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Competition in Digital Markets

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Competition in Digital Markets

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Contents

Chapter 1

Introduction

The rise of the digital era has created new challenges for competition policies. We observe big tech giants who dominate in their core markets with strong network effects. When network effects are strong, each user receives significantly higher utility from more users using the same network. Hence, the market is likely to tip in favor of one firm. Furthermore, mergers and acquisitions can be used to strengthen monopoly power. The acquired product or service may complement the acquirer's existing businesses while eliminating a potential competitor in the process. Digital firms also possess a vast amount of consumer data. These data, combined with the developments in algorithms in the past decade, allow for personalized practices with an unprecedented level of accuracy. How does personalization affect competition? What can competition policy do to ensure healthy competition in digital markets?

In this thesis, I study how three features and regulations in digital markets affect competition among firms. Chapter 2 studies the effects of interoperability in a market with network effects on price competition and innovation. In Chapter 3, my co-author–Jasper van den Boom–and I study the effects of complementarity and economies of scope on the long-run foreclosure effects of conglomerate mergers involving a digital ecosystem. In Chapter 4, I analyze how (inaccurate) product recommendations affect price competition.

To ensure competition in digital markets with strong network effects, many regulators have proposed to impose interoperability–the ability for two or more networks to exchange information–in these markets. However, opponents argue against interoperability based on the ground that it reduces each firm's incentive to invest in its quality since the investment also benefits the interoperable networks of its competitors.

Chapter 2 studies the effects of interoperability from a different perspective that has not received much attention. Specifically, how does interoperability affect competition in a market at an early stage before the monopoly is entrenched? I show that while interoperability is effective in ensuring that multiple firms can coexist, the effect of interoperability must be carefully analyzed when the market structure–a monopoly or a duopoly–is endogenously determined. A regulator cannot base its analysis solely on one market structure. Welfare under a duopoly with full interoperability can be lower than welfare under a monopoly without interoperability.

Furthermore, if the market is likely to become a duopoly, I find that interoperability increases investment in interaction quality, i.e., the quality consumers receive from interacting with other consumers in a network. Because interoperability allows each firm to charge a higher price under a duopoly, they earn greater profits for their investments.

In Chapter 3, my co-author Jasper van den Boom and I focus on the effects of complementarity and economies of scope, which are prominent characteristics in digital markets, in conglomerate mergers involving a digital ecosystem. A high degree of complementarity and economies of scope creates several linkages between products and services in a digital ecosystem, unlike traditional conglomerate mergers. In this paper, we develop a simple economic model and review six merger cases before the European Commission. Each of these cases involves one of the Big Five ecosystems (Apple, Amazon, Facebook, Google and Microsoft).

We argue that complementarity and economies of scope can lead to foreclosure in the long run. Hence, competition authorities should weigh between short-run benefits and long-run harms. The presumption that conglomerate mergers are less likely to produce anti-competitive effects should not apply.

We propose that mergers involving a digital ecosystem deserve their own standard of assessment that incorporates horizontal effects. Despite horizontal elements, the Horizontal Merger Guideline is not appropriate due to two reasons. First, because of high linkages across products and services in an ecosystem, defining and assessing all relevant markets of a merger is costly. Second, an entrant does not typically compete head-on against an incumbent: competition is often indirect. As such, a standard of assessment for mergers involving a digital ecosystem should be akin to horizontal mergers while recognizing issues specific to digital markets.

Furthermore, we propose the use of flexible remedies that only trigger once certain harms identified by a competition authority materialize. Flexible remedies help mitigate problems associated with uncertainties of an ex-ante assessment of long-run effects.

In Chapter 4, I look at the effect of (inaccurate) product recommendations on price competition. In many digital markets, consumers may receive information about a product through a product recommendation. When the accuracy of product recommendations improves, consumers benefit by experiencing the increased likelihood of seeing a product they like. But how does higher accuracy affect the prices that consumers pay?

I develop a search model where each consumer receives a recommendation sent by a recommendation system. A consumer may receive a correct recommendation for her preferred product variety or an incorrect recommendation for the mismatched variety. In the existing literature, an established result is that improvements in the information on consumers' preferred products always increase the equilibrium price. However, this result is not always true under higher *allocative* accuracy, which is a type of the accuracy of a recommendation system I propose in this paper. A recommendation system is *allocatively* more accurate when a consumer who highly dislikes the mismatched variety is more likely to receive a correct recommendation, but a consumer who slightly dislikes the mismatched variety is more likely to receive an incorrect recommendation.

Higher allocative accuracy decreases the equilibrium price when the search cost is low but increases the equilibrium price when the search cost is high. Higher allocative accuracy improves social welfare by reducing total search costs and mismatches between consumer preferences and recommended varieties. Consumer surplus also increases under allocative accuracy with a low search cost because the consumers also pay a lower price. However, the effect on consumer surplus is ambiguous when the equilibrium price increases.

Chapter 2

Interoperability, Competition, and Investments

2.1 Introduction

Network effects are common in modern industries. Due to network effects, each consumer receives higher utility from more consumers using the same network. Consequently, a market with strong network effects typically tips in favor of one firm, although several firms compete at the early stage. There is no competition ex-post. Policymakers worldwide have recognized this problem. Thus, they have brought forward proposals to impose interoperability in several markets, such as the Digital Markets Act (DMA)[1] in the EU and the ACCESS Act[2] in the US. Interoperability is the ability for two or more networks to exchange information and then use the exchanged information. For example, Zoom users can talk directly with Microsoft Teams users if the two networks are interoperable. Because interoperability gives a firm access to the networks of its competitors, all firms are more likely to survive. However, opponents from the private sector argue that providing such access creates a free-riding problem. An investment by one firm to improve its quality benefits interoperable networks offered by its competitors. Therefore, interoperability may lower the investment incentive.[3]

The DMA and the ACCESS Act focus on big tech firms with entrenched monopoly power in their core markets. However, I look at the effects of interoperability from a different angle that may not have received enough consideration. In particular, what are the effects of interoperability at the early stage–ex-ante competition–before the monopoly is entrenched? This paper focuses on horizontal interoperability–interoperability between competing networks–in the short and long run.[4] In the short run, the quality of each firm is fixed, where different firms may have different qualities. Is interoperability an effective

[1]Proposal for a Digital Markets Act, Brussels, 15.12.2020, COM(2020) 842 fin. (https://eur-lex.europa.eu/legal-content/en/TXT/?qid=1608116887159&uri=COM%3A2020%3A842%3AFIN)

[2]Augmenting Compatibility and Competition by Enabling Service Switching Act of 2021, 117th Congress, 11.06.2021, H.R.3849. (https://www.congress.gov/bill/117th-congress/house-bill/3849/text)

[3]Among these discussions, there are concerns that interoperability forces networks to be homogeneous, which hinders consumer preferences for product differentiation, and markets may get stuck in an outdated interoperable standard (Farrell & Simcoe (2012) and Kerber & Schweitzer (2017a)).

[4]Formally, Kerber & Schweitzer (2017a) define *horizontal* interoperability as "the interoperability of competing products, services, or platforms." In contrast, *vertical* interoperability is "the interoperability of a product, service, or platform with complementary products and services." Because the objective of this paper is to study whether interoperability can bring in competitors, I focus on horizontal interoperability, instead of vertical interoperability.

tool for maintaining competition in a market? Can Zoom and Microsoft Teams coexist? And more importantly, is interoperability socially desirable? Even without the effect on investment, is it good to have a duopoly or oligopoly instead of a monopoly in the presence of network effects?

Furthermore, this paper studies whether interoperability increases or decreases investment in *interaction qualit*y by each (symmetric) firm in the long run. When a network has high interaction quality, each consumer enjoys each interaction with another consumer in the same network more. Thus, higher interaction quality leads to higher direct network effects for a given network size. For example, a communication app can develop new features, such as text messaging, group chat, and video calls. These features represent interaction quality since they make each interaction between consumers in the messaging app more enjoyable.

In addition to the DMA and ACCESS Act, there are other policies and initiatives to impose interoperability in markets where multiple firms are competing. The first example is the European Electronic Communication Code (EECC), which was adopted in December 2019.[5] To capture changes in consumer behavior, the EECC was designed such that the code covers interpersonal communications services (e.g., WhatsApp, Messenger, Zoom, Microsoft Teams, and Skype). In the second paragraph of Article 1, the code aims to "implement an internal market in electronic communications networks and services that will result in [...] sustainable competition [and] interoperability of electronic communications services." Furthermore, according to the first paragraph of Article 59, national regulatory agencies shall ensure interoperability "in a way that promotes efficiency, [...] efficient investment and innovation."[6]

Another example is self-driving cars. Interoperability enables cars from different manufacturers to interact by exchanging information with one another. Despite still being in an early stage of development, regulatory agencies are pushing for interoperability. In Europe, the European Commission financed a study of interoperability in the adoption of autonomous driving.[7] The aim is to enhance interoperability between vehicles as well as traffic infrastructures such as roadside equipment, traffic control centers, and other devices. In addition, the US Department of Transport is working with the automotive industry to advance vehicle-to-vehicle (V2V) communication.[8] V2V communication enables vehicles to wirelessly exchange information relevant to driving–speed, location, and heading. Such communications offer a wide range of benefits, including better traffic flow,[9] lowering time spent in cars,[10] reducing accidents,[11] saving energy, and reducing pollution.[12] Currently, multiple firms, such as Waymo and Tesla, are active in the market.

Even at an early stage, a firm with significantly higher quality than other firms may become a monopolist in the market without interoperability. However, when interoperability is imposed, other firms may become active due to higher network effects from access to the firm's high-quality network. Therefore, the market structure becomes a duopoly or oligopoly. So, the two types of market structure are not two

[5] Proposal for a Directive establishing the European Electronic Communication Code, Brussels, 12.10.2016, COM(2016) 590 fin. (https://digital-strategy.ec.europa.eu/en/library/proposed-directive-establishing-european-electronic-communications-code)

[6] See Graef (2015a) and Brown & Marsden (2013) for legal discussions regarding interoperability in number-independent interpersonal communications services and social media platforms.

[7] See European Commission (n.d.b).

[8] See U.S. Department of Transport (n.d.b).

[9] See Hyldmar *et al.* (2019) and Overtoom *et al.* (2020).

[10] See Bertoncello & Wee (2015).

[11] See U.S. Department of Transport (n.d.a).

[12] See Government of the Netherlands (2015).

separate situations. However, the effects of interoperability when the market structure is endogenously determined have not been systematically analyzed in a unified framework. One of the contributions of this paper is that it uses a unified framework to study the effects of interoperability in a market where the structure is endogenously determined.

In contrast, if a market is a duopoly or oligopoly without interoperability, imposing interoperability will not change the market structure into a monopoly. However, the effects of interoperability on investments in this type of market are not sufficiently studied. In contrast, the effect of interoperability on investments under a monopoly is well established among policymakers and the academic circle.[13] That is, competition as a result of interoperability increases innovation. The second contribution of this paper is to fill in this gap by studying the investment in interaction quality.

In the existing literature, interaction quality is always assumed to be exogenous.[14] However, interaction quality is a strategic variable in practice, and this paper allows for this possibility. For example, besides standard one-to-one text messaging in messaging apps, firms can develop additional features that enable consumers to send pictures or videos, create group chats, and make phone or video calls. Even though the size of the network remains unchanged, additional features enable consumers to get more network effects as they enjoy using the apps more. Furthermore, self-driving cars need software to interpret the information they receive. Higher quality software improves self-driving capabilities, which increases consumer utility. Clearly, firms have to invest in developing the software. To the best of my knowledge, this paper is the first to investigate the effect of interoperability on investment in interaction qualities.

This paper builds a model to study the effect of interoperability on price competition and investments in interaction qualities (hereafter, *qualities*). In particular, consumers are looking to buy a network good. They choose between two horizontally differentiated firms that compete on quality and price. Consumer utility consists of two elements–the intrinsic preference for the chosen firm à la Hotelling and network effects.

Network effects that consumers of one firm receive come from two sources. The first source is the network effects generated from their home network, which are the product of the size of their own network and the quality of the firm. For the second source, consumers receive network effects from interoperability, provided that the networks are interoperable. Similarly, the network effects from interoperability are the product of the size of the competing network and the interaction quality from interoperability (hereafter, *quality from interoperability*) adjusted by the interoperability level between the two networks. The higher the interoperability level, the higher the network effects from interoperability.

The analysis is divided into the short run and the long run. In the short run, the qualities are fixed. This paper characterizes whether a monopoly or a duopoly outcome arises in equilibrium for any given values of qualities, qualities from interoperability, and an interoperability level.[15] Furthermore, it in-

[13] See, for example, European Commission (2019), Stigler Committee on Digital Platforms (2019), OECD (2021), and Scott Morton & Kades (2021).

[14] Crémer *et al.* (2000), Baake & Boom (2001), Doganoglu & Wright (2006), and Chen (2018) assume that interaction quality interacts linearly with the network size–an additional consumer creates a constant amount of network effects. Note that interaction quality is typically called the *strength of network effects* in these papers. Alternatively, Katz & Shapiro (1985), Farrell & Saloner (1986), and Alexandrov (2015) assume that network effects follow a fixed functional form given the network size. In other words, an additional consumer may create a different amount of network effects from a previous consumer, but how much network effects each consumer generates is exogenously given.

[15] Appendix 2.9 supplements the short-run analysis. I introduce two additional assumptions (Assumptions 4 and 5). The equilibrium outcomes specified in Proposition 1 are the unique equilibrium outcomes that are supportable by SPE that satisfy

vestigates how interoperability affects price competition when the market structure is endogenously determined. The long-run analysis includes the investment stage.

In the short run, an equilibrium with the monopoly outcome arises when the quality of at least one firm is high, and the interoperability level is low. When the quality is high, the monopolist can generate sufficient network effects to cover the market. Even the consumer who has the lowest preference toward the monopolist is willing to buy from it. When the interoperability level is low, the other firm cannot compete against the monopolist because the other firm does not receive enough network effects from interoperability. In contrast, the duopoly outcome, where both firms have positive demands, arises when the interaction qualities of both firms are low, or the interoperability level is high.

There exists a threshold of interoperability such that when the interoperability level is lower than the threshold, an equilibrium with the monopoly outcome arises. In contrast, when the interoperability level is higher than the threshold, the market structure becomes the duopoly in equilibrium.

A higher interoperability level decreases the monopoly price as long as the market structure remains the monopoly. Since interoperability increases the network effects of the competitor, it is easier for the competitor to become active in the market. Consequently, the monopolist has to charge a lower price to keep the competitor out. Even though the competitor is inactive, interoperability acts as a constraint preventing the monopolist from setting a high price.

Once the interoperability level reaches the threshold where the equilibrium outcome becomes the duopoly, a higher interoperability level increases the duopoly prices.[16] Intuitively, when one firm lowers its price, it increases its network size as more consumers join its network. But the competitor also benefits from the price reduction since it receives the network effects from interoperability. Thus, the firms have a lower incentive to compete by lowering their prices.

Social welfare under the duopoly with full interoperability can be lower than social welfare under the monopoly. It happens when the monopolist has sufficiently higher quality than the competitor, and consumers do not highly value having an alternative network (a low degree of horizontal differentiation, Figure 2.5b). When some consumers switch to the alternative network, their utility increases, but the increase is limited due to low horizontal differentiation. However, consumers who still participate in the monopolist's network lose huge network effects. The significant decrease in network effects dominates the limited gain from having an alternative network.

These results highlight the importance of having a complete analysis that unifies both types of equilibrium outcomes under a single framework. Interoperability creates different effects on the equilibrium prices and welfare depending on whether the monopoly or the duopoly outcome arises in equilibrium. It is not always good (or bad) to increase the interoperability level to the perfect level.

In the long run, when the firms decide to invest in qualities, suppose that the cost of investment increases rapidly such that none of the firms has a high enough quality to become the monopolist. Then, a higher interoperability level encourages the firms to invest more. When the networks are interoperable, the firms make more profits from their investments because interoperability allows them to charge higher prices, as argued earlier. Thus, when consumer utilities are larger due to higher interaction qualities, the firms can extract these utilities because they can charge higher prices and gain profits from their investments. Accordingly, the firms have more incentives to invest in the interaction qualities when the

these two assumptions.

[16]Note that this result is in line with the existing literature (Shy (2001), Baake & Boom (2001), and Garcia & Vergari (2016)).

interoperability level is higher.

Based on the results of this paper, the policy recommendations are as follows. First, a regulator can use interoperability to ensure that there is competition when a market is at an early stage. However, imposing full interoperability is not always desirable. When the degree of product differentiation is low, social welfare is higher when all consumers are in one network. In contrast, when the degree of product differentiation is high, imposing interoperability is desirable as it increases social welfare. Finally, suppose the cost of investment in quality is increasing quickly such that none of the firms can have a high enough quality to become the monopolist. Then, interoperability increases investment in interaction quality by each firm.

The rest of the paper is organized as follows. Related literature is discussed in the next section. It reviews papers related to network compatibility. In Section 2.3, I set up the model to study the effects of interoperability on price competition and the firms' incentive to invest in (interaction) qualities. Section 2.4 looks at the short-run effects of interoperability on the equilibrium outcomes. It allows for a shift in the equilibrium from a monopoly outcome to a duopoly outcome when the interoperability level increases. The effects of interoperability on both types of equilibrium outcomes are analyzed. Then, the long-run effects of interoperability on the investments in (interaction) qualities under a duopoly setting are in Section 2.5. Policy implications derived from the results in this paper are summarized in Section 2.6. Finally, Section 2.7 concludes the paper.

2.2 Literature review

Kerber & Schweitzer (2017a) argue that interoperability is a "sub-category" of compatibility. Briefly speaking, compatibility only requires two systems or components to be able to work together to perform their functions. Interoperability has the additional requirement that the systems must exchange information. For example, an electric car produced by a manufacturer can be compatible with a charging station built by another manufacturer. However, the electric car and the charging station are not interoperable.[17]

Crémer *et al.* (2000) analyze the effect of compatibility when firms have different installed bases (locked-in consumers). They extend the model by Katz & Shapiro (1985) by assuming that one firm has more existing consumers than the other. The firms compete on quantity for remaining consumers. The firm with a larger installed base has an advantage because it already has larger existing network effects. Thus, it can attract more consumers in equilibrium.

Under Cournot competition, Katz & Shapiro (1985) and Crémer *et al.* (2000) show that the firm with more consumers loses its profit from compatibility, while the firm with fewer consumers benefits from it. Despite both firms gaining larger network effects when their networks are compatible, the strategic effects that each firm experiences are opposite. Compatible networks equalize the network effects across firms; the firms are less differentiated. Consequently, it diminishes the comparative advantage of the firm with more consumers. The smaller firm can compete on a more equal footing. Chen (2018) simulates Nash equilibria for infinitely many periods when consumers face switching costs. When a consumer

[17]Li (2019) develops and estimates a structural econometric model to analyze the effect of compatibility in electric vehicle charging stations. She finds that compatibility reduces investment in these charging stations. Note that investment in charging stations are different from investment in interaction qualities in this paper as they are related to different types of network effects. In the charging station case, it is indirect network effects. Consumer utilities increase when there are more charging stations, and vice versa.

wants to switch to a new firm in the next period, she incurs the switching cost. He shows similar results to Katz & Shapiro (1985) and Crémer *et al.* (2000).

Baake & Boom (2001) and Garcia & Vergari (2016) study the effect of compatibility under price competition in the context of vertical differentiation. Consumers differ in their willingness to pay for quality. However, consumers have the same preferences for the network effects. Particularly, a consumer's utility u when she buys a product from firm i is

$$u = \alpha v_i + \beta \left(D_i + tD_j\right) - p_i, \tag{2.1}$$

where α represents a consumer's preference towards the intrinsic quality v_i of firm i's product. The parameter α is normally assumed to follow a certain distribution $F(\alpha)$. Thus, different consumers value the intrinsic quality of the network differently. β is the strength of network effects which is exogenous. It is also assumed to be the same for all firms. Note that the strength of the network effects β is the same as (interaction) quality in this paper. D_i and D_j are the demands for firm i and firm j, respectively, and t is the interoperability level. If it equals one, then firm i's network is fully interoperable with firm j's network. The networks are not interoperable when the interoperability level is zero. Lastly, p_i is the price.

These papers show that compatibility relaxes the degree of price competition. When one firm cuts its price to increase its demand, compatibility increases network effects that consumers in the other firm receive as well. So, the competitor's network is more attractive. This spillover reduces the incentive of the firms to compete aggressively.

There are four main differences between the existing papers and this paper. First, the existing papers study price competition under vertical differentiation, while this paper studies price competition under horizontal differentiation. This difference results in different specifications of the utility functions. In this paper, consumers do not value intrinsic quality differently. However, they have preferences for one firm compared to another (See Equation 2.2 for the utility function used in this paper). As shown in Section 2.4, the main result that compatibility (or interoperability) relaxes the degree of price competition extends to horizontal differentiation when both firms are active in the market (duopoly).

Second, the existing papers assume that the quality β is sufficiently small: no firm can generate significant network effects to dominate the market. Consequently, they rule out an equilibrium with a monopoly outcome. They only look at the equilibrium with a duopoly outcome. In contrast, this paper allows for the possibility that the quality β is large. So, the effects of interoperability under a monopoly and a duopoly can be studied in a unified framework.

Third, the existing papers assume that the qualities β are symmetric across firms. Thus, it is a special case of this paper that allows for different interaction qualities β between firms. The analysis later shows that some results are different. Lastly, this paper also studies the decision to invest in the interaction quality β. So, it is no longer an exogenous variable but a strategic choice of firms.

An element of innovation related to compatibility is typically in the form of new product adoption. A firm has a better new product than an original product (a higher value or a lower cost). However, existing papers in this area assume that a new product is exogenous given. There is no investment stage. Farrell & Saloner (1986) study the adoption of a new product with network effects by new consumers. They assume that the new product is introduced at a given date. The firm does not choose when to introduce the product. The paper also assumes that the introduced product is incompatible with the existing product, and the firms perfectly compete so that they price at marginal costs. Farrell & Saloner (1986) show that, depending on the existing installed base, either *excess inertia* (tendency against the new product) or *excess momentum* (favor for the new product) could arise.

Katz & Shapiro (1992a) analyze a richer game than Farrell & Saloner (1986). The introduction date of a new product and the prices are strategic choices. The new product may or may not be compatible with the existing one. However, product development (cost reduction) is an exogenously given process. They show that the private incentive to introduce the new product coincides with the social welfare objective when the products are compatible. However, the new product will be introduced too soon under incompatibility. This is because the firm that supplied the new product does not take into account the lost utilities of consumers who buy the old product. These consumers are stranded: they no longer receive network effects from new consumers who buy the new product.

Endogenous product development appears in Baake & Boom (2001), discussed earlier. They study firms' decisions to choose their intrinsic quality v_i in Equation 2.1. They show that the firm that chooses a low quality in equilibrium increases its quality level under compatibility compared to under incompatibility. Note first that the well-known result under vertical differentiation is that firms have an incentive to provide different intrinsic qualities v_i as much as possible. This is because the quality difference relaxes price competition (Choi & Shin (1992) and Wauthy (1996b)). In Baake & Boom (2001), because compatibility partly relaxes the degree of price competition, the low-quality firm can afford to intensify price competition by lowering the quality difference.

In addition to network compatibility, there is another strand of literature on compatibility which is component compatibility. In this setting, each consumer consumes a bundle of products. These products are strictly complementary, e.g., Nikon camera bodies and Canon camera lenses. There are no network effects in component compatibility. More consumers using these cameras do not increase the utility of each consumer who uses them. Matutes & Regibeau (1988) find that compatibility lowers firms' incentive to cut their prices. To see this, suppose that there are two complementary products, and two firms supply both of the products. When there is no compatibility, consumers have to buy both products from either of the firms. Under compatibility, the consumers can mix and match. They can buy one product from one firm and another product from the other firm. Compatibility reduces the firms' incentive to cut prices. When one firm cuts the price for one of its products, it also boosts the demand for the complementary product sold by the other firm. Thus, the firm that cuts the price does not receive the full benefits. Hahn & Kim (2012) extend the analysis to asymmetric firms with different production costs or product qualities. Finally, Miao (2009) studies component compatibility under the two-sided market setting.

2.3 A Model of interoperability and interaction qualities

A unit mass of consumers is uniformly distributed over a unit-length interval $[0,1]$. Each consumer chooses to join one of the two networks supplied by firm A or firm B. Firm A is located at the left endpoint, and firm B is at the right endpoint. Consumers have heterogeneous preferences towards the firms à la Hotelling. A consumer located at point x who joins firm A's network or firm B's network receives utility

$$\begin{aligned} u_A(x) &= 1-\tau x \qquad +\beta_A D_A + t\tilde{\beta}_A(\beta_A,\beta_B) D_B - p_A, \\ u_B(x) &= 1-\tau(1-x)+\beta_B D_B + t\tilde{\beta}_B(\beta_B,\beta_A) D_A - p_B, \end{aligned} \tag{2.2}$$

respectively. The parameter τ represents the degree of horizontal differentiation, β_i is the (interaction) quality of firm i, and D_i is the demand of firm i. The interoperability level is denoted by $t \in [0,1]$, and $\tilde{\beta}_i(\beta_i,\beta_j)$ is the quality from interoperability of firm i from being interoperable with firm j's network. The explicit functional forms of quality from interoperability $\tilde{\beta}_i$ are specified in Section 2.3.1. Finally, p_i is the price charged by firm i. Section 2.3.2 discusses the interpretation of each element in the utility function.

Each firm makes a decision to invest in its quality β_i. Firm i has to invest $I(\beta)$ to achieve the quality level β. $I(\cdot)$ is convex with $I(0)=0$. In addition, the marginal costs of both firm A and firm B are normalized to zero. Accordingly, firm i's operating profit π_i (before the investment cost) is

$$\pi_i = p_i \cdot D_i,$$

and the net profit is $\pi_i - I(\beta)$. The firms compete on price.

The timing of the game is as follows (Table 2.1). Given an interoperability level t, each firm simultaneously invests in developing its quality β_i. Observing the investment levels, each firm sets its price p_i simultaneously. Finally, each consumer decides which firm to join. The solution concept is subgame perfect equilibrium (SPE) in pure strategies.

Table 2.1: Timing of the model

Period	Description
1	Each firm independently invests in its interaction quality β_i, where $i \in \{A,B\}$, given an interoperability level t.
2	Each firm simultaneously chooses its price p_i, where $i \in \{A,B\}$.
3	Consumers decide whether to buy and from whom.

To ensure competition between the two firms, Assumption 1 restricts the degree of horizontal differentiation τ to be sufficiently small. If the degree of horizontal differentiation τ is large, each firm acts as a local monopolist on its part of the market. There is no competition between them.

Assumption 1. *The degree of horizontal differentiation τ is sufficiently small such that*

$$\tau \leq \frac{2}{3}.$$

2.3.1 Two specifications of qualities from interoperability $\tilde{\beta}_i$

This paper investigates two types of interoperability–information interoperability and services interoperability. Information interoperability allows information generated by one network to be transferred and utilized by users in another network. When the two networks are interoperable, information can always be exchanged.

Under services interoperability, a user of network i can use services provided by network j to interact with users of network j. However, the services by network j must be supported by network i. Even though the networks are interoperable, there is still a limitation on which services can be used across networks.

2.3.1.1 Information interoperability

Under *information interoperability*, information is transferred between the two networks. Furthermore, information interoperability can be further divided into two categories: information processing and information collection.

■ **Information processing:**

Under information processing, the information received from interoperability needs further processing before it is useful to the receiving network. An example of this category is self-driving cars. When a Tesla car receives information from other Tesla or Waymo cars, the information has to be processed by Tesla's self-driving AI. Whether the information comes from cars in the same network or the other network, the information is processed based on the same AI quality β_i.

Accordingly, firm i's quality from interoperability $\tilde{\beta}_i$ is its quality β_i under information processing, i.e.

$$\tilde{\beta}_i = \beta_i. \tag{2.3}$$

■ **Information collection:**

Under information collection, information received from interoperability is useful without further processing. Only collected information matters. For example, professional social media platforms (e.g., LinkedIn, Meetup, and Xing) collect user information.[18] They invested in their abilities to collect many aspects of user information: education, work experience, skills, activities, etc. The higher ability to collect various data, the higher the quality β_i.

Suppose that LinkedIn is interoperable with Meetup. Then, users of Meetup can access information collected by LinkedIn with quality β_j. So, the quality from interoperability $\tilde{\beta}_i$ that Meetup users received is based on LinkedIn quality β_j. Hence, quality from interoperability $\tilde{\beta}_i$ under information collection is

$$\tilde{\beta}_i = \beta_j. \tag{2.4}$$

■ **Combining information processing and collection**

To capture both information processing and information collection simultaneously, the quality from interoperability $\tilde{\beta}_i$ is the weighted average between both firms' qualities (β_i and β_j). Specifically, information interoperability is represented by

$$\tilde{\beta}_i = w\beta_j + (1-w)\beta_i, \tag{2.5}$$

where $w \in [0,1]$.

When $w = 0$, Equation (2.5) becomes $\tilde{\beta}_i = \beta_i$. This is the same as information processing (Equation (2.3)). On the other hand, information collection is represented by $w = 1$. In this case, Equation (2.5)

[18] Professional social media platforms also have features related to indirect network effects or two-sided markets. For example, the platforms allow users to connect with potential employers. However, I focus on the direct network effects that these platforms generate. For example, they allow users to see profiles of other users. The users can also communicate and form groups with other users with similar profiles.

becomes $\tilde{\beta}_i = \beta_j$, which is the same as quality from interoperability $\tilde{\beta}_i$ under information collection (Equation (2.4)).

A higher weight w indicates that firm i's quality from interoperability relies more on the quality of firm j. Thus, the network effects from interoperability that firm i's consumers receive are based more on the investment made by firm j. In contrast, when the weight w is low, firm i generates the network effects from interoperability by relying more on its investment in quality β_i.

2.3.1.2 Services interoperability

The second specification is motivated by interpersonal communication apps. Suppose that one app allows users to send text messages and make video calls, while the other app only allows users to send text messages. When the two apps are interoperable, it is unlikely that the users of the app with both text messaging and video calling will be able to make video calls with users who use the app with text messaging only. Consumers would be able to use the minimum level of the services provided by the two apps. Accordingly, the qualities from interoperability $\tilde{\beta}_i$ are the minimum of the qualities β_i, i.e.,

$$\tilde{\beta}_i = \min\{\beta_A, \beta_B\}. \tag{2.6}$$

Suppose that firm A's quality is higher than firm B's quality, i.e., $\beta_A > \beta_B$. Then, an investment by firm B to increase β_B increases firm A's quality from interoperability $\tilde{\beta}_A$. Thus, the investment still benefits the competitor.

Note. The results in the main part of the paper are based on *information interoperability* (Equation (2.5)). The effects of information processing ($w = 0$) and information collection ($w = 1$) will be studied through the weight w. Appendix 2.8.1 contains the same analysis based on *services interoperability*. Generally, the results under the two types of interoperability are comparable with similar intuitions. The reason that I focus on information interoperability is that the results are simpler than services interoperability. The effect of the weight w can also be studied directly.

2.3.2 Discussion and interpretation of the model

I discuss the key modeling choices–single-homing, horizontal differentiation, the continuous interoperability level–in Section 2.3.2.1. Furthermore, the interpretation of each network's intrinsic value is in Section 2.3.2.2.

2.3.2.1 Discussion on the modeling choices

An important modeling choice is that each consumer participates in one network only (single-home). Single-homing is appropriate in some situations, e.g., self-driving cars. However, consumers may join several networks (multi-home) in other cases, such as communication apps.

Suppose the model is adjusted such that consumers can choose to multi-home. As long as not all consumers multi-home, each firm still has an incentive to compete for single-homing consumers. Accordingly, the main result that interoperability relaxes the degree of price competition can be expected to remain the same. However, interoperability might relax price competition to a lesser extent under multi-homing since there are fewer consumers the firms can compete to attract.

The second modeling choice is horizontal differentiation. In practice, some consumers may prefer a car by brand *A* more than brand *B*. European citizens prefer WhatsApp, while WeChat is more prevalent in China. Professional social media platforms may also target different groups of users with different requirements or characteristics. For example, Xing, based in Hamburg, targets more of the German market than other platforms.

Nevertheless, several products, such as cars, are differentiated vertically as well as horizontally . Even though I do not have vertical differentiation in the model, the papers reviewed in Section 2.2 show that interoperability also relaxes the degree of price competition under vertical differentiation as well.

Finally, the paper assumes the interoperability level can be any value between zero and one. For example, a self-driving car may send only some of the information it collects to self-driving cars by other brands. In addition, there could be some delay or degraded signal when a user of one communication talks with other users who use another app. In these cases, the interoperability level is not necessarily zero or one. Suppose there is a restriction such that interoperability is either perfect or none at all. Then, while the analysis in this paper is still applicable, some results will be different. A consumer surplus which is maximized at an intermediate level of interoperability (Figure 2.4) is no longer valid.

2.3.2.2 Interpretation of the intrinsic quality of a network

According to consumer utility in Equation 2.2, each network has an intrinsic quality of one. In relation to three examples mentioned earlier–self-driving cars, professional social media platforms, and interpersonal communication apps, the intrinsic value can be interpreted as follow.

In self-driving cars, the intrinsic values are the values of the cars. Even though there are no network effects, consumers benefit from being able to use them for transportation. For professional social media platforms, the intrinsic values could represent the Sales Intelligence Solution (SIS) services. The SIS service makes customized matches between platform users with appropriate sales professionals. Users benefit from matches tailored to their circumstances.[19] Finally, interpersonal communication apps are not limited to communications between personal users. Private users can communicate with public agencies and companies. More importantly, some apps have other functions. For example, WeChat allows users to make payments and order in restaurants. These functions create utility for users without interacting with other users.

In addition, I assume that the intrinsic values of both networks are the same. In practice, the intrinsic values could be different. For example, a car manufacturer might use better materials, include more functions, and have better safety features than the other manufacturer. This assumption does not affect the results in a crucial way. For example, suppose that the monopolist's intrinsic quality is higher than the competitor. Then, an interoperability-level threshold that changes the market structure from a monopoly to a duopoly still exists. However, the threshold will be higher than when the intrinsic qualities of both firms are the same. This is because it is harder for the competitior to compete: it needs more network effects from interoperability to survive.

[19] In the European Commission's assessment of the merger between Microsoft and LinkedIn, users reported that they appreciated such services. See, Case M.8124, 'Microsoft/LinkedIn' C(2016) 8404 Final, Commission Decision of 6.12.2016, par. 206 - 217. For further discussions on the importance of SIS services, see van den Boom & Samranchit (2020)

2.4 Short run: interoperability and price competition

This section analyzes the effects of interoperability on equilibrium outcomes in the short run. That is, the qualities β_i and the qualities from interoperability $\tilde{\beta}_i$ are fixed. The firms can only adjust their prices p_i. So, there is only the price effect of interoperability.

Appendix 2.9 supplements the analysis in Section 2.4.1. The appendix shows that the short-run equilibrium outcomes in Proposition 1 are the unique outcomes supportable by SPE that satisfy two additional assumptions–Assumptions 4 and 5 in Appendix 2.9.

Assumption 5 is crucial. Briefly, the assumption states that when firm A or firm B can be the monopolist, all consumers choose firm A when $\beta_A \geq \beta_B$ and firm B when $\beta_B > \beta_A$.[20] A justification for this selection is that social welfare is higher when the firm with higher quality is the monopolist (Lemma 1). More discussions on the equilibrium selection are included in Appendix 2.9. Without the assumption, there are infinitely many subgame perfect equilibria with different equilibrium outcomes.

Furthermore, when outcomes such that a monopoly with a covered market and a non-covered market (some consumers do not choose either firm) are possible equilibrium outcomes, Assumption 4 requires that all consumers coordinate on the monopoly outcome. This assumption is for a technical purpose to lower the complexity of the analysis. I believe that Assumption 4 does not affect the equilibrium outcomes. Because Assumption 1 restricts the degree of horizontal differentiation τ to be small, prices that can create a non-covered market must be high. As shown in Proposition 1, the unique equilibrium prices under the monopoly and the duopoly are nowhere near such level. See more discussions in Section 2.9.1.

In this section, whenever the phrase "the unique equilibrium outcomes" is used, it refers to the unique outcomes supportable by SPE that satisfy Assumptions 4 and 5. Section 2.4.1 describes the unique equilibrium outcomes for any possible values of parameters. Section 2.4.2 discusses the effects of the interoperability level on consumer surplus and social welfare. This section focuses on the interoperability level t that could influence whether a monopoly or a duopoly arises in equilibrium.

2.4.1 Short-run equilibrium

I begin with the characterization of the equilibrium outcomes for each combination of qualities β_i, qualities from interoperability $\tilde{\beta}_i$, and an interoperability level t. For each combination, the equilibrium outcome is either a monopoly or a duopoly. The main objective of this section is to show that the market structure is a monopoly when the interoperability level t is low, but the outcome becomes a duopoly when the interoperability level t is sufficiently high.

First, it is helpful to introduce the term *adjusted quality* $\beta_i - t\tilde{\beta}_j$. Whether the market structure is a monopoly or a duopoly depends on adjusted quality $\beta_i - t\tilde{\beta}_j$. Suppose that firm i has high quality β_i. Firm i is attractive to consumers because it can generate high network effects. However, if $t\tilde{\beta}_j$ is also high, firm j receives high network effects from interoperability from firm i's network. Hence, the high firm i's quality β_i benefits its competitor. It is harder for firm i to cover the market. Adjusting for the spillover to its competitor, firm i has low adjusted quality $\beta_i - t\tilde{\beta}_j$.

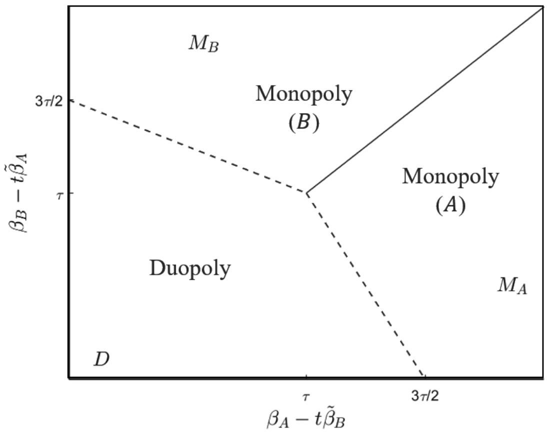
Under the *monopoly outcome,* all consumers participate in the monopolist's network. The other firm receives zero demand. This happens when the monopolist's *adjusted* quality $\beta_i - t\tilde{\beta}_j$ is high, i.e., areas

[20]It is straightforward to show that $\beta_i - t\tilde{\beta}_j \geq \beta_j - t\tilde{\beta}_i$ if and only if $\beta_i \geq \beta_j$.

M_A and M_B in Figure 2.1. Because of its high quality β_i, the monopolist can generate high network effects to cover the whole market. At the same time, the other firm does not receive sufficient network effects from interoperability (low $t\tilde{\beta}_j$) to co-exist with the monopolist.

The *duopoly outcome* arises when the adjusted quality $\beta_i - t\tilde{\beta}_j$ of both firms are low. Each firm cannot generate sufficient network effects to attract consumers who have a strong preference for the other firm. The duopoly outcome arises in areas D in Figure 2.1.

Figure 2.1: Equilibrium outcome as functions of adjusted qualities $\beta_i - t\tilde{\beta}_j$



Note: The figure shows the three areas with different market structures in equilibrium. In area M_i, where $i \in A, B$, firm i is the monopolist. The other firm is inactive. In area D, the market structure is the duopoly, where both firms are active.

Proposition 1 states the unique equilibrium outcomes for each area M_A, M_B, and and D. The areas in Figure 2.1 are defined formally in Definition 1 in Appendix 2.8.

Proposition 1. *The characterization of the unique equilibrium outcomes supportable by SPE that satisfy Assumptions 4 and 5*

1. *In area M_i, i.e., $\left(\beta_A - t\tilde{\beta}_B, \beta_A - t\tilde{\beta}_B\right) \in M_i$ where $i \in \{A, B\}$ and $i \neq j$. The unique equilibrium outcome is that firm i is the monopolist ($D_i = 1$), while firm j is inactive ($D_j = 0$). The monopolist charges the monopoly price p_i^m, where*

$$p_i^m(\beta_i, \beta_j; t) = \beta_i - t\tilde{\beta}_j - \tau. \tag{2.7}$$

The monopolist profit is $\pi_i^m = \beta_i - t\tilde{\beta}_j - \tau$. The inactive firm sets the price $p_j = 0$ with zero profit.

2. *In area D, i.e., $\left(\beta_A - t\tilde{\beta}_B, \beta_A - t\tilde{\beta}_B\right) \in D$. Both firms are active in the market. Firm i sets the duopoly prices p_i^d, where*

$$p_i^d(\beta_i, \beta_j; t) = \frac{3\tau - (\beta_i + 2\beta_j) + t\left(2\tilde{\beta}_i + \tilde{\beta}_j\right)}{3}, \tag{2.8}$$

with the profit

$$\pi_i^d(\beta_i,\beta_j;t)=\frac{\left(3\tau-(\beta_i+2\beta_j)+t\left(2\tilde{\beta}_i+\tilde{\beta}_j\right)\right)^2}{9\left(2\tau-\beta_i-\beta_j+t\left(\tilde{\beta}_i+\tilde{\beta}_j\right)\right)},\tag{2.9}$$

where $i\in\{A,B\}$ *and* $i\neq j$.

Proof. See the supplementary analysis in Appendix 2.9. □

In area M_i, firm i has high adjusted quality $\beta_i-t\tilde{\beta}_j$. So, firm i can generate enough network effects to cover the market. Because of Assumption 5 introduced in Appendix 2.9, all consumers choose firm i over firm j in area M_i. Therefore, firm i becoming the monopolist with the monopoly price p_i^m is the unique equilibrium outcome.

In area D, both firms have low adjusted quality $\beta_i-t\tilde{\beta}_j$. So, none of them can generate enough network effects such that all consumers want to be in the same network. Consumers who have a stronger preference for firm A (low x) chooses firm A, and vice versa. Thus both firms are active with the duopoly price p_i^d.

According to Proposition 1, the market structure is determined by adjusted qualities $\beta_i-t\tilde{\beta}_j$. Can a higher interoperability level t turn the market structure from the monopoly to the duopoly? And what happens to the equilibrium price? The following corollary shows that, even though firm i's quality β_i is high, the market structure becomes the duopoly when the interoperability level is high enough.

Corollary 1. *Suppose* $(\beta_A,\beta_B)\in M_i$, *there exists a threshold* $\bar{t}$ *where*

$$\bar{t}\equiv\frac{2\beta_i+\beta_j-3\tau}{\tilde{\beta}_i+2\tilde{\beta}_j}.\tag{2.10}$$

1. *When the interoperability level is sufficiently low, i.e.,* $t\leq\bar{t}$, *the equilibrium outcome is the monopoly. If* $t\leq\bar{t}$, *the equilibrium price decreases with interoperability.*

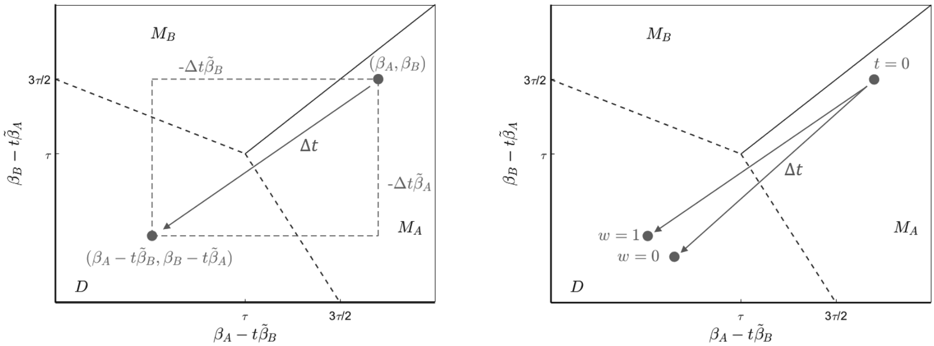
2. *When the interoperability level is sufficiently high, i.e.,* $t>\bar{t}$, *the equilibrium outcome is the duopoly. If* $t>\bar{t}$, *the equilibrium price increases with interoperability.*

Proof. See Appendix 2.8.2.1. □

When the interoperability level increases, the adjusted quality $\beta_i-t\tilde{\beta}_j$ of each firm decreases. Both networks are more similar in their ability to generate network effects. Thus, both firms can co-exist. Figure 2.3a illustrates a situation where the interoperability level t increases from zero by Δt. Firm i's adjusted quality $\beta_i-t\tilde{\beta}_j$ decreases by $\Delta t\tilde{\beta}_j$. Therefore, the pair of adjusted qualities $\left(\beta_A-t\tilde{\beta}_B,\beta_B-t\tilde{\beta}_A\right)$ moves south-west with the slope $\tilde{\beta}_A/\tilde{\beta}_B$. Once the interoperability level t is high enough, area D (duopoly outcome) is reached.

Figure 2.3b displays the role of the weight w. Suppose $\beta_A>\beta_B$. When the weight w equals one ($w=1$), the quality from interoperability that firm B receives is the quality of firm A ($\tilde{\beta}_B=\beta_A$), but the quality from interoperability that firm A receives is the quality of firm B ($\tilde{\beta}_A=\beta_B$). Thus, interoperability benefits firm B more than firm A. On the contrary, when the weight is zero ($w=0$), the quality from

Figure 2.2: Interoperability t and the equilibrium outcomes



(a) The effect on the adjusted qualities (b) The effects under different weights w

Note: The figures are based on $\beta_A = 1.7$, $\beta_B = 1.5$, and $\Delta t = 0.7$. The weight w in Figure (a) is 0.5.

interoperability is own quality ($\tilde{\beta}_B = \beta_B$ and $\tilde{\beta}_A = \beta_A$). Firm A can generate more network effects from interoperability than firm B. A higher weight w favors firm B more than when the weight w is low.[21]

Because a higher interoperability level t can alter the market structure, its effect on the equilibrium price depends on which market structure prevails. A higher interoperability level t decreases the equilibrium price under the monopoly ($t \leq \bar{t}$, defined in Equation (2.10)), But higher interoperability level t increases the equilibrium price under the duopoly ($t > \bar{t}$).

The monopolist sets the monopoly price $p_i^m(\beta_i, \beta_j; t)$ such that the last consumer is willing to participate. The monopolist does not have an incentive to increase the monopoly price further and lose some demand. Because of its high quality β_i, even a small fraction of consumers generate high network effects. If they leave, the monopolist's network loses a lot of network effects. In turn, more consumers will want to leave.

In the monopoly area M_i, when the interoperability level t is higher, the inactive firm receives more network effects from the monopolist. Thus, it is easier for the inactive firm to become active. The monopolist responds by competing more aggressively by lowering its price. So, the monopolist acts as a *constrained* monopolist. Interoperability is effective in lowering the monopoly price without an active competitor when the market is in an early stage.

On the contrary, interoperability relaxes the degree of price competition under the duopoly area D. This is because when firm i cuts its price p_i (to increase its demand D_i), its competitor, firm j, also receives network effects from interoperability $\left(t\tilde{\beta}_j D_i\right)$. The higher the interoperability level t, the higher the network effects from interoperability the competitor receives. Therefore, a decrease in firm i's price p_i increases its demand D_i less when the interoperability level t is high. In other words, the demand function becomes more inelastic with higher interoperability t. In fact, firm i's demand function is

[21] Since firm B benefits more from interoperability when the weight w is high, a question arises whether it is possible that firm B will replace firm A as a monopolist for some interoperability levels. However, Lemma 4 in Appendix 2.8 shows that such a situation cannot happen. If firm i is the monopolist in area M_i ($\beta_i > \beta_j$), it is not possible to have an equilibrium outcome such that firm j becomes the monopolist for any interoperability level t.

$$D_i\left(p_i,p_j\right)=\frac{1}{2}+\frac{1}{2}\cdot\underbrace{\frac{1}{2\tau-\beta_i-\beta_j+t\left(\tilde{\beta}_i+\tilde{\beta}_j\right)}}_{\text{slope}}\left[\underbrace{\Delta_i\beta+t\Delta_i\tilde{\beta}}_{\text{Qualities}}-\underbrace{2\Delta_i p}_{\text{price}}\right], \tag{2.11}$$

where $\Delta_i X \equiv X_i - X_j$.

