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BY

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by

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To My Wife

ABSTRACT

Digital signal processing is becoming increasingly important, and is finding applications in speech processing and telecommunications in the area of 1-D signal processing. One of the important branches in digital signal processing is digital filtering.

Among the numbers of structure of digital filters, the recursive(IIR) filter is known for its computational efficiency compared to the FIR counterparts.

In this thesis, an alternative approach to the direct design of 1-D recursive digital filters satisfying prescribed magnitude specifications with or without constant group delay characteristic using two all-pass filters is presented. It is known that, by this approach, the most computationally efficient realization can be obtained among IIR filters for meeting the filter specifications. The method uses unconstrained optimization techniques for the filter design to approximate both the group delay and the magnitude response of the desired filter simultaneously if the constant group delay characteristic is required.

Two different approaches are chosen for the stability of the filter. In the first approach, a new stability test is used to generate the stable polynomials. In the second approach, one-variable Hurwitz polynomials(HPs) using properties of positive definite matrices are generated. Bilinear transformations are applied to the HPs to obtain the stable polynomials in z domain. The polynomials generated using the approaches explained above are imposed on the filter's denominator polynomials through the variable substitution method, hence ensuring the stability of the designed filter. The designed filters using this method are stable in nature and neither stability check nor stabilization procedure is required. To illustrate the usefulness of the technique, the results obtained are compared with a well known direct method design using a general 1-D IIR transfer function.

Once the infinite precision filter is obtained, through a procedure based on discretization and reoptimization technique we discretize all coefficients to integer values. By this algorithm, the error caused by truncating the filter coefficients is minimized. Examples are given with comparisons in order to demonstrate the usefulness of the algorithm.

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To my parents and wife, I extend my sincerest thanks.

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Chapter I
INTRODUCTION

1.1 DIGITAL SIGNAL PROCESSING

A signal can be defined as a function that conveys information, generally about the state or behaviour of a physical system. Digital signal processing is concerned with the representation of signals by sequences of numbers or symbols and the processing of these sequences which can be represented by a unique function of frequency. The purpose of such processing may be to estimate characteristic parameters of a signal or to transform a signal into a form which is in some sense more desirable.

Filtering, which is an important branch of signal processing, is a process by which the frequency spectrum of a signal can be modified, reshaped, or manipulated according to some desired specification. It may entail amplifying or attenuating a range of frequency components, rejecting or isolating one specific frequency component, etc. The uses of filtering are manifold, e.g., to eliminate signal contamination such as noise, to remove signal distortion, to separate two or more distinct signals, to resolve signals into their frequency components, to demodulate signals, to convert discrete-time signals into continuous-time signals, and to bandlimit signals.

The digital filter is a digital system that can be used to filter discrete-time signals while the analog filter is for continuous-time signals.

The proliferation of the use of digital signal processing can be witnessed by its appearance in a variety of scientific endeavours such as biomedical engineering, seismic and geophysical research, radar and sonar detection and countermeasures, acoustics and speech research, ECG and telecommunications. In the case of speech, for example, it may be used to extract the information corresponding to the identity of a speaker.

1.2 CHARACTERIZATION OF DIGITAL FILTERS

A system or filter is essentially an algorithm for converting input sequence x(n) into output sequence y(n). If the input is the impulse sequence δ(n), the resulting output is called the impulse response of the filter and is denoted by h(n). Any linear shift-invariant(time-invariant) system is completely characterized by its unit-sample response h(n) as follows.

$$y(n) = T[\, x(n) \,] \tag{1.1}$$

$$= T[\, \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) \,] \tag{1.2}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, T[\delta(n-k)] \tag{1.3}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \tag{1.4}$$

$$= \sum_{k=-\infty}^{\infty} x(n-k)\, h(k) \tag{1.5}$$

$$= x(n) * h(n) \tag{1.6}$$

This is commonly called the convolution sum.

A discrete-time filter is stable if a bounded input sequence produces a bounded output sequence.[44] For a linear time-invariant filter, the stability holds if, and only if,

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty. \qquad (1.7)$$

This can be shown as follows. If eqn.(1.7) is true and x is bounded, i.e., $|x(n)| < M$, for all n, then from eqn.(1.5)

$$|y(n)| = |\sum_{k=-\infty}^{\infty} h(k)\, x(n-k)| \le M \sum_{k=-\infty}^{\infty} |h(k)| < \infty. \qquad (1.8)$$

Thus y is bounded. And for necessity condition, if we assume eqn.(1.7) is not true; i.e.,

$$\sum_{k=-\infty}^{\infty} |h(k)| = \infty \qquad (1.9)$$

Consider the bounded sequence x(n) defined by

$$x(n) = \begin{cases} 1 & \text{if } h(-n) \ge 0 \qquad (1.10) \\ -1 & \text{if } h(-n) < 0 \qquad (1.11) \end{cases}$$

Then from eqn.(1.4) the output at n=0 is

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = \sum_{k=-\infty}^{\infty} |h(-k)| = \sum_{k=-\infty}^{\infty} |h(k)| = \infty. \qquad (1.12)$$

Thus y(0) is not bounded. Therefore, the condition for stable system is that its impulse response to be absolutely summable.

A discrete-time filter is causal or realizable if the output at $n = n_0$ is dependent only on values of the input for $n \le n_0$. For linear time-invariant filter, this implies that the impulse response h(n) is 0 for $n < 0$.

The z-transform X(z) of a sequence x(n) which is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \qquad (1.13)$$

where z is a complex variable may be viewed as a unique

representation of that sequence in the complex z plane.
If y(n) is the convolution of the two sequences x(n) and h(n), then

$$Y(z) = X(z)\ H(z). \qquad (1.14)$$

If h(n) is the impulse response, its z-transform H(z) is often referred to as the system function. The system function evaluated on the unit circle (i.e., for $|z| = 1$) is the frequency response of the system. From eqn.(1.30), it can be also written as

$$H(z) = Y(z) \ / \ X(z) \qquad (1.15)$$

when X(z) and Y(z) are z-transform of input and output sequences respectively.

When the system, i.e.,filter is describable by a linear constant-coefficient difference equation, the system function is a ratio of polynomials. If we consider a system for which the input and output satisfy the general Nth-order difference equation

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{N} b_r x(n-r) \qquad (1.16)$$

By the application of z-transform,

$$\sum_{k=0}^{N} a_k\ z^{-k} Y(z) = \sum_{r=0}^{N} b_r\ z^{-r} X(z). \qquad (1.17)$$

From eqn.(1.17),

$$H(z) = \sum_{r=0}^{N} b_r\ z^{-r} \ / \ \sum_{k=0}^{N} a_k\ z^{-k}. \ = A(z) \ / \ B(z) \qquad (1.18)$$

eqn.(1.18) expresses the functional form of the system function, and it is noted that the coefficients in the numerator and denominator polynomials correspond, respectively, to the coefficients on the right- and left-hand sides of the difference equation (1.16).

1.3 CLASSIFICATION OF DIGITAL FILTERS

1.3.1 IIR AND FIR FILTERS

As we saw in the previous section, digital filters are characterized in terms of difference equations,

$$y(n) = \sum_{r=0}^{M} b_r x(n-r) - \sum_{k=1}^{N} a_k y(n-k) \qquad (1.19)$$

which shows that the present output value y(n) can be computed from the present and M past input values and N past output values. If past output values are actually used in the computation of the present output, i.e., if the filter implementation contains feedback, then the implementation is said to be recursive. Otherwise, the filter implementation is nonrecursive.
Recursive filter is also called IIR(infinite impulse response) filter because the impulse response is of infinite duration and nonrecursive filter, FIR(finite impulse response) whose impulse response is of finite duration.

1.3.2 SOME COMPARISONS OF IIR AND FIR DIGITAL FILTERS

The main advantage of IIR filters is that they require much less multiplications than FIR digital filters, hence they are cheaper to implement and have faster response time. In the comparison of optimum(minimax) FIR lowpass filters and elliptic IIR filters by Rabiner, et al, it was proved[16] that the order of optimum FIR filter should be at least three times greater than that of elliptic IIR filter to meet the same frequency response specification. Generally IIR filters are much more efficient in achieving given specifications on the magnitude response than FIR filters. The FIR filters have the additional useful properties, however, that their phase can be exactly linear; i.e., there is no group delay distortion, and there is no inherent stability problem. Therefore, it has been of great interest to design stable IIR filters with constant group delay specification in the band of interest.

1.4 DESIGN OF DIGITAL FILTERS

1.4.1 SOME ELEMENTARY PROPERTIES OF DIGITAL FILTERS

An analog filter characterized by a continuous-time transfer function H(s) has a steady-state sinusoidal response of the form

$$\lim_{t \to \infty} y(t) = M(w) \sin[wt + \theta(w)] \qquad (1.20)$$

$$\text{where} \quad M(w) = |H(jw)|, \quad \theta(w) = \arg H(jwT)$$

M(w) is the gain and θ(w) is the phase shift of the filter at frequency w. For a digital filter, the transfer function H(z) can be expressed as

$$H(e^{jwT}) = H(z)\big|_{z=\exp(jwT)} \qquad (1.21)$$

$$= M(w)\, e^{j\theta(w)} \qquad (1.22)$$

where $M(w) = |H(e^{jwT})|$ = magnitude or gain of the filter

$\theta(w)$ = Phase shift of the filter.

The magnitude response of the filter is defined as

$$|H(e^{jwT})| = \sqrt{\,\mathrm{Re}[\,H(e^{jwT})\,]^2 + \mathrm{Im}[\,H(e^{jwT})\,]^2} \qquad (1.23)$$

and the phase response is defined as

$$\theta(w) = \tan^{-1}\{\,\mathrm{Im}[H(e^{jwT})] \,/\, \mathrm{Re}[H(e^{jwT})]\,\} \qquad (1.24)$$

which may be written in the alternate form

$$\theta(w) = \ln\{ H(e^{jwT}) / H(e^{-jwT}) \}/ 2j \qquad (1.25)$$

As we can see, the magnitude response is a symmetric function of w and phase response is an antisymmetric function of w. It is well known that phase accuracy is extremely important in image processing filters[21] as well as speech processing filters[19].

The group delay of a filter is a measure of the average delay of the filter as a function of frequency and is defined as

$$\tau(w) = -d\theta(w)/dw = -jz\, d\theta/dz\big|_{z=\exp(jwT)} \qquad (1.26)$$

Using eqn. (1.25), $\tau(w)$ can be written as

$$\tau(w) = -\text{Re}\{ z\,(dH(z)/dz)\,/H(z) \}\big|_{z=\exp(jwT)} \qquad (1.27)$$

$$= -\text{Re}\{ z\, d[\ln H(z)]\, /\, dz \}\big|_{z=\exp(jwT)} \qquad (1.28)\ [8]$$

$$= \text{Re}\{ z\,[D'(z)/D(z) - N'(z)/N(z)]\}\big|_{z=\exp(jwT)} \qquad (1.29)$$

where $H(z) = N(z)/D(z)$

$$N'(z) = dN(z)/dz \text{ and } D'(z) = dD(z)/dz \qquad (1.30)$$

A desirable group delay characteristic of a filter is one that is approximately constant over the band(s) of frequency that the filter passes. If it is not constant, i.e., the phase is not linear, we have what is known as delay distortion. To visualize delay distortion more clearly, we recall from Fourier analysis that any signal is made up of different frequency components. An ideal transmission system should delay each frequency component equally. If the frequency components are delayed by different amounts, the reconstruction of the output signal from its Fourier components would produce a signal of different shape as the input. For pulse application, delay distortion is an essential design consideration. Therefore, the design of digital filters that approximates both magnitude and group delay responses is of great importance in some applications in signal processing.

1.4.2 THE FILTER DESIGN PROBLEM WITH STABILITY CONSIDERATION

In the most general sense, a digital filter is a linear shift-invariant discrete-time system that is realized using finite-precision arithmetic. The design of digital filters involves three basic steps: (1) the specification of the desired properties of the system; (2) the approximation of these specifications using a causal discrete-time system; and (3) the realization of the system using a causal finite-precision arithmetic. In a narrow sense, the design of a digital filter is to find the filter coefficients such that the filter's response approximates a prescribed behaviour. As such, the "filter design problem" is basically a mathematical approximation problem.

For FIR filters, the impulse response is only defined over a bounded limit, this type of filter is always stable.

For IIR filter described by its transfer function (1.18),

$$H(z) = \sum_{r=0}^{N} b_r z^{-r} \Big/ \sum_{k=0}^{N} a_k z^{-k}. = A(z) / B(z) \ , \ z = e^{jwT}$$

to be stable [18,20]

$$B(z) = 0 \qquad |z| \geq 1 \qquad (1.31)$$

Similarly, for an analog filter [18,20]

$$H(s) = N(s) / D(s) \text{ to be stable,}$$

$$D(s) = 0 \qquad Re\{s\} \geq 0 \qquad (1.32)$$

Stability can be checked for both analog and digital filters by means of pole location routine. Methods are available in the literature regarding the location of the roots of a polynomial in z domain within the unit circle such as Schur-Cohn[9], Jury test [28], the inners test[15], the use of inverse bilinear transformation[26] and the new stability test[46]. Also, stability tests have been formulated based on the z-domain continued fraction expansion relative to the bilinear function $(z-1)/(z+1)$[25,27] or in terms of $(z-1)$ and $(1-z^{-1})$ factors[37]. The Routh-Hurwitz Criterion[44], which is explained in chapter 2.5.1 is available for s domain stability test. Stabilization[6,7] can be carried out if any poles are found to be outside of the unit circle in z-domain or RHS(right hand side) of s-plane in analog domain by replacing them with their mirror images with respect to unit circle and jw axis respectively. But it has a disadvantage in that it distorts the phase response due to the pole replacements. We can use constrained optimization technique to design a stable IIR filter. But it is always preferable to design a filter whose stability is guaranteed in nature so that we can utilize the unconstrained optimization technique which is more efficient than the constrained one.

1.4.3 DESIGN OF IIR DIGITAL FILTERS

This thesis deals with a new scheme of 1-D IIR filter design as shown by its name. In this section, we will overview the design techniques of 1-D IIR digital filters.

There are generally three ways of designing IIR filters. First, the most popular technique for designing 1-D IIR filter is to digitize an analog filter that satisfies the design specification through well-known transformations.[18,20,28] A second method is direct closed form design in the z plane.[18]

Beginning with the desired response of the filter, one can often decide where to place poles and zeros to approximate this response directly. A third way in which IIR filters are often designed is by using optimization procedures to place poles and zeros at appropriate positions in the z plane to approximate in some sense the desired response.[18,20,28] Iterative techniques are generally used to arrive at the desired filter. In this section, the first and third method of designing techniques will be briefly discussed.

1.4.3.1 DESIGN OF IIR DIGITAL FILTERS FROM ANALOG FILTERS

This is the traditional approach to the design of IIR digital filters. In this approach, the design of filters is accomplished in two steps. First a prototype normalized analog lowpass filter which is designed through one of the analog lowpass approximation techniques, including Butterworth, Chebyshev, elliptic and Bessel approximations is transformed into a denormalized lowpass, highpass, etc., analog filter employing the standard analog-filter transformations. The desired frequency selective digital filter can, then, be obtained from the analog filter by an analog-digital transformation. Among the most widely used procedures for digitizing the analog filters are impulse invariance transformation, bilinear transformation, matched z transformation and mapping of differentials.[20] When the specification is not suitable for the application of the transformation approach, one must resort to computer aided methods.

1.4.3.2 COMPUTER AIDED DESIGN OF IIR DIGITAL FILTERS

Although the analog-digital transformation approach of designing digital filter is applicable to the majority of situations encountered in practice, it cannot be used if analytical procedures do not exist for the design of either analog or digital filters to match arbitrary frequency response specifications or other types of specifications. In such cases, the desired discrete-time transfer function can be generated directly from the given specifications through the use of iterative methods based on linear or non-linear programming. Using this iterative procedure, either the error eventually reaches a minimum value or a specified maximum number of iterations is performed and the procedure terminates. Steiglitz has proposed an IIR filter design procedure based on minimization of the mean-square error of magnitude response in the frequency domain.[10] Deczky has generalized the procedure in a number of ways.[11] Instead of minimizing the average squared error, a weighted average of the error raised to the pth power was minimized. Also, this technique was applied to both the magnitude and the group delay. Linear programming technique[13,35] was also utilized for the approximation of an IIR filter satisfying a prescribed magnitude response with or without a prescribed phase characteristic. In most cases, one of the available optimization algorithms is proven adequate for approximating unusual frequency domain specifications.

