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DISCUSSION

Discussion of "Probabilistic seismic slope stability analysis and design"[1]

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Introduction

The discussers welcome the paper by Burgess et al. (2019) and appreciate their efforts to develop, presumably, the first probabilistic seismic slope stability design aids using the "random finite element method" (RFEM). Although they conducted a parametric study, their charts provide geotechnical engineers with a preliminary estimate of failure probability of simple slopes subjected to seismic excitations. However, some considerations need to be pointed out that may affect the results achieved in the paper under discussion.

Random limit equilibrium method (RLEM) results

The "random limit equilibrium method" (RLEM) was first introduced by Javankhoshdel et al. (2017), which was a combination of circular "limit equilibrium method" (LEM) and random field theory developed by Fenton and Vanmarcke (1990). In the RLEM simulations, a random field is first generated using the local average subdivision (LAS) method developed by Fenton and Vanmarcke (1990) and then mapped onto a grid of elements (mesh). Each mesh element in the random field has different values of soil properties through realizations, and cells close to one another have values that are closer in magnitude, based on the value of the spatial correlation length. In each realization, a search is carried out to find the mesh elements intersected by the slip surface. The random soil property values are assigned to the slices whose base mid-point falls within that element. A limit equilibrium approach is then used to calculate the factor of safety (FS) for each realization. The probability of failure is calculated as the ratio of the number of simulations resulting in FS <1 to the total number of simulations.

Javankhoshdel et al. (2017) showed that even circular RLEM has the ability of finding the same answer in the case of cohesive soil slopes with simple geometries and with isotropic spatial variability of soil properties.

The noncircular RLEM, introduced by Cami et al. (2018), is a combination of the Cuckoo search method, the Surface Altering Optimization technique, and the LEM (Morgenstern-Price method) together with the random field theory. The Cuckoo search is a very fast and efficient global optimization method (metaheuristic approach), which is used for locating critical noncircular slip surfaces. In noncircular RLEM, the Cuckoo search is used together with a local optimization called the Surface Altering optimization technique. When used in conjunction with a noncircular search, this optimization method can be very effective and efficient at locating (searching out) slip surfaces with lower safety factors. Cami et al. (2018) showed that noncircular RLEM was able to find the weakest failure path using local and global optimization techniques similar to the failure path using the RFEM approach. The run time for the noncircular RLEM was shown to be at least 40% less than running the same example using the RFEM method with the same output.

In this discussion, the first step is to duplicate results of the RFEM using noncircular RLEM in the Slide2 2018 software (Rocscience Inc. 2018) for one case scenario (several case scenarios were duplicated, but only one of them is shown in this paper). Figure D1 FD1 shows results of the comparison between the noncircular RLEM and RFEM for the case with coefficient of variation $\nu = 0.3$, stability factor $\lambda = 0.40$, correlation length / slope height $\theta/H = 0.2$, friction angle $\phi = 20°$, and seismic coefficient $k = 0.2$, presented by Burgess et al. (2019). It can be seen in this figure that, for the range of slope angle used in this figure, noncircular RLEM gives a higher probability of failure. Thus, RFEM results cannot be an upper bound as mentioned by Burgess et al. (2019) compared to RLEM. The difference between the probabilities of failure can be about 10%–15%.

In the following sections, additional factors that can affect the results of Burgess et al. (2019) are discussed.

Influence of mesh size on results

Accurate modeling of a spatially variable domain in a digital circumstance entails minimizing the discretization error (i.e., the difference between solutions in a continuous and discretized medium), which would be achieved by an appropriate mesh size associated with the variability of the soil. Ching and Phoon (2013*a*, 2013*b*), Huang and Griffiths (2015), Ching and Hu (2016), Cami et al. (2018), and Tabarroki and Ching (2019) explored the effect of the mesh density used in finite element models with spatially variable soil properties. Ching and Phoon (2013*b*) introduced a critical ratio (i.e., scale of fluctuation (δ) / domain size), beyond which the discretization error would be minimum, and reported the effect of the auto-correlation model (i.e., single or squared exponential model), discretization method (i.e., element-level averaging and midpoint strategy), spatial variability pattern (i.e., isotropic and anisotropic), and stress states (i.e., pure shear or compression) on this ratio. Ching and Phoon (2013*a*, 2013*b*) showed that for a specific scale of fluctuation, as the element size decreases, the disparity among continuous and discretized solutions will diminish.

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Fig. D1. Comparison between RLEM and RFEM slope stability analyses results.