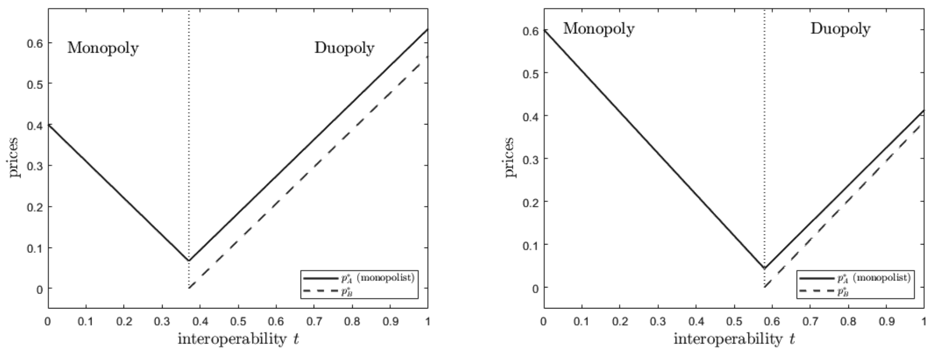
A higher interoperability level t leads to a steeper demand function. Consequently, both firms have an incentive to set high prices. This result is in line with the existing papers on the effects of interoperability under the duopoly and vertical differentiation setting (Baake & Boom (2001) and Garcia & Vergari (2016)).

Note that higher quality β_i makes the demand function more elastic. In other words, higher quality intensifies the degree of price competition. This observation is confirmed by Equation 2.8. Higher quality β_i leads to a lower duopoly price. When interaction qualities β_i are high, additional consumers that firm i can attract significantly increase the attractiveness of firm i's network. In turn, more consumers prefer moving to firm i's network. Hence, the firms have higher incentives to lower the prices to attract as many consumers as possible.

As shown in corollary 1, the effect of interoperability t on the equilibrium price under the monopoly and duopoly outcomes cannot be analyzed separately. When the interoperability level t is sufficiently high, the equilibrium outcome will turn from the monopoly to the duopoly.

Figure 2.3 illustrates the effect of the interoperability level t on the equilibrium price under a high and a low degree of horizontal differentiation τ. Notice that the monopoly price with no interoperability ($t = 0$) is lower than the duopoly price with perfect interoperability ($t = 1$) when the degree of horizontal differentiation τ is high (Figure 2.4a). In contrast, when the degree of horizontal differentiation τ is low (Figure 2.4b), the monopoly price with no interoperability is higher than the duopoly price with perfect interoperability.

Figure 2.3: The effect of interoperability t on the prices



(a) High horizontal differentiation $\tau = 0.6$ (b) Low horizontal differentiation $\tau = 0.4$

Note: The figures are based on $\beta_A = 1.0$, $\beta_B = 0.8$, and $w = 0.8$. The values of the degree of horizontal differentiation τ in Figure (a) and Figure (b) are 0.6 and 0.4, repsectively. The equilibrium duopoly prices with full duopoly could be lower or higher than the monopoly price without interoprerability. The difference is due to the degree of horizontal differentiation τ.

With high horizontal differentiation τ, the consumer who is located the furthest from the monopolist's location receives low utility from joining the monopolist's network. Consequently, the monopolist has to keep its monopoly price low to attract the last consumer. On the contrary, under the duopoly outcome, a high degree of horizontal differentiation τ relaxes the degree of price competition. Both firms charge high duopoly prices. Eventually, consumers end up paying more with perfect interoperability. The intuition in the case of low horizontal differentiation is the exact opposite. Corollary 2 summarizes this result.

Corollary 2. *Suppose $\beta_i > \beta_j$. Then, firm i's monopoly price p_i^m with no interoperability ($t = 0$) is lower than firm's i duopoly price p_i^d with perfect interoperability ($t = 1$) if and only if the degree of horizontal differentiation τ is sufficiently high such that*

$$\tau > \frac{(2+w)\,\beta_A + (1-w)\,\beta_B}{6}.$$

Proof. See Appendix 2.8.2.2. □

A regulator who wants to increase the number of competitors in a market may be successful by using interoperability. However, a regulator must be aware that imposing a too high interoperability level will lead to a higher price once the market structure becomes a duopoly. In markets with strong horizontal differentiation, comprehensively analyzing the effect of interoperability across different market structures is particularly important. Imposing full interoperability in such markets can lead to duopoly prices that are higher than a monopoly price.

Corollary 2 is based on information interoperability. A similar result also holds for services interoperability $\tilde{\beta}_i = \min\left\{\beta_i, \beta_j\right\}$, as summarized in Corollary 5 in Appendix 2.8.1.

2.4.2 The short-run effects of interoperability on welfare

Moving to the effects of interoperability t on consumer surplus and social welfare. Lemma 1 derives the consumer surplus and social welfare functions.

Lemma 1. *For given qualities β_i and an interoperability level t, consumer surplus CS is*

$$CS = \underbrace{\left[1 + \beta_A D_A^* + t\tilde{\beta}_A D_B^* - p_A^*\right] D_A^*}_{\text{network } A} + \underbrace{\left[1 + \beta_B D_B^* + t\tilde{\beta}_B D_A^* - p_B^*\right] D_B^*}_{\text{network } B} \underbrace{- \frac{\tau}{2}\left[1 - 2D_A^* D_B^*\right]}_{\text{mismatch}}. \tag{2.12}$$

Social welfare W is

$$W = \left[1 + \beta_A D_A^* + t\tilde{\beta}_A D_B^*\right] D_A^* + \left[1 + \beta_B D_B^* + t\tilde{\beta}_B D_A^*\right] D_B^* - \frac{\tau}{2}\left[1 - 2D_A^* D_B^*\right]. \tag{2.13}$$

Under the monopoly, consumer surplus CS^m and social welfare W^m are reduced to

$$CS^m = 1 + \frac{\tau}{2} + t\tilde{\beta}_j, \tag{2.14}$$

$$W^m = 1 + \beta_i - \frac{\tau}{2}. \tag{2.15}$$

Proof. See Appendix 2.8.2.3. □

Starting with consumer surplus, it consists of three components–the network effects generated by firm A and firm B net the prices, and the horizontal differentiation component. The last term exists because consumers in the middle of the Hotelling line have some mismatches in preferences with both firms A and B. Note that the mismatch component is minimized when each firm gets half of the market. Adding the firms' profits to consumer surplus yields social welfare.

The interoperability level t affects consumer surplus and social welfare directly and indirectly. The direct network effect is always positive since it increases the total network effects. In addition, it indirectly affects social welfare through the change in demands. The indirect channel could be negative when interoperability makes consumers switch to an inferior network.

For social welfare W^m under the monopoly , an increase in the interoperability level t does not affect social welfare, as long as the adjusted qualities $\beta_i - t\tilde{\beta}_j$ remain in the monopoly area M_i. Because there is only one active firm, there is no role for interoperability to generate more network effects from interoperability. In addition, social welfare increases with its own quality β_i since the monopolist can generate more network effects from its network.

Consumer surplus CS^m under the monopoly increases with the interoperability level t. Recall that the monopoly price p^m (Equation (2.7)) decreases with interoperability. The (constrained) monopolist must be more aggressive to deter entry. Thus, consumers pay a lower price with higher interoperability. In other words, interoperability is also useful under a monopoly from the perspective of consumer surplus

However, recall that when the interoperability level t is higher than the threshold $\bar{t}$, the equilibrium regime switches from the monopoly to the duopoly. Figure 2.4 plots consumer surplus and social welfare when there is a regime switch.

In the duopoly, an increase in the interoperability level t from the threshold level $\bar{t}$ *decreases* social welfare at first. In other words, around the point when the duopoly outcome replaces the monopoly outcome, higher interoperability is detrimental to social welfare (see Figures 2.4a and 2.4b). However, when the qualities of the two firms are equal, social welfare shifts downward when the equilibrium outcome shifts to the duopoly (see Figure 2.5ac).

Proposition 2 formalizes these results for information interoperability. Note that similar results hold under services interoperability as shown in Proposition 7 in Appendix 2.8.1.

Proposition 2. *Suppose* $(\beta_A, \beta_B) \in M_i$ *and* $\left(\beta_A - \tilde{\beta}_B, \beta_B - \tilde{\beta}_A\right) \in D$*. An increase in the interoperability level* t *above the threshold* $\bar{t}$ *initially decreases social welfare. Specifically,*

1. *if* $\beta_i > \beta_j$ *and* $(\beta_A, \beta_B) \in M_i$*, then*

$$W^m = W^d(\bar{t})$$

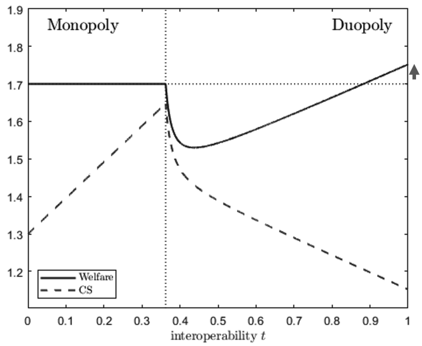
and

$$\left.\frac{dW^d}{dt}\right|_{t=\bar{t}} < 0.$$

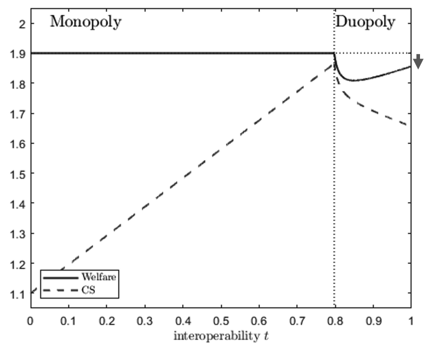
2. *If* $\beta_A = \beta_B$*, then* $W^m > W^d(\bar{t})$*. Furthermore, social welfare when the interoperability level is one* $W^d(\beta, \beta; t = 1)$ *(welfare under the duopoly) is higher than social welfare with no interoperability* W^m *(welfare under the monopoly).*

Proof. See Appendix 2.8.2.4. □

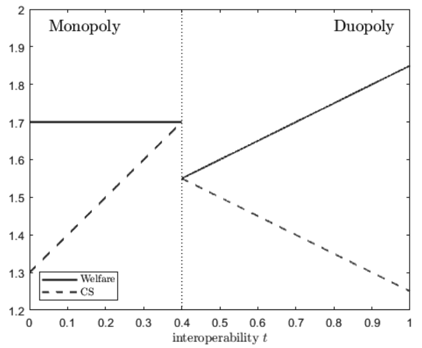
Figure 2.4: The effect of interoperability t on consumer surplus and social welfare



(a) High horizontal differentiation $\tau = 0.6$



(b) Low horizontal differentiation $\tau = 0.4$



(c) Equal qualities $\beta_A = \beta_B$

Note: Figures (a) and (b) are based on $\beta_A = 1.0$, $\beta_B = 0.8$, and $w = 0.8$. The values of the degree of horizontal differentiation τ in Figure (a) and Figure (b) are 0.6 and 0.4, repsectively. For Figure (c), the parameters are $\beta_A = \beta_B = 1.0$ and $\tau = 0.6$

When $\beta_A > \beta_B$, firm A has higher quality than firm B. Once the interoperability level t slightly increases beyond the threshold $\bar{t}$, a small proportion of consumers who have the least preferences towards firm A switch to firm B. Because the level of interoperability is relatively low, firm A's network effects from interoperability are low. Hence, the total network effects that consumers in firm A's network received drop. The consumers who switch to firm B do not take this drop in network effects into account when they decide. Consequently, total network effects diminish, leading to lower social welfare.

If the firms have the same level of qualities ($\beta_A = \beta_B$), half of the consumers suddenly switch to the other firm when the interoperability level t is higher than the threshold $\bar{t}$. Consequently, social welfare shifts down due to a sudden decrease in network effects because of imperfect interoperability.

However, a higher interoperability level t leads to higher social welfare when the level t is sufficiently high, as indicated in Figure 2.4. Intuitively, even though more consumers switch to firm B's low-quality network, high interoperability increases the network effects from interoperability that consumers in both networks enjoy. Furthermore, consumers who switch to firm B's network benefit from participating in their preferred network: the disutility from horizontal differentiation is lower. This observation suggests that when the degree of horizontal differentiation τ is large relative to the qualities β_i, a lower mismatch from horizontal differentiation could dominate lower network effects. As such, social welfare when the interoperability level t is one is higher than social welfare under monopoly. The following remark sums up the observation.

Remark. When the degree of horizontal differentiation τ is relatively high compared to the qualities β_i, social welfare under the duopoly with perfect interoperability ($t = 1$) is higher than social welfare under the monopoly, i.e., $W^d(\beta_A, \beta_B; t = 1) > W^m$ (see Figure 2.5a). On the other hand, if the degree of horizontal differentiation τ is relatively low compared to the qualities β_i, then $W^d(\beta_A, \beta_B; t = 1) < W^m$ (see Figure 2.5b).

In a special case where firm A's quality and firm B's quality are equal ($\beta_A = \beta_B$), social welfare under the duopoly with perfect interoperability ($t = 1$) is always higher than social welfare when the market structure is the monopoly. Because the two networks can generate the same network effects, the total network effects under the monopoly outcome are the same as the sum of the network effects under duopoly with perfect interoperability. In addition, each consumer can choose a network that is closer to her preference, leading to lower disutility from horizontal differentiation under the duopoly.

Moving to consumer surplus CS, the effects of interoperability on the equilibrium prices play an important role. In contrast to social welfare W, consumer surplus CS increases with interoperability t when the adjusted qualities $\beta_i - t\tilde{\beta}_j$ are in area M_i (Lemma 1). However, Figure 2.4 indicates that when the interoperability level increases above the threshold $\bar{t}$, so that the adjusted qualities are in area D, a higher interoperability level *decreases* consumer surplus. This is because interoperability relaxes the degree of price competition. A higher interoperability level t increases the duopoly prices (Equation 2.8). The exact properties of the consumer surplus function when the adjusted qualities are in area D are complicated to infer analytically. Nevertheless, Lemma 2 shows that consumer surplus with the interoperability level at the threshold $\bar{t}$ (monopoly) is always higher than consumer surplus with perfect interoperability under the duopoly outcome. Lemma 6 in Appendix 2.8.1 provides a similar result under services interoperability.

Lemma 2. *Consumer surplus when $t = \bar{t}$ (monopoly) is larger than consumer surplus when $t = 1$ (duopoly), i.e., $CS^m(t = \bar{t}) > CS^d(t = 1)$.*

Proof. See Appendix 2.8.2.5. □

The analysis of social welfare and consumer surplus so far assumes that the qualities β_i are high enough, i.e., $(\beta_A, \beta_B) \in M_i$, such that the monopoly outcome is possible with a low interoperability level. However, it is possible that the qualities β_i are too low for any firm to be the monopolist, i.e., $(\beta_A, \beta_B) \in D$. In this case, full interoperability maximizes social welfare, but consumer surplus is at the maximum level when there is no interoperability. These results are formalized in Corollary 4 in Appendix 2.8

Because higher interoperability increases network effects from interoperability, it enlarges social welfare. However, consumers suffer from higher interoperability due to lower price competition. Thus, there is a trade-off between social welfare and consumer surplus. If a regulator sets perfect interoperability, it maximizes social welfare but minimizes consumer surplus. In contrast, consumer surplus is maximized with no interoperability, but social is minimized.

Remark. The key messages from the short-run analysis

1. Interoperability is an effective tool to increase competition by increasing the number of players in a market that is in an early stage (ex-ante competition).

2. However, a higher interoperability level *relaxes* the degree of price competition in the duopoly regime. Consumers are worse off due to higher prices.

3. Social welfare under the duopoly with perfect interoperability could be lower than under the monopoly. This is because consumers do not take network externalities into account when choosing a network.

4. Consumer surplus is maximized at an intermediate interoperability level. It is the level that the inactive firm is about to become active. At this interoperability level, the (constrained) monopolist is forced to set the lowest monopoly price.

2.5 Long run: interoperability and investments

In this section, I analyze how interoperability affects the firms' decisions to innovate by investing in qualities β_i. Section 2.5.1 analyzes the equilibrium qualities given an interoperability level t. Then, Section 2.5.2 looks for the implications of interoperability t on social welfare. In addition to Assumption 1 that the degree of horizontal differentiation is sufficiently small, I further assume that the investment cost function $I(\beta)$ is infinitely increasing when the quality β approaches τ (Assumption 2).

Assumption 2. *The investment cost function $I(\beta)$ satisfies*

$$\lim_{\beta \to \tau} I'(\beta) = \infty.$$

Accordingly, the qualities β_i that the firms invest can only be in the range $[0, \tau)$. As a result, the adjusted qualities will always be in area D (duopoly), i.e., $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in D$ for all $t \in [0,1]$.

It is well-established among policymakers and the academic circle that when a market tips in favor of one firm due to strong network effects, interoperability that increases competition encourages investment. However, there is a gap in the literature about the investment effect when the market does not tip. So, the first reason I restrict β to be lower than τ is to fill this gap. Second, Assumption 2 also greatly simplifies the analysis. Of course, it would be interesting to relax Assumption 2, and investigate what happens with the investment in this case.

2.5.1 The investments in interaction qualities β_i

Recall the operating profit $\pi_i^d(\beta_i, \beta_j; t)$ under the duopoly as a function of the qualities β_i and the interoperability level t (Equation 2.9). Firm i chooses its quality β_i by maximizing its net profit $\pi_i^d(\beta_i, \beta_j; t) - I(\beta_i)$ taking β_j as given. To ensure that there is a unique maximum, I make another assumption on the investment cost function. Assumption 3 requires that the investment cost is increasing sufficiently fast in quality β_i.

Assumption 3. *The marginal investment cost increases sufficiently fast such that*

$$\frac{\partial^2 I(\beta_i)}{\partial \beta_i^2} \geq \frac{2(1+t(1-2w))^2(\tau-(1-t)\beta_j)^2}{9(2\tau-(1-t)(\beta_i+\beta_j))^3},$$

for any values of β_j, t, and w.

First, the qualities–β_i and β_j–are strategic substitutes. In other words, if firm j sets a higher quality β_j, firm i responds by setting a lower level of its quality β_i. Lemma 3 formalizes this result.

Lemma 3. *Let $\beta_i^{br}(\beta_j)$ be the firm i's best-response function for a given level of firm j's quality β_j. Particularly, $\beta_i^{br}(\beta_j)$ satisfies*

$$\frac{\partial \pi_i^d\left(\beta_i^{br}(\beta_j), \beta_j; t\right) - I(\beta_i)}{\partial \beta_i} = 0.$$

The best-response function $\beta_i^{br}(\beta_j)$ is decreasing in β_j, i.e.,

$$\frac{d\beta_i^{br}(\beta_j)}{d\beta_j} < 0.$$

Proof. See Appendix 2.8.2.6. □

Intuitively, recall that higher quality β_i intensifies price competition (as seen in the demand function in Equation 2.11). Therefore, when one firm sets higher quality, the other firm has an incentive to lower the degree of competition by choosing a lower quality level.

Given the best-response functions of both firms, Proposition 3 summarizes the condition for the equilibrium quality β^* given the interoperability level t and the weight w. The similar result under services interoperability is in Proposition 8 in Appendix 2.8.1.

Proposition 3. *The unique equilibrium qualities β_i^*, for $i \in \{A, B\}$, satisfy*

$$I'(\beta_i^*) = \frac{t}{3}\left(\frac{5}{4} - w\right) - \frac{1}{12}. \tag{2.16}$$

Proof. See Appendix 2.8.2.7. □

Following Proposition 3, Corollary 3 shows that the equilibrium qualities β_i^* are increasing in the interoperability level t. But they decrease with the weight w. A similar result for services interoperability is in Corollary 8 in Appendix 2.8.1.

Corollary 3. *The equilibrium qualities β_i^* are increasing in the interoperability level t. On the other hand, a higher weight w leads to lower equilibrium qualities β_i^*.*

Proof. See Appendix 2.8.2.8. □

The intuition for the result that a higher interoperability level t leads to higher investments follows from the short-run result that interoperability relaxes price competition. Interoperability allows the firms to charge higher prices and make more profits for any values of qualities β_i. They can raise the prices to extract more from the network effects. The higher the level of interoperability t, the lower the degree of competition. The firms get more profits from their investments under a higher interoperability level t. Thus, interoperability encourages the firms to invest more.

In contrast, when the weight w is high, firm i's quality from interoperability relies more heavily on its competitor's investment. Thus, a higher weight w reduces the investments because it allows firm j to benefit more from the investment in quality made by firm i. Firm j free-rides from firm i's investment. A higher weight w is comparable to a higher free-riding effect. Consequently, the firms have lower incentives to invest in their qualities when the weight w is high.

2.5.2 Long-run impact on welfare

Having analyzed the private incentives for investment in qualities, I turn to the optimal level of qualities β_i that maximizes social welfare given an interoperability level t. And whether the firms over-invest or under-invest in their qualities compared to the social welfare perspective. Under the equilibrium qualities, which are symmetric $\beta_A = \beta_B = \beta$, and the welfare function in Equation 2.13, the maximization problem becomes

$$\max_{\beta} 1 - \frac{\tau}{4} + \frac{\beta}{2}(1+t) - 2I(\beta). \tag{2.17}$$

Note that this maximization problem is identical for both information interoperability and services interoperability because the qualities from interoperability are the same under both specifications. Proposition 4 derives the condition on the welfare-maximizing quality β^w.

Proposition 4. *For an interoperability level t, the welfare-maximizing quality β^w is given by*

$$I'(\beta^w) = \frac{1+t}{4}. \tag{2.18}$$

The welfare-maximizing quality β^w is increasing with the interoperability level t.

Proof. See Appendix 2.8.2.9. □

Accordingly to Proposition 4, it is socially optimal for the firms to provide positive qualities regardless of the interoperability level t, because $I'(0) = 0$. The higher qualities β_i increase network effects, while the marginal cost to innovate for a very low value of quality ($\beta \to 0$) is small. Therefore, the benefits outweigh the costs, at least at the beginning of the development in the qualities β_i. In addition, the welfare-maximizing quality β^w is increasing in the interoperability level t. This is because interoperability creates more network effects. It compensates for a higher investment cost.

The next step is to compare the investment incentives between the private and the welfare perspectives. The following proposition shows that the firms always under-invest in their qualities β_i for any level of interoperability t.

Proposition 5. *For any interoperability level t and weight w, the firms always under-invest in the qualities compared to the welfare-maximizing level, i.e., $\beta^* < \beta^w$.*

Proof. See Appendix 2.8.2.10. □

Intuitively, higher qualities β_i increase network effects as well as social welfare. But they also intensify price competition between the firms, leading to lower prices and profits. Thus, the firms receive lower benefits from the investments compared with the welfare point of view: the firms under-invest.

Finally, what is the interoperability level that maximizes social welfare? Recall Proposition 7 which states that perfect interoperability maximizes social welfare, provided the qualities are sufficiently low such $(\beta_A, \beta_B) \in D$. Would it be desirable to have perfect interoperability ($t = 1$) when the firms can invest in qualities? The answer is it depends on how fast the investment cost increases.

Note that the derivative of the welfare function before the investment costs with respect to interoperability t, evaluated at $t = 1$, is

$$\left.\frac{dW^d\left(\beta^*(t), \beta^*(t); t\right)}{dt}\right|_{t=1} = \frac{\beta^*(1)}{2} + \frac{d\beta^*(t)}{dt}\left(1 - 2I'\left(\beta^*(1)\right)\right). \tag{2.19}$$

If the derivative is negative, social welfare improves when the interoperability level t is lower than one. Proposition 6 summarizes the results.

Proposition 6. *The welfare-maximizing interoperability level t^w is lower than one if the investment cost increases sufficiently fast such that*

$$I'(\beta(1)) > \frac{1}{2} + \frac{6\beta(1) I''(\beta(1))}{5 - 3w}.$$

Proof. See Appendix 2.8.2.11. □

If the investment cost increases sufficiently fast, a higher interoperability level might encourage the firms to invest in qualities too much. Lowering the interoperability level reduces costly investments. Thus, the welfare-maximizing interoperability t^w is not necessarily the full interoperability.

2.6 Implications on interoperability policies

In this section, I combine and synthesize the results of this paper that have implications for (horizontal) interoperability policies. Note that the recommendations in this section apply to a market with network effects in an early stage. That is, there are firms competing in the market (ex-ante competition) before the monopoly is entrenched.

Regulators and legislators should consider the four recommendations below when deciding whether to impose interoperability in a particular market. The first three recommendations are based on fixed qualities. The fourth recommendation concerns investment in quality.

The parameters that the first three recommendations are based on are such that the qualities are high, i.e., $(\beta_A, \beta_B) \in M_i$. So, absent interoperability, firm i becomes the monopolist. Furthermore, both firms can co-exist when the interoperability level t is high enough. That is, there exists $t > \bar{t}$ such that $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in D$.

Recommendation 1. If a regulator's objective is to ensure that competitors can co-exist in a targeted market, imposing a sufficiently high interoperability level is an effective tool.

Because of interoperability, competitors have access to each other's networks. As a result, it is less likely that the network by one firm has significantly higher network effects than the other networks. So, one firm is not dominated by another firm.

Recommendation 2. Suppose that a regulator wants to maximize consumer surplus. Then, the regulator should set interoperability at the level to which a competitor is about to become active. This is an intermediate value of the interoperability level.

A higher interoperability level forces the constrained monopolist to lower the monopoly price to prevent its competitor from becoming active. However, once the market structure is a duopoly, higher interoperability relaxes the degree of price competition. Consumers have to pay a higher price.

Recommendation 3. Suppose that a regulator wants to maximize social welfare.

1. If the degree of product differentiation is high, then the regulator should impose perfect interoperability.
2. If the degree of product differentiation is low, then any interoperability level (including zero) in which the market remains the monopoly maximizes social welfare.

When the degree of product differentiation is high, consumers receive high utility from having an alternative. So, it is better to have several networks for consumers to choose from. The interoperability should be perfect for maximizing network effects across networks. While measuring the degree of product differentiation is an empirical question, the car market may have high horizontal differentiation. Since there are currently many brands in the car market, it is arguable that different consumers have strong heterogeneous preferences toward these brands.

When the degree of product differentiation is low, consumers do not value having an alternative highly. Therefore, having only one network maximizes network effects. No consumer ends up in a network with lower quality. In fact, a regulator does not have to impose interoperability at all if it wants to maximize social welfare. Potential examples of products with low horizontal differentiation are Zoom/Microsoft Team and professional social media (e.g., LinkedIn, Meetup, Xing). There is no clear reason why one consumer should prefer an app to another.

For recommendation 4, based on Assumption 2, the cost of investment is increasing fast enough such that each firm never invests so that its quality β_i is larger or equal to τ, i.e., $\beta_i < \tau$ for $i \in \{A, B\}$.

Recommendation 4. When network effects cannot be sufficiently strong for a monopoly outcome to emerge, a regulator could encourage firms to invest in their quality by imposing a higher interoperability level. However, the regulator should be aware that too high interoperability can lead to too much investment if the investment cost quickly increases.

Because interoperability relaxes the degree of price competition, the firms make more profit from the investments in the qualities. Therefore, a higher interoperability level leads to higher qualities. However, too much interoperability could lead to too high qualities. Hence, firms spend more on investment costs than optimal. Thus, when a regulator is setting an interoperability level, it has to consider the dynamic efficiency.

2.7 Conclusion

In several modern industries, network effects are common. For example, users of a messaging app with a large user base receive higher utilities since they can interact with more people. Another example is self-driving cars. These cars can interact with each other to exchange information. The information is used by self-driving software to improve self-driving performances, resulting in fewer accidents, better traffic flows, and reduced pollution.

Firms in industries with network effects compete on interaction qualities, in addition to traditional price competition. Firms that offer messaging apps compete by providing more features (text messaging, voice calls, and video calls) that make their consumers enjoy using the apps more. In addition, better self-driving software is necessary to improve the self-driving mode. Improving interaction quality requires investment.

Regulators and legislators in many countries are proposing to impose interoperability in several markets with network effects (e.g., Digital Market Act, ACCESS Act, and European Electronic Communication Code). This paper investigates how these proposals could affect investments and competition in a market with network effects at an early stage before the monopoly is entrenched. The novelty of this paper is twofold. First, it studies the effects of interoperability on price competition when either a monopoly or a duopoly outcome could arise endogenously in equilibrium. Second, the paper studies firms' investment incentives in interaction qualities.

Given fixed interaction qualities, the paper shows that a firm becomes a monopolist when two conditions are met. First, the firm's quality is high. And second, the interoperability level is low. In contrast, a duopoly outcome occurs when the qualities are low, or the interoperability level is high. When the interoperability level keeps increasing, the market structure in equilibrium switches from a monopoly to a duopoly. In other words, the market structure is endogenously determined.

The effects of interoperability on the equilibrium price are the opposite under the monopoly and the duopoly. A higher interoperability level decreases the monopoly price but increases the duopoly prices. Consequently, consumer surplus increases with interoperability under the monopoly outcome, but it decreases with interoperability under the duopoly outcome.

Perfect interoperability is not always good in terms of social welfare either. When the degree of horizontal differentiation is low relative to the monopolist's interaction quality, social welfare under the duopoly with perfect interoperability is lower than social welfare under the monopoly. The effects of interoperability on consumer surplus and social welfare highlight the necessity to have a unified framework to study the effects of interoperability under both the monopoly and the duopoly simultaneously.

When firms make decisions to invest in interaction qualities, interoperability encourages firms to invest in interaction qualities. The intuition follows from the short-run result that interoperability relaxes the degree of price competition. When firms improve interaction qualities, there are higher consumer utilities that the firms can extract. Because interoperability allows the firms to charge higher prices, they can reap more profits from their investments. Thus, it is more profitable for firms to invest in interaction qualities when there is interoperability than when there is no interoperability.

However, full interoperability could encourage firms to invest too much in interaction qualities. When investment costs increase sufficiently fast with interaction qualities, inducing lower investments by setting a lower interoperability level will save considerable investment costs. Thus, imperfect interoperability increases social welfare compared to full interoperability. Therefore, when a regulator sets the interoperability level, it must consider the dynamic efficiency of investments.

2.8 Appendix: additional results and definitions

Definition 1. The sets of the adjusted qualities $\beta_i - t\tilde{\beta}_j$ for different types of equilibrium

1. $M_A = \left\{ \left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \middle| \beta_A - t\tilde{\beta}_B \geq 3\tau - \beta_A - \beta_B + t\left(\tilde{\beta}_A + \tilde{\beta}_B\right) \text{ and } \beta_A - t\tilde{\beta}_B \geq \beta_B - t\tilde{\beta}_A \right\}$,
2. $M_B = \left\{ \left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \middle| \beta_B - t\tilde{\beta}_A \geq 3\tau - \beta_A - \beta_B + t\left(\tilde{\beta}_A + \tilde{\beta}_B\right) \text{ and } \beta_B - t\tilde{\beta}_A > \beta_A - t\tilde{\beta}_B \right\}$

for $i \in \{A, B\}$,

3. $D = \left\{ \left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \middle| \beta_i - t\tilde{\beta}_j < 3\tau - \beta_i - \beta_j + t\left(\tilde{\beta}_i + \tilde{\beta}_j\right) \text{ for both } i, j \in \{A, B\} \text{ and } i \neq j \right\}$,

Lemma 4. *Suppose $\beta_i > \beta_j$ and $(\beta_A, \beta_B) \in M_i$. Then,* $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \notin M_j$ for all t.

Proof. Without loss of generality, suppose that $\beta_A > \beta_B$. If $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in M_A$ for some t, then it must be that $\beta_i > \tau$ for both $i \in \{A, B\}$. Furthermore, it is straightforward to show that the adjusted qualities move along the line

$$y - \beta_B = \frac{\tilde{\beta}_A}{\tilde{\beta}_B}(x - \beta_A), \tag{2.20}$$

where x and y represent the adjusted qualities of firms A and B, respectively. Notice that if there exists an interoperability level t that makes $(y, x) \in M_A$, then y must be greater than τ when x equals τ. See Figure 2.2 for an illustration.

The most likely case that this could happen is when the slope $\tilde{\beta}_A/\tilde{\beta}_B$ is as low as possible. The slope is the lowest when $w = 1$, so $\tilde{\beta}_A/\tilde{\beta}_B = \beta_B/\beta_A$. Consequently, when $x = \tau$, $y = \frac{\beta_B}{\beta_A}(\tau - \beta_A) + \beta_B$. Then, $y > \tau$ if and only if $\beta_B > \beta_A$ which is a contradiction. So, it is not possible to have $y > \tau$ when $x = \tau$. □

Corollary 4. *Suppose that the qualities β_A and β_B are low such that $(\beta_A, \beta_B) \in D$, then*

1. *social welfare $W^d(\beta_A, \beta_B; t)$ is maximized when the interoperability level is one ($t = 1$).*

2. *In contrast, consumer surplus $CS^d(\beta_A, \beta_B; t)$ is maximized when the interoperability level is zero ($t = 0$).*

Proof. Starting with social welfare, the result that $t = 1$ maximizes social welfare is proved through a series of results and arguments.

First, through direct calculations, it can be checked that social welfare when interoperability is one is greater or equal to $W^d(\beta_A, \beta_B; 1) \geq W^d(\beta_A, \beta_B; 0)$. In fact, social welfare between the two scenario is the same only when $\beta_A = \beta_B = 0$.

Second, the welfare function is either concave or convex for any $t \in [0, 1]$. In other words, for any values of β_A, β_B, and w, the second order derivative $d^2W^d(\beta_A, \beta_B; t)/dt^2$ is either positive or negative for all values of t. Specifically,

$$\frac{d^2W^d(\beta_A, \beta_B; t)}{dt^2} = -\frac{2(\beta_A - \beta_B)^2(\tau - \beta_A - \beta_B + w(2\tau - \beta_A - \beta_B))K}{9(2\tau - \beta_A - \beta_B + t(\beta_A + \beta_B))^4},$$

where

$$\begin{aligned} K = \tau^2 &+ (\beta_A + \beta_B)(5\tau - 2(\beta_A + \beta_B)) + 2t(\beta_A + \beta_B)(\tau + \beta_A + \beta_B) \\ &+ w\left[2t\left((\beta_A + \beta_B)^2 + \beta_B(\tau + \beta_B)\right) + 4\beta_A(\tau - \beta_B) + (\tau - \beta_A)(\tau + 2\beta_A) + \beta_B(5\tau - 2\beta_B)\right]. \end{aligned}$$

Because $\tau > \beta_i$ for both $i \in \{A, B\}$, $K > 0$. Thus, the sign of dW^2/dt^2 is the same for any values of the interoperability level t given a combination of β_A, β_B, and w.

If $W^d(\beta_A, \beta_B; t)$ is convex in t, then the maximum is attained at $t = 0$ or $t = 1$. However, we have $W^d(\beta_A, \beta_B; 1) \geq W^d(\beta_A, \beta_B; 0)$. So, the maximum is when the interoperability level is one.

If $W^d(\beta_A,\beta_B;t)$ is concave, the first-order derivative of the welfare function with respect to t is decreasing in t. Hence, if $dW^d/dt\big|_{t=1} > 0$, then $dW^d/dt > 0$ for all t. Consequently, the welfare is always increasing with t. To show this, notice that

$$\left.\frac{dW^d(\beta_A,\beta_B;t)}{dt}\right|_{t=1} = \frac{\left[3(\tau-\beta_A-\beta_B)(1-w)-\tau\left(1+2w^2\right)\right](\beta_A-\beta_B)^2+9\tau^2(\beta_A+\beta_B)}{36\tau^2}.$$

If $3(\tau-\beta_A-\beta_B)(1-w)-\tau\left(1+2w^2\right)>0$, then the derivative is clearly positive. On the other hand, if it is negative, then

$$\begin{aligned}\left.\frac{dW^d(\beta_A,\beta_B;t)}{dt}\right|_{t=1} &\geq \frac{\left[3(\tau-\beta_A-\beta_B)(1-w)-\tau\left(1+2w^2\right)\right]\tau^2+9\tau^2(\beta_A+\beta_B)}{36\tau^2}\\ &\geq \frac{\left[3(\tau-\beta_A-\beta_B)-3\tau\right]\tau^2+9\tau^2(\beta_A+\beta_B)}{36\tau^2}\\ &= \frac{\beta_A+\beta_B}{6}\geq 0.\end{aligned}$$

Accordingly, $dW^d/dt\big|_{t=1}\geq 0$ for any values of β_A, β_B, and w. Thus, social welfare reaches the maximum when the interoperability is perfect ($t=1$).

Finally, it remains to be shown that consumer surplus is maximized when there is no interoperability. By direct calculations, it can be shown that

$$\frac{dCS^d(\beta_A,\beta_B;t)}{dt}\leq 0$$

for any values of the parameters. Note also that the derivative equals to zero only when $\beta_A=\beta_B=0$. □

2.8.1 Results under services interoperability

In appendix 2.8.1, I analyze the results based on services interoperability. That is, the quality from interoperability $\tilde{\beta}_i$ is the minimum of the two qualities: $\tilde{\beta}_i=\min\{\beta_A,\beta_B\}$.

Corollary 5. *Suppose the quality of firm i is higher than firm j, i.e., $\beta_i>\beta_j$. Firm i's monopoly price with no interoperability ($t=0$) is lower than firm's i duopoly price with perfect interoperability ($t=1$) if and only if the degree of horizontal differentiation τ is sufficiently high such that*

$$\tau>\frac{4\beta_i-\beta_j}{6}.$$

Proof. When $t=0$, the monopoly price is $p_i^m=\beta_i-\tau$. The duopoly price when $t=1$ is $p_i^d=(3\tau-\beta_i-2\beta_j+2\tilde{\beta}_i+\tilde{\beta}_j)/3$. Using services interoperability, the direct calculation yields the result. □

Proposition 7. *Suppose $(\beta_A,\beta_B)\in M_i$. Under services interoperability, an increase in the interoperability level t from the threshold $\bar{t}$ decreases social welfare. In particular,*

1. *without loss of generality, suppose $\beta_A>\beta_B$ and $(\beta_A,\beta_B)\in M_i$, then*

$$\left.\frac{dW^d(\beta_A,\beta_B;t)}{dt}\right|_{t=\bar{t}}<0.$$

2. *If $\beta_A = \beta_B$, then $W^m < W^d(\beta_A, \beta_B; \bar{t})$.*

Proof. Starting with the first part of the proposition, the direct calculation yields

$$\left.\frac{dW^d(\beta_A, \beta_B; t)}{dt}\right|_{t=\bar{t}} = -\frac{\left(\tilde{\beta}_A + 2\tilde{\beta}_B\right)\left(2\beta_B(\beta_A - \beta_B) + 2\tau\tilde{\beta}_A + \tau\tilde{\beta}_B\right)}{3\beta_B(\beta_A - \beta_B)} < 0,$$

since $\beta_A > \beta_B$ by assumption. The proof of the second part of the proposition is identical to Proposition 2 which shows the same results under information interoperability. □

Corollary 6. *Suppose $(\beta_A, \beta_B) \in M_i$ and $\left(\beta_A - \tilde{\beta}_B, \beta_B - \tilde{\beta}_A\right) \in D$. Under services interoperability, consumer surplus when $t = \bar{t}$ (monopoly) is larger than consumer surplus when $t = 1$ (duopoly), i.e., $CS^m(t = \bar{t}) > CS^d(t = 1)$.*

Proof. Without loss of generality, suppose that $\beta_A \geq \beta_B$. The direct calculation gives

$$CS^m(\bar{t}) - CS^d(1) = \frac{\tau[(2(\beta_A - \beta_B) - 3\tau)(4(\beta_A - \beta_B) - 9\tau)]}{9(2\tau - \beta_A + \beta_B)^2}.$$

Hence $CS^m(\bar{t}) - CS^d(1) > 0$ if $\beta_A - \beta_B < 3\tau/2$. However, for $\left(\beta_A - \tilde{\beta}_B, \beta_B - \tilde{\beta}_A\right) \in D$, it must be that $\beta_A - t\tilde{\beta}_B < 3\tau - \beta_A - \beta_B + t\left(\tilde{\beta}_A + \tilde{\beta}_B\right)$. Because $\tilde{\beta}_i = \min\{\beta_A, \beta_B\}$. It guarantees that $\beta_A - \beta_B < 3\tau/2$. □

Lemma 5. *Under services interoperability, there is no asymmetric pure strategy equilibrium for the qualities β_i. Specifically, any combination of $\left(\hat{\beta}_A, \hat{\beta}_B\right)$ where $\hat{\beta}_i \neq \hat{\beta}_j$ cannot be equilibrium qualities.*

Proof. Suppose without loss of generality that $\beta_A > \beta_B$, it can be shown that firm A can increase its profit be decreasing its quality β_A. To show this, notice that when $\beta_A > \beta_B$, a slight change in β_A does not affect the minimum of the qualities $\tilde{\beta}$. Hence, we have

$$\frac{d\pi_A^d(\beta_A, \beta_B, t)}{d\beta_A} = -\frac{(\tau - \beta_A + \beta_B t)(3\tau + 3\beta_B t - \beta_A - 2\beta_B)}{9\left(2\tau - \beta_i - \beta_j + 2\tilde{\beta} \cdot t\right)^2} < 0.$$

The inequality sign comes from the assumption that $\beta_i < \tau$. Hence, if firm A slightly decreases its quality from $\hat{\beta}_A$, its operating profit π_A increases. At the same time, firm A saves the investment cost $I(\beta_A)$. As such, a combination of $\left(\hat{\beta}_A, \hat{\beta}_B\right)$ where $\hat{\beta}_A \neq \hat{\beta}_B$ cannot be an equilibrium. □

Corollary 7. *Suppose $(\beta_A, \beta_B) \in D$. Under services interoperability, for given levels of qualities β_A and β_B,*

1. *social welfare $W^d(\beta_A, \beta_B; t)$ is maximized when the interoperability level is one ($t = 1$).*
2. *In contrast, the consumer surplus $CS(\beta_A, \beta_B; t)$ is maximized when the interoperability level is zero ($t = 0$).*

Proof. Suppose without loss of generality that $\beta_A \geq \beta_B$, so $\tilde{\beta}_i = \beta_B$. Note first that the welfare function in this case is convex in t, i.e.,

$$\frac{d^2W^d(\beta_A, \beta_B; t)}{dt^2} = \frac{2\beta_B^2(\beta_A - \beta_B^2)(13\tau - 5\beta_A - 5\beta_B + 10t\beta_B)}{9(2\tau - \beta_A - \beta_B + 2t\beta_B)^4} \geq 0.$$

Hence, the maximum is attained either at $t = 0$ or $t = 1$. By direct calculations, $W^d(\beta_A, \beta_B; 1) \geq W^d(\beta_A, \beta_B; 0)$. Note that they are equal only when $\beta_A = \beta_B = 0$. So, social welfare under the first specification is maximized when $t = 1$.

Furthermore, the consumer-surplus function under the first specification is also convex in t, as

$$\frac{d^2 CS^d(\beta_A, \beta_B; t)}{dt^2} = \frac{2\beta_B^2 \left(\beta_A - \beta_B^2\right)(3\tau - \beta_A - \beta_B + 2t\beta_B)}{3(2\tau - \beta_A - \beta_B + 2t\beta_B)^4} \geq 0.$$

So, the first-order derivative of the consumer-surplus function is highest when $t = 1$. Yet, it can be shown that

$$\left.\frac{dCS^d(\beta_A, \beta_B; t)}{dt}\right|_{t=1} = -\beta_B \left(\frac{1}{2} + \frac{2(\beta_A - \beta_B)^2}{9(2\tau - \beta_A + \beta_B)^2} + \frac{(\beta_A - \beta_B)^3}{18(2\tau - \beta_A + \beta_B)^2}\right) \leq 0.$$

Therefore, $dCS^d(\beta_A, \beta_B; t)/dt \leq 0$ for any values of β_A, β_B, and t. Since the consumer surplus is always decreasing with interoperability, the consumer surplus is maximized when $t = 0$. □

There are two reasons why Lemma 5 holds. For the first reason, suppose that quality of firm A is higher than firm B ($\beta_A \geq \beta_B$), so $\tilde{\beta}_A = \beta_B$. Firm A can slightly reduce its quality β_A without affect its quality from interoperability $\tilde{\beta}_A$. On the one hand, firm A can generate lower network effects from its own network. On the other hand, it does not suffer from lower quality from interoperability. Thus, while the overall firm A's network effects decrease, they do not decrease too much. One the other hand, firm A is compensated by a higher price–the second reason. As argued earlier, higher interaction qualities intensify price competition. Lowering β_A allows firm A to charge a higher price. Therefore, it is beneficial for firm A to lower its quality β_A when $\beta_A > \beta_B$.

Because of Lemma 5, any pure strategy equilibrium must be symmetric. The following proposition shows that for a given interoperability level t, there are multiple equilibria when t is sufficiently high. And there is a unique equilibrium that nobody invests when t is low.

Proposition 8. *Under services interoperability, the equilibrium qualities in pure strategy β_i^* must be symmetric. The equilibrium is characterized as follows:*

1. When the interoperability level is sufficiently high such that $t > 1/6$, there exist multiple equilibria. In particular, any combination of (β^*, β^*) such that

$$I'(\beta^*) \leq \frac{1}{2}\left(t - \frac{1}{6}\right) \tag{2.21}$$

   is an equilibrium.

2. When the interoperability level is sufficiently low such that $t \leq 1/6$, the equilibrium qualities are zero, i.e., $\beta_A^*(t) = \beta_B^*(t) = 0$.

Proof. Because of the min function $\tilde{\beta} = \min\{\beta_A, \beta_B\}$, there is a kink in the equilibrium profits as functions of qualities β_i. The first step is to show that each firm does not have an incentive to *increase* its quality from the equilibrium. Because the equilibrium qualities must be symmetric, an increase in the quality by firm i does not affect the minimum of the qualities, i.e., $\tilde{\beta} = \beta_j$. Hence, the derivative of firm

i's profit π_i with respect to firm i's interaction quality β_i evaluated at a symmetric level is

$$\left.\frac{d\pi_i^d(\beta_i,\beta_j,t)}{d\beta_i}\right|_{\beta=\beta_i=\beta_j} = -\frac{1}{12}.$$

Hence, both firms do not have any incentives to increase their qualities from any symmetric combination of the qualities.

Second, it must be that no firm wants to unilaterally *decrease* its quality either. When firm i decreases β_i, the minimum of the qualities becomes β_i, i.e., $\tilde{\beta} = \beta_i$. Hence, we have

$$-\left.\frac{d\pi_i^d(\beta_i,\beta_j,t)}{d\beta_i}\right|_{\beta=\beta_i=\beta_j} = \frac{1}{12} - \frac{t}{2}.$$

When firm i decreases β_i, it also saves on investment cost. Thus, for (β^*,β^*) to be an equilibrium, firm i must make a lower profit from decreasing its quality. Hence, it must be that

$$-\left(\left.\frac{d\pi_i^d(\beta_i,\beta_j,t)}{d\beta_i}\right|_{\beta^*=\beta_i=\beta_j} - I'(\beta^*)\right) \leq 0.$$

This is equivalent to

$$I'(\beta^*) \leq \frac{1}{2}\left(t - \frac{1}{6}\right).$$

By assumption, $I'(\beta) > 0$ for any $\beta \in [0,\tau)$. Hence, the preceding inequality can hold only in the case where $t \geq 1/6$. This completes the first part of the proposition.

For the second part of the proposition, notice that, when $t < 1/6$, the right-hand-side of the inequality is negative. The firms always find it profitable to lower their qualities. Therefore, the equilibrium arise when $\beta^* = 0$. □

In the second part of the proposition, when the interoperability level t is too low, an increase in qualities greatly intensify price competition, which reduces their profits. So, no firm has an incentive to invest. Once the interoperability level t is sufficiently high, the first part of Proposition 8 indicates that a pair of higher qualities can be supported as an equilibrium when the interoperability level t is higher. Corollary 8 formalizes this result.

Corollary 8. *Under services interoperability, $\tilde{\beta} = \min\{\beta_A,\beta_B\}$, a higher level of interoperability t (weakly) increases the highest level of the equilibrium qualities β^* that is supportable as an equilibrium.*

The result that the equilibrium qualities increase with the interoperability level t is similar to the result in Corollary 3, where the qualities from interoperability is the weighted average of the two qualities. The intuition is similar. Interoperability relaxes price competition, allowing the firms to charge higher prices from their investments.

Next, I investigate whether the firms under-invest or over-invest in qualities β^* compared to the welfare-maximizing qualities β^w. In Proposition 5, it shows that the firms always under-invest in qualities β^* compared to the welfare-maximizing level β^w. However, this result is not always true under services interoperability. Proposition 9 summarizes the result.

