Essays in Family Economics

by

Francisco Javier Rodríguez Román

A dissertation submitted in partial fulfilment of the requirements

for the degree of Doctor of Philosophy in Economics

Universidad Carlos III de Madrid

Advisor:

Matthias Kredler

April 2021

To my parents.

ACKNOWLEDGEMENTS

First and foremost I thank Matthias for his excellent guidance and support throughout this process. I also received valuable feedback and help from Andrés Erosa, Luisa Fuster, Felix Wellschmied, Juanjo Dolado, Emir Yurdagul, Ludo Visschers and the rest of the faculty at Carlos III. The second chapter of this thesis was written together with Lidia Cruces, who also deserves a huge thank you. I owe much of my sanity to the graduate students with whom I shared this process: Rubo, Isma, Miche, Tomás, Bea, Liza, Salvo, Fede Curci, Fede Masera, Jose, Iván, Belén, Álvaro and the rest. I must thank my family, especially my parents Francisco and Ana and my siblings Rubén and Valeria, and all the other friends that have been together with me from the distance.

Contents

List of Figures

List of Tables

Chapter 1

The Sex Ratio, Marriage and Bargaining: A Look at China

1.1 Introduction

Between 1990 and 2010, weekly hours devoted to paid work and to housework decreased by 14% and 47% respectively among young (20 to 35 years old) married women in China, while paid work remained constant and housework decreased much less for their male counterparts. As a result, the female-to-male leisure ratio for young married people increased. For singles however this ratio remained constant.

This chapter studies how changes in the sex ratio could be partly responsible for these trends. One of the most striking instances of the sex ratio deviating from its natural level has been going on in China since the 1980s. Census data paints a clear picture of an increasing gap between the number of males and females. Table 1.1 shows the sex ratio at birth for the last four census years. It goes up from an already somewhat high 1.085 boys per girl in 1982 to almost 1.2 in 2010. Naturally this imbalance does not have an immediate impact the marriage market, but does so 20 to 30 years ahead, when people in those cohorts start getting married.

In Figure, 1.1 the data from the 2000 census is projected both backward and forward to obtain sex ratios for marriageable-age population (individuals aged 20 to 35 years old)[1]. As can be observed, there was an increase between 1990 and 2010, and the imbalance is going to keep widening all the way to 2020.

The underlying causes of China's skewed sex ratio at birth, although fascinating in their own right, are

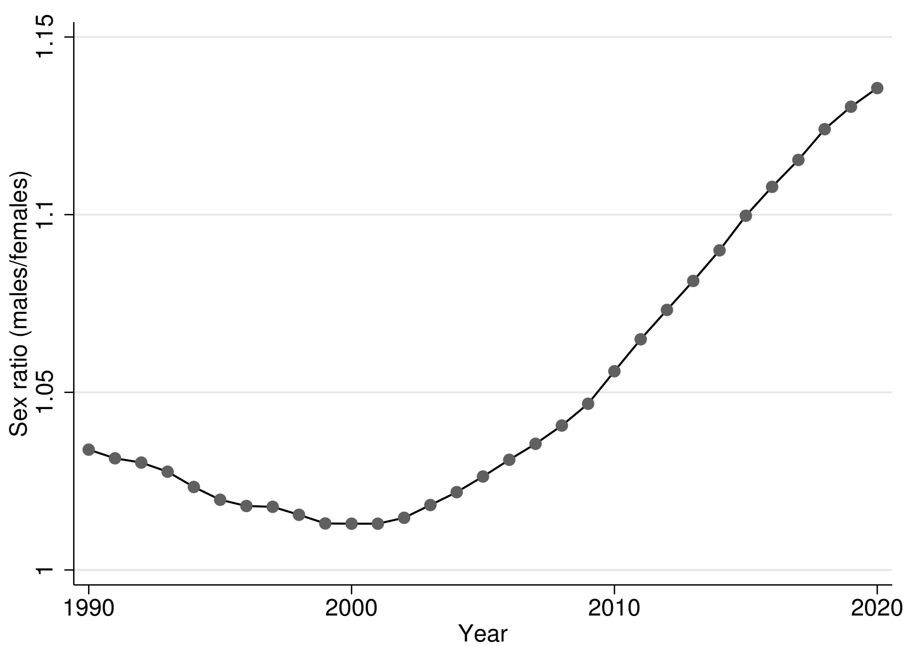
[1]The reader might notice that the sex ratios at birth are higher than the sex ratios for the marriageable age population 20 years later up to 2000. There are intercensal inconsistencies and other issues around the measurement of the sex ratio. However, although there is disagreement on the real scale of the imbalance, it seems clear that an increase has indeed occurred: "the recorded sex imbalance is not a statistical artefact, but a reality", see Cai, 2013.

Table 1.1: Sex ratio at birth (male births over female births), 1982-2010

Year	Sex ratio at birth
1982	1.085
1990	1.113
2000	1.169
2010	1.179

Source: Tabulation on the Population Census of the People's Republic of China, National Bureau of Statistics.

Figure 1.1: Projected sex ratio in China for population aged 20-35, 1990-2020



Source: Author's work using data from the 2000 Population Census of the People's Republic of China.

not explored in this chapter. Demographics are taken as exogenous and focus instead on analysing the effects that the surplus of men has on marriage market outcomes and the allocation of resources by couples within the household.

Conditions in the marriage market affect the outside option of getting married. If the sex ratio goes up, the value of remaining single for a woman increases, as she is more likely to meet potential spouses, while the opposite is true for men. Therefore, the bargaining position of women improves when negotiating the allocation of resources within marriage. Moreover, the sex ratio affects the types of households that are formed, i.e. the degree of assortative mating and female/male hypergamy (when females/males marry a spouse of higher status then themselves), as women become choosier-and men less so-when there is an excess of males.

To study the mechanisms mentioned above, a model is developed that allows for agents to be heterogeneous based on their education. People are divided into three skill groups according to their education attainment: low (primary or less), medium (high school) and high (college). Moreover, in the model marriage decisions are endogenous and bargaining between spouses determines resource allocation. The bargaining positions of women and men depend on the conditions in the marriage market.

To avoid tractability problems that emerge when studying two-sided search models with heterogeneous agents, while keeping marital sorting endogenous, marriage markets are structured in two stages. People may meet potential spouses of different types at the beginning of their life as singles, while in subsequent periods they only meet people of their own type. This is as an extension of the structure used in Fernández et al. (2005). In terms of time allocation, the model works in a very similar fashion as the one in Knowles (2013). People allocate time between three competing uses: paid work, housework, and leisure. Through paid work households generate resources to purchase a private consumption good and home equipment. A combination of the latter and housework time yields a home-produced good. Married households decide via bargaining on how to allocate time and split the private consumption good. As mentioned earlier, the sex ratio affects the decisions of married couples as it increases the outside value of wives.

The model is calibrated so that in steady-state equilibrium the time allocation and marital sorting moments match their data counterparts for China in 1990. Indeed, the model is able to reproduce the data almost perfectly for that year. Next the sex ratio, the distribution of people across skill groups, the wage structure, and the parameters associated to home production are changed one at a time to their 2010 values. The model does a good job replicating qualitatively the changes in time allocation and marital sorting that occurred between 1990 and 2010. In the model, as in the data, paid work time decreases and leisure increases for married women, while the time allocation does not change very much for married men. The degree of assortative mating increases both in the model and in the data, although more so in the latter.

A decomposition exercise reveals that the increase in the sex ratio alone accounts for 38%-52%[2] of the decrease in paid work time and for 26%-47% of the increase in leisure for married women between 1990 and 2010. Moreover, it accounts for 18%-61% of the increase in assortative mating. Decomposing further the effect of the sex ratio on married people's time allocation shows that most of it operates via bargaining, and very little of it through the changes in marital sorting (different households being formed as a result of the excess men in the marriage market).

As the skill distribution changed substantially between 1990 and 2010, the effects of the changes in the sex ratio were not felt equally among all skill groups. In the model, the sex ratio among singles in steady state equilibrium increased the most between 1990 and 2010 for low and medium-skilled people, while actually decreased for the high skilled ones. This happened for two reasons. First, the skill distribution became more equal across genders, with the fraction of women with high skill actually overcoming that of men's in 2010. Second, the fact that it is quite common that women marry men with higher skill than themselves (while the opposite is very rare) means that women in the marriage markets face competition from women with lower skill levels. Similarly, men face competition from higher skilled men. Therefore, when the sex ratio increases, more women marry up and more men marry down. However, that is not possible for women in the highest skill group and for men in the lowest. In the model, women in the first two skill groups experience the largest decrease in paid work and increase in leisure.

Finally, the model implies that the increase in the sex ratio leads to 7%-10% higher private consumption for married women, at the expense of a decrease for married men of 4.5%-8.5%.

This chapter contributes to several lines of research. First, the literature on marriage. Economists have been interested in it since the seminal work by Becker (1973, 1974). Within the marriage markets literature, one strand may be described as a general equilibrium approach as in Chiappori and Weiss (2006), and the other as the search frictions approach, as in Greenwood et al. (2016). The model developed here belongs to the latter. Moreover, this paper speaks to the growing literature on the importance of intra-household bargaining on aggregate outcomes, of which Knowles (2013) is a prominent example, and a forerunner of this paper. The main contribution to this literature is that this chapter studies steady state equilibria with heterogeneous agents, while the literature in general avoids having both at the same time.

Secondly, the paper adds to the substantial literature that explores the effect sex ratios have on various socio economic outcomes. Most of these papers are empirical and employ reduced-form specifications to capture local effects. A classic example is Angrist (2002). He exploits the variation in sex ratios among different ethnic groups of immigrants in the United States, and finds that higher sex ratios increase the

[2]The percentages of the changes that are explained by the sex ratio are given as ranges because the model is not linear and therefore the results vary with the order of the decomposition. This is discussed more in depth in section 1.5.

likelihood of marriage and decrease the labour force participation of females. Grosjean and Khattar (2018) report similar findings for Australia, where sex ratios were heavily male-biased historically as a result of the British policy of sending convicts there during the 19th century. Moreover, they find that the effect is persistent, i.e. women work fewer hours outside the house in areas where the sex ratio was more male-biased, even today with a normal sex ratio. Abramitzky et al. (2011) present evidence on the improvement of the position of males in the marriage market due to their relative scarcity due to WWII in France. In regions where male mortality was higher, marriage rates went up for men and down for women. In addition, males in those areas were less likely to marry females of lower social classes. For the case of China, Wei and Zhang (2011) explore the hypothesis that the rising sex ratio in China creates a competitive savings motive for parents with a son, increasing the household savings rate.

Two papers stand at the intersection of marriage literature and the socio economic effects of variations in the sex ratio. Seitz (2009) develops a dynamic model of marriage and labour force participation to assess the extent to which differences in those outcomes between blacks and whites in the United States are due to lower sex ratios among the former. There are however some important differences between the approach in that paper and mine. On one hand, she has the advantage of having a detailed panel data set to estimate the structural parameters of her model. On the other hand, this study relies on a calibration strategy using aggregate moments. Moreover, by comparing two populations, Seitz (2009) has the advantage of being able to use the current and future sex ratios of whites when performing counterfactual exercises for blacks. There is not an obvious way of pinning down expectations on future sex ratios in the counterfactual exercises presented here, which generates an important additional challenge.

Wang (2018) provides a quantitative model with imbalanced sex ratios, marriage, divorce and intra-household bargaining in China. The main result is on welfare: females are 9.5% better off in consumption units and men are 14.42% worse due to the high sex ratio. There are also some important differences between this paper and mine. First, the author does not include housework in the model, as is done here. This is important, since a reduction in paid work does not mean that married women are enjoying more leisure. Secondly, the two-sided search model with heterogeneous agents in that paper features endogenous expectations about the distribution of education among singles, which make the model's equilibrium properties hard to assess. In this chapter, this is avoided with the two-stage marriage markets discussed earlier. Incidentally, this structure allows the model to better capture the marital sorting patterns.

The rest of this chapter is organized as follows. Section 1.2 discusses the data and lays out some facts about relevant changes that China experienced during the two decades between 1990 and 2010. Section 1.3 is devoted to describe the model. In section 1.4, the calibration strategy is explained and the model fit showcased. Section 1.5 is devoted to the decomposition and counterfactual exercises and its results. Section

1.6 concludes.

1.2 Data and empirical facts

This section and the rest of this chapter rely mainly on the various waves of the China Health and Nutrition Survey (CHNS)[3] and the findings of Ge and Yang (2014) on the changes in China's wage structure. A detailed description of the data can be found in appendix 1.A.

In what follows a series of empirical facts for China between the years 1990 and 2010 are presented. These facts concern changes in time allocation, distribution of skills, wages and assortative mating. The focus is on people aged 20-35 years old, what can be called the marriageable age, as they are the ones that are primarily affected by the changes in the sex ratio during this period.

Figure 1.2, plots the time devoted to paid work and to housework[4] by married people. Several things stand out. First, women spend less time doing both paid work and housework in 2010 than in 1990. This means that their leisure time must have increased. Second, men's paid work and housework are quite similar in 1990 and 2010. Moreover, the time trend for both is quite flat. Therefore, leisure time for married men has remained roughly constant during the period. These changes imply that the female-to-male ratio of paid work and housework has decreased, while it has increased for leisure. This ratio, on the other hand, does not show an upward trend among single people, as shown in Figure 1.3.

The skill distribution based on educational attainment registers important changes between 1990 and 2010, as shown in Figure 1.4. For both men and women, there has been an increase in the fraction of high skilled. However, it was steeper for women. While in 1990 4.47% of men and only 3.69% of women were classified as high skill, the percentages were 28.5% and 31.0% respectively in 2010. That is, women surpassed men in college attainment. At the same time, the fraction of low skilled went down for both genders. In 1990, 32.8% of men and 46.9% of women fell in this category. By 2010, the percentages had fallen to 13.6% and 17.8%. Looking at the relative proportions it becomes apparent that the reduction was steeper for women. In 1990, the relative fraction of low skilled women to men was 1.42, while in 2010 it was 1.30. Finally, the fraction of medium skill people fell for men and rose for women.

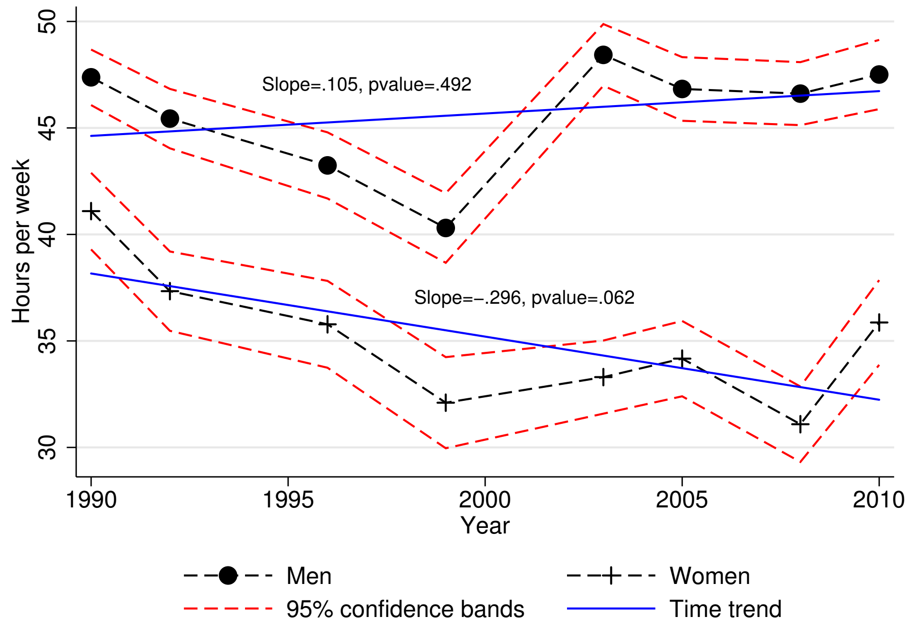
In terms of wages, dramatic changes occurred in the two decades between 1990 and 2010. It is well known that after the Opening of China that started with Deng Xiaoping's reforms in 1978, China has seen spectacular economic growth. According to the International Monetary Fund, GDP per capita rose on

[3] The CHNS is an international collaborative project between the Carolina Population Center at the University of North Carolina at Chapel Hill and the National Institute for Nutrition and Health at the Chinese Center for Disease Control and Prevention.

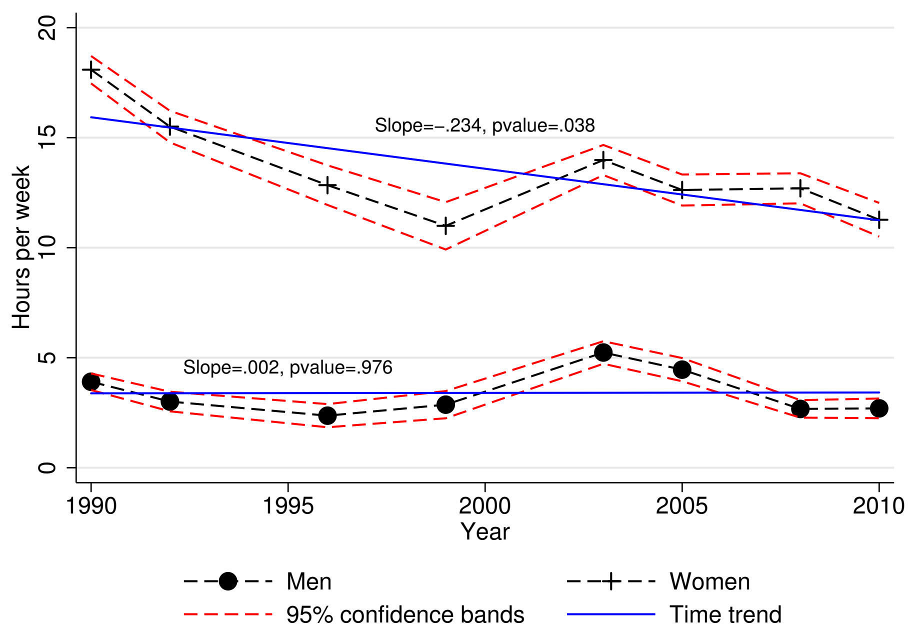
[4] For a detailed explanation of how the data for housework is constructed see Appendix 1.A.

Figure 1.2: Time allocation for Chinese married people aged 20-35, 1990-2010

(a) Paid work

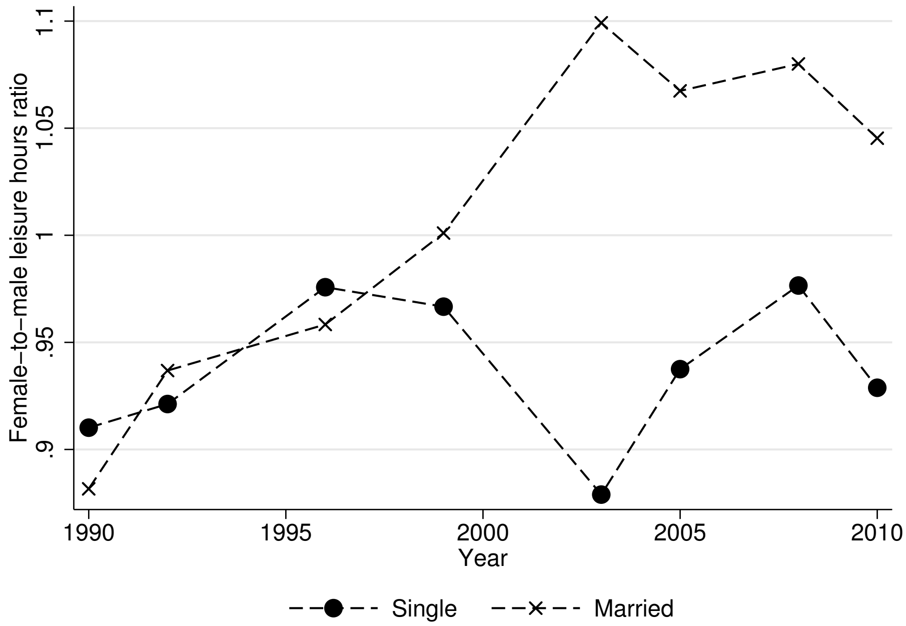


(b) Housework



Source: Author's work using the China Health and Nutrition Survey.

Figure 1.3: Female-to-male leisure ratio for population aged 20-35, 1990-2020



Source: Author's work using the China Health and Nutrition Survey.

average 8.95% per year between 1980 and 2010 and by 9.58% between 1990 and 2010[5]. This growth has translated into quite large wage gains, although some groups benefited more than others.

The changes in China's wage structure by skill and sex are presented in Table 1.2[6]. Overall, the growth in wages has followed closely GDP per capita growth. However, medium and high skilled wages grew faster than low skilled, leading to an increase in the skill premium. Moreover, male wages grew faster than female's. In 1992, the gender wage ratio (female over male wages) was 0.83, but in 2007 it had fallen to 0.75.

Finally, three different methods to measure assortative mating are used, based on Greenwood et al. (2014)[7]. The results are presented in Figure 1.5. For the three measures considered, a higher value means more assortative mating, and for all of them the values for 2010 are higher than for 1990. Therefore, it seems that assortative mating rose in China among young people during the period of interest.

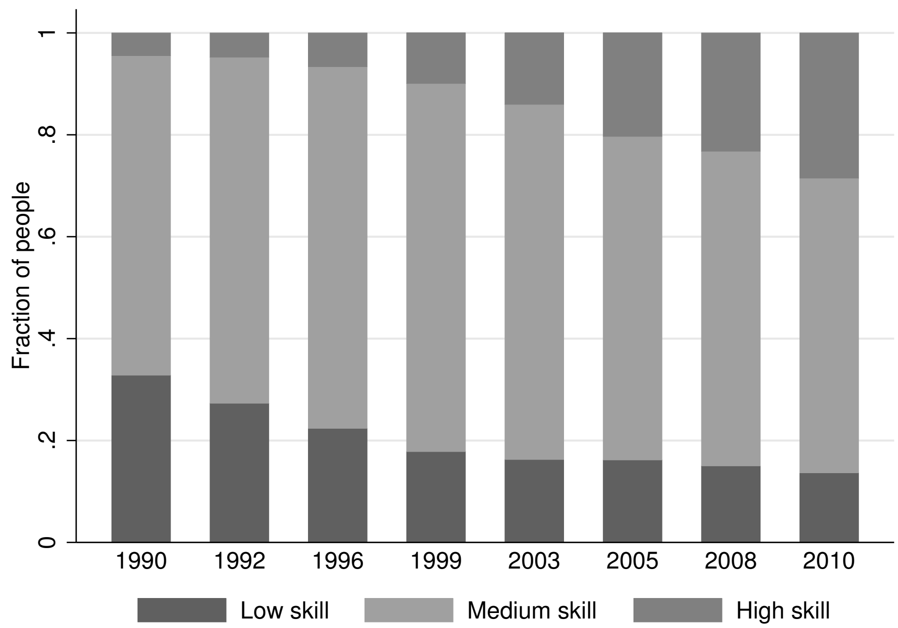
[5]Growth rate of Gross Domestic Product per capita, constant prices in 2011 international dollars (PPA adjusted) as reported in the World Economic Outlook Database, October 2018.

[6]For a more in depth discussion of the data in Ge and Yang (2014) and some of my own estimates on the changes in the wage structure using the CHNS data, see Appendix 1.A.

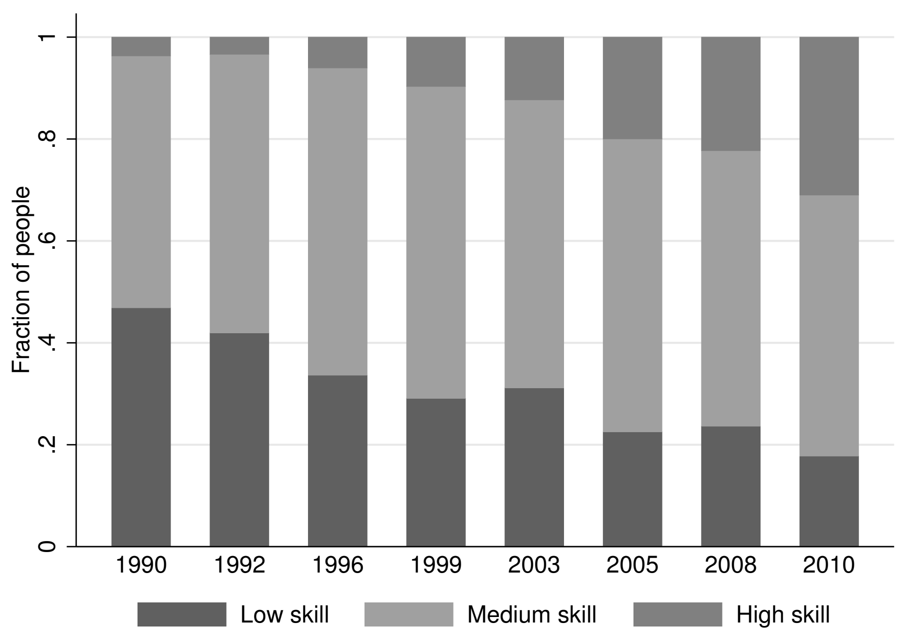
[7]For a detailed explanation of how these measures are constructed, see Appendix 1.A.

Figure 1.4: Skill distribution for Chinese people aged 20-35, 1990-2010

(a) Males



(b) Females



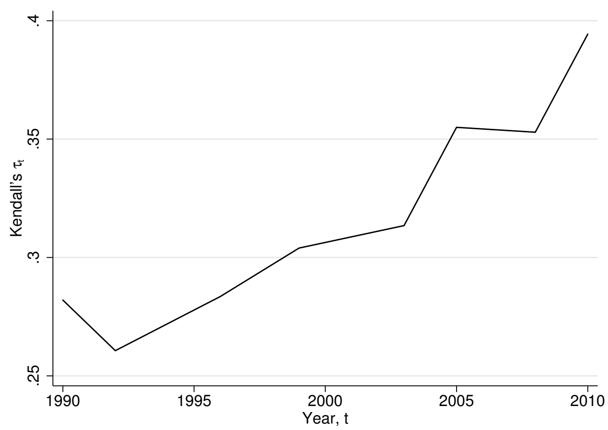
Source: Author's work using the China Health and Nutrition Survey.

Note: Skill levels are defined based on educational attainment (highest grade attained). Low skill individuals are those with primary or less, medium skill are those with at least some middle-high school and no college and high skill those with some college or more.

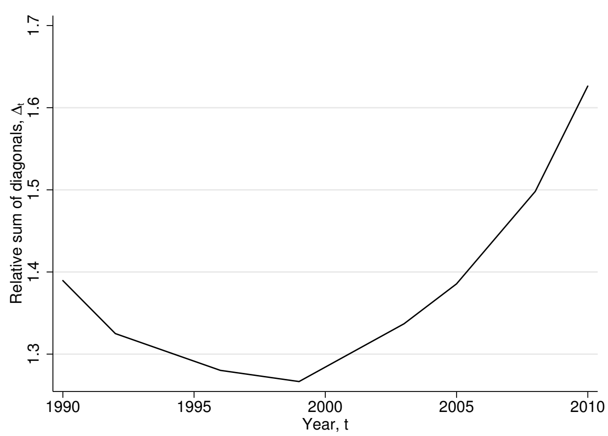
Figure 1.5: Assortative mating in China among people aged 20-35, 1990-2010

(a) Regression approach

(b) Rank correlation approach



(c) Contingency table approach



Source: Author's calculations using the China Health and Nutrition Survey.

Note: For panel a, the dashed lines represent 95% confidence intervales. These are not available for panels b and c.

Table 1.2: Changes in wage structure in China, 1992-1997

Classification	Annual growth 1992-1997	Premium 1992	Premium 1997
Overall	7.6%	-	-
By skill			
Low	5.9%	-	-
Medium	6.9%	6.44%	22.46%
High	8.5%	28.63%	86.08%
By sex			
Female	7.2%	-	-
Male	7.9%	20.01%	33.04%

Source: Author's work using the data presented in Table 1 of Ge and Yang (2014).

Note: Skill premium is calculated with respect to low skill wages. The gender wage premium is calculated with respect to female wages.

1.3 The model

It is hard to rationalize the time allocation patterns among married people under a unitary household framework. A decrease in the gender wage ratio would imply that women substitute paid work with housework, and men do the opposite. This means that the female to male paid work ratio decreases, while the housework ratio increases. In the data we observe that they both decrease. Moreover, wealth effects stemming from the rapidly rising wages combined with growing productivity in home production[8] can account for the increasing leisure that married women enjoy, but would also imply increasing leisure for men.

Moreover, as suggested by the results in Abramitzky et al. (2011), the sex ratio may affect marital sorting. Assortative mating increased in China between 1990 and 2010. Was the rise in the sex ratio partially responsible for that? Or alternatively, would assortative mating had increased even more if the sex ratio had remained constant?

To understand the effect the sex ratio has on resource allocation and marital sorting, it would be useful to have a model featuring a marriage market with search frictions (people have difficulties contacting potential spouses), bargaining within households, time allocation, and home production decisions. The search frictions imply that the sex ratio affects the probability of finding a viable spouse, which in turn affects the outside value of marriage (remaining single), the bargaining position of spouses and ultimately the resource allocation (including time).

