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Optimal sales-mix and generation plan in a two-stage electricity market

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Abstract

A bi-level stochastic programming problem is used to model the optimal decision of a risk averse electricity producer, interacting in a two-stage market with cost minimizer competitors. His decision variables include the distribution of production (which plant of different technologies and variable costs to operate) and the sales-mix (how much generation to commit to bilateral contracts and spot market). To enhance computation times, the bi-level problem is transformed into a Mixed-Integer Linear Problem (MILP) by applying sophisticated linearization techniques. Electricity demand, Renewable Energy Sources (RES) generation and production costs are different sources of uncertainty. A copula method is used to generate scenarios under different correlations values (between RES generation and demand), to analyze the impact of correlation on the optimal solution. The model is tested through extensive numerical simulations based on data from the Spanish electricity market. The results show that correlation and risk aversion have a relevant impact on how sales-mix and generation plan decisions should combine optimally.

Keywords: CVaR, demand uncertainty, electricity industry, futures market, renewable uncertainy, risk aversion, spot market.

1. Introduction

Risk management in the electricity sector is a difficult task. Probably it is the toughest compared to all other industrial sectors, because of the extreme volatility of many key variables, in particular spot price of electricity, weather conditions and demand. At the origin of such vulnerability there is probably a unique circumstance, differentiating the electricity sector from all the others, that is the lack of a significant storage system. With no doubts, if large quantities of electricity could be stored, the volatility of weather conditions, demand or events like outages and transmission disruptions would not hit so violently the spot price

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of electricity, as we are used to see. Negative prices, a kind of economic singularity of such markets, is an astonishing example of such violence.

An opportunity largely used among the producers in the electricity sector, to reduce the impact of the volatility of electricity prices, is that of committing part of their generation through bilateral contracts. Various forms exist to this purpose: customized over-the-counter forwards, standardized futures of different maturities. In all these cases, the common feature is that of fixing a price to supply electricity for a given period of time. The benefit of reducing the volatility of electricity prices is of course welcome also to the retailers, since buying electricity at fixed price help them to better plan their distribution business. However, introducing these financial instruments in the profit function of a producer is not a trivial decision from a risk management point of view. Some major difficulties arise because the spot price of electricity depends strictly on other key variables, namely fuel prices and electricity demand, which in its turn depends on weather conditions. Other relevant difficulties also originate because such financial instruments must coordinate with technical decision variables, namely the distribution of generation over the producing plants that usually are diversified across different technologies. To make an example of these links, consider a mix of plants composed by Renewable Energy Sources (RES) such as wind and conventional. During weeks with good wind conditions, RES plants generate large output levels (at virtually zero cost), placing a negative pressure on the spot price of electricity. Taking short positions on a futures contract during those weeks would therefore offer a profit protection to an electricity producer. However, if wind happened to be relatively low and spot price relatively high, a producer would prefer to sell his wind generation on the spot market rather than commit it with a fixed price contract. Uncontrolled choices about how to share the power generation between fixed price contracts and spot market can be the origin of high inefficiency. Such a decision is referred as the *sales-mix*.

Next to this, generators also face routinely the problem of planning the capacity of their dispatchable plants. Although sales-mix is usually decided before than the production plan, it would be a mistake to consider that the two decisions can be taken independently. Planning how much each plant should produce and how the corresponding generation should be directed in the sales-mix affects the market supply curve, the distribution of the possible profits and ultimately that of the sales-mix itself. The two decisions can interact in complex ways. To understand this interaction provides far-reaching insights relevant both to the purposes of optimal management and market regulation. The objective of this paper is therefore to solve and analyze the optimal combination of the sales-mix and the production plan (generation plan) decisions for a large generator (i.e. his choice affects to some extent the market price).

Additionally, this work argues that this problem is largely affected by the correlation between RES generation and demand. There is a consistent empirical evidence that such a correlation changes not only across different markets, but also during the period of the year. For example, in markets where wind is the prevailing RES technology, such correlation tends to be negative overall; where photovoltaic prevails, such correlation tends to be positive especially during summer periods, since sunny hot days increase both generation and the use of air conditioners. To provide some evidence about different types of dependence between these two variables, figures 1 and 2 present a sample of the joint distribution of electricity demand and RES generation for the case of Spain and German electricity markets. The two cases show clearly different patterns. In particular, the German market shows a non linear dependence between the two variables, turning from negative (for low levels of the demand) to positive (for higher levels of the demand).

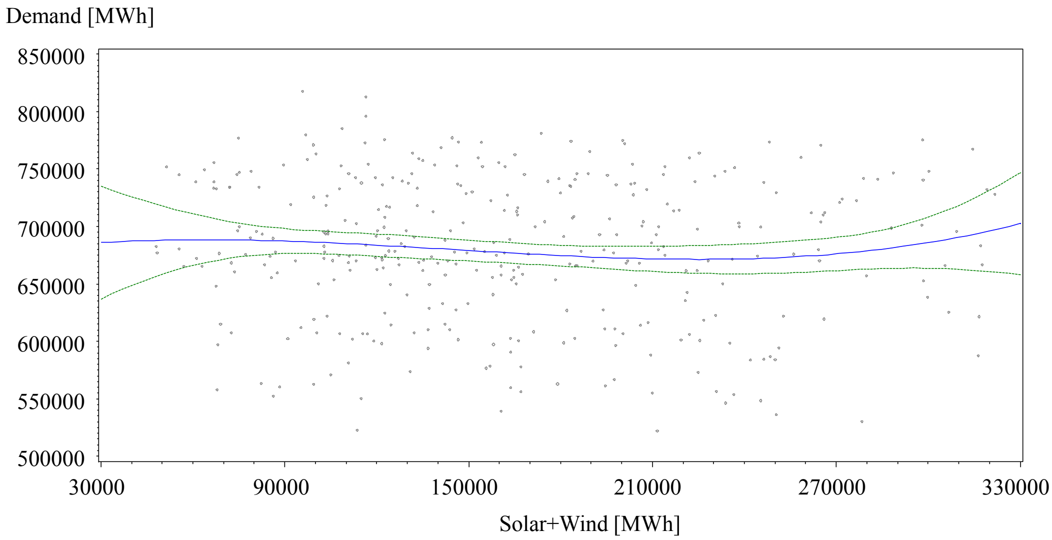


Figure 1: Joint distribution of the power demand and RES generation in the Spanish market, daily data, October 2015 to October 2016

Moreover, this correlation is expected to play a key role in the future electrical system. Under the new paradigm of *smart grids*, distributed generation (mostly based on RES technologies) is being incorporated to the electrical systems by individual consumers (*prosumers*). In such a setting, if weather conditions are favorable during a given period, prosumers can self-generate their electricity and decrease their demand from the central grid, resulting in a negative correlation between demand and RES generation. To this aim, this paper compares optimal solutions under different correlation levels between those two variables. The objective of the large generator consists of a linear utility function combining expected profit and conditional value at risk. Such function includes a risk aversion parameter to model both the case of a risk neutral and a risk averse producer. A two-stage stochastic programming problem is adapted to the case. It is then turned into an equivalent MILP formulation, to speed up calculations and extend the complexity of the model to a real scale size, through ad hoc variable transformations. To analyze thoroughly the properties of the model, it has been applied to a consistent number of realistic simulated scenarios.

The paper is organized as follows. Section 2 reviews the relevant bibliography on the subject. Section 3 develops the model for the decision problem, showing how it can be reformulated from the initial two-stage stochastic programming to a MILP formulation. Section 4 applies the model and comments on the results. Section 5 concludes.

2. Bibliography review and contributions

The presence of correlations among various key factors characterizing the electricity markets has been analyzed in several studies. Alvarez et al. (2010) study time varying correlations

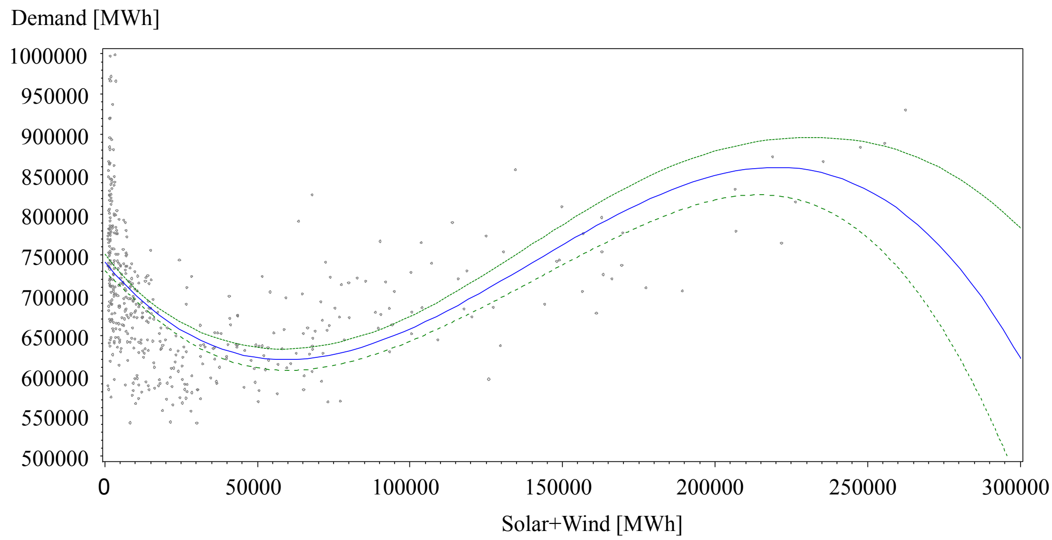


Figure 2: Joint distribution of the power demand and RES generation in the German market, daily data, January 2015 to October 2016

between electricity prices and demand to the purpose of improving price forecasting and risk management for an electricity producer. Same type of correlation has also been analyzed in Boroumand et al. (2015) to the purpose of setting up an optimal hedging portfolio management for retailers seeking to meet demand in different intraday time segments.

The impact of renewables on the power sector has also been analyzed under different aspects. Bell et al. (2015) find a small correlation between electricity demand and wind speed in Australia, tending to increase during their observation period (2010-2012). They analyze the benefit of meeting electricity demand in the different Australian states (without the use of storages) resulting from this increasing correlations. Still focusing on the Australian market Cutler et a. (2011) also find strong evidence of negative correlation between wind generation and electricity price. In Chaiamarit and Nuchprayoon (2014) the authors assess the impact of several renewable sources on the demand of electricity and, consequently, find meaningful implication on operating costs. Large evidence of a negative impact of RES on electricity prices in Germany has been found by Dilig et al. (2016).

The correlation among the key variables of electricity markets is also central to the problem of diversifying the technology portfolio (also referred as the energy-mix problem). Adabi et al. (2016) and Delarue et al. (2011) resort to the theory of financial portfolios. In particular, the latter paper combines short term generation and long term investment optimization. Interestingly such work shows that wind capacity can help to lower the risk on generation cost. However that paper focuses just on the cost side the problem, so it misses the correlation between electricity prices and producing factors (usually gas cost), which drives the risk of electricity generators. Vice versa, a paper explicitly considering the opportunity to reduce risk leveraging on the correlation between electricity price and generation costs is that of Falbo et al. (2010). The connection of this paper with this work is that in both cases (part of) the solution of the optimization problem is set with the sales-mix between spot market and bilateral contracts. However, in that work the decision is taken by individual price-taker producer, while here a game theoretic approach is considered between a producer and more competitors, so that electricity price is an outcome depending on the solution adopted in the game.