Proposition 9. *Under services interoperability, $\tilde{\beta}_i = \min\{\beta_A,\beta_B\}$,*

1. *the firms under-invest in qualities β_i^* compared to the welfare-maximizing level β^w when the interoperability level t is low such that $t \leq 2/3$.*

2. *But there exist equilibria which the firms over-invest in qualities β^* compared to the welfare-maximizing level β^w.*

Proof. Recall Proposition 8, the equilibrium qualities β^* under services interoperability is characterized by Equation 2.21. Because $I'' > 0$, the welfare-maximizing qualities β^w is higher than the largest equilibrium qualities when the right-hand side of Equation 2.18 is greater than the right-hand side of Equation 2.21. The direct comparison yields the results in the proposition. □

Under services interoperability, when t is sufficiently large, the firm loss a lot when reducing its own quality. This is because it brings down its own quality as well as both qualities from interoperability. Under services interoperability, this greatly increases the degree of competition. So, even qualities that are higher than the welfare-maximizing level are still supportable as an equilibrium.

Finally, in-line with Proposition 6, under services interoperability, the welfare-maximizing interoperability level t^w could be lower than full interoperability if the investment cost is increasing fast enough in quality, as summarized in the following proposition.

Proposition 10. *Under information interoperability, the welfare-maximizing interoperability level t^w is lower than one if the investment cost increases sufficiently fast such that*

$$I'(\beta(1)) > \frac{1}{2} + \frac{6\beta(1)I''(\beta(1))}{5-3w}.$$

Proof. If $W^d(\beta^*(t), \beta^*(t); t)/dt\big|_{t=1} < 0$, lowering the interoperability level from one improves social welfare. Using Equation 2.19, this happens when

$$I'(\beta^*(1)) > \frac{1}{2} + \left(\frac{d\beta^*(t)}{dt}\right)^{-1} \frac{\beta^*(1)}{2}. \tag{2.22}$$

Using the implicit function theorem, under the first specification, we have

$$\frac{d\beta^*(t)}{dt} = \frac{1}{2I''(\beta^*(t))}.$$

Putting this equation in Equation 2.22 proves the proposition. □

2.8.2 Proofs of the lemmas and propositions

2.8.2.1 Proof of Corollary 1

Proof. According to Proposition 1, when $(\beta_A, \beta_B) \in M_i$, the monopoly outcome arises. Furthermore, the duopoly outcome arises when the adjusted qualities fall in area D. This happens when $\beta_i - t\tilde{\beta}_j < 3\tau - \beta_i - \beta_j + t\left(\tilde{\beta}_i + \tilde{\beta}_j\right)$. Re-arranging this inequality yields the condition in Corollary 1.

The effects of interoperability t on the equilibrium price under the monopoly and the duopoly follow directly from Equation (2.7) and Equation (2.8), respectively. □

2.8.2.2 Proof of Corollary 2

Proof. When $t = 0$, the monopoly price is $p_i^m = \beta_i - \tau$. The duopoly price when $t = 1$ is $p_i^d = (3\tau - \beta_i - 2\beta_j + 2\tilde{\beta}_i + \tilde{\beta}_j)/3$. Using the information interoperability specification for $\tilde{\beta}_i$, a direct calculation yields the result. □

2.8.2.3 Proof of Lemma 1

Proof. Using the utility functions in Equation 2.2, social welfare is given by

$$W(\beta_A, \beta_B; t) = \int_0^{\hat{x}} \left(1 - \tau x + \beta_A D_A + t\tilde{\beta}_A D_B\right) dx + \int_{\hat{x}}^1 \left(1 - \tau(1-x) + \beta_B D_B + t\tilde{\beta}_B D_A\right) dx,$$

where $\hat{x}$ is the marginal consumer. Calculating the above equation out yields the social welfare function as stated in the lemma. For the consumer surplus, it follows from the definition that $CS = W - \pi_A^* - \pi_B^*$.

When the adjusted qualities $\beta_i - t\tilde{\beta}_i$ is in area M_i (the monopoly regime), the demand of the monopolist is one ($D_i = 1$), while the demand of the other firm is zero ($D_j = 0$). This simplifies the expressions of consumer surplus and social welfare, as shown in the lemma. □

2.8.2.4 Proof of Proposition 2

Proof. Starting with the first part of the proposition, the direct calculation yields

$$\left.\frac{dW^d(\beta_A, \beta_B; t)}{dt}\right|_{t=\bar{t}} = -\frac{\left(\tilde{\beta}_A + 2\tilde{\beta}_B\right)\left((\beta_A - \beta_B)(2\beta_A w + \beta_B) + 2\tau\tilde{\beta}_A + \tau\tilde{\beta}_B\right)}{3(\beta_A - \beta_B)((\beta_A + \beta_B - \tau)(1-w) + \tau w)} < 0,$$

since $(\beta_A, \beta_B) \in M_i$ and $\beta_A > \beta_B$ by assumption.

In the second part of the proposition, note that when $\beta_A = \beta_B = \beta$ and $t = \bar{t}$, the social welfare function reduces to

$$W^d(\beta, \beta; \bar{t}) = 1 + \beta - \frac{3\tau}{4}.$$

According to Lemma 1, $W^m = 1 + \beta - \tau/2$. Hence, $W^m > W^d(\beta, \beta; \bar{t})$.

Finally, using Equation (2.15), social welfare in area M_i is $W^m = 1 + \beta - \frac{\tau}{2}$. Additionally, by direct calculations, social welfare under area D when $t = 1$ is $W^d(\beta, \beta; t = 1) = 1 + \beta - \frac{\tau}{4}$. So, $W^d(\beta, \beta; t = 1) > W^m$. □

2.8.2.5 Proof of Lemma 2

Proof. Without loss of generality, suppose that $\beta_A \geq \beta_B$. The direct calculation gives

$$\begin{aligned} CS^m(\bar{t}) - CS^d(1) = {} & [3\tau - (1-w)(\beta_A - \beta_B)][(\beta_A - \beta_B)(5w^2(\beta_A - \beta_B) \\ & + 3w(2\beta_A + 3\beta_B) - 15\tau w + \beta_A + 2\beta_B) + 21\tau\beta_A + 6\tau\beta_B)] \\ & \Big/ 36\tau(\beta_A + 2\beta_B + w(7\beta_A + 2\beta_B)). \end{aligned}$$

□

It can be shown that both terms in the numerator are positive. Starting with the first term, the most likely case that it could be negative is when $w = 0$. So, it must be that $\tilde{\beta}_i = \beta_i$. It is possible for the duopoly outcome to arise when $\beta_A - t\tilde{\beta}_B < 3\tau - \beta_A - \beta_B + t\left(\tilde{\beta}_A + \tilde{\beta}_B\right)$. This condition implies $3\tau - (\beta_A - \beta_B) > 0$.

For the second term in the nominator, notice that the only negative term is $15(\beta_A - \beta_B)\tau w$. However, notice that

$$\begin{aligned} 21\tau\beta_A - 15(\beta_A - \beta_B)\tau w &\geq 21\tau\beta_A - 15w\tau\beta_A \\ &= 3\tau\beta_A(7 - 5w) \\ &> 0. \end{aligned}$$

The denominator is always positive. Thus, $CS^m(\bar{t}) > CS^d(1)$.

2.8.2.6 Proof of Lemma 3

Proof. Using the implicit function theorem, we have

$$\frac{d\beta_i^{br}(\beta_j)}{d\beta_j} = -\frac{\partial\left[\frac{\partial\pi_i^*(\beta_i,\beta_j;t) - I(\beta_i)}{d\beta_i}\right] \Big/ \partial\beta_j}{\partial\left[\frac{\partial\pi_i^*(\beta_i,\beta_j;t) - I(\beta_i)}{d\beta_i}\right] \Big/ \partial\beta_i}.$$

First, note that the denominator is negative because of Assumption 3. Furthermore, it can be shown that the numerator is also negative as

$$\frac{\partial^2\pi_i^d(\beta_i,\beta_j;t)}{d\beta_i d\beta_j} = -\frac{2(1+t(1-2w))^2(\tau-(1-t)\beta_A)(\tau-(1-t)\beta_B)}{9(2\tau-(1-t)(\beta_A+\beta_B))^3} < 0.$$

□

2.8.2.7 Proof of Proposition 3

Proof. The first-order conditions of the net profits evaluated at $\beta = \beta_A = \beta_B$ yield the equilibrium condition as specified in the proposition. Note that the second-order condition is satisfied because of Assumption 3. Furthermore, Assumption 3 makes the net profit functions strictly concave. So, the equilibrium is unique. □

2.8.2.8 Proof of Corollary 3

Proof. Using the implicit function theorem on Equation 2.16, the derivatives of the equilibrium qualities β^* with respect to interoperability t and the weight w, respectively, are

$$\begin{aligned} \frac{d\beta_i^*}{dt} &= \frac{\frac{1}{3}\left(\frac{5}{4} - w\right)}{I''(\beta_i^*)} > 0, \\ \frac{d\beta_i^*}{dw} &= -\frac{t/3}{I''(\beta_i^*)} < 0. \end{aligned}$$

□

2.8.2.9 Proof of Proposition 4

Proof. The first-order condition of Equation 2.17 yields the condition in the proposition. The second-order condition is also satisfied as the second-order derivative is $-2I''(\beta) < 0$. □

2.8.2.10 Proof of Proposition 5

Proof. Recall Corollary 3, the highest equilibrium qualities $\beta^*$for each interoperability level t is when the weight w is zero. Similarly to the first part, it can be shown that the right-hand side of Equation 2.5 given $w = 0$ is always lower than the right-hand side of Equation 2.18 for all $t \leq 1$. □

2.8.2.11 Proof of Proposition 6

Proof. If $W^d(\beta^*(t), \beta^*(t); t)/dt\big|_{t=1} < 0$, lowering the interoperability level from one improves social welfare. Using Equation 2.19, this happens when

$$I'(\beta^*(1)) > \frac{1}{2} + \left(\frac{d\beta^*(t)}{dt}\right)^{-1} \frac{\beta^*(1)}{2}. \tag{2.23}$$

Under the specification for information interoperability, we have

$$\frac{d\beta^*(t)}{dt} = \frac{1}{12}\left(\frac{5-3w}{I''(\beta^*(t))}\right),$$

which proves the the proposition. □

2.9 Appendix: supplementary analysis for Proposition 1

This appendix supplements the proof of Proposition 1 and solves for SPE in pure strategies. I solve the same model specified in Section 2.3. The equilibrium concept is subgame perfect equilibrium (SPE) in pure strategies. The game consists of two stages.

1. In the first stage, each firm simultaneously sets its price $p_i \geq 0$, where $i \in \{A, B\}$.
2. In the second stage, each consumer decides whether to join firm A, firm B, or none at all, given p_A and p_B.

I introduce two additional assumptions. First, suppose that the two following outcomes are possible: (1) a monopoly and (2) a non-covered market. Assumption 4 requires all consumers to choose the monopoly outcome. This assumption significantly reduces the complexity of calculations. I believe that Assumption 4 does not impact the results because a non-covered market can arise only when both firms set very high prices. The unique equilibrium prices (Proposition 12) cannot be such high prices.

Assumption 5 is crucial. It assumes that when firm A or firm B can be a monopolist, each consumer chooses firm A when $\beta_A \geq \beta_B$ and firm B when $\beta_B > \beta_A$. Note that this is equivalent to each consumer choosing based on adjusted quality $\beta_i - t\tilde{\beta}_j$.

I show that the equilibrium outcomes described in Proposition 1 are the unique outcomes supportable by SPE that satisfy Assumptions 4 and 5. Without Assumption 5, there are many equilibrium prices supportable by SPE with a duopoly (Lemma 9) or a monopoly (Lemma 10).

Nash equilibria in the second stage are derived in Section 2.9.1. Then, Section 2.9.2 shows that there are infinitely many subgame perfect equilibria with different outcomes. By introducing Assumptions 4 and 5, there are still multiple subgame perfect equilibria. However, the strategies are the same along the equilibrium path, leading to the unique equilibrium outcomes supportable by SPE that satisfy Assumptions 4 and 5.

2.9.1 Nash equilibria in the second stage

This section solves for the equilibrium decisions of consumers for each combination of $\left\{p_i, \beta_i, \tilde{\beta}_i, t\right\}_{i\in\{A,B\}}$. For conciseness, consumer x refers to the consumer who is located at point x. Denote $\mathscr{P} = \left\{\beta_A, \beta_B, \tilde{\beta}_A, \tilde{\beta}_B, t\right\}$ as the set of the primitives. In Lemma 6, I discuss possible types of equilibrium outcomes.

Lemma 6. *For a given $\mathscr{P}$, any Nash equilibrium outcome in the second stage has the following properties:*

1. *If consumer x' chooses firm A in equilibrium, any consumer x whose $x < x'$ chooses firm A in equilibrium.*
2. *If consumer x'' chooses firm B in equilibrium, any consumer x whose $x'' < x$ chooses firm B in equilibrium.*

Proof. Starting with the first case, suppose $x_1' < x_2'$. If consumer x_2' chooses firm A in equilibrium, she must receive higher utility from firm A than firm B. So, for given network sizes D_A and D_B, it must be that $u_A(x_2') \geq \max\{0, u_B(x_2')\}$, i.e.,

$$1 - \tau x_2' + \beta_A D_A + \tilde{\beta}_A D_B - p_A \geq \max\left\{0, 1 - \tau\left(1 - x_2'\right) + \beta_B D_B + \tilde{\beta}_B D_A - p_B\right\}.$$

Because $x_1' < x_2'$, we have $u_A(x_1') > \max\{0, u_B(x_1')\}$, i.e.,

$$1 - \tau x_1' + \beta_A D_A + \tilde{\beta}_A D_B - p_A > \max\left\{0, 1 - \tau\left(1 - x_1'\right) + \beta_B D_B + \tilde{\beta}_B D_A - p_B\right\}.$$

Hence, any consumer $x_1' < x_2'$ must choose firm A.

The proof of the second case is similar. Suppose $x_1'' < x_2''$ and consumer x_1'' chooses firm B in equilibrium. So, $u_B(x_1'') \geq \max\{0, u_A(x_1'')\}$, i.e.,

$$1 - \tau\left(1 - x_1''\right) + \beta_B D_B + \tilde{\beta}_B D_A - p_B \geq \max\left\{0, 1 - \tau x_1'' + \beta_A D_A + \tilde{\beta}_A D_B - p_A\right\}.$$

Since $x_1'' < x_2''$, we have

$$1 - \tau\left(1 - x_2''\right) + \beta_B D_B + \tilde{\beta}_B D_A - p_B > \max\left\{0, 1 - \tau x_2'' + \beta_A D_A + \tilde{\beta}_A D_B - p_A\right\}.$$

So, consumer $x_2'' > x_1''$ must choose firm B in equilibrium. □

Note that Lemma 6 rules out an outcome such that a consumer x_1 does not choose either firm, while consumer x_2, where $x_1 < x_2$, chooses firm A. In equilibrium, consumer x_1 must choose firm A as well. However, it is possible to have an outcome such that any consumer x, where $x_1 < x < x_2$ does not choose any firm even though some consumers to the left of x_1 choose firm A, and some consumers to the right of x_2 choose firm B.

Following Lemma 6, there are four main possible types of NE outcomes in the second stage:

1. Monopoly A: Firm A receives all consumers.

2. Monopoly B: Firm B receives all consumers.

3. Duopoly d with a covered market: given $\hat{x} \in (0,1)$, Firm A receives consumers $x \leq \hat{x}$, while firm B receives consumers $x > \hat{x}$.

4. Non-covered market n: given $\hat{x}_A$ and $\hat{x}_B$, where $\hat{x}_A < \hat{x}_B$, Firm A receives consumers $x \leq \hat{x}_A$, while firm B receives consumers $x \geq \hat{x}_B$. None of the firms gets any consumers x whose $\hat{x}_A < x < \hat{x}_B$.

   (a) Duopoly with a non-covered market: $0 < \hat{x}_A < \hat{x}_B < 1$.
   (b) Monopoly A with a non-covered market: $\hat{x}_A > 0$ and $\hat{x}_B \geq 1$
   (c) Monopoly B with a non-covered market: $\hat{x}_B < 1$ and $\hat{x}_A \leq 0$.
   (d) Zero demands for both firms: $\hat{x}_A \leq 0$ and $\hat{x}_B \geq 1$.

In the second stage, the analysis has to be divided into two main areas. I denote these areas as R^H and R^L. In area R^H, the sum of the adjusted qualities $\beta_i - t\tilde{\beta}_i$ of both firms is high. In contrast, the sum of the adjusted qualities $\beta_i - t\tilde{\beta}_j$ is low in area R^L. Formally, R^H and R^L are defined in Definition 2.

Definition 2. The areas R^H and R^L are as follows.

1. $R^H = \left\{ \left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \middle| \left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) \geq 2\tau \right\}$,
2. $R^L = \left\{ \left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \middle| \left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) < 2\tau \right\}$.

To shorten notations subsequently, Definition 3 introduces what I call 'decision DE_k^r' by a consumer, where $k \in \{A, B, d, n\}$ and $r \in \{H, L\}$. A decision DE_k^r indicates *how* a consumer decides to choose either firm A, firm B, or none at all for area R^r. Each decision DE_k^r corresponds to each type of possible NE outcome discussed earlier. Note that decision DE_k^r is not a consumer strategy or a NE in the second stage. Decision DE_k simply indicates how a consumer makes a decision.

Definition 3. The definition of decision DE_k^r, where $k \in \{A, B, d, n\}$ and $r \in \{H, L\}$

1. DE_A^r: Decision DE_A^r is such that consumer x, for all $x \in [0,1]$, chooses firm A.

2. DE_B^r: Decision DE_B^r is such that consumer x, for all $x \in [0,1]$, chooses firm B.

3. DE_d^r: Decision DE_d^r is such that , for a given $\hat{x} \in (0,1)$, consumer x chooses firm A if $x \leq \hat{x}$ and firm B if $x > \hat{x}$.

4. DE_n^r: Decision DE_n^r is such that, given $\hat{x}_A$ and $\hat{x}_B$, where $\hat{x}_A < \hat{x}_B$, consumer x chooses firm A if $x \leq x_A$ or firm B if $x \geq x_B$.

(a) If $0 < \hat{x}_A < \hat{x}_B < 0$, there exist consumers who join A, B, or none of the firms.

(b) If $\hat{x}_B \geq 0$ and $\hat{x}_A > 0$: Monopoly A with a non-covered market

(c) If $\hat{x}_A \leq 0$ and $\hat{x}_B < 1$: Monopoly B with a non-covered market

(d) If $\hat{x}_A \leq 0$ and $\hat{x}_B \geq 1$: zero demands for both firms

In what follows, I will show that each consumer does not have an incentive to deviate from following decision DE_k^r when all consumers follow DE_k^r under a certain region of parameters $\left\{p_i, \beta_i, \tilde{\beta}_i, t\right\}_{i \in \{A,B\}}$. In particular, I will find a certain region of prices (p_A, p_B) given the primitives $\mathscr{P}$ such that each player choosing DE_k^r is a best response when all consumers choose DE_k^r in this region. This will give us a NE for each price pair (p_A, p_B) given $\mathscr{P}$. Different price pairs (p_A, p_B) may lead to different DE_k^r being the best-response.

Before that, I make Assumption 4 which states that each consumer chooses a monopoly over a non-covered market.

Assumption 4. *For any set of the primitives $\mathscr{P}$,*

1. *If p_A and p_B are such that a monopoly by firm A, a monopoly by firm B, and a non-covered market n are possible Nash equilibrium outcomes in the second stage, then each consumer chooses the same monopoly outcome by firm $i \in \{A,B\}$.*

2. *If p_A and p_B are such that a monopoly by firm $i \in \{A,B\}$ and a non-covered market n are possible Nash equilibrium outcomes in the second stage, then each consumer chooses the monopoly outcome by firm $i \in \{A,B\}$.*

Note that a monopoly with a non-covered market falls under a non-covered market n. A justification of Assumption 4 is as follows. Under a non-covered market, there is a positive mass of consumers who receive zero utility. In contrast, only the last consumer with a zero mass (e.g., $x = 1$ if firm A is the monopolist) gets zero utility. All other consumers receive positive utility. Furthermore, this assumption significantly reduces the complexity of the analysis.

Let $Z_k^r(\mathscr{P}) \subseteq [0,\infty] \times [0,\infty]$ be a set of prices (p_A, p_B), $k \in \{A,B,d,n\}$ and $r \in \{H,L\}$, which depends on the primitives $\mathscr{P} = \left\{\beta_i, \tilde{\beta}_i, t\right\}_{i \in \{A,B\}}$. When $(p_A, p_B) \in Z_k^r(\mathscr{P})$, none of the consumers has an incentive to deviate from decision DE_k^r when all consumers follow DE_k^r. Lemma 7 summarizes the set $Z_k^H(\mathscr{P})$ for each DE_k^r for area R^H. Similarly, Lemma 8 finds $Z_k^L(\mathscr{P})$ for each decision DE_k^L for area R^L.

Note that I find all possible price pairs (p_A, p_B) for a given $\mathscr{P}$ such that DE_k^r is the best response for each consumer when all consumers follow DE_k^r for $k \in \{A,B,d\}$. In other words, for $k \in \{A,B,d\}$, some consumers have an incentive to deviate from DE_k^r for any price pair $(p_A, p_B) \notin Z_k^r(\mathscr{P})$.

However, Assumption 4 allows me to restrict $Z_n^r(\mathscr{P})$ to be the set of prices that are not in $Z_A^r(\mathscr{P})$ or $Z_B^r(\mathscr{P})$. Furthermore, a non-covered market n and a duopoly d with a covered market are two opposite cases. So, $Z_n^r(\mathscr{P})$ cannot overlap with $Z_d^r(\mathscr{P})$. Consequently, Assumption 4 allows the construction of $Z_n^r(\mathscr{P})$ to be such that

$$Z_n^r(\mathscr{P}) = [0,\infty] \times [0,\infty] \setminus (Z_A^r \cup Z_B^r \cup Z_d^r).$$

In other words, I do not find all possible price pairs (p_A, p_B) that DE_n^r is applicable. Such exercise is complex.[22] Instead, I restrict $Z_n^r(\mathscr{P})$ to be the set of prices that are not in other sets $Z_k^r(\mathscr{P})$, where $k \in \{A,B,d\}$. So, a monopoly or a duopoly with a covered market cannot arise in equilibrium for any price pair $(p_A, p_B) \in Z_n^k(\mathscr{P})$.

I believe that the construction of $Z_n^r(\mathscr{P})$ in this way does not affect the equilibrium outcomes. Recall Assumption 1 which restricts the degree of horizontal differentiation τ to be small. Thus, a non-covered market requires the prices of both firms to be very high. Because of Assumption 1, the market is covered when we have a duopoly in equilibrium. So, prices that lead to a non-covered market can never be equilibrium outcomes.

A monopoly outcome may also arise when the monopolist's adjusted quality is large. So, the prices of both firms must be extremely high to overcome high network effects to create a non-covered market. Additionally, the equilibrium SPE outcome with a monopoly is that one firm is a monopolist who covers the whole market, and the inactive firm sets a zero price. Because the price of the inactive firm is zero, Assumption 4 that deals with a non-covered market with high prices is highly unlikely to affect the equilibrium outcomes.

Lemma 7 summarizes a *specific region* $Z_k^H(\mathscr{P})$ of parameters such that DE_k^H is the best response of each consumer when all consumers follow DE_k^r for area R^H. Similarly, Lemma 8 finds a specific region $Z_k^L(\mathscr{P})$ for each decision DE_k^L for area R^L.

Nash equilibria in the second stage for area R^H and R^L can be constructed from Lemmas 7 and 8, respectively. By construction, $Z_n^r(\mathscr{P})$ does not overlap with the other sets. So, Nash equilibria can be constructed as follows.

- When the parameters are in the region such that $Z_A^r(\mathscr{P}) \cap Z_B^r(\mathscr{P}) \cap Z_d^r(\mathscr{P}) \neq \emptyset$, each and every consumer follows DE_k^r, where $DE_k^r \in \{DE_A^r, DE_B^r, DE_d^r\}$,
- When the parameters are in the region such that $\left(Z_{k_1}^r(\mathscr{P}) \cap Z_{k_2}^r(\mathscr{P})\right) \setminus Z_{k_3}^r(\mathscr{P}) \neq \emptyset$, each and every consumer follows DE_k^r, $DE_k^r \in \{DE_{k_1}^r, DE_{k_2}^r\}$, where $k_1,k_2,k_3 \in (A,B,d)$ and $k_1 \neq k_2 \neq k_3$.
- When the parameters are in the region such that $Z_{k_1}^r(\mathscr{P}) \setminus \left(Z_{k_2}^r(\mathscr{P}) \cup Z_{k_3}^r(\mathscr{P})\right) \neq \emptyset$, each and every consumer follows $DE_{k_1}^r$, where $k_1,k_2,k_3 \in (A,B,d)$ and $k_1 \neq k_2 \neq k_3$.
- When the parameters are in $Z_n^r(\mathscr{P})$, each and every consumer follows DE_n^r.

It is possible that the sets $Z_k^r(\mathscr{P}) \in \{Z_A^r(\mathscr{P}), Z_B^r(\mathscr{P}), Z_d^r(\mathscr{P})\}$ overlap with each other. Then, we have multiple Nash equilibria. Because of Assumption 4, Nash equilibria following the above characterization are a subset of all Nash equilibria.

Lemma 7 summarizes the set of parameters Z_k^H, where $k \in \{A,B,d,n\}$, when each decision D_k^H is applicable. Please refer to Figure 2.5 for visualizations of these sets Z_k^H.

Lemma 7. *In area R^H, i.e., $\left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) \geq 2\tau$, we have the following.*

1. *Decision DE_i^H, where $i \in \{A,B\}$ (each consumer chooses firm i):*

   Suppose $(p_A, p_B) \in Z_i^H(\mathscr{P})$, for $i \in \{A,B\}$, where

[22] The scenarios that have to be covered are (1) $\hat{x}_A < \hat{x}_B$, $\hat{x}_A > 0$, and $\hat{x}_B < 1$, (2) $\hat{x}_A \leq 0$ and $0 < \hat{x}_B < 1$, (3) $\hat{x}_B > 1$ and $0 < \hat{x}_A < 1$, and (4) $\hat{x}_A \leq 0$ and $\hat{x}_B \geq 1$. In each of these four cases, there are sub-cases on whether a certain combination of parameters is positive or negative.

$$Z_i^H(\mathscr{P}) = \left\{ (p_A, p_B) | \, p_i \le \max\left\{1, p_j - t\tilde{\beta}_j\right\} + \beta_i - \tau, \text{ where } i \in \{A, B\} \right\}.$$

When $(p_A, p_B) \in Z_i^H(\mathscr{P})$, each consumer does not have an incentive to deviate from DE_i^H when all consumers follow DE_i^H.

2. *Decision DE_d^H ($x \le \hat{x}$ chooses A, and $x > \hat{x}$ chooses B):*

   Suppose $(p_A, p_B) \in Z_d^H(\mathscr{P})$, where

$$Z_d^H(\mathscr{P}) = \left\{ \begin{array}{r} (p_A, p_B) | \, p_i < p_j + \beta_i - t\tilde{\beta}_j - \tau \text{ for both } i \in \{A, B\}, i \ne j, \text{ and} \\ \left(\beta_B - t\tilde{\beta}_B - \tau\right) p_A + \left(\beta_A - t\tilde{\beta}_A - \tau\right) p_B \le \\ \left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) - 2\tau + (\beta_A - \tau)(\beta_B - \tau) - \tilde{\beta}_A\tilde{\beta}_B t^2 \end{array} \right\}. \quad (2.24)$$

   Then, there exists $\hat{x} \in (0,1)$, where

$$\hat{x} = \frac{\beta_B - t\tilde{\beta}_A - \tau + p_A - p_B}{\left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) - 2\tau}. \quad (2.25)$$

   When $(p_A, p_B) \in Z_d^H(\mathscr{P})$, each consumer does not have an incentive to deviate from DE_d^H when all consumers follow DE_d^H.

3. *Decision DE_n^H (non-covered market):*

   Suppose $(p_A, p_B) \in Z_n^H(\mathscr{P})$, where

$$Z_n^H(\mathscr{P}) = \left\{ (p_A, p_B) | \, p_i > 1 + \beta_i - \tau \text{ and } p_j > \min\left\{1 + \beta_j - \tau, p_i + \left(\beta_j - t\tilde{\beta}_i - \tau\right)\right\}, \text{ where } i \in \{A, B\} \right\}. \quad (2.26)$$

   There exist $\hat{x}_A$ and $\hat{x}_B$, such that

$$\hat{x}_A = \frac{(\tau - \beta_B)(1 - p_A) + t\tilde{\beta}_A(1 - p_B)}{\tau(\tau - \beta_A - \beta_B) + \beta_A\beta_B - \tilde{\beta}_A\tilde{\beta}_B t^2},$$
$$\hat{x}_B = 1 - \frac{(\tau - \beta_A)(1 - p_B) + t\tilde{\beta}_B(1 - p_A)}{\tau(\tau - \beta_A - \beta_B) + \beta_A\beta_B - \tilde{\beta}_A\tilde{\beta}_B t^2},$$

   where $\hat{x}_A < \hat{x}_B$. Then, decision DE_n^H is such that consumer x chooses firm A if $x \le \hat{x}_A$, and she chooses firm B if $x \ge \hat{x}_B$. Consumer x does not buy from any firm if $\hat{x}_A < x < \hat{x}_B$.

Proof. 1) Let's begin with DE_i^H, where $i \in \{A, B\}$. All consumers join firm i. Without loss of generality, let's focus on the case where firm A is the monopolist. For all consumers to join firm A, two conditions must be satisfied. First, the last consumer ($x = 1$) must receive higher or equal utility from firm A than firm B, provided that all consumers join firm A. Second, the last consumer ($x = 1$) must receive positive utility. Combining the two conditions yields

$$1 - \tau + \beta_A - p_A \ge \max\left\{0, 1 + t\tilde{\beta}_B - p_B\right\}.$$

This is equivalent to

$$p_A \leq \max\left\{1, p_B - t\tilde{\beta}_B\right\} + \beta_A - \tau,$$

as stated in the lemma.

2) For DE_d^H, we have consumer x joins firm A if $x \leq \hat{x}$ or firm B if $x > \hat{x}$. In other words, $\hat{x}$ is the location of the marginal consumer who is indifferent between joining firm A and firm B, i.e.,

$$u_A(\hat{x}) = u_B(\hat{x}).$$

Solving the above equation yields $\hat{x}$ as stated in Equation (2.25). Any consumer whose location x is to the left of $\hat{x}$ strictly prefers firm A. In contrast, any consumer to the right of $\hat{x}$ prefers firm B.

In addition, for the marginal consumer $\hat{x}$ to exist, we need the utility of the marginal consumer to be greater or equal to zero, i.e., $u_A(\hat{x}) = u_B(\hat{x}) \geq 0$. This condition is equivalent to Equation (2.24) as stated in the lemma.

3) For DE_n^H, notice that the prices of both firms are high when $(p_A, p_B) \in Z_n^H(\mathscr{P})$. Following Assumption 4, $Z_n^H(\mathscr{P})$ is constructed using

$$Z_n^H(\mathscr{P}) = [0,\infty] \times [0,\infty] \setminus \left(Z_A^H \cup Z_B^H \cup Z_d^H\right).$$

An outcome where the market is covered with a duopoly or a monopoly by one firm cannot happen. So, we have the case with a non-covered market. Consequently, there exist $\hat{x}_A$ and $\hat{x}_B$, where $\hat{x}_A < \hat{x}_B$, such that $u_A(\hat{x}_A) = 0$ and $u_B(\hat{x}_B) = 0$. Any consumer whose $x \leq \hat{x}_A$ joins firm A, and any consumer whose $x \geq \hat{x}_B$ joins firm B. So, we have

$$u_A(\hat{x}_A) = 1 - \tau\hat{x}_A + \beta_A\hat{x}_A + t\tilde{\beta}_A(1-\hat{x}_B) - p_A = 0, \tag{2.27}$$

$$u_B(\hat{x}_B) = 1 - \tau(1-\hat{x}_B) + \beta_B(1-\hat{x}_B) + t\tilde{\beta}_B\hat{x}_A - p_B = 0. \tag{2.28}$$

Solving Equations (2.27) and (2.28) simultaneously yields $\hat{x}_A$ and $\hat{x}_B$ as stated in the lemma. □

To plot all of the sets $Z_k^H(\mathscr{P})$ on an $[0,\infty] \times [0,\infty]$ plane, four sub-cases have to be divided for area R^H. Table 2.2 summarizes these four sub-cases. They are the combinations of the following situations:

1. Two cases with $\beta_j - t\tilde{\beta}_j \geq \tau$ or $\beta_j - t\tilde{\beta}_j < \tau$, while $\beta_i - t\tilde{\beta}_i \geq \tau$.
   (a) Notice the same subscripts for β_j and $\tilde{\beta}_j$.
   (b) If $\beta_j - t\tilde{\beta}_j \geq 0$, the line representing $u_A(\hat{x}) = u_B(\hat{x}) = 0$ is downward sloping [Cases 1 and 2], and vice versa [Cases 3 and 4].
2. Two cases $\beta_j - t\tilde{\beta}_i \geq \tau$ or $\beta_j - t\tilde{\beta}_i < \tau$, while $\beta_i - t\tilde{\beta}_j \geq \tau$.
   (a) Notice the different subscripts for β_j and $\tilde{\beta}_i$.
   (b) If $\beta_j - t\tilde{\beta}_i \geq \tau$, the line $p_j = p_i + \left(\beta_j - t\tilde{\beta}_i - \tau\right)$ (the above upward parallel line) cuts the y-axis above zero [Cases 1 and 3], and vice versa [Cases 2 and 4].

Table 2.2: Four sub-cases for area R^H

	$\beta_i - t\tilde{\beta}_j \geq \tau$, and $\beta_j - t\tilde{\beta}_i \geq \tau$	$\beta_i - t\tilde{\beta}_j \geq \tau$, and $\beta_j - t\tilde{\beta}_i < \tau$
$\beta_i - t\tilde{\beta}_i \geq \tau$, and $\beta_j - t\tilde{\beta}_j \geq \tau$	Case 1	Case 2
$\beta_i - t\tilde{\beta}_i \geq \tau$, and $\beta_j - t\tilde{\beta}_j < \tau$	Case 3	Case 4

These four sub-cases have different regions from each other. See Figure 2.5 for illustrations of these regions Z_k^H for these four cases. These figures are based on $\beta_A - t\tilde{\beta}_B > \beta_B - t\tilde{\beta}_A$. Note that the argument $(\mathscr{P})$ is dropped from $Z_k^H(\mathscr{P})$ in the figures. The horizontal and vertical axes represent p_A and p_B, respectively.

Having constructed the region $Z_k^H(\mathscr{P})$ for which none of the consumers has an incentive to deviate from decision DE_k^H when all consumers follow DE_k^H, Nash equilibria in the second stage can be constructed as argued earlier. Corollary 9 summarizes Nash equilibria for area R^H.

Corollary 9. *Nash equilibria for area R^H, i.e., $\left(\beta_A - t\tilde{\beta}_B, \beta_A - t\tilde{\beta}_B\right) \in R^H$ are as follows.*

1. *When $(p_A, p_B) \in \bar{Z}_{ABd}^H(\mathscr{P})$, where*

$$\bar{Z}_{ABd}^H(\mathscr{P}) = Z_A^H(\mathscr{P}) \cap Z_B^H(\mathscr{P}) \cap Z_d^H(\mathscr{P}), \tag{2.29}$$

   the Nash equilibria are each consumer choosing DE_A^H, each consumer choosing DE_B^H, and each consumer choosing DE_d^H.

2. *When $(p_A, p_B) \in \bar{Z}_{k'k''}^H(\mathscr{P})$, where*

$$\bar{Z}_{k'k''}^H(\mathscr{P}) = \left(Z_{k'}^H(\mathscr{P}) \cap Z_{k''}^H(\mathscr{P})\right) \backslash Z_{k'''}^H(\mathscr{P}), \tag{2.30}$$

   for $k', k'', k''' \in (A, B, d)$ and $k' \neq k'' \neq k'''$, the Nash equilibria are each consumer choosing $DE_{k'}^H$ and each consumer choosing $DE_{k''}^H$, where $k', k'' \in (A, B, d)$ and $k' \neq k''$.

3. *When $(p_A, p_B) \in \bar{Z}_{k'}^H(\mathscr{P})$, where*

$$\bar{Z}_{k'}^H(\mathscr{P}) = Z_{k'}^H(\mathscr{P}) \backslash \left(Z_{k''}^H(\mathscr{P}) \cup Z_{k'''}^H(\mathscr{P})\right), \tag{2.31}$$

   for $k', k'', k''' \in (A, B, d)$ and $k' \neq k'' \neq k'''$, the NE is each consumer choosing $DE_{k'}^H$, where $k' \in (A, B, d)$.

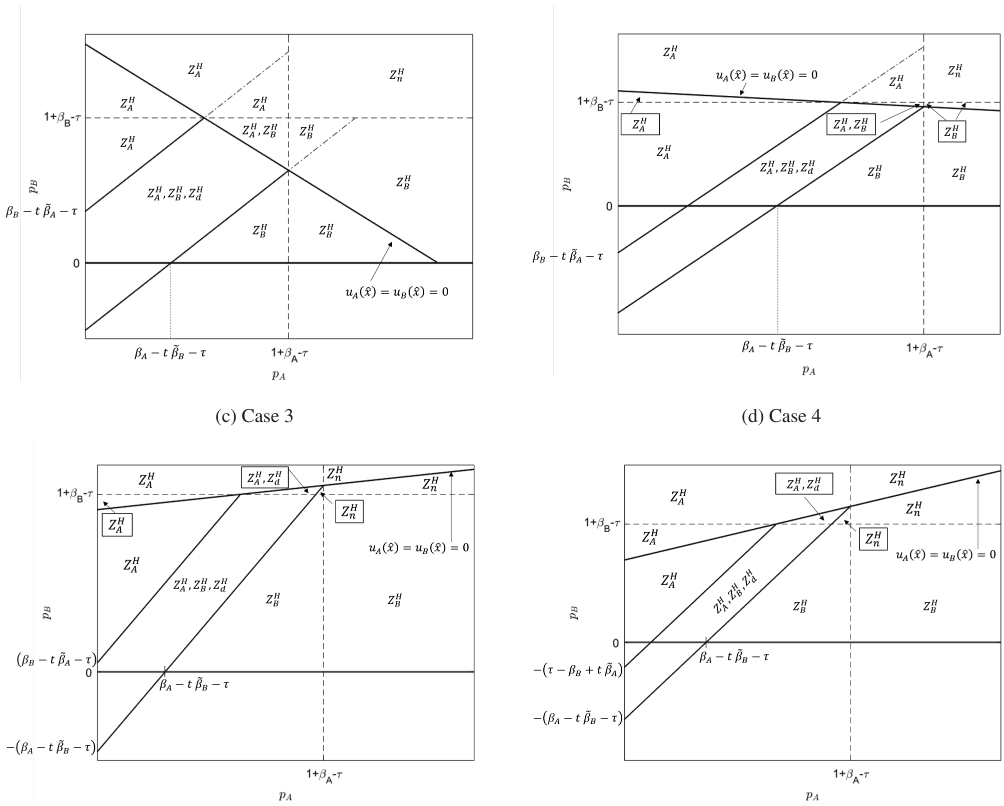
4. *When $(p_A, p_B) \in Z_n^H(\mathscr{P})$, defined in Equation (2.26), the NE is each consumer choosing DE_n^H.*

There are multiple Nash equilibria for area R^H. For example, in all Figures 2.5a - 2.5d, there is a region $\bar{Z}_{ABd}^H(\mathscr{P}) \neq \emptyset$ with three equilibria: each consumer choosing DE_A^H, each consumer choosing DE_B^H, and each consumer choosing DE_d^H. As shown in Lemmas 9 and 10, this multiplicity leads to infinitely many subgame perfect equilibria in pure strategies without further assumption.

Furthermore, some sets $\bar{Z}_{k'k''}^H(\mathscr{P})$ and $\bar{Z}_{k'}^H(\mathscr{P})$ are empty in all four sub-cases, such as $\bar{Z}_d^H(\mathscr{P})$. There is no region such that only decision $DE_d^H(\mathscr{P})$ is applicable in that region. Some sets $\bar{Z}_k^H(\mathscr{P})$ are

Figure 2.5: The regions for Z_k^H for area R^H

(a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4



non-empty in certain sub-cases, but they are empty in others. For example, the set $\bar{Z}_{AB}^H(\mathscr{P})$ is non-empty in Cases 1 and 2, but it is empty in Cases 3 and 4.

Moving to area R^L, Lemma 8 derives the set of parameters Z_k^L such that each consumer does not have an incentive to deviate from decision DE_k^L when all consumers follow DE_k^L. See also Figure 2.6 for an illustration of these sets.

Lemma 8. *In area R^L, i.e., $\left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) < 2\tau$, we have the following.*

1. *Decision DE_i^L, where $i \in \{A,B\}$ (all consumers choose firm i):*

   Suppose $(p_A, p_B) \in Z_i^L(\mathscr{P})$, for $i \in \{A,B\}$, where

   $$Z_i^L(\mathscr{P}) = \left\{ (p_A, p_B) |\, p_i \leq \max\left\{1, p_j - t\tilde{\beta}_j\right\} + \beta_i - \tau, \text{ where } i \in \{A,B\} \right\}.$$

   Then, if all consumers follow decision DE_i^L, where $i \in \{A,B\}$, none of the consumers has an incentive to deviate from DE_i^L.

2. *Decision DE_d^L ($x \leq \hat{x}$ chooses A, and $x > \hat{x}$ chooses B):*

Suppose $(p_A, p_B) \in Z_d^L(\mathscr{P})$, *where*

$$Z_d^L(\mathscr{P}) = \left\{ (p_A, p_B) | \, p_i > p_j - \left(\tau - \beta_i + t\tilde{\beta}_j\right) \text{ for both } i \in \{A,B\}, i \neq j, \text{ and } \right.$$
$$\left. \left(\tau - \beta_B + t\tilde{\beta}_B\right) p_A + \left(\tau - \beta_A + t\tilde{\beta}_A\right) p_B \leq 2\tau - \left(\beta_A - t\tilde{\beta}_B\right) - \left(\beta_B - t\tilde{\beta}_A\right) - (\tau - \beta_A)(\tau - \beta_B) + \tilde{\beta}_A \tilde{\beta}_B t^2 \right\}. \quad (2.32)$$

Then, there exists $\hat{x} \in (0,1)$, *where*

$$\hat{x} = \frac{\tau - \beta_B + t\tilde{\beta}_A - p_A + p_B}{2\tau - \left(\beta_A - t\tilde{\beta}_B\right) - \left(\beta_B - t\tilde{\beta}_A\right)}. \quad (2.33)$$

If all consumers follow decision DE_d^L, *none of the consumers has an incentive to deviate from* DE_d^L.

3. *Decision* DE_n^L *(*$x \leq \hat{x}_A$ *chooses A, and* $x \geq \hat{x}_B$ *chooses B):*

   Suppose $(p_A, p_B) \in Z_d^H(\mathscr{P})$, *where*

$$Z_n^L(\mathscr{P}) = \{ (p_A, p_B) | \, p_i > 1 + \beta_i - \tau, \text{ for } i \in \{A,B\}, \text{ and Equation (2.32) does not hold}\}$$
$$\cup \left\{ (p_A, p_B) | \, p_i > 1 + \beta_i - \tau \text{ and } p_j > p_i + \left(\tau - \beta_i + t\tilde{\beta}_j\right), \text{ where } i \in \{A,B\} \right\}.$$

   There exist $\hat{x}_A$ *and* $\hat{x}_B$, *such that*

$$\hat{x}_A = \frac{(\tau - \beta_B)(1 - p_A) + t\tilde{\beta}_A (1 - p_B)}{\tau(\tau - \beta_A - \beta_B) + \beta_A \beta_B - \tilde{\beta}_A \tilde{\beta}_B t^2},$$
$$\hat{x}_B = 1 - \frac{(\tau - \beta_A)(1 - p_B) + t\tilde{\beta}_B (1 - p_A)}{\tau(\tau - \beta_A - \beta_B) + \beta_A \beta_B - \tilde{\beta}_A \tilde{\beta}_B t^2},$$

   where $\hat{x}_A < \hat{x}_B$. *Then, decision* DE_n^L *is such that consumer x chooses firm A if* $x \leq \hat{x}_A$, *and she chooses firm B if* $x \geq \hat{x}_B$. *Consumer x does not buy from any firm if* $\hat{x}_A < x < \hat{x}_B$.

Each of the sets $Z_A^L(\mathscr{P})$, $Z_B^L(\mathscr{P})$, $Z_d^L(\mathscr{P})$, *and* $Z_n^L(\mathscr{P})$ *does not overlap with each other. Specifically,* $Z_{k_1}^L(\mathscr{P}) \cap Z_{k_2}^L(\mathscr{P}) = \emptyset$ *for any* $k_1, k_2 \in \left\{Z_A^L(\mathscr{P}), Z_B^L(\mathscr{P}), Z_d^L(\mathscr{P}), Z_n^L(\mathscr{P})\right\}$ *and* $k_1 \neq k_2$.

Proof. (1) The proof for DE_i^L and the corresponding set $Z_i^L(\mathscr{P})$ is identical to the proof for DE_i^H and $Z_i^H(\mathscr{P})$ in Lemma 7.

(2) For DE_d^L, $\hat{x}$ must satisfy $u_A(\hat{x}) = u_B(\hat{x}) \geq 0$. Solving $u_A(\hat{x}) = u_B(\hat{x})$ yields $\hat{x}$ as stated in Equation (2.33). The condition specified in Equation (2.32) makes sure that the marginal consumer receives positive utility.

Furthermore, for the duopoly to exist, we need $0 < \hat{x} < 1$. This is equivalent to $p_i > p_j - \left(\tau - \beta_i + t\tilde{\beta}_j\right)$ for both $i \in \{A,B\}, i \neq j$, as stated in the lemma.

(3) Finally, when $(p_A, p_B) \in Z_n^L(\mathscr{P})$, an outcome where the market is covered with a duopoly or a monopoly by one firm cannot happen. So, we have the case with a non-covered market. Following Assumption 4, the set $Z_n^L(\mathscr{P})$ is constructed by

$$Z_n^r(\mathscr{P}) = [0,\infty] \times [0,\infty] \setminus \left(Z_A^L(\mathscr{P}) \cup Z_B^L(\mathscr{P}) \cup Z_d^L(\mathscr{P})\right).$$

Consequently, there exist $\hat{x}_A$ and $\hat{x}_B$, where $\hat{x}_A < \hat{x}_B$, such that $u_A(\hat{x}_A) = 0$ and $u_B(\hat{x}_B) = 0$. Any consumer whose $x \leq \hat{x}_A$ joins firm A, and any consumer whose $x \geq \hat{x}_B$ joins firm B. So, we have

$$u_A(\hat{x}_A) = 1 - \tau\hat{x}_A + \beta_A\hat{x}_A + t\tilde{\beta}_A(1-\hat{x}_B) - p_A = 0, \tag{2.34}$$

$$u_B(\hat{x}_B) = 1 - \tau(1-\hat{x}_B) + \beta_B(1-\hat{x}_B) + t\tilde{\beta}_B\hat{x}_A - p_B = 0. \tag{2.35}$$

Solving Equations (2.34) and (2.35) simultaneously yields $\hat{x}_A$ and $\hat{x}_B$ as stated in the lemma.

