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Fractional-Order Impedance Identification for Inductors

Ravneel Prasad, Utkal Mehta, Sela Bloomfield and Navneet Charan*

Electrical and Electronics Engineering, The University of the South Pacific (USP), Laucala Campus, Suva, Fiji

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Abstract: The accuracy of a model is required to represent the integrated device in any electrical system for quality assurance. This article intends to measure the accurate and realistic impedance of the inductor, called pseudo-inductance. For this purpose, we have applied fractional-order modeling and a simple scheme to estimate the impedance value of the inductor (or inductive coils). Also, the approach does not require any high-end measurement device like the impedance meter. The complexity of the fractional-order derivative for identification purpose is simplified using the block-pulse-operational-matrix. The complexity of the fractional-order derivative for identification purpose is simplified using the block-pulse operational matrix to approximate the time response behaviour in terms of the inductor model's parameters. The study goes on to highlight the importance of fractional derivative and its interpretation in terms of the phase difference. The proposed model can be used to accurately model any inductor based on its time response data. The purpose of the study is validated through the experimental results.

Keywords: Fractional calculus, fractional-order inductor, pseudo-inductance, impedance model, operational matrix.

1 Introduction

Electrical passive elements are widely used in various systems such as wireless or wired systems, power transfer systems, transformers for high frequencies, intrabody communications, and many more [1]. The most common passive element is an inductor that is usually designed by winding the wire into a coil around a core, which can act as a storage device for energy, line filter for powerline and frequency selector for oscillator [2, 3]. In principle, it is activated by applying voltage or current source and takes advantage of the wire windings to induce a magnetic field within the core resulting in a flow of current through the core windings. Thus, the inductor's impedance varies with the resistance of the wire or material type used or the input signal. It is found commonly the impedance of an inductor can be modelled by a simple series circuit, a resistor (R) and an inductor (L). It is written as [4].

$$Z(s) = Ls + R \tag{1}$$

In theory, an inductor introduces an input/output phase shift of $\pi/2$ rads but this is not always true in reality. Therefore, it is a practical question to know the inductor's impedance equivalently same as its real value. With the knowledge of fractional theory in recent years, the integer order transfer function of a circuit can be converted equivalently to any real order for both the inductor and capacitor based circuits [5]. Fractional calculus has become a very popular concept nowadays in system modeling, especially in science and engineering. The mathematical phenomena of fractional-order calculus describe real systems more accurately compared to the normal integer-order methods [6, 7]. It considers complex systems involving impedance modeling, quantum mechanics, and memristor systems as fractional-order system has an unlimited memory [8]. The fractional transfer function determined from the response of the system closely features the original response [9, 10]. In fact, fractional calculus helps to improve accuracy in control engineering and better portrays dynamic behavior [11]. Taking this into account, the impedance model represented in (1) for the inductor can be rewritten using fractional calculus as given below.

$$Z(s) = L_\alpha s^\alpha + R, \tag{2}$$

* Corresponding author e-mail: utkal.mehta@usp.ac.fj

where, α is a real number $(0 < \alpha \leq 1)$, s^{α} is defined as fractional derivative and L_{α} is a fractional inductance. In theory, L_{α} is called the pseudo-inductance in units of $H/\sec^{1-\alpha}$. It is found in the literature that there is a huge ongoing effort to build fractal elements for various applications [5, 6].