Several works have analyzed the direct economic impact of introducing futures or forward transactions in markets. The relationship between futures and spot markets has been studied by Allaz (1992) and Allaz and Vila (1993) which, in the contest of a duopoly model, show that market prices may decrease with the introduction of futures trading. This model is extended by Dong and Liu (2007) to a supply chain where players are risk averse. Mendelson and Tunca (2007) analyze the efficiency of a supply chain when fixed-price contracts are introduced. They show that supply and demand uncertainty play a key role in the relationship between forward and spot trading. The impact of demand uncertainty in the optimal sales volume in forward and spot markets is also explored by Popescu and Seshadri (2013). It is observed that demand elasticity is a major factor affecting the shares of forward and spot trading.

Focusing on electricity markets, Niu et al. (2005) propose a supply function equilibrium model to derive the optimal bidding strategy of firms considering forward contracts. An oligopolistic model is used by Anderson and Hu (2008) to study how futures trading may increase social welfare when retailers have market power. Bushnell (2007) extend the Allaz Vila model to multiple firms and increasing costs to show that suppliers' market power plays a key role in the interaction between forward and spot trading. A stochastic programming framework is presented by Carrión et al. (2007) to derive the optimal futures and spot trading by a risk-averse and price-taking retailers. Similar to the proposed model, Conejo et al. (2008) analyze the optimal involvement of a price-taker electricity producer in the futures market considering risk aversion and exogenous prices. An equilibrium model is proposed by Aïd et al. (2011) to show that vertical integration plays a key role in the relationship between spot and forward markets with demand uncertainty. Another equilibrium model is presented in Ruiz et al. (2012) to analyze the equilibria in futures and spot markets with oligopolistic generators and conjectural variations. Different types of futures contracts, and their impact on the market outcomes, are tested in a game theoretical setting by Oliveira et al. (2013). Fanzeres et al. (2015) derive the optimal contracting strategy of a price-taker and risk averse energy trading company which operates with a renewable portfolio. Considering a medium-term model, Marí et al. (2017) propose a heuristic algorithm to derive the optimal involvement in pool and bilateral contracts of a generation company.

This paper contributes to the state-of-the-art on this topic by presenting a new complex and computationally efficient tool for decision making under uncertainty. Power producers can benefit from it in order to decide which is the optimal level of commitment in a futures market, together with the most adequate strategy (generation plan) to supply that energy. Similarly, market operators can use it to evaluate the impact of competition and risk aversion in two-stage markets, and infer how these may evolve in the coming years under the smart grid paradigm. More specifically, the contributions of this work are sixfold:

1) To derive the simultaneous optimal sales-mix and generation plan of a generation company with different types of technologies: nuclear, renewables and conventional generation units.

2) To model the multivariate risk generated by the interaction of by demand load, renew-

able generation, and generation costs uncertainties.

3) To account for the generators market power by the endogenous formation of spot prices.

4) To formulate a bi-level stochastic optimization model to derive the optimal strategy in 1). The model is converted into an equivalent single level Mathematical Problem with Equilibrium Constraints (MPEC) that is latter linearized without approximation into an equivalent Mixed-Integer Linear Problem (MILP) easy to solve by available branch-and-cut solvers.

5) To extend the model formulation in 4) to account for risk via the Conditional Value-at-Risk (CVaR).

6) To analyze the impact of correlation between RES and electricity demand in a set of realistic numerical simulations based on realistic data (Spanish Electricity Market).

It is worth observing that the model is of theoretical nature, as its properties derive from its hypotheses, not from empirical speculation. The reference to the case of Spanish market, adopted in the application, is just used to give results a realistic flavor. Indeed, given the linear form of our model, similar results and conclusions would have been obtained using laboratory data, or data from other markets with a similar oligopolistic structure and similar RES penetration levels. Examples of this kind are common and include the markets of Germany, Italy, Denmark or Norway, or markets that can expected to develop similarly in the future as, for example, UK, France, Australia, Japan or Canada.

3. Problem settings

We consider the optimization problem of an electricity producer (also referred as *our producer* or *the producer*) competing in an economy populated with other rival producers. He is large enough to condition (to some extent) the spot market, so we can also refer to him as a *strategic* producer. The rival producers can manage up to J types of plants (producing units). So index j with $j = 1, \ldots, J$ identifies the j-th generating unit belonging to a rival producer. To distinguish the generating units of our producer from those of his competitors we will index them with i, with $i = 1, \ldots, I$. These generating units, for both the reference and the rival producers, may be based on different generation technologies, such as wind, solar, combined cycle, nuclear, coal, etc.

We model a risk averse player. More precisely, we start with a risk neutral objective in formulation (1), and introduce risk aversion in the final problem formulation (10), which is actually applied. The producer combines two types of control variables: the *sales-mix*, that is the decision about how much generation to distribute between futures contracts and spot market, and the *generation plan*, that is the decision about how to match the sales-mix using the production from the different plants.

We take fixed contracts price p^f as given. In this way we model the situation where other players have already closed their fixed contract position, equilibrium price of futures market has been revealed and, on this market, our producer behaves as price taker. In a natural way, in this model the demand arriving on the spot market is therefore the residual quantity after that satisfied through bilateral agreements. The two control variables have a strategic value for our producer since, on the spot market, he can affect the shape of the market supply curve (through the generation plan) and the expected value of the residual demand (through the sales-mix). Additionally, both the producer and its rivals bid on the spot all the residual capacity (i.e. not committed to futures contracts) at the marginal cost of the different technologies. The interaction between the producer and its competitors occurs therefore only on the spot market.

We first proceed by presenting the risk neutral version of the bi-level problem in Section 3.1. Then its stochastic counterpart is formulated in Section 3.2 which is latter transformed into an equivalent single-level Mathematical Problem with Equilibrium Constraints (MPEC) that is linearized without approximation in Section 3.3. Finally, the proposed framework is generalized by considering a risk averse producer in Section 3.4.

3.1. Bi-level formulation, risk neutral producer

Mathematically, the interaction between the reference producer, his competitors and the Independent System Operator (ISO) is modeled through a bi-level stochastic optimization problem. The upper level problem represents the profit maximization of the producer which identifies the optimal production quantities to trade in the futures market anticipating possible realizations of the subsequent spot market, represented within the lower-level problem. The lower-level problem represents the optimal market clearing performed by the ISO which seeks to satisfy the electricity demand by minimizing the social cost.

In particular, the problem faced by the producer can be recast as the following bi-level formulation:

$$\max_{q^{ft},q_i^f,q_i^s,\mathcal{Z}} \quad \mathbb{E}\left[p^f q^{ft} + p^s \sum_{i=1}^{I} q_i^s - \sum_{i=1}^{I} \widetilde{c}_i^s (q_i^f + q_i^s)\right] \tag{1a}$$

s.t.

$$q^{ft} = \sum_{i=1}^{I} q_i^f \tag{1b}$$

$$0 \le q_i^f \le \widetilde{Q}_i^s \quad \forall i \tag{1c}$$

$$(q_i^s, \mathcal{Z}) \in \arg\min \sum_{i=1}^{I} \widetilde{c}_i^s q_i^s + \sum_{j=1}^{J} \widetilde{c}_j^{sr} q_j^{sr} - \sum_{p=1}^{P} \widetilde{c}_p^{sd} q_p^{sd} \tag{1d}$$

s.t.

$$\sum_{p=1}^{P} q_p^{sd} = \sum_{i=1}^{I} q_i^s + \sum_{j=1}^{J} q_j^{sr} \quad (p^s) \tag{1e}$$

$$0 \le q_i^s \le \widetilde{Q}_i^s - q_i^f \qquad (\mu_i^{\min}, \mu_i^{\max}) \qquad \forall i \tag{1f}$$

$$0 \le q_j^{sr} \le \widetilde{Q}_j^{sr} \qquad (\nu_j^{\min}, \nu_j^{\max}) \qquad \forall j \tag{1g}$$

$$0 \le q_p^{sd} \le \widetilde{Q}_p^{sd} \qquad (\gamma_p^{\min}, \gamma_p^{\max}) \qquad \forall p \tag{1h}$$

where $\mathcal{Z} = \left\{q_j^{sr}, q_p^{sd}, p^s, \mu_i^{\min}, \mu_i^{\max}, \nu_i^{\min}, \nu_i^{\max}, \gamma_p^{\min}, \gamma_p^{\max}\right\} \forall i, \forall j, \forall p$ includes both primal and dual lower-lever decision variables.

The upper level objective function (1a) represents the expected profit for the producer where the first term is the revenue from selling the energy production q^{ft} in the futures market at a price p^f. The second term is the revenue from selling the total production $\sum_{i=1}^{I} q_i^s$ in the spot market at a price p^s. The last term accounts for the total production costs for both the productions traded in the futures and in the spot market. Constraint (1b) guarantees that the total production traded in the futures market need to be distributed over the different units with a maximum capacity imposed by (1c).

The lower level problem is represented within (1d)-(1h) and constraints the upper level problem as the set of variables $\mathcal{Z}$ are also upper level decision variables. At the lower level, the ISO minimizes in (1d) the overall production costs associated to the reference producer units (first term) plus the production costs for the rival units (second term) minus the consumers utility (third term). These costs are calculated as the product of the offered price (that coincide with the variable cost) times the output from the corresponding plant: $\widetilde{c}_i q_i^s$ for the i-th plant of the reference producer and $\widetilde{c}_j q_j^{sr}$ for the j-th plant of the competitors. Similarly, we consider that the consumers place demand through a decreasing step-wise curve from which their utility can be computed as the products of the different ask prices times the energies dispatched: $\widetilde{c}_p^{sd} q_p^{sd}$. Therefore, the elasticity of the demand is modeled explicitly in the proposed formulation. It can be shown that such a minimization coincides with the minimization of the social cost for the energy, that is the classical objective for an ISO.

The energy balance (demand = total generation) is imposed by equality (1e) whose associated dual variable p^s represents the spot market clearing price. Constraint (1f) sets the maximum capacity for the spot production for the reference producer units. Note that this spot capacity is decreased if unit i is used to supply the futures market contract. Constraints (1g) and (1h) impose upper bounds ($\widetilde{Q}_j^{sr}$ and $\widetilde{Q}_p^{sd}$) for the rival units spot productions and for the demand blocks, respectively. Dual variables are indicated at their corresponding constraint between parenthesis.

The randomness of the model parameters is indicated by the symbol "$\sim$". The sources of uncertainty in the model are the variable cost (€$/MWh$) of conventional plants (mainly driven by the cost of fuel), the output level of non-conventional (green) plants (MWh) and the level of the demand (MWh).

The considered bi-level optimization framework can be viewed as an extension of classical Cournot or Bertrand competition models, as the producer, even though he offers his generation competitively, can accurately anticipate the impact of its decisions in the market clearing prices. Indeed, the proposed model resembles a Stackelberg (Stackelberg 1952) game where the leader (strategic producer) anticipates te reaction of the followers (ISO and competitive rivals). More specifically, the producer seeks to maximize a risk functional calculated on probability distribution of his profit function in (1a). Such profit depends on the generation plan decision variables q_i^f, with $i = 1, \ldots, I$, where $q^{ft} = \sum_{i=1}^{I} q_i^f$, showing how much generation obtained from the i-th plant to commit with a futures. As mentioned before, the remainder of the generation, $\widetilde{Q}_i^s - q_i^f$ in (1f), i.e., that is not committed in the futures market, is offered on the spot market at a marginal cost $\widetilde{c}_i$. This energy will be paid at the resulting spot market price p^s, which is determined endogenously as the Lagrangian multiplier associated to the power balance constraint (1e) within the lower level problem in.