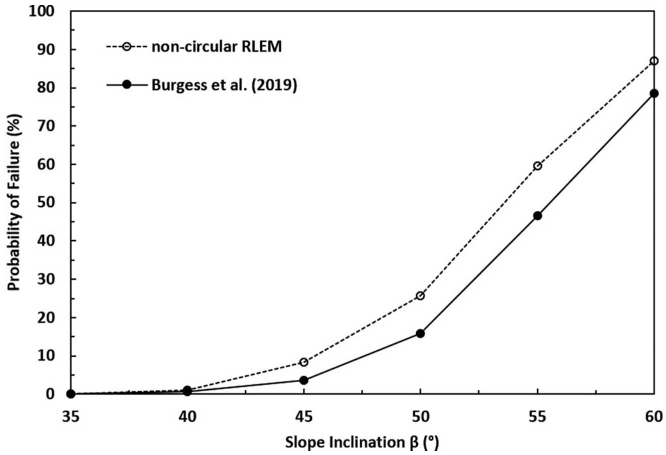
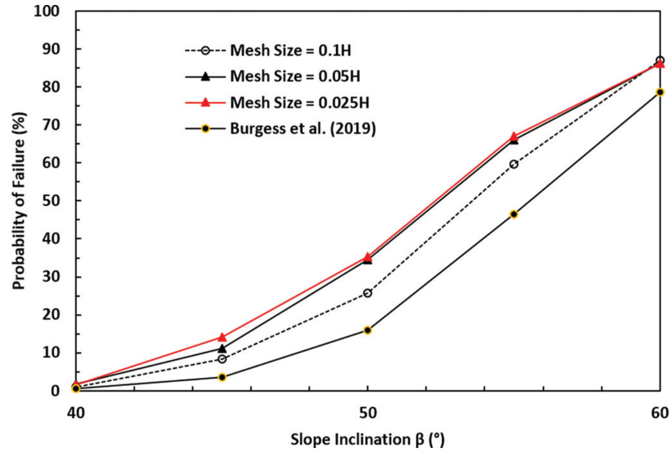


Fig. D2. Mesh size influence on accuracy of probability of failure estimation. [Colour online.]



Considering a biaxial compression test, Huang and Griffiths (2015) found that a larger element size is associated with a greater failure load for a typical simulation (i.e., a particular θ), which is unconservative, and introduced the most critical θ (associated with the largest failure load or the minimum mean overall undrained shear strength) that would be achieved when θ is equal to the typical dimension of the system, L (i.e., $\theta = L$). In fact, they believed that the mesh size could be cautiously increased to this point to have a time-efficient and conservative analysis. They also pointed out that when $\theta > 2L$, the element size becomes ineffective. Finally, the element size was recommended to be less than half of the correlation length in random finite element analyses. Numerical challenges arising from small correlation lengths have been discussed in Javankhoshdel et al. (2017).

In contrary to other studies that were associated with shear strength random fields, Ching and Hu (2016) estimated a critical element size beyond which the discretization error may exceed a predetermined value for a Young's modulus random field. For an isotropic correlation condition, they came to the conclusion that by adopting element-level averaging, even coarse elements yield the least discretization error for various scales of fluctuation while the allowable mesh size for anisotropic correlation case was slightly smaller. By contrast, the midpoint strategy proved to produce greater discretization errors.

Through a series of sensitivity analyses, Cami et al. (2018) investigated the effect of correlation length and mesh size on the reliability of the slope. By adopting RLEM, a worst-case mesh size was detected between 0.1 and 0.5 m (related to the maximum probability of failure for that cases of $\theta/H = 0.5$, 0.2, and 0.1) which was not tried through the RFEM software (MRSLOPE2D) of Fenton and Griffiths (2008), presumably due to the convergence problems.

Tabarroki and Ching (2019) observed that as the mesh size increases, the extremum of the sample mean value of compressive strength or active lateral force graphs (related to different normalized mesh sizes) tends to their nominal values in which undrained shear strength S_u was treated as a single random variable (homogeneous S_u value) and the worst-case "scale of fluctuation" ~~SOF; is~~ essential in design, especially in the case of limited soil data for determining the site-specific SOF) changes. In a sense, the denser the mesh size of finite element and random field becomes, the more accurate worst-case SOF that would be achieved. They also asserted that such changes arising from the mesh size variation depend on the type of discretization method (e.g., midpoint or spatial averaging) and the adopted auto-correlation model (e.g., single exponential model or squared exponential model).

