Borderline Contradictions

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I, Caitlin Rose Canonica, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

[redacted] September 30, 2021

In loving memory of

Valeria Marta Canonica

Abstract

Borderline contradictions have long been considered to be false, but recent developments in the intersection of the fields of experimental philosophy and linguistics have lead to a consensus that ordinary speakers of natural language find borderline contradictions to be true. Furthermore, speakers are more likely to agree to *disjunctive* borderline contradictions ($\neg(A \vee \neg A)$) than their *conjunctive* counterparts ($A \wedge \neg A$). We focus our attention on a series of studies of this inequality, culminating in Égré and Zehr's 2016 algorithmic account, which invokes *strict* and *tolerant* operators to predict that while speakers are more likely to agree to disjunctive contradictions than to conjunctive contradictions, they are also more likely to agree to both of these than to their positive and negative subparts (A and $\neg A$). We present the results of three new studies, one a replication of Égré and Zehr's work, the results of which suggest that speakers only find the positive subsentence to be false, suggesting that some leading accounts of how speakers interpret borderline contradictions may require modification.

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Impact Statement

Imagine you find yourself standing before a bushel of apples. You are instructed to sort the apples into piles according to colour. You grab an apple. Red. You start a red pile. You grab another apple. Yellow. You start a yellow pile. Next. Red. Red pile. Next Green. You start a green pile. Next you grab an apple that is not as green as the first green apple, but it's definitely not yellow. Green pile. The next apple is certainly more yellow than the second green apple, but is it yellow enough for the yellow pile? It might be. What do you do? Do you arbitrarily choose a pile? Do you start a new pile of apples that are both green and yellow? But if you start a new pile, aren't those apples neither green nor yellow? Could that mean the apple you are holding is *both green and not green*, *neither yellow nor not yellow*?

The aim of this body of research is to enhance our understanding of how humans reason about the liminal areas between concepts and how they express this using natural language. We use borderline objects, such as the apple, to discover what strategies speakers use to interpret contradictory statements like *the apple is green and not green*, and why it is often the case that speakers judge these contradictions to be true. We then try to square this information with theoretical models of how language cognition works in order to improve those models.

The applications of this are both profound and myriad. Better models of the human language apparatus improve methods of training better performing natural user interfaces, and this is especially important as we move toward a future in which humans increasingly interact with interfaces such as virtual assistants. For just one example, healthcare systems around the world are increasingly turning to automation and artificial carers, both of which must be equipped to evaluate liminal cases and interpret patients' borderline language to provide optimal care.

Moreover, the complex of cognitive phenomena surrounding borderline language and judgments, including reasoning about borderline contradictions, is inherent to and ubiquitous within natural language. It therefore constitutes a crucial key to understanding how we use language to make sense of the world. In this sense, a better understanding of how speakers perceive and communicate about borderline cases impacts our ability as a society to create and interpret just laws, communicate public information perspicuously, and analyse ourselves, our decisions, and our opinions.

Finally, vagueness is a deeply perplexing issue because it is vital to how we think and communicate given the finiteness of our minds, but it also poses paradoxes that we are unable to fully come to grips with. Contradictions are at the heart of how we think we know what we know; contradictions that are true require us to push the limits of how we think about what we know, with deep philosophical implications.

Contents

Preface

More than two thousand years ago, Aristotle formulated two laws, which together establish a relationship between propositions and their negations that has underpinned much of logical inquiry during the two millenia since. The law of noncontradiction (LNC) avers that a proposition and its negation cannot both be true, while the law of excluded middle (LEM) maintains that for any proposition, either it is true or its negation is true. Thus, it cannot be the case that *it is raining and it is not raining*; nor can it be the case that *it is neither raining nor is it not raining*. Both are classical contradictions, and as such, are trivially false.

Subsequent centuries have borne witness to great advancements in the field of logical inquiry, and this progress has been in no small part made possible by virtue of the power of classical contradictions. Indeed, such contradictions have proven to be an invaluable tool not only in the hands of mathematicians and philosophers, but also in the hands of researchers operating throughout the gamut of the modern sciences.

However, the view that classical logic, along with its classical contradictions, is the logical foundation underlying cognitive and natural language phenomena, or more cautiously stated, that it is the logic best suited to the gross analysis of these phenomena, is not uncontested. Is it possible that the facts of the world could be such, and that our perception of the facts of the world could be such, and that our conception of the world based upon those perceptions could be such, that as competent reasoners and speakers of natural language we could agree that the sentence *it is raining and it is not raining* is true. Or perhaps we could agree that it is not entirely false? If so, then what process of evaluation brings us to that conclusion? And if we agree that the sentence is not trivially false, to what extent can it be considered a genuine contradiction? Are there true contradictions?

One might imagine any number of experiments aimed at probing for answers to these questions, in which conditions are manipulated, and a resulting truth judgment is recorded. Such experiments are, of course, considerably more easily conceived of than put into practice, but that hasn't stopped a sizeable and growing group of researchers from attempting to tackle the problem head on. On that front, the growing consensus is that given specific conditions, and taking on board a wealth of assumptions concerning the relationships between natural language, logic, and mental states, certain contradictions are true. More precisely stated, under specific circumstances, naïve speakers of a language agree to characterise sentences bearing propositional content which is formally and classically contradictory, as true.

So what are the specific circumstances to which we have so far only alluded? The primary requisite condition we are concerned with here is borderlineness, an integral feature of vagueness in natural language, although we make no claim as to whether this condition is unique with regard to licensing true judgments for classical contradictions. In order to avoid commitment to any particular analysis of borderline phenomena this early on, what is meant by "borderlineness" is perhaps better introduced at this point by means of an example than by means of a technical definition. We will get to definitions shortly.

Imagine you are looking out a window on a dreary afternoon of on again off again drizzles, and considering whether the time is right to make a dash outside to the corner shop to buy an umbrella. Compared to an earlier downpour, precipitation seems to have momentarily lessened to a heavy, vertically descending mist. You are clearly not going to get your umbrella without becoming a little damp, but at least you won't be soaked before you can shield yourself with it for the dash back. But is it raining? If someone turned to you and said, "Best stay indoors. It's raining," would you agree with that last sentence? Would you be willing to turn to her and declare that it's not raining? Although it is not at all guaranteed that all speakers would agree, it is probably safe to assume that a majority of speakers would experience some hesitation before committing to either of these possibilities. This contrasts with how the majority of speakers might feel about the aforementioned earlier cloudburst, during which it was clearly the case at the time that *it is raining* was true, because it was clearly raining. But for now, as you look out the window and don outerwear, *it is raining* is a borderline proposition; not clearly true, not clearly false. It is in just these conditions that the borderline contradiction, *it is raining and it is not raining*, might plausibly be deemed by a competent speaker of the language be true.

The example is mundane enough, but it demonstrates a fact that has some profound implications because not only does it bring into question the appropriateness of classical logic to the analysis of vague language use, but given that vagueness is widely held to be a pervasive feature of natural language, it also brings into question the ultimate effectiveness of an approach to natural language meaning that is committed to classical logic. Furthermore, the issue extends beyond natural language use. It is rather an issue pertaining to how we perceive and organise our conception of the world. There is a sense in which, beyond the question of whether or not the sentence *it is raining and it is not raining* can be said to be true, the core question is whether or not the propositional content of the sentence can be true on a conceptual level. Do we on some level conceive of the world as a place where propositions and their negations may both be true to an extent, or conversely, where neither a proposition nor its negation might be true? And if we do, how deeply do these classical violations seep into the processes by which we conceptualise and reason about the world? How do these questions bear upon how we organise the information we perceive into objects and categories available to cognitive and linguistic manipulation? What flexibility might such a view of the world afford in how we draw distinctions, apply linguistic expressions accordingly, and use these expressions to modify how other speakers draw distinctions and apply linguistic expressions? On the other hand, if we truly are committed to the notion that at some deep level, the basis of our cognition and language use is ultimately bivalent, then how do we account for the apparently widespread intuition that certain classical contradictions in certain circumstances are true, as well as our apparent inability to establish at what exact point one drop of rain tips our assessment of the weather from *it is not raining* to *it is raining*? These are far-reaching questions, and we will not even attempt to address them fully here, but it is worth taking a moment to appreciate the breadth of the puzzle of true contradictions.

The puzzle we *will* be concerned with begins by asking how we can reconcile the mismatch between those intuitions the standard semantic tradition would expect speakers to report regarding this special class of contradictions, and what they actually report. We should emphasise that rigorous empirical study of these contradictions has only come to the fore within the last two decades or so; the longer, historical tradition has been to assume that speakers do not find these contradictions true, and reports to the contrary are attributable to some error or accident that has been incurred, either on the part of the speakers or on the part of whoever is collecting the speakers' judgments. We should note that this view has not been replaced. It persists in parallel to the view that speakers' intuitions are not mistaken. But if we take the latter view, and seriously entertain the possibility that speakers' intuitions are not mistaken, then what kind of semantics undergirds them? What modifications to the semantics are required in order to account for them and what independent evidence exists to motivate said modifications? What existing theories of vagueness are equipped to predict them? How can we differentiate between distinct theories in case they cast identical predictions and in what sense would such a differentiation be meaningful?

Happily, there are more facets to this puzzle than just the existence of true borderline contradictions. In fact, there is a host of closely related borderline sentences whose relative truth has the potential to depict a distinct pattern. Furthermore, not all borderline contradictions are equal, and the ways in which they fail to be so has served, and promises to continue to serve, as a valuable tool in differentiating between those theories which are equipped to account for the full pattern of observations, and those that require supplementation in order to do so.

But the puzzle does not end there, because as well motivated supplementation is supplied to each theory in the terms best adapted to each, the task of meaningfully differentiating between theories in terms of their predictive power once again becomes increasingly difficult. At a certain level of abstraction, approaches to vagueness begin to appear like mere reflections of each other; different translations, same story. In some cases it can even be argued that in light of the limited data that we have access to, and perhaps fundamentally due to the limits on what data we might conceivably someday have access to, certain theories are provably functionally identical. Thus, on a meta-theoretical level, we need to ask what could possibly distinguish one line of theory from another, and possibly, which theory proffers the most versatile and elegant solution to the puzzle. There may be multiple solutions...

Part I

Vagueness and the Borderline Region

Chapter 1

The basics of borderlines

1.1 Introduction

It seems that we all know what vagueness is, or more critically restated, we have a good grasp of the effects of vagueness even if we do not name it as such. We know and accept, for instance, that some objects are green, some are not, and some are difficult to identify as either green or as some alternative to green. The appropriateness of characterising a member of this third category as *green* may be cast as unclear, debatable, context dependent, or unstable. As an example, let us say we are talking about apples. We can all likely remember or imagine an apple that we and others, as speakers and observers of the apple, might hesitate to characterise either as *green* or as an alternative to *green*, e.g., *not green*, *greenish*, *yellow*, *greenish-yellow*, etc. In this case, we would not necessarily expect for each speaker's characterisation of the apple to be exactly aligned with any other speaker's characterisation, that is, speakers might faultlessly disagree with one another. We may also expect speakers' characterisations to change in relation to context, for instance relative to a comparison class, e.g., some Golden Delicious apples are green, but in the context of a comparison to Granny Smith apples they are not green. We may even expect speakers' characterisations of the apple to spontaneously change with no outwardly apparent reason for doing so, for example in the following dialogue:

A: What colour is a Golden Delicious apple?

B: Golden yellow, of course. Well, sometimes green actually. To be honest, I'm not quite sure.

The features we have been describing are all features of vagueness in language, which is the starting point of our inquiry. But briefly, before beginning, we should take a moment to describe what is meant by 'vagueness'. The term 'vagueness' is colloquially used to refer to a large array of linguistic and cognitive phenomena that touch on some notion of unclarity, however when linguists and philosopher discuss 'vagueness' it is usually in a narrow sense associated with one specific type of unclarity.

Vagueness is often succinctly defined in terms of that which it is not. Within the field of semantics, the problem is typically cast as a lack of precise truth conditions. Vague predicates lack precise boundaries to their extensions, and thus borderline cases of these predicates arise, instances in which it is unclear whether the object in question belongs to the vague predicate's extension or not. Thus for a borderline case of a predicate, speakers may disagree over whether the predicate applies, and individual speakers may demonstrate instability in their own judgments. But this is deeply troubling insofar as meaning is bound up in truth conditions, for if speakers can neither agree upon nor be internally consistent with regard to the truth conditions of the words that they use, and indeed may even be unconvinced of the very existence of precise truth conditions, then to what extent can they be said to be competent speakers of the language to which those words belong? A careful characterisation of borderline cases is therefore integral to an understanding of how vagueness affects language use, and it is to this end that this investigation is aimed.

The investigation to follow will be organised as follows: in this chapter we will define in more precise terms the phenomena we will be concerned with for the bulk of the dissertation and provide practical examples in order to get a better grasp of the rather small part of the puzzle we will attempt to address. We will then review an excerpt of the evidence, results from experiments that support the claim that ordinary speakers accept borderline contradictions to be true, in Chapter 2. Chapter 3 will continue in this vein, reporting on three new experiments. The discussion to follow in Chapter 4 will take stock of what we have learned and how a theorist might supplement his or her preferred theory in order to provide an account for the data.

1.2 Borderline cases and the borderline region

A drizzly situation in which it is difficult to tell whether, in the provided context, either *it is raining* or *it is not raining* is true, is a *borderline case* of the predicate *is raining* in that context. The paradigmatic example of a borderline case is that associated with the paradigmatic example of a vague predicate, namely, the individual of average height who is somewhat *tall*, somewhat *not tall*, and not clearly either. In this case, and assuming the appropriate context, we say the individual, let us call him *Sam*, is a borderline case of the vague predicate *tall*, $tall(Sam)$ represents the borderline proposition that *Sam is tall*, and 'Sam is tall' is a borderline sentence.

We now turn to the semantics of the gradable predicate to give us a better idea of what it means to be a borderline case, and there are a few options here. Before proceeding, however, it is important to recognise that a the majority of the proposed semantics for gradable predicates are not theory neutral with regard to how borderline cases are characterised, and we will devote some time to grappling with this issue. What we would ideally like to capture is the unadorned intuition that there is some borderline region where unclear cases of the predicate reside.

We contend that the simplest representation of the idea of a borderline region requires an initial notion of tripartition: in a given context, some individuals are clearly tall, some are clearly not tall, and some are borderline tall. However, this picture appears to pose a couple of serious drawbacks. First, it appears to allow us to divorce the existence of borderline cases from the contrast between vagueness and precision. No loss of precision is entailed in the move from bipartition to tripartition; the sharp distinction between extension membership and non-membership is instead distributed into two sharp distinctions: one between extension membership and borderline membership, and the other between borderline membership and non-membership. Yet, the idea that a predicate may be precise and have borderline cases runs counter to the intuition that the presence of borderline cases is indicative of the predicate's being vague. Precision ought to be degraded in the presence of borderline cases, and this notion goes hand in hand with the association between vagueness and a lack of sharp transitions. Indeed, most if not all theories of vagueness entertain some notion that on some level, there ought to be no identifiable sharp transitions. For example, a fuzzy theory of vagueness is certainly motivated by a desire to represent vagueness in terms of smooth transitions between extension membership and non-membership, regardless of whether or not it is ultimately successful in that aim. Meanwhile the epistemic approach to vagueness, while committed to sharply bounded extensions at the level of the underlying semantics, certainly does not contend that a speaker's epistemic relation with that boundary is sharply delineated.

All this being said, precise tripartition may not be a misguided first step toward a basic definition of borderline cases, with the potential to be refined in more theory specific terms to capture a more natural notion of imprecision. It may not be the case that the move from bipartitioning to tripartitioning does not incur a loss precision, depending upon how precision is defined. That is, we might argue that the characterisation of precision in terms of sharp transitions is misguided, or more accurately, the opposition of vagueness to a notion of precision defined as the presence of sharp transitions is misguided. For example, if the transition from extension membership to non-membership is represented as continuous, that is, there is a continuum of borderline P-ness into which individuals may be mapped, then an equivalent representation of this continuum is in terms of a non-terminating process of tripartitioning. In this case, it would be difficult to isolate any single instance of tripartitioning as the instance which transforms a discrete series of borderline categories into a continuum of borderline-ness. Rather, it seems more likely that every finer tripartitioning shifts the transition towards a limit, that of being continuous. In this sense, loss of precision in the presence of a smoother and smoother transition appears to be an illusion brought on by evaluating the transition at finer and finer granularities. Vagueness might therefore be characterised as standing in opposition to course transitions, that is, consisting of relatively few sharp cut-offs, rather than to sharp cut-offs, per se. In this sense, the first shift in granularity, from bipartition to tripartition, while incurring no decrease in the sharpness of individual cut-offs, is nevertheless a shift toward a higher degree of vagueness.

Another drawback is that tripartition invites us to characterise the borderline region in one of three ways: either as an overlap between the extension of P and the extension of *not* P, as an underlap between the same, or as some region of instability with regard to the first two options. This contrasts with fuzzy theories where intermediate truth values do not overtly invoke notions of either overlap or underlap. The fuzzy theorist would instead characterise the borderline region as a transitional region, and while we may be able to capture this notion of transition by characterising the region as one of half truth, we do so at the cost of abandoning the smoothness afforded by a fuzzy set of truth values, while at the same time forfeiting the potential predictive benefits of an overlap/underlap characterisation.

There is no doubt that a complete account of the borderline region both ought to account for the intuition of smooth transitions and ought to characterise the relationship between the positive and negative extension of a vague predicate in a manner that accurately describes and predicts speakers' use of the predicate, whether as a gap, or a glut, or some possibility we have so far neglected to discuss. However, we will argue that the tripartite picture is the fundamental first step away from bivalence, in appearance at least[1], upon which these and other more refined characterisations may be compositionally

[1]Although the notion of tripartition might seem to exclude classical theories from the discussion of borderline-ness, it need not, provided that the tripartition is not on the notion of truth. Any realistic account of vagueness which adheres to bivalence must still invoke the notion of tripartition in order to agree with speakers' basic intuitions and usage. For instance, epistemic approaches commonly invoke a tripartition with respect to knowledge in order to define borderlineness: in a given context there are those propositions which are known to be true, those known to be false, and those about whose truth values we are ignorant.

built. After all, even a continuous transition can be understood as the application and re-application of tripartitioning at every extension boundary, all the way down.

That is to say that approaches to the semantics of predicates have the potential to differ in their granularity, insofar as they differ in the fineness of the distinctions they are able to capture over the course of the transition between a predicate's extension and non-extension. A bipartite approach is coarsest; a continuous approach (infinitely many partitionings) is finest. A tripartite approach, in whatever manner tripartition is realised in terms of a specific theory, is the coarsest approach which satisfies the minimal requisite to accommodate vagueness, namely, a notion of borderlineness. It is for this reason that we shall take the notion of tripartition as the fundamental first step for a coarse definition of borderlineness.

One way we might attempt to provide this coarse picture is via a semantics for gradable adjectives based in degrees [57, 14, 66, 8, 29, 31][2]. The assumptions here are that a gradable predicate is associated with a totally ordered set of abstract units of measurement on an appropriate dimension, in other words, a one-dimensional scale composed of degrees, and that the predicate is a measure function mapping individuals to degrees on that scale[3].

[2]We leave aside alternative semantics for vague predicates, including the delineation approach [34, 26, 32], as we are confident that either approach would be adequate to account for our conclusions with respect to borderline contradictions.

[3]Following Cresswell [14], we take it that for any predicate P there is an equivalence relation $\approx_P$, defined for any individuals x, y and z as $x \approx_P y$ iff for all z, $x \prec z$ iff $y \prec z$ and $z \prec x$ iff $z \prec y$. This relation with respect to P-ness induces a partitioning

Of course, not all gradable predicates are one-dimensional. A characterisation of borderline cases of multi-dimensional adjectives must be afforded, as well. On that front, a promising proposal is that an averaging function over a set of weighted dimensions, both of multi-dimensional adjectives and, to a possibly greater extent, of nouns, could be key to the evaluation of constructions of comparison, and possibly also to borderline contradictions involving multi-dimensional concepts (see Sassoon [52, 53]). Thus, it may be possible to apply concepts from a one-dimensional approach to the analysis of borderline cases and borderline contradictions even when the relevant predicate is not one-dimensional, although there is assuredly much more to the story in these cases. In what follows though, we will outline a degree-based approach along a single dimension, with the expectation that the borderline regions of most gradable adjectives have at least the potential to be defined analogously.

Following the standard in degree-based semantics of gradable predicates we define the meaning of a gradable predicate as a measure function from individuals to degrees ($\langle e, d \rangle$), incorporating a partial ordering relation. Thus, continuing with the example of *tall*: $[\![tall]\!] = \lambda d \lambda x \in D_{\langle e \rangle}.f_{tall}(x) \succeq d$, where f_{tall} is a measure function from individuals to degrees on a scale of height, and $\succeq$ a binary relation on degrees which is reflexive, antisymmetric, and transitive. The predicate is mapped to a property of individuals

of the domain into the equivalence classes of P. Degrees of P are then just P-equivalent classes, and accordingly, the scale associated to P is just the totally ordered set of these equivalence classes.

by some manner of degree morphology, such as a comparative or a measure phrase. For example, the measure phrase *183cm* might have the denotation $[\![183cm]\!] = \lambda g \in D_{\langle e,d\rangle}\lambda x.g(x) \succeq 183cm$, relating the degree corresponding to *183cm* to some degree determined by a function from individuals to degrees. When *183cm* is applied to *tall*, we then have: $[\![183cm]\!]([\![tall]\!]) = \lambda x.f_{tall}(x) \succeq 183cm$, relating an individual's degree of height to the degree supplied by the degree morphology in such a way that should the relation be borne out, the expression evaluates to be true. In the case of the positive form, which does not appear in most languages to display overt degree morphology, the degree morphology is commonly proposed to be saturated by the null morpheme *pos*, which introduces some context dependent degree or interval of degrees, serving as a standard[4]. Alternatively, we could suppose that the standard is introduced by a free variable whose value is set by a delineation function within context[30, 10]. Since a thorough account of the semantics of gradable predicates is not our primary goal, we will take a shortcut along the lines of the latter approach for our purposes. For gradable predicate P, a threshold θ_P is a contextually determined degree, such that should the degree to which f_P maps an individual either equal or exceed θ_P, then the individual counts as P. For example, in a context which determines θ_{tall} to be equal to 180cm, $[\![tall]\!]([\![Sam]\!])$ is true iff $f_{tall}(Sam) \succeq 180cm$ iff there is a degree on the scale of height to which Sam is mapped which is

[4] A commonly cited example of the null morpheme appearing in overt morphology is that of unstressed 'hěn' in Mandarin Chinese (see Liu [36]). A far less well known contender is an iconic classifier found in Italian Sign Language (see Aristodemo [5, 4]).

equal to or exceeds 180cm. Generally, the denotation of any gradable predicate may be represented as follows in (1).

(1) $[\![P]\!] = \lambda x.f_P(x) \succeq \theta_P$

A *borderline region* for vague predicate P can then be defined as the set of degrees in the interval $[\theta_P \pm \varepsilon]$ on the associated scale, where ε is a contextually determined measure. The intuition is that those individuals whose degree of P-ness does not significantly differ from θ_P, cannot reliably count as either P or *not* P. There is therefore an interval on the scale of P, centering on θ_P, such that should f_P map an individual to a degree in that interval, then the individual is a borderline case of P. We therefore simultaneously define a borderline case of P and the set of borderline cases B_P, to which it belongs:

(2) An individual x is a borderline case of P iff $x \in B_P$ iff $\theta_P - \varepsilon \preceq f_P(x) \preceq \theta_P + \varepsilon$.

The contextually determined measure ε is some minimal value or degree on the relevant scale that licenses discrimination between individuals which count as P and those that count as *not* P. It is therefore strongly connected with Williamson's *margin for error principle*, roughly: if individuals a and b do not differ beyond a contextually appropriate *margin for error* with respect to vague predicate P, then whenever a counts as P, b ought not count as *not* P. Thus, if the difference in *Sam*'s and *Bill*'s heights does not exceed this small measure, then in Williamson's epistemically oriented terms, it cannot be known that *Sam* is *tall* and *Bill* is *not tall* (although it may not be known whether or not either is *tall*, i.e., one or both could be a borderline case of *tall*).

We might also associate ε to the significant value invoked by the *principle of tolerance*: if individuals a and b do not differ by at least some significant value with respect to vague predicate P, be it relevant to perceptual discriminability or to some other contextually relevant parameter, then whenever a counts as P, b ought to, as well; and conversely, whenever a counts as *not* P, b ought to count as *not* P, as well[5]. Thus, if the difference in *Sam*'s and *Bill*'s heights does not exceed this significant value, then whenever *Sam* counts as tall, *Bill* must count as tall, too[6].

Reasonably, the contextually determined measure ε should not only be cast as a defining feature of the borderline region, but should also be incorporated into the denotation of the predicate, itself, i.e., $[\![P]\!] = \lambda x.f(x) \succeq \theta_P + \varepsilon$, in keeping with the observation that the use of vague predicates is generally reserved for those cases which are not borderline [7]. For example, if

[5] The contextually determined measure ε may be compared to the significant difference between ordered objects that must be exceeded to move an otherwise weak semi-order into a partial order [65].

[6] The impression might be that the principle of tolerance is a stronger principle than that of the margin for error, since the latter allows that *Bill* be a borderline case of *tall*, but as stated here, the former does not. Notably though, the principle of tolerance may be restated in terms of dual forces of interpretation of the predicate to allow borderline cases to be related to clear cases in a manner analogous to the the margin for error principle: whenever a counts as clearly P, b cannot count as clearly *not* P, rather it must count as at least somewhat P (and potentially somewhat *not* P, as well).

Sam is a borderline case of *tall*, it would be misleading in most commonplace exchanges to say '*Sam is tall*', since in keeping with Gricean pragmatics, this would likely be interpreted to mean that Sam's height sufficiently exceeds the threshold to both be noteworthy and to leave no doubt as to the veracity of the sentence.

Another aspect of incorporating a contextually determined value into the denotation of the predicate is that this measure might be expected to vary across contexts, according to the degree of vagueness that is appropriate to the context, and thus a degree of vagueness is built into the denotation of the predicate. This flexibility affords us the possibility of casting the breadth of the borderline region as directly proportionate to the contextually appropriate degree of precision. Leaving ε out of the denotation of the predicate robs us of the entailment that where $\varepsilon = 0$, the predicate is both "precise" (in the sense of the absence of vagueness) and consequently has no borderline cases[7].

The immediately apparent problem with this method of incorporation is that it builds an extension gap into the semantics of the predicate and we wish to avoid favouring any particular characterisation of the borderline region as necessarily a gap or glut or anything else at this point. This puts us in a difficult position, however, since the semantics given in (1) is not neutral

[7] A possible alternative might to be to define the threshold value not as a degree, rather as an interval, in which case the borderline region of P might be defined as equal to θ_P, thereby allowing for borderline cases despite precision. This alternative however still poses the problem of assuming the presence of a gap.

with respect to whether gradable predicates are fundamentally precise and classical; according to (1), they are, and the source of vagueness therefore lies somewhere outside the underlying semantics of the predicate.

A step in the direction of a solution to this dilemma could be to provide dual denotations: $[\![P_t]\!] = \lambda x.f(x) \succeq \theta_P - \varepsilon$ and $[\![P_s]\!] = \lambda x.f(x) \succeq \theta_P + \varepsilon$, the former corresponding to a weak interpretation of the predicate, and the latter to a strict interpretation. On this approach the borderline region is neither assumed to be a gap or a glut, rather its characterisation is dependent upon force of interpretation at play. At the same time, a decrease in the value of ε also entails a narrowing of the borderline region up to and including the situation in which ε is equal to zero, in which case the dual denotations collapse and the predicate is devoid of borderline cases, altogether.

