each intermediate is produced by a single firm – its market leader – which acts as a monopolist. Therefore, each monopolist chooses labor input (and thus quality supplied), its price, and R&D expenditure in order to maximize its profits. I focus first on the intra-temporal labor-quality-price decision of the monopolist. Taking as given (2.5), each monopolist solves

$$\begin{aligned} \max_{p_j, k_j} \quad & p_j k_j - w l_j \\ \text{s.t.} \quad & k_j = \bar{q} l_j, \\ & p_j = L^\beta [m_j q_j]^\beta k_j^{-\beta}, \end{aligned}$$

which yields the optimal quantity supplied

$$k_j = \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1}{\beta}} L m_j q_j, \quad (2.6)$$

at the price

$$p_j = \frac{1}{1-\beta} \frac{w}{\bar{q}}. \quad (2.7)$$

Note that all monopolists set the same price – a direct consequence of Assumption 1 – but sell different quantities in equilibrium. Intermediate firms' profits in market j are then given by

$$\pi_j = \beta(1-\beta)^{\frac{1-\beta}{\beta}} \left(\frac{\bar{q}}{w} \right)^{\frac{1-\beta}{\beta}} L m_j q_j, \quad (2.8)$$

for w to be determined.¹³ This implies that differences in profits across firms are given by differences in quality, size (number of products), or both.

Innovation

Incumbent firms can invest in R&D activities to improve the quality of their product portfolio by conducting internal R&D. Moreover, incumbent firms can expand their product portfolio by conducting external R&D. Potential entrants can invest in external R&D to enter and become the leaders of a variety. Figure 2.6 shows the evolution of quality under each type of innovation. The quality of each variety is represented by the vertical bars, and the quality jump obtained after each type of successful innovation is represented by the black bars on top of each variety that is innovated upon. I assume that innovation allows firms to obtain a perpetual patent on a variety-quality pair, i.e., no other firm can supply the same variety at the same level of quality. Given the assumptions made on competition, a successful innovator can become the market leader of a variety and prevent any other firm from supplying the same variety-quality pair to the market. However, the quality of that variety becomes the state-of-the-art or frontier quality upon which other firms can innovate.¹⁴

Internal innovation I start by focusing on an incumbent firm's decision to improve the quality of one of the goods it supplies to the market. To do that, the firm needs to conduct internal R&D. I assume that internal R&D is directed, i.e., the firm can specifically select in which good it will invest to increase its quality. To create a Poisson flow rate $z_{nj} \geq 0$ of improving product j quality, the firm must incur the flow cost

$$C_z(z_{nj}, q_j) = \hat{\chi} z_{nj}^{\hat{\psi}} q_j,$$

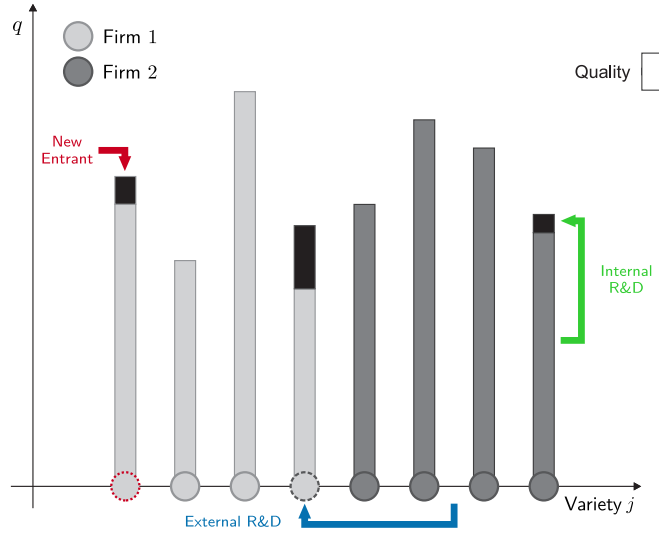
in terms of the final good of the economy, where $\hat{\chi} > 0$ is a scale parameter and $\hat{\psi} > 1$ implies that the cost function is convex in z_{nj} . If R&D is successful, the firm is able

¹³Anticipating a later result, each monopolist per-variety profit is given by

$$\pi_j = \underbrace{\beta^\beta (1 - \beta)^{2-2\beta} L \left(\frac{\bar{q}}{\bar{Q}} \right)^{1-\beta}}_{\pi} m_j q_j.$$

Profits thus depend on a constant term across firms π , which increases in \bar{q} and decreases in \bar{Q} , the average provider-adjusted quality of the economy, to be defined later.

¹⁴Patents can therefore be considered a protective tool that firms use to pre-empt competitors from entering their product market. As a consequence, by construction, this model exhibits a tight relationship between patents and innovation. However, previous work shows that the relationship between patents and innovation is complex. See for example [Argente, Baslandze, Hanley and Moreira \(2020\)](#).

Figure 2.6: Innovation types. Cf. (Akcigit and Kerr, 2018, Figure 5).

produce good j with incremental quality $q_{jt+\Delta t} = q_{jt}(1 + \lambda_z) \equiv \Lambda_z$, where $\lambda_z > 0$ is assumed to be constant.

External innovation An incumbent firm can also perform external R&D to improve the quality of a product outside its quality portfolio, and become the market leader of that good. I assume that external R&D is undirected. An incumbent firm with product portfolio size $n \geq 1$ chooses the Poisson flow rate $X_n \geq 0$ with which it improves the quality of an already existing product not currently owned. For a given flow cost in external R&D C_x , the flow rate of external innovations is given by the innovation function

$$X_n = \chi(C_x/\bar{q})^\psi n^\sigma,$$

which delivers the R&D flow cost function

$$C_x(x_n, n, \bar{q}) = \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q}, \quad (2.9)$$

in terms of the final good, where $x_n = X_n/n$ is the per-product flow rate of external R&D and $\tilde{\chi} = \chi^{-\frac{1}{\psi}} > 0$ is a scale parameter, $\tilde{\psi} = 1/\psi$ and $\tilde{\sigma} = (1 - \sigma)/\psi$. The relationship between $\psi > 0$ and $\sigma > 0$ determines the returns to scale of the innovation function. In particular, if $\psi + \sigma = 1$ it exhibits constant returns to scale as in (Klette and Kortum, 2004), while if $\psi + \sigma < 1$ it exhibits decreasing returns to scale as in (Akcigit and Kerr, 2018). If R&D is successful, a firm can produce any randomly drawn product (not previously owned) with incremental quality $q_{jt+\Delta t} = q_{jt}(1 + \tilde{\lambda}_x) \equiv q_{jt}\Lambda_x$, with $\tilde{\lambda}_x > 0$ drawn from an exponential distribution with parameter λ_x^{-1} . Unlike most previous work

in the literature, in this environment a firm that innovates and improves the quality of a product does not necessarily become its new producer.¹⁵ The innovator will only become the new market leader if it can offer the highest provider-adjusted quality to price ratio. As a consequence, firms with larger product portfolios, and firms receiving higher quality improvements, are more likely to become becoming the market leader.

In what follows, I describe how quality improvements interact with provider-driven complementarity in determining the market leader of each variety. Let I denote a firm obtaining a quality improvement in some product line, and let L denote the current market leader in that product line. Moreover, let m_I and m_L denote the provider-driven complementarity effect generated by the innovating firm and the current market leader, respectively. I assume that the economy follows the behavioral through which the innovating firm I (which could either be a brand-new entrant or an incumbent with outstanding market leadership in at least one variety) will become the new producer of a variety if it can price-out the incumbent by offering a higher provider-adjusted quality to price ratio, i.e., if

$$\frac{m_I}{m_L} \Lambda_x > 1.$$

Equivalently,

$$\Lambda_x \geq \gamma^{\frac{1}{n_I+1} - \frac{1}{n_L}} \equiv \gamma^{\Delta(n_I+1, n_L)}. \quad (2.10)$$

This expression implies that the quality jump needed to price-out the incumbent must be sufficiently large to offset the provider-driven complementarity gap. Note that the left-hand side is always bigger than one, while the right-hand side may be bigger, equal, or smaller than 1 and is a function of the number of products currently produced by the incumbent and the innovator. Consequently, for a given quality jump Λ_x , firms already selling many different varieties find it easier to become market leaders than smaller firms. Put it differently; for a given size of an incumbent market leader, smaller firms need bigger quality jumps than bigger firms to become market leaders.

To provide further intuition, in Figure 2.7 I consider two possible situations where, without loss of generality, I consider an economy with two firms competing to supply eight different varieties. First, panels 2.7a and 2.7b depict a situation where both firms are initially supplying the same number of goods. Panel 2.7a shows that, under provider-driven complementarity, the provider-adjusted quality enjoyed by the consumer is composed of two parts: the variety-specific quality, represented as in the Baseline framework with a solid bar, and the provider-driven complementarity, represented by the four hatched bars for each variety. Now, suppose that Firm 2 is successful in conducting external R&D,

¹⁵For another recent example, see De Ridder (2020).

being now able to produce one of Firm 1 varieties with higher quality. Suppose the quality jump over Firm 1 in that variety is represented by the solid black bar in panel Panel 2.7b. Note that, as both firms have the same initial size, the quality jump is not key to determine the new market leader of that variety, because the consumer has a double incentive to switch providers for that variety: Firm 2 quality is higher, and it increases the provider-driven complementarity effect obtained by the consumer not only in that variety but also in all the remaining varieties supplied by Firm 2. This increase in provider-driven complementarity of Firm 2 is represented by the white hatched bar. Note also that the consumer enjoys less provider-driven complementarity in all the varieties still supplied by Firm 1 so that she would not be willing to pay the same price as before. This is a novel price-effect given by provider-driven complementarity. Firm 1 will necessarily re-optimize prices to keep maximizing its profits.

Panels 2.7c and 2.7d depict a situation where both firms are initially supplying a different number of varieties. As before, Panel 2.7c represents an initial situation where Firm 1 is supplying six varieties while Firm 2 is supplying the remaining two. Again, suppose that Firm 2 is successful in conducting external R&D, being now able to produce one of Firm 1 varieties with higher quality. Now the interaction between the quality jump and each firm's size becomes key to determine the new market leader. The black bar in Panel 2.7d represents the quality jump that would make the consumer indifferent between sticking with the lower quality version supplied by Firm 1 or switching to the brand-new better quality version offered by Firm 2. In other words, any quality jump bigger than the one represented in Panel 2.7d would make Firm 2 become the new market leader, while any quality jump smaller would allow Firm 1 to keep its market leadership.

As the quality increase is stochastic, the probability of condition (2.10) being satisfied is characterized by¹⁶

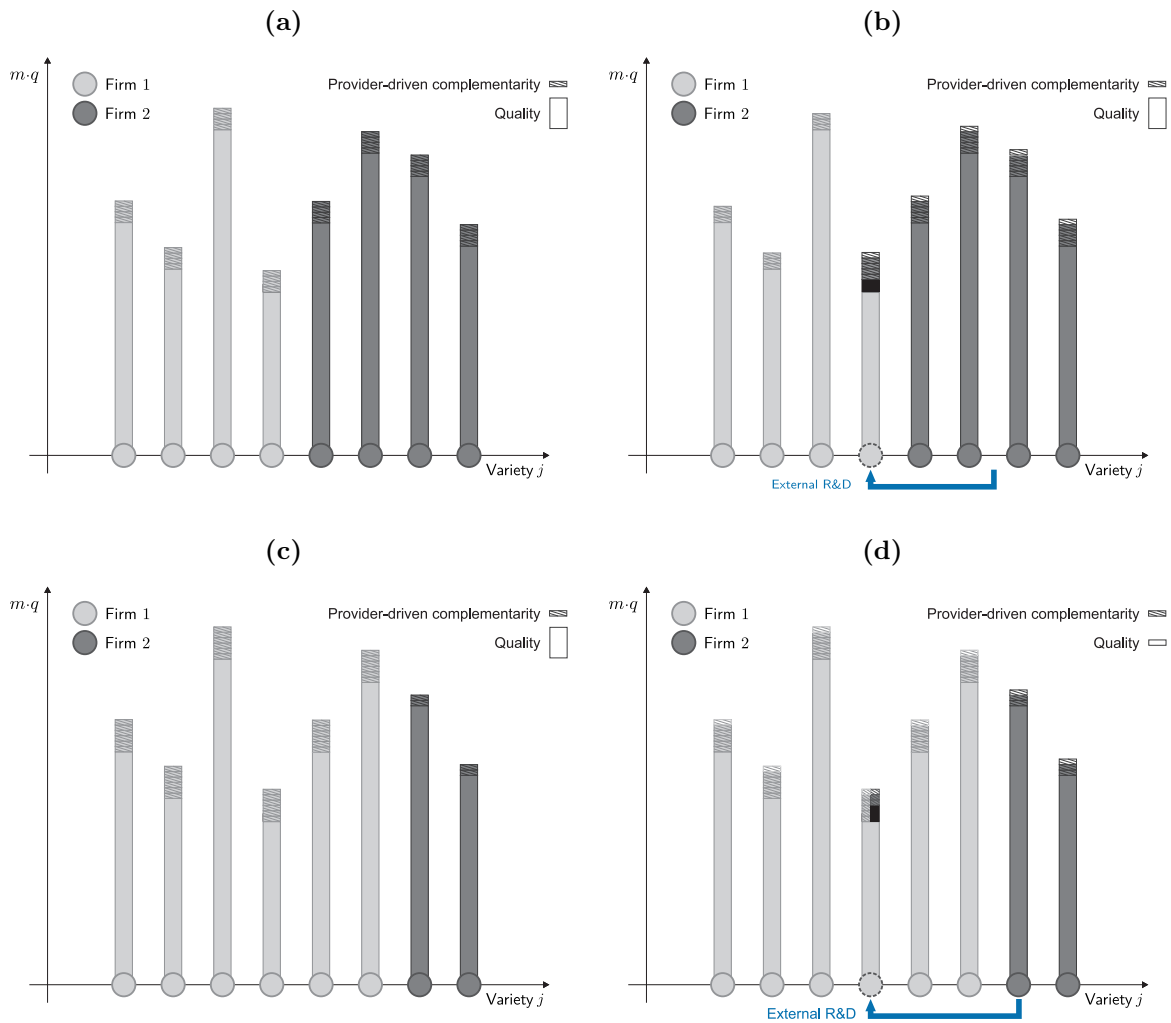
$$\Pr(\Lambda_x \geq \gamma^{\Delta(n_I+1, n_L)}) = \exp\left(-\frac{1}{\lambda_x} [\gamma^{\Delta(n_I+1, n_L)} - 1]\right). \quad (2.11)$$

Note that the fact that firms endogenously increase or decrease their size over time makes condition (2.10) dynamic. I assume that once a firm has been priced out of a product market, it is no longer a competitor in that product market.¹⁷ This assumption avoids the possibility of losing the market leadership of a product even if no other firm improves the quality of that product, but simply because a second firm that produces

¹⁶For an exponential distribution with parameter λ , $\Pr(X < x) = F_X(x; \lambda) = 1 - e^{-\lambda x}$, and thus $\Pr(X > x) = e^{-\lambda x}$.