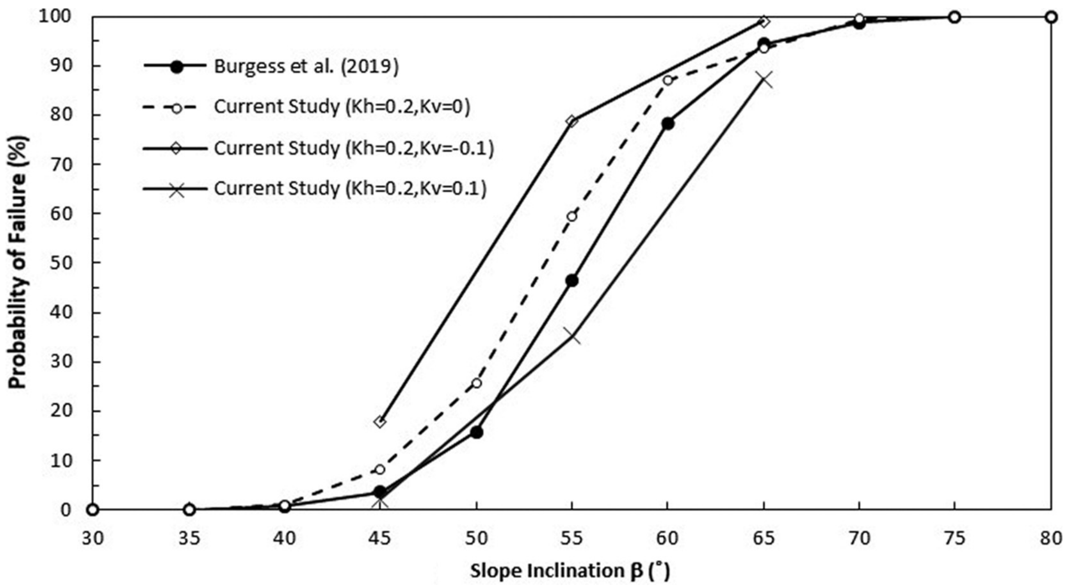
In the paper under discussion, Burgess et al. (2019) have limited their results to a specific mesh size (i.e., 0.5 × 0.5) for a wide range of correlation lengths (i.e., $\theta/H = 0.1$, 0.2, 0.3, 0.5, 0.7, 2, 3, 5, 7, 10). As is evident in recently published papers (e.g., Huang and Griffiths 2015; Cami et al. 2018), mesh size plays a primary role in the determination of the worst-case correlation length as well as in the thorough consideration of spatial variability of the soil. Regarding the relationship introduced by Huang and Griffiths (2015) between the correlation length and the element size, for the smallest correlation length (i.e., $\theta = 0.5$), the corresponding element sizes (i.e., $\theta/2 = 0.25$, 0.15) will result in drastic changes in the results, which have been neglected in the paper under discussion. The discussers have tried to evaluate the influence of the mesh size on the accuracy of the probability of failure estimation as illustrated in Fig. D2. In this figure, the cases in Fig. D1 (noncircu- FD2
lar RLEM and RFEM) that have a mesh size of $0.1H$ are compared with two cases with mesh sizes of $0.05H$ and $0.025H$ using noncircular RLEM analysis. It is evident from the finding of the noncircular RLEM analyses that finer mesh sizes are expectedly yielding more conservative failure assessments. The case with the mesh size of $0.05H$ has higher probability of failure for the same slope angle compared to the case with the mesh size of $0.1H$. However, probability of failure does not change much when mesh size changes from $0.05H$ to $0.025H$, which confirms that the assumption of $0.05H$ for this example is more accurate than $0.1H$. In Fig. D2, the difference between the probability of failure for the case with a mesh size of $0.05H$ using the noncircular RLEM method and the case with a mesh size of $0.1H$ using RFEM for a slope angle of 50° is about 20%!

Vertical seismic coefficient

The second aspect of the paper that will be examined further is the effect of the vertical seismic component which was not included in the analyses of the paper under discussion. According to Khazai and Sitar (2000), the inertial effects of the earthquake are represented by horizontal and vertical forces that are the so-called pseudo-static forces. They are assumed to be proportional to the weight of the slope multiplied by the seismic coefficients in the horizontal and vertical directions.

Innumerable studies have been found that consider the seismic slope stability analysis accounting for the lateral ground excitation; however, research including the vertical component of the acceleration is rather limited. It is worth noting that in general, the vertical acceleration component is much smaller in magnitude and more intense in high frequencies than the horizontal component (Sarma and Scorer 2009). Burgess et al. (2019) may have followed some past papers that ignore the vertical seismic component under the assumption that its effect on slope stability is small. However, although the design charts provided by the

Fig. D3. Influence of vertical earthquake acceleration component. k_h, horizontal seismic component; k_v, vertical seismic component.



authors are quite useful, the discussers maintain that incorporating both the vertical and horizontal earthquake components into the design would expectedly yield more realistic charts. Hence, it is necessary to take simultaneous account of the cross-seismic acceleration components along with the stochastic analyses.

Mixed effects have been reported on the influence of the vertical earthquake acceleration component. Gazetas et al. (2009) investigated a rigid block numerical model to comprehend the simultaneous influences of the vertical and horizontal seismic accelerations. They demonstrated that even the most robust vertical seismic components did not affect the sliding block slippage. Simultaneously, Sarma and Scorer (2009) asserted that joint consideration of the horizontal and vertical earthquake components in cohesive soils may not have a serious effect on the stability of slopes with any general slip surface in contrast to its influence on cohesionless, dry soils. On the contrary, some scholars hold the opposite view including Sun et al. (2012) who used a real case, the Donghekou landslide induced by the Wenchuan earthquake. They arrived at the conclusion that the vertical seismic force plays a key role in the stability of slopes, and the combined horizontal and vertical peak accelerations have led to the premature occurrence of failure. So, the corresponding vertical force was recommended by them to be considered in future rock avalanche stability analysis. Zhang et al. (2015) showed that the effect of the vertical seismic component is negligible if a pseudo-static approach is employed compared to a dynamic approach, which appropriately illustrated the reduction in the slope factor of safety due to the combined effect of the vertical and horizontal earthquake components. In fact, the dominance of the dynamic analysis over the static one was proved by their numerical investigations.

To illustrate the effect of including the vertical seismic component, some supplementary RLEM analyses have been carried out
FD3 by the discussers and the results are presented in Fig. D3. Depending on the direction of the vertical earthquake component, in some geometric conditions even 40% deviation between the probabilities of failure estimation is to be expected. This signifies that the vertical earthquake component cannot be overlooked.