However, two-sided search models with heterogeneous agents are known to be intractable. The origin of

[8] Greenwood et al. (2005) argue that what they call the revolution of consumer durable during the 20th century was an important factor in liberating women's time devoted to housework in the United States. Many of these durable goods became widely available in China much more recently than in the U.S.

this intractability is expectations: since the distribution of skills among single people is endogenous, agents need to form expectations about this multi-dimensional object in the future that affect their behaviour today. The properties of the equilibrium, whether it exists and whether it is unique[9] are uncertain, not to mention the properties of a potential steady-state.

A common workaround for this issue is the assumption that whenever a couple gets married, two identical agents flow into singlehood to replace them. However, it implies that the primitives of the model are the distribution over types and sex ratio *among singles*, not in the general population. This is problematic for the interest here is in generating counterfactual steady-state equilibria featuring different sex ratios among the general population.

The structure of marriage markets proposed here deals with this issue. Young agents that enter the marriage market are matched randomly with potential partners of any skill level. However, in subsequent periods they may only meet potential partners with the same skill as themselves. Therefore, agents face a trade-off between accepting matches with high quality but low earnings potential. This is inspired by Fernández et al. (2005). Agents therefore do not need to form expectations about the skill distribution among singles, which is the source of intractability. The model however, allows agents to marry outside of their skill level, meaning that marital sorting is still endogenous.

Moreover, single and married households must allocate their time between paid work (which allows for consumption of market goods), housework (which allows for home production), and leisure (which generates utility directly). A combination of inputs for home production must also be chosen. The allocation of resources within married households responds to conditions in the marriage market, notably the sex ratio, via bargaining.

1.3.1 The Setup

The economy is populated by agents that are characterized by a gender $i \in \{f, m\}$ (female or male). At the time agents start looking for a spouse, they are also characterized by a type $z \in \mathcal{Z}$ that remains constant throughout their life.

Time is discrete and infinite. All agents discount the future at a rate β and face a constant probability

[9] Burdett and Coles (1997) describe the issue of multiplicity of steady-state equilibria in a two-sided search model under fully rational expectations and random search. Essentially, the cause is the presence of a sorting externality, which may lead to steady-state equilibria with distributions of singles that have more or less mass at the top of the distribution. The intuition goes as follows: suppose different types of agents can be arranged from most to least desirable, and that every agent agrees on this ordering (as is the case with skill). If those at the top of the distribution expect an abundance (paucity) of agents at the top on the other side, they will be more (less) selective, thus spending more time searching and generating the expected abundance (paucity). Moreover, this effect cascades down the distribution of types in complex ways. When an agent expects those on top of her to be more (less) selective, there are two opposing effects on her own selectiveness. On one hand, the probability of meeting someone with a higher type increases (decreases). On the other, conditional on meeting, the probability of that agent wanting to marry them decreases (increases).

of dying equal to δ. Exogenous measures of 1 of females and θ_0 of men enter the economy every period to replace those who die. Therefore, the overall measures of females and males in the general population are $\frac{1}{\delta}$ and $\frac{\theta_0}{\delta}$. Moreover, the sex ratio among entrants and in the general population is θ_0. Finally, the probability distributions over types for each sex, $\mathcal{P}_i(z) : z \in \mathcal{Z} \to \mathbb{R}^+$, $\sum_{z\in\mathcal{Z}} \mathcal{P}_i(z) = 1$ for $i \in \{f, m\}$, are also exogenous.

1.3.2 The time allocation and home production problem

In every period, households need to decide how to allocate their time and how to produce home goods. The problem is almost identical to the one in Knowles (2013). Each agent is endowed with one unit of time that has to be allocated across leisure l, paid work n, and housework h. Thus, the time constraint for each agent is

$$l + n + h = 1.$$

Utility over leisure, consumption of market and home goods takes a CES form:

$$u\left(c, l, g\right) = \frac{\sigma_c}{1-\sigma} c^{1-\sigma} + \frac{\sigma_l}{1-\sigma} l^{1-\sigma} + \frac{\sigma_g}{1-\sigma} g^{1-\sigma}.$$

The home production function is a Cobb-Douglas that takes as inputs housework time h and home equipment e_q:

$$G\left(h, e_q\right) = A_G \left[e_q^{1-\alpha_G}\right] h^{\alpha_G}.$$

For the case of married couples, effective housework time h is a function of wife's (h_f) and husband's (h_m) individual housework times:

$$H(h_f, h_m) = \left[\eta_f h_f^{1-\eta} + \left(1-\eta_f\right) h_m^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

1.3.3 Wages and the price of home equipment

Wages are an exogenous function of an agent's type and sex, which we denote as $\omega_i(z) : \mathcal{Z}_i \to \mathbb{R}$ for $i \in \{f, m\}$. The price of home equipment is also an exogenous parameter denoted by p_e.

1.3.4 Solution to the time allocation and home production problem

Since there are no savings decisions and home equipment is a jump variable, the time allocation and home production problem is static.

Singles

The indirect utility flow of a single person of sex i and type z is

$$U_i^S(z; p_e, \omega_i(z)) = \max_{c,l,h,n,e_q,g} u(c, l, g) \tag{1.1}$$

$$\text{subject to}$$

$$l + n + h = 1$$

$$g = G(h, e_q)$$

$$c = \omega_i(z)\, n - p_e e_q.$$

The closed-form demand functions for market goods, leisure and home produced goods are given by

$$[c_i(z), l_i(z), g_i(z)] = \left[\left(\frac{\sigma_c}{\lambda_i(z)}\right)^{\frac{1}{\sigma}}, \left(\frac{\sigma_l}{\lambda_i(z)\omega_i(z)}\right)^{\frac{1}{\sigma}}, \left(\frac{\sigma_g}{\lambda_i(z)D_i(z)}\right)^{\frac{1}{\sigma}}\right],$$

while the inputs for home production are proportional to $g_i(z)$:

$$[h_i(z), e_{qi}(z)] = \left[\frac{g_i(z)}{x_i^g(z)}, \frac{x_i^e(z)g_i(z)}{x_i^g(z)}\right],$$

where $\lambda_i(z)$ is the Lagrange multiplier associated to the budget constraint, $D_i(z)$ is the effective marginal price of home-produced goods, $x_i^e(z)$ is the ratio of home equipment to housework and $x_i^g(z)$ is the ratio of of home production to housework. Closed-form expressions for these objects are derived in Appendix 1.B.

In steady state, the indirect utility associated to the problem in 1.1 is denoted just by $U_i^S(z)$.

Married households

The allocations for married households are the outcome of an auxiliary problem that finds the allocation on the Pareto frontier, taking as given the Pareto weight associated to the wife's utility χ_f[10] (and thus the one associated to the husband's utility, $1-\chi_f$). This weight is the result of bargaining, and thus responds to the conditions in the marriage market (in particular, to the sex ratio) in a way that is explained further down this section.

Married households therefore maximize a welfare function that consists of a weighted sum of the utilities of each spouse. The problem faced by a married household with wife's type z_f and husband's type z_m is therefore:

$$\max_{c_f,c_m,l_f,l_m,h_m,h_f,n_f,n_m,e_q,g} \left\{\chi_f u_f\left(c_f,l_f,g\right)+\left(1-\chi_f\right)u_m\left(c_m,l_m,g\right)\right\} \tag{1.2}$$

subject to

$$l_f+h_f+n_f=1$$

$$l_m+h_m+n_m=1$$

$$h=H\left(h_f,h_m\right)$$

$$g=G\left(h,e_q\right)$$

$$c_m+c_f=\omega_f(z_f)n_f+\omega_m(z_m)n_m-p_e e_q.$$

The closed form demands for market goods, leisure and home production are

$$[c_i(z_f,z_m,\chi_f),l_i(z_f,z_m,\chi_f),g(z_f,z_m,\chi_f)]=$$
$$\left[\left(\frac{\chi_f\sigma_c}{\lambda(z_f,z_m,\chi_f)}\right)^{\frac{1}{\sigma}},\left(\frac{\left(1-\chi_f\right)\sigma_l}{\lambda(z_f,z_m,\chi_f)\omega_i(z_i)}\right)^{\frac{1}{\sigma}},\left(\frac{\sigma_g}{\lambda(z_f,z_m,\chi_f)D(z_f,z_m,\chi_f)}\right)^{\frac{1}{\sigma}}\right]$$

for $i\in\{f,m\}$ and $\chi_m=1-\chi_f$. Home production inputs again are proportional to $g(z_f,z_m,\chi_f)$:

$$[h_f(z_f,z_m,\chi_f),h_m(z_f,z_m,\chi_f),e_q(z_f,z_m,\chi_f)]=$$
$$\left[\frac{x^f(z_f,z_m,\chi_f)g(z_f,z_m,\chi_f)}{x^g(z_f,z_m,\chi_f)},\frac{g(z_f,z_m,\chi_f)}{x^g(z_f,z_m,\chi_f)},\frac{x^e(z_f,z_m,\chi_f)g(z_f,z_m,\chi_f)}{x^g(z_f,z_m,\chi_f)}\right].$$

[10]The Pareto weight for the wife may vary with the wife and husband education types, but to save in notation we just denote it by χ_f instead of $\chi_f\left(z_f,z_m\right)$.

Closed-form expressions for $\lambda(z_f, z_m, \chi_f)$, $D(z_f, z_m, \chi_f)$, $x^g(z_f, z_m, \chi_f)$, $x^e(z_f, z_m, \chi_f)$, $x^f(z_f, z_m, \chi_f)$ are derived in Appendix 1.B.

The indirect utility flow accrued by an agent of sex i in a marriage of type (z_f, z_m) with Pareto weights $(\chi_f, 1-\chi_f)$ is denoted by $U_i^M(z_f, z_m, \chi_f; \omega_f(z_m), \omega_f(z_f), p_e)$. That is, the value of the above problem in 1.2 is given by

$$\chi_f U_f^M(z_f, z_m, \chi_f; \omega_f(z_m), \omega_f(z_f), p_e) + (1-\chi_f)\, U_m^M(z_f, z_m, \chi_f; \omega_f(z_m), \omega_f(z_f), p_e).$$

In steady state these indirect utility flows are denoted just by $U_i^M(z_f, z_m, \chi_f)$ for $i \in \{f, m\}$.

1.3.5 Marriage markets

Agents may participate in two marriage markets in the model. The first one is a pooled market, in which they only stay for one period upon entry. In this market, they may meet randomly one agent of the opposite sex and of any skill level. All single agents that are not entrants participate in segregated markets in which they may only meet agents of the opposite sex and same skill level. They may participate in this markets for several periods. That is, agents may only marry someone with a different type upon entry.

The functioning of the same-skill marriage markets will be described first, and then the one of the pooled market.

Marriage markets for same-skill agents

Agents face search frictions when looking for potential spouses. Denote by $S_f(z)$, $S_m(z)$ and $\theta_S(z) = \frac{S_m(z)}{S_f(z)}$ the measures of females, males and the sex ratio among singles of type $z \in \mathcal{Z}$, respectively. In each period, single agents meet at most one agent of the opposite sex and the same type, where the measure of meetings is given by

$$X = \min\left\{A_X S_m(z)^{\alpha_X} S_f(z)^{1-\alpha_X}, S_f(z), S_m(z)\right\},$$

and thus the probabilities of meeting a potential spouse are obtained dividing $X(z)$ by $S_i(z)$ for $i \in \{f, m\}$:

$$\pi_i\left(\theta_S(z)\right) = \begin{cases} \min\left\{A_X\left(\frac{1}{\theta_S(z)}\right)^{1-\alpha_X}, 1, \frac{1}{\theta_S(z)}\right\} & \text{if } i = m, \\ \min\left\{A_X\theta_S(z)^{\alpha_X}, \theta_S(z), 1\right\} & \text{if } i = f. \end{cases}$$

Upon meeting a potential spouse, agents draw a match quality q from a distribution with cdf $Q_{z,z}$. This represents the flow value of companionship that each partner will enjoy during the marriage. After observing q, agents must decide whether they want to get married.

Since the values of q is constant, and in steady-state so are the wages $\omega_i(z)$ for $i \in \{f, m\}$ and price of home equipment p_e, in steady-state equilibrium there is no divorce. Moreover, it is assumed that if either partner dies, the other dies as well (there are no widowers). Therefore the value of a marriage between agents of type $z \in \mathcal{Z}$ for an agent of sex $i \in \{f, m\}$ and a given Pareto weight is

$$V_i^M\left(z, z, \chi_f, q\right) = \sum_{t=0}^{\infty}\left[\beta\left(1-\delta\right)\right]^t\left[U_i^M(z, z, \chi_f) + q\right] = \frac{U_i^M(z, z, \chi_f) + q}{1-\beta\left(1-\delta\right)}. \tag{1.3}$$

Single agents face a probability of leaving the marriage market and become lifelong singles of ρ. This means that people can expect to spend on average $\frac{1}{\rho}$ periods being eligible for marriage. Moreover, it ensures that the sex ratio among singles does not explode due to the accumulation of men that were unlucky for many periods[11]. Apart from the utility derived from private goods consumption, leisure and the consumption of home-produced goods that results from solving problem 1.1, single agents experience a fixed utility flow per period ψ_i. This represents an intrinsic value of being single. Note that ψ_i could be negative. The value of remaining single in steady-state is thus

$$V_i^S\left(z, \theta_S^E(z)\right) = U_i^S\left(z\right) + \psi_i + \beta\left(1-\delta\right)\left\{\rho\frac{U_i^S\left(z\right) + \psi_i}{1-\beta\left(1-\delta\right)} + \right.$$
$$\left.\left(1-\rho\right)\left[\left[1-\pi_i(\theta_S^E(z))\left(\theta_S\right)\right]V_i^S\left(z, \theta_S^E(z)\right) + \pi_i\left(\theta_S^E(z)\right)\int_{q\in\mathcal{Q}} V_i^X\left(z, z, q\right) dQ_{z,z}\right]\right\},$$

where $\theta_S^E(z)$ is the expected sex ratio among singles of type z and $V_i^X\left(z, z, q\right)$ is the value of a match between agents of type z for an agent of type i. The first two terms represent the current period's total utility flow. The term in curly brackets is the expected value of next period, which depends on the probability of exiting the singles market ρ, the discounted utility flow of being single for the rest of her life $\frac{U_i^S(z)+\psi_i}{1-\beta(1-\delta)}$ the

[11] The age difference among spouses doesn't change much in the data, staying close to 2 years.

probability of meeting a potential spouse $\pi_i(\theta_S^E(z))$, the value of a match and the value of being single.

Now, let $q_r\left(z,z,\theta_S^E(z)\right)$ be the lowest possible value for the draw of q such that marriage occurs when agents of types z meet, i.e.,

$$V_i^M\left(z,z,\chi_f,q\right)\geq V_i^S\left(z,\theta_S^E(z)\right)\text{ for both }i\in\{f,m\}\text{ for }q\geq q_r$$

then:

$$V_i^X\left(z,z,q\right)=\begin{cases}V_i^S\left(z,\theta_S^E(z)\right) & \text{if } q<q_r\left(z,z,\theta_S^E(z)\right)\\ V_i^M\left(z,z,\chi_f,q\right)=\frac{U_i^M(z,z,\chi_f)+q}{1-\beta(1-\delta)} & \text{if } q\geq q_r\left(z,z,\theta_S^E(z)\right).\end{cases}$$

Substituting into the value of being single:

$$\begin{aligned}&V_i^S\left(z,\theta_S^E(z)\right)=\\&U_i^S\left(z\right)+\psi_i+\beta\left(1-\delta\right)\left\{\rho\frac{U_i^S\left(z\right)+\psi_i}{1-\beta\left(1-\delta\right)}\right.\\&+\left(1-\rho\right)\left[\left[1-\pi_i\left(\theta_S^E(z)\right)+\pi_i\left(\theta_S^E(z)\right)Q_{z,z}\left[q_r\left(z,z,\theta_S^E(z)\right)\right]\right]V_i^S\left(z,\theta_S^E(z)\right)\right.\\&\left.\left.+\pi_i\left(\theta_S^E(z)\right)\left[1-Q_{z,z}\left[q_r\left(z,z,\theta_S^E(z)\right)\right]\right]\frac{U_i^M(z,z,\chi_f)+\mathbb{E}\left[q\mid q>q_r\left(z,z,\theta_S^E(z)\right)\right]}{1-\beta\left(1-\delta\right)}\right]\right\}.\end{aligned}\tag{1.4}$$

The right hand side of the equation above has four terms. The first one is the flow value of being single. Next, there are three terms that are discounted at an effective rate $\beta\left(1-\delta\right)$. The second term is the present value of being single for the rest of their life multiplied by the probability of leaving the marriage market. The last two terms are multiplied by the probability of staying in the marriage market. The third one is the value of being single multiplied by the total probability of remaining single, i.e. the probability of not meeting anyone $(1-\pi_{iz}\left(\theta_{Sz}\right))$ plus the probability of meeting someone but marriage not happening $(\pi_i\left(\theta_S^E(z)\right)Q_{z,z}\left[q_r\left(z,z,\theta_S^E(z)\right)\right])$. The final term is the expected value of getting married next period multiplied by the probability of marriage occurring.

The value of being single in the marriage market for types z can be obtained by solving for $V_i^S\left(z,\theta_S^E(z)\right)$ in equation 1.4.

The marriage decision of entrants

New entrants are matched randomly with other agents, that can potentially be of a different type than themselves. If the sex ratio among entrants θ^0 is above one, then some men are going to be unmatched. In that case, the probability of being matched for men is $\frac{\theta^0-1}{\theta^0}$ and for women is 1. Conditional on being matched, the probability for a man to meet with a women of type z_f is $P_f(z_f)$, while the probability for a woman to meet with a man of type z_m is $P_m(z_m)$. If such a meeting happens, marriage will follow if there exists a χ_f such that:

$$V_i^M(z_f, z_m, \chi_f, q) \geq V_i^S\left(z_i, \theta_S^E(z_i)\right) \text{ for both } i \in \{f, m\},$$

where $V_i^S\left(z_i, \theta_S^E(z_i)\right)$ for $i \in f, m$ corresponds to the value of being single in the marriage market for types z_i, that was derived in the previous section.

The Pareto weights

Following Knowles (2013), the Pareto weights are a function of the surplus from marriage, defined as:

$$W_i\left(z_f, z_m, \chi_f, q, \theta_{Sz}^E\right) = V_i^M(z_f, z_m, \chi_f, q) - V_i^S\left(z, \theta_{Sz}^E\right).$$

In this chapter, the focus is on the Egalitarian Bargaining solution, i.e. the mapping from functions $W_f()$ and $W_m()$ to the Pareto weight for the wife that equalises the surplus, i.e. χ_f solves:

$$W_f\left(z_f, z_m, \chi_f, q, \theta_S^E(z)\right) = W_m\left(z_f, z_m, \chi_f, \theta_S^E(z)\right)$$

1.3.6 Flows

The policy functions of entrants imply endogenous flows of new singles into each of the same-type markets, given by

$$S_i^0\left(z, \left\{\theta_S^E(z)\right\}_{z\in\mathcal{Z}}\right) =$$

$$\begin{cases} \mathcal{P}_f(z) \sum_{z_m \in \mathcal{Z}} \mathcal{P}_m(z_m) Q\left[q_r\left(z, z_m, \theta_S^E(z), \theta_S^E(z_m)\right)\right] & \text{if } i = f, \\ \mathcal{P}_m(z) \left\{\theta^0 - 1 + \sum_{z_f \in \mathcal{Z}} \mathcal{P}_f(z_f) Q\left[q_r\left(z_f, z, \theta_S^E(z_f), \theta_S^E(z)\right)\right]\right\} & \text{if } i = m, \end{cases}$$

while the policy function of singles in each of the same-type markets imply marriage rates given by

$$MR_i\left(z, \theta_S(z_i), \theta_S^E(z)\right) = \begin{cases} \pi_f\left(\theta_S(z)\right)\left[1 - Q\left[q_r\left(z, z, \theta_S^E(z)\right)\right]\right] & \text{if } i = f, \\ \pi_m\left(\theta_S(z)\right)\left[1 - Q\left[q_r\left(z, z, \theta_S^E(z)\right)\right]\right] & \text{if } i = m. \end{cases}$$

Therefore, the *actual* sex ratio among singles in market for type z is defined by

$$\begin{aligned} \theta_S(z) &= \frac{\frac{S_m^0\left(z, \left\{\theta_S^E(z)\right\}_{z \in \mathcal{Z}}\right)}{1-(1-\delta)\left(1-MR_m\left(z, \theta_S(z), \theta_S^E(z)\right)\right)}}{\frac{S_f^0\left(z, \left\{\theta_S^E(z)\right\}_{z \in \mathcal{Z}}\right)}{1-(1-\delta)\left(1-MR_f\left(z, \theta_S(z), \theta_S^E(z)\right)\right)}} \\ &= \frac{S_m^0\left(z, \left\{\theta_S^E(z)\right\}_{z \in \mathcal{Z}}\right)\left[1-(1-\delta)\left(1-MR_f\left(z, \theta_S(z), \theta_S^E(z)\right)\right)\right]}{S_f^0\left(z, \left\{\theta_S^E(z)\right\}_{z \in \mathcal{Z}}\right)\left[1-(1-\delta)\left(1-MR_m\left(z, \theta_S(z), \theta_S^E(z)\right)\right)\right]}. \end{aligned}$$

1.3.7 Equilibrium

Suppose all agents believe that the sex ratios among singles in the same-type marriage markets are $\{\theta_S^E(z)\}_{z \in \mathcal{Z}}$.

Definition 1 *A steady-state equilibrium with Egalitarian Bargaining (SSEB), consists on reservation match qualities $q_r(z_f, z_m)$, Pareto weights for the wives $\chi_f(z_f, z_m)$, and values of being married $V_i^M(z_f, z_m, \chi_f, q)$ for all $\{z_f, z_m\} \in \mathcal{Z}_f \times \mathcal{Z}_m$, sex ratios among singles $\theta_S(z)$, expectations on the sex ratios among singles $\theta_S^E(z)$, and values of being single $V_i^S\left(z, \theta_S^E(z)\right)$ for all $z \in \mathcal{Z}$ and $i \in \{f, m\}$ such that:*

1. *The value functions solve the Bellman equations for men and women, i.e.:*
   - *$V_i^M(z_f, z_m, \chi_f(z_f, z_m), q)$ satisfies equation 1.3 for all $\{z_f, z_m\} \in \mathcal{Z}_f \times \mathcal{Z}_m$,*
   - *$V_i^S\left(z, \theta_S^E(z)\right)$ satisfies equation 1.4 for all $z \in \mathcal{Z}$ and $i \in \{f, m\}$.*
2. *The reservation match qualities set the marriage surplus to zero, i.e.:*

$$\begin{aligned} & W_f\left(z_f, z_m, \chi_f(z_f, z_m), q_r(z_f, z_m), \theta_S^E(z_f)\right) \\ & + W_m\left(z_f, z_m, \chi_f(z_f, z_m), q_r(z_f, z_m), \theta_S^E(z_m)\right) = 0 \quad \forall \{z_f, z_m\} \in \mathcal{Z} \times \mathcal{Z} \end{aligned}$$

3. The allocations for married people implied by the Pareto weights equal those generated by Egalitarian Bargaining, i.e.:

$$
\begin{aligned}
&W_f\left(z_f, z_m, \chi_f(z_f, z_m), q, \theta_S^E(z_f)\right) \\
=&W_m\left(z_f, z_m, \chi_f(z_f, z_m), q, \theta_S^E(z_m)\right), \quad \forall q \geq q_r\left(z_f, z_m\right), \quad \forall \{z_f, z_m\} \in \mathcal{Z} \times \mathcal{Z}
\end{aligned}
$$

4. Expectations are correct, i.e.:

$$\theta_S^E(z) = \theta_S(z), \quad \forall z \in \mathcal{Z}$$

1.4 Calibration

To perform quantitative exercises, the model parameters should be chosen to replicate relevant features of the Chinese marriage market and time allocation behaviour. The year 1990 is chosen as a baseline year for the calibration. A three-step strategy is then followed. First, some basic parameters are taken from previous literature. Then, a second set of parameters is chosen to match a set of data moments that do not require solving the model. Finally, the remaining parameters are chosen jointly to match another set of data moments in steady-state equilibrium. The exogenous objects can then be changed to their 2010 values and assess the fit of the model.

1.4.1 Bringing the model to the data

The exogenous objects of the model that need to be obtained from the data are the sex ratio among entrants θ_0, the distribution of skills, the wages, the price of home equipment p_e and the productivity of home production A_g.

For the sex ratio among entrants, the number of males divided by the number of females in the CHNS data is used. This value is 1.071 in 1990 and 1.139 in 2010.

People are divided into three types based on their reported educational attainment: Low skill (primary or less), Medium skill (high school) and High skill (college or more). That is, $\mathcal{Z} = \{\text{Low skill}, \text{Medium skill}, \text{High skill}\}$. The distributions of new entrants over types ($\mathcal{P}_f$ and $\mathcal{P}_m$) are taken from the CHNS data for people aged 20-35.

Wages are constructed based on Ge and Yang (2014)[12]. Low skilled male wages in 1990 to 1 are normalised to 1, and it is assumed that the gender wage ratio is constant across skill levels within the same year.

[12] More precisely, the data in Table 1 is used: Changes in wage and employment structures in China, 1992-2007

Table 1.3: Exogenous objects in the model, 1990 and 2010

Object	1990	2010	Male		Female	
			1990	2010	1990	2010
Sex ratio, θ_0	1.07	1.14	-	-	-	-
Home production						
p_e	1.82	1.06	-	-	-	-
A_g	1.00	6.23	-	-	-	-
Skill distribution						
Low skill	-	-	0.33	0.14	0.47	0.18
Medium Skill	-	-	0.63	0.58	0.49	0.51
High skill	-	-	0.04	0.29	0.04	0.31
Wages						
Low skill	-	-	1.00	2.35	0.83	1.77
Medium Skill	-	-	1.06	2.88	0.89	2.16
High skill	-	-	1.29	4.37	1.07	3.29

Source: Author's work using the China Health and Nutrition Survey and Table 1 of Ge and Yang (2014).

Note: Home production productivity normalized to 1 in 1990. Wages for low skilled men in 1990 normalized to 1.

Since there does not seem to be any data for home equipment prices in China, the values are taken from Knowles (2013), i.e. this study uses the relative prices for the United States. Concretely, BEA Table 2.3.4 is used. The price index for furnishings and durable household equipment is divided by the index for personal consumption expenditure. The relative price for home equipment thus comes to 1.82 for 1990 and 1.06 for 2010. Finally, the productivity of home production is normalized to 1 in 1990 ($A_g = 1$). It is assumed that it grew at the same rate as GDP per capita between 1990 and 2010, 9.58% per annum. Therefore, the value for A_g in 2010 is $1.0958^{20} = 6.232$.

The numbers used for the exogenous objects in the model are presented in Table 1.3.

1.4.2 Parameters externally calibrated

A group of parameters are chosen externally. This includes the discount rate β, which is set to 0.96 as is standard for a period of 1 year. The value for the death rate δ is assigned so that the life expectancy is 49 years, assuming people enter the marriage market at age 20, so that total life expectancy is 69 years as reported by the United Nations Population Division for China in 1990. The value for ρ is set to have the expected number of periods that a person stays in the marriage market to 15, as marriageable ages are taken to be between 20 and 35.

In the model, since the home production technology is Cobb-Douglas, $1 - \alpha_g$ is the share of home production expenditures over the total cost of home production. This value of this parameter is set following Knowles (2013). He finds that the spending share of home equipment in the United States fluctuates between 4 and 6%. Therefore, the value of α is set to 0.95, to have the share of home equipment expenditure of households in the model be 5%. For the other home production parameter, the elasticity of the time aggregator for married couples η, the exact same value as Knowles (2013) of 0.33 is used.

1.4.3 Parameters calibrated without solving the model

Another group of parameters are chosen to match the data before solving the model. Recall that married households face a restriction when aggregating their hours of work that depends on η and η_f. From the first order conditions of the married couple's problem, we have:

$$\frac{h_f}{h_m} = \left[\frac{\eta_f}{(1-\eta_f)} \frac{w_m}{w_z} \right]^{\frac{1}{\eta}}$$

The value of η_f is set so that exactly match the ratio of wife to husband's housework in 1990, using the gender wage ratio of that year (0.833). The resulting value is slightly above 0.5, meaning that wives' time is more valuable for home production, but not by much.

The parameters of the utility function for single people can also be chosen without solving the model.

The elasticity of substitution in the utility function is set to 1.25. Knowles (2013) uses the same value for the United States as in Attanasio et al. (2008). However, this value seems to be too high for China, as it would imply a huge decrease in hours worked that is not supported in the data. A value of 1 for σ would imply that wealth effects and substitution effects cancel out, as utility become logarithmic. This other extreme is also unsatisfactory, as paid hours do fall slightly. An intermediate value for σ is chosen.

Finally, the values for the weights of consumption σ_c, leisure σ_l and the home produced good σ_g in the utility function for singles are set. This is done so that paid work and housework time exactly match the data in 1990, using the average wages separately for men and women. It is imposed that the sum of the weights is equal to one, so that the parameters are exactly identified.