¹⁷Recall that the incumbent is already behaving like a monopolist due to the two-stage price-bidding game.

Figure 2.7: External innovation under provider-driven complementarity.



this product has improved the quality of a different product becomes its market leader. I assume that this is also the case if an innovation is not successful in pricing out an incumbent. In other words, even if a firm can improve the quality of a product, that quality improvement is lost if it is not big enough to become the market leader. The underlying idea implies that: 1. Once a firm has been priced out of a market, it dismantles the production capacity for a product and can no longer supply it; 2. If a product is not sufficiently good to be demanded by consumers, a firm has to start a new R&D project from scratch (which in this context is the quality of the leader) and improve quality upon it.

Finally, I assume that incumbent firms incur adjustment costs when their size changes due to external innovation. This adjustment cost is assumed to be an increasing function

of the per-product external innovation rate x_n , the firm's aggregate quality, and its size. In other words, increasing the portfolio of a firm is costlier for firms with high-quality products. Formally, firms pay

$$C_a(x_n, n, \mathbf{q}_i) = x_n \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j,$$

with Ω_n (weakly) decreasing in n . This assumption serves two purposes. First, it allows obtaining closed-form solutions to the value function of the firm. Second, it ensures that there exists a balanced growth path.¹⁸

Entry As is standard in the literature, a mass of entrants invest in R&D in order to become intermediate producers of a variety. Entrants choose an innovation flow rate $x_e > 0$ with an R&D cost

$$C_e(x_e, \bar{q}) = \nu x_e \bar{q},$$

in terms of the final good, where $\nu > 0$.

Creative destruction Successful innovation by entrants and incumbent firms can cause other incumbent firms to lose production of the goods that were innovated upon. The rate at which this happens is the creative destruction rate, τ_n . As the probability of obtaining a successful innovation depends on the interplay between quality improvement and provider-driven complementarity, which is ultimately a function of size, the rates of creative destruction suffered by firms of different sizes will differ. These rates are endogenous and are determined from the optimal R&D decisions of firms. The per-product rate of creative destruction suffered by a firm with $n > 0$ products is

$$\tau_n = x_e \Pr(\Lambda_x \geq \gamma^{\Delta(1,n)}) + \sum_{s=1}^{\infty} F \mu_s X_s \Pr(\Lambda_x \geq \gamma^{\Delta(s+1,n)}).$$

The first term in this expression captures the creative destruction rate suffered by a firm of size n by new entrants in the economy. This is given by the entrant innovation intensity times the probability of the quality jump being sufficiently big to offset the provider-driven complementarity effect of a size n firm. Similarly, the summation captures the creative

¹⁸Without this assumption, the incentives to conduct external R&D not only depend on the size of a firm but also on its specific quality portfolio. That implies that all else equal, big high-quality firms have more incentives to innovate than big small-quality firms. The adjustment cost breaks the dependence of R&D on the quality portfolio of the firm. Specifically, it ensures that the incentive to conduct external R&D depends on the profits associated with becoming a market leader of a new product incorporated into the firms' product portfolio, as is standard in this literature.

destruction rate suffered as a consequence of the innovation activity of already existing firms producing $n \geq 1$ products, where μ_s is the invariant share of firms producing s products (to be determined in equilibrium).

Finally, an incumbent firm exits the economy if it loses the market leadership of its only product.

Aggregate variables

To conclude the exposition of the environment and before turning to the characterization of firms' dynamic decisions, it remains to derive some key aggregate variables. I start by deriving the equilibrium wage. Substituting the optimal intermediate quantity supplied (2.6) in the labor demand optimality condition of the final good producer's problem yields

$$w = \beta \frac{L^{\beta-1}}{1-\beta} \int_0^1 [m_j q_j]^\beta \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1-\beta}{\beta}} L^{1-\beta} [m_j q_j]^{1-\beta} dj,$$

which can be expressed as

$$w = \beta^\beta (1-\beta)^{1-2\beta} \bar{q}^{1-\beta} \bar{Q}^\beta, \quad (2.12)$$

where

$$\bar{Q} = \int_0^1 m_j q_j dj, \quad (2.13)$$

denotes the average provider-adjusted quality of the economy.¹⁹

Aggregate output can be obtained by substituting the optimal intermediate quantity supplied (2.6) and the equilibrium wage (2.12) into the final good production function (2.2), obtaining

$$Y = \beta^{\beta-1} (1-\beta)^{1-2\beta} L \bar{q}^{1-\beta} \bar{Q}^\beta. \quad (2.14)$$

The share of workers in the production of the final good can be obtained from the labor market clearing condition

$$L + L^k = 1,$$

where

$$L^k = \int_0^1 l_j dj = \int_0^1 \frac{k_j}{\bar{q}} dj.$$

Substituting the optimal intermediate quantity supplied (2.6) and the equilibrium wage (2.12) in the previous expression yields

$$L^k = \frac{(1-\beta)^2}{\beta} L.$$

¹⁹Note that $\bar{Q} \geq \bar{q}$ with equality if and only if $m_j = 1, \forall j$.

Therefore, from the labor market clearing condition one obtains that employment in the final good sector represents a constant share of total employment given by

$$L = \frac{\beta}{\beta + (1 - \beta)^2}. \quad (2.15)$$

Along a balanced growth path, aggregate variables will grow a constant rate g . As L is constant, from (2.14) it is clear that output will grow at the same rate as $\bar{q}^{1-\beta}\bar{Q}^\beta$. The rest of this Section characterizes the growth rate of these two components. First, I derive the growth rate of the average quality \bar{q} . Recovering the time subscript, and defining

$$z_t = \sum_{n=1}^{\infty} F\mu_n n z_n,$$

$$\tau_t = \sum_{n=1}^{\infty} F\mu_n n \tau_n,$$

the evolution of \bar{q}_t during any interval $[t, t + \Delta t]$ can be expressed as

$$\bar{q}_{t+\Delta t} = \bar{q}_t(1 + \lambda_x)\tau_t\Delta t + \bar{q}_t(1 + \lambda_z)z_t\Delta t + \bar{q}_t(1 - \tau_t - z_t)\Delta t + o(\Delta t),$$

i.e., in expectation the average quality of the economy can either increase by the factor λ_x due to creative destruction τ_t (which occurs for a fraction τ_t of products), or it can increase by the fixed factor λ_z due to internal innovation z_t (which occurs for a fraction z_t of products), or may remain unchanged (which occurs for a fraction $1 - \tau_t\Delta t - z_t\Delta t$ of products). Re-arranging the previous expression and taking limits as $\Delta t \rightarrow 0$ yields

$$g \equiv \lim_{\Delta t \rightarrow 0} \frac{\bar{q}_{t+\Delta t} - \bar{q}_t}{\Delta t} = \lambda_z z_t + \lambda_x \tau_t. \quad (2.16)$$

Finally, the next proposition establishes the growth rate of the average provider-adjusted quality \bar{Q} .

Proposition 2. *Along the balanced growth path*

$$\frac{\partial \ln \bar{Q}}{\partial t} = \frac{\partial \ln \bar{q}}{\partial t} = g.$$

Proof. Appendix 2.6.2. ■

The previous proposition anticipates an important equilibrium result. Along the balanced growth path, the equilibrium firm size distribution will be constant. Consequently, along the balanced growth path, provider-adjusted quality grows at the same rate as average quality, and so does aggregate output.

2.3.2 Equilibrium

In this subsection, I analyze the dynamic R&D decisions of firms. To facilitate the exposition, I divide the Subsection into two parts. The first part considers a simplified version of the model that restricts the analysis to external innovations with quality improvements that always offset provider-driven complementarity. This allows introducing notation, simplifies the characterization of firms' decisions and equilibrium outcomes, and helps obtain some key insights regarding how provider-driven complementarity affects firms' optimal decisions. The second part extends the analysis to the more general case where quality and provider-driven complementarity interact and jointly determine each variety's market leader.

Simplified framework

I start the analysis of the dynamic R&D decisions of firms under provider-driven complementarity by restricting to a simple case where external innovations improve the quality of a variety so that the effect of provider-driven complementarity is always offset. The following assumption ensures this result.

Assumption 2. *Upon external innovation, quality improves by a fixed step λ_x s.t. $\Lambda_x > \gamma$.*

The value γ is an upper bound for the provider-driven complementarity effect. Therefore Assumption 2 implies that after successful external R&D, a firm can produce any randomly drawn product (not previously owned) with incremental quality $q_{jt+\Delta t} = q_{jt}(1 + \lambda_x) \equiv q_{jt}\Lambda_x$, with $\lambda_x > 0$ being a constant. This is the only modification concerning the previous Subsection environment but is key for the results. The next proposition shows that under Assumption 2, provider-driven complementarity is not relevant in determining the market leader of a variety.

Proposition 3. *Under Assumption 2, a firm that is successful in conducting external R&D and improves the quality of a product, becomes its new producer independently of the number of products it supplies to the market.*

Proof. It is straightforward to show that under Assumption 2, the firm providing the highest quality of a variety is always able to offer the highest provider-adjusted quality to price ratio, as the incremental quality jump obtained through external R&D (λ_x) is always bigger than the maximum possible effect of provider-driven complementarity (γ). ■

Proposition 3 implies that the probability of condition (2.10) being satisfied – obtaining a successful innovation that allows to replace a incumbent – is always equal to 1,

and specifically does not depend on the size of the innovator or the incumbent. As a consequence the rate of creative destruction suffered by a firm with $n \geq 1$ is now given by

$$\tau = x_e + \sum_{s=1}^{\infty} F\mu_s s x_s. \quad (2.17)$$

In other words, the rate of creative destruction is constant across firms of different sizes. This implies that any firm is equally likely to lose a product through creative destruction.

Taking as given the values of r , g , and τ , the value functions of firms determine their optimal dynamic R&D decisions. I start with the value function of an incumbent firm. Consider a firm $i \in \mathcal{F}$ being the market leader of $n \geq 1$ varieties. Such a firm chooses the per-product external, x_n , and internal, z_{nj} , innovation rates in order to maximize²⁰

$$\begin{aligned} rV_n(\mathbf{q}_i) = \max_{x_n, \{z_{nj}\}_{j=1}^n} & \left\{ \sum_{q_j \in \mathbf{q}_i} \left[\pi\gamma^{1-n-1} q_j + z_{nj} (V_n(\mathbf{q}_i \setminus \{q_j\} \cup_+ \{q_j\Lambda_z\}) - V_n(\mathbf{q}_i)) - \hat{\chi} z_{nj}^{\hat{\psi}} q_j \right. \right. \\ & \left. \left. + \tau (V_{n-1}(\mathbf{q}_i \setminus \{q_j\}) - V_n(\mathbf{q}_i)) \right] \right. \\ & \left. + nx_n (\mathbb{E}_{h,\lambda_x} [V_{n+1}(\mathbf{q}_i \cup_+ \{q_h\Lambda_x\})] - V_n(\mathbf{q}_i) - \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j) - \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} \right\} \\ & + \dot{V}_n(\mathbf{q}_i) \end{aligned}$$

The first line on the right-hand side collects the profits obtained per each variety in which the firm is the market leader and the change in value if internal innovation is successful for each of those varieties. The second line captures the change in value experienced from losing each of the varieties through creative destruction. The third line captures the increase in value due to external innovation. If the firm's R&D is successful, it improves the quality of any product h outside its quality portfolio and becomes the new market leader of that variety. Given that R&D is undirected, the new product's quality is unknown and is captured by the expected value \mathbb{E}_{h,λ_x} , which is an expectation over quality level and innovation step. Finally, the last line captures the change in firm value due to the economy's growth along its balanced growth path. The following proposition characterizes the closed-form solution of the value function.

Proposition 4. *There exists an equilibrium with adjustment cost $\Omega_n = A_{n+1} - A_n$, and positive entry, where for $n \geq 1$ an incumbent's value function has the form*

$$V_n(\mathbf{q}_i, \bar{q}) = A_n \sum_{q_j \in \mathbf{q}_i} q_j + B_n \bar{q}, \quad (2.18)$$

²⁰A detailed derivation can be found in Appendix 2.6.2.

and the coefficients A_n , B_n and Ω_n satisfy the recursions

$$(r + \tau n)A_n = \pi\gamma^{1-n-1} + \hat{\chi}(\hat{\psi} - 1) \left(\frac{A_n \lambda_z}{\hat{\chi} \hat{\psi}} \right)^{\frac{\hat{\psi}}{\hat{\psi}-1}} + \tau(n-1)A_{n-1}, \quad (2.19)$$

$$B_{n+1} = B_n - A_{n+1} \Lambda_x + \tilde{\psi} \tilde{\chi}^{\frac{1}{\tilde{\psi}}} n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}}} \left(\frac{(\rho + n\tau)B_n - n\tau B_{n-1}}{\tilde{\psi} - 1} \right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}}, \quad (2.20)$$

with $A_0 = B_0 = 0$. Moreover, the per-product external and internal innovation rates are given by

$$x_n = \left(\frac{A_{n+1} \Lambda_x + B_{n+1} - B_n}{\tilde{\psi} \tilde{\chi} n^{\tilde{\sigma}-1}} \right)^{\frac{1}{\tilde{\psi}-1}}, \quad (2.21)$$

$$z_{nj} = z_n = \left(\frac{A_n \lambda_z}{\hat{\chi} \hat{\psi}} \right)^{\frac{1}{\hat{\psi}-1}}. \quad (2.22)$$

Proof. Appendix 2.6.2. ■

The value function (2.18) consists of two parts. The first part A_n captures the expected discounted stream of profits obtained by being the variety's market leader. Its dependence on size stems from the profit function (2.8). Besides, it also incorporates the value of improving the quality of each variety through internal R&D. The second part B_n relates to the value of extending the product portfolio of the firm through external R&D. The equilibrium per-product internal innovation rate depends on size as, all else equal, bigger firms generate more provider-driven complementarity, yielding a higher marginal gain from improving the quality of each one of their varieties. However, it is independent of quality as both the profit and internal cost function are linear in quality. Size also affects the equilibrium per-product external innovation rate for two reasons. First, it directly affects the production function of external innovations but, most importantly, affects the firm's provider-driven complementarity. As a firm grows, it generates more complementarity, which affects its incentives to conduct external R&D.

It remains to characterize the R&D decision of an outside entrepreneur (potential entrant). Denote by V_0 the value of a firm with size 0, i.e., a firm with an empty quality portfolio. Taking as given the values of r , g and τ , an outside entrepreneur chooses the entry rate x_e in order to maximize

$$rV_0 - \dot{V}_0 = \max_{x_e} \{x_e (\mathbb{E}_{h,\lambda_x} [V_1(\{q_h \Lambda_x\}) - V_0]) - \nu x_e \bar{q}\}, \quad (2.23)$$

where $V_1(\{q_h \Lambda_x\})$ denotes the value of a firm that owns a single product line with quality $q_h \Lambda_x$ and $\dot{V}_0 \equiv \partial V_0 / \partial t$ captures the change in the value of being an outside entrepreneur

due to the growth of the economy along its balanced growth path. The expected value \mathbb{E}_{h,λ_x} is an expectation over quality level q_h and innovation step $\tilde{\lambda}_x$. Therefore, an outside entrepreneur's value is determined by the expected value from obtaining an external innovation and replacing an incumbent.