Cross-correlation between soil properties

Another factor not considered in the study of Burgess et al. (2019) is the cross-correlation between the strength parameters. For this reason, the discussers have provided a comparison between the cross-correlated and independent strength parameters' influences on the stability of stochastic slopes.

By definition, soil properties at a given location tend to correlate with one another at the same location or at close locations, which is termed "cross-correlation" (Le 2014). While some existing probabilistic models assume independence between the random variables by ignoring all possible correlations as a matter of mathematical convenience (Lumb 1970; Schultze 1975; Alonso 1976; Tobutt 1982; Nguyen and Chowdhury 1984; Huang et al. 2010), others have drawn much attention to cross-correlation among geotechnical parameters in their numerical formulations (Nguyen and Chowdhury 1985; Tamimi et al. 1989; Fenton and Griffiths 2003; Ferson and Hajagos 2006; Youssef Abdel Massih et al. 2008; Griffiths et al. 2009; Cho and Park 2009; Lü and Low 2011). Wu (2013) considered the cross-correlation between soil strength parameters as a key factor to have accurate evaluation of the reliability in slope stability analyses. Wang and Akeju (2016) believed that ignoring the cross-correlation between effective cohesion c' and effective friction angle ϕ' may lead to a biased approximation of failure probability. On the contrary, there were investigators who were not sympathetic to this view. In effect, the negligible influence of the cross-correlation through their studies was demonstrated (e.g., Fenton and Griffiths 2003; Jamshidi Chenari et al. 2015).

A prolonged discussion exists on the correlation between various soil properties. Only a number of scholars regarded a positive dependency between the soil strength parameters including Griffiths et al. (2009, 2011) and Le (2014), while negative correlation between cohesion c and friction angle ϕ have been reported from laboratory measurements (Lumb 1970; Yucemen et al. 1973; Wolff 1985; Cherubini 1997; Forrest and Orr 2010; Hata et al. 2012). Available data on the correlation between c and unit weight γ and also ϕ and γ endorsed by experimental tests are limited and have only been reported by Matsuo and Kuroda (1974) and Parker et al. (2008). A positive value has most often been assumed, which has been reported in several studies (Chowdhury and Xu 1992; Low and Tang 1997; Sivakumar Babu and Srivastava 2007). However, a negative correlation has been reported by Lumb (1970) and Wolff (1985). A comprehensive set of cross-correlation values has been assembled by Wu (2013). As a result, a negative correlation between c and ϕ and a positive coefficient between two other pairs of properties (i.e., c and γ, and ϕ and γ) would be preferable. Nevertheless, few researchers have focused on a positive correlation between c and ϕ due to its destabilizing effect on slope stability, which yields a higher probability of slope failure; hence, it would be more conservative (e.g., Griffiths et al. 2009, 2011; Le 2014).

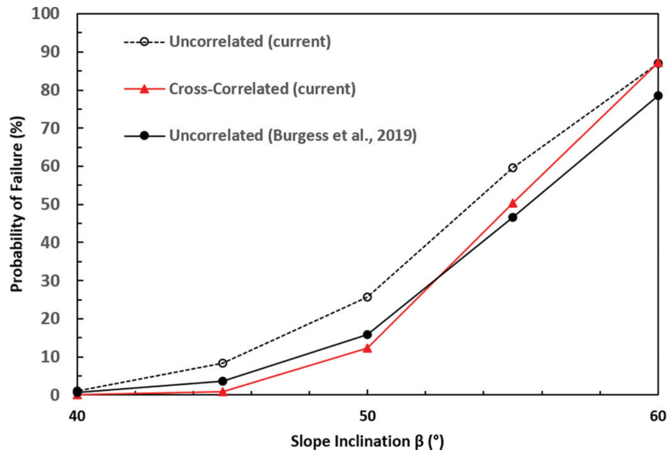
Employing FEM combined with first-order reliability method (FORM), which ignores spatial variation of the random variables, Griffiths et al. (2009) showed that the assumption of positively correlated c and ϕ (i.e., cross-correlation coefficient $\rho = 0.5$) ends in a conservative design when $P_f < 0.5$ while a negative correlation (i.e., $\rho = -0.5$) gives higher probabilities of failure for $P_f > 0.5$ as opposed to RFEM, which always gives the highest probability of failure for $\rho = 0.5$, regardless of P_f. In contrast, making use of RFEM, they realized that ignoring the spatial variability of soil overestimates the reliability of heterogeneous slopes, if the shear strength variability degree exceeds a certain critical value. In addition, Le (2014) came to the conclusion that simultaneous variation of both heterogeneous shear strength parameters has a greater influence on the reliability of slopes compared to solely varying one parameter even if they are uncorrelated.

It is worth mentioning from the literature that negative correlation between c and ϕ have been reportedly confirmed from laboratory measurements and is thought to be more realistic (Lumb 1970; Yucemen et al. 1973; Cherubini 1997, 2000; Forrest and Orr 2010; Hata et al. 2012). In the same vein, negatively correlated soil strength properties have been considered in several studies (e.g., Rackwitz 2000; Cherubini 2000; Li et al. 2011; Lü and Low 2011; Soubra and Mao 2012; Ranjbar Pouya et al. 2014). Cho and Park (2009) and Cho (2010) asserted that the assumption of uncorrelated shear strength parameters renders conservative results in the case where actual correlation was negative, but it would underestimate the probability of failure if the actual correlation was positive. It was proved that for a specific anisotropic correlation length, as the cross-correlation between c and ϕ becomes more negative, the uncertainty in factor of safety will decrease (Cho 2010).