1.5 ORGANIZATION OF THESIS

In this thesis, non-linear programming technique will be studied for the design of IIR filters. IIR filters are preferred in terms of efficiency, i.e., faster speed of filtering, smaller memory requirements and easier implementation compared to FIR filters. Chapter II of this thesis presents the new class of IIR filters using two all-pass filters which is more efficient than conventional Elliptic filters. Also, two different methods of generating stable polynomials will be considered, namely the polynomials using New Stability Test and Hurwitz Polynomials.
The benefits of the new class of filter will be discussed and stable digital IIR filter with linear phase characteristic in the passband region of this class will be implemented.

Also filters designed using the all-pass method will be compared to those of direct design using a general 1-D IIR transfer function to prove the superiority of the all-pass method.

In Chapter III, integer programming which is simple and effective, will be introduced. This algorithm minimizes the performance error caused by truncating the coefficients. The usefulness of this technique is proved by the filter design with the filters designed in chapter II as original infinite precision coefficients filters. Comparisons will be made between the original filters and the integer coefficient filters. Chapter IV, the final chapter, is the conclusion of the thesis.

Chapter II

DESIGN OF 1-D DIGITAL FILTERS USING TWO ALL-PASS FILTERS

2.1 INTRODUCTION

It is known that IIR digital filters are generally computationally efficient in meeting given specifications compared to their FIR counterparts. A measure of computational efficiency of recursive digital filters is the average number of multiplications required in computing each output sample. All recursive filters of a given order do not require the same number of computations per output sample because the amount of computation depends on the method of realization and any exploitable properties of filter coefficients. A recursive filter with a transfer function of the form

$$H(z) = \sum_{i=0}^{N} a_i z^{-i} \Big/ \left(1 + \sum_{i=1}^{N} b_i z^{-i}\right), \; b_N = 0 \qquad (2.1)$$

in general, requires (2N + 1) multipliers and 2N adders. The usual cascade realization of an elliptic filter of order N requires $[(3N + 2) / 2]_I$, rather than 2N + 1 multipliers where $[\ ]_I$ denotes the integer part of the argument. The reduction results not from any imposed structure on the filter but simply because the optimum minimax solution to amplitude approximation turns out to be one with zeros on the unit circle. Another approach for the computationally efficient filter design can be made by the use of the structure with an inherent saving, i.e., the use of the all-pass filter. The recursive filters which are realized as a parallel connection of two all pass sections were first proposed in 1981 by R. Ansari and B. Liu.[32] It is known[51] that the new computationally efficient scheme for recursive filters uses approximately 2/3 as many multiplications as a conventional cascade elliptic filter realizations which are known to be optimal among recursive filters of the same order, for meeting the same amplitude specifications. This method allows us to design low-pass, high-pass, band-pass and, in general, multiband filters. It is proven [51] that the odd-order classical digital low-pass filters (Butterworth, Chebyshev and elliptic design) derived from the corresponding analog filters via the bilinear transformation can be implemented as a sum of two all-pass filters. It is reported that the necessary and sufficient conditions for a digital filter to be implementable as a sum of two all-pass filters can be derived directly in the z-plane.[52] This structure has also been known to have a very low pass-band sensitivity.[54] This new scheme with an approximately linear phase characteristics has been studied in 1986 by M. Refors and T Saramaki[55], but they did not consider the problem related to the stability during the design process.

2.2 ALL-PASS FILTERS AND ITS REALIZATION

The digital all-pass filter is a computationally efficient signal processing building block which is quite useful in many signal processing applications. The magnitude of the frequency response of an all-pass filter $A(e^{jw})$ is unity at all frequencies, i.e.,

$$| A(e^{jw}) | = 1, \text{ for all } w, \qquad (2.2)$$

and only the phase response changes as the pole and zero positions vary. The transfer function of such a filter has all poles and zeros occurring in conjugate reciprocal pairs, and takes the form

$$A(z) = e^{j\theta} \prod_{k=1}^{M} (\sigma^*_k - z^{-1}) / (1 - \sigma_k z^{-1}). \qquad (2.3)$$

For stability reasons we assume $| \sigma_k | < 1$ for all k to place all the poles inside the unit circle. If A(z) is constrained to be a real function, it can be expressed in the form

$$A(z) = z^{-M} D(z^{-1}) / D(z) \qquad (2.4)$$

where M is the order of the filter.
In effect, the numerator polynomial is obtained from the denominator polynomial by reversing the order of the coefficients. For example,

$$A(z) = (a_2 + a_1 z^{-1} + z^{-2}) / (1 + a_1 z^{-1} + a_2 z^{-2}) \qquad (2.5)$$

is a second-order all-pass function of the form(2.4) above, since the numerator coefficients appear in the reverse order of those in the denominator. In this case, the numerator and denominator polynomials are said to form a mirror-image pair.

From the definition of an all-pass function in (2.2), setting A(z) = Y(z) / X(z) reveals

$$| Y(e^{jw}) |^2 = | X(e^{jw}) |^2, \qquad \text{for all } w. \ (2.6)$$

Upon integrating both sides from $w = -\pi$ to π and applying Parseval's relation, we can obtain the relation that the output energy equals the input energy for all finite energy inputs as follows.

$$\sum_{n=-\infty}^{\infty} | y(n) |^2 = \sum_{n=-\infty}^{\infty} | x(n) |^2 \qquad (2.7)$$

It is called lossless. If the all-pass filter is stable as well, it is termed Lossless Bounded Real (LBR).[45]

The mirror-image symmetry relation between the numerator and denominator polynomials of an all-pass transfer function can be exploited to obtain a computationally efficient filter realization with a minimum number of multipliers. To see this, consider the second-order all-pass function of (2.5) which, upon expressing $A(z) = Y(z) / X(z)$, corresponds to the second-order difference equation

$$y(n) = a_2 [x(n) - y(n-2)] + a_1 [x(n-1) - y(n-1)] + x(n-2) \quad (2.8)$$

in which terms have been grouped in such a way that only two multiplications are required. A similar strategy can be applied to an arbitrary Mth-order all-pass filter, such that only M multiplications are required to compute each output sample.[17,60]

The difference equation as expressed in (2.8) requires four delay elements to be realized. Since the difference equation is of second order, this does not represent a canonic realization. However, minimum multiplier delay-canonic all-pass filter structures can be developed using the multiplier extraction approach[17,59]. By this approach numerous first-order and second-order all-pass filter structures have been catalogued[17,51,59], many with roundoff noise expressions as functions of the pole locations, and with the minimum multiplier property. Another useful structure for realizing all-pass functions is the Gray and Markel lattice filter[14]. The synthesis procedure uses the following recursing[58];

$$z^{-1} A_{m-1}(z) = (A_m(z) - k_m) / (1 - k_m A_m(z)) \quad (2.9)$$
$$m = M, M-1, \ldots, 1$$

where $k_m = A_m(\infty)$

and $A_{m-1}(z)$ is stable all-pass filter with one order lower. This structural interpretation of (2.9) for the first step in the recursion is as Fig.2.1 where $A_{M-1}(z)$ is an (M-1)th-order all-pass function. The recursion of (2.9) then continues on $A_{M-1}(z)$, and so on, which leads to the cascade lattice realization of Fig.2.2, where the constraining multiplier A_0 has unit magnitude. With the lattice structures above, stability of the filter is equivalent to the condition that $|k_m| < 1$ for all m.[58]

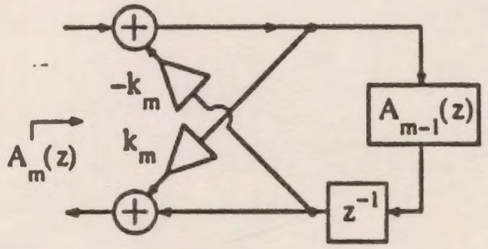


Fig.2.1

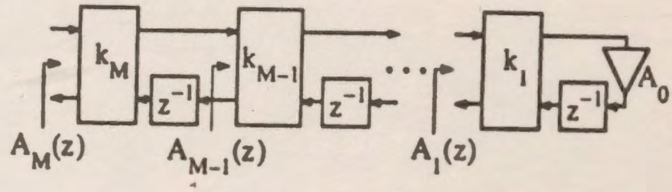


Fig.2.2

2.3 THE TRANSFER FUNCTION OF ALL-PASS METHOD

The general form of the M-th order filter of all-pass method is

$$H(z) = \{A_1(z) + (-1)^I z^M A_2(z)\} / 2, \qquad (2.10)$$

where $A_i(z) = z^{-N} D_i(z^{-1}) / D_i(z)$. (2.11)

i.e., $A_1(z)$ and $A_2(z)$ are stable Nth order all-pass filters. It is clear that on the unit circle,[54]

$$H(e^{jw}) = \{e^{j\theta 1(w)} \pm e^{j\theta 2(w)}\} / 2 \qquad (2.12)$$

where $\theta_1(w)$ and $\theta_2(w)$ are real-valued function of w. Thus,

$$|\ H(e^{jw})\ | = \tfrac{1}{2}\ |\ 1 \pm e^{j[\theta 2(w) - \theta 1(w)]}\ | \qquad (2.13)$$

which shows that $|\ H(e^{jw})\ |$ cannot be all-pass unless $\theta_2(w) = \theta_1(w)$ for all w. Equivalently, unless $A_1(z) = A_2(z)$, H(z) is not all-pass.

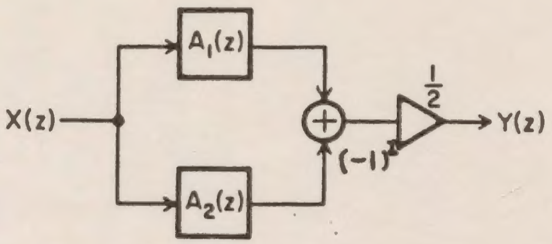


Fig.2.3

We can decide the filter class by examining the values of $|H(z)|$ when z is 1 or -1, i.e., s is 0 or ∞ in analog domain respectively as follows.

$$|H(+1)| = (1 + (-1)^{I}) / 2 \text{ and } |H(-1)| = (1 + (-1)^{I+M}) / 2 \quad (2.14.1)$$

when the order of all-pass filters is even.

$$|H(+1)| = (1 + (-1)^{I}) / 2 \text{ and } |H(-1)| = | -1 - (-1)^{I+M} | / 2$$

$$= (1 + (-1)^{I+M}) / 2 \quad (2.14.2)$$

when the order of all-pass filters is odd. In both cases, the same equations are derived. We note that $|H(+1)|$ and $|H(-1)|$ takes on only two values, 0 or 1. By inspection we see that, for different filter responses, the following constraints should be satisfied:

low-pass : $|H(+1)| = 1$, $|H(-1)| = 0$

high-pass : $|H(+1)| = 0$, $|H(-1)| = 1$

band-pass : $|H(+1)| = 0$, $|H(-1)| = 0$

band-stop : $|H(+1)| = 1$, $|H(-1)| = 1$.

These restrictions lead to a set of constraints on I and M as summarized in table 2.1.

Even if we assume the general form of this scheme as above, the following can be applicable for implementation and the value of I and M can be decided by the magnitude of the transfer function when

$z = \pm 1$.

Filter Type	I	M
Low-pass	0	1
High-pass	1	1
Band-pass	1	0
Band-stop	0	0

Table 2.1 Set of constraints for the transfer function

$$H(z) = (z^L + (-1)^I z^M A(z)) / 2 \qquad (2.15)$$

$$H(z) = (z^M A_0(z) + (-1)^I A_1(z)) / 2 \qquad (2.16)$$

where L is the order of all-pass filter.

2.4 THE BENEFITS OF ALL-PASS METHOD

First of all, as we explained, it is the most computationally efficient scheme at the present time. It uses fewer multiplications than conventional elliptic filter realizations for meeting filter specifications.

Secondly, the complementary filter is obtained from the original one by simply changing the sign of one of the all-pass sections. Therefore, a complementary filter pair (i.e., a lowpass/highpass or a bandstop/bandpass filter pair) can be implemented with the cost of a single filter. This complementary filter pair is called doubly complementary which are all-pass complementary

$$|H_1(e^{jw}) + H_2(e^{jw})| = 1, \text{ for all } w \qquad (2.17)$$

as well as power complementary

$$|H_1(e^{jw})|^2 + |H_2(e^{jw})|^2 = 1, \text{ for all } w. \qquad (2.18)$$

where $H_1(z) = \{A_1(z) + z^M A_2(z)\} / 2$ and

$H_2(z) = \{A_1(z) - z^M A_2(z)\} / 2$.

Another advantage of this form of filters is that it has a very low pass-band sensitivity characteristic because of its LBR(lossless bounded real) structure.[54] It is clear that $|H(e^{jw})|$ can never exceed unity for any value of w because the all-pass filters remain all-pass in spite of parameter quantization due to their mirror image structures. That is, the magnitude function $|H(e^{jw})|$ is structurally bounded by one. Suppose now that H(z) is designed such that at specific frequencies, to be denoted w_k, the passband amplitude achieves the upper bound of unity, i.e., $|H(e^{jw})| = 1$.(Fig.2.4) Regardless of the sign of any multiplier perturbation, i.e. $m_i \rightarrow m_i + \Delta m_i$, (due to quantization) the magnitude of the transfer function at $w = w_k$ can only decrease. Orchard's argument[4] can be applied at these frequencies (the so-called points of maximum power transfer) to establish the low-passband sensitivity behaviour.
Therefore, the structural losslessness of A(z) induces structural boundedness of H(z) which leads to a simple low passband sensitivity implementation.

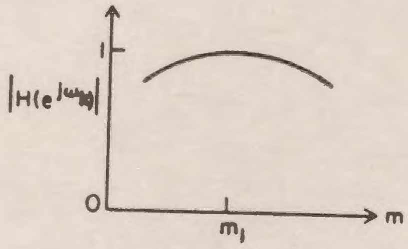


Fig.2.4

2.5 DESIGN EXAMPLES WITH NEW STABILITY TEST

2.5.1 STABILITY CONSIDERATION

In 1976, Schussler[23] formulated a set of conditions to determine the stability of a discrete system which can be stated as follows;

Schussler's Theorem:

Let D(z), a polynomial of degree m having real coefficients, be described as

$$D(z) = a_m z^m + a_{m-1} z^{m-1} + \ldots\ldots + a_1 z + a_0$$

$$= \sum_{i=0}^{m} a_i z^i \qquad (2.19)$$

where a_i is always positive.

D(z) can be decomposed as the sum of the mirror-image polynomial,

$$F_1(z) = \tfrac{1}{2} [D(z) + z^m D(z^{-1})] \qquad (2.20)$$

and the anti-mirror image polynomial,

$$F_2(z) = \tfrac{1}{2} [D(z) - z^m D(z^{-1})] \qquad (2.21)$$

For D(z) to have all zeros inside the unit circle, the necessary and sufficient conditions are :

i) The zeros of $F_1(z)$ and $F_2(z)$ are located on the unit circle,

ii) They are simple,

iii) They separate each other, and

iv) $| a_0 / a_m | < 1$.

The fourth condition was added later.[34]

The properties of $F_1(z)$ and $F_2(z)$ were studied by Ramachandran and et al.[46] In order to generate a 1-D polynomial of order m which has all its zeros inside the unit circle, it has been shown that such a polynomial can be obtained using the following relationship.

For m even

$$D(z) = k_e \prod_{i=1}^{n} (z^2 - 2\,\alpha_i\, z + 1) + (z^2 - 1) \prod_{i=1}^{n-1} (z^2 - 2\,\beta_i\, z + 1),$$

$$n = m/2,\ k_e > 1 \ \text{ and } \qquad (2.22)$$

$$1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 \ldots > \beta_{n-1} > \alpha_n > -1. \qquad (2.22\text{-}1)$$

For m odd

$$D(z) = k_o\,(z+1) \prod_{i=1}^{n} (z^2 - 2\,\alpha_i\, z + 1) + (z-1) \prod_{i=1}^{n} (z^2 - \beta_i\, z + 1)$$

$$n = (m-1)/2 \ \text{ and } \qquad (2.23)$$

$$1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 \ldots > \alpha_n > \beta_n > -1. \qquad (2.23\text{-}1)$$

And also in both cases $| a_0 / a_m | > 1$.

2.5.2 FORMULATION OF THE DESIGN PROBLEM

The design method used here involves the application of the optimization techniques in the course of searching for coefficients of the filter transfer function such that it approximates the given specifications. For stability, eqn.(2.22) or eqn.(2.23) is assigned to the denominators of a 1-D transfer function stated as

$$H(z) = \{A_0(z) + (-1)^I z^M A_1(z)\} / 2, \qquad (2.24)$$

where $A_i(z) = z^{-M} D_i(z^{-1}) / D_i(z)$. (2.25)

and $D_i(z)$ is described in Eqn.(2.22) or Eqn.(2.23).
We let the ideal magnitude response of the filter be denoted as

$$| H_I(e^{jwT}) | = M_I(w) \qquad (2.26)$$

and the designed magnitude response of the filter as

$$| H_D(e^{jwT}) | = M_D(w) \qquad (2.27)$$

while the ideal and designed group-delay response are $\tau_I(w)$ and $\tau_D(w)$, respectively.