To conclude the equilibrium's characterization, I still need to derive the invariant distribution of the number of products across firms. Recall that, as laid out before, μ_n denotes the share of firms producing n goods, and it satisfies $\sum_{n=1}^{\infty} \mu_n = 1$. The invariant distribution depends on the flow equations

n	Inflows	Outflows
0	$F\mu_1\tau$	x_e
1	$F\mu_2 2\tau + x_e$	$F\mu_1(\tau + x_1)$
\vdots	\vdots	\vdots
n	$F\mu_{n+1}(n+1)\tau + F\mu_{n-1}(n-1)x_{n-1}$	$F\mu_n n(\tau + x_n)$

The first row shows the inflows and outflows into outside entrepreneurs. An inflow happens whenever a firm with one product loses it through creative destruction, either to another outside entrepreneur or an incumbent firm. Outflows happen when an outside entrepreneur obtains a successful innovation and becomes an incumbent with one single product. The second row shows the inflows and outflows of firms with market leadership in one variety. Inflows occur when outside entrepreneurs obtain a successful innovation and whenever a firm with two products loses one through creative destruction. Outflows happen if a firm loses its only product and becomes an outside entrepreneur or when a firm with one product obtains a successful innovation and increases its portfolio of varieties. The last row is simply a generalization for firms with any number of varieties $n > 1$. Finally, the following proposition derives a closed-form solution of the invariant product number distribution.

Proposition 5. *For $n \geq 1$, the invariant distribution μ_n is given by*

$$\mu_n = \frac{1}{n} \frac{x_e}{F x_n} \prod_{s=1}^n \frac{x_s}{\tau}. \quad (2.24)$$

Proof. Appendix 2.6.2. ■

Before introducing the definition of the balanced growth path equilibrium of this economy, which will conclude this subsection, it is helpful to obtain an expression for aggregate R&D expenditure R . This is given by the sum of external R&D expenditure of

incumbents x_n (including adjustment costs) and potential entrants x_e , and internal R&D expenditure of incumbents z_n . Therefore, R can be obtained as

$$R = \sum_{i=1}^n F \mu_n \left[x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} + \sum_{j=1}^n \left(n \Omega_n + \hat{\chi} z_n^{\hat{\psi}} \right) q_j \right] + \nu x_e \bar{q}. \quad (2.25)$$

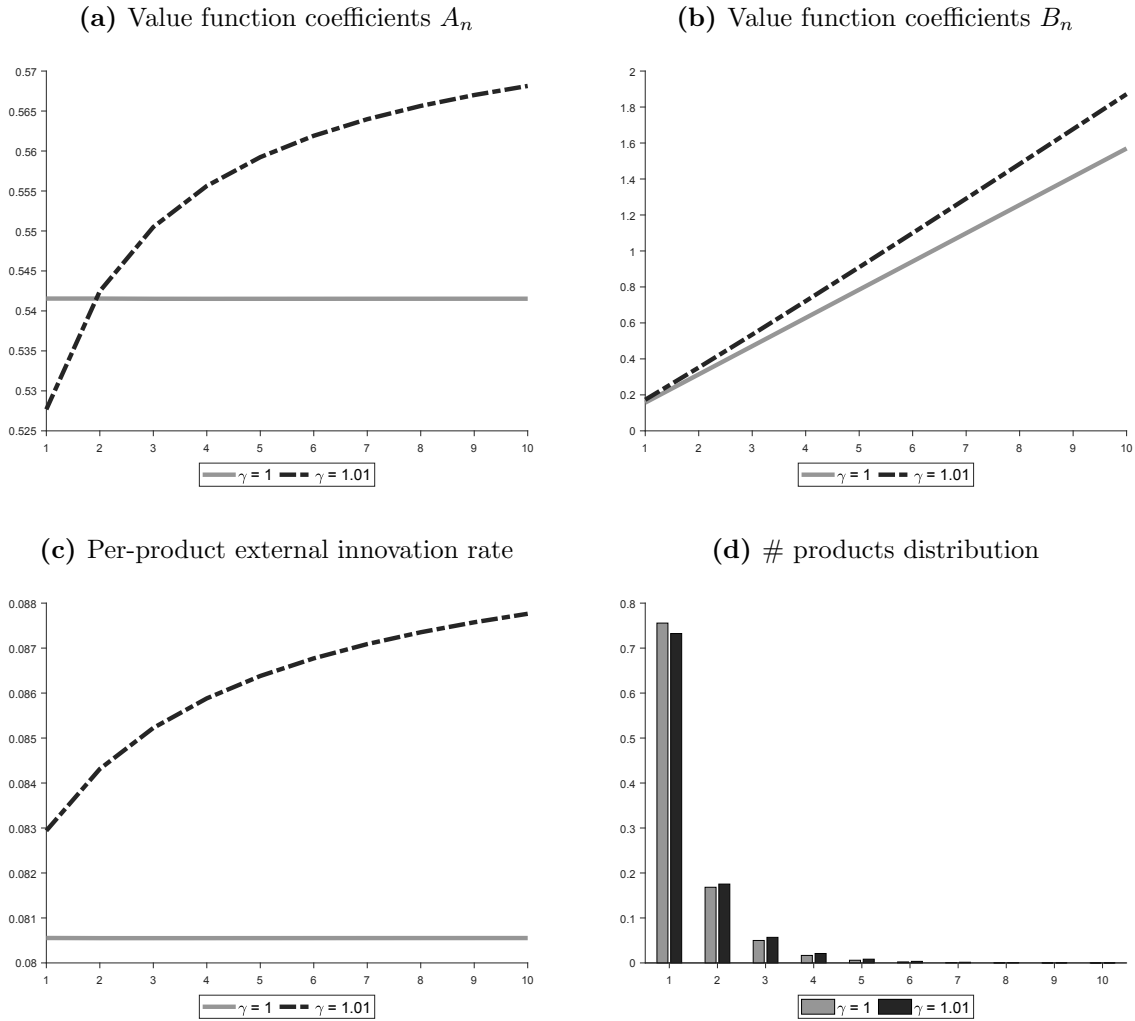
Next, a balanced growth path equilibrium and its properties are defined.

Definition 2.3.1. *For every t , $j \in [0, 1]$, n , \bar{q} and q_j , a balanced growth path equilibrium consists of allocations $\{k_j, l_j, \pi_j, x_n, z_n, x_e\}$, provider-driven complementarity $\{m_j\}$, aggregate variables $\{Y, C, R, A, L, L^x, F, \bar{Q}\}$, value function coefficients $\{A_n, B_n\}$, distribution of number of products $\{\mu_n\}$, rates $\{\tau, g\}$ and prices $\{p, w, r\}$ such that*

1. $\{k_j, l_j, \pi_j, p\}$ are the solution to the intra-temporal labor-quality-price decision of intermediate firms,
2. per-product external $\{x_n\}$ and internal $\{z_n\}$ flow rates satisfy (2.21) and (2.22),
3. $\{w\}$ satisfies (2.12),
4. share of workers producing the final good L and intermediate goods L^k satisfy (2.15) and $L^k = 1 - L$,
5. creative destruction τ satisfies (2.17),
6. entry flow rate x_e and measure of incumbents F solve (2.23), and satisfy the free-entry condition $V_0 = 0$,
7. distribution of number of products satisfies (2.24),
8. given μ_n , provider-driven complementarity $\{m_j\}$ and average provider-adjusted quality \bar{Q} satisfy (2.3) and (2.13),
9. aggregate R&D, R , satisfies (2.25), aggregate output Y satisfies (2.2) and aggregate consumption satisfies the market clearing condition $C = Y - R$,
10. the growth rate satisfies (2.16),
11. the value function coefficients $\{A_n, B_n\}$ satisfy (2.19) and (2.20),
12. the interest rate satisfies the Euler equation (2.1).

The effects of provider-driven complementarity In this Section, I explore the equilibrium implications of provider-driven complementarity. Figure 2.8 compares an economy without provider-driven-complementarity ($\gamma = 1$) with an economy with provider-driven-complementarities ($\gamma = 1.01$). Otherwise, the parametrization is the same for both economies and is chosen for illustrative purposes.

Figure 2.8: The effects of provider-driven complementarity



Under provider-driven complementarity, the per-variety profit function (2.8) shows that the profits of a firm increase in the number of products supplied to the market. As a consequence, the first term A_n in the value function 2.18 also does, as it captures the discounted stream of profits obtained by a market leader. Panel 2.8a shows this result. Note that the value of being an incumbent with one product if firms generate provider-driven complementarity is lower than there is no provider-driven complementarity. This

happens due to the different rates of creative destruction between both economies. As the value of being an incumbent firm increases in firm size, the franchise value of expanding into more product lines, captured by the second term in the value function 2.18, is slightly convex as Panel 2.8b shows. Hence, the per-product external innovation rates increase in firm size, leading to a higher creative destruction rate in equilibrium. This increase implies that incumbents expect to be replaced faster, reducing their discounted stream of benefits. This higher equilibrium creative destruction rate is compensated for by the direct effect of provider-driven complementarity on the profit function for firms that are market leaders in more than one variety. This is not the case for firms with one variety that do not generate provider-driven complementarity.

The effects of provider-driven complementarities on the R&D decisions of firms lead to significant changes in the equilibrium invariant firm size distribution as shown in Panel 2.8d. In particular, the increase in the creative destruction rate leads to a decline in new firms' entry rate as, upon entry, the discounted stream of benefits of an entrant is lower. Consequently, the measure of incumbent firms is also reduced and, most importantly, incumbent firms become bigger.

Baseline framework

The previous discussion highlights that provider-driven complementarity shapes R&D decisions of firms even in a simple model where complementarity does not interact with quality in determining the market leader of each variety. Recall that in the simplified framework both the probability of obtaining a successful innovation and the creative destruction rates experienced by firms are independent of their size (number of varieties in which they are market leaders). This Section considers a generalized version of the model where condition 2.10 – which determines the market leader of each variety – depends both on the quality jump obtained by an innovator, and on the relationship between its size and that of the incumbent.

Taking as given the values of r , g , $\{\tau_n\}$, and $\{\mu_n\}$, an incumbent firm $i \in \mathcal{F}$ chooses the per-product external, x_n , and internal, z_{nj} , innovation rates in order to maximize²¹

$$rV_n(\mathbf{q}_i) = \max_{x_n, \{z_{nj}\}_{j=1}^n} \left\{ \sum_{q_j \in \mathbf{q}_i} \left[\pi \gamma^{1-n-1} q_j + \tau_n [V_{n-1}(\mathbf{q}_i \setminus \{q_j\}) - V_n(\mathbf{q}_i)] \right. \right. \\ \left. \left. + z_{nj} [V_n(\mathbf{q}_i \setminus \{q_j\} \cup_+ \{q_j \Lambda_z\}) - V_n(\mathbf{q}_i)] - \hat{\chi} z_{nj}^{\hat{\psi}} q_j \right] \right\}$$

²¹A detailed derivation can be found in Appendix 2.6.2.

$$\begin{aligned}
& + nx_n \mathbb{E}_{h, \lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(n+1, j)}\}} \left[V_{n+1}(\mathbf{q}_i \cup_+ \{q_h \Lambda_x\}) - V_n(\mathbf{q}_i) - \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j \right] \right] \\
& - \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} \Big\} + \dot{V}_n(\mathbf{q}_i).
\end{aligned}$$

There is a key difference with respect to the simplified framework. Now, to become the new producer of a variety, the innovation step has to be sufficiently big so that it can offset the difference in provider-driven complementarity offered by the incumbent and the innovator. This is captured by the indicator function inside the expectation operator $\mathbb{E}_{h, \lambda_x}$, which now depends crucially on the equilibrium distribution of the numbers of products across firms. This implies that to form this expectation correctly, each firm needs to know the firms' equilibrium distribution across products. As a consequence, this model is computationally more intensive, although it remains tractable. The following proposition characterizes the closed-form solution of the value function, closely resembling that of the simplified framework.

Proposition 6. *There exists an equilibrium with adjustment cost $\Omega_n = A_{n+1} - A_n$, and positive entry, where for $n \geq 1$ an incumbent's value function has the form (2.18), and the coefficients A_n , B_n and Ω_n satisfy the recursions*

$$\begin{aligned}
(r + \tau_n n) A_n - \tau_n (n-1) A_{n-1} &= \pi \gamma^{1-n^{-1}} + \hat{\chi} (\hat{\psi} - 1) \left(\frac{A_n \lambda_z}{\hat{\chi} \hat{\psi}} \right)^{\frac{\hat{\psi}}{\hat{\psi}-1}}, \\
\frac{\mathbb{E}_{h, \lambda_x} [\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(n+1, j)}\}} (A_{n+1} \Lambda_x + B_{n+1} - B_n)]}{\tilde{\psi} \tilde{\chi}^{\frac{1}{\tilde{\psi}}} n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}}}} &= \left(\frac{(\rho + n\tau) B_n - n\tau B_{n-1}}{\tilde{\psi} - 1} \right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}},
\end{aligned}$$

with $A_0 = B_0 = 0$, and per-product R&D intensities given by

$$x_n = \left(\frac{\mathbb{E}_j [\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(n+1, j)}\}} (A_{n+1} \Lambda_x + B_{n+1} - B_n)]}{\tilde{\psi} \tilde{\chi} n^{\tilde{\sigma}-1}} \right)^{\frac{1}{\tilde{\psi}}}, \quad (2.26)$$

$$z_{nj} = z_n = \left(\frac{A_n \lambda_z}{\hat{\chi} \hat{\psi}} \right)^{\frac{1}{\hat{\psi}-1}}. \quad (2.27)$$

Proof. The proof follows closely that of Proposition 4. ■

The value function in the baseline framework also consists of two parts. Note that the key difference with the simplified framework is that the probability of obtaining a successful innovation is now endogenous and appears both explicitly and implicitly through its effect in the creative destruction rate τ_n . The term A_n , in addition to its size dependence due to the shape of the profit function, now additionally depends on size due to the effect

of provider-driven complementarity on creative destruction. This is a direct consequence of the fact that all else equal, firms with market leadership over a higher number of goods are less likely to lose their leadership through creative destruction. The probability of obtaining a successful innovation also appears in the recursion for B_n , which relates to the value of extending the product portfolio of the firm through external R&D, and in the per-product external innovation rates x_n . As firms increase their quality portfolio, the probability of obtaining successful innovations increases, but at a declining rate. In other words, the marginal probability increase is declining in size (asymptotically converging to 0 as the probability approaches 1).