3.2. Bi-level stochastic formulation, risk neutral producer

In order to account for the uncertainty of the model parameters, we approximate their probability distribution by a discrete set of plausible realizations, i.e., scenarios. We use the scenario index ω with $\omega = 1, \dots, \Omega$ and let σ_ω be the probability associated with each scenario. Moreover, the lower level primal and dual variables in $\mathcal{Y}$ are considered scenario dependent, as these decisions are taken after the uncertainty realization (*wait-and-see* decisions). This is equivalent to consider multiple lower level problems (1d)-(1h), one per scenario:

$$\max_{q^{ft}, q^f_{i\omega}, q^s_{i\omega}, \mathcal{Y}} \quad \sum_{\omega=1}^{\Omega} \sigma_\omega \left(p^f q^{ft} + p^s_\omega \sum_{i=1}^{I} q^s_{i\omega} - \sum_{i=1}^{I} c^s_{i\omega}(q^f_{i\omega} + q^s_{i\omega}) \right) \tag{2a}$$

s.t.

$$q^{ft} = \sum_{i=1}^{I} q^f_{i\omega} \qquad \forall \omega \tag{2b}$$

$$0 \le q^f_{i\omega} \le Q^s_{i\omega} \quad \forall i, \forall \omega \tag{2c}$$

$$(q^s_{i\omega}, \mathcal{Y}) \in \{\arg\min \sum_{i=1}^{I} c^s_{i\omega} q^s_{i\omega} + \sum_{j=1}^{J} c^{sr}_{j\omega} q^{sr}_{j\omega} - \sum_{p=1}^{P} c^{sd}_{p\omega} q^{sd}_{p\omega} \tag{2d}$$

s.t.

$$\sum_{p=1}^{P} q^{sd}_{p\omega} = \sum_{i=1}^{I} q^s_{i\omega} + \sum_{j=1}^{J} q^{sr}_{j\omega} \quad (p^s_\omega) \tag{2e}$$

$$0 \le q^s_{i\omega} \le Q^s_{i\omega} - q^f_{i\omega} \qquad (\mu^{\min}_{i\omega}, \mu^{\max}_{i\omega}) \qquad \forall i \tag{2f}$$

$$0 \le q^{sr}_{j\omega} \le Q^{sr}_{j\omega} \qquad (\nu^{\min}_{j\omega}, \nu^{\max}_{j\omega}) \qquad \forall j \tag{2g}$$

$$0 \le q^{sd}_{p\omega} \le Q^{sd}_{p\omega} \qquad (\gamma^{\min}_{p\omega}, \gamma^{\max}_{p\omega}) \qquad \forall p\} \quad \forall \omega \tag{2h}$$

where $\mathcal{Y} = \{q^{sr}_{j\omega}, q^{sd}_{p\omega}, p^s_\omega, \mu^{\min}_{i\omega}, \mu^{\max}_{i\omega}, \nu^{\min}_{i\omega}, \nu^{\max}_{i\omega}, \gamma^{\min}_{p\omega}, \gamma^{\max}_{p\omega}\} \, \forall \omega, \forall i, \forall j, \forall p$ includes both primal and dual lower-lever decision variables.

In the proposed model (2) the total production quantity traded in the futures q^{ft} is scenario independent, as it is a first-stage decision variable. However, the way in which this production is distributed among the producer's generating units can be decided within the second stage, once the uncertainty is resolve and the energy delivery takes place. Hence, we allow $q^f_{i\omega}$, i.e., the generation of unit i destined to supply the futures contract, explicitly depend on scenario ω. The result is that each scenario would present a different generation plan to supply the futures contract, which will depend on the particular realization of costs, demand and the availability of renewable resources, but ensuring the total futures quantity q^{ft} through constraint (2b). Then, the remaining capacity for each generating unit is used to participate in the spot market, as imposed by constraint (2f). Hence, the formulation of the lower level problem (2d)-(2h) allows representing the flexibility of the energy system to adapt to the uncertain demand realization and the availability of generating resources.

3.3. Single level MPEC, risk neutral producer

The lower level problem (2d)-(2d) is linear and thus convex and thus it can be replaced by its first-order optimality conditions (in the form of Karush-Kuhn-Tucker conditions). The resulting single level model results in a Nonlinear Programing Problem (NLP).

$$\max_{q^{ft}, q^f_{i\omega}, q^s_{i\omega}, \mathcal{Y}} \quad \sum_{\omega=1}^{\Omega} \sigma_\omega \left(p^f q^{ft} + p^s_\omega \sum_{i=1}^{I} q^s_{i\omega} - \sum_{i=1}^{I} c^s_{i\omega} (q^f_{i\omega} + q^s_{i\omega}) \right) \tag{3a}$$

s.t.

$$q^{ft} = \sum_{i=1}^{I} q^f_{i\omega} \qquad \forall \omega \tag{3b}$$

$$0 \leq q^f_{i\omega} \leq Q^s_{i\omega} \quad \forall i, \forall \omega \tag{3c}$$

$$\sum_{p=1}^{P} q^{sd}_{p\omega} = \sum_{i=1}^{I} q^s_{i\omega} + \sum_{j=1}^{J} q^{sr}_{j\omega} \quad \forall \omega \tag{3d}$$

$$c^s_{i\omega} - p^s_\omega + \mu^{\max}_{i\omega} - \mu^{\min}_{i\omega} = 0 \quad \forall i, \forall \omega \tag{3e}$$

$$c^{sr}_{j\omega} - p^s_\omega + \nu^{\max}_{j\omega} - \nu^{\min}_{j\omega} = 0 \quad \forall j, \forall \omega \tag{3f}$$

$$p^s_\omega - c^{sd}_{p\omega} + \gamma^{\max}_{p\omega} - \gamma^{\min}_{p\omega} = 0 \quad \forall p, \forall \omega \tag{3g}$$

$$0 \leq Q^s_{i\omega} - q^s_{i\omega} - q^f_{i\omega} \perp \mu^{\max}_{i\omega} \geq 0 \qquad \forall i, \forall \omega \tag{3h}$$

$$0 \leq q^s_{i\omega} \perp \mu^{\min}_{i\omega} \geq 0 \qquad \forall i, \forall \omega \tag{3i}$$

$$0 \leq Q^{sr}_{j\omega} - q^{sr}_{j\omega} \perp \nu^{\max}_{j\omega} \geq 0 \qquad \forall j, \forall \omega \tag{3j}$$

$$0 \leq q^{sr}_{j\omega} \perp \nu^{\min}_{j\omega} \geq 0 \qquad \forall i, \forall \omega \tag{3k}$$

$$0 \leq Q^{sd}_{p\omega} - q^{sd}_{p\omega} \perp \gamma^{\max}_{p\omega} \geq 0 \qquad \forall p, \forall \omega \tag{3l}$$

$$0 \leq q^{sd}_{p\omega} \perp \gamma^{\min}_{p\omega} \geq 0 \qquad \forall p, \forall \omega \tag{3m}$$

where $\perp$ indicates complementarity, so that $0 \leq a \perp b \geq 0$ implies: $a \geq 0$, $b \geq 0$ and $ab = 0$.

The nonlinear and nonconvex structure of problem (3) hinders the convergence to optimality of standard NLP solvers, specially when the number of scenarios Ω is high. However, due to its characteristic structure, it can be linearized without approximation and be transformed into a Mixed-Integer Linear Problem (MILP), which can be solved efficiently with available branch-and-cut solvers. The nonlinearities included in (3) can be treated as indicated below.

- The complementarity constraints (3h)-(3m) can be linearized, as proposed by Fortuny-Amat and McCarl. (1981), by introducing auxiliary binary variables ($u^{\max}_{i\omega}, u^{\min}_{i\omega}, v^{\max}_{j\omega}$,

$v_{j\omega}^{\min}, z_{p\omega}^{\max}$ and $z_{p\omega}^{\min}$) which results in the following set of disjunctive conditions:

$$0 \leq Q_{i\omega}^{s} - q_{i\omega}^{s} - q_{i\omega}^{f} \leq u_{i\omega}^{\max} M \qquad \forall i, \forall \omega \tag{4a}$$
$$0 \leq \mu_{i\omega}^{\max} \leq (1 - u_{i\omega}^{\max}) M \qquad \forall i, \forall \omega \tag{4b}$$
$$0 \leq q_{i\omega}^{s} \leq u_{i\omega}^{\min} M \qquad \forall i, \forall \omega \tag{4c}$$
$$0 \leq \mu_{i\omega}^{\min} \leq (1 - u_{i\omega}^{\min}) M \qquad \forall i, \forall \omega \tag{4d}$$
$$0 \leq Q_{j\omega}^{sr} - q_{j\omega}^{sr} \leq v_{j\omega}^{\max} M \qquad \forall j, \forall \omega \tag{4e}$$
$$0 \leq \nu_{j\omega}^{\max} \leq (1 - v_{j\omega}^{\max}) M \qquad \forall j, \forall \omega \tag{4f}$$
$$0 \leq q_{j\omega}^{sr} \leq v_{j\omega}^{\min} M \qquad \forall j, \forall \omega \tag{4g}$$
$$0 \leq \nu_{j\omega}^{\min} \leq (1 - v_{j\omega}^{\min}) M \qquad \forall j, \forall \omega \tag{4h}$$
$$0 \leq Q_{p\omega}^{sd} - q_{p\omega}^{sd} \leq z_{p\omega}^{\max} M \qquad \forall p, \forall \omega \tag{4i}$$
$$0 \leq \gamma_{p\omega}^{\max} \leq (1 - z_{p\omega}^{\max}) M \qquad \forall p, \forall \omega \tag{4j}$$
$$0 \leq q_{p\omega}^{sd} \leq z_{p\omega}^{\min} M \qquad \forall p, \forall \omega \tag{4k}$$
$$0 \leq \gamma_{p\omega}^{\min} \leq (1 - z_{p\omega}^{\min}) M \qquad \forall p, \forall \omega \tag{4l}$$
$$\left\{u_{i\omega}^{\max}, u_{i\omega}^{\min}, v_{j\omega}^{\max}, v_{j\omega}^{\min}, z_{p\omega}^{\max}, z_{p\omega}^{\min}\right\} \in \{0, 1\} \qquad \forall i, \forall j, \forall \omega. \tag{4m}$$

where M is a sufficiently large constant.