The last step is to show that the sets $Z_A^L(\mathscr{P})$, $Z_B^L(\mathscr{P})$, $Z_d^L(\mathscr{P})$, and $Z_n^L(\mathscr{P})$ do not overlap with each other. First, $Z_n^L(\mathscr{P})$ does not overlap by construction. The conditions for each of the remaining sets are

$$\begin{aligned}
Z_d^L(\mathscr{P}) &: p_A > p_B - \left(\tau - \beta_A + t\tilde{\beta}_B\right) \text{ and } p_B > p_A - \left(\tau - \beta_B + t\tilde{\beta}_A\right), \\
Z_A^L(\mathscr{P}) &: p_A \leq p_B - \left(\tau - \beta_A + t\tilde{\beta}_B\right), \\
Z_B^L(\mathscr{P}) &: p_B \leq p_A - \left(\tau - \beta_B + t\tilde{\beta}_A\right).
\end{aligned}$$

It is clear that the conditions for $Z_d^L(\mathscr{P})$ contradict with the conditions for $Z_A^L(\mathscr{P})$ and $Z_B^L(\mathscr{P})$. Hence, $Z_d^L(\mathscr{P}) \cap Z_i^L(\mathscr{P}) = \emptyset$ for both $i \in \{A,B\}$. The last step is to show that $Z_A^L(\mathscr{P}) \cap Z_B^L(\mathscr{P}) = \emptyset$. The condition for $Z_B^L(\mathscr{P})$ can be written as $p_A \geq p_B + \left(\tau - \beta_B + t\tilde{\beta}_A\right)$. Combining the preceding inequality with the condition for $Z_A^L(\mathscr{P})$ yields $\left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) \geq 2\tau$ which contradicts the condition for area R^L. Thus, $Z_A^L(\mathscr{P}) \cap Z_B^L(\mathscr{P}) = \emptyset$. □

In Figure 2.6, I illustrate the regions where the sets $Z_A^L(\mathscr{P})$, $Z_B^L(\mathscr{P})$, $Z_d^L(\mathscr{P})$, and $Z_n^L(\mathscr{P})$ are applicable. Because these sets do not overlap, only one Decision D_k^r is possible for each pair of (p_A, p_B) when $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^L$. So, there is a unique NE for each price pair (p_A, p_B) for area R^L.

Nevertheless, there are also four sub-cases for area R^L, as summarized in Table 2.3. They are the combinations of the following conditions.

1. Two cases with $\beta_j - t\tilde{\beta}_j \geq \tau$ or $\beta_j - t\tilde{\beta}_j < \tau$, while $\beta_i - t\tilde{\beta}_i \geq \tau$.
   (a) This division is similar to area R^H.
   (b) If $\beta_j - t\tilde{\beta}_j \geq 0$, the line representing $u_A(\hat{x}) = u_B(\hat{x}) = 0$ is downward sloping [Cases 1 and 3], and vice versa [Cases 2 and 4].
2. Two cases for $2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) < 3\tau$ and $2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) > 3\tau$
   (a) These two cases do not look very different visually when the sets Z_k^L are plotted in Figure 2.6.
   (b) However, they have different outcomes in equilibrium. In the former, the equilibrium outcome for the overall game is a duopoly. In the latter, we have a monopoly as an SPE outcome.

Table 2.3: Four sub-cases for area R^L

	$\tau \geq \beta_i - t\tilde{\beta}_i$, and $\tau \geq \beta_j - t\tilde{\beta}_j \geq$	$\tau \geq \beta_i - t\tilde{\beta}_i$, and $\tau < \beta_j - t\tilde{\beta}_j$
$2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) < 3\tau$	Case 1	Case 2
$2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) \geq 3\tau$	Case 3	Case 4

Using Lemma 8, I summarize the Nash equilibrium in the second stage for area $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^L$ in Corollary 10. Because the sets $Z_A^L(\mathscr{P}), Z_B^L(\mathscr{P}), Z_d^L(\mathscr{P})$, and $Z_n^L(\mathscr{P})$ do not overlap, the NE in the second stage is unique for area R^L.

Corollary 10. *The unique NE for area R^L, i.e., $\left(\beta_A - t\tilde{\beta}_B, \beta_A - t\tilde{\beta}_B\right) \in R^L$ is as follows.*

1. *When $(p_A, p_B) \in Z_i^L(\mathscr{P})$, where $i \in \{A, B\}$, each and every consumer follows DE_i^L.*

2. *When $(p_A, p_B) \in Z_d^L(\mathscr{P})$, each and every consumer follows DE_d^L.*

3. *When $(p_A, p_B) \in Z_n^L(\mathscr{P})$, each and every consumer follows DE_n^L.*

Notice that there is a region $Z_d^L(\mathscr{P})$ where the duopoly is the only NE outcome in the second stage. This allows for a possibility that a duopoly arises as a unique SPE outcome for the overall game without Assumption 5 introduced in the next section. This possibility is shown formally in Proposition 12.

2.9.2 Equilibrium price under SPE in pure strategies

In this section, I move to the first stage of the game, where each firm sets its price. One of the main conclusion is that when $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^H$, there are infinitely many price pairs (p_A, p_B) that are supportable as an SPE with a different equilibrium outcome. In Lemma 9, I show that there are infinitely many subgame perfect equilibria where both firms are active with different equilibrium duopoly prices. Lemma 10 shows that there are also numerous subgame perfect equilibria where one firm is a monopolist with different monopoly price levels.

Starting with duopoly outcomes, I focus on region R^{multi} in Figure 2.7 to show the existence of multiple equilibria. Lemma 9 states that any price pair (p_A, p_B) in R^{multi} is supportable as SPE prices.

Lemma 9. *Suppose $\beta_i - t\tilde{\beta}_j - \tau > 0$ for both $i \in \{A, B\}$. Then, any price pair (p'_A, p'_B) where $p'_i \in \left[0, \beta_i - t\tilde{\beta}_j - \tau\right]$ is supportable as an SPE with a duopoly outcome.*

Furthermore, each pair of equilibrium prices (p'_A, p'_B) are supportable by countless subgame perfect equilibria. An SPE that supports (p'_A, p'_B) is characterized by

1. *in the first stage, each firm i sets p'_i where $i \in \{A, B\}$.*

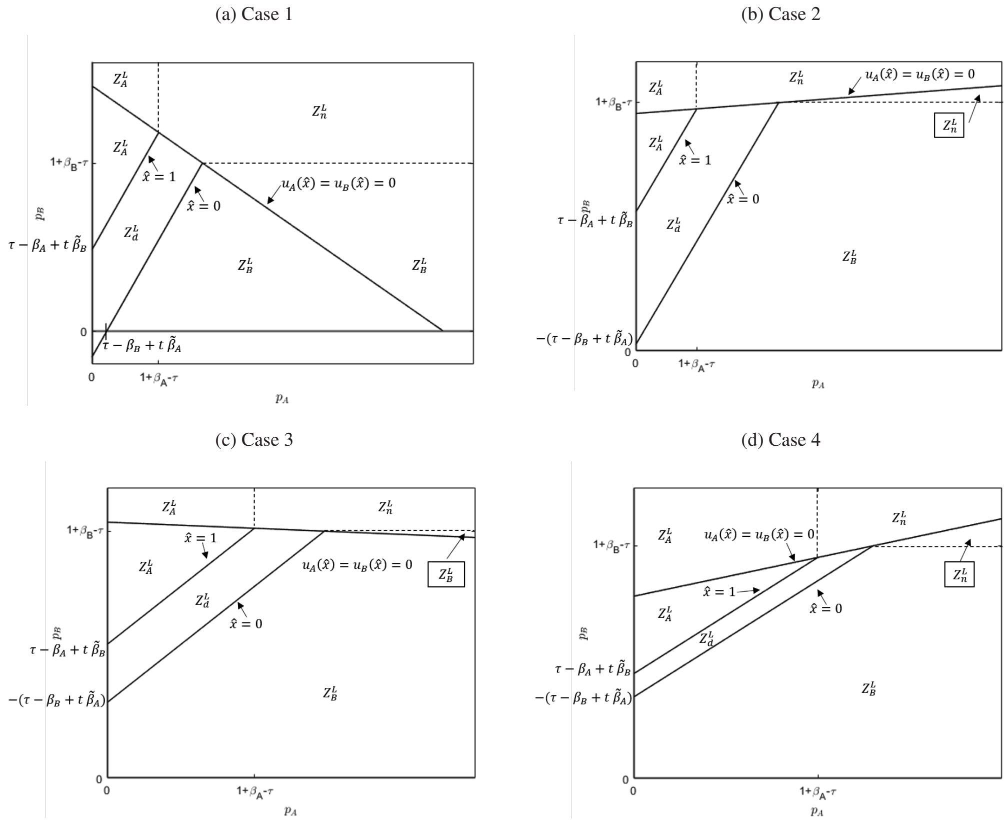
2. *In the second stage,*

   (a) *when consumer x observes (p'_A, p'_B), she chooses firm A if $x \leq \hat{x}$ and firm B if $x > \hat{x}$ $\left(DE_d^H\right)$, where $\hat{x}$ is defined in Equation (2.25).*

   (b) *When consumer x observes $p_i = p'_i$ and $p_j \neq p'_j$, she chooses firm i, for all $x \in [0,1]$ $\left(DE_i^H\right)$.*

   (c) *For any other price pair (p_A, p_B), all consumers follow DE_k^H, where $DE_k^H \in \left\{DE_A^H, DE_B^H, DE_d^H, DE_n^h\right\}$, provided that DE_k^H is applicable, i.e., $(p_A, p_B) \in Z_k^H$.*

Figure 2.6: The region Z_k^L for area R^L



Proof. According to 2.(b), if firm $j \in \{A,B\}$ unilaterally deviates from p'_j, its demand drops to zero. Therefore, each firm does not have an incentive to deviate. In the second stage, each consumer follows a Nash equilibrium. Therefore, p'_A and p'_B are equilibrium prices under SPE. In addition, many strategies can be composed that satisfy 2.(c). Hence, there are multiple equilibria. □

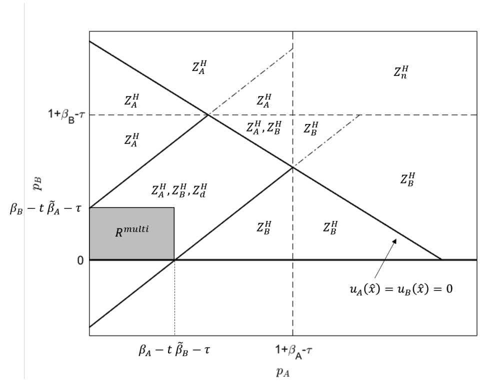
Next, I will argue that there are also infinitely many subgame perfect equilibria with various monopoly outcomes when $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^H$. For simplicity let's focus on the case where $\beta_A - t\tilde{\beta}_B > \beta_B - t\tilde{\beta}_A$, and firm A is the monopolist. Lemma 10 shows that any $p_A \leq \beta_A - t\tilde{\beta}_B - \tau$ is supportable as an SPE outcome where firm A is the monopolist.

Lemma 10. *Suppose* $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^H$ *and* $\beta_A - t\tilde{\beta}_B > \beta_B - t\tilde{\beta}_A$. *Then, any price pair* (p''_A, p''_B) *such that*

1. $p''_A < p''_B + \left(\beta_A - t\tilde{\beta}_B - \tau\right)$ *and* $p''_B < p''_A + \left(\beta_B - t\tilde{\beta}_A - \tau\right)$,
2. $p''_A \leq \beta_A - t\tilde{\beta}_B - \tau$,

is supportable as an SPE outcome where firm A is the monopolist.

Figure 2.7: Multiple equilibria with different duopoly outcomes



Each pair of equilibrium prices (p_A'', p_B'') are supportable by countless subgame perfect equilibria. An SPE that supports (p_A'', p_B'') is characterized by

1. *in the first stage, each firm i sets p_i'' where $i \in \{A, B\}$.*

2. *In the second stage,*

   (a) *when each consumer x observes (p_A'', p_B''), she chooses firm A , for all $x \in [0,1]$ $\left(DE_A^H\right)$.*

   (b) *When each consumer x observes $p_A = p_A''$ and $p_B \neq p_B''$, she chooses firm A, for all $x \in [0,1]$ $\left(DE_A^H\right)$.*

   (c) *When each consumer x observes $p_A \neq p_A''$ and $p_B = p_B''$,*

      i. *if $p_A < p_A''$, she chooses firm A, for all $x \in [0,1]$ $\left(DE_A^H\right)$.*

      ii. *If $p_A > p_A''$, she chooses firm B, for all $x \in [0,1]$ $\left(DE_B^H\right)$.*

   (d) *For any other price pair (p_A, p_B), all consumers follow DE_k^H, where $DE_k^H \in \left\{DE_A^H, DE_B^H, DE_d^H, DE_n^h\right\}$, provided that DE_k^H is applicable, i.e., $(p_A, p_B) \in Z_k^H$.*

Proof. For firm B, its equilibrium profit is zero. However, by deviating to $p_B \neq p_B''$, firm B's demand is still zero. So, there is no incentive to deviate. According to 2.(c), if firm A unilaterally deviates by choosing $p_A > p_A''$, all consumers switch to firm B. Firm A's profit drops to zero. On the other hand, if $p_A < p_A''$, firm A's demand remains the same, but it charges a lower price. So, the profit also drops. Hence, firm A does have an incentive to deviate either.

In the second stage, each consumer follows a NE strategy. Therefore, p_A'' and p_B'' are equilibrium prices under SPE. In addition, many strategies can be composed that satisfy 2.(c). Hence, there are multiple equilibria. □

According to Lemmas 9 and 10, there are infinitely many equilibria with numerous equilibrium outcomes. To avoid infinitely many SPE outcomes, I introduce Assumption 5. The assumption states that when either firm A or firm B can be a monopolist, each consumer chooses firm A if $\beta_A \geq \beta_B$. Otherwise, each consumer chooses firm B when $\beta_B > \beta_A$. Note that it is straightforward to show that $\beta_i \geq \beta_j$ if and only if $\beta_i - t\tilde{\beta}_j \geq \beta_j - t\tilde{\beta}_i$ under both information and services interoperability. So, a firm with higher quality β_i also has higher adjusted quality $\beta_i - t\tilde{\beta}_j$.

Assumption 5. *In area R^H, $\left(\beta_A - t\tilde{\beta}_B - \tau\right) + \left(\beta_B - t\tilde{\beta}_A - \tau\right) \geq 2\tau$, for any p_A and p_B such that either firm A or firm B can be the monopolist,*

1. *if $\beta_A \geq \beta_B$, each and every consumer follows DE_A^H.*
2. *If $\beta_B > \beta_A$, each and every consumer follows DE_B^H.*

Note that Assumption 5 is necessary for area R^H. But it is not necessary for area R^L. This is because there is no price pair (p_A, p_B) such that either firm A or firm B can be a monopolist in area R^L, i.e., $Z_A^L \cap Z_B^L = \emptyset$.

A justification of the assumption is that when all consumers choose a firm with higher quality β_i, social welfare is higher. This result is shown in Lemma 1. However, this does not mean that all consumers are better off. For example, suppose that both firms can be a monopolist and $\beta_A > \beta_B$. If all consumers choose firm A, the last consumer $(x = 1)$ receives zero utility. Hence, this consumer would prefer that all consumers choose firm B, so that she receives positive utility.

In Proposition 11, I show the existence of an SPE outcome satisfying Assumptions 4 and 5 for both areas R^H and R^L. Then, Proposition 12 argues that the equilibrium outcomes in Proposition 11 are unique.

Proposition 11. *There exist equilibrium outcomes supportable by SPE that satisfy Assumptions 4 and 5 as follows.*

1. *Suppose $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^H$. When $\beta_i > \beta_j$ (or $\beta_i \geq \beta_j$ when $i = A$), firm i is a monopolist ($D_i = 1$ and $D_j = 0$), while firm j is inactive. The monopolist charges the monopoly price p_i^m, where*
$$p_i^m = \beta_i - t\tilde{\beta}_j - \tau. \tag{2.36}$$
The inactive firm sets a zero price $p_j = 0$.

2. *Suppose $\left(\beta_A - t\tilde{\beta}_B, \beta_B - t\tilde{\beta}_A\right) \in R^L$.*
   (a) *When $2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) < 3\tau$, for $i \in \{A, B\}$ and $i \neq j$, both firms are active. The duopoly price p_i^d for firm i, where $i \in \{A, B\}$ and $i \neq j$, is*
$$p_i^d = \frac{3\tau - \left(\beta_i - t\tilde{\beta}_j\right) - 2\left(\beta_j - t\tilde{\beta}_i\right)}{3}, \tag{2.37}$$
   with the demand
$$D_i^d = \frac{1}{3}\left(\frac{3\tau - \left(\beta_i - t\tilde{\beta}_j\right) - 2\left(\beta_j - t\tilde{\beta}_i\right)}{2\tau - \left(\beta_i - t\tilde{\beta}_j\right) - \left(\beta_j - t\tilde{\beta}_i\right)}\right). \tag{2.38}$$
   (b) *When $2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) \geq 3\tau$, for $i \in \{A, B\}$ and $i \neq j$, firm i is a monopolist ($D_i = 1$ and $D_j = 0$). Firm i sets the monopoly price $p_i^m = \beta_i - t\tilde{\beta}_j - \tau$. Firm j is inactive with $p_j = 0$.*

Proof. I begin with the first part of the lemma (area R^H), followed by the second part (area R^L).

Part 1: R^H

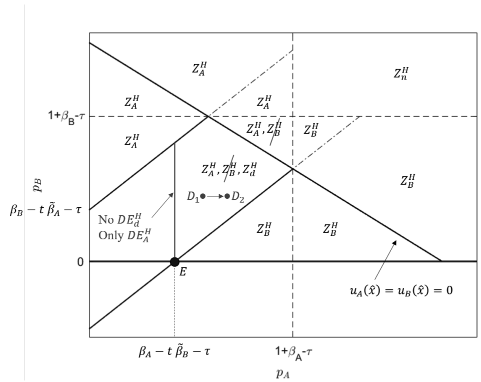
Without loss of generality, I focus on the case where $\beta_A \geq \beta_B$. Under area R^H, we have $\left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right) \geq 2\tau$. Since $\beta_A - t\tilde{\beta}_B \geq \beta_B - t\tilde{\beta}_A$ when $\beta_A \geq \beta_B$, it must be that $\beta_A - t\tilde{\beta}_B \geq \tau$.

The proof will be done in two steps. First, I argue that consumers cannot follow DE_d^H when a price pair is (p_A^m, p_B) where $p_B > 0$. The second step argues that no one has an incentive to deviate from the monopoly price p_A^m and a zero price ($p_B = 0$) by the inactive firm B.

Step 1: Suppose that there exists (p_A^m, p_B') where $p_B' > 0$ such that each consumer follows DE_d^H (both firms have positive demands). Then, firm B has an incentive to deviate from $p_B = 0$ to $p_B = p_B'$. Firm B then earns a positive profit. Therefore, consumers following DE_d^H when she observes $(p_A^m, p_B > 0)$ cannot be an equilibrium. In Figure 2.8, point E is the proposed equilibrium prices ($p_A = p_i^m, p_B = 0$). So, the first step requires that DE_d^H is not played anywhere along the vertical line starting from point E.

Note that consumers do not play DE_B^H along this vertical line as DE_B^H is ruled out by Assumption 5. With abuse of notation, Z_B^H is crossed out to represent that DE_B^H is ruled out in these regions.

Figure 2.8: Existence of an SPE for R^H



Step 2: Following step 1, when a consumer observes $(p_A^m, p_B > 0)$, she must follow DE_A^H. Firm B still receives zero demand. Hence, firm B does not have an incentive to deviate. Firm A does not have an incentive to decrease its price from p_A^m either. If it decreases its price, the profit always drops regardless of whether each consumer follows DE_d^H (lower demand and lower price) or DE_A^H (same demand but lower price). Hence, $(p_A^m, p_B = 0)$ are equilibrium prices.

<u>Part 2:</u> R^L

In the second part of the proposition (area R^L), the parameters are such that $2\tau > \left(\beta_A - t\tilde{\beta}_B\right) + \left(\beta_B - t\tilde{\beta}_A\right)$. Given $\hat{x}$, the demand function of firm i can be written as

$$D_i(p_i, p_j) = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2\tau - \left(\beta_i - t\tilde{\beta}_j\right) - \left(\beta_j - t\tilde{\beta}_i\right)} \left[\Delta_i \beta + t\Delta_i \tilde{\beta} - 2\Delta_i p\right], \tag{2.39}$$

where $\Delta_i X \equiv X_i - X_j$. Each firm maximizes its profit $\pi_i(p_i, p_j) = p_i \cdot D_i(p_i, p_j)$. Given p_j, the profit function $\pi_i(p_i, p_j)$ is strictly concave in p_i, since

$$\frac{\partial^2 \pi_i(p_i, p_j)}{\partial p_i^2} = -\frac{2}{2\tau - \left(\beta_i - t\tilde{\beta}_j\right) - \left(\beta_j - t\tilde{\beta}_i\right)} < 0.$$

Solving the first-order conditions of $\pi_A(p_A, p_B)$ with respect to p_A and $\pi_B(p_B, p_A)$ with respect to p_B yields the duopoly price p_i^d and the duopoly demand D_i^d as stated in Equations (2.37) and (2.38), respectively.

Case 2.(a): For a duopoly outcome to arise in equilibrium, the duopoly demand D_i^d must be between zero and one, i.e., $D_i^d \in (0,1)$. Taking into account the constraints, it must be that

$$2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) < 3\tau, \tag{2.40}$$

for both $i \in \{A, B\}$ and $i \neq j$.

Assumption 1 in the main text ($\tau \leq 2/3$) guarantees that the marginal consumer $\hat{x}$ exists, i.e., $u_A(\hat{x}) = u_B(\hat{x}) \geq 0$ for any parameters. Each firm does not have an incentive to deviate as long as $\hat{x} \in [0,1]$ because the profit function is strictly concave in this region. However, Firm A may raise its price so much such that $\hat{x} < 0$. But, firm A's demand becomes zero. Similarly, firm B may increase its price such that $\hat{x} > 1$, but its demand also drops to zero. Hence, there is no local or global profitable deviation. When condition (2.40) is satisfied, the duopoly price p_i^d (Equation (2.37)) is an equilibrium price under SPE in pure strategies.

Case 2.(b): In contrast, when condition (2.40) is violated, i.e., $2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) \geq 3\tau$. Firm i's demand is truncated at one. Substituting $2\left(\beta_i - t\tilde{\beta}_j\right) + \left(\beta_j - t\tilde{\beta}_i\right) = 3\tau$ into p_i^d yields the monopoly price $p_i^m = \beta_i - t\tilde{\beta}_j - \tau$, and the price of the inactive firm j is $p_j^d = 0$. If firm j raises its price, its demand is still zero. □

Under Assumptions 4 and 5, the following proposition states that there is only one equilibrium outcome under SPE in pure strategies for each combination of the parameters. When firm i's adjusted quality $\beta_i - t\tilde{\beta}_j$ is high, firm i becomes a monopolist. When the adjusted qualities of both firms are low, we have a duopoly. Proposition 12 summarizes the results.

Proposition 12. *The equilibrium prices (p_i^m and p_i^d), and the associated equilibrium demands, specified in Proposition 11 are the unique equilibrium outcomes supportable by SPE that satisfy Assumptions 4 and 5.*

Proof. First, a monopoly with a non-covered market or zero demands for both firms can never be an SPE outcome under both areas R^H and R^L. Suppose otherwise that there are equilibrium prices (p'_A, p'_B) such that firm A is the monopolist who does not cover the market. However, firm B can profitably deviate by setting its price slightly above zero. Then, firm B will have a positive demand with a positive profit. Similarly, suppose there are equilibrium prices (p'_A, p'_B) such that both firms' demands are zero. Then, either firm can deviate by setting its price slightly above zero to get a positive demand.

(1) Starting with area R^H, we must have that $(p_A^m, p_B = 0)$ is the only equilibrium price pair. First, I will argue that a duopoly cannot arise as an SPE outcome. Second, I will show that $(p_A^m, p_B = 0)$ is the only equilibrium price pair with a monopoly outcome.

Step 1: suppose otherwise that there exists a price pair (p_A^d, p_B^d) where both firms are active. However, firm A always has an incentive to increase its price, say, to p'_A, provided that

$$p'_A \leq \max\left\{1 + \beta_A - \tau, p_B^d + \left(\beta_A - t\tilde{\beta}_B - \tau\right)\right\}.$$

Observing (p'_A, p^d_B), each consumer may follow DE^H_A or DE^H_d. Note that DE^H_B is ruled out by Assumption 5.

Using Figure 2.8 as an illustration, suppose point D_1 represents the price pair (p^d_A, p^d_B). Then, firm A has an incentive to move to point D_2, where only DE^H_A and DE^H_d are applicable. However, firm A's profit increases with either DE^H_A or DE^H_d. Suppose all consumers follow DE^H_A, firm A still serves the whole market with a higher price. Suppose instead that all consumers follow DE^H_d. Then, the demand for firm A under p'_A is higher than p^d_A as $\hat{x}$ (Equation (2.25)) increases. Therefore, any duopoly outcome cannot be an equilibrium.

Step 2: suppose otherwise that there is another equilibrium price pair (p''_A, p''_B) where firm A is a monopolist. Then, firm A must set p''_A as high as possible such that it remains a monopolist. So, it must be that $p''_A = p''_B + \left(\beta_A - t\tilde{\beta}_B - \tau\right)$ (the lower parallel line in Figure 2.8). However, firm B can slightly lower its price by $\varepsilon > 0$ such that $p''_B - \varepsilon < p_A - \left(\beta_A - t\tilde{\beta}_B - \tau\right)$. Then, each consumer follows DE^H_B: firm B becomes a monopolist with a positive profit. There is a contradiction.

Hence, in area R^H where $\beta_A \geq \beta_B$. The unique equilibrium outcomes under SPE that satisfy Assumptions 4 and 5 is that firm A is a monopolist. It charges the monopoly price $p^m_A = \beta_A - t\tilde{\beta}_B - \tau$. Firm B is inactive with the price $p_B = 0$. The same arguments apply in the case where $\beta_B > \beta_A$. Then, firm B is a monopolist with $p^m_B = \beta_B - t\tilde{\beta}_A - \tau$, and firm A is inactive with $p_A = 0$.

(2) Moving to area R^L, for case 2.(a) in Proposition 12, we have the duopoly outcome. Because the profit function is strictly concave, there cannot be another duopoly equilibrium.

Suppose that there exists a monopoly outcome where firm B is the monopolist, say, point A in Figure 2.9a. Recall that this is the case where the adjusted quality of one firm is not significantly higher than the other firm, i.e., $2\left(\beta_i - t\tilde{\beta}_j\right) < 3\tau - \left(\beta_j - t\tilde{\beta}_i\right)$. Then, it is possible for firm A to lower its price such that area Z^L_d is reached. The market becomes a duopoly. Firm A gets a positive profit. Similarly, if $(p_A, p_B) \in Z^L_A$, where firm A is the monopolist. Firm B can lower its price such that a duopoly outcome arises.

Suppose that the prices are such that $\left(p_A = 0, p_B = \beta_B - t\tilde{\beta}_A + \tau\right)$ (point B in figure 2.9a). Firm B is the monopolist. If $\beta_B - t\tilde{\beta}_A + \tau < 0$, then $\left(p_A = 0, \beta_B - t\tilde{\beta}_A + \tau < 0\right)$ cannot be an equilibrium. Suppose instead that $\beta_B - t\tilde{\beta}_A + \tau > 0$. However, firm B has an incentive to decrease its price to move into area Z^L_d. Firm B's profit increases since it is at the decreasing part of the profit function. Thus, a monopoly outcome cannot arise in equilibrium.

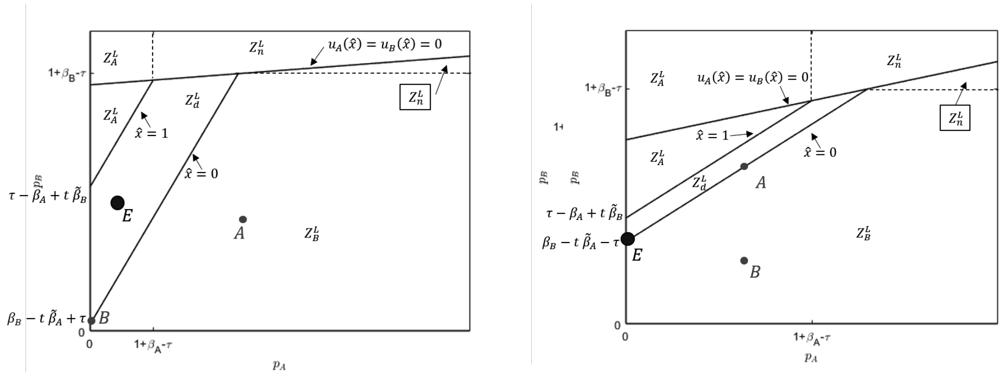
For case 2.(b), where one firm is the monopolist. Suppose without loss of generality that firm B is the monopolist. The equilibrium prices are $\left(p_A = 0, p_B = \beta_B - t\tilde{\beta}_A - \tau\right)$, i.e., point E in Figure 2.9b. For a duopoly outcome to arise, firm B must increase its price. However, the profit function is in the increasing part. By lowering its price, firm B's profit decreases. A duopoly outcome is not possible.

To show that there is no other monopoly outcome, suppose that there exists another monopoly outcome where $(p'_A, p'_B) \in Z^L_B$, say point A in figure 2.9b. Then, firm A can lower its price such that it has a positive demand. So, this situation cannot be an equilibrium outcome.

Suppose the prices are at point B in figure 2.9b. Firm A cannot decrease its price such that it has a positive demand. However, firm B has an incentive to increase its price. Firm B remains the monopolist, but it charges a higher price. So, there is a profitable deviation. Therefore, the prices $\left(0, \beta_B - t\tilde{\beta}_A - \tau\right)$ are the unique equilibrium outcomes. □

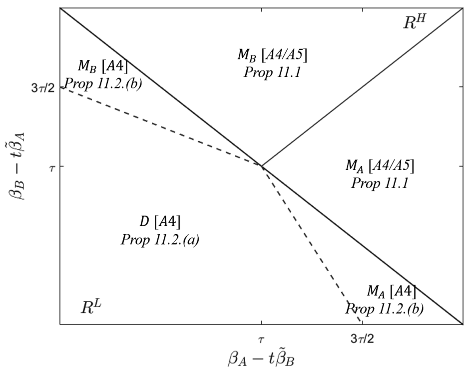
Figure 2.9: Uniqueness of the SPE outcomes for area R^L

(a) Case 2.(a) (b) Case 2.(b)



Using Propositions 11 and 12, Figure 2.10 illustrates the market outcome for each area. Note that each axis represents adjusted quality $\beta_i - t\tilde{\beta}_j$, not a price p_i as in the previous figures.

Figure 2.10: The equilibrium outcomes



Propositions 11 and 12 are based on the equilibrium selection (Assumption 5) that is justified by using social welfare. An alternative equilibrium selection is by using consumer surplus. According to Lemma 1, consumer surplus when firm i is the monopolist is $CS^m = 1 + \frac{\tau}{2} + t\tilde{\beta}_j$. In this case, firm i is chosen when $\tilde{\beta}_i < \tilde{\beta}_j$. However, using consumer surplus adds more complexity. This is because the weight w has to be taken into account. Specifically, suppose $\beta_i > \beta_j$. Then, $\tilde{\beta}_i \leq \tilde{\beta}_j$ if and only if $w \geq 0.5$. Nevertheless, even though consumer surplus is used as a selection device, the last consumer still receives zero utility. In the future, finding a better criterion or justification for the equilibrium selection is sensible to see how the results are affected.

2.9.3 Conclusion

In this appendix, I have shown that the equilibrium outcomes in Proposition 1 are the unique outcomes supportable by SPE that satisfy Assumptions 4 and 5. Specifically, there are three possible types of market structures in equilibrium (Figure 2.10). These market structures are a monopoly by firm *A* or firm *B* and a duopoly with a covered market. The subsequent analysis following Proposition 1 in the main part of the paper rely on these unique equilibrium outcomes.

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Chapter 3

Assessing the Long-run Competitive Effects of Digital Ecosystem Mergers

This chapter is co-authored with Jasper van den Boom.

3.1 Introduction

This article studies the roles of complementarity and economies of scope in the creation of long-run anti-competitive effects following mergers involving a digital ecosystem. As complementarity between products and services in a digital ecosystem implies that they are not substitutes, such mergers are considered conglomerate mergers. In EU competition law, conglomerate mergers are traditionally viewed as creating efficiencies with limited anti-competitive effects.[1] We argue otherwise; the combined effects of complementarity and economies of scope could lead to foreclosure and subsequent long-run harms when a digital ecosystem is involved in a merger. Hence, the benefits from complementarity and economies of scope should be weighed against long-run harms. The presumption in EU competition law that conglomerate mergers are unlikely to create anti-competitive effects should not be applied to mergers involving a digital ecosystem.[2]

This article focuses on multi-product digital ecosystems. Specifically, we define a digital ecosystem as 'a collection of platforms, products or services operated by a single undertaking that exhibits linkages through shared standards, inputs or data with some degree of complementarity and economies of scope

[1] Commission Notice – Guidelines on the Assessment of Non-horizontal Mergers under the Council Regulation on the control of concentrations between undertakings, DG COMP, 28 October 2010, par. 92

[2] Commission Notice – Guidelines on the Assessment of Non-horizontal Mergers under the Council Regulation on the control of concentrations between undertakings, DG COMP, 28 October 2010, par. 92

when jointly produced and supplied.' This is not to be confused with a multi-actor ecosystem consisting of a platform operator and its third-party complementors.[3]

Three aspects distinguish conglomerate mergers involving a digital ecosystem from traditional conglomerate mergers. First, a digital ecosystem consists of several complementary products and services. So, a merger that directly affects one product may indirectly affect many other products in the ecosystem. Second, unlike firms in non-digital markets, digital ecosystems rely heavily on user-generated data. Data generated by one product can be used to improve other products in the ecosystem. Because of these linkages, it is costly to define and assess all relevant markets of a merger involving a digital ecosystem. Third, an entrant does not typically compete with an incumbent head-on by offering the same products and services as the incumbent. Instead, an entrant normally enters by offering different products. After the entrant has sufficient users, it may expand its ecosystem to compete in the incumbent's core markets. So, the competition faced by an incumbent is typically indirect.

In particular, we look at two characteristics abundantly found in digital ecosystems – complementarity and economies of scope. Bourreau and de Streel (2019) make a similar observation that these two characteristics are one of the main reasons why traditional conglomerates are different from digital ones.[4]

Complementarity means that the value of products when consumed together is greater than the sum of the values when these products are consumed separately. Several products and services in a digital

[3] While the term digital ecosystem does not yet have a formal definition, this economic definition of an ecosystem helps understand how it creates value. The definition formulated here however, fits well within the frameworks of modern conceptualizations of the digital ecosystem used in policy and academia. See to this extent the discussions by Lianos I & Carballa B., 'Economic Power and New Business Models in Competition Law and Economics: Ontology and New Metrics', *CLES Research Paper Series 3/2021* (2021), provides an extensive definition of the ecosystem concept, this paper argues that "the concept of ecosystem reflects the emergence of business environments marked by modularity in production, co-evolution, and decisional complexity, where innovation must be coordinated across different hierarchies, markets, and industries. They form "intentional communities" of economic actors who to a large extent co-evolve their goods and services with aligned visions and "whose individual business activities share in some large measure the fate of the whole community". Furthermore, the paper argues that ecosystems are defined by the existence of non-generic complementarities.; Alexiadis P. & De Streel A., 'Designing an Intervention Standard for EU Digital Platforms' *EUI Working Paper Series RSCAS 2020/14* (2020), refers to the works of M. Bourreau and A. de Streel, *Digital Conglomerates and EU Competition Policy*, March 2019, pp 12-13 and Koca, *Product Release Strategies in the Digital Economy*, PhD Thesis, Imperial College London, 2018, arguing that "product ecosystems exist when products bought together by a customer generate synergies between those products. In turn, those synergies might facilitate the leveraging of market power between products and/or services". This definition refers to the presence of complementarity between ecosystem services; Policy reports such as Crémer et al (2019) and the Stigler Center Study of the Economy and the State, 'Stigler Committee on Digital Platforms', *Final Report* (2019), do not clearly define digital platforms but merely refer to them as a form of conglomeration that produces economies of scope and where data is shared. These reports focus in particular on the function of ecosystem to entrench the dominant position of large digital platform operators. While an overarching definition of digital ecosystems does not exist, it is clear that economies of scope, complementarity, interoperability and a shared goal or regime are central tenets of ecosystem creation. It should also be noted that Because mergers that we are interested in are related to complementarity, they differ from portfolio effects in a fundamental way. This is because portfolio effects concern with weak substitute; see Neven, D., 'The analysis of conglomerate effects in EU merger control', Handbook of Antitrust Economics, MIT Press, Cambridge, MA (2008)

[4] Bourreau M. & De Streel, A., 'Digital Conglomerates and EU Competition Law' (2019)

ecosystem are intrinsically complementary.[5] Consider an Apple Watch, an iPhone and an iPad, which are all part of Apple's ecosystem. An Apple Watch can be synced with an iPhone and an iPad. Files on an iPhone and an iPad can be exchanged via Apple's Airdrop, which is an exclusive feature for Apple devices. Users of these products benefit from complementarity between them. In contrast, an Apple Watch cannot be synced with an Android phone. So, an Apple Watch and Android phone user does not enjoy the same complementarity as an Apple Watch and iPhone user. There are many more examples like this that exist in the Apple ecosystem. So, there are more linkages than discussed in this example.

Economies of scope refer to the costs saved by supplying multiple products or services at the same time. In other words, it is less expensive for a firm to provide a set of products or services together than to supply each of them separately. This is usually due to common inputs, such as software integrations, personnel and algorithms. For example, once Apple develops a new version of its operating system (iOS), it can be used on both iPhones and iPads. Furthermore, data is an important source of economies of scope in digital markets as it is a shareable input that can be reused across different products offered by the undertaking. Access to extensive datasets reduces the costs for the ecosystem operator to improve the quality of products and services it already offers. Extensive datasets also allow the ecosystem operator to develop a new product or service at a reduced cost.[6]

The presence of complementarity and economies of scope provides larger digital ecosystems with an efficiency advantage over entrants that have a smaller ecosystem. A larger ecosystem can generate higher values from more linkages between products and services at lower costs. Hence, acquisitions of additional products or services make large digital ecosystems more efficient. While this is beneficial when looking from the short-run perspective, this article argues that such mergers create larger entry barriers. If the mergers result in entry foreclosure, the detriment to competition and consumers related to long-run harms may outweigh the short-run efficiencies generated by the mergers.

Can efficiencies be bad for competition? In addition to this article, Argentesi et al. (2021) provide evidence to show that efficiencies from mergers related to digital markets could create anti-competitive effects.[7] The authors conduct two ex-post reviews on the *Facebook/Instagram* and *Google/Waze* mergers, approved by the Office of Fair Trading in the UK. For *Facebook/Instagram*, they argue that the efficiencies derived from the merger have considerably improved Facebook's ability for targeted advertising. However,

[5] Jacobides, M. G., Cennamo, C., & Gawer, A., 'Towards a theory of ecosystems', *Strategic Management Journal, 39(8), 2255–2276* discuss generic and non-generic complementarities in digital ecosystems. These examples relate to non-generic complementarities as they require the creation of a specific structure or relationships and alignments to create value; i.e., hardware relies on the presence of an operating system to create value, and an operating system needs applications to create value. Complementarities may also exist in weaker forms, such as the inclusion of a person-to-person messaging service with a social media network.

[6] ibid, p. 11-12

[7] Argentesi, E., Buccirossi, P., Calvano, E., Duso, T., Marrazzo, A., & Nava, S. (2021). 'Merger policy in digital markets: an ex post assessment.' *Journal of Competition Law & Economics*, 17(1), 95-140.

this improvement has significantly reduced the ability of Facebook's rivals to compete in targeted advertising. The advertising revenues of its rivals are significantly lagging behind Facebook and Instagram.

Argentesi et al. (2021) recognise that high efficiencies were created from the merger of *Google/Waze*. Combining data from Google Maps and Waze improves the quality and accuracy of both apps. However, the authors argue that the UK authority inadequately assessed the negative consequences of the merger. The authority did not sufficiently investigate whether Waze could grow to become Google's competitor in navigation services. So, the merger might lead to a loss in potential competition. In addition, the authority did not investigate the markets in which Waze generated its revenue.

While Argentesi et al. (2021) and this paper have similar conclusions, they differ in two ways. First, among other policy recommendations suggested by Argentesi et al. (2021), they argue that competition authorities should take into account the multi-sidedness of a merger in digital markets. In particular, competition authorities should investigate how a merger affects the monetising strategy of the merged entity (online advertising for Facebook/Instagram). In contrast, we propose that competition authorities should look further than the multi-sidedness of digital platforms and that competition authorities should look at a digital ecosystem as a whole. Because of complementarity and economies of scope, the effect of a merger will permeate throughout the ecosystem. Second, Argentesi et al. (2021) assess the negative effects on existing competitors. In our paper, we focus on the effects on potential entrants and long-run harms.

The first section of this paper formulates our theory of harm based on the combined effects of complementarity and economies of scope in mergers involving a digital ecosystem. The main mechanism is that a conglomerate merger creates a competitive advantage for the acquiring ecosystem that competitors cannot duplicate. As such, the merger diminishes the profitability of entry into the market. Subsequently, the section explains how this foreclosure ultimately leads to (empirically observed) long-run harms. We also provide an overview of relevant existing theories of harm to clarify how our theory of harm differs from existing ones formulated by other authors.

The second section contains a simple economic model that allows us to incorporate both complementarity and economies of scope into a merger between an incumbent and an existing firm. It shows that both characteristics provide a sufficient condition for an increase in entry barriers post-merger. The stronger the synergy, the more likely entry foreclosure will occur. The incumbent has an incentive to merge to increase its ecosystem's efficiency and possibly deter entry in the long run.

The model distinguishes between the assessment of a merger by a myopic and a foresighted competition authority. The myopic competition authority assesses the merger without taking into account the long-run entry dynamics, while the foresighted competition authority fully considers long-run entry. The model shows that the myopic competition authority always clears the merger due to the short-run efficiencies. However, the foresighted competition authority balances the trade-offs between short-run efficiencies and long-run harms to entry and consumers. We show that the foresighted competition authority blocks the merger due to long-run considerations in some situations. Thus, the myopic competition authority always underenforces. The model highlights that short-run efficiencies alone are insufficient to clear a merger.

Section 3 provides empirical insights by reviewing six mergers involving digital ecosystems assessed by the European Commission (hereafter, the Commission). These cases are *Google/DoubleClick (2008)*, *Microsoft/Skype (2011)*, *Facebook/WhatsApp (2014), Microsoft/LinkedIn (2016)*, *Apple/Shazam (2018) and Google/Fitbit.*[8] We focus on the considerations made by the Commission to decide on the merger cases concerning complementarity, economies of scope and potential long-run effects. The case studies use the knowledge of hindsight to assess events happened in the relevant markets post-merger. Finally, we explain why certain foreclosure effects or practical consequences of the mergers were not fully appreciated at the time of the merger decision and how this translates into (potential) consumer harms.

We argue that conglomerate mergers involving a digital ecosystem should be treated differently from traditional conglomerate mergers. Section 4 makes three policy suggestions that allow competition authorities to take more actions against potential long-run effects from conglomerate mergers involving a digital ecosystem. First, since conglomerate mergers in digital markets lead to more horizontal effects due to indirect competition, horizontal effects should be incorporated into the assessment. Additionally, the presumption that conglomerate mergers are less likely to produce anti-competitive effects should not apply in digital markets. Second, potential long-run harms to competition in dynamic digital markets must be assessed. Competition authorities must balance between short-run efficiencies and long-run harms. To mitigate the problems associated with uncertainties of the ex-ante assessment of long-run effects, we propose the use of flexible remedies that only trigger once certain harms materialise. Flexible remedies strike a balance between the need to intervene against long-run harms and preventing undue burdens on undertakings. Third, we reiterate that mergers involving digital markets are different from those in non-digital markets. Hence, a distinct standard of assessment for mergers involving a digital ecosystem is advisable.

[8] Case No. Comp/M.4731, *'Google/DoubleClick* – C(2008) 927 final', Commission Decision of 11 March 2008 (*Google/Doubleclick)*; Case M. 6281, 'Microsoft/Skype C(2011) 7279 Final, *Commission Decision of 07/10/2011 (Microsoft/Skype)*; Case M.7217, 'Facebook/WhatsApp C(2014) 7239 Final, *Commission Decision of 3.10.2014 (Facebook/WhatsApp)*; Case M.8124, 'Microsoft/LinkedIn' C(2016) 8404 Final, Commission Decision of 6.12.2016 (*Microsoft/LinkedIn);* Case M.8788, 'Apple/Shazam – C(2018) 5748 final', Commission Decision of 6 September 2018 *(Apple/Shazam)*; Case M.9660 – Google/Fitbit, Commission Decision of 17 December 2020

3.2 Theories of harm

The first part of this section proposes a theory of harm that could arise from complementarity and economies of scope. It discusses how the two characteristics can lead to foreclosure in the long run. We also identify four conditions in which foreclosure is more likely to occur. In the second part, we discuss related literature and existing theories of harm related to digital markets proposed elsewhere.

3.2.1 Developing a theory of harm related to complementarity and economies of scope

We propose that, under certain conditions, complementarity and economies of scope lead to long-run foreclosure, limiting the degree of competition. To see this, suppose that there is an incumbent ecosystem and a potential entrant that can offer a competing ecosystem. The entrant has to pay an entry cost to enter. The incumbent has an opportunity to acquire a stand-alone firm prior to an entry decision by the entrant. If the incumbent acquires the stand-alone firm, its ecosystem will become more efficient. It generates a higher value because of the complementarities, which increase the utility to consumers. At the same time, the incumbent's cost does not increase substantially due to economies of scope.

When the incumbent becomes more efficient post-merger, the profit the entrant can earn if it enters diminishes. It becomes less likely that the profit will be enough to compensate for the entry cost. In other words, the merger between the incumbent and the stand-alone firm raises a barrier to entry. Hence, the incumbent could potentially use the merger to maintain its monopoly position.

Accordingly, a competition authority that decides whether the merger should be allowed must consider the dynamic effect on the market structure. If the competition authority is myopic, meaning it does not consider the possibility of entry, it may allow the merger when it should not. Suppose the competition authority allows the merger because of the efficiency argument. Then, the entry may be prevented because of this efficiency. Consumers benefit in the short run, but they may be harmed in the long run. This does not mean that consumers will always be harmed: complementarity and economies of scope could be high such that consumers benefit more even if the incumbent remains the monopolist. Hence, short-run benefits must be weighed against long-run harms.

We identify four circumstances in which the foreclosure effect is more likely to be pronounced. The first circumstance is when the degree of complementarity is high. That is when consumers derive more utility when consuming a larger bundle of goods and services. Second, when economies of scope are strong, the incumbent with a larger ecosystem attains a more comparative advantage over the entrant. Third, when the ecosystem of the incumbent is already expansive, it is unlikely that the entrant can develop an ecosystem that can compete with the incumbent. Fourth, if competition between the incumbent and the entrant is more likely to be intense, the entrant has a lower incentive to enter because competition will dilute its profit.

Sections 3.3 and 3.4 support our proposed theory of harm from two perspectives. Section 3.3 develops an economic model to analyse the situation theoretically. Furthermore, we also look from a legal perspective in Section 3.4 which empirically reviews six merger cases related to digital ecosystems decided by the Commission.

3.2.2 Other theories of harm and related literature

The concerns about anti-competitive effects from conglomerate mergers are not new. The Commission expressed these concerns in several landmark cases even before the digital era, such as *Tetra Laval/Sidel* and *General Electric/Honeywell.*[9] While the Commission has laid out several theories of harm from conglomerate mergers, Neven (2008) argues that the primary anti-competitive effect comes from tying and bundling, leading to foreclosure.[10]

There is long-standing economic literature on tying and bundling. In the early economic analysis, several Chicago School economists theorize that a monopolist in one market cannot profitably leverage its market power in its monopolized market to another competitive market. This assertion is famously known, among competition policy scholars, as the single-monopoly-profit theory.[11] However, the Chicago School's argument relies on a crucial assumption that tying or bundling must not create an externality that affects market structure.[12]

Subsequent studies demonstrate that a monopolist may have an incentive to bundle if it leads to the foreclosure of its competitors. The seminal work by Whinston (1990) shows that a monopolist has an incentive to tie to foreclose entry when products are independent, but not when the products are complementary. His model assumes that the monopolist has monopoly power in market A, while it faces a potential entrant only in market B.[13] Furthermore, Carlton and Waldman (2002) argue that a monopolist has an incentive to bundle complementary products when the monopolist faces a potential entrant in market A as well. In their model, both products A and B are perfect complements. By bundling, the entrant in market B is automatically foreclosed. Without the entrant into market B, the potential entrant in market

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[9] Case COMP/M.2220, General Electric/Honeywell, Commission Decision of 3 July 2001; Case Comp/M.2416, Tetra Laval/Sidel, Commission Decision of 1 January 2003;

[10] Neven (2008)

[11] To see the main argument of this theory, suppose there are two products, say, A and B where both products must be consumed together. A monopolist monopolizes market A, while there is perfect competition in market B. The cost of production for both products is 1 euro. Suppose further that each consumer is willing to pay 10 euro for a bundle of product A and product B. So, the maximum profit that a firm can extract is 8 euro per bundle. Since market B is perfectly competitive, the price for product B will be driven down to cost. Then, the monopolist can charge 9 euro for product A. As a result, with tying, the monopolist earns the net profit of 8 euro for each bundle which is also equal to the maximum profit possible. Hence, tying cannot possibly increase the monopolist's profit.

