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PAPERS

Model of fractional-order resonant wireless power transfer system for optimal output

Ravneel Prasad, Krishneel Sharma, Bhavish Gulabdas, Utkal Mehta[1]

Wireless Power Transfer (WPT) technology has recently gained popularity in applications and research topics. It enables the transfer of electrical energy from a source to a load without connecting wires physically. The WPT system is commonly studied classically using integer order capacitors and inductors. Nonetheless, such integer order based systems have drawbacks, such as low output power, poor transmission efficiency and sensitivity to parameter variations. This paper proposes a fractional order resonant WPT circuit whereby both the transmitting and receiving ends are composed of a fractional capacitor and inductor to overcome such problems. In this paper, the overall performance is studied based on its output power and efficiency considering a series-parallel topology. The effect of fractional order in fractal elements will be analyzed to observe the optimal combination of components to achieve the maximum output power with higher efficiency. Through a comparative analysis of the results, several combinations of circuit parameters can provide a theoretical understanding for implementing an experimental system.

K e y w o r d s: wireless power transfer, fractal elements, power efficiency, impedance model, series-parallel topology

1 Introduction

With fast innovations in mobile products, wireless power transfer and related technology have gained immense attention in recent decades. It becomes now what is informally known as the game-changing" solution for charging electronic devices [1]. The scale of WPT is not just limited to consumer electronics, but ever since electrical vehicles (EVs) have grown in popularity and demand, opportunities in the wireless charging market have significantly increased. Due to more and more portable electronic devices, the traditional method of recharging the batteries through cables is not the best option. To avoid the inconvenience of connecting cables and enhancing safety and flexibility, researchers have developed various ways and means to transmit electrical energy from a transmitter to a receiver wirelessly.

The wireless charging technology is advancing in two significant directions, *ie*, non-radiative and radiative WPT. Non-radiative techniques can be further classified into three categories, *ie*inductive coupling, magnetic resonance coupling, and capacitive coupling methods [2]. On the other hand, the radiative technique involves microwaves to transmit power long distances. Some of the previous great works have led to deeper insights and ideas about WPT. For instance, in 2007, scientists at MIT proposed a highly efficient mid-range wireless power transmission system, more widely known as WiTricity. This is based on the magnetic resonance coupling method capable of transmitting power up to more than two meters to light up a bulb [3]. In theory, two standard methods of WPT are namely near-field and far-field. A near-field transmission works as per electromagnetic field technique [4], while the far-field technique uses optical and microwaves to transfer energy to long distances [5]. Both have led to many applications such as, but are not limited to, biomedical implants, consumer electronics, EVs, low-power sensor networks, space, and military applications [6].

In [7], a 1MW inductive power transfer (IPT) system was designed for a high-speed train to power vehicles in real-time without the use of any energy storage devices and achieved an efficiency of 82.7% when 989 kW was transmitted. This study proved that IPT systems have some potential for railroad applications. Such systems typically work at frequencies in the range of kHz, as confirmed by [2]. The critical characteristics of the IPT system are the kHz operating frequency delivering high power (kW - MW), and greater efficiency at short distances [7]. One of the significant drawbacks of this approach is that as the transmission distance increases, the power delivered to the load drops significantly leading to low efficiency, and for this reason, IPT systems are generally only used for very short distances [4], [8]. In [9], a selective method is proposed to transfer power to multiple loads using different resonant frequencies.

Extensive research occurred on the near-field methods in recent years, and the results are undoubtedly promising. How- ever, there exist certain drawbacks in using an integer-order model, such as low output power, high resonant frequency, less transmission efficiency, and frequency splitting [10]. Hence, the fractional-order WPT

[1] Electrical and Electronics Engineering Discipline, The University of the South Pacific, Laucala campus, Fiji.ail: ravprasad8@gmail.com; sharmakrishneel940@gmail.com; bhavishgulabdas@ymail.com; utkal.mehta@usp.ac.fj

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(FOWPT) was introduced to solve the problems mentioned above. In [10], analysis on the effect of fractional orders on power transmission and efficiency was performed. It also demonstrated that resonant frequency and coupling coefficient can be adjusted by changing the fractional order. Similarly, in [11], a generalized FOWPT was established, and analysis of its performance and efficiency was done in [12] investigating the two-port network concept in the analysis of the FOWPT circuits whereby both the series-series (SS) and series-parallel (SP) topologies were analyzed. It was found that the SP topology had better efficiency and performance than the SS topology when the coil size is different. In [13] was proposed a fractional-order (FO) resonant WPT system which contained a FO resonant circuit on the transmitter side. By choosing the appropriate parameters in a fractional circuit, one can achieve a constant current output (independent of load size). In this case, the constant current was obtained by neglecting internal resistance. Nevertheless, this cannot be ignored in an actual application. Thus, it may cause lower current accuracy. One of the several reasons for the low efficiency and partially stable operation of IPT systems is the phenomenon of frequency bifurcation. Authors in [14] have conducted a study on frequency bifurcation using the fractional-order inductive power transfer (FOIPT) system. A frequency bifurcation is known to occur under certain conditions, such as coil misalignment (coupling coefficient changes) and load changes. By controlling the fractional order, the boundary of critical distance and critical load, the frequency bifurcation phenomena can be eliminated. A fractional coupled model of FOWPT was obtained using generalized fractional coupled-mode equations in [15]. It confirms the correctness of the fractional couple model to build the circuit.

In [16], a study with the FOWPT system discussed the sensitivity of the resonant frequency. The resonant frequency is easily disturbed by metal or non-resistive load, which occurs in real-world applications. A FOWPT system is proposed whereby the order of the capacitor is more than one. When the order is less than one, the resistance of FOC is positive, which brings additional losses in the system. On the other hand, when the order is greater than one, the resistance of FOC becomes negative, and the capacitor produces active power. In the case of integer-order WPT (IOWPT), there is no degree of freedom to adjust the loss; thus, efficiency is highly sensitive to the resonant frequency. It is evident that fractional element circuits have more freedom to adjust the efficiency, counter to the classical approach.

While numerous research have studied various WPT systems, there still exist many practical issues such as low output power, high resonant frequency, low efficiency, and a few others [16]. The traditional WPT systems are comprised of integer-order elements (capacitors and inductors). However, as per [17], [18], integer-order elements do not ideally exist, *ie* , in practical applications, the order of most of the capacitors and inductors are close to 1. Hence, their fractional-order characteristics are disregarded. To effectively avoid the problems above, many researchers have directed their studies using fractional elements [16], [15]. It was found that the analysis and dynamic behaviour of systems cannot be well explained by the integer order calculus [15], [19]. Thus, research in FC started to grow rapidly whereby it is now applied in various systems including, but are not limited to, wireless power transfer systems [15], circuit theory [20], chaotic systems [21], electromagnetics [22], DC-DC convert Riemann, Grunwald, and Caputo [24].