In any case, and regardless of whether we take the semantics of the predicate to be fundamentally classical or dual in the manner just described, the borderline region may be defined as the interval $[\theta_P \pm \varepsilon]$, and precisely so. We therefore adhere to the original definition of the borderline region as this interval with the acknowledgment that this rudimentary definition results in a vagueness which is paradoxically precise in its tripartition. Indeed, in the absence of some notion of either a higher order partitioning or of permanent instability in how a partitioning is achieved, our tripartite first step toward defining the borderline region must be precise. However, we at least now have a rudimentary means of defining a borderline case of P, as well as the collection of P's borderline cases, B_P: the set of individuals whose heights map to a degree within the interval $[\theta_P \pm \varepsilon]$.

1.3 Borderline expressions

Borderline contradictions are sentences of the form $A \wedge \neg A$, or a classical equivalent of this, where A is itself a borderline sentence. They are formal contradictions, although certain versions of them may be satisfiable relative to logical and semantic frameworks which accommodate borderline cases. Moreover, mounting experimental evidence suggests that a majority of natural language speakers are likely to believe these sentences to be true when predicated of a borderline case, suggesting that the primary mode of reasoning when making such judgments is unlikely to correspond directly to classical logic, but if it does, then there must be a compelling reason behind the discrepancy between truth value and reported truth judgment. Furthermore, there are various syntactic forms in which a borderline contradiction might be presented, and while these versions are generally taken to be logically equivalent to each other, classically, they are not necessarily equivalent through the lens of the leading theories of vagueness.

As we will see, the level of speakers' agreement to borderline contradictions appears to correlate with the specific version presented, revealing a pattern of judgments which some theories may be better equipped to account for than others. Thus, insofar as a thorough and explanatory account of vagueness should correctly predict the facts, evidence of such patterns could serve as an invaluable tool in teasing apart those theories which make the cut (with regard to borderline contradictions, at least), and those which might require some supplementation in order to do so. What is more, careful consideration of the effective purpose of this supplementation may be revealing of the commonalities shared by viable accounts, as well as the subtleties by which they differ, thereby setting into relief the basic machinery that makes an adequately predictive theory of vagueness adequately predictive, at least with regard to those predictions which have borne experimental scrutiny, while at the same time laying out the path for future experiments.

1.4 A typology of borderline contradictions

First, a few practical examples are merited in order to illustrate some basic characteristics of the sentences we are concerned with.

(3) a. *Sam is tall and not tall.*

b. $A \wedge \neg A$

(4) a. *Sam is neither tall nor not tall.*

b. $\neg(A \vee \neg A)$

The logical form of the English sentence in (3a) is presented in (3b), as is that of (4a) in (4b). The reader can confirm for him or herself that the two logical forms are classically equivalent per a straightforward instance of de Morgan's Laws, and are both classical contradictions. In the case of the English sentences, if the situation is such that the predicate *tall* is vague, and the individual denoted by *Sam* is a borderline case of *tall*, then we shall refer to these sentences as borderline contradictions.

1.4.1 Conjunction, disjunction

Henceforth, we shall also make a distinction between two main types of borderline contradictions, a *conjunctive borderline contradiction*, exemplified in (3a) and (3b), and a *disjunctive borderline contradiction* exemplified in (4a) and (4b). The significance of this distinction will be taken up in detail later, but at this point it may be worth taking a moment to clarify that the distinction we have in mind is not determined by the conjunctive and disjunctive connectives which incidentally appear in these examples. Although the connectives certainly relate to the distinction, to categorise borderline contradictions purely in terms of overt connectives would be misleading in some cases and possibly inappropriate in all cases, since if they are taken to be classical connectives, then a number of equivalences arise which may not be entirely welcome, especially considering that the very question of whether or not they are equivalent is more or less what is at issue.

Instead, the generalisation undergirding the distinction that we would like to make is that a conjunctive borderline contradiction appears to assert that the borderline individual is a member of both of two extensions, while the disjunctive borderline contradiction appears to deny that the individual is a member of either of two extensions. If we define a complementary pair of extensions such that every individual is a member of one or the other (as opposed to contrary extensions, which are consistent with violations of LEM and permit individuals to be members of neither), then a conjunctive contradiction appears to assert that an individual is somehow located in two "places" at once, while a disjunctive contradiction seems to deny an individual's location in any place, at all. At least, these are the situations which would have to obtain in order for the contradictions to be true. Thus, while both present a violation of bivalence, the conjunctive does so in the manner of a violation of the LNC, the disjunctive in the manner of a violation of the LEM; and this is a distinction which is not governed by the incidental choice between syntactic connectives, it is a semantic distinction.

This touches on a crucial point, that what distinguishes a borderline contradiction from any other contradiction is a semantic distinction and not a formal syntactic distinction. It is the borderline-ness of the contradiction's individual conjuncts in (3b), or disjuncts in (4b), A and $\neg A$, which confer to it this special status. To demonstrate this, consider the case in which the individual *Sam* is 210cm tall, and so is actually a very tall man and hence not a borderline case of *tall*. In this case, the sentence "Sam is tall and not tall" is not a borderline contradiction because there is nothing borderline about Sam's height, and presumably most speakers would agree that it is false.[8]

Similarly, what distinguishes a conjunctive borderline contradiction from a disjunctive one is not necessarily expressed overtly in the syntax, and indeed, sentences may be ambiguous. For example, the A and $\neg A$ clauses in the sentence, "Sam is not tall and he's not not tall" are coordinated by a conjunctive connective, and we might represent the logical form of the sen-

[8]It might be tempting to add truth judgments to the definition of borderline contradictions, since these judgments often appear to mirror the distinction between borderline contradictions and non-borderline contradictions. But while it is certainly essential to recognise this generalisation, it would be a mistaken to define the distinction purely in terms of truth judgments for at least a couple of reasons. First, these truth judgments are not universal, indeed some speakers reject any and all contradictions, out of hand, so truth judgments are not a reliable definitional criterion. Second, not all formal contradictions which appear to be true (or not categorically false) involve borderline cases. For example, "Pegasus neither does nor doesn't have a white hind leg" presents a formal contradiction which might plausibly be judged true, however such a true judgment could be licensed by virtue of "Pegasus"' failure to refer (and likewise for a false judgement), rather than by virtue of vagueness in the property of having a white hind leg.

tence fairly straightforwardly as $\neg A \wedge \neg\neg A$. But this conjunctive connective does not mean that the borderline contradiction must be interpreted conjunctively, for there is a sense in which the sentence can be interpreted as a denial of membership to either of two complementary extensions rather than assertion of membership to both. Paraphrased, this interpretation runs along the lines of, ‘it is not the case that *Sam* belongs to the extension of *tall* and it is also not the case that he belongs to the extension of *not tall*’. That is, by distributing negation, $\neg A \wedge \neg\neg A$ yields a possible interpretation of the form $\neg(A \vee \neg A)$, which we have already defined as disjunctive. If, instead, double negation in $\neg A \wedge \neg\neg A$ is simplified, the resultant expression, $\neg A \wedge A$, is clearly equivalent to our default example of a conjunctive borderline contradiction, $A \wedge \neg A$, and is thus clearly conjunctive. In this case we might paraphrase the interpretation along the lines of, ’*Sam* is a member of the extension of *not tall* and he is also a member of the complement of the extension of *not tall*’, i.e. he is in two ‘places’, at once[9].

These example serve to highlight both that syntactic form and overt choice of connective do not necessarily determine whether a borderline contradiction is conjunctive or disjunctive, and indeed, that some sentences may

[9]As a note, we hypothesise that “both” and “either” have the potential to be used as a diagnostic, or disambiguative, tool, “either” selecting the disjunctive interpretation, and “both” the conjunctive interpretation. Contrast the following, where a disjunctive interpretation is most salient in (1a), while a conjunctive interpretation is perhaps most salient in (1b), although a disjunctive interpretation may still be accessible:

(1) a. *Sam is not tall and not not tall either.*
    b. *Sam is both not tall and not not tall.*

be ambiguous between the two. Like the distinction between borderline and non-borderline contradictions, the distinction between conjunctive and disjunctive borderline contradictions is primarily semantic, and not syntactic. However, the syntax may play a prominent role in a different way, in that the options for interpretation may be limited by the syntax.

As a final note, borderline contradictions, whether conjunctive or disjunctive, can theoretically be expressed at any syntactic level from the phrase up, so long as the borderline proposition $A \wedge \neg A$ is put across. Arguably, according to this definition, any linguistic unit of the form $A \wedge \neg A$ could count as a borderline contradiction, provided it is predicated of a borderline concept.

The broadness of this definition of a borderline contradiction means that a myriad of more compositionally complex sentences also fall under the label of borderline contradictions, in addition to these primary exemplars. Thus, for example the following English sentences are also borderline contradictions.

(5) a. $[[$*Sam is tall*$]_{A'}$ *and*$[$*not tall*$]_{\neg A'}]_A$ *and* $[$*it is not the case that* $[$*Sam is tall*$]_{A'}$ *and* $[$*not tall*$]_{\neg A'}]_{\neg A}$.

b. $[$*Sam is tall and not tall or he's a fictional character*$]_A$ *and* $[$*it is not the case that Sam is tall and not tall or he's a fictional character*$]_{\neg A}$.

c. [*Sam is tall or not tall*]$_A$ *and* [*it is not the case that Sam is tall or not tall*]$_{\neg A}$.

d. [*Sam is tall and not tall and it is not the case that Sam is tall and not tall*]$_A$ *and* [*it is not the case that Sam is tall and not tall and it is not the case that Sam is tall and not tall*]$_{\neg A}$.

For the time being, we leave aside the question of whether a borderline contradiction can come in the form of $A \wedge \neg A'$, where A and A' are classically equivalent but not syntactically equivalent. For example, *Sam is tall and not tall and it is not the case that Sam is neither tall nor not tall* can be converted to a form equivalent to $A \wedge \neg A'$, where on a deeper level, the A conjunct in this definition consists of a conjunctive borderline contradiction while the A' conjunct consists of a disjunctive borderline contradiction. Since gappy and glutty approaches to vagueness differentiate between conjunctive borderline contradictions and disjunctive borderline contradictions, this is a considerably more thorny question than it might at first appear.

1.4.2 Elision

We would like to take this opportunity to address the fact that our main examples of borderline contradictions involve elision of the subject in the second clause. A concern here is that elision might have an important effect on discerning genuine contradiction from what merely appears to be contradiction. For example, in the non-elided sentence, “Sam is tall and Sam is not tall” each instance of the name “Sam” could have a different referent, and so there are readings of the sentence in which it is not necessarily a contradiction. However, to the extent that we can reasonably fix the referent of “Sam”, we do not foresee problems arising from elision of the subject. A related and intriguing question arises in the case that referents both differ and are both borderline cases of *tall*, as this is not a problem of reference, but rather of consistency in how the predicate is interpreted. Of much greater concern with regard to elision is the possibility for shifts in the predicate.

Experimental work reported by Ripley [48] (see Section 2.3) seeks to address this very problem. Ripley's concern was with shifts in the meaning of the predicate from one property to another, in the manner of what he refers to as Soames' ‘indexical contextualism’ [60, 61]. For example, in the non-elided sentence “Lucy is smart$_1$ and Lucy is not smart$_2$”, $smart_1$ might refer to the property of having mathematical skill, while $smart_2$ might refer to the property of street savviness, in which case, the sentence is both true and non-contradictory if Lucy turns out to be a maths whiz who is not particularly adept at sniffing out a snake oil salesman. Ripley compared non-elided sentences of this form to elided versions in which it is the predicate which is elided. For example, his elided sentences were of a form equivalent to, “Lucy both is and isn't smart”, in which the predicate makes only one appearance, and presumably may therefore only express either $smart_1$ or $smart_2$. Ripley's results indicate that the effect of this type of elision is non-significant. He takes this as evidence that the predicate does not shift indexically, and thus agreement to borderline contradictions is not licensed by indexical shifts, but he also acknowledges that subtle shifts in context might be responsible for licensing agreement in what he calls the 'non-indexical contextualist approach' promoted most notably by Raffman [45, 46].

On this approach, shifts in the predicate need not be overt. Instead, subtle, transient shifts in context, such as subjective evaluation from moment to moment, may license borderline contradictions while not requiring that these shifts be signalled explicitly. We would further suggest a middle ground between these two possibilities, in that the single appearance of *smart* in the sentence "Lucy both is and isn't smart", might plausibly convey both $smart_1$ and $smart_2$, simultaneously, especially if the contradiction is followed up by some manner of clarification: "I mean, in the sense that she's a brilliant mathematician but she's an ideal mark for a con man"[10].

It therefore seems that elision of the predicate does not reliably exclude indexical contextualist interpretations; both types of contextualism are plausible, regardless of elision. Now, our paradigmatic borderline sentence, "Sam is tall and not tall" does not involve elision of the predicate, but we contend that this lack of elision neither necessarily favours nor rules out an indexi-

[10]For an example from the wild, consider, "Instead, support coalesced around Bolsonaro, **who both is and isn't an outsider**: He has served for nearly three decades in Congress, but he has often been at the margins of that institution and he painted himself as just the strong man Brazil needed to dismantle a failing system" [16]. Despite elision, two different senses of "an outsider" are intended, as explained in the follow-up clarification.

cal contextualist account. That is, a discrepancy in agreement to "Sam is tall and not tall" and "Sam is and isn't tall" while of great interest, cannot be taken as evidence either for or against either type of contextualism. We therefore set aside the various different elided forms for the time being, in favour of focusing on factors which might differentiate between the explanatory powers of contextualism, on a whole, and other accounts of agreement to borderline contradictions.

1.4.3 Higher order borderline contradictions

It is worth a moment to briefly touch on "higher order borderline contradictions". There are a couple of ways of framing what these might be. One is to focus on the individual of whom the contradiction is predicated. For example, a contradiction of the form $A \wedge \neg A$, where A is of the form $A \wedge \neg A$, is maximally true just in case the individual of which it is predicated is a second order borderline case. So, "(Sam$_i$ is tall and not tall) and (he$_i$'s not tall and not tall)" is maximally true just in case "Sam" is a borderline case of a borderline case. Theoretically, such embedding could be repeated *ad infinitum* in the case of every higher order borderline case, although as with higher order borderline cases, there would seem to be a low limit to the practicality of such expressions.

A second tack is to shift focus to a the meta-linguistic concept of a borderline case of a borderline contradiction, whether predicated of a second order borderline case or not. A straightforward example of the first possibility is a first order borderline contradiction predicated of a second order borderline case: “Sam is tall and not tall” when the individual “Sam” refers to neither clearly belongs to the borderline region of *tall*, nor clearly doesn’t belong to the borderline region. In such a case, the truth value of the borderline contradiction is expected to be degraded. In this case it is a borderline case of a borderline contradiction due to its non-strict adherence to the criteria by which we have defined a borderline contradiction. Another possible type of borderline case of a borderline contradiction might be a contradiction in which the predicate is not clearly sufficiently vague to license a borderline contradiction, for example, “Lucy is pregnant and not pregnant” is arguably a borderline case of a borderline contradiction in that the extent to which *pregnant* can be interpreted vaguely is severely limited. Another possibility for a borderline case of a borderline contradiction, is a contradiction which is not clearly contradictory. Such might be the case of a contradiction that may or may not involve subtle contextual shifts in the meaning of the predicate, rendering it neither clearly contradictory, nor clearly non-contradictory, an interpretation which is very much up for debate. In this sense, the concept of such borderline cases of borderline contradictions touch directly upon the fundamental question of whether there are true contradictions.

1.5 The information of borderline contradictions

A crucial feature of borderline contradictions is type and quantity of information conveyed by them. Unlike a positive sentence such as, "Sam is tall", the borderline contradiction, "Sam is tall and not tall" conveys specific information about the measure of *Sam*'s height, and unlike a sentence with an explicit measure phrase such as, "Sam is 183cm tall", the borderline contradiction conveys specific information about context and how the predicate *tall* is to be interpreted within context.

Consider the situation in which speaker A knows nothing of *Sam*'s physical aspect, while speaker B has decent knowledge of *Sam*'s appearance, including a good understanding of *Sam*'s height.

(6) A: Tell me about Sam. What does he look like?
B: He's tall. He has dark hair and wears glasses.

In this exchange, speaker B's utterance of "he's tall" conveys that $f_{tall}(Sam) \succeq \theta_{tall} + \varepsilon$, per our semantics outlined in section 1.2. Like a comparative, the main contribution of the utterance is therefore to establish an order, and in order to gain an insight into *Sam*'s height, speaker A must draw upon some implicit cue as to what the measure of θ_{tall} and what the measure of ε are. Otherwise, the utterance conveys nothing about the actual measure of *Sam*'s height. Now consider a variation of the conversation in (7).

(7) A: Tell me about Sam. What does he look like?

B: He's about five foot eleven. He has dark hair and wears glasses.

The measure phrase, "five foot eleven", anchors *Sam*'s height: $f_{tall}(Sam) = 5'11"$, and therefore conveys specific information about the measure of *Sam*'s height, but conveys no information regarding the order between *Sam*'s height and $\theta_{tall} + \varepsilon$, leaving speaker A to depend upon some implicit cue as to whether *Sam* counts as *tall*. So, while the exchange in (6) provided the order, it provided no anchor for *Sam*'s height, and while the exchange in (7) anchored *Sam*'s height, it provided no information about the order. Furthermore, neither exchange conveyed the measure of either θ_{tall} nor ε (see Barker 2002 for details on the information conveyed about the threshold vis à vis information about the individual)[7]. Now consider the final exchange in (8).

(8) A: Tell me about Sam. What does he look like?

B: He's neither tall nor not tall. He has dark hair and wears glasses.

In this case, three types of information are conveyed by the borderline contradiction. First, speaker B's utterance conveys that *tall* is a vague predicate, and concomitantly, that since the borderline region is non-empty the semantics at play is not bivalent. That is, $\varepsilon \neq 0$. To appreciate this contribution, consider what information would be conveyed had speaker A retorted with an analogous tautology. Speaker A's retort that, "he's either tall or not tall," would serve to "correct" speaker B's assertion that *tall* is vague in context, and would re-assert bivalence; that $\varepsilon = 0$. In a sense, the borderline contradiction and its tautological twin are implicit instructions as to whether vagueness is appropriate to the context of the exchange, that is, whether a non-zero measure of ε is permissible.

A second, but intimately related piece of information that is conveyed by the borderline contradiction is the order $\theta_{tall} + \varepsilon \succeq f_{tall}(Sam) \succeq \theta_{tall} - \varepsilon$, since whatever degree the output of the measure function applied to *Sam* is, it must be within the interval of the borderline region. The borderline contradiction therefore conveys at least as much information about the order between *Sam's*'s height and the threshold of *tall* as does the positive statement in (6). As a contrastive reference, the borderline contradiction's tautological partner does not deliver a unique order.

Furthermore, there is the potential for an informative interaction between what is known about *Sam*'s height and what is known about θ_{tall}. The more that is known about Sam's height, the better the degree of the threshold can be pinpointed, and vice-versa. So, in a context in which the speakers have *Sam* within their sight, and therefore have an idea of the degree of his height, speaker B's utterance of the borderline contradiction conveys that the difference between θ_{tall} and $f_{tall}(Sam)$ does not exceed ε. The range of possible values for $f_{tall}(Sam)$ is thus constricted, enhancing speaker A's knowledge of it. Speaker A's knowledge of the contextually determined measure ε is also augmented, as it similarly cannot be smaller than the difference between θ_{tall} and $f_{tall}(Sam)$. So, not only is it asserted that *tall* is vague in context, but the degree to which it is vague, that is, the size of the appropriate margin of error, is also potentially communicated. Symmetrically, if speaker A has some notion of what θ_{tall} is and what margin of error might be appropriate to the context, then the speaker can also form a good approximation of the degree of *Sam*'s height. Thus, with limits, the borderline contradiction provides a complex of information relating both to order and to measurement. It also provides cues as to what degree of vagueness is appropriate to the evaluation of the predicate within the given context.

Part II

Borderline Experiments

Chapter 2

A survey of notable experiments

Until about two decades ago, the majority of the study of vagueness was carried out 'in the armchair', with little concern for the intuitions of ordinary speakers. It was only beginning with Bonini et al.'s 1999 experiment that the experimental study of vague phenomena truly took off[1], although this must be qualified, for not all vague phenomena have been afforded an equal amount of experimental attention. In particular, excluding studies into effects of hysteresis and enhanced contrast with regard to soritical series

[1] This is not entirely true, as some early studies in related phenomena had already been conducted at around the same time, including a very small exploratory exploratory experiment into fuzzy intuitions by Parikh [41], a protoypicality experiment by Hampton [23], and a colour hysteresis study by Raffman, Brown, and Lindsey (reported in Raffman 2013 [47]).

of stimuli [20, 47], very little has been done in the way of testing speakers intuitions regarding the truth of the premises of the Sorites, nor directly into the Principle of Tolerance. The majority of work has actually centred around borderline contradictions, sometimes drawing upon related borderline sentences for comparison. We review here the series of experiments which directly culminates in our three experiments to be presented in the following Chapter. Thus, this survey is neither intended to be exhaustive, nor is it limited to so-called "borderline contradiction" experiments. We are also interested in early data that sparked researchers toward their hypotheses.

Of course, there is always a major concern that is raised for these types of experiments, namely, how researchers should interpret participants' responses relative to the framing of the response they are asked to provide. Some experimenters ask whether a sentence is true, some ask whether it is true to utter the sentence, some ask about agreement with the sentence. But it is difficult to be sure that a response to any of these reflects truth. For example, if a participant is allowed to indicate how true they think a sentence is by choosing a score between 0 and 100, we cannot be sure that the score they choose reflects how true they think it is, how true they are willing to say that it is, or something else, such as how assertible they find it, or how informative they find it. Since borderline contradictions are highly informative, there is good reason to use them in everyday speech, but not everything that is informative and common to everyday speech need be true, logically or otherwise, in order for speakers to agree with it. Cultural biases may also accompany participants' responses, a possibility pointed out in particular by Ripley [48]. Frequency might also have a significant effect on whether speakers find a given sentence acceptable, a question that might be cleared up to some extent by corpora studies[2].

Despite this inherent concern, when a pattern of judgments remains across various strategies of providing stimuli and requesting responses, an argument can be made for the validity of a conclusion, as a whole. One such conclusion is that ordinary speakers do not consider borderline contradictions to be outright false. Another is that speakers do not generally consider a disjunctive borderline contradiction to be less true than its conjunctive counterpart. Beyond these two conclusions, results from the studies to be discussed greatly diverge, however the ways in which they diverge may be enlightening, a theme we will return to at the conclusion of the survey.

[2]This task is hindered somewhat by the size of available corpora, as borderline contradictions are not a common construction, and by the need to for the researcher to interpret instances of formal analogues of "$predicate_i$ and not $predicate_i$" as actually contradictory, i.e., other constructions are also likely to contain these strings. Additionally, since it is not settled whether borderline contradictions are actually contradictory, that is, it could be that they mask implicit contextual shifts, it is not clear whether instances in which such a strategy is explicitly marked can be counted as borderline contradictions, not to mention instances where a contextual shift may be implied but not made explicit.

2.1 The Bonini, Osherson, Viale, and Williamson (1999)

One of the first attempts to investigate the borderline region experimentally was a 1999 experiment conducted and reported by Nicolao Bonini, Daniel Osherson, Riccardo Viale, and Timothy Williamson. While the experiment did not probe borderline contradictions per se, it signalled a shift toward the idea that philosophical notions about the borderline region could be justified not only theoretically, but also through experiments into the psychology of speakers.

The researchers were interested in comparing three theoretical characterisations of the borderline region: a trivalent characterisation, which they further broke down into either a truth gap or a truth glut characterisation; a fuzzy characterisation, invoking thresholds for assertibility and deniability; and the epistemic characterisation, according to which speakers lack beliefs about the truth values of borderline cases. They additionally adopted the assumption (which does not apply to the epistemic characterisation), that speakers typically know the truth value of vague predicates applied to common objects and assent to a vague expression if they consider it to be true. They further assumed in the case of the fuzzy characterisation, that the threshold for assertibility is typically associated with a degree of truth higher than .5, and symmetrically, with a degree less than .5 for deniability.

Out of these foundations they developed a pattern of predictions regarding the boundaries of the borderline region, summarised as follows:

1. if x is the least degree of the relevant property of vague predicate P, such that P applied to an individual mapping to that, or a greater degree of the property, is true; and y is the greatest degree of the relevant property, such that P applied to an individual mapping to that, or a lesser degree counts as false, then:

    (a) if the borderline region is a truth gap, then x is appreciably greater than y;

    (b) if the borderline region is a truth glut, then x is appreciably less than y;

    (c) if the borderline region is a fuzzy spectrum of truth, the x is appreciably greater than y;

    (d) if speakers lack beliefs about truth values in the borderline region, then x is appreciably greater than y.

Their final prediction relies upon the assumption that in a state of ignorance regarding the truth values of borderline expression, speakers prefer to commit errors of omission rather than commission. Speakers are therefore likely to conservatively set the lower boundary of the predicate's positive extension at the degree at which they are confident that the predicate applies, and to set the upper boundary of the predicate's negative extension at the degree at which they are confident that it does not apply. An unfortunate result of this pattern of predictions is that if all that is probed experimentally is the relative degrees associated to x and y, then the most that can be concluded is either that the truth gap characterisation is supported, or that the truth glut characterisation is uniquely not supported (or neither in the highly unlikely case that $x = y$). Essentially, what is presented as four separate predictions actually boils down to two. Further, in the less interesting but equally plausible case that x and y reflect no clear and stable ordering relation, all that can be concluded is tripartition, in other words, it can be inferred merely that the predicate is vague.

All studies were carried out via questionnaire. A total of 737 social science undergraduates from Italian universities were recruited to participate, none in more than one study, with roughly half providing judgments just about x and half providing judgments just about y, never both. In study 1 participants provided a measurement for x, the lower bound of the positive extensions of *tall*, *mountain*, and *old*[3], and a measurement for the upper bound y of the negative extension of these predicates. A Mann-Whitney U-test allowed the authors to reject the hypothesis of no difference between the median measurements provided by the two groups, revealing x to be substantially greater that y. Study 2 replicated these results with participants responding to an additional three vague predicates, *long*, *high*, and *far apart*.

[3]Rather, what the authors have translated as "tall", "mountain", and "old" in their report. All of their studies were carried out in Italian.

Study 3 in turn replicated study 2 with six further vague predicates, *late for an appointment*, *poor*, *dangerous*, *expensive*, *high*, and *populous*. Study 4 was identical to the three previous experiments, except that rather than being questioned meta-linguistically in terms of when it is true or false to say that the predicate applies, participants were asked directly whether or not the predicate applies. Study 5 replicated the results of study 4 using shortened questions and fewer instructions. Study 6 explicitly introduced tripartition into the instructions, referring to the positive extension, a 'medium' extension, and the negative extension of the predicates. Finally, study 7 probed participants' intuitions about the boundaries of the average value of the predicates, rather than the predicates, themselves.[4] All studies revealed a significant gap between x and y, in nearly all cases.

Although the authors' analysis of the results favours an epistemic characterisation[5], there is nothing inherent to the data they collected which requires this analysis, and both their methods and conclusion have been sub-

[4]The authors assumed that it is common knowledge that averages have definite cut-off points, although the value of the cut-off is not usually known. They took gaps in response to these questions about averages as evidence that gaps are expected in case of a definite but unknown cut-off. Of course, it might successfully be argued that both *tall* and *of average height* are vague predicates both lacking bivalent cut-off points, as pointed out by Serchuk et al. (see section 2.2, to follow).