[12] See Elhauge, E., 'Tying, bundled discounts, and the death of the single monopoly profit theory', *Harv. L. Rev., 123* (2009) for the summary of conditions where the single-monopoly-profit theory does not hold.

[13] Whinston M., 'Tying, Foreclosure and Exclusion' *American Economic Review Vol. 80/4, pp. 837-859* (1990)

A cannot sell its product neither since consumers cannot buy product B from elsewhere.[14] Thus, the monopolist has an additional incentive to prevent entry in market B. Even though the strategy seems unprofitable when considered only from market B perspective, the monopolist might use the strategy to keep its market power in market A.[15]

Note that the settings in Whinston (1990) and Carlton and Waldman (2002) have the commitment issue: the incumbent has an incentive to de-bundle if the entrant does enter.[16] Nalebuff (2004) shows that the incumbent does not have the commitment issue when it can be used as a price discrimination device.[17]

The rise of digital platforms with two-sided markets and the network effect provides new incentives for the monopolist to use tying or bundling strategy to foreclosure competitors. When negative pricing is not practical,[18] Jeon and Choi (2020) point out that a platform can use a tying strategy to mimic a negative price. A competitor who sells only one product will not be able to compete.[19]

In addition to tying and bundling, there is also a situation called "killer acquisitions," where an incumbent acquires an entrant to stop it from entering the market. In Motta and Peitz (2020), an entrant could become a competitor if it successfully develops its product. Nevertheless, the product can be developed if the entrant has enough resources (e.g., funding or data).[20] They show that the incumbent has an incentive to acquire the entrant to maintain its monopoly position, where the incumbent may or may not keep developing the acquired product. Cunningham et al. (2021) estimate that 5.3 percent to 7.4 percent of acquisitions in the U.S. pharmaceutical market are killer acquisitions.[21]

Existing economic literature tends to focus the analysis on horizontal mergers where the efficiency gain comes from economies of scale or a lower marginal cost. And, when applicable, the anti-competitive effect comes from the exit of existing competitors. In addition to the papers that are already mentioned, other examples are Farrell and Shapiro (1990), Gowrisankaran (1999), Motta (2004), Nocke and Whinston (2010), and Varma et al. (2020). Mergers related to digital ecosystems normally exhibit both economies

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[14] Carlton D.W. & Waldman M., 'The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries'. *The RAND Journal of Economics 33, 194–220* (2002)

[15] This rationale was observed in the case *US v. Microsoft,* where Microsoft tied its Windows OS with Internet Explorer (IE) to prevent Netscape from entering the OS market.

[16] Whinston (n. 23); Carlton & Waldman (n. 24)

[17] Nalebuff, B., 'Bundling as an entry barrier', *The Quarterly Journal of Economics, 119(1), 159-187* (2004)

[18] A negative price may not be practical due to a moral hazard problem. For example, suppose that consumers can buy a product at a negative price on an e-commerce platform. To get free money, some consumers may order them even though they do not derive utility from them.

[19] Jeon D.S. & Choi J.P., 'A leverage theory of tying in two-sided markets with non-negative price constraints', *American Economic Journal: Microeconomics* (2020)

[20] Motta & Peitz (2020) (n. 4)

[21] Cunningham, C., Ederer, F., & Ma, S., 'Killer acquisitions', *Journal of Political Economy, 129(3),* 649-702 (2021)

of scope and complementarity.[22] The economies of scope can be related to a lower cost of production that is relatively similar to existing literature. However, the role of complementarity is not evident in horizontal mergers since, by definition, firms sell the same product.

Another interesting theory of harm is related to the one-stop-shopping feature, proposed by Rhodes and Zhou (2019). When there is a search cost, consumers prefer to go to a larger ecosystem because they can buy several products and services in one go. Such an advantage favours a large ecosystem. It is a feature that a small competitor cannot duplicate.[23] So, the main mechanism in Rhodes and Zhou (2019) is a lower total search cost. They did not look at complementarity and economies of scope. Interestingly, this harm is what we observe in the merger between Google and DoubleClick, which we review in Section 3.4.1.

Furthermore, the harm may not manifest as a foreclosure in the absence of competition. It may come from a reduction in quality. Anderson and Coate (2005) analyse the length of advertisements in television broadcasting. They show that when the degree of competition is lower, there will be more advertisements. While their situation is not exactly related to digital platforms, it shows a general result that firms with more market power could decrease the quality of their products or services.[24] In fact, we also observe the reduction in quality following Google/DoubleClick merger, where Google subsequently altered its contracts with advertisers. More details can be found in Section 3.4.

Argentesi et al. (2021) provide a summary of seven theories of harms related to digital markets. We review three theories of harms that we have not discussed earlier and are relevant to this article. The most relevant theory of harm is the *loss of potential competition*. Such harm may arise when merging parties do not directly compete against one another pre-merger. However, they may enter each other's market(s) and become direct competitors in the future if there is no merger. Therefore, the merger eliminates potential competition from an *existing* firm. Note that this theory of harm is different from ours. We do not look at the loss of potential competition from existing firms. Instead, we look at an entrant who is not in the market at the time of the merger.

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[22] Farrell, J. & Shapiro C., 'Horizontal Mergers: An Equilibrium Analysis', *American Economic Review, 80(1), 107-126* (1990); Gowrisankaran, G., 'A dynamic model of endogenous horizontal mergers' *The RAND Journal of Economics, pp.56-83* (1999); Motta, M., 'Competition policy: theory and practice', *Cambridge University Press* (2004); Nocke, V. and Whinston, M.D., 'Dynamic merger review', *Journal of Political Economy*, *118(6), pp.1200-1251* (2010); Varma, G.D. and De Stefano, M., Entry Deterrence, Concentration, and Merger Policy (August 8, 2020). Available at SSRN: https://ssrn.com/abstract=3626734 or http://dx.doi.org/10.2139/ssrn.3626734; Bourreau & De Streel (2019) (n. 6)

[23] Rhodes, A. and Zhou, J., 'Consumer search and retail market structure', *Management Science, 65(6),* pp. 2607-2623 (2019)

[24] Anderson, S.P. and Coate, S., 'Market provision of broadcasting: A welfare analysis', *The review of Economic studies, 72(4), pp. 947-972* (2005)

The next theory of harm is the *loss of competition in markets for attention*. Many digital firms generate revenue from online advertising. They need consumers to spend time on their products or services. The markets for these products and services are called markets for attention. When two firms merge, the merged entity has access to a larger group of consumers. Consequently, some users who multi-home are now exclusive under a single entity that supplies both products. More consumer data also increase the merged entity's ability in targeted advertising. So, other firms that compete in an online advertising market cannot compete against the merged entity. The merged entity has higher bargaining power over advertisers, allowing it to charge higher advertising prices. This is the main theory of harm used by Argentesi et al. (2021) to show the anti-competitive effect of Facebook/Instagram described earlier.

The last theory of harm is *big data as an essential input to compete*. A merger allows the merged entity to combine two sources of data. Data is a crucial source for digital firms in several aspects, e.g., to develop their algorithm, infer consumers' preferences, and make more accurate recommendations. Hence, the combination of the datasets increases the merged entity's competitive advantage. This advantage could lead to foreclosure. In our review of Microsoft/LinkedIn and Apple/Shazam, we also rely on this theory of harm. Note that the last two theories of harms rely on efficiencies that created anti-competitive effects. Accordingly, the mechanism is the same as the mechanism for our theory of harm.

3.3 A model of entry barrier by complementarity and economies of scope

We build an economic model that simultaneously incorporates complementarity and economies of scope to support our theory of harm developed in Section 3.2. Since all of the merger cases we reviewed were approved by the Commission, we do not have any baseline for possible comparisons between when a merger is and is not approved. Therefore, we purposely develop a simple set-up, yet rich enough, to provide us with counterfactuals of what could happen if the mergers were prevented. It also provides the conditions when mergers should be approved even though the long-run perspective is considered.

3.3.1 Model

In the main part of the paper, we present a reduced form of our full model to simplify the analysis. In the appendix, we introduce a full model. The assumptions made in this section are justified by the results from the full model.

Players:

There are two ecosystems offered by an incumbent (I) and a potential entrant (E). The incumbent's ecosystem consists of a set of products and services denoted by Θ_I. Similarly, the entrant's ecosystem offers a set of products and services denoted by Θ_E. The potential entrant is not yet active. If the potential entrant wants to enter, it incurs an entry cost F.

Additional to the two ecosystems, firm A is a stand-alone firm that supplies product A. Firm A is a monopolist in the market for product A. The incumbent's and the entrant's ecosystems do not contain product A, i.e., $A \notin \Theta_A$ and $A \notin \Theta_B$.

Timing:

The timing of the game is as follows. First, the incumbent decides whether to merge with firm A. The incumbent makes a take-it-or-leave-it offer T to firm A. If the incumbent acquires firm A, the incumbent's set of products and services becomes $\Theta_I \cup \text{A}$.

Next, the entrant decides whether to enter with the entry cost F. If the entrant does not enter, the incumbent remains the monopolist. If the entrant enters, the market structure becomes a duopoly. As firm A is the monopolist for product A, firm A does not compete with the incumbent or the entrant.

Payoffs – profits π and consumer surplus CS:

Suppose that the incumbent does not acquire firm A. Then, firm A's profit is denoted by π_A^m. In addition, consumer surplus generated by firm A is CS_A. If firm A is acquired by the incumbent, the payoff of firm A is the offer T made by the incumbent. Consumer surplus from firm A is zero as it is no longer active.

The incumbent's payoffs, without the take-it-or-leave-it offer T, are as follows. First, suppose that the entrant does not enter. Then, the incumbent's monopoly profit given its ecosystem Θ_I' is $\pi_I^m(\Theta_I')$. On the other hand, if the entrant enters, the incumbent's duopoly profit is given by $\pi_I^d(\Theta_I', \Theta_E)$. Note that $\Theta_I' = \Theta_I$ if the incumbent does not acquire firm A, and $\Theta_I' = \Theta_I \cup A$ if it does. If the entrant does not enter, its profit is zero. If the entrant enters, its duopoly profit before the entry cost F is $\pi_E^d(\Theta_E, \Theta_I')$. So, the entrant's profit net the entry cost is $\pi_E^d(\Theta_E, \Theta_I') - F$.

Finally, consumer surplus when the incumbent is the monopolist is $CS^m(\Theta_I')$. And consumer surplus under the duopoly is $CS^d(\Theta_I', \Theta_E)$. The timing and the payoffs of the incumbent and the entrant are summarized in Figure 1.

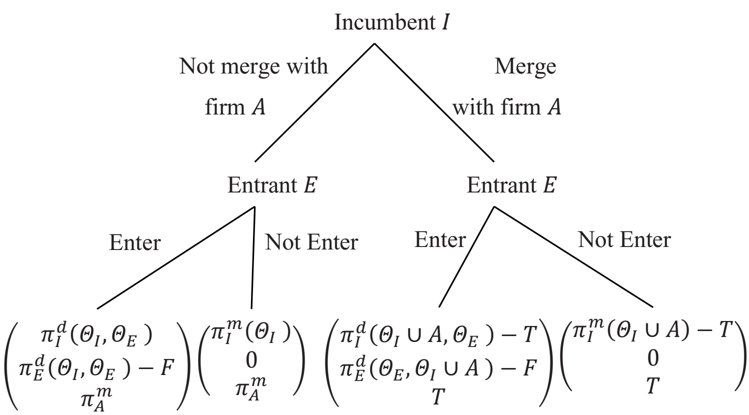
The information is perfect. All parties observe the decisions made in the previous stages. The solution concept is subgame perfect Nash equilibrium (SPNE).

Assumptions due to complementarity and economies of scope:

We make assumptions on the relationships between the profits and consumer surplus as motivated by complementarity and economies of scope. To understand the motivations behind the assumptions, it is helpful to briefly discuss how we model complementarity and economies of scope in our full model. Specifically, there are consumers who choose to join one of the ecosystems. Each consumer receives a value $v(\Theta)$ if they buy from a firm that supplies a bundle of product and services Θ in its ecosystem. The associated marginal cost is $c(\Theta)$.

Due to complementarity, the value of the combination of the products is greater than the combination of the values of separate products. Additionally, economies of scope allow the ecosystems to supply a bundle of the products at a lower marginal cost than to supply each of them separately. Assumption 1 formalizes these two characteristics.

Figure 1: Summary of the timing and the payoffs



Note: The payoffs of the incumbent, the entrant, and firm A are in the first, second, and third row of each bracket, respectively.

Assumption 1: Let Θ and Θ' be disjoint sets of products and services ($\Theta \cap \Theta' = \emptyset$). Then, by complementarity and economies of scope, respectively, we have

$$v(\Theta \cup \Theta') > v(\Theta) + v(\Theta'), \text{ and} \tag{3.1}$$

$$c(\Theta \cup \Theta') < c(\Theta) + c(\Theta'). \tag{3.2}$$

Note that complementarity and economies of scope, defined in Equations (3.1) and (3.2), are not specific to digital ecosystems. They also apply generally to conglomerate mergers. However, as argued earlier, complementarity and economies of scope are particularly strong in digital ecosystems where there are strong linkages between products and services due to the expansive nature of a digital ecosystem.[25]

Given Assumption 1, the incumbent is stronger after the merger with firm A: its ecosystem can generate a higher value at a lower cost. Accordingly, the incumbent' monopoly profit with the merger $\pi_I^m(\Theta_I \cup A)$ is assumed to be higher than its monopoly profit without the merger $\pi_I^m(\Theta_I)$. Similarly, the incumbent's duopoly profit with the merger $\pi_I^d(\Theta_I \cup A, \Theta_E)$ is higher than its profit without the merger $\pi_I^d(\Theta_I, \Theta_E)$. On the other hand, the entrant's duopoly profit is lower if the incumbent and firm A merge, i.e., $\pi_E^d(\Theta_E, \Theta_I \cup A) < \pi_E^d(\Theta_E, \Theta_I)$. The assumptions on the profits are summarized in Assumption 2.

Assumption 2: The incumbent's and the entrant's profits are as follows.

i. *Monopoly profit:* $\pi_I^m(\Theta_I \cup A) > \pi_I^m(\Theta_I)$

ii. *Duopoly profit:* $\pi_I^d(\Theta_I \cup A, \Theta_E) > \pi_I^d(\Theta_I, \Theta_E)$ *and* $\pi_E^d(\Theta_E, \Theta_I \cup A) < \pi_E^d(\Theta_E, \Theta_I)$.

For consumer surplus, we assume that consumer surplus under the monopoly with the merger is greater or equal to consumer surplus from the incumbent and firm A without the merger, i.e., $CS^m(\Theta_I \cup A) \geq CS^m(\Theta_I) + CS_A$. Similarly, consumer surplus under the duopoly with the merger is higher or equal to consumer surplus from the ecosystems and firm A without the merger, i.e., $CS^d(\Theta_I \cup A, \Theta_E) \geq CS^d(\Theta_I, \Theta_E) + CS_A$. However, we do not make any assumption on the relationship between consumer surplus under the monopoly and the duopoly. That is, given the incumbent's ecosystem Θ_I' and the entrant's ecosystem Θ_E, consumer surplus under the duopoly $CS^d(\Theta_I', \Theta_E)$ may be higher or lower than consumer surplus under the monopoly $CS^m(\Theta_I', \Theta_E)$.

Assumption 3: Consumer surplus under the monopoly and the duopoly is as follows.

i. *Under the monopoly:* $CS^m(\Theta_I \cup A) \geq CS^m(\Theta_I) + CS_A$

ii. *Under the duopoly:* $CS^d(\Theta_I \cup A, \Theta_E) \geq CS^d(\Theta_I, \Theta_E) + CS_A$

iii. *Comparison between the duopoly and the monopoly:* $CS^d(\Theta_I', \Theta_E) \gtreqless CS^m(\Theta_I', \Theta_E)$

Note that the relationships between the profits and consumer surplus assumed in Assumptions 2 and 3 hold under the full model as shown in Lemma 4 and Lemma 6, respectively, in Appendix A2.

Furthermore, it is normally expected that consumer surplus under the duopoly $CS^d(\Theta_I', \Theta_E)$ is higher than consumer surplus under the monopoly $CS^m(\Theta_I', \Theta_E)$ due to a higher degree of competition. However, we provide an example and the discussion to show that this expectation is not generally true in Appendix

[25] Bourreau M. & De Streel, A., 'Digital Conglomerates and EU Competition Law' (2019)

A2. It is possible that consumer surplus under the duopoly is lower than consumer surplus under the monopoly.

Discussion on the modelling choices

An assumption made in the model is perfect information. The incumbent has perfect knowledge about its value $v(\Theta_I \cup A)$ post-merger, the possibility of an entrant entering, and the entrant's value $v(\Theta_E)$. For the first point, based on our review of the case law, the big tech firms always have a clear idea of what they want to do with the acquired product. Hence, they should have a good idea about $v(\Theta_I \cup A)$. For example, the acquisition of DoubleClick by Google allowed it to expand its business from search advertising to display advertising quickly. In addition, Microsoft promptly integrated LinkedIn with other Microsoft software right after its commitments with the Commission expired. These firms know precisely how to benefit from the acquired product.

Second, how can the incumbent know that there is an entrant ready to enter? It is well documented that one of the motives of acquiring young firms is to eliminate the possibility of them growing to become competitors.[26] Since these mergers occur occasionally, this suggests that big tech firms are aware of and monitor potential threats regularly. It does not seem far-fetched to assume that the incumbent expects an entrant to enter.

Third, the entrant's value $v(\Theta_E)$ could indeed be hard for the incumbent to know perfectly. In that case, the incumbent could form an expectation about the possible values of $v(\Theta_E)$. If the incumbent expects a high $v(\Theta_E)$, then the incumbent has a higher incentive to acquire firm A, and vice versa. Of course, this could lead to a situation where the incumbent acquires firm A, but $v(\Theta_E)$ turns out to be low. Consequently, the benefit of the merger is not as high as the incumbent expected.

Another assumption is that only the incumbent could acquire firm A. The first motivation is asymmetrical financial positions between the incumbent and the entrant. The incumbent could be a big tech firm that has vast financial resources. On the other hand, the entrant, who is presumably small, may not have the sufficient financial capability to finance a big acquisition. Second, product A might not fit well with the entrant's ecosystem. The products and services in the incumbent's and the entrant's ecosystems are not necessarily the same.

Nevertheless, suppose the model is adjusted such that the entrant could acquire firm A if the incumbent does not. Because the acquisition will make the entrant stronger, the incumbent will have a higher incentive to acquire firm A, either to weaken the entrant or to prevent the entrant from entering. Hence, allowing the entrant to acquire firm A makes the result of this paper stronger. Instead, suppose that both firms can bid to acquire firm A. Then, there are two contrasting effects. On the one hand, an ecosystem that benefits more

[26] See, for example, Motta and Peitz (2020) and Argentesi et al. (2021).

from complementarity and economies of scope has an incentive to bid higher. On the other hand, the other ecosystem will have to compete against a much stronger competition if it loses the bid. Therefore, it has an incentive to bid aggressively as well. So, which firm has a higher incentive to bid is likely to depend on the specificity of a situation.

3.3.2 Solving the model

The game is solved by using backward induction. In the last stage of the game, there are two subgames. These subgames are whether the incumbent acquires firm A or not. Given the incumbent's ecosystem Θ_I', the entrant will enter when its operating profit is greater or equal to the entry cost: $\pi_E^d(\Theta_E, \Theta_I') \geq F$. This gives the first proposition.

Proposition 1: Given the incumbent's ecosystem Θ_I', the entrant will enter if and only if

$$F \leq \bar{F}(\Theta_I', \Theta_E) \equiv \pi_E^d(\Theta_E, \Theta_I').$$

The entry cost F must be lower than the threshold $\bar{F}(\Theta_I', \Theta_E)$ for the entrant to enter. In this context, $\bar{F}(\cdot)$ represents the barrier to entry. The lower the threshold $\bar{F}(\cdot)$, the higher the barrier to entry.

According to Assumption 2.*ii*., the entrant's duopoly profit is lower if the incumbent merges with firm A, i.e., $\pi_E^d(\Theta_E, \Theta_I \cup A) < \pi_E^d(\Theta_E, \Theta_I)$. Thus, $\bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E)$. The barrier to entry is higher after the merger because it is harder for the entrant to compete with the incumbent. The entrant's profit is lower, so the entry cost must be sufficiently low to incentivize the entry.

Next, we move to the first stage of the game where the incumbent decides whether to acquire firm A. Because firm A makes the monopoly profit π_A^m, firm A is willing to accept a take-it-or-leave-it offer T by the incumbent if the offer is at least π_A^m. In addition, if the incumbent wants to acquire firm A, the incumbent wants to make the take-it-or-leave-it offer T as low as possible. Thus, the incumbent will make the offer T equals π_A^m, which is accepted by firm A. This observation is summarized in the following lemma.

Lemma 1: Suppose that the incumbent wants to acquire firm A. Then, the incumbent makes the leave-it-or-leave-it offer T equals π_A^m.

The incumbent is willing to acquire firm A if and only if the incumbent gets an additional profit higher than the acquisition cost. To see when the incumbent will acquire firm A, the analysis is divided into three cases. First, when $F \leq \bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E)$, the merger does not prevent entry. The entrant will enter regardless of the merger decision. Hence, the incumbent compares the duopoly profits pre- and post-merger, i.e., $\pi_I^d(\Theta_I \cup A, \Theta_E)$ and $\pi_I^d(\Theta_I, \Theta_E)$. Second, when $\bar{F}(\Theta_I \cup A, \Theta_E) < F \leq \bar{F}(\Theta_I, \Theta_E)$, the merger will block the entry. The incumbent will acquire firm A if its monopoly profit after the merger $\pi_I^m(\Theta_I \cup A)$

minus the acquisition cost is greater than the duopoly profit without the merger $\pi_I^d(\Theta_I, \Theta_E)$. Third, when the entry cost is high such that $\bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E) < F$, the entrant never enters. The incumbent compares the monopoly profits between the two options, i.e., $\pi_I^m(\Theta_I \cup A)$ and $\pi_I^m(\Theta_I)$. The incumbent's strategy is summarized in the following proposition.

Proposition 2: The incumbent acquires firm A by offering π_A^m *provided that the increase in the profit is larger or equal to* π_A^m*. The condition is divided into three cases as follows.*

i. *When* $F \leq \bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E)$*, the entrant always enters. The incumbent acquires firm A when* $\pi_I^d(\Theta_I \cup A, \Theta_E) - \pi_I^d(\Theta_I, \Theta_E) \geq \pi_A^m$.

ii. *When* $\bar{F}(\Theta_I \cup A, \Theta_E) < F \leq \bar{F}(\Theta_I, \Theta_E)$*, the acquisition prevents the entry. Otherwise, the entrant will enter. The incumbent acquires firm A when* $\pi_I^m(\Theta_I \cup A) - \pi_I^d(\Theta_I, \Theta_E) \geq \pi_A^m$.

iii. *When* $\bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E) < F$*, the entrant never enters. The incumbent acquires firm A when* $\pi_I^m(\Theta_I \cup A) - \pi_I^m(\Theta_I) \geq \pi_A^m$.

Intuitively, when complementarity and economies of scope are strong, the incumbent's monopoly and duopoly profits with the merger are significantly higher than the profits without the merger. As such, the merger is more likely to occur. On the other hand, if the incumbent receives a small benefit from complementarity or economies of scope, the merger is less likely to happen.

Case *ii.* in Proposition 2 is the most problematic case. The incumbent could use the merger to prevent the entry. Without the merger, the entrant could profitably enter the market, i.e., $F \leq \bar{F}(\Theta_I, \Theta_E)$. In contrast, the merger raises the entry barrier such that the entrant could not enter profitably, i.e., $\bar{F}(\Theta_I \cup A, \Theta_E) < F$. When the monopoly profit with the merger $\pi_I^m(\Theta_I \cup A)$ is high compared to the duopoly profit without the merger $\pi_I^d(\Theta_I, \Theta_E)$, the incumbent has an incentive to acquire firm A to prevent entry. Hence, the merger prevents entry in the long run, allowing the incumbent to maintain its monopoly power.

The model analysed so far assumes that the entrant makes the entry decision with the entry cost F. Nevertheless, there is an alternative interpretation. The model can be used to describe a situation where the incumbent and the entrant are already in the market. F, in this case, is a fixed operating cost. With this interpretation, the entrant is forced to exit the market when $F > \pi_E^d(\Theta_E, \Theta_I')$. So, when the incumbent acquires firm A, it lowers the threshold, similar to Proposition 1. The incumbent can potentially force the entrant out of business by acquiring firm A. This alternative interpretation is relevant to the Google/DoubleClick case, which we will discuss in Section 3.4.1.

3.3.3 When should the merger be allowed?

This section discusses whether the merger between the incumbent and firm A should be allowed. We focus on the case that a competition authority has the power to either allow or prevent the merger. We assume that the competition authority bases its decision on the effect of the merger on consumers. In particular, the merger will be allowed if it does not decrease consumer surplus.[27]

We consider two types of competition authority – myopic and foresighted. The myopic competition authority does not take into account the potential effect of the merger on the market structure. It does not consider that the entry might occur if the merger is prohibited, while the entry could be prevented otherwise. Note that this does not necessarily mean the competition authority is short-sighted. It could be due to a requirement of a merger control that a competition authority has to assess the potential impacts of a merger for a short period of time into the future. As we will discuss in the case law, the Commission tends to look into the future for around two to three years. Though, the integration of products and services in digital ecosystems could take three to five years after a merger. As a result, the long-run effect of entry foreclosure may not be properly included in merger assessments by design. In contrast, the foresighted competition authority takes the long-term effect of entry into the assessment.

From our model's perspective, the short-run effect refers to the effect on consumer surplus after the merger assuming that the market structure does not change. That is, the incumbent remains the monopolist regardless of the merger decision. On the other hand, the long-run effect is the consequence on consumer surplus given the entrant has an opportunity to decide if it will enter or not.

Consumer surplus before the merger is generated by the incumbent and firm A. So, total consumer surplus before the merger is $CS^m(\Theta_I) + CS_A$. After the merger, the myopic competition authority believes that consumer surplus is $CS^m(\Theta_I \cup A)$. Hence, the myopic competition authority will allow the merger because $CS^m(\Theta_I \cup A) \geq CS^m(\Theta_I) + CS_A$. according to Assumption 3.

Proposition 1: The myopic competition authority allows the merger.

Next, we consider the foresighted competition authority, which takes into account the long-term perspective of the change in market structure in its assessment. The analysis is divided into three cases depending on how the merger affects the market structure. Similar to the analysis when the incumbent decides whether to merge with firm A, the three cases are (1) $F \leq \bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E)$, (2) $\bar{F}(\Theta_I \cup A, \Theta_E) < F \leq \bar{F}(\Theta_I, \Theta_E)$, and (3) $\bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E) < F$.

[27] While the discussion on whether the consumer surplus standard is the most appropriate one is warranted, it seems to be the standard adopted by some competition authorities, including the European Commission and the Federal Trade Commission.

The foresighted competition authority compares consumer surplus when the merger is and is not allowed considering the effect on market structure. Consumer surplus pre- and post-merger under different cases is summarized in Table 1. Proposition 4 states the decisions made by the foresighted competition authority.

Proposition 2: The foresighted competition authority's decision is as follows:

i. *When* $F \leq \bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E)$, *the merger is always allowed (same as the myopic competition authority).*

ii. *When* $\bar{F}(\Theta_I \cup A, \Theta_E) < F \leq \bar{F}(\Theta_I, \Theta_E)$, *the merger is allowed if*

$$CS^m(\Theta_I \cup A) \geq CS^d(\Theta_I, \Theta_E) + CS_A.$$

iii. *When* $\bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E) < F$, *the decision coincides with the competition authority*

The decisions of both types of competition authorities are the same when the entry cost is low or high. In the case of the high entry cost, the result is straightforward since the market structures in the short run and the long run are identical. So, both types of competition authorities use the correct measure of consumer surplus under the duopoly.

Table 1: Consumer surplus with and without the merger

Entry cost F	CS with the merger		CS without the merger
$F \leq \bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E)$	$CS^d(\Theta_I \cup A, \Theta_E)$	$>$	$CS^d(\Theta_I, \Theta_E) + CS_A$
$\bar{F}(\Theta_I \cup A, \Theta_E) < F \leq \bar{F}(\Theta_I, \Theta_E)$	$CS^m(\Theta_I \cup A)$	$\gtreqless$	$CS^d(\Theta_I, \Theta_E) + CS_A$
$\bar{F}(\Theta_I \cup A, \Theta_E) < \bar{F}(\Theta_I, \Theta_E) < F$	$CS^m(\Theta_I \cup A)$	$>$	$CS^m(\Theta_I) + CS_A$

When the entry cost is low, the merger does not prevent entry. Therefore, the merger is allowed by both types of competition authorities to harvest the efficiencies from complementarity and economies of scope. Note that even though the decisions are the same, the myopic competition authority uses the wrong criteria in assessing the merger. It uses consumer surplus under the monopoly CS^m rather than the duopoly CS^d.

However, the myopic competition authority may clear the merger when it should not when the entry cost is in the intermediate region. The entrant will enter if there is no merger, while the entry is blocked otherwise. When $CS^m(\Theta_I \cup A) < CS^d(\Theta_I, \Theta_E) + CS_A$, the merger prevents consumer surplus from increasing to $CS^d(\Theta_I, \Theta_E) + CS_A$ in the long run. This case highlights the limitation of the efficiency argument in a merger with strong complementarity and economies of scope. That is, efficiency gains are not sufficient to clear a merger case. The competition authority must make sure that these gains have no adverse consequences thereafter.

In Appendix A3, we provide numerical examples of when the foresighted competition authority allows and does not allow the merger under case *ii.* So, even though the merger allows the incumbent to maintain its monopoly power, there are circumstances that the merger is beneficial to consumers. Note that the situations in cases *i.* and *iii*. could easily arise when the entry cost F is sufficiently low (case *i.*) or high (case *iii.*).

To be clear, we are not arguing that all mergers that prevent an entry should be prohibited. It is possible that efficiencies from complementarity and economies of scope are adequately strong to compensate a lower of degree of competition. In particular, when $CS^m(\Theta_I \cup A) \geq CS^d(\Theta_I, \Theta_E) + CS_A$, the merger should be cleared even though the incumbent remains the monopolist.

Two key lessons for competition policy can be drawn from the model. First, any conglomerate merger that exhibits complementarity and economies of scope inevitably increases the entry barrier. The stronger this synergy is, the higher the entry barrier. The competition authority should assess how likely a potential entrant will be foreclosed following the merger. The long-run effect on the market structure has to be analysed. Second, the short-run efficiency argument is not sufficient to pass a merger. The competition authority has to weigh between the efficiencies generated by complementarity and economies of scope and the foreclosure effect of the merger.

3.4 Ex-post review of merger decisions involving digital ecosystems

In this section we discuss the landmark cases before the European Commission involving digital ecosystem mergers: *Google/DoubleClick* (2008), *Microsoft/Skype* (2011), *Facebook/WhatsApp* (2014), *Microsoft/LinkedIn* (2016), *Apple/Shazam* (2018) and *Google/Fitbit* (2020).[28] For each of these cases, we look into the long-run harms to competition resulting from the mergers due to complementarity and economies of scope between the acquired products or services and the ecosystems.

The ex-post review of these mergers must be viewed in light of the acquirers being operators of large digital ecosystems. The acquisition of an undertaking that operates in a complementary market that should not be considered problematic in non-digital markets may be more problematic in digital markets involving digital ecosystems for several reasons.

[28] Case No. Comp/M.4731, C(2008) 927 final', Commission Decision of 11 March 2008 (Google/DoubleClick); Case M. 6281, C(2011) 7279 Final, Commission Decision of 07/10/2011 (Microsoft/Skype); Case M.7217, C(2014) 7239 Final, Commission Decision of 3.10.2014 (Facebook/WhatsApp); Case M.8124, C(2016) 8404 Final, Commission Decision of 6.12.2016 (Microsoft/LinkedIn); Case M.8788, C(2018) 5748 final', Commission Decision of 6 September 2018 (Apple/Shazam)

Firstly, digital markets are not easily delineated, especially when platforms are involved. Platforms – rather than being markets in themselves – often connect multiple markets on different sides of the platform.[29] The platform may also integrate and offer multiple products at the same time: a smartphone Operating System does not simply offer one service in one market but rather allows users to access a multitude of device-based and web-based applications to end-users and important distribution channels to third-party complementors (often termed professional users).[30] The modularity of digital goods makes them easily integrated and offered as a cluster. As integration happens, the otherwise complementary apps become relevant for horizontal competition.[31] Similar observations can be made for other platforms. For instance, social media platforms can integrate new functionalities as to better compete with entrants, expanding their on-platform offering.[32]

Secondly, digital product ecosystems are characterized by ecosystem linkages. The operator of an expansive digital ecosystem does not necessarily need to bundle the goods and services to create a competitive advantage across markets.[33] The data generated in one of the markets can be used to increase efficiency across a wide range of markets, for instance by allowing for more fine-grained advertising, better

[29] Filistrucchi L., Geradin D.A.A.G., Van Damme E.E.C. & Affeldt P., 'Market Definition in Two-Sided Markets: Theory and Practice' (2014) 10 Journal of Competition Law and Economics 293 discusses how all sides of the market must be assessed in determining the function of the market, as well as the applicability of general tests used in competition policy. This work also distinguishes between transaction and non-transaction markets, which may determine whether the platforms and the multiple sides must be viewed as a single market with multiple sides or multiple connected markets. Other authors distinguish between more types of platform, including matching and attention platforms or innovation platforms, see Bundeskartellamt, Working Paper – Market Power of Platforms and Networks B6-113/15 (2016); Mandrescu D., 'Applying (EU) Competition Law to Online Platforms: Reflections on the Definition of the Relevant Market(s)' *41 World Competition 453* (2018)

[30] Tiwana A., 'Platform Ecosystems: Aligning Architecture, Governance, and Strategy' (2013) discusses how the platform operator and its complementors develop an ecosystem of multiple actors, where the business (or professional users) depend on the platform to supply their services; Competition & Markets Authority, 'Online Platforms and Digital Advertising Market' (2019) <www.nationalarchives.gov.uk/doc/open-government-> discusses how advertising in digital markets happens by creating a collection of goods and services while offering advertisement on the platform at the same time, integrating multiple offerings into a platform; Competition & Market Authority, 'Mobile Ecosystems Market Study - Interim Report' (2021) Market Study Interim Report discusses how the integrated offering of both the products produced by the ecosystem operator and third-parties determine the value of the offering; Bourreau M., 'Some Economics of Digital Ecosystems', Note for the OECD (2020); Bourreau M. , 'Digital Conglomerates and EU Competition Policy' (2019) discuss the modularity of digital goods and the reasons why conglomeration in digital markets is more likely, this also includes considerations on the ease with which digital products can be integrated or de-intgrated from the offering on a certain platform. Mandrescu D, 'Tying and Bundling by Online Platforms – Distinguishing between Lawful Expansion Strategies and Anti-Competitive Practices' *40 Computer Law and Security Review* (2021) distinguishes between on-platform and cross-platform expansion as a part of competitive strategies in digital markets.

[31] Marc Bourreau, Pinar Dogan and Matthieu Manant, 'Modularity and Product Innovation in Digital Markets' *6 Review of Network Economics* (2019); Bourreau (2019)

[32] Gallagher B., Copycat: How Facebook Tried to Squash Snapchat (Wired, 16 February 2018); Frier S. & Stone B., Mark Zuckerberg Is Blowing Up Instagram to Try and Catch TikTok (Bloomberg, 25 May 2022)

[33] Jacobides M.G. and Lianos I., 'Ecosystems and Competition Law in Theory and Practice', *1 UCL Center for Law, Economics and Society Research Paper Series* (2021) <https://www.ucl.ac.uk/cles/research->.

personalization or better insights into user demand for new or improved services.[34] Similarly, acquiring a start-up with a large installed base allow the operator of the ecosystem to fuse the network of the acquired entity with their own network. The ecosystem operator can leverage these users across the markets they operate through shared logins, nudges or cross-platform functionalities.[35] As such, acquisitions of seemingly loosely connected products and services may have more pronounced effects in digital markets than it did in non-digital markets.

Thirdly, competitive pressures in digital markets tend to arise from indirect competition. Indirect competition exists when entrants can capture an installed base in one market as to develop their services in a way that allows them to challenge the incumbent in their core markets. By sustaining a competitive advantage in a market from which indirect competitive pressures may arise, the incumbent ecosystem operator insulates itself from competition.[36] As such, conglomerate mergers in digital market may involve the elimination of a potential competitor in a more meaningful way than they did in non-digital markets.[37]

The review of the merger decisions by the Commission happens in light of the knowledge developed in academic literature on competition in digital markets.[38] As such, there are two important focal points for the analysis in terms of studying the role of complementarity and economies of scope in creating long-run harms to competition. The first point of focus is whether the acquisition forecloses competition in the markets that are relevant to the merger itself. High complementarity and economies of scope between the incumbent ecosystem operator's products and the acquired nascent competitor may create an efficiency

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[34] Fast V, Schnurr D, Wohlfarth M. Regulation of Data-driven Market Power in the Digital Economy: Business Value Creation and Competitive Advantages from Big Data. Journal of Information Technology. August 2022. doi:10.1177/02683962221114394; Krämer, Jan & Schnurr, Daniel. (2021). Big Data and Digital Markets Contestability: Theory of Harm and Data Access Remedies. Journal of Competition Law & Economics. 10.1093/joclec/nhab015; Jens Prüfer and Christoph Schottmüller, 'Competing with Big Data' (Tilburg University, Tilburg Law and Economic Center 2017) <https://EconPapers.repec.org/RePEc:tiu:tiutil:29de4480-00db-473b-a0ee-b387e748c5a4>; Nestor Duch-Brown, Bertin Martens and Frank Mueller-Langer, 'The Economics of Ownership, Access and Trade in Digital Data' (2017) 1 JRC Digital Economy Working Paper; Fortuny et al., 'Mining Massive Fine-Grained Behavior Data to Improve Predictive Analytics' (2016) 40 MIS Quarterly 869.

[35] Mandrescu (2021)

[36] ACCC, 'Digital Platform Inquiry' (2019) argues that Google has been able to insulate itself from dynamic competition through a series of acquisitions that protect its core markets; Luigi Zingales and others, 'Stigler Committee on Digital Platforms - Final Report' (2019) <https://research.chicagobooth.edu/stigler/media/news/committee-on-digital-> similarly observe how dominant digital undertakings use acquisitions to diminish entry into connected markets and thereby protect themselves from competition; A Ezrachi and ME Stucke, How Big-Tech Barons Smash Innovation—and How to Strike Back (HarperCollins 2022) <https://books.google.nl/books?id=yXlHEAAAQBAJ> discuss how incumbent digital undertakings use acquisitions as part of anti-competitive strategies to absorb innovation and diminish any threats to their core markets.

[37] Commercial and Administrative Law of the Committee on the Judiciary Subcommittee on Antitrust, 'House Investigation of Competittion in Digital Markets' (2020) also pays attention to the elimination of nascent competitors as a source of competitive harms. The idea that nascent competitors must be protected to ensure the efficiency of digital markets is at the core of several proposed US bills including Senator Warren's 'Prohibiting Anti-competitive Mergers Act of 2022'.

[38] See notes 30-37

advantage that entrenches the incumbent's position in those markets.[39] The second point of focus is on the complementarity and economies of scope between the acquired products and the broader ecosystem operated by the incumbent. Products that may not seem directly related to the markets operated by the incumbent may be relevant to strengthen their ecosystem.[40] For instance, the acquisition of LinkedIn by Microsoft may seem as only loosely related to its other services but may be explained by the complementarity between operating a professional network service and Microsoft's focus on productivity and enterprise software in its other ventures.

The ex-post review of these merger decisions will make use of the knowledge of hindsight to determine where the merger may have led to foreclosure or long-run harms in unexpected ways. The conclusions of this review do not indicate that the Commission erred in their analysis or application of law at the time of the decision. Rather, its aim is to provide tools to identify potential long-run harms and provide suggestions to better capture these harms in the analysis of mergers in the future.

3.4.1 Google/DoubleClick

Google/DoubleClick serves as a starting point for this paper to analyse the Commission's assessment of acquisitions involving digital ecosystems. With this acquisition, Google combined its proprietary key search advertising and ad placing technologies with DoubleClick's key display advertising and ad serving technologies.[41] The Commission noted that there were high levels of complementarity between the services offered by Google and DoubleClick and that the undertakings had a largely overlapping customer base. However, the complementary nature of the services – rather than substitutability - meant that the merger would be assessed on the basis of its conglomerate effects. As a result, the Commission did not look at potential foreclosure effects as a result of an efficiency advantage of the merged entity, but instead focused on whether Google would have the ability and incentive to foreclose competition post-merger, rather than weighing efficiencies of the mergers against potential foreclosure effects.[42]

The Commission found that, despite the parties' leadership position in their respective markets, they did not possess any superior technologies that would allow them to enter each other's markets to durably compete. In fact, the Commission viewed Yahoo and Microsoft as having a competitive advantage over

[39] ACCC (2019); Zingales and others (2019); Digital Competition Expert Panel, 'Unlocking Digital Competition', (2019)

[40] Jacobides & Lianos (2021); Mandrescu (2021)

[41] Note: keyword advertising is the use of sponsored results in any kind of search that blends in with the organic results, while display advertising comes in audio-visual forms (pictures, videos or spoken ads) that describe and market the product. Ad placing technology refers to Google's system for placing (targeted) advertisements on websites and ad serving technology refers to the auctioning of advertisements and deciding which advertiser gets published.

[42] Google/DoubleClick, par. 215-288; The Commission formulated three theories of harm: (i) foreclosure scenarios based on DoubleClick's position in ad serving; (ii) foreclosure scenarios based on Google's market position in search advertising and online ad intermediation and (iii) foreclosure scenarios based on the combination of Google and DoubleClick's datasets

Google and DoubleClick concerning integrated advertising technologies, while Google and DoubleClick were only leading in their respective stand-alone product markets.[43] Concerning the combination of the datasets, the Commission noted that privacy and confidentiality were important to publishers and advertisers. Therefore, they would have no incentive to accept changes in Google's terms and conditions to allow more data combination or re-usage.[44] With the lack of ability to foreclose competition by leveraging its position from one market, and the perceived countervailing buyer power of advertisers and publishers, the Commission cleared the merger without requiring any commitments.

The merger's effects on competition were arguably stronger than the Commission had foreseen in its assessment. Post-merger, Google quickly captured a large share of the market and became dominant in digital advertising. Its newly acquired services allowed it to provide a one-stop-shop service for digital advertising which included the auctioning, targeting and publishing of both keyword and display advertising. DoubleClick's technologies for ad publishing and tracking provided Google with know-how that strengthened its search and advertising divisions as well.[45] Moreover, as noted by Google in its notification of the merger to the SEC, the combination of the services gave it access to new customers through improved accessibility. This allowed Google to serve more customers in the long tail of advertising: Google's advertising solution proved particularly popular among small (and often local) entrepreneurs. These small players together created a valuable share of the market.[46]

As Google captured an increasingly large market share, the market share held by Yahoo and Microsoft diminished. The latter became a niche player in online search advertising and the first left the market completely. The long-run harms to consumers as a result of foreclosure exhibited themselves after 2016: while advertisers originally enjoyed the efficiencies of the merger, they no longer had the countervailing buyer power to stop Google from changing its terms and conditions to allow for the combination and re-use of its data. The risk of the creation of 'super profiles' as noted by the Commission became reality, with little to no pushback from advertisers.[47] Moreover, blockaded entry has been observed in recent reports that note that Google is able to extract economic rents without much competitive threat and without entry into

[43] Ibid., par. 222-284

[44] Ibid., par. 359-366

[45] DoubleClick's technologies involved targeting on the basis of cookies and were significantly more efficient than Google's targeting technologies, see Srinisvan D., 'Why Google Dominates Advertising Markets - Competition Policy Should Lean on the Principles of Financial Market Regulation', Stanford Technology Law Review Vol 24.1 (2020); *Why Did Google Buy DoubleClick* (Towards Data Science, 6 May 2020) available at: <https://towardsdatascience.com/why-did-google-buy-doubleclick-22e706e1fb07>

[46] See Srinisvan (2020); Anderson C., 'The Long Tail – How Endless Choice is Creating Unlimited Demand', *Unabridged ed.* (2009)

[47] Oracle's submission to the ACCC's submission for the Digital Platform Inquiry provides an in-depth insight into how Google's policy concerning publishers has changed in attachment B of their submission, which can be found here <https://www.accc.gov.au/system/files/Oracle%20Corporation%20%28March%202019%29.PDF>

to the advertising market.[48] In conclusion, consumers enjoyed short-run efficiencies through improved targeted search advertising. However, the long-run effects of the merger were reduced privacy, fewer consumer choices, weakened countervailing buyer power and higher prices for advertisers.

With respect to foreclosure in the markets relevant to the merger, we argue that the unexpected effects of the merger are explained by the high level of complementarity between two services and the economies of scope derived from operating both services jointly. In the context of Google/DoubleClick's merger, the high levels of complementarity between all activities surrounding advertisements facilitated the creation of a bundled one-stop-shop service. Moreover, the technologies and data acquired with DoubleClick created economies of scope with Google's existing activities in advertising. Access to more data and better technologies led to better targeted advertising and Google's provision of both display and keyword advertising allowed it to develop new 'richer' search advertisement services: Google Shopping and the use of display elements in search.[49]

3.4.2 Microsoft/Skype

Three years after *Google/DoubleClick*, the Commission assessed the *Microsoft/Skype* merger. Microsoft's acquisition of Skype's video calling consumer communication service (CCS), facilitated its entry into the CCS market. The Commission's review of this merger predominantly revolved around the possibility for Microsoft to integrate Skype's CCS functionalities with its proprietary enterprise communications software Lync.[50]

The Commission's concerns revolved around potential foreclosure effects if Microsoft integrated the Skype and Lync functionalities and/or networks.[51] However, the Commission dismissed these concerns on the basis that Skype was not a 'must have' software and that access to the Skype network for enterprises was not dependent on using Lync software. Instead, professional clients could use both Skype and Lync next to one another. Moreover, network effects in the CCS video-calling were not considered to be particularly strong, as most consumers only use Skype to communicate with a small group of friends and family.[52]

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[48] Stigler Committee on Digital Platforms, 'Final Report', Stigler Center (2020), p. 43; Digital Expert Competition Panel, 'Unlocking Digital Competition', (2019) (Furman report), p. 41-42,75; Crémer, de Montjoye and Schweitzer p. 112; ACCC, 'Digital Platform Inquiry – Final Report' (2019) , p. 7-9

[49] The Commission's views on display advertising and Google Shopping are discussed extensively in the *Google Search (Shopping)* case; European Commission, 'Case AT.39740, Google Search (Shopping) – C2017 4444 final, Official Publication of the European Union (2017) (Google Shopping/Google Search);

[50] Case M. 6281, 'Microsoft/Skype C(2011) 7279 Final, *Commission Decision of 07/10/2011 (Microsoft/Skype),* par. 181; Lync offered a combination of instant messaging, video-calling and complex functions such as automatic call distribution, call type detection, call authorization codes, centralized administration and monitoring as well as centralized management and maintenance

[51] Ibid., par. 191-213

[52] Ibid., par. 130, 219

Four years after the merger, Microsoft rebranded Lync as Skype-for-business and integrated the consumer communications and enterprise communications networks with one another. As the Commission noted in their decision, by integrating the networks, Microsoft now offered a communication tool that consumers are familiar with, but that lived up to the standards of quality and safety required for enterprise communications tools.[53]

We do argue however that the complementarity between ECS and CCS services was significantly weaker than those between keyword advertising and display advertising as observed in *Google/DoubleClick* because the products are not as close to one another. The consumer demands and methods for attracting potential customers are too different between these products to create strong complementarities.

We argue this is due to the differences in uses between Skype as a CCS and Skype-for-business as an ECS, which minimizes overlapping use. Skype-for-business is predominantly used for professional communications related to internal or external communications of businesses. Skype, on the other hand, is used for personal communications. As such, even when a user uses both programs, there is little to no benefits that spill-over from using them both personally and professionally. As such, synergies derived from the complementary functions between the two are low. Alternatively, businesses will likely discourage employees from communicating professionally with their personal accounts, that are less secure or lower quality than professional accounts.[54] As such, we argue that the integration of these networks alone provided no reason to block the merger.