1.4.4 Parameters calibrated by moment matching

The remaining parameters are jointly set to target the time allocation for married couples and the marital sorting in 1990. These parameters are the weights in the utility function of married couples, the relative disutility of being single for women ψ_f and the means of the distribution of match quality draws for all different combinations of potential wife and husband education levels $\left\{\mu_{z_f,z_m}\right\}_{z_f,z_m \in \mathcal{Z}\times\mathcal{Z}}$.

All the parameters involved in principle can affect all of the targeted moments. However, the utility weights play a more significant role in the determination of the time allocation of married couples. As the parameters of the aggregator of effective housework time are set to exactly match the relative housework, this will be correct in the model. Therefore, the weight of home produced good consumption σ_g in utility of married couples will help get the absolute amount of time spent doing housework right, while the weight of market goods consumption σ_c helps get the total amount of paid work right. Comparing the values of the utility weights for married households with those for singles, we see that married people value more home produced goods, and less leisure and private goods consumption.

Moreover, from the first order conditions of the married household problem,

$$\frac{l_f}{l_m} = \left[\frac{\omega_m}{\omega_f}\frac{\chi_f}{1-\chi_f}\right]^{\frac{1}{\sigma}}$$

since the leisure ratio within the household depends on the Pareto weight of the wife, the relative (dis)utility of being single affects women's outside value, and therefore their Pareto weights within marriage, which helps get the leisure ratio right. The calibration implies a negative value for ψ_f, meaning that women experience disutility from being single relative to men.

Finally, the means of the match quality draws affect directly how likely it is that people marry upon meeting, which helps the model reproduce the marital sorting observed in the data. This is measured using the contingency tables that introduced in section 1.2, i.e. the contingency tables generated by the model are compared with the ones observed in the data.

The list of calibrated parameters can be found in tables 1.4 and 1.5.

1.4.5 Results

The results regarding time allocation patterns are presented in table 1.6. Model housework, paid work and leisure times for married couples are very close to their corresponding data counterparts in 1990. Of course, the values for singles are almost exactly matched, as was discussed before.

The contingency matrices for marital sorting for both the data and the model are presented in table 1.7. In general all entries for the mode matrix are close to their data counterparts. The assortative mating measure described in section 1.2 is also calculated. Most of the entries of the contingency matrix generated by the model are very close to the ones observed in the data. However, off-diagonal elements are slightly

Table 1.4: Calibrated parameters

Parameter	Value	Source
Parameters externally calibrated		
Parameter	Value	Source
β	0.960	Standard
δ	0.020	Life expectancy of 49 years
ρ	0.067	Expected 15 years searching for spouse
α_g	0.950	Knowles (2014)
η	0.330	Knowles (2014)
Parameters calibrated before solving the model		
Parameter	Value	Set to match
η_f	0.580	Gender housework ratio, married people in 1990
σ	1.250	Leisure increase for singles between 1990 and 2010
Single women		
σ_c	0.391	
σ_l	0.570	Single women time allocation
σ_g	0.040	
Single men		
σ_c	0.387	
σ_l	0.607	Single men time allocation
σ_g	0.006	
Parameters jointly calibrated by moment matching		
Parameter	Value	Target
Married people		
σ_c	0.365	
σ_l	0.573	Married households time allocation
σ_g	0.062	
ψ_f	-0.373	Husband to wife leisure ratio
M	See Table 1.5	Marital sorting contingency matrix

Table 1.5: Calibrated means of the match quality draw by potential spouse's skill level

Female skill level	Male skill level		
	Low	Medium	High
Low	0.753	1.483	-0.004
Medium	0.813	-0.253	4.078
High	-0.698	0.608	-0.785

Table 1.6: Fit of the model, time allocation 1990 (hours per week)

Statistic	Model	Data
Married women housework	18.09	18.13
Married women paid work	41.10	41.06
Married women leisure	58.82	58.81
Married men housework	3.91	3.81
Married men paid work	47.38	47.48
Married men leisure	66.71	66.71
Single women housework	7.39	7.39
Single women paid work	48.00	48.01
Single women leisure	62.61	62.60
Single men housework	1.66	1.66
Single men paid work	47.55	47.56
Single men leisure	68.79	68.78

Table 1.7: Fit of the model, marital sorting 1990

Wife	Husband					
	Low skill		Medium skill		High skill	
	Data	Model	Data	Model	Data	Model
Low skill	0.251	0.263	0.247	0.206	0.006	0.004
Medium skill	0.074	0.072	0.371	0.400	0.023	0.022
High skill	0.001	0.001	0.011	0.013	0.017	0.020

smaller while the converse is true for diagonal elements, which means that the model generates a bit too much assortative mating, at 1.46 versus 1.39 in the data.

Altogether, the fit of the model is quite satisfactory, and the objective of having a model that reproduces the time allocation and marital sorting patterns in China in 1990 is achieved.

1.4.6 Fit of the model

As external validation for the model, the results for the 2010 model steady state against the data counterparts are presented here. Recall that all parameters are chosen or calibrated to reproduce the 1990 time allocation and marital sorting. The only parameter that targets changes in time is σ, but his was chosen so that wealth effects for single people are not too strong, so it was not calibrated using any data for married couples in 2010.

In terms of time allocation, the model correctly predicts the directions of changes in housework, paid work and leisure time. For married women, the leisure time is remarkably close to the data. However, the split between housework and paid work is slightly off, with too much housework and a too little paid work. For married men, the model predicts a bit too little paid work and a little too much leisure. The same pattern is true for singles. This can be seen in Table 1.8.

Table 1.8: Fit of the model, time allocation 2010 (hours per week)

Statistic	Hours per week		Δ% 1990-2010	
	Data	Model	Data	Model
Married women housework	11.27	15.06	-47.32%	-18.58%
Married women paid work	35.87	31.59	-13.61%	-26.22%
Married women leisure	70.86	71.36	18.64%	19.34%
Married men housework	2.70	2.56	-37.02%	-39.67%
Married men paid work	47.51	45.42	0.28%	-4.44%
Married men leisure	67.79	70.02	1.60%	4.84%
Single women housework	4.50	5.54	-49.69%	-28.75%
Single women paid work	45.06	43.31	-6.32%	-10.31%
Single women leisure	68.44	69.15	8.91%	9.95%
Single men housework	1.59	1.23	-4.68%	-30.02%
Single men paid work	42.73	41.68	-10.70%	-13.19%
Single men leisure	73.69	75.09	6.88%	8.78%

Marital sorting results are presented in table 1.9. Here, the issue is the converse to that of 1990: too little assortative mating. Assortative mating increases in the model to 1.52, but in the data it does so more, to 1.62.

Table 1.9: Fit of the model, marital sorting 2010

Wife	Husband					
	Low skill		Medium skill		High skill	
	Data	Model	Data	Model	Data	Model
Low skill	0.099	0.100	0.101	0.072	0.002	0.010
Medium skill	0.051	0.036	0.433	0.362	0.068	0.143
High skill	0.005	0.005	0.079	0.114	0.162	0.158

In sum, the model predicts most changes in terms of time allocation and marital sorting correctly qualitatively, while it makes small mistakes quantitatively in terms of time allocation, and it generates an increase in assortative mating that is too small. This could be due to a myriad of other factors that affect preferences for partner between 1990 and 2010.

1.5 Quantitative experiments

An important advantage of having a model is that it allows to perform quantitative experiments. The main objective of this section is to quantify the effect that changes in the sex ratio have on marriage and resource allocation within the household. Moreover, whether the effects of sex ratio operate mainly through the bargaining or the marital sorting channel is also of interest. This objectives are attained by changing the primitives of the model and comparing the outcomes of interest across steady-states.

1.5.1 Decompositions

The first exercise performed consists on two decompositions. The first one is denoted as the forward decomposition. Starting with the model in steady state in 1990, one by one the sex ratio among entrants, the distributions of skills, the wage structure, and finally the price of home equipment and efficiency of home production are changed. The second one, which is called the backward decomposition, goes the other way around: it starts with the model in steady state in 2010 and changes one by one the factors mentioned before to their 1990 level. This decompositions allows to separate the effect of each of these forces and better understand and explain the mechanisms behind the model.

Naturally, as there are all sort of non-linearities and complementarities involved, the order of the decomposition may matter. Since there are four different factors that change in time, there are 24 possible decompositions.

The results of the decompositions in levels are presented in tables 1.10 and 1.11. Figures 1.6 and 1.7 show the decomposition of the effects of the four factors on time allocation for married people in percentage terms. In the forward decomposition, when a statistic increases between 1990 and 2010, the total effect is -100%, when it decreases it is 100%. The opposite holds for the backward decomposition.

It is evident that the sex ratio plays a very limited role in explaining the changes in housework time for both men and women. In fact, in the model, the Pareto weights play no role on the determination of housework allocation within the couple as long as the solution is interior, that is, as long as the wife is working a positive number of hours. Changes in marital sorting induced by changes in the sex ratio may affect average housework across married people, by changing the distribution of relative wages couples face. Buy it seems that the importance this channel plays is limited in the model.

However, the changes in the sex ratios play an important role in explaining changes in paid work and leisure. In the case of married women, the increase in the sex ratio explains between 38 and 52 percent of the changes in paid work between 1990 and 2010, and between 26 and 45 percent of the changes in leisure time. For married men, the percent contributions seem very high, but that is a consequence of married men's time allocation not changing much. This in turn is a result of the effects of the increase in the sex ratio and the changes in the wages structure largely cancelling each other, as is discussed below.

For married women, the effect of the changes in wages goes in the same direction as the effect in the sex ratios. This is explained by two separate channels. First, the elasticity of substitution between consumption of private goods, home produced goods and leisure is below one ($\sigma > 1$). Therefore, as wages increase the wealth effect dominates the substitution effect and people work less, enjoy more leisure and substitute paid work for housework. Moreover, as the gender wage ratio decreased between 1990 and 2010, couples substitute part ot the wife's for more of the husband's paid work time.

For married men, the effect of changes in the sex ratios and the wage structure go in opposite directions. As the sex ratio increases, the Pareto weights in marriage shift towards the wife. This causes married men to work more. On the other hand, wealth effects mean that married men work less, while the decrease in the gender wage ratio implies more husband paid work. However, wealth effects seem to be stronger, as the net effect of the change in the wage structure is negative in the forward decomposition and positive in the backward one.

Since the sex ratio does play an important role in determining housework, the analysis of the effect of changes in the sex ratio on leisure time is analogous to the analysis it plays on paid work. It is worth noting though that the sex ratios explain a lower fraction of the changes in leisure time for women, as changes in home production technology and prices affect leisure time more than paid work.

The results of the decomposition on assortative mating are presented in figure 1.8. The large gap between the fraction of the changes in assortative mating that are explained by the sex ratio in the forward and backward decompositions seems a bit puzzling at fist glance. In the forward decomposition it appears that the changes in the skill distribution are the major factor driving the changes in assortative mating, while in the backward decomposition the sex ratio and the changes in skill distribution seem roughly equally important. The fact that changes in skill distribution drive the increase in assortative mating is no surprise, given that the distribution of skills is much more similar across genders in 2010 than in 1990. This is precisely the reason why the sex ratio has a larger effect in assortative mating in the backward decomposition.

The forward decomposition starts at the 1990 steady state, with a distribution of skills that is quite mismatched across genders (with males having higher skill level on average) and a relatively low assortative mating. As the sex ratio increases, there quite is a lot of margin for males to avoid unfavourable marriage market conditions within their own skill level by marrying below. This is also an attractive option for women, as they are marrying men with higher earnings. This limits the positive effect the increase in the sex ratio has on assortative mating. The backward decomposition starts at the 2010 steady state, with skill distributions that are more balanced across genders. The channel of marrying up for women (down for men) is weaker, thus allowing for a stronger negative effect when the sex ratio drops to its 1990 level.

It is clear though that in the model an increase in the sex ratio drives assortative mating up. This result is in line with Wang (2018).

Finally, although we do not have data to corroborate this finding, the model has implications for private consumption of married females and males. Changes in the sex ratio alone increase consumption for married females between 7 and 10 percentage points, at the expense of a reduction between 4.5 and 8.5 percentage points for married males.

1.5.2 Future additional increases in the sex ratio

What should we expect to happen to marriage and household resource allocation as the sex ratio increases further? As discussed in previous sections, the sex ratio among people of marriageable age may reach 1.2 men per woman around 2020. How big of an additional impact will this have for Chinese people? To answer this question, a steady state with all the fundamental values of 2010, but with a sex ratio of 1.2 ($\theta_0 = 1.2$) is computed. The results are presented in table 1.12.

The model predicts that in a steady state with a sex ratio of 1.2 in 2010, paid hours for married women are almost 9% lower and 5% higher for men. This is between one third and one half of the percent change between the 1990 and 2010 model steady states in both married female and male paid hours. Moreover, leisure time increases for women and decreases for men, about three and a half percentage points each. Assortative mating goes up very slightly. Finally, consumption for married women increases 4.65% at the expense of a reduction of married men consumption of 3.14%. An increase in the sex ratio has no effect on single people's time allocation, by construction of the model. Incorporating savings decisions would activate a channel by which the sex ratio would affect singles.

1.5.3 The role of the gender wage gap

In all previous exercises, it is assumed that the wages of men rose more than the wages of women between 1990 and 2010, as found by Ge and Yang (2014). This means that the gender wage gap increased between 1990 and 2010. As discussed in section 1.2, this result may include composition effects, which are not relevant in this setting. The gender gap that is relevant in this study is on potential hourly earnings. In fact, the conditional gender wage gap computed with the CHNS data is constant, albeit closer to the lower value in 2010. Therefore, the steady state equilibrium in 2010 is computed using the 1990 gender wage gap in order to assess how this changes results. In particular, it is of interest to know if a constant gender wage gap helps the model perform better in relation to the data in 2010.

The result of this exercise is presented in table 1.13. A constant gender wage gap does bring the steady state time allocation a bit closer to the data, but only for married women. Notably, the leisure time for married people remains almost unchanged, as the lower gender wage gap with respect to the baseline mainly generates a reallocation of housework and paid work between wives and husbands. The fit of the model

Table 1.10: Forward decomposition results

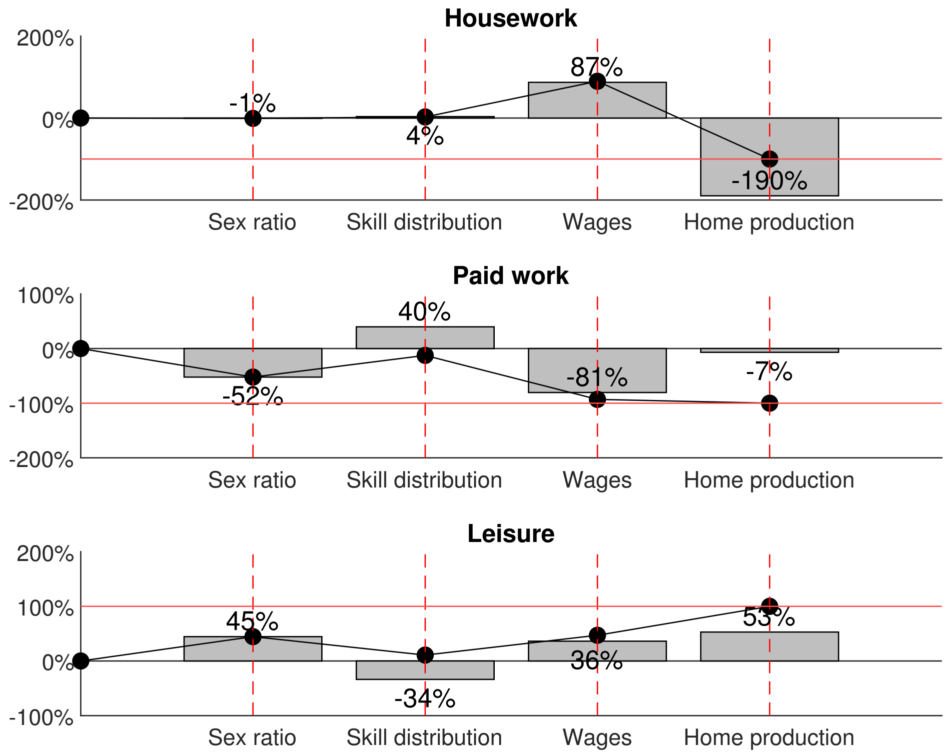
Statistic	Model 1990	Sex ratio	Skill distributions	Wages	Home production
Married women housework	18.130	18.102	18.220	21.420	15.056
Married women paid work	41.058	35.799	39.727	32.166	31.587
Married women leisure	58.812	64.099	60.053	64.413	71.357
Married men housework	3.808	3.812	3.892	3.565	2.561
Married men paid work	47.482	51.903	47.782	42.877	45.421
Married men leisure	66.710	62.285	66.326	71.559	70.018
Single women housework	7.387	7.387	7.424	7.900	5.541
Single women paid work	48.012	48.012	47.621	42.481	43.308
Single women leisure	62.600	62.600	62.955	67.619	69.150
Single men housework	1.665	1.665	1.671	1.788	1.233
Single men paid work	47.557	47.557	47.232	41.500	41.679
Single men leisure	68.779	68.779	69.096	74.712	75.088
Married women consumption	0.309	0.337	0.330	0.746	0.831
Married men consumption	0.411	0.384	0.433	1.102	1.080
Average wife Pareto weight	0.414	0.461	0.423	0.396	0.435
Assortative mating measure	1.466	1.476	1.544	1.534	1.521

Table 1.11: Backward decomposition results

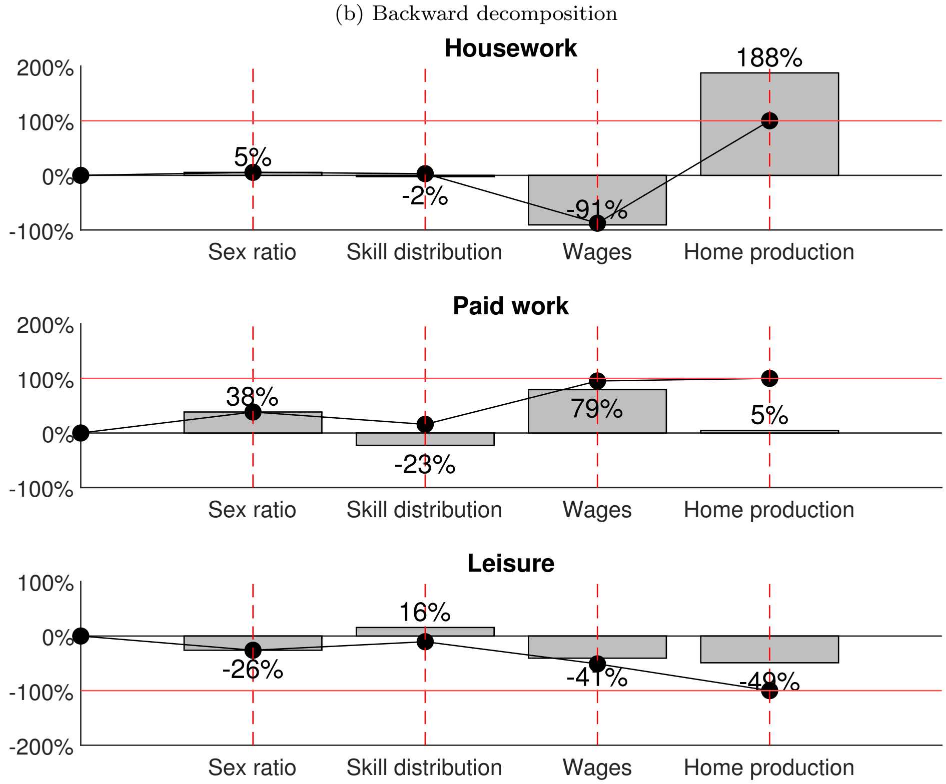
Statistic	Model 2010	Sex ratio	Skill distributions	Wages	Home production
Married women housework	15.056	15.210	15.142	12.794	18.130
Married women paid work	31.587	34.940	32.927	40.550	41.058
Married women leisure	71.357	67.850	69.931	64.656	58.812
Married men housework	2.561	2.521	2.254	2.689	3.808
Married men paid work	45.421	42.422	45.764	50.038	47.482
Married men leisure	70.018	73.057	69.982	65.274	66.710
Single women housework	5.541	5.541	5.481	5.175	7.387
Single women paid work	43.308	43.308	44.248	48.902	48.012
Single women leisure	69.150	69.150	68.272	63.923	62.600
Single men housework	1.233	1.233	1.223	1.147	1.665
Single men paid work	41.679	41.679	42.427	47.752	47.557
Single men leisure	75.088	75.088	74.350	69.101	68.779
Married women consumption	0.831	0.781	0.712	0.340	0.309
Married men consumption	1.080	1.122	0.933	0.402	0.411
Average wife Pareto weight	0.435	0.404	0.421	0.449	0.414
Assortative mating measure	1.521	1.484	1.445	1.452	1.466

Figure 1.6: Contributions to changes in married female time allocation, 1990-2010

(a) Forward decomposition



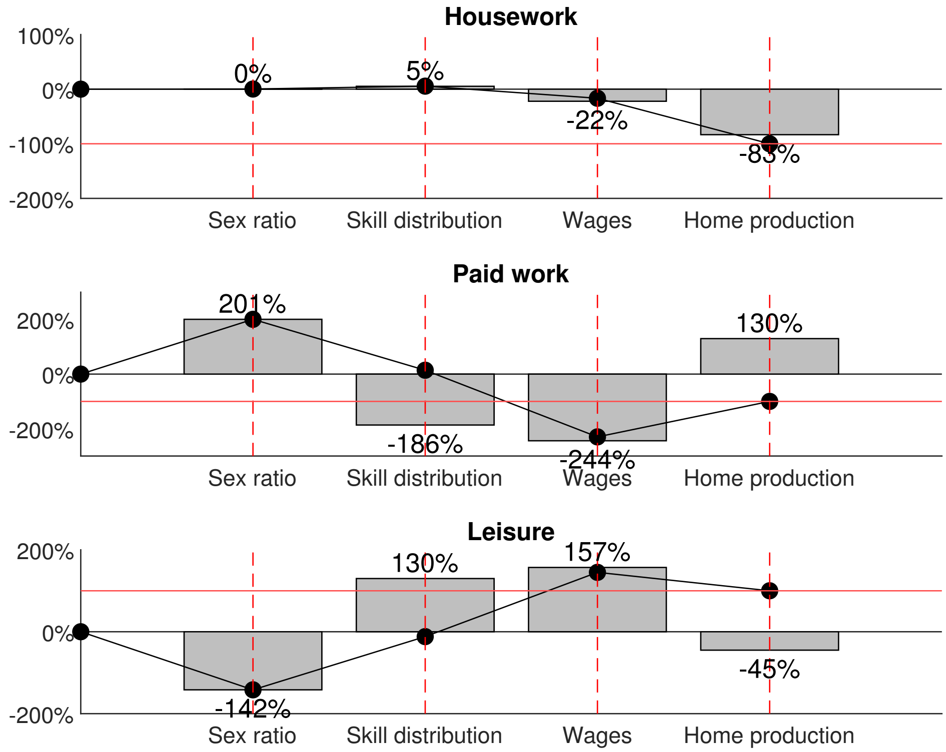
(b) Backward decomposition



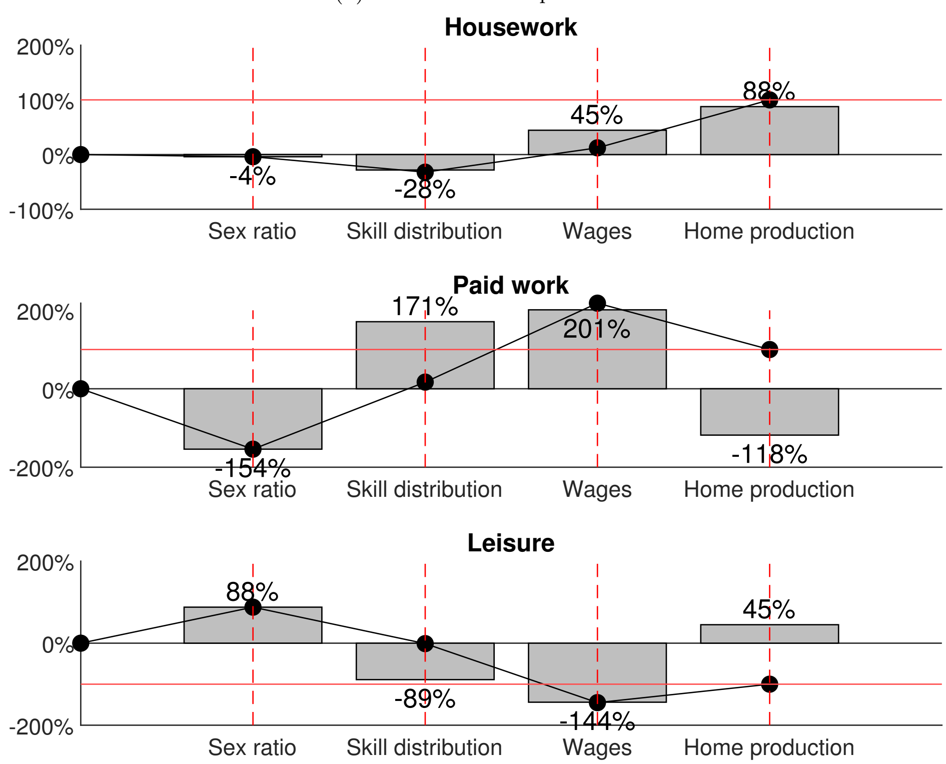
Note: The solid line with circle markers represents the cumulative change between 1990 and 2010, up to that factor. The bars represent the contribution of that factor alone to the total change. Forward decomposition starts at the 1990 steady state. Backward decomposition starts at the 2010 steady state.

Figure 1.7: Contributions to changes in married male time allocation, 1990-2010

(a) Forward decomposition



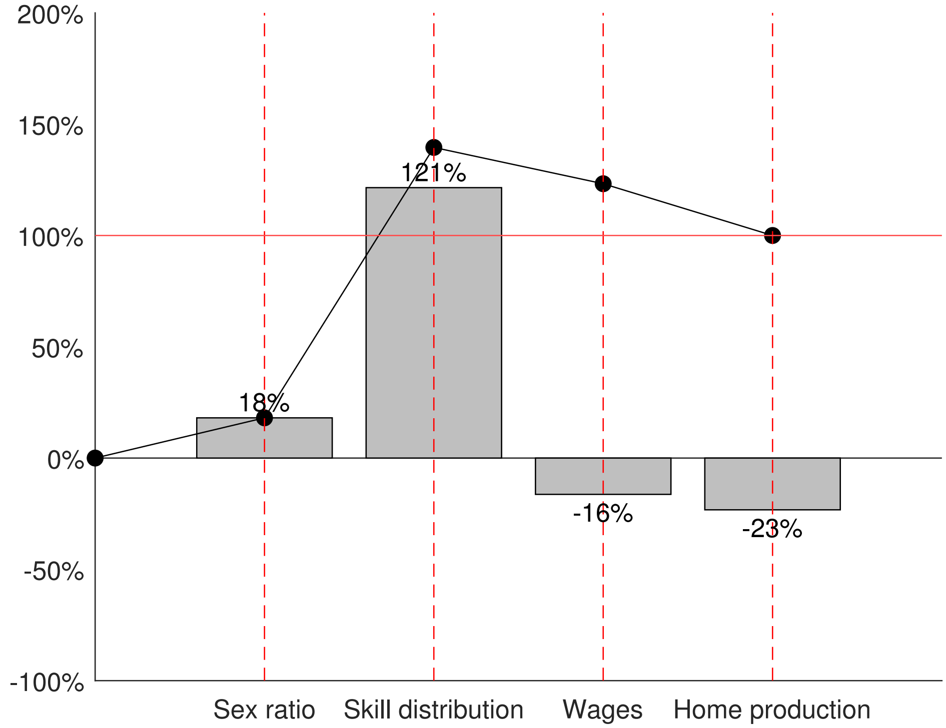
(b) Backward decomposition



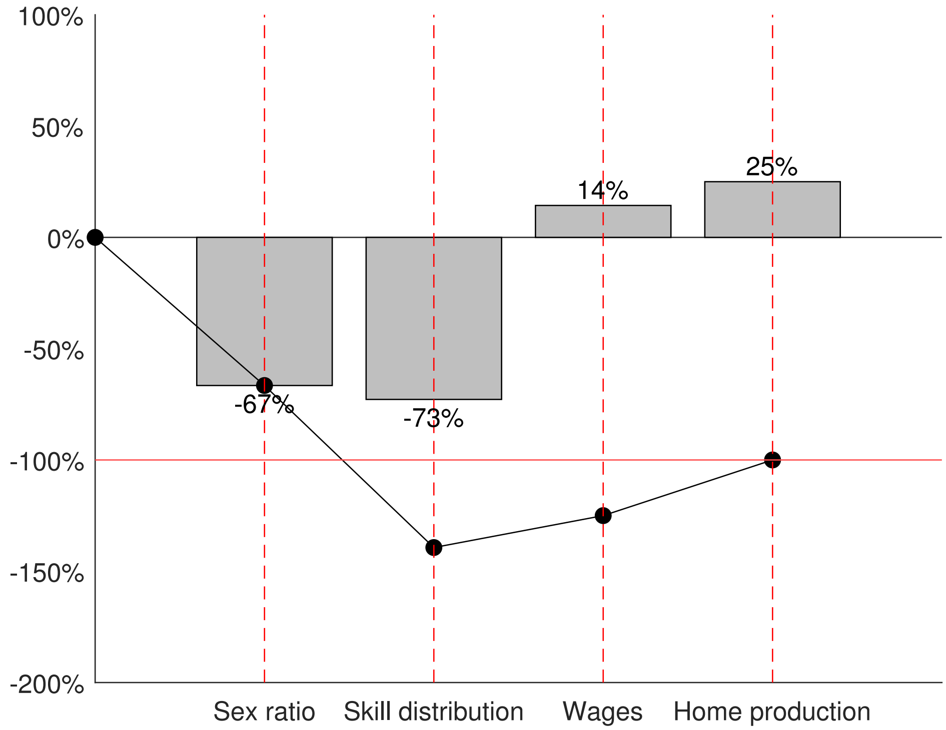
Note: The solid line with circle markers represents the cumulative change between 1990 and 2010, up to that factor. The bars represent the contribution of that factor alone to the total change. Forward decomposition starts at the 1990 steady state. Backward decomposition starts at the 2010 steady state.