[5]They argued against non-epistemic trivalent characterisations on the grounds of the 'psychological implausibility' of higher-order vagueness and against a fuzzy logic characterisation on the grounds of the implausibility of conjunctive contradictions of the form $A \wedge \neg A$ having the same truth value as conjunction such as $A \wedge \neg B$, when all components have a truth value of .5. They also argued that the meta-linguistic studies should have revealed larger gaps than the non meta-linguistic studies on these approaches, an argument which is convincingly refuted by Alxatib and Pelletier in their 2011 response [2]. They argued for the epistemic characterisation primarily on the grounds that it preserves LNC and LEM.

jected to a good share of criticism over the twenty years since publication. However, their gappy result has repercussions for later experimentalists, in particular Alxatib and Pelletier [2]. Of course, this gap could be analysed in a number of ways, however as we will see in the results of the next experiment, it is likely not a reliable observation.

2.2 Serchuk, Hargreaves, and Zach (2011)

Serchuk, Hargreaves, and Zach published a report of a series of four experiments into vagueness adjacent topics, two of which are directly relevant to borderline contradictions. The first was an intentionally partial replication Bonini et al.'s results, accompanied by a revised experiment which improved upon their methodology. We will not go into all of the fine points of the authors' critique of the appropriateness of Bonini et al.'s methodology to testing epistemicism, nor their critique of the epistemic interpretation of results, but we will concern ourselves with their methodological objections and improvements.

The primary methodological problem Serchuk et al. pointed to was that although Bonini et al. were interested in, for the example, the lowest bound of *tall*, the question they actually posed to participants could have been understood as asking for just some height above which an individual counts as *tall*, but not necessarily the least height at which an individual counts as tall. Serchuk et al. therefore designed a replication experiment with the ambiguous wording intact, and a revised experiment, corrected to ask only for the least bound. There were two versions, one using 'vague' predicates, *tall*, *old*, and *long*, and one intended to replicate study 7 using averages and therefore 'crisp' predicates. As participants, 713 undergraduates were recruited from the student populations of the University of Calgary and the University of Toronto.

The results replicated a statistically significant gap for the 'vague' predicates with Bonini et al.'s wording intact, however results from the revised wording saw the gap disappear, and in the case of *tall*, reverse to reflect a glut. In the case of the 'crisp' predicates, the gap could not be replicated for two of the three predicates. Revised wording revealed two statistically significant gluts, instead. The authors' summary of results are reproduced in Table 2.3.

Table 2.1: Serchuk et al. vs. Bonini et al.

	predicate					
study	*tall*	*average height*	*old*	*average age*	*long*	*average length*
original	gap	gap	gap	gap	gap	gap
replication	gap	gap	gap	n.s.	gap	n.s.
revised	gap	glut	n.s.	glut	n.s.	n.s.

Although we refrain from repeating much of Serchuk et al.'s criticism of Bonini et al.'s statistical methods[6], as well as their many criticisms of

[6] In a nutshell, they objected that since Bonini et al.'s hypothesis concerned the means of a predicates upper and lower bounds, the statistical test should have been one that

Bonini et al.'s theoretical arguments for epistemicism, it is worth pointing to two main take-aways from their study and analysis. One, there is not strong evidence for a statistically significant gap rather than a glut in the results of a study such as this, and two, it is unclear what the finding of a statistically significant gap or glut would mean theoretically, since neither epistemicism nor tripartite theories require that such an effect be statistically significant, nor would they be unequipped to plausibly account for any outcome of such an experiment. Any result would therefore be relatively unilluminating with regard to theory.

The second relevant experiment, which Alxatib and Pelletier [2] would come to draw from, was concerned with disambiguating between three possible forces of negation: strong (or what they compare to Kleene negation), weak negation, and intuitionistic/Gödel negation; a truth table for all three of which is shown in Table 2.2, where, following Alxatib and Pelletier, 'O' stands in for "other"/"unknown".

Table 2.2: Three forces of negation

p	strong $\sim p$	weak $\neg p$	Gödel$-p$
T	F	F	F
O	O	T	F
F	T	T	T

The experimenters' hypothesis was simply that there are at least two types of negation, with an auxiliary hypothesis that the word, 'not', typically

compares the means of distributions, not the medians.

conveys strong negation, while the phrase, 'it is not the case that', typically conveys weak negation. 350 University of Calgary undergraduates were recruited, roughly half of whom were assigned to the *heavy* group, and half of which were assigned to the *rich* group. Participants were asked to reflect upon the truth of each of six sentences, two of which were borderline tautologies and two of which were borderline contradictions, and were provided the response options: 'True', 'False', 'Neither', 'Partially', 'Both', and 'Don't know' (abbreviated in Table 2.3: T, F, N, P, B, and D/k, respectively). The preamble and set of sentences for the *rich* group are reproduced below.

> Imagine that on the spectrum of rich women, Susan is somewhere between women who are clearly rich and women who are clearly non-rich. We are interested in your opinion about the status of the following twelve[7] sentences. Please check *one* box only for each.

(1) Susan is not rich.

(2) It is not the case that Susan is rich.

(3) Either Susan is rich or Susan is not rich.

(4) Susan is rich or it is not the case that Susan is rich.

(5) Susan is rich and Susan is not rich.

(6) Susan is rich and it is not the case that Susan is rich.

[7] The experiment also included sentences related to a separate experiment, which are omitted here.

Of interest to us with regard to the results for sentences (1) and (2) is that a majority of participants chose a response that was not 'True' or 'False', and neither sentence elicited a response consistent with Gödel negation. With regard to the results for sentences (3) and (4), the borderline tautologies, a key result is that 39% of participants found the (3) to be false, compared to approximately 32% who found it true. Moreover, this means that approximately 68% of participants did not judge this tautology to be true (nearly 62% if a 'Both' response counts for true), suggesting that truth judgments about borderline tautologies are not classical. This effect was less stark for (4), around 40% of responses to (4) were 'True' responses, with 'False' responses trailing not far behind.

Table 2.3: Responses to Serchuk et al.'s borderline contradictions

		Answer to (6)						
		T	N	P	F	B	D/k	Total
Answer to (5)	T	13	1	10	33	0	9	66
	N	2	9	0	10	5	2	28
	P	1	2	3	18	3	3	30
	F	4	6	1	173	2	9	195
	B	2	1	3	10	3	2	21
	D/k	3	0	0	3	0	4	10
	Total	25	19	17	247	13	29	350

In the case of the borderline contradictions (5) and (6), the most frequent response was 'False', with nearly 50% of respondents rejecting both contradictions (see Table 2.3). In contrast, not even 4% of participants answered 'True' to both contradictions. Participants were particularly averse to the periphrastic borderline contradiction in (6), with 247 of 350 answering 'False' and only 25 'True', a result which was entirely unexpected, given the hypothesis that periphrastic negation is weak. Puzzlingly, participants also rejected the responses that would best represent a truth gap or glut, with only 9 participants choosing 'Neither' and a negligible 3 choosing 'Both'. These results are unique in the study of borderline contradictions and are worth serious consideration.

2.3 Ripley (2011)

The next experiment of note was David Ripley's 2009 investigation which directly probed speakers' intuitions regarding borderline contradictions and is a direct precursor of the shape-colour experiment reported in section 3.2. Ripley recruited 149 University of North Carolina undergraduate students and presented them with a slide projection depicting seven square/circle pairs of shapes, each pair incrementally instantiating a different distance between the two shapes. See Figure 2.1.

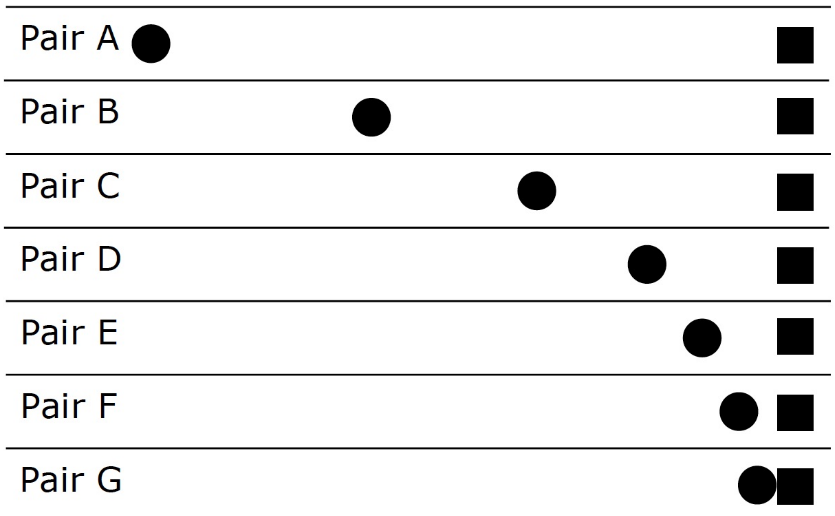

Figure 2.1: Ripley's stimulus slide

Participants were randomly assigned to respond to one of four contradiction types: conjunctive non-elided, conjunctive elided, disjunctive non-elided, and disjunctive elided; and asked rate their agreement to the contradiction as an apt description of each of the seven pairs on a seven point Likert scale, see Table 2.4.

Table 2.4: Ripley's target sentences

Conjunctive	
non-elided	The circle is near the square and it isn't near the square.
elided	The circle both is and isn't near the square.
Disjunctive	
non-elided	The circle neither is near the square nor isn't near the square.
elided	The circle neither is nor isn't near the square.

The crucial result of Ripley's experiment was that mean ratings indicated at least partial agreement to all pairs, with agreement peaking at just over the midpoint of the scale at pair C, the pair that is presumed to be the maximally borderline case of *near*, in this context. Furthermore, Ripley was able to analyse the vast majority of participants' responses as falling into one of four response types: hump, flat, slope up, and slope down. Just over half of participants provided a hump response, meaning that their rating peaked somewhere between pair A and pair G, never falling before the peak, and never rising after the peak. The next most common response was flat (the same rating for all pairs), followed by slope up (rising from pair A to pair G), and finally slope down (falling from pair A to pair G).

No significant contrast in participants' maximal ratings to elided versus non-elided sentences were revealed, nor to conjunctive versus disjunctive sentences. Nor did elision have a significant effect on response type, however conjunctive contradictions were found to significantly correlate with slope up responses, disjunctive contradictions with slope down responses.

Ripley's experiment was exploratory, but the post-hoc hypothesis that he proffers is a dialetheic account of borderline contradictions along the lines of the paraconsistent Logic of Priest [43], primarily motivated by the observation that while mean responses barely rose above the midpoint of the scale, a majority of participants' responses peaked at full agreement, suggesting that these participants found the borderline contradictions not only partially, but fully true.

2.4 Alxatib and Pelletier (2011)

Alxatib and Pelletier [2] took as their jumping off point the results of Bonini et al.'s experiment, however they objected to the dismissal of the gap interpretation of these results, proposing instead, an account which invokes precisifications to derive the gap. On this account, truth is supertruth, and a statement such as "x is tall" is supertrue just in case the individual x is a member of the extension of *tall* in every precisification. Similarly "x is not tall" is true just in case x is not a member of the extension of *tall* in every precisification. If x is a borderline case, then there are some precisifications in which it is a member of *tall*'s extension, and some precisifications in which it is not, therefore neither of the statements, "x is tall" and "x is not tall" is true. The result is a gap between the lower bound of what counts as *tall* and the upper bound of what counts as *not tall*.

There are further predictions to be made involving negation. Alxatib and Pelletier were interested speakers's truth judgements, or lack of judgments, regarding both statements "x is tall" and "x is not tall", and they considered the three forces of negation that interested Serchuk et al.: strong negation (which they alternatively refer to as Łukasiewicz negation), weak negation, and Gödel/intuitionistic negation.

Since according to the epistemic account of vagueness there is true bivalence, whenever p is true, $\neg p$ is false, and vice-versa. Therefore, in a borderline case of *tall*, speakers are expected to be as likely to judge "x is tall" as true, as "x is not tall" as false. However, on the supervaluationist account, "x is tall" cannot be supertrue in a borderline case, and neither can it be superfalse; and likewise, "x is not tall" can neither be supertrue nor superfalse. Speakers are therefore expected to be more likely to judge "x is tall" as false than "x is not tall" as true, and more likely to judge "x is not tall" as false than "x is tall" as true, in line with Gödel negation.

To test this prediction, Alxatib and Pelletier recruited 76 undergraduate students from Simon Fraser University and presented them with an American style police line-up of 5 suspects of varying heights in random order (see Figure 2.2, identified by number. Participants responded to a questionnaire consisting of four statements, repeated once per suspect, for a total of 20 questions per participant.

Table 2.5: Alxatib & Pelletier's target sentences

positive	# 1 is tall.
negated	# 1 is not tall.
conjunctive	# 1 is tall and not tall.
disjunctive	# 1 is neither tall nor not tall.

Three response options were provided: 'True', 'False', and 'Can't Tell'.

The results of the study bore out the authors' predictions, revealing that in the case of # 2, the maximally borderline suspect, significantly more

Figure 2.2: Alxatib and Pelletier's stimulus image

participants were inclined to deny both "#2 is tall" and "#2 is not tall" than to affirm the negation of these statements. As Table 2.6 shows, a greater percentage of participants gave 'False' responses to the positive statement than gave 'True' responses to its negation, and likewise a greater percentage of participants gave a 'False' response to the negated statement than gave a 'True' response to its positive counterpart.

Table 2.6: Percentage responses to the borderline case

	'True'	'False'
# 2 is tall.	46.1%	44.7%
# 2 is not tall.	25%	67.1%

Alxatib and Pelletier were able to replicate Ripley's hump effect in the 'True' responses to both contradictions, observing a peak at the maximally borderline suspect. An inverted hump was additionally observed in the 'False' responses to the contradictions (see Figure 2.3). In addition, Alxatib and Pelletier found that around two thirds of participants who judged both both "x is tall" and "x is not tall" to be 'False' also judged the conjunctive contradiction 'True', and around one third of participants who judged the conjunctive contradiction to be 'True' found those component statements to be 'False'. There was therefore a sizeable proportion of participants who answered non-classically and their judgments did not reflect the rule of $\wedge$-elimination.

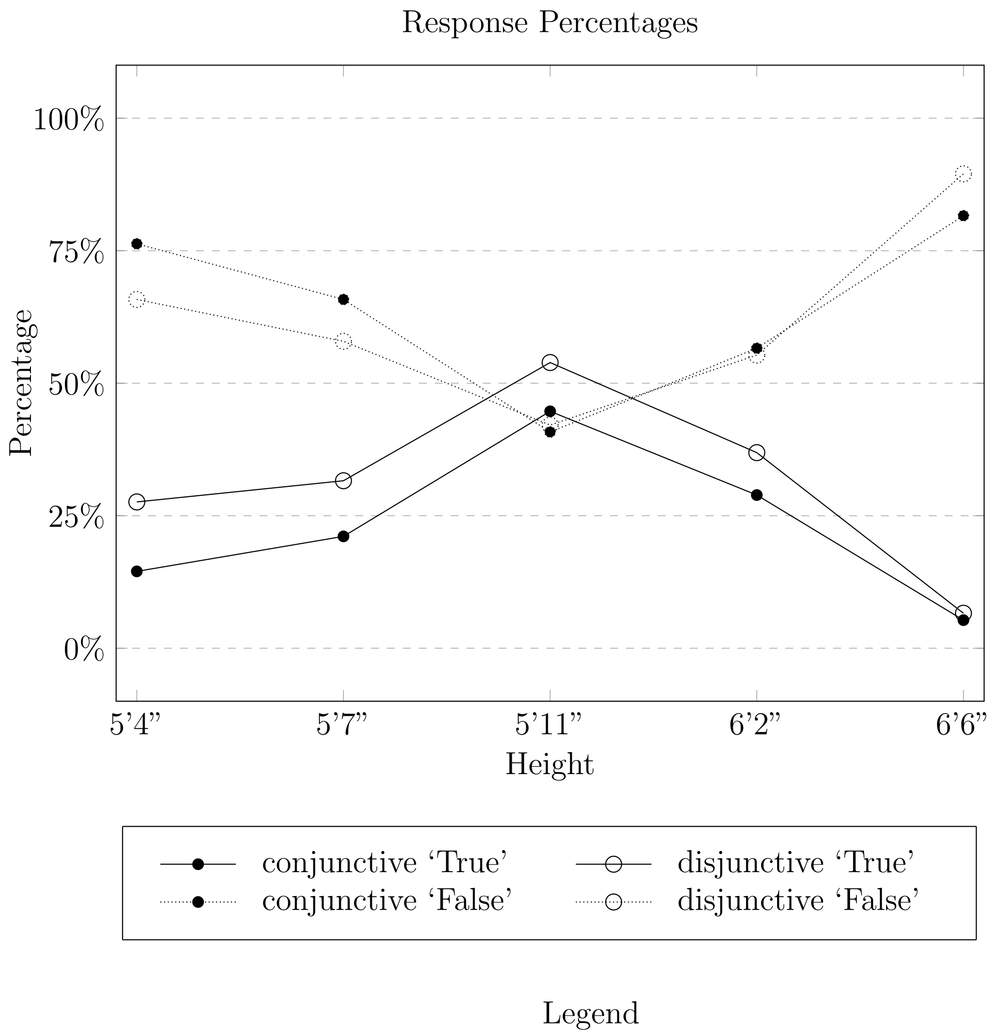

Figure 2.3: Percentage of 'True' and 'False' responses by height

The authors accounted for the pattern of rejection of the simple statements coupled with acceptance of a borderline contradiction by invoking a version of the Strongest Meaning Hypothesis [15], coupled with the notion that vague expressions are ambiguous between a sub-interpretation, in which the set of tall individuals is composed of those individuals that are in the extension of *tall* in at least one precisification, and a super-interpretation, in which the set of tall individuals is composed of just those individuals that are in the extension of *tall* in all precisifications. The Gricean Maxim of Quantity requires that "x is tall" and "x is not tall" be disambiguated toward the super-interpretation, and in case x is a borderline case, these statements are therefore likely to evaluate to 'False'. But in the case of the conjunctive contradiction, the super-interpretation is empty, thus the Maxim of Quality requires a disambiguation toward to the sub-interpretation, which contains just those individuals which are borderline cases. Hence, the conjunctive contradiction carries the implication that the individual it is predicated of is a borderline case. Although Alxatib and Pelletier did not make a point of it, their story also plausibly accounts for a relatively higher acceptance rate for the disjunctive contradiction compared to the conjunctive, in that the disjunctive can be construed to be true of a borderline case under the strongest interpretation of *tall* and *not tall* (super-interpretation), but the same is not true in the case of the conjunctive, which can only be true under a weakened, and therefore non-optimal, interpretation (sub-interpretation).

2.5 Sauerland (2011)

Sauerland's contention was that an account of borderline contradictions rooted in fuzzy logic approach is inadequate to deal with the rate of acceptance of borderline contradictions evidenced in Ripley's, as well as Alxatib and Pelletier's, experimental results. The facts at odds with the fuzzy account are the relatively low acceptance of A and $\neg A$ relative to $A \wedge \neg A$ revealed by Alxatib and Pelletier, as well as the relatively high acceptance rate of borderline contradictions revealed in both experiments, facts which Sauerland attributes to a pragmatic contribution of the contradiction. His experiment was therefore aimed at differentiating between borderline contradictions of the form $A \wedge \neg A$ and $\neg(A \vee \neg A)$ and non-contradictory analogues to them, ($A \wedge \neg B$ and $\neg(A \vee \neg B)$ hypothesising that speakers would more frequently agree with the former than the latter.

The experiment involved a pretest conducted via Amazon MTurk, in which 50 incentivised participants were asked to name a specific value at which an individual is a borderline case of a vague predicate on the relevant scale. The median response for each predicate was then used to generate atomic borderline sentences for each predicate, in order to then be combined into contradictions and non-contradictory analogues for the main experiment. Sauerland's atomic sentences are reproduced in Table 2.7.

Table 2.7: Sauerland's A and B sentences

pair	A/B	sentence
1	A	A 5'10"-guy is tall.
	B	A guy with \$100,00 is rich.
2	A	A car driving at 70mph is fast.
	B	A town 45 miles away is far.
3	A	A 83 degree Fahrenheit day is hot.
	B	A town 45 miles away is far.
4	A	A 2 hour flight is long.
	B	A 50 year old guy is old.
5	A	A 3280-foot mountain is high.
	B	A 10 day vacation is long.

Sauerland recruited 100 subjects for the main experiment, also carried out via Amazon MTurk. Participants were tested on five pairs of sentences under eight conditions per pair (A, B, $\neg A$, $\neg B$, $A \wedge \neg B$, $B \wedge \neg A$, $A \wedge \neg A$, and $B \wedge \neg B$, and each participant never saw each basic item A or B of each pair more than once over the course of the experiment. For example, a questionnaire might contain A and B of pair 1, $\neg A$ and $\neg B$ of pair 2, $A \wedge \neg B$ of pair 3, $B \wedge \neg A$ of pair 4, plus $A \wedge \neg A$ and $B \wedge \neg B$ of pair 5. Participants indicated how true they felt the items were on a 0-100 point scale.

In order to ensure that the analysis of responses to the conjunctive sentences did not include cases which were not sufficiently borderline, Sauerland restricted his analysis to items for which mean agreement was in the 40%-60% range for A, B, $\neg A$, and $\neg B$ components. Only pair 1 satisfied this requirement[8] (mean scores for this pair are shown in Table 2.8[9]). The results indicate a lower than predicted score for both $A \wedge \neg A$ and $B \wedge \neg B$, 48.15 and 46.5, respectively, and no significant contrast between the scores of these two borderline contradictions and the A, B, $\neg B$, and $\neg A$ sentences they are composed of. However, the borderline contradictions benefitted from significantly higher levels of agreement than their non-contradictory analogues, bearing out Sauerland's hypothesis.

Table 2.8: Mean scores for pair 1 (and standard error)

	A	$\neg A$	B	$\neg B$	$A \wedge \neg A$	$B \wedge \neg B$	$A \wedge \neg B$	$B \wedge \neg A$
mean	45	42	45.75	47.25	48.15	46.5	43.2	25.65
SE	(6.7)	(6.0)	(7.0)	(7.0)	(4.3)	(6.1)	(4.3)	(5.4)

2.6 Égré and Zehr (2016)

Égré and Zehr built upon Alxatib and Pelletier's [3] experimental results and accounts that invoke the Strongest Meaning Hypothesis to resolve ambiguity between a strict and a tolerant interpretation of vague expressions (such as the TCS framework [11]), in order to hypothesise a testable contrast between conjunctive and disjunctive borderline contradictions. They observed

[8] As an interesting side note, mean agreement to the conjuncts of Sauerland's contradictions did not significantly differ from agreement to the contradictions, as determined by a signed rank test. That is, even though the scores for A and $\neg A$ differed greatly for certain pairs, the means of A and $\neg A$ pairs did not differ from the scores for corresponding $A \wedge \neg A$, and the same goes for Sauerland's B data; an observation that runs very much counter the fuzzy logic account of conjunction.

[9] No explanation is offered to account for the discrepancy between scores to $A \wedge \neg B$ versus $B \wedge \neg A$.

that disjunctive borderline contradictions are true when interpreted strictly, that is, if *Sam* is a borderline case of *tall*, then it is neither the case that he's strictly *tall*, nor the case that he's strictly *not tall*. But conjunctive borderline contradictions are necessarily false when strictly interpreted. It cannot be the case that *Sam* is strictly *tall* and strictly *not tall*, simultaneously. Since the Strongest Meaning Hypothesis would seem to favour speakers' use of strict interpretation over a less informative tolerant interpretation, they hypothesised that truth value judgments for conjunctive borderline contradictions could be expected to be degraded relative to those for their disjunctive counterparts.

To flesh out this hypothesis they devised an algorithm to model how speakers evaluate truth in borderline cases. The algorithm is based upon a trivalent logic with truth values $\{0, ½, 1\}$, supplemented with tolerant and strict operators that disambiguate strength of interpretation, and it is governed by a few explicit assumptions. The application of the tolerant operator T to a borderline expression with truth value of ½ converts this value to 1. Conversely, the strict operator S converts this value to 0. The operators have no effect on expressions assigned a truth value of 0 or 1.

The T and S operators are assumed to take wide scope over negation, except in marked circumstances.[10] Thus, the strict interpretation of the

[10] As we will discuss in far greater detail in the chapter to follow, speakers may be required to interpret *not not tall* as $\neg S(\neg tall)$, since otherwise this would be equivalent to just *tall*, and presumably speakers would wish to avoid undue prolixity.

sentence 'Sam is not tall' is $S(\neg tall(Sam))$, rather than $\neg S(tall(Sam))$, and is true just in case *Sam* is not a borderline case of *tall*.[11]

Another assumption is referred to as bottom-up strengthening, meaning that the smallest syntactic components are interpreted strongly, then composed into larger constituents, and then these constituents undergo a check for meaning. If the meaning is trivial, defined as necessarily false or tautological, then the smallest syntactic components are assigned the next strongest available meaning, recomposed, and again checked; otherwise, the algorithm proceeds. The process continues until the algorithm ends and produces a verdict on truth.

A final assumption is that speakers prefer simple computational procedures to complex ones, and are licensed to optionally halt the algorithm on a trivial meaning, rather than perform the more taxing operation of reinterpretation. It is therefore not necessary that the verdict be nontrivial. The prediction that falls out from this is that nontrivial verdicts that require complex computations are less frequently reached than nontrivial verdicts resulting from simple computations.

These conditions result in a three-way pattern of predictions for borderline sentence types, exemplified in Table 2.9, in which 'outcome' corresponds

[11] A complication here is that if the disjunctive contradiction is transformed into a conjunction per de Morgan's laws, strict adherence to this assumption would derive the clearly non-equivalent $S(\neg tall) \wedge S(\neg\neg tall)$. What would be preferable in this case is for operators to take wide scope to derive $T(tall) \wedge T(\neg tall)$, per the dual relationship between strength of interpretation and negation.

to some measure of acceptability, be it strength of agreement or frequency of 'True' judgments. If an outcome is degraded, then its acceptability is expected to be somewhere between that of borderline sentences types whose outcomes are 'True' and that of those whose outcomes are 'False'. Although they do not centre in Égré and Zehr's experiment, we include predictions for positive atomic sentences, their negated forms, and non-contradictory analogue forms of the borderline contradictions, as they will be useful to us in the next Chapter. For simplicity, negation is dropped from the B component of analogue sentences, as it makes no contribution to the algorithmic process. Trivial meaning is indicated by an asterisk.

Table 2.9: Demonstration of Égré and Zehr's algorithm

		interpretation	re-interpretation	outcome
$A = 1/2$	$\longrightarrow$	$strict = 0$	$\longrightarrow$	False
$\neg A = 1/2$	$\longrightarrow$	$strict = 0$	$\longrightarrow$	False
$A \wedge \neg A = 1/2$	$\longrightarrow$	$strict = 0^* \rightarrow$	$tolerant = 1 \rightarrow$	degraded
$\neg(A \vee \neg A) = 1/2$	$\longrightarrow$	$strict = 1$	$\longrightarrow$	True
$A \wedge B = 1/2$	$\longrightarrow$	$strict = 0$	$\longrightarrow$	False
$\neg(A \vee B) = 1/2$	$\longrightarrow$	$strict = 0$	$\longrightarrow$	False

Égré & Zehr selected eight target predicates: *rich*, *tall*, *old*, *heavy*, *fast*, *large*, *loud*, and *wide*, of which the first four are characterised as human-oriented, and the last four as object-oriented. The predicates are all gradable, relative, non-evaluative, and individual-level. For each predicate they created a scenario in which a borderline case of the predicate was described. An example scenario is provided in (9).

(9) *A survey on heights has been conducted in your country. In the population there are people of a very high height, and people of a very low height. Then there are people who lie in the middle between these two areas.*

Imagine that Sam is one of the people in the middle range. Comparing Sam to other people in the population, is it true to say the following?