In the broader context of the ecosystem and product development however, the acquisition gave rise to significant scope economies in the development of new products. Microsoft launched Microsoft Teams in 2016. This software product runs on Skype technologies, integrates the features of Lync and provides novel features such as document sharing and group chats. Microsoft's acquisition of Skype technologies helped it develop a novel and more comprehensive product.[55]

Finally, Microsoft has attempted to create complementarities between its different services by tying its enterprise communications software and other productivity software. In particular, through Microsoft Outlook, it is possible by default to plan a Skype meeting, but this possibility is not offered for other meetings. With recent updates to Teams, notifications for meetings will automatically redirect users to Teams, even if there is an invitation to a conference via another enterprise communications tool in the invitation itself. Issues related to tying were given more attention in the *Microsoft/LinkedIn* merger discussed *infra.*

[53] Ibid., par. 173-176

[54] Ibid., par. 14, 35

[55] Day M., Microsoft Replacing Skype for Business with Teams (*The Seattle Times, 28 September 2017)*, can be accessed at: https://phys.org/news/2017-09-microsoft-skype-business-teams.html

We conclude that *Microsoft/Skype* did not give rise to the same competitive harms as *Google/DoubleClick* due to the absence of strong complementarities between the relevant markets. While there are significant economies of scope between the acquired service and proprietary technology, the ECS market has remained open to entry and competition. This is showcased by the stable market share of other ECS providers such as Cisco and entry by undertakings such as Zoom. It seems that both some levels of economies of scope and complementarity are required to create independent long-run foreclosure effects in the markets relevant to the merger. In the absence of one of these conditions, foreclosure effects are more likely to be created through anti-competitive leveraging practices such as tying and bundling, the effects of which are already assessed in conglomerate mergers.

3.4.3 Facebook/WhatsApp

The *Facebook/WhatsApp* case was a merger where complementarities between the proprietary service and the acquired service are significantly higher than in *Microsoft/Skype*. This merger involved two services that operated in the CCS market: WhatsApp as a number-dependent consumer communications service and Facebook Messenger as a number-independent service. In this case, there was a limited assessment of the complementarities between these two services as the Commission deemed this merger horizontal.

In its assessment, the Commission argued that potential anti-competitive effects are unlikely as the services had different qualities (Facebook had a richer environment, one is number-dependent and the other number-independent and the privacy policies were different) and consumers showed a tendency to multi-home between the different services. Therefore, the Commission cleared the merger as there was no elimination of a potential competitor and no significant strengthening of market power.[56]

Had the Commission looked at the complementarities between the two services and the economies of scope derived from this merger, its conclusion may have been different. The Commission noted that instant messaging CCS competes on several parameters: reliability of the service, privacy and security, the size of the network and price. When it is accepted that WhatsApp and Facebook Messenger do not compete with one another due to their differences, the Commission could have looked at foreclosure effects similar to portfolio effects. By employing the two largest complementary networks, Facebook offers two free and ad-free services with high reliability, large networks and which differ in privacy and security.[57] Considering these competitive parameters, it can be argued that the acquisition of these products alone provided Facebook with a dominant position in the supposedly separate markets. Moreover, operating both services creates scope economies in the form of know-how and shared inputs and resources (technologies, server

[56] Case M.7217, 'Facebook/WhatsApp C(2014) 7239 Final, Commission Decision of 3.10.2014, par. 1-107

[57] WhatsApp data was not to be used by Facebook and WhatsApp offered end-to-end encryption while Facebook Messenger does not; see *Facebook/WhatsApp,* par. 87-91

space, etc.). By the Commission's focus on the horizontal effects, these conglomerate effects remained underassessed in the decision.

The importance of these conglomerate effects was demonstrated post-merger. Throughout the merger investigation, Facebook had argued that there were technical obstacles to integrating Facebook Messenger and WhatsApp and matching user identities. In August 2016 however, Facebook announced changes to the WhatsApp privacy policies that allowed for the linking of user identities between the two platforms, leading the Commission to address a statement of objections to Facebook. The European Commission fined Facebook EUR 110 million in 2017 for providing misleading information.[58]

After the merger, entry into the consumer communications market for instant messaging seems to be foreclosed. There are still niche operators in the CCS IM market, but they do not have a credible strategy to monetize their services. Moreover, due to a combination of network effects and the quality difference between Facebook-ecosystem products and stand-alone products, countervailing buyer power seems limited. This was once again demonstrated with Facebook's recent announcement of changes in its privacy policy. This evoked a negative response among consumers which threatened to switch to Signal. However, even in light of this protest by consumers the switch to Signal has remained limited in numbers.[59] It should be noted that if these niche operators were completely absent, the changes to the privacy policy might have been even further reaching.[60]

We argue that the high levels of complementarity and scope economies contributed to Facebook's dominant position allowing it to create significant network effects, which are crucial in these markets. Accordingly, users are reluctant to switch from Facebook to other providers. Despite the absence of anti-competitive conduct, it resulted in foreclosure. The merger between Facebook and WhatsApp created efficiencies in the short run, but ultimately led to long-run consumer harms including the degradation of quality in the form of diminished privacy and data protection.

We also observe that the foreclosure effect in the CCS market post-*Facebook/WhatsApp* was greater than in the ECS market post-*Microsoft/Skype.* This is likely partly due to the higher levels of

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[58] European Commission, 'Mergers: Commission fines Facebook €110 million for providing misleading information about WhatsApp takeover', Press Release of 18 May 2017

[59] Lomas N., 'Facebook ordered not to apply controversial WhatsApp T&Cs in Germany' (Tech Crunch, 11 May 2021), can be accessed <https://techcrunch.com/2021/05/11/facebook-ordered-not-to-apply-controversial-whatsapp-tcs-in-germany/>; 'Hern A., WhatsApp to Force Users to Accept Changes of Terms to Service' (The Guardian, 14 May 2021) < https://www.theguardian.com/technology/2021/may/14/whatsapp-to-force-users-to-accept-changes-to-terms-of-service>; Statt N., WhatsApp clarifies it's not giving all your data to Facebook after surge in Signal and Telegram users (The Verge, 12 January 2021), can be accessed: < https://www.theverge.com/2021/1/12/22226792/whatsapp-privacy-policy-response-signal-telegram-controversy-clarification>

[60] '*Facebook admits defeat over controversial WhatsApp privacy policy'* (BGR, 7 May 2021), it should be noted that admitting defeat is an overstatement, Facebook rather gave users the option to opt out over an otherwise automatically implemented change of terms and services.

complementarity between two similar CCS products than between differentiated CCS and ECS products, and the more horizontal relationship between different products in the market for consumer communications services. While a number-dependent and number-independent messaging services may be seen as complementary, both WhatsApp and Facebook Messenger operated in the broader market for Consumer Communications Services. This differs from the acquisition of Skype by Microsoft, where it only established linkages between its CCS and ECS segment. This affirms the idea that the acquisition of highly complementary services in digital markets may have effects akin to horizontal mergers and may have to be assessed as such.

There are however other factors that may play a role. For instance, consumer demands in the ECS market are more heterogeneous than in the CCS market. Users in the ECS markets look for products with more heterogeneous functions and have different demands surrounding the quality and functionalities of a product (for instance, they may want to have communications tools that are heavily focused on stability and high quality, data protection or to broadcast to larger audiences). In order to acquire the optimal product, enterprises have a higher willingness to pay than end consumers. As such, the foreclosure effect that arises in the CCS market by proliferating free products may not occur in ECS markets. It is arguable that the foreclosure effects of conglomerate mergers absent tying are stronger in more homogeneous markets, especially those where price competition is not possible.

3.4.4 Microsoft/LinkedIn

The *Microsoft/LinkedIn* merger marked a change of course for the Commission in their assessment of mergers related to digital ecosystems. This was the first case where the Commission required commitments to clear the merger. It also distinguished itself through its identification of a wide range of markets[61] that were impacted by the merger and by its more elaborate assessment of the potential effects of combining datasets.[62]

The Commission noted several issues that may arise from the merger. First, the Commission focused on the possibility to pre-install LinkedIn on the Windows OS. This would trigger consumers' pre-installation bias and cement LinkedIn's position as the dominant PSN service provider. The Commission considered this to be an anti-competitive form of leveraging and has addressed this by prohibiting it in its commitments.

[61] Case M.8124, 'Microsoft/LinkedIn' C(2016) 8404 Final, Commission Decision of 6.12.2016; (i) PC operating systems, (ii) productivity software, (iii) customer relationship management (CRM) software, (iv) sales intelligence solutions, (v) online communications services, (vi) professional social networking, (vii) online recruitment and online advertising services.; the Commission noted that the market for professional social networks was characterized by high switching costs as users had to invest time into creating and curating their online profile.

[62] Ibid., par 210 et seq.

Secondly, the Commission expressed concerns about the integration of LinkedIn into other productivity software such as Outlook. Were Microsoft to integrate LinkedIn functionalities into its productivity software but deny access to APIs that allowed third-party competitors to do so, this would again provide LinkedIn with a stronger competitive position through leveraging. The Commission again dealt with these forms of leveraging through the commitments required for the clearance of this merger.

Third, the Commission assessed the possible effect of Microsoft's acquisition of Sales Intelligence Solution service (SIS services) technologies held by LinkedIn with Microsoft's existing Customer Relationship Management Software.[63] According to the decision, SIS services complement CRM software as they provide useful insights which can increase the productivity and effectiveness of sales forces. This complementarity was confirmed as industry reports noted that users appreciated sales intelligence solutions that connected directly to their CRM programs and the numerous partnerships that already existed between SIS and CRM service providers.[64] While several notifying parties expressed concerns that Microsoft may have the ability and incentive to foreclosure competitors by tying its SIS and CRM software, a high level of uncertainty remained regarding the profitability of such an endeavour. Ultimately, the Commission decided that the merger did not give rise to serious concerns, as there were still stronger – stand-alone – competitors in the SIS market such as Salesforce, SAP and Oracle.[65]

The assessment of effects on the SIS and CRM markets also connected closely to the Commission's increased assessment of the effects of merging data and datasets. As both SIS and CRM rely heavily on access to data, the Commission investigated whether Microsoft's acquisition of LinkedIn data would allow it to deny access to that data to others, leading to a form of input foreclosure. Pre-merger, LinkedIn did not share its full dataset with any third parties: its full dataset was the driver for its in-house SIS service, while CRM providers were allowed access to a subset of LinkedIn's data.[66] The data in this case was special, as datasets from the SIS and CRM market demonstrate non-generic complementarities with one another. As CRM software is used to manage the ongoing sales processes with existing customers, access to SIS data may help the seller create better insights into additional demands by their customers. A proper CRM and SIS integration may thus better help the seller identify what and when their client wants to make additional purchases. For this reason, both are heavily data-dependent: the company needs information on the client's

[63] Ibid*.,* par. 29; CRM software solutions help companies of various industry sectors manage
their customer interactions by organising, automating, and synchronising data from various sources, such as sales, marketing, customer database, customer service and technical functions. CRM software solutions collate sets of data and display them in a user-friendly manner. This enables companies, in particular the sales department, to improve customer relationships, to better manage accounts, to enhance sales effectiveness, to optimise data quality, and to mitigate regulatory compliance risks.

[64] *Microsoft/LinkedIn,* par. 206 - 217

[65] Ibid par. 218- 245

[66] Ibid., par. 246-250

business and current activities (CRM software) and potential expansions and interests (SIS software).[67] Privileged access to such data may thus give rise to an integrated service of two highly complementary services, both technologically and data-wise.

While notifying parties expressed views that LinkedIn may share its full dataset with third parties to monetize its service, the Commission argued that this was unlikely due to potential privacy concerns by users. Rather, the Commission expressed optimism concerning Microsoft's acquisition of LinkedIn – and Microsoft's subsequent access to data – could lead to pro-competitive efficiencies as it allowed for the development of new products and improving existing products on the market.[68] The Commission's concerns for competition on the basis of data focused squarely on the potential input foreclosure that occurred if Microsoft denied all access to the data. These concerns were ultimately dismissed due to the strong position of competitors and the prevalence of other sources of data for competing providers.[69]

The Commission's assessment of *Microsoft/LinkedIn* showed increased scrutiny of the potential anti-competitive effects of tying or bundling different software services. Through its remedies, the Commission addressed many short-run concerns including pre-installation, the integration of LinkedIn functionalities with other productivity software and access to APIs. While the Commission took note of the acquisition of highly complementary services (CRM and SIS services) and the economies of scope from merging datasets, it still viewed them simply as pro-competitive efficiencies and was not concerned with the long-run effects.

While this paper agrees with the assessment of the Commission that Microsoft's position in the relevant markets in this case was not strong enough to warrant commitments at the time of the merger, as it competed with a dominant undertaking, looking at the ecosystem dimension highlights the necessity to take better account for long-run harms in digital markets is emphasized by two aspects of the decision.

First, Microsoft's increased access to professional services and data allows it to generate economies of scope and complementarities that fortify its position as the dominant provider of enterprise software. This may again lead to entry foreclosure in markets such as the CRM and SIS markets. It is worth monitoring the developments in these markets in the coming year to determine if Microsoft has gained the ability to foreclose long-run entry and extract additional rents from its user base.

Secondly, the commitments regarding the integration of LinkedIn with other Microsoft software were only applicable for five years after the merger.[70] In 2021, when the commitments expired, Microsoft rapidly integrated its LinkedIn services into its other productivity software such as Microsoft Outlook, Microsoft

[67] Ibid., par. 210 *et seq.*
[68] Ibid., par. 249
[69] Ibid., par. 251-277
[70] Ibid., par. 410

Dynamics 365 and Microsoft Teams.[71] By integrating its access to a network of corporate entities, professionals and jobseekers with the services that it offers in enterprise- and consumer communications services and other productivity services, Microsoft has cemented its position as the provider of professional and productivity software. This makes Microsoft central in enterprise operations: they do not only set standards and formats for desktop-based production, but also operate as an intermediary between enterprises and labourers with significant insights into the conditions of the labour market. As digital markets are characterized by the utilization of long-term strategies by digital ecosystem operators and harms that exhibit themselves only in the long run, the five-year applicability of commitments is likely to be insufficient.

While the Commission had expanded its analysis of the relevance of the acquired data in *Microsoft/LinkedIn*, this article argues that in order to capture the full effects of mergers involving digital ecosystems, one must pay attention to the economies of scope and complementarity of data, technologies and the acquired network in light of effects that permeate throughout the ecosystem. A focus on an efficiency advantage in the markets relevant to the merger may be indicative of the existence of long-run harms, but not fully capture the effects of the merger.

3.4.5 Apple/Shazam

Apple/Shazam concerned with the acquisition of Shazam into the Apple ecosystem. Shazam was a provider of Audio Recognition technology that is also active in online advertising. The Commission assessed both the potential competitive effects of the merger of these services and the potential effects of integrating their datasets. The Commission again noted limited horizontal overlap and mostly considered these two undertakings as providers of complementary services, thus focusing on conglomerate effects.

In Apple/Shazam, the Commission applied a similar test as in *Microsoft/LinkedIn.* It assessed the potential anti-competitive behaviour post-merger more strictly when it related to integration, tying or foreclosing access to functionalities or important datasets. However, the Commission found that Apple would have limited abilities and incentives to foreclose competitors post-merger as a result of their position in the market.[72]

The most important aspect of this case is the Commission's increased scrutiny of data-driven effects. First, the Commission studied whether data could be used anti-competitively by leveraging commercially sensitive data. The Commission determined that access to the types of data that Apple would have post-merger was not important enough to actively foreclose competitors such as Spotify. The second – and more

[71] See Microsoft's Official Website, 'Bring your LinkedIn network to Outlook', available online: https://support.microsoft.com/en-us/office/bring-your-linkedin-network-to-outlook-98253fdc-a3c2-47e4-8852-ebb4fbed0bc5

[72] Ibid., par. 231-312

significant – expansion of the assessment was the Commission's considerations on whether Apple would gain a 'big data advantage' by exploiting data generated through Shazam to develop new functionalities or improve existing functionalities. With this assessment, the Commission has introduced a new test in the assessment of conglomerate mergers: it introduced foreclosure based on the efficiency advantage derived from data-driven economies of scope.[73] Shazam data did not generate competitive concerns as it was not more comprehensive than other datasets available on the market and generated data at lower speed and lower user engagement than alternative sources of data. However, the introduction of this novel test ensures that the Commission assessment better reflects the economic realities of digital markets.[74]

The review of the *Apple/Shazam* merger shows that the Commission again increased its scrutiny over mergers involving digital ecosystems. The Commission expanded its test to better account for economies of scope that can be derived from data to improve existing products or develop new products. In light of our proposed theory of harm, this is a necessary step to capture the effects of mergers involving digital ecosystems. The assessment of this merger by the Commission could potentially be improved by better taking into account the long-run effects of economies of scope and complementarities derived from the acquisition of new technologies in relation to the broader ecosystem. Shazam supplied Audio Content Recognition (ACR) software.

The Commission looked into the complementarity between ACR technologies in relation to Apple's music streaming service and online advertising endeavours but did not study whether the acquisition of these technologies results in economies of scope. Moreover, the Commission touches upon the importance of ACR software for the development of voice assistants but did not seem to have the analytical tools to assess whether this would lead to long-run harms.[75] The assessment of Apple/Shazam by the Commission shows increased attention to the economies of scope and complementarity derived from both the acquisition of technologies and data. These issues are however still discussed distinctly, while this article argues in favour of treating them by looking at their combined effects. While the *Apple/Shazam* merger is unlikely to lead to serious competitive harms, these pitfalls in the analysis of mergers involving digital ecosystems become more evident when looking at *Google/Fitbit*.

3.4.6 Google/Fitbit

The *Google/Fitbit* merger marked a merger where data was considered to be a valuable asset and where increased access to data could potentially lead to foreclosure. More importantly, this merger decision highlights the differences how the Commission assessed the roles of complementarity and economies of

[73] Ibid., para. 231-292; in par. 313-325 the Commission discusses the four V's (variety, velocity, volume and value) as indicators on whether the acquired datasets are likely to provide a big data advantage.

[74] Ibid., par. 313-329

[75] Ibid., para 341-348

scope in creating long-run foreclosure between acquired data and acquired products or services. In particular, the Commission argued that efficiencies created from acquired data could cause horizontal foreclosure in the long run. In contrast, regarding the conglomerate effects from acquiring technologies and data, the Commission paid little attention to any efficiency advantages in creating long-run foreclosure. Instead, it looked at traditional leveraging behaviours such as tying when assessing the acquired technologies.

The Commission looked into the relationship between five markets,[76] but our analysis is focused primarily on the markets for wearables, Operating Systems and online advertising, as the assessment of these markets highlight how the Commission's viewed horizontal and conglomerate effects in this merger. In their assessment of the merger's effects on competition, the Commission distinguished between traditional and non-traditional effects on competition. The first category can broadly be understood as referring to the acquisition of technologies and non-data-related assets. Non-traditional effects are then those that relate to the acquisition of data and datasets. A discussion of the Commission's assessment helps to illustrate the difference between these two types of effects.

The assessment of the horizontal effects was discussed first, as this highlighted both the Commission's concerns surrounding the acquisition of technologies and access to data. In its assessment of the traditional effects, the Commission identified a market for all wrist-worn wearables and the second market for wearable Operating Systems. According to the assessment there was a single market for fitness trackers and smart watches, as well as a single market for licensable and non-licensable wearable Operating Systems. The market OS for other smart devices including smartphones were however distinct from the wrist-worn wearable OS market. As a result of this assessment, Apple was considered to be dominant with around 50% of the market share for devices and Operating Systems with its non-licensable OS, Samsung held around 20% of the market share with its devices and its licensable OS and Fitbit only held 0-5% of the market share in both markets. In short, Google's position in the wearable and the wearable OS markets was small. Therefore, the traditional horizontal effect on competition from acquired (non-data related) technologies was considered negligible.[77]

The Commission expressed more concerns about the non-traditional (data-related) effects. Google's access to Fitbit-generated data would give them access to a wide range of sensitive data about Fitbit users. It could use this data to strengthen its advantage in targeted online advertising, further diminishing competition in this market. The Commission desired commitments that would prevent the use of Fitbit data for the purpose of targeted advertising. Google responded by arguing that its increased efficiency should

[76] These markets were (i) the market for wearables; (ii) Operating Systems; (iii) Search engines; (iv) Online advertising and ad tech services and (v) Digital healthcare services.

[77] Ibid., par. 316-384

not be cause for concern.[78] The Commission however responded that "in the long term, given the lack of contestability in these markets, Google would likely raise its prices to both advertisers and publishers (in the case of intermediation services) and would likely reduce its innovation efforts". These negative long-run effects that arose as a consequence of increased barriers to entry and decreased countervailing buyer power are likely to outweigh the short-term benefits enjoyed by advertisers.[79]

This affirms the ideas set out in this article and demonstrates that the Commission is increasingly conscious of the long-run effects associated with economies of scope and complementarity in digital ecosystems, especially when acquisitions of data are involved. In its reasoning, the Commission highlighted that the combination of the possessed and acquired datasets may help Google to enrich the dimensions of the dataset and help Google further granulate its advertising.[80] This highlights that Fitbit data and Google's existing datasets are complementary to one another. Moreover, Fitbit data would strengthen its position in online advertising as well as other ad tech services, highlighting the importance of data as a reusable input that provides economies of scope in the proliferation of multiple (again complementarity) services.[81] Due to the focus on horizontal effects however, the Commission's commitments are limited to the market for online advertising.

The assessment of conglomerate effects focused on anti-competitive leveraging, both through traditional means (tying and bundling or degrading interoperability) and non-traditional means (using commercially sensitive data to foreclose competition). The Commission raised concerns about Google's ability to degrade interoperability between Android and other wrist-worn wearables, and the Commission acted here by imposing commitments.[82] With regards to non-traditional effects, the Commission dismissed concerns by Strava that Google would gain access to commercially sensitive data by acquiring Fitbit. The Commission noted here that Google already had access to this information through installed apps, that the access to this data was not unique to Fitbit and that it would only cover a small portion of Android users. As such, there was unlikely to be a material impact on competition as a consequence of the merger.[83] This demonstrates that the Commission does continue to consider data access as a potential source of competitive advantages even in their assessment of conglomerate effects. However, the Commission's study on the potential economies of scope related to data access from both smartphones and wearables was not as elaborate as in *Apple/Shazam.*

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[78] Google described this reasoning by the Commission as a type of 'efficiency offense', see Vande Walle, S., 'The European Commission's Approval of Google / Fitbit – A Case Note and Comment, *Concurrences Competition Law Review*, Nr. 3-2021; Google/Fitbit, par. 431-446.

[79] Ibid., par. 428-466, 467

[80] Ibid., par. 444-446, 454

[81] Ibid., par. 430-434, 461-466

[82] Ibid., par. 717-791, 944 et seq.

[83] Ibid., par. 835-844

Google's access to data through its integration into wearables may be greater than anticipated at the time of the merger. Google provides the licensable Wear OS for wrist-worn wearables. The Commission anticipated that – as Fitbit only held a marginal market share – other market players such as Apple and Samsung would have more access to data. Unexpectedly, Samsung returned to relying on Google Wear for its new generation of smartwatches around a year after the merger decision was published.[84] As Samsung has around 20% of the market for wearable Operating Systems (aside from Apple's 50-60% market share in non-licensable Operating Systems), Google has gained access to a significant portion of user-generated data in the market for licensable Operating Systems. This access to data may again result in efficiencies advantages related to market entry. Specifically, Google may use its privileged access to data to vertically integrate to take over popular app markets, improve the quality of its wearables and apps and for innovation or expand its ecosystem in other ways.

This may result in similar outcomes as described in the Commission's assessment of horizontal effects in the merger: the short-term efficiencies derived from economies of scope and complementarity may not outweigh a potential duopoly between Apple and Google where the first dominates the market for non-licensable and the latter for licensable wearable Operating Systems. While such considerations are anticipatory, lessons derived from previous mergers demonstrate that there is a serious risk of foreclosure in an otherwise intensely competitive market as a result of this merger.[85] It cannot however be desired from the Commission that they take into account these types of competitive risks, as they had no possibility to foresee this evolution at the time of the merger. This demonstrates that while some long-run effects can be identified, there are clear limits to the ability to anticipate market evolutions.

In conclusion, we argue that the Commission could extend its analysis as used in the assessment of horizontal effects to conglomerate effects. The Commission should consider the efficiencies derived from economies of scope and complementarity as potential sources of foreclosure in the long run and weigh these harms to competition against short-run benefits for consumers. In this respect, we argue in favour of a more elaborate and strict merger assessment. However, as showcased by the example involving Samsung, we cannot expect that *all* scenarios that result in long-run harm can be identified in the merger decision.

3.4.7 Foreclosure effects and lessons for competition policy

Based on the case studies, we argue that *Google/DoubleClick*, *Facebook/WhatsApp*, *Microsoft/LinkedIn* and *Google/Fitbit* demonstrate both strong economies of scope and complementarities. In contrast, *Microsoft/Skype* and *Apple/Shazam* exhibit only strong economies of scope and weak complementarities. Table 2 categorizes these cases.

[84] Heater B., Samsung Returns to Wear OS with the Galaxy Watch 4', (Techcrunch, 11 August 2021)
[85] See Commission Decision in Google/Fitbit, par. 844 et seq.

In the first category where both complementarity and economies of scope are high, the foreclosure effect becomes more pronounced over a relatively short period of time. In *Google/DoubleClick* and *Facebook/WhatsApp*, the mergers have passed for a sufficient period of time such that both foreclosure and consumer harms have been observed. In contrast, Microsoft/LinkedIn and Google/Fitbit are more recent. Thus, we have not observed the full impacts of the mergers. Yet, we have identified similar trends that may result in long-run harms.

The Commission has shown its ability to correctly predicted long-run harms in many cases. However, *Google/Fitbit* demonstrated that it is unreasonable to expect the competition authority to come up with all possible scenarios, especially those influenced by a third party. The ability of the competition authority to act foresighted is (reasonably) limited.

In *Microsoft/Skype* and *Apple/Shazam*, only economies of scope are strong. We have not observed significant foreclosure effects in these cases. There is still sufficient competition in the markets for ECS services. Microsoft has not obtained a leading position in CCS services for video-calling either. Apple/Shazam has happened more recently, so long-run effects cannot yet be assessed. However, the nature of the obtained service and the value of the potential data generated are less likely to create strong foreclosure effects across markets. Instead, the merger created efficiencies for Apple in their voice-driven digital assistants or their ability to personalize services, e.g., Apple Music. While these technologies exhibit economies of scope as they can be reused across products, user synergies are not significantly enhanced. It does not strengthen value-driven lock-in. The risk of foreclosing competitors in markets directly relevant to the merger, e.g., music streaming and voice-driven services, seems to be lower. As such, short-run benefits from efficiencies are more likely to outweigh long-run harms. This observation seems to reoccur in both mergers where only one of the efficiency-enhancing effects is pronounced.

Table 2: Assessing the degree of complementarity and economies of scope

		Economies of scope	
		Strong	Weak
Complementarity	Strong	Google/DoubleClick Facebook/WhatsApp Microsoft/LinkedIn Google/Fitbit	N/A
	Weak	Microsoft/Skype Apple/Shazam	N/A

Compared with Argentesi et al. (2021), we found several similar conclusions. First, Argentesi et al. (2021)'s main criticism is that the UK Office of Fair Trading failed to incorporate the online advertising market. This is in line with our assessment that a competition authority should consider a digital ecosystem as a whole instead of the directly related markets to a merger.

Second, they found that the anticompetitive effects are created by efficiencies from combined user-generated data from two products in the same ecosystem. The combined dataset increases Facebook's competitive advantage in the online advertising market, which negatively affects other players. We observe a similar anticompetitive effect in our assessment. The acquisition of DoubleClick significantly increases Google's competitive advantage. The merger negatively affects its competitors, such as Yahoo and AOL. Similarly, the combined consumer data from the merger between Facebook and WhatsApp allow the merged entity to strengthen its power, preventing other firms from entering the CCS market.

Argentesi et al. (2021) make some similar recommendations to this paper. Firstly, a competition authority must weigh efficiencies from a merger against its anticompetitive effects. Secondly, they also argue that the timeframe of two years is too short to properly assess the effects of a merger in a digital market. Thirdly, a competition authority should be aware that uncertainties from long-run assessments are unavoidable.

We identify three lessons from the case studies in relation to the assessment of mergers involving a digital ecosystem. First, an acquisition of new data and technologies that demonstrates high levels of complementarity and economies of scope with the existing products and services in the ecosystem carries a high risk of entry foreclosure and long-run harms to consumers. In a merger where a competition authority identifies both effects to be strong, there should be an in-depth investigation into potential long-run harms. In a merger where only one effect is strong, a more limited assessment of long-run harms will likely suffice. The current approach of assessing whether the merger provides the acquiring entity with an ability and incentive to foreclose, does not capture the potential harms associated with conglomerate mergers involving digital ecosystems.

Second, the effects of complementarity and economies of scope that permeate throughout an ecosystem are hard to capture fully due to the expansive nature of digital ecosystems. Thus, defining and assessing all relevant markets in a merger can be extremely challenging. To step away from the market definition, the Commission may have to adopt a distinct approach to assessing digital ecosystems. A more formalistic approach is warranted. The Commission does not have to prove the effects within a certain market but will rather have to formulate a convincing theory of long-run harm.[86] Other authors have argued that competition authorities should no longer look at a merger as a one-off situation. Instead, it should be taken into account whether the acquisition is a part of a larger series of acquisitions or part of an anti-competitive strategy.[87]

[86] Zingales and others (2019) for instance argue in favour of an adjusted burden of proof for mergers involving digital ecosystems. Herbert Hovenkamp, 'Antitrust and Platform Monopoly' (2021) 130 Yale Law Journal <http://ssrn.com/abstract=3639142.Electroniccopyavailableat:https://ssrn.com/abstract=3639142> explores whether market definition is helpful in determining potential harms to competition in digital markets, or whether we should rely on 'direct measurements' of market power to determine the ability to harm competition

[87] Digital Competition Expert Panel (2019); Zingales and others (2019); Hemphill S. & Wu T., 'Nascent Competitors' 168 U. Pa. L. Rev. 1879 (2019)

Third, the case studies show that the Commission was able to identify several specific long-run harms in their assessments that have materialized, such as the creation of super profiles, the merging of datasets or functionalities and the changing of privacy policies. Yet, the Commission did not require any commitments from the undertakings. So, a more proactive approach may be required to mitigate long-run harms, and the competition authority must balance the short-run efficiencies against the potential long-run harms.[88]

The next section contains policy suggestions that aim to improve the Commission's ability to combat long-run harms that result from an efficiency advantage held by digital ecosystems, while mitigating issues such as the uncertainty of long-run effects. These recommendations should provide competition authroities with tools and ability to address those potential long-run harms.

3.5 Policy recommendations

We make three policy recommendations in this section. First, competition authorities should treat conglomerate mergers involving digital ecosystems as if they are horizontal. Second, competition authorities must consider long-run effects and mitigate potential anti-competitive effects that arise from the merger. We recommend that competition authorities use flexible remedies that trigger when an identified scenario of potential harms materializes. Flexible remedies are a novel tool that complements competition authorities' power to require commitments from merging parties. The third recommendation is to develop a distinct policy to control mergers in digital markets. We discuss the scope of application for these recommendations and how efficiencies and harms can be balanced in digital markets. This allows competition authorities to consider the specifics of mergers involving a digital ecosystem without convoluting the merger control framework for non-digital mergers.

3.5.1 Applying standards of assessment for horizontal mergers in digital ecosystem mergers

Competition authorities can assess conglomerate mergers involving a digital ecosystem more akin to horizontal mergers. This is because efficiencies from these acquisitions may remove potential competition by foreclosing nascent, indirect or future competitors. These acquisitions may also insulate the core markets of an ecosystem from competitive pressures.

By relying on a standard of assessment that is more akin to the assessment of horizontal mergers, the competition authority can weigh the perceived efficiencies of a merger against the potential anti-

[88] Pierre Larouche and Alexandre de Streel, 'The European Digital Markets Act: A Revolution Grounded on Traditions' (2021) 12 19 also argue that it seems that maintaining a long-run perspective is necessary in digital markets, by weighing short-run efficiencies against long-run potential dynamic competition the competition authority can protect dynamism in digital markets. This may lead to consumers being better served in the long-run.

competitive effects. These anti-competitive effects may be related to the foreclosure of a potential competitor or entrant.[89] Here, complementarity and economies of scope between the acquired goods and services and the services offered by the incumbent ecosystems may be indicators that the competition authority must exercise increased scrutiny.

Digital markets are highly dynamic. The used products or the ways for which products are used can change rapidly.[90] As a result, activities in nascent markets have a more horizontal relationship to one another than they did in non-digital markets. This is exemplified by the one-stop-shopping solution developed after Google acquired DoubleClick. Here, the integration of activities in digital markets creates seemingly horizontal effects that impede entry across markets, both in adjacent markets (the integrated ad space, - intermediation and -serving markets) and throughout the ecosystem. The integration of the adjacent markets leads to higher entry barriers as a competing provider of advertising would have to develop a one-stop-shop solution of its own to provide a product of equal or greater value. The integration of new advertising technologies also permeates throughout the entirety of the ecosystem, and the expansiveness of the ecosystem determines the value of the integrated advertising services. Google's position in search advertising is predominantly determined by its dominant position in search. However, the ecosystem as a whole consists of building blocks that help it to strengthen its competitive advantage in search through economies of scope and complementarity.

Nevertheless, bundling or integration is not necessary to achieve these quasi-horizontal effects. Facebook's acquisition of WhatsApp has strengthened its position in the market for communications as it allows Meta to offer both complementary number-independent and number-dependent messaging services that operate on shared technologies. As a result, entry into the broader market for consumer communications is more difficult as entrants will have to offer a service with a higher value than those offered by Facebook, irrespective of whether Facebook's current users use one or both of these services. This indirectly impedes an entrant's ability to challenge Facebook's position in its core market by diminishing the risks of indirect competition in other markets, such as social media.[91]

So, the presumption that conglomerate mergers are less likely to lead to competitive harms but are efficiency-enhancing should not apply to mergers involving a digital ecosystem. Instead, competition authorities have to weigh efficiencies against potential anti-competitive effects, rather than limiting

[89] European Commission, Guidelines on the assessment of horizontal mergers under the Council Regulation on the control of concentrations between undertakings, OCJ 031, 5 February 2004

[90] Ezrachi & Stucke (2022)

[91] Pierre Larouche and Alexandre de Streel, 'The European Digital Markets Act: A Revolution Grounded on Traditions' (2021) 12 19; A Ezrachi and ME Stucke, How Big-Tech Barons Smash Innovation—and How to Strike Back (HarperCollins 2022) both argue that Facebook's acquisition of WhatsApp was aimed at preventing WhatsApp from growing out into a competing social media that would threaten to displace Facebook. The acquisition had a dual effect: it removed a potentially direct competitor in social media (horizontal) and strengthened Meta's competitive advantage in the markets where this competitive threat had arisen (quasi-horizontal).

themselves to determining whether the incumbent has the ability and incentive to behave anti-competitively as a result of the merger. The acquiring ecosystem would, in turn, have to prove that the created efficiencies are specific to the merger, lowering the evidentiary burden for the competition authority to intervene.[92] This moves the analysis away from the ability and incentive to act anti-competitively and more towards a balancing test.

3.5.2 Assessing long-run effects, uncertainty & flexible remedies

As demonstrated by the ex-post reviews of case law, the anti-competitive effects of digital mergers may exhibit themselves at a later stage which currently falls outside of the scope of the Commission assessment, or exhibit themselves after the time that the commitments have expired. Thus, the assessment of these long-run effects is highly important in preventing harms to competition.

In assessing long-run effects, uncertainties are unavoidably involved. Hence, we recommend that competition authorities use flexible remedies to help strike a balance between safeguarding the long-run competitiveness in digital markets and dealing with uncertainty. The competition authority can introduce preliminary commitments that may trigger if specific scenarios – or activities with the same *de facto* outcome – identified in the review of the merger materialize. The exact remedies to the anti-competitive effects (or harms) that arise from the materialized scenarios can then be determined at a later time.

Flexible remedies can encompass a range of different interventions by the competition authority. The competition authority can prohibit the merged entity from merging certain datasets, changing their terms and conditions vis-à-vis professionals or end-users if these are viewed as exploitative, prohibiting the merger of complementary networks or even mandating the ex-post divestiture of parts of the merged entity.

Which remedies are imposed to resolve the long-run anti-competitive effects should be determined once the scenario has materialized. Having more time to assess the actual effects of the merger provides the competition authority with more information on suitable remedies. To safeguard legal certainty, it is desirable if the competition authority lays out certain categories of remedies that it will consider in case the long-run effects materialize. For instance, issues related to a potential merger of networks can be remediated by a requirement to separate the networks if the merger leads to foreclosure effects, facilitate competitors with access to the users in the network or mandate the ex-post divestiture of parts of the merged entity related to the harmful activities.

Flexible remedies have three advantages. First, the competition authority will not have to start a new and potentially lengthy investigation into the harms and anti-competitive effects of the behaviour, but it can rely on the limitations set out in the merger decisions. Second, it provides ex-ante clarity for the undertaking about which behaviour is allowed or disallowed post-merger. Third, the competition authority has the

[92] Hemphill & Wu (2019)

benefit of delaying its decision-making. As noted earlier, the types of harm and harmful effects may become clear only after the fact. Flexible remedies provide the competition authority with parameters for the data collection on effects on price, quality or innovation in markets post-merger, allowing for a more informed decision on whether remedies are necessary and which types of remedies are suitable.

Due to the forward-looking nature of merger assessment, competition authorities that consider an acquisition of competitors in connected markets before the fact are in the unusual position that delay may, in some cases, increase the accuracy of the decision.[93] For instance, when the Commission had to calculate the fine on Facebook for providing misleading information in its acquisition of WhatsApp, the necessary information could be observed after the conclusion of the merger.

Hemphill & Wu (2019) argue that – when faced with uncertainty – the potential loss of value associated with the foreclosure of innovative entrants creates a bias *towards* action. While there is an uncertainty that durable competition will emerge as a result of the intervention by competition authorities, even a modest probability that high-impact innovations are realized would warrant action.[94] Similar considerations apply in assessing potential long-run harms to competition as a result of the competitive advantage secured by an incumbent following a merger. Even if the merger does not necessarily result in market foreclosure and harms to consumers, a modest threat of long-run harms to competition may warrant the imposition of remedies or even the blocking of an acquisition by an incumbent ecosystem operator.

Therefore, it suffices that the competition authority can create a reasonable account as to which types of long-run harms may occur. In the current treatment of conglomerate mergers, the competition authority must prove that a merger bestows the ability and incentive to act anti-competitively after the merger. When dealing with incumbent ecosystem operators, simply proving the ability may suffice, as the intent and methods by which foreclosure occurs often exhibit themselves at a later stage and are not easily captured in an *ex-ante* assessment. The presumption of conglomerate mergers that they are more likely to create efficiencies than anti-competitive effects create an unwarranted burden on competition authorities to prove the latter and fails to manage costly 'false negatives' associated with mergers involving a digital ecosystem.

3.5.3 Scope of application and balancing efficiencies and harms

We recommend that a distinct standard of assessment for conglomerate mergers involving a digital ecosystem should be developed. In Section 3.5.1, we argue that horizontal elements should be incorporated in assessing mergers involving a digital ecosystem. However, directly using the horizontal merger guideline may not be the best option.

[93] Hemphill & Wu (2019)
[94] Hemphill & Wu (2019)

The reasons for a distinct standard are as follows. First, because of high linkages in a digital ecosystem due to complementarity and economies of scope, there are potentially many directly or indirectly relevant markets to a merger involving a digital ecosystem. Defining and assessing all relevant markets are likely to be time-consuming and costly exercises. Second, as mentioned earlier, the competition faced by a digital ecosystem operator is usually indirect. Competitors are unlikely to compete by offering the same products or services as the incumbent. Hence, the horizontal merger guideline may not be applicable in all situations.

With a standard of assessment specific to conglomerate mergers involving a digital ecosystem, the treatment of conglomerate mergers more akin to horizontal mergers, difficulties with the market definition in changing and potentially integrated markets, alternative thresholds for intervention and the use of flexible remedies can be reserved specifically for the digital sector. This diminishes the risks of developing divergent standards for the assessment of mergers and creates legal certainty for digital and non-digital undertakings by providing clarity about which regime applies to them.

Recommendations to create a merger policy specifically for digital markets are not unique to this article. Uwe-Franck, Monti & De Streel (2021) have also discussed the desirability and legal feasibility of alternative or additional merger rules for digital markets. The policy options set out by these authors include recommendations to set out new rules on mergers in the Digital Markets Act, or to create a merger regime specifically for digital markets that runs parallel to the general Merger Control Regulation.[95] The insights produced in this paper support the idea that alternative rules may be necessary to capture the full effects on competition of conglomerate mergers. Lianos & Carballa (2021) describe extensively how the contemporary approach to defining markets has issues with capturing the different effects that arise in complex networks and ecosystems. These authors argue in favour of divergent standards for merger policy in digital markets and the treatment of ecosystems specifically due to the feedback loops that arise between various groups of users and across markets.[96]

Even though traditional conglomerate mergers in non-digital markets may also exhibit some degree of complementarity and economies of scope, we do not extend our recommendations or comments on the desirability of using this analytical framework in non-digital markets. As discussed earlier, there are three main differences between traditional conglomerate mergers and mergers involving a digital ecosystem. To briefly repeat them here, first, complementarity and economies of scope of significantly stronger when a digital ecosystem is involved due to its expansive nature. Second, data generated in digital markets is an important source that is less applicable in non-digital markets. Finally, the competition faced by digital ecosystem operators is typically indirect.

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[95] Uwe-Franck, J, Monti G., De Streel A., 'Article 114 as the Legal Basis for Strengthened Control of Acquisitions by Digital Gatekeepers', Legal Opinion commissioned by the German Federal Ministry for Economic Affairs and Energy (2021)

[96] Lianos & Carballa (2021), p. 14

The policy suggestions set out in this section help to capture sources of barriers to entry, foreclosure and long-run harms. They help strike a balance between the risk of overenforcement and underenforcement when faced with the complexity of interactions in digital markets and uncertainty related to the assessment of long-run effects.

3.6 Conclusion

This paper looks at the long-run effects of conglomerate mergers involving a digital ecosystem on potential entry. Given the two characteristics abundantly found in a digital ecosystem – complementarity and economies of scope – the possibility of foreclosure is more likely than in a traditional conglomerate merger. Hence, the anti-competitive effects are more conspicuous than is the case for a conglomerate merger in a non-digital market. Given we have observed ubiquitous mergers by big-tech companies in the past decade, the reassessment of merger control on conglomerate mergers involving a digital ecosystem is warranted.

We develop a simple economic model that allows us to incorporate both complementary and economies of scope into the analysis. These characteristics guarantee that when an incumbent ecosystem acquires a stand-alone firm, the entry barrier is unavoidably increased. A more potent synergy between the ecosystem and the stand-alone firm leads to a higher entry barrier. As such, this provides an additional incentive for the incumbent to merge. In the short run, the efficiency of the ecosystem increases. However, the merger may prevent entry and preserve the incumbent's monopoly power in the long run. As such, a competition authority should consider the dynamic aspect of the merger in its assessment. If it fails to incorporate the dynamic consideration, the merger might be cleared when it should be blocked.

Our review of the case studies demonstrates that the long-term effects derived from the digital ecosystem perspective were not given a fair weight. While some effects were hard to foresee at the time, other long-run problems that were identified by the Commission were – in hindsight - dismissed too easily. The changing of privacy policies by Google and Facebook, the value changes of LinkedIn's data, Apple's dominant position in smartwatches and Skype's integration into Lync were all neglected in the assessments. Moreover, the Commission tends to pay significant amounts of attention to issues arising from network effects, while it did not pay attention to economies of scope until *Apple/Shazam*, and only implicitly discussed complementarity under concerns on tying and bundling. *Google/Fitbit* already shows increased attention by the Commission for both long-run effects and economies of scope. However, the merger regime may still have to become stricter to keep digital markets competitive.

Firstly, we argue that conglomerate mergers involving a digital ecosystem have effects that are more akin to horizontal mergers than conglomerate mergers in non-digital markets. The variety of products offered on a platform, their connected markets and the existence of product ecosystems have heightened the cross-market effects of a merger. Moreover, it seems that indirect competition – or the role of nascent competitors – is more important in digital markets. As a result, conglomerate mergers involving digital ecosystems should be assessed in a similar manner as horizontal mergers, and the presumption that conglomerate mergers are less likely to lead to anti-competitive effects should not apply.

Secondly, the fast-changing nature of digital markets makes it more important to weigh short-run efficiencies against long-run harms to competition. In order to capture the full effects of a merger, the competition authority must act foresighted rather than myopic. Consequently, a determination that the short-run efficiencies outweigh short-run harms is insufficient, and the future of market development must be considered. This article recognizes the element of uncertainty that is inherent to *ex ante* – and more so long-term – assessments. In order to strike a balance between the inherent uncertainty and the need to develop tools to deal with long-run harms, this article proposes the introduction of flexible commitments, which only trigger once specific scenarios (or activities with the same *de facto* outcomes) materialize. This allows the competition authority to set clear limitations to the behaviour of a firm without unduly burdening the firm with potentially unnecessary commitments.

Finally, we recommend a distinct policy for mergers involving a digital ecosystem. Such a distinct policy allows the assessment to incorporate elements of mergers specific to digital markets without affecting the assessment of mergers in non-digital markets. We have noted that the observations of this paper and recommendations are limited to mergers in digital markets and that we remain agnostic to the potential applicability of our recommendations to non-digital markets due to their different intrinsic nature.

3.7 Appendix

An objective of the appendix is to justify the relationships between profits and consumer surplus as assumed in Assumptions 2 and 3. We introduce the full model where consumers have heterogeneous preferences toward the incumbent and the entrant à la Hotelling. The incumbent and the entrant compete on price to attract consumers.

A1 The set-up of the full model

There are two ecosystems. The incumbent's ecosystem Θ_I has an intrinsic value $v(\Theta_I)$. The marginal cost of supplying Θ_I is $c(\Theta_I)$. Similarly, the potential entrant's ecosystem Θ_E provides an intrinsic value $v(\Theta_E)$ with the associated marginal cost $c(\Theta_E)$. Define $s(\Theta_i) \equiv v(\Theta_i) - c(\Theta_i)$ as the net surplus of ecosystem i. Each ecosystem sets its price p_i, where $i \in \{I, E\}$.[97] If the potential entrant wants to enter, it incurs an entry cost, denoted by F.

Similar to Section 3.3, the ecosystems have the two characteristics which are complementarity and economies of scope. So, Assumption 1 also holds in the full model.

Consumers are heterogeneous in their preference toward the ecosystems à la Hotelling. Specifically, a unit mass of consumers is uniformly located on a unit-length line. The locations of the incumbent and the entrant (if enters) are at points 0 and 1, respectively. For simplicity, the location of the incumbent is assumed to remain the same even though the entrant does not enter. The utility of a consumer at point d on the line if she buys from the incumbent (u_I) or the entrant (u_E), respectively, are

$$u_I(\Theta_I, p_I) = v(\Theta_I) - td - p_I \text{ and } u_E(\Theta_E, p_E) = v(\Theta_E) - t(1-d) - p_E,$$

where t is a disutility parameter. In other words, consumers who are located closer to the left prefer the incumbent's ecosystem, holding other factors constant, and vice versa, .

In addition, firm A is a monopolist in market A. The product supplied by firm A has an intrinsic value $v(A)$ and the marginal cost $c(A)$. Assume that $s(A) \equiv v(A) - c(A) > 0$. For simplicity, all consumers have the same preference towards firm A. They receive gross utility $v(A)$ if they consume the product. Firm A sets the price p_A if it is active.

The incumbent has an option to acquire firm A before the entrant decides to enter. The incumbent makes a take-it-or-leave-it offer T to firm A. If the merger is successful, the incumbent's set of products and services becomes $\Theta_I \cup$ A.

[97] In reality, digital ecosystems may set different prices for different products and services and sell them separately. Consumers may only buy a subset of products and services. Nevertheless, we assume that each ecosystem sets only one price: that is, it bundles all of its products together. This greatly simplifies the analysis. In addition, as argued by Neven (2008), the biggest concern for conglomerate mergers is also tying and bundling. The price here can be explicit (retail price) or implicit (consumer data that the firm collected).