Further, $\{w_i,\ i = 1,2,\ldots,\ N\}$ is the discrete set of frequencies, equally or nonequally spaced, at which $E_M(jw_i)$, the error in magnitude response, and $E_\tau(jw_i)$, the error in group-delay response, between the designed and ideal values, are evaluated:

$$E_M(w_i) = |M_I(w_i) - M_D(w_i)| \qquad (2.28)$$

and

$$E_\tau(jw_i) = \tau_I - \tau_D(jw_i) \qquad (2.29)$$

In order to formulate the design problem, two cases will be considered.

1. For the approximation of the magnitude response only.

   In this case, we use the least mean-square error criterion (l_2 norm) using Eqn.(2.28) in the following relationship:

$$E_{G1}(w_i,\ \alpha,\ \beta,\ k) = \sum_{i \in I_{ps}} E_M^2(w_i) \qquad (2.30)$$

where I_{ps} is the set of discrete frequency points in the passband and stopband region. Now the problem is to calculate the filter coefficients α, β and k in such a say so as to minimize $E_{G1}(w_i, \alpha, \beta, k)$ in Eqn. (2.30). This is a simple nonlinear optimization problem and can be implemented using any of the existing nonlinear optimization routines.

2. For the approximation of the magnitude and group-delay response of the filter.

In this case, the general mean-square error E_{G2} is calculated using equations (2.28) and (2.29) in the following manner:

$$E_{G2}(w_i, \alpha, \beta, k) = \sum_{i \in I_{ps}} E_M^2(jw_i) + \sum_{i \in I_p} E_\tau^2(jw_i) \quad (2.31)$$

where I_{ps} is as previously defined, and I_p is the set of discrete frequency points in the passband region.

Again, in the design of a 1-D filter satisfying prescribed magnitude and constant group-delay response, α, β and k should be calculated in such a way that E_{G2} in Eqn. (2.31) is minimized.

The design problem now becomes that of calculation (α, β, k), the numerator and denominator coefficients, in such a way that E_G in equation (2.30) or (2.31) is minimized, subject to linear constraint (2.22-1) or (2.23-1), depending on whether the filter has even or odd order. This new formulation is also a simple nonlinear constraint optimization problem, which can be solved either by utilization of any suitable optimization with a constraint, or by transforming the problem to an unconstrained optimization problem by using following N-variable substitution method. For example, if m is even we may write

$$\alpha_n = \cos[\ \pi \exp(-\theta_1^2)\ ]$$

$$\beta_{n-1} = \cos[\ \pi \exp(-(\theta_1^2 + \theta_2^2))\ ]$$

$$a_{n-1} = \cos[\ \pi \exp(\ -\ (\theta_1^2 + \theta_2^2 + \theta_3^2)\ )]$$

$$\vdots$$

$$a_1 = \cos[\ \pi \exp(\ -\sum_{i=1}^{2n-1} \theta_i^2\ )] \qquad (2.32)$$

Similar conditions can be derived if m is odd. Now any unconstrained optimization technique can be applied to compute the new filter coefficients θ and k to minimize E_{G1} in Eqn.(2.30) or E_{G2} in Eqn.(2.31).

2.5.3 EXAMPLES

To illustrate the usefulness of the proposed method, several examples of IIR digital filters satisfying a prescribed magnitude response with or without constant group delay characteristics are given.

In all examples, the order of the filter is considered to be equal to eight and the Hooke and Jeeve method[2,65] is used to minimize the cost function. Also, the sampling frequency w_s is 2π rad/sec and T is 1 second.

i) Design of a lowpass filter without linear phase characteristic which has the following specifications:

$$|\ H_D(jw_i)\ | = 1 \qquad \text{for} \quad 0 \le w \le 0.8$$
$$= 0 \qquad \text{for} \quad 1.0 \le w \le w_s/2 = \pi$$

As a sixteenth order low-pass filter, the transfer function is

$$H(z) = \{A_1(z) + z\ A_2(z)\}\ /\ 2\ \Big|_{z=\exp(jwnT)}$$

where $A_i(z) = z^{-8}\ D_i(z^{-1})\ /\ D_i(z)$

and $D_i(z)$ is a stable eighth order polynomial described in Eqn.(2.22) when m is eight. Table (2.2) shows the values of θ, k_e, α and β of the filter's transfer function obtained by minimizing the least mean square error of Eqn.(2.30) while Fig. 2.5 shows the magnitude response of the designed filter.
Because the coefficients of the numerators are easily obtained by reversing the order of the coefficients, the coefficients of denominators are given only.

value of θ, Denominator coeff. of $A_1(z)$ and $A_2(z)$			
θ_{11} = 0.511648	θ_{21} = 0.465082	α_{11} = 0.736251	α_{21} = 0.798816
θ_{12} = 0.367777	θ_{22} = 0.387899	β_{11} = 0.643678	β_{21} = 0.581869
θ_{13} = 0.367715	θ_{23} = 0.315256	α_{12} = 0.497853	α_{22} = 0.293313
θ_{14} = 0.509287	θ_{24} = 0.442832	β_{12} = 0.146792	β_{22} =-0.049265
θ_{15} = 0.551923	θ_{25} = 0.490935	α_{13} =-0.270768	α_{23} =-0.389671
θ_{16} = 0.431298	θ_{26} = 0.541289	β_{13} =-0.515260	β_{23} =-0.569779
θ_{17} = 0.398925	θ_{27} = 0.621481	α_{14} =-0.749441	α_{24} =-0.819039
ke_1 = 1.000000	ke_2 = 1.045603		

Table 2.2 Coefficients for the lowpass filter using new stability test without linear phase

ii) Design of lowpass filter with linear phase characteristic which has following specifications:

$$|H_D(jw_i)| = 1 \quad \text{for} \quad 0 \le w \le 0.3$$
$$= 0 \quad \text{for} \quad 0.5 \le w \le w_s/2 = \pi$$

The transfer function is same as that of example (i).
Table (2.3) shows the values of θ, k_e, α and β of the filter's transfer function obtained by minimizing the least mean square error of Eqn.(2.31) where τ_1 is set to 22. Fig. 2.6 a,b show the magnitude response and the group delay response of the designed

filter respectively.

Denominator coeff. of $A_0(z)$ and $A_1(z)$

θ_{11} = 0.372733	θ_{21} = 0.314706	α_{11} = 0.994363	α_{21} = 0.994376
θ_{12} = 1.039060	θ_{22} = 0.744511	β_{11} = 0.966831	β_{21} = 0.994172
θ_{13} = 0.000001	θ_{23} = 1.075096	α_{12} = 0.939062	α_{22} = 0.993157
θ_{14} = 0.565770	θ_{24} = 0.897259	β_{12} = 0.781085	β_{22} = 0.973658
θ_{15} = 0.808268	θ_{25} = 0.821960	α_{13} = 0.598783	α_{23} = 0.870508
θ_{16} = 0.553599	θ_{26} = 0.283421	β_{13} = 0.598783	β_{23} =-0.063753
θ_{17} = 0.942596	θ_{27} = 0.133703	α_{14} =-0.918115	α_{24} =-0.956443
ke_1 = 1.032954	ke_2 = 1.031355		

Table 2.3 Coefficients for the lowpass filter using the new stability test with linear phase

iii) Design of highpass filter with linear phase characteristic which has following specifications:

$$| H_D(jw_i) | = 0 \quad \text{for } 0 \le w \le 1.1$$
$$= 1 \quad \text{for } 1.5 \le w \le w_s/2$$

The transfer function of eighth order high-pass filter is

$$H(z) = \{A_1(z) - z\, A_2(z)\} / 2,$$

where $A_i(z) = z^{-8}\, D_i(z^{-1}) / D_i(z)$

and $D_i(z)$ is a stable eighth order polynomial described in Eqn.(2.22) when m is eight. The filter coefficients are calculated based on the cost function described by equation (2.31) where τ_1 is equal to 4. Table (2.4) shows the values of θ, k_e, α and β of the filter's transfer function and Fig. 2.7 a,b show the magnitude and the group delay responses of the designed filter respectively.

Denominator coeff. of $A_0(z)$ and $A_1(z)$

θ_{11} = 0.510683	θ_{21} = 0.467197	α_{11} = 0.705441	α_{21} = 0.786566

θ_{12} = 0.629195	θ_{22} = 0.507874	β_{11} = 0.422068	β_{21} = 0.476785
θ_{13} = 0.450963	θ_{23} = 0.576342	α_{12} = 0.356210	α_{22} = 0.332007
θ_{14} = 0.005554	θ_{24} = 0.356953	β_{12} = 0.239163	β_{22} = 0.332007
θ_{15} = 0.311210	θ_{25} = 0.000000	α_{13} = 0.239123	α_{23} = 0.170135
θ_{16} = 0.247184	θ_{26} = 0.371119	β_{13} =-0.058300	β_{23} =-0.371433
θ_{17} = 0.604374	θ_{27} = 0.691607	α_{14} =-0.751019	α_{24} =-0.816168
ke_1 = 1.133429	ke_2 = 1.000003		

Table 2.4 Coefficients for the highpass filter using new stability test with linear phase

iv) Design of bandpass filter without linear phase characteristic which has following specifications:

$$| H_D(jw_i) | = 0 \quad \text{for } 0 \le w \le 1.2$$
$$= 1 \quad \text{for } 1.4 \le w \le 2.6$$
$$= 0 \quad \text{for } 2.8 \le w \le w_s/2$$

The transfer function is obtained by setting I = 1 and M = even number as

$$H(z) = \{A_0(z) - A_1(z)\} / 2,$$
$$\text{where } A_i(z) = z^{-5} D_i(z^{-1}) / D_i(z)$$

and $D_i(z)$ is a stable eighth order polynomial described in Eqn.(2.22) when m is eight. Table (2.5) shows the values of θ, k_e, α and β of the filter's transfer function while Fig. 2.8 shows the magnitude response of the designed filter.

value of θ, Denominator coeff. of $A_0(z)$ and $A_1(z)$

θ_{11} = 0.420393	θ_{21} = 0.391452	α_{11} = 0.300492	α_{21} = 0.286560
θ_{12} = 0.411793	θ_{22} = 0.180593	β_{11} = 0.198442	β_{21} = 0.056146
θ_{13} = 0.572652	θ_{23} = 0.414162	α_{12} = 0.008626	α_{22} = 0.030353

θ_{14} =-0.028683	θ_{24} = 0.548869	β_{12} =-0.028670	β_{22} =-0.055128
θ_{15} = 0.153612	θ_{25} = 0.232437	α_{13} =-0.029986	α_{23} =-0.586532
θ_{16} = 0.361268	θ_{26} = 0.130008	β_{13} =-0.606176	β_{23} =-0.861379
θ_{17} = 0.282887	θ_{27} = 0.410095	α_{14} =-0.873271	α_{24} =-0.902036
ke_1 = 1.421711	ke_2 = 1.261749		

Table 2.5 Coefficients for the bandpass filter using the new stability test without linear phase

v) Design of bandstop filter with linear phase characteristic which has following specifications:

$$|H_D(jw_i)| = 1 \quad \text{for} \quad 0 \le w \le 1.0$$
$$= 0 \quad \text{for} \quad 1.4 \le w \le 2.6$$
$$= 1 \quad \text{for} \quad 3.0 \le w \le w_s/2$$

The transfer function is obtained by setting I = 0 and M = even number as

$$H(z) = \{A_0(z) + A_1(z)\} / 2,$$
$$\text{where } A_i(z) = z^{-8} D_i(z^{-1}) / D_i(z)$$

and $D_i(z)$ is a stable eighth order polynomial described in Eqn.(2.22) when m is eight. Table (2.6) shows the values of θ, k_e, α and β of the filter's transfer function and Fig. 2.9 a,b show the magnitude response and the group delay response of the designed filter respectively. τ_1 is set to 7.5.

value of θ, Denominator coeff. of $A_0(z)$ and $A_1(z)$

θ_{11} = 0.454964	θ_{21} = 0.312411	α_{11} = 0.908182	α_{21} = 0.910328
θ_{12} = 0.464496	θ_{22} = 0.331433	β_{11} = 0.646968	β_{21} = 0.657536
θ_{13} = 0.318018	θ_{23} = 0.461838	α_{12} = 0.394431	α_{22} = 0.236553
θ_{14} = 0.557794	θ_{24} = 0.318241	β_{12} = 0.206273	β_{22} =-0.285430

θ_{15} = 0.395843	θ_{25} = 0.577967	α_{13} =-0.285669	α_{23} =-0.468620
θ_{16} = 0.543610	θ_{26} = 0.667357	β_{13} =-0.468604	β_{23} =-0.828901
θ_{17} = 0.834938	θ_{27} = 0.832422	α_{14} =-0.832386	α_{24} =-0.957631
ke_1 = 1.016295	ke_2 = 1.110829		

Table 2.6 Coefficients for the bandstop filter using new stability test with linear phase

vi) Design of multi-band filter without linear phase characteristic which has following specifications:

$$
\begin{aligned}
| H_D(jw_i) | &= 0 && \text{for } 0 \le w \le 0.8 \\
&= 1 && \text{for } 1.0 \le w \le 1.5 \\
&= 0 && \text{for } 1.7 \le w \le 2.2 \\
&= 1 && \text{for } 2.4 \le w \le 2.7 \\
&= 0 && \text{for } 2.9 \le w \le w_s/2
\end{aligned}
$$

The transfer function is same as that of band-pass filter shown in example (iv). Table (2.7) shows the values of θ, k_e, α and β of the filter's transfer function obtained by minimizing the least mean square error of Eqn.(2.30) while Fig.(2.10) shows the magnitude response of the designed filter.

value of θ, Denominator coeff. of $A_0(z)$ and $A_1(z)$			
θ_{11} = 0.369134	θ_{21} = 0.456008	α_{11} = 0.488356	α_{21} = 0.583673
θ_{12} = 0.259767	θ_{22} = 0.385014	β_{11} = 0.163873	β_{21} = 0.509964
θ_{13} = 0.303739	θ_{23} = 0.004605	α_{12} = 0.021788	α_{22} = 0.185623
θ_{14} = 0.419719	θ_{24} = 0.344443	β_{12} =-0.378771	β_{22} =-0.373911
θ_{15} = 0.484722	θ_{25} = 0.587235	α_{13} =-0.693178	α_{23} =-0.588629
θ_{16} = 0.311024	θ_{26} = 0.538527	β_{13} =-0.836970	β_{23} =-0.588667
θ_{17} = 0.531085	θ_{27} = 0.291703	α_{14} =-0.920987	α_{24} =-0.831038
ke_1 = 3.061915	ke_2 = 2.475582		

Table 2.7 Coefficients for multiband filter using new stability test without linear phase

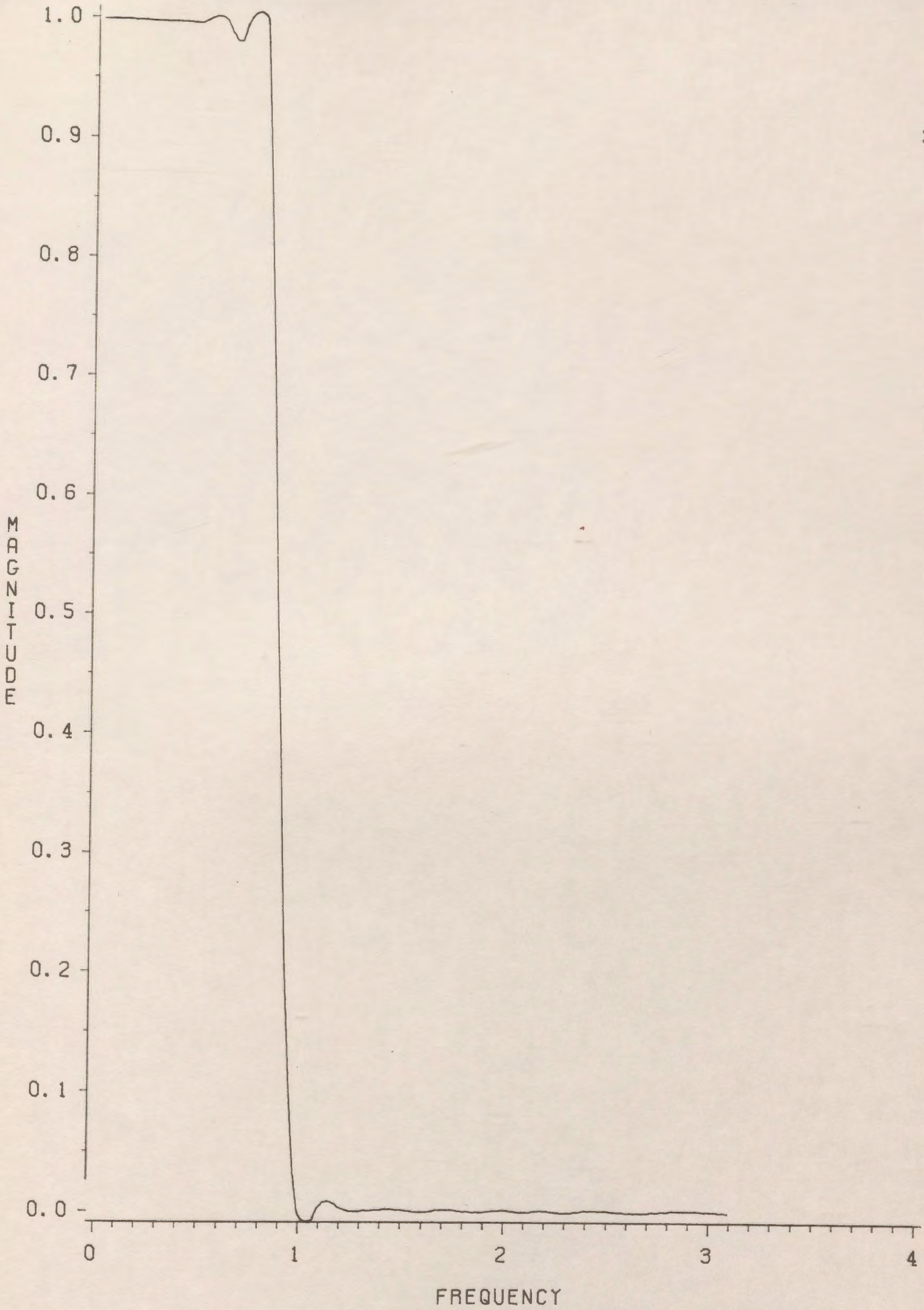


Figure 2.5 Magnitude response of eighth order lowpass filter using new stability test without linear phase.