- The bilinear products $p_{\omega}^{s} \sum_{i=1}^{I} q_{i\omega}^{s}$ in the objective function (3a) can be linearized as follows:

  Multiplying all terms of (3d) by p_{ω}^{s} and rearranging renders

$$p_{\omega}^{s} \sum_{i=1}^{I} q_{i\omega}^{s} = p_{\omega}^{s} \sum_{p=1}^{P} q_{p\omega}^{sd} - p_{\omega}^{s} \sum_{j=1}^{J} q_{j\omega}^{sr} \qquad \forall \omega. \tag{5}$$

  From (3f) and (3g) we obtain that:

$$p_{\omega}^{s} = c_{j\omega}^{s} + \nu_{j\omega}^{\max} - \nu_{j\omega}^{\min} \quad \forall i, \forall \omega \tag{6a}$$
$$p_{\omega}^{s} = c_{p\omega}^{sd} - \gamma_{p\omega}^{\max} + \gamma_{p\omega}^{\min} \quad \forall p, \forall \omega. \tag{6b}$$

  Replacing (6a) and (6b) in the first and second right hand p_{ω}^{s} in (5), respectively, we obtain

$$p_{\omega}^{s} \sum_{i=1}^{I} q_{i\omega}^{s} = \sum_{p=1}^{P} c_{p\omega}^{sd} q_{p\omega}^{sd} - \sum_{p=1}^{P} \gamma_{p\omega}^{\max} q_{p\omega}^{sd} + \sum_{p=1}^{P} \gamma_{p\omega}^{\min} q_{p\omega}^{sd}$$
$$- \sum_{j=1}^{J} c_{i\omega}^{s} q_{j\omega}^{sr} - \sum_{j=1}^{J} \nu_{j\omega}^{\max} q_{j\omega}^{sr} + \sum_{j=1}^{J} \nu_{j\omega}^{\min} q_{j\omega}^{sr} \qquad \forall \omega. \tag{7}$$

  Moreover, from (3l), (3m), (3j) and (3k) we know that $\gamma_{p\omega}^{\max} q_{p\omega}^{sd} = \gamma_{p\omega}^{\max} Q_{p\omega}^{sd}$, $\gamma_{p\omega}^{\min} q_{p\omega}^{sd} = 0$, $\nu_{j\omega}^{\max} q_{j\omega}^{sr} = \nu_{j\omega}^{\max} Q_{j\omega}^{sr}$, and $\nu_{j\omega}^{\min} q_{j\omega}^{sr} = 0$, respectively. Hence, replacing these expressions in (7) we obtain

$$p^s_\omega \sum_{i=1}^{I} q^s_{i\omega} = \sum_{p=1}^{P} c^{sd}_{p\omega} q^{sd}_{p\omega} - \sum_{p=1}^{P} \gamma^{\max}_{p\omega} Q^{sd}_{p\omega} - \sum_{j=1}^{J} c^s_{j\omega} q^{sr}_{j\omega} - \sum_{j=1}^{J} \nu^{\max}_{j\omega} Q^{sr}_{j\omega} \qquad \forall \omega, \tag{8}$$

which is a linear formulation of the bilinear products $p^s_\omega \sum_{i=1}^{I} q^s_{i\omega}$.

To summarize, the resulting MILP formulation, equivalent to stochastic model (2) and to the associated single-level MPEC (3) is:

$$\max_{q^{ft}, q^f_{i\omega}, q^s_{i\omega}, \mathcal{W}} \quad \sum_{\omega}^{\Omega} \sigma_\omega \left(p^f q^{ft} + X_\omega - \sum_{i=1}^{I} c^s_{i\omega} (q^f_{i\omega} + q^s_{i\omega}) \right) \tag{9a}$$

s.t.

$$X_\omega = \sum_{p=1}^{P} c^{sd}_{p\omega} q^{sd}_{p\omega} - \sum_{p=1}^{P} \gamma^{\max}_{p\omega} Q^{sd}_{p\omega} - \sum_{j=1}^{J} c^s_{j\omega} q^{sr}_{j\omega} - \sum_{j=1}^{J} \nu^{\max}_{j\omega} Q^{sr}_{j\omega} \quad \forall \omega \tag{9b}$$

$$\text{(3b)-(3g) and (4)} \tag{9c}$$

where $\mathcal{W} = \mathcal{Y} \cup \left\{ u^{\max}_{i\omega}, u^{\min}_{i\omega}, v^{\max}_{j\omega}, v^{\min}_{j\omega}, z^{\max}_{p\omega}, z^{\min}_{p\omega} \right\}$

3.4. Risk modeling

The risk neutral formulation (9) can be extended to account for risk by including a risk measure. For this work, as (9) is a MILP problem, it admits a linear formulation of the Conditional Value at Risk (CVaR). In particular, we employ the linear formulation presented in Rockafellar and Uryasev (2002) which is also valid under a MILP framework (Schultz et al. (2006)).

$$\max_{q^{ft}, q^f_{i\omega}, q^s_{i\omega}, \mathcal{W}, \xi, \eta_\omega} \quad (1-\phi) \sum_{\omega} \sigma_\omega \Pi_\omega + \phi \text{CVaR} \tag{10a}$$

s.t.

$$\text{CVaR} = \quad \xi - \frac{1}{1-\alpha} \sum_{\omega=1}^{\Omega} \sigma_\omega \eta_\omega \tag{10b}$$

$$\eta_\omega \geq -\Pi_\omega + \xi \qquad \forall \omega \tag{10c}$$

$$\eta_\omega \geq 0 \qquad \forall \omega \tag{10d}$$

$$\Pi_\omega = p^f q^{ft} + X_\omega - \sum_{i=1}^{I} c^s_{i\omega} (q^f_{i\omega} + q^s_{i\omega}) \quad \forall \omega \tag{10e}$$

$$X_\omega = \sum_{p=1}^{P} c^{sd}_{p\omega} q^{sd}_{p\omega} - \sum_{p=1}^{P} \gamma^{\max}_{p\omega} Q^{sd}_{p\omega} - \sum_{j=1}^{J} c^s_{j\omega} q^{sr}_{j\omega} - \sum_{j=1}^{J} \nu^{\max}_{j\omega} Q^{sr}_{j\omega} \quad \forall \omega \tag{10f}$$

$$\text{(3b)-(3g) and (4)} \tag{10g}$$

The objective function (10a) represents a trade-off between the expected profit and the CVaR regulated by the pre-specify parameter $\phi \in [0, 1]$. Note that for $\phi = 0$, problem (10) is equivalent to the risk neutral problem (9), while for $\phi = 1$ the reference producers is fully risk averse. Given a profit distribution, the CVaR_α is defined as the expected value conditioned to those cases where the probability distribution is lower than $1 - \alpha$. η_ω and ξ are auxiliary variables. In particular, ξ equals the Value-at-Risk (VaR_α) at the optimal solution of (10). The used of the CVaR is extensively adopted within the literature, specially to model the decision making process of all the agents participating in the electricity market (García-González et al. 2007, Conejo et al. 2010, Ehrenman and Smeers 2011, Boroumand et al. 2015, Inzunza et al. 2016).

4. Numerical application

The energy system has been characterized with five electricity generation technologies: nuclear-, wind-, solar-, coal- and gas-based. We assume that 25% of the overall (power) capacity is owned by the strategic producer and distributed over $I = 8$ generating units that adopt one of these five technologies. Similarly, the remainder capacity (75%) is owned by the rival producers and is provided by $J = 8$ generating units that also adopt one of the five generation technologies.

The analysis of the model has been developed through a rather sophisticated simulation. It consisted of a generation of sets of 1000 simulated scenarios. Each scenario (ω) consisted of a realization of a random vector of fifteen random variables:

- RES (solar and wind) generation capacity (MWh): $\widetilde{Q}_i^s$ for $i = 2, 3, 4$ and $\widetilde{Q}_j^{sr}$ for $j = 2, 3, 4$.
- Marginal generating cost (€$/MWh$): $\widetilde{c}_i^s$ and $\widetilde{c}_j^{sr}$ for coal-based ($i = 5, 6$ and $j = 5, 6$) and gas-based technologies ($i = 7, 8$ and $j = 7, 8$).
- Total electricity demand, in terms of the size of the first demand block (MWh): $\widetilde{Q}_{p=1}^{sd}$.

The univariate distribution and the corresponding moments of the fifteen random variables included in the vector have been estimated from historical data of the Spanish Electricity Market (OMIE 2018). In particular, we have considered hourly data, including market prices, demand and total generation per technology, from 25/11/2015 to 25/11/2016, which is publicly available at ESIOS (2018). As indicated in Section 2, we use the Spanish market as a reference case for many other electricity markets worldwide, because of its characteristic oligopolistic structure and its high level of renewable energy penetration. Nevertheless, the aim of this case study is not to reproduce the functioning of any specific electricity market but rather provide general conclusions that can be extended to the entire electricity sector.

A summary of the data employed in the numerical simulations is presented in Table 1: the marginal costs (second column), capacity (third column) and cumulative capacity (fourth column) for each generating unit, and the price bid (sixth column), size (seventh column), and cumulative demand (eighth column) for each demand block in the spot market. Moreover, the generating units belonging to the strategic (index i) and rival producers (index j) are ordered by increasing marginal costs. Similarly, demand blocks are ordered from the highest to the lowest bid prices. Those costs, or capacity values, that are considered uncertain in the simulations are summarized by their mean (μ) and standard deviation (σ). By the intersection of the offer and demand curves, it can be expected that the market price will be determined by the marginal costs of coal or gas technologies. However, as both the renewable capacity and the demand level are random, this market price can vary dramatically within the scenarios.

Table 1: Generating units and demand curve data

Generating Units	Marg. Costs (€/MWh)	Capacity (MWh)	Exp. Cumul. Cap. (GWh)	Demand blocks	Price (€/MWh)	Size (MWh)	Exp. Cumul. Dem. (GWh)
$i = 1$ (nuclear)	0	1250	1.25	$p = 1$	180	$\mu = 18431.77$ $\sigma = 4416.17$	18.43
$j = 1$ (nuclear)	0	3750	5.00	$p = 2$	100	100	18.53
$i = 2$ (wind)	0	$\mu = 692.64$ $\sigma = 411.21$	5.69	$p = 3$	80	100	18.63
$i = 3$ (wind)	0	$\mu = 692.64$ $\sigma = 411.21$	6.38	$p = 4$	60	100	18.73
$j = 2$ (wind)	0	$\mu = 2077.92$ $\sigma = 1233.62$	8.46	$p = 5$	50	100	18.83
$j = 3$ (wind)	0	$\mu = 2077.92$ $\sigma = 1233.62$	10.54	$p = 6$	40	100	18.93
$i = 4$ (solar)	0	$\mu = 353.07$ $\sigma = 418.58$	10.89	$p = 7$	30	100	19.03
$j = 4$ (solar)	0	$\mu = 1059.22$ $\sigma = 1255.75$	11.95	$p = 8$	20	100	19.13
$j = 5$ (coal)	$\mu = 32.99$ $\sigma = 1.21$	1500	13.45				
$i = 5$ (coal)	$\mu = 36.64$ $\sigma = 1.16$	500	13.95				
$j = 6$ (coal)	$\mu = 39.78$ $\sigma = 1.24$	1500	15.45				
$i = 6$ (coal)	$\mu = 41.67$ $\sigma = 1.13$	500	15.95				
$j = 7$ (gas)	$\mu = 43.43$ $\sigma = 0.95$	9000	24.95				
$i = 7$ (gas)	$\mu = 45.44$ $\sigma = 0.89$	3000	27.95				
$j = 8$ (gas)	$\mu = 48.83$ $\sigma = 1.13$	9000	36.95				
$i = 8$ (gas)	$\mu = 56.68$ $\sigma = 1.88$	3000	39.95				

Each set of 1000 scenarios have been simulated adopting different Gaussian copulas (ρ) between the total RES generation and the first block of the aggregate demand curve, i.e., between $\sum_{i=2,3,4} \widetilde{Q}_i^s + \sum_{j=2,3,4} \widetilde{Q}_j^{sr}$ and $\widetilde{Q}_{p=1}^{sd}$). Furthermore, the total RES generation resulting from this simulation is assumed to be proportionally distributed between the strategic (25%) and the rival producers (75%). In particular, six rank correlation values have been fixed for such copulas (ρ): $-0.5, -0.3, -0.1, 0.1, 0.3$ and 0.5. The remainder random variables have been assumed independent. Hence, the couple (ρ, ω_ρ) identifies the unit element of the simulation.