Inductors are being conventionally modeled with resistance in series and parallel. These resistances represent coil and core losses. Also, there is a parasitic capacitance that is formed in separating insulating layers for the coil windings. Since there is a frequency dependent resistance loss induced, it makes the conventional model less easy to analyze regarding the amplitude response interval [12]. Similar work conducted by Abuaisha and Kertzscher [13] studied the conventional and fractional method of modeling an interchangeable core for an electrical coil. It is made known that to precisely model electrical machines, the present electrical coil (non-linear inductor) must also be accurately modeled over a broad range of frequencies. Moreover, the conventional method of modeling was sufficient for only a lower range of frequency due to its existing losses. The analysis of the conventional and fractional order methods concluded that through the calculation of the errors, the fractional-order modeling was a better method and shows the realistic behavior of the electrical coils. Correspondingly, parallel research states that according to the losses in a ferromagnetic inductor coil, there is a certain saturation of the hysteresis model that can accurately represent the behavior of the nonlinear coil [14]. The fractional-order model of the ferromagnetic coil has more range and accuracy compared to the conventional (integer) model. In [15], a non-linear inductor was modelled focusing on the characteristics and behavior of the inductor. The research used an intelligent electrical impedance spectroscopy technique. Schafer and Kruger [12], states coils that possess a much higher and significant loss can be described realistically through a fractional-order model. The focus on fractional-order modeling is beneficial for electrical machines and magnetic core coils, as it looks at non-linear systems. The impedance is considered to be very significant for electrical machines and elements. Hence, considering all losses such as hysteresis, copper core, core, and eddy current, fractional calculus is suitable for describing realistic and accurate circuit models.

In describing or modeling the impedance of a system, it is essential to know accurate physical parameters, but often some parameters are unknown [16]. These parameters can be identified with impedance meters and electrical impedance spectroscopy, but these measuring equipment are expensive and may not be feasible for individuals/organisations that are financially constrained. Therefore, we have opted to employ the identification scheme to model the unknown parameter values by analysing available data that can also increase modeling precision in the design, simulation and implementation of circuits especially in the field of controls. The technique utilised in this article involves interpreting and analyzing the time domain response of the inductor under measurement. In this short article, a fractional inductor model is presented, known as pseudo-inductance, through the use of block pulse operational matrix. The approach further verifies the physical interpretation of fractional order, α, used in the modeling of the inductor through time domain response. An indirect merit of the presented work is its electronic tuning capability.

2 Fractional-order derivative using block-pulse operational matrix

The block pulse function (BPF) has ascertained the quality in systems' identification and synthesis [17, 18]. The key idea was to convert the fractional derivative function into a simple algebraic format so that the long, complex calculations faced due to fractional calculus' long memory property can minimize. The BPF composes of a set of orthogonal piecewise functions of constant values, which are specified over the time interval $[0, \tau]$ as

$$\phi_i(t) = \begin{cases} 1, & \frac{i-1}{M}\tau \leq t \leq \frac{i}{M}\tau \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

where M is the number of elementary functions used and $i = 1, 2, \ldots, M$. Let us consider a function $x(t)$. It is differential over the time interval $[0, \tau]$. When it is represented in terms of BPF, we get

$$x(t) \cong X^T \phi_{(M)}(t) = \sum_{i-1}^{M} X_i \phi_i(t), \tag{4}$$

where T represents the transpose, $X^T(t) = [X_1, X_2, \ldots, X_M]$ is coefficient vector and $\phi^T{}_{(M)}(t) = [\phi_1(t), \phi_2(t), \ldots, \phi_M(t)]$ is the BPF vector. As per this theory, we can obtain the fractional-order derivative (FOD)in matrix format as below.

$$(\partial^{\alpha} \phi_{(M)})(t) \approx B^{\alpha} \phi_{(M)}(t). \tag{5}$$

The above equation (5) has a generalized operational matrix B^α with FOD of order α. It also satisfies $B^\alpha = (F^\alpha)^{-1}$ and F^α is represented as below.

$$F^{\alpha}_{M\times M} = \left(\frac{\tau}{M}\right)^{\alpha} \frac{1}{\Gamma(\alpha+2)} \begin{pmatrix} m_1 & m_2 & m_3 & \dots & m_M \\ 0 & m_1 & m_2 & \dots & m_{M-1} \\ \vdots & \ddots & m_1 & \dots & m_{M-2} \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & m_1 \end{pmatrix}_{(M\times M)}, \tag{6}$$

where $m_1 = 1, m_r = r^{\alpha+1} - 2(r-1)^{\alpha+1} + (r-2)^{\alpha+1}$ and $r = 2, 3, \dots, M$.