As in the simplified framework, the value V_0 of being an outside entrepreneur is the expected value from entering and successfully replacing a incumbent. This value satisfies

$$rV_0 - \dot{V}_0 = \max_{x_e} \left\{ x_e \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(1,j)}\}} [V_1(\{q_h \Lambda_x\}) - V_0] \right] - \nu x_e \bar{q} \right\}.$$

Similarly to the case of an incumbent, to become the new producer, the innovation step has to be sufficiently big to offset the difference in provider-driven complementarity offered by the incumbent and the innovator. The indicator function again captures this.

The invariant measure of firms producing n goods, μ_n , needs to be re-defined to incorporate the probability of obtaining a successful innovation. The invariant distribution now depends on the flow equations

n	Inflows	Outflows
0	$F\mu_1\tau_1$	$x_e \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(1,j)}\}} \right]$
1	$F\mu_2 2\tau_2 + x_e \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(1,j)}\}} \right]$	$F\mu_1\tau_1 + F\mu_1 x_1 \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(2,j)}\}} \right]$
\vdots	\vdots	\vdots
n	$F\mu_{n+1}(n+1)\tau_{n+1}$ $+ F\mu_{n-1}(n-1)x_{n-1} \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(n,j)}\}} \right]$	$F\mu_n n \tau_n$ $+ F\mu_n x_n \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(n+1,j)}\}} \right]$

The following proposition derives a closed-form solution of the invariant product number distribution.

Proposition 7. *For $n \geq 1$, the invariant distribution μ_n is given by*

$$\mu_n = \frac{1}{n} \frac{x_e}{F x_n} \prod_{s=1}^n \frac{x_s}{\tau_s} \mathbb{E}_{h,\lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(s,j)}\}} \right].$$

Proof. The proof follows closely that of Proposition 5. ■

The definition of the balanced growth path equilibrium of the baseline framework economy concludes this section. It follows closely Definition 2.3.1, which can be extended to accommodate the baseline framework. Specifically, the definition needs to include the probability matrix (2.11) and the sequence of creative destruction rates $\{\tau_n\}$.

2.4 Quantitative Analysis

In this Section, I use the theory of provider-driven complementarity to perform a quantitative experiment. The experiment is motivated by the influential contribution of Bloom et al. (2020) arguing that ideas have become harder to find during the last decades. Specifically, I decrease the size of the quality jump obtained after a successful innovation, which is treated as exogenous to the model. This decline can be also be interpreted as a reduction in the probability of obtaining a ‘radical’ innovation.²²

I start by considering a version of the model with a mild level of provider-driven complementarity, which I calibrate by targeting average moments of the data between 1985 and 1990. Then I decrease the parameter governing the quality jump’s size after a successful innovation (λ). To do that, I explicitly target the decline in the U.S. growth rate between 1985-1990 and 2010-2015. I show that this decline has important effects on business dynamism: the entry rate declines, the concentration of sales increases, and the equilibrium R&D expenditure increases even as the economy’s aggregate growth rate declines. To highlight the importance of provider-driven complementarities in explaining the dynamics of firms observed in the data, I conduct a second exercise where I consider a version of the model without provider-driven complementarity. I henceforth refer to this second version of the model as standard quality ladder model. I compare both versions of the model along their respective balanced growth paths with constant rates of growth. Contrary to the provider-driven complementarity framework, the standard quality ladder model cannot replicate the evolution of business dynamism observed in the data.

In what follows, I start by briefly describing the numerical solution algorithm and the model’s calibration before discussing the results.

²²Within this framework, ‘radical’ innovation is any external innovation (either from an entrant or an incumbent) that allows it to become a market leader. This definition follows that of Acemoglu and Cao (2015).

2.4.1 Solution

The solution method is based on computing the equilibrium firm value functions and R&D decisions. This equilibrium is obtained as a fixed point in a space that includes the rate of growth along the economy's balanced growth path, stationary creative destruction rates, and stationary distribution of the number of products. After obtaining the solution for a set of parameters, the model is simulated to obtain the model-based moments of interest.²³

2.4.2 Calibration

Here I present the baseline calibration. The model has 11 structural parameters that need to be calibrated, which I partition into two sets. The values of the first set are determined without solving the model by relying on previous literature. The second set of values are determined by solving the model and targeting several moments from the data.

Externally calibrated parameters

I start by setting the discount rate ρ to 2%, a standard value in this literature. The theory I propose relies on the asymmetries generated by provider-driven complementarities across firms of different sizes. As laid out in the previous section, firms' behavior depends on the number of varieties they supply to the market and on the distribution of the number of products across firms. To focus on the effects of provider-driven complementarity, I assume constant returns to scale in the R&D technology for external innovations, i.e., $\sigma = 1 - \psi$, as in (Klette and Kortum, 2004).²⁴ The parameters $\tilde{\psi}$ and $\hat{\psi}$ govern the curvature of the external and internal R&D cost functions. To assign a value to these parameters, I follow Akcigit and Kerr (2018) that, relying on previous literature, set both equal to 0.5.²⁵ This implies that both cost functions are quadratic in the Poisson (external or internal) rates of innovation.

The key parameter for the calibration is γ , which measures the strength of provider-driven complementarities. This effect is not trivial to measure in the data, thus given the lack of an estimate for this parameter, I fix a value of $\gamma = 1.0245$, which implies a mild effect of provider-driven complementarity. Later on, I consider alternative values in a sensitivity analysis. As shown in Section 2.3 with a theoretical example for the simplified

²³Further details can be found in Appendix 2.6.4.

²⁴This assumption has important effects on the relationship between R&D and size. In Appendix 2.6.5 I relax this assumption allowing for decreasing returns to scale as in (Akcigit and Kerr, 2018).

²⁵This implies that $\psi = 2$.

model, introducing provider-driven complementarity yields a higher weight of the right tail of the size distribution. With provider-driven complementarity, firms can profit more from each product line obtained through external innovation. Under the assumption of constant returns to scale in the external innovation's production function, the equilibrium innovation rates increase with size, leading to bigger firms in equilibrium. It is important to remark that provider-driven complementarity does not play any role in determining the market leader of each variety by assumption in the simplified model. Consequently, the effects of provider-driven complementarity on firm size are reinforced in the baseline framework when provider-driven complementarity interacts with the quality dimension in determining the equilibrium market leader of each variety.

Internally calibrated parameters

It remains to assign parameters to the step size of external and internal innovations, scale parameters of external (for incumbents and entrants) and internal cost functions, and per-product profitability. For simplicity, I assume that the average innovative step obtained from external innovations is the same as the fixed step of internal innovations.²⁶ This additional restriction leaves me with the following five parameters that need to be determined:

1. Mean value of innovative step size (λ),
2. Scale parameter of external innovation cost function ($\tilde{\chi}$),
3. Scale parameter of internal innovation cost function ($\hat{\chi}$),
4. Scale parameter of entrant cost function (ν),
5. (Inverse) Elasticity of substitution across varieties (β).

Although all the parameters jointly determine the endogenous equilibrium outcomes of interest, each of them is tightly related to a moment of interest. I discuss these connections in what follows.

As is well known, the source of endogenous growth in a quality ladder model is the successful improvement of quality. Therefore, the mean value of innovative step size for external and internal (λ) innovations is closely related to the economy's equilibrium growth rate. The scale parameters of the external and internal innovation cost functions ($\tilde{\chi}$ and $\hat{\chi}$, respectively) will determine the equilibrium expenditure of firms in external

²⁶In terms of the parameters shown in previous sections, that implies $\tilde{\lambda}_x = \lambda_z \equiv \lambda$.

and internal R&D. Besides, their interaction determines the ratio of external to internal innovation. Consequently, both scale parameters are connected to the equilibrium ratios of R&D expenditure over total cost and sales. I use my sample of firms from Compustat to target these ratios. The scale parameter of the entrant cost function (ν) determines the entry cost and is closely related to new firms' entry rate. As measuring entry in Compustat is not straightforward, I use data from Business Dynamics Statistics to target the entry rate. Finally, the elasticity of substitution across varieties (β^{-1}) determines the equilibrium per-product profitability of each variety and is closely related to the average profitability of firms. Again, using my Compustat sample of firms, I target the average ratio of total sales minus total operating expenditures before depreciation to total sales.

Based on the previously established connections, I initially calibrate the model to the average moments of interest between 1985 and 1990. Specifically, I compute model implied moments and compare them to their counterpart moments in the data. Technically, I solve the simple minimum distance equation given by

$$\min \sum_{j=1}^5 \frac{|\text{model}(j) - \text{data}(j)|}{|\text{data}(j)|}.$$

Fit of the Model

Table 2.1 shows the full set of calibrated parameter values for the provider-driven complementarity model and the standard quality ladder model as well as the fit of both models to the targeted moments.

Table 2.1: Calibrated parameters

Parameter	PDC	ST	Target	Data	PDC	ST
λ	0.113	0.112	GDPpc growth (%)	2.5	2.5	2.5
$\tilde{\chi}$	4.914	4.59	(Mean) R&D / Sales (%)	6.4	6.8	6.2
$\hat{\chi}$	0.593	0.592	(Mean) R&D / Total Cost (%)	7.4	7.8	7.2
ν	0.799	0.823	Entry Rate (%)	11.9	11.9	11.9
β	0.195	0.198	Mean profitability (%)	12.5	12.5	13.4

Note. PDC: Provider-driven complementarity model features. ST: Standard quality ladder model. Second and third columns report values for the internally calibrated parameters. Values of the externally calibrated parameters: $\rho = 0.02$, $\psi = 0.5$, $\sigma = 0.5$, $\gamma = 1.0245$ (PDC), $\gamma = 1$ (ST).

Both models do an excellent job in replicating the targeted moments for the 1985-1990

period. Introducing a mild provider-driven complementarity effect does not turn into a significant change in the parameters' values with respect to those of the standard quality ladder model. Moreover, the values of the parameters I find are similar to those of [Akcigit and Kerr \(2018\)](#), even though they use a different sample of firms. In what follows, I provide intuition on how the parameters differ across the provider-driven complementarity framework and the standard quality ladder model.

According to the predictions of the theory highlighted in [Section 2.3](#), under provider-driven complementarity, incumbent firms have an extra incentive to innovate - both externally and internally - to increase their profits. Consequently, to be able to match the desired ratios between R&D and total cost or total sales, a lower cost to conduct R&D is needed in the standard quality ladder model. In turn, this implies that in the standard quality ladder model, the scale parameters of the external ($\tilde{\chi}$) and internal ($\hat{\chi}$) innovation cost functions are slightly lower than their counterparts in the provider-driven complementarity model. The calibrated values imply that around 55% of total growth comes from external innovation, while 20% comes from internal innovation (the remaining 25% is attributed to entry). These results are comparable to those of [Akcigit and Kerr \(2018\)](#). However, in their recent contribution, [Garcia-Maza et al. \(2019\)](#) show that up to 70% of growth can stem from internal innovation. In [Appendix 2.6.5](#) I show that my main results are preserved even adjusting the calibration to capture that alternative relationship between external and internal R&D. The theory I propose in this paper also implies that entrants are less likely to become market leaders as they do not generate provider-driven complementarity upon entry while incumbents do. Consequently, the calibrated value of the scale parameters of the entrant cost function (ν) needed to obtain the entry rate of firms observed in the data must be smaller under provider-driven complementarity.

The growth rate in both models is 2.5%, the same as observed in the data. By construction, the source of endogenous growth in a quality ladder model is the successful improvement of quality. In particular, the economy grows as additional successful rungs of the quality ladder are attained. The differences in the needed step sizes of innovation (λ) across models are minor but highlight an important equilibrium outcome: under provider-driven complementarity, there are fewer active firms in equilibrium, and incumbents tend to be bigger. This is why, although incumbent firms innovate more in this framework, a bigger step size of innovation is needed to obtain the growth rate observed in the data.

Finally, both models closely match the average profitability of firms found in the data. It is important to note that the ratio between profits and sales in the model includes R&D expenditures. Consequently, a relatively higher value is needed under provider-

driven complementarity compared to the value obtained for the standard quality ladder model. One relevant remark about profitability in the model is that, by construction, markups are exogenous and fixed. Therefore, the model would only generate big changes in firms' profitability if markups were endogenous or, at least, time-varying. This would be reflected in the model through exogenous changes in the (inverse) elasticity of substitution across varieties β . Although I abstract from these changes in the main exercise, recent literature has pointed out that markups have indeed changed during the last decades, and are important to explain recent macroeconomic trends, see for example Barkai (2020), De Loecker et al. (2020b) or Feijoo-Moreira (2020). In Appendix 2.6.5 I show that my main results are preserved even if an increasing trend in markups is introduced into the model.

2.4.3 Results

For the quantitative exercise, I re-calibrate λ to deliver the average aggregate growth rate between 2010 and 2015 and leave all the remaining parameter values unchanged.

I now discuss the implications of a decline in the 'radicalness' of innovation on business dynamism, which is the quantitative analysis's primary objective. This exercise is motivated by the literature arguing that ideas have become harder to find during the last decades, which I capture in the model by reducing the average innovation step size (λ) obtained after a successful innovation.

Table 2.2: Quantitative analysis Results. Changes in the U.S. between 1985-1990 and 2010-2015.

Moment	Data	PDC	ST
GDP _{pc} growth (p.p.)	-0.99	-1.00	-0.99
(Mean) R&D / Sales (p.p.)	3.71	0.97	-0.16
(Mean) R&D / Total Cost (p.p.)	4.32	1.01	-0.17
Entry Rate (p.p.)	-2.89	-0.28	0.06
Top 20% Sales (p.p.)	1.68	1.52	-2.19

Note. PDC: Baseline provider-driven complementarity model. ST: Standard quality ladder model.

To perform the exercise, I re-calibrate λ to match the observed decline in the U.S. growth rate between 1985-1990 and 2010-2015, leaving the remaining parameters constant. This delivers a value of λ equal to 0.073 with provider-driven complementarity and 0.072

in the standard quality ladder model. In what follows, I compare both models along their respective balanced growth paths. The results of the quantitative exercise are summarized in Table 2.2.

The first row of Table 2.2 shows that both models can accommodate the decline in the economy's growth rate, reproducing the observed decline in the data closely. From (2.16) it follows that reducing the quality jump, everything else constant, decreases the increase in quality upon successful innovation therefore reducing the growth rate of the economy.