One of the initial applications of FC to WPT was the electrical energy transmission using fractional- order components [19]. As research was further developed over the years, it was found that FOWPT systems could deliver better output power, transmission efficiency, and resonant frequency by adjusting the orders of the fractional-order elements [15]. Although fractal components are not standard market- oriented parts, many have been fabricated in labs [25]. The circuits having fractional-order elements are undoubtedly the solution to many practical problems generally encountered in WPT. Hence, this paper presents following contributions. Firstly, the new magnetic resonance coupling technique is built in the fractional domain for the SP topology. The modified expressions of transmission efficiency and output power are developed to see the effects of varying fractional orders in capacitor and inductor. Moreover, the proposed scheme can verify power transfer with different resonant frequency using the exact expressions provided. The power transmission efficiency can be optimized by adjusting orders of the fractional elements. Finally, the comparisons will show that the output power and efficiency of the FOWPT system are much more than the classical WPT circuit in literature due to flexible nature of the circuit. The various conditions are studied numerically to prove the effectiveness of the proposed scheme.

2 Proposed series-parallel FOWPT model

The proposed circuit model for the WPT system is illustrated in impedance form in Fig.1. The schematic with fractional elements, namely fractional-order capacitor and fractional-order inductor, will be used in series-parallel topology. The simplified fractional circuit in Fig. 1 has $(L_{1\beta}, L_{2\beta})$ pseudo-inductance values with (β_1, β_2) fractional-orders of the inductor at the transmitter and receiver, respectively. Also, it has $(C_{1\alpha}, C_{2\alpha})$ pseudo-capacitance values with (α_1, α_2) fractional-orders of the capacitor at the transmitter and receiver, respectively. The source power supply U_s can be obtained from an inverter. The proposed circuit in complex notation $\mathcal{U} = U\angle\phi_U,\ \mathcal{J} = I\angle\phi_I,\ \mathcal{Z} = Z\angle\phi_Z$, can be described as

$$\begin{aligned}
\mathcal{U}_s &= \mathcal{J}_1 R_1 + \mathcal{Z}_{\mathrm{C1}}\mathcal{J}_1 + \mathcal{J}_1\mathcal{Z}_{\mathrm{L1}} - \mathcal{J}_2\mathcal{Z}_M,\\
0 &= \mathcal{J}_2\mathcal{Z}_{\mathrm{L2}} + \mathcal{J}_2 R_2 + \mathcal{Z}_{\mathrm{C2}}(\mathcal{J}_2 - \mathcal{J}_3) - \mathcal{J}_1\mathcal{Z}_M, \qquad (1)\\
0 &= \mathcal{Z}_{\mathrm{C2}}(\mathcal{J}_3 - \mathcal{J}_2) + \mathcal{J}_3 R_{\mathrm{L}},
\end{aligned}$$

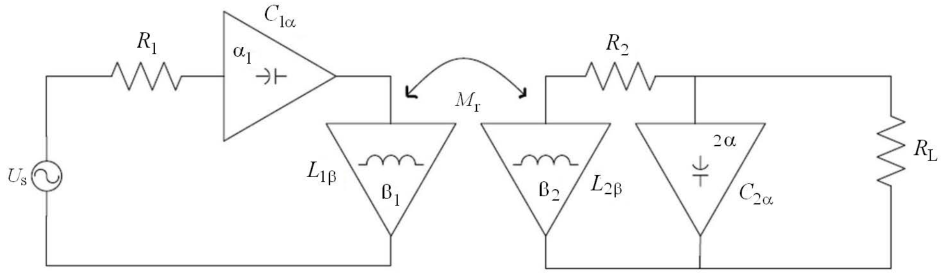

Fig. 1. Equivalent circuit of series-parallel wireless power transfer

where

- $\mathcal{U}_s$ is the high-frequency voltage source.
- R_1 is the internal resistance of the primary side.
- R_2 is the internal resistance of the secondary side.
- $\mathcal{Z}_{\rm C1}$ is the equivalent impedance of the primary compensation capacitor.
- $\mathcal{Z}_{\rm C2}$ is the equivalent impedance of the secondary compensation capacitor.
- $\mathcal{Z}_{\rm L1}$ is the equivalent impedance of the transmitting coil.
- $\mathcal{Z}_{\rm M}$ is the equivalent mutual impedance.
- $\mathcal{Z}_{\rm L2}$ is the equivalent impedance of the receiving coil.
- $R_{\rm L}$ is the load resistance.
- $\mathcal{J}_1$ is the current following in the primary side.
- $\mathcal{J}_2$ is the current following in the secondary side.
- $\mathcal{J}_3$ is the current following in the second loop of the secondary side.

Here, $\mathcal{Z}_{\rm C1}, \mathcal{Z}_{\rm C2}, \mathcal{Z}_{\rm L1}, \mathcal{Z}_{\rm L2},$, and $\mathcal{Z}_{\rm M}$, are defined as follows

$$\begin{aligned} \mathcal{Z}_{\rm C1} &= R_{\rm C1} - jX_{\rm C1}, \quad \mathcal{Z}_{\rm L1} = R_{\rm L1} + jX_{\rm L2}, \\ \mathcal{Z}_{\rm C2} &= R_{\rm C2} - jX_{\rm C2}, \quad \mathcal{Z}_{\rm L2} = R_{\rm L2} + jX_{\rm L2}, \\ & \qquad \qquad \qquad \quad \mathcal{Z}_{\rm M} = R_{\rm M} + jX_{\rm M}. \end{aligned} \tag{2}$$

- $R_{\rm C1}$ and $R_{\rm L1}$ are equivalent resistances of the fractional-order capacitor and inductor, respectively, of the transmitter side.
- $X_{\rm C1}$ and $X_{\rm L1}$ are equivalent reactance of the transmitter sides fractional-order capacitor and inductor, respec tively.
- $R_{\rm C2}$ and $R_{\rm L2}$ are equivalent resistances of the receiver side of the fractional-order capacitor and inductor.
- $X_{\rm C2}$ and $X_{\rm L2}$ are equivalent reactance of the receiver sides fractional-order capacitor and inductor, respectively.
- $R_{\rm M}$ and $X_{\rm M}$ are equivalent resistance and equivalent reactance of the fractional-order mutual inductance, respectively.