Two risk aversion profiles have been compared: according to the specification of problem (10), the parameter $\phi = 0$ has been used for the risk neutral producer, and $\phi = 1$ for the risk averse. The threshold of the value at risk α has been fixed to 0.9.

The solution of the problem (10) has been calculated for each (ρ, ω_ρ) and for the two risk aversion profiles, for a total of $6 \times 1000 \times 2 = 12000$ solutions. Each solution is characterized first of all by the optimal values of the control variables: $q^{*ft}, q^{*f}_{i\omega}$ and $q^{*s}_{i\omega}$. Notice that q^{*ft}

is not scenario dependent as it represents the optimal amount of futures contracts (under a fixed correlation value) to maximize the objective, which is an expectation. So it is an overall solution, unconditioned from any information about what scenario will eventually realize. Vice versa the $q^{*f}_{i\omega}$ and $q^{*s}_{i\omega}$ represent the optimal production levels for each technology i committed to futures contracts and to spot market in each scenario. Several other variables result as a (mathematical) consequence of the optimal solution: for each ω, it is possible to determine the optimal capacity committed to the spot market, the profit resulting from each technology and the equilibrium price on the spot market.

Recall that p^f, the sale price of electricity committed with futures, is a parameter in the model and it has been set to 42 €$/MWh$.

The body of the 12'000 solutions has been partitioned in 12 distributions (i.e. corresponding to the combinations of 6 copula values × 2 risk profiles) of 1000 elements each, that is $\omega_\rho = 1, ..., 1000$. A first analysis was based on the visual inspection of these distributions. In particular the distribution have been represented through box plots, which allow to highlight: average, median, 25th and 75th percentiles. Additionally, to get some economic insights of the solution, the box plots have been superimposed with the 5th and 95th percentiles of four key variables shown as white and gray markers, respectively. In particular we use triangles (△) for profit, squares (□) for spot price, circles (◯) for demand and stars (★) for RES generation.

Since both the visual inspection and the data analysis have been applied separately for the 12 distributions, as well as all the variables characterizing the optimal solutions, the resulting output was quite large. So we organize the comments focusing on the major results.

The model has been implemented in JuMP ver. 0.16.2 (Dunning et al 2017) under the open source Julia Language ver. 0.5.1 (Bezanson et al. 2017) and solved by using the Gurobi solver ver. 7.5.1 (Gurobi 2018), on a computer with an i7-6700 CPU (8 cores at 3.40 GHz) and with 32 Gb of RAM memory running Windows 10. We should remark that the computational times to solve model (10) with $\Omega = 1000$ scenarios, for each correlation level ρ, never exceeded 10 minutes.

In the following we present several numerical analysis which, for the sake of clarity, have been divided into three main parts:

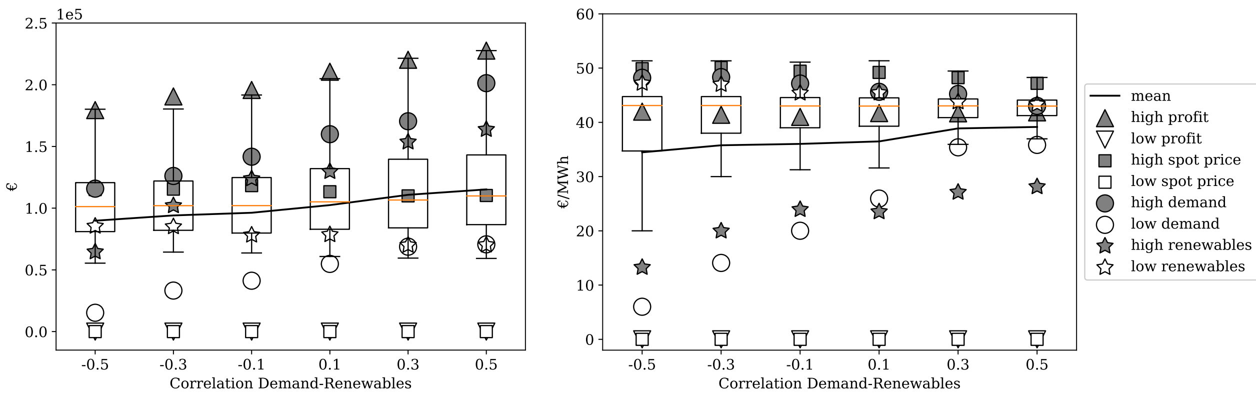
- As a base case, in Section 4.1 we analyze a market with no futures where competitive producers trade all their energy in a spot market. We study the impact of correlation between RES and demand in profits, market prices and dispatched generating units.

- The market with futures trading is introduced in Section 4.2. In particular, the total profits (Section 4.2.1), futures (Section 4.2.2) and spot (Section 4.2.3) market outcomes, together with their respective optimal generation plan, are analyzed in detail for different correlation and risk aversion levels.

- The third part of the study (Section 4.3) consists in the application of some methods of Data Analysis: cluster analysis and discriminant analysis in particular. The purpose of such approach was to discover if and under what conditions the distribution of the solutions tended to form groups following similar patterns.

4.1. Solution on a market with no futures

Let's initially observe some major aspects of the optimal solution in a market with no futures contracts. We first analyze the distribution of profits in Figure 3a. Obviously the highest profit scenarios (i.e. upper dark triangles) are placed at the top of the upper whiskers. The most relevant aspect to explain the distribution of profits is however the level of the demand. Higher demand percentiles (i.e. gray circles markers) are placed on the upper whiskers, but not all at the same depth. When correlation is negative high demand does not fully match best profit scenarios. This is not surprising. Under negative correlation, high demand scenarios couple with low renewable generation (white stars), that is the technology offering the highest profit margin. Demand must therefore be satisfied with more costly technologies, the conventional ones especially. On the contrary, when correlation turns to positive, highest demand scenarios match highest RES production (gray stars), which can therefore be easily absorbed to provide large profit levels.



(a) Profits

(b) Spot prices

Figure 3: Profit for the strategic producer and spot prices with no futures market

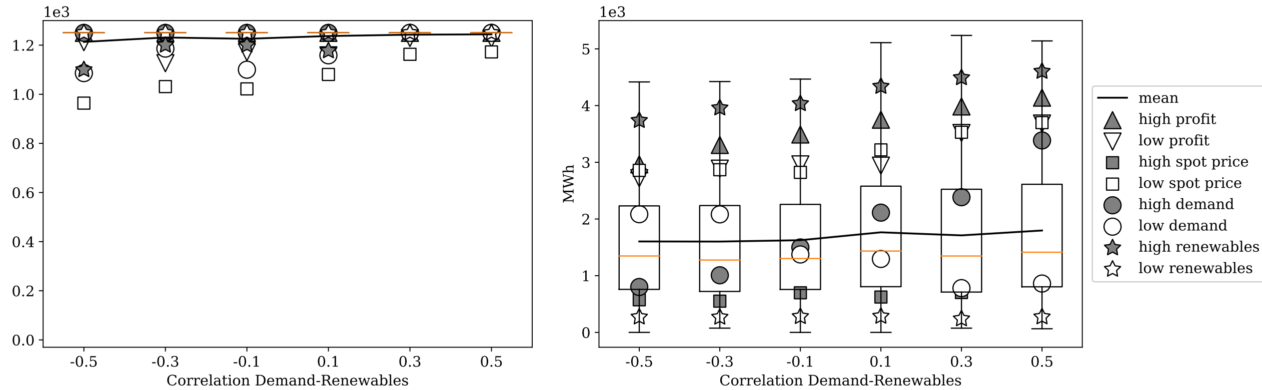
The optimal solution of the generation plan problem is general fixed with respect to nuclear generation, that is constantly all offered independently form the level of correlation see Figure 4a. Optimal offer of RES also has a very high priority, even though its distribution is relatively dispersed, as it can be concluded from the position of the 95th high RES generation markers in Figure 4b. Those gray stars tell us that when RES generation is large the producer prefers to sell it, even when demand is low (see the white circles under the negative correlation) and the price tends to low. The optimality of such strategy can be understood observing that the best profit opportunities (upper gray triangles) tend to match with the high RES generation scenarios.

Conventional generation tends finally to take a residual priority role. As it can be seen in Figure 5, conventional generation is mostly offered when RES generation is low and price is high. Vice versa it is significantly reduced when RES generation is large.

4.2. Solution on a market with futures

4.2.1. Profits.

Let's therefore start with the distribution of the profits accruing to the producer (see Figure 6). Comparing the two risk aversion types in the figure, we see that on average the



(a) Nuclear generation

(b) Renewable generation

Figure 4: Nuclear and renewable generation by the strategic producer in the spot market with no futures market

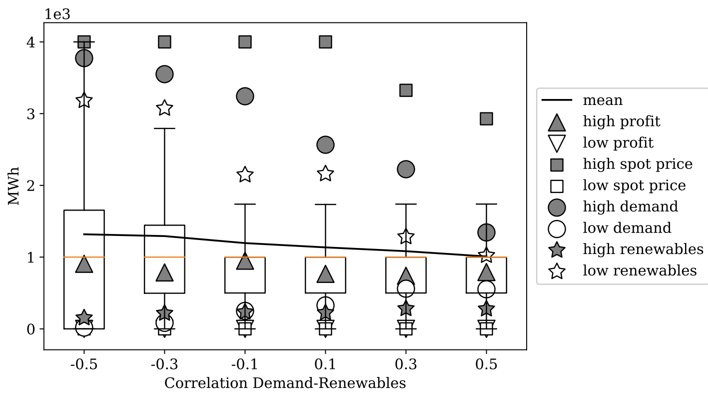


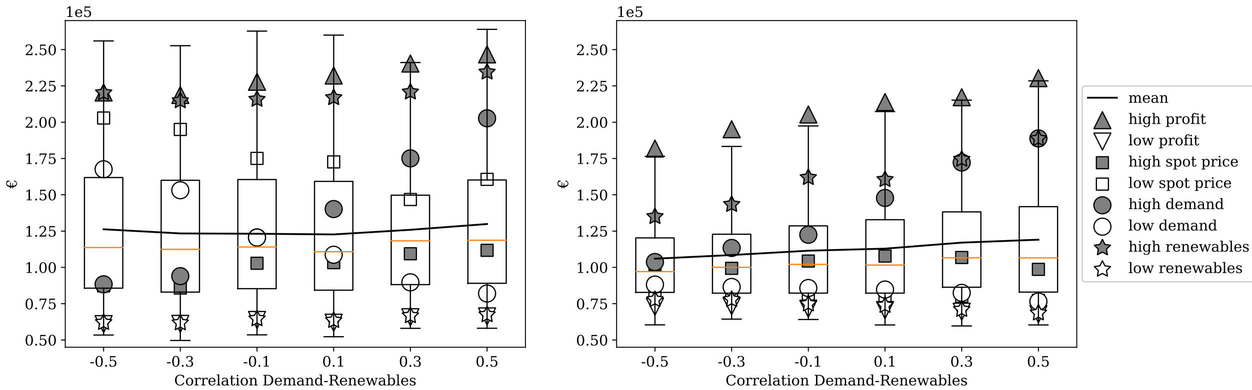
Figure 5: Conventional generation by the strategic producer in the spot market with no futures market

risk neutral producer earns more, but the risk of lower results is higher for him. Besides this, different levels of correlation ρ seem to matter only slightly for the risk averse producer. However, the 95th percentiles of demand (gray circle) and price (gray square) show that something relevant is occurring underneath. Indeed, their positions change significantly across the different values of ρ as well as across the two risk attitudes. Notice for example that, for negative values of ρ in the risk neutral case, the best profit solutions tend to occur along with low demand and low spot price percentiles; high demand percentile matches highest profits in the presence of positive correlation.