The following four sentences were then presented in a cumulative sequence so that each only appeared after an answer had been provided for the previous sentence, and sentences no sentence disappeared from view until all four had been displayed. Participants were additionally allowed to change their responses to any of the sentences for each scenario up until the point that they moved on to the next scenario. Responses were forced choice, indicated by ticking a box for *Yes* or *No*.

(10) a. *Sam is neither tall nor not tall*

b. *Sam is tall and not tall*

c. *Sam is taller than at least one person*

d. *Sam is taller than everybody else*

148 participants were recruited through the Amazon Mechanical Turk platform, which directed them to an Ibex[12] experiment server hosting the

[12] Software for running internet based experiments, developed by Alex Drummond, hosted by Ibex Farm, 2010-2021.

experiment. Participants were assigned to two groups: one group provided truth value judgments for target descriptions before performing a filler exercise, the other provided judgments after the filler. The filler was a duplicate of the main test, in which negated adjectives were replaced by the corresponding lexical antonym: *short* for *not tall*, etc. Each group was further divided into two subgroups. One subgroup was presented with four object-oriented target items first, followed by four human-oriented target items; the other was presented with these in the inverse order.

The results showed a clear preference for *disjunctive* borderline contradictions over *conjunctive* borderline contradictions, with the mean acceptance rate of the former at around 85-90%, and the mean acceptance rate of the latter at between 25-30%. For reference, the true control had a mean acceptance rate of just under 100%, and the false control at just over 0%. Participants rarely rejected both types of contradiction, but if they rejected one and accepted the other, they overwhelmingly chose to reject the conjunctive contradiction.

A regression model treating group as a factor revealed that participants who completed the filler portion first were significantly less likely to agree to conjunctive contradictions. Similarly, a regression model treating subgroup as a factor revealed a significantly higher acceptance of conjunctive contradictions for object-oriented predicates. Furthermore, three mixed effect regression models, each treating either the disjunctive contradiction, the true control, or the false control as the intercept, revealed highly significant contrasts between all four sentence types (with group and predicate included in the random effects part of the model).

2.7 Summary

As mentioned in the introduction to this survey, the two main takeaways from this body of experimental work is that there is convincing evidence that borderline contradictions are not necessarily judged to be false by ordinary speakers, and that furthermore, conjunctive and disjunctive contradictions appear to be treated differently. Specifically, even though Alxatib, Pelletier, Égré, and Zehr's studies point to a preference for disjunctive descriptions, while Ripley's points to no preference between the two, it at least seems the case that a disjunctive description is generally not judged less true than its conjunctive counterpart. Additionally, it appears that borderline contradictions are generally judged to be more true/more frequently true than their conjuncts/disjuncts, in isolation. All of this supports a tripartite account which characterises the borderline region as alternatively a gap in a strict interpretation or a glut in a tolerant interpretation.

Although the aim of this survey has not been to provide a thorough account of investigators' theoretical motivations, nor to critique their theoretical conclusions, there are a few issues that are worth addressing before moving on. First, there is the issue of Serchuk et al.'s unusual results. Their participants overwhelmingly rejected both forms of the conjunctive contradiction, even when afforded the response options ‘Neither’, ‘Partially’, ‘Both’, and ‘Don't know’ as alternatives to ‘True’ and ‘False’. The low ‘Don't know’ response does not appear to be consistent with an epistemic account of vagueness and/or a preference for errors of omission theory, because if a participant were in a state of ignorance, he or she could express that without making an error of commission. An epistemicist might counter-argue that some circumstance of the study predisposed participants to avoid neutral answers or to avoid admitting a lack of knowledge, but this is hard to square with the relatively high proportion of participants who answered ‘False’ to the tautology. Surely, if a participant is merely ignorant of where the cut-off is, then of the two non-neutral responses the better one to have chosen would be ‘True’, and there should have been virtually no ‘False’ responses.

The low preference for ‘Neither’, ‘Both’, and ‘Partially’ responses are also problematic for defenders of truth gluts and truth gaps, the alternation in the form of negation might offer an explanation. About 25% of just the ‘True’ and ‘False responses to (5) were ‘True’, and while this is certainly much less than we'd expect given the results of similar studies of borderline contradictions, it is not a negligible percentage, especially when we consider that had response options been a forced choice between ‘True’ and ‘False’, it is more likely that participants who chose ‘Neither’, 'Partially', or ‘Both’ (and thus, did not interpret the contradiction either strictly or classically) would have chosen 'True' rather than 'False'. So, the overall response to (5) might not be entirely at odds with the tolerant/strict account (and is actually just in line with results from two of our experiments, discussed in Chapter 3). The same cannot be said for (6), for which less than 10% of only 'True' and 'False' responses were 'True'. However the conjuncts in (6) were not presented as diametric equals, in fact, if Serchuk et al.'s hypothesis that periphrastic negation is weak is to be accepted, then the interpretation of the sentence is equivalent either to 'Susan is strictly rich and tolerantly not rich', a contradiction, or to 'Susan is tolerantly rich and tolerantly not rich'. But the periphrastic could also be interpreted simply as a retraction of the first conjunct, in other words, denying the truth of whatever was just asserted. In this case the sentence is a contradiction on either interpretation. So while a gappy/glutty theorist is still challenged to explain the relatively low agreement to (5), the response to (6) is less mysterious.

Another issue for concern is the lack of significant contrast between the conjunctive and disjunctive contradictions exhibited in Ripley's data, compared to Alxatib, Pelletier, Égré and Zehr's. In the case of Égré and Zehr's results, the explanation might actually be an account for why they did find such a stark contrast. We would like to suggest that a combination of the linguistic modality of the experiment and the explicit explanation that a population can be sorted into three categories with respect to the predicate, may have predisposed participants toward agreeing with disjunctive contradictions (more on this in Chapter 3 Section 3.2). While Alxatib and Pelletier did not emphasise the difference in responses to conjunctive versus disjunctive predicates, Égré and Zehr report that a McNemar-Bowker test revealed the contrast to be significant. This is puzzling, since both studies presented participants with perceptual stimuli. It could be that the larger spread of borderline cases coupled with the larger set of response options offered by Ripley contributed to a weaker effect, in which case follow-up studies might prove to be revealing.

Chapter 3

Three experiments

3.1 Replica descriptive scenario experiment

We carried out a study in order to replicate Égré and Zehr's results and to test one additional condition,: a borderline contradiction achieved through double negation, $\neg A \wedge \neg\neg A$, since this is logically equivalent to both the conjunctive contradiction through double negation elimination, and to the disjunctive contradiction by an instance of De Morgan's Laws.

The hypothesis was that assuming that Égré and Zehr's finding of a distinct preference for disjunctive borderline contradictions over conjunctive borderline contradictions is replicable, then in line with Égré and Zehr's algorithm, truth judgments for double negation contradictions are not predicted to exceed those for conjunctive borderline contradictions.

There are a few of possibilities for the application of the algorithm depending upon the interaction of scope between the operators and negation. One possiblity is to strictly adhere to the assumption that operators always take wide scope over negation, thus application of strict operators yields $S(\neg A) \wedge S(\neg\neg A)$. Since both A and $\neg A$ have a truth value of $1/2$ and the two instances of negation within the scope of the operator in the second conjunct cancel each other, this leaves us with a truth value of ½ strengthened to 0 for both conjuncts. The contradiction is therefore trivially false, just as the conjunctive is trivially false on a strict interpretation. Tolerant reinterpretation weakens the truth value to 1, resulting in a nontrivial verdict of 1. On this version of the algorithm, the process of evaluating truth for the double negation contradiction is cast as essentially identical to that for the conjunctive contradiction, and barring a degradation in overall truth judgments resulting from the greater complexity inherent to processing the extra negation, truth judgments in an analogous experimental context can be predicted to be more or less the same across both of these contradiction types.

The problem with this take on the algorithm is that a case of double negation likely qualifies as a marked circumstance, in which negation takes wide scope over the operator. This is because if it didn't, then $S(\neg\neg A)$ would be logically equivalent to $S(A)$. Given the option of uttering either the former or the latter, a competent speaker should utter the latter, in line with the Gricean Maxim of Manner that requires speakers to avoid both prolixity and unnecessary burdens to processing. But if double negation is interpreted as $\neg S \neg A$, then the contribution is to signal the hearer to skip the strong interpretation and proceed immediately to the weak interpretation, as $\neg S(A)$ is equivalent to $T(A)$ per the dual relationship between the operators through negation. In spoken language, this state of affairs would most likely be signalled through word stress. Lacking clear and overt signalling via word stress, and this will be the case for the experiment, we can only suppose that speakers may infer this interpretation.

Assuming this tolerant interpretation of double negation, there are a few possibilities for how the algorithm might proceed. One possibility is as follows: strict operators are applied to yield $S(\neg A) \wedge \neg S(\neg A)$, or equivalently as $S(\neg A) \wedge T(\neg A)$. This is not a contradiction. In fact the first conjunct logically entails the second conjunct: strict truth entails tolerant truth [11]. It also cannot true in a borderline situation. In an experimental situation we therefore expect a speaker who interprets the sentence this way to judge the sentence, 'Sam is not tall and not not tall' as false if *Sam* is a borderline case of *tall*. However, because of the order in which the conjuncts are presented, coupled with the entailment relation, the sentence also violates the Maxim of Manner, as the second conjunct is redundant. A speaker would therefore be justified in rejecting this interpretation (as we also do).

The more plausible option is that $S(\neg A) \wedge \neg S(\neg A)$ is indeed perceived to be equivalent to $S(\neg A) \wedge T(A)$ immediately, prompting back-tracking and re-interpretation to weaken the interpretation of the first conjunct, yielding $T(\neg A) \wedge T(A)$, same as the conjunctive contradiction, excluding order.

One final possibility is that the strict operators are applied as follows: $S(\neg A) \wedge \neg(S(\neg A))$, the extra brackets in the second conjunct intended to highlight an interpretation of the outside negation as periphrastic negation, taking scope over the entire second conjunct, so that the meaning of the conjunct is akin to "it is not the case that *strictly* not A". The sentence is a grave contradiction the same way that Serchuk et al.'s "Susan is rich and it is not the case that Susan is rich" is. The sentential negation is interpreted as a retraction of the initial assertion so that no matter the force of interpretation, the sentence is incoherent. As for violating Gricean Maxims, it would appear to violate all of them. We therefore do not expect this to be a likely interpretation. It must be noted though, as with all interpretations that we find implausible on pragmatic grounds, that we cannot be sure that some speakers will not choose such an interpretation in the context of an experiment, as they are not provided intonational clues for interpretation and may view the experiment as a test of their logical abilities, rather than their linguistic intuitions.

We have explored four options for how a speaker's interpretation might interact with the algorithm to produce a verdict of truth, the optimal of which, we have argued is an initial tolerant interpretation of the second conjunct followed by tolerant re-interpretation of the first conjunct to yield a verdict, in an analogous process to that for the conjunctive contradiction. We have also explored an alternative way to arrive at this same verdict, although it involves a less plausible interpretation of double negation, as well as re-interpretation of both conjuncts. In either case, tolerant re-interpretation is required. Just in consideration of computational complexity then, we might expect the double negation and conjunction contradictions to be judged to be equally true. However, we have an inkling that a couple of factors might result in some differences.

First, the presence of negation increases the burden of processing, and the presence of double negation can only be expected to compound this. As a result, we could expect more mistakes or, since Égré and Zehr assume that speakers can optionally choose to halt the algorithm on a trivial verdict, we might expect speakers to choose to halt the algorithm on one of the less plausible interpretations that does not result in a true verdict.

Second, according to the process of interpretation we see as optimal, a tolerant interpretation for the second conjunct is derived immediately in the case of the double negation contradiction. Re-interpretation is then only necessary for the first conjunct to bring it into line with the second. In the case of the conjunctive contradiction both conjuncts must be re-interpreted and this could have an effect on truth judgments. What effect it might have is not clear. It could be that it is easier to compute a re-interpretation when less of the sentence must be re-interpreted, thereby favouring the double negation contradiction, or it could be that it is easier to compute re-interpretations uniformly over an entire sentence, thereby favouring the conjunctive contradiction.

Finally, the motivation for re-interpretation is different. The conjunctive contradiction can be re-interpreted to avoid a trivial meaning, but the initial interpretation of the double negation contradiction is not trivial, instead it is just false in a borderline case. A speaker must therefore reject it and re-interpret based not on semantic necessity but on pragmatic likelihood. We should therefore be aware that if speakers perceive of an experiment as testing their logical ability rather than linguistic, they might be less sensitive to pragmatic cues, resulting in degraded truth judgments for the double negation contradiction. In all, we have a couple of reasons to expect that the double negation contradiction faces some additional hurdles on its path to a tolerant, true verdict, so while we do not expect truth judgments for it to exceed those for the conjunctive, we would not be surprised if truth judgments fall short of those for the conjunctive.

3.1.1 Methods

The study consisted of eight blocks, presented in randomised order, each corresponding to one of Égré and Zehr's carefully selected eight adjectives: *wide*, *tall*, *rich*, *old*, *large*, *fast*, *heavy*, and *loud*. In each block, the participant saw a verbal description of a borderline case of the adjective in more or less identical wording to Égré and Zehr's eight scenarios. At the same time they saw a set of five sentences in random order.[1] They were asked to indicate whether it would be true to say the sentences given the scenario as described, and asked to select either 'Yes' or 'No'.

Following Égré and Zehr, two of the sentences within each block were control sentences. Failure to correctly answer these control questions would indicate that the participant had not understood a basic element of the set-up and that the data-set provided by the participant was unlikely to be reliable. It was deemed acceptable for a participant to answer one control sentence incorrectly, but no more. The target sentences were the same two borderline conjunctive and disjunctive borderline contradictions tested by Égré and Zehr, plus the double negation borderline contradiction.

The study was carried out online, but hosted by the Qualtrics platform and distributed via Prolific to a predominantly UK based subject pool. Participants were required to be native speakers of English, to be at least 18 years of age, and to have an approval rating of 75% or more on Prolific. Responses were provided by 33 participants, three of whom answered at least two control questions incorrectly and whose data was consequently omitted. Following Égré and Zehr, a pre-questionnaire was included, modified from their original to suit a UK based subject pool. These seven questions tested general knowledge, for example, the identity of the current Prime Minister,

[1] And some extras that we will address in the analysis.

and served to indicate whether subjects were likely to answer the test items responsibly, as well as to ensure that results were not influenced by technical problems or confusion stemming from the instructions. A post-questionnaire was also included.

While the study adhered closely to Égré and Zehr's in format, some changes were made that could conceivably have impacted the results, the lack of filler questions that exposed participants to antonymic versions of the sentences being one example. In light of these changes, it is fair to say that the study cannot technically replicate Égré and Zehr's original results, however most changes were minor. Moreover, to the extent that some of these changes were intended to control potential confounds, a replication of results would indicate the robustness of the effect that Égré and Zehr originally observed. The most significant changes were to the order in which blocks and questions were presented and to the manner in which questions within each block were revealed. To be clear, each block here corresponds to a scenario concerning any one adjective, along with its five accompanying sentences. Not only was the order in which blocks were shown randomised, but the order in which the sentences accompanying each scenario were presented was randomised. This was intended to minimise the possibility that participants might identify a pattern and answer formulaically without actually reading the sentences. Within each block, participants were also shown the scenario and the five sentences simultaneously, rather than one by one. This choice was made for reasons involving the survey platform rather than by purposeful design, but it may have highlighted to participants the option of changing their answers, or had some effect in allowing them to answer questions within each block in any order they wished. In any case, participants were able to change their answers for a given scenario as they pleased before moving on to the next scenario, just as Égré and Zehr's participants had been.

3.1.2 Results

The results showed significant differences between all sentence types except for between conjunctive borderline contradictions and double negation borderline contradictions. The percentage of total responses to the former that were *Yes* responses was 50.8%, while to the latter it was 55.8%. The percentage of *Yes* responses to the disjunctive contradictions was 86.7%, clearly higher than for both other types of borderline contradictions, yet also clearly lower than the percentage of *Yes* responses to the true control.

In Table 3.1 we report the result of a mixed effect regression model fitted on a subset of the data which excludes the control sentences, with variability owing to participants' tendencies and variability owing to predicate folded into the random effects portion of the model. Our formula was identical to Égré and Zehr's with the exception that 'group' did not figure into the random effects, as we had no such distinction in our experiment (no filler block). The model estimated response ('Yes' = 1) predicted by contradiction type ('condition'), with varying intercepts conditioned on participant ('subject') and varying intercepts and slopes conditioned on predicate ('predicate'). The model converged on the default optimizer *bobyqa*. Thus, our formula was glmer((response == "1") $\sim$ condition + (1|subject) + (1 + condition|predicate), data = contradictions, family = binomial).

Table 3.1: Output of the mixed effects regression model

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	0.1682	0.5925	0.284	0.776
disjunctive	3.6959	0.4168	8.867	$<$ 2e$-$16***
double neg	0.4429	0.2766	1.602	0.109

Mean response according to sentence type is visually displayed in Figure 3.1.

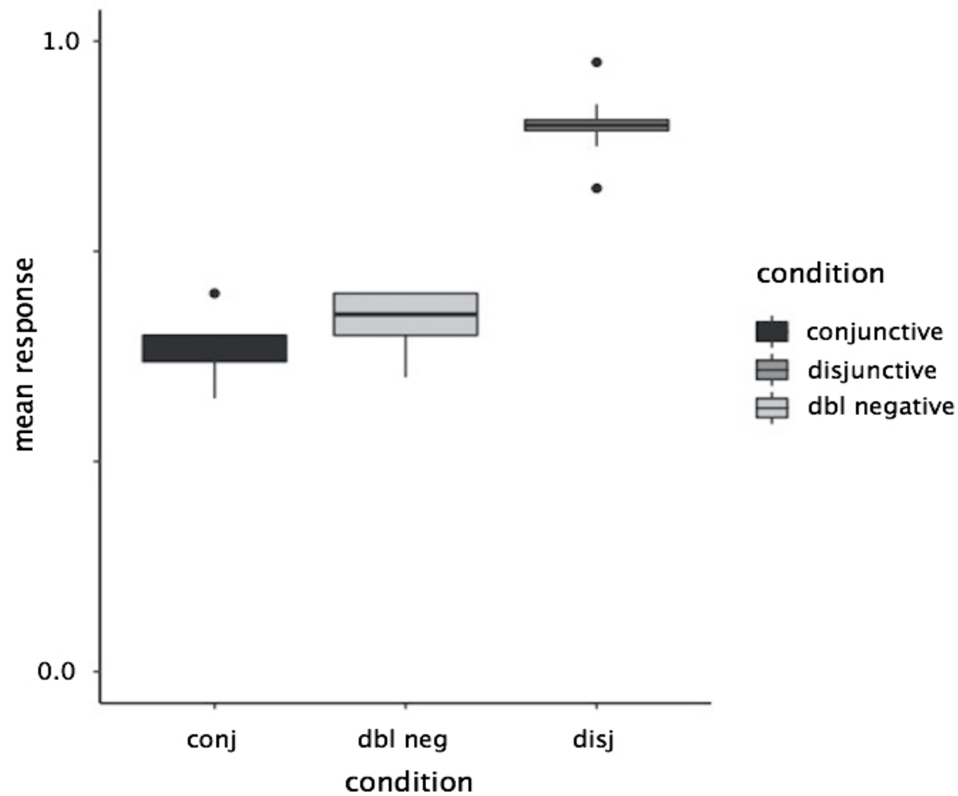

Figure 3.1

3.1.3 Analysis and discussion

The experimental hypothesis is supported by the results, which shows that significantly more 'Yes' responses were given to disjunctive borderline contradictions than to either of the other contradiction types, but that there was no significant difference between the number of 'Yes' responses given to the conjunctive borderline contradictions and the number of 'Yes' responses given to the double negation borderline contradictions.

Additionally, Égré and Zehr's results are replicated, in that disjunctive borderline contradictions were judged to be true significantly more often than their conjunctive counterparts, although the actual proportions are rather different. The proportion of 'Yes' responses to disjunctive borderline contradictions matches Égré and Zehr's results nicely, both in the 85-90 percentile range. However, whereas Égré and Zehr recorded that only 25-30% of responses to the 'conjunctive' were 'Yes' responses, in our study, this percentage was considerably higher at 50.8%. It is unclear why our results differ from Égré and Zehr's in this respect, but they are also not out of keeping with Ripley, Alxatib, and Pelletier's results, which were similarly higher than Égré and Zehr's.

Thus far we have actually ignored an aspect of the experiment, which is that it also included a small exploratory study carried out on 20 of the 33 participants. These participants were supplied with two additional sentence types, positive and negated, for a total of 7 sentence types shown for each scenario. Thus for the *tall* scenario, they saw the sentences "Sam is tall" and "Sam is not tall" in addition to the contradictory sentences and controls. Recall that Égré and Zehr's algorithm would predict both of these to be judged to be true of a borderline case less often than the borderline contradictions, as per the algorithm, they are both strictly false. This prediction appears to be borne out in the case of the positive sentences, but not the negated ones. In fact, the mean response to the positive sentences was only .05625 (‘No’ = 0, ‘Yes’ = 1) with only 4 different participants providing a total of 9 ‘Yes’ responses. The mean response to the negated sentences was .66875, with only one of the 20 participants rejecting it uniformly across all eight scenarios (see Figure 3.2). A Wilcoxon signed rank test comparing the responses to these two sentences reveals a very significant contrast ($W = 4960$, p-value $< 2.2e{-16}$).

This is clearly an unexpected result with regard to the algorithm and runs counter Alxatib and Pelletier's finding of degraded truth judgements for both of these sentence types. What's further puzzling about this result is that unlike Alxatib and Pelletier's police line-up, in which participants were able to form their own judgments concerning which individual was a borderline case, here participants were instructed to imagine a borderline case and presumably a fairly typical borderline case, too. So these results do not appear to be a case of mistaken identity. Moreover, it was exactly for these imagined borderline cases that a plurality of participants observed the borderline contradictions side by side with these positive and negated sentences, accepted both contradictions and the negated sentences, and rejected the positive sentence. It is clear that for these participants, the positive and negated sentences are not peers.

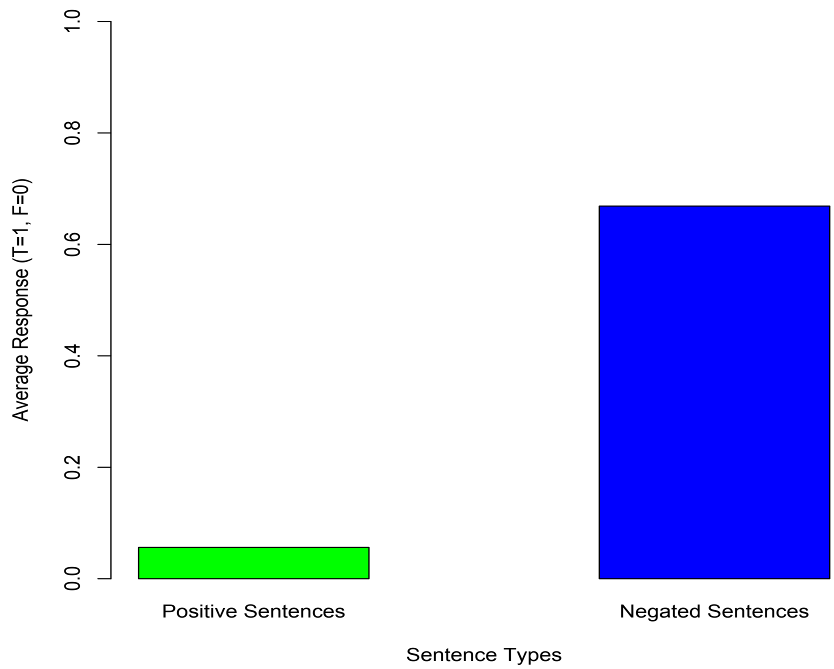

Figure 3.2: Mean responses to positive sentences and negated sentences

In accounting for the very low acceptability of the positive sentence, it still seems that the Strongest Meaning Hypothesis, requiring a strict interpretation, is the best bet. But we would not expect the default strength of interpretation to differ for positive and negated sentences, so surely if the Strongest Meaning Hypothesis is responsible for a strict interpretation of the positive sentence we would expect its negated counterpart to be interpreted strictly, as well. Assuming that to be the case, the relatively high acceptability of the negated sentences could be due to an inverted scope relation between the strict operator and negation.[2] If we discard the assumption that negation takes narrow scope by default, we get an interpretation of "Sam is not tall" equivalent to $\neg S(tall(Sam))$, or $T(\neg tall(Sam))$, which would be true exactly in a borderline case where it is false that *Sam* is strictly *tall*.[3]

But if that is the case, then the default interpretation of the conjunctive borderline contradiction is $S(A) \wedge \neg S(A)$, or equivalently $S(A) \wedge T(\neg A)$, which is trivially false. Instead, it can only be interpreted as true if the first conjunct is tolerantly re-interpreted in a similar manner to the double negation contradiction.

In the case of the disjunctive contradiction, a strict interpretation with the strict operator taking narrow scope under negation results in $\neg(S(A) \vee \neg S(A))$, equivalent to $\neg(S(A) \vee T(\neg A))$, which is also trivially false. It can only be interpreted as true after undergoing re-interpretation. On this default view of negation then, both conjunctive and disjunctive contradictions can only attain a non-trivial meaning after re-interpretation. But if both must be re-interpreted, then the algorithmic account's ability to predict the conjunctive contradiction's lower rate of acceptability is effectively eliminated.

Nevertheless, there is one important difference between the re-interpretation

[2] Note that this understanding of the strongest meaning is bottom up, assigning strongest meaning to the "leaves" in the sense of Égré and Zehr's original assumptions.

[3] We can think of this in terms of Gödel negation such that it has the contribution of selecting the complement of the predicate's strongly interpreted extension. In Alxatib and Pelletier's precisificational terms, the negation of the predicate would be true of those individuals that are not members of predicate's extension in at least one precisification.

required in the case of the disjunctive borderline contradiction and that required in the case of the conjunctive and double negation borderline contradictions, namely, the re-interpretation is from tolerant to strict, rather than the other way around. That is, the trivially false initial interpretation of the disjunctive contradiction, $\neg(S(A) \vee T(\neg A))$, is correctly re-interpreted by strengthening the interpretation of the second disjunct to obtain $\neg(S(A) \vee S(\neg A))$. This provides two advantages.

First, we may have reason to believe that in order for informativity to increase over the course of a given discourse, the natural progression of the meaning of an expression in that discourse is toward becoming narrower, that is stricter, not the other way around (compare this to the updating process in DRT [27]). More concretely, in the case of the conjunctive borderline contradiction, the first conjunct was first interpreted strictly, thus the extension of the predicate was interpreted to be just the set of individuals about whom it is true to say that they are strictly *tall*. Tolerant re-interpretation of the conjunct results in an extension consisting of the union of that original strict set with the set of individuals about whom it is true to say that they are (at least) tolerantly tall, resulting in a re-interpreted extension that is at least as large as the original extension. Informativity can therefore only be lost. Vice-versa, in the case of the disjunctive contradiction, strict re-interpretation of the tolerant disjunct results in an extension that is no larger than the original interpretation, although it could be smaller, and thus informativity can only increase.[4]

The second advantage has to do with order. Re-interpretation of the first conjunct of the conjunctive and double negation borderline contradictions can only occur once the second conjunct has been interpreted. Re-interpretation therefore requires some amount of backtracking for reanalysis, a procedure that has been shown to increase both processing time [35] and susceptibility to arriving at sub-optimal analyses [21, 42] (compare this to re-analysis of semantically ambiguous garden-path sentences). Since re-interpretation of the disjunctive borderline contradiction occurs in the second disjunct it may not require backtracking in order to obtain an optimal interpretation. If we take it that what is meant by the assumption that speakers prefer simple computations to complex ones is that speakers prefer to avoid backtracking, then the assumption licenses speakers to halt the algorithm at a sub-optimal interpretation, an outcome which would then be expected to occur more frequently for conjunctive and double negation contradictions than for disjunctive contradictions.