Figure 1.8: Contributions to changes in assortative mating, 1990-2010

(a) Forward decomposition



(b) Backward decomposition



Note: The solid line with circle markers represents the cumulative change between 1990 and 2010, up to that factor. The bars represent the contribution of that factor alone to the total change. Forward decomposition starts at the 1990 steady state. Backward decomposition starts at the 2010 steady state.

Table 1.12: The effects of a sex ratio of 1.2 in 2010

Statistic	Baseline 2010	$\theta_0 = 1.2$	% change
Married women housework	15.06	14.97	-0.54%
Married women paid work	31.59	28.90	-8.90%
Married women leisure	71.36	74.13	3.81%
Married men housework	2.56	2.58	0.91%
Married men paid work	45.42	47.81	5.12%
Married men leisure	70.02	67.61	-3.50%
Single women housework	5.54	5.54	0.00%
Single women paid work	43.31	43.31	0.00%
Single women leisure	69.15	69.15	0.00%
Single men housework	1.23	1.23	0.00%
Single men paid work	41.68	41.68	0.00%
Single men leisure	75.09	75.09	0.00%
Married women consumption	0.83	0.87	4.65%
Married men consumption	1.08	1.05	-3.14%
Average wife Pareto weight	0.44	0.46	5.35%
Assortative mating measure	1.52	1.55	2.15%

becomes a little bit worse for single females. Altogether, the conclusion is that a constant gender wage gap does not dramatically improve the model fit to the data in 2010.

Finally, one can go a step further and ask what would happen if there were no gender wage gap at all. The previous exercise hints at the direction of the changes, but to properly answer the question the results for the steady state in 2010 with no gender wage gap next to the results for the baseline model in 2010 are presented in table 1.14.

Not surprisingly, the removal of the sex ratio leads to housework time becoming more equal across husband and wives, although a gap of about seven hours per week persists. This is a result of the weight of female housework time being larger than one half ($\eta_f = 0.58$) in the housework time aggregator for married couples. And again, leisure time remains largely unchanged for both females and males.

Married women consumption goes up substantially. Interestingly married men consumption also increases slightly. As both leisure and consumption rise for the latter, it seems that a reduction of the gender wage gap benefits both men and women in the model. This is less trivial than it appears at first. The drop in the gender wage gap affects time allocation via two channels: substitution of husband for wife paid work, and an increase in the bargaining position of females as the value of their outside option increases, as reflected by the larger Pareto weight in the steady state with no gender wage gap. These two channels operate in opposite directions, and therefore ex ante is uncertain which one will prevail. It seems therefore that policies

Table 1.13: The model in 2010 with a constant gender wage gap

Statistic	Data 2010	Baseline model 2010	Constant gender wage gap 2010
Married women housework	11.27	15.06	13.87
Married women paid work	35.87	31.59	32.59
Married women leisure	70.86	71.36	71.54
Married men housework	2.70	2.56	3.15
Married men paid work	47.51	45.42	44.60
Married men leisure	67.79	70.02	70.25
Single women housework	4.50	5.54	5.58
Single women paid work	45.06	43.31	42.75
Single women leisure	68.44	69.15	69.67
Single men housework	1.59	1.23	1.23
Single men paid work	42.73	41.68	41.68
Single men leisure	73.69	75.09	75.09
Married women consumption		0.83	0.90
Married men consumption		1.08	1.08
Average wife Pareto weight		0.44	0.46
Assortative mating measure	1.63	1.52	1.52

that attempt to reduce the gender wage gap would be beneficial for married people in general.

1.6 Conclusions

This chapter explores the effect changes in the sex ratio has on marital sorting, bargaining and time allocation between spouses. It does so by building a model of marriage that allows for people with different education levels to marry. Moreover, it allows for bargaining over its gains to affect the amount of paid work, housework and leisure each spouse does or enjoys.

The model is calibrated to reproduce time allocation and marital sorting data for China in 1990. It does so successfully. A set of quantitative exercises are subsequently performed.

It is found that the sex ratio accounts for between one third and one half of the changes in paid work and leisure time for married women between 1990 and 2010. Moreover, most of the change operates via bargaining and very little via marital sorting. A counterfactual exercise reveals that had the sex ratio reached 1.2 men per woman in 2010, paid hours would have been almost 9% lower for married females and 5% higher for their male counterparts. Eliminating the gender wage gap would have a similar impact on women, but would decrease men's paid work hours by 6%.

A number of extensions or improvements could be made to the present work. The most obvious would be

Table 1.14: The model in 2010 with no gender wage gap

Statistic	Baseline 2010	No gender wage gap	% change
Married women housework	15.06	11.82	-24.17%
Married women paid work	31.59	34.37	8.45%
Married women leisure	71.36	71.80	0.63%
Married men housework	2.56	4.50	56.41%
Married men paid work	45.42	42.81	-5.91%
Married men leisure	70.02	70.68	0.95%
Single women housework	5.54	5.64	1.77%
Single women paid work	43.31	41.77	-3.61%
Single women leisure	69.15	70.59	2.06%
Single men housework	1.23	1.23	0.00%
Single men paid work	41.68	41.68	0.00%
Single men leisure	75.09	75.09	0.00%
Married women consumption	0.83	1.04	22.51%
Married men consumption	1.08	1.09	1.12%
Average wife Pareto weight	0.44	0.50	14.10%
Assortative mating measure	1.52	1.52	0.16%

to endogeneize the education decision. Agents could face a trade-off in a pre-marriage-market period between acquiring more education and exerting effort. Second, adding savings decisions would create interactions between he sex ratio and single people's time allocation that are not present in the model, albeit it would considerably increase the difficulty of defining, computing the equilibrium and calibrating the model. Finally, being able to make finer refinements (geographically, in terms of skills) could improve our knowledge of time allocation patterns across different types of households and allow for better testing of the rich dynamics in the model.

1.A Data and empirical facts

Data description

This paper uses data from the China Health and Nutrition Survey (CHNS), a collaborative project between the Carolina Population Center at the University of North Carolina at Chapel Hill and the National Institute for Nutrition and Health at the Chinese Center for Disease Control and Prevention. There are ten years in the study: 1989, 1991, 1993, 1997, 2000, 2004, 2006, 2009, 2011 and 2015. The sample comes from fifteen provinces: Beijing, Chongquing, Guanxi, Guizhou, Heilongjiang, Henan, Hubei, Hunan, Jiangsu, Liaoning, Shaanxi, Shangdong, Shanghai, Yunnan and Zhejiang.

For measurement of housework, the time allocation data in the CHNS is used. Time spent buying and preparing food for the household, washing clothes and caring for children is considered. Data for another housework category, cleaning, was incomplete and therefore could not be used. However, it is hard to believe that time spent cleaning the house increased so much during the period so as to change the downward trend. Even more, probably this category of housework also decreased, although it never accounted for a large fraction of housework time. In sum, if anything the decrease in housework is slightly underestimated.

Wages in the CHNS

Figure 1.9 shows estimates of the hourly gender wage ratio. This was obtained controlling for province, education and age. Contrary to Ge and Yang (2014), there is constant gender wage ratio close to 0.75 for the whole period.

Figure 1.10, plots the results obtained for the skill premium on hourly wages. Like Ge and Yang (2014), they shows a marked increase in the skill premium, although for 1990 the skill premium seems too low. The estimated used to obtain this figure also control for age and province.

Three measures of assortative mating

For the first measure, I regress wife's education level on her husband's:

$$EDU^w_{my} = \alpha + \beta \times EDU^h_{my} + \sum_{t \in T} \gamma_t \times EDU^h_{my} \times YEAR_{ty} + \sum_{t \in T} \theta_t \times YEAR_{ty} + \epsilon_{my}$$

In the specification above, EDU^y_{mw} and EDU^y_{mh} represent wife's and husband's years of education in marriage m and year y, and $YEAR_{ty}$ is a time dummy that takes the value of 1 when $t = y$ and 0 otherwise, with $T = \{1992, 1996, 1999, 2003, 2005, 2008, 2010\}$. The coefficient β measures the correlation between wife's and husband's education in the base year (1990), the θ_t's control for the secular rise in education. The interest is in the γ_t's which measure the difference between wife and husband's correlation in year t and the baseline year. If γ_t rises with t, there's evidence of increasing assortative mating over time.

For the next two measures, the levels of education are collapsed into three categories: low skill (primary or less), medium skill (High school) and high skill (college).

The second measure of assortative mating used is Kendall's τ rank correlation. A value of 1 means perfect positive rank correlation, that is, the man with the highest education level is married with the woman with the highest education level, the man with the second highest education level is married with the woman with the second highest education level and so forth. A value of -1 means the opposite, i.e perfect negative rank correlation. The closer Kendall's τ is to 1, the higher the assortative mating. The value of τ_t is measured for $t \in \{1990, 1992, 1996, 1999, 2003, 2005, 2008, 2010\}$. Again, if τ_t rises with t, this points to increasing assortative mating over time.

Finally, a measure of assortative mating based on contingency tables is computed, as the one shown in table 1.15 for 1990. Each cell has two entries: the one on the left gives the observed fraction of married households with the combination of wife and husband's education for that row and column. The second gives the fraction of households there would be in that cell if matching was random. This is obtained by multiplying the total fraction of women in her education category (the sum of elements in that row) by the total fraction of men in his (the sum of elements in that column). The values along the diagonal are the fractions of marriages where both spouses have the same education level. To measure assortative mating, take the sum along the diagonal for both the observed and random matches, and divide the first by the second. This is denoted Δ_t, and it is computed again for $t \in \{1990, 1992, 1996, 1999, 2003, 2005, 2008, 2010\}$. Values of Δ_t above 1 mean that there is positive assortative mating, as the observed fraction of matches in which spouses have the same education is larger than the one random matching would produce. Once more, if Δ_t rises with t, there's evidence of increasing assortative mating.

Table 1.15: Contingency table for marriages in China, 1990

Wife	Husband: Low skill		Medium skill		High skill		Marginal
Low skill	**0.251**	**0.164**	0.247	0.317	0.006	0.023	0.504
Medium skill	0.074	0.153	**0.371**	**0.294**	0.023	0.021	0.468
High skill	0.001	0.009	0.011	0.018	**0.017**	**0.001**	0.028
Marginal	0.326		0.629		0.046		

Source: Author's work with data from the China Health and Nutrition Survey.

1.B Solution of the time allocation and home production problem

Single agents

The problem of a single agent of sex $i \in \{f, m\}$ and type z can be written as:

$$U_S^i(z) = \max_{c,l,h,e_q} u(c, l, G(h, e_q))$$

$$\text{subject to}$$

$$c = \omega_i(z)(1 - l - h) - p_e e_q.$$

The corresponding Lagrangian is:

$$\mathcal{L} : \frac{\sigma_c}{1-\sigma} c^{1-\sigma} + \frac{\sigma_l}{1-\sigma} l^{1-\sigma} + \frac{\sigma_g}{1-\sigma} \left[A_G \left[e_q^{1-\alpha_G}\right] h^{\alpha_G}\right]^{1-\sigma} + \lambda_i(z) \left[\omega_i(z)(1 - l - h) - p_e e_q\right].$$

The first order conditions are given by:

$$\sigma_c c^{-\sigma} - \lambda_i(z) = 0 \implies c = \left(\frac{\sigma_c}{\lambda_i(z)}\right)^{\frac{1}{\sigma}} \quad \text{(SFOC 1)}$$

$$\sigma_l l^{-\sigma} - \lambda_i(z)\omega_i(z) = 0 \implies l = \left(\frac{\sigma_l}{\lambda_i(z)\omega_i(z)}\right)^{\frac{1}{\sigma}} \quad \text{(SFOC 2)}$$

$$\sigma_g g^{-\sigma} A_G \alpha_G \left[e_q^{1-\alpha_G}\right] h^{\alpha_G - 1} - \lambda_i(z)\omega_i(z) = 0 \quad \text{(SFOC 3)}$$

$$\sigma_g g^{-\sigma} A_G (1 - \alpha_G) \left[e_q^{-\alpha_G}\right] h^{\alpha_G} - \lambda_i(z) p_e. \quad \text{(SFOC 4)}$$

From SFOC 1 and SFOC 2 we have derived demands for market goods and leisure. From SFOC 3 and SFOC 4 we can derive an expression for the ratio between home equipment use and housework:

$$\frac{\sigma_g g^{-\sigma} A_G \alpha_G \left[e_q^{1-\alpha_G}\right] h^{\alpha_G - 1}}{\omega_i(z)} = \frac{\sigma_g g^{-\sigma} A_G (1 - \alpha_G) \left[e_q^{-\alpha_G}\right] h^{\alpha_G}}{p_e}$$

$$\iff x_i^e(z) \equiv \frac{e_q}{h} = \frac{(1 - \alpha_G)\,\omega_i(z)}{\alpha_G p_e}.$$

Furthermore, the ratio of home production to housework can be expressed as a function of $x_i^e(z)$:

$$x_i^g(z) \equiv \frac{g}{h} = \frac{A_G\left[e_q^{1-\alpha_G}\right]h^{\alpha_G}}{h} = A_G\left(\frac{e_q}{h}\right)^{1-\alpha_G} = A_G\left[x_i^e(z)\right]^{1-\alpha_G}.$$

From SFOC 3 we can derive an expression for the demand of home produced goods that is a function of $x_i^e(z)$ and the Lagrange multiplier:

$$g = \left(\frac{\sigma_g}{\lambda_i(z)D_i(z)}\right)^{\frac{1}{\sigma}},$$

where $D_i(z) = \frac{\omega_i(z)}{A_G\alpha_G\left[x_i^e(z)\right]^{1-\alpha_G}}$ is the effective marginal price of home goods. Moreover, $h = \frac{g}{x_i^g(z)}$ and $e_q = \frac{x_i^e(z)g}{x_i^g(z)}$.

Substituting the expressions for c, l, h and e_q in the budget constraint:

$$\left(\frac{\sigma_c}{\lambda_i(z)}\right)^{\frac{1}{\sigma}} = \omega_i(z)\left[1 - \left(\frac{\sigma_l}{\lambda_i(z)\omega_i(z)}\right)^{\frac{1}{\sigma}} - \frac{g}{x_i^g(z)}\right] - p_e\frac{x_i^e(z)g}{x_i^g(z)}.$$

Substituting the expression for g, we can solve for the Lagrange multiplier:

$$\left(\frac{\sigma_c}{\lambda_i(z)}\right)^{\frac{1}{\sigma}} = \omega_i(z)\left[1 - \left(\frac{\sigma_l}{\lambda_i(z)\omega_i(z)}\right)^{\frac{1}{\sigma}} - \frac{\left(\frac{\sigma_g}{\lambda_i(z)D_i(z)}\right)^{\frac{1}{\sigma}}}{x_i^g(z)}\right] - p_e\frac{x_i^e(z)\left(\frac{\sigma_g}{\lambda_i(z)D_i(z)}\right)^{\frac{1}{\sigma}}}{x_i^g(z)}$$

$$\Longrightarrow \lambda_i(z) = \left[\frac{\sigma_c^{\frac{1}{\sigma}} + \omega_i(z)\left(\frac{\sigma_l}{\omega_i(z)}\right)^{\frac{1}{\sigma}} + \frac{\omega_i(z)}{x_i^g(z)}\left(\frac{\sigma_g}{D_i(z)}\right)^{\frac{1}{\sigma}} + \frac{p_e x_i^e(z)}{x_i^g(z)}\left(\frac{\sigma_g}{D_i(z)}\right)^{\frac{1}{\sigma}}}{\omega_i(z)}\right]^{\sigma}.$$

Notice that the above expressions for $x_i^g(z)$, $D_i(z)$, $x_i^e(z)$ (and thus $\lambda_i(z)$) depend only on parameters of the model. Thus, we have found a closed-form solution for the time allocation problem. This solution will be always interior for singles, as the marginal utilities of market goods consumption, leisure and home produced goods all go to infinity when paid work, leisure or housework time go to zero, respectively.

Married households

The problem of a married household with wife's type z_f and husband's type z_m can be written as:

$$\max_{c_f,c_m,l_f,l_m,h_m,h_f,eq}\left\{\chi_f u_f\left(c_f,l_f\right)+\left(1-\chi_f\right)u_m\left(c_m,l_m\right)+\frac{\sigma_g}{1-\sigma}G\left[H\left(h_f,h_m\right),e_q\right]^{1-\sigma}\right\}$$

subject to

$$c_f+c_m=\omega_f(z_f)\left(1-l_f-h_f\right)+\omega_m(z_m)\left(1-l_m-h_m\right)-p_e e_q.$$

The Lagrangian of this problem is:

$$\mathcal{L}:$$
$$\chi_f\left(\frac{\sigma_c}{1-\sigma}c_f+\frac{\sigma_l}{1-\sigma}l_f\right)+\left(1-\chi_f\right)\left(\frac{\sigma_c}{1-\sigma}c_m+\frac{\sigma_l}{1-\sigma}l_m\right)+\frac{\sigma_g}{1-\sigma}G\left[H\left(h_f,h_m\right),e_q\right]^{1-\sigma}$$
$$+\lambda(z_f,z_m)\left[\omega_f(z_f)\left(1-l_f-h_f\right)+\omega_m(z_m)\left(1-l_m-h_m\right)-p_e e_q-c_f-c_m\right].$$

The first order conditions are:

$$\chi_f\sigma_c c_f^{-\sigma}-\lambda(z_f,z_m)=0\implies c_f=\left(\frac{\chi_f\sigma_c}{\lambda(z_f,z_m)}\right)^{\frac{1}{\sigma}} \tag{MFOC 1}$$

$$\left(1-\chi_f\right)\sigma_c c_m^{-\sigma}-\lambda(z_f,z_m)=0\implies c_m=\left(\frac{\left(1-\chi_f\right)\sigma_c}{\lambda(z_f,z_m)}\right)^{\frac{1}{\sigma}} \tag{MFOC 2}$$

$$\chi_f\sigma_l l_f^{-\sigma}-\lambda(z_f,z_m)\omega_f(z_f)=0\implies l_f=\left(\frac{\chi_f\sigma_l}{\lambda(z_f,z_m)\omega_f(z)}\right)^{\frac{1}{\sigma}} \tag{MFOC 3}$$

$$\left(1-\chi_f\right)\sigma_l l_m^{-\sigma}-\lambda(z_f,z_m)\omega_m(z_m)=0\implies l_m=\left(\frac{\left(1-\chi_f\right)\sigma_l}{\lambda(z_f,z_m)\omega_m(z_m)}\right)^{\frac{1}{\sigma}} \tag{MFOC 4}$$

$$\sigma_g g^{-\sigma}G_h H_{h_f}-\lambda(z_f,z_m)\omega_f(z_f)=0 \tag{MFOC 5}$$

$$\sigma_g g^{-\sigma}G_h H_{h_m}-\lambda(z_f,z_m)\omega_m(z_m)=0 \tag{MFOC 6}$$

$$\sigma_g g^{-\sigma}G_{e_q}-\lambda(z_f,z_m)p_e, \tag{MFOC 7}$$

where:

$$G_h=\frac{\partial G}{\partial h}=\alpha_G A_G\left[e_q^{1-\alpha_G}\right]h^{\alpha_G-1}=\alpha_G A_G\left[\frac{e_q}{h}\right]^{1-\alpha_G}$$
$$G_{e_q}=\frac{\partial G}{\partial e_q}=\left(1-\alpha_G\right)A_G\left[e_q^{-\alpha_G}\right]h^{\alpha_G}=\left(1-\alpha_G\right)A_G\left[\frac{e_q}{h}\right]^{-\alpha_G}$$

$$
\begin{aligned}
H_{h_f} &= \frac{\partial H}{\partial h_f} = \frac{1}{1-\eta}\left[\eta_f h_f^{1-\eta} + (1-\eta_f)\, h_m^{1-\eta}\right]^{\frac{1}{1-\eta}-1}(1-\eta)\,\eta_f h_f^{-\eta} \\
&= h^{\eta}\eta_f h_f^{-\eta} \\
H_{h_m} &= \frac{\partial H}{\partial h_m} = h^{\eta}\,(\eta_f)\, h_m^{-\eta}.
\end{aligned}
$$

Now, define $x^g(z_f, z_m) \equiv \frac{g}{h}$, $x^e(z_f, z_m) \equiv \frac{e_q}{h_m}$, $x^f(z_f, z_m) \equiv \frac{h_f}{h_m}$ and $x^h(z_f, z_m) \equiv \frac{h}{h_m}$. From MFOC 7:

$$
g = \left(\frac{\sigma_g G_{eq}}{\lambda(z_f, z_m) p_e}\right)^{\frac{1}{\sigma}} = \left[\frac{\sigma_g\,(1-\alpha_G)\, A_G \left(\frac{e_q}{h}\right)^{-\alpha_G}}{\lambda(z_f, z_m) p_e}\right]^{\frac{1}{\sigma}}.
$$

Since $\frac{e_q}{h} = \frac{x^e(z_f, z_m)}{x^h(z_f, z_m)}$, then:

$$
g = \left[\frac{\sigma_g\,(1-\alpha_G)\, A_G \left(\frac{x^e(z_f, z_m)}{x^h(z_f, z_m)}\right)^{-\alpha_G}}{\lambda(z_f, z_m) p_e}\right]^{\frac{1}{\sigma}} = \left(\frac{\sigma_g}{\lambda(z_f, z_m) D(z_f, z_m)}\right)^{\frac{1}{\sigma}},
$$

where $D = \frac{p_e\left(\frac{x^e(z_f, z_m)}{x^h(z_f, z_m)}\right)^{\alpha_G}}{A_G(1-\alpha_G)}$ is the effective marginal price of home goods.

From MFOC 5 and MFOC 6 we can derive a closed form expression for $x^f(z_f, z_m)$:

$$
\begin{aligned}
&\frac{\sigma_g g^{-\sigma} G_h H_{h_f}}{\omega_f(z_f)} = \frac{\sigma_g g^{-\sigma} G_h H_{h_m}}{\omega_m(z_m)} \\
\Longrightarrow\ &\frac{H_{h_f}}{\omega_f(z_f)} = \frac{H_{h_m}}{\omega_m(z_m)} \\
\Longrightarrow\ &\frac{h^{\eta}\eta_f h_f^{-\eta}}{\omega_f(z_f)} = \frac{h^{\eta}\,(\eta_f)\, h_m^{-\eta}}{\omega_m(z_m)} \\
\Longrightarrow\ &x^f(z_f, z_m) \equiv \frac{h_f}{h_m} = \left[\frac{\eta_f \omega_m(z_m)}{(1-\eta_f)\,\omega_f(z_f)}\right]^{\frac{1}{\eta}}.
\end{aligned}
$$

Using the expression above, we can derive one for $x^h(z_f, z_m)$:

$$
h_f = \left[\frac{\eta_f \omega_m(z_m)}{(1-\eta_f)\,\omega_f(z_f)}\right]^{\frac{1}{\eta}} h_m.
$$

Substituting into the expression for h:

$$h=\left\{\eta_f\left[\frac{\eta_f\omega_m(z_m)}{(1-\eta_f)\,\omega_f(z_f)}\right]^{\frac{1-\eta}{\eta}}h_m^{1-\eta}+(1-\eta_f)\,h_m^{1-\eta}\right\}^{\frac{1}{1-\eta}}$$

$$\Longrightarrow x^h(z_f,z_m)\equiv\frac{h}{h_m}=\left[\eta_f\left(\frac{\eta_f}{1-\eta_f}\frac{\omega_m(z_m)}{\omega_f(z_f)}\right)^{\frac{1-\eta}{\eta}}+1-\eta_f\right]^{\frac{1}{1-\eta}}.$$

From MFOC 6 and MFOC 7 we can derive an expression for $x^e(z_f,z_m)$ that depends on $x^h(z_f,z_m)$ (for which we already have a closed form solution):

$$\frac{\sigma_g g^{-\sigma}G_hH_{h_m}}{\omega_m(z_m)}=\frac{\sigma_g g^{-\sigma}G_{e_q}}{p_e}$$

$$\Longrightarrow\frac{\alpha_GA_G\left[\frac{e_q}{h}\right]^{1-\alpha_G}h^{\eta}\,(\eta_f)\,h_m^{-\eta}}{\omega_m(z_m)}=\frac{(1-\alpha_G)\,A_G\left[\frac{e_q}{h}\right]^{-\alpha_G}}{p_e}$$

$$\Longrightarrow\frac{x^e(z_f,z_m)}{x^h(z_f,z_m)}x^h(z_f,z_m)^{\eta}=\frac{(1-\alpha_G)\,\omega_m(z_m)}{\alpha_Gp_e\,(1-\eta_f)}$$

$$\Longrightarrow x^e(z_f,z_m)=\frac{(1-\alpha_G)\,\omega_m(z_m)x^h(z_f,z_m)^{1-\eta}}{\alpha_Gp_e\,(1-\eta_f)}.$$

We derive also an expression for $x^g(z_f,z_m)$ that depends on $x^e(z_f,z_m)$ and $x^h(z_f,z_m)$:

$$g=A_G\left(e_q^{1-\alpha_G}\right)h^{\alpha_G}=A_Gx^e(z_f,z_m)^{1-\alpha_G}x^h(z_f,z_m)^{\alpha_G}h_m$$

$$\Longrightarrow x^g(z_f,z_m)\equiv\frac{g}{h_m}=A_Gx^e(z_f,z_m)^{1-\alpha_G}x^h(z_f,z_m)^{\alpha_G}.$$

We are almost ready to derive an expression for the Lagrange multiplier. We just have to notice that $h_m=\frac{g}{x^g(z_f,z_m)}$, $h_f=\frac{gx^f(z_f,z_m)}{x^g(z_f,z_m)}$ and $e_q=\frac{gx^e(z_f,z_m)}{x^g(z_f,z_m)}$, and substitute these along with the expressions for c_f, c_m, l_f, l_m and g in the budget constraint:

$$\left(\frac{\chi_f\sigma_c}{\lambda(z_f,z_m)}\right)^{\frac{1}{\sigma}}+\left(\frac{(1-\chi_f)\,\sigma_c}{\lambda(z_f,z_m)}\right)^{\frac{1}{\sigma}}=\omega_f(z_f)\left[1-\left(\frac{\chi_f\sigma_l}{\lambda(z_f,z_m)\omega_f(z_f)}\right)^{\frac{1}{\sigma}}-\frac{gx^f(z_f,z_m)}{x^g(z_f,z_m)}\right]$$

$$+\,\omega_m(z_m)\left[1-\left(\frac{(1-\chi_f)\,\sigma_l}{\lambda(z_f,z_m)\omega_m(z_m)}\right)^{\frac{1}{\sigma}}-\frac{g}{x^g(z_f,z_m)}\right]+p_e\frac{x^e(z_f,z_m)g}{x^g(z_f,z_m)}.$$

Finally, the closed form expression for the Lagrange multiplier is:

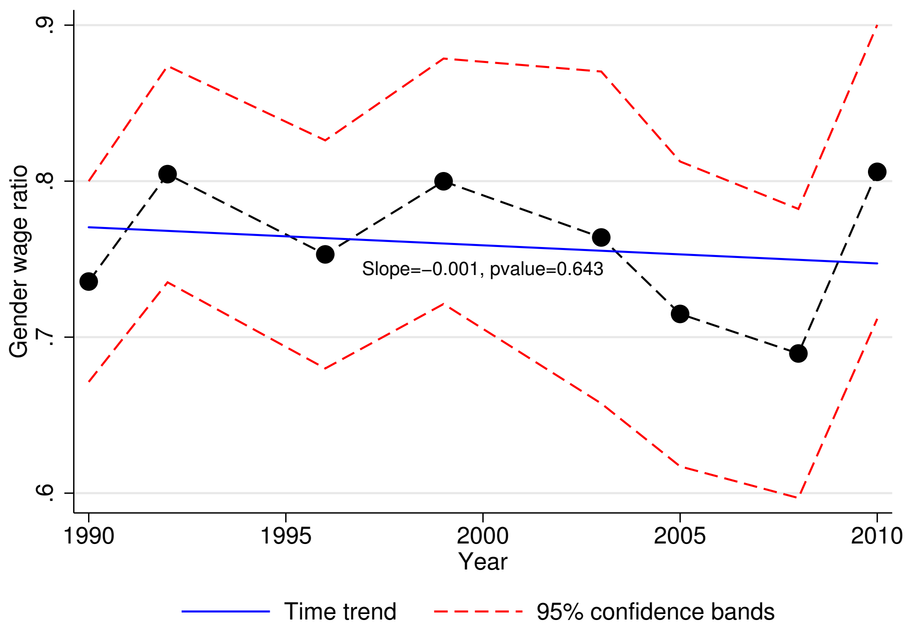
$$\lambda(z_f, z_m) = \left(\frac{B(z_f, z_m)}{\omega_f(z_f) + \omega_m(z_m)} \right)^{\sigma},$$

where:

$$B(z_f, z_m) = (\chi_f \sigma_c)^{\frac{1}{\sigma}} + [(1-\chi_f)\,\sigma_c]^{\frac{1}{\sigma}} + \omega_f(z_f) \left(\frac{\chi_f \sigma_l}{\omega_f(z_h)} \right)^{\frac{1}{\sigma}} + \omega_m(z_m) \left[\frac{(1-\chi_f)\,\sigma_l}{\omega_m(z_m)} \right]^{\frac{1}{\sigma}}$$
$$+ \left(\frac{\sigma_g}{D(z_f, z_m)} \right)^{\frac{1}{\sigma}} \left[\frac{\omega_f(z_f)}{x^g(z_f, z_m)} x^f(z_f, z_m) + \frac{\omega_m(z_m)}{x^g(z_f, z_m)} + \frac{p_e x^e(z_f, z_m)}{x^g(z_f, z_m)} \right].$$

Solutions may not always be interior for the married household problem. In particular, for some parameter configurations h_f, h_m, n_f or n_m could optimally be zero, i.e., there may be solutions in which one of the spouses doesn't do paid work or housework. Whenever solutions are not interior, numerical methods are used to find the optimal quantities for married households.

