MMF Design using Evolutionary Algorithms

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Abstract: *We review the latest advances on the design of few-mode fibres guiding up to 21 polarisation modes with low differential mode delay over the C-band, suitable for long-haul transmission.*

Keywords: *Few-Mode Fibres, Differential Mode Delay, Refractive Index Profile*

I. INTRODUCTION

Space-division multiplexing (SDM) has emerged as a solution to overcome the capacity limit of single-mode fibres (SMFs) [1]. Among the possible SDM approaches, few-mode fibres (FMFs) and coupled-core multi-core (CC-MCFs) offer the highest spatial information density. They exhibit low nonlinear coefficients and high pumping efficiency allowing for high spatial-density integration in transponders [2] and add-drop multiplexers [3]. However, the multitude of spatial modes introduces new linear impairments, namely: group delay (GD) spread [4-10] stemming from the interplay between differential mode delay (DMD) and linear mode coupling (LMC). The GD spreading can be undone using of multiple-input multiple-output (MIMO) equalization **Error! Reference source not found.**, however complexity scales with the total time spread. According to [10], to keep equalisation complexity treatable for long-haul transmission *DMD* should be ~10 ps/km over the transmission band.

In this paper, we review our results [11] on the design of FMFs with low DMD over the C-band with up to 21 linearly polarised (LP) modes (counting $LP_{\mu\nu}$ as two modes for non-zero μ). Two different types of refractive index profiles are considered: graded-index (GI) profile with a cladding trench and a multi-step-index (multi-SI) profile. The optimization results show that the multi-SI profiles present lower DMD than the minimum achieved with a GI profile. Regarding the multi-SI profiles, it is shown that 64 steps are required to obtain a DMD improvement considering 21 LP modes.
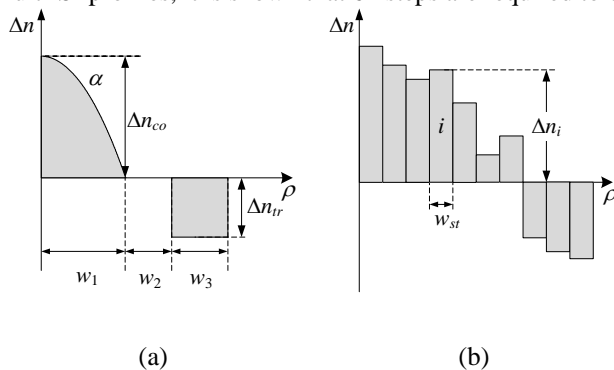

Fig. 1. (a) GI profile with cladding trench.
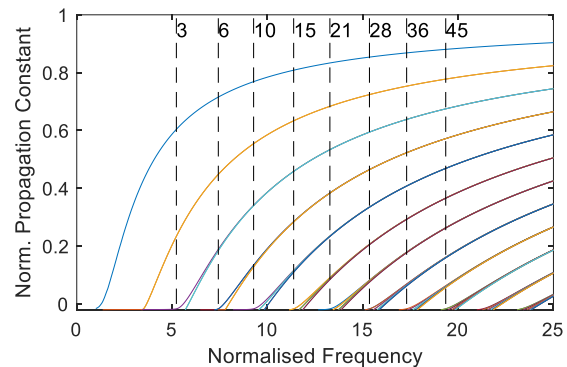(b) Multi-SI profile with constant w_{st}.

Fig. 2. Normalised mode effective index as a function of *V*.

II. FIBER PROFILE DESCRIPTION AND ANALYSIS

Arbitrary fibre profiles can be described using refractive index relative difference (Δn). For azimuthally symmetric fibres, Δn is a function of the radial coordinate ρ and is given by: $\Delta n(\rho) = [n(\rho)^2 - n_{cl}]/n(\rho)^2$, where $n(\rho)$ is the refractive index at ρ and n_{cl} is the cladding refractive index. The linearly polarised guided modes ($LP_{\mu\nu}$) and their characteristics are calculated solving the Maxwell equations numerically using the method described in [12]. The $LP_{\mu\nu}$ mode characteristics calculated are the effective index $\bar{n}^{LP\mu\nu}$, effective group index $\bar{n}_g^{LP\mu\nu}$ and DMD. The DMD of mode $LP_{\mu\nu}$ is calculated as $DMD_{LP_{\mu\nu}}(\lambda) = [\bar{n}_g^{LP_{\mu\nu}}(\lambda) - \bar{n}_g^{LP_{01}}(\lambda)] / c$, where λ stands for wavelength and c for speed of light in vacuum. The dispersion properties of pure, Ge-doped or F-doped silica have been modelled using the Sellmeier coefficients provided in [13].