Then, the FOD of any differential functional can be approximated using (4) and (5) as below.

$$(\partial^\alpha x)(t) \approx X^T B^\alpha \phi_{(M)}(t). \tag{7}$$

It has been shown from the study that the FOD in form (7) will simplify the intricate calculations of fractional derivatives into an easy algebraic operation of matrix multiplication. As studied in [17], this matrix format also allows identification of the fractional-order system with any real-order α.

3 Impedance model and estimation

The impedance model for the inductor is considered as (2) and the concept for measurement is demonstrated by the circuit in Fig. 1. The inductor under test is connected in series with a known resistor, R_L. The pseudo-inductance L_α and resistance, R, are unknown parameters for identification. Considering a sinusoidal input voltage as V_{in} and the output voltage as V_{out} measured across R_L, then the circuit transfer function of Fig. 1 is given by

$$\frac{V_{out}}{V_{in}}(s) = \frac{R_L}{R_L + R + L_\alpha s^\alpha}. \tag{8}$$

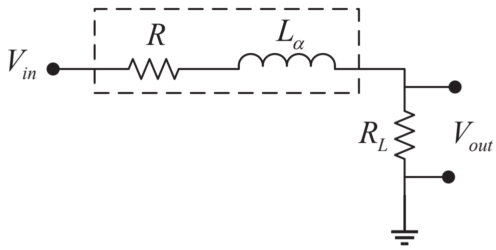

Fig. 1: Fractional-order inductor model

Taking inverse Laplace and simplifying in terms of input-output (I/O) time domain signals, one can obtain,

$$R_L v_{in}(t) = v_{out}(t)[R_L + R] + L_\alpha(\partial^\alpha v_{out})(t). \tag{9}$$

Above relation can be useful to estimate impedance parameters if the I/O data is available from the circuit. However, it is critical to handle the signal due to the non-integer derivative ∂^α presents in expression. As discussed in Section 2, the BPF is applied to simplify the relation and arbitrary order differentiation into easy algebraic matrix multiplication steps. Considering BPF property (7), v_{in}, v_{out} and ∂^α can be represented as:

$$v_{in}(t) = {V_{in}}^T \phi_m(t), \tag{10}$$

$$v_{out}(t) = {V_{out}}^T \phi_m(t), \tag{11}$$

$$\partial^{\alpha} v_{out}(t) = {V_{out}}^T B^{\alpha} \phi_m(t), \tag{12}$$

where ${V_{in}}^T$ and V_{out}^T are the transpose of the vectors v_{in} and v_{out} respectively; and ϕ_m is the BPF. Then using (10), (11) and (12), (9) can be converted as,

$$R_L({V_{in}}^T \cdot \phi_m(t)) = ({V_{out}}^T \cdot \phi_m(t))(R_L + R + LB^{\alpha}). \tag{13}$$

Rearranging and simplifying equation (13) takes the following form,

$${V_{out}}^T = {V_{in}}^T \times \frac{R_L}{R_L + R + LB^{\alpha}}. \tag{14}$$

Finally, using equation (11), the time response is obtained for the estimating voltage signal across the output load resistor. The output expression can be written as,

$$v_{out}(t) = {V_{in}}^T \phi_m(t) \times \frac{R_L}{R_L + R + LB^{\alpha}}. \tag{15}$$

The above equation contains the three unknown parameters namely L_{α}, α and R which needs to be identified to estimate the response of the inductor. Thus, the output signal can be estimated to fit with the actual response signal, with more flexibility to fit phase shift induced by the inductor in the circuit. To understand the fractional inductor, known as pseudo-inductance when $\alpha \neq 1$, the relationship in (15) is simulated in Fig. 2 revealing the true sense of pseudo-inductance. All the variables were kept constant for the inductor model except for α that is varied. The figure was simulated for $L = 3.75H$, $R = 10$, $R_L = 5000$ and $v_{in} = 5V@1Hz$ and plotted output is normalized for simplicity. It is noted from the figure that the sinusoidal wave starts to lag by an angle of $\frac{\alpha\pi}{2}$ rads. In theory, the ideal phase difference induced should be $\frac{\pi}{2}$ but due to unknown magnetic phenomena or material properties, this angular difference changes by a factor α. In this work, we have attempted to capture the actual inductor behavior by taking a flexible impedance model.