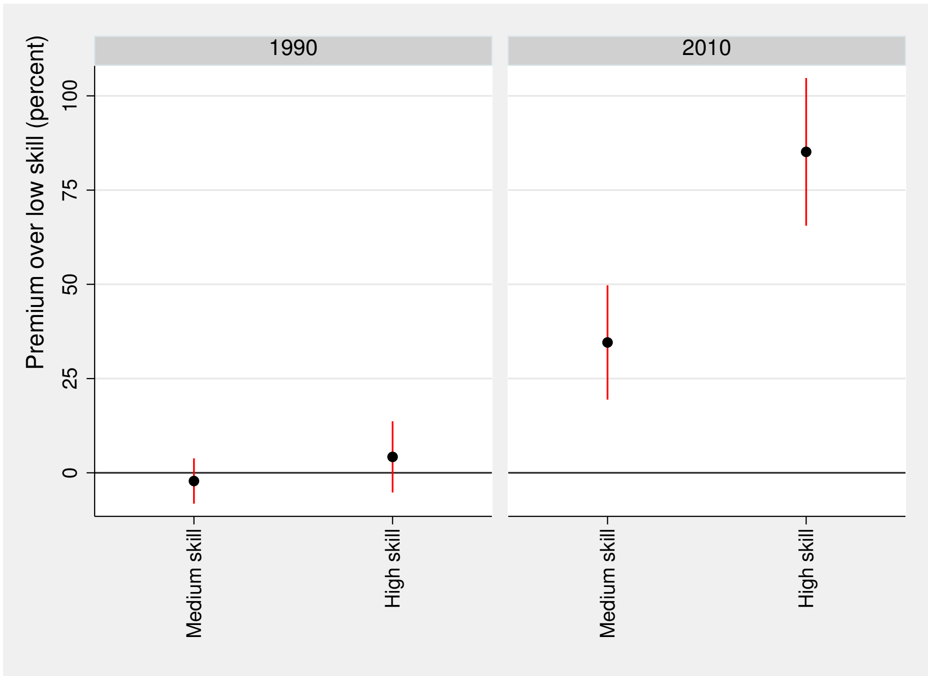
Figure 1.9: Gender hourly wage ratio in China, 1990-2020



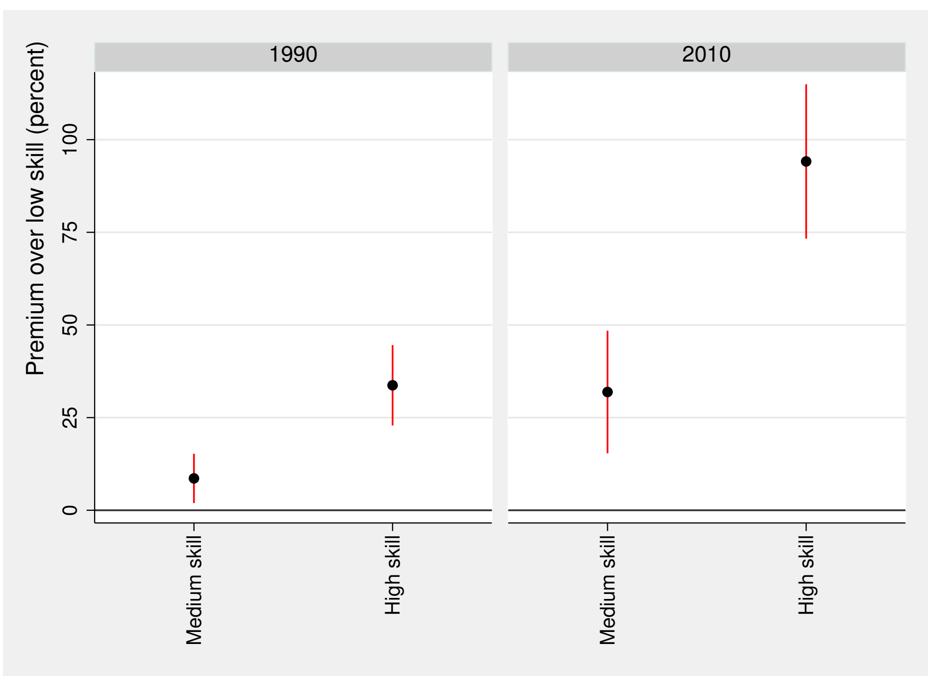
Source: Author's work with data from the China Health and Nutrition Survey.

Figure 1.10: Skill distribution for Chinese people aged 20-35, 1990-2010

(a) Males



(b) Females



Source: Author's work with data from the China Health and Nutrition Survey.

Note: Skill levels are defined based on educational attainment (highest grade attained). Low skill individuals are those with primary or less, medium skill are those with middle-high school and high skill those with some college or more.

Chapter 2

Quantifying the Impact of Childcare Subsidies on Social Security

2.1 Introduction

Higher female labour force participation and decreased fertility levels are two of the most significant changes in women's socio economic behaviour in developed countries in recent decades. Social security systems are directly affected by them, especially those that rely on pay-as-you-go schemes. The increase of female labour force participation brings in more contributions to Social Security in the short run. In the future, this implies additional pension entitlements once these women retire. Moreover, a drop in fertility levels causes the old-age dependency ratio to increase eventually (absent migration). As the share of pensioners over contributors rise, pressure on the system's finances builds.

Naturally, changes in female labour force participation and fertility are not independent of each other. They are jointly determined. Any policy that affects this joint decision affects the finances of social security.

Using a variety of data sources, including administrative, household panel and time use surveys, this chapter provides evidence suggesting that childcare policies may boost fertility and strengthen mother's labour market attachment in Spain. A number of facts underlie this claim. First, like in most developed countries, there is a family gap in female labour force participation. In other words, women with children participate less in the labour market than childless women, and are more likely to be part-time workers if they do. Second, there is government provided universal coverage of preschool for children 3-6, and families use it extensively. In contrast, preschool usage for children 0-3, which is paid mostly out of pocket by families, is much lower. Moreover, access to informal unpaid childcare provided by other family members[1] is limited as well. Taken together, this suggests families with young children face important constraints when deciding how to cover their childcare needs. Finally, there is a positive gap between desired and realised fertility. That is, women have fewer children than they would like to have. Moreover, the number one reason women say prevents them from attaining their desired fertility is lack of compatibility between career and family. One can hypothesize then that the introduction of a policy that subsidises childcare for children aged 0-3 could improve the social security's budget balance by jointly stimulating fertility and female labour force participation.

To quantify the effect of the introduction of such a policy, an overlapping generations model is built in which women choose fertility levels at the beginning of life. In subsequent periods they decide on labour force participation, and if they have young children, childcare allocation among maternal, informal and out-of-pocket alternatives. Simultaneously, the government runs a Social Security PAYG system financed with a payroll tax and pays retirement benefits.

The model is calibrated to Spanish data and social security regulations. A series of policy experiments are subsequently performed. First, two policies that have actually been implemented by the Spanish government in the past and have been evaluated in the literature are tested in the model, namely universal childcare for children 3-6 and cash transfers upon birth. Then, the model is used to experiment with the introduction of partial (50%) and full (100%) childcare subsidies for women with children aged 0 to 3, a new policy.

For the first two policies, the findings are similar qualitatively with respect to previous literature. The newly proposed policies have small effects on female labour force participation and fertility decisions, which translates into a negative impact on the present value of the budget balance of Social Security.

In particular, the subsidies on childcare for children aged 0-3 induce an increase in part-time and a decrease in full-time work among women aged 33-39 with children in this ages, with a small net positive effect on total participation. The total fertility rate rises marginally, from 1.25 in the baseline economy to 1.26 under partial subsidies and 1.27 under full subsidies. These results are mainly driven by couples who did not have any children in the baseline economy, and under the new scenario, they choose to have one. Therefore, it suggests that introducing childcare subsidisation in Spain has a small positive effect on the number of children in the extensive margin for couples without any in the baseline economy.

The combined effects of the subsidies on female labour force participation and fertility are to worsen the Social Security budget balance in all future periods, which lowers its present value. The effect of fertility is too small to compensate for the immediate cost that the subsidies entail. Therefore, the policy fails to deliver the benefits hypothesised above.

[1] For instance, grandparents.

This chapter contributes to different strands of the literature. It is broadly related to the extensive body of research on the relationship between fertility, women's labour market outcomes, and policies relevant to it. Among the papers that rely on structural, dynamic models to study the issues at hand are Erosa et al. (2002), Da Rocha and Fuster (2006) and Adda et al. (2017). Unlike these, the labour market is not modelled here in detail because there are no search frictions or occupational choices. In contrast, a detailed Social Security system is included. The model economy is similar to the one in Bick (2016), Hannusch (2019) and Laun and Wallenius (2021). Women's labour force and childcare utilisation decisions during their child-rearing years are heavily inspired by Bick (2016). The contributions consist on modelling non-child rearing years, retirement benefits, and changing the way decisions regarding the number of children are made, by relying on desired fertility data. Differently from Hannusch (2019), women are allowed to decide to use less than their endowment of informal childcare. It may be that some mothers who value maternal childcare time highly do not use all the informal childcare endowment. The arrival of children to the household conditional on the fertility level follows a deterministic schedule, as in Laun and Wallenius (2021). That is, the model does not study the timing and spacing of multiple children, only total fertility. Even though the last two papers introduce pensions, they do not quantify family policies' effect on the system's finances.

This chapter relates as well to the literature on Spanish child-related policies. Sánchez-Mangas and Sánchez-Marcos (2008) examine the introduction of a reform of Spain's income tax in 2003. The reform introduced two main changes. First, it increased children's taxable income deduction [2]. Second, it raised the additional monthly cash benefit/tax deduction to working mothers with children under three years[3]. Through a difference-in-differences-in-differences (DDD) estimator, they find that this policy increased labour market participation rate of eligible mothers by three percentage points compared to non-policy-eligible ones. González (2013) studies the impact of a sizeable universal child benefit introduced in Spain between 2007 and 2010 on fertility and early maternal labour supply. Following a regression discontinuity design (RDD), she found that this policy encourages fertility, and mothers to stay out of the labour force for longer after childbirth. Nollenberger and Rodríguez-Planas (2015) investigate the effect of the introduction in Spain of universal public preschool provision for children aged three and older in the 1990s on mother's labour force participation. The strategy relies on a DDD approach for this natural experiment. They find that two mothers enter the labour market for every ten additional children enrolled in public childcare.

Versions of the last two policies are implemented in the structural model to compare the results as an external validity test. Namely, a one-time payment of 2500 euros for mothers giving birth and a counterfactual

[2] In particular, before the policy, families could deduct 1200 euros each for the first and second child, and 1800 for each subsequent child. The tax reform increased these deductions to 1400 for the first, 1500 for the second, 2200 for the third and 2300 euros for the subsequent children.

[3] From 300 euros to 1200 euros per child.

analysis in which public preschool provision is not present. The results are qualitatively similar. However, they differ in magnitude[4].

Finally, this chapter builds on a long tradition in macroeconomics research on sustaining social security systems. This literature uses life-cycle models, pioneered by Auerbach and Kotlikoff (1987). This study contributes to the research that proposes and quantifies potential policy reforms that seek to help finance social security systems. This proposals include partial privatisation (Nishiyama and Smetters, 2007), mandatory fully funded social security systems financed with a consumption tax (Kotlikoff et al., 2007), retirement age delay (Díaz-Giménez and Díaz-Saavedra (2009) in Spain, Imrohoroğlu and Kitao (2012) and Kitao (2014) in the US), increasing the payroll tax, decreasing the wage replacement rate or making the benefits fall one-to-one with income above a threshold level (Kitao, 2014). Cruces (2021) shows that women's increased labour force participation in Spain has helped finance social security in the short run, but will generate pressure in the long run. Her model takes fertility as exogenous, however. This chapter's novelty is that it assesses childcare subsidisation as a policy to achieve sustainability, which has not been considered before.

The rest of the chapter is structured as follows. Section 2 discusses some empirical facts on female labour force participation around childbearing years, childcare utilisation and fertility choices, as well as the state of child-related policies in Spain. Section 3 describes the model. Section 4 discusses the calibration and model performance. Section 5 presents the policy experiments. Section 6 concludes.

2.2 Data, empirical facts and policy environment

This section briefly discusses Spanish policies related to labour gender equality and work/life balance. Then, women's labour market behaviour when they become mothers and the usage of childcare in Spain is discussed. This section and the remainder of the chapter relies mainly on data from the *Encuesta de Condiciones de Vida* (ECV henceforth) of the Spanish National Statistics Institute, *INE*. The ECV is an annual survey that consists of a rotating panel in which families are interviewed for four years. *INE* provides cross-sectional and longitudinal weights that allow for computing statistics representing the whole Spanish population. The years 2011 to 2019 of the survey are used.

2.2.1 Family policies in Spain

The main regulatory instrument to facilitate the compatibility between family and work is a billed from 1999 called Spanish family and workers' labour life reconciliation. This law includes different mechanisms that

[4]The long-run effects of family policies obtained through structural models may be smaller than the short-run effects found by quasi-experimental models, as pointed out by Adda et al. (2017).

aim to balance work and family, mainly regarding parental leaves. The current legislation entitles mothers with 16 and fathers with 4.3[5] paid leave weeks with full wage replacement. This parental leave includes full job protection during the first year. For the next two years, only a return to a similar or to a job of the same category is guaranteed. Besides, parents are entitled to unpaid leave to take care of their children. Returning to the same position after a short leave[6] is guaranteed, while only a return to a similar position is guaranteed after a long one.

The Spanish legislation currently does not consider direct cash transfers to families with children. However, between 2007-2010 the government provided a cash benefit of 2500 euros for each child born or adopted. This policy was faded out as part of the austerity measures after the 2008 economic crisis.

Childcare provision depends on the age of the child. For children younger than three years old, regional authorities are in charge, while for older children, it is the Spanish government who supplies it. The difference in who the supplier is, implies an unbalanced provision across ages and regions. The enrolment rate in early childhood education and care services from 0 to 2-year-old is low compared to three-year-old children. There exists unequal public kindergarten provision across regions. This fact is well-documented in González (2004). The lack of public childcare provision for younger children in Spain puts significant pressure on parents, especially on women. Families are forced to use informal childcare, to enrol their kids in private kindergartens and/or to change their labour market behaviour when children are born.

2.2.2 Female labour force participation

The *ECV* includes a labour force participation variable for each month over the last year. Taking full-time work to be 1, part-time work to be 0.5 and being out of the labour force to be zero, average labour force participation is computed for each woman in the sample in each year. If this average is above 0.75, the woman is assigned to full-time work, if it is between 0.25 and 0.75, she is assigned to part-time work, and if it is below 0.25, she is assigned to be out of the labour force for that year.

Panels (a) and (b) of Figure 2.1 show the 2011-2019 average fractions of part-time and full-time working women by age. Panel (a) shows those fractions among mothers and panel (b) among childless women. Both full-time and part-time work fall with age among mothers and childless women. Women in older cohorts with lower labour force attachment partially explain this result. Panel (c) plots the difference or gap between full and part-time work between mothers and childless women, plus the participation gap, which is the sum of the two. Unsurprisingly, there is a large (-15 percentage points) participation gap around age 30 that closes steadily and becomes nearly zero after age 50. This is the well known “family gap” in female labour force

[5] The number of weeks assigned was equated across genders in 2021.
[6] An unpaid leave is considered as a short one if it is less than one year.

participation. The part-time gap is positive, meaning that mothers are more likely to work part-time. The full-time gap is negative, which means the opposite. Both close with time, although older mothers are still somewhat more likely to work part-time and less likely to work full time than childless women.

2.2.3 Childcare

Figure 2.2 plots the average hours per week spent at preschool for children aged 0 to 3 and 3 to 6 by labour force participation of the mother. There are several things to notice. First, as expected, the children of women who work full time spend more time in preschool than those who work part-time, and they spend more time than those out of the labour force. Second, children aged 3 to 6 spend more time at preschool than children aged 0 to 2 regardless of their mother's labour force participation status. Third, the difference in average preschool hours between children of working mothers and non-working mothers is much smaller among children aged 3 to 6. Children of women who work full-time aged 0 to 3 spend almost twice as many hours in preschool on average than children of women who are not in the labour force. Among children aged 3 to 6, the difference is only about 10%.

This difference is mostly a result of universal public preschool coverage for children older than three, introduced in the late 1990s. It comes as no surprise that most children between 3 and 6 spend almost 30 hours per week at preschool since public preschools are in general open for six hours per day[7].

Preschool is not the only source of childcare tapped by working mothers, as it covers only about 15 of the 30-40 hours per week, and about 10 of the 15-20 hours per week required by full-time and part-time working mothers with children aged 0 to 3, respectively. The *ECV* includes information on childcare received without any payment in return. Grandparents, aunts, uncles, older siblings, extended family or neighbours might provide this childcare. This is called here informal childcare. Average hours spent under informal childcare for children aged 0 to 3 and 3 to 6 by the mother's labour force participation are shown in Figure 2.3.

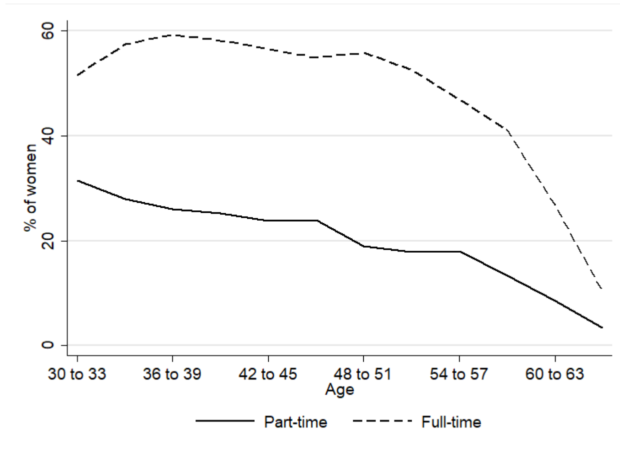
The results are surprisingly low. Adding the average preschool and informal care hours still leaves some required childcare unaccounted for. Likely the informal childcare is under-reported in the survey. Father's childcare time is not measured separately in the *ECV*, although it is unlikely that this solves the issue since most fathers work full time. There are other childcare sources measured in the *ECV*, but none of them is significant on average.

To get a better idea of the father's contribution to childcare, *INE's* occasional *Encuesta del uso del tiempo*, i.e. Time Use Survey (TUS), is used. The last iteration of the survey ran between 2009 and 2010. Contrary

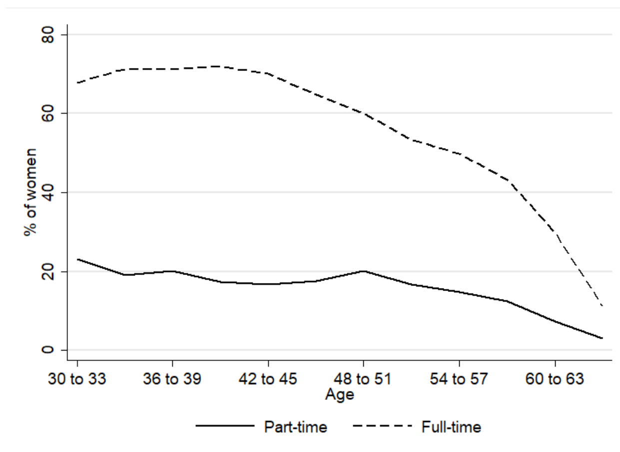
[7] Average preschool hours are below 30 because some children aged 3 to 6 were not yet eligible at the time of the survey to attend public preschools. The reason is that they were not aged three by January 1st. Moreover, others may be already attending primary schooling. When computing average preschool hours for children aged 4 and 5 at the time of the survey, who are eligible for public preschool and not for primary, an average of 30 hours for all levels of labour force participation of the mother is obtained.

Figure 2.1: Labour force participation of woman by age (30-65) and motherhood status in Spain, 2011-2019

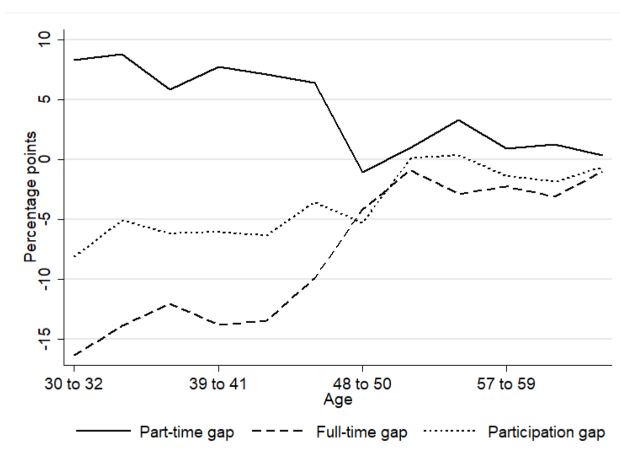
(a) Mothers



(b) Childless

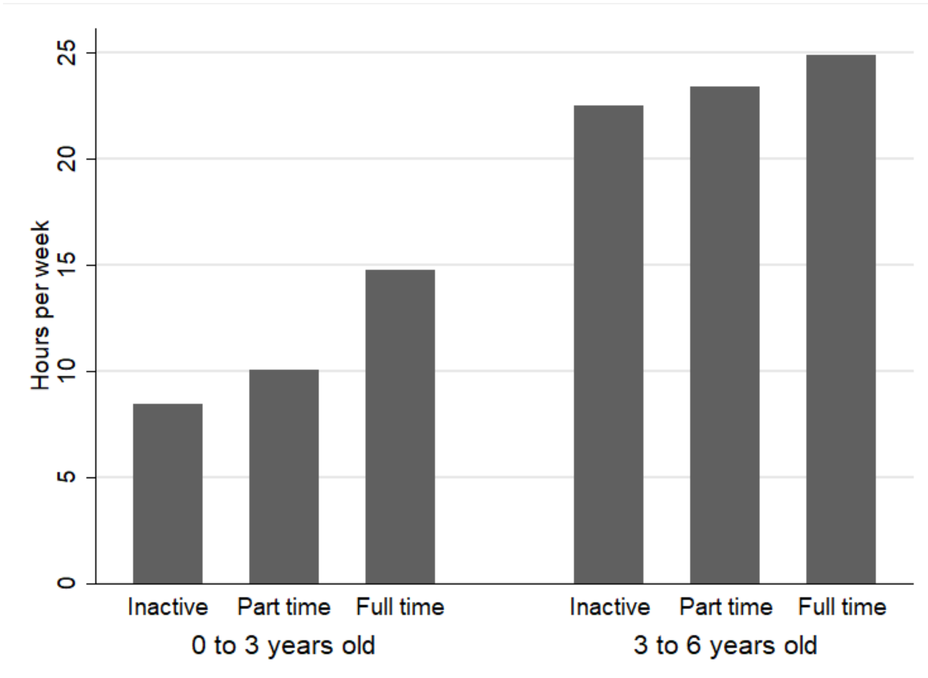


(c) Gap



Source: Author's work with data from *INE's Encuesta de Condiciones de Vida*.

Figure 2.2: Preschool usage by age of child and labour force participation status of mother (2011-2019)



Source: Author's work with data from *INE's Encuesta de Condiciones de Vida*.

to the American Time Use Survey (ATUS), the Spanish one collects data for each household member[8]. Thus, it allows to study how parents split chilcare duties.

The average gender gap was −86.57%[9]. In other words, mothers provide 86.57% more of childcare compared to fathers. On average mothers spend 183 minutes per day looking after children while fathers spend 98 minutes. After controlling by labour market participation, three results stand out. First, the childcare gender gap is very close to the average among two-earner households[10], at −80.42%. Second, this gap dramatically increases to −174.09% in households where mothers are out of the labour force. Third, in households where women are the sole income provider, men perform slightly more childcare than women, as the gap among these households is 14.53%. A great majority of the households are two-earner, meaning that on average mothers do a great deal more childcare than fathers.

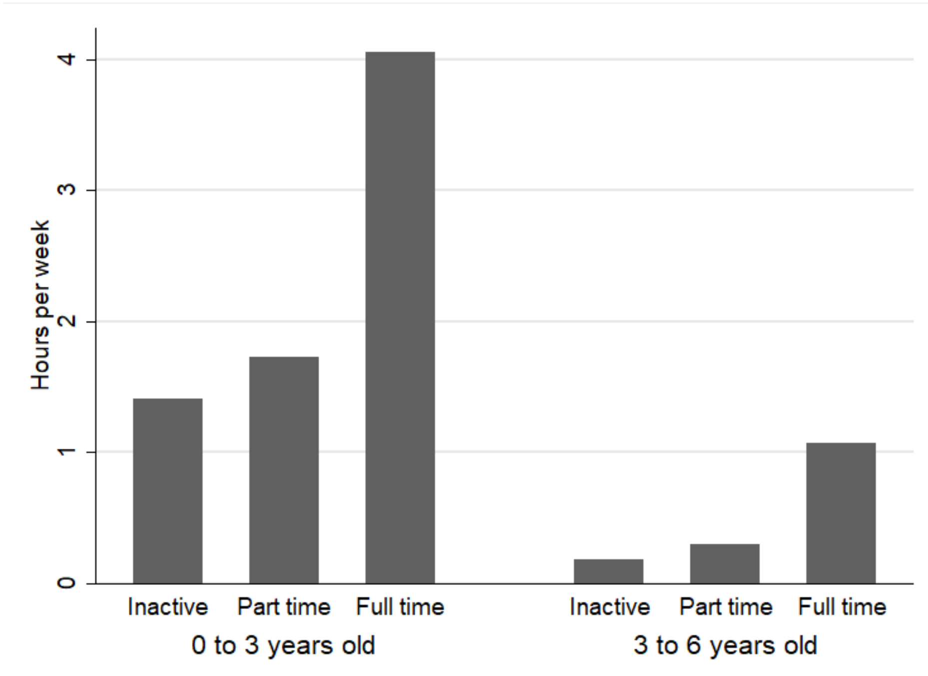
To sum up, the evidence points that mothers of children aged 0 to 3 are in a problematic childcare position. This stems from insufficient access to affordable options and persistent gender norms measured through the childcare gender gap.

[8] The ATUS collects data for one individual of each household.

[9] Defined as $(1 - \frac{\text{Mother's time}}{\text{Father's time}}) \times 100\%$.

[10] Households in which both spouses participate in the labour market.

Figure 2.3: Informal childcare usage by age of child and labour force participation status of mother (2011-2019)



Source: Author's work with data from *INE's Encuesta de Condiciones de Vida*.

2.2.4 Desired vs. realised fertility

Like in most European countries, Spanish women would like to have more children than they do. In 2018, the Spanish Statistics Institute ran a new iteration of its fertility survey *Encuesta de Fecundidad*, henceforth *EdF* (the previous one is dated from 1999). Table 2.2 shows the realised and desired fertility for women aged 40 to 44, and the difference between them. This age group was chosen because most of these women already completed their fertility at this point, meaning that the difference with desired fertility is definitive. The most popular fertility choice is two children, both in desired and realised terms.