Considering more negative correlation values, Li et al. (2011) observed that the reliability of the slope increases. Allahverdizadeh et al. (2015) investigated the influence of the cross-correlation coefficient between the soil strength properties (i.e., $\rho = -0.5$, 0, 0.5) on the probability of failure of drained slopes. Due to the unreality of positive correlation between c' and $\tan\phi'$ and the lower probability of failure arising from a negative correlation, which may be more realistic, these authors asserted that the independence of the soil strength properties would yield more conservative stability evaluations.

Javankhoshdel and Bathurst (2014) developed circular-RLEM-based probabilistic design charts that included the effect of negative dependency of the shear strength parameters. Later, Javankhoshdel and Bathurst (2015) improved the corresponding charts through the inclusion of maximum cross-correlation between the soil parameters by assuming maximum variability of the soil input parameters (i.e., negative correlation of −0.7 between c and ϕ and a positive value of 0.7 between c and γ, and ϕ and γ as confirmed by Sivakumar Babu and Srivastava (2007)). Considering the dependency between the soil random variables led to the decrease of the probability of failure for $P_f < 50\%$ compared to the uncorrelated case. It was also realized that the cross-correlation between c and ϕ controls the probability of failure if the variability of c and ϕ is higher than that of γ (Javankhoshdel and Bathurst 2015). Nevertheless, the design charts introduced by the authors disregarded the random variability of the soil parameters in their charts, which might be less realistic. Subsequently, Javankhoshdel et al. (2017) considered interdependence and spatial heterogeneity of the soil undrained variables in a comparative study on RLEM and RFEM capabilities. They found that assuming a positive correlation between S_u and γ ends in a negligible increase in the reliability of slopes for isotropic correlation condition as opposed to the anisotropic case, in which the corresponding increase was more significant, particularly for the RLEM approach. In all cases, whether isotropic or anisotropic, Javankhoshdel et al. (2017) noticed that reducing the spatial variability is more effective in increasing the slope reliability than increasing the cross-correlation coefficient provided that P_f is less than 30% for the isotropic condition.

Fig. D4. Influence of cross-correlation between soil properties. [Colour online.]



The overwhelming majority of studies in the literature advocate the existence of negative correlation between c and ϕ, and positive correlation between c and γ as well as ϕ and γ. The discussers would like to explore the impact of these correlations, as the correlations reported in literature are empirically based.

Figure D4 shows the comparison between the results of noncircular RLEM with and without cross-correlation and the RFEM results presented by Burgess et al. (2019) with the assumption of uncorrelated soil properties. The uncorrelated cases using noncircular RLEM and RFEM are the cases presented in Fig. D1. It can be seen in this figure that, by considering the cross-correlation and using the noncircular RLEM approach, the probability of failure decreases. However, the noncircular cross-correlated RLEM approach compared with the uncorrelated RFEM approach gives different results. For the slope angles of less than 52°, RFEM tends to give higher probability of failure compared to the cross-correlated RLEM. In contrast, for steeper slopes ($\beta > 52°$), cross-correlated RLEM probabilities of failure are higher than the corresponding value of the uncorrelated RFEM, i.e., results of the cross-correlated RLEM can be even more conservative compared to the uncorrelated RFEM results. FD4

Variability of seismic coefficient

Earthquake records have long been recognized to be of random nature. This means that earthquake time series contain time variation of both the intensity and frequency content, deemed to be typical of real earthquake ground motions as addressed by Conte and Peng (1997). The earthquakes being nonstationary random time series is another issue that has drawn the attention of many researchers in the literature. Burgess et al. (2019) did not consider the random nature of the seismic coefficients by adopting a representative constant pseudo-static earthquake coefficient. They mentioned that the pertinent coefficient is a random variable due to the unknown nature of the maximum seismic force and called their calculations a conditional approach based on the maximum probable seismic coefficient. However, the discussers have shown that applying random variability to the earthquake coefficient would definitely bring about conservative estimations of the probability of failure of the heterogeneous sloped deposit. This implies that overlooking the random nature of the earthquake coefficients may lead to a compromise on the accuracy of the probabil-

Fig. D5. Influence of probabilistic variation of earthquake coefficient. [Colour online.]

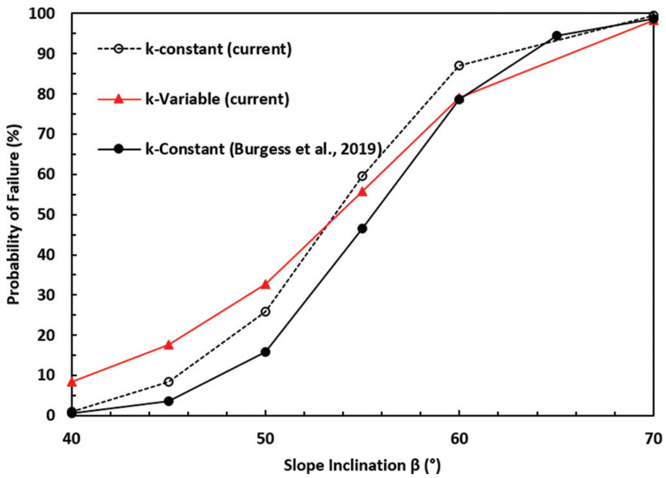


Table D1. Summary of quantitative influences of different effects.