The GI profile with cladding trench is presented in Fig. 1 (a), where w_1 is the core radius, w_2 is the radial distance between the end of the core and the beginning of the trench, w_3 is the trench width, $\Delta n_{co} = \Delta n(0)$, and Δn_{tr} is Δn at the trench, and α is the core grading index exponent. When designing a GI fibre to have a given number of modes, one can start by choosing the normalized frequency (V) value: $V = 2\pi w_1/\lambda \cdot [n_{co}^2 - n_{cl}^2]^{1/2}$, where $n_{co} = n(0)$. For each target number of modes fibre, we choose the highest V that guarantees the guidance of the first n^{th}-modes while cutting off the next higher-order modes, as in [14] – for example considering the highest α in the search domain to be defined, e.g. with $\alpha = 2.3$ and $\Delta n_{tr} = 0$. See Fig. 2 for normalised effective index as a function of V, including the target V for set number of

modes (dashed vertical lines). The highest V allows for the highest possible $\vec{n}^{LP\mu\nu}$, such that modes are strongly guided are more core confined. The w_1 for each target fibre is then readily available from the earlier V definition – considering the lowest λ of the C-band (1530 nm) to guarantee that the number of modes does not change across the bandwidth of interest. The impact of α and Δn_{tr} on the DMD of a GI with cladding trench profile is the following [11]: α allows controlling the DMD average and Δn_{tr} allows controlling the DMD slope, as shown in Fig. 3.
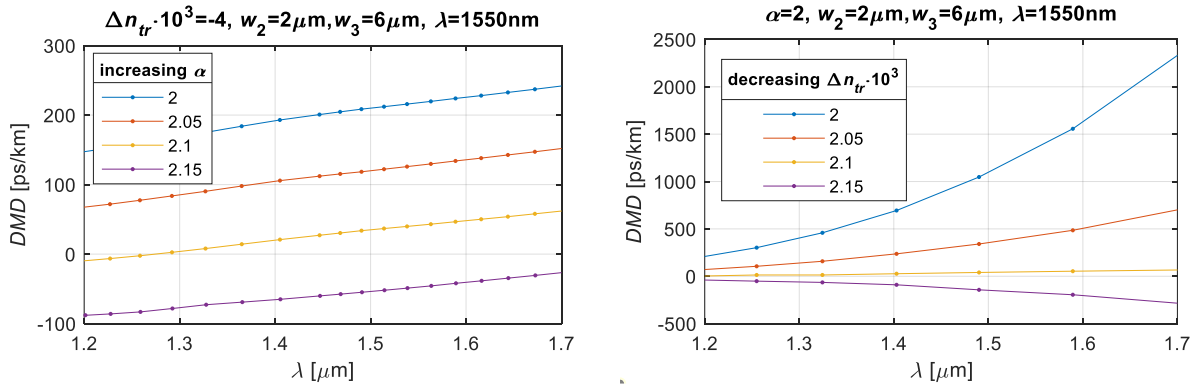

Fig. 3. a) *DMD* as a function of λ for decreasing Δn_{tr}. b) DMD as a function of λ for decreasing

The multi-SI profile is shown in Fig. 1 (b), where each step i has an arbitrary Δn value (Δn_i) and step width w_{st}. The number of steps is N_{st}. Such profile allows evaluating increasingly arbitrary profiles by simultaneously scaling N_{st} upwards and w_{st} downwards. In this way, one can evaluate to what extension are GI profiles close to an optimum solution.

III. OPTIMIZATION FUNCTION AND ALGORITHMS

The optimization parameters of each profile can be gathered in a parameter vector (pv): $pv = [\alpha, \Delta n_{co}, \Delta n_{tr}, w_2, w_3]$ for the GI profile and $pv = [w_{st}, \Delta n_1, ..., \Delta n_{N_{st}}]$ for the multi-SI profile. The *optimization function* equals the maximum *DMD* among the guided modes and over the defined λ range given by $maxDMD(pv) = \max_\lambda \left(\max_{\mu\nu} |DMD(\lambda, \mu\nu)| \right)$.