Nevertheless, the data shows that the fraction of childless women is more than double the fraction of women who declare not wanting to have any children. There is gap of almost ten percentage points between the fraction of women with a single child and the fraction that declares wanting only one child. Less than half of the women that want three children have three children. These results show that there is a considerable number of women whose completed fertility is lower than their ideal one.

An additional question of interest is whether most of the gap between desired and realised fertility is due to some women being unable to have children (the extensive margin), or whether some mothers would like to have more children (the intensive margin). Table 2.2 shows realised and desired fertility for mothers

Table 2.1: Desired and realised fertility, women aged 40 to 44 in 2018 in Spain

Number of children	Realised	Desired	Difference
0	18.99%	7.90%	11.09%
1	24.96%	15.20%	9.76%
2	43.79%	49.75%	-5.96%
3 or more	12.26%	27.15%	-14.89%

Source: *Encuesta de Fecundidad 2018, INE.*

Table 2.2: Desired fertility by motherhood status, women aged 40 to 44 in 2018 in Spain

Number	Mothers			Childless		
of children	Realised	Desired	Difference	Realised	Desired	Difference
0	0.00%	0.22%	-0.22%	100.00%	40.65%	59.35%
1	30.81%	13.97%	16.84%	0.00%	20.47%	-20.47%
2	54.06%	54.14%	-0.08%	0.00%	31.00%	-31.00%
3 or more	15.13%	31.67%	-16.54%	0.00%	7.88%	-7.88%

Source: *Encuesta de Fecundidad 2018, INE.*

and childless women aged 40 to 44, and the difference between the two. First, notice that very few mothers declare not wanting children at all. This result shows that few women go beyond their desired fertility level. This seems reasonable because contraception and abortion are legal and widely available in Spain. Second, the number of desired children is much higher for mothers than for childless women. Thus, even among mothers, there is a difference between realised and the desired number of children. The fact that the number of women with two children coincides with the number of women declaring to want two children does not mean that every woman in this category is satisfied with her fertility. Likely a fraction of them would like to have three children, while a fraction of those with one would like to have two.

There are many potential reasons why women may choose not to have their desired number of children. The *EdF* asks women why they have fewer children than their desired number. The most common reasons given by women aged 40 to 44 are "work and work-family balance reasons" and "economic reasons".

Given the evidence laid out here, it seems that there is room for a childcare subsidisation policy targeted at women with children aged 0 to 3 years old to boost mother's labour force participation and fertility levels. These changes, in turn, would affect the social security budget. To quantify the magnitude of these changes, the model described in the next section is developed.

2.3 The model

This section studies an economy populated by overlapping generations of households that derive utility from consumption, leisure, the number of children they have, and the time spent with their children (if any).

Each household consists at least of a woman and her husband. At the beginning of life a fertility level is chosen. Conditional on this choice, children arrive and leave[11] the household following a fixed schedule. That is, households choose total fertility, but not the timing of births.

Working-age households make labour force participation decisions. Young children impose childcare needs upon parents that they need to cover with a combination of their own time, out-of-pocket childcare purchased in the market, and unpaid childcare time available to them. Therefore, households with young children need to balance labour force participation decisions and childcare needs.

Households eventually retire from the workforce. There are no savings, but the government runs a social security PAYG scheme financed with contributions from current working-age households in the form of a payroll tax. Retired households receive benefits that depend on women and husbands' income back when they were in the labour force. In the rest of the section, the model is described in detail.

2.3.1 Demographics, endowments and heterogeneity

At each period t, a new generation of individuals is born and another one dies. Since the prices faced by each cohort are the same, the life-cycle decisions are the same for households with identical states. Therefore, the time subscript is omitted from household variables.

By the age that modelling of household decisions begins, it is assumed that individuals are already sorted into couples formed by a woman $i = f$ and her husband $i = m$. The model abstracts from singles[12], divorce and remarriage. Spouses are assumed to be the same age. At age $j = J_0$ households make a fertility decision and start working-age life. Mandatory retirement occurs at $j = J_R$, and the household dies with probability one at $j = J$, i.e. life expectancy is deterministic. The life cycle of the household is therefore divided into $W = J_R - J_0$ working-age, and $R = J - J_R + 1$ retirement periods.

Households start working-age with no assets, and they are unable to save their period's income or borrow from the future. They are endowed with one unit of time in each period, which can be devoted to work, leisure and looking after children. At age $j = J_0$, spouses receive a pair of correlated income shocks $(\epsilon^f_{J_0}, \epsilon^m_{J_0})$. This shocks then evolve following an AR(1) process:

[11] Children eventually grow up and leave.

[12] This excludes 10.1% of Spanish households that were mono parental in 2019, out of which 82% were composed of women and her children. This households would likely respond very differently than two-parent households to child related policies, but introducing them would considerably complicate the model and it is left for future work.

$$\begin{aligned} \epsilon_j^f &= \phi^f \epsilon_{j-1}^f + \nu_j^f \\ \epsilon_j^m &= \phi^m \epsilon_{j-1}^m + \nu_j^m \end{aligned} \quad \text{with} \quad \begin{bmatrix} \nu_j^f \\ \nu_j^m \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_{\nu^f}^2 & \rho \\ \rho & \sigma_{\nu^m}^2 \end{bmatrix}\right), \tag{2.1}$$

Apart from having different income shocks, households differ in their preferences over children. There are two sources of heterogeneity regarding fertility preferences: over the desired number of children and over the intensity of those preferences. The exact functional form this takes in the model is discussed in subsection 2.3.3.

2.3.2 Timing and choice variables

The length of each model period corresponds to three years. Period $j = J_0$ is set to start around age 30, while $W = 18$ and $R = 13$. That is, the working-age stage in the model spans ages 30-66, while the retirement stage goes from ages 66 to 84. For households with children, the working-age stage is further subdivided into two stages: child-rearing and non child-rearing. The former lasts while there are children younger than 12 years old present in the household.

In what follows, the choice variables in each stage are described.

Prior to $j = J_0$. Before starting the first period and after a couple is formed, the household decides on the number of children they will have. This decision is denoted by b, with $b \in \{0, 1, 2, 3\}$. That is, couples can have up to three children. If $b > 0$, the first child arrives at $j = 1$. If $b > 1$, a child is born in every subsequent period until the chosen number of children is reached. This implies a spacing between siblings of three years.

Working-age stage. In every working-age period, that is for $j < J_R$, households face a discrete labour supply choice. The alternatives available are full-time employment, part-time employment and staying out of the labour force. It is assumed that husbands are always employed full time. The labour force supply decision of the woman in period j is therefore simply denoted by $n_j \in \{0, \frac{1}{4}, \frac{1}{2}\}$. Taking non-sleeping hours to be 16 per day, this corresponds to full and part time employment taking 8 and 4 hours per day, respectively.

Child-rearing stage. If the household chooses a positive number of children, it enters working-age, child-rearing stage at $j = J_0$. The number of periods spent in this stage depends only on the chosen fertility, since the arrival of children is pre-determined. Let $W_c(b)$ be the number of child-bearing periods. Then,

$$W_c(b) = \begin{cases} 4 & \text{if } b = 1 \\ 5 & \text{if } b = 2 \\ 6 & \text{if } b = 3 \end{cases} \cdot$$

The youngest child imposes a time constraint on parents. They must be looked after at all times. Therefore, additionally to deciding on the woman's labour supply, child-rearing households must decide how to cover their childcare needs. Various sources are considered: mother's own time, other relatives' time (including the father and grandparents), formal institutions such as kindergartens and schools. These are aggregated into three types of childcare: mother's own time (h_j^m), unpaid childcare (h_j^u) and out-of-pocket childcare purchased in the market (h_j^p). By choosing two of them, the household is implicitly choosing the third via the time constraint. Therefore, from here on h_j^m and h_j^p are the two additional choice variables in the child-rearing stage.

2.3.3 Preferences

The basic preference structure follows Bick (2016), who in turn points to Greenwood et al. (2003) and Jones et al. (2010) as previous examples. However, important differences regarding utility related to children are introduced here.

As in Bick (2016), the utility that the household maximises is that of the woman, which can be interpreted either as her being the sole decision maker, or her having full bargaining power. Therefore, the gender subscript i is dropped for all variables in the utility function. Mother's and childless women's utility functions are different. The specification for the former will be described first.

Mothers. At age j, a woman that has children ($b > 0$) derives utility from consumption c_j, leisure l_j, and a component related with children $\Gamma\left(b, h_j^m; b^*\right)$:

$$u_j\left(c_j, l_j, h_j^m; b, b^*\right) = \frac{\left(\frac{c_j}{\psi_j(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} + \Gamma(h_j^m; b, b^*), \tag{2.2}$$

where $\psi_j(b)$ is an age-dependent consumption equivalence scale, and:

$$\Gamma(h_j^m; b, b^*) = \begin{cases} -\delta_2 |b - b^*|^{\gamma_2} + \zeta + \delta_3 \left(h_j^m\right)^{\gamma_3} & \text{if } j \leq W_c(b) \\ -\delta_2 |b - b^*|^{\gamma_2} + \zeta & \text{if } j > W_c(b) \end{cases}. \tag{2.3}$$

That is, utility is the sum of two constant relative risk aversion terms for consumption and leisure, and a child-related term. The first two are standard, but the last one deserves some discussion.

Utility from children may have two or three components, depending on the age of children. The first component depends negatively on the absolute value of the difference between desired and realised fertility. That is, there is a utility penalty from having a number of children different to the ideal one. The parameter $\delta_2 > 0$ determines how large this penalty is, while γ_2 determines the curvature of the penalty with respect to the difference between desired and realised fertility. The second term is a parameter ζ, that could be positive or negative, which is a fixed utility cost or gain from motherhood. The final term applies only to women of child-bearing age, and it depends on the amount of time spent taking care of the children.

Childless women. At age j, a childless woman's utility is:

$$u_j\left(c_j, l_j; b^*\right) = \frac{\left(\frac{c_j}{\psi(0)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 b^{*\gamma_2}. \tag{2.4}$$

The first two terms are identical to those for mothers, with the equivalence scale evaluated at $b = 0$. The third term is equivalent to the first component of the child-related term for mothers, as $-\delta_2|0 - b^*|^{\gamma_2} = -\delta_2 b^{*\gamma_2}$. That is, childless women do experience disutility from the children that they didn't have but want to have. Notice that this term is 0 if $b^* = 0$. That is, women that do not want to have children and do not have them do not experience any utility penalty. The term is negative if $b > 0$. This can be interpreted as fertility regret or dissatisfaction.

Preference heterogeneity. Parameters γ_0, δ_1, γ_1, δ_2, δ_3 and γ_3 are the same for all women. However, δ_2, ζ, and desired fertility b^* are allowed to vary across women. Notice that if all women had the same preferences for children, either with this or other utility additively separable specification, all variation in fertility would have to come from differences the initial income shock vector, as is the case in Bick (2016). This is a bit too restrictive, as variation in fertility within households with similar incomes and/or education levels is indeed observed.

2.3.4 Income and constraints

During working-age stage, full-time gross income depends on gender i, accumulated experience x_j^i and the current shock ϵ_j^i. Since husbands are assumed to work full time every period, $x_j^m = j - J_0$, $\forall j < J_R$, and the gender subscript for women's experience can be dropped, $x_j^f = x_j$, $\forall j < J_R$. Full-time gross income for a woman and her husband at age j, with experience x_j and income shocks $\left(\epsilon_j^f, \epsilon_j^m\right)$ is:

$$\begin{aligned} \ln y_j^f &= \eta^f + \eta_1^f x_j + \eta_2^f x_j^2 + \epsilon_j^f \\ \ln y_j^m &= \eta^m + \eta_1^m (j - J_0) + \eta_2^m (j - J_0)^2 + \epsilon_j^m. \end{aligned} \tag{2.5}$$

As there are no savings, consumption will be equal to net income minus out-of-pocket childcare costs, if any. Net income is equal to actual gross earnings minus taxes. For women, actual gross earnings are full-time gross earnings multiplied by the labour supply choice times two as $n_j = 1/2$ denotes full-time work. That is, a woman working part time earns half as much as she would if she worked full time. If retired, the household consumes its pension benefits, which are defined at time J_R and depend on prior average earnings at the time of retirement, $\bar{y}_{J_R}^f$ and $\bar{y}_{J_R}^m$. Moreover, in the case of the woman, the pension depends on how many years she worked, i.e. her experience at age J_R, x_{J_R}. Pension benefits are denoted by $p_f\left(\bar{y}_{J_R}^f, x_{J_R}\right)$ and $p_m\left(\bar{y}_{J_R}^m\right)$. The budget constraint in each stage of life is therefore:

$$c_j = \begin{cases} 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j\left(h_j^p, b\right) & \text{if } b > 0 \text{ and } j < J_0 + W_c(b) \\ 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) & \text{if } b = 0 \text{ or } b > 0 \text{ and } J_0 + W_c(b) \leq j < J_R \\ p_f\left(\bar{y}_{J_R}^f, x_{J_R}\right) + p_m\left(\bar{y}_{J_R}^m\right) & \text{if } j \geq J_R \end{cases},$$

where $T\left(y_j^f, y_j^m, n_j\right)$ is the tax payable by the household, and $\lambda_j\left(h_j^p, b\right)$ is the cost of out-of-pocket childcare, which takes the form:

$$\lambda_j\left(h_j^p, b\right) = q\left(1 - \Upsilon\right)\sqrt{b}\,h_j^p.$$

The cost of out-of-pocket childcare is therefore a concave function of the number of children, where the price per unit of time is denoted by q, and $\Upsilon \in [0, 1]$ denotes the fraction of the cost that is subsidised.

The time constraints faced by a working-age woman depends on whether she has children, and if she does on whether she is in child-bearing stage. Retired women don't work and can use all of the time for leisure. In general, the endowment of time is normalised to 1 in every period. The sum of hours spent working, enjoying leisure and taking care of children (if any) must be 1:

$$\begin{cases} l_j + n_j + h_j^m = 1 & \text{if and } j < J_0 + W_c(b) \\ l_j + n_j = 1 & \text{if } b = 0 \text{ or } b > 0 \text{ and } J_0 + W_c(b) \leq j < J_R \\ l_j = 1 & \text{if } j \geq J_R \end{cases}$$

Moreover, there is an additional constraint imposed by young children on child-bearing mothers. So, if $b > 0$:

$$h_j^m + h_j^u + h_j^p = 1 \quad \forall j < J_0 + W_c(b).$$

There is an endowment of unpaid childcare available to the household that sets an upper bound for h_j^u. Likewise, mandatory schooling sets a lower bound for unpaid childcare use. Both the endowment and the amount of mandatory schooling time depend on the age of the children, which is a function of age of the household and number of children. Formally:

$$m_j(b) \leq h_j^u \leq \kappa_j(b),$$

where $m_j(b)$ is the amount of mandatory schooling and $\kappa_j(b)$ the endowment of unpaid childcare.

2.3.5 Social security and population dynamics

There is a pay-as-you-go social security system that taxes current working-age population's earnings and provides pension benefits to retired individuals. The social security tax rate, Ω is proportional to gross earnings up to a cap, d_1. The tax rate that is levied on the worker is denoted by τ. Total social security contributions attributable to a household with full-time earnings y_j^f and y_j^m, and choice of labour supply for the woman n_j are:

$$\mathcal{C}\left(y_j^f, y_j^m, n_j\right) = \Omega\left[\min\left(2n_j y_j^f, d_1\right) + \min\left(y_j^m, d_1\right)\right],$$

while the contributions payable by the household, which should be deducted from gross income in the household's budget constraint, are:

$$T\left(y_j^f, y_j^m, n_j\right) = \tau\left[\min\left(2n_j y_j^f, d_1\right) + \min\left(y_j^m, d_1\right)\right].$$

The retirement income for a household is the sum of the woman's and her husband's pensions, which depends on $\bar{y}_{J_R}^f$, x_{J_R} and $\bar{y}_{J_R}^m$.

The formula to compute the pensions reflects Spanish Social Security rules. In general, the pension is equal to a fraction (called the wage replacement rate) of the average taxable earnings over the last N_R periods of working-age life. The wage replacement rate depends on the number of years worked over the life cycle. Since it is assumed that all men work full time in every period, the wage replacement rate is the same for all men, and it is denoted θ_m. For women, it depends on the experience accumulated upon retirement, and it is denoted $\theta\left(x_{J_R}\right)$. Moreover, there are maximum and minimum pensions, denoted by $\bar{p}$ and $\underline{p}$, respectively.

The first step to compute the pensions is to establish the average taxable earnings:

$$\begin{aligned}
\bar{y}_{J_R}^f &= \frac{\sum_{j=R-N_R-1}^{R-1} \min\left\{d_1, 2n_j y_j^f\right\}}{N_R} \\
\bar{y}_{J_R}^m &= \frac{\sum_{j=R-N_R-1}^{R-1} \min\left\{d_1, y_j^m\right\}}{N_R}.
\end{aligned}$$

Then, the actual pensions are computed as:

$$\begin{aligned}
p\left(\bar{y}_{J_R}^f, x_{J_R}\right) &= \min\left\{\bar{p}, \max\left\{\theta_f\left(x_{J_R}\right)\bar{y}_{J_R}^f, \underline{p}\right\}\right\} \\
p\left(\bar{y}_{J_R}^m, x_{J_R}\right) &= \min\left\{\bar{p}, \max\left\{\theta_m \bar{y}_{J_R}^m, \underline{p}\right\}\right\}
\end{aligned}$$

To describe aggregate social security revenue and expenditure per time period, it is necessary to describe first how population evolves in time. It is assumed that there is an initial population, the fraction of which is aged j at time $t = 0$ is given by $\mathcal{P}_0^j$. Let $\mathcal{F}(b)$ denote the fraction of women that have $b \in \{1, 2, 3\}$ children.

Notice that there is no time subscript, since the same fraction of women are going to have each number of children in every cohort. Starting from $\left\{\mathcal{P}_0^j\right\}_{j=1}^J$, population evolves according to:

$$\mathcal{P}_{t+1}^j = \begin{cases} \sum_{k=0}^{2}\left[\sum_{b=k+1}^{3}\mathcal{F}\left(b\right)\mathcal{P}_t^{J_0+k}\right] & \text{if } j=1 \\ \mathcal{P}_t^{j-1} & \text{if } 1<j\leq J \end{cases}.$$

That is, as a result of the sequential arrival of children to the household, the size of the new cohort $j=1$ at time $t+1$ is equal to the fraction of women in cohort J_0 at time t having children, $\sum_{b=1}^{3}\mathcal{F}\left(b\right)\mathcal{P}_t^{J_0}$ plus the fraction of women from cohort J_0+1 at time t that chose to have two and three children $\sum_{b=2}^{3}\mathcal{F}\left(b\right)\mathcal{P}_t^{J_0+1}$ plus the fraction of women from cohort J_0+2 at time t that chose to have three children $\mathcal{F}\left(3\right)\mathcal{P}_t^{J_0+2}$. The size of cohort aged j at $t+1$ is just the size of cohort aged $j-1$ at t, as there is no inter-year mortality. Notice that $\sum_{j=1}^{J}\mathcal{P}_t^j\neq 1$ in general. That is, total population can grow or fall depending on fertility, but it is normalised to 1 at $t=0$.

Let $\bar{\mathcal{C}}_j$ be the average contribution to social security attributable to households aged $j\in[J_0,J_R-1]$. Aggregate social security revenue in period t is then:

$$SS_t^R=\sum_{j=J_0}^{J_R-1}\mathcal{P}_t^j\bar{\mathcal{C}}_j.$$

Likewise, let $\bar{\mathcal{B}}_j$ be the average pension received by households aged $j\in[J_R,J]$. Aggregate social security expenditure in period t is:

$$SS_t^X=\sum_{j=J_R}^{J}\mathcal{P}_t^j\bar{\mathcal{B}}_j.$$

Finally. the balance of the social security in period t is:

$$SS_t=SS_t^R-SS_t^X.$$

2.3.6 Household problem in recursive form

In this subsection the household problem in recursive form is formally written and discussed, starting from the last stage of life and going backwards.

Retirement stage. Households need not to make any decisions in the retirement stage. In each period, the household consumes its pension income, which depends on x_{J_R}, $\bar{y}^f_{J_R}$ and $\bar{y}^m_{J_R}$. Women don't work or take care of any children, so all time is devoted to leisure, i.e. $l_j = 1$ for $j \in [J_R, J]$. Therefore, the value of retirement at time J_R is:

$$\begin{aligned} V_{J_R}\left(x_{J_R}, \bar{y}^f_{J_R}, \bar{y}^m_{J_R}; b\right) &= \sum_{j=J_R}^{J} \beta^{j-J_R} u_j \left[p_f\left(\bar{y}^f_{J_R}, x_{J_R}\right) + p_m\left(\bar{y}^m_{J_R}\right), 1; b, b^*\right] \\ &= \frac{1-\beta^R}{1-\beta} u_j \left[p_f\left(\bar{y}^f_{J_R}, x_{J_R}\right) + p_m\left(\bar{y}^m_{J_R}\right), 1; b, b^*\right] \end{aligned}$$

Working-age non child-rearing/working-age childless stage. In each period, women observe theirs and their husband's income shocks and decide on their labour supply, by optimally choosing to be either out of labour force, work part time or work full time. By working more, they are able to consume more in the current period, accumulate more labour market experience that potentially increases income in future periods and increase their average earnings for a higher pension (if the current period counts towards its calculation, i.e. if $j \geq J_R - N_R$). However, the leisure component of utility goes down. The problem solved by households in this stage is:

$$V_j\left(\epsilon^f_j, \epsilon^m_j, x_j, \bar{y}^f_j, \bar{y}^m_j; b\right) = \max_{\substack{c_j \geq 0 \\ n_j \in \{0, \frac{1}{4}, \frac{1}{2}\}}} \quad u_j\left(c_j, l_j; b\right) + \beta\,\mathbb{E}\left[V_{j+1}\left(\epsilon^f_{j+1}, \epsilon^m_{j+1}, x_{j+1}, \bar{y}^f_{j+1}, \bar{y}^m_{j+1}; b\right)\right]$$

$$\begin{aligned} &\text{s.t.} \\ &c_j = 2n_j y^f_j + y^m_j - T\left(y^f_j, y^m_j, n_j\right) \\ &l_j + n_j = 1 \\ &\ln y^f_j = \eta^f + \eta^f_1 x_j + \eta^f_2 x^2_j + \epsilon^f_j \\ &\ln y^m_j = \eta^m + \eta^m_1 (j - J_0) + \eta^m_2 (j - J_0)^2 + \epsilon^m_j \\ &\epsilon^f_j = \phi^f \epsilon^f_{j-1} + \nu^f_j \\ &\epsilon^m_j = \phi^m \epsilon^m_{j-1} + \nu^m_j \end{aligned}$$

$$x_{j+1} = x_j + 2n_j$$

$$\bar{y}^f_{j+1} = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}^f_j N_R + \min\left\{d_1, n_j y^f_j\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases}$$

$$\bar{y}^m_{j+1} = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}^m_j N_R + \min\left\{d_1, y^m_j\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases}$$

Working-age child-rearing. The problem faced by child-rearing women is similar to the one solved by women past this stage, but it is more complicated. Additionally to choosing labour supply, they face an additional constraint regarding childcare. Providing it themselves provides utility, but reduces leisure. Purchasing it out-of-pocket reduces consumption. Notice that, conditional on a labour supply decision, childcare allocation is a static problem. Appendix 2.A shows that it has a unique solution under reasonable parameter values, and characterises this solution. The problem solved by households in this stage is:

$$V_j(\epsilon_j, \epsilon^h_j, x_j, \bar{y}^f_j, \bar{y}^m_j; b) = \max_{\substack{c_j \geq 0 \\ n_j \in \{0, \frac{1}{4}, \frac{1}{2}\} \\ h^m_j, h^p_j, h^u_j \geq 0}} \quad u_j\left(c_j, l_j, h^m_j; b\right) + \beta\,\mathbb{E}\left[V_{j+1}\left(\epsilon_{j+1}, \epsilon^h_{j+1}, x_{j+1}, \bar{y}^f_{j+1}, \bar{y}^m_{j+1}; b\right)\right]$$

s.t.

$$c_j = 2n_j y^f_j + y^m_j - T\left(y^f_j, y^m_j, n_j\right) - \lambda_j\left(h^p_j, b\right)$$

$$\lambda_j\left(h^p_j, b\right) = q\left(1 - \Upsilon\right)\sqrt{b}h^p_j$$

$$l_j + n_j + h^m_j = 1$$

$$h^m_j + h^p_j + h^u_j = 1$$

$$m_j(b) \leq h^u_j \leq \kappa_j(b)$$

$$\ln y^f_j = \eta^f + \eta^f_1 x_j + \eta^f_2 x^2_j + \epsilon^f_j$$

$$\ln y^m_j = \eta^m + \eta^m_1\left(j - J_0\right) + \eta^m_2\left(j - J_0\right)^2 + \epsilon^m_j$$

$$\epsilon^f_j = \phi^f \epsilon^f_{j-1} + \nu^f_j$$

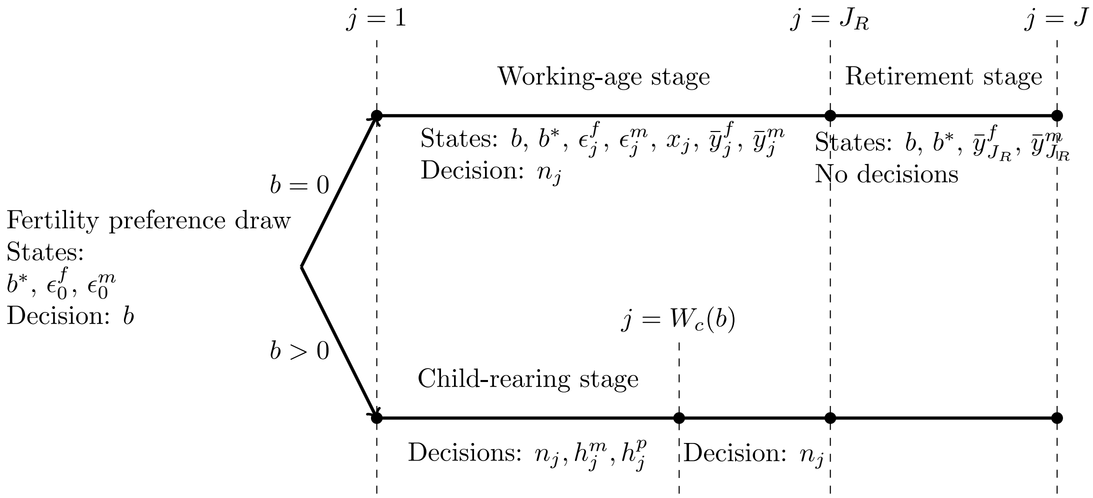
$$\epsilon^m_j = \phi^m \epsilon^m_{j-1} + \nu^m_j$$

$$x_{j+1} = x_j + 2n_j$$

$$\bar{y}^f_{j+1} = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}^f_j N_R + \min\left\{d_1, n_j y^f_j\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases}$$

!h

Figure 2.4: States and decisions through the life cycle



$$\bar{y}_{j+1}^m = \begin{cases} 0 & \text{if } j < J_R - N_R \\ \frac{\bar{y}_j^m N_R + \min\left\{d_1, y_j^m\right\}}{N_R} & \text{if } j \geq J_R - N_R \end{cases}$$

Fertility decision. Prior to age $j = J_0$, the woman draws her fertility preferences, observes hers and her husband's initial income shocks, and decides the total number of children she will bear. From then on, b becomes a state for the household. The problem at this stage is:

$$\max_{b \in \{0,1,2,3\}} V_{J_0}\left(\epsilon_0^f, \epsilon_0^n, x_0, \bar{y}_0^f, \bar{y}_0^m; b\right)$$

where $x_0 = 0$, $\bar{y}_0 = 0$ and $\bar{y}_0^h = 0$.

Figure 2.4 shows states and decisions through the life cycle.

2.4 Calibration

The calibration strategy followed here can be divided in steps. First, a set of parameters is chosen exogenously, either taken from previous literature, directly from the data, or from Spanish law, as is the case for the parameters governing social security. A second set of parameters is estimated without solving the model, notably the income process and the price of childcare. Finally, the remaining parameters are chosen by solving the model in equilibrium. Since there is no closed-form solution, this relies on the method of simulated moments.

2.4.1 Parameters chosen exogenously

The set of parameters chosen exogenously includes the subjective discount factor β, the functional form of the equivalence scale $\psi(\cdot)$, the initial population $\left\{\mathcal{P}_0^j\right\}_{j=1}^{J}$, the probability distribution for the draw of desired number of children b^* and the parameters for social security contributions and benefit calculations.

The subjective discount factor is set to be $\beta = (1/1.04)^3 = 0.89$, which is the standard time discount factor used in macro since Kydland and Prescott (1982). The equivalence scale for household consumption used here is the OECD scale, i.e. $\psi(b) = 1.7 + \sqrt{b}$. The initial population is taken from *INE* for the year 2020. Cohorts are aggregated into brackets of 3 years, to match the model period's length. The probability distribution for the desired number of children draw used is the fraction of women aged 40-44 that declares wanting 0, 1, 2 and 3 or more children in *INE*'s 2018 *Encuesta de Fecundidad* presented in Table 2.1.