The timing of the game is as follows. (1) The incumbent decides whether to acquire firm A by offering a take-it-or-leave-it offer T. (2) The entrant decides if it will enter with the entry cost F. (3) All active firms set their prices simultaneously. (4) Each consumer decides which ecosystem to join (single homing) and whether to buy product A if firm A is active. Then, the payoffs are realized. The information is perfect. The solution concept is the SPNE.

Before we start solving the model, it will be useful to show that the combination of complementarity and economies of scope is sufficient to guarantee that a more extensive ecosystem could generate more surplus than a smaller one. Lemma 2 formalizes this result.

Lemma 2: If $s(\Theta') > 0$, complementarity and economies of scope guarantee that the surplus of the merged ecosystem – $s(\Theta \cup \Theta')$ – is larger than the surplus pre-merger – $s(\Theta)$. That is,

$$s(\Theta \cup \Theta') > s(\Theta).$$

Proof: By definition, we have $s(\Theta \cup \Theta') = v(\Theta \cup \Theta') - c(\Theta \cup \Theta')$. Due to complementarity and economies of scope, $v(\Theta \cup \Theta') > v(\Theta) + v(\Theta')$ and $c(\Theta \cup \Theta') < c(\Theta) + c(\Theta')$. Hence,

$$s(\Theta \cup \Theta') > \big(v(\Theta) - c(\Theta)\big) + \underbrace{\big(v(\Theta') - c(\Theta')\big)}_{>0} > v(\Theta) - c(\Theta) \equiv s(\Theta). \qquad \blacksquare$$

Intuitively, the result is derived from the characteristics that the merged entity can create higher value at a lower cost. On the contrary, if one of them does not hold, the surplus post-merger could be lower.

A2 The relationship between the profits and consumer surplus

In this section, we solve the model for the equilibrium prices, profits, and consumer surplus. The game is solved by using backward induction.

Before going into competition between both ecosystems, we first analyse the strategy of firm A when it is not acquired by the incumbent. Given that all consumers value product A at $v(A)$, firm A, which is a monopolist, will charge the price equal to the consumers' valuation, regardless of the entry decision. All consumers are willing to buy. Since firm A extracts all of the surpluses from the consumers, consumer surplus generated by firm A is zero, i.e., $CS_A = 0$. This observation is summarized in the following lemma.

Lemma 3: When firm A is active, it charges the monopoly price $p_A = v(A)$. All consumers are willing to buy. The profit of firm A is $\pi_A = s(A) \equiv v(A) - c(A)$. And consumer surplus from product A is zero ($CS_A = 0$).

At the last stage of the game, the consumers decide which ecosystem they will participate. There are four subgames. These include whether the incumbent acquires firm A and whether the entrant enters. Consider the two subgames where the entrant does not enter, while firm A is or is not acquired. In these subgames, the incumbent is a monopolist. Let the set of incumbent's products and services is Θ_I', where $\Theta_I' = \Theta_I$ if the incumbent does not merge with firm A and $\Theta_I' = \Theta_I \cup A$ if they merge. Given the price p_I, the consumers will participate in the incumbent's ecosystem if $v_I(\Theta_I') - td - p_I \geq 0$, or

$$d \leq \max\left\{\frac{(v_I(\Theta_I') - p_I)}{t}, 1\right\}.$$

The incumbent chooses p_I to maximize its profit. By solving the first-order condition, it is straightforward to get the equilibrium price under the monopoly p_I^m given Θ_I', which is

$$p_I^m(\Theta_I') = \begin{cases} \dfrac{v(\Theta_I') + c(\Theta_I')}{2} ; if\ s(\Theta_I') < 2t \\ v_I(\Theta_I') - t ; if\ s(\Theta_I') \geq 2t. \end{cases}$$

The corresponding monopoly profit is

$$\pi_I^m(\Theta_I') = \begin{cases} \dfrac{s(\Theta_I')^2}{4t} ; if\ s(\Theta_I') < 2t \\ s(\Theta_I') - t ; if\ s(\Theta_I') \geq 2t. \end{cases} \tag{3.3}$$

Now, we analyse the other two remaining subgames where the entrant enters. Given the prices set by the incumbent and the entrant, each consumer decides which ecosystem she will participate. That is, each consumer compares the utility from joining the entrant's ecosystem $u_E(\Theta_E, p_E)$ with $u_I(\Theta_I, p_I)$ if the incumbent and firm A did not merge or $u_I(\Theta_I \cup A, p_I)$ if they merge.

Under the duopoly, the incumbent and the entrant must have an incentive to operate. They must earn a positive profit margin, i.e., $p_i^d - c_i \geq 0$ for $i \in \{I, E\}$. Accordingly, the value of the entrant's ecosystem must be sufficiently large to compete with the incumbent under both cases whether the merger occurs or not. Similarly, the incumbent, merged or unmerged, must also be able to compete with the entrant's ecosystem if it enters. These conditions are satisfied when the values of the two ecosystems are not too far apart from each other. Hence, we make the following assumption.

Assumption 4: The difference between the surpluses of the incumbent's ecosystem and the entrant's ecosystem is not too different such that

$$|s(\Theta_I \cup A) - s(\Theta_E)| \leq 3t,$$

and

$$|s(\Theta_I) - s(\Theta_E)| \leq 3t.$$

Each ecosystem sets its price to maximize its profit taking the price of another ecosystem as given. Under Assumption 4, the equilibrium price for ecosystem i, where $i \in \{I, E\}$, is

$$p_i^d(\Theta_i, \Theta_j) = c(\Theta_i) + t + \frac{s(\Theta_i) - s(\Theta_j)}{3} > c(\Theta_i),$$

where $i \neq j$. The duopoly profit excluding the entry cost (when applicable) is

$$\pi_i^d(\Theta_i, \Theta_j) = \frac{\left(3t + s(\Theta_i) - s(\Theta_j)\right)^2}{18t}. \quad (3.4)$$

Intuitively, the equilibrium price and the equilibrium profit under the duopoly of each ecosystem depend positively on its surplus $s(\Theta_i)$ and negatively on the other ecosystem's surplus $s(\Theta_j)$. Furthermore, notice that the profits of both ecosystems increase with the disutility parameter t. When t is low, a consumer who is located far away from ecosystem i still derives high utility from the ecosystem. If p_i is low enough, these consumers will participate in ecosystem i. Thus, ecosystem i has a higher incentive to compete more aggressively to get the consumers. Consequently, ecosystem j responds by keeping p_j low as well. The disutility parameter t represents the degree of competition: the lower the parameter, the higher the degree of competition.

Given the monopoly profit $\pi_I^m(\Theta_I')$ and the duopoly profit $\pi_i^d(\Theta_i, \Theta_j)$ in Equations (3.3) and (3.4), respectively, we can summarize the relationships between them to show that Assumption 2 is justified.

Lemma 4: The relationships between the equilibrium profits are as follows.

i. *The monopoly profit with the merger $\pi_I^m(\Theta_I \cup A)$ is higher than the monopoly profit without the merger $\pi_I^m(\Theta_I)$, i.e., $\pi_I^m(\Theta_I \cup A) > \pi_I^m(\Theta_I)$.*
ii. *Incumbent's duopoly profit with the merger $\pi_I^d(\Theta_I \cup A, \Theta_E)$ is higher than its duopoly profit without the merger, i.e., $\pi_I^d(\Theta_I \cup A, \Theta_E) > \pi_I^d(\Theta_I, \Theta_E)$.*
iii. *Entrant's duopoly profit with the merger $\pi_E^d(\Theta_E, \Theta_I \cup A)$ is lower than its duopoly profit without the merger $\pi_E^d(\Theta_E, \Theta_I \cup A)$, i.e., $\pi_E^d(\Theta_E, \Theta_I \cup A) < \pi_E^d(\Theta_E, \Theta_I)$.*

Proof: According to Equation (3.3), the monopoly profit $\pi_I^m(\Theta_I')$ is increasing in $s(\Theta_I')$. In addition, due to Lemma 2, we have $s(\Theta_I \cup A) > s(\Theta_I)$. Therefore, $\pi_I^m(\Theta_I \cup A) > \pi_I^m(\Theta_I)$ as stated in the first part of the lemma.

According to Equation (3.4), incumbent's duopoly profit $\pi_I^d(\Theta_I', \Theta_E)$ is increasing in $s(\Theta_I')$. And entrant's duopoly profit $\pi_E^d(\Theta_E, \Theta_I')$ is decreasing in $s(\Theta_I')$. Since $s(\Theta_I \cup A) > s(\Theta_I)$, we have $\pi_I^d(\Theta_I \cup A, \Theta_E) > \pi_I^d(\Theta_I, \Theta_E)$ and $\pi_E^d(\Theta_E, \Theta_I \cup A) < \pi_E^d(\Theta_E, \Theta_I)$. ■

To support the assumptions on consumer surplus (Assumption 3), we first calculate consumer surplus given the equilibrium price. Consumer surplus depends on the eventual market structure and the appropriate sets of products and services of the incumbent and the entrant.

Lemma 5: Consumer surplus under the monopoly (CS^m) and the duopoly (CS^d) given the sets of products and services Θ_I' and Θ_E, respectively, are

$$CS^m(\Theta_I') = \begin{cases} \dfrac{s(\Theta_I')^2}{8t} \ ; if \ s(\Theta_I') < 2t \\ \dfrac{t}{2} \ if \ s(\Theta_I') \geq 2t, \end{cases} \tag{3.5}$$

and

$$CS^d(\Theta_I', \Theta_E) = \frac{1}{36t}\left[\left(s(\Theta_I') - s(\Theta_E)\right)^2 + 9t\left(2\left(s(\Theta_I') + s(\Theta_E) - 5t\right)\right)\right]. \tag{3.6}$$

Proof: For the monopoly outcome, when $s(\Theta_I') < 2t$, the market is not covered. Only consumers whose $d \leq \frac{s(\Theta_I')}{2t}$ buy from the monopolist. Thus, consumer surplus is

$$\int_0^{\frac{s(\Theta_I')}{2t}} \left(v(\Theta_I') - td - \frac{v(\Theta_I') + c(\Theta_I')}{2}\right) dd.$$

When $s(\Theta_I') \geq 2t$, all consumers buy from the monopolist. Accordingly, consumer surplus can be calculated as

$$\int_0^1 \left(v(\Theta_I') - td - \left(v(\Theta_I') - t\right)\right) dd.$$

By direct calculations, we get the first part of Lemma 5.

For the duopoly outcome where the entrant is active in the market, recall that consumers whose $d \leq d(p_I^d, p_E^d)$ will buy from the incumbent, while others will buy from the entrant. So, the consumer surplus is

$$CS^d(\Theta'_I, \Theta_E) = \int_0^{d(p_I^d, p_E^d)} \left(v(\Theta_I') - td - p_I^d\right) dd + \int_{d(p_I^d, p_E^d)}^1 \left(v(\Theta_E) - t(1-d) - p_E^d\right) dd,$$

Integrating out the above equation yields the second part of the proposition. ■

Given consumer surplus under the monopoly $CS^m(\Theta_I')$ and under the duopoly $CS^d(\Theta'_I, \Theta_E)$, in Equations (3.5) and (3.6), respectively, the relationships between consumer surplus are summarized in the following lemma. Noted that $CS_A = 0$ according to Lemma 3.

Lemma 6: The relationships between consumer surplus are as follows.

i. *The merger between the incumbent and firm A (weakly) increases consumer surplus under the monopoly, i.e.,* $CS^m(\Theta_I \cup A) \geq CS^m(\Theta_I)$.

ii. *The merger between the incumbent and firm A (strictly) increases consumer surplus under the duopoly, i.e.,* $CS^d(\Theta_I \cup A, \Theta_E) \geq CS^d(\Theta_I, \Theta_E)$.

Proof: According to Equation (3.5), $CS^m(\Theta_I')$ is strictly increasing in $s(\Theta_I')$ when $s(\Theta_I') < 2t$, and $CS^m(\Theta_I')$ remains constant when $s(\Theta_I') \geq 2t$. Furthermore, we have $s(\Theta_I \cup A) > s(\Theta_I)$ from Lemma 2. Hence, $CS^m(\Theta_I \cup A) \geq CS^m(\Theta_I)$ as stated in the lemma.

Under the duopoly, it is straightforward to see in Equation (3.6) that $CS^d(\Theta_I', \Theta_E)$ is strictly increasing in $s(\Theta_I')$. Since $s(\Theta_I \cup A) > s(\Theta_I)$, we have $CS^d(\Theta_I \cup A, \Theta_E) \geq CS^d(\Theta_I, \Theta_E)$. ■

When the market is not covered under the monopoly ($s(\Theta_I') < 2t$), the monopoly price increases less than the created value, i.e., $p_I^m(\Theta_I \cup A) - p_I^m(\Theta_I) < v(\Theta_I \cup A) - v(\Theta_I)$, and more consumers are buying from the incumbent. So, consumer surplus increases. On the other hand, if the market is already covered ($s(\Theta_I) \geq 2t$), the merger does not affect consumer surplus. The incumbent can increase the price equal to the created value, while the number of consumers is still bounded at one. Under the duopoly, the incumbent cannot increase the price equal to the created value since the incumbent must compete with the entrant. Consumer surplus under the duopoly always increases with the merger.

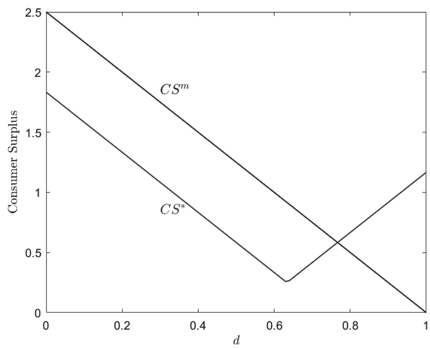
However, there is no guarantee that consumer surplus under the duopoly $CS^d(\Theta_I', \Theta_E)$ is more or less than consumer surplus under the monopoly $CS^m(\Theta_I')$. In most cases, $CS^d(\Theta_I', \Theta_E)$ is higher than $CS^m(\Theta_I')$. However, this is not always true. For example, let the values of the parameters to be as follows: $v(\Theta_I') = 10, c(\Theta_I') = 5, v(\Theta_E) = 6, c(\Theta_E) = 3$, and $t = 2.5$. These values yield $CS^d(\Theta_I', \Theta_E) = 0.92$ and $CS^m(\Theta_I') = 1.25$. The surplus that each consumer receives for both the monopoly and the duopoly cases is plotted in Figure 2.

The entry can lead to lower consumer surplus when (1) the value of the entrant's ecosystem $v(\Theta_E)$ is significantly lower than the incumbent's ecosystem $v(\Theta_I')$, and (2) the disutility parameter t is high. The intuition is that when $v(\Theta_E)$ is much lower than $v(\Theta_I')$, the entrant could not create enough competitive pressure on the incumbent. Accordingly, the incumbent, who knows that it will lose some demand from consumers who have strong preferences for the entrant anyway, increases the price to extract more surplus from its remaining consumers.

Only consumers whose location are very close to one gain more surplus. However, as $v(\Theta_E)$ is too low, it is not enough to compensate for the decrease in the surplus from consumers on the left. (See Figure 2)

Additionally, when the disutility parameter t is large, each firm has more market power over its respective groups of consumers. This exacerbates the situation as it allows both firms to keep their prices high.

Figure 2: Surplus from Each Consumer under Monopoly and Duopoly



Note: The values of the parameters are $v(\Theta_I') = 10, c(\Theta_I') = 5, v(\Theta_E) = 6, c(\Theta_E) = 3$, and $t = 2.5$.

A3 Numerical Examples for Proposition 4 when the merger blocks entry

In Appendix A3, I provide numerical examples to show that case *ii.* in Proposition 4 is possible. It is the case where the merger prevents the entry. Hence, the foresighted competition authority compares between consumer surplus with the merger $CS^m(\Theta_I \cup A)$ and without the merger $CS^d(\Theta_I, \Theta_E) + CS_A$. In what follows, I provide two sets of parameters to show that the possibility of the foresighted competition authority to allow and not allow the merger are non-empty.

Scenario 1: The foresighted competition authority allows the merger when the merger blocks the entry

Consider the following set of parameters: any set of $\{v(\Theta_I), c(\Theta_I), v(\Theta_I \cup \text{A}), c(\Theta_I \cup \text{A}), v(\Theta_E), c(\Theta_E)\}$ such that $s(\Theta_I) = 5$, $s(\Theta_I \cup A) = 9$, $s(\Theta_E) = 3$, $t = 2.5$, $F = 0.5$, and $CS_A = 0$.

Then, the entry barriers with and without merger, respectively, are $\bar{F}(\Theta_I \cup A, \Theta_E) = 0.05$ and $\bar{F}(\Theta_I, \Theta_E) = 0.67$. Given the entry cost F at 0.5, the entrant will not enter if the merger is allowed. But the entrant will enter if the merger is not allowed.

According to Equations (3.5) and (3.6), we have $CS^m(\Theta_I \cup \text{A}) = 1.25$ and $CS^d(\Theta_I, \Theta_E) + CS_A = 0.92$. So, consumer surplus if the merger is allowed $CS^m(\Theta_I \cup \text{A})$ is higher than consumer surplus if the merger is not allowed $CS^d(\Theta_I, \Theta_E) + CS_A$. Even though, the merger allows the entrant to maintain its monopoly power, consumers are better off with the merger. ■

Scenario 2: The foresighted competition authority stops the merger when the merger blocks the entry.

Consider the following set of parameters: any set of $\{v(\Theta_I), c(\Theta_I), v(\Theta_I \cup \mathrm{A}), c(\Theta_I \cup \mathrm{A}), v(\Theta_E), c(\Theta_E)\}$ such that $s(\Theta_I) = 2$, $s(\Theta_I \cup A) = 3$, $s(\Theta_E) = 3$, $t = 1.0$, $F = 0.7$, and $CS_A = 0$.

The merger under Scenario 2 blocks the entry since $\bar{F}(\Theta_I \cup A, \Theta_E) = 0.5$ and $\bar{F}(\Theta_I, \Theta_E) = 0.89$. So, we have $\bar{F}(\Theta_I \cup A, \Theta_E) < F$. If the merger is blocked, the entrant will enter ($F < \bar{F}(\Theta_I, \Theta_E)$).

If the merger is allowed, the incumbent remains the monopolist. So, consumer surplus $CS^m(\Theta_I \cup A)$ is 0.5. On the other hand if the merger is block, consumer surplus under the duopoly $CS^d(\Theta_I, \Theta_E) + CS_A$ is 1.28. Since consumer surplus without the merger $CS^d(\Theta_I, \Theta_E) + CS_A$ is higher than consumer surplus with the merger $CS^m(\Theta_I \cup A)$. The foresighted competition authority will prevent the merger. ■

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Chapter 4

Price Competition under Search with Inaccurate Recommendations

4.1 Introduction

The Diamond paradox (Diamond, 1971) asserts that perfectly competitive sellers charge a monopoly price in equilibrium when consumers have search costs, and the prices are unobservable ex-ante.[1] Several papers build on Diamond (1971)'s idea by introducing two types of consumers–informed and uninformed consumers. Informed consumers have perfect information on sellers' prices or on which sellers sell their preferred product. In contrast, uninformed consumers do not have such information. When more informed consumers have knowledge about their *preferred product*, Anderson & Renault (2000a) find an *increase* in the equilibrium price. However, when there are more informed consumers who know about sellers' prices, Varian (1980) shows that the equilibrium price always *decreases*.[2] Hence, existing literature has shown a monotonic relationship between better information (more informed consumers) and the equilibrium price.

In this paper, I show that the monotonic relationship between better information and the equilibrium price found in the existing literature is not a general result. Rather than relying on exogenous groups of informed and uninformed consumers, I introduce a product recommendation system that sends one recommendation to each consumer. The recommendation system is not always accurate: some consumers receive a recommendation for a product that does not fit their preferences.[3] I show that improved information–in the form of a more accurate recommendation system–could either increase or decrease the equilibrium price. To the best of my knowledge, this is the first paper that shows such a

[1]To understand the intuition behind the paradox, suppose two sellers sell a homogeneous product with value v to consumers. Each seller sets its price that is not observable by the consumers before they inspect the seller. The first inspection is free. The consumers incur a search cost of $s > 0$ to inspect the second seller.

The unique equilibrium is that both sellers set the monopoly price v. Neither seller has an incentive to lower the price. A price cut is not observable to consumers who have not inspected the sellers. Therefore, the price cut does not increase the seller's demand. On the other hand, suppose that the sellers set their prices at $p < v$. Then, a seller has an incentive to increase the price to $p + \varepsilon$ where $\varepsilon < s$. The seller does not lose the demand because consumers are worse off paying the search cost s to inspect and buy from another seller at price p. Hence, only the monopoly price v is an equilibrium.

[2]Technically, the seller equilibrium strategy is a mixed strategy in Varian (1980). More informed consumers lower the equilibrium *average* price. Further discussions on Varian (1980) and Anderson & Renault (2000a) are in the literature review.

[3]Product recommendations are also inaccurate in practice. A report by Boston Consulting Group (Abraham *et al.*, 2019) claims that most retailers are in the early stage of personalization development. They still cannot provide personalized experiences to reach the level of consumer expectation.

non-monotonic relationship. It is also the first to introduce inaccurate product recommendations to avoid the Diamond paradox.

Examples of an (inaccurate) product recommendation are as follows. First, a consumer could look for a recommendation from a consumer organization[4] when looking for a product. Second, a government agency may recommend a fertilizer that a farmer might be interested in using. Finally, a traveler may look for a flight on Skyscanner, which, in turn, shows recommended flights and the associated prices. The traveler can then visit the airline's website to complete the purchase. There are many reasons why recommendations are not perfectly accurate. A sender of recommendations may not have sufficient consumer data to deduce the correct recommendation for each consumer. Even with adequate data, the processes or statistical techniques to determine consumer preferences are imperfect.

In this paper, I develop a search model to study price competition under inaccurate recommendations. Each consumer has a unit demand for a product with two varieties–A and B. Consumers are heterogeneous in two dimensions. The first dimension is the variety each consumer prefers. Some consumers prefer variety A, while others prefer variety B. Each consumer receives lower utility from the mismatched variety (e.g., variety B for consumers who prefer variety A) than the utility she would receive from her preferred variety. The second dimension is the disutility toward the mismatched variety. Some consumers dislike the mismatched variety more than others.

Each product variety is sold by two sellers.[5] Each seller sets its price. However, consumers do not yet observe these prices ex-ante.

An exogenous recommendation system sends one recommendation to each consumer. Each recommendation contains a recommended seller and its price. Nevertheless, the recommendation system is not always accurate. Some consumers receive a recommendation for a seller who sells their preferred variety (hereafter, correct recommendations). Other consumers receive a recommendation for a seller who sells the mismatched variety (hereafter, incorrect recommendations).

A consumer may be dissatisfied with her recommendation when she receives an incorrect recommendation, and she highly dislikes the mismatched variety. Alternatively, dissatisfaction could result from the recommended seller's high price. Then, the consumer can search to inspect another seller. She learns about the inspected seller's price. The consumer can choose which seller to inspect. Multiple searches are possible with a search cost per search.

I study how the equilibrium price depends on the accuracy of the recommendation system. But how should "more accurate" be defined? I propose two possibilities–total and allocative accuracy. First, a *totally* more accurate recommendation system is more likely to send a correct recommendation to all consumers compared with a totally less accurate one.

In contrast, under higher *allocative* accuracy, whether a consumer is more likely to receive a correct or an incorrect recommendation correlates with her level of disutility toward the mismatched variety. A consumer who highly dislikes the mismatched variety is more likely to receive a correct recommendation. Conversely, a consumer who slightly dislikes the mismatched variety is less likely to receive a correct recommendation. However, the total numbers of consumers who receive a correct recommendation are the same under a more accurate recommendation system and a less accurate one.

Allocative accuracy may not sound natural. However, I argue that it could occur in practice. Typi-

[4]For example, Consumer Reports in the US and Consumentenbond in the Netherlands.

[5]Note that it is straightforward to extend the model to any number of sellers more than two without affecting the results.

cally, recommendation algorithms rely on several dimensions of consumer data, such as past purchases, demographic information, and consumer interests. When the developer of a recommendation system has a lot of consumer data, standard techniques increase computational burdens exponentially. One method to address these burdens is by reducing the amount of data or dimensions used (Linden *et al.*, 2003).

By reducing the number of data or dimensions, a recommendation algorithm might have a challenge differentiating between consumers *A* and *B*, who slightly dislike their mismatched variety. For example, suppose that a recommendation algorithm now disregards demographic data and only uses past purchase behaviors. Some consumers *A* who slightly dislike variety *B* may buy variety *B* more often than consumers *A* who highly dislike variety *B* in the past. So, the recommendation algorithm might falsely infer that consumers *A* who slightly dislike variety *B* are consumers *B*.

Under a symmetric equilibrium where all sellers set the same price, the equilibrium price is lower when a marginal consumer is more likely to receive an incorrect recommendation. The marginal consumer is the consumer who is indifferent between leaving and buying from the recommended seller who sells her mismatched variety. When the marginal consumer receives an incorrect recommendation, the recommended seller has an incentive to keep its price low to prevent the marginal consumer from leaving. The higher the probability of the marginal consumer receiving an incorrect recommendation, the higher the incentive for a low price.

Higher total accuracy always increases the equilibrium price. This is because all consumers, including the marginal consumer, are less likely to receive an incorrect recommendation.

In contrast, higher *allocative* accuracy decreases the equilibrium price when the search cost is low but increases the equilibrium price when the search cost is high. When the search cost is low, consumers who slightly dislike the mismatched variety will search for another seller. Therefore, the marginal consumer is the consumer who slightly dislikes the mismatched variety. However, a consumer who slightly dislikes the mismatched variety is more likely to receive an incorrect recommendation under allocative accuracy. Thus, the marginal consumer is more likely to receive an incorrect recommendation. Conversely, when the search cost is high, the marginal consumer is the consumer who highly dislikes the mismatched variety. Higher allocative accuracy decreases the probability that a consumer who highly dislikes the mismatched variety receives an incorrect recommendation. So, the marginal consumer is more likely to receive an incorrect recommendation when the search cost is high.

Both higher total and allocative accuracy increase social welfare. Under allocative accuracy, more consumers who slightly dislike their mismatched variety consume it. Even though this creates disutility, the negative impact on social welfare is low because the total disutility is limited. The negative impact is dominated by a lower total search cost from fewer searches by consumers who highly dislike their mismatched variety. Furthermore, fewer consumers buy a mismatched variety, and fewer consumers search with higher total accuracy. There are only positive effects on social welfare.

The effect of higher accuracy on consumer surplus is ambiguous when higher accuracy increases the equilibrium price. The only case where higher accuracy always leads to higher consumer surplus is under higher allocative accuracy with a low search cost. It is the only case that the equilibrium price decreases with higher accuracy.

The rest of the paper is organized as follows. Related literature is discussed in Section 4.2. I focus on Varian (1980) and Anderson & Renault (2000a) to understand why they find a monotonic relationship between better information and price. The setup of the model is described in Section 4.3. In Section 4.4, I solve the model given a restricted parameter space such that some consumers search in equilibrium. The effects of more accuracy on the equilibrium price, social welfare, and consumer surplus are discussed in Section 4.5. The definitions of a more accurate recommendation system are also specified. In Section 4.6, I relax the restriction made in Section 4.4. Section 4.7 concludes the paper.

4.2 Literature review

The paper by Anderson & Renault (2000a) is closely related to this paper since the information is about consumers' preferred varieties. They show that improved information increases the equilibrium price. There are two sellers who sell horizontally differentiated products. Each seller sets its price that is unobservable to all consumers before an inspection.

A large number of consumers are heterogeneous in their valuations of the products sold by the sellers. The product values that each consumer receives are randomly drawn from a distribution. Further, the consumers are separated into two groups–informed and informed consumers. Each informed consumer knows her valuations of the products sold by both sellers ex-ante, but she does not know about the prices. On the contrary, each uninformed consumer does not know both her valuations and the prices. When an informed consumer searches, new information she learn is the price of an inspected seller. An uninformed consumer learns about her valuation and the price of an inspected seller when she searches.

Under a symmetric equilibrium, all consumers expect both sellers to set the same price. So, each informed consumer searches and buys from a seller that gives her the highest valuation. Hence, informed consumers are the source of inelastic demand. On the other hand, an uninformed consumer randomly searches for a seller. Each seller might successfully convince the uninformed consumer to buy if it sets a sufficiently low price. Hence, uninformed consumers are the source of elastic demand.

The authors find that the equilibrium price increases when there are more informed consumers and fewer uninformed consumers. This is because the demand is more inelastic when there are more informed consumers. Hence, Anderson & Renault (2000a) find that improved information–in the sense that there are more informed consumers–monotonically increases the equilibrium price.

In Varian (1980), information is about prices. There are many sellers that sell a homogeneous product. Informed consumers know the prices set by all sellers. In contrast, uninformed consumers do not have information on any of the prices. Each consumer has a unit demand for the product with a maximum willingness to pay of v. The informed consumers will buy from a seller who sets the lowest price provided that the price is lower than v. Each uninformed consumer randomly buys from a seller as long as the seller's price is lower than v. There is no consumer search in Varian (1980).

Each seller faces a trade-off when setting its price. On the one hand, the seller will successfully attract all informed consumers if it manages to set the lowest price. On the other hand, each seller has an incentive to keep its price high to extract surplus from uninformed consumers who randomly match with it. Varian (1980) shows that the equilibrium must be a mixed strategy for prices.

An important result is that the equilibrium average price decreases when there are more informed consumers. Intuitively, the demand is more elastic when there are more informed consumers. So, improved information (more informed consumers) on price visibility leads to a lower (average) equilibrium price.

The crucial difference between Anderson & Renault (2000a) and Varian (1980) is what the informed consumers know. If the information is about product valuations, more informed consumers lead to a higher degree of product differentiation which increases the equilibrium price, as in Anderson & Renault (2000a). In contrast, More informed consumers will intensify price competition if the information is about price transparency, as in Varian (1980). I summarize similarities and differences between Anderson & Renault (2000a), Varian (1980), and this paper in Table 4.1.

Table 4.1: Summary of the papers

Paper	Information	Feature	Improved info.
Anderson & Renault (2000a)	Preferred Product	informed/uninformed consumers	↑ price
Varian (1980)	Price	informed/uninformed consumers	↓ price
This paper	Preferred product	product recommendations	↑ / ↓ price

Armstrong (2015a) discusses the effect of improved information on an equilibrium price in these two directions. He focuses on a situation where the presence of informed consumers creates externalities on uninformed consumers. That is, a larger group of informed consumers makes uninformed consumers better off or worse off. When informed consumers know about prices, uninformed consumers benefit from more informed consumers because of a lower equilibrium price. Armstrong (2015a) refers to this situation as "search externalities." On the other hand, "ripoff externalities" refers to situations where a larger number of informed consumers makes uninformed consumers worse off due to a higher equilibrium price. Ripoff externalities occur when informed consumers are aware their preferred product.

In sum, the existing literature has shown that improved information in the form of more informed consumers has a monotonic relationship with the equilibrium price, depending on what information informed consumers know. In contrast, I will show that improved information in the form of a more accurate recommendation system may increase or decrease the equilibrium price. The monotonic relationship is not generally true.

4.3 Model

Consumers and sellers:

A product exists in two varieties–A and B. Let $\alpha \in \{A,B\}$ denote the product variety. Consumers have heterogeneous preferences toward the varieties α. The consumer types are (i,θ) where $i \in \{A,B\}$ and $\theta \in [0,1]$. Hereafter, consumers $i \in \{A,B\}$ refer to consumers with type i for any values of $\theta \in [0,1]$. Each consumer knows her type (i,θ).

Each consumer has a unit demand for the product. If consumer (i,θ) consumes variety α at price p, her utility $u_{i,\theta}(p,\alpha)$ is

$$u_{i,\theta}(p,\alpha) = \begin{cases} v - p & \text{if } \alpha = i \\ v - \theta - p & \text{if } \alpha \neq i, \end{cases}$$

where $v \geq 1$. For each type $i \in \{A,B\}$ of consumers i, i.e., $\{(i,\theta) \mid \theta \in [0,1]\}$, θ is uniformly distributed on the interval $[0,1]$ ($\theta \sim U[0,1]$). So, there are two unit masses of consumers.

There are four sellers. Sellers a_1 and a_2 sell variety A, and sellers b_1 and b_2 sell variety B. Let $S_A = \{a_1, a_2\}$ and $S_B = \{b_1, b_2\}$ be the sets of sellers who sell varieties A and B, and $S = S_A \cup S_B$. The products sold by sellers who sell the same variety are identical. Thus, consumer (i, θ) receives the same gross utility buying from sellers a_1 or a_2 (or sellers b_1 or b_2).

Each seller $k \in S$ sets a price p_k. The production cost of each seller is normalized to zero. Suppose $D_k(p_k)$ is seller k's demand at price p_k. Then, seller k's profit is $p_k D_k(p_k)$.

Recommendations:

Each consumer receives a single recommendation from an exogenous recommendation system r. A recommendation contains the identity of a recommended seller $k \in S$ and its price p_k. Thus, a recommendation is a pair (k, p_k).

The recommendation system r is not always accurate. Consumer (i, θ) receives a correct recommendation for her preferred variety with probability $r(\theta)$ and an incorrect recommendation for the mismatched variety with probability $1 - r(\theta)$. Given that a recommendation is correct, the recommendation is equally likely for one of the two sellers who sell her preferred variety. Similarly, an incorrect recommendation is equally likely for one of the two sellers who sell her mismatched variety.

Furthermore, define

$$R(\theta) \equiv \int_0^{\theta} r(y) dy.$$

Thus, the mass $R(1)$ of consumers i receives a correct recommendation. Incorrect recommendations are received by the mass $1 - R(1)$ of consumers i.

Information, search, and timing:

In the first stage of the game, all sellers set their prices p_k simultaneously. The consumers do not observes these prices.

Next, each consumer receives a recommendation (k, p_k). However, the consumer still does not know the prices of the other sellers.

Then, each consumer decides if she will buy from her recommended seller or search for another seller. If a consumer decides to search, she can choose the next seller to inspect. Each search provides full information on the price of the inspected seller. The prices of non-inspected sellers remain unknown. The consumer can keep searching as many times as she wants with a search cost s per search.

Let S^I be the set of inspected sellers and $S^{NI} = S \backslash S^I$ be the set of non-inspected sellers. Note that the inspected set S^I always includes the recommended seller k. The consumer can come back to buy from any previously inspected seller without an additional cost. She cannot buy from a seller whom she has not inspected.

The recommendation system r, the variety of the product that the sellers sell (S_A and S_B), the value v, and the search cost s are common knowledge. Consumers are aware of which variety each seller sells.

In sum, the information that consumer (i, θ) has at each stage of the search includes her type (i, θ), the sets of sellers (S_A and S_B), the set of inspected sellers S^I, the prices p_k of inspected sellers $k \in S^I$, and the set of non-inspected sellers S^{NI}. So, the information set I is

$$I = \left\{ (i,\theta), S_A, S_B, S^I, S^{NI}, \left\{ p_k | k \in S^I \right\} \right\}. \tag{4.1}$$

The summary of the timing and the structure of the game is in Table 4.2.

Table 4.2: Summary of the stages of the game

Stage	Action
0	A recommendation system r, the value v, and the search cost s are common knowledge. Each consumer knows her type (i,θ), where $i \in \{A,B\}$ and $\theta \in [0,1]$.
1	Each seller set its price p_k simultaneously. Consumers do not observe the prices.
2	Each consumer receives a random recommendation from r. A recommendation is a pair (k, p_k) for the recommended seller k and its price p_k.
3	Each consumer decides whether to buy from the recommended seller k or search for another seller. If the consumer buys from the recommended seller k, the consumer receives utility from consuming the product. If the consumer decides to search, the game moves to stage 4.
4	The consumer incurs the search cost s. The consumer chooses a seller in the non-inspected set S^{NI} to inspect. The consumer observes the price $p_{k'}$ of the inspected seller k'.
5	The consumer decides whether to buy from a seller in the inspected set S^I or keep searching. If the consumer decides to buy, the consumer receives utility from consuming the product. If the consumer keeps searching, Step 4 repeats.

Equilibrium concept

In line with the existing literature (Wolinsky (1983) and Anderson & Renault (2000a)), the equilibrium concept used in this paper is a symmetric perfect Bayesian equilibrium (PBE) in pure strategies. Hence, it is assumed that, in stage 1, the prices p_k of all sellers are deterministic and the same.

Furthermore, for each piece of information h that contains the sellers that have been inspected or recommended and their prices at each stage, consumer (i,θ) forms a belief about the price $p^e(k|h,i,\theta)$ charged by each non-inspected seller $k \in S^{NI}$. When consumer (i,θ) has to decide if she will inspect seller $k \in S^{NI}$, she makes the decision based on the expected price $p^e(k|I,i,\theta)$.

The consumer equilibrium strategy–consisting of searching and buying strategies–maximizes (the expected) utility of each consumer for a given information set I at each stage of the game. In addition, each seller maximizes its profit given the consumer strategy and the expected prices of other sellers. In equilibrium, the expected price $p^e(k|h,i,\theta)$ equals the actual price p_k for each seller k in the non-inspected set S^{NI}, i.e., $p^e(k|h,i,\theta) = p_k$ for all $k \in S^{NI}$.

The beliefs $p^e(k|h,i,\theta)$ follow the "no-signal-what-you-don't-know" condition, specified by Fudenberg & Tirole (1991). So, if a consumer sees a seller whose price differs from an equilibrium price, the consumer still believes that all other sellers charge the equilibrium price. In the literature, this is called the passive beliefs assumptions. Since each seller sets its price independently, a deviation by one seller cannot reflect decisions made by other sellers.[6]

Further assumptions and definitions

Assumption 6. *$r(\theta)$ is continuously differentiable and increasing in θ for the whole support $\theta \in [0,1]$, i.e.,*

$$r'(\theta) \geq 0.$$

Assumption 6 states that consumers i who dislike the mismatched variety $\alpha \neq i$ more (i.e., who have higher θ) are more likely to receive a correct recommendation.

Furthermore, yo avoid a degenerated outcome that consumers never search for any recommendation system r, Assumption 7 requires that the search cost s is lower than one–the maximum value of disutility θ from a mismatched variety.[7]

Assumption 7. *The search cost s is sufficiently low such that $s < 1$.*

Before proceeding further, it is useful to define the set of the best search alternatives $S^s_{i,\theta}(h) \subseteq S^{NI}$ (Definition 4) and the set of best buying options $S^b_{i,\theta}(h) \subseteq S^I$ (Definition 5). For the former, the set of the best search alternatives $S^s_{i,\theta}(h) \subseteq S^{NI}$ is the set of sellers for which consumer (i,θ) receives the highest expected utility from searching given a set of non-inspected sellers S^{NI}. The latter–the set of the best buying option $S^b_{i,\theta}(h) \subseteq S^I$–is the set of sellers that consumer (i,θ) receives the highest utility buying from them given a set of inspected sellers S^I.

Definition 4. The set of the best search alternatives $S^s_{i,\theta}(h)$

Given consumer (i,θ) and a piece of information h, the set $S^s_{i,\theta}(h) \subseteq S^{NI}$ is given by

$$S^s_{i,\theta}(h) = \arg\max_{z \in S^{NI}} u_{i,\theta}\left(p^e(z|h,i,\theta), \alpha_z\right).$$

Definition 5. The set of the best buying options $S^b_{i,\theta}(h)$:

Given consumer (i,θ) and a piece of information h, the set $S^b_{i,\theta}(h) \subseteq S^I$ is given by

$$S^b_{i,\theta}(h) = \arg\max_{w \in S^I} u_{i,\theta}\left(p_w, \alpha_w\right).$$

The set of the best search alternatives $S^s_{i,\theta}(h)$ or the set of the best buying options $S^b_{i,\theta}(h)$ may contain more than one seller. That is, consumer (i,θ) receives the same (expected) utilities from all sellers in the same set. In this case, I assume that consumer (i,θ) randomly decides to search or to buy from one of the sellers in the respective sets. Assumption 8 formalizes this requirement.

Assumption 8. *Let $|X|$ be the number of sellers in set X. Suppose that $S^s_{i,\theta}(h) \neq \emptyset$ and $S^b_{i,\theta}(h) \neq \emptyset$.*

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[6]Note that there is no Bayesian updating in this paper. According to Fudenberg & Tirole (1991), a condition for a PBE is that beliefs are updated according to Bayes rule *whenever possible*. Since sellers do not have private types in this paper, the update according to Bayes rule is unnecessary. The PBE has been use in other settings that do not have Bayesian updating. For example, Rey & Tirole (2007) use the PBE to study a vertical relationship between an upstream firm that does not have a private type and downstream firms. A contract between the upstream firm and a downstream firm is not observable to other downstream firms. So, downstream firms have to form beliefs on contracts that the upstream firm offered to their rivals.

[7]Suppose that the search cost s is higher than the maximum value of θ. Even the consumer who dislikes the mismatched variety the most ($\theta = 1$) are not willing to search in equilibrium. The payoff from searching $v - s - p^*$ is lower than the payoff buying from the mismatched variety $v - \theta - p^*$ for any consumer.

1. *Then, when consumer (i,θ) decides to search, she chooses each seller in the set of the best search alternatives $S^s_{i,\theta}(h)$ with probability $1/\left|S^s_{i,\theta}(h)\right|$.*
2. *When consumer (i,θ) decides to buy, she chooses each seller in the set of the best buying options $S^b_{i,\theta}(h)$ with probability $1/\left|S^b_{i,\theta}(h)\right|$.*

Before starting to solve the model, consider a special case where the accuracy is perfect. Every consumer receives a correct recommendation, i.e., $r(\theta)=1$ for all $\theta \in [0,1]$. The following proposition shows that the sellers will charge the monopoly price v.

Proposition 13. *Under a perfectly accurate recommendation system, i.e., $r(\theta)=1$ for all θ, the unique equilibrium price p^* is*

$$p^* = v.$$

None of the consumers searches in equilibrium.

Proof. See Appendix 4.8.1.1 □

The intuition is as follows. Given any price p' that is lower than v, each seller can increase its profit by raising its price by $\varepsilon < s$ such that $p'+\varepsilon < v$. Consumers do not have an incentive to leave. This allows other sellers to slightly increase their prices without losing their demand. Thus, the prices keep increasing until the sellers can extract all of the surpluses from consumers.

In addition, none of the sellers has an incentive to lower its price. As consumers do not observe the prices prior to inspections, cutting the price does not affect consumer's belief. The consumers do not search for the seller. Its demand remains the same. Note that Proposition 13 is similar to the Diamond paradox (Diamond, 1971).

Proposition 13 suggests that when a recommendation system r is highly accurate–high $r(\theta)$–the equilibrium price will be high. However, if the equilibrium price is is too high, the utility from searching $v-s-p^*$ will be negative. None of the consumers searches in equilibrium.

To make sure that some consumers search, I make a technical assumption (Assumption 9) which requires that the recommendation system r is not too accurate. As shown in Section 4.3, the assumption guarantees that the equilibrium price p^* is sufficiently low.

Assumption 9. *The recommendation system is sufficiently inaccurate such that*

$$r(s) \leq 1 - \frac{1}{v-s}.$$

Assumption 9 is indeed restrictive. For example, the assumption never holds when the recommendation system r is perfect, i.e., $r(\theta)=1$ for all $\theta \in [0,1]$. I assume Assumption 9 throughout Sections 4.4 and 4.5. The relaxation of Assumption 9 is discussed in Section 4.6.

4.3.1 Discussions on the model

An essential element of the model is an "exogenous" recommendation system r. An interpretation is that there is a third party, who is not a seller or a consumer, that operates the recommendation system r. Recall the examples in the introduction. The roles of consumer organizations are to review and recommend products. They do not sell products themselves. Furthermore, government agencies do not sell fertilizers to maximize their profits either. Skyscanner does not operate an airline.

Furthermore, two elements of the model differ from the features observed in reality. The first feature is that many recommendation systems show several recommendations, instead of one, to each consumer. For the second feature, some sellers may sell several varieties in practice. This is true in the case of consumer organizations: many sellers recommended by a consumer organization sell several product varieties. In the model, each seller sells only one variety. Though, this element is true in some cases. For example, KLM has only one flight from Amsterdam to Bangkok each day.

I assume these two elements–single recommendation and one variety per seller–to keep the model simple. If time permits, it is desirable to expand the model to incorporate the two features. I could introduce the third type of sellers who sell both varieties. In addition, the recommendation system could send two recommendations to each consumer. Some consumers receive one recommendation for variety A and another for variety B. Other consumers may receive recommendations for the same variety. These recommendations could be for the sellers who sell both types of varieties as well.

4.4 Analysis

The analysis begins with a consumer strategy in Section 4.4.1. Then, I derive an equilibrium price p^* given the consumer strategy in Section 4.4.2.

4.4.1 Consumer strategy

Proposition 14 states an equilibrium consumer strategy, for any prices p_k and the beliefs $p^e(k|h,i,\theta)$, consisting of the search strategy and the buy strategy under Assumption 8.

Proposition 14. *For any piece of information h, an equilibrium strategy of consumer (i,θ) is as follows.*

1. *If the set of the best search alternatives is empty ($S^s_{i,\theta}(h)=\emptyset$), consumer (i,θ) stops searching.*

2. *If the set of the best search alternatives is non-empty ($S^s_{i,\theta}(h)\neq\emptyset$), consumer (i,θ) searches for seller $z\in S^s_{i,\theta}(h)$ with probability $1/\left|S^s_{i,\theta}(h)\right|$ provided that $u_{i,\theta}(p^e(z|h,i,\theta);\alpha_z)-s\geq \max_{w\in S^I} u_{i,\theta}(p_w;\alpha_w)$. On the other hand, if $u_{i,\theta}(p^e(z|h,i,\theta);\alpha_z)-s< \max_{w\in S^I} u_{i,\theta}(p_w;\alpha_w)$, consumer (i,θ) stops searching.*

3. *Once the consumer stops searching and the set of the best buying options is non-empty ($S^b_{i,\theta}(h)\neq\emptyset$), consumer (i,θ) buys from seller $w\in S^b_{i,\theta}(h)$ with probability 1/$\left|S^b_{i,\theta}(h)\right|$ if $u_{i,\theta}(p_w;\alpha_w)\geq 0$. Otherwise, consumer (i,θ) does not buy from any seller.*

Proof. See Appendix 4.8.1.2 □

The equilibrium consumer strategy is straightforward. Each consumer keeps searching as long as the expected payoff from searching is higher or equal to the highest payoff she currently has. In addition, it is possible that a consumer runs out of a search option ($S^s_{i,\theta}(h)=\emptyset$). Consequently, the consumer stops searching by construction. Once the consumer stops searching, she buys from the a seller who gives her the highest payoff. According to Assumption 8, the consumer randomly picks one seller to search for or to buy from if there are equivalent alternatives.

Notice that there are two decisions to which a consumer can be indifferent. First, a consumer may be indifferent between staying or searching. In this case, the consumer strategy in Proposition 14 requires that the consumer always searches. Second, once a consumer has decided to search, regardless of whether she is indifferent or strictly prefers to search, she might be indifferent between the best search alternatives $\left|S^s_{i,\theta}(h)\right| \geq 2$. Then, the consumer randomizes between the best search alternatives (Assumption 8).

An important implication of Proposition 14 is that when an equilibrium price p^* is higher than $v-s$, consumers never search in equilibrium. Assumption 9 rules out the case that $p^* > v-s$. On the other hand, if $p^* \leq v-s$. Consumer (i,θ) who receives an incorrect recommendation will search for a seller who sells her preferred variety if $\theta \leq s$.

4.4.2 Equilibrium price

I derive the equilibrium seller strategy in this section. I focus on an equilibrium outcome in which some consumers search. The reason is that each seller has to compete on price to prevent consumers from leaving when some consumers search. Therefore, price competition under inaccurate recommendations can be studied.

Without loss of generality, I focus on seller $a_1 \in S_A$. Because I am looking for a symmetric equilibrium in pure strategies, I calculate seller a_1's best response p^s when all other sellers also charge the same price $p = p^s$ assuming that consumers could search. The best response p^s is derived in Lemma 11. Then, I will argue that the price p^s is an equilibrium price p^* under Assumption 9 in Proposition 15.

I start by deriving seller a_1's demand function. Let $D_{a_1}(p_{a_1})$ be the mass of consumers who buy from seller a_1 at the price p_{a_1} given the other sellers charge p^s. And the expected prices $p^e(k|h,i,\theta)$ of all sellers, including seller a_1, are $p^e(k|h,i,\theta) = p^e = p^s$ for all $k \in S$. Lemma 11 summarizes seller a_1's demand $D_{a_1}(p_{a_1})$ and the best response of seller a, i.e., the profit maximizing price p^s.