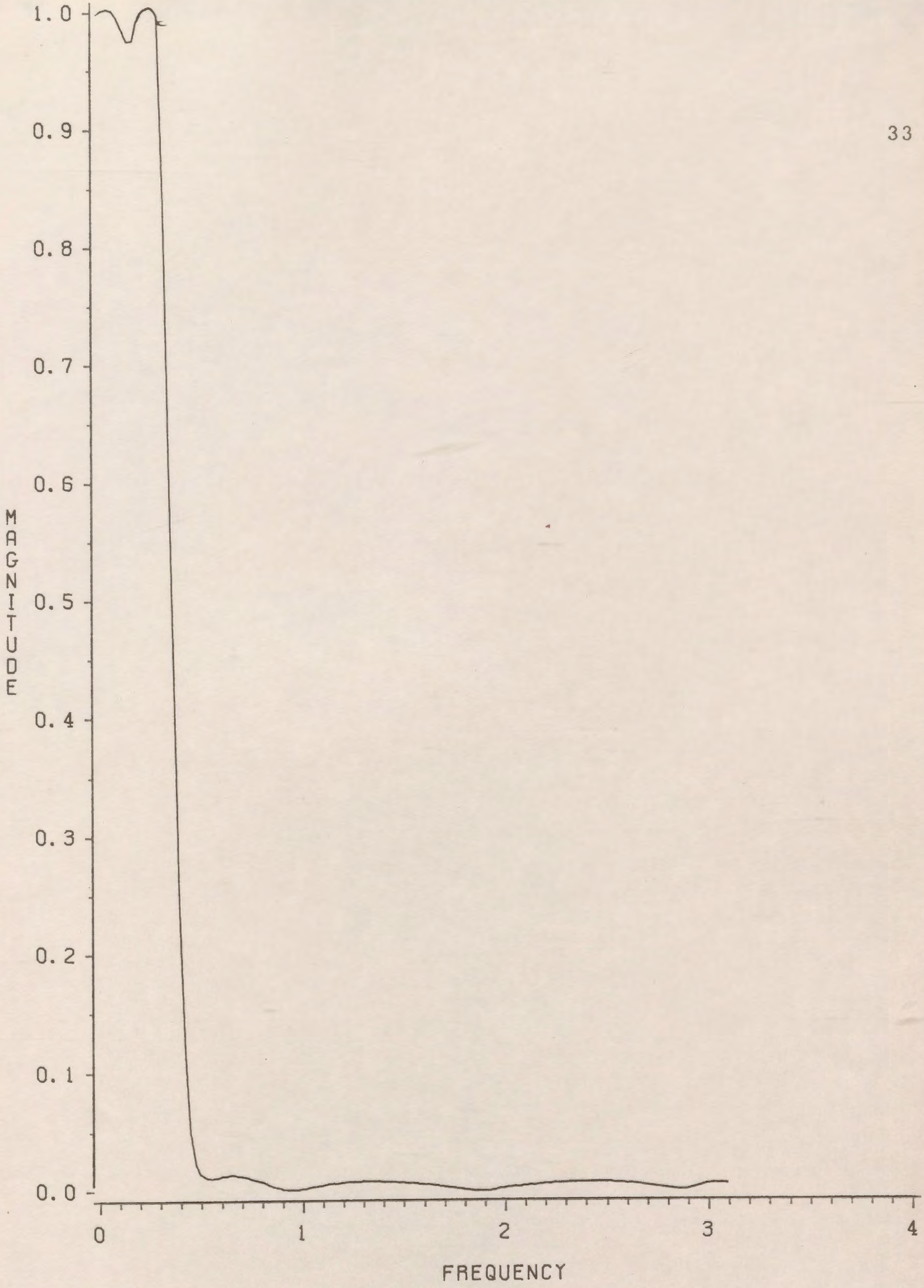


Figure 2.6 a Magnitude response of eighth order lowpass filter using new stability test with linear phase.

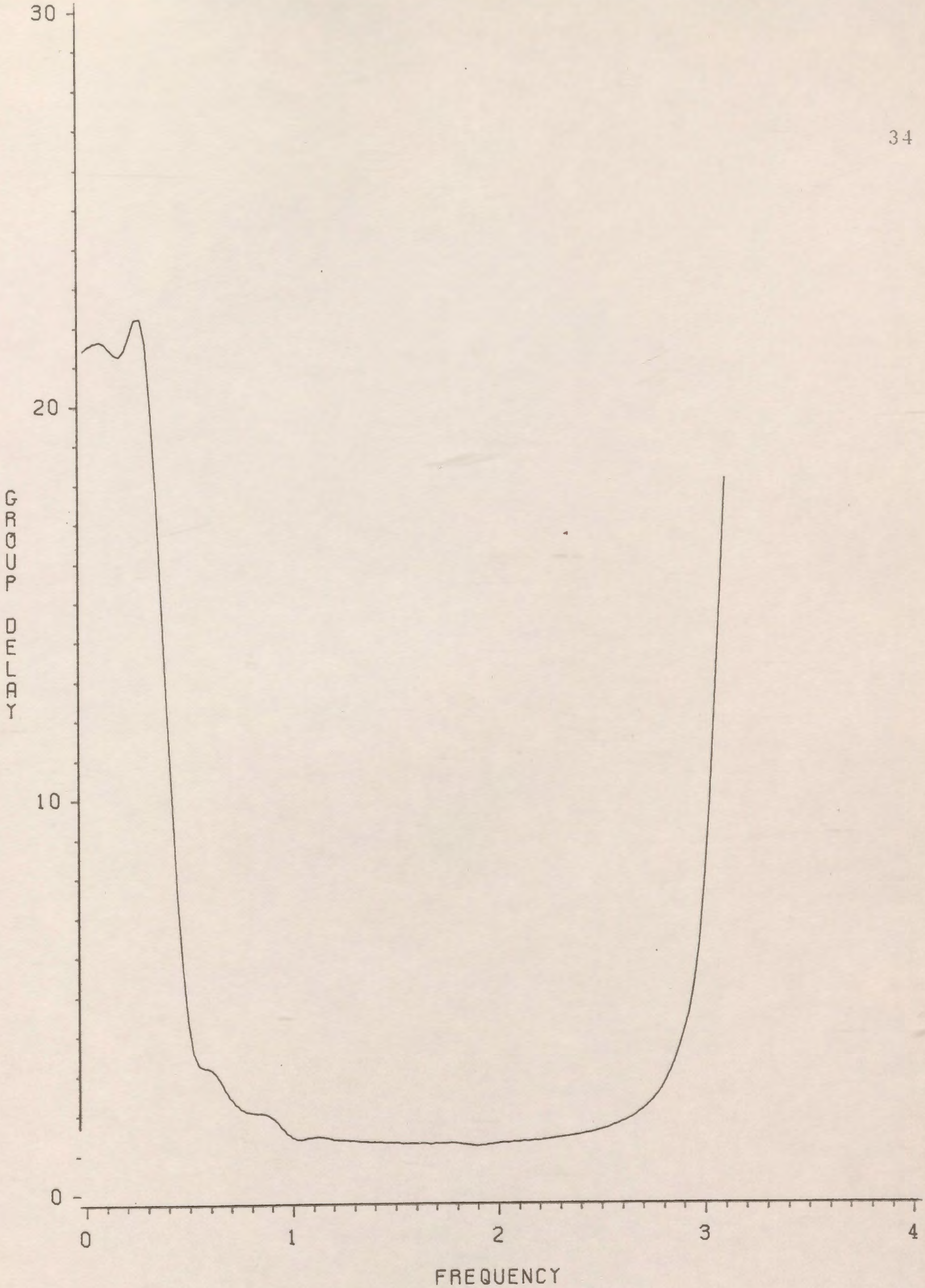


Figure 2.6 b Group delay response of eighth order lowpass filter using new stability test with linear phase.

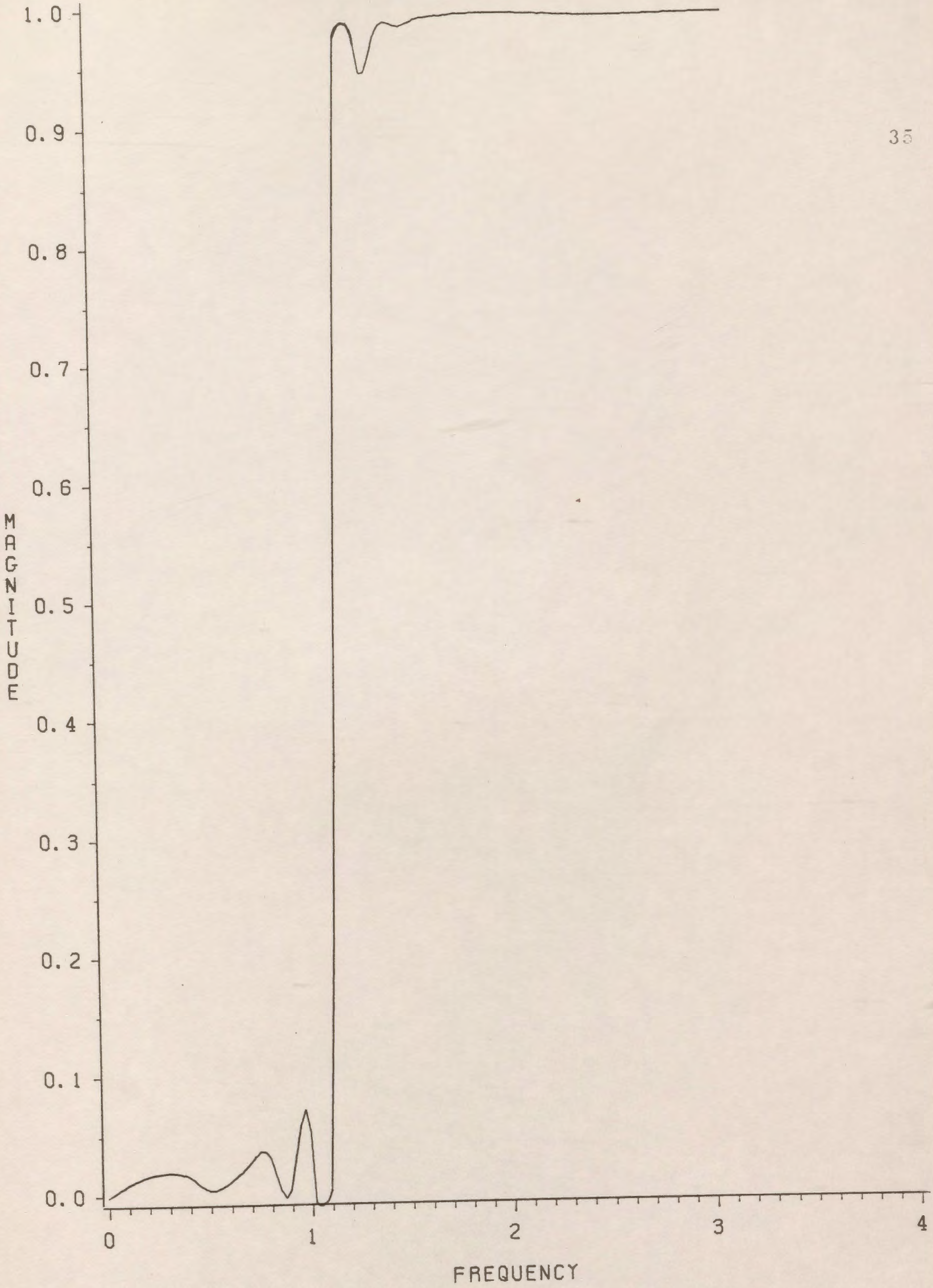


Figure 2.7 a Magnitude response of eighth order highpass filter using new stability test with linear phase.

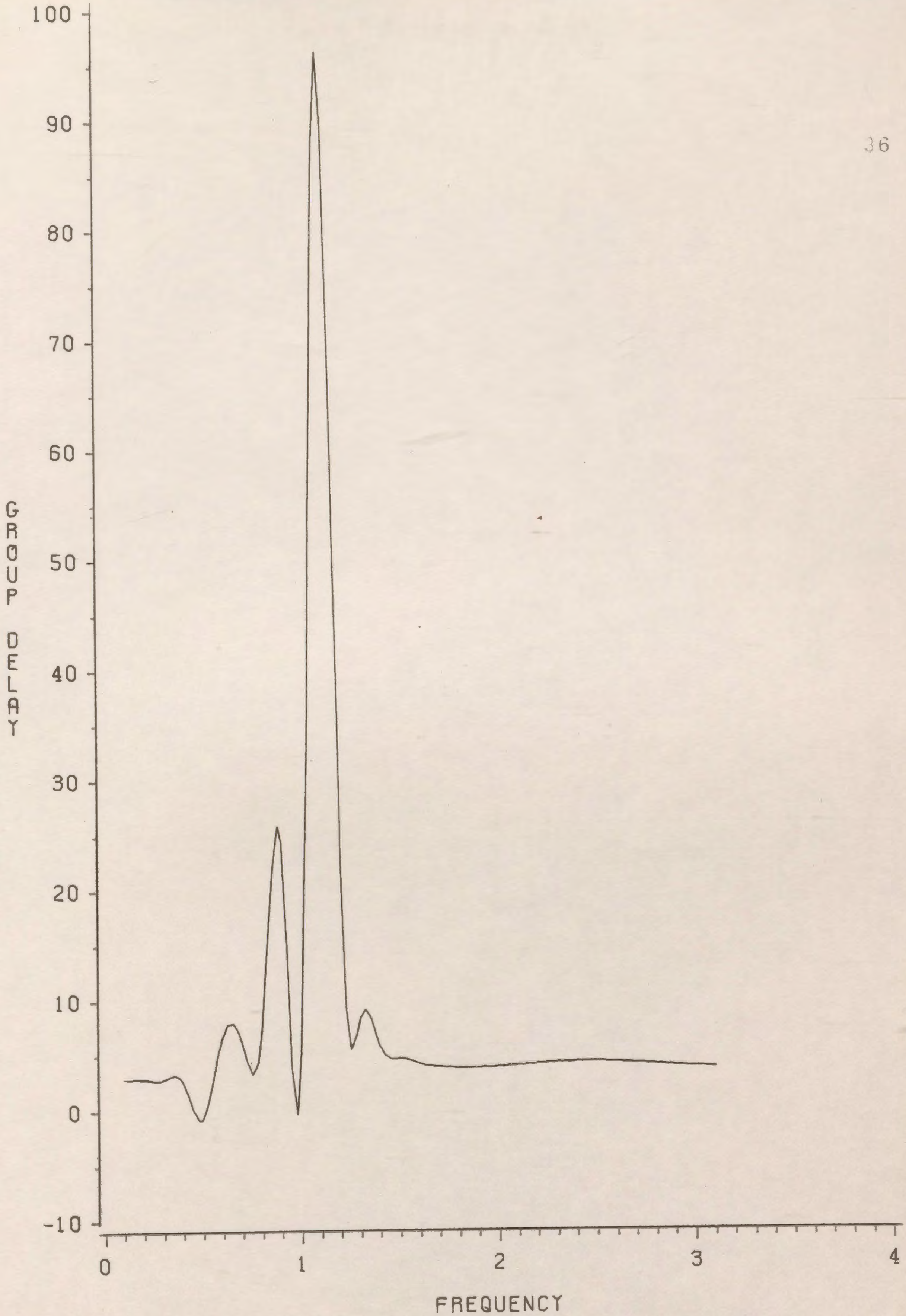


Figure 2.7 b Group delay response of eighth order highpass filter using new stability test with linear phase.