The equivalent resistances and reactances of the transmitter and receiver side are

$$\begin{aligned} R_{{\rm L}k} &= \omega^{\beta k} L_{k\beta} \cos \frac{\beta_k \pi}{2}, \\ R_{\rm M} &= \omega^{\gamma} M_{\gamma} \cos \frac{\gamma \pi}{2}, \\ R_{{\rm C}k} &= \frac{1}{\omega^{\alpha k} C_{k\alpha}} \cos \frac{\alpha_k \pi}{2}, \\ X_{{\rm L}k} &= \omega^{\beta k} L_{k\beta} \sin \frac{\beta_k \pi}{2}, \\ X_{{\rm C}k} &= \frac{1}{\omega^{\alpha k} C_{k\alpha}} \sin \frac{\alpha_k \pi}{2}, \end{aligned} \tag{3}$$

for $k = \{1, 2\}$ as described in [10]. From (1), one can derive the currents flowing through the fractional-order capacitor and inductor of the transmitter and receiver circuit

$$\begin{aligned} \mathcal{J}_1 &= \frac{\mathcal{J}_2(\mathcal{P} - \mathcal{Q})}{\mathcal{Z}_{\rm M}}, \\ \mathcal{J}_2 &= \frac{\mathcal{Z}_{\rm M} \mathcal{U}_2}{\mathcal{Z}_1(\mathcal{P} - \mathcal{Q}) - \mathcal{Z}_{\rm M}^2}, \\ \mathcal{J}_3 &= \frac{\mathcal{Z}_{\rm C2} I_2}{\mathcal{Z}_{\rm C2} + R_{\rm L}}, \quad \mathcal{J}_{\rm C2} = \mathcal{J}_2 - \mathcal{J}_3, \end{aligned} \tag{4}$$

$$\mathcal{P} = \mathcal{Z}_{\rm L2} + R_2 + \mathcal{Z}_{\rm C2}, \quad \mathcal{Q} = \frac{\mathcal{Z}_{\rm C2}^2}{\mathcal{Z}_{\rm C2} + R_{\rm L}}. \tag{5}$$

When the system meets the condition of resonance, the angular frequency of the receiver and transmitter is equal to the angular frequency of the supply. The angular frequency of the SP-WPT is the same as the angular frequency of the SS-WPT, [26]

$$\omega_0 = \sqrt[\alpha_k + \beta_k]{\frac{\sin \frac{\alpha_k \pi}{2}}{L_k C_k \sin \frac{\beta_k \pi}{2}}}, \tag{6}$$

again for for $k = \{1, 2\}$. Hence, the value of the transmitter and receiver compensation capacitor can be calculated from (6).

Then, the input power $P_{\rm in}$, the output power $P_{\rm out}$ and efficiency η at the resonance are

$$\begin{aligned} P_{\rm in} &= I_1^2 \left[R_1 + A_1 + B_1 \right] + \\ &+ I_2^2 \left[R_2 + B_2 \right] + C + \\ &+ I_{\rm C2}^2 \left[A_2 \right] + I_3^2 R_{\rm L} + C, \\ P_{\rm out} &= I_2^2 R_{\rm L}, \quad \eta = \frac{P_{\rm out}}{P_{\rm in}} \times 100, \end{aligned} \tag{7}$$

where

$$A_k = \frac{1-\operatorname{sgn}(\alpha_k - 1)}{2} R_{Ck},$$
$$B_k = \frac{1-\operatorname{sgn}(\beta_k - 1)}{2} R_{Lk}, \qquad (8)$$
$$C = \frac{1-\operatorname{sgn}(\gamma - 1)}{2} R_{M} \operatorname{Re}\{\mathcal{J}_1 \mathcal{J}_2\}.$$

Remark: The fractional-order components have negative resistance property thus, to eliminate the negative resistance, the signum function

$$\operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x \le 0 \end{cases}, \qquad (9)$$

was used.

Note: when the values of all fractional-orders are unity, the equations for power and efficiency are the same as those of the IO-WPT system.

3 Methodology to verify performances

For the presented schematic of SPWPT in Fig. 1, we first assumed the fractional order of mutual inductance to be unity, $ie \ = 1$ with no loss of generality. We study the circuits performance with four fractional-order components, inductors and capacitors.The circuit's input- output power and thus, efficiency have different variations with respect to different fractional orders. To fully understand how the system performance is affected with varying fractional orders, four cases including one special case are considered as following.

Case 1: classical WPT model

It is required to check the effect of fractal elements with respect to classical components $\alpha_{1,2} = 1, \beta_{1,2} = 1$. In all cases, at least one component in SP-WPT behaves in the integer domain. The output power and efficiency for respective circuit are calculated for the analysis.

Case 2: fractional-order capacitors and integer inductors

Now orders α_1 and α_2 vary with fixed $\beta_{1,2} = 1$ The rationale of the study is twofold: First, the efficiency and output power are determined with a fix one α_1 for varying the order of the compensation capacitor (α_2) in the receiving end. Second, similar analysis is continued with one value of α_2 for varying the order of capacitor at the transmitting end.

Case 3: full fractional-order circuit parameters

Here $\beta_{1,2}$ vary for $0 < \alpha_1 < 2$ and $\alpha_2 = 1.2$. Upon analyzing the previous cases, the order of $\alpha_2 = 1.2$ was found to produce the maximum efficiency. Hence, α_2 was kept at optimal fixed value while the order of the series compensation capacitor in the transmitting end was varied from 0 to 2. Finally, the efficiency and output power were graphed, and the best combi- nations were identified. In a similar manner, the efficiency and output power were determined for varying α_2 in the receiving capacitor with fix capacitor order α_1 in the transmitting end. At the end, the efficiency and output power were graphed, and the best combinations were identified. After analyzing all the cases, the best combinations of $\alpha_1, \alpha_2, \beta_{1,2}$ were selected

Special case: Effect of the load resistance R_L. It is vital to analyse the effect of output load resistance value on efficiency. Such analysis is not presented in the previous literature. It can suggest a favourable range of load value with respect to optimal efficiency and peak output power.

4 Analysis of SPWPT with fractional-order element

The generalized output power equation and efficiency relation at the resonance provide a valuable result for the analysis of FOWPT system. According to the fractional-order SPWPT model, the fractal component has ability to adjust the loss by varying the order. Note it again, it is not possible with integer classical model.

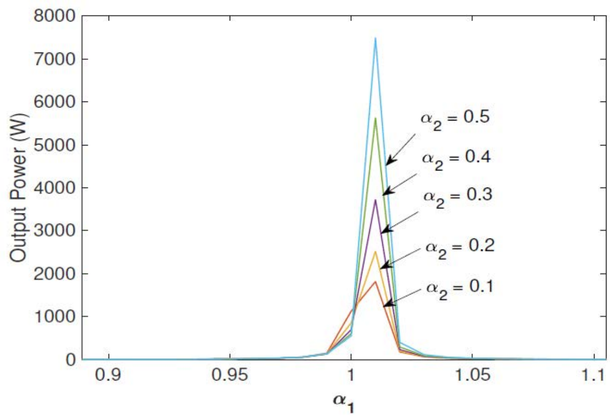

Fig. 2. Output power versus α_1 when $0 < \alpha_2 \le 0.5$ at $\beta_{1,2} = 1$

The systematic analysis shows below that the lossy system can be improved by varying resistances in $(L_{1\beta}, L_{2\beta})$ and $(C_{1\alpha}, C_{2\alpha})$. It is necessary to choose the right value and combination of (β_1, β_2) and (α_1, α_2). In analysis, the circuit parameters such as $U_s = 100\,\text{V}$, $f = 250\,\text{kHz}$, $R_1 = R_2 = 1.3\,\Omega$, $L_{1\beta} = 159.2\,\mu\text{H}$, 4 $L_{2\beta} = 15.9\,\mu\text{H}$, $M_\gamma = 15.9\,\mu\text{H}$ and $R_L = 1\,\text{k}\Omega$ have been used to illustrate the transmission efficiency of the fractional-order SPWPT system. Let us take the fractional-order capacitors for varying α_1 and α_2, but inductors could be integer or fractional order but, same for both as $\beta_{1,2} = \beta$.