A. GI profile

The search space in the GI case is: $1.53 \leq \lambda[nm] \leq 1.65$; $1.85 \leq \alpha \leq 2.1$; $-5 \leq \Delta n_{tr} \cdot 10^3 \leq 0$; $1 \leq \Delta n_{co} \cdot 10^3 \leq 5$; $0 \leq w_2 \leq w_1/2$; $0 \leq w_3 \leq w_1$. The optimization algorithm consists of brute force search over w_2 and w_3, but a double golden section search over $(\alpha, \Delta n_{tr})$ which has been shown to be a convex space [15]. multi-SI profile.

B. Multi-SI profile

In the multi-SI case, the search space is: $-5 \leq \Delta n_i \cdot 10^3 \leq 5$; $N_{st} = \{16 \quad 32 \quad 64\}$; $0.5 \leq w_{st}[\mu m] \leq 5$. w_{st} is constrained such that the total length of the profile ($w_{st} \cdot N_{st}$) takes values close tp the total length considered for the GI profile ($w_1 + w_2 + w_3$). In this case a genetic algorithm (GA) is used due to the high number of optimization parameters. The individual genes correspond to the profile parameter vector, and the individual fitness is given by the inverse of *objective function* presented above. The initial population is randomly generated within the Δn_i and w_{st} constraints. The Δn_i genes are generated using a uniform distribution and the w_{st} gene is generated using a Gaussian distribution with mean equal to $(w_1+w_2+w_3) / N_{st}$, considering the optimum GI parameters obtained with $\Delta n_{co} = 0.001$, and standard deviation equal to $[\max(w_{st}) - \min(w_{st})]/6$. The size of the initial population is 2000. After the first generation the population size is reduced to 200. The following generations are created using crossover, mutation and selection operators.

The crossover operator randomly selects the jth gene in the solution structure and, given two parents $X = [w_{st}{}^X, \Delta n_1{}^X, ..., \Delta n_{Nst}{}^X]$ and $Y = [w_{st}{}^Y, \Delta n_1{}^Y, ..., \Delta n_{Nst}{}^Y]$, the generated offspring are constructed as: $U = [w_{st}{}^X, \Delta n_1{}^X..., \Delta n_{j-1}{}^X, \Delta n_j{}^Y, ..., \Delta n_{Nst}{}^Y]$ and $W = [w_{st}{}^Y, \Delta n_1{}^Y..., \Delta n_{j-1}{}^Y, \Delta n_j{}^X, ..., \Delta n_{Nst}{}^X]$. The selection of the individuals for the crossover operation is executed using a roulette wheel selection scheme. In this scheme, the probability of an individual being selected is proportional to its fitness. The size of the offspring generated is 200.

The mutation operation is executed adding to the lth gene (randomly selected) a Gaussian random variable with zero mean and standard deviation σ_l. σ_l is weighed by the domain of the lth gene, in order to compensate for the different orders of magnitude between w_{st} and Δn_i. σ_l is given by: $[\max(w_{st}) - \min(w_{st})]/6$ for $l = 1$ and $[\max(\Delta n_i) - \min(\Delta n_i)]/6$ for $l > 1$. In each generation, the mutation operation is applied to the offspring with a probability of 50 %. Additionally, 20 new individuals are added to the offspring selecting the 20 best parents (parents with higher fitness) and applying the mutation operator (the original parents are kept in the population).

Finally, the next generation is composed of 10 % of the best individuals (individuals with higher fitness), and the remaining population is selected using the roulette wheel selection scheme already used in the crossover operation.

IV. Results

The optimisation results are shown in this section for the GI and multi-SI profiles. Fig. 4 shows *maxDMD* for the optimum GI profiles as a function of Δn_{co}. It can be seen that *maxDMD* decreases with decreasing Δn_{co} for all numbers of modes – as expected since the difference $|\vec{n}^{LP\mu\nu} - \vec{n}^{LP01}|$ is bounded by n_{co} and n_{cl}. The drawback of the utilization of a low Δn_{co} is related to macro-bending loss [11]. Moreover, it can be concluded that *maxDMD* cannot be reduced to negligible levels (*maxDMD* < 0.1 ps/km) for more than three modes.