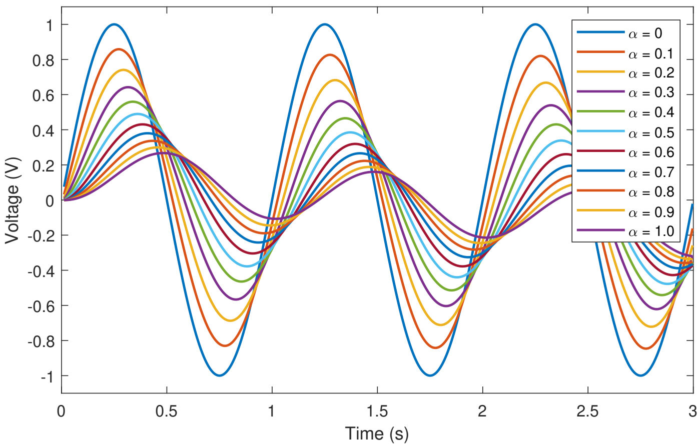

Fig. 2: Effect of changing the α value in modeling

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4 Experimental study

The presented approach is simple and requires a simple measurement setup. As per the model considered in Fig.1, an actual inductor test setup shown in Fig. 3. The data was collected across the known load resistor, R_L using an Embest DSO2300 USB Digital Intelligent Oscilloscope, with a sampling rate of 25000 samples per second. A sinusoidal input of 5V peak was supplied to the input of the circuit. The study was conducted on various inductor values of 3.8mH, 3.75H and 3.8H. To study the presence of fractional behaviour in an inductor, the time domain responses of the inductor are collected for estimation purpose. The curve fitting method is applied to check the actual output with the calculated output from the

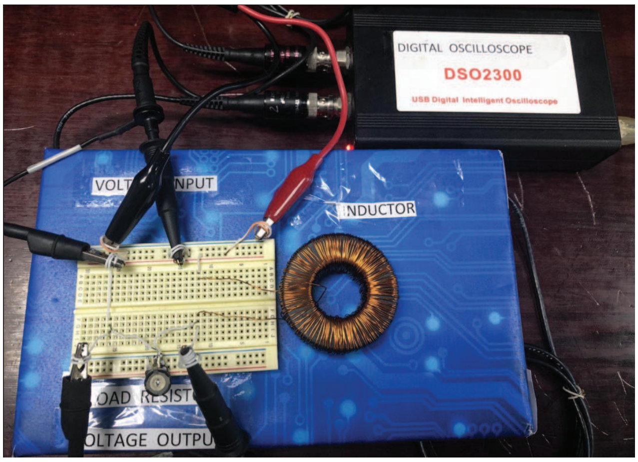

Fig. 3: Experimental Setup

relation (15). In order to estimate the best parameter values, the non-linear least squares (NLS) approach as below was used in this work.

$$\min_{\eta} \left\| \hat{v}_{out}(\eta) - v_{out} \right\|_2^2 = \min_{\eta} \sum_{j}^{n} \left(\hat{v}_{out}(\eta)_j - v_{outi} \right)^2, \tag{16}$$

where $\eta = (L_\alpha, \alpha, R)$ is the vector of unknown parameters, $\hat{v}_{out}$ is the identified output signal and v_{out} is the measured signal. The subscript j demonstrates the current sample value from measured samples n.

Now in order to verify the method, a test data was collected for a 3.75H inductor with $v_i = 5V@50Hz$ and a load resistance of 800 Ω. The estimated fractional-order model showed a good agreement with the actual response data as shown in Fig. 4, yielding parameters of $L_\alpha = 3.88H$, $\alpha = 0.45$ and $R = 23.14\Omega$. On the same figure, it can be seen that a classical integer impedance model ($\alpha = 1$), is unable to estimate the inductor behavior since it is not able to account for the actual phase shift introduced by the inductor. The logarithmic absolute error performance of the proposed model against the classical model illustrates the significance of fractional order modeling. Fig. 5 shows the better performance of fractional-impedance model since the estimation error is very small.