Figure 2 shows how the output power is affected by varying α_2 from 0 to 0.5 with increments of 0.1 with respect to changing α_1 while keeping $\beta_{1,2} = 1$. The figure depicts that as the order of α_2 increases from 0.1 to 0.5, the output power increases also. Figure 3 illustrates how the output power is affected with α_2 from 0.5 to 1. Continuing from Fig. 2, the output power decreases for α_2 ranging from 0.5 to 0.6 and then it starts to increase from

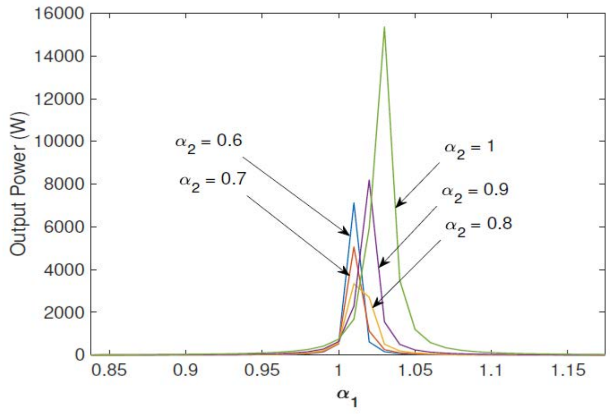

Fig. 3. Output power versus α_1 when $0.5 < \alpha_2 \leq 1$ at $\beta_{1,2} = 1$

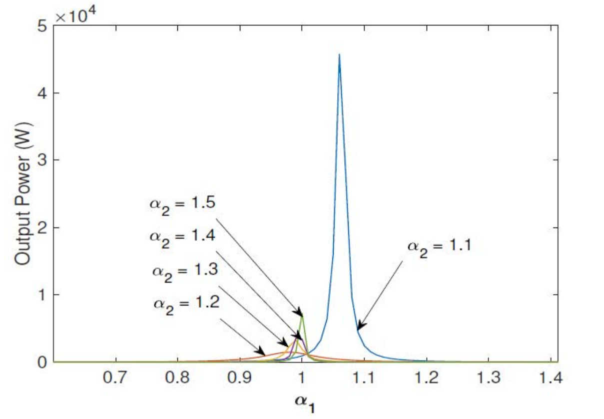

Fig. 4. Output power versus α_1 when $1 < \alpha_2 \leq 1.5$ at $\beta_{1,2} = 1$

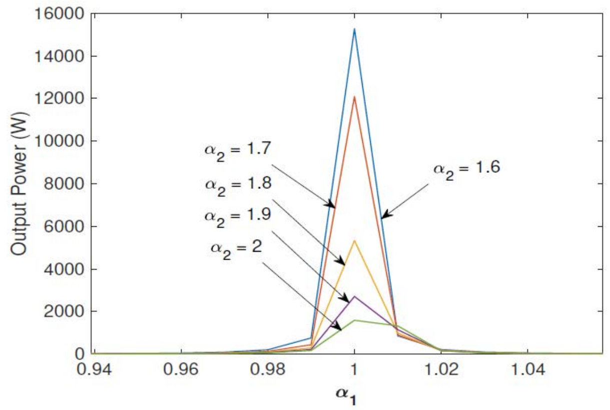

Fig. 5. Output power versus α_1 when $1.5 < \alpha2 \leq 2$ at $\beta_{1,2} = 1$

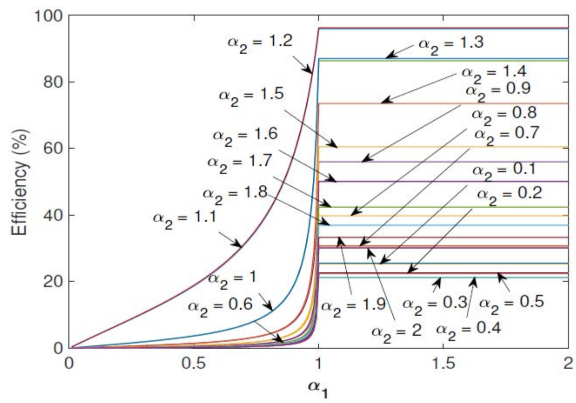

Fig. 6. Efficiency versus α_1 when $0 < \alpha_2 \leq 2$ at $\beta_{1,2} = 1$

0.7 to 1. Figure 4 is with α_2 varied from 1 to 1.5. As for the order of α_2 at 1.1, the output power has significantly increased three times more than what it was at $\alpha_2 = 1$ from previous values in Fig. 3. Conversely, the output power decreases almost twice the value when the order of α_2 is increased from 1.1 to 1.2. Further increase of the order of α_2 from 1.2 to 1.5 shows a gradual increase in the output power.

Figure 5 also shows similar effect when α_2 changes from 1.5 to 2. When $\alpha_2 = 1.6$, the output power increases twice more than what it was at $\alpha_2 = 1.5$. On the other hand, increasing the order of α_2 from 1.7 to 2 shows a decreasing trend of the output power. At the same time, the efficiency is calculated for varying order α_2. Figure 6 is showing the efficiency variations with α_2 changes from 0 to 2 against α_1. An efficiency of 96.23% is achieved, which is the maximum efficiency of the system when the order of α_1 and α_2 are 1 and 1.2, respectively.

The analysis is continued with different combinations. Figure 7 portrays how the output power is affected by varying α_1 from 0 to 2 with increments of 0.1 against 2. The figure depicts that as the order of α_1 increases from 0.1 to 0.9, the output power steadily increases. However, as α_1 is increased to 1, the output power drastically increases and then it has a sudden drop to around 2.5 kW at $\alpha_1 = 1.1$. Thereafter, it steadily decreases as the order of α_1 increases from 1.2 to2) With the calculated output power, the efficiency is studied by varying α_1 from