Although this account of the pattern of results is somewhat ad hoc, in that it does not seem to be an appropriate account of results from Alxatib and Pelletier's experiment, it does seem to be in line with Serchuk et al.'s

[4] It must be noted that this does not accord exactly with the Strongest Meaning Hypothesis as it was originally conceived, inasmuch as Dalrymple et al. supposed that meaning is generally weakened from the default strict interpretation, although this has been contested [69]. Arguably though, negation constitutes a special case, as the tolerant interpretation paradoxically *is* the output of the strict interpretation's interaction with negation.

data that indicated appreciable agreement to negated sentences alongside relatively low agreement to conjunctive borderline contradictions. It also manages to account for the facts of this particular case with only minimal changes to the assumptions and algorithm. For now it remains to be seen whether the contrasts between simple positive sentences and their negated counterparts can be reproduced, and if so, why such a result was not produced in Alxatib and Pelletier's experiment. As we move forward into the next two experiments we will therefore put this modified account aside, while remaining keen to consistently test these component sentences alongside the borderline contradictions they compose.

3.2 Shape-colour experiment

The results of the verbal description experiment were promising, but its pitfalls prompt a follow-up experiment seeking to minimise them, if at all possible. If the pattern of results still stands, then we will have even more compelling evidence that disjunctive borderline contradictions are more likely to be judged true than their conjunctive counterparts, and therefore evidence that discrepant processes of the kind proposed by Égré and Zehr are involved in the derivation of truth value judgments for these two sentence types.

Chief among pitfalls is the inadvertent instruction of participants to not only tripartition the domain into the sets P, B_P, and $\neg P$, but to also treat these sets as exclusive. Ideally we would like to avoid suggesting any kind of domain partitioning. If that is not avoidable, then we are at least required to abstain from any suggestion that intermediate regions be treated as extension under- or overlaps.

Perhaps, instead of introducing three categories for each property and situating *Sam* in the intermediate category, we could describe a scenario involving a relevant comparison class of individuals along with their measurements along the appropriate scale, as well as *Sam*'s measurement. Indeed, a hybrid version of this methodology will be the focus of section 8.3, concerning the height-nationality experiment. But there are a number of reasons we might not want to use this as our starting point. For one, using explicit measurements along an existing scale introduces variables that would need to be controlled for, namely: how familiar the participant is with the scale and its units; how able the participant is to estimate and/or recall the measurements of objects against which to compare; and how accurately the participant is able to gauge and evaluate verbally communicated numerical measurements, both for the specific scales chosen and in general. We have already seen examples of the first two in Sauerland's 2011 experiment: familiarity with the given units of measurement, e.g., Celsius versus Fahrenheit for "A 83 degree Fahrenheit day is hot", and ability to estimate or recall measurements, e.g., the heights of various mountains for "A 3280-foot mountain is high"[54]. As for the third variable, evidence suggests that people's mental states and attitudes, as well as attributes of the scale they are using, affect their ability to veridically represent reality [39, 44, 6], thus we should be cautious to assume that participants are able to accurately and consistently represent measurements, even if they are thoroughly familiar with them.

While we could attempt to control for these variables through careful selection and calibration, and presumably their effect might even out over a large number of trials, the surest way to control for them would be to simply avoid explicit scales and measurements. But there is an additional compelling reason to take this route: it plainly cannot be the case that the use of explicit measurements, corresponding to a standardised scale, is a requisite for the use of vague language and the evaluation of borderline cases. Indeed, speakers regularly use vague language with access neither to explicit measurements nor to a standardised scale, either because they are not aware of a standardised scale that exists, or because the property at hand does not lend itself to standardised measurement. For example, although colours in the visible light spectrum are perfectly measurable and amenable to standard scalar evaluation in terms of wavelength, conscious consideration of wavelength almost certainly does not play a role in the average speaker's evaluation of a borderline blue hue. And although scales of attractiveness have underpinned more than a few bad dating jokes, most rational people would agree that establishing a standardised scale of beauty would be as frivolous as it is futile, both because as a property, beauty is highly qualitative and multidimensional, and as an experience, beauty is highly subjective. But these deficits in no way prevent speakers from evaluating objects and individuals to be borderline blue or borderline beautiful. We can think of countless more examples of ever more abstract vague properties, up to and including vagueness, itself, as in "the property of being vague is vague", which seem particularly antagonistic toward explicit or standardised measurement. The point to be made is that many if not most vague properties are learned, reflected upon, and expressed without reference to standard measurements. To be clear, the point is of course not that vague properties are not associated with scales; they are (see our definitions of the semantics of gradable predicates in chapter 1), but merely that the scales associated with them need neither be standardised nor explicit. They may instead, for example, be conceptualised along the lines of an analog magnitude scale [18]. But without providing explicit measurements or explicit instructions as to how to consider the intended borderline case, there seem to be few options for verbally communicating the idea that the borderline case is borderline, aside, of course, from describing it via a borderline contradiction.

Thus, we turn to the option of running a more natural experiment in which participants' senses are exposed directly to the stimuli about which they are to make linguistic judgments. In a few words, instead of describing *Sam* to participants we could show *Sam* to them, free of instruction or comment, so that they can evaluate him just as they would if they had seen him on the street. We could thereby avoid creating the very effect we hope to observe by bypassing any theoretically non-neutral characterisation of the borderline case, while at the same time allowing participants to integrate perception and evaluation in a manner we hold to be common to all vague predicates, not just those associated to a standard scale. It is exactly this that the shape-colour experiment strives to accomplish, with the exception of course, that instead of showing participants *Sam*, we shall show them more abstract objects (for reasons we will address in section 2.2.1).

In addition to addressing this major concern, the geometric shapes experiment is motivated by four desiderata. The first is to confirm that participants are interpreting the case intended to be borderline as borderline. For this purpose, we can make use of Ripley's hump effect to generalise that participants judge borderline contradictions to be true of borderline cases more frequently than they judge them to be true of non-borderline cases, and moreover that the frequency of true responses correlates directly with perceived similarity to a maximally borderline case. Thus, if participants are tested on a spread of cases ranging from non-borderline instances of a property, to borderline instances, to non-borderline non-instances, then we expect to observe Ripley's hump effect in the aggregate response. Failure to observe it might indicate a problem with the experimental set-up. Any skewing, distortion, or otherwise anomalous changes to Ripley's hump effect could additionally provide a strong indication of what went wrong. For example, a participant displaying a consistently skewed hump effect is likely to consider the boundary of the predicate to be either higher or lower than do her fellow participants, perhaps because her comparison class differs from theirs, among other possibilities. Moreover, establishing the hump effect facilitates more accurate cross trial and cross experiment comparison by anchoring the highest mean truth judgement to the case that is maximally borderline.

The second desideratum is to establish a pattern of truth judgments for subpart positive and negative sentences, reflecting atomic A and $\neg A$ that form the conjuncts and disjuncts of borderline contradictions. In doing so, we would like to observe how the parts of a borderline contradiction contribute to its whole, what the relationship is between the parts and the whole. Is it an inverse relationship? Can we predict the degree to which a borderline contradiction is judged true based upon the truth judgments of its subparts?

This desideratum is bound up with the third desideratum, to establish a non-contradictory baseline for comparison. The subpart sentences can provide this, but so might non-contradictory analogues to the borderline contradictions. That is, analogue sentences in which one conjunct (or disjunct) contains a borderline property of the individual, while the other conjunct (or disjunct) contains a property of the individual which can be non-controversially established to be either true or false.

However, non-contradictory analogues cannot be assumed to behave well, and uncovering how they might misbehave constitutes the fourth desideratum. For example, if *Sam* is a borderline case of “tall” and a bachelor, then “Sam is tall and married” is truth functionally false, regardless of the truth of the borderline conjunct. However, it is possible that for some reason, speakers’ truth judgments are not determined strictly truth-functionally and are instead determined by an averaging function, allowing the sentence to be true to some non-zero degree which would be reflected in the average response. Naturally, along with the extra conditions of non-borderline cases, subpart sentences, and non-contradictory analogues comes a host of new patterns of predictions which will need to be filtered in formulating our hypotheses, and it is to that endeavour which we now turn.

3.2.1 Working hypothesis and alternative hypotheses

Our working hypothesis at this point is that there are two forces of interpretation available, a strict interpretation and a tolerant interpretation, and that the default interpretation is the stricter. The disjunctive borderline contradiction $\neg(A \vee \neg A)$ evaluates to true on a strict interpretation, but its conjunctive counterpart $A \wedge \neg A$ does not, rather it can only be at most tolerantly true. Since the latter can only be true on a weaker strength of interpretation, its truth is degraded. In the setting of an experiment, where participants are required to provide binary truth judgments for these sentences and therefore the relative truth of each is reflected in the number of recorded "True" responses, and assuming all else to be equal, the frequency of "True" responses to the conjunctive version of a borderline contradiction will be lower than that in response to its disjunctive counterpart. So far this is in line with Égré and Zehr's algorithm and assumptions, and we expect this result to be borne out as it has been in past experiments.

In addition to testing this hypothesis, we are also interested in probing a larger pattern of predictions which Égré and Zehr's algorithm casts for non-contradictory subpart sentences, A and $\neg A$, as well as for non-contradictory analogue sentences, both in reference to a borderline case and to a non-borderline case $A \wedge \neg B$ and $\neg(A \vee B)$[5].

In the case of the non-contradictory analogues, we specify that the state of the world and the truth value of the B component are such that, provided the truth value of the A component makes the sentence true, the whole sentence is true. This means that the B component is intended to be true in the case of the conjunctive analogue, and false in the case of the disjunctive analogue. The truth value of the non-contradictory analogues therefore hinges on the truth value of the potentially borderline component, while the "non-vague" component remains fixed.

(11) If j is a borderline case of the vague predicate in A and is not a borderline case of the predicate in B, then:

a. the sentence Aj is strictly false and tolerantly true.

b. the sentence $\neg Aj$ is strictly false and tolerantly true.

c. the sentence $Aj \wedge \neg Aj$ is strictly false, but tolerantly true.

d. the sentence $\neg(Aj \vee \neg Aj)$ is strictly true, but tolerantly false[6].

[5]Although the B disjunct of the analogue should technically be negated for a faithful analogue, we have chosen to drop negation as its absence is not expected to impact an evaluation of truth so long as the experiment items are suitably manipulated. A drawback to this is that the analogue is potentially simpler to process.

[6]What we mean here is that it is true if the strict operator applies to the leaves Aj and

e. when Bj is true, the sentence $Aj \wedge Bj$ is strictly false, but tolerantly true.

f. when Bj is false, the sentence $\neg(Aj \vee Bj)$ is true, and strictly so.

This pattern of predictions is summarised in Tables 3.2-3.4. Table 3.2 summarises (11a)-(11d), the predicted truth judgments for the contradictions and their subparts, following application of the T and S operators (collapsed into O), given each possible base value for A: true (1), 'borderline' (½), and false (0). Highlighting indicates the predictions for just the case in which j is a borderline case, the darker shade indicating which of the two possible interpretations is predicted to be preferred and therefore more often observed. In the case of the conjunctive borderline contradiction, we expect to see a non-negligible proportion of truth judgments to coincide with both possible truth values, since both strict and tolerant interpretations are expected to be accessible. However, there is no specific prediction to be inferred from Égré and Zehr's algorithm concerning whether the sentence is expected to be judged more often true than false, and vice versa. The only prediction is that whatever the frequency of 'True' responses to the conjunction borderline contradiction, it should not exceed the frequency of 'True' responses to its disjunctive counterpart.

$\neg Aj$, not to the sentence as a whole.

Table 3.2: Predictions (11a)-(11d).

A	Operator	$O(A)$	$O(\neg A)$	$O(A \wedge \neg A)$	$O(\neg(A \vee \neg A))$
1	T S	1	0	0	0
½	T	1	1	1	0
	S	0	0	0	1
0	T S	0	1	0	0

The predictions in (11e) and (11f) are summarised in Tables 3.3 and 3.4, respectively. Since in the case of (11e), we have specified that j is a borderline case of the predicate in A and is not a borderline case of the predicate in B, and since the conjunctive analogue is true just in case B is true, the possible values are just those indicated by highlighting in the corresponding table where these conditions coincide, of which, the darker shade indicates the predicted truth judgment. Similarly in Table 3.4, since the disjunctive analogue is true just in case B is false, the possible truth values are just those highlighted in the table, of which the darker is the predicted judgment.

Table 3.3: Prediction (11e)

$\bigwedge$		**A** 1		½		0	
		T	S	T	S	T	S
B 1	T	1	1	1	0	0	0
	S	1	1	1	0	0	0
½	T	1	1	1	0	0	0
	S	0	0	0	0	0	0
0	T	0	0	0	0	0	0
	S	0	0	0	0	0	0

Table 3.4: Prediction (11f)

$\bigvee$		**A** 1		½		0	
		T	S	T	S	T	S
B 1	T	0	0	0	0	0	0
	S	0	0	0	0	0	0
½	T	0	0	0	0	0	0
	S	0	0	0	1	1	1
0	T	0	0	0	1	1	1
	S	0	0	0	1	1	1

Note that there are a total of 36 possible combinations of the analogue subparts with three possible base truth values, transformed by two operators, and this is reflected in the tables. However, we are only really interested for the purpose of the experiment in a very small subset of these. For one thing, we do not intend to probe truth judgments in case j is a borderline case of both A and B predicates, although we could well have chosen to probe these. The primary reason for not probing these is that the algorithm informing the hypothesis makes no distinction in terms of predicted truth values between this case and the case in which only one subsentence is true. The addition of a second borderline property additionally creates more possible confounds that would need to be controlled for, with little payoff per the lack of distinction already mentioned.

Then there is the notion of mixed operators, indicated in the tables by the dark cells. We see no *a priori* reason to discount the possibility of implicit mixed interpretation, as mixed interpretation would seem to be possible to achieve explicitly, as in the sentence, “Sam is tolerantly tall, but he is strictly not rich”. However, the algorithm behind the hypothesis only distinguishes between mixed and uniform operators in the case that both subsentences are borderline true, and since we have decided not to probe these, we expect to see no effect on truth judgments hinging upon whether or not a speaker interprets the subsentences at uniform force. Hence, we set them aside. Incidentally, the fact that the algorithm makes a distinction when both subsentences have a base truth value of ½ is not a compelling reason to probe these cases in the hope that we might be able to determine whether implicit mixing of operators occurs, and this is because the algorithm results in no *unique* predictions for mixed interpretations. For example, the only distinction in predictions for the disjunctive analogue is between mixed interpretation and uniform strict interpretation, but not between mixed and tolerant interpretations. Thus, if the sentence is judged true, we can conclude that interpretation is uniformly strict, but if it is judged false, we have no way of determining whether this is due to mixed or uniformly tolerant interpretation.

(12) If j is a clear case of the vague predicate in A and is not a borderline case of the predicate in B, then truth is classical, thus:

a. the sentence Aj is true, and strictly so.

b. the sentence $\neg Aj$ is false, and strictly so.

c. the sentence $Aj \wedge \neg Aj$ is false, and strictly so.

d. the sentence $\neg(Aj \vee \neg Aj)$ is false, and strictly so.

e. when Bj is true, the sentence $Aj \wedge Bj$ is true, and strictly so.

f. when Bj is false, the sentence $\neg(Aj \vee Bj)$ is false, and strictly so[7].

Table 3.5: Predictions (12a)-(12d)

A	Operator	$O(A)$	$O(\neg A)$	$O(A \wedge \neg A)$	$O(\neg(A \vee \neg A))$
1	T S	1	0	0	0
½	T	1	1	1	0
	S	0	0	0	1
0	T S	0	1	0	0

[7] $S(A) = 1$ entails $T(A) = 1$, $S(A) = 0$ entails $T(A) = 0$, and this is reflected in the highlighted cells of Tables 3.6-3.7.

Table 3.6: Prediction (12e)

$\wedge$		**A**: 1		½		0	
B		T	S	T	S	T	S
1	T	1	1	1	0	0	0
	S	1	1	1	0	0	0
½	T	1	1	1	0	0	0
	S	0	0	0	0	0	0
0	T	0	0	0	0	0	0
	S	0	0	0	0	0	0

Table 3.7: Prediction (12f)

$\vee$		**A**: 1		½		0	
B		T	S	T	S	T	S
1	T	0	0	0	0	0	0
	S	0	0	0	0	0	0
½	T	0	0	0	0	0	0
	S	0	0	0	1	1	1
0	T	0	0	0	1	1	1
	S	0	0	0	1	1	1

(13) If j is a clearly not a case of the vague predicate in A and is not a borderline case of the predicate B, then;

a. the sentence Aj is false, and strictly so.

b. the sentence $\neg Aj$ is true, and strictly so.

c. the sentence $Aj \wedge \neg Aj$ is false, and strictly so.

d. the sentence $\neg(Aj \vee \neg Aj)$ is false, and strictly so.

e. when Bj is true, the sentence $Aj \wedge Bj$ is false, and strictly so.

f. when Bj is false, the sentence $\neg(Aj \vee Bj)$ is true, and strictly so.

Table 3.8: Prediction (13a)-(13d)

A	Operator	$O(A)$	$O(\neg A)$	$O(A \wedge \neg A)$	$O(\neg(A \vee \neg A))$
1	T S	1	0	0	0
½	T	1	1	1	0
	S	0	0	0	1
0	T S	0	1	0	0

Table 3.9: Prediction (13e)

$\wedge$		A: 1, T	A: 1, S	A: ½, T	A: ½, S	A: 0, T	A: 0, S
B: 1	T	1	1	1	0	0	0
	S	1	1	1	0	0	0
B: ½	T	1	1	1	0	0	0
	S	0	0	0	0	0	0
B: 0	T	0	0	0	0	0	0
	S	0	0	0	0	0	0

Table 3.10: Prediction (13f)

$\vee$		A: 1, T	A: 1, S	A: ½, T	A: ½, S	A: 0, T	A: 0, S
B: 1	T	0	0	0	0	0	0
	S	0	0	0	0	0	0
B: ½	T	0	0	0	0	0	0
	S	0	0	0	1	1	1
B: 0	T	0	0	0	1	1	1
	S	0	0	0	1	1	1

This extended pattern of predictions, as well as the hypothesis' prediction of in terms of relative frequency of 'True' responses to the borderline contradictions coalesce into the following pattern of relative truth at the borderline case, where $<$ indicates that the left hand item is relatively less true than the item to its right.

$$\begin{array}{c} Aj \\ \neg Aj \\ Aj \wedge Bj \end{array} \quad < \quad Aj \wedge \neg Aj \quad < \quad \begin{array}{c} \neg(Aj \vee \neg Aj) \\ \neg(Aj \vee Bj) \end{array}$$

3.2.2 Methods

The experiment could have been carried out using a stimulus to any sensory apparatus, however for the majority of modes of sensation this would best be achieved within the controlled environment of the lab. In the interest of carrying out the experiment online, it was decided that the mode of representation would be visual, as this could best be controlled remotely. In an effort to minimise the effect of any preconceived comparison class and the influence of the participant's personal circumstances on the comparison class against which the borderline individual is evaluated, e.g., his or her personal wealth in the case of "A guy with \$100,000 is rich"[54], we chose to present participants with abstract images of geometric shapes, about which they would presumably harbour no preconceived notions.

3.2.2.1 Capturing the distance variable in images

Following Ripley 2011, the test images each displayed two geometric shapes, a square and a circle. Both shapes were either red or blue, set within an off-white square frame. The use of two colours allowed the number of test images we were able to generate to be doubled, however, only one colour was ever presented in any one image in order to avoid a potential increase in processing load. The shapes differed from each other slightly in area, but their own areas remained constant and the length of the side of the square was always equal to the diameter of the circle, lending an impression that the shapes were of roughly the same size.

Also in line with Ripley's 2011 experiment, the relevant borderline property to be represented was proximity of the shapes to each other. Thus within each colour scheme, five distinct distances were represented, constituting the five levels of the factor of distance. The sequence of represented distances is quadratic, meaning that the difference between differences in proximity from one pair of shapes to the next is constant. This results in a clustering of shorter distances, maximising the use of the small space afforded by the frame, while maintaining perceptual distinctness between distances (on the assumption that small differences in small distances are perceptually as distinct as larger differences in larger distances, i.e. the difference between 3cm and 4cm is perceptually more distinct than the difference between 99cm and 100cm). Images were generated from a grid in which each shape fully occupies a four grid cell square, thus the length of the side of the square/diameter of the circle is equal to the length of two grid units. Distances are reckoned from the centre of the circle to the centre of its square partner. To be precise, based on a unit of measurement u equal to the diagonal of a cell (the diagonal of the frame then measuring $24u$), and where n refers to the index of each distance in the sequence, the formula for the sequence is of distances is $n^2 - n + 2$, resulting in the sequence $(2u, 4u, 8u, 14u, 22u)$. Figure 3.3 displays a composite drawing of all five levels using the blue scheme, each indicated by its index number 1-5.

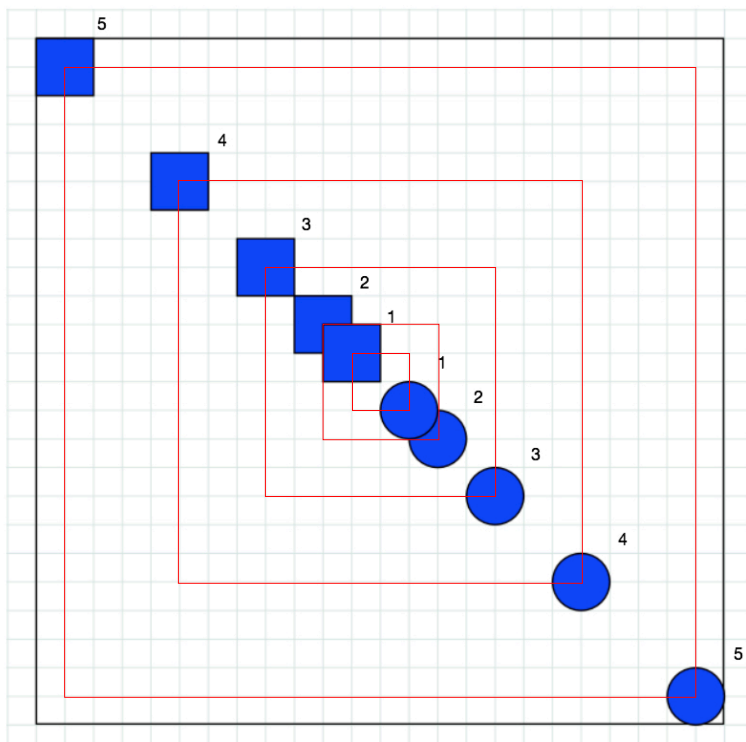

Figure 3.3: Composite drawing of geometric images

Pairs 1 and 5 were intended to instantiate cases of clear non-borderlineness. Pair 3 was intended to instantiate a borderline pair. Pairs 2 and 4 were intended to represent an intermediate degree of borderlineness, possibly construed as second-order. The original reason behind the inclusion of these second-order borderline cases was to disguise the contrast between borderline and non-borderline cases, since feedback from an exploratory pilot suggested that participants were quick to recognise this contrast early on and therefore to provide their answers formulaically. Of course, the inclusion of second-order borderline cases does not preclude formulaic responses, however at the very least, the second-order borderline cases were also expected to provide a more thorough representation of Ripley's hump effect.

One image of each colour scheme was generated per level, resulting in a total of 10 images. Additionally, a random three of these ten were displayed as mirrored images, with the aim of disrupting monotony from the perspective of the participant. Neither mirroring of images nor colour scheme were considered to be significant factors. All target images can be found in the appendix to this chapter.

3.2.2.2 Capturing the condition variable in sentences

A single target vague predicate was chosen, *near*, along with two auxiliary predicates, *red* and *blue*, such that the target predicate always appeared in the A component and the auxiliaries in the B component of the target sentences. As discussed before, target sentences came in six types: conjunctive contradiction, disjunctive contradiction, conjunctive analogue, disjunctive analogue, positive subsentence, negated subsentence. Two sentences were generated per type, alternating the syntactic role taken on by the square and the circle, resulting in a total of 12 test sentences, grouped into six levels. All target sentences are illustrated in Table 3.11.

Table 3.11: Shape-colour Sentences

Conjunctive Contradiction: $A \wedge \neg A$	
1	The circle is near the square and not near the square.
2	The square is near the circle and not near the circle.
Disjunctive Contradiction: $\neg(A \vee \neg A)$	
3	The circle is neither near the square nor not near the square.
4	The square is neither near the circle nor not near the circle.
Conjunctive Analogue: $A \wedge B$	
5	The circle is near the square and blue.
6	The square is near the circle and blue.
Disjunctive Analogue: $\neg(A \vee B)$	
7	The circle is neither near the square nor blue.
8	The square is neither near the square nor blue.
Positive Subsentence: A	
9	The circle is near the square.
10	The square is near the circle.
Negated Subsentence: $\neg A$	
11	The circle is not near the square.
12	The square is not near the circle.

What shape took on which syntactic role was not considered a significant factor, the alternation serving merely as a further measure to disrupt monotony. In order to control for any random effect of the alternates, each alternate sentence would have ideally appeared once per distance, however this would result in a total of 60 test items, requiring considerable endurance on the part of participants. Therefore, the experimental items were not repeated, and alternates were selected per condition and distance in a manner that would appear to be random. The experiment therefore had a 6×5 factorial design.

3.2.2.3 Participant recruitment and prescreening

34 participants were recruited through the Prolific platform and the online experiment was hosted by Ibex Farm. The experiment was designed for participation via desktop, laptop, tablet, and smartphone. Participants were prescreened by Prolific for English as a first language and were asked to confirm this explicitly at the start of the experiment. With a sample of this size coupled with a relatively large number of target items, it was especially important to ensure that participants were fully aware of the task confronting them and remained attentive throughout. Controls were therefore distributed throughout the experiment. These were formally identical to a subset of the test items, but were not vague, and a conscientious participant could be expected to easily discern between true and false controls. Participants were afforded only one mistake on these items (see the Appendix to this section for a full example of control items). Data from one participant was omitted

on these grounds. Furthermore, data from each participant was examined for anomalies. In order for a response to be deemed anomalous, it had to violate a logical entailment. An example of this is a reversal such as a "True" response to the positive subsentence, "the circle/square is near the square/circle" for the two nearest pairs (Pairs 1 and 2), a "False" response for the next two nearest pairs (Pairs 3 and 4), and a "True" response for the pair furthest apart from each other (Pair 5). Such anomalous responses could be due to a lack of attention, haste, or simply to an error in the manual selection of the participants' desired response, and as such, their occurrence is expected. However, the commission of frequent errors indicates a level of attention that does not meet the requirements of an experiment of this size. Data was therefore omitted if a participant's data clearly displayed three or more of these anomalies. Three sets of data were omitted on these grounds. No items were repeated. Therefore, including controls, each participant was asked to respond to 40 items.

3.2.3 Results

The mean results for all conditions are presented in Table 3.12 and visually represented in Figure 3.4. Recall that Pair 1 refers to the two shapes that were closest to each other and Pair 5 to those farthest from one another.