Social security. The parameters governing social security contribution and retirement pension calculation are set to match as closely as possible the Spanish *Régimen General de la Seguridad Social* (RGSS), which is the regime that covers most of the workers in the country. The rules used for the baseline calibration are the ones in place in 2013, which excludes the latest pension reforms. This is done to avoid changing legislation over time, as the reforms have been introduced progressively. Moreover, since some of the dispositions will only be implemented by 2027, the reforms have little impact for a large share of women making labour force participation decisions during the period covered by the data.

The retirement age set by the RGSS is 65 years. As J_0 is set to start at age 30 and there are 18 three-year working-age periods, retirement in the model J_R happens when the person turns 66 years old, which does not perfectly match the law, but is close enough. The number of contribution periods that count towards retirement pension computation in the law is 15 years, which fits perfectly with the last 5 working-age periods in the model. That is, $N_R = 5$.

The replacement rate defined in the RGSS $\theta_{RGSS}(\chi)$ depends on the number of years that the worker has contributed χ, where part-time work for a year counts as half of full-time work over the same period, according to the schedule:

$$\theta_{RGSS}(\chi) = \begin{cases} 0 & \text{if } \chi < 15 \\ 0.5 + 0.03(\chi - 15) & \text{if } 15 \leq \chi < 25 \\ 0.8 + 0.02(\chi - 25) & \text{if } 25 \leq \chi < 35 \\ 1 & \text{if } \chi \geq 35 \end{cases}.$$

Again, since there are 12 three-year working-age periods and it is assumed that husbands work full time always, all men have the same replacement rate $\theta_m = 1$. For women, it is a function of accumulated labour market experience x_j and follows the following schedule:

$$\theta_f(x_j) = \begin{cases} 0 & \text{if } x_j < 5 \\ 0.5 + 0.03(x_j - 5) & \text{if } 5 \leq x_j \leq 8 \\ 0.8 + 0.02(x_j - 8) & \text{if } 8 < x_j < 12 \\ 1 & \text{if } x_j = 12, \end{cases}$$

where again, the fit between model periods and the kinks in the schedule that determines the replacement rate is not perfect, but is the closest possible given the three year model periods.

The RGSS collects a proportional tax on covered earnings in order to finance pensions. Covered earnings are defined as the worker's gross labour income with a cap above and a tax-exempt minimum. Following Díaz-Giménez and Díaz-Saavedra (2009), the tax-exempt minimum is not included. In 2019, the total payroll tax was 28.3%, where 11.7% was levied on the employer and the remaining 16.6% was levied on workers. Therefore, Ω is set to 28.3% and τ to 16.6%. The cap on earnings for contribution calculation was 56 981 euros per year. As in the model one period represents three years, d_1 is set to 170 944. Finally, the minimum pension set out in the law is 9 081 euros per year, which comes to 27 243 euros in three years, while the maximum one is 37 566 euros per year, or 112 698 per model period. Table 2.3 summarises the social security parameters used.

Table 2.3: Social security parameters

Parameter	**Description**	**Value**
J_R	Retirement age (model periods)	22
N_R	Regulatory base (model periods)	5
Ω	Total payroll tax (%)	28.3
τ	Payroll tax levied on workers (%)	16.6
d_1	Maximum taxable period earnings (euros/model period)	170 944
$\underline{p}$	Minimum retirement pension (euros/model period)	27 243
$\bar{p}$	Maximum retirement pension (euros/model period)	112 698

2.4.2 Parameters estimated without solving the model

The next set of parameters is estimated using data from various sources, without having to solve the model. This includes the income process and the price of childcare. The estimation is discussed in that order.

The income process. The income process described by equations 2.1 and 2.5 is estimated using the *Muestra Continua de Vidas Laborales* (MCVL), the Spanish Social Security database on working life histories.

The methodology followed is based on Bick (2016). Mincer (1974) equations are estimated separately for men and women to obtain the parameters of the income process, and then the residuals of the Mincer regressions are regressed on the first lag to estimate the persistence of the income shocks. Since the data is at the individual level, it is not possible to directly estimate the correlation between wife and husband's income shocks, ρ. As in Bick (2016) and Attanasio et al. (2008), the value of 0.25 that Hyslop (2001) estimated for the United States is used. For the numerical solution of the model, the income shock process is discretised using the methodology proposed by Tauchen and Hussey (1991). Table 2.4 presents the results of the estimation.

Table 2.4: Income process parameters

Parameter	**Value**	**Parameter**	**Value**
η_0^f	9.2860	η_0^m	9.5330
η_1^f	0.0326	η_1^m	0.0318
η_2^f	-0.0003	η_2^m	-0.0004
ϕ^f	0.6550	ϕ^m	0.6540
σ^f	0.0342	σ^m	0.0370

The price of childcare. Following Sánchez-Mangas and Sánchez-Marcos (2008), the price of out-of-pocket childcare q, is computed so that a couple aged J_0 with average income shocks in which both spouses work full time and that has one child attending full time spends 33% of their earnings on childcare. That is, q solves:

$$q = 2 \times 0.33 \left[3 \left(e^{\eta_0^f} + e^{\eta_0^m} - T \left(e^{\eta_0^f}, e^{\eta_0^m}, \frac{1}{2} \right) \right) \right].$$

The result of this calculation comes down to 37 053 euros per model period, or 12 351 euros per year.

2.4.3 Parameters estimated by solving the model

Since the utility function is additively separable, the utility term associated with children's mere presence does not affect labour supply decisions conditional on the number of children chosen. To take advantage of this, the rest of the parameters, namely γ_0, γ_1, γ_3, δ_1 and δ_3, are calibrated separately. Taking the observed distribution of children across women, the method of simulated moments is used to choose these parameters to replicate several labour supply moments along the life cycle. Even though all parameters affect all moments, it is worth to discuss the chosen targeted moments and how they relate to specific parameters.

The curvature of consumption γ_0 is informative about differences in labour force participation between women with different number of children through the potential monetary costs that childcare entails and through consumption adjustment via the equivalence scale. Therefore the fraction of women with children aged 6 to 12 who work full time and the fraction of childless women the same age working full-time are included as targets.

The curvature of leisure γ_1 is informative about women's labour supply decisions with different household income levels at the intensive margin (part-time versus full time). As a target for this parameter, the fraction of women with children aged 6 to 12 working part-time, and the fraction of childless women the same age working part-time are included.

The utility weight of leisure δ_1 is informative for labour force participation decisions, i.e. labour supply at the extensive margin. Thus total labour force participation for childless women (lifetime average) is included as a target.

The parameters associated with childcare δ_3 and γ_3 govern the utility mothers derive from spending time with children, and are therefore informative about labour force supply for women with children and their childcare utilization. So, labour force participation of women with children aged 0-2 and the paid childcare utilization of women that do not work are included as targets.

Once the labour supply parameters are chosen, it is possible to compute lifetime utility, excluding the term related to children. This term will be different for each fertility level because of the equivalence scale and the time constraints imposed by children.

To finish the calibration, a set of parameters needs to be set such that the fraction of households that choose to have 0, 1, 2 and 3 children coincides with the number of households that have them in the data.

Intuitively, having more children lowers lifetime utility because they lower equivalent consumption and leisure. Fertility parameters need to be such that households are compensated for this difference, just in the right amounts on average so that their fertility decisions are close to the observed ones.

If all households have the same fertility parameters, variation in fertility decisions is based only on the initial income shocks and differences in the desired number of children. This is not enough to generate the observed distribution of children. Therefore, each household's parameters δ_2 and ζ are drawn from an exponential distribution. That is, households are heterogeneous in how much they care for departures from their ideal level of fertility b^* and how much they care about having children at all. The parameters that estimated here are the underlying means of those draws.

2.4.4 Calibration results

The calibration results for the baseline economy are shown in Table 2.5. Parameters and moments related to labour force participation and childcare, and those related to fertility decisions are shown separately.

The estimated curvature parameters for consumption and leisure γ_0 and γ_1 are relatively close to 1. This implies that the consumption and leisure utility terms are close to being logarithmic. Therefore, changes in the earnings potential have a muted effect on labour force participation, as income and substitution effects would roughly cancel each other.

The model is able to reproduce quite closely the total labour force participation of women whose youngest child is 0 to 3 years old, and the childcare usage of women that do not work and that have a child aged 0 to 3 years old. In the baseline economy, the share of women working full time is lower than in the data, while the share of women working part-time is higher for both women with the youngest child aged 6-12 and women aged 48-59. However, the total labour force participation is close for women with children aged 6-12. Childless women work too much with respect to the data. These results seem to suggest that γ_1 should be set at a higher value and δ_1 at a lower one, according to the previous discussion as to how each parameter affects each of the moments. However, in reality, all parameters affect all moments. The model is over identified as there are more moments than parameters. Changing γ_1 and δ_1 affects its ability to reproduce the labour force participation and childcare usage of women with children 0-2, which are very important given the nature of the question of this chapter.

It is also likely that a few things are missing in the current setting. The effect of experience on young women is probably underestimated. Currently, the cost of non-participation is just missed experience. However, early-career interruptions can be particularly costly. Moreover, the high prevalence of zero-hour contracts in Spain could increase the value of labour force participation early on. Working full time may increase the likelihood of converting one's contract into an indefinite contract in the future, which would factor in the labour force participation decision. Allowing for these kinds of effects within the model is left for future work. Reflecting this in a tractable manner within the framework laid out here has proven to be challenging.

In terms of fertility, the model matches the share of childless women closely, but overestimates the fraction of women with one child and underestimates the fraction of women with two with respect to the data. This is partly compensated by a slightly higher fraction of women with three children so that the total fertility rate is only slightly below the one in the data.

Table 2.5: Baseline calibration results

Parameter	Value	Target(s)	Model	Data
Labour force participation and childcare				
γ_0	0.968	Share full-time work (women with youngest child 6-12)	32.72%	47.03%
		Share full-time work (women aged 48-59)	30.39%	41.68%
γ_1	1.011	Share part-time work (women with youngest child 6-12)	44.04%	20.32%
		Share part-time work (women aged 48-59)	44.77%	13.41%
γ_3	0.730	Labour force participation (women with youngest child 0-3)	69.81%	71.28%
δ_1	1.062	Average lifetime LFP (childless women)	83.03%	66.49%
δ_3	2.330	Weekly paid childcare hours (non-working women with child 0-2)	9.74	8.80
Fertility				
		Share of women with:		
ζ	0.10	0 children	24.50%	25.50%
		1 child	35.40%	27.40%
δ_2	1.60	2 children	30.50%	39.80%
γ_2	3.00	3 children	9.60%	7.30%
		Total fertility rate	1.25	1.29

2.5 Policy experiments

In this section, the model is used to test three different policies. The first two have been implemented in the past by the Spanish government. The third one has not. The first to be discussed are the one implemented in the past.

2.5.1 Universal childcare coverage for children aged 3-6 and baby checks

In an attempt to compare the quantitative model to empirical evidence of child-related policies' effectiveness, two policies that have been assessed in the empirical literature are tested in it.

The first one is the universal offering of public childcare for 3-6-year-old children in Spain's late 1990s. This policy's impact was studied by Nollenberger and Rodríguez-Planas (2015) using its staggered implementation across Autonomous Communities as a natural experiment. They find that the policy raised maternal employment by 9.6% for women with children in this age group, with the results for women older than 30 and with two or more children being larger, 15.3% and 14.9% respectively, and no effect on fertility.

Since in the baseline calibration there is a provision of 30 hours per week of schooling for children aged 3-6, such provision is removed and the model is solved with the same parameters used in the baseline. The differences in labour force participation for women with children aged 3-6 and in fertility between the economy with no provision of schooling and the baseline economy are then computed. This can be compared to the results in Nollenberger and Rodríguez-Planas (2015). There are some caveats, though. In the model, women with children aged 3-6 are either 33 to 36, 36 to 39 or 39 to 42 years old by definition. Moreover, those aged 33 to 36 have one or two children. Those aged 36 to 39 have two or three, and those aged 39 to 42 have three children.

Table 2.6 presents the results of the exercise. Labour force participation of older women (which are also the ones that have more children) reacts strongly to the removal of the schooling provision. However, the magnitude of the effect is much larger than in Nollenberger and Rodríguez-Planas (2015) though. The effect on fertility in the model is very small, while they don't find any.

The second policy that tested in the model is the "cheque bebé" (baby check). This was a one-time cash transfer upon the birth of 2500 euros offered to parents by the Spanish government between 2007 and 2010. González (2013) evaluates this policy empirically, and finds negatives effect on labour force participation after childbirth and a significant increase in fertility.

The policy is tested in the model by introducing the cash benefit mentioned above in the period in which a child is born. Similarly, the model is solved with the same parameters used in the baseline. The difference in labour force participation and fertility outcomes with respect to the baseline is computed.

Table 2.6: The effect of removing free provision of pre-schooling for children aged 3 to 6.

Outcome	**Baseline**	**No free scholing 3-6**	**Difference**
Labour force participation			
(women with child 3 to 6):			
Aged 33-36	69.67%	71.99%	2.33%
Aged 36-39	65.59%	48.51%	-17.08%
Aged 39-42	50.00%	35.00%	-15.00%
Fertility			
Share of women with:			
0 children	24.50%	24.30%	-0.20%
1 child	35.40%	35.50%	0.10%
2 children	30.50%	30.20%	-0.30%
3 children	9.60%	10.00%	0.40%
Total fertility rate	1.25	1.26	0.01

Again, there are some caveats to this. The most important is that there are no savings in the model, and therefore the cash benefit has to be consumed within the period in which it is received. This may not a very serious caveat for two reasons. First, if the families are credit constraint and expect their earnings to increase in the future, they would consume the cash benefit immediately. Second, the model period is three years, so the cash benefit is spread out at least over that period.

The results of the exercise are reported in Table 2.7. The baby check has a negligible effect on the youngest mothers' labour force participation, but increases part-time relative to full-time employment for mothers aged 33 to 36 and then lowers labour force participation for mothers aged 36-39 and 39 42. In a sense, this is similar to the previous exercise: older mothers are more affected than young ones. Moreover, the results are qualitatively similar to González (2013), at least for older mothers. The baby check leads to a tiny increase in total fertility, driven by more women having three children. In her study, González finds that the baby check increased births by 6 per cent. In the model, this is much lower. One reason may be that part of the effect captured by her is anticipated births. This effect would be more substantial if people expected the policy to end (which it did, although they did not have the certainty it would at the time). In the model, households cannot anticipate births, which would lead to a difference in results even if they expected the policy to end.

The main take from these two exercises is that female labour force participation and fertility responses to the two policies tested in the model are qualitatively similar to the empirical literature's effects. However, there are quantitatively important differences. The model features stronger labour force participation effects than Nollenberger and Rodríguez-Planas (2015), but more muted fertility effects than González (2013).

Table 2.7: The effect of a cash benefit per child born.

Outcome	**Baseline**	**Baby check**	**Difference**
Labour force participation			
Part time share (all mothers):			
Aged 30-33	53.77%	54.19%	0.41%
Aged 33-36	43.97%	49.08%	5.11%
Aged 36-39	51.13%	39.27%	-11.86%
Aged 39-42	37.48%	30.50%	-6.99%
Aged 42-45	47.81%	46.86%	-0.96%
Aged 45-48	49.14%	49.48%	0.34%
Full time share (all mothers):			
Aged 30-33	19.60%	19.50%	-0.10%
Aged 33-36	25.70%	21.60%	-4.10%
Aged 36-39	16.16%	15.71%	-0.45%
Aged 39-42	36.03%	37.57%	1.54%
Aged 42-45	38.54%	38.61%	0.07%
Aged 45-48	33.91%	34.03%	0.12%
Fertility			
Share of women with:			
0 children	24.50%	24.30%	-0.20%
1 child	35.40%	35.50%	0.10%
2 children	30.50%	30.20%	-0.30%
3 children	9.60%	10.00%	0.40%
Total fertility rate	1.25	1.26	0.01

2.5.2 Childcare subsidies for children aged 0-3

The main policy experiments considered in this chapter consist of subsidising childcare for mothers with children aged 0-3 years old. Partial and full subsidisation are implemented, i.e. the government covering 50% and 100% of childcare costs for up to 30 hours per week. Table 2.8 presents the results of this policy experiment.

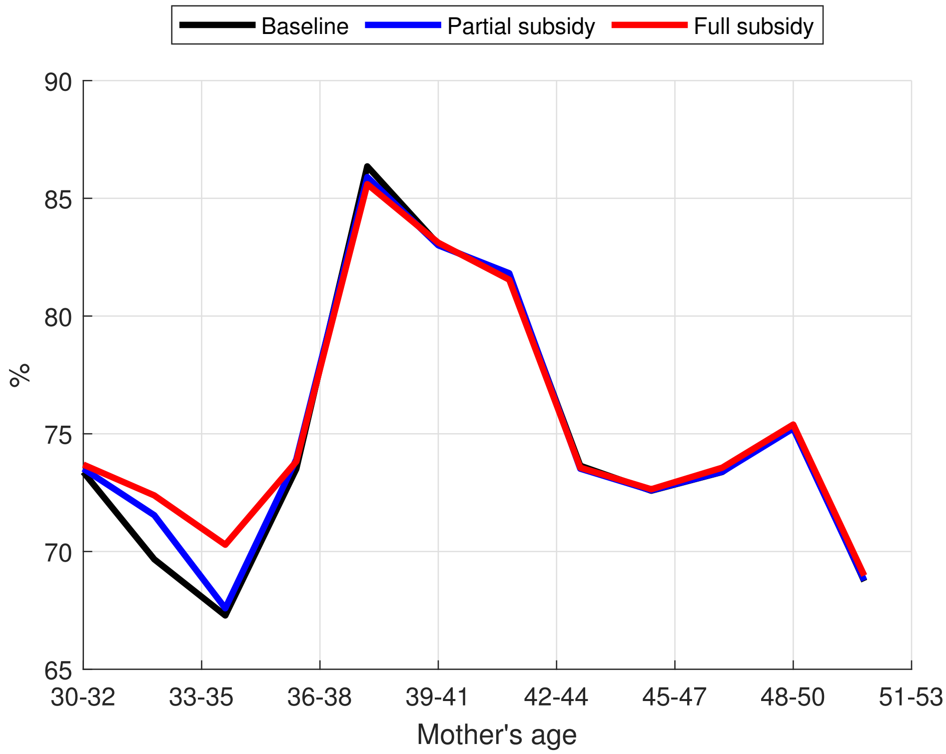
Table 2.8: The effect os subsidising childcare for children aged 0 to 3

Outcome	**Baseline**	**Partial**	**Full**
Female labour force participation			
Part time share			
(women with child 0 to 2):			
Aged 30-33	53.77%	53.89%	54.19%
Aged 33-36	42.14%	45.77%	57.07%
Aged 36-39	15.63%	34.38%	59.18%
Full time share			
(women with child 0 to 2):			
Aged 30-33	19.60%	19.63%	19.50%
Aged 33-36	27.93%	27.61%	17.62%
Aged 36-39	25.00%	10.42%	7.14%
Childcare use			
Hours per week (women with child 0 to 2):			
Aged 30-33	25.88	26.52	27.14
Aged 33-36	26.18	27.47	27.05
Aged 36-39	20.14	19.71	24.12
Fertility			
Share of women with:			
0 children	24.50%	24.10%	23.60%
1 child	35.40%	35.70%	36.10%
2 children	30.50%	30.60%	30.50%
3 children	9.60%	9.60%	9.80%
Total fertility rate	1.25	1.26	1.27
Present value of SS budget			
% of average working-age family's income	40.10%	39.53%	38.88%

In terms of female labour force participation, the subsidies lead to a shift from full-time to part-time work among older mothers with younger children. This leads to a minimal increase in total labour force participation for mothers aged 33 to 39, as shown in Figure 2.5.

Unsurprisingly, the childcare subsidies lead to an increase in childcare usage among women with young children for all age brackets. The increase is not very large, being less than 10% in all cases except for women aged 36-39 when moving from no subsidies to full subsidies.

Figure 2.5: Labour force participation of mothers by age under different childcare subsidisation policies for children aged 0-3.



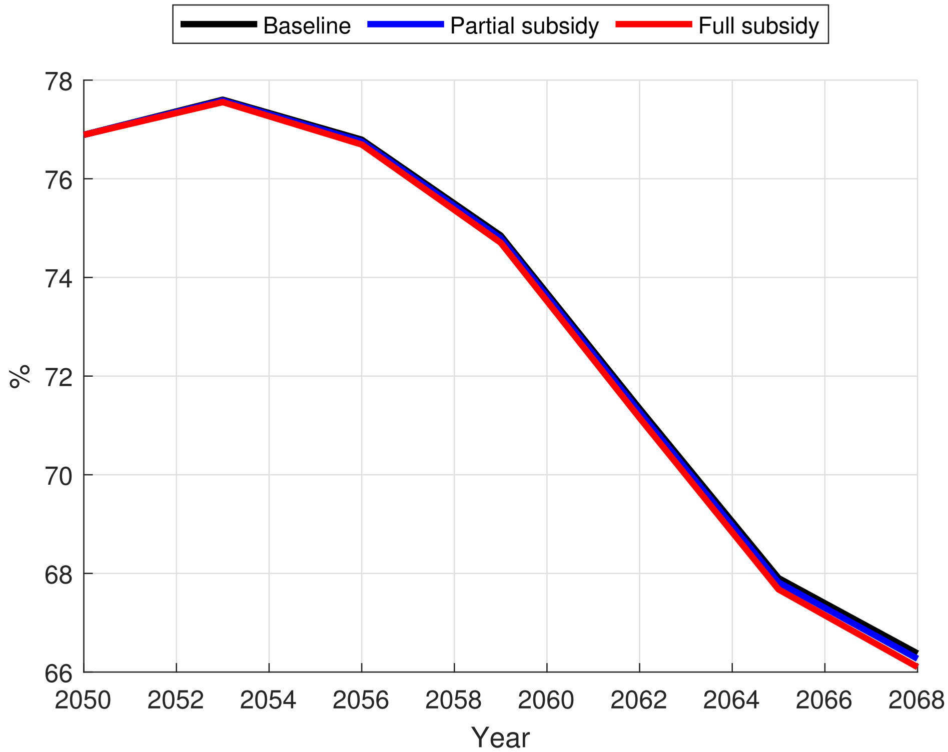
The effect of childcare subsidies on fertility is small. The fraction of childless women falls with respect to the baseline under both partial and full subsidies. The fraction of women with three children increases very slightly with full subsidies. Total fertility rate increases only marginally, from 1.25 children per woman in the baseline to 1.27 in the full subsidies case. The subsidies may lead to welfare benefits by allowing for a lower mismatch between desired and realised fertility. Moreover, if the subsidies were budget-neutral or budget-positive for the social security, it would be possible to claim unequivocally that they raise welfare.

However, the policy is not budget neutral. The present value of the social security budget balance falls as a percentage of the average working-age family's income[13]. Expenditures for the social security administration increase immediately, albeit not by a lot since the subsidies' cost is small compared to the regular outlays of social security (the retirement benefits). This lowers the net budget in the present and near-present periods. On the other hand, the increase in the total fertility rate induced by the subsidies barely affects the future's old-age dependency ratio. This can be seen in Figures 2.6 and 2.7.

Unfortunately, the experiments' main takeaway is that childcare subsidies do not achieve any of the goals

[13]The present value of the Social Security balance is positive in the baseline economy. In reality, this value is negative. Payroll taxes laid out in the law (28.3%) are used to calculate social security revenues. This is likely too high. However, estimating the effective contribution to Social Security as a function of gross income using the administrative in the *MCVL* data set was beyond the scope of the chapter.

Figure 2.6: Projected old dependency ratio under different childcare subsidisation policies (2050-2068)



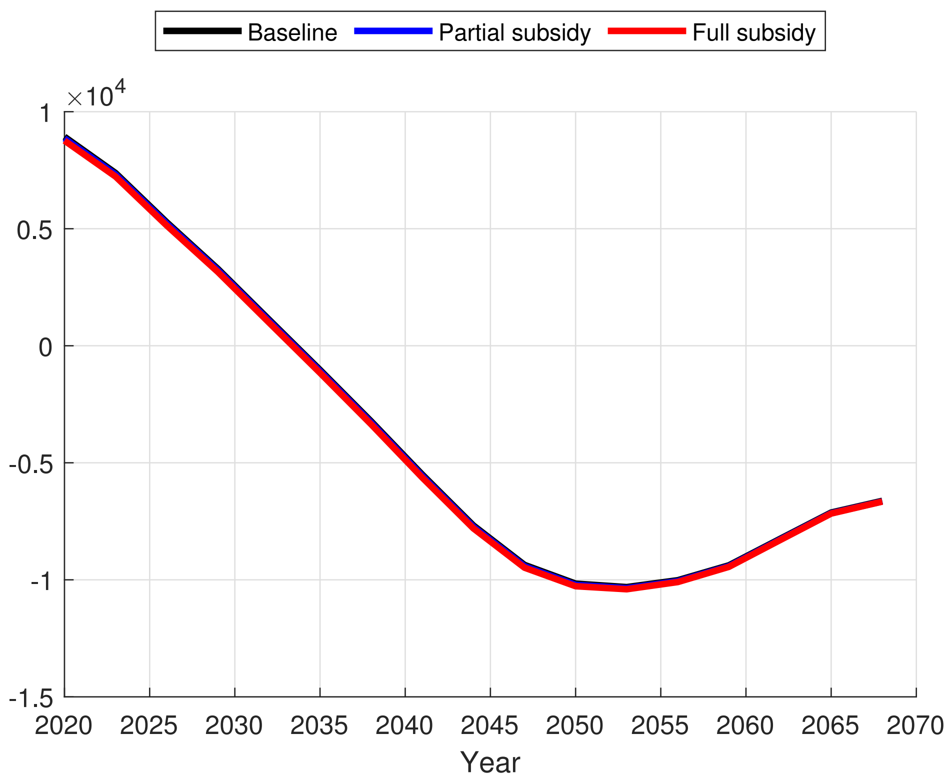


Figure 2.7: Social Security budget balance

hypothesised before. The response of the mother's labour force participation is not the expected one, and the subsidies fail to generate a large enough fertility response to be budget-neutral for the social security administration.

2.5.3 Discussion

What drives the findings of THE policy experiments? The childcare subsidies induce an income effect and a substitution effect. On the one hand, they increase households' real income with children aged 0-3, which should lead to higher consumption, leisure, and time spent with children. On the other hand, they make childcare less expensive. This increases the price of leisure relative to consumption, inducing the household to decrease leisure, increasing consumption, and increasing the price of time spent with children relative to leisure, which should decrease time spent with children and increase leisure. Notice that, at the margin, childcare is still as expensive as before for women that want to consume many hours of it since the subsidy is only 30 hours per week. Therefore, it is costly for women working full time. Since γ_0 and γ_1 are both close to one, the income and substitution effects should roughly cancel each other out for leisure.

The childcare subsidies have a negligible effect on the labour force participation of women aged 30-33 with children aged 0-3. Since childcare also increases, this means that time spent with children decreases. Therefore, for these women, the substitution effect dominates the income effect on their children's time.

For older women, the subsidies induce women to increase time spent with children. They would tend to have higher consumption since they are older and therefore, husbands' incomes and their own earnings potential are higher. Therefore, they care more about the time spent with children.

The effect of the subsidies on fertility depends heavily on the fertility parameters. These are chosen to match the distribution of children across households. Intuitively, δ_2 controls how much households care more about departures from their desired fertility, while ζ matters for the decision between having children or not. The latter seems to be playing a more important role here, that is, the subsidies can convince some (small) fraction of women to have children, but not many to have more children.

2.6 Conclusions

This chapter quantifies the effect of childcare subsidisation policies on fertility, female labour force participation, and social security budgets. To this end, an overlapping generations economy is studied, in which women initially choose fertility and then the labour force participation and childcare along their life-cycle. The model is calibrated to Spanish data and used to experiment by introducing partial (50%) and full (100%) childcare subsidies for women with children aged 0-3, since childcare for children 3-6 is already free and universal in Spain.

The chapter's main takeaway is that the childcare subsidies do not have a strong positive effect on the mother's labour force participation and fertility. This somewhat consistent with previous literature. Moreover, the subsidies affect the budget balance of Social Security negatively: more is spent on them than is recovered through future lower old-age dependency ratios. Mainly, the positive impact of the subsidies on fertility is too small to offset their immediate cost.

The analysis presented here has two major caveats. First, women choose the number of children at age 30, and they are matched with men in a deterministic way. Therefore, this framework is not suitable for analysing how childcare subsidies affect the timing of childbirth. Second, the mode abstracts from important features of the Spanish labour market, like high unemployment and the pervasiveness of fixed-term contracts.

These two features, potentially interact in non-trivial ways to determine women's response to childcare subsidies. Since the effects of shifts in childbirth timing on social security contributions over the life cycle are probably small compared to changes in total fertility, the first caveat may be minor. Besides, to the extent that the income process imposed here accounts for the effects of fixed-term contracts and high unemployment, it can also be claimed that the model is missing only second-order effects through the second.