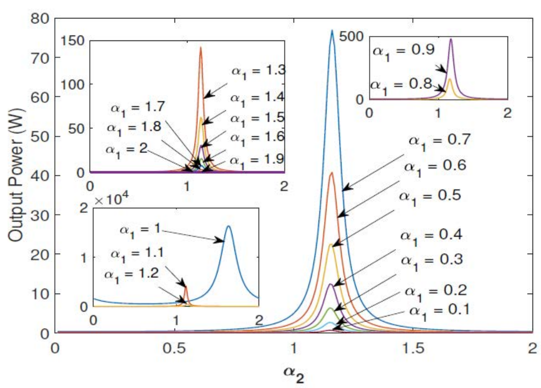

Fig. 7. Output power versus α_2 when $0 < \alpha_1 \leq 2$ at $\beta_{1,2} = 1$

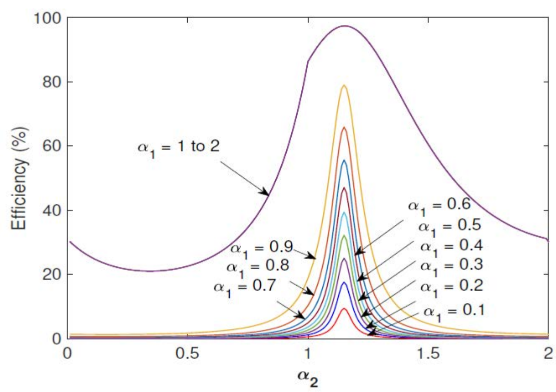

Fig. 8. Efficiency versus α_2 when $0 < \alpha_1 \leq 2$ at $\beta_{1,2} = 1$

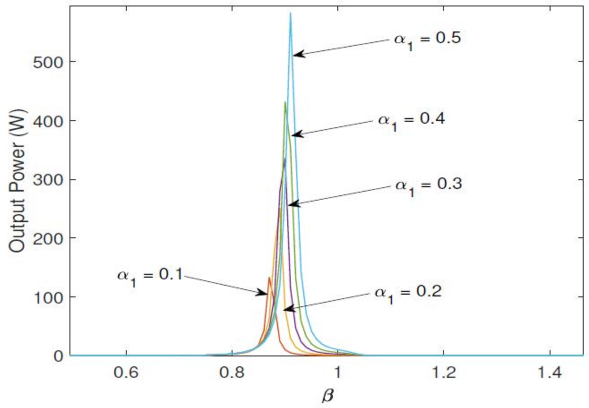

Fig. 9. Output power versus $\beta 1,2$ when $0 < \alpha_1 \leq 0.5$ at $\alpha_2 = 1.2$

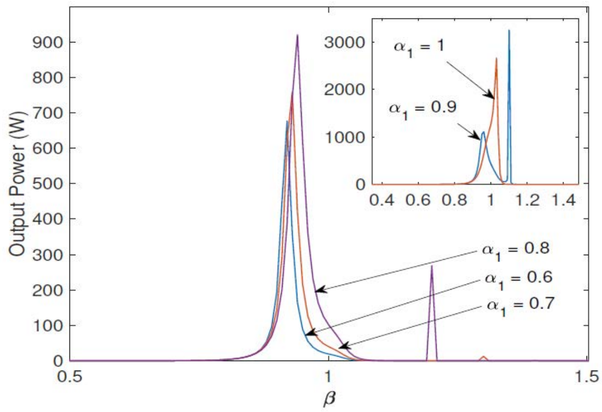

Fig. 10. Output power versus $\beta_{1,2}$ when $0.5 < \alpha_1 \leq 1$ at $\alpha_2 = 1.2$

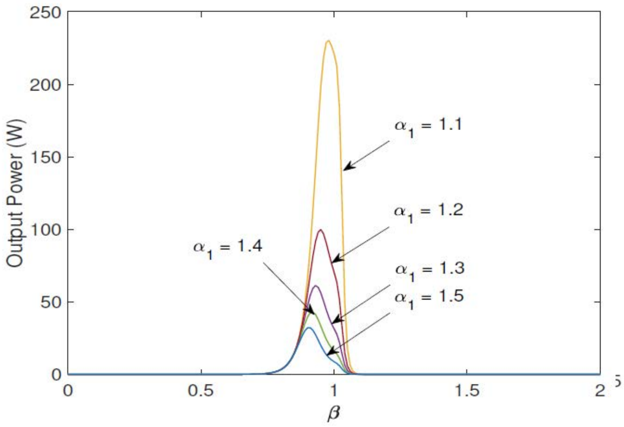

Fig. 11. Output power versus $\beta_{1,2}$ when $1 < \alpha_1 \leq 1.5$ at $\alpha_2 = 1.2$

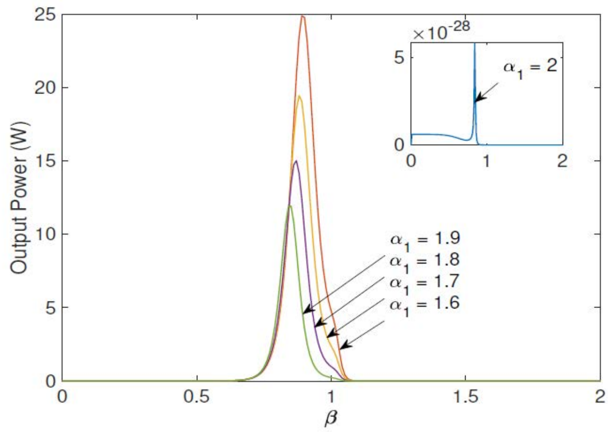

Fig. 12. Output power versus $\beta_{1,2}$ when $1.5 < \alpha_1 \leq 2$ at $\alpha_2 = 1.2$

Table 1. Optimal combination of $\alpha 1$, $\alpha 2$ and $\beta_{1,2}$

Test	α_1	α_2	$\beta_{1,2}$	P_{out} (kW)	η (%)	C_α (nF)
Case 2	1.2	1.0	1.0	1.28	96.23	$C_{1\alpha} = 13.96$ $C_{2\alpha} = 2.54$
Case 3.1	1.0	1.2	1.03	2.66	97.09	$C_{1\alpha} = 1.66$ $C_{2\alpha} = 910.75$
Case 3.2	1.0	1.2	1.02	2.02	97.22	$C_{1\alpha} = 1.90$ $C_{2\alpha} = 10.50$

0 to 2 with respect to changing α_2 and keeping $\beta_{1,2}$ still constant. Fig. 8 delineates how the efficiency is affected by varying orders. It can be noted that efficiency of 96.23% is achieved, which is the maximum efficiency of the system when the order of α_1 is in the range of 1 to 2 and α_2 is 1.2. The numerical values studied from Fig. 2 to Fig. 6, one can note that an efficiency of 96.23% is achieved with a power output of 1.3 kW and the fractional order $\alpha_1 = 1.2$. On the other hand, the system could produce a maximum output power about 15 kW, however, at the cost of reduced efficiency.

Similarly, Fig. 7 and Fig. 8 presented that the α_2 value was critical to set the maximum output power. It was found that the maximum output power about 15 kW was achieved with low efficiency as 50.07% when $\alpha_1 = 1, \alpha_2 = 1.6$ and $\beta_{1,2} = 1$. This is very important observation while designing the power transfer circuit. Furthermore, as seen from Fig. 7 and 8, the maximum efficiency of 96.23% was achieved with an output power of 1.28 kW when $\alpha_1 = 1, \alpha_2 = 1.2$ and $\beta_{1,2} = 1$. Then, the full fractional order case is considered with positive real orders in fractal elements. Firstly, the output power is calculated versus $\beta_{1,2}1$ and by varying $0 < \alpha_1 2$ and fix $\alpha_2 = 1.2$. The reason to keep $\alpha_2 = 1.2$ is derived from previous analysis where the power efficiency is maximum.