Effect	Absolute expected deviation (%)	
	Minimum	Maximum
Analysis method (RLEM or RFEM)	0	13
Mesh size	0	7.5
Vertical earthquake component	6	25
Cross-correlation between soil parameters	0	13.5
Probabilistic nature of earthquakes	0	10

ity of failure estimations. Youssef Abdel Massih et al. (2008) and Johari et al. (2015) considered a truncated exponential probability density function for the earthquake acceleration coefficients to incorporate the stochastic nature of the earthquakes. Therefore, it is better to take this effect into account appropriately.

Figure D4 shows the results of noncircular RLEM and RFEM
FD5 analyses presented in Fig. D1. Superimposed on Fig. D5 are the results of noncircular RLEM with the variability of horizontal seismic coefficient k. It can be noted from this figure that noncircular RLEM with variable k gives higher probability of failure compared to the noncircular RLEM with constant k when $\beta < 52°$ and also gives RFEM with constant k when $\beta < 60°$.

Concluding remarks

This discussion encompassed some important issues that are deemed influential when conducting seismic slope stability analyses. The methodology of slope stability analysis, appropriate selection of the mesh size when discretizing the slope stability analysis domain, earthquake vertical acceleration component, cross-correlation between the strength parameters involved, and the probabilistic nature of the propagating earthquake waves are considered important when estimating the probability of failure
T1 of naturally occurred slopes. Table D1 provides summary information on the quantitative contributions of the above-mentioned effects. It is observed that, depending on the geometry of the slope under study, at most 25% deviation in the estimation of the probability of failure of the slope may be expected.

References

Allahverdizadeh, P., Griffiths, D., and Fenton, G. 2015. The Random Finite Element Method (RFEM) in probabilistic slope stability analysis with consideration of spatial variability of soil properties. *In* Proceedings of the International Foundations Congress and Equipment Expo 2015 (IFCEE 2015). San Antonio, Texas. American Society of Civil Engineers. pp. 1946–1955. doi:10.1061/9780784479087.178.

Alonso, E.E. 1976. Risk analysis of slopes and its application to slopes in Canadian sensitive clays. Géotechnique, **26**(3): 453–472. doi:10.1680/geot.1976.26.3.453.

Burgess, J., Fenton, G.A., and Griffiths, D.V. 2019. Probabilistic seismic slope stability analysis and design. Canadian Geotechnical Journal, **56**(12): 1979–1998. doi:10.1139/cgj-2017-0544.

Cami, B., Javankhoshdel, S., Bathurst, R.J., and Yacoub, T. 2018. Influence of mesh size, number of slices, and number of simulations in probabilistic analysis of slopes considering 2D spatial variability of soil properties. *In* Proceedings of IFCEE 2018. pp. 186–196. doi:10.1061/9780784481585.019.

Cherubini, C. 1997. Data and considerations on the variability of geotechnical properties of soils. *In* Proceedings of the International Conference on Safety and Reliability (ESREL97), Lisbon, Vol. 2. pp. 1583–1591.

Cherubini, C. 2000. Reliability evaluation of shallow foundation bearing capacity on $c'–\phi'$ soils. Canadian Geotechnical Journal, **37**(1): 264–269. doi:10.1139/t99-096.

Ching, J., and Hu, Y.G. 2016. Effect of element size in random finite element analysis for effective Young's modulus. Mathematical Problems in Engineering, **2016**: article ID 8756271. doi:10.1155/2016/8756271.

Ching, J., and Phoon, K.-K. 2013*a*. Mobilized shear strength of spatially variable soils under simple stress states. Structural Safety, **41**: 20–28. doi:10.1016/j.strusafe.2012.10.001.

Ching, J., and Phoon, K.-K. 2013*b*. Effect of element sizes in random field finite element simulations of soil shear strength. Computers & Structures, **126**: 120–134. doi:10.1016/j.compstruc.2012.11.008.

Cho, S.E. 2010. Probabilistic assessment of slope stability that considers the spatial variability of soil properties. Journal of Geotechnical and Geoenvironmental Engineering, **136**(7): 975–984. doi:10.1061/(ASCE)GT.1943-5606.0000309.

Cho, S.E., and Park, H.C. 2009. Effect of spatial variability of cross-correlated soil properties on bearing capacity of strip footing. International Journal for Numerical and Analytical Methods in Geomechanics, **34**: 1–26. doi:10.1002/nag.791.

Chowdhury, R.N., and Xu, D.W. 1992. Reliability index for slope stability assessment-two methods compared. Reliability Engineering & System Safety, **37**: 99–108. doi:10.1016/0951-8320(92)90002-3.

Conte, J.P., and Peng, B.F. 1997. Fully nonstationary analytical earthquake ground-motion model. Journal of Engineering Mechanics, **123**(1): 15–24. doi:10.1061/(ASCE)0733-9399(1997)123:1(15).