The remaining rows of Table 2.2 show the effects of the decline in the other moments of interest, which are all non-targeted. The decline in the innovative step size generates opposite effects across both versions of the model in:

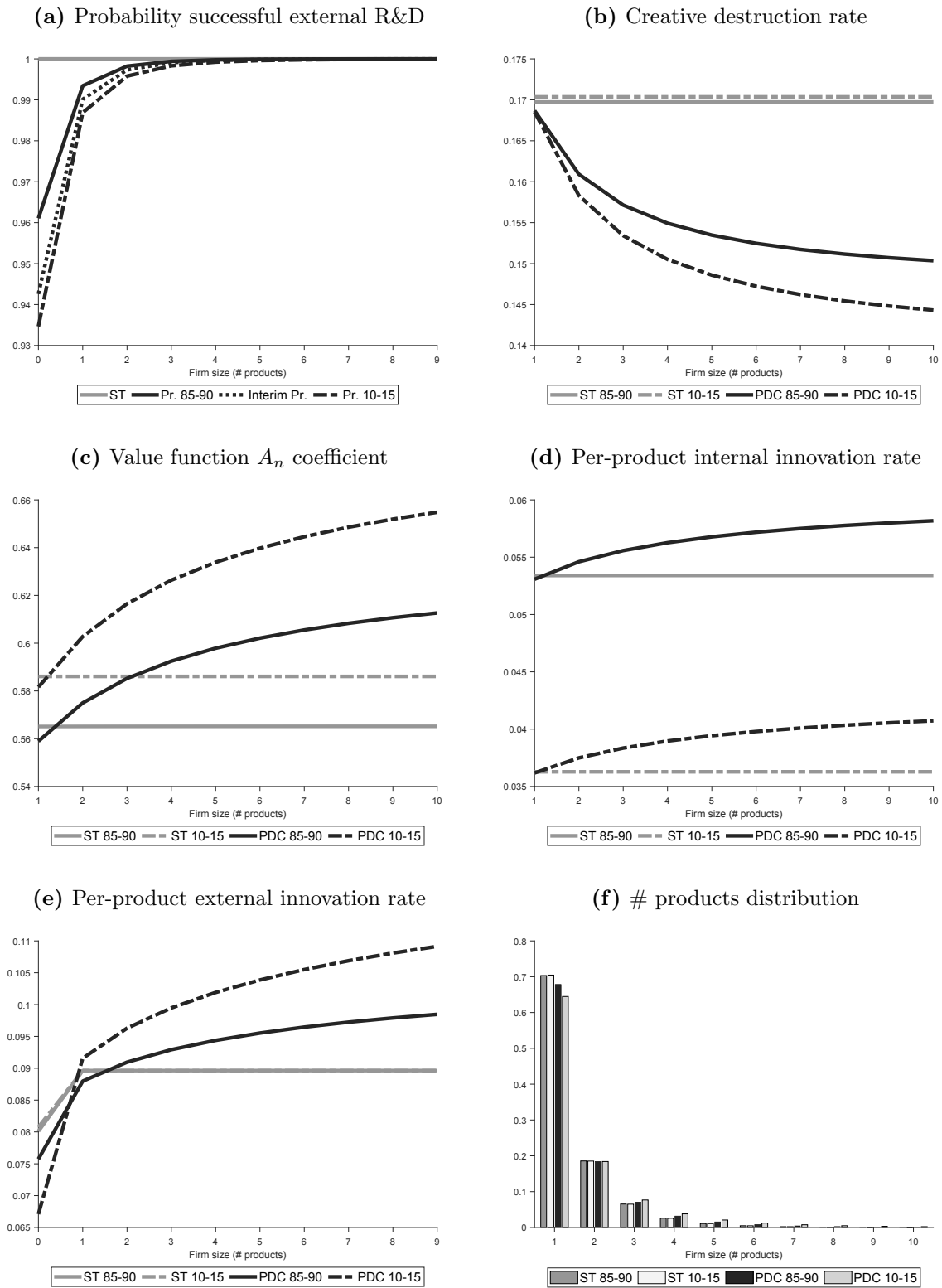
1. the equilibrium R&D expenditure relative to the total sales or total cost of firms,
2. the equilibrium entry rates,
3. the share of sales accounted for by the 20% biggest firms of the economy.

To provide intuition on the reasons underlying this opposite behavior and help better understand the general equilibrium effects of the experiment, in what follows, I start by discussing the implications of the decline within the standard quality ladder model. This model is a particular case of the baseline framework when $\gamma = 1$, thus many of its implications carry over to the provider-driven complementarity framework. In Figure 2.9 I show a battery of theoretical results for both models, which I now turn to discuss in detail.

Standard quality ladder model

Consider the effects of the decline in the innovation step size for the standard quality ladder model. Figure 2.9c shows that when the innovation step size declines, the value function's coefficient capturing the stream of discounted profits of being a market leader increases (ST 85-90 vs. ST 10-15). This is a direct consequence of the decline in the economy's growth rate, which drives down the general equilibrium interest rate according to the household's Euler equation (2.1). I henceforth refer to this result as the *market effect*, which enhances the incentives to conduct R&D. However, its quality portfolio also determines the profitability of a firm. As the innovative step size declines, a firm expects to obtain smaller quality improvements over after a successful innovation (internal or external), which hinders its incentives to conduct R&D. I henceforth refer to this result as the *quality effect*.

Figure 2.9: Quantitative analysis results



Note. PDC: provider-driven complementarity model. ST: standard quality ladder model. Note: along all panels, x -axis represents number of products - 0 refers to a potential entrant. Panel 2.9a represents the expected probability of replacing a incumbent through external innovation conditional on the innovator current size. Interim Pr. depicts the probability of replacing a incumbent after declining the step size innovation, but fixing the size distribution to its 1985-1990 level.

In Figure 2.9d I show that the equilibrium per-product internal innovation rates decline sharply, implying that the *quality effect* dominates the *market effect*. This is an intuitive result: internal R&D is a mechanism to increase the quality of a variety the firm already produces, i.e., a firm conducting internal R&D is already the market leader of that variety. As a consequence, the marginal benefit of increasing the quality of that variety is dominated by the expected quality increase after innovating.²⁷ As the average innovative step size declines, its incentive to conduct internal R&D also does. This naturally leads to less internal R&D in equilibrium, reducing R&D expenditure.

The dynamics of external R&D, represented in Panel 2.9e, are substantially different. After the innovative step size decline, the per-product external innovation rates are barely affected. Opposite to the internal R&D decision, a firm conducts external R&D to become the market leader of a variety outside its quality portfolio. As a consequence, for the external R&D decision, the increase in the stream of discounted profits associated with being a market leader (the *market effect*) is now as important as the decline in the expected quality increase after innovating (the *quality effect*). Provided that the per-product external innovation rates remain constant, both effects cancel out for incumbent firms. However, for potential entrants, I find that the *market effect* slightly dominates the *quality effect*, leading to a small increase in the innovation rate of entrants. This increase has three important implications: 1. it leads to an increase in the equilibrium entry rate (see 2.2); 2. it generates a small increase in the rate of creative destruction of the economy, as shown in Figure 2.9b; and 3. it yields an increase in the measure of incumbent firms in equilibrium. Finally, Figure 2.9f shows that the distribution of the number of products across firms slightly compresses. As the distribution of the number of products across firms and the size distribution of firms are tightly linked,²⁸ it follows that the share of sales of the biggest firms in the economy declines as shown in Table 2.2.

Provider-driven complementarity

Before discussing the implications of provider-driven complementarity, it is crucial to note in this framework the probability of obtaining a successful innovation is a function of size, as Figure 2.9a shows. This is opposed to the standard quality ladder model, where this probability is equal to one, i.e., any successful quality improvement always find its way to the market. According to (2.3), as firms increase their portfolio, they generate a higher

²⁷Loosely speaking, the productivity of investing in internal R&D declines, and the marginal profit gain is now lower.

²⁸This is a common feature of this type of models which is also highlighted in Akcigit and Kerr (2018) or Cavenaile and Roldan-Blanco (2019).

level of complementarity. Therefore small firms, all else equal, find it more challenging to become market leaders as they provide less complementarity to the consumer. When ideas get harder to find, more quality innovations that would be successful in the absence of provider-driven complementarity do not find their way into the markets. Even though the decline in the innovative step size reduces this probability for any firm of the economy, its effects are more severe for small firms than for big firms. In other words, all firms are less likely to become market leaders, but small firms are relatively more unlikely.

The decline in the probability of obtaining a successful innovation can be separated into two components. The first one is purely mechanical: reducing the average step size innovation implies that firms are less likely to obtain a successful innovation even in the absence of general equilibrium effects. The second component that affects the decline in the probability of obtaining a successful innovation is the change in firms' distribution, which is a general equilibrium outcome. Ultimately, the change in firms' distribution alters the level of complementarity offered by every firm in equilibrium. The dotted line in Panel 2.9a (Interim Pr.) is intended to capture the first component by representing the counterfactual probability of obtaining a successful innovation after the decline in the innovative step size but had remained the firm size distribution as in the period 1985 - 1990. The dashed line (Pr. 10-15) represents the actual probability obtained in the new equilibrium in 2010 - 2015. Thus the difference between the dotted and the dashed line captures the second component. An important and direct consequence of the equilibrium effects of the probability of obtaining a successful innovation can be observed in the rate of creative destruction represented in Panel 2.9b, which becomes decreasing in firm size. As a firm becomes the market leader of more markets, it generates a higher provider-driven complementarity effect. In turn, this becomes a barrier to entry for smaller firms: it is more unlikely that a big firm loses a product through creative destruction.

Most of the effects of the decline in the innovative step size in the standard quality ladder model are preserved under provider-driven complementarity. Specifically, Panel 2.9c shows that the stream of discounted profits of being a market leader also increases; however, under provider-driven complementarity, it increases in firm size. The increase can be separated into two parts. First, there is a symmetric effect across all firms stemming from the equilibrium interest rate decline, precisely as in the standard quality ladder model. Second, a non-linear equilibrium effect is inherited from the decline in the creative destruction rate following the decline in the innovation step size. In other words, as big firms suffer a lower rate of creative destruction, their discounted stream of benefits increases as they expect to remain longer as market leaders. The decline in the innovation

step size affects internal R&D in the same way as in the standard quality ladder model, the *quality effect* still dominates the *market effect*.

However, under provider-driven complementarity, the effects on external R&D are different from those obtained from the standard-quality ladder model. On the one hand, as the probability that an entrant obtains a successful innovation declines sharply, this drives down the incentives to conduct R&D for entrants. On the other hand, the incentives to conduct R&D for an incumbent increase and are also increasing in firm size, following the non-linear increase in the profitability of being an incumbent. In other words, for an entrant, the *quality effect* dominates the *market effect*, while the opposite happens for an incumbent. The asymmetric effect of provider-driven complementarities between entrants and incumbents is key for the results. In particular, entrants do not generate provider-driven complementarity, which implies that not only do they find it difficult to become market leaders but, upon entry, they are more likely to lose their market leadership due to creative destruction. Incumbents, in turn, generate provider-driven complementarity, which not only increases the profitability of all their product lines but drives down the probability of losing a product through creative destruction.

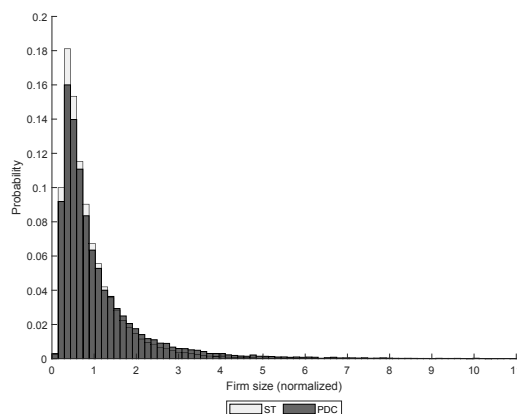
In summary, the standard quality ladder model predicts a decline in internal innovation R&D rates, accompanied by small, almost negligible, effects on external R&D rates. As a consequence, R&D expenditure declines. In contrast, under provider-driven complementarity, the increase in external R&D innovation rates compensates for the decline in internal R&D, leading to an increase in R&D expenditure. As highlighted before, the decline in the innovation step size also affects firms' equilibrium size distribution. In the standard quality ladder model, the small increase in entrants' innovation rate leads to an increase in new firms' entry rate and the equilibrium measure of incumbent firms. Finally, the firm size distribution slightly compresses, which yields a decline in the concentration of sales. Under provider-driven complementarity, I find the opposite. Specifically, the strong decline in the entrants innovation rate leads to a decline in new firms' entry rate, as shown in Table 2.2. Specifically, the mechanism proposed in this paper accounts for roughly a 10% of the decline in the entry rate observed in the data. This decline and the increase in the external R&D innovation rates of incumbents lead to a decline in the number of incumbents in equilibrium. Moreover, as 2.9f shows the firm's size distribution shifts to the right, implying that a substantial share of firms become bigger in equilibrium. In turn, this implies an increase in the concentration of sales as shown in Table 2.2.

2.4.4 Non-targeted moments

The quantitative experiment results show the importance of the asymmetry between entrants and incumbents generated by provider-driven complementarity. In this Subsection, I highlight the performance of the model along several non-targeted dimensions.

As it is well known in the literature, the standard quality ladder model generates tails of the distribution slimmer than the data. Figure 2.10 represents the simulated invariant firm size distribution combined with the quality margin. The provider-driven complementarity framework improves upon the standard quality ladder by generating a thicker right tail of the sale distribution, although the tails of the distribution are still not as fat as in the data.

Figure 2.10: Firm size distribution, standard quality ladder vs. provider-driven complementarity



Note: Normalized quality relative to average quality.

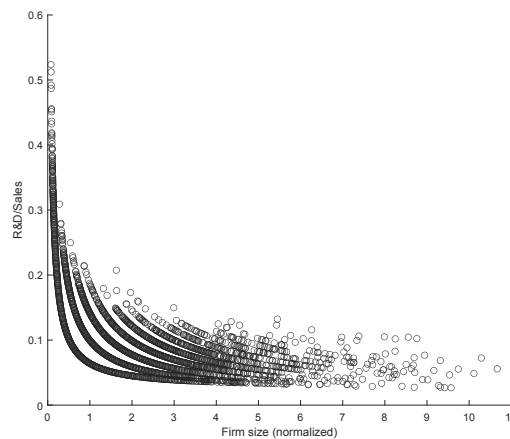
Finally, I review the relationship between R&D intensity and firm size. Recent literature has documented that smaller innovative firms usually exhibit higher R&D intensity on average.²⁹ The preceding Subsections show that bigger firms conduct higher per-product innovation rates (both internal and external) under provider-driven complementarity, even under the assumption of constant returns to scale in the production of external innovations. However, the relationship between R&D intensity and firm size is ultimately determined by the growth of R&D expenditure relative to the growth of sales as the size of firms increase.

Figure 2.11 shows that the provider-driven complementarity framework correctly predicts the declining relationship between R&D intensity and firm size. Recall that provider-

²⁹See, among others, [Akcigit and Kerr \(2018\)](#), [Cavenaile and Roldan-Blanco \(2019\)](#).

driven complementarity is assumed to be increasing in firm size. From (2.6) the optimal quantity supplied is proportional to the level of provider-driven complementarity, thus intermediate sales also are. As the (normalized) firm size increases, the declining relationship between R&D intensity and firm size disappears, resulting in constant returns to scale in the external innovation production function.