Figure 9 shows the output power by varying α_1 from 0 to 0.5 with changing $\beta_{1,2}$ and $\alpha_2 = 1.2$. The figure depicts that as the order of $\alpha - 1$ increases from 0.1 to 0.5, the output power gradually increases.

Figure 10 is generated the same way with higher α_1. Continuing from Fig. 9, the output power further increases for α_1 ranging from 0.6 to 0.9, however, with a slight drop when the order is 1.

Figure 11 illustrates further results for α_1 varied from 1 to 1.5. As for the order of α_1 at 1.1, the output power has significantly decreased by almost ten times more than what it was at $\alpha_1 = 1$. Thereafter, the output power gradually decreases for α_1 ranging from 1.2 to 1.5.

Figure 12 also depicts the output power for α_1 changed between 1.5 to 2. When α_1 increased more than 1.6, the output power gradually decreased to a value equivalent to zero at $\alpha_1 = 2$.

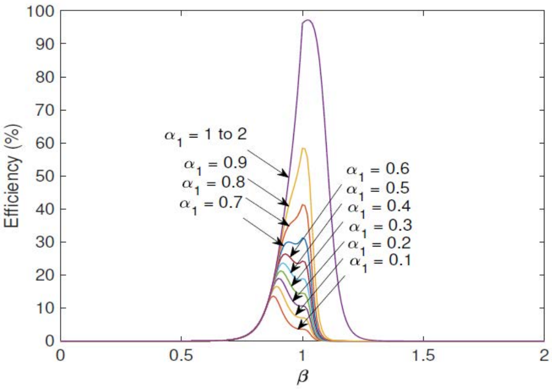

Fig. 13. Efficiency versus $\beta_{1,2}$ when $0 < \alpha_1 \leq 2$ and $\alpha_2 = 1.2$

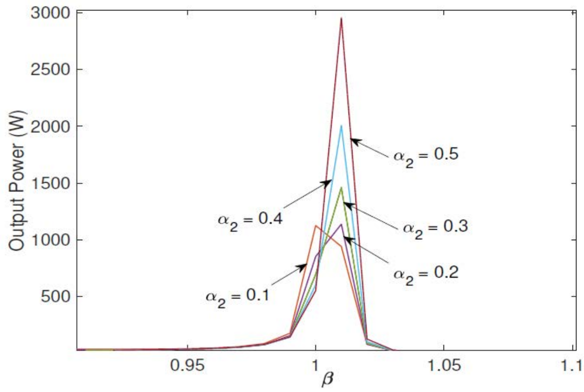

Fig. 14. Output power versus $\beta_{1,2}$ when $0 < \alpha_2 \leq 0.5$ at $\alpha_1 = 1$

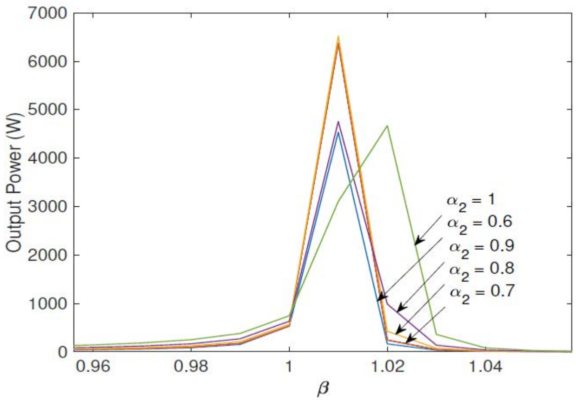

Fig. 15. Output power versus $\beta_{1,2}$ when $0.5 < \alpha_2 \leq 1$ at

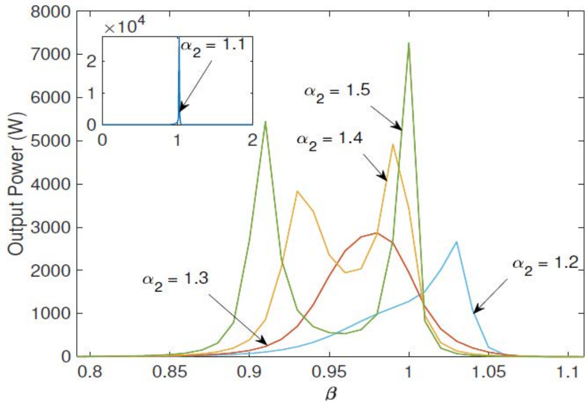

Fig. 16. Output power versus $\beta_{1,2}$ when $1 < \alpha_2 \leq 1.5$ at $\alpha_1 = 1$

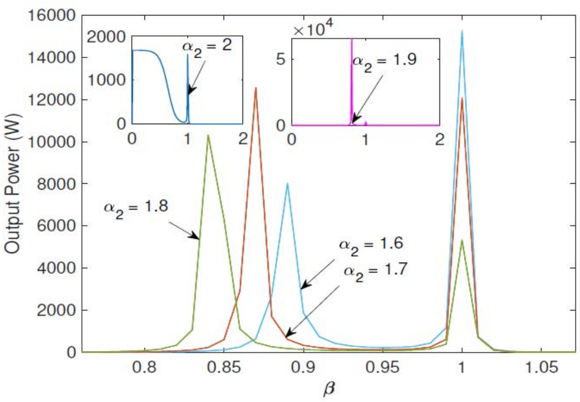

Fig. 17. Output power versus $\beta_{1,2}$ when $1.5 < \alpha_2 \leq 2$ at $\alpha_1 = 1$

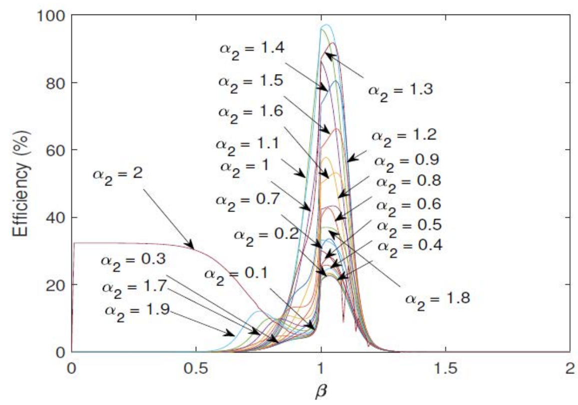

Fig. 18. Efficiency versus $\beta1, 2$ when $0 < \alpha2 \leq 2$ at $\alpha1 = 1$

Figure 13 represents the efficiency after varying α_1 from 0 to 2 with respect to various $\beta_{1,2}$. An efficiency of 97. 22% is achieved, which is the maximum efficiency of the system when the order of $\beta_{1,2} = 1.02$. After studied the effect of $\beta_{1,2}$ on output power and its respective efficiency, Fig. 9 to Fig. 11. are obtained. It was noticed that the maximum output power about 2.66 kW was achieved at $\beta = 1.03$. That value gives an efficiency 97. 09% when α_1=1 and $\alpha_2 = 1.2$. The useful results are obtained as shown in Fig. 12 to Figure 13. The value of $\beta_{1,2}$ was 1.02 and has given the maximum efficiency of 97. 22% with power of 2.02 kW. Note that the order of α_1 and α_2 were 1 and 1.2, respectively.