Therefore, these mechanisms are outside the scope of the questions that are asked in this chapter. Nevertheless, further research is necessary to understand them in their own right.

2.A The childcare allocation problem

Given a choice of labour force participation n_j, and state variables ϵ_j^f, ϵ_j^m, x_j and b the childcare allocation problem faced by a working-age child rearing mother at age j is:

$$\max_{h_j^m, h_j^p, h_j^u} \frac{\left(\frac{c_j}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 |b - b^*|^{\gamma_2} - \zeta + \delta_3 \left(h_j^m\right)^{\gamma_3}$$

subject to

$$c_j = 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j\left(h_j^p, b\right)$$

$$\lambda_j\left(h_j^p, b\right) = q\left(1-\Upsilon\right)\sqrt{b}h_j^p$$

$$l_j + n_j + h_j^m = 1$$

$$h_j^m + h_j^p + h_j^u = 1$$

$$m_j(b) \leq h_j^u \leq \kappa_j(b)$$

$$\ln y_j^f = \eta^f + \eta_1^f x_j + \eta_2^f x_j^2 + \epsilon_j^f$$

$$\ln y_j^m = \eta^m + \eta_1^m (j - J_0) + \eta_2^m (j - J_0)^2 + \epsilon_j^m,$$

where h_j^m is the childcare provided by the mother, h_j^p is paid childcare purchased in the market and h_j^u in unpaid childcare (childcare provided by the father or other family members, time spent at school, etc.).

Consumption c_j is equal to the household's net income $2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right)$ minus childcare costs $\lambda_j(h_j^p, b)$.

Leisure l_j is equal to one minus paid labour time n_j and mother provided childcare time. The amount of unpaid childcare h_j^u is bounded below by the amount of mandatory schooling $m_j(b)$. This is a function of the age (and hence the age of the children) and the number of children. It is bounded above by the total unpaid childcare available to the mother, $\kappa_j(b)$. Since mandatory schooling is included in total available unpaid childcare, $m_j(b) \leq \kappa_j(b)$. Mother childcare provision and paid childcare utilization must be non negative. Consumption and leisure must be strictly positive. Finally, total childcare time must be equal to 1, that is, children must be looked after every moment of the day.

It is possible that for a given labour force participation choice n_j the mother is unable to satisfy the children's time constraint. Define $\bar{h}_j^p$ as:

$$\bar{h}_j^p = \max\{0, \sup\{h_j^p : 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j(h_j^p, b) > 0\}\},$$

that is, $\bar{h}_j^p$ is the maximum amount of paid childcare that the household can afford. Likewise, $\bar{h}_j^m = 1 - n_j$ is the maximum childcare time that the mother can provide given that her labour force participation is n_j. If:

$$\bar{h}_j^m + \bar{h}_j^p + \kappa_j(b, a) \leq 1,$$

then the choice of labour force participation n_j is unfeasible for the mother, and the childcare problem is not well defined.

From here on, it is assumed that:

$$\bar{h}_j^m + \max\{0, \bar{h}_j^p\} + \kappa_j(b, a) > 1.$$

Lemma 2.A.1 *If $\{h_j^m, h_j^p, h_j^u\}$ is a solution to the childcare allocation problem, then either:*

1. $h_j^p = 0$ *and* $h_j^u < \kappa_j(b, a)$, *or*
2. $h_j^p = 0$ *and* $h_j^u = \kappa_j(b, a)$, *or*
3. $h_j^p > 0$ *and* $h_j^u = \kappa_j(b, a)$.

Proof. Assume that $\{h_j^m, h_j^p, h_j^u\}$ solves the childcare allocation problem, with $h_j^p > 0$ and $h_j^u < \kappa_j(b)$. There exists ϵ such that $\epsilon > 0$ and $\epsilon < \min\{h_j^p, \kappa_j(b, a) - h_j^u\}$. Allocation $\{h_j^m, \epsilon, h_j^u + \epsilon\}$ provides a higher utility to the household and satisfies all constraints. The result follows by contradiction. ■

Lemma **??** just states that the mother would first exhaust all available unpaid childcare before drawing on paid childcare, as the former does not affect utility in any way, while the latter decreases it by reducing consumption.

An implication of lemma 2.A.1 is that the childcare allocation problem can be written in terms of mother provided childcare only:

$$\max_{h_j^m} \quad u_{n_j}\left(h_j^m\right) \tag{2.6}$$

subject to

$$0 \leq h_j^m \leq 1 - m_j(b)$$

$$1 - \bar{h}_j^p - \kappa_j(b) < h_j^m < 1 - n_j$$

where:

$$u_{n_j}\left(h_j^m\right) = \frac{\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{1-\gamma_0} - 1}{1-\gamma_0} + \delta_1 \frac{l_j\left(h_j^m\right)^{1-\gamma_1} - 1}{1-\gamma_1} - \delta_2 |b - b^*|^{\gamma_2} - \zeta + \delta_3\left(h_j^m\right)^{\gamma_3}$$

and

$$c_j(h_j^m) = \begin{cases} 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j(1 - h_j^m - \kappa_j(b), n) & \text{if } h_j^m \leq 1 - \kappa_j(b) \\ 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) & \text{if } h_j^m > 1 - \kappa_j(b) \end{cases}$$

$$l_j\left(h_j^m\right) = 1 - n_j - h_j^m.$$

Define the following interval from the constraints of the problem:

$$I_{h_j^m} = \left[\max\{0, 1 - \bar{h}_j^p - \kappa_j(b)\}, \min\{1 - m_j(b), 1 - n_j\right].$$

Lemma 2.A.2 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the derivative of $u_{n_j}\left(h_j^m\right)$ exists almost everywhere in the interval $I_{h_j^m}$.*

Proof. If $h_j^m > 1 - \kappa_j(b)$, then:

$$u'_{n_j} = -\delta_1 l_j(h_j^m)^{-\gamma_1} + \delta_3 \gamma_3 (h_j^m)^{\gamma_3 - 1}.$$

For $\gamma_1 > 0$, the first term on the right hand side exists as long as:

$$l_j = 1 - n_j - h_j^m > 0 \iff h_j^m < 1 - n_j,$$

and for $0 < \gamma_3 < 1$, the second term on the right hand side exists as long as $h_j^m > 0$.

If $h_j^m < 1 - \kappa_j(b)$, then

$$u'_{n_j} = \left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0} \frac{1}{\psi(b)} \frac{\partial \lambda_j\left(h_j^p, b\right)}{\partial h_j^p} - \delta_1 l_j(h_j^m)^{-\gamma_1} + \delta_3 \gamma_3 (h_j^m)^{\gamma_3 - 1}.$$

It is already known that the second and third terms exist almost everywhere in $I_{h_j^m}$. For $\gamma_0 > 0$, the first term on the right hand side exists as long as:

$$c_j(h_j^m) > 0 \iff h_j^m > 1 - \bar{h}_j^p - \kappa_j(b, a).$$

Finally, u'_{n_j} does not exist at $h_j^m = 1 - \kappa_j(b, a)$.

Therefore, u'_{n_j} exists everywhere in $I_{h_j^m}$ except at $h_j^m = 1 - n_j$, $h_j^m = 0$, $h_j^m = 1 - \bar{h}_j^p - \kappa_j(b, a)$ and $h_j^m = 1 - \kappa_j(b)$ whenever they belong to $I_{h_j^m}$. ■

Lemma 2.A.3 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the second derivative of $u_{n_j}\left(h_j^m\right)$ exists and is negative almost everywhere in $I_{h_j^m}$.*

Proof. If $h_j^m > 1 - \kappa_j(b)$, then:

$$u''_{n_j} = -\gamma_1 \delta_1 (1 - n_j - h_j^m)^{-\gamma_1 - 1} + \delta_3 \left(\gamma_3 - 1\right) \gamma_3 (h_j^m)^{\gamma_3 - 2}.$$

For $\gamma_1 > 0$, the first term on the right hand side exists as long as

$$1 - n_j - h_j^m > 0 \iff h_j^m < 1 - n_j.$$

For $0 < \gamma_3 < 1$, the second term on the right hand side exists as long as $h_j^m > 0$. Both elements are negative for $\gamma_1 > 0$ and $0 < \gamma_3 < 1$ as long as the expression is valid, therefore the whole expression exists and is negative everywhere except at $h_j^m = 1 - n_j$ and $h_j^m = 0$ in $I_{h_j^m}$.

If $h_j^m < 1 - \kappa_j(b)$, then:

$$\begin{aligned} u''_{n_j} = & -\gamma_0 \left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0 - 1} \frac{1}{\psi(b)} \frac{\partial \lambda_j\left(h_j^p, b\right)}{\partial h_j^p} \\ & -\gamma_1 \delta_1 (1 - n_j - h_j^m)^{-\gamma_1 - 1} + \delta_3 \left(\gamma_3 - 1\right) \gamma_3 (h_j^m)^{\gamma_3 - 2}. \end{aligned}$$

It is already known that the last two terms on the left hand side exist and are negative almost everywhere in $I_{h_j^m}$, so only with the first term is of interest. For $\gamma_0 > 0$ the expression exists as long as

$$c_j(h_j^m) > 0 \iff h_j^m > 1 - \bar{h}_j^p - \kappa_j(b),$$

and is negative as long as it exists since $\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0 - 1} > 0$, $\frac{1}{\psi(b)} > 0$ and $\frac{\partial \lambda_j(h_j^p, n)}{\partial h_j^p} > 0$. Therefore, the whole expression exists and is negative everywhere except at $h_j^m = 1 - n_j$, $h_j^m = 0$ and $h_j^m = 1 - \bar{h}_j^p - \kappa_j(b)$ in $I_{h_j^m}$.

Finally, the second derivative does not exist at $h_j^m = 1 - \kappa_j(b)$. ■

Lemma 2.A.4 *If $\gamma_0 > 0$, $\gamma_1 > 0$ and $0 < \gamma_3 < 1$, the first derivative of $u_{n_j}\left(h_j^m\right)$ is strictly decreasing in $I_{h_j^m}$.*

Proof. It is known from lemma 2.A.3 that u''_{n_j} is negative almost everywhere in $I_{h_j^m}$, with the notable exception of $h_j^m = 1 - \kappa_j(b)$. If $1 - \kappa_j(b)$ is not an interior point of $I_{h_j^m}$, the result follows automatically. Suppose that $1 - \kappa_j(b)$ is an interior point of $I_{h_j^m}$. Since

$$\left(\frac{c_j(h_j^m)}{\psi(b)}\right)^{-\gamma_0} \frac{1}{\psi(b)} \frac{\partial \lambda_j\left(h_j^p, n\right)}{\partial h_j^p} > 0,$$

then,

$$\lim_{h_j^m \to 1-\kappa_j(b)^-} u'_{n_j} > \lim_{h_j^m \to 1-\kappa_j(b)^+} u'_{n_j}.$$

Thus, u'_{n_j} is strictly decreasing in $I_{h_j^m}$. ■

The preceding lemmas are now used to show that the solution to the childcare allocation problem is unique and to characterise that solution.

Theorem 2.A.5 *If $\gamma_0 > 1$, $\gamma_1 > 1$ and $0 < \gamma_3 < 1$, the childcare allocation problem has a unique solution in $I_{h_j^m}$.*

Proof.

Suppose that $\exists\, h_j^m \in I_{h_j^m} : u'_{n_j}(h_j^p) = 0$. From lemma 2.A.4 it is known that this point is unique, and by standard marginal arguments it follows that it necessarily is a maximum of u_{n_j} in $I_{h_j^m}$.

Now suppose there is no such point. Notice that:

$$\lim_{h_j^m \to \inf I_{h_j^m}{}^+} u'_{n_j} = \begin{cases} \lim_{h_j^m \to 0^+} u'_{n_j} = \infty & \text{if } \; 0 \geq 1 - \bar{h}_j^p - \kappa_j(b, a) \\ \lim_{h_j^m \to 1-\bar{h}_j^p - \kappa_j(b)^+} u'_{n_j} = \infty & \text{if } \; 0 < 1 - \bar{h}_j^p - \kappa_j(b). \end{cases}$$

Suppose further that $1 - n_j < 1 - m_j(b, a)$. Since

$$\lim_{h_j^m \to 1-n_j+} u'_{n_j} = -\infty,$$

then $1-\kappa_j(b) < 1-n_j$, for otherwise $u'_{n_j}(h_j^p) = 0$ for some $h_j^p \in I_{h_j^m}$ as u'_{n_j} would be a strictly decreasing continuous function that takes both positive and negative values in $I_{h_j^m}$. Moreover, using the same argument it follows that:

$$\begin{aligned} u'_{n_j}(h_j^m) > 0 \quad \forall h_j^m \in \left(\inf I_{h_j^m}, 1 - \kappa_j(b)\right) \\ u'_{n_j}(h_j^m) < 0 \quad \forall h_j^m \in (1 - \kappa_j(b, a), 1 - n_j), \end{aligned}$$

which implies that $h_j^m = 1 - \kappa_j(b)$ is the unique solution to the childcare allocation problem.

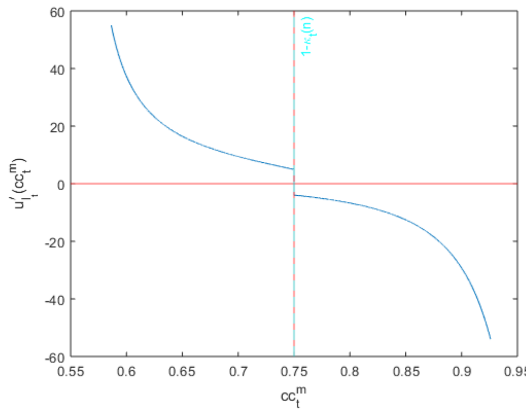
Assume now that $1-m_j(b, a) < 1-n_j$. There are two possibilities: $u'_{n_j}(1-m_j(b)) < 0$, or $u'_{n_j}(1-m_j(b)) \geq 0$. Suppose first that the former is true. Then, by the same argument used twice before, it follows that $1 - \kappa_j(b) < 1 - m_j(b)$, that:

$$\begin{aligned} u'_{n_j}(h_j^m) > 0 \quad \forall h_j^m \in \left(\inf I_{h_j^m}, 1 - \kappa_j(b)\right) \\ u'_{n_j}(h_j^m) < 0 \quad \forall h_j^m \in (1 - \kappa_j(b), 1 - m_j(b)), \end{aligned}$$

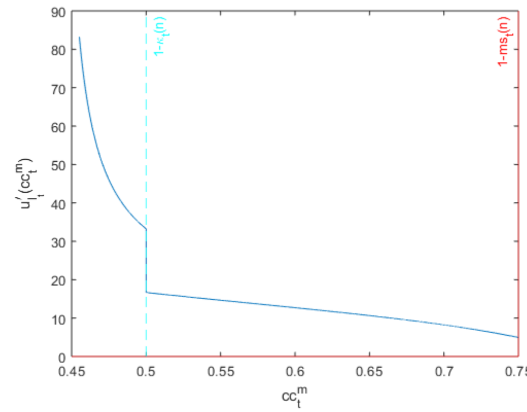
and hence that $h_j^m = 1 - \kappa_j(b)$ is the unique solution to the childcare allocation problem.

Finally, if $u'_{n_j}(1 - m_j(b)) \geq 0$, then it follows directly that the unique solution to the childcare allocation problem is $h_j^m = 1 - m_j(b)$. ■

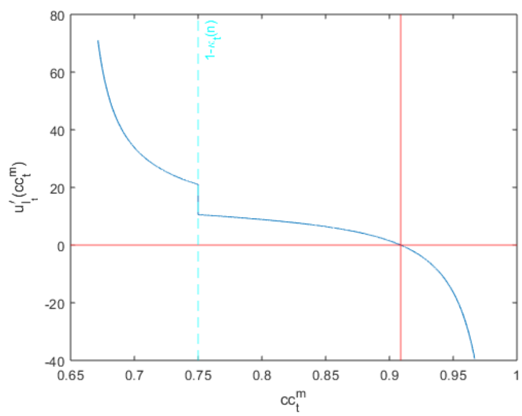
The proof of theorem 2.A.5 characterises all the forms the unique solution to the childcare allocation problem can take. Figure 2.8 illustrates this. Panel (a) shows a situation in which the solution is at the kink of the utility function, where marginal utility experiences a discrete jump. In this situation, the marginal utility of mother's childcare is positive right before reaching the unpaid childcare endowment, as it includes the term related to marginal consumption, but is negative right after. Panel (b) shows the strange but theoretically possible situation in which the mother would like to spend more time with her children but she cannot because of the limit imposed by mandatory schooling. Panels (c) and (d) show situations in which the solution is interior. In panel (c), marginal utility crosses zero after the point in which the household does



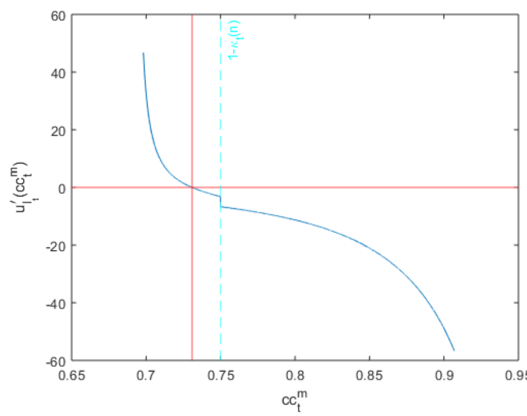
(a) Kink solution



(b) Corner solution



(c) Interior solution without paid childcare



(d) Interior solution with paid childcare

Figure 2.8: Solutions to the childcare allocation problem

not need to use paid childcare, while in panel (d) it does so before. Therefore, the household does not need to use paid childcare in the former but does in the latter.

2.B Algorithms

```
Inputs   : Model parameters
Outputs: Value and policy functions
begin
    Discretise the AR(1) process for the income shocks (ϵ_j^f, ϵ_j^m) using the Tauchen method;
    Construct a grid for the state space {ϵ_j, ϵ_j^h, x_j, ȳ_j^f, ȳ_j^m, b}, using the grid values for (ϵ_j, ϵ_j^m) ;
    for j = J_R : −1 : J_0 do
        if j = J_R then
            for Each point of the state space grid for {x_{J_R}, ȳ_{J_R}^f, ȳ_{J_R}^m, b} do
                Compute the value of retirement V_{J_R}(x_{J_R}, ȳ_{J_R}^f, ȳ_{J_R}^m; b) ;
            end
        end
        else if j ∈ [J_0, J_R − 1] then
            for b = 0 : 3 do
                if b > 0 and j ≤ W_c(b) then
                    for Each point of the state space grid for {ϵ_j, ϵ_j^h, x_j, ȳ_j^f, ȳ_j^m} do
                        Solve the static childcare problem for each possible value of n_j ;
                        Compute the flow utility for the period and retrieve the continuation value
                         according to the rules of motion for the state variables;
                        Select the choice of n_j that returns the largest value;
                        Record both the value V_j(ϵ_j, ϵ_j^h, x_j, ȳ_j^f, ȳ_j^m; b) and the policy ;
                    end
                end
                else
                    for Each point of the state space grid for {ϵ_j, ϵ_j^h, x_j, ȳ_j^f, ȳ_j^m} do
                        Compute the flow utility for the period and retrieve the continuation value
                         according to the rules of motion for the state variables;
                        Select the choice of n_j that returns the largest value;
                        Record both the value V_j(ϵ_j, ϵ_j^h, x_j, ȳ_j^f, ȳ_j^m; b) and the policy ;
                    end
                end
            end
        end
    end
end
```

Algorithm 1: Solving the working-age stages of the life cycle model

Inputs : state variables $\{x_j, \epsilon_j, \epsilon_j^m, b\}$, labour force participation choice n_j
Outputs: value of utility v_{n_j}, optimal choice of childcare $\{h_j^m, h_j^p, h_j^u\}$

begin
- Calculate the maximum mother provided childcare time and paid childcare;
- $\bar{h}_j^m = 1 - n_j$;
- $\bar{h}_j^p = \max\{0, \sup\{h_j^p : 2n_j y_j^f + y_j^m - T\left(y_j^f, y_j^m, n_j\right) - \lambda_j(h_j^p, b) > 0\}\}$;
- **if** $\bar{h}_j^m + \bar{h}_j^p + \kappa_j(b) \leq 1$ **then** choice of labour force participation n_j is unfeasible
  - Set $h_j^m = NaN$, $h_j^p = NaN$, $h_j^u = NaN$ and $v_{n_j} = -\infty$;
- **else**
  - Attempt to find a root for u'_{n_j} in $I_{h_j^m}$;
  - **if** $\exists x \in I_{h_j^m} : u'_{n_j}(x) = 0$ **then** x is the unique solution for the childcare allocation problem, which is interior with or without paid childcare
    - Set $h_j^m = x$, $h_j^u = \min\{1 - x, \kappa_j(b)\}$, $h_j^p = 1 - h_j^m - h_j^u$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^u\right)$;
  - **else**
    - **if** $1 - n_j \leq 1 - m_j(b)$ *or* $1 - m_j(b) < 1 - n_j$ *and* $u'_{n_j}(1 - ms_j(b)) < 0$ **then** there is a unique kink solution to the childcare allocation problem
      - Set $h_j^m = 1 - \kappa_j(b)$, $h_j^p = 0$, $h_j^u = \kappa_j(b)$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
      - Verify that $\lim_{h_j^m \to 1-\kappa_j(b)^-} u'_{n_j} > 0$ and $\lim_{h_j^m \to 1-\kappa_j(b)^+} u'_{n_j} < 0$;
    - **else**
      - There is a unique corner solution to the childcare allocation problem;
      - Set $h_j^m = 1 - m_j(b)$, $h_j^p = 0$, $h_j^u = m_j(b)$ and $v_{n_j} = u_{t,b}\left(h_j^m, h_j^p, h_j^i\right)$;
      - Verify that $u'_{n_j}\left(1 - m_j(b)\right) > 0$;
    - **end**
  - **end**
- **end**

end

Algorithm 2: Solving the childcare allocation problem

Bibliography

Abramitzky, Ran, Adeline Delavande, and Luis Vasconcelos, "Marrying Up: The Role of Sex Ratio in Assortative Matching," *American Economic Journal: Applied Economics*, July 2011, *3* (3), 124–57.

Adda, Jérôme, Christian Dustmann, and Katrien Stevens, "The Career Costs of Children," *Journal of Political Economy*, 2017, *125* (2), 293–337.

Angrist, Josh, "How Do Sex Ratios Affect Marriage and Labor Markets? Evidence from America's Second Generation," *The Quarterly Journal of Economics*, 2002, *117* (3), 997–1038.

Attanasio, Orazio, Hamish Low, and Virginia Sánchez-Marcos, "Explaining Changes in Female Labor Supply in a Life-Cycle Model," *American Economic Review*, September 2008, *98* (4), 1517–52.

Auerbach, Alan J. and Laurence J. Kotlikoff, *Dynamic fiscal policy*, Cambridge University Press, 1987.

Becker, Gary S., "A Theory of Marriage: Part I," *Journal of Political Economy*, 1973, *81* (4), 813–846.

_ , "A Theory of Marriage: Part II," *Journal of Political Economy*, 1974, *82* (2), S11–S26.

Bick, Alexander, "The quantitative role of child care for female labor force participation and fertility," *Journal of the European Economic Association*, 2016, *14* (3), 639–668.

Burdett, Ken and Melvyn G. Coles, "Marriage and Class," *The Quarterly Journal of Economics*, 1997, *112* (1), 141–168.

Cai, Yong, "China's New Demographic Reality: Learning from the 2010 Census," *Population and Development Review*, 2013, *39* (3), 371–396.

Chiappori, Pierre-Andre and Yoram Weiss, "Divorce, Remarriage, and Welfare: A General Equilibrium Approach," *Journal of the European Economic Association*, 05 2006, *4* (2-3), 415–426.

Cruces, Lidia, "Gender gaps in the labor market and pension sustainability," *Working Paper*, 2021.

Da Rocha, José María and Luisa Fuster, "Why Are Fertility Rates and Female Employment Ratios Positively Correlated across O.E.C.D. Countries?," *International Economic Review*, 2006, *47* (4), 1187–1222.

Díaz-Giménez, Javier and Julián Díaz-Saavedra, "Delaying retirement in Spain," *Review of Economic dynamics*, 2009, *12* (1), 147–167.

Erosa, Andrés, Luisa Fuster, and Diego Restuccia, "Fertility Decisions and Gender Differences in Labor Turnover, Employment, and Wages," *Review of Economic Dynamics*, 2002, *5* (4), 856 – 891.

Fernández, Raquel, Nezih Guner, and John Knowles, "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," *The Quarterly Journal of Economics*, 02 2005, *120* (1), 273–344.

Ge, Suqin and Dennis Tao Yang, "Changes in China's Wage Structure," *Journal of the European Economic Association*, 2014, *12* (2), 300–336.

González, Maía José, "La escolarización de la primera infancia en España: desequilibrios territoriales y socioeconómicos en el acceso a los servicios," *El Estado del Bienestar en España, Madrid, Tecnos*, 2004, pp. 291–312.

González, Libertad, "The Effect of a Universal Child Benefit on Conceptions, Abortions, and Early Maternal Labor Supply," *American Economic Journal: Economic Policy*, 2013, *5* (3), 160–188.

Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorukoglu, "Engines of Liberation," *The Review of Economic Studies*, 2005, *72* (1), 109–133.

_ , Nezih Guner, and John A. Knowles, "More on Marriage, Fertility, and the Distribution of Income," *International Economic Review*, 2003, *44* (3), 827–862.

_ , _ , Georgi Kocharkov, and Cezar Santos, "Marry Your Like: Assortative Mating and Income Inequality," *American Economic Review*, May 2014, *104* (5), 348–53.

_ , _ , _ , and _ , "Technology and the Changing Family: A Unified Model of Marriage, Divorce, Educational Attainment, and Married Female Labor-Force Participation," *American Economic Journal: Macroeconomics*, January 2016, *8* (1), 1–41.

Grosjean, Pauline and Rose Khattar, "It's Raining Men! Hallelujah? The Long-Run Consequences of Male-Biased Sex Ratios," *The Review of Economic Studies*, 05 2018, *86* (2), 723–754.

Hannusch, Anne, "Taxing families: The impact of child-related transfers on maternal labor supply," *Unpublished Manuscript, Mannheim University*, 2019.

Hyslop, Dean R., "Rising U.S. Earnings Inequality and Family Labor Supply: The Covariance Structure of Intrafamily Earnings," *American Economic Review*, September 2001, *91* (4), 755–777.

Imrohoroğlu, Selahattin and Sagiri Kitao, "Social security reforms: Benefit claiming, labor force participation, and long-run sustainability," *American Economic Journal: Macroeconomics*, 2012, *4* (3), 96–127.

Jones, Larry E., Alice Schoonbroodt, and Michèle Tertilt, "Fertility Theories: Can They Explain the Negative Fertility-Income Relationship?," in "Demography and the Economy" NBER Chapters, National Bureau of Economic Research, Inc, 2010, pp. 43–100.

Kitao, Sagiri, "Sustainable social security: Four options," *Review of Economic Dynamics*, 2014, *17* (4), 756–779.

Knowles, John, "Why are Married Men Working So Much? An Aggregate Analysis of Intra-Household Bargaining and Labour Supply," *Review of Economic Studies*, 2013, *80* (3), 1055–1085.

Kotlikoff, Laurence J, Kent Smetters, and Jan Walliser, "Mitigating America's demographic dilemma by pre-funding social security," *Journal of monetary Economics*, 2007, *54* (2), 247–266.

Kydland, Finn E. and Edward C. Prescott, "Time to build and aggregate fluctuations," *Econometrica: Journal of the Econometric Society*, 1982, pp. 1345–1370.

Laun, Tobias and Johanna Wallenius, "Having It All? Employment, Earnings, and Children*," *The Scandinavian Journal of Economics*, 2021, *123* (1), 353–381.

Mincer, Jacob, *Schooling, Experience, and Earnings*, National Bureau of Economic Research, Inc, 1974.

Nishiyama, Shinichi and Kent Smetters, "Does social security privatization produce efficiency gains?," *The Quarterly Journal of Economics*, 2007, *122* (4), 1677–1719.

Nollenberger, Natalia and Núria Rodríguez-Planas, "Full-time universal childcare in a context of low maternal employment: Quasi-experimental evidence from Spain," *Labour Economics*, 2015, *36*, 124–136.

Sánchez-Mangas, Rocio and Virginia Sánchez-Marcos, "Balancing family and work: The effect of cash benefits for working mothers," *Labour Economics*, 2008, *15* (6), 1127–1142.

Seitz, Shannon, "Accounting for Racial Differences in Marriage and Employment," *Journal of Labor Economics*, 2009, *27* (3), 385–437.

Tauchen, George and Robert Hussey, "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 1991, *59* (2), 371–396.

Wang, Xiaodi, "Essays on family economics and intra-household bargaining." PhD dissertation, University of Mannheim 2018.

Wei, Shang-Jin and Xiaobo Zhang, "The Competitive Saving Motive: Evidence from Rising Sex Ratios and Savings Rates in China," *Journal of Political Economy*, 2011, *119* (3), 511–564.