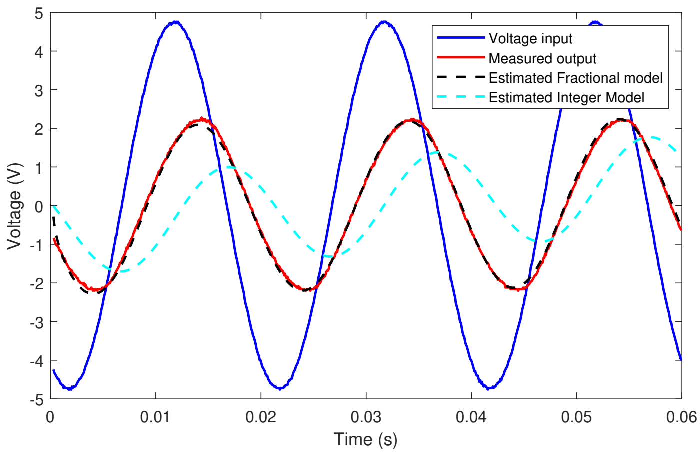

Fig. 4: Voltage response of 3.75H inductor

Table 1 presents the results verified for other values of inductors. All tests resulted in fractional behaviour with close estimation of inductance value to the actual rating of the inductors. A trend was seen through the results collected which

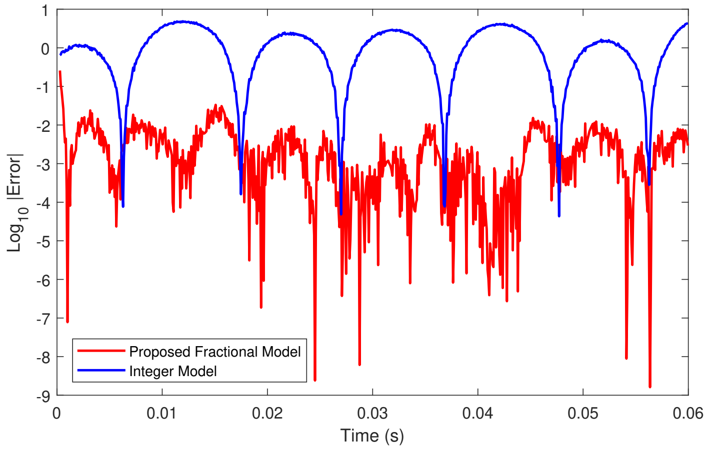

Fig. 5: Logarithmic Absolute Error plot of 3.75H inductor

Table 1: Identified fractional-order model parameters from various inductors

Tested Inductors	Estimated Parameters		
	L_α	α	R
3.8mH @ 50Hz	0.005	0.23	0.01
3.75H @ 50Hz	3.88	0.45	23.14
3.75H @ 60Hz	3.78	0.43	21.79
3.75H @ 80Hz	3.70	0.48	19.59
3.80H @ 50Hz	4.11	0.23	8.76
3.80H @ 60Hz	4.06	0.25	8.53

indicated that with the increase in frequency the estimated inductance and the internal resistance decreased. This is another reason for the fractional order inductor representation.

5 Conclusion

A fractional order inductor for accurate impedance calculation is presented for inductor coils. The method is developed using BPF which is simple but effective to handle computationally. The model demonstrates that the voltage and resulting current in the circuit are not always out of phase by $\frac{\pi}{2}$ rads. The fractional order α is basically the multiple factor of $\frac{\pi}{2}$ rads by which the voltage and current are out of phase.Though the research demonstrates that with frequency the time response parameters changes, there is a need to further study this to establish the effect of frequency change in the input wave form on the model parameters. The presented work can be further extended on study the behaviour with different input waveforms.

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