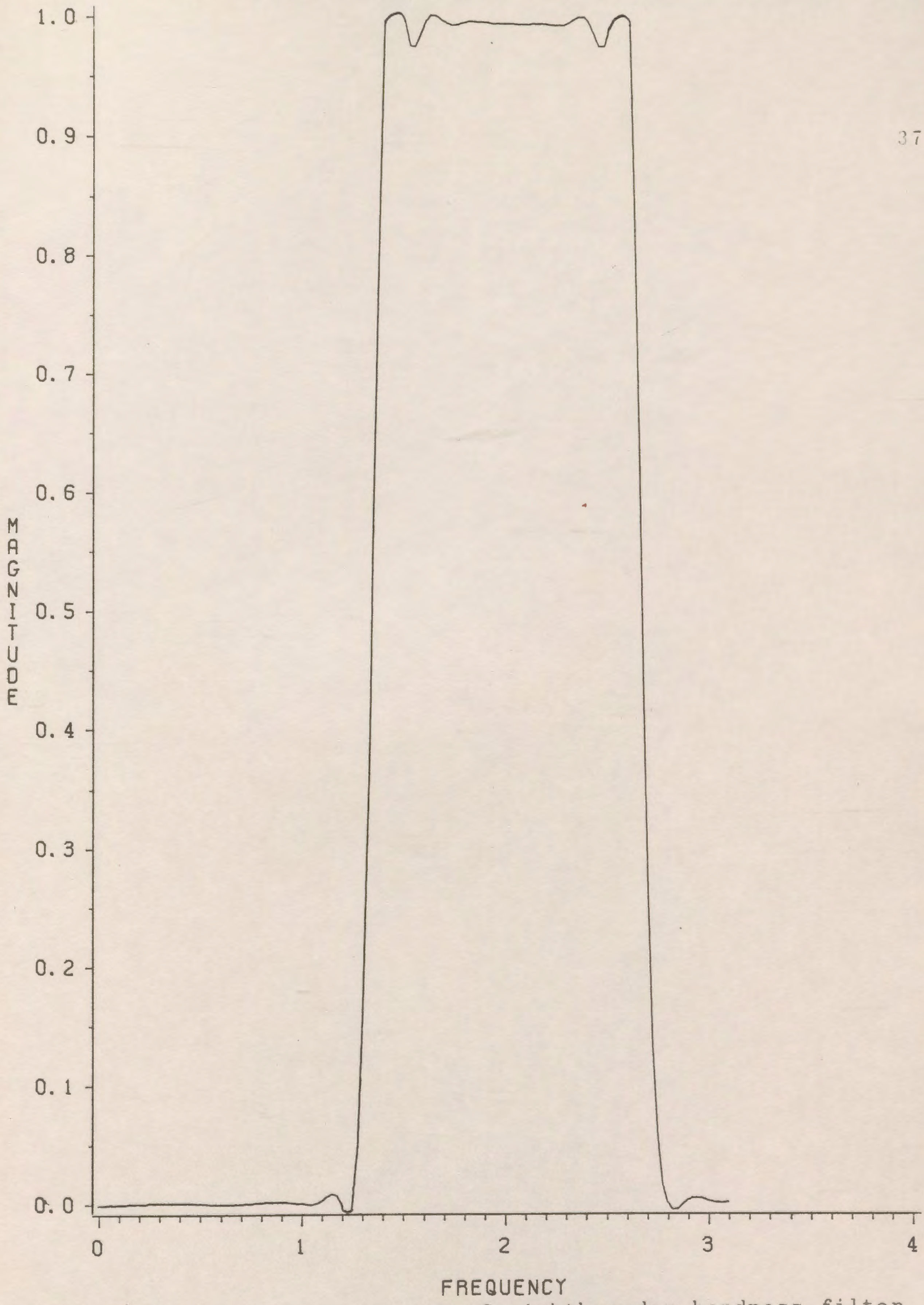


Figure 2.8 Magnitude response of eighth order bandpass filter using new stability test without linear phase.

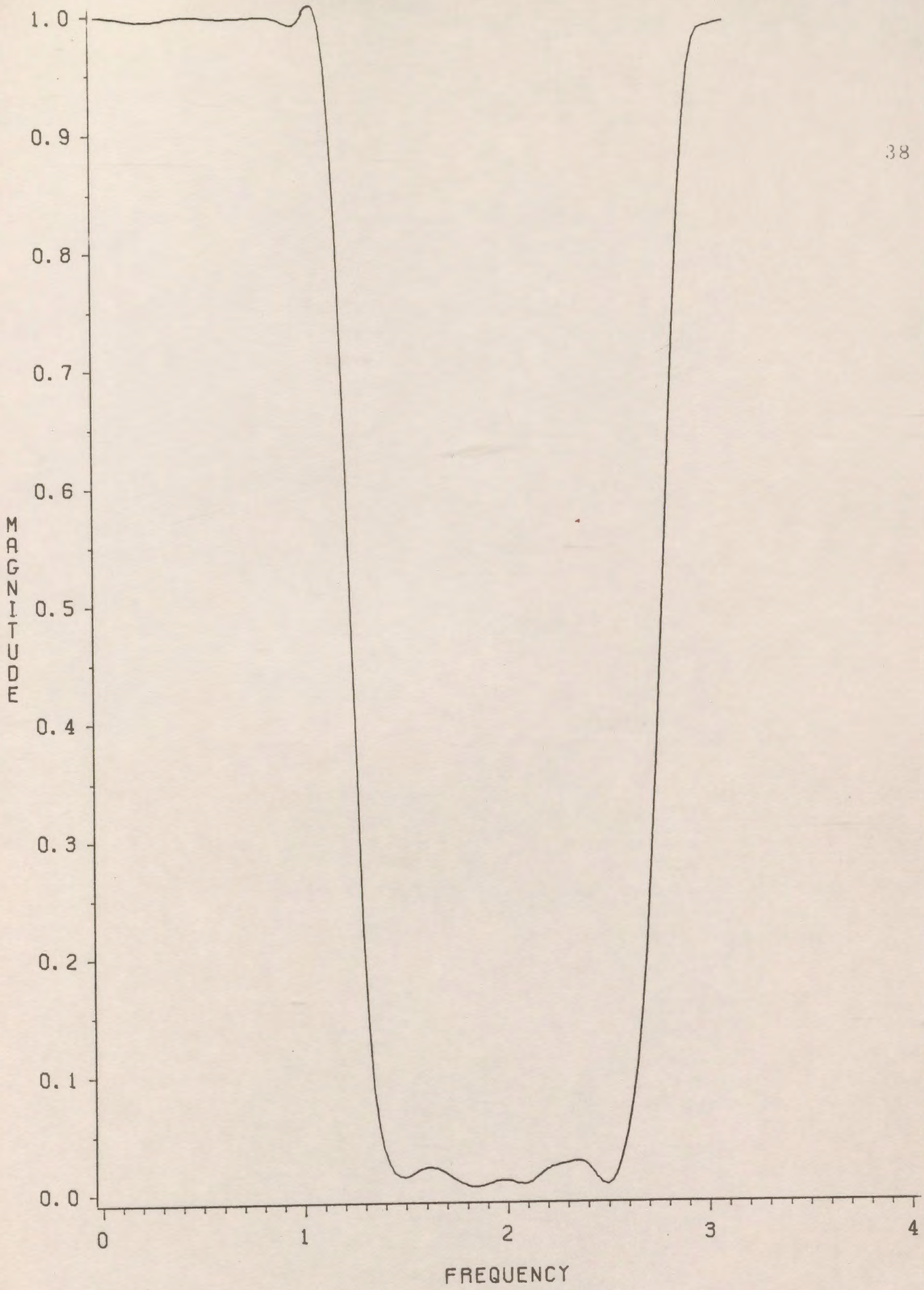


Figure 2.9 a Magnitude response of eighth order bandstop filter using new stability test with linear phase.

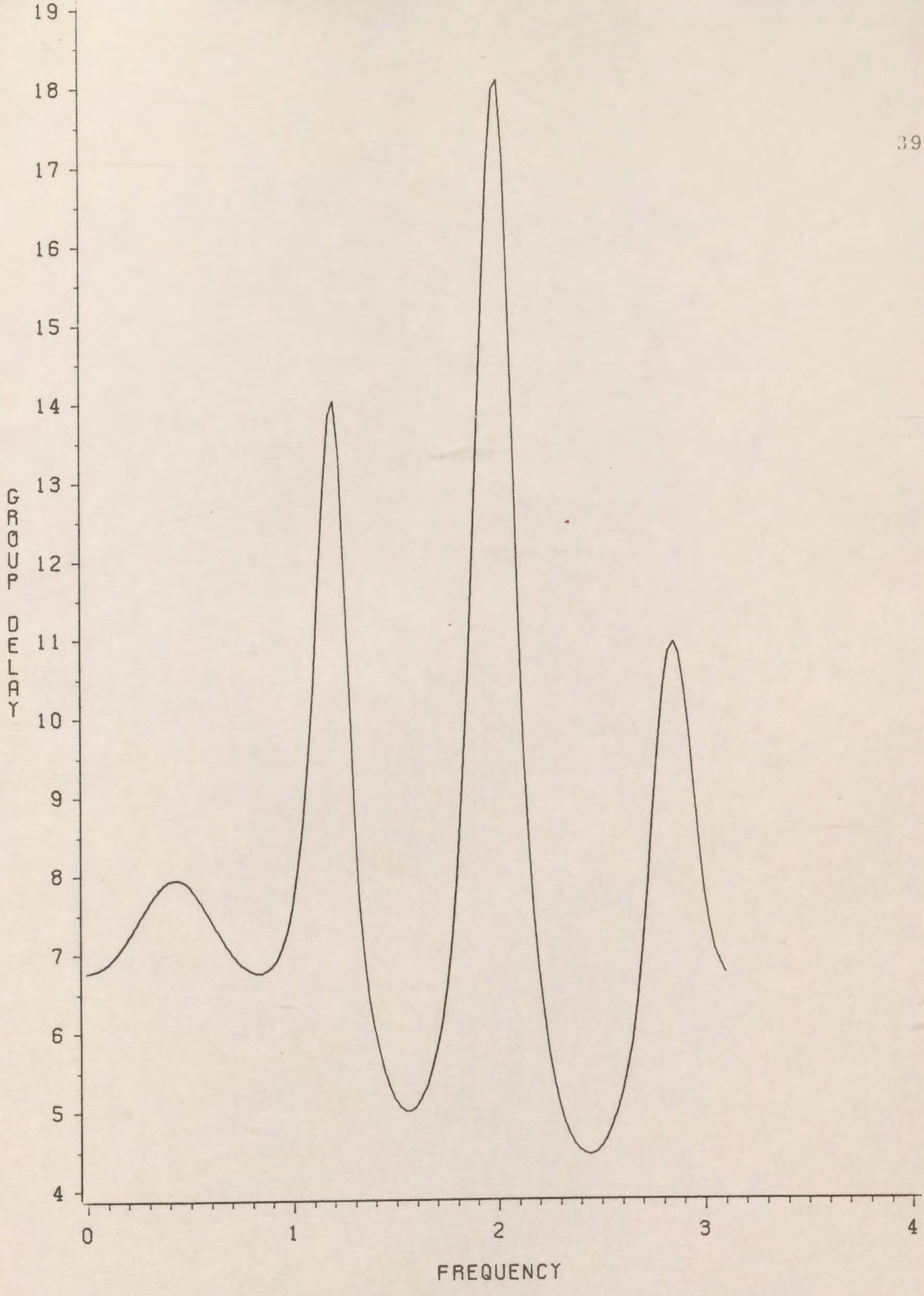


Figure 2.9 b Group delay response of eighth order bandstop filter using new stability test with linear phase.

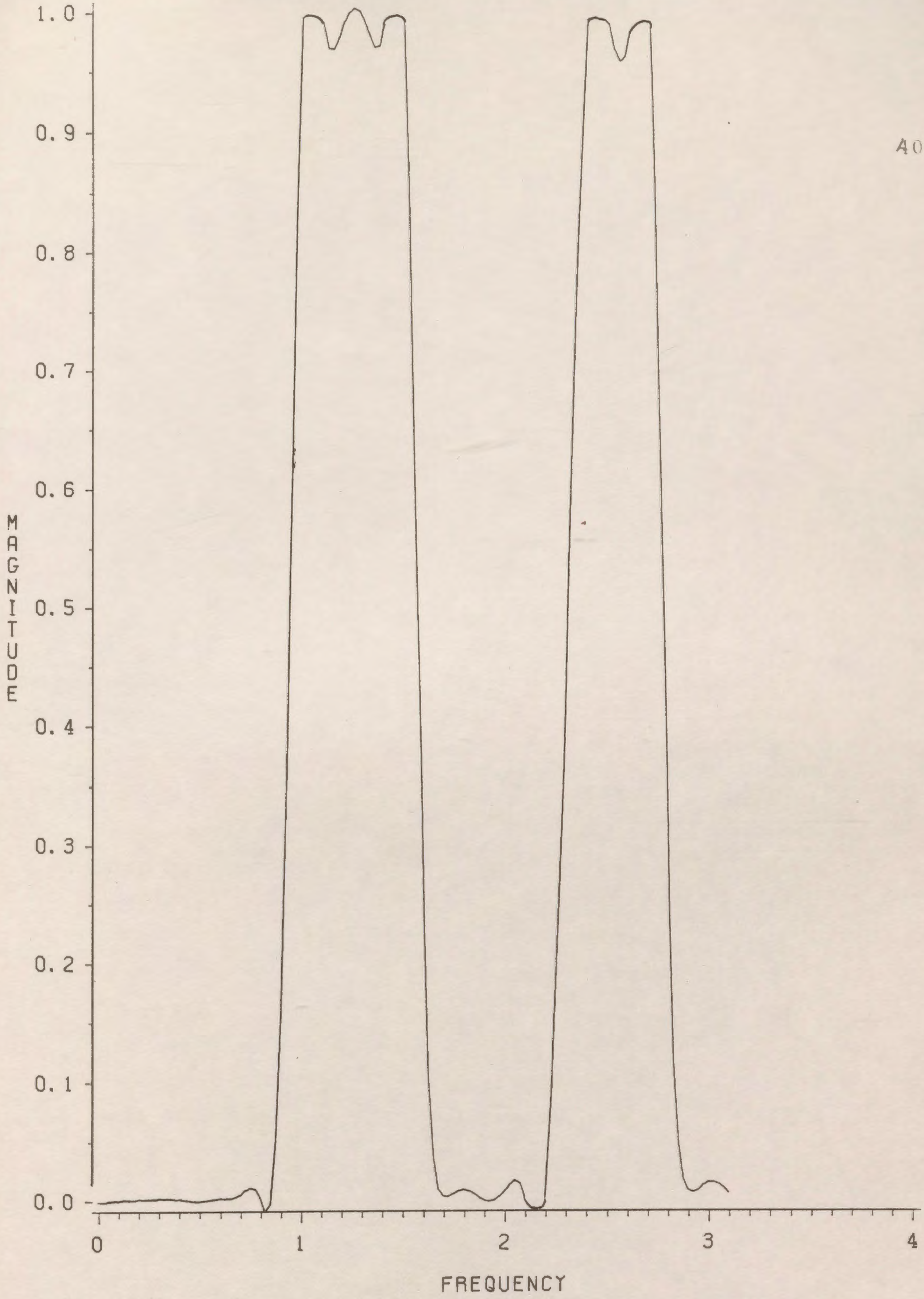


Figure 2.10 Magnitude response of eighth order multiband filter using new stability test without linear phase.