Figure 2.11: Relationship between R&D/Sales and firm size



Note: Normalized quality relative to average quality.

2.4.5 Sensitivity

In this Section, I report on a series of sensitivity exercises regarding the strength of provider-driven complementarity. First, to stress the interaction between innovative step size and provider-driven complementarity and its effects on the creative destruction rate, I conduct the baseline exercise using the simplified model in the first sensitivity exercise. In the second exercise, I show how sensitive the main experiment results are to changes in γ , the parameter governing the strength of provider-driven complementarity.

I start by describing the results of the simplified framework. To conduct this sensitivity exercise, I recalibrate all the model's parameters following the same procedure as before, and I conduct the same exercise based on declining the step size of innovation. In the simulation of the model, this step size is assumed to be constant and bigger than the maximum possible value of provider-driven complementarity γ . The results are shown in the first part of Table 2.3.

In the simplified framework, the effect of provider-driven complementarity is always offset by a successful quality increase. This implies that potential entrants, small firms

and big firms are equally likely to obtain a successful innovation that allows them to become the market leader of a variety. In other words, provider-driven complementarity is irrelevant in determining the market leader of each variety. Consequently, provider-driven complementarity affects firms' decisions and outcomes through its effect on the return function, but it does not affect the rate of creative destruction experienced by firms of different sizes. This ultimately implies that the industrial organization of firms is not relevant for the individual decisions of firms. As a consequence, the results of the simplified model are both qualitatively and quantitatively very close to the predictions of the standard quality ladder model, which is in line with the theoretical predictions laid out in Section 2.3. Although there is marginally more entry as the step size innovation declines in the simplified framework, the concentration of sales declines less than in the standard quality ladder model. This is given by the higher skewness of the firm size distribution of firms in the simplified framework, a feature highlighted in Section 2.3. Therefore, even under more entry, the simplified model can sustain a higher level of concentration. This highlights that the key mechanism driving the baseline provider-driven complementarity model results is the interaction of the quality dimension with the complementarity dimension.

The second sensitivity exercise is based on the specification of γ , the parameter governing provider-driven complementarity. Now I report on the sensitivity of the main experiment results to changes in the value of this parameter, which can be understood as varying the strength of provider-driven complementarity. As it was highlighted in the model's calibration, the effect of provider-driven complementarity is not readily measurable in the data. To avoid this complication, I assumed a mild level of provider-driven complementarity by fixing a value $\gamma = 1.0245$. Now I consider the alternative values $\gamma = 1.0215$ and $\gamma = 1.0275$. In each case, I recalibrate all the model's parameters following the same procedure as before and conduct the same exercise based on declining the step size of successful innovation. The results are shown in the second part of Table 2.3. As the strength of provider-driven complementarity declines (increases), the increase in the equilibrium R&D expenditure is smaller (bigger), the entry rates declines less (more), and the share of sales of the biggest firms increases less (more). In summary, the absolute value of the moments of interest is monotone in the value of γ .

It can be shown that as $\gamma \rightarrow 0$, the results converge to those of the standard quality-ladder model. Interestingly, this implies that for low levels of provider-driven complementarity, it is possible to obtain qualitatively aligned results with those of the standard quality ladder model. This is not surprising as, as stressed before, the interaction between

Table 2.3: Sensitivity Analysis Results. Changes in the U.S. between 1985-1990 and 2010-2015.

Simplified vs. Baseline			
Moment	ST	Simplified	PDC
GDPpc growth (p.p.)	-0.99	-0.99	-1.00
(Mean) R&D / Sales (p.p.)	-0.16	-0.15	0.97
(Mean) R&D / Total Cost (p.p.)	-0.17	-0.16	1.01
Entry Rate (p.p.)	0.06	0.05	-0.28
Top 20% Sales (p.p.)	-2.19	-2.07	1.52

Sensitivity with respect to γ			
Moment	PDC	$\gamma = 1.0215$	$\gamma = 1.0275$
GDPpc growth (p.p.)	-1.00	-0.99	-1.00
(Mean) R&D / Sales (p.p.)	0.97	0.70	1.32
(Mean) R&D / Total Cost (p.p.)	1.01	0.73	1.38
Entry Rate (p.p.)	-0.28	-0.17	-0.38
Top 20% Sales (p.p.)	1.52	0.59	2.55

Note. The results for the standard-quality ladder model (ST) in the first part and the provider-driven complementarity model (PDC) in the second are obtained from Table 2.2 and represented here for comparative purposes. In the simplified model the value of γ is 0.0245 as in the baseline provider-driven complementarity model (PDC).

quality and complementarity in determining each market leader is key for the results. If γ declines, even very small quality improvements can be enough to offset the provider-driven complementarity effect, independently of the incumbent's size. As a consequence, as γ declines, provider-driven complementarity becomes less important and ultimately negligible.

2.5 Conclusion

This paper explores the role of provider-driven complementarity as a mechanism to explain several salient facts about declining business dynamism. In particular, the main interest lies in increasing R&D expenditures and concentration yet decreasing entry rates and

economic growth that have taken place during the last two decades.

I propose a theory where provider-driven complementarity makes seemingly independent products become complements when provided by a single firm. In particular, I build on the [Akcigit and Kerr \(2018\)](#) quality ladder model of endogenous growth through R&D, which I extend to explore the effects of this kind of complementarity. The model remains tractable and allows closed-form solutions. The main difference between a standard quality ladder model and the provider-driven complementarity model is that complementary acts as a barrier to entry. In the absence of product complementarities, successful R&D improving the quality of any good enables the innovating firm to displace the previous lower-quality incumbent. However, when firms generate provider-driven complementarity, consumers do not necessarily switch to the state-of-the-art higher quality product but may remain attached to the lower-quality incumbent if the product complementarity effect is sufficiently large. Therefore, small firms need to develop sufficiently ‘radical’ quality improvements so that the products they produce can find their way into the markets.

I then conduct a quantitative experiment motivated by the recent literature arguing that ideas have become harder to find during the last decades. I show that a mild level of provider-driven complementarity can speak to declining business dynamism within this environment. In particular, as innovation gets less radical and the economy’s growth rate declines, the model predicts an increase in R&D expenditure given by the reaction of incumbents, which ultimately yields a decline in the entry rate of new firms and an increase in the concentration of sales. I show that a standard quality ladder model without provider-driven complementarity yields the reverse predictions.

It is worth noting that the theory developed here does not assume limit-pricing, an otherwise frequent assumption in quality ladder models. Limit-pricing opens the door for markup heterogeneity, a widely documented fact in the data. However, allowing for limit-pricing under provider-driven complementarities gives rise to complicated although exciting and novel markup dynamics beyond this paper’s scope. Additionally, the theory does not allow for mergers and acquisitions between firms. However, the provider-driven complementarity is a suitable framework to explain the increased number of mergers and acquisitions observed in the data. Both issues are left for future research.

2.6 Appendix

2.6.1 Data

Data arrangements

I perform the following series of adjustments in the sample of firms obtained from Compustat. First, I restrict the sample to firms that are observed for at least five consecutive years. Second, I restrict my attention to firms with positive sales that report values of Cost of Goods Sold (COGS) and Selling, Administrative and General Expenditure (SGA). Even though I am mainly interested in firms that conduct Research and Development (R&D) activities, I do not exclude firms from the sample that do not conduct R&D. However; I exclude firms that do not report R&D expenditure. I also exclude firms that report a level of R&D expenditure such that its share of R&D as a function of operating expenditure (the sum of COGS and SGA) is above 1.

Aggregate data on R&D expenditure

The increase in R&D can also be observed by using aggregate data from OECD. The next Figure shows that the share of gross domestic expenditure on R&D has increased during the last decades.

Figure 2.12: Share of GDP of Gross domestic expenditure on R&D



Note: Gross domestic expenditure on R&D (GERD) includes expenditure on research and development by business enterprises, higher education institutions, as well as government and private non-profit organisations. Source: OECD.

2.6.2 Proofs and derivations

Proposition 2

Proof. For any distribution of products across firms $\{\mu_n\}_{n \geq 1}$, it is always possible to find a sequence of real numbers $\{i_1, i_2, \dots\}$, such that the $[0, 1]$ product space can be arranged in a way that varieties $j \in [0, i_1)$ are produced by firms with $n = 1$; $i \in [i_1, i_2)$ are produced by firms with $n = 2$; \dots ; and, finally, $i \in [i_{N-1}, 1)$ are produced by very big firms with $n = N$ and $N \rightarrow \infty$. Therefore, one can write

$$\int_0^1 m_j q_j \, dj = \lim_{N \rightarrow \infty} \sum_{n=1}^N \gamma^{1-\frac{1}{n}} \int_{i_{n-1}}^{i_n} q_j \, dj, \quad (2.28)$$

where $i_0 = 0$, $\lim_{n \rightarrow \infty} i_n = 1$, and

$$\bar{q}_n = \int_{i_{n-1}}^{i_n} q_j \, dj$$

denotes the average quality of each subset of products. Following the same argument laid out in the main text, each subset grows at an expected rate

$$g_n = F \mu_n (\lambda_z n z_n + \lambda_x n \tau_n),$$

As the equilibrium satisfies an invariant distribution of firms (see Proposition 7), the implied provider-driven complementarity effect also is. This implies that along the balanced growth path the only source of growth is the increase in quality through R&D. Therefore the law of large numbers assures that the sum in (2.28) asymptotically grows at the same rate as that of average quality, g . ■

Value Functions

Consider an innovator that upon entry would have quality portfolio of size $i \geq 1$ if the incumbent in a given variety has a quality portfolio of size $j \geq 1$. Re-define the indicator function that control which innovations are successful in replacing the incumbent as

$$\xi_{i,j} \equiv \mathbf{1}_{\{\Lambda_x \geq \gamma^{\Delta(i,j)}\}}.$$

For the Simplified framework, $\xi_{i,j} = 1, \forall i, j$. Bellman's principle states that value function for an incumbent firm of size $1 \leq n = |\mathbf{q}_i|$ at some point in time \bar{t} can be obtained as

$$V_{n, \bar{t}-\Delta t}(\mathbf{q}_i, \bar{q}_{\bar{t}-\Delta t}) = \max_{\substack{x_{n, \bar{t}-\Delta t}, \\ \{z_{jn, \bar{t}-\Delta t}\}_{j=1}^n}} \left\{ \sum_{q_j \in \mathbf{q}_i} \left[\pi_{\bar{t}-\Delta t} \gamma^{1-n-1} - \hat{\chi} z_{jn, \bar{t}-\Delta t}^{\hat{\psi}} \right] q_j \Delta t - \tilde{\chi} x_{n, \bar{t}-\Delta t}^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q}_{\bar{t}} \Delta t \right.$$

$$\begin{aligned}
& + e^{-r_{\bar{t}}\Delta t} \left(\sum_{q_j \in \mathbf{q}_i} \left[z_{jn, \bar{t}-\Delta t} \Delta t V_{n, \bar{t}}(\mathbf{q}_i \setminus \{q_j\} \cup_+ \{q_j \Lambda_z\}, \bar{q}_{\bar{t}}) \right. \right. \\
& \quad + \tau_{n, \bar{t}-\Delta t} \Delta t V_{n-1, \bar{t}}(\mathbf{q}_i \setminus \{q_j\}, \bar{q}_{\bar{t}}) \\
& \quad + x_{n, \bar{t}-\Delta t} \Delta t \mathbb{E}_{h, \lambda_x} [\xi_{n+1, j} \{V_{n+1, \bar{t}}(\mathbf{q}_i \cup_+ \{q_h \Lambda_x\}, \bar{q}_{\bar{t}}) - \Omega_n q_j\}] \left. \right. \\
& \quad \left. \left. + \left(1 - \sum_{q_j \in \mathbf{q}_i} [z_{jn, \bar{t}-\Delta t} + \tau_{n, \bar{t}-\Delta t} + x_{n, \bar{t}-\Delta t}] \Delta t \right) V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) \right) \right\}
\end{aligned}$$

where to ease notation I avoid including second or higher order terms in the Poisson arrival rates for internal and external innovation or creative destruction rates. Performing a first-order Taylor expansion of the continuation value yields

$$\begin{aligned}
V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) + & \left[-r_{\bar{t}} V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) + \sum_{q_j \in \mathbf{q}_i} \left[z_{jn, \bar{t}} V_{n, \bar{t}}(\mathbf{q}_i \setminus \{q_j\} \cup_+ \{q_j \Lambda_z\}, \bar{q}_{\bar{t}}) \right. \right. \\
& \left. \left. + \tau_{n, \bar{t}} V_{n-1, \bar{t}}(\mathbf{q}_i \setminus \{q_j\}, \bar{q}_{\bar{t}}) + x_{n, \bar{t}} \mathbb{E}_{h, \lambda_x} [\xi_{n+1, j} \{V_{n+1, \bar{t}}(\mathbf{q}_i \cup_+ \{q_h \Lambda_x\}, \bar{q}_{\bar{t}}) - \Omega_n q_j\}] \right) \right] \Delta t.
\end{aligned}$$

Substituting this expression in the original value function, it remains to divide by Δt both sides, and take limits as $\Delta t \rightarrow 0$ to obtain

$$\begin{aligned}
r_{\bar{t}} V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) = & \max_{x_{n, \bar{t}}, \{z_{nj, \bar{t}}\}_{j=1}^n} \left\{ \sum_{q_j \in \mathbf{q}_i} \left[\pi_{\bar{t}} \gamma^{1-n-1} q_j + \tau_{n, \bar{t}} [V_{n-1, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}} \setminus \{q_j\}) - V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}})] \right. \right. \\
& \left. \left. + z_{nj, \bar{t}} [V_{n, \bar{t}}(\mathbf{q}_i \setminus \{q_j\} \cup_+ \{q_j \Lambda_z\}, \bar{q}_{\bar{t}}) - V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}})] - \hat{\chi} z_{nj, \bar{t}}^{\hat{\psi}} q_j \right] \right. \\
& \left. + n x_{n, \bar{t}} \mathbb{E}_{h, \lambda_x} \left[\mathbf{1}_{\{\Lambda_x \geq \gamma \Delta(n+1, j)\}} [V_{n+1, \bar{t}}(\mathbf{q}_i \cup_+ \{q_j \Lambda_x\}, \bar{q}_{\bar{t}}) - V_{n, \bar{t}}(\mathbf{q}_i, \bar{q}_{\bar{t}}) - \Omega_n \sum_{q_j \in \mathbf{q}_i} q_j] \right] \right. \\
& \left. - \tilde{\chi} x_{n, \bar{t}}^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q}_{\bar{t}} \right\}
\end{aligned}$$