Fenton, G.A., and Griffiths, D.V. 2003. Bearing-capacity prediction of spatially random $c–\phi$ soils. Canadian Geotechnical Journal, **40**(1): 54–65. doi:10.1139/t02-086.

Fenton, G.A. and Griffiths, D.V. 2008. Risk assessment in geotechnical engineering. John Wiley & Sons, New York, USA. Available from http://courses.engmath.dal.ca/rfem/.

Fenton, G.A., and Vanmarcke, E.H. 1990. Simulation of random fields via local average subdivision. Journal of Engineering Mechanics, **116**(8): 1733–1749. doi:10.1061/(ASCE)0733-9399(1990)116:8(1733).

Ferson, S., and Hajagos, J.G. 2006. Varying correlation coefficients can underestimate uncertainty in probabilistic models. Reliability Engineering & System Safety, **91**: 1461–1467. doi:10.1016/j.ress.2005.11.043.

Forrest, W.S., and Orr, T.L. 2010. Reliability of shallow foundations designed to Eurocode 7. Georisk, **4**(4): 186–207. doi:10.1080/17499511003646484.

Gazetas, G., Garini, E., Anastasopoulos, I., and Georgarakos, T. 2009. Effects of near-fault ground shaking on sliding systems. Journal of Geotechnical and Geoenvironmental Engineering, **135**(12): 1906–1921. doi:10.1061/(ASCE)GT.1943-5606.0000174.

Griffiths, D.V., Huang, J., and Fenton, G.A. 2009. Influence of spatial variability on slope reliability using 2-D random fields. Journal of Geotechnical and Geoenvironmental Engineering, **135**(10): 1367–1378. doi:10.1061/(ASCE)GT.1943-5606.0000099.

Griffiths, D.V., Huang, J., and Fenton, G.A. 2011. Probabilistic infinite slope analysis. Computers and Geotechnics, **38**: 577–584. doi:10.1016/j.compgeo.2011.03.006.

Hata, Y., Ichii, K., and Tokida, K. 2012. A probabilistic evaluation of the size of earthquake induced slope failure for an embankment. Georisk, **6**(2): 73–88. doi:10.1080/17499518.2011.604583.

Huang, J., and Griffiths, D.V. 2015. Determining an appropriate finite element size for modelling the strength of undrained random soils. Computers and Geotechnics, **69**: 506–513. doi:10.1016/j.compgeo.2015.06.020.

Huang, J., Griffiths, D.V., and Fenton, G.A. 2010. System reliability of slopes by RFEM. Soils and Foundations, **50**(3): 343–353. doi:10.3208/sandf.50.343.

Jamshidi Chenari, R., Kamyab Farahbakhsh, H., and Zamanzadeh, M. 2015. Investigation into the effect of the inherent heterogeneity of soil on the long-term stability of earth slopes. Sharif Journal of Science and Research, **31.2**(3.1): 3–17.

Javankhoshdel, S., and Bathurst, R.J. 2014. Simplified probabilistic slope stability design charts for cohesive and cohesive-frictional ($c–\phi$) soils. Canadian Geotechnical Journal, **51**(9): 1033–1045. doi:10.1139/cgj-2013-0385.

Javankhoshdel, S., and Bathurst, R.J. 2015. Influence of cross correlation between soil parameters on probability of failure of simple cohesive and $c–\phi$ slopes. Canadian Geotechnical Journal, **53**(5): 839–853. doi:10.1139/cgj-2015-0109.

Javankhoshdel, S., Luo, N., and Bathurst, R.J. 2017. Probabilistic analysis of simple slopes with cohesive soil strength using RLEM and RFEM. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, **11**(3): 231–246. doi:10.1080/17499518.2016.1235712.

Johari, A., Mousavi, S., and Nejad, A.H. 2015. A seismic slope stability probabilistic model based on Bishop's method using analytical approach. Scientia Iranica Transaction A Civil Engineering, **22**(3): 728.

Khazai, B., and Sitar, N. 2000. Assessment of seismic slope stability using GIS modeling. Geographic Information Sciences, **6**(2): 121–128. doi:10.1080/10824000009480540.

Le, T.M.H. 2014. Reliability of heterogeneous slopes with cross-correlated shear strength parameters. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, **8**(4): 250–257. doi:10.1080/17499518.2014.966117.

Li, D., Chen, Y., Lu, W., and Zhou, C. 2011. Stochastic Response Surface Method for reliability analysis of rock slopes involving correlated non-normal variables. Computers and Geotechnics, **38**(1): 58–68. doi:10.1016/j.compgeo.2010.10.006.

Low, B.K., and Tang, W.H. 1997. Efficient reliability evaluation using spreadsheet. Journal of Engineering Mechanics ASCE, **123**: 749–752. doi:10.1061/(ASCE)0733-9399(1997)123:7(749).

Lü, Q., and Low, B.K. 2011. Probabilistic analysis of underground rock excavations using response surface method and SORM. Computers and Geotechnics, **38**: 1008–1021. doi:10.1016/j.compgeo.2011.07.003.

Lumb, P. 1970. Safety factors and the probability distribution of soil strength. Canadian Geotechnical Journal, **7**: 225–242. doi:10.1139/t70-032.