Next, the third case is considered by varying β and α_2 and fix α_1. Figure 14 shows the results on output power. As the order α_2 varies from 0.1 to 0.5, the output power gradually increases. This pattern continued further with α_2 changes from 0.5 to 1 as shown in Fig. 15. But, it starts to drop when α_2 crosses 0.8 value.

Figure 15 and 16 show the output power variations for higher value of α_2. Interestingly, the output power spikes at $\alpha_2 = 1$ to almost 30 kW and then drastically drops below 3 kW at $\alpha_2 = 1.2$. Also, the value of α_2 between 1.3 to 1.5 shows more output power. Continuing from Fig. 16, the output power further increases for $\alpha_2 = 1.5$ to 1.7 as shown in Fig. 17. It has a slight drop at $\alpha_2 = 1.8$ before a sudden dramatic rise to almost 80 kW occurs at 1.9. Also, it shows large drops in output below 2 kW

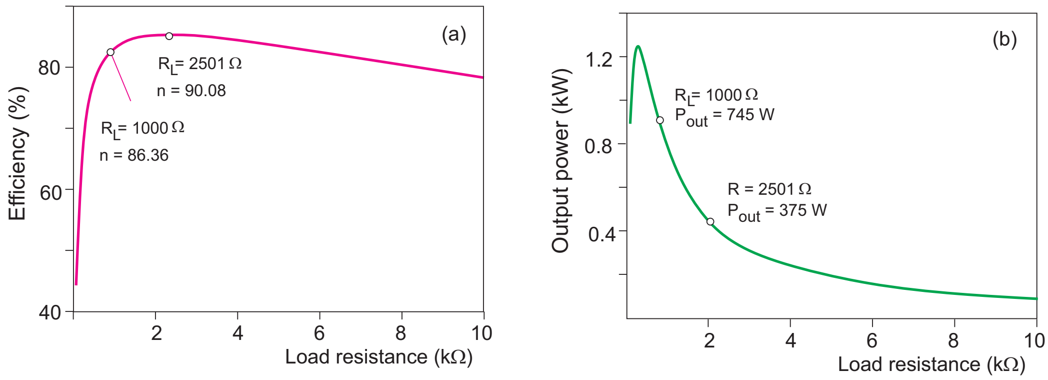

Fig. 19. Results from classical WPT system

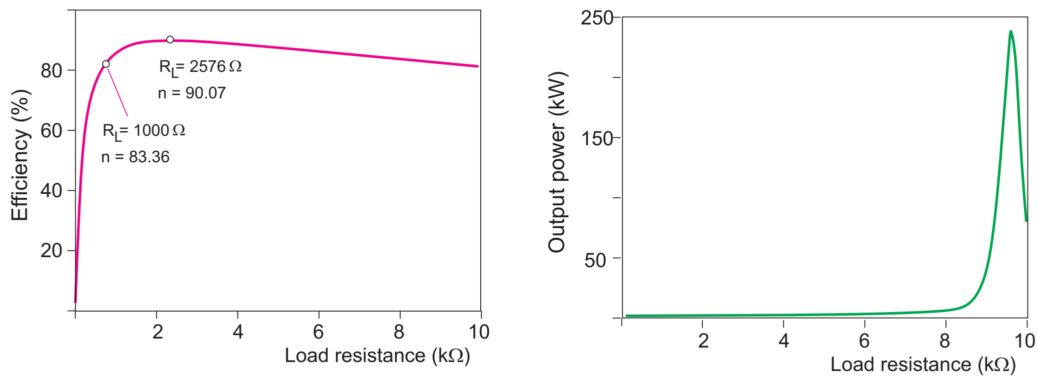

Fig. 20. Results from FOWPT system (Case 2)

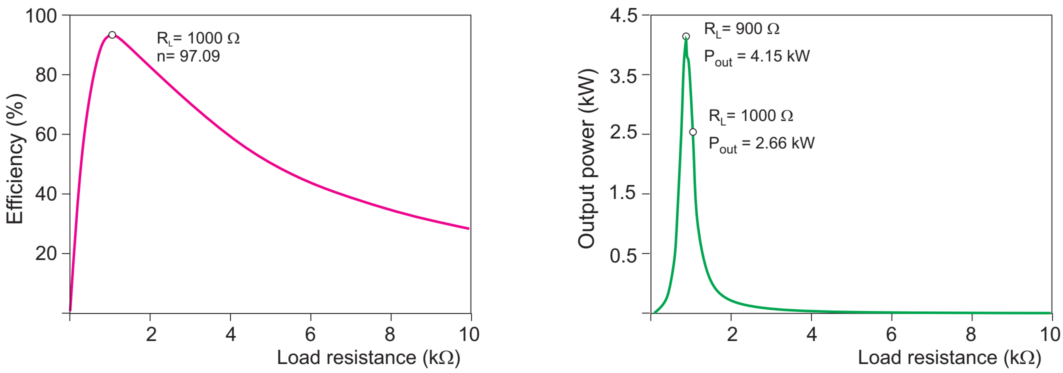

Fig. 21. Results from FOWPT system (Case 3)

at 2. There are two possibilities of output powers for this range of orders as well. The efficiency is calculated with variations in α_2 from 0 to 2 with respect to $\beta_{1,2}$ and constant α_1. An efficiency of 97. 22% is achieved at $\alpha_2 = 1.2$, which is the maximum efficiency of the system when the order of $\beta_{1,2}$ is 1.02. The sensitive changed was observed from Fig. 14 to 17 with slight variations in $\beta_{1,2}$. It was captured with the maximum output power of 27.81 kW and $\beta_{1,2} = 1.02$ with an efficiency of 94. 34% when the order of α_1 and α_2 is 1 and 1.1, respectively.

On the other hand, the maximum efficiency of 97. 09% was achieved with an output power of 2.66 kW when $\beta_{1,2} = 1.03$ and $\alpha_2 = 1.2$. Same way from Figure 18 the efficiency was observed with respect to variation in $\beta_{1,2}$. The peak efficiency of 97. 22% was achieved at $\beta_{1,2} = 1.02$, with an output power of 2.02 kW.

In summary, the estimated fractal elements with most opti mal fractional orders are obtained for maximum efficiency and output power. Table 1 is provided with values and combination parameters. This result provides interesting insight on the optimal range of fractional orders, that can be useful to achieve better efficiency in transfer power.

In a special case, we have also analyzed the fractional WPT circuit with effect of load resistance value. Let us take the integer classical WPT system. Figure 19(a) shows that as the load resistance increases from 0Ω to 2.5 kΩ , the efficiency increases. However, increasing the load resistance further decreases the efficiency. Same way, the output power is also affected with load resistance as seen from Fig. 19(b). As it increases from 0Ω to 300Ω, the output power increases. However, increasing the load resistance further decreases the output power. As per optimal range of values from Tab. 1, the second condition was verified on the fractional WPT system for maximum efficiency and output power with respect to load resistance. Figure 20(a) shows that the maximum efficiency is achieved of 97.09% when $R_{\mathrm{L}} = 1\,k\Omega$.