Proposition 4

Proof. Plugging the guess

$$V_n(\mathbf{q}_i, \bar{q}) = A_n \sum_{q_j \in \mathbf{q}_i} q_j + B_n \bar{q},$$

in the value function of an incumbent yields³⁰

$$\begin{aligned}
 rA_n \sum_{q_j \in \mathbf{q}_i} q_j + rB_n \bar{q} = & \max_{x_n, \{z_j\}_{j=1}^n} \left\{ \pi \gamma^{1-n^{-1}} \sum_{q_j \in \mathbf{q}_i} q_j + \sum_{q_j \in \mathbf{q}_i} z_{nj} A_n q_j \lambda_z - \hat{\chi} \sum_{q_j \in \mathbf{q}_i} z_{nj}^{\tilde{\psi}} q_j \right. \\
 & + \tau \left[(n-1)A_{n-1} \sum_{q_j \in \mathbf{q}_i} q_j - nA_n \sum_{q_j \in \mathbf{q}_i} q_j + n(B_{n-1} - B_n) \bar{q} \right] \\
 & + nx_n \left[(A_{n+1} - A_n) \sum_{q_j \in \mathbf{q}_i} q_j + A_{n+1} \bar{q} \Lambda_x + (B_{n+1} - B_n) \bar{q} \right] \\
 & \left. - x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} - nx_n (A_{n+1} - A_n) \sum_{q_j \in \mathbf{q}_i} q_j \right\} + B_n \bar{q} g
 \end{aligned}$$

which trivially reduces to

$$\begin{aligned}
 rA_n \sum_{q_j \in \mathbf{q}_i} q_j + rB_n \bar{q} = & \max_{x_n, \{z_j\}_{j=1}^n} \left\{ \pi \gamma^{1-n^{-1}} \sum_{q_j \in \mathbf{q}_i} q_j \sum_{q_j \in \mathbf{q}_i} z_{nj} A_n q_j \lambda_z - \hat{\chi} \sum_{q_j \in \mathbf{q}_i} z_{nj}^{\tilde{\psi}} q_j \right. \\
 & + \tau \left[(n-1)A_{n-1} \sum_{q_j \in \mathbf{q}_i} q_j - nA_n \sum_{q_j \in \mathbf{q}_i} q_j + n(B_{n-1} - B_n) \bar{q} \right] \\
 & \left. + nx_n [A_{n+1} \bar{q} \Lambda_x + (B_{n+1} - B_n) \bar{q}] - \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \bar{q} \right\} + B_n \bar{q} g
 \end{aligned}$$

On the one hand, collecting terms with \bar{q} yields

$$(r-g)B_n = n\tau(B_{n-1} - B_n) + \max_{x_n} \left\{ nx_n [A_{n+1} \Lambda_x + B_{n+1} - B_n] - \tilde{\chi} x_n^{\tilde{\psi}} n^{\tilde{\sigma}} \right\}. \quad (2.29)$$

The FOC that characterizes external innovation of a firm with size $n \geq 1$ is given by

$$n(A_{n+1} \Lambda_x + B_{n+1} - B_n) - \tilde{\psi} \tilde{\chi} x_n^{\tilde{\psi}-1} n^{\tilde{\sigma}} = 0,$$

where rewriting yields

$$x_n = \left(\frac{A_{n+1} \Lambda_x + B_{n+1} - B_n}{\tilde{\psi} \tilde{\chi} n^{\tilde{\sigma}-1}} \right)^{\frac{1}{\tilde{\psi}-1}}.$$

Define

$$\mathcal{A} \equiv A_{n+1} \Lambda_x + B_{n+1} - B_n,$$

Substituting x_n in (2.29) yields

$$(r-g)B_n = n\tau(B_{n-1} - B_n) + n \frac{\mathcal{A}^{\frac{1}{\tilde{\psi}-1}+1}}{(\tilde{\psi} \tilde{\chi} n^{\tilde{\sigma}-1})^{\frac{1}{\tilde{\psi}-1}}} - \tilde{\chi} \left(\frac{\mathcal{A}}{\tilde{\psi} \tilde{\chi} n^{\tilde{\sigma}-1}} \right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}} n^{\tilde{\sigma}}, \quad (2.30)$$

³⁰Note that

$$\dot{V}_n(\mathbf{q}_i, \bar{q}) = B_n \dot{\bar{q}} = B_n \bar{q} g.$$

and simplifying the RHS gives

$$\begin{aligned}
(r - g)B_n &= n\tau(B_{n-1} - B_n) + \mathcal{A}^{\frac{\tilde{\psi}}{\tilde{\psi}-1}} \left[n \left(\frac{1}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}} \right)^{\frac{1}{\tilde{\psi}-1}} - \tilde{\chi}n^{\tilde{\sigma}} \left(\frac{1}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}} \right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}} \right] \\
&= n\tau(B_{n-1} - B_n) + (\tilde{\psi} - 1)\tilde{\chi}n^{\tilde{\sigma}} \left(\frac{A_{n+1}\Lambda_x + B_{n+1} - B_n}{\tilde{\psi}\tilde{\chi}n^{\tilde{\sigma}-1}} \right)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}} \\
&= n\tau(B_{n-1} - B_n) + \frac{(\tilde{\psi} - 1)}{\tilde{\psi}^{\frac{\tilde{\psi}}{\tilde{\psi}-1}}\tilde{\chi}^{\frac{1}{\tilde{\psi}-1}}n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}-1}}} (A_{n+1}\Lambda_x + B_{n+1} - B_n)^{\frac{\tilde{\psi}}{\tilde{\psi}-1}}.
\end{aligned}$$

Rewriting yields

$$B_{n+1} = B_n - A_{n+1}\Lambda_x + \tilde{\psi}\tilde{\chi}^{\frac{1}{\tilde{\psi}}}n^{\frac{\tilde{\sigma}-\tilde{\psi}}{\tilde{\psi}}} \left(\frac{(\rho + n\tau)B_n - n\tau B_{n-1}}{\tilde{\psi} - 1} \right)^{\frac{\tilde{\psi}-1}{\tilde{\psi}}}.$$

On the other hand, collecting terms with q_j yields

$$rA_n = \pi\gamma^{1-n^{-1}} + \max_{\{z_{nj}\}} \left\{ z_{nj}A_n\lambda_z - \hat{\chi}z_j^{\hat{\psi}} \right\} + \tau[(n-1)A_{n-1} - nA_n]. \quad (2.31)$$

The FOC that characterizes internal innovation is given by

$$A_n\lambda_z - \hat{\chi}\hat{\psi}z_{nj}^{\hat{\psi}-1} = 0,$$

where rewriting yields

$$z_{nj} = \left(\frac{A_n\lambda_z}{\hat{\chi}\hat{\psi}} \right)^{\frac{1}{\hat{\psi}-1}}.$$

Substituting this expression in (2.31) obtain

$$(r + \tau n)A_n = \pi\gamma^{1-n^{-1}} + \hat{\chi}(\hat{\psi} - 1) \left(\frac{A_n\lambda_z}{\hat{\chi}\hat{\psi}} \right)^{\frac{\hat{\psi}}{\hat{\psi}-1}} + \tau(n-1)A_{n-1}.$$

■

Proposition 5

Proof. By induction, I first check it holds for $n = 1$ and $n = 2$. For $n = 1$

$$\mu_1 = \frac{x_e}{F\tau_1},$$

which is the same condition obtained from the first flow equation. For $n = 2$

$$\mu_2 = \frac{1}{2} \frac{x_e}{F} \frac{x_1 x_2}{x_2 \tau_1 \tau_2} = \frac{1}{2} \frac{x_e}{F} \frac{x_1}{\tau_1 \tau_2},$$

where re-arranging and adding and subtracting $x_e\tau_1$ in both sides yields

$$F\mu_2 2\tau_2\tau_1 + x_e\tau_1 = x_e(x_1 + \tau_1).$$

Dividing through by τ_1 write

$$F\mu_2 2\tau_2 + x_e = \frac{x_e}{\tau_1}(x_1 + \tau_1) = F\mu_1(x_1 + \tau_1),$$

which is the same condition obtained from the second flow equation.

Now suppose it holds for n and $n - 1$, and prove it also holds for $n + 1$. In this case

$$\mu_{n-1} = \frac{1}{n-1} \frac{x_e}{F x_{n-1}} \prod_{s=1}^{n-1} \frac{x_s}{\tau_s},$$

and

$$\mu_n = \frac{1}{n} \frac{x_e}{F x_n} \prod_{s=1}^n \frac{x_s}{\tau_s}.$$

From the third flow equation write

$$F\mu_{n+1}(n+1)\tau_{n+1} + F \frac{1}{n-1} \frac{x_e}{F x_{n-1}} \prod_{s=1}^{n-1} \frac{x_s}{\tau_s} (n-1)x_{n-1} = F \frac{1}{n} \frac{x_e}{F x_n} \prod_{s=1}^n \frac{x_s}{\tau_s} n(\tau_n + x_n),$$

where simplifying, re-arranging and multiplying and dividing in the RHS by x_{n+1} yields

$$\mu_{n+1} = \frac{1}{n+1} \frac{x_e}{F x_{n+1}} \prod_{s=1}^{n+1} \frac{x_s}{\tau_s}.$$

■

2.6.3 Extension: Limit-pricing

In this Appendix I explore an alternative environment that relaxes Assumption 1. In particular, I extend the model by allowing for limit-pricing and endogenous markups, in a similar fashion as in (Peters, 2018). This Section shows, under very restrictive assumptions, how to introduce provider-driven complementarity in an otherwise standard limit-pricing framework of quality ladder models.

Consider the final good production function (as opposed to (2.2))

$$\ln Y_t = \int_0^1 \ln \left[\left(\sum_{i \in \mathcal{F}} [m_{ijt} q_{ijt} k_{ijt}]^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right] dj, \quad (2.32)$$

where for simplicity labor is not a factor of production. Assuming $\theta = \infty$ implies that, once adjusted by quality and provider-driven complementarity, different vintages of each

variety j supplied by different firms $i \in \mathcal{F}$ are perfect substitutes. This implies that only the variety with the highest adjusted-quality to price ratio will be demanded in equilibrium. We can therefore reduce the previous expression to

$$\ln Y_t = \int_0^1 \ln(m_{jt}q_{jt}k_{jt}) dj.$$

Normalizing $P_t^Y = 1, \forall t$, and dropping time subscripts (just to ease notation), the solution to the final good producer problems yields the inverse demand function

$$p_{jt} = \frac{Y_t}{k_{jt}}.$$

Assume production of intermediates is linear in labor, that is $k_{jt} = l_{jt}$, thus the marginal cost of production is the wage rate w_t and is constant across all varieties. Furthermore, assume that intermediate producers compete à la Bertrand. As (2.32) exhibits unitary demand elasticity of substitution, these assumptions imply that it is optimal for the market leader will to limit-price its closest competitor. Suppose that the current market leader in variety j has size n_j and can produce variety j with quality q_j , while its closest follower has size n_{-j} and can produce variety j with quality q_{-j} . The markup set by the market leader of variety j is

$$\mu_{jt} = \frac{\gamma^{1-\frac{1}{n_j}} q_j}{\gamma^{1-\frac{1}{n_{-j}}} q_{-j}} = \gamma^{\Delta(n_j, n_{-j})} \Lambda_j,$$

where $\Delta(n_j, n_{-j}) \equiv (n_j - n_{-j})/n_j n_{-j}$ and $\Lambda_j \equiv q_j/q_{-j}$. This implies that the equilibrium price of variety j is

$$p_{jt} = \gamma^{\Delta(n_j, n_{-j})} \Lambda_j w_t.$$

Finally, per-variety profits are given by

$$\pi_{jt}(\Delta(n_j, n_{-j}); \Lambda_j) = p_{jt}k_{jt} - w_t k_{jt} = (p_{jt} - w_t)k_{jt} = \left(1 - \frac{1}{\gamma^{\Delta(n_j, n_{-j})} \Lambda_j}\right) Y_t.$$

Consequently, the market leader of each product needs to keep track of the size of its closest follower and its quality, as both jointly determine the equilibrium price and ultimately the profitability of each product. Note that contrary to the general framework analyzed in the main text, any change in a follower's size leads to a change in the equilibrium markup. Consider the following example. Suppose that there are 3 firms in the economy, denoted as A , B and C , and a total of 6 varieties. Without loss of generality, suppose that firm A is the market leader of 4 varieties, B is the market leader of the remaining 2, and C is a potential entrant. Now, suppose that C obtains a successful innovation over

one of the products of B and becomes the market leader in that variety. In the baseline framework, the profitability of firm A would not change - only its incentives to conduct R&D through due to the change in the firm size distribution. However, with limit pricing, profitability changes, as now a follower has 1 product less, which implies that A can now charge a higher price. This example highlights that creative destruction outcomes with limit-pricing and provider-driven complementarity are much richer, but more complicated than in the baseline model.

In the remainder of this Section, I characterize the value function of an incumbent firm. I start by assuming that the innovation technology is now characterized by a general cost function $\Gamma(\{x_n, z_{nj}\}_{j=1}^n)$ where x_n and z_{nj} are the per-product external and internal innovation rates. If successful, quality improves as in the general framework, i.e. a firm can produce any randomly drawn product (not previously owned) with incremental quality $q_{jt+\Delta t} = q_{jt}(1 + \tilde{\lambda}_x) \equiv q_{jt}\Lambda_x$, with $\tilde{\lambda}_x > 0$ drawn from an exponential distribution with parameter λ_x^{-1} . This allows to characterize the probability in the exact same way as in the general framework. Incumbent firms are subject to a rate of creative destruction τ_n which is size-dependent, and is also defined as in the general framework. Finally, I assume that the market leader of each variety can also improve the quality distance with respect to its closest competitor by investing in internal R&D. If successful, this firm can produce variety j with quality $q_{jt+\Delta t} = q_{jt}(1 + \tilde{\lambda}_x) \equiv q_{jt}\Lambda_x$, with $\tilde{\lambda}_x > 0$, so that the quality jump with respect to its closest competitor is now given by $\Lambda_j(1 + \lambda_x)$.