The optimization results for multi-SI profiles in [11] showed that the *maxDMD* of the optimum profile decreases with N_{st} as expected, and that 64 steps are required to reduce *maxDMD* below that of the correspondent optimum GI profile with $\Delta n_{co} = 1\cdot10^{-3}$. For $N_{st} = 64$, the *maxDMD* reduction ranges from 19 % to 31 % - but note that obtained at the expense of a more complex profile. Fig. 5 shows the superposition of the optimum multi-SI profile with $N_{st} = 64$ and the optimum GI profile with $\Delta n_{co} = 1\cdot10^{-3}$, for {6, 10, 15, 21} modes. From Fig. 5 it can be concluded that the optimum multi-SI profiles are similar to correspondent optimum GI profile. The similarity between profiles holds for smaller N_{st}.
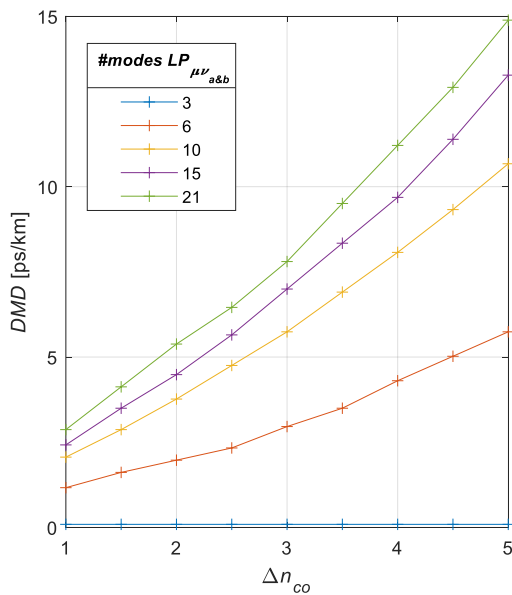

Fig. 4. *maxDMD* [ps/km] optimum values as a function of Δn_{co} for different numbers of modes.
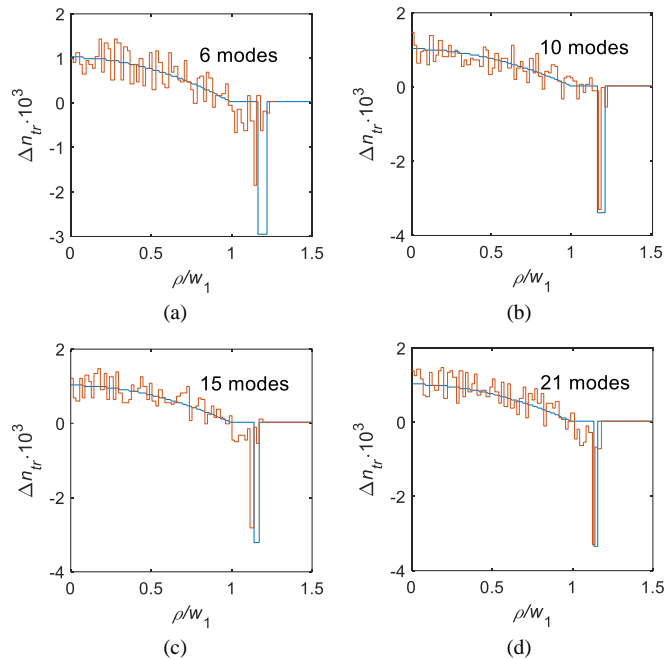

Fig. 5. Optimum multi-SI profile with 64 steps and optimum GI profile considering $\Delta n_{co} = 1\cdot10^{-3}$ as a function of ρ/w_1, for: (a) 4M, (b) 6M, (c) 9M and (d) 12M.

V. Conclusions

In this work, the design of FMFs with low DMD over the C-band was investigated. Two different profiles have been considered: a GI profile and a multi-SI profile. The profiles parameters were optimized obtaining the lowest *maxDMD* achievable for 2M to 12M. The results have shown that the multi-SI optimisation converges to solutions that resemble a GI core with cladding trench – with similar *maxDMD*. Evolutionary algorithms may play a role in the design of fibres to be fabricated using the stack-and-draw method.

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