2.6 DESIGN EXAMPLES WITH HURWITZ POLYNOMIALS

2.6.1 HURWITZ POLYNOMIALS

For a analog transfer function to be stable, its poles must be restricted to the left-half plane of the jw axis. Moreover, the poles on the jw axis must be simple. The denominator polynomial of the analog transfer function described above belongs to a class of polynomials known as Hurwitz polynomial.

2.6.1.1 DEFINITION AND PROPERTIES

A polynomial P(s) is said to be Hurwitz if the following conditions are satisfied:[9]

1. P(s) is real when s is real.
2. The roots of P(s) have real parts which are zero or negative.[1]

As a result of these conditions, the Hurwitz polynomials have the following properties:

1. All the coefficients of P(s) are nonnegative.
2. Both the odd and even parts of a Hurwitz polynomial P(s) have roots on the jw axis only.
3. As a result of property 2, if P(s) is either even or odd, all its roots are on the jw axis.
4. The continued fraction expansion of the ratio of the odd to even parts or the even to odd parts of a Hurwitz polynomial yields all positive quotient terms.

By using the property 4, we can determine if any polynomial is Hurwitz or not.[44] For example, let us test whether the polynomial

$$F(s) = s^4 + s^3 + 5s^2 + 3s + 4 \qquad (2.33)$$

is Hurwitz. The even and odd parts of F(s) are

$$m(s) = s^4 + 5s^2 + 4 \qquad (2.34)$$

$$n(s) = s^3 + 3s \qquad (2.35)$$

We now perform a continued fraction expansion of $\Phi(s) = m(s)/n(s)$ then the continued fraction expansion of $\Phi(s)$ can be written as

$$\begin{array}{l} s^3 + 3s \;)\; s^4 + 5s^2 + 4 \;(\; s \\ \qquad\quad \underline{s^4 + 3s^2} \\ \qquad\qquad 2s^2 + 4 \;)\; s^3 + 3s \;(\; s/2 \\ \qquad\qquad\qquad\quad \underline{s^3 + 2s} \\ \qquad\qquad\qquad\qquad s \;)\; 2s^2 + 4 \;(\; 2s \\ \qquad\qquad\qquad\qquad\quad \underline{2s^2} \\ \qquad\qquad\qquad\qquad\qquad 4 \;)\; s \;(\; s/4 \\ \qquad\qquad\qquad\qquad\qquad\quad \underline{s} \\ \qquad\qquad\qquad\qquad\qquad\quad 0 \end{array}$$

so that the continued fraction expansion of $\Phi(s)$ is

$$\Phi(s) = m(s)/n(s) = s + \cfrac{1}{s/2 + \cfrac{1}{2s + \cfrac{1}{s/4}}}$$

Since all the quotient terms of the continued fraction expansion are positive, F(s) is Hurwitz.

2.6.1.2 GENERATION OF 1-D HURWITZ POLYNOMIALS

It has been known[64] that a network described by symmetric positive definite (or positive semi definite) immittance matrix is always physically realizable. It is further known that any positive definite matrix **P** can always be decomposed as a product of two matrices **Q** and $\mathbf{Q}^T$, where **Q** is either an upper-triangular matrix or a lower-triangular matrix. Therefore, the matrix

$$\mathbf{D} = \mathbf{A}\,\mathbf{U}\,\mathbf{A}^T s + \mathbf{G} \qquad (2.36)$$

where **A** are upper-triangular matrices, **U** is a diagonal matrix with non-negative elements, and G is a skew-symmetric matrix, is realizable reactance network. The matrices are shown below.

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \cdot & \cdot & a_{1n} \\ 0 & 1 & a_{23} & \cdot & \cdot & a_{2n} \\ 0 & 0 & 1 & \cdot & \cdot & a_{3n} \\ \vdots & \vdots & \cdot & \cdot & \cdot & \vdots \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{bmatrix} \qquad (2.37.a)$$

$$U = \begin{bmatrix} u_1^2 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & u_2^2 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & u_3^2 & \cdot & \cdot & 0 \\ \vdots & \vdots & \cdot & \cdot & \cdot & \vdots \\ 0 & 0 & \cdot & \cdot & \cdot & u_n^2 \end{bmatrix} \qquad (2.37.b)$$

$$G = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdot & \cdot & g_{1n} \\ -g_{12} & 0 & g_{23} & \cdot & \cdot & g_{2n} \\ -g_{13} & -g_{23} & 0 & \cdot & \cdot & g_{3n} \\ \vdots & \vdots & \cdot & \cdot & \cdot & \vdots \\ -g_{1n} & -g_{2n} & \cdot & \cdot & \cdot & 0 \end{bmatrix} \qquad (2.37.c)$$

Since **D** is a physically realizable matrix, we have

$$D(s) = \det \mathbf{D} + K \{ (\det \mathbf{D}) / s \} \qquad (2.38)$$

as a **Hurwitz polynomial**, where K is nonnegative constants. Similarly, other HPs can be formed using higher-order derivatives.

As an example, let us generate a second order 1-D HP. Using Eqn.(2.36) for n = 2 gives

$$\mathbf{D} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1^2 & 0 \\ 0 & u_2^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} s + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \qquad (2.39)$$

$$= \begin{bmatrix} (u_1^2 + a^2 u_2^2) s & a u_2^2 s + g \\ a u_2^2 s - g & u_2^2 s \end{bmatrix} \qquad (2.40)$$

Taking the determinant of D in Eqn.(2.40) yields

$$\det D = u_1^2 u_2^2 s^2 + g^2 \tag{2.40}$$

To generate a 1-variable HP, we use the Eqn.(2.38) with the value of K set to two. This gives

$$D(s) = u_1^2 u_2^2 s^2 + 4u_1^2 u_2^2 s + g^2 \tag{2.41}$$

which is a second order HP. A fourth order 1-variable HP can be generated by either cascading two second order Hurwitz polynomials or directly from Eqn.(2.36) and (2.37) with n, the order of the matrices = 4.

Another method of generating 1-D Hurwitz polynomials is a modification of the first one by introducing the concept of a resistance matrix. We now define the matrix

$$\mathbf{D} = \mathbf{A}\ \mathbf{U}\ \mathbf{A}^{\mathbf{T}}\ s + \mathbf{G} + \mathbf{R}\ \boldsymbol{\Sigma}\ \mathbf{R}^{\mathbf{T}} \tag{2.42}$$

where matrices **A**, **U** and **G** are as defined in Eqn.(2.37.a-c). **R** is an upper triangular matrix with unity elements in its diagonal expressed as

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & r_{13} & \cdot & \cdot & r_{1n} \\ 0 & 1 & r_{23} & \cdot & \cdot & r_{2n} \\ 0 & 0 & 1 & \cdot & \cdot & r_{3n} \\ \vdots & \vdots & \cdot & \cdot & \cdot & \vdots \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{bmatrix} \tag{2.43}$$

while **Σ** is a diagonal matrix with non-negative elements shown as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \sigma_2^2 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \vdots & \vdots & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \sigma_n^2 \end{bmatrix} \tag{2.43}$$

It can also be shown that a 1-D HP can be generated by simply taking the determinant of D in Eqn.(2.42) and therefore avoiding the calculation of the partial derivatives.

As an example, we generate a second order HP using the Eqn.(2.42) as follows

$$D = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1^2 & 0 \\ 0 & u_2^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} s + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ r & 1 \end{bmatrix} \qquad (2.44)$$

$$= \begin{bmatrix} (u_1^2 + a^2 u_2^2)s + (\sigma_1^2 + r^2\sigma_2^2) & a u_2^2 s + r\sigma_2^2 + g \\ a u_2^2 s + r\sigma_2^2 - g & u_2^2 s + \sigma_2^2 \end{bmatrix}$$

Taking the determinant of Eqn.(2.44.a) gives

$$D(s) = u_1^2 u_2^2 s^2 + \{ \sigma_1^2 u_2^2 + \sigma_2^2 u_1^2 + [a - r]^2 u_2^2 \sigma_2^2 \}s + \{ \sigma_1^2 \sigma_2^2 + g^2 \} \qquad (2.45)$$

which is a second order 1-D HP. Higher order HPs can be generated as mentioned by either raising the order of the matrices in eqn.(2.42) or by cascading lower order HPs.

2.6.2 FORMULATION OF THE DESIGN PROBLEM

In this method using HP, two different ways of designing are considered and compared, namely, direct design and all-pass design. The fourth order transfer function of the direct design is

$$H(z) = H(s) \Big|_{s = [2(1-z^{-1})]/[T(1+z^{-1})]} \qquad (2.46)$$

$$H(s) = \left(\sum_{i=0}^{4} n(i)\, s^i\right) / \left(D_{01}(s)\, D_{02}(s)\right) \qquad (2.47)$$

where $D_1(s)$ and $D_2(s)$ are second order HP as described in Eqn.(2.45) as follows

$$D_{i1}(s) = u_{i1}^2 u_{i2}^2 s^2 + \{ \sigma_{i1}^2 u_{i2}^2 + \sigma_{i2}^2 u_{i1}^2 + [a_{i1} - r_{i1}]^2 u_{i2}^2 \sigma_{i2}^2 \}s + \{ \sigma_{i1}^2 \sigma_{i2}^2 + g_{i1}^2 \} \quad (2.48)$$

$$D_{i2}(s) = u_{i3}^2 u_{i4}^2 s^2 + \{ \sigma_{i3}^2 u_{i4}^2 + \sigma_{i4}^2 u_{i3}^2 + [a_{i2} - r_{i2}]^2 u_{i4}^2 \sigma_{i4}^2 \}s + \{ \sigma_{i3}^2 \sigma_{i4}^2 + g_{i2}^2 \} \quad (2.49)$$

where i is 0.

The transfer function of all-pass design is

$$H(z) =\{A_1(z) + (-1)^I z^M A_2(z)\} / 2 \quad (2.50)$$

$$A_1(z) = z^{-4} D_1(z^{-1}) / D_1(z) \quad (2.51)$$

$$A_2(z) = z^{-4} D_2(z^{-1}) / D_2(z) \quad (2.52)$$

$$D_1(z) = D_{11}(s) D_{12}(s) \Big|_{s = [2(1-z^{-1})]/[T(1+z^{-1})]} \quad (2.53)$$

$$D_2(z) = D_{21}(s) D_{22}(s) \Big|_{s = [2(1-z^{-1})]/[T(1+z^{-1})]} \quad (2.54)$$

where $D_{i1}(s)$ and $D_{i2}(s)$ are shown above where i is set to 1 and 2 respectively.

The value of I and M are determined according to the type of the filter as explained in chapter 2.3.

The method explained in chapter 2.4.2 is used here for the minimization of error function for magnitude characteristic, which is the least mean-square error criterion(l_2 norm).

However, constant group delay characteristic is obtained by minimizing E_τ shown below.

$$E_\tau = w\,(\tau_{max} - \tau_{min}) / (\tau_{max} + \tau_{min}) \qquad (2.55)$$

where w : weighting factor

τ_{min} : minimum group delay in passband region

τ_{max} : maximum group delay in passband region

This error function for group delay is obtained from the fact that the measure of flatness of the group delay is expressed as[31]

$$Q = [100\,(\tau_{max} - \tau_{min})] / [2\,\tau_{AV}] \qquad (2.56)$$

where

$$\tau_{AV} = (\tau_{max} + \tau_{min}) / 2 \qquad (2.57)$$

τ_{min} and τ_{max} are described above.

With this cost function for group delay characteristic, all we have to concern is the weighting factor, w by which we could control the preciseness of magnitude or group delay responses. For the conventional cost function for group delay responses using the least mean-square error criterion, many ideal group delay values should be tried even the range of them are known[35]. The cost function we explained above enables us to find the optimal function value at one step of optimization by which time efforts can be saved.

Therefore, we can summarize the designing procedure as follows.

1. A 1-D HP is generated using the Eqn.(2.41) or (2.45).

2. The discrete version of the 1-D HP is obtained by the application of the bilinear transformation and assigned to the denominator of Eqn.(2.51) and Eqn.(2.52).

3. The proper form of digital filter is obtained using Eqn.(2.50) with the consideration of the value I and M in table 2.1.

4. Generate the cost function for the magnitude response to be minimized as follows.

$$E(jw_{mT}, \Phi) = \sum_{m \in I_{ps}} E_M^2(jw_m T, \Phi) \qquad (2.58)$$

$$E_M(jw_mT,\ \Phi) = |\ H(e^{jwmT},\ \Phi)\ | - |\ H_I(e^{jwmT})\ | \qquad (2.60)$$

where $|\ H_I\ |$ is the magnitude response of the ideal filter
$|\ H\ |$ is the magnitude response of the desired filter
Φ is the coefficient vectors

5. Use proper non-linear optimization technique for the evaluation of coefficients vector Φ so that E in eqn.(2.58) is minimized.

6. If the linear phase characteristic is required, generate the cost function for the magnitude and group delay responses as follows.

$$E(jw_mT,\ \Phi) = \sum_{m \in I_{ps}} E_M^2(jw_mT,\ \Phi) + E_\tau(jw_mT,\ \Phi) \qquad (2.59)$$

where I_{ps} is the set of discrete frequency points in the passband and stopband region.

$$E_\tau = w\ (\tau_{max} - \tau_{min}) / (\tau_{max} + \tau_{min}) \qquad (2.61)$$

where w : weighting factor
τ_{min} : minimum group delay in passband region
τ_{max} : maximum group delay in passband region

$$\tau(w) = -d\theta(w)/dw = -jz\ d\theta/dz|_{z=\exp(jwT)} \qquad (2.62)$$

$$= -\ \mathrm{Re}\{\ z\ (dH(z)/dz)\ /H(z)\ \}|_{z\,=\,\exp(jwT)} \qquad (2.63)$$

$$= -\ \mathrm{Re}\{\ z\ d[\ \ln H(z)\ ]\ /\ dz\ \}|_{z\,=\,\exp(jwT)} \qquad (2.64)$$

$$= \mathrm{Re}\{\ z\ [D'(z)/D(z) - N'(z)/N(z)]\}|_{z\,=\,\exp(jwT)} \qquad (2.65)$$

where $H(z) = N(z)/D(z)$
$N'(z) = dN(z)/dz$ and $D'(z) = dD(z)/dz$

7. Now utilize any suitable non-linear optimization technique for calculation of vector Φ so that E in Eqn.(2.59) is

minimized.

MINOS[24,29] which is composed of Quasi-Newton method and conjugate gradient method is used for minimization of the magnitude error function expressed in Eqn.(2.58). For the design which needs constant group delay characteristic (Eqn.(2.59)), the Hooke and Jeeve method[65] is used with the initial parameter values obtained by MINOS. MINOS requires the computation of the gradients. If μ is a parameter in $H_D(z)$, then by differentiating E in Eqn. (2.58) and simplifying we get

$$E(jwT, \Phi) / \mu = \sum_{m=1}^{M} \{ Re[H_D(jw_mT, \Phi)] Re[H_D'(jw_mT, \Phi)] + Im[H_D(jw_mT, \Phi)] Im[H_D'(jw_mT, \Phi)] \} E(jw_mT, \Phi) / | H_D(jw_mT, \Phi) | \quad (2.66)$$

where

$$H_D'(jw_mT, \Phi) = dH_D(jw_mT, \Phi) / d\mu \quad (2.67)$$

and M is the number of frequency points for evaluation. Programs have been written to carry out this optimization. The program converges reasonably fast. Some results of design based on this program are given in the next section.

2.6.3 EXAMPLES

To illustrate the usefulness of the proposed method, several examples of recursive digital filters satisfying prescribed magnitude responses with or without constant group delay characteristics are given. This includes lowpass, highpass bandpass and bandstop filters. The order of the filters are four and the denominators of the filters are obtained by cascading two second order Hurwitz polynomials illustrated in Eqn.(2.48) and Eqn.(2.49) to ensure the stability. The weighting factor w in Eqn.(2.61) is set to one for the design where the linear phase characteristics are required. The specifications of the filters are shown below.

For lowpass filter;

$$|H_I(w_i)| = \begin{cases} 1 & \text{for} \quad 0 \le w \le 0.7 \\ 0 & \text{for} \quad 1.4 \le w \le w_s/2. \end{cases}$$

For highpass filter;

$$|H_I(w_i)| = \begin{cases} 1 & \text{for} \quad 1.4 \le w \le w_s/2 \\ 0 & \text{for} \quad 0 \le w \le 0.7. \end{cases}$$

where $w_s = 2\pi$ rad/sec and $T = 1$ sec.