Moreover, large RES generation (gray star) appear regularly on the upper tails of profits: that is to say, profits tend to originate from renewables.

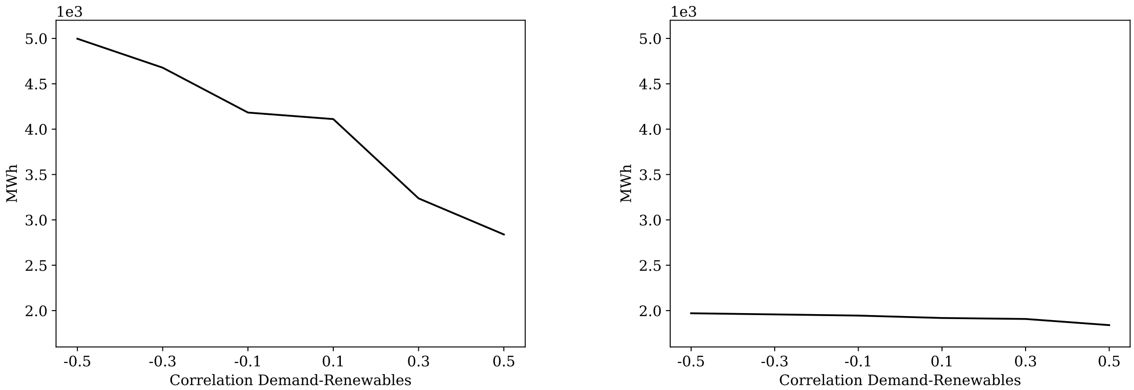
4.2.2. Futures market outcomes.

Turning to the optimal solution, let' start with the number of futures (i.e. q^{*ft}, the units are representative of 1 MWh) to commit with futures contracts (see Figure 7). Let's recall that is a single value optimal solution, calculated to optimize the expectation (in the objective function) over all the 1000 scenarios. Here we see a clear impact of different correlation levels. The output shows that the futures contracts should be reduced as the correlation ρ changes



(a) Risk-neutral (b) Risk-averse

Figure 6: Profits of strategic producer



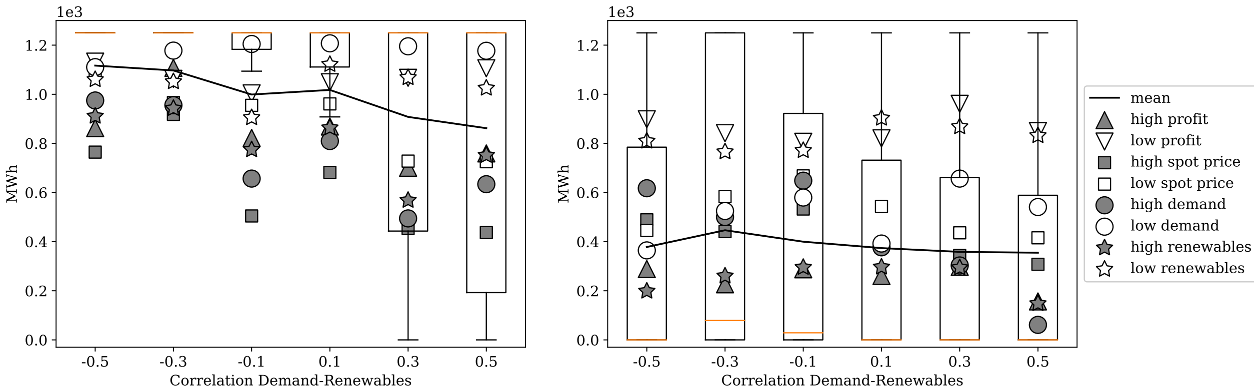
(a) Risk-neutral (b) Risk-averse

Figure 7: Futures trading

from negative to positive. Perhaps even more surprisingly, the risk averse producer commits a neatly lower volume of energy with futures. For the latter also observe in Figure 7 that the relative impact of different correlation levels is small: as ρ passes from -0.5 to 0.5, futures trading declines from 1960 to 1840 MWh, that is about a 6% reduction.

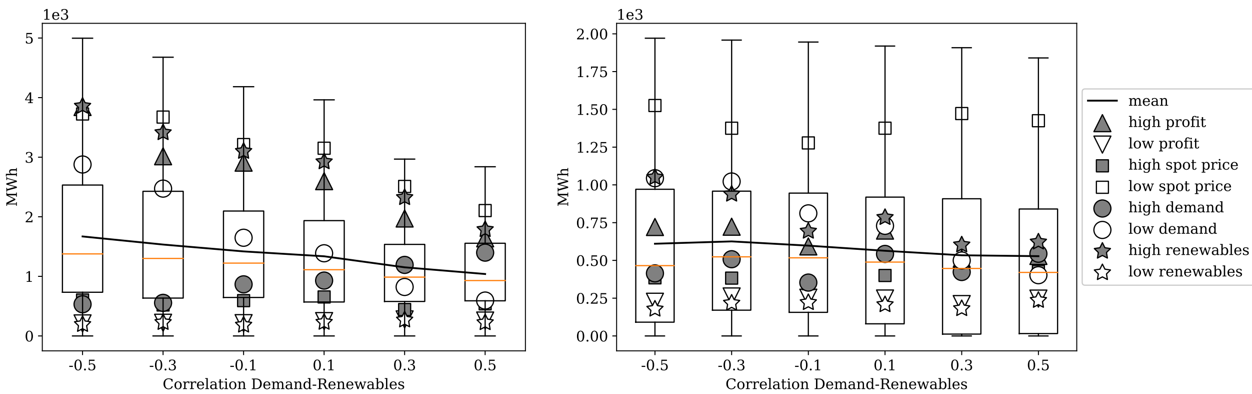
A natural interest is then about how the other components of the optimal solution change under the different scenario. In particular, let's see how much of the production available from the different technologies it is optimal to commit to futures contracts (i.e. $q^{*f}_{i,\omega}$) and how much to direct to the spot market (i.e. $q^{*s}_{i,\omega}$) to satisfy the spot demand. To synthesize the analysis, results have been grouped under three major classes: renewables (solar+wind), nuclear and conventional (coal+gas). Starting with the futures, the following figures (8, 9 and 10) give some direct answers. Notice preliminarily that the risk neutral producer follows a strategy deeply different from the risk averse. The former changes significantly the generation plan used to meet his futures commitment depending on the level of ρ, while the latter does not.

For the risk neutral, if the correlation ρ is negative or close to zero, nuclear has clearly a priority: it is almost invariably directed to futures (both the interquartile and the whiskers are squeezed to the value of 1250, that is the maximum nuclear capacity). Interestingly, the



(a) Risk-neutral (b) Risk-averse

Figure 8: Nuclear generation to futures

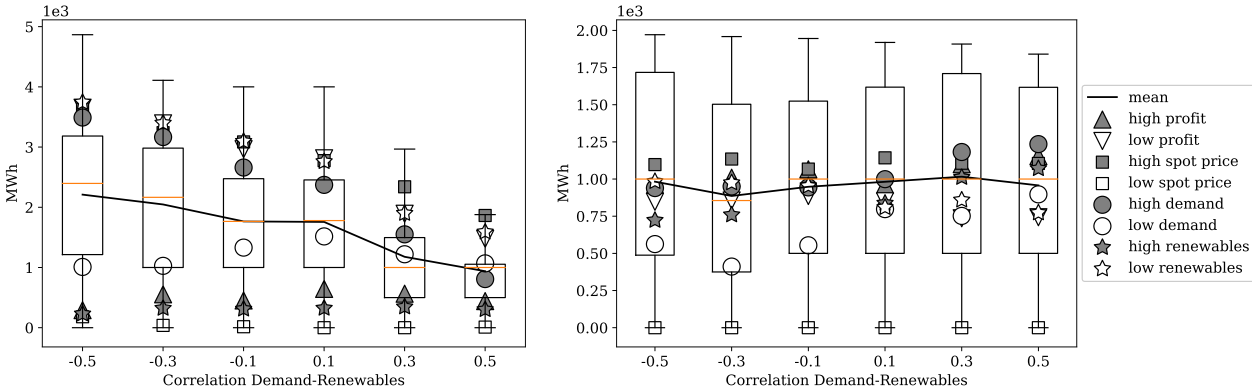


(a) Risk-neutral (b) Risk-averse

Figure 9: Renewable generation to futures

few exceptions to this rule occur when renewables generation is either very high or very low. The first case is obviously explained: when ρ is negative high renewable generation couples with low demand and in such condition renewable generation would (strongly risk to) be sold at zero price on the spot market. In the opposite scenarios (high demand and low RES) part of nuclear is subtracted from satisfying futures and sold at a high price on the spot market, eventually replacing it with conventional generation. Similar level of priority is given to RES: graphs show (see the position of the gray and the white star in Figure 9) that satisfaction of the futures commitment proceeds according with the level of RES generation. In accordance with this findings, when the correlation is negative ($\rho = -0.5, -0.3$), it is worth also noticing in figures 11 and 12 that the risk neutral producer directs little or zero nuclear and RES generation to the spot market. As it will be discussed later, nuclear and RES generation undergo a significantly different strategy when ρ turns to positive. The least priority is for conventional technologies: they cover the commitment of futures when RES generation is low.

The risk averse producer follows a more stable strategy, not really changing as ρ passes from negative to positive values. This strategy, as we will observe more precisely later on, points to selling most of RES generation on the spot market at the best price. Indeed, as we can start observing in Figure 9 this strategy tends to commit RES generation to futures not when it is high, but when the spot price is very low (see the position of the white squares). In this case conventional contribution (to future sales) drops regularly to zero (see the white square in Figure 10). Nuclear generation is directed to futures as second priority choice, when RES generation is low (see the white star markers in Figure 8). Overall, conventional generation seems to play a passive role, even if on average it covers about 60% of futures sales.



(a) Risk-neutral

(b) Risk-averse

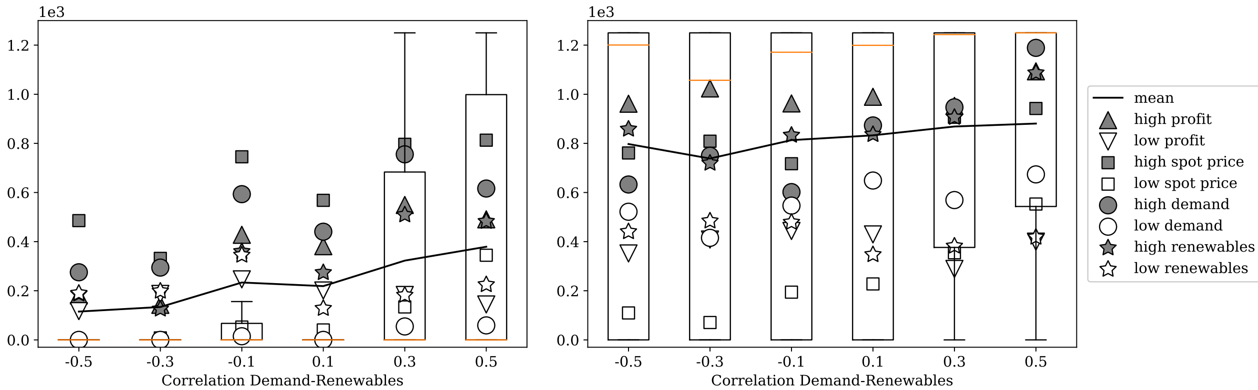
Figure 10: Conventional generation to futures

4.2.3. Spot market outcomes.

A second natural question at this point is about how the generation of different technologies distributes on the spot market. Starting with nuclear plants, the risk neutral producer has a clear strategy. In the case of negative correlation (and correspondingly to what it happens to the futures trading) he sells zero or little quantity of electricity on the spot market (see Figure 11). He changes this basic rule only in the scenarios where the spot price is very high (see the gray squares). If the correlation ρ turns to positive, the sales of nuclear to the spot market increase. For the risk averse producer, on the contrary, the nuclear energy to be directed to the spot market follows a more variable strategy largely depending, as we will see shortly, from the level of the RES generation and decreasing to zero when spot price is low.