The state of a size n incumbent firm and is determined by its size, the collection of size gaps with respect to each of its followers $\mathbf{\Delta} \equiv \{\Delta(n_j, n_{-j})\}_{j=1}^n = \{\Delta n_{-j}\}_{j=1}^n$,³¹ and the corresponding quality jump in each variety $\mathbf{\Lambda} \equiv \{\Lambda_j\}_{j=1}^n$. Following the notation laid out in the characterization of the baseline model, the value function of such an incumbent is now given by

$$\begin{aligned} rV_n(\mathbf{\Delta}; \mathbf{\Lambda}) &= \sum_{j=1}^n \pi_j(\Delta n_{-j}; \Lambda_j) \\ &+ \sum_{j=1}^n \tau_n \left[V_{n-1}(\mathbf{\Delta} \setminus \{\Delta(n_{-j})\}; \mathbf{\Lambda} \setminus \{\Lambda_j\}) - V_n(\mathbf{\Delta}; \mathbf{\Lambda}) \right. \\ &+ \sum_{i \neq j} \left(V_{n-1}(\mathbf{\Delta} \setminus \{\Delta n_{-j}\} \cup_+ \{\Delta n_{-j} - 1\}; \mathbf{\Lambda}) - V_n(\mathbf{\Delta}; \mathbf{\Lambda}) \right) \left. \right] \\ &+ \sum_{j=1}^n \tau_{n-j} \left[V_n(\mathbf{\Delta} \setminus \{\Delta n_{-j}\} \cup_+ \{\Delta n_j + 1\}; \mathbf{\Lambda}) - V_n(\mathbf{\Delta}; \mathbf{\Lambda}) \right] \end{aligned}$$

³¹In the latter notation, $\Delta n_{-j} = n - n_{-j}$, i.e., a firm only needs to track the its size difference with its closest competitor in each variety.

$$\begin{aligned}
& + \sum_{j=1}^n n_{-j} \tilde{x}_{n-j} \mathbb{E}_i [\xi_{n-j+1, i}] \left(V_n(\Delta \setminus \{\Delta_{n-j}\} \cup_+ \{\Delta_{n-j} - 1; \Lambda\}) - V_n(\Delta; \Lambda) \right) \\
& + \max_{\{z_{nj}, x_n\}_{j=1}^n} \left\{ \sum_{j=1}^n z_{nj} \left(V_n(\Delta; \Lambda \setminus \{\Lambda_j\} \cup_+ \{\Lambda_j(1 + \lambda_x)\}) - V(\Delta; \Lambda) \right) \right. \\
& + n x_n \left(\mathbb{E}_i \left[\xi_{n+1, i} \left(V_{n+1}(\tilde{\Delta} \cup_+ \{\Delta_{n-i}\}; \Lambda \cup_+ \{\Lambda_i\}) \right) \right] - V_n(\Delta; \Lambda) \right) \\
& \left. - \Gamma(\{x_n, z_{nj}\}_{j=1}^n) \right\} + \dot{V}_t(\Delta; \Lambda)
\end{aligned}$$

where

$$\xi_{n, i} \equiv \Pr(1 + \lambda_x \geq \gamma^{\Delta(n, i)}),$$

and

$$\tilde{\Delta} \equiv \Delta \setminus \{\Delta_{n-j}\}_{j=1}^n \cup_+ \{\Delta_{n-j} + 1\}_{j=1}^n.$$

The value of a firm with size n and portfolio (Δ, Λ) is characterized by the following elements. The first line captures the instantaneous profits for each variety in which the firm is the outstanding market leader. The second and third lines capture two direct effects of creative destruction on the value of the firm. On the one hand, the second line captures the standard loss of a variety through creative destruction. On the other hand, the third line captures the fact that the firm must re-adjust its limiting-price in all the remaining varieties after that loss, as now it generates less provider complementarity. In this framework, creative destruction also affects the firm's profitability through the effects on the products of the incumbents. In this sense, the fourth and fifth lines capture two indirect effects of creative destruction. Specifically, the fourth captures the firm's increase in profits if any of the followers in each product line loses one product due to creative destruction of a third firm. Moreover, the fifth line represents the decline in the firm's profits if any of the followers in each product line increases its product portfolio at the expense of a third firm. The remaining three lines characterize the R&D decisions of the firm. The sixth line captures the change in value if internal innovation is successful. The sixth line captures the increase in value due to external innovation. If the firm's R&D is successful, it improves the quality of any product i and becomes the new market leader of that variety. Moreover, it can increase its profits in all the remaining product lines as now it generates more complementarities. Given that R&D is undirected, the quality of the new product is unknown and is captured by the expected value \mathbb{E}_i , which is an expectation over varieties –which determine the size of the current market leader in variety i – and innovation step Λ_i . The quality jump has to be sufficiently big to

offset the incumbent's provider-driven complementarity to become the new producer. The probability term again captures this.

The analysis of the value function highlights that this framework allows for richer firm dynamics, but it hinders the tractability of the model. I leave this interesting avenue for further research.

2.6.4 Solution algorithm

I solve the generalized framework as a nested fixed point over the balanced growth path rate of the economy, along which the equilibrium relationship between \bar{q} and \bar{Q} remains constant. The algorithm follow the next steps:

1. Guess $M > 1$ such that along the balanced growth path, $\bar{Q} = M\bar{q}$.
 - (a) Guess a growth rate g .
 - i. Guess a firm size distribution and a sequence of creative destruction rates $\{\tau_n\}$.
 - ii. Given the distribution, characterize the probability sequence of successful innovation, i.e., the probability of a firm of each size to obtain a successful innovation conditional on the distribution of incumbents.
 - iii. Solve for the sequences $\{A_n\}$, $\{B_n\}$, $\{x_n\}$ and $\{z_n\}$ in Proposition 6.³²
 - iv. Compute the implied firm-size distribution and creative destruction rates. If they are the same as the initial guess, go to the next step. If not converged, go back to step i. by using as new guess a linear interpolation between the old and new guess.
 - (b) Compute the implied growth rate of the economy, if not converged, go back to (a) and update the guess until convergence.
2. Simulate an economy and verify that $\bar{Q} = M\bar{q}$, otherwise, go back to 1. and update the guess.

To simulate the model, I assume an economy composed by 50,000 varieties, with initial quality normalized to 1. Moreover, I assume that initially, each good is produced by a single firm. I simulate the economy for 2,000 periods of length $\Delta t = 0.1$. As time is

³²The critical step is solving for the sequence $\{B_n\}$. As provider-driven complementarities vanish asymptotically, and the external R&D cost function will be assumed to have constant returns to scale, as n grows, B_n must converge to a line.

continuous and the number of varieties is kept constant, at each simulated instant, I draw a vector of shocks that determines whether there could be entry in a new variety (a new firm obtains a quality innovation for this variety), external innovation (an incumbent in a different variety obtains a quality innovation) or internal innovation (the incumbent obtains a quality innovation). If any of the two first events happens, a quality jump is drawn from the exponential distribution, and the new market leader on the variety is chosen.³³ Over time, this fixed size economy converges to its balanced growth path where all aggregate varieties grow at the rate g and the firm size distribution is constant.

2.6.5 Robustness

In this Appendix, I perform a series of alternative experiments to the baseline quantitative analysis carried out in Section 2.4. Specifically, I recalibrate all the model's parameters following the same procedure as described in the main text, and I conduct the same exercise based on declining the step size of innovation.

Returns to scale in external R&D

I start by reviewing the relationship between $\psi > 0$ and $\sigma > 0$ in the external R&D production function, which determines the returns to scale the production function. In the baseline experiment I impose $\psi + \sigma = 1$, so that the production function exhibits constant returns to scale as in (Klette and Kortum, 2004). In this Section, I choose $\psi + \sigma < 1$ so that it exhibits decreasing returns to scale as in (Akcigit and Kerr, 2018). Specifically, the value of ψ remains as in the baseline exercise but following (Akcigit and Kerr, 2018) I fix a value of $\sigma = 0.395$. The results are summarized in Table 2.4.

Allowing for decreasing returns to scale in the external innovation production function does not change the qualitative nature of the baseline quantitative analysis results. In the absence of provider-driven complementarity, any assumption regarding the cost function's curvature yields the same results. However, with decreasing returns to scale, the innovative step size decline is strengthened if firms generate provider-driven complementarity. The reasons underlying these results are the same as in the baseline experiment, i.e., when the innovative step size declines, the probability of obtaining a successful innovation also does. As potential entrants do not generate provider-driven complementarity until their product portfolio grows, they now find it more challenging to become market leaders. Under decreasing returns to scale, it is more expensive to grow through external R&D

³³In the unlikely event of ties, equal probabilities are assigned to both firms, and a coin is tossed to decide the market leader.

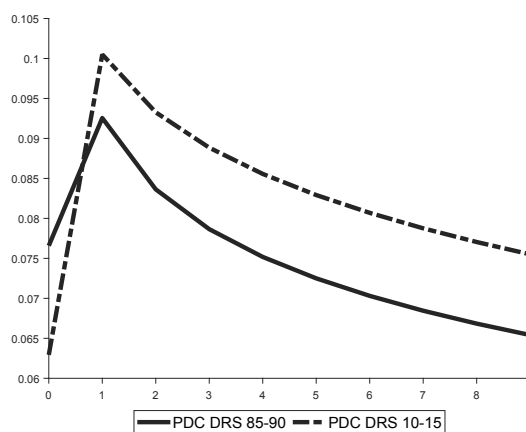
upon entry. Therefore, a stronger effect of provider-driven complementarity is needed to obtain comparable results to the baseline. In that case, even though big incumbents are subject to a relatively higher cost, they find that compensated through the effect of provider-driven complementarity, which increases the profitability of all their product lines and drives down the probability of losing a product through creative destruction for an incumbent firm.

Table 2.4: Robustness: Returns to scale in R&D production function.

Moment	Baseline		DRS	
	PDC	ST	PDC	ST
GDPpc growth (p.p.)	-1.00	-0.99	-0.98	-1.00
(Mean) R&D / Sales (p.p.)	0.97	-0.16	1.24	-0.19
(Mean) R&D / Total Cost (p.p.)	1.01	-0.17	1.30	-0.18
Entry Rate (p.p.)	-0.28	0.06	-0.64	0.07
Top 20% Sales (p.p.)	1.52	-2.19	2.12	-2.29

Note. DRS: Decreasing returns to scale. PDC: Provider-driven complementarity model. ST: Standard quality ladder model. Columns 1 and 2 are obtained from Table 2.2.

Figure 2.13: Per-product external innovation rates with decreasing returns to scale



Note: x -axis represents number of products - 0 denotes a potential entrant.

It is instructive to look at the profile of per-product innovation rates as a function of size, represented in Figure 2.13. As the product portfolio of a firm increases, the probability of losing a product through creative destruction declines, which increases

the profitability of being an incumbent. Consequently, even if the external innovation production function exhibits decreasing returns to scale when the innovative step size declines, bigger firms increase their external innovation rates relatively more with respect to smaller firms. As a final remark, in the main text, I show that the provider-driven complementarity framework correctly predicts the declining relationship between R&D intensity and firm size, even assuming constant returns to scale. As under decreasing returns to scale, the innovation rates decline with size, this relationship is only reinforced.

Relationship between internal and external R&D

The baseline quantitative analysis's calibrated values imply that around 55% of total growth comes from external innovation, while 20% comes from internal innovation and 25% is due to the entry of new firms. While these results are comparable to those of [Akcigit and Kerr \(2018\)](#), in this Section, I consider an alternative calibration where internal R&D is the main source of growth, accounting for 60% of total growth. This is similar to the results of [Garcia-Maza et al. \(2019\)](#), which show that even up to 70% of growth can stem from internal innovation.

Table 2.5: Robustness: Relationship between internal and external R&D.

Moment	Baseline		Ext. < Int.	
	PDC	ST	PDC	ST
GDPpc growth (p.p.)	-1.00	-0.99	-1.00	-0.99
(Mean) R&D / Sales (p.p.)	0.97	-0.16	0.68	-0.38
(Mean) R&D / Total Cost (p.p.)	1.01	-0.17	0.71	-0.40
Entry Rate (p.p.)	-0.28	0.06	-0.10	0.27
Top 20% Sales (p.p.)	1.52	-2.19	2.53	-1.21

PDC: Provider-driven complementarity model. ST: Standard quality ladder model. Columns 1 and 2 are obtained from [Table 2.2](#).

[Table 2.4](#) shows that the qualitative nature of the results is preserved. Relative to the baseline exercise, when internal innovation is the main channel of firm growth, the decline in the innovative step size of innovation generates a stronger increase in the entry rate in the standard quality ladder model and a milder decline in the entry rate in the provider-driven complementarity framework. This happens because in this exercise conducting external R&D becomes relatively more expensive (than in the baseline), which favors potential entrants relative to active incumbents.

Focusing on the provider-driven complementarity framework, as the innovative step size declines the probability of obtaining a successful innovation declines and an incumbent firm's profits increase through the same mechanism described in the main text. However, incumbent firms react by increasing less their external R&D rates as it is relatively costlier now. This observation has two important implications. On the one hand, the firms' equilibrium distribution shifts less to the right than in the baseline exercise, i.e., firms become bigger, but less so than in the baseline. Consequently, although probability of obtaining a successful innovation declines, it declines less than in the baseline, specifically for potential entrants, so that the entry rate declines less than in the baseline. On the other hand, the R&D expenditure of firms increases less than in the baseline. Interestingly, even though quantitatively, these results are milder than in the baseline, the concentration of sales in the economy increases. This happens because in this economy, firms conduct more internal R&D than in the baseline. Therefore, even though the distribution of firms shifts less to the right, big firms in this alternative economy can concentrate a higher share of the total sales of the economy as they innovate more in internal R&D, which ultimately implies that on average bigger firms concentrate a larger amount of higher quality, more complementary – and ultimately, more expensive – products.

Increasing trend in markups

In the baseline quantitative exercise, firms' markups are assumed to be constant. However, recent papers as Barkai (2020), De Loecker et al. (2020b) or Feijoo-Moreira (2020) show that markups have increased during the last decades and are important to explain recent macroeconomic trends. In this Section, I consider an alternative calibration where markups increase by 25% between 1985-1990 and 2010-2015. The results are summarized in Table 2.6.

Introducing an increasing trend in markups simultaneously that the step size of innovation declines reinforces the results obtained in the baseline framework. This happens because in this economy, the profits associated with being an incumbent increase not only as a consequence of the decline in the growth rate of the economy – and thus the equilibrium interest rate – but also because now firms can charge higher markups. Consequently, incumbent firms have a much higher incentive to conduct external R&D to increase their quality portfolio. However, as in the baseline exercise the reaction of potential entrants is different between the standard quality ladder model and the provider-driven complementarity framework. In the standard quality model, potential entrants invest relatively more in R&D to obtain market leadership. Together with the increase in the external

R&D rates of incumbents, the aggregate expenditure in R&D increases. Since there are more firms in the economy, the concentration of sales declines. In contrast, the opposite happens in the provider-driven complementarity framework. The strong increase in the external R&D of incumbents increases not only the creative destruction rate suffered by small firms relative to big firms, but also drops down the probability of obtaining a successful innovation. The consequence is a strong decline in the entry rate of new firms in the economy. Moreover, this increasing R&D effort of incumbents is reflected in a stronger increase in R&D expenditure. Finally, the combination of declining entry and the shift to the right of the size distribution implies an increase in sales concentration.

Table 2.6: Robustness: Declining step size of innovation with increasing trend in markups

Moment	Baseline		Increasing	
	PDC	ST	PDC	ST
GDPpc growth (p.p.)	-1.00	-0.99	-0.99	-1.00
(Mean) R&D / Sales (p.p.)	0.97	-0.16	1.55	0.69
(Mean) R&D / Total Cost (p.p.)	1.01	-0.17	2.17	0.23
Entry Rate (p.p.)	-0.28	0.06	-1.31	2.55
Top 20% Sales (p.p.)	1.52	-2.19	2.98	-3.41

PDC: Provider-driven complementarity model. ST: Standard quality ladder model. Columns 1 and 2 are obtained from Table 2.2.

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