For bandpass filter;

$$|H_I(w_i)| = \begin{cases} 0 & \text{for} \quad 0 \le w \le 1 \text{ rad/sec} \\ 1 & \text{for} \quad 2 \le w \le 3 \text{ rad/sec} \\ 0 & \text{for} \quad 4 \le w \le w_s/2 \end{cases}$$

For bandstop filter;

$$|H_I(w_i)| = \begin{cases} 1 & \text{for} \quad 0 \le w \le 1 \text{ rad/sec} \\ 0 & \text{for} \quad 2 \le w \le 3 \text{ rad/sec} \\ 1 & \text{for} \quad 4 \le w \le w_s/2 \end{cases}$$

where $w_s = 10$ rad/sec and $T = \pi/5$ sec.

For lowpass and bandpass filters, the results are compared with the direct method filter design using general 1-D IIR transfer function expressed in Eqn.(2.46). The design using direct design for the purpose of comparison is from [64] which proved the usefulness of the direct design using HP over the conventional design with Elliptic filter cascaded with all-pass filter.

Table 2.8 and 2.9 shows the coefficients of the lowpass filter transfer function using all-pass method with or without the constant group delay characteristics respectively. Table 2.10 and table 2.15 contains the coefficients of the lowpass and bandpass filter transfer function using direct method with linear phase respectively. By the same way, table 2.11 and 2.12, table 2.13 and 2.14 and table 2.16 and 2.17 show the highpass, bandpass and bandstop filter with or without the constant group delay characteristics respectively. Figure 2.11 through 2.18 a,b are the frequency responses of the filters. In the plots with comparison, the solid line with the character 'A' illustrates the responses of the filter using all-pass method and the dotted line with 'D' illustrates those of the flter using the direct method which are explained above.

u_{11} = 1.47028	u_{13} = 26.38531
u_{12} = 3.89266	u_{14} = 1.75729
σ_{11} = 3.27065	σ_{13} = 11.74771
σ_{12} = 41.12621	σ_{14} = 0.19632
a_{11} = 4.67212	a_{12} = 0.40833
r_{11} = 1.98954	r_{12} = 3.70914
g_{11} = 8.84125	g_{12} = 18.26448
u_{21} = 0.39441	u_{23} = 14.76238
u_{22} = 4.43515	u_{24} = 11.09443
σ_{21} = 11.37613	σ_{23} = 12.22905
σ_{22} = -5.37913	σ_{24} = 4.76599
a_{21} = 2.72014	a_{22} = 8.36952
r_{21} = 8.55726	r_{22} = 9.79558
g_{21} = 29.02797	g_{22} = 38.76263

Table 2.8 coefficients for lowpass filter using HP without linear phase

u_{11} = 1.96350	u_{13} = 24.07001
u_{12} = 3.93000	u_{14} = 2.00000
σ_{11} = 4.07200	σ_{13} = 9.93000
σ_{12} = 27.73375	σ_{14} = -1.00000
a_{11} = 4.00000	a_{12} = 2.94500
r_{11} = 2.00000	r_{12} = 1.61499
g_{11} = 43.94450	g_{12} = 20.07100
u_{23} = 3.88599	u_{23} = 14.01749
u_{24} = 2.93000	u_{24} = 11.07200
σ_{21} = 11.92600	σ_{23} = 12.00000
σ_{22} =-20.81250	σ_{24} = 5.00000
a_{21} = 4.00000	a_{22} = 8.00000
r_{21} = 9.00000	r_{22} = 9.00000
g_{21} = 81.22599	g_{22} = 46.92999

Table 2.9 Coefficients for lowpass filter using HP with linear phase

n_0 = 2.114	u_{01} = 1.717	u_{03} = 0.9516
n_1 = -0.1062	u_{02} = 1.123	u_{04} = 1.131
n_2 = -1.706	σ_{01} = 0.9953	σ_{03} = 0.7125
n_3 = -0.2938	σ_{02} = 0.9344	σ_{04} = 0.6969
n_4 = -0.1891	a_{01} = 1.194	a_{02} = 1.225
	r_{01} = 0.8219	r_{02} = 0.9250
	g_{01} = 0.7688	g_{02} = 1.081

Table 2.10 Coefficients for lowpass filter with linear phase using direct design

u_{11} = 1.18236	u_{13} = 26.21999
u_{12} = 3.33656	u_{14} = 1.07605
σ_{11} = 3.93642	σ_{13} = 11.93851
σ_{12} = 41.13332	σ_{14} = 0.18339
a_{11} = 3.63917	a_{12} = 0.41248
r_{11} = 3.02249	r_{12} = 3.70499
g_{11} = 8.84235	g_{12} = 18.38373
u_{21} = 0.79257	u_{23} = 10.65307
u_{22} = 2.04794	u_{24} = 5.02551
σ_{21} = 11.92353	σ_{23} = 10.29856
σ_{22} = -5.32574	σ_{24} = -0.30300
a_{21} = 4.24272	a_{22} = 10.03650
r_{21} = 7.03468	r_{22} = 8.12860
g_{21} = 29.08598	g_{22} = 41.05048

Table 2.11 Coefficients for highpass filter using HP without linear phase

u_{11} = 0.00067	u_{13} = 27.12123
u_{12} = 2.94281	u_{14} = 1.46105
σ_{11} = 4.32342	σ_{13} = 10.93226
σ_{12} = 40.60526	σ_{14} = -0.05386
a_{11} = 4.00017	a_{12} = 0.76685
r_{11} = 2.81249	r_{12} = 3.31518
g_{11} = 9.53148	g_{12} = 19.10260

u_{21} = -0.00067	u_{23} = 10.16307
u_{22} = 2.60794	u_{24} = 5.24151
σ_{21} = 11.97903	σ_{23} = 10.99081
σ_{22} = -5.60574	σ_{24} = 0.03425
a_{21} = 3.96272	a_{22} = 10.75838
r_{21} = 7.41968	r_{22} = 7.49498
g_{21} = 29.42361	g_{22} = 41.19922

Table 2.12 Coefficients for highpass filter using HP with linear phase

u_{11} = 5.45017	u_{13} = 27.77431
u_{12} = 21.13387	u_{14} = 0.99128
σ_{11} = 9.27320	σ_{13} = 9.14055
σ_{12} = 35.96005	σ_{14} = 0.45904
a_{11} = 3.32999	a_{12} = 0.48235
r_{11} = 3.32999	r_{12} = 3.62765
g_{11} = 8.84417	g_{12} = 17.39709
u_{21} = -6.27417	u_{23} = 13.08469
u_{22} = 4.92103	u_{24} = 2.02689
σ_{21} = 12.42151	σ_{23} = 6.49338
σ_{22} = -4.56132	σ_{24} = 0.80434
a_{21} = 5.09685	a_{22} = 3.48518
r_{21} = 6.17315	r_{22} = 14.67682
g_{21} = 29.25192	g_{22} = 41.769287

Table 2.13 Coefficients for bandpass filter using HP without linear phase

u_{11} = 4.34300	u_{13} = 28.73500
u_{12} = 21.89500	u_{14} = 1.00000
σ_{11} = 9.01000	σ_{13} = 16.80749
σ_{12} = 36.73950	σ_{14} = 0.29200
a_{11} = 3.01900	a_{12} = 1.67375
r_{11} = 3.00000	r_{12} = 2.45138
g_{11} = 74.94376	g_{12} = 16.03551
u_{21} = -6.56300	u_{23} = 12.31500
u_{22} = 5.50750	u_{24} = 1.85600
σ_{21} = 14.14375	σ_{23} = 7.41425
σ_{22} = -4.94750	σ_{24} = 1.00000
a_{21} = 7.24499	a_{22} = 3.00000
r_{21} = 6.00000	r_{22} = 14.00000
g_{21} = 19.60225	g_{22} = 40.59250

Table 2.14 Coefficients for bandpass filter using HP with linear phase

n_0 = 6.295	u_{01} = 0.6954	u_{03} = 3.620
n_1 = 3.570	u_{02} = 1.347	u_{04} = 0.2748
n_2 = 18.07	σ_{01} = 1.441	σ_{03} = 0.03785
n_3 = 0.3827	σ_{02} = 0.5336	σ_{04} = -0.4346
n_4 = 0.06109	a_{01} = 0.9045	a_{02} = 0.7771
	r_{01} = -0.2047	r_{02} = 1.299
	g_{01} = -0.4215	g_{02} = -2.196

Table 2.15 Coefficients for bandpass filter using HP with linear phase using direct design

u_{11} = 3.81093	u_{13} = 27.88622
u_{12} = 15.49588	u_{14} = 1.61605
σ_{11} = 9.43035	σ_{13} = 9.12750
σ_{12} = 38.34715	σ_{14} = 0.60453
a_{11} = 3.33083	a_{12} = 0.45093
r_{11} = 3.33083	r_{12} = 3.66654
g_{11} = 8.84554	g_{12} = 17.23302
u_{21} = -5.63975	u_{23} = 9.34464
u_{22} = 5.27174	u_{24} = 1.75503
σ_{21} = 12.60878	σ_{23} = 9.99789
σ_{22} = -4.55507	σ_{24} = -0.42986
a_{21} = 3.78703	a_{22} = 3.41296
r_{21} = 7.49037	r_{22} = 14.75214
g_{21} = 29.22672	g_{22} = 41.79152

Table 2.16 Coefficients for bandstop filter using HP without linear phase

u_{11} = 7.13927	u_{13} = 26.70387
u_{12} = 19.02568	u_{14} = 1.70355
σ_{11} = 6.99135	σ_{13} = 11.51516
σ_{12} = 35.04593	σ_{14} = 0.73228
a_{11} = 3.63483	a_{12} = -1.06118
r_{11} = 3.82083	r_{12} = 5.68057
g_{11} = 11.93870	g_{12} = 15.23614

$u_{21} = -7.67275$	$u_{23} = 10.12776$
$u_{22} = 7.30256$	$u_{24} = 1.93503$
$\sigma_{21} = 11.45214$	$\sigma_{23} = 11.60094$
$\sigma_{22} = -2.43921$	$\sigma_{24} = -0.88586$
$a_{21} = 4.91141$	$a_{22} = 5.34589$
$r_{21} = 6.75537$	$r_{22} = 13.49414$
$g_{21} = 27.26541$	$g_{22} = 41.72433$

Table 2.17 Coefficients for bandstop filter using HP with linear phase

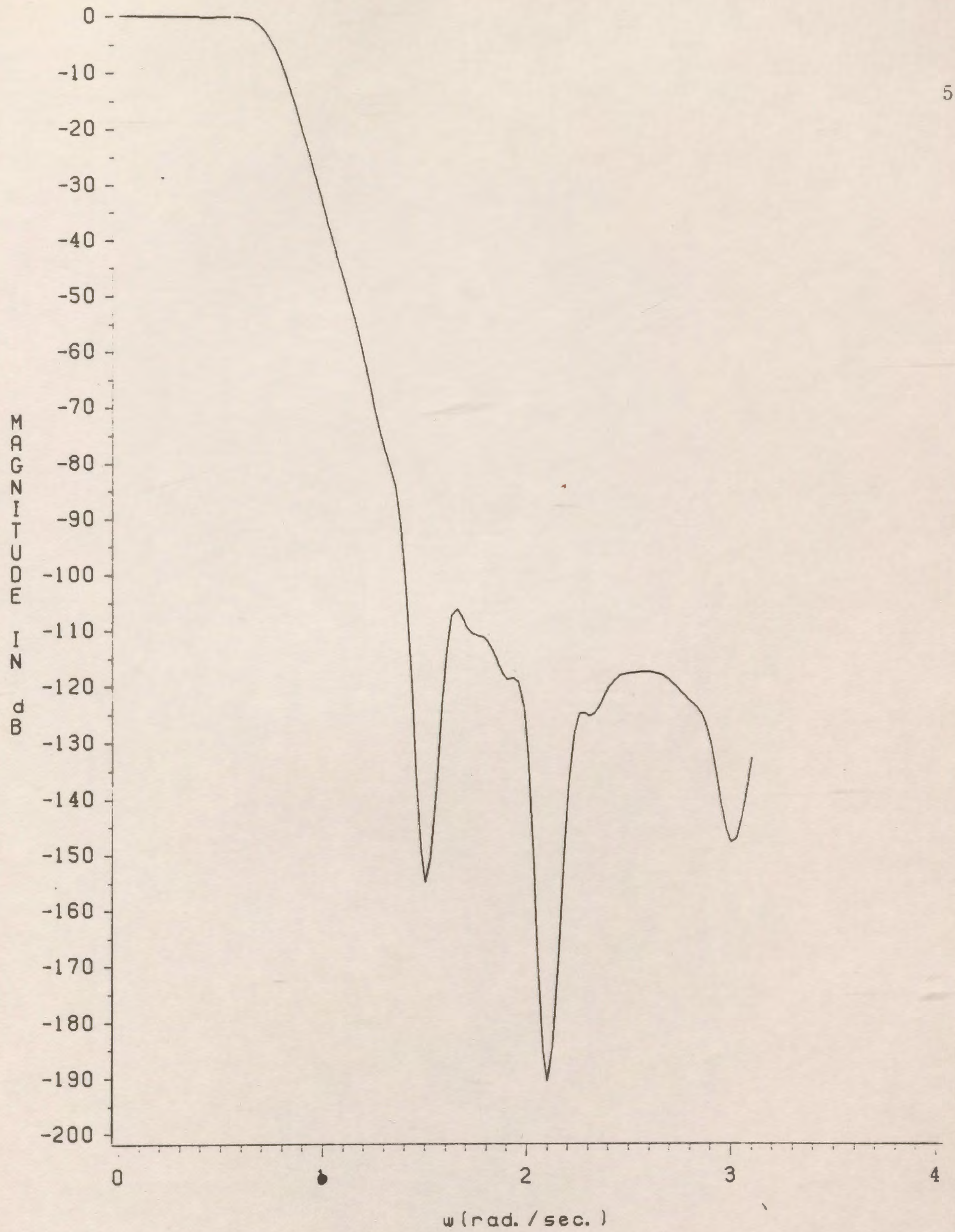


Figure 2.11 Magnitude response of the lowpass filter using HP without linear phase

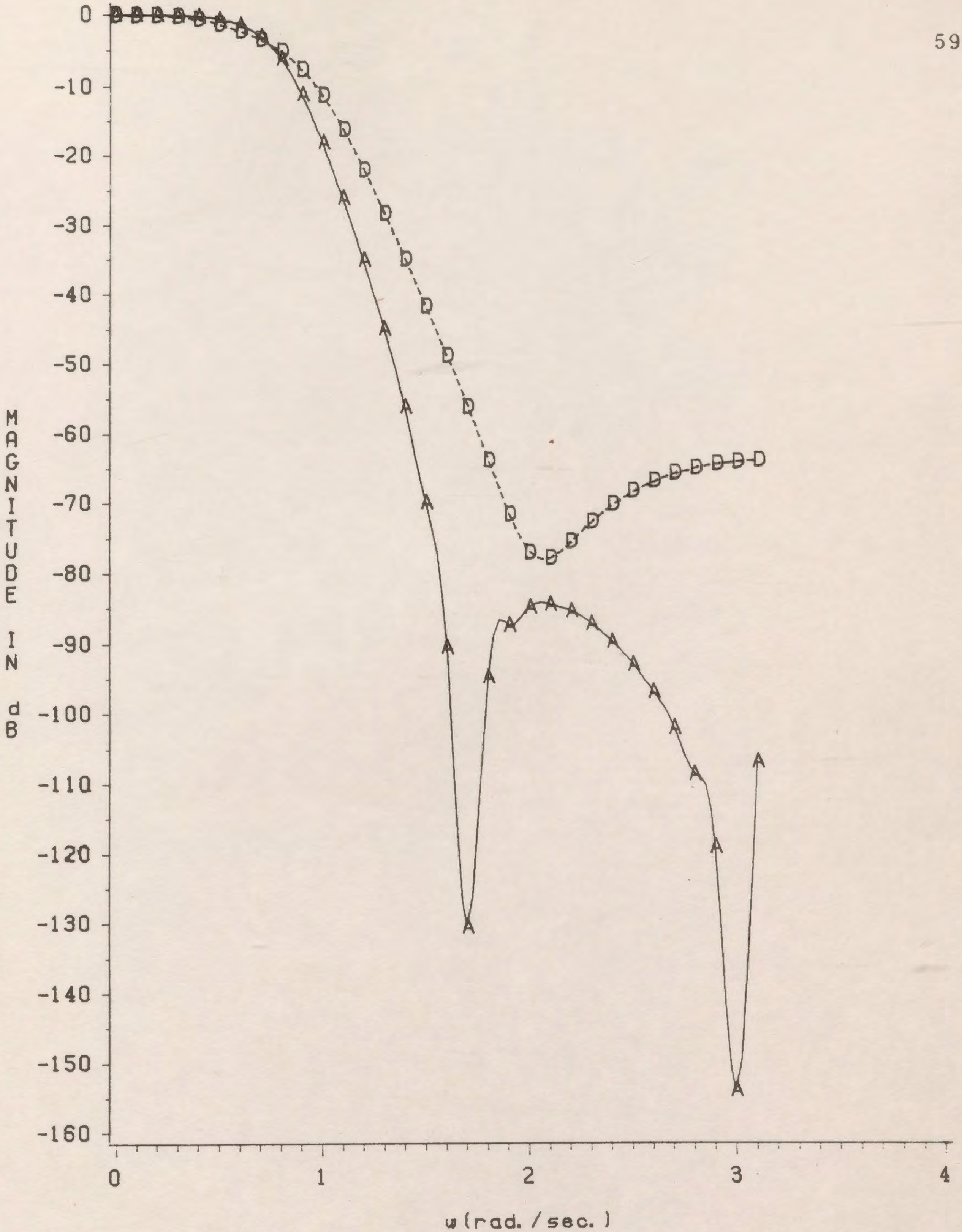


Figure 2.12 a Magnitude response of the lowpass filter using HP with linear phase compared with direct method