Matsuo, M., and Kuroda, K. 1974. Probabilistic approach to design of embankments. Soils and Foundations, **14**: 1–17. doi:10.3208/sandf1972.14.2_1.

Nguyen, V.U., and Chowdhury, R.N. 1984. Probabilistic study of spoil pile stability in strip coal mines-two techniques compared. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, **21**: 303–312. doi:10.1016/0148-9062(84)90363-2.

Nguyen, V.U., and Chowdhury, R.N. 1985. Simulation for risk analysis with correlated variables. Géotechnique, **35**: 47–58. doi:10.1680/geot.1985.35.1.47.

Parker, C., Simon, A., and Thorne, C.R. 2008. The effects of variability in bank material properties on riverbank stability: Goodwin Creek. Mississippi. Geomorphology, **101**: 533–543. doi:10.1016/j.geomorph.2008.02.007.

Rackwitz, R. 2000. Reviewing probabilistic soils modelling. Computers and Geotechnics, **26**: 199–223. doi:10.1016/S0266-352X(99)00039-7.

Ranjbar Pouya, K., Zhalehjoo, N., and Jamshidi Chenari, R. 2014. Influence of random heterogeneity of cross-correlated strength parameters on bearing capacity of shallow foundations. Indian Geotechnical Journal, **44**(4): 427–435. doi:10.1007/s40098-013-0096-9.

Rocscience Inc. 2018. Slide Version 2018 – 2D limit equilibrium slope stability analysis. Toronto, Ontario, Canada. Available from http://rocscience.com.

Sarma, S.K., and Scorer, M. 2009. The effect of vertical accelerations on seismic slope stability. *In* Proceedings of the International Conference on Performance-Based Design in Earthquake Geotechnical Engineering, Tokyo, 2009. *Edited by* T. Kokusho, U. Tsukamoto, and M. Yoshimine. pp. 889–896. doi:10.13140/2.1.2814.3045.

Schultze, E. 1975. Some aspects concerning the application of statistics and probability to foundation structures. *In* Proceedings of the 2nd International Conference on the Application of Statistics and Probability in Soil and Structural Engineering, Aachen 4. pp. 457–494.

Sivakumar Babu, G.L., and Srivastava, A. 2007. Reliability analysis of allowable pressure on shallow foundation using response surface method. Computers and Geotechnics, **34**(3): 187–194. doi:10.1016/j.compgeo.2006.11.002.

Soubra, A.-H., and Mao, N. 2012. Probabilistic analysis of obliquely loaded strip foundations. Soils and Foundations, **53**(3): 524–538. doi:10.1016/j.sandf.2012.05.010.

Sun, P., Yin, Y.P., Wu, S.R., and Chen, L.W. 2012. Does vertical seismic force play an important role for the failure mechanism of rock avalanches? A case study of rock avalanches triggered by the Wenchuan earthquake of May 12, 2008, Sichuan, China. Environmental Earth Sciences, **66**(5): 1285–1293. doi:10.1007/s12665-011-1338-8.

Tabarroki, M., and Ching, J. 2019. Discretization error in the random finite element method for spatially variable undrained shear strength. Computers and Geotechnics, **105**: 183–194. doi:10.1016/j.compgeo.2018.10.001.

Tamimi, S., Amadei, B., and Frangopol, D.M. 1989. Monte Carlo simulation of rock slope reliability. Computers & Structures, **33**(6): 1495–1505. doi:10.1016/0045-7949(89)90489-6.

Tobutt, D.C. 1982. Monte Carlo simulation methods for slope stability. Computers & Geosciences, **8**: 199–208. doi:10.1016/0098-3004(82)90021-8.

Wang, Y., and Akeju, O.V. 2016. Quantifying the cross-correlation between effective cohesion and friction angle of soil from limited site-specific data. Soils and Foundations, **56**(6): 1055–1070. doi:10.1016/j.sandf.2016.11.009.

Wolff, T.F. 1985. Analysis and design of embankment dam slopes: a probabilistic approach. PhD. Thesis, Purdue University, Lafayette, IN.

Wu, X.Z. 2013. Trivariate analysis of soil ranking-correlated characteristics and its application to probabilistic stability assessments in geotechnical engineering problems. Soils and Foundations, **53**(4): 540–556. doi:10.1016/j.sandf.2013.06.006.

Youssef Abdel Massih, D.S., Soubra, A.-H., and Low, B.K. 2008. Reliability-based analysis and design of strip footings against bearing capacity failure. Journal of Geotechnical and Geoenvironmental Engineering, **134**(7): 917–928. doi:10.1061/(ASCE)1090-0241(2008)134:7(917).

Yucemen, M.S., Tang, W.H., and Ang, A.H.S. 1973. A probabilistic study of safety and design of earth slopes. Civil Engineering Studies, Structural Research Series, 402, University of Illinois, Urbana, Ill. Available from http://hdl.handle.net/2142/13816.

Zhang, Y., Zhang, J., Chen, G., Zheng, L., and Li, Y. 2015. Effects of vertical seismic force on initiation of the Daguangbao landslide induced by the 2008 Wenchuan earthquake. Soil Dynamics and Earthquake Engineering, **73**: 91–102. doi:10.1016/j.soildyn.2014.06.036.