Table 3.12: Shape-colour response means

	Pair				
	1	2	3	4	5
positive subsentence	1	.967	.333	.1	0
negated subsentence	0	.067	.7	1	.967
conjunctive contradiction	.133	.233	.333	.167	.2
disjunctive contradiction	.1	.167	.567	.3	.267
conjunctive analogue	1	.967	.367	.067	.067
disjunctive analogue	.167	.367	.9	.967	.967

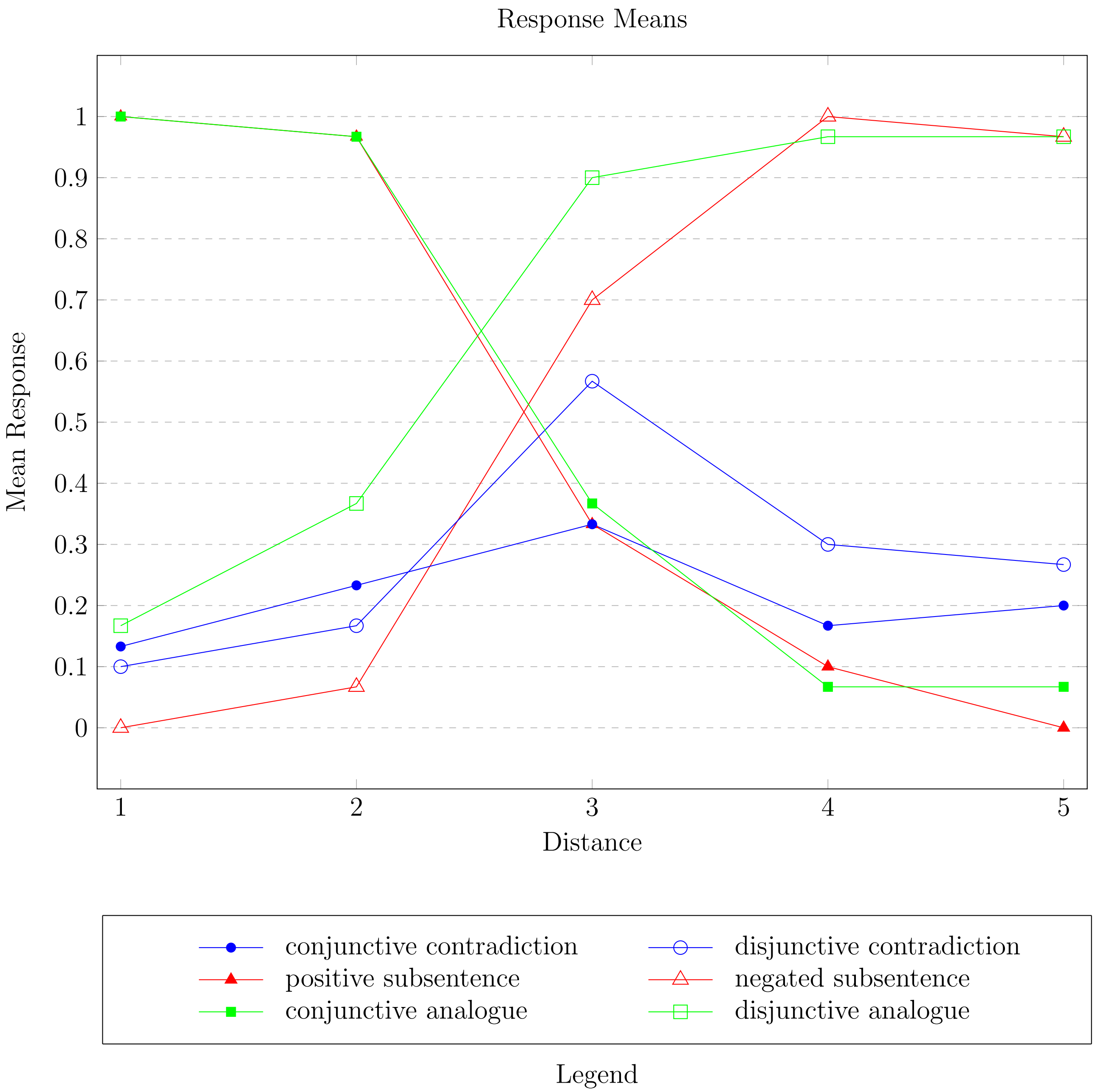

Figure 3.4: Shape-colour response means

The first observation to be made is that responses to the disjunctive contradiction weakly hint at a hump effect centring on pair 3, indicating that of the five shape pairs, pair 3 was interpreted to be the maximally borderline case of the predicate, as intended. This is somewhat supported by a series of one tailed Wilcoxon signed ranks tests applied pairwise between borderline distances 1 and 3, and 3 and 5, revealing that "True" responses to distance 3 were significantly more frequent than "True" responses to both of the other distances. We say that the effect is only somewhat supported since correction for multiple comparisons has the potential to erase the observed hump (see the footnote Table 3.13). No pairwise contrasts of distance were significant regarding the conjunctive contradiction, thus despite a superficial likeness to a hump on visual inspection of the means, in the case of this admittedly limited data-set, the conjunctive contradiction curve can only be characterised as a "flat" response, to use Ripley's terminology.

Table 3.13: Shape-colour contrasts for Ripley's hump effect

Condition	Contrasted Distance Pairs	W	p[8]
disjunctive contradiction	$3 \sim 1$	540	$< .001$
disjunctive contradiction	$3 \sim 5$	630	$< .01$
conjunctive contradiction	$3 \sim 1$	540	ns
conjunctive contradiction	$3 \sim 5$	510	ns

[8]Bonferroni corrected for four comparisons (.05/4). Admittedly, Bonferroni correction is a very conservative method, which is paradoxically used here assuming only a subset of the total number of comparisons actually performed over the course of the analysis. The correction has therefore been used in a manner which somewhat undermines itself. A more conservative adjustment would erase the observation of a hump effect. In total, 12 pairwise comparisons were analysed for the purpose of examining the hump effect across both contradiction types, as well as 9 additional pairwise comparisons between sentence

No meaningful observations can be made regarding whether the intended 2nd-order borderline cases at distances 2 and 4 were indeed interpreted as 2nd order borderline cases. This is due to the small size of the sample coupled with the size of the margin between responses to maximally borderline cases and responses to clear cases. More explicitly, in order for a response to an intended 2nd-order borderline case to be characterised as indicative of a 2nd-order borderline interpretation, the response to this case would have to be distinct from both the response to the maximally borderline case and the response to its adjacent clear case. The best candidate for this kind of contrast might be a response to the disjunctive contradiction at Pair 2, since of the contrasts represented in Table 3.13, that at the disjunctive contradiction between Pairs 1 and 3 is the most pronounced. However, given the responses to Pairs 1 and 3, it would be necessary for exactly 7 or exactly 8 of the 30 participants to agree to the contradiction at Pair 2 in order for a Wilcoxon one tailed test to identify a statistically significant difference between this hypothetical response and the responses to both Pair 1 and Pair 3. What's more, the hypothetical contrast disappears when controls for multiple comparisons are taken into account. It follows that given the responses displayed in Table 3.13, no response from the 30 participants to either con-

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type at distance three. If as few as all 12 possible comparisons for the hump effect are taken into account, the only contrast which is significant is that between distances 3 and 1 in the case of the disjunctive contradiction, which does not support the observation of a hump response. We nevertheless again emphasise that while we do not want to read to deeply into these weak results, we also do not want to turn a blind eye to an effect that might be hinted at, but cannot be confirmed in such a small sample. This caveat holds of all Bonferroni corrected p-values presented in this study.

tradiction at Pairs 2 and 4 can be reliably characterised as indicative of a 2nd-order borderline interpretation, and it is inappropriate to analyse the data as bearing any meaningful insight on this question. This shortcoming highlights the problem that, given that we are interested in what are likely rather small effects, the size of the sample is insufficient to support strong conclusions, an issue which will arise more than a few times and which we will address more thoughtfully in the next subsection.

At most we can conservatively note where there is a strong contrast between the responses to Pairs 2 and 4 on the one hand, and the maximally borderline and clear cases on the other. This exclusively concerns the response to the disjunctive contradiction at Pair 2, which is distinct from the response at Pair 3 and not distinct from that at Pair 1 ($W = 630$, $p < .01$; $W = 480$, $p = 1$; respectively, with Bonferroni correction for 12 comparisons). Bearing in mind that we are unable to affirm the interpretation of this case as 2nd-order for the reasons described above, we can nevertheless tentatively posit that the interpretation of Pair 2 is better characterised as non-borderline.

Before diving into whether our predictions in section 1.1 were borne out, it will be useful to define what can be considered to be a "True" and a "False" response in the data as a whole. We propose that an overall response which tends more toward "True" than could be expected by chance is appropriately characterised as a "True" response, and likewise, an overall response that tends more toward “False” than could be expected by chance, is appropriately characterised as a “False” one. In this case we use a binomial sign test to determine that a response is unlikely to be attributable to chance. Six sentence types across five distances results in 30 comparisons, and the results we report are corrected for this ($\alpha = .05/30$). All results of these tests are reported in full in the Appendix to this section.

In order to evaluate the effect of the operators on truth we must assume that if a response to a sentence can be characterised as “True” or “False”, and there is no theoretical motivation for the sentence to be interpreted tolerantly, we are licensed to characterise the response along a strict interpretation. Otherwise, we must look to the overall response to the sentence relative to the overall response to other sentences in order to interpret the results as possibly indicating a tolerant interpretation, including in those cases where a sentence cannot be characterised as “True” or “False”.

Regarding the borderline case and the corresponding Predictions (11a)-(11d) visualised in Table 3.2, only Prediction (11c): *the sentence* $A \wedge \neg A$ *is strictly false, but tolerantly true* was consistent with the results, as the response to the conjunctive contradiction could not be characterised as either a “True” or a “False” response. Prediction (11a): *the sentence* $\neg(A \vee \neg A)$ *is strictly true* was not borne out, as the response to the disjunctive contradiction was equally inconclusive. Predictions (11a): *the sentence* Aj *is strictly false* and (11b): *the sentence* $\neg Aj$ *is strictly false* were also inconclusive.

It is notable that in the case of the latter the trend was toward an unexpected "True" response ($p < .05$ without correction), a trend which might be supported by evidence from a larger trial.

Prediction (11e): *when* Bj *is true, the sentence* $Aj \wedge Bj$ *is strictly false*, was not borne out, as the response was not conclusively "True" or "False". Prediction (11f): *when* Bj *is false, the sentence* $\neg(Aj \vee Bj)$ *is strictly true* was supported by the data ($p < .01$).

Moving on to Predictions (12a)-(12f), the data clove very closely to the expected truth judgments. Since we effectively have no second-order borderline case here, the distances relevant to evaluating these predictions are distances 1 and 2. For distance 1 both contradictions were judged "False", the positive subsentence was judged "True", and its negated counterpart "False", the conjunctive analogue was judged "True", and its disjunctive counterpart judged "False". Results for distance 2 differed with regard to the conjunctive contradiction and the disjunctive analogue, which were both inconclusive. Thus, as expected, in a non-borderline, clear case of the predicate *near*, responses were classical. The results are less clear for the "non-borderline" case at distance 2.

Similarly, Predictions (13a)-(13f) were completely borne out in the data, not only for the clear case at distance 5, but also for distance 4. At these distances, both contradictions were rejected, the positive subsentence accepted, its negated counterpart rejected, the conjunctive analogue rejected, and its disjunctive counterpart rejected. Thus, in the non-borderline case of clearly not *near*, responses were classical.

As for any contrast between the two contradictions, a Wilcoxon signed rank test revealed no significant contrast between the responses to the disjunctive and conjunctive contradictions at the borderline case (W = 555, p = .65718, with Bonferroni correction for 9 comparisons at the borderline case, see the appendix to this section for details). However, given the sample size, we should be careful not to afford this observation undeserved gravity. To put this in perspective, had just one participant who gave a "False" response to both contradictions decided to instead give a "True" response to the disjunctive contradiction (or had the inverse occurred), the test would have indicated a significant contrast in favour of the disjunctive without correction. A different response from as few as three participants would similarly support a contrast in the far more conservative corrected results. Although we err toward a conservative reticence to over-interpret, it is clearly not ideal to afford such weight to each participant's response and a larger sample is therefore likely merited in order to lend support to this result.

Regarding other contrasts at the borderline case, two are of particular note. First, although responses to the atomic subsentences could neither be characterised as "True" or "False", a signed rank test revealed a significant contrast between them ($p < .05$, $\alpha = .05/9$). Second, in addition to the finding that the disjunctive analogue is strictly true at the borderline case, we can add that it also contrasts significantly with its conjunctive counterpart ($p < .001$). All other pairwise comparisons at the borderline case were not significant. Of lesser note is that the contrast between the disjunctive contradiction and its analogue was on the cusp of significance ($W = 585$, $p = .098$) at the borderline case.

In summary, a signed rank test revealed no significant contrasts between the conjunctive contradiction, its analogue, and the positive simple subsentence at the borderline case, and none of these three sentences could be characterised as either "True" or "False". Similarly, no significant contrasts were observed between the disjunctive contradiction, its analogue, and the negated subsentence at the borderline case. Interestingly however, although neither the disjunctive contradiction nor the negated subsentence could be characterised as either "True" or "False", the disjunctive analogue could be characterised as "True". Remarkably, responses to the positive and negated subsentences were not "False", as expected under a strict interpretation, and in fact, neither was judged less true than the conjunctive contradiction.

This leaves us with a two tiered pattern of relative truth judgments at the borderline case, rather than the three tiered pattern we had initially predicted at the conclusion of section 1.1:

$$\left.\begin{array}{c}(Aj < \neg Aj) \\ Aj \wedge \neg Aj \\ \neg(Aj \vee \neg Aj) \\ Aj \wedge \neg Bj\end{array}\right. \quad < \quad \neg Aj \vee \neg Bj$$

Finally, the pronounced contrast found between the positive/conjunctive non-contradictions and their disjunctive/negated counterparts at distance 3 indicates that negation may play a crucial role in how these non-contradictory sentences are evaluated in a borderline case.

3.2.4 Analysis and interim discussion

The few small forays we have made into the limits of the data-set to provide meaningful insights serves to emphasise the caution with which it ought to be analysed. The results of this experiment are fragile. Furthermore, they are not uniformly fragile, as responses at the maximally borderline case are just those responses whose analysis is most likely to change drastically with small changes in participants' responses. And while the coincidence of a certain few of these results with those of previous experiments that drew upon much larger samples, coupled with good theoretical motivations for why we expect to see the particular results that we do see, justify some confidence in our results, caution must be taken not to read too deeply into them. The flip side of this coin is that we also cannot have confidence that the fragile effects that we observe here are not a glimpse of what might prove to be a robust effect, which would be more clearly in evidence were the data-set larger. In this sense, although we set out with some definite questions to be answered by the data, we should not lose sight of the questions that the data has the potential to prompt. Hence we shall dare to speculate upon what lies over the horizon, while keeping firmly in view the limits of our sight.

In that spirit, and with utmost caution, it is worth considering the alternative of how the pattern of relative truth judgments might have turned out. Indeed, the small size of the sample, accompanied with a rather conservative method of correction, potentially flattened what might prove to be a three tiered pattern. Particularly, the negated subsentence was only just not characterised as true, and the positive subsentence along with the conjunctive contradiction and analogue were narrowly not characterised as false (by a margin of less than five participants, even corrected for 30 comparisons). Thus, the pattern was very nearly three tiered:

$$\begin{array}{c} Aj \\ Aj \wedge \neg Aj \\ Aj \wedge \neg Bj \end{array} \quad < \quad \neg(Aj \vee \neg Aj) \quad < \quad \begin{array}{c} \neg Aj \\ \neg Aj \vee \neg Bj \end{array}$$

Although this pattern is not exactly what would be predicted by our ad hoc account discussed in Section 3.1, it is surprisingly not far from the mark, with only the relatively low acceptance of the conjunctive contradiction straying from predictions. Moreover, we hold it a crucial finding that, despite the weakness of our methods to reveal contrasts, the two clear contrasts that we could make out were between the positive subsentence and negated subsentence, and between the disjunctive analogue and the conjunctive analogue, both findings in line with the exploratory results of the replica experiment, again suggesting that negated components are interpreted tolerantly, not strictly, per Égré and Zehr's algorithm. Two places where the results do not coincide are in the level of agreement to the positive subsentence, to which almost no participants agreed in the replica experiment, and the level of agreement to the negated subsentence with respect to the level of agreement to the disjunctive borderline contradiction, recall that the disjunctive had a percentage of agreement $> 85\%$ and the negated subsentence as percentage $< 60\%$ in the results of the replica experiment. If average response to the disjunctive analogue can acceptably serve as a proxy to the negated subsentence, then this interaction is reversed in the current results. We suspect that these difference might be attributable to the perceptual mode of stimulus used in the latter experiment, but more tests would be needed to confirm this.

A surprising high proportion of individual responses from the shape-colour experiment were classed as "slope up" responses, an effect which might be attributable to a lack of context, but which is not in line with Ripley's results (keeping in mind that because of how we ordered the data, what we refers to as "slope down" is what we refer to as "slope up". If participants are given a visual representation of a distance with no context for reference, (for example, whether the representation is intended to be interpreted one to one or one to 1,000,000, as could be the case for a map; and additionally, what the purpose of the evaluation of distances is), save for the possible implication that other distances or the frame might be relevant to determining a comparison class, then we might expect an unusual degree of heterogeneity across participants' responses. In particular, if participants took the representation to be one to one, there could be a tendency to evaluate all distances as representative of "near" relative to a comparison class of commonly encountered real measurements, since the actual length of the distances displayed on participants' device screens would presumably have been rather small by this standard. In this case we might wonder whether participants who gave a "slope up" response would have resolved the slope into a hump, had the spread of distances been greater. This is, however, very unlikely, as in all cases save one, participants found the negated subsentence "True" for distance 5 and "False" for distance 1, with the reverse pattern holding for the positive subsentence, and all participants additionally found the negated subsentence "True" for distance 4, with the reverse pattern holding for the positive subsentence (with 3 exceptions), strongly indicating that distance 5 was interpreted nearly universally as a clear case of "not near".

Another discrepancy with Ripley's observations pertains to the proportion of individual participants' responses classed as "hump", "slope", "flat" and "other". Ripley's data indicated a strong preference for a hump response (76 of 149 participants). Our data indicates a far less pronounced preference for a hump response, with a pronounced preference for a "flat" response in the case of the conjunctive contradiction and no preference for a hump response with regard to contradictions, overall. In this case, a "flat" response is characterised as either a "True" or as "False" response at all distances, however in practice, "True" responses across the board are very rare. Of the 30 participants, only one gave a flat "True" response to the conjunctive contradiction[9]. Thus, this milder hump effect may merely be indicative of a higher rejection rate of contradictions in comparison to Ripley's study.

The following table displays the number of participants, as well as the percentage of participants, whose responses fell into any of the four patterns described by Ripley, sorted by contradiction type. For example, 11 of 30 participants, or 36.67% of participants, showed a "hump" response to the disjunctive contradiction, and 19 responses out of 60, or 31.67% or responses showed a "hump" effect, overall.

Table 3.14: Individual responses classified into Ripley's 4 types

	hump	slope up	slope down	flat	other
disjunctive contradiction	11/36.67%	5/16.67%	1/3.33%	8/26.67%	5/16.67%
conjunctive contradiction	8/26.67%	4/13.33%	2/6.67%	15/50%	1/3.33%
Total (of 60)	19/31.67%	9/15%	3/5%	23/38.33%	6/10%

Despite the lower prevalence of individual hump responses, we contend that the overall contrasts observed for the disjunctive contradiction between

[9]This same participant's response type to the disjunctive contradiction was classed as "other": "True" for all but distance 2, possibly indicating that the participant did not make a strong distinction between the two contradiction types.

distance 3 on the one hand, and distances 1 and 5 on the other, is sufficient to conclude that distance 3 was interpreted to be the maximally borderline case of *near*, as intended, and to justify our analysis of contrasts in responses to this case as truly reflecting judgments about borderline cases.

3.3 Height-nationality experiment

The height-nationality experiment was conceived as a daughter experiment to the shape-colour experiment, differing chiefly in the level of concreteness present in the visual stimuli. Whereas the shape-colour stimuli were designed to be abstract to such an extent as to be divorced from an explicit scale, and to thereby be evaluated according to unmitigated perception, the height-nationality stimuli were deliberately associated to a proportionate representation of a familiar scale of height. In this way, the degree to which participants were free to build their own representations of the vague predicate *tall* and its boundaries, and especially their representations of the relevant context, was constrained, while at the same time providing no overt indication of how the target predicate was to be mapped to individuals. If the shape-colour experiment was aimed at extreme freedom of interpretation, the height-nationality experiment was aimed at moderation of the same.

3.3.1 Hypotheses

As a direct descendent of the shape-colour experiment, all hypotheses were identical to the hypotheses of the parent experiment. We refer the reader to Section 3.2.1 and we will not repeat them here. We expect the baseline analogues to be false in a manner analogous to false controls.

3.3.2 Methods

The American style police line-up of the stimulus images is inspired by Alxatib and Pelletier's experiment on conjunctive and disjunctive borderline contradictions [3]. Although the design differs in some respects from Alxatib and Pelletier's, it is similar enough that a replication of their results could be anticipated, just as well as a replication of the results of the shape-colour experiment. Additionally, contrasts with the results of either of these two experiments were expected to be informative as to how small changes in design elements have the potential to influence results.

3.3.2.1 Capturing the Height Variable in Images

The stimulus images consisted of a line-up of five fairly realistic silhouettes overlaid upon a scale of height marked for feet and inches, with horizontal grid lines spaced at half-foot intervals to allow participants to estimate the silhouettes' heights to a fairly specific degree. These were the five "suspects", each identified by a number, "1"-"5", and the national flag corresponding to a well-known (primarily) anglophone country, see Figure (3.5) for a sample image. Heights covered a range from approximately 5'1.5" to 6'10.5" with the borderline case (suspect number 4 in Figure 3.5) representing a height of approximately 5'10.5", just at the upper range of mean heights for men of the represented countries born in the last half century [40, 51][10].

Figure 3.5: Sample stimulus image

[10]The borderline case would ideally have been represented an inch or two shorter to fit comfortably within the range of actual world means, however, relative to the range of heights represented in the images, the borderline case is just short of both the median height and the mean height of the represented suspects. For now we can assume that these small discrepancies should have little effect on results; the objective is merely to depict a borderline case and this can be confirmed in the results by the appearance of a hump effect.

Colour predicates of the shape-colour experiment correspond to nationality predicates in the height-nationality experiment, cued by the associated flag. Since accurate results therefore hinge upon correct identification of national flags, all participants were screened for their familiarity with the flags by means of a pretest (one participant's results were excluded on these grounds).

The order of suspects in the line-up was not constant, however the nationality and height associated with each silhouette was. Each suspect was therefore a "character" which could appear in any numbered slot "1" through "5", although in practice, not all "characters" appeared in every one of the slots, and some appeared repeatedly in the same slot. Five such line-up orders were created. As a precautionary measure to avoid unreliable results in the case that a particular silhouette or nationality/height combination should prove unexpectedly problematic, a B version of the line-up was created using a different set of "characters", that is, a different set of silhouettes was used and the nationality/height pairs were scrambled with respect to the A version. Half of participants were exposed to the set of suspects in version A and half to those in version B, the idea being that if all went well, results for both versions should match. All ten stimulus images are included in the appendix to this section.

3.3.2.2 Capturing the condition variable in sentences

As in the shape-colour experiment, target sentences came in six formal types: conjunctive contradiction, disjunctive contradiction, conjunctive analogue, disjunctive analogue, positive subsentence, and negated subsentence. As before, the A component of analogue sentences featured the target predicate, *tall* and the B component an auxiliary nationality predicate: *Australian*, *Irish*, *Canadian*, *British*, or *American*.

However, in addition to the familiar six formal types, two logical alternates to the analogue sentences were distinguished: a conjunctive analogue baseline and a disjunctive analogue baseline. In these cases the analogue sentences were paired with images such that the truth value that participants were expected to assign to the B component would preclude the sentence's being truth functionally true, regardless of the evaluation of the vague A component. Thus, analogue sentences came in two flavours, one in which truth hinges on the evaluation of the target predicate and one in which truth hinges on the evaluation of the auxiliary. Baseline analogues could provide an indication of the error rate associated with conjunctive and disjunctive analogues. We might interpret an elevated error rate as indicative of higher processing demands, which we would like to take into account when interpreting the body of results. The distinction between the two types is illustrated by comparing the examples in Table 3.15 to the sample image of Figure 3.5.

Table 3.15: Height-nationality sentences

1	**Conjunctive Contradiction**	
	Suspect ID is tall and not tall.	
Example	Suspect 4 is tall and not tall.	
2	**Disjunctive Contradiction**	
	Suspect ID is neither tall nor not tall.	
Example	Suspect 4 is neither tall nor not tall.	
3	**Conjunctive Analogue**	**Conjunctive Analogue Baseline**
	Suspect ID is tall and NATIONALITY.	
Example	Suspect 4 is tall and British.	Suspect 4 is tall and Irish.
4	**Disjunctive Analogue**	**Disjunctive Analogue Baseline**
	Suspect ID is neither tall nor NATIONALITY.	
Example	Suspect 4 is neither tall nor Irish.	Suspect 4 is neither tall nor British,
5	**Positive Subsentence**	
	Suspect ID is tall.	
Example	Suspect 4 is tall.	
6	**Negated Subsentence**	
	Suspect ID is not tall.	
Example	Suspect 4 is not tall.	

With sentence type encompassing eight levels (the original six, plus the extra two logical subtypes), tested across five levels of height, and carried out once for each of the two versions of the experiment, the experiment had a $8 \times 5 \times 2$ factorial design.

3.3.2.3 Participant recruitment and prescreening

Thirty-three participants were recruited through the Prolific platform and the online experiment was hosted by Ibex Farm. Participants who had participated in a previous borderline study or pilot were excluded. A participant's data were omitted from the results in case the participant was unable to correctly identify all five national flags and to pass at least 90% of control items (found in the Appendix to this section). Three sets of data were omitted on these grounds.

3.3.3 Results

The mean results for all conditions is presented in Table 3.16 and visually displayed in Figure 3.6, while the isolate results of the contradictions is displayed in Figure 3.7. In order to facilitate comparison to the corresponding plot for the shape-colour experiment, the plot shows heights in descending order so that the average responses in reaction to tallest suspect are the leftmost column of responses (1), the the shortest, the rightmost (5). Therefore in both cases, left corresponds to the case best exemplifying the vague predicate. A two way ANOVA mixed revealed no effect for Version, so results from both versions are collapsed into one set of results.