When we considered the maximum output power as give in Fig. 20(b), it captured when $R_L = 0.9\,k\Omega$ and at $R_L = 1\,k\Omega$ the output power is measured as 2.66 kW. Another case for high efficiency for fractional orders as per Tab. 1, Case 3.2 are plotted in Figure 21. According to the study provided in this paper, the simulation results on series-parallel type FOWPT system have great advantages over classical WPT system. In the fractional wireless power transfer circuit, the primary and secondary side fractional-order inductor and capacitors can provide more flexibility to obtain higher transmission efficiency.

5 Conclusions

The analysis can conclude that a proper selection of fractional-order components can achieve optimal efficiency. The presented fractional circuit showed promising results and had better characteristics than the integer WPT system. The circuit performance is also checked with load resistance together classical model. The theoretical and simulation verification depicted that the proposed structure could exhibit several advantages, requiring low resonant frequency and producing high output power and efficiency. Such flexible systems for WPT can be used for medium to high power applications. This article may promote the development of fractal elements for power applications. Future work might include experimental study and real-time realization of WPT.

References

[1] N. Iqteit, K. Yahya, and S. A. Khan, "Wireless power charging in electrical vehicles", *IntechOpen*, 2021.

[2] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han,, "Wireless charging technologies: Fundamentals, standards, and network applications", *IEEE*, vol. 18, no. 2, 2015,.

[3] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, "Wireless power transfer via strongly coupled magnetic resonances", *Science*, vol. 317, no. 5834, pp. 8386, 2007.

[4] Z. Zhang, H. Pang, A. Georgiadis, and C. Cecati, "Wireless power transferan overview", *IEEE Transactions on Industrial Electronics*, vol. 66, no. 2, pp. 10441058, 2018.

[5] Z. Zhang, K. Chau, C. Qiu, and C. Liu, "Energy encryption for wireless power transfer", *IEEE Transactions on Power Electronics*, vol. 30, no. 9, pp. 52375246, 2014.

[6] J. Guo, L. Tan, H. Liu, W. Wang, and X. Huang, "Stabilization control of output power in double-source wireless power transfer systems without direct output feedback", *IEEE Microwave and Wireless Components Letters*, vol. 26, no. 11, pp. 960962, 2016.

[7] J. H. Kim, B.-S. Lee, J.-H. Lee, S.-H. Lee, C.-B. Park, S.-M. Jung, S.-G. Lee, K.-P. Yi, and J. Baek, "Development of 1-mw inductive power transfer system for a high-speed train", *IEEE Transactions on Industrial Electronics*, vol. 62, no. 10, pp. 62426250, 2015.

[8] T. C. Rao and K. Geetha, "Categories, standards and recent trends in wireless power transfer: A survey", *Indian Journal of Science and Technology*, vol. 9, pp. 20, 2016.

[9] Y. Zhang, T. Lu, Z. Zhao, F. He, K. Chen, and L. Yuan, "Selective wireless power transfer to multiple loads using receivers of different resonant frequencies", *IEEE Transactions on Power Electronics*, vol. 30, no. 11, pp. 60016005, 2014.

[10] X. Shu and B. Zhang, "The effect of fractional orders on the transmission power and efficiency of fractional-order wireless power transmission system", *Energies*, vol. 11, no. 7, pp. 1774, 2018.

[11] G. Zhang, Z. Ou, and L. Qu, "A fractional-order element (foe)-based approach to wireless power transmission for frequency reduction and output power quality improvement", *Electronics*, vol. 8, no. 9, pp. 1029, 2019.

[12] D. A. Fathi, M. E. Fouda, L. A. Said, N. R. Khafagy, and A. G. Radwan, "Two-port network analysis of equal fractional-order wireless power transfer circuit", *2020 32nd International Conference on Microelectronics (ICM*,). IEEE, 2020, pp. 14.

[13] Y. Jiang, B. Zhang, and J. Zhou, "A fractional-order resonant wireless power transfer system with inherently constant current output", *IEEE Access*, vol. 8, pp. 23 31723 323, 2020.

[14] X. Shu, B. Zhang, C. Rong, and Y. Jiang, "Frequency bifurcation in a series-series compensated fractional-order inductive power transfer system", *Journal of Advanced Research*, vol. 25, pp. 235242, 2020.

[15] C. Rong, B. Zhang, and Y. Jiang, "Analysis of a fractional-order wireless power transfer system", *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 10, pp. 17551759, 2019.

[16] Y. Jiang and B. Zhang, "A fractional-order wireless power transfer system insensitive to resonant frequency", *IEEE Transactions on Power electronics*, vol. 35, no. 5, pp. 54965505, 2019.

[17] J. T. Machado and A. M. Galhano, "Fractional order inductive phenom- ena based on the skin effect", *Nonlinear Dynamics*, vol. 68, no. 1, pp. 107115, 2012.

[18] H. Samavati, A. Hajimiri, A. R. Shahani, G. N. Nasserbakht, and T. H. Lee, "Fractal capacitors", *IEEE Journal of solid-state circuits*, vol. 33, no. 12, pp. 20352041, 1998.

[19] B. Zhang, R. Huang, and D. Qiu, "Fractional order series resonance system for wireless electric energy transmission", *Apr. 11 2017, US Patent 9,620,965*.

[20] A. G. Radwan and K. N. Salama, "Fractional-order RC and RL circuits", *Circuits, Systems, and Signal Processing*, vol. 31, no. 6, pp. 19011915, 2012.

[21] M. Chen, S.-Y. Shao, P. Shi, and Y. Shi, "Disturbance-observer -based robust synchronization control for a class of fractional-order chaotic systems", *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 4, pp. 417421, 2016.

[22] A. G. Radwan, A. Shamim, and K. N. Salama, "Theory of fractional order elements-based impedance matching networks", *IEEE Microwave and Wireless Components Letters*, vol. 21, no. 3, pp. 120122, 2011.

[23] X. Chen, Y. Chen, B. Zhang, and D. Qiu, "A modeling and analysis method for fractional-order dcdc converters", *IEEE Transactions on Power Electronics*, vol. 32, no. 9, pp. 70347044, 2016.

[24] K. Kothari, U. Mehta, and R. Prasad, "Fractional-order system modeling and its applications", *Journal of Engineering Science and Technology Review*, vol. 12, no. 6, pp. 110, 2019.

[25] Y. Jiang and B. Zhang, "High-power fractional order capacitor with $1 < \alpha < 2$ based on power converter", *IEEE Transactions on Industrial Electronics*, vol. 65, no. 4, pp. 31573164, 2017.

[26] M. Rehman, P. Nallagownden, and Z. Baharudin, "Efficiency investigation of SS and SP compensation topologies for wireless power transfer", *International Journal of Power Electronics and Drive Systems*, vol. 10, no. 4, pp. 2157, 2019.

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