The strategy of the risk neutral producer is even more clear observing the solution for the RES generation (Figure 12). Negative values of ρ vanish his profit opportunities: when RES generation is low, there is simply no RES generation to direct to the spot market. Vice versa when RES provide a large quantity of electricity, then demand tends to depress along with spot prices. So, in both cases spot market is not a good opportunity for RES. Committing RES to futures is therefore a (much) preferable choice. However, as ρ increases, the chances to couple high (low) demand and large (low) RES generation increase. In both cases the spot price will in general keep high, and so directing RES to the spot market is a good profit solution (see the gray triangles and gray stars in Figure 12).

The main difference with the risk averse producer gets clear observing further the same figure. The latter is able to keep a similar strategy over all the correlation levels, consisting of directing to spot market relevant amounts of renewable generation (see the gray star marker

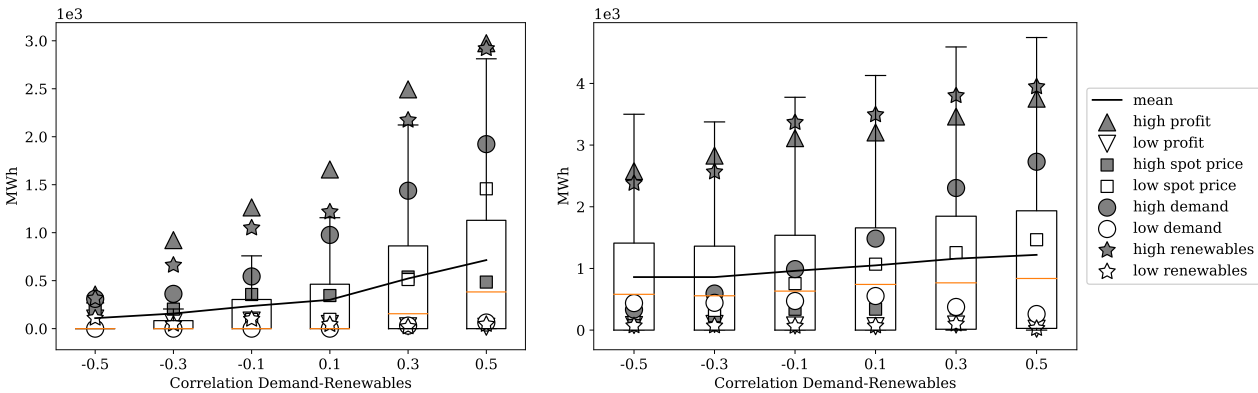


(a) Risk-neutral

(b) Risk-averse

Figure 11: Nuclear generation to spot market

placed in the upper whiskers). Notice that such strategy matches closely with the highest profit percentile (i.e. gray triangle).



(a) Risk-neutral

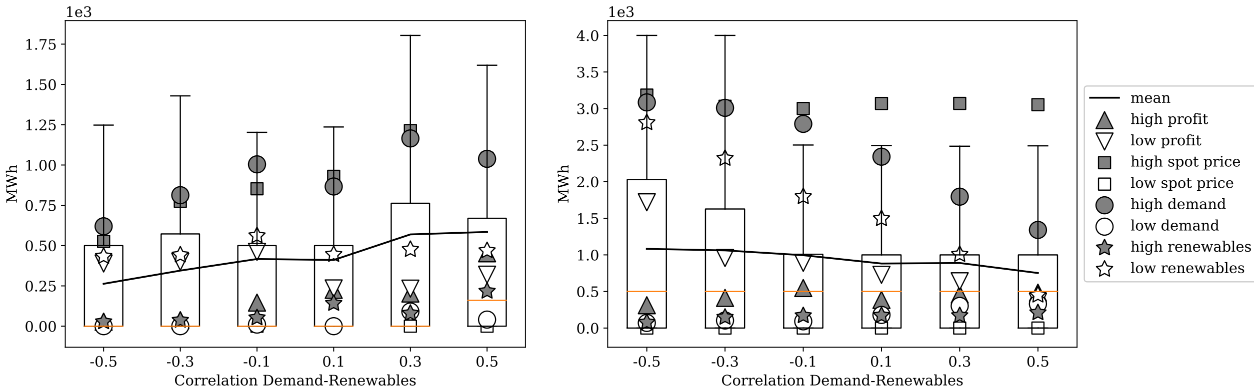
(b) Risk-averse

Figure 12: Renewable generation to spot market

Also for the sales on the spot market, it seems that the conventional plants play a residual role. For both the risk neutral and the risk averse producers, conventional generation is sold on the spot market when both demand and electricity price are sufficiently high (see the gray circles and gray squares in Figure 13). This is of course perfectly in line with standard economic theory.

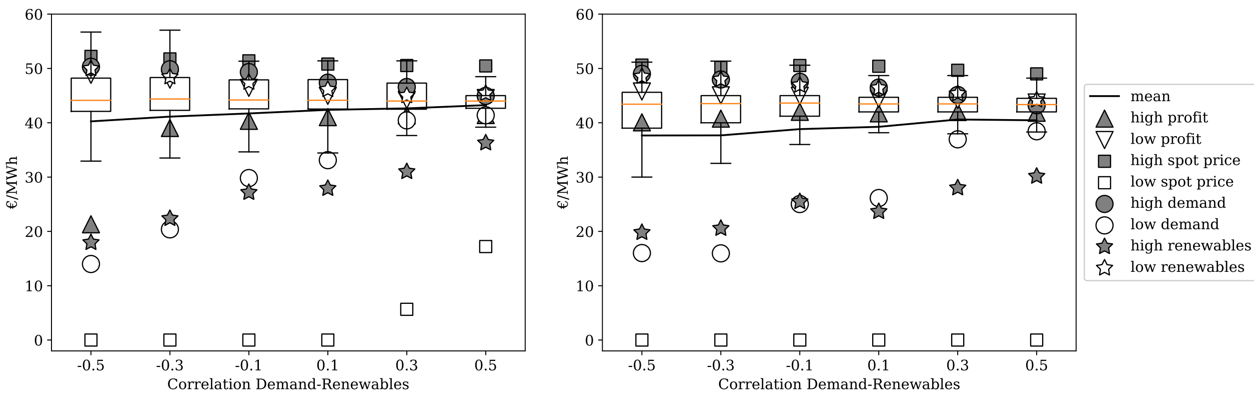
An important outcome of the analysis is also connected with the equilibrium price of electricity (see Figure 14).

Spot price tends to be more volatile in the presence of negative ρ, as a consequence of the extreme mismatch between RES generation and demand discussed previously. A first striking evidence is that the best profit percentiles (i.e. 95th) are regularly placed, for both types of producers, at or below the 25th percentiles of the spot price distributions. In other words, higher profits tend to come with low spot prices. The opposite occurs for lowest profit percentiles. This is explained by the (relevant) share of RES: as already observed, large RES generation offers the best profit opportunity but places downward pressure to the spot prices.



(a) Risk-neutral (b) Risk-averse

Figure 13: Conventional generation to spot market



(a) Risk-neutral (b) Risk-averse

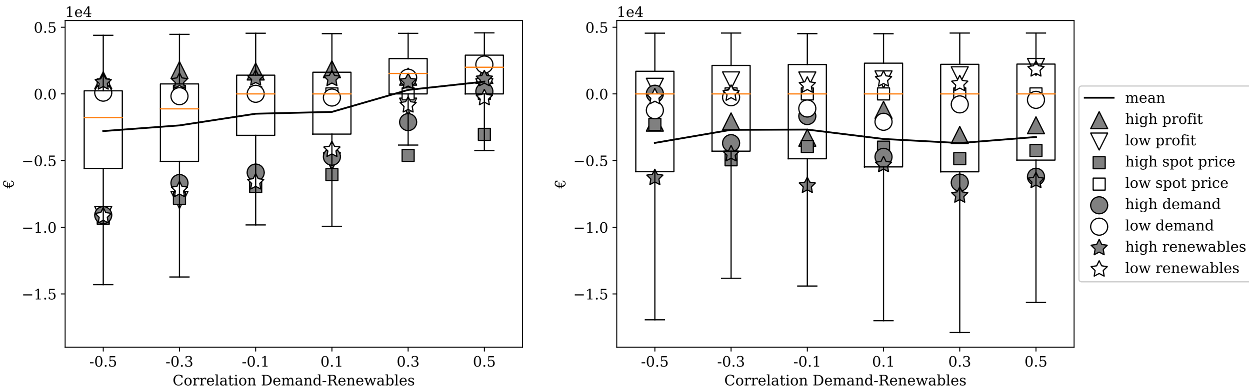
Figure 14: Spot price

It is interesting to observe at this point the difference with the no futures market. When RES generation is very high, futures absorb good part of it, guaranteeing some profit (actually the best profit opportunities for the risk averse producer). Vice versa, in the absence of futures large RES generation depresses dramatically spot prices and profit. This can be seen comparing Figure 3b and Figure 14. In the former figure, zero price matches with zero profit, as it is expected; in the latter, low profit scenarios matches surprisingly with above than average spot prices.

The median price remains constantly well above the level of the futures price, that is 42 €$/MWh$.

A final comment to the profit resulting from conventional generation committed to the futures (fig 15). Such evidence is perhaps the most surprising result and, at the same time, a highly revealing one.

We remember from the previous discussion that committing conventional generation to futures contracts appeared as a residual and partly unexplained strategy. Now we learn from Figure 15 that such strategy even generates consistent losses on average, especially to the risk averse producer. Such losses seem implausible, since there is not apparent necessity to incur them. The explanation of such puzzle arrives observing the placement of 95th



(a) Risk-neutral

(b) Risk-averse

Figure 15: Profit of conventional generation to futures

percentile scenarios of spot price and demand (gray squares and gray circles). Indeed such marks are placed at or just below the 25th percentile. This is showing that the generation of conventional plants is strategically sold through futures contracts, to sustain the spot price in the scenarios of high demand. The resulting benefit is collected, in particular, selling the RES and nuclear generation on the spot market, as we have seen previously.

4.3. Data analysis

As anticipated, the second part of this model is based on Data Analysis and it consists in applying a cluster analysis (Ward aggregation method), to each group of 1000 scenarios resulted for every level of correlation (ρ) and for two types of risk aversion. The motivating idea was to check if the solutions tended to group following some few patterns, even though the 1000 random scenarios tended to distribute statistically according to their theoretical densities. To magnify the differences resulting from the clusters, a canonical discriminant analysis was also applied, using the cluster membership to discriminate the solutions.