Table 3.16: Height-nationality response means

	6'10.5"	6'4.5"	5'10.5"	5'7.5"	5'1.5"
positive subsentence	1	.96	.3	.1	0
negated subsentence	0	.03	.56	.86	.96
conjunctive contradiction	.06	.3	.5	.4	.1
disjunctive contradiction	.06	.06	.56	.56	.26
conjunctive analogue (false B conjunct)	.1	.03	0	0	0
conjunctive analogue (true B conjunct)	1	.96	.53	.2	0
disjunctive analogue (false B disjunct)	0	.06	.13	.16	.26
disjunctive analogue (true B disjunct)	.2	.3	.66	.93	.83

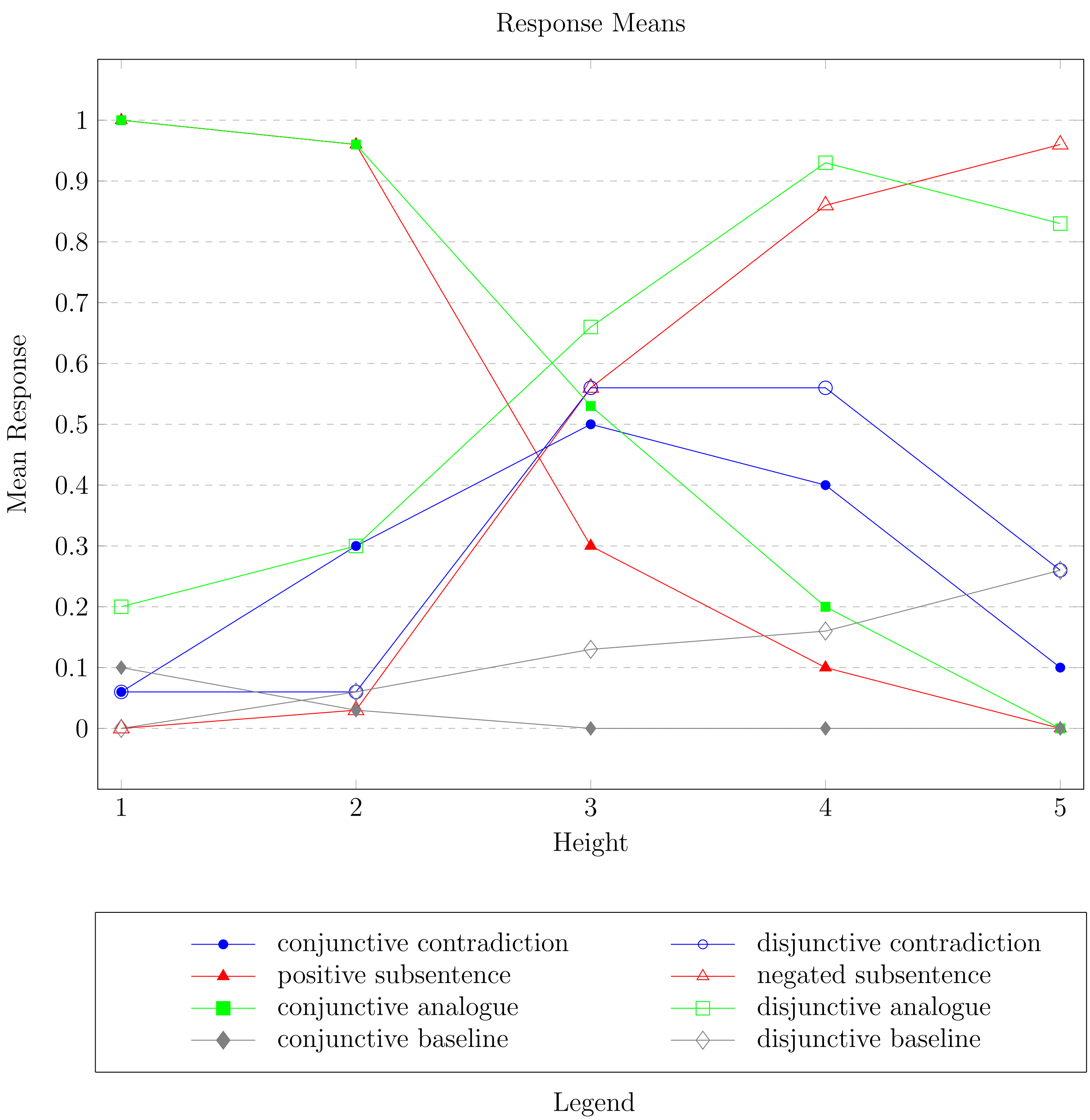

Figure 3.6: Height-nationality response means

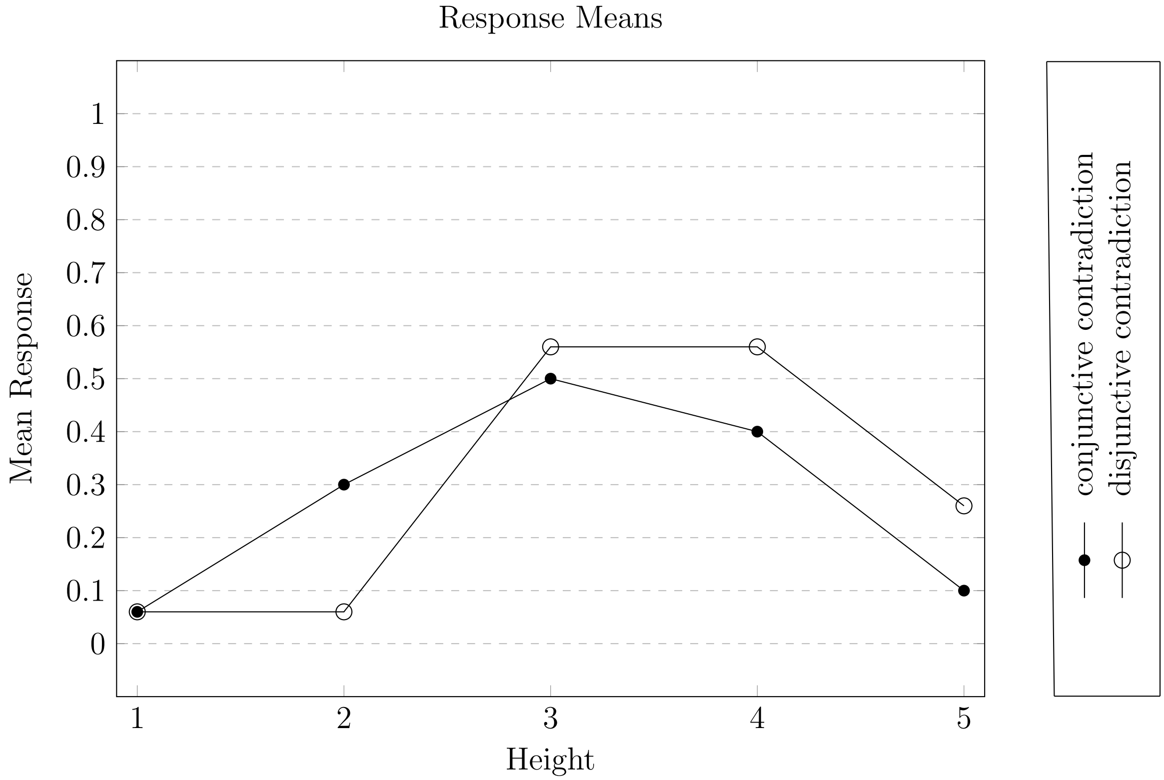

Figure 3.7: Contradiction response means

Visually speaking, the mean response to the conjunctive borderline contradiction displays a smooth and relatively symmetrical transition across the border. In contrast, the mean response to the disjunctive contradiction plateaus at the lower edge of the presumed border before descending abruptly to a very low plateau above the border.

The hump effect was observed for both contradictions. The results of a Wilcoxon signed ranks test of pairwise contrasts between the maximally borderline height and clear cases of *tall* and *not tall* are reported in Table 3.17 (p-values are corrected by the Bonferroni method for four comparisons (.05/4)).

Table 3.17: Shape-colour contrasts for Ripley's hump effect

Condition	Contrasted Distance Pairs	W	p[11]
disjunctive contradiction	$3 \sim 1$	675	$< .001$
disjunctive contradiction	$3 \sim 5$	585	$< .05$
conjunctive contradiction	$3 \sim 1$	645	$< .01$
conjunctive contradiction	$3 \sim 5$	530	$< .01$

An analysis was carried out as it had been for shape-colour experiment to determine whether it is appropriate to interrogate the results for 2nd-order borderline cases. The best candidate for a 2nd-order case, based upon the responses to its clear case and maximally borderline case neighbours, was height 2 with respect to the disjunctive borderline contradiction. Now, in reality, the overall response to this height was identical to that for height 1, and thus it is better characterised as a clear case. Regardless, the analysis revealed that the margin between the response to height 1 and height 3 was too narrow to afford a meaningful characterisation of a hypothetical 2nd-order borderline case between them. It follows that the margin was also too narrow for the remaining three potential 2nd-order borderline cases, thus it would be inappropriate to attempt to characterise any case as either 2nd-order borderline or not 2nd-order borderline. It is possible to observe that responses to heights 2 and 4 for the disjunctive contradiction were identical

[11]Bonferroni corrected for four comparisons (.05/4).

to heights 1 and 3, respectively, and are thus perhaps best characterised as non-borderline and borderline, respectively.

As before, in determining whether our predictions in Section 3.2.1 were borne out, we consider an overall response which tends more toward "True" than could be expected by chance as appropriately characterised as a "True" response, and an overall response that tends more toward "False" than could be expected by chance, appropriately characterised as a "False" one. We again use a binomial sign test to determine that a response is unlikely to be attributable to chance. Eight sentence types across five heights results in 40 comparisons, and the results we report are corrected for this ($\alpha = .05/40$). All results of these tests are reported in full in the Appendix to this section.

Of predictions (11a)-(11d) (refer to Table 3.2 in Section 3.2.1), only Prediction (11c): *the sentence* $A \wedge \neg A$ *is strictly false, but tolerantly true* was consistent with the results. It was not characterised as either true or false, which is consistent with the possibility of its being trivially false on a strict interpretation, and optionally true on a tolerant interpretation. Excluding responses to the analogue baseline sentences, all responses to the borderline case could not be characterised as true or false, and since all other predictions for the borderline case were predicted to be either strictly true or strictly false, none of the predictions about them were borne out. Results weakly diverged from the results fo the shape-colour experiment with regard to the positive and negative subsentences, for whereas previously the negated subsentence was only narrowly not determined to be a false response, this time no such narrow margin appeared. Instead, the positive subsentences was only narrowly not determined to be false ($p < .05$ without correction). Thus the results for the subsentences, while not incompatible with those from the shape-colour experiment, are both shifted toward a lower response.

Predictions (12a)-(12f) were clearly borne out. In a non-borderline, clear case of *tall*, responses were classical. Similarly, Predictions (13a)-(13f) were clearly borne out in the data. Thus, in the non-borderline case of clearly not *tall*, responses were classical.

As before, a Wilcoxon signed rank test revealed no significant contrast between the responses to the disjunctive and conjunctive contradictions at the borderline case ($W = 480$, $p \approx .6$, without correction). Again, given the sample size was likely insufficient to reveal a marked contrast for this type of experiment. With that said, the consistent lack of significant contrast between contradiction types we observed in this and the previous experiment, compared with the clear contrast observed in the replica experiment, points to an effect for type of stimulus method. No other contrasts at the borderline case were statistically significant, although we would like to point out that the contrast between the positive subsentence and negative subsentence was found to be significant before being Bonferroni corrected ($W = 570$, $p < .05$), weakly echoing the contrast between these revealed by the shape-colour experiment.

Chapter 4

General discussion

The results of our experiments are not ground breaking, but they do provide small hints toward refining our picture of how speakers conceive of the borderline region and what their intuitions are about borderline cases. As expected, borderline contradictions are not necessarily judged to be false, although the degree to which they are judged to be true appears to vary greatly depending upon experimental methods. Truth judgments appear to be particularly high when speakers are asked to provide judgments about a borderline case that they are linguistically instructed to imagine, as in the case of Égré and Zehr's experiment and our replication, although Serchuk et al.'s data does not indicate this as strongly.

In a 2013 study on hysteresis, Égré et al. [20] found an effect of enhanced contrast as speakers were led along a soritical series of colour patches and asked to name the colour patch according to a set of colour predicate options. However an identical test in which participants were asked to choose a choose a colour patch that best matched the target colour patch, this effect disappeared. In effect, speakers' judgments appeared to be more accurate when they were given perceptual stimuli for comparison to the target, rather than being required to access their semantic memories in order to compare the predicate to the target. An analogous effect might be responsible for the enhanced contrasts revealed by the purely linguistic tasks, for which speakers are required to access some semantic memory of a prototypical borderline case. If this is the case, then their responses to linguistic tasks might better exemplify their linguistic intuitions about borderline cases, or inversely, it might be their intuitions about what types of expressions ought to be true of a prototypical borderline case that informs their semantic image of that prototype. In perceptual tasks, however, there is no assumption that any stimulus is a prototypical borderline case, no set of intuitions about what sort of expressions should be true of any of the cases, they are evaluated on their face value. This might explain relatively lower agreement to contradictions.

There is a caveat here, which pertains to the manner in which the questions were posed, for in both the original and replication experiments, participants were asked whether or not it is true "to say" the target sentence, and not whether the target sentence was true. It is therefore admissible to suppose that participants provided an opinion on the assertibility of the sentence, rather than its truth, in these experiments. In contrast Ripley's, Alxatib and Pelletier's, and the present shape-colour and height-nationality experiments, all asked for an opinion on truth, so while we cannot rule out the possibility that assertibility may have influenced participants' responses (they may have interpreted the question as such, or concerns of assertibility may have impacted the very act of providing a response), it is fair to suppose that as measures of truth judgments, the last experiments might be more reliable.

Although contrasts between contradiction types was not marked in our visual experiments, perhaps also due to the perceptual nature of the stimuli, they were not necessarily inconsistent with previous results such as Ripley's, and it seems likely that the size of our experiments was a limiting factor on our ability to discern contrasts. However, the result that deserves the most attention was surprisingly one that was explored somewhat subordinately to the expressions that were the focus of our investigation, the high acceptability of cases of simple negated borderline sentences and their relation to their positive counterparts. This contrast was especially striking in the replication experiment, and it is all the more striking if speakers are, indeed, accessing some semantic memory of a prototypical borderline case in order to provide these judgments. While we have provided an ad hoc account of how a tolerant interpretation of negated expressions can be reconciled with the pattern of predictions stemming from accounts that describe the contrasts between conjunctive and disjunctive contradiction types in terms of two forces of interpretation, we recognise that the initial observation needs bolstering from further empirical studies.

4.1 Conclusion

In concluding we would like to highlight the great need for more experimental work in the area of borderline contradictions. Ideally, we would also hope that a large scale study could bolster our results regarding judgments to the positive and negated subsentences, and if replicated, that this pattern of judgments could be disseminated to serve as a platform to build a more accurate representation of how speakers think of the borderline region and communicate about it. We would also like to bring attention to the need for studies that attempt to investigate the pragmatic aspect of borderline contradictions, both how pragmatic factors might trigger reanalysis, per our ad hoc account, and also speaker use contradictions. For example, it would be useful to attempt studies that reverse the stimulus/response relationship we have so far seen, say, by providing scenario/image as response types to see what information speakers infer from borderline expressions.

In this work we have attempted to provide a very general and theory neutral way of talking about the borderline region and borderline cases. We have defined borderline contradictions in terms of their semantics and pragmatics, and examined brefly how syntactic form bears upon these, hopefully in a manner that elucidates some of the distinctions that experimental work has sought to bring into relief. We have added to the body of experimental work with three new experiments which both support previous findings, and bring to light a discrepancy that may warrant a modification of some leading accounts of borderline contradictions. We have offered a minimal modification with the caveat that it is somewhat uniquely suited to our results, which in turn are in need of support.

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4.2 Appendix to Section 3.2: Shape-colour experiment

4.2.1 Sequence of the experiment

(Estimated time: 5-10 minutes; limit: 15 minutes; 10 practice items; 30 target items)

1. Welcome

   (a) Introduction: Here participants were greeted, provided a summary of the time frame and aims, and were asked to acknowledge the privacy notice and conditions. Participants ticked a box to verify that they had read and agreed to the privacy notice and conditions of their participation. Acceptance of these conditions was obligatory.

(b) English Form: Participants were required to tick a box for "yes" or "no" when asked whether English was their first language. This was obligatory. Participants were additionally asked to enter the name of their first language into a free text box. Data from participants who ticked the box for "no" or who entered the name of a language other than English, was discarded (although these participants were nevertheless paid for their participation).

2. Instructions and Practice

(a) Participants were provided with the following instructions.

INSTRUCTIONS:

For this experiment you will be asked to assess a number of items. Each item consists of a picture and an associated judgment task.

For each item you will see the picture and a sentence relating to the picture.
For any given picture-sentence pair, you will be asked for a judgment:
Do you consider the sentence to be TRUE or do you consider it to be FALSE, relative to the picture?

If you judge the sentence to be TRUE, click on the box bearing the label of T.

If you judge the sentence to be FALSE, click on the box bearing the label of F.

There is no time limit on this task, but we ask that you devote your full attention to it and that you INDICATE YOUR ANSWER AS SOON AS YOU HAVE MADE YOUR DECISION. We are interested in your basic intuitions about the sentences, so there are no wrong answers.

It is worth repeating that you must follow the instructions you will see at the conclusion of the experiment regarding the COMPLETION CODE in order to be paid.

   (b) Participants were informed that they would have an opportunity to practice on 10 practice items, which were then presented. They were then told to press continue when ready to proceed to the main experiment.

3. Main Experiment

   (a) 5 sentences corresponding to distance condition X 6 sentence type conditions, for a total of 30 target items. All target items were provided in random, shuffled order.
   All items are provided in the appendix to this report

4. Conclusion

   (a) When all 30 target items had been completed, a thank you page appeared, urging participant to click "Submit" to send the results to the server.

(b) Participants were then provided with a Prolific Completion Code and reminded to copy and paste it into a new browser window in order to be redirected to Prolific and paid for their participation.

4.2.2 Controls

The practice portion was composed of items which were formally identical to a subset of the test items, however through the conditions of the specific images and through substitution of the B component predicate for that on which the truth of the sentence hinges, the sentences were expected to score unequivocally true or false. For example: where both shapes were red, the sentence, "neither the square nor the circle is blue" should always score a "True" response. Each of the five distances was shown twice in this portion.

The practice portion served a number of purposes. Foremost among these is that it exposed participants to the full range of distances they would see in the test portion. This exposure was crucial in order to establish the context within which participants were intended to evaluate the target items. That is, in choosing to isolate participants from preconceived notions of the comparison class against which to evaluate the sentences, participants were also deprived of a central parameter to their decisions, which then needed to be resupplied. Having seen all five cases of the *near* twice, and having seen how each case relates to the frame, participants should have formed a good understanding of the full range of distance relevant to the context, i.e. they should have eliminated the possibility that *near* might be interpreted at any distance exceeding the size of the frame.

Additional purposes were to familiarise participants with the procedure, as well as to reveal whether the procedure or the intended contextual clues had been misinterpreted. The final purpose was to have a baseline for truth and falsity based on a “non-vague” component against which to compare responses to sentences whose truth hinged on the vague component.

The practice items were never randomised.

4.2.3 Results of Statistical Tests

Table 4.1: Results of the sign test to determine "True" and "False" responses

Item	Estimate	.95 CI	p-value	Corrected	.99833 CI
conjContra_d1	.133	.038 - .307	p < .001	p < .01	.014 - .417
conjContra_d2	.233	.099 - .423	p < .01	ns	.053 - .532
conjContra_d3	.333	.173 - .528	ns	ns	.108 - .633
conjContra_d4	.167	.056 - .347	p < .001	p < .01	.025 - .457
conjContra_d5	.2	.077 - .386	p < .01	p < .05	.038 - .496
disjContra_d1	.1	.021 - .265	p < .001	p < .001	.006 - .374
disjContra_d2	.167	.056 - .347	p < .001	p < .01	.025 - .457
disjContra_d3	.567	.374 - .374	ns	ns	.278 - 0.826
disjContra_d4	.3	.147 - .494	p < .05	ns	.088 - .601
disjContra_d5	.267	.123 - .459	p < .05	ns	.07 - .567
pos_d1	1	.884 - 1	p < .001	p < .001	.79 - 1
pos_d2	.967	.828 - .999	p < .001	p < .001	.726 - 1
pos_d3	.333	.173 - .528	ns	ns	.108 - .633
pos_d4	.1	.021 - .265	p < .001	p < .001	.006 - .374
pos_d5	0	0 - .116	p < .001	p < .001	0 - .21
neg_d1	0	0 - .116	p < .001	p < .001	0 - .21
neg_d2	.0667	.008 - .221	p < .001	p < .001	.001 - .327
neg_d3	.7	.506 - .853	p < .05	1	.399 - .912
neg_d4	1	.884 - 1	p < .001	p < .001	.79 - 1
neg_d5	.967	.828 - .999	p < .001	p < .001	.726 - 1
conjAnaT_d1	1	.884 - 1	p < .001	p < .001	.79 - 1
conjAnaT_d2	.967	.828 - .999	p < .001	p < .001	.726 - 1
conjAnaT_d3	.367	.199 - .561	ns	ns	.129 - .664
conjAnaT_d4	.0667	.008 - .221	p < .001	p < .001	.001 - .327
conjAnaT_d5	.0667	.008 - .221	p < .001	p < .001	.001 - .327
disjAnaT_d1	.2	.077 - .386	p < .01	p < .05	.038 - .496
disjAnaT_d2	.333	.173 - .528	ns	1	.108 - .633
disjAnaT_d3	.867	.693 - .962	p < .001	p < .01	.583 - .986
disjAnaT_d4	.933	.779 - .992	p < .001	p < .001	.673 - .999
disjAnaT_d5	.933	.779 - .992	p < .001	p < .001	.673 - .999

Estimate is on a scale of truth from 0, False, to 1, True. All figures are rounded to the third decimal place, where appropriate. CI = Confidence Interval, from lower bound to upper bound. Method of correction is Bonferroni with alpha adjusted to .00167 (.05/30) and confidence interval adjusted accordingly ($1 - .00167$).

Table 4.2: Pairwise Comparisons at Distance 3 (Wilcoxon signed rank test)

Contrast	W	p-value	Bonferroni Corrected
disjContra vs. conjContra	555	ns (.073)	ns
Neg vs. Pos	615	$p < .01$	$p < .05$
disjAna vs. conjAna	675	$p < .001$	$p < .001$
conjContra vs. Pos	450	ns	ns
Neg vs. disjContra	510	ns	ns
Pos vs. conjAna	435	ns	ns
disjAna vs. Neg	525	ns	ns
conjContra vs. conjAna	435	ns	ns
disjAna vs. disjContra	585	$p < .05$	ns

Two tailed test. For correction, alpha was adjusted to .0056 ($\alpha/9$).

Table 4.3: Pairwise Distance Comparisons for Hump Effect (Wilcoxon signed rank test)

Contrast	W	p-value	Bonferroni Corrected
conjContra_d3 vs. conjContra_d1	540	$p < .05$	ns
conjContra_d3 vs. conjContra_d5	510	ns	ns
disjContra_d3 vs. disjContra_d1	660	$p < .001$	$p < .001$
disjContra_d3 vs. disjContra_d5	585	$p < .01$	$p < .05$

One tailed test. For correction, alpha was adjusted to .0125 ($\alpha/4$).

4.3 Appendix to Section 3.3: Height-nationality experiment

4.3.1 Sequence of the experiment

(Estimated time: 10 minutes; limit: 30 minutes; 5 practice items; 5 controls; 40 target items)

1. Welcome

    (a) Introduction: Here participants were greeted, provided a summary of the time frame and aims, and were asked to acknowledge the privacy notice and conditions. Acceptance of these conditions was obligatory.

    (b) English Form: Participants were required to tick a box for "yes" or "no" when asked whether English was their first language. This was obligatory. Participants were additionally asked to enter the name of their first language into a free text box. Data from participants who ticked the box for "no" or who entered the name of a language other than English, was discarded (although these participants were nevertheless paid for their participation).

    (c) Pretest: The pretest portion ensured that participants were able to match five national flag images to their nationality predicates.

Participants were informed that they would need to pass the pretest with 100% accuracy and were informed that they were free to consult outside resources in order to achieve that.

2. Instructions, Practice, and Attentiveness Controls

   (a) Participants were provided with the following instructions.

      INSTRUCTIONS:

      There are 50 items in this experiment, including practice items. For each item you will see a police line-up of 5 suspects.

      The height of each suspect, in feet and inches, is provided via the scale in the background. The nationality of each suspect is indicated by the flag directly below his feet. The suspects are numbered 1-5 for identification.

      A sentence describing one of the suspects, identified by number, appears below each police line-up.

      Your task is to indicate whether you think the sentence is true or false with respect to the current police line-up.

   (b) Practice Portion: Participants were required to complete five practice items. Practice items presented comparative and superlative uses of *tall*, effectively eliminating vagueness. For example, "Suspect 1 is not the tallest suspect" was paired with an image in which Suspect 1 was the shortest suspect.

(c) Controls: the five practice items were repeated in randomised order over the course of the main experiment as controls for attentiveness.

3. Main Experiment

(a) All target items were provided in random, shuffled order.
All items are provided in the appendix to this report.

4. Conclusion

(a) When all 30 target items had been completed, a thank you page appeared, urging participant to click "Submit" to send the results to the server.

(b) Participants were then provided with a Prolific Completion Code and reminded to copy and paste it into a new browser window in order to be redirected to Prolific and paid for their participation.

There were 5 practice items, presented in non-random order before any of the target items. These were each repeated once, in random order, and randomly shuffled between target items so that each participant responded to each of these items twice over the course of the experiment. These were intended to ensure that the participants understood and followed directions, were attentive, and were consistent in their responses. In only two cases were controls failed catastrophically (however, consistently: item numbers 14/20 and 16/22, and also 21, in one case. In addition 12,15, and 18 each received one mistaken response from different participants).

4.3.1.1 Pretests

A pretest was necessary in order to establish that participants were capable of identifying the flags that served as indicators of nationality in the main experiment.

Figure 4.1: Nationality pretest item

The flags were visually distinctive and represented primarily anglophone nations. They were presented in the following order:

Figure 4.2: National flags

One participant's dataset was excluded based upon failure in this task.

4.3.1.2 Main experiment

There were 2 Versions of the experiment, Version A and Version B, which were virtually identical except for the silhouettes. Ten male silhouettes were generated, each in a slightly different pose and wearing slightly different clothes so that five could be allocated to each version.

Version A:

> Five images were created for version A, each depicting 5 suspect "characters". While the line-up order varies randomly between images, the characters remain stable, that is, each silhouette is associated with a unique height and unique flag.

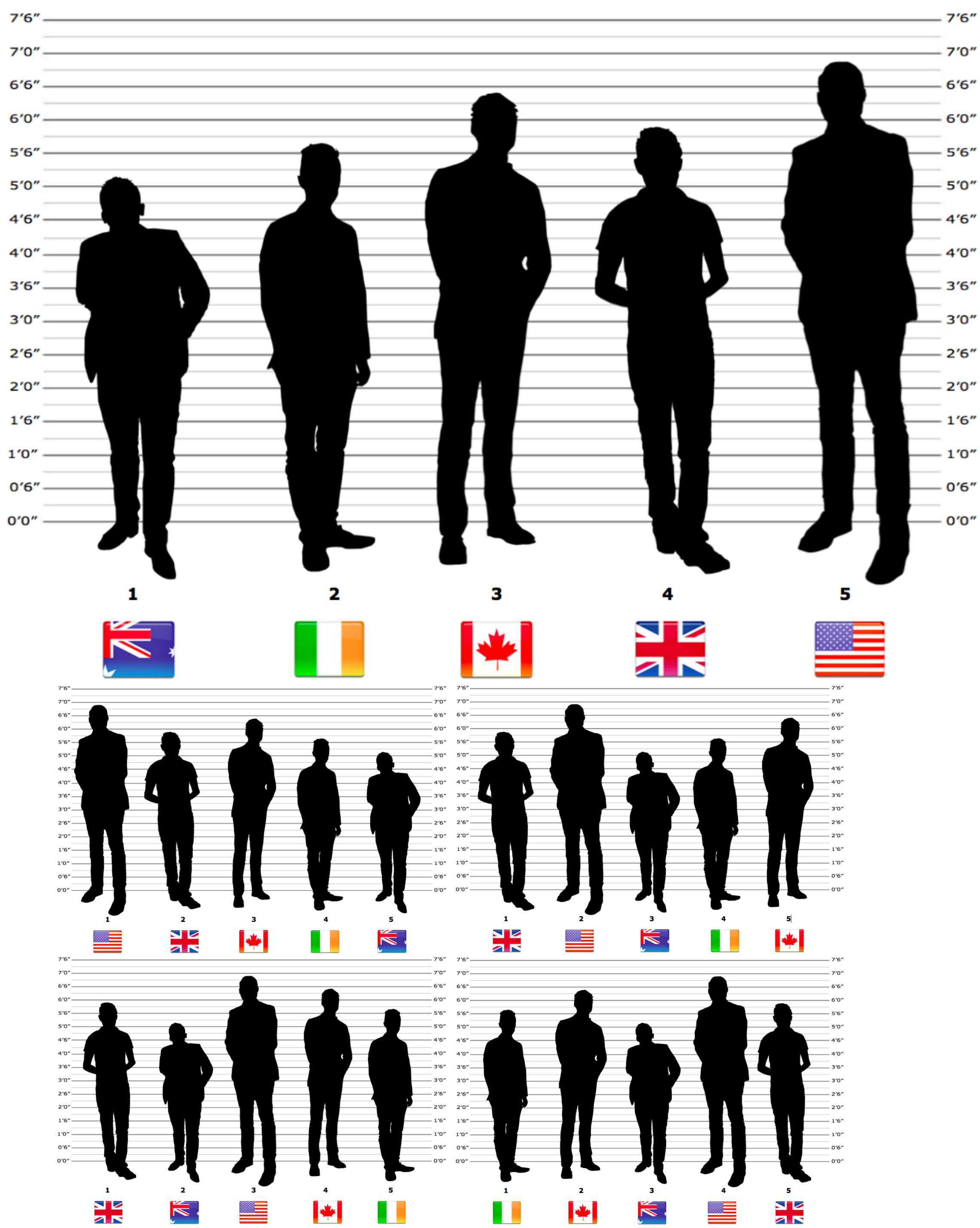

Figure 4.3: Version A Images

Table 4.4: Suspect Profiles

For each of the five characters:		
Borderline Status	Height	Nationality
Non-borderline	5’0”	Australian
Intermediate	5’6”	Irish
Borderline	5’11”	British
Intermediate	6’4”	Canadian
Non-borderline	6’10”	American

Version B:

Five images were created for version B, each depicting 5 suspect “characters”. While the line-up order varies randomly between images, the characters remain stable, that is, each silhouette is associated with a unique height and unique flag.