Lemma 11. *Suppose all other sellers, except seller a_1, set the price $p^s(s,r)$. And $p^e(k|h,i,\theta) = p^e = p^s(s,r)$ for all $k \in S$. Then, seller a_1's demand D_{a_1} at price p_{a_1} is*

$$D_{a_1}(p_{a_1}) = \begin{cases} \frac{1}{2}\left(1+R(s)-R(p^e-p_{a_1}+s)+p^e-p_{a_1}\right) & \text{if } p_{a_1} < p^e+s \\ 0 & \text{if } p_{a_1} \geq p^e+s. \end{cases} \tag{4.2}$$

Furthermore, $p^s(s,r)$ is also seller a_1's best response when

$$p^s(s,r) = \frac{1}{1-r(s)}. \tag{4.3}$$

Proof. See Appendix 4.8.1.3 □

When $p_{a_1} \geq p^e+s$, all consumers i are willing to pay the search cost s and leave seller a_1. Doing so allows them to buy from other seller who sells their preferred variety $\alpha = i$ at a lower price $p^s = p^e$. So, the demand of seller a_1 is zero.

When $p_{a_1} < p^e+s$, seller a_1's demand comes from four sources. The first source is consumers A who receive a recommendation for seller a_1. They do not have an incentive to leave seller a_1 if $p_{a_1} < p^e+s$. This is because the best alternative is to search for seller $a_2 \in S_A$ with the expected price p^e. However, the expected payoff from searching $v - p^e - s$ is lower than the payoff from seller a_1. So, one half of the proportion $R(1)$ of consumers A buy from seller a_1.

Secondly, consumers A who receive a recommendation for seller b_1 or seller b_2 search for seller a_1 or seller a_2 when they are not satisfied with variety B. This happens with consumer (A, θ) with $\theta \geq s$. In addition, as long as $p_{a_1} < p^e + s$, consumers A who search for seller a_1 will buy from it. Because the consumers randomly search for a seller who sells variety A, seller a_1 receives a half of such consumers.

Third, consumers B will stay and buy from seller a_1 if the price p_{a_1} is not too high and they do not have a strong preference against variety A (low θ). Specifically, consumers B buy from seller a_1 when $\theta < p^e - p_{a_1} + s$.

The last source is consumers A who receive a recommendation for seller a_2 and search for seller a_1. However, these consumers A do not have an incentive to pay the search cost s when they expect both sellers a_1 and a_2 to charge the same price. The fourth source of the the demand of seller a_1 is zero. Summing up the four sources of the demand gives the demand function in Equation 4.2.

The price $p^s(s,r)$ is the best response of seller a_1 assuming that some consumers search. In Proposition 15, I show that $p^s(s,r)$ is an equilibrium price p^* under Assumption 9

Proposition 15. *Under Assumption 9, the price $p^s(s,r)$, defined in Equation (4.3), is the equilibrium price $p^*(s,r)$, i.e.,*

$$p^*(s,r) = p^s(s,r) = \frac{1}{1 - r(s)}. \tag{4.4}$$

Proof. See Appendix 4.8.1.4. □

When Assumption 9 holds, the price $p^s(s,r)$ is lower than or equal to $v - s$. Hence, $p^s(s,r)$ is the equilibrium price $p^*(s,r)$ because there are consumers who search at this price level as required when $p^s(s,r)$ is derived. And $p^s(s,r)$ maximizes the profit of all sellers.

The equilibrium price $p^*(s,r)$ is inversely related to the probability that a marginal consumer receives an incorrect recommendation $(1 - r(s))$. A marginal consumer is indifferent between buying from the recommended seller or searching. For seller a_1, the marginal consumer is consumer (B,s) who receives an incorrect recommendation for seller a_1. So, the equilibrium price $p^*(s,r)$ is determined by a specific point of the recommendation function, i.e., $1 - r(s)$.

Intuitively, seller a_1 has an incentive to keep its price low to prevent the marginal consumer (B,s) from leaving. However, the marginal consumer (B,s) may or may not receive a recommendation for seller a_1. If the marginal consumer (B,s) does not receive a recommendation for seller a_1, there is no point in trying to prevent the marginal consumer from leaving. When $1 - r(s)$ is high, the marginal consumer (B,s) has a higher chance to receive a recommendation from seller a_1. Thus, seller a_1 has a higher incentive to keep its price low. In contrast, if $1 - r(s)$ is small, the marginal consumer (B,s) is less likely to receive a recommendation for seller a_1. Seller a_1 has a lower incentive to keep the price low.

Corollary 11. *Under Assumption 9, the equilibrium price $p^*(s,r)$ is increasing in the search cost s.*

When the search cost s is high, consumers, including the marginal consumer, are less likely to leave their recommended seller. Therefore, the degree of competition between sellers is weaker. The sellers set a higher price in equilibrium.[8]

[8] A higher price from a higher search cost is a standard result when consumers do not observe prices before inspections.

4.5 More accurate recommendation systems

In this section, I study the effects of a more accurate recommendation system r_1 over a less accurate one r_0. In Section 4.5.1, I introduce the two types of higher accuracy used in this paper . The effects of accuracy on the equilibrium price $p^s(s,r)$ and social welfare W are studied in Sections 4.5.2 and 4.5.3, respectively.

4.5.1 Definitions of a more accurate recommendation system

First, I start by defining a *totally* and an *allocatively* more accurate recommendation system. For conciseness, higher total or allocative accuracy refers to a totally or an allocatively more accurate recommendation system, respectively. Higher total accuracy (Definition 6) is such that all consumers are more likely to receive a correct recommendation.

Definition 6. Recommendation system r_1 is *totally* more accurate than recommendation system r_0 when

$$r_1(\theta) \geq r_0(\theta)$$

for all θ, with strict inequality for some θ. Both $r_1(\theta)$ and $r_2(\theta)$ are continuously differentiable and increasing for all $\theta \in [0,1]$.

In words, any consumer (i,θ) is equally or more likely to receive a correct recommendation under higher total accuracy. Consequently, the mass of consumers who receive a correct recommendation is higher under recommendation system r_1 than recommendation system r_0, i.e., $R_1(1) > R_0(1)$. Figure 4.2a illustrates an example of higher total accuracy.

Next, allocative accuracy is defined in Definition 7.

Definition 7. Recommendation system r_1 is *allocatively* more accurate than recommendation system r_0 if there exists $\bar{\theta} \in (0,1)$ such that $R_0(1) = R_1(1)$, $r_0(\bar{\theta}) = r_1(\bar{\theta})$, and

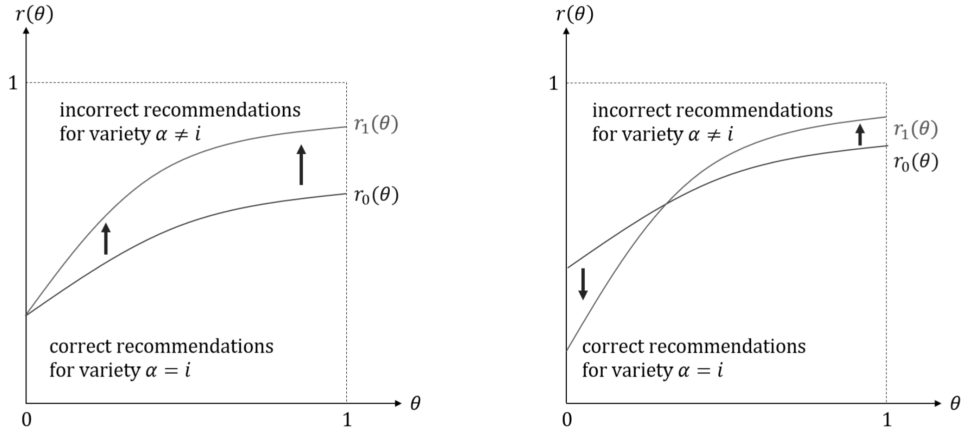
$$r_0(\theta) > r_1(\theta) \text{ for all } \theta < \bar{\theta},$$
$$r_0(\theta) < r_1(\theta) \text{ for all } \theta > \bar{\theta}.$$

Both $r_1(\theta)$ and $r_2(\theta)$ are continuously differentiable and increasing for all $\theta \in [0,1]$.

In contrast to total accuracy, an allocatively more accurate recommendation system r_1 sends the same amount of correct recommendations as a less accurate one r_0, i.e., $R_1(1) = R_0(1)$. Instead, higher accuracy comes from a better allocation of correct and incorrect recommendations. A consumer who highly dislikes the mismatched variety ($\theta > \bar{\theta}$) is more likely to receive a correct recommendation for her preferred variety. However, a consumer who slightly dislikes the mismatched variety ($\theta < \bar{\theta}$) is less likely to receive a correct recommendation. Therefore, the total disutility θ from incorrect recommendations is lower under higher allocative accuracy. Figure 4.2b illustrates an example of allocative accuracy. Note that allocative accuracy is similar to the single-crossing property. The function $r_0(\theta)$ crosses the function $r_1(\theta)$ only once.

Another strand of literature which assumes that consumers observe prices in advance yields the opposite result. This is because sellers use the prices in order to attract more consumers. The competition intensifies when the search cost is higher since the consumers are less likely to search. So, if a seller that can attract the consumers first, it will get a high demand. See, for example, Armstrong (2017a) and Choi *et al.* (2018a).

Figure 4.1: A more accurate recommendation system



(a) Total accuracy

(b) Allocative accuracy

Note: Recommendation system r_1 is totally and allocatively more accurate than recommendation system r_0 in Sub-figure (a) and Sub-figure (b), respectively.

Discussions on the two types of accuracy

In computer science, the most common measures of the accuracy of a recommendation system are the mean-square error (MSE) and the root-mean-square error (RMSE).[9] In the context of this paper, the MSE can be calculated as the weighted averaged of disutility square θ^2 conditional on consumers receiving an incorrect recommendation. That is,

$$MSE = \int_0^1 \theta^2 (1 - r(\theta)) \, d\theta. \tag{4.5}$$

and the RMSE is the square root of MSE.

It can be shown that both higher total and allocative accuracy lower MSE as defined in Equation (4.5). Therefore, both types of higher accuracy proposed in this paper are in line with the accuracy measures used in computer science.

The two types of accuracy focus on different dimensions of accuracy. The idea behind higher total accuracy is straightforward since every consumer is less likely to receive an incorrect recommendation. On the contrary, higher allocative accuracy enhances accuracy from a better allocation of correct and incorrect recommendations while maintaining the number of correct recommendations.

4.5.2 More accuracy and the equilibrium price

The effects of higher total and allocative accuracy on the equilibrium price $p^*(s,r)$ are summarized in Proposition 16 and Proposition 17, respectively. Proposition 16 states that the equilibrium price $p^*(s,r)$ always increases with higher total accuracy.

[9]See, for example, Marchand & Marx (2020) and Jannach *et al.* (2012).

Proposition 16. *Suppose that recommendation system r_1 is totally more accurate than recommendation system r_0. Under Assumption 9, higher total accuracy (weakly) increases the equilibrium price $p^*(s,r)$, i.e.,*

$$p^*(s,r_1) \geq p^*(s,r_0)$$

with strict inequality if $r_1(s) > r_0(s)$.

Proof. See Appendix 4.8.1.5. □

Any consumer (i,θ), including the marginal consumer (i,s), is less likely to receive an incorrect recommendation under higher total accuracy. Therefore, each seller has a lower incentive to keep its price low to prevent the marginal consumer (i,s) from leaving. Improved information, in the form of higher total accuracy, always increases the equilibrium price $p^*(s,r)$.

Recall that Anderson & Renault (2000a) also find a similar result that improved information, in the form of more informed consumers who know about their preferred product, always increases the equilibrium price. However, this positive relationship is not general result. In Proposition 17, I show that improved information in the form of higher allocative accuracy decreases the equilibrium price $p^*(s,r)$ when the search cost s is low.

Proposition 17. *Suppose that recommendation system r_1 is allocatively more accurate than recommendation system r_0.*

1. *If the search cost s is low such that $s \leq \bar{\theta}$, higher allocative accuracy decreases the equilibrium price, i.e., $p^*(s,r_1) \leq p^*(s,r_0)$.*
2. *If the search cost s is high such that $s > \bar{\theta}$, higher allocative accuracy increases the equilibrium price, i.e., $p^*(s,r_1) > p^*(s,r_0)$.*

Proof. See Appendix 4.8.1.6. □

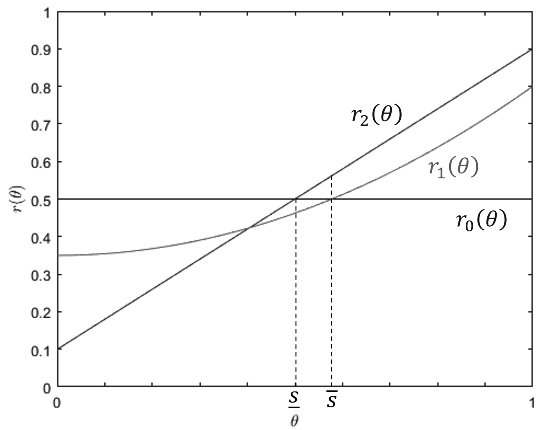
According to Proposition 17, the size of the search cost s determines whether the equilibrium price $p^*(s,r)$ increases or decreases with higher allocative accuracy. To understand the intuition, note that there are two mechanisms at play–the location of the marginal consumer ($\theta = s$) and the probability that the marginal consumer receives an incorrect recommendation $1 - r(\theta)$. Let's look from seller a_1's perspective. When the search cost s is low, the marginal consumer is a consumer B with low θ. And higher allocative accuracy increases the chance of consumers B with low θ receiving an incorrect recommendation for seller a_1. Hence the marginal consumer is more likely to receive an incorrect recommendation for seller a_1. Seller a_1 has a higher incentive to keep its price low to prevent the marginal consumer from leaving.

In contrast, when the search cost s is high, only consumers B who highly dislike variety A (high θ) leave seller a_1. So, the marginal consumer has high θ. The equilibrium price $p^*(s,r)$ increases since higher allocative accuracy decreases the chance that consumers B with high θ receive an incorrect recommendation for seller a_1.

So far, I have shown that higher allocatively accuracy may increase or decrease the equilibrium price $p^*(s,r)$ depending on different values of the search cost s. Next, I will show that higher allocative accuracy has a *non-monotonic* relationship with the equilibrium price $p^*(s,r)$ for a given search cost s.

In Figure 4.2, I provide a numerical example of three recommendation systems (r_0, r_1, and r_2). The three recommendation systems are such that r_2 is allocatively more accurate than r_1, and r_1 is allocative more accurate than r_0. According to Figure 4.2, we have $r_1(\theta) < r_0(\theta) < r_2(\theta)$ for any $s \in (\underline{s}, \bar{s})$. Hence, the relationship between the equilibrium prices is $p^*(s, r_1) < p^*(s, r_0) < p^*(s, r_2)$. The improvement in allocative accuracy reduces the equilibrium price at first (r_0 to r_1), but it increases the equilibrium price later (r_1 to r_2).

Figure 4.2: Non-monotonic relationship under allocative accuracy



Note: The specifications of the three recommendation systems are (1) $r_0(\theta) = 0.5$, (2) $r_1(\theta) = 0.35 + 0.45\theta^2$, and (3) $r_2(\theta) = 0.1 + 0.8\theta$. The value $R(1)$ of each recommendation systems is 0.5.

Remark. Improved information in the form of higher allocative accuracy has a non-monotonic relationship with the equilibrium price $p^*(s, r)$.

4.5.3 The welfare effects of accuracy

To study the effects of accuracy on social welfare W and consumer surplus CS, I derive these two measures in Lemma 12.

Lemma 12. *Given a recommendation system r and a search cost s, social welfare W and consumer surplus CS, respectively, are*

$$W = 2\left(v - s\left(1 - \frac{s}{2}\right) + sR(1) - \int_0^s R(\theta)\, d\theta\right), \tag{4.6}$$

$$CS = W - 2\left(\frac{1}{1 - r(s)}\right). \tag{4.7}$$

Proof. See Appendix 4.8.1.7 □

Given social welfare W and consumer surplus CS, it can be shown that both higher total and allocative accuracy increase social welfare W. Proposition 18 formally states the result.

Proposition 18. *Under both total and allocative accuracy, a more accurate recommendation system* r_1 *increases social welfare* W.

Proof. See Appendix 4.8.1.8 □

First, higher allocative accuracy creates two opposing effects on social welfare W. The negative effect is that it increases the number of incorrect recommendations to consumers who have low disutility θ for their mismatched variety. And these are consumers who buy the mismatched variety. However, the total disutility from mismatches $\int_0^s \theta (1 - r(\theta))\, d\theta$ is limited since θ is small. On the other hand, the positive effect comes from a lower total search cost $\int_s^1 s (1 - r(\theta))\, d\theta$. Higher allocative accuracy reduces the number of incorrect recommendations when θ is large. So, fewer consumers search. Since the negative effect is small, the positive effect dominates the negative effect. Higher allocative accuracy increases social welfare W.

Under higher total accuracy, each consumer (i, θ) is more likely to receive a correct recommendation. Therefore, higher total accuracy reduces both the total disutility from mismatches $\int_0^s \theta (1 - r(\theta))\, d\theta$ and the total search cost $\int_s^1 s (1 - r(\theta))\, d\theta$.

For consumer surplus, the effect of accuracy on the equilibrium price $p^*(s, r)$ must be taken into account. Only higher allocative accuracy with a low search cost s increases consumer surplus CS with certainty. It is the only case that the equilibrium price $p^*(s, r)$ decreases with higher accuracy. In all other cases, higher accuracy increases $p^*(s, r)$. The following remark summarizes this observation.

Remark. Consumer surplus CS is higher under an allocatively more accurate recommendation system when the search cost s is low. However, consumer surplus may increase or decrease with higher accuracy in all other cases (total accuracy and allocative accuracy with a high search cost s).

4.6 Relaxing Assumption 9: $p^s > v - s$

So far, I assume that Assumption 9 holds. It restricts the price $p^s(s, r)$, defined in Equation (4.3), to be less than or equal to $v - s$. Assumption 9 is relaxed in this section. I investigate what happens when

$$r(s) > 1 - \frac{1}{v - s}. \tag{4.8}$$

When Condition (4.8) holds, the price $p^s(s, r)$ exceeds $v - s$. Consequently, $p^s(s, r)$ cannot be an equilibrium because no consumer searches at this price level (Proposition 14). The demand used to derive $p^s(s, r)$ is no longer valid.

When consumers are not allowed to search, each seller acts as a monopolist over consumers who receive a recommendation for it. Let p^{ns} be the best response of each seller assuming that consumers cannot search. The price p^{ns} is derived in Lemma 13.

Lemma 13. *Suppose that consumers cannot search. Then, the best response of each seller* $k \in S$ *is to set the price* $p^{ns}(r)$ *defined as follows.*

1. *If* $1 + \frac{1}{1 - r(1)} \leq v$*, then* $p^{ns}(r) = v - 1$.

2. *If* $\frac{R(1)}{1-r(0)} < v < 1+\frac{1}{1-r(1)}$*, there exists a unique* $p^{ns}(r) \in (v-1, v)$ *that satisfies*

$$p^{ns}(r) = \frac{R(1) + \int_0^{v-p^{ns}(r)} (1-r(\theta))\, d\theta}{1-r(v-p^{ns}(r))}. \tag{4.9}$$

3. *If* $\frac{R(1)}{1-r(0)} > v$*, then* $p^{ns}(r) = v$*.*

Proof. See Appendix 4.8.1.9 □

According to the first part of Lemma 13, the lowest value of p^{ns} is $v-1$. This value is the utility of the consumer who receives the lowest utility from her mismatched variety. So, there is no reason for a seller to set a lower price than $v-1$. This case happens when the recommendation system r is sufficiently inaccurate, i.e., $r(1) \leq 1-1/(v-1)$. The largest value of p^{ns} is v which is the maximum willingness to pay. This case arises when the recommendation is highly accurate, i.e., $r(0) > 1-R(1)/v$.

For the rest of this section, I will argue that $p^{ns}(r)$ is an equilibrium price when Assumption 9 is violated, $v-s < p^s(s,r)$, and when $v-s < p^{ns}(r)$ in Proposition 19. However, when $p^{ns} \leq v-s < p^s(s,r)$, neither $p^{ns}(r)$ and $p^s(s,r)$ can be equilibrium prices. Two possible equilibrium candidates are $v-s$ and v. The former is the highest price such that some consumers search. The latter is the highest price each seller could set with a positive demand. Nevertheless, I will show that none of them can be equilibrium. So, I conjecture that an equilibrium when $p^{ns} \leq v-s < p^s(s,r)$ is likely a mixed strategy equilibrium or an asymmetric equilibrium.

Proposition 19. *Suppose that Condition (4.8) holds (*$p^s(s,r) > v-s$*) and* $p^{ns}(r) > v-s$*. Then,* $p^{ns}(r)$ *is the equilibrium price* $p^*(s,r)$ *, i.e.,*

$$p^*(s,r) = p^{ns}(r).$$

.

Proof. See Appendix 4.8.1.10. □

When $p^{ns}(r)$ is an equilibrium price $p^*(s,r)$, a higher number of correct recommendations $R(1)$ or a lower chance of consumers receiving an incorrect recommendation $1-r(\theta)$ increases the equilibrium price $p^*(s,r)$. Intuitively, a higher $R(1)$ or a smaller $1-r(\theta)$ increases the number of consumers who receive a correct recommendation. These consumers are willing to pay v for the product. As such, the sellers can charge a higher price when more consumers are willing to pay v.

Note that the equilibrium price is truncated at v when $r(0)$ is sufficiently high, as stated in third case of Lemma 13. Therefore, this result is in line with Proposition 13, which states that the equilibrium price is v under perfect accuracy, i.e., $r(\theta) = 1$ for all $\theta \in [0,1]$.

For a given set of parameters $(v, s, r(\theta))$, one might expect that the price $p^{ns}(r)$ with no consumer search is higher than the price $p^s(s,r)$ with consumer search. In the former, each seller acts as a monopolist over consumers who receive a recommendation for it. In the latter, the sellers have to compete with each other to prevent consumers from leaving. However, the intuition is not true when Condition (4.8) holds. In fact, I show that $p^{ns}(v,r) \leq p^s(s,r)$ in Corollary 12.

Corollary 12. *Under Condition (4.8) (*$p^s(s,r) > v-s$*), the price* $p^{ns}(v,r)$ *is lower or equal to the price* $p^s(s,r)$*, i.e.,* $p^{ns}(r) \leq p^s(s,r)$*.*

Proof. See Appendix 4.8.1.11. □

When consumers are not allowed to search, consumers who have a low willingness to pay decide whether to buy from their recommended seller. The consumers cannot leave. So, each seller has to set a low price $p^{ns}(r)$ to convince these consumers to buy. On the other hand, when consumers are allowed to search, a consumer could search for a seller who sells their preferred variety under $p^s(s,r)$. Consumers are willing to pay more for the product. Each seller can charge a higher price to extract the surplus.

A problem emerges when $p^{ns}(r) \leq v - s < p^s(s,r)$. So, the price $p^{ns}(v,r)$, which requires that no consumer searches, is not high enough to prevent all consumers from searching in equilibrium. At the same time, the price $p^s(s,r)$, which requires that some consumers search, is too high such that no consumer searches in equilibrium.

When $p^{ns}(r) \leq v - s < p^s(s,r)$, a reader might conjecture that an equilibrium price is v or $v - s$ when the number of correct recommendations $R(1)$ is sufficiently high or low, respectively. Suppose that all sellers set their price at v. Then, there is no consumer search. Each seller can sell to consumers who receive a correct recommendation for the seller. The profit of each seller is $vR(1)/2$. On the other hand, if all sellers set their price at $v - s$, which is the highest price that there is consumer search. The demand of each seller is a half (Equation (4.2)), and the corresponding profit is $(v - s)/2$. If $R(1)$ is high, the profit from setting the price at v is also high. In contrast, if $R(1)$ is low, the sellers might want to increase their demand by lowering their price to $v - s$ to convince the consumers to search.

However, I show that neither v nor $v - s$ can be an equilibrium price in Appendix 4.8.2. I investigate a specific case when the recommendation system r is a horizontal line, i.e., $r(\theta) = \bar{r}$, where $\bar{r} \in [0,1]$. If all sellers set their price at v, then each seller has an incentive to deviate by lowering its price for any value of $\bar{r}$ (Example 1 in Appendix 4.8.2). Furthermore, if all sellers set the price at $v - s$, then each seller has an incentive to increase its price (Example 2 in Appendix 4.8.2).

To conclude, when Assumption 9 is violated ($p^s > v - s$), the price $p^{ns}(r)$ is the equilibrium price $p^*(s,r)$ when $p^{ns}(r) > v - s$. However, when $p^{ns}(r) \leq v - s < p^s(s,r)$, neither $p^s(s,r)$ not $p^{ns}(r)$ is an equilibrium. I speculate that an equilibrium is a mixed strategy or an asymmetric equilibrium.

4.7 Conclusion

Consumers can easily receive product recommendations through online and offline settings. However, these recommendations are not always accurate. Some consumers are satisfied with the product sold by recommended sellers. In contrast, consumers who dislike the recommended products may search for other products.

This paper contributes to the literature on the effect of improved information on an equilibrium price. In standard set-ups, there are exogenous proportions of informed and uninformed consumers. Informed consumers have information about prices or product valuations. The existing literature always finds that an improvement in the information (more informed and fewer uninformed consumers) always leads to a decrease or an increase in the equilibrium price depending on whether improved information reveals price visibility or product valuations, respectively (Armstrong, 2015a).

In contrast, I do not have informed consumers in the model. Instead, I introduce a recommendation system that sends a recommendation to each uninformed consumer. So, different uninformed consumers have different information. Improved information in this paper is in the form of higher accuracy of a recommendation system, rather than more informed consumers.

I propose two types of accuracy studied in this paper. First, a *totally* more accurate recommendation system is more likely to send a correct recommendation to every consumer. In contrast, an *allocative* more accurate recommendation system is more likely to send a correct recommendation to consumers who highly dislike their mismatched variety. However, a consumer who slightly dislikes her mismatched variety is more likely to receive an incorrect recommendation. Hence, the two types of accuracy look at different aspects. There are fewer incorrect recommendations under higher *total* accuracy in aggregate. In contrast, the number of incorrect recommendations remains the same under allocative accuracy. The improvement comes from a better allocation of correct and incorrect recommendations.

Introducing a recommendation system provides novel insight into the relationship between improved information and the equilibrium price. I show that a more accurate recommendation system does not always increase the equilibrium price. In particular, the equilibrium price is lower under higher allocative accuracy with a low search cost. In contrast, higher allocative accuracy increases the equilibrium price when the search cost is high. Even under a fixed search cost, the equilibrium price may decrease initially and increase later as the recommendation system keeps improving. Conversely, higher total accuracy always increases the equilibrium price. The monotonic relationship found in the existing literature holds under total accuracy only.

Higher accuracy is beneficial in terms of social welfare. Even though higher allocative accuracy slightly increases the total disutility from consumers who consume the mismatched variety, it lowers the total search cost. In addition, higher total accuracy reduces both the mismatches and the total search cost as fewer consumers search. However, consumers may be worse off from higher prices in the cases of total accuracy and allocative accuracy with a high search cost.

A recommendation system with higher allocative accuracy can be called a more accurate recommendation system according to a measure of the accuracy of a recommendation system (MSE) commonly used in computer science. However, a question of whether higher allocative accuracy can be called improved information is warranted. I argue that it can be, at least from the aggregated level. Even though some consumers are more likely to receive bad information (an incorrect recommendation), they do not suffer a lot from it. In contrast, consumers who suffer a lot from bad information are more likely to receive good information (a correct recommendation). Hence, in aggregate, consumers receive higher gross utility from the recommendation system.

4.8 Appendix:

4.8.1 Proofs of lemmas and propositions

4.8.1.1 Proof of Proposition 13

Proof. When $r(\theta) = 1$ for all θ, all consumers receive a recommendation for a seller who sells their preferred variety. When $p^* = p^e = v$, all consumers have a negative expected payoff from searching, i.e., $v - p^* - s < 0$. Hence, the consumers buy from their recommended seller at the price $p^* = v$, yielding a zero payoff.

To show that the sellers do not have an incentive to deviate, suppose a seller increases its price to

$v+\varepsilon$ where $\varepsilon > 0$. The price exceeds the consumers' willingness to pay. So, they will never buy from the seller. The profit drops to zero.

On the other hand, suppose a seller decreases its price to $v-\varepsilon$. However, its demand does not change. The consumers who see the recommendation for this seller still buy from it. However, consumers who receive a recommendation for other seller do not observe the price cut. They will not search for this seller. Therefore, its profit drops from the price cut.

To show that the equilibrium is unique, suppose otherwise that there exists another symmetric equilibrium $p' = p^e < v$. Then a seller can increase its profit by raising the price to $p'+\varepsilon \leq v$ where $\varepsilon < s$. The consumer will not search for other sellers since $v-(p'+\varepsilon) > v-p^e-s$. Hence, the seller gets the same demand at a higher price. □

4.8.1.2 Proof of Proposition 14

Proof. First, if $S^s_{i,\theta}(h) = \emptyset$, there is no seller left for consumer (i,θ) to inspect. So, the consumer stops searching by construction.

Second, if $S^s_{i,\theta}(h) \neq \emptyset$, consumer (i,θ) expects to get higher or equal utility from search when the expected gain from searching is at least equal the search cost s, i.e., $u_{i,\theta}(p^e(z|h,i,\theta);\alpha_z) - \max_{w\in S^I} u_{i,\theta}(p_w;w) \geq s$. Consequently, consumer (i,θ) searches for seller $z \in S^s_{i,\theta}(h)$ with probability $1/\left|S^s_{i,\theta}(h)\right|$ by Assumption 8. In contrast, when the best search alternative provides lower expected utility subtracting the search cost s than the existing options in the inspected set S^I, there is no expected gain from search. As such, consumer (i,θ) stops searching.

Once consumer (i,θ) stops searching, she can buy from a seller in the inspected set S^I. Because consumer (i,θ) has a single-unit demand, she chooses to buy from a seller that gives him the highest payoff provided that it is more than zero. So, she buys from each of the sellers in the set of the best buying options $S^b_{i,\theta}(h)$ with probability $1/\left|S^b_{i,\theta}(h)\right|$ by Assumption 8. □

4.8.1.3 Proof of Lemma 11

Proof. For the second case when $p_{a_1} \geq p^e + s$, all consumers leave seller a_1. Consumers A will search and buy from seller $a_2 \in S_A$. Their payoff is $u_{A,\theta}(p^e;A) = v-p^e-s > v-p_{a_1} = u_{A,\theta}(p_{a_1};A)$. So, seller a_1 never gets the demand from consumers A. Consumers B get the payoff $u_{B,\theta}(p^e;B) = v-p^e-s$ if they search for any seller $b \in S_B$. Their payoff is higher than buying from seller a_1, i.e., $v-\theta-p_{a_1}$. So seller a_1 does not receive any demand from consumers B either.

Suppose that $p_{a_1} < p^e + s$, then seller a_1's demand comes from four sources. First, consumers A who receive a recommendation for seller a_1 will stay and buy from seller a_1. Doing so yields the payoff $v-p_{a_1}$. If the consumers search, the next best alternative is to search for seller $a_2 \in S_A$. Consumers A receive the net expected payoff from search which equals $v-p^e-s$. However, the expected payoff from searching is lower than buying from seller a_1 when $p_{a_1} < p^e + s$. Since the proportion of consumers A who receive a recommendation for variety A is $R(1)$, seller a_1 receives

$$D^1_{a_1} = \frac{1}{2}R(1). \tag{4.10}$$

Second, consumers A who receive a recommendation for seller b_1 or seller b_2 may search and buy from seller a_1. Consumers A will search for a seller who sells variety A when $v-\theta-p^s \leq v-p^e-s$, or

$$\theta \geq p^e - p^s + s = s.$$

By Assumption 8, consumers A whose $\theta \geq s$ will randomly choose between sellers a_1 and a_2. So, seller a_1 receives a half of such consumers. Once the consumers search, they will buy from seller a_1 at price p_{a_1} if $v - p_{a_1} \geq v - \theta - p$, which is equivalent to

$$\theta \geq p_{a_1} - p.$$

So, if $p_{a_1} < p^e + s$, all consumers A who search for seller a_1 buy from seller a_1. Accordingly, the second source of the demand is

$$D_{a_1}^2 = \frac{1}{2}\int_s^1 (1 - r(\theta))\, d\theta. \tag{4.11}$$

The third source of the demand is from consumers B who receive a recommendation for seller a_1. They will stay and buy from seller a_1 when $v - \theta - p_{a_1} > v - p^e - s$. So, the third source of the demand is

$$D_{a_1}^3 = \frac{1}{2}\int_0^{p^e - p_{a_1} + s} (1 - r(\theta))\, d\theta. \tag{4.12}$$

The last source of the demand is consumers A who receive a recommendation for seller a_2. However, notice that the argument for the first source of the demand is also applicable for seller a_2. Therefore, when seller a_2 set the price $p^s = p^e$, no such consumers will search for seller a_1. Thus,

$$D_{a_1}^4 = 0. \tag{4.13}$$

Combining the four sources (Equations 4.10-4.13) yields the demand $D_{a_1}(p_{a_1})$ as stated in the lemma.

Having derive the demand function, we can find seller a_1's best response p^s. Given the profit $\pi_{a_1}(p_{a_1}) = p_{a_1} D_{a_1}(p_{a_1})$, the first-order condition of the profit with respect to p_{a_1} evaluating at $p_{a_1} = p^e = p^s$ is

$$p^s (1 - r(s)) = 1.$$

Rearranging the first-order condition yields the best response p^s as stated in the lemma.

To make sure that p^s maximizes the profit function, I check the second-order condition. Given $p_{a_1} = p^e = p^s$,

$$\left.\frac{\partial^2 \pi_{a_1}(p_{a_1})}{\partial p_{a_1}^2}\right|_{p_{a_1}=p^e=p^s} = -p^s r'(s) - (1 - r(s)) < 0,$$

since $r'(\cdot) \geq 0$ and $r(s) < 1$ by Assumption 9.

The second-order condition implies that any $p_{a_1} \in [0, p^s + s)$ gives a lower profit to seller a_1 than p^s. Because p_{a_1} in this region does not change the functional form of the demand. Globally, seller a_1 could still deviate to $p_{a_1} \geq p^e + s$. However, the demand drops to zero. So, it is not a profitable deviation. □

4.8.1.4 Proof of Proposition 15

Proof. Under Assumption 9, the equilibrium price $p^*(s,r)$ is less than or equal to $v-s$, i.e., $p^*(s,r) \leq v-s$. Therefore, there are some consumers who search in equilibrium. The demand function used to derive the price $p^s(s,r)$ is valid. Therefore, $p^s(s,r)$ is an equilibrium price. □

4.8.1.5 Proof of Proposition 16

Proof. By Definition 6, $r_1(\theta) \geq r_0(\theta)$ for all θ when recommendation system r_1 is totally more accurate than recommendation system r_0. Therefore, we have

$$p^*(s,r_1) = \frac{1}{1-r_1(s)} \geq \frac{1}{1-r_0(s)} = p^*(s,r_0).$$

If $r_1(s) > r_0(s)$, then we get the strict inequality. □

4.8.1.6 Proof of Proposition 17

Proof. By Definition 7, $r_0(s) \geq r_1(s)$ when $s \leq \bar{\theta}$. Consequently, the equilibrium prices with consumer search under both recommendation systems are such that

$$p^*(s,r_1) = \frac{1}{1-r_1(s)} \leq \frac{1}{1-r_0(s)} = p^*(s,r_0).$$

On the other hand, $r_0(s) > r_1(s)$ when $s > \bar{\theta}$. Thus, the opposite outcome arises, i.e., $p^*(s,r_1) > p^*(s,r_0)$. □

4.8.1.7 Proof of Lemma 12

Proof. In equilibrium, consumer (i,θ) buys from a seller who sells variety $\alpha = i$ with probability $r(\theta)$. Therefore, the surplus is v. Furthermore, consumer (i,θ) receives an incorrect recommendation for a seller who sells variety $\alpha \neq i$ with probability $1-r(\theta)$. Consumers who receive an incorrect recommendation and $\theta < s$ stay at their recommended seller. Hence, the surplus is $v-\theta$. The consumers whose $\theta \geq s$ search for a seller who sells variety $\alpha = i$. They generate the surplus $v-s$.

Because there are both consumers A and consumers B, social welfare W is given by

$$W = 2\left(\int_0^1 vr(\theta)\,d\theta + \int_0^s (v-\theta)(1-r(\theta))\,d\theta + \int_s^1 (v-s)(1-r(\theta))\,d\theta\right).$$

Rearranging the previous equation and using the integration by part yields social welfare W in Equation (4.6).

For consumer surplus CS, notice that all consumers A and B buy at the equilibrium price $p^*(s,r)$. Therefore, consumer surplus CS equals social welfare W minus two times $p^*(s,r)$ as stated in Equation (4.7). □

4.8.1.8 Proof of Proposition 18

Proof. Suppose recommendation system r_1 is allocatively or totally more accurate than r_0. And denote W_1 and W_0 as social welfare under r_1 and r_0, respectively. Then, the direct calculation yields

$$W_1 - W_0 = s\left(R_1(1) - R_0(1)\right) - \int_0^s \left(R_1(\theta) - R_0(\theta)\right) d\theta.$$

Starting with total accuracy, according to Definition 6, we have $1 - r_1(\theta) \leq 1 - r_0(\theta)$ with strict inequality for some θ. In addition, the welfare function can be re-arranged as

$$W = v - s\int_s^1 \left(1 - r(\theta)\right) d\theta - \theta \int_0^s \left(1 - r(\theta)\right) d\theta.$$

So, social welfare W is decreasing in $1 - r(\theta)$. Thus, higher total accuracy increases social welfare, i.e., $W_1 > W_0$.

Moving to allocative accuracy. By Definition 7, we have $R_1(1) = R_0(1)$. So, the difference reduces to $W_1 - W_0 = \int_0^s \left(R_0(\theta) - R_1(\theta)\right) d\theta$. It can be shown that $R_0(\theta) > R_1(\theta)$ for all $\theta \in [0,1)$ under allocative accuracy as follows.

First, because $r_0(\theta) > r_1(\theta)$ for all $\theta < \bar{\theta}$, we have $R_0(\theta) > R_1(\theta)$ for all $\theta < \bar{\theta}$. For $\theta \geq \bar{\theta}$, suppose otherwise that there exists $\theta' \in \left[\bar{\theta}, 1\right)$ such that $R_0(\theta') = R_1(\theta')$ and $R_0(\theta'') \leq R_1(\theta'')$ for all $\theta'' \in [\theta', 1]$. Furthermore, by the single-crossing property, we have

$$\int_{\theta'}^1 r_0(\theta) d\theta < \int_{\theta'}^1 r_1(\theta) d\theta.$$

Then,

$$R_1(1) = R_1\left(\theta'\right) + \int_{\theta'}^1 r_1(\theta) d\theta > R_0\left(\theta'\right) + \int_{\theta'}^1 r_0(\theta) d\theta = R_0(1).$$

So, there is a contradiction since $R_1(1) = R_0(1)$ under allocative accuracy. Hence, $R_0(\theta) > R_1(\theta)$ for all θ. We have $W_1 - W_0 = \int_0^s \left(R_0(\theta) - R_1(\theta)\right) d\theta > 0$ for all s. □

4.8.1.9 Proof of Lemma 13

Proof. The first step is to derive the demand of each seller. Without loss of generality, let's focus on the demand $D_{a_1}^{ns}(p_{a_1})$ of seller a_1 at price p_{a_1} given that consumers do not search. First, if $p_{a_1} > v$, the price exceeds the highest amount that any consumer is willing to pay. Therefore, the demand is zero. Suppose that $p_{a_1} \leq v$. First, all consumers A are willing to buy. So, the demand from consumers A is $R(1)/2$.

Second, consumer (B, θ) buys from seller a_1 if $v - \theta - p_{a_1} \geq 0$, or $\theta \leq v - p_{a_1}$. Accordingly, seller

a_1's demand from consumers B is $\left(\int_0^{v-p_{a_1}}(1-r(\theta))\,d\theta\right)/2$. Combining the two sources yields

$$D_{a_1}^{ns}(p_{a_1}) = \begin{cases} \frac{1}{2}\left(R(1)+\int\limits_0^{v-p_{a_1}}(1-r(\theta))\,d\theta\right) & \text{if } p_{a_1}\le v \\ 0 & \text{if } p_{a_1}>v, \end{cases} \tag{4.14}$$

with the corresponding profit $\pi_{a_1}^{ns}(p_{a_1}) = p_{a_1}D_a^{ns}(p_{a_1})$.

The first-order derivative of seller a_1's profit function $\pi_{a_1}^{ns}(p_{a_1})$ is

$$\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}} = \frac{1}{2}\left(R(1)+\int_0^{v-p_a}(1-r(\theta))\,d\theta - p_{a_1}(1-r(v-p_{a_1}))\right), \tag{4.15}$$

and the second-order derivative is

$$\frac{\partial^2 \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}^2} = -\frac{1}{2}\left(p_{a_1}r'(v-p_{a_1})+2(1-r(v-p_{a_1}))\right)<0.$$

Therefore, the first-order derivative of the profit in Equation (4.15) is continuously and strictly decreasing. There are three possible cases:

1. $0 \ge \left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v-1} > \left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v}$,

2. $\left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v-1} > 0 > \left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v}$, and

3. $\left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right| > \left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v} \ge 0$.

In the first case, the first-order derivative is always non-positive. Hence, $p^{ns}(r)=v-1$ which equals the utility of the consumer who dislikes the mismatched variety the most. At this price, all consumers who receive a recommendation from seller a_1 is willing to buy. The condition that $0 \ge \left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v-1}$ is satisfied when $1+\frac{1}{1-r(1)} \le v$ as stated in the first part of the lemma.

Let's skip to the third case. The first-order derivative is always non-negative. Therefore, each seller sets the highest price possible, i.e., $p^{ns}(r)=v$. The condition that $0 > \left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=v}$ is equivalent to $v < \frac{R(1)}{1-r(0)}$.

In the second case where $\frac{R(1)}{1-r(0)} < v < 1+\frac{1}{1-r(1)}$, the first-order derivatives evaluated at $p_{a_1}=v-1$ and $p_{a_1}=v$ are positive and negative, respectively. Because the second-order derivative is negative, the intermediate value theorem guarantees that there exists a unique $p^{ns}(r)\in[v-1,v]$ such that the first-order condition is satisfied, i,e.,

$$\left.\frac{\partial \pi_{a_1}^{ns}(p_{a_1})}{\partial p_{a_1}}\right|_{p_{a_1}=p^{ns}(r)} = 0.$$

The above equation is equivalent to Equation (4.9) in the second part of the lemma. □

4.8.1.10 Proof of Proposition 19

Proof. In equilibrium, the expected price of each seller equals the equilibrium price: $p^e(k|h,i,\theta) = p^*(s,r) > v-s$ for all $k \in S$. So, no consumer searches at the price $p^{ns}(r)$, and $p^{ns}(r)$ maximizes the profit of each seller when no consumer searches. Therefore, $p^{ns}(r)$ is an equilibrium price. The equilibrium is unique because there is no other price that a seller could charge to earn a higher profit. □

4.8.1.11 Proof of Corollary 12

Proof. Suppose otherwise that $p^{ns}(r) > p^s(s,r)$. So, we have

$$p^{ns}(r) = \frac{R(1) + \int\limits_0^{v-p^{ns}(r)} (1-r(\theta))\,d\theta}{1-r(v-p^{ns}(r))} > \frac{1}{1-r(s)} = p^s(s,r).$$

Because $R(1) + \int\limits_0^{v-p^{ns}(r)} (1-r(\theta))\,d\theta \leq 1$, it is necessary that

$$\frac{1}{1-r(v-p^{ns}(r))} \geq \frac{1}{1-r(s)}.$$

The above inequality is satisfied when $p^{ns}(r) \leq v-s$. Recall that, under Condition (4.8), we have $p^s(s,r) > v-s$. So, $p^{ns}(r) \leq v-s < p^s(s,r)$. There is a contradiction. Therefore, $p^{ns}(r) \leq p^s(s,r)$. □

4.8.2 Check whether v or $v-s$ can be an equilibrium

Suppose Assumption 9 is violated, and $p^{ns}(r) \leq v-s < p^s(s,r)$. Then, I show that neither v nor $v-s$ is an equilibrium price in this appendix. It is easiest to show with a specific functional form of a recommendation system. In particular, I assume that $r(\theta)$ is a horizontal line, i.e., $r(\theta) = \bar{r}$ for $\bar{r} \in [0,1]$.

Example 1. Suppose $p^{ns}(v,r) \leq v-s < p^s(s,r)$, and $r(\theta) = \bar{r}$ where $\bar{r} \in [0,1]$. Then, the price p' equals v cannot be an equilibrium for any value of $\bar{r}$.

To show that $p' = v$ cannot be an equilibrium, suppose otherwise that $p' = v$ is an equilibrium. So, consumers do not search at price p'. Each seller could sell to consumers who receive a correct recommendation for it, which equals $\bar{r}/2$. Hence, the non-deviating profit π^{non} is $v\bar{r}/2$.

I will show that any seller has an incentive to deviate by lowering its price from v. Again, without loss of generality, let's look at seller a_1. Suppose that seller a_1 decreases its price by $\varepsilon \geq 0$. So, the deviating price p^{dev} is $v-\varepsilon$. Using Equation (4.14), we have the demand of seller a_1 at price p^{dev}. And the corresponding deviating profit is

$$\pi_{a_1}^{dev} = \frac{1}{2}(v-\varepsilon)(\bar{r} + (1-\bar{r})\varepsilon).$$

To show that each seller has an incentive to deviate by lowering its price ($\varepsilon > 0$), we need

$$\left.\frac{d\pi_{a_1}^{dev}}{d\varepsilon}\right|_{\varepsilon=0} = \frac{1}{2}(v(1-\bar{r}) - \bar{r}) > 0.$$

Under $p^{ns}(v,r) \leq v-s$, it must be that $v(1-\bar{r})-\bar{r} \geq 2s$. So, the above inequality always holds. If all sellers set the price $p' = v$, then each seller has an incentive to lower its price. Thus, $p' = v$ cannot be an equilibrium for any $\bar{r}$.

Example 2. Suppose $p^{ns}(v,r) \leq v-s < p^{s}(s,r)$, and $r(\theta) = \bar{r}$ where $\bar{r} \in [0,1]$. Then, the price p'' equals $v-s$ cannot be an equilibrium for any value of $\bar{r}$.

Suppose otherwise that $p'' = v-s$ is an equilibrium. At price p'', some consumers search in equilibrium. According to Lemma 11, the demand of all sellers is 1/2. Hence, the non-deviating profit π^{non} is $(v-s)/2$.

Without lost of generality, I will show that seller a_1 has an incentive to deviate by increasing its price above $p'' = v-s$. Suppose that seller a_1 sets the deviating price p^{dev} at $v-s+\varepsilon$, where $\varepsilon \in [0,s]$. Using Equation (4.2), we get the demand of seller a_1. And the deviating profit $\pi_{a_1}^{dev}$ is

$$\pi_{a_1}^{dev} = \frac{1}{2}(v-s+\varepsilon)(1-(1-\bar{r})\varepsilon).$$

Seller a_1 has an incentive to deviate by increasing its price ($\varepsilon > 0$) when

$$\left.\frac{d\pi_{a_1}^{dev}}{d\varepsilon}\right|_{\varepsilon=0} = \frac{1}{2}(1-(1-\bar{r})(v-s)) > 0.$$

Under $v-s < p^{s}(s,r)$, it is required that $1-(1-\bar{r})(v-s) > 0$. Hence, the above inequality is satisfied. So, $p'' = v-s$ cannot be an equilibrium for any value of $\bar{r}$.

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This thesis contains three essays on different topics related to competition in digital markets. The first essay analyzes the effects of interoperability on price competition in a market with network effects. It looks at a market in an early stage of development before the monopoly is entrenched. The second essay studies the roles of complementarity and economies of scope in conglomerate mergers involving a digital ecosystem. It shows that these two characteristics lead to foreclosure in the long run. The last essay investigates the effect of improved information, in the form of more accurate product recommendations, on price competition.

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