For sake of synthesis, we only report a few output. As an example, the following figure (Figure 16) plots the position of the 1000 solutions over the 1^{st} and the 3^{rd} discriminant axes for the correlation $\rho = -0.3$ and the risk averse producer. As well known, the interpretation of these plots requires some familiarity with the two statistical techniques. However, the main results can be observed matching the position of the five clusters (each identified with a different color/level of gray) with the position of the variables names, that happened to show relevant discriminating power. More precisely, the position of the labels on each quadrant of Figure 16 is proportional to the correlation of the same variable with the canonical asex.

It is rather evident that the strategies tend to form quite well separated groups. One in particular is well isolated on the left of the graph. This group is discriminated by a joint combination of high volume of RES committed to futures and high levels of unoffered nuclear and RES generation. Since labels “Spot demand” and “Spot price” appear on the opposite side the graph, this group can also discriminated by low level of demand and spot price. This group corresponds to the column clus1 in the following Table 3; the other four clusters are associated to the other columns. What this analysis really helps us to catch, is the strategy to adapt the solution to the changing scenarios. The two following tables (Table 2 and Table 3) contain all the necessary elements to characterize the five groups of solutions. The two tables

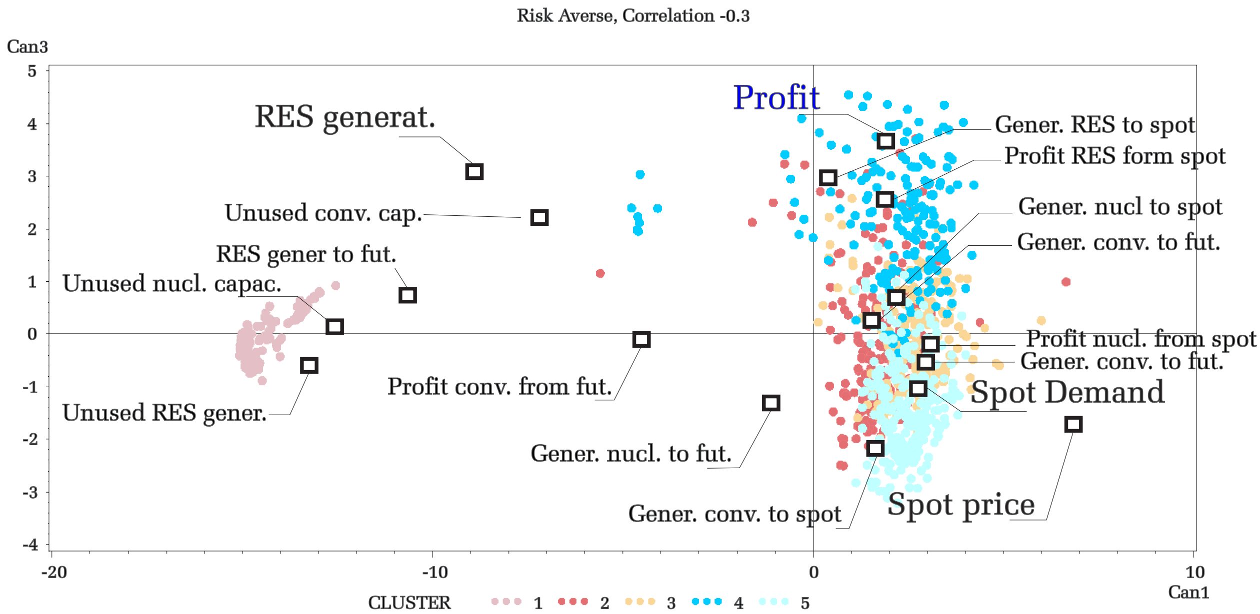


Figure 16: Discriminant analysis: Risk averse case and $\rho = -0.3$

compare the risk averse and the risk neutral producers, under the same level of correlation $\rho = -0.3$.

In both tables, column clust1 happens to describe the group of solutions under scenarios with (very) low demand and (very) high RES generation. In this case we see that the spot prices drops to zero, so that the only origin of profit for both types of producers is selling energy through futures contracts. The difference between the two producers in terms of resulting expected profit is however huge: extremely positive for the risk neutral (189'799 €), very poor for the risk averse (82'990 €). This can be explained mainly by the different levels of the futures commitment between the two players: large for the risk neutral and much lower for the latter.

Table 2: Risk neutral, 5 cluster of strategies, average values.

		clust1	clust2	clust3	clust4	clust5
Summary	Expected Profit (€)	189'799	176'550	102'825	92'424	96'185
	Spot price (€)	0.0	41.8	43.4	48.5	48.8
	Spot Demand (MWh)	13'178	18'541	15'036	22'567	21'510
	Total renewable (MWh)	13'927	11'601	4'864	3'568	3'857
Sales-mix (MWh)	Futures_trading	4'541	4'541	4'541	4'541	4'541
	Total spot	71	501	68	1'553	1'551
Profit from futures and for technology (€)	Futures nuclear	40'937	50'332	52'485	51'985	4'596
	Futures renewable	148'709	107'532	49'151	28'401	30'906
	Futures conv.	153	1'829	-1'230	-4'972	-7'605
	Futures conv. cheap	153	2'139	2'787	1'798	2'236
	Futures conv. expen.	0	-310	-4'017	-6'770	-9'841
Profit from spot and for technology (€)	Spot nuclear	0	2'159	18	591	55'670
	Spot renewable	0	14'174	2'252	10'458	11'035
	Spot conv	0	525	150	5'961	1'583
	Spot conv. cheap	0	525	126	3'145	1'575
	Spot conv. expen.	0	0	24	2'816	9
Generation to futures and for technology (MWh)	Futures nuclear	975	1'198	1'250	1'238	109
	Futures renewable	3'541	2'560	1'170	676	736
	Futures conv.	26	783	2'121	2'627	3'696
	Futures conv. cheap	26	692	984	675	831
	Futures conv. expen.	0	90	1'137	1'952	2'865
Generation to spot and for technology (MWh)	Spot nuclear	10	52	0	12	1'141
	Spot renewable	61	340	46	216	228
	Spot conv.	0	110	22	1'325	182
	Spot conv. cheap	0	110	15	325	166
	Spot conv. expen.	0	0	6	1'000	16
Unused or unsold capacity for technology (MWh)	Nuclear	266	0	0	0	0
	Renewable	535	0	0	0	0
	Conv.	6'974	6'108	4'857	3'048	3'123
	Conv. cheap	974	198	1	0	4
	Conv. expen.	6'000	5'910	4'856	3'048	3'119

We therefore discover here another piece of the risk averse strategy (still look at Table 3): in most extreme scenarios like in clust1 (too few evidently alter significantly the position of the 95th percentile of the highest RES generation in Figure 12) the risk averse player does not commit RES to spot, but towards futures contracts. The excess RES generation is just not offered. However, as soon as the RES lowers and/or demand increases (as in the solutions in groups clus2, clus4, clus5), the spot price reverts to standard levels and RES (or alternatively nuclear) is promptly directed to the spot market to sustain consistently expected profits.

Things go quite differently for the risk neutral. His larger commitment to the futures turns out as a heavy opportunity cost exactly when the spot market price is high (see in Table 2 the significant generation of RES and nuclear to futures in the scenarios clust3 and clust4; or the generation of the expensive conventional plants to futures, resulting in a heavy loss).

It is exactly at this point we therefore see why a (relatively) reduced commitment to the futures contracts benefit a risk averse player. Moreover, these considerations let us also see that a higher futures price, would, ceteris paribus, increase both the profitability and the risk of committing the sales of electricity to the futures. So it can be expected that the difference in the strategy of the two types of producer should increase.

Table 3: Risk averse, 5 cluster of strategies, average values

		clust1	clust2	clust3	clust4	clust5
Summary	Expected Profit (€)	82'990	102'967	104'033	151'798	100'154
	Spot price (€)	0.0	44.5	42.7	38.7	45.9
	Spot Demand (MWh)	13'831	18'832	15'950	18'961	21'473
	Total renewable (MWh)	12'388	4'134	4'733	11'089	4'143
Sales-mix (MWh)	Futures trading	1'976	1'976	1'976	1'976	1'976
	Total spot	1'138	2'423	1'700	3'690	3'860
Profit from futures and for technology (€)	Futures nuclear	19'228	51'725	5'445	2'176	4'801
	Futures renewable	63'726	14'446	31'527	27'709	14'201
	Futures conv.	-1	891	1'018	-6'889	-8'534
	Futures conv. cheap	-1	1'175	2'618	711	1'094
	Futures conv. expen.	0	-284	-1'600	-7'600	-9'628
Profit from spot and for technology (€)	Spot nuclear	14	707	47'703	46'284	51'954
	Spot renewable	22	29'264	17'865	80'847	30'659
	Spot conv.	0	5'934	474	1'671	7'073
	Spot conv. cheap	0	3'339	443	1'671	3'553
	Spot conv. expen.	0	2'595	30	0	3'520
Generation to futures and for technology (MWh)	Futures nuclear	458	1'232	130	52	114
	Futures renewable	1'517	344	751	660	338
	Futures conv	1	400	1'096	1'264	1'524
	Futures conv. cheap	1	374	799	304	415
	Futures conv. expen.	0	27	297	960	1'108
Generation to spot and for technology (MWh)	Spot nuclear	328	18	1'120	1'198	1'136
	Spot renewable	809	690	433	2'113	698
	Spot conv.	0	1'715	147	379	2'027
	Spot conv. cheap	0	544	135	376	548
	Spot conv. expen.	0	1'171	12	4	1'479
Unused or unsold capacity for technology (MWh)	Nuclear	464	0	0	0	0
	Renewable	1'601	0	0	0	0
	Conv.	6'999	4'884	5'758	5'356	3'450
	Conv. cheap	999	82	67	320	j,e
	Conv. expen.	6'000	4'802	5'691	5'036	3'413

5. Conclusions

The problem analyzed here has provided several important insights about management of power generation.

The first is that the various levels of correlation between RES generation and the demand of electricity occurring in the markets (and/or in different periods of the year) affects significantly the optimal production plans as well as the optimal sales-mix decision. It also affects the profit opportunities, as well as the volatility of spot prices.

The second is concerned with risk aversion. Not surprisingly, including risk aversion in the objective function of a producer has a significant impact on the optimal strategy. However, the key point in this discussion is that the channel of futures/bilateral contracts is a relevant opportunity for a risk neutral producer more than for a risk averse one. This is somewhat more surprising, since such contracts are supposed to be mainly hedging instruments. Indeed, the optimal strategy of risk averse producers gives the spot market a clear preference in the sales-mix decision, independently from the level of correlation (between RES generation and demand of electricity).

Other results are even more surprising. In particular, we have seen that conventional plants enter in the optimal solution with a particular strategic role. Their generation is sold through bilateral contracts, even if the price fixed in those contracts is lower than the marginal cost. If the producer can affect (at least partially) the equilibrium price on the spot market, it turns out that such losses are functional to sustain the profit of selling lower cost generation (such as those of RES or nuclear plants).

Key to these findings has been the approach to combine two types of decisions, that are usually treated separately in the literature of power generation management, namely the sales-mix and the production planning (generation plan). Moreover, another key assumption in this work has been to let futures price fixed. The relaxation of such hypothesis, as already commented, can be the object of future extension. However, the relevance and the impact of different levels of correlations, as well as that of the risk aversion, outlined in this work can be expected to remain valid. As well known, spot and futures prices tend to move in the same direction. So, a variable futures price will affect attractiveness of bilateral contracts and spot market at the same time, without posting theoretical reasons as to why the impact of the two factors discussed in this paper (namely correlation RES-demand and risk aversion) should significantly change.

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