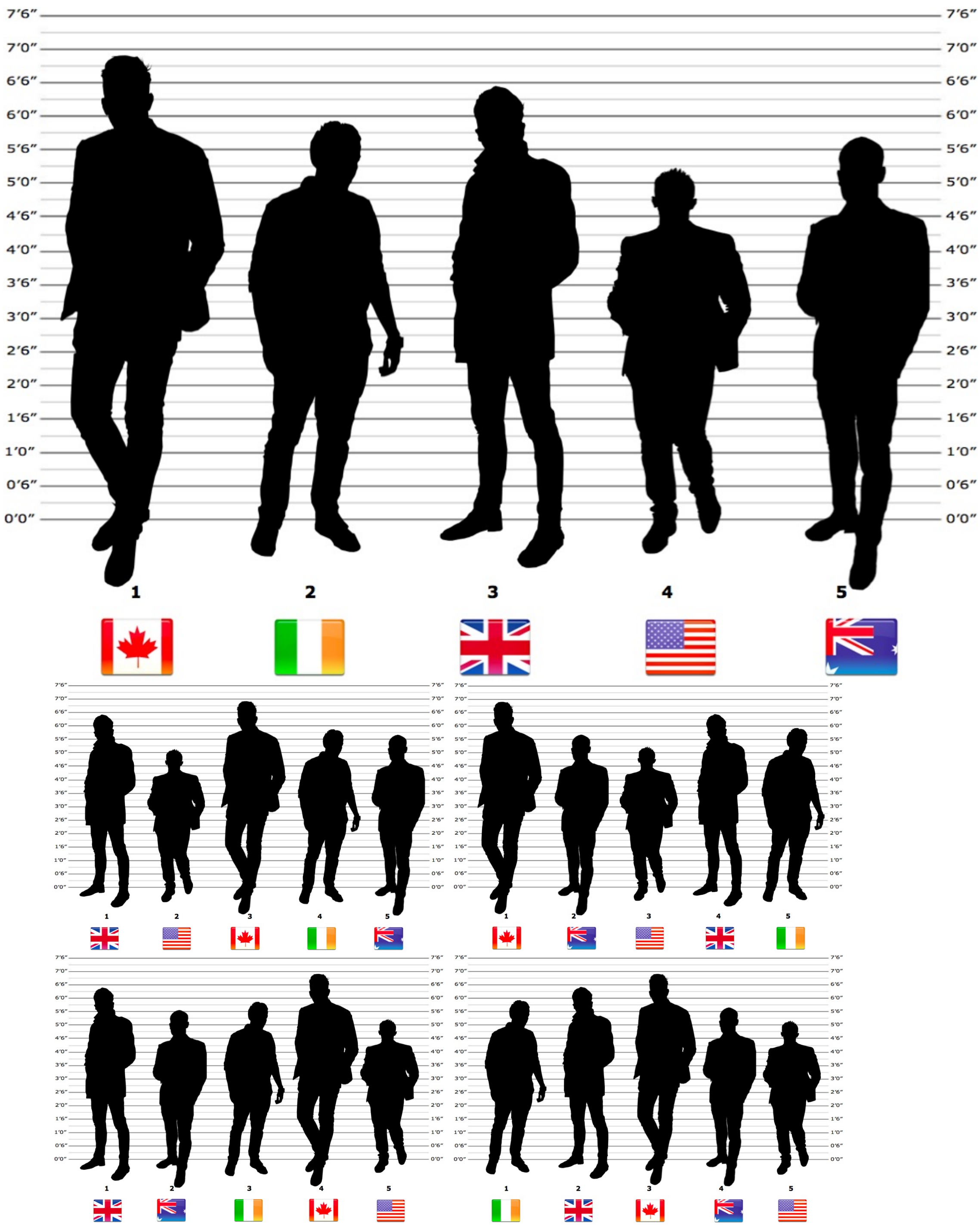

Figure 4.4: Version B Images

Table 4.5: Suspect Profiles

For each of the five characters: Borderline Status	Height	Nationality
Non-borderline	5’0”	American
Intermediate	5’6”	Australian
Borderline	5’11”	Irish
Intermediate	6’4”	British
Non-borderline	6’10”	Canadian

4.3.1.3 Target items

There were 40 target items, 5 for each sentence type (and one for each height). The 8 sentence types were:

Table 4.6: Police Line-up Target Sentences

Sentence Type	Sentence
Positive	“Suspect n is tall”
Negated	“Suspect n is not tall”
Conjunctive Contradiction	“Suspect n is tall and not tall”
Disjunctive Contradiction	“Suspect n is neither tall nor not tall”
Conjunctive Baseline	“Suspect n is tall and *nationality*”
Conjunctive Analogue	“Suspect n is tall and *nationality*”
Disjunctive Baseline	“Suspect n is neither tall nor *nationality*”
Disjunctive Analogue	“Suspect n is neither tall nor *nationality*”

All target items were presented in random order. 33 sets of data were collected. 3 sets were discarded due to failure of controls/pretest.

4.3.2 Practice and control items

There were 5 practice items, presented in non-random order before any of the target items. These were each repeated once, in random order, and randomly shuffled between target items so that each participant responded to each of these items twice over the course of the experiment. These were intended to ensure that the participants understood and followed directions, were attentive, and were consistent in their responses. In only two cases were controls failed catastrophically (however, consistently: item numbers 14/20 and 16/22, and also 21, in one case. In addition 12,15, and 18 each received one mistaken response from different participants).

4.3.3 Results of Statistical Tests

Table 4.7: Results of the sign test to determine "True" and "False" responses Part I

Item	Estimate	.95 CI	p-value	Corrected	.99875 CI
conjContra_h1	.1	.021 - .265	p < .001	p < .01	.006 - .382
conjContra_h2	.4	.227 - .594	p < .01	ns	.146 - .700
conjContra_h3	.5	.313 - .687	ns	ns	.218 - .782
conjContra_h4	.3	.147 - .494	p < .001	p < .01	.085 - .608
conjContra_h5	.067	.008 - .221	p < .01	p < .05	.001 - .334
disjContra_h1	.267	.123 - .459	p < .001	p < .001	.067 - .575
disjContra_h2	.567	.374 - .745	p < .001	p < .01	.272 - .831
disjContra_h3	.567	.374 - .745	ns	ns	.272 - .831
disjContra_h4	.067	.008 - .221	p < .05	ns	.001 - .334
disjContra_h5	.067	.008 - .221	p < .05	ns	.001 - .334
pos_h1	0	0. - .116	p < .001	p < .001	0 - .218
pos_h2	.1	.021 - .265	p < .001	p < .001	.006 - .382
pos_h3	.3.	.147 - .494	ns	ns	.085 - .608
pos_h4	.967	.828 - .999	p < .001	p < .001	.718 - 1
pos_h5	1	.884 - 1	p < .001	p < .001	.782 - 1
neg_h1	.967	.828 - .999	p < .001	p < .001	.718 - 1
neg_h2	.867	.693 - .962	p < .001	p < .001	.575 - .987
neg_h3	.567	.374 - .745	p < .05	1	.272 - .831
neg_h4	.033	.001 - .172	p < .001	p < .001	.272 - .831
neg_h5	0	0 - .116	p < .001	p < .001	0 - .218

Estimate is on a scale of truth from 0, False, to 1, True. All figures are rounded to the third decimal place, where appropriate. CI = Confidence Interval, from lower bound to upper bound. Method of correction is Bonferroni with alpha adjusted to .00167 (.05/30) and confidence interval adjusted accordingly ($1 - .00125$).

Table 4.8: Results of the sign test to determine “True” and “False” responses Part II

Item	Estimate	CI	p-value	Corrected	Adjusted CI
conjAnaT_h1	0	0 - .116	p < .001	p < .001	0 - .218
conjAnaT_h2	.2	.077 - .386	p < .001	p < .001	.036 - .50
conjAnaT_h3	.533	.343 - .717	ns	ns	.244 - .807
conjAnaT_h4	.967	.828 - .999	p < .001	p < .001	.718 - 1
conjAnaT_h5	1.	.884 - 1	p < .001	p < .001	.782 - 1
conjAnaF_h1	0	0 - .116	p < .001	p < .001	0 - .218
conjAnaF_h2	0	0 - .116	p < .001	p < .001	0 - .218
conjAnaF_h3	0	0 - .116	ns	ns	0 - .218
conjAnaF_h4	.033	.001 - .172	p < .001	p < .001	0 - .282
conjAnaF_h5	.1.	.021 - .265	p < .001	p < .001	.006 - .382
disjAnaT_h1	.833	.653 - .944	p < .01	p < .05	.535 - .977
disjAnaT_h2	.933	.779 - .992	ns	1	.666 - .999
disjAnaT_h3	.667	.472 - .827	p < .001	p < .01	.360 - .896
disjAnaT_h4	.3	.147 - .494	p < .001	p < .001	.09 - .608
disjAnaT_h5	.2	.077 - .386	p < .001	p < .001	.036 - .504
disjAnaF_h1	.267	.123 - .459	p < .01	p < .05	.067 - .575
disjAnaF_h2	.167	.056 - .347	ns	1	.024 - .465
disjAnaF_h3	.133	.038 - .307	p < .001	p < .01	.013 - .425
disjAnaF_h4	.067	.001 - .221	p < .001	p < .001	.001 - .334
disjAnaF_h5	0	0 - .116	p < .001	p < .001	0 - .218

Estimate is on a scale of truth from 0, False, to 1, True. All figures are rounded to the third decimal place, where appropriate. CI = Confidence Interval, from lower bound to upper bound. Method of correction is Bonferroni with alpha adjusted to .00167 (.05/30) and confidence interval adjusted accordingly ($1 - .00125$).

Table 4.9: Pairwise Comparisons at Distance 3 (Wilcoxon signed rank test)

Contrast	W	p-value	Bonferroni Corrected
disjContra vs. conjContra	480	ns	ns
Neg vs. Pos	570	$p < .05$	ns
disjAna vs. conjAna	510	ns	ns
conjContra vs. Pos	540	ns	ns
Neg vs. disjContra	450	ns	ns
Pos vs. conjAna	345	ns	ns
disjAna vs. Neg	495	ns	ns
conjContra vs. conjAna	435	ns	ns
disjAna vs. disjContra	495	ns	ns

Two tailed test. For correction, alpha was adjusted to .0056 (.05/9).

Table 4.10: Pairwise Distance Comparisons for Hump Effect (Wilcoxon signed rank test)

Contrast	W	p-value	Bonferroni Corrected
conjContra_d3 vs. conjContra_d1	630	$p < .001$	$p < .01$
conjContra_d3 vs. conjContra_d5	645	$p < .001$	$p < .01$
disjContra_d3 vs. disjContra_d1	585	$p < .01$	$p < .05$
disjContra_d3 vs. disjContra_d5	675	$p < .001$	$p < .001$

One tailed test. For correction, alpha was adjusted to .0125 (.05/4).

4.4 On a theoretical note

As this was a primarily experimentally oriented work, our aim here was not to explore every aspect of every theory available to us. However, we would like to make a few notes on theory, particularly in supporting our position that all theories known to us ultimately invoke at multi-partite explanation for borderline phenomena, generally and most simplistically, an explanation grounded in tripartition.

We would also like to make a special argument that approaches that aim to prove that truth is bivalent are unsuited to experimental investigation. That is not to say they lack validity, but that the source of that validity must come from elsewhere. In the case of most supporters of these bivalent approaches, this point seems to be well appreciated. By and large, their motivation is through conviction in the necessity of bivalence. Thus, their position starts from a logical conviction and from this stance seeks to explain why the truth of the matter is not obvious. But there are those who, as we have seen, have sought to find support for a mathematical and perhaps even meta-physical truth about the world, from studies on human language use, and we take issue with this strategy.

4.4.1 Tripartition

Approaches to the characterisation of borderline cases can be organised into three broad categories according to how they cast the special status of borderline cases. First, there are the approaches which modify the logic in order to assign to borderline cases some non-classical truth status, either through the introduction of additional truth values, or through modification to the very notion of truth. Then there are the approaches which instead make modifications to the semantics, most prominently, the precisificational approaches. And then, there is a non-homogenous category of approaches which take a completely different tack in characterising borderline cases, those that reframe the question of what it means to be borderline, of particular note, the epistemic and the contextualist approaches.

We have seen how the most simplistic view of borderline cases, as inhabitants of a borderline region sandwiched between two non-borderline regions, supplemented with a few assumptions about strength of interpretation and possibly some interaction with negation, provides the necessary tools to account for speakers' intuitions about borderline expressions. We would like to argue that this view is compatible with most if not all theories of vagueness, a fact which we believe is appreciated by most theorists, but which we would like to make the case for explicitly.

We do not think we need to make the case for trivalent logics, nor for theories that invoke two opposing non-classical notions of truth such as the Tolerant, Classical, Strict Framework [11]), nor for s-valuationism, which derives a trivalent picture through notions of sub- and super-truth. We consider the parallels between these to be self evident [1].

In the case of epistemicism, tripartition is also fairly clear: there are propositions that can be known to be false, propositions that can be known to be true, and propositions whose truth cannot be known. In the case of

[1] A far less frequently cited but honourable mention is an account of borderlines in terms of ortho-pairs [33].

fuzzy logic with assertability thresholds, there are propositions whose degree of truth licenses their assertion, propositions whose degree of truth does not license the assertion of their negation, and propositions whose degree of truth licenses neither their assertion nor the assertion of their negation. In contextualism there must similarly be limits to a region of instability. That is, there is an unstable region where borderline cases reside, and more or less stable regions where they do not reside, and we might suppose that statements about affairs that are stable are more assertible than statements about unstable states of affairs. Of course, we might want to fine tune these assertibility intuitions per the theoretical approach they are attached to, but seeing as we can paint a picture of each of these accounts in terms of assertibility, for the sake of argument let us continue in this vein. We shall consider the role of assertibility to an epistemic account, assuming that these considerations apply analogously to other accounts.

We believe that the guiding principles behind the algorithmic approach we have focused so closely on is compatible with epistemicism supplemented with some notion of assertibility. For example, the epistemicist could argue, and some have argued, that a speaker should not assert what she does not know to be true, meaning that she should not assert that *Sam is tall* and should equally not assert that *Sam is not tall*, assuming a strong interpretation of negation. This conveniently allows the speaker to reject both propositions on the grounds that neither breaches the threshold of assertibility, and thus allows her to agree with the disjunctive borderline contradiction while maintaining a stable and conservative threshold of assertibility. On the other hand, deriving a non-trivial meaning for the conjunctive contradiction thrusts the speaker into the uncomfortable position of weakening her threshold of assertibility, resulting in an utterance which has possibly sacrificed something in the way of the Gricean notions of quality for the sake of quantity. Perhaps, this payoff licenses her use of the contradiction given a certain context, perhaps it doesn't. This is but a small, very broadly conceived example, but we think it makes the point that given the right machinery, a tripartite account can get the job done.

The key is the tripartition, and this is exactly why these accounts such as fuzzy logic and epistemicism must call upon flexible thresholds of something else besides truth in order to deal with speakers' judgments.

As a final point, we do not mean to say that tripartition is the answer to all questions about vague language use at the borderline region. We just mean to say that tripartition of some notion that can be cast as corresponding to true and false judgments is highly compatible with the data we have seen. Our data is course, and our accounts of it are, as well. It is the minimal machinery required. As we mentioned in Section 1, we do believe that theories of vagueness can be ordered in terms of a notion of theoretical granularity. Tripartition is a course solution. We believe that issues of higher order vagueness and blurring of borders can be dealt with using more finely partitioned theoretical treatments. Five partitions is a more suitable way to treat 2nd-order borderline cases. Even more partitions may be suited to treating even finer distinctions, especially penumbral connections[2]. We believe that a particularly good example of a finely partitioned theoretical treatment is a probabilistic treatment [18], which we also see as a version of contextualism. That is, we do not believe that there must be a sharp cut-off between stability and instability, rather, the boundary between these two might be modelled best in terms of the probabilistic location of a cut-off. But we are also not certain that such fine grained, probabilistic treatments could not be extended to other accounts, in fact we think there is very good reason to think that they could[3].

4.4.2 Why speakers cannot tell us what truth is

The classical approaches are unified by an adherence to classical notions of truth, validity, and logical consequence. The main contenders here are epistemicism and contextualism. Supporters of these approaches typically explain apparent violations of LNC and LEM in psychological terms, pointing to the absurdity of abandoning the inferences of classical logic, in support of their position. Because of this psychological aspect they are also very difficult to examine empirically, insofar as what is testable is generally not the

[2]These might be accounted particularly well via a fine grained version of s-valuationism, in which these connections can be modelled in term of how many precisifications a proposition is true in relative the another.

[3]See Akiba for an example of how multivalent logics and s-valuation can be unified and extended to finer granularities [1]

theoretical object, rather the psychological assumptions that may accompany it, and these effects may also be subtle. Thus, while they are generally not the motivating theory behind experimental work, they are practically never to be excluded as a possible truth underpinning what is observed in experimental work. In what follows we will argue for this with reference to epistemicism, but we are convinced that the same argument holds of any account of vagueness that is committed to bivalence.

The family of epistemic treatments of vagueness are related by two core values: the complete preservation of classical logic, and the attribution of the phenomena of vagueness to a 'lack of knowledge' regarding the nature of the precise boundary of a vague predicate, or alternatively and more specifically, 'ignorance' concerning sharp borders[62, 67]. On the epistemic approach, the cut-off of a vague predicate is identical to the cut-off of a precise predicate (to the extent that any predicate can be considered to truly be free of vagueness), the difference between the two being the epistemic state of speakers with regard to the cut-off. Typically, borderline cases are cast as cases that fall within a small margin of error around this cut-off such that, although there is a fact in the matter as to whether or not they are members of the predicate's extension, this fact cannot be known. We can think of this in more or less similar terms to our interval $[\theta_P \pm \varepsilon]$ by casting it in epistemic terms. For individuals whose measure of P-ness falls within the interval, we cannot know whether they count as P or not.

Epistemic proponents must attempt to account for the empirical phenomena of vagueness by appealing to possible behaviours speakers might exhibit in a state of ignorance, and this extends to borderline contradictions. For example, in a borderline situation a language user might agree independently to both a vague proposition and its negation, for in her state of ignorance she considers both to be plausibly true; or perhaps she might disagree with both, since she is more inclined to refrain from affirming that of which she holds doubt; or she might agree only to the negated proposition, since she might be inclined to reject an affirmative statement of which she holds doubt and consequently, to agree to its rejection, insofar as she interprets affirmation of the negative to be rejection of the positive. Given the right assumptions regarding how the language user is likely to act, an epistemic account could accommodate any one of these possibilities. So, because the predictions cast by these theories involve an additional layer of assumptions concerning the behaviour of language users under specific conditions, it is useful to distinguish between two levels of theory in considering how the epistemic view might be empirically examined and compared with non-epistemic treatments of vagueness. On the one hand there is the epistemic state of speakers and hearers: what propositions they know and believe to be true, as well as how and why they know, or do not know, and believe them to be true. On the other hand there is a set of predictions regarding the behaviours we expect to observe from language users when they find themselves in a specific epistemic state.

As a basic example of the distinction, consider a case in which a speaker of the language is asked the question, “is it snowing outside?”, and that she does not know the answer to this question. Her state of ignorance: the reason for it (she is in a sound-proof room with no windows), how it might be changed or influenced (she has a weather app on her smartphone), as well as her own knowledge and belief concerning her state of ignorance (she believes it is snowing because it was snowing when she last checked, but she is aware that given the variable forecast, it is perfectly plausible that her belief is now wrong), are all a matter pertaining to her epistemic state. However, our predictions regarding what she might decide to say in response to the question are contingent upon assumptions about how she is probable to *behave* given her state of ignorance, i.e., is she likely to assert an answer that she believes to be true but does not know to be true, or is she more likely in this case to abstain from asserting a statement she knows might very well turn out to be false? If we are to transparently assess the viability of an epistemic theory of vagueness in light of behaviours recorded experimentally, then we must as a matter of necessity be explicit about the additional assumptions we posit to connect subjects’ responses to their mental states.

An apt example of this explicitness is the assumption that in the presence of a sharp cut-off of which speakers are ignorant, speakers prefer to risk committing errors of false negatives over errors of false positives[4]. Thus,

[4]This is actually the assumption which Bonini et al. [9] set forth as the basis for their predictions, as discussed in Chapter II.

faced with borderline case, a truth-judger who is ignorant of the true cut-off of the vague predicate will tend not to judge the corresponding vague proposition to be true. With the added assumption that the same holds true of the proposition's negation, a testable prediction is derived, namely, that in a borderline case, a speaker of the language is likely to decline to agree with both the positive proposition and its negation. Therefore, although the cut-off is sharp and unlimited in its sensitivity, a truth gap is predicted to appear to arise around it. The strength of such an experimental result toward supporting the epistemic account is thus tempered vis-à-vis the credibility of the initial assumptions. If we find reason to refute those assumptions, then the evidence becomes far less compelling [5].

Therein lies the gravest problem facing the epistemicist who would seek to bolster his or her view with experimental evidence: given the right assumptions, the epistemic view can be argued to be consistent with just about any set of data concerning speakers' judgments and use of vague language. Conversely, there is no specific type of judgment or use of vague language that is uniquely incompatible the existence of sharp, unknowable boundaries, with one possible exception, which is that speakers may doubt the existence of

[5] Serchuk et al.[56] provide an argument counter Bonini et al.'s assumption. They point out that Bonini et al. would require evidence to suggest that speakers actively avoid Type I errors (false positives) when using vague language, and that moreover, regardless of whether the epistemicist were to predict truth gaps or truth gluts based on behavioural assumptions, evidence of these gaps and gluts might just as well provide support for a non-epistemic theory that also predicts gaps or gluts. Put slightly differently, there is nothing about the presence of truth gaps or gluts that the epistemic view is uniquely equipped to account for.

such a boundary. But this counterpoint is actually just another example of how the right form of epistemicism, supplemented by the right assumptions, might prove not to be incompatible with this doubt. So it is nigh impossible to design an experiment whose results could support a competing hypothesis while falsifying every possible epistemic hypothesis. All auxiliary assumptions cast aside, bare epistemicism is untestable [49].

This conclusion should not be surprising, after all, the kernel that unites the various incarnations of epistemicism is just bivalence, the principle that all vague predicates are ultimately precise; and ignorance, speakers of the language are ignorant of the position of precise cut-offs and possibly even their very existence. It is not clear what we would expect to see were we able to directly observe such a situation. We cannot observe epistemicism's underlying precise world. Without a doubt, such a world without the phenomena of vagueness can be glimpsed only through our conception of it as a theoretical object. And while we could posit that speakers' state of ignorance directly informs their behaviour, i.e. they always truthfully report their ignorance, this, too, would be an assumption. The big question is then, what do speakers of the language do when they do not know the fact of the matter? What machinery must they invoke to structure their world into something useful, despite incomplete knowledge of the world?

As it turns out, these questions are major concerns for non-epistemic theories of vagueness, such as those which modify the semantics or the logic, as well. However, unlike the epistemic view, other theories of vagueness are generally concerned exclusively with this machinery. For these theories of vagueness, concerns about what the fact of the matter is, and even whether there is a fact of the matter, do not necessarily play a significant role.

As an example, suppose in the spirit of the epistemic view that for some vague predicate F, and individuals a, b, c, d, and e there is a sharp cut-off in the extension of F such that individuals a, b, and c are members of the it, and d and e are not. This state of affairs, which is classical and precise, and which we shall suppose happens to correspond to the actual world, is a precisification of the predicate. Let us call it P_{actual}.

$P_{actual} : [\![F]\!] = \{a, b, c\}$, $[\![\neg F]\!] = \{d, e\}$

Now imagine some speaker of the language who, in line with the epistemic view, is ignorant of the fact that P_{actual} corresponds to the actual world. She imagines it to be perfectly plausible that the cut-off of F excludes individual c, as in P_1.

$P_1 : [\![F]\!] = \{a, b\}$, $[\![\neg F]\!] = \{c, d, e\}$

Since she does not know which precisification is actual, she takes both into account in her use of the predicate,[6] and since the cut-off is not stable

[6]If we like, we can shift this modification from the semantics to the logic by representing truth values relative to a set of precisifications. Thus, while Fa has a truth value of 1 relative to P_{actual}, for the speaker it has a truth value of $(1_{actual}, 1_1)$ relative to $\{P_{actual}, P_1\}$. Similarly, the speaker's truth value for e is $(0_{actual}, 0_1)$, and her truth value for c is $(1_{actual}, 0_1)$. Averaging these values yields the familiar set of trivalent values $\{1, 1/2, 0\}$, a set that can be extended even to a set of fuzzy truth values, by increasing the size

across her set of precisifications, from her point of view a set of borderline cases of F arises, in this case $\{c\}$. Along the lines of an s-valuationist account, she therefore might treat Fc as sub-true but not super-true, in whatever way that treatment might be indicated through her use of language. The point here is that her recourse to a set of precisifications is compatible with both the epistemic view and an s-valuationist treatment, the difference being that in the case of the former, it is fundamental that there is a unique precisification which represents the actual state of the world, and in the case of the latter, the issues surrounding whether there is such a unique precisification, which precisification it might then be, and how such a fact might be determined, hold little relevance. To consider these questions on a philosophical level and to hold convictions about them, is a decidedly separate endeavour from refining a theoretical framework to model the use of vague language.

of the set of precisifications. Precision obtains when the relevant set of truth values is classical, corresponding to the set $\{P_{actual}\}$ of precisifications, the resolution representing the actual world. Again, there is no incompatibility between the existence of a fact of the matter concerning the cut-off and the invocation of a multivalent logic to model speakers' uncertainty around that fact.

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