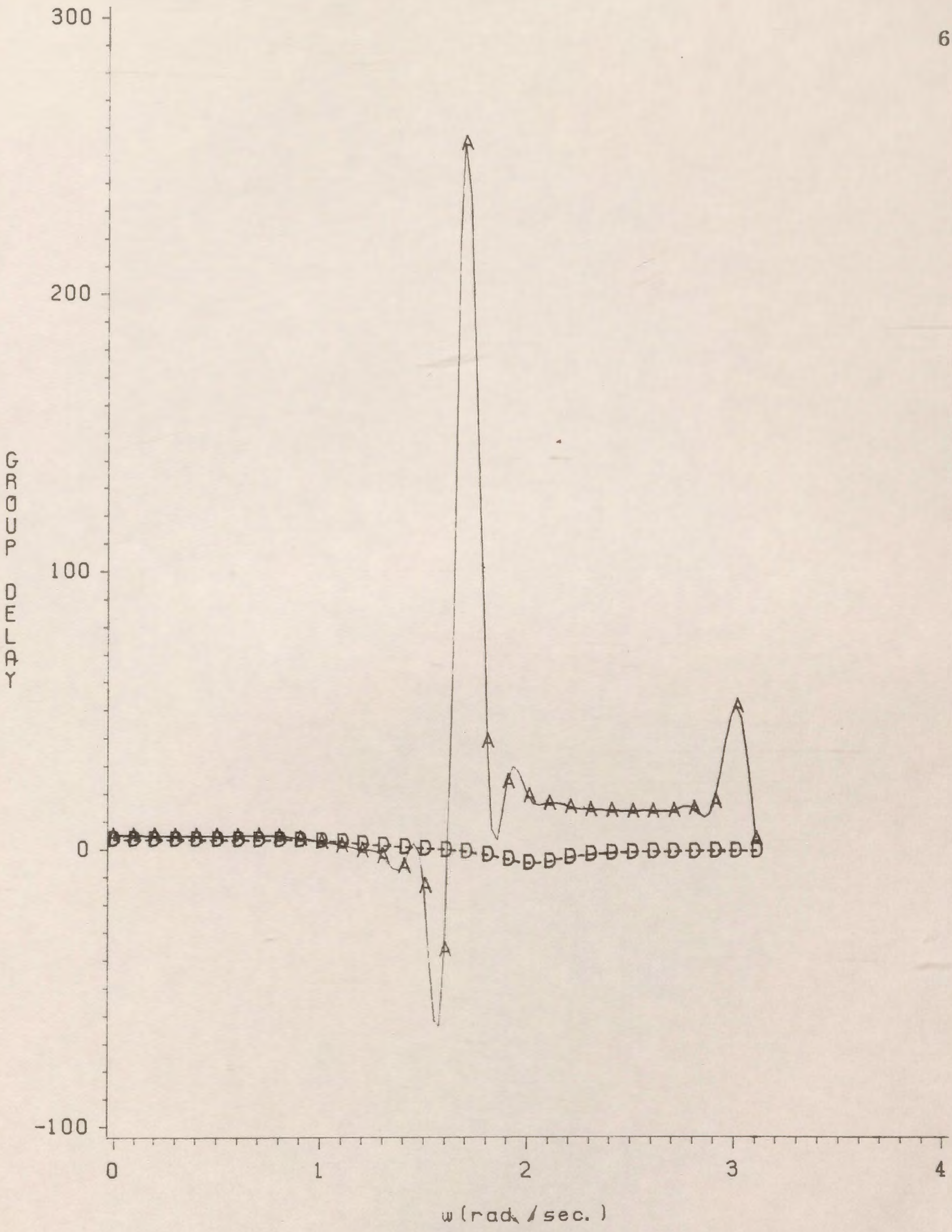


Figure 2.12 b Group delay response of the lowpass filter using **HP** with linear phase compared with direct method

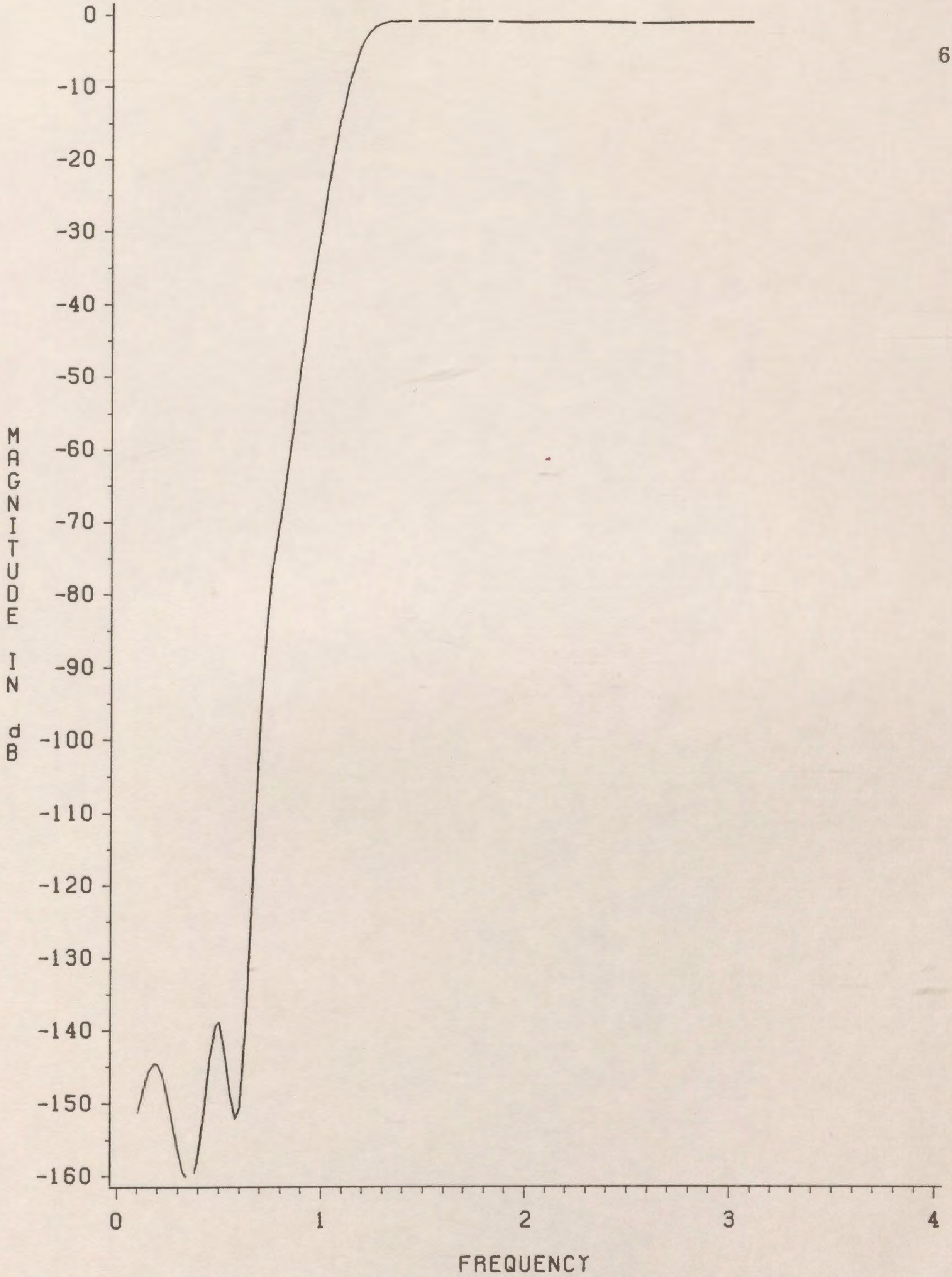


Figure 2.13 Magnitude response of the highpass filter using MP without linear phase

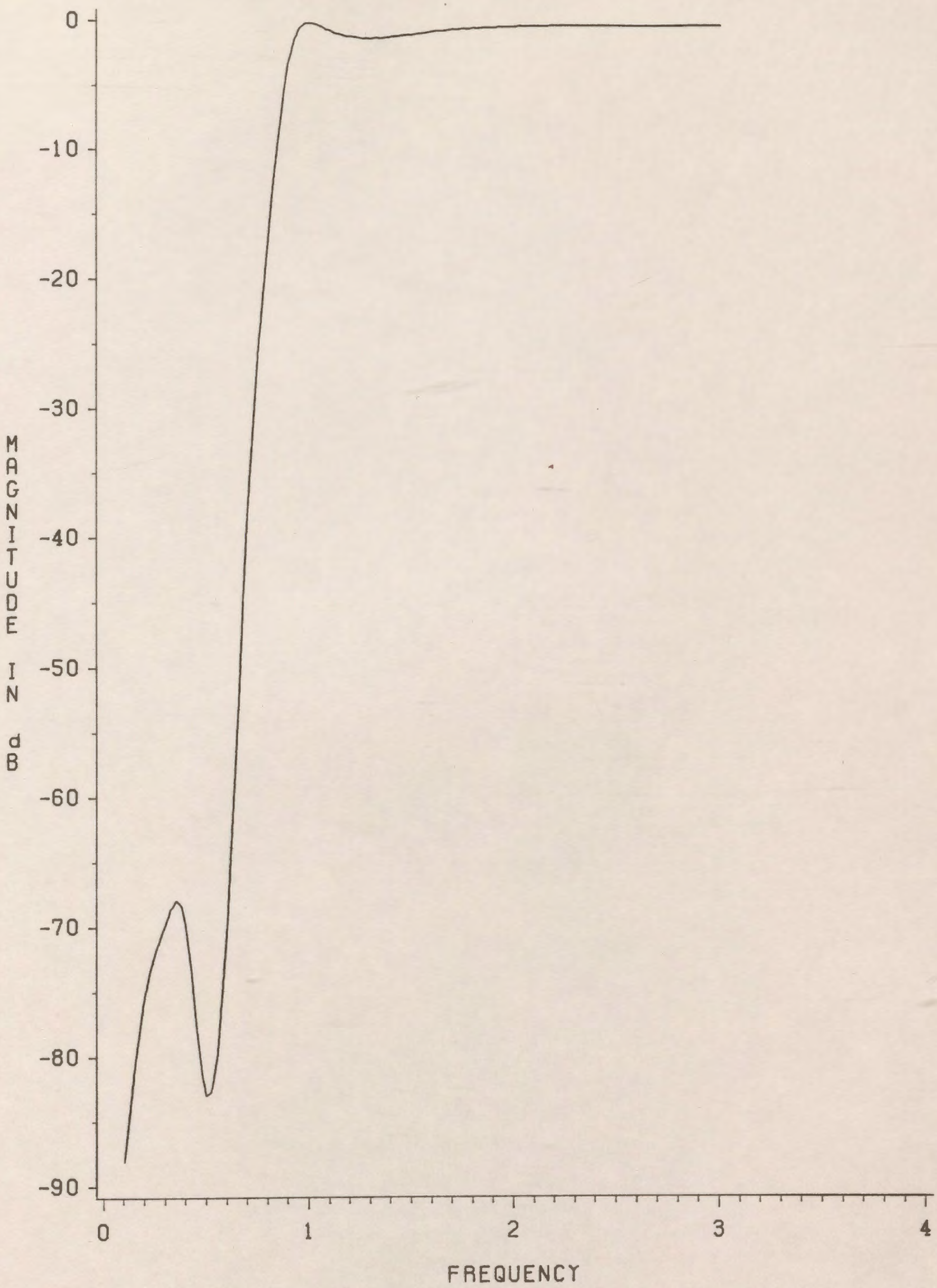


Figure 2.14 a Magnitude response of the highpass filter using HP with linear phase

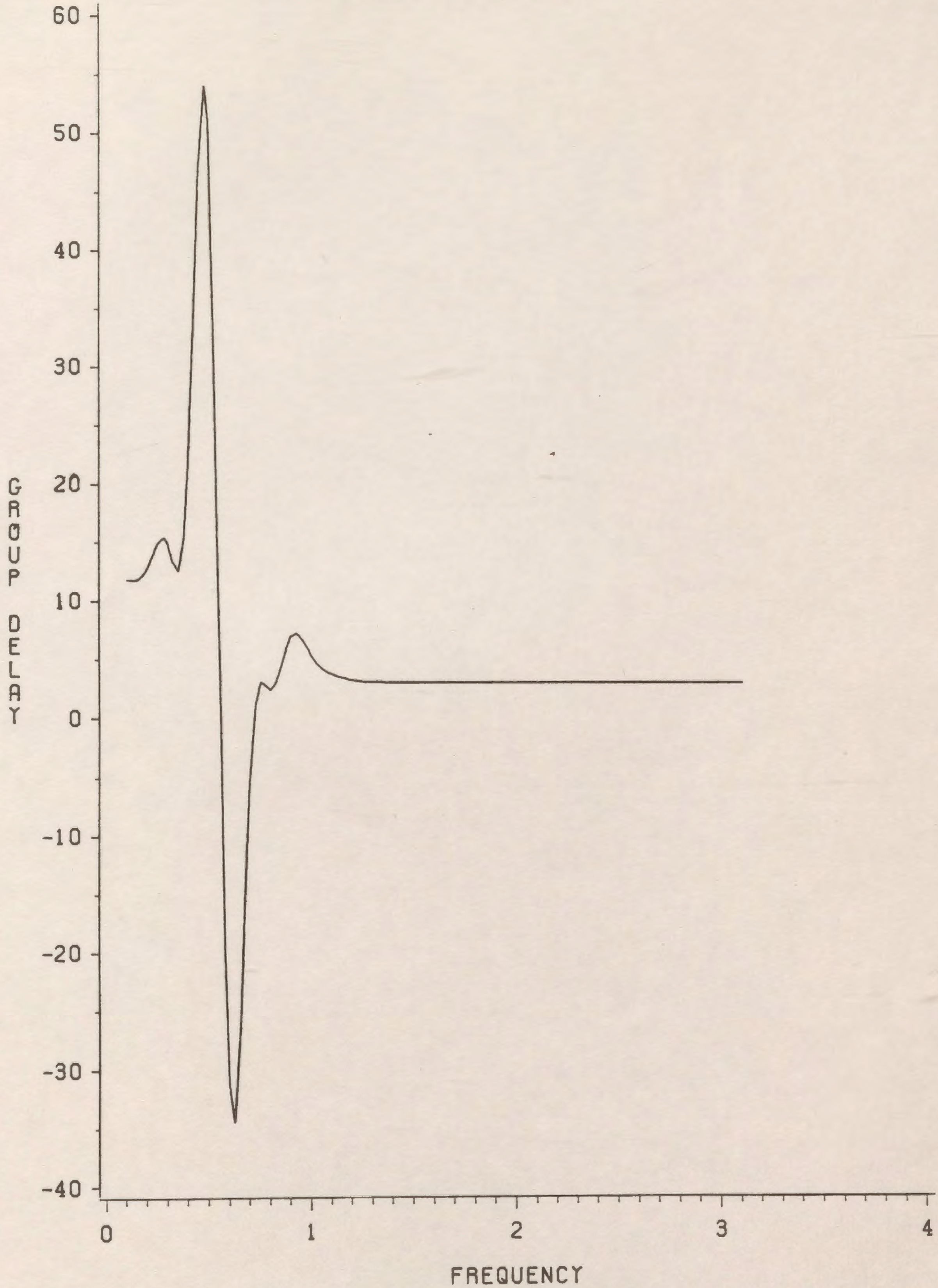


Figure 2.14 b Group delay response of the highpass filter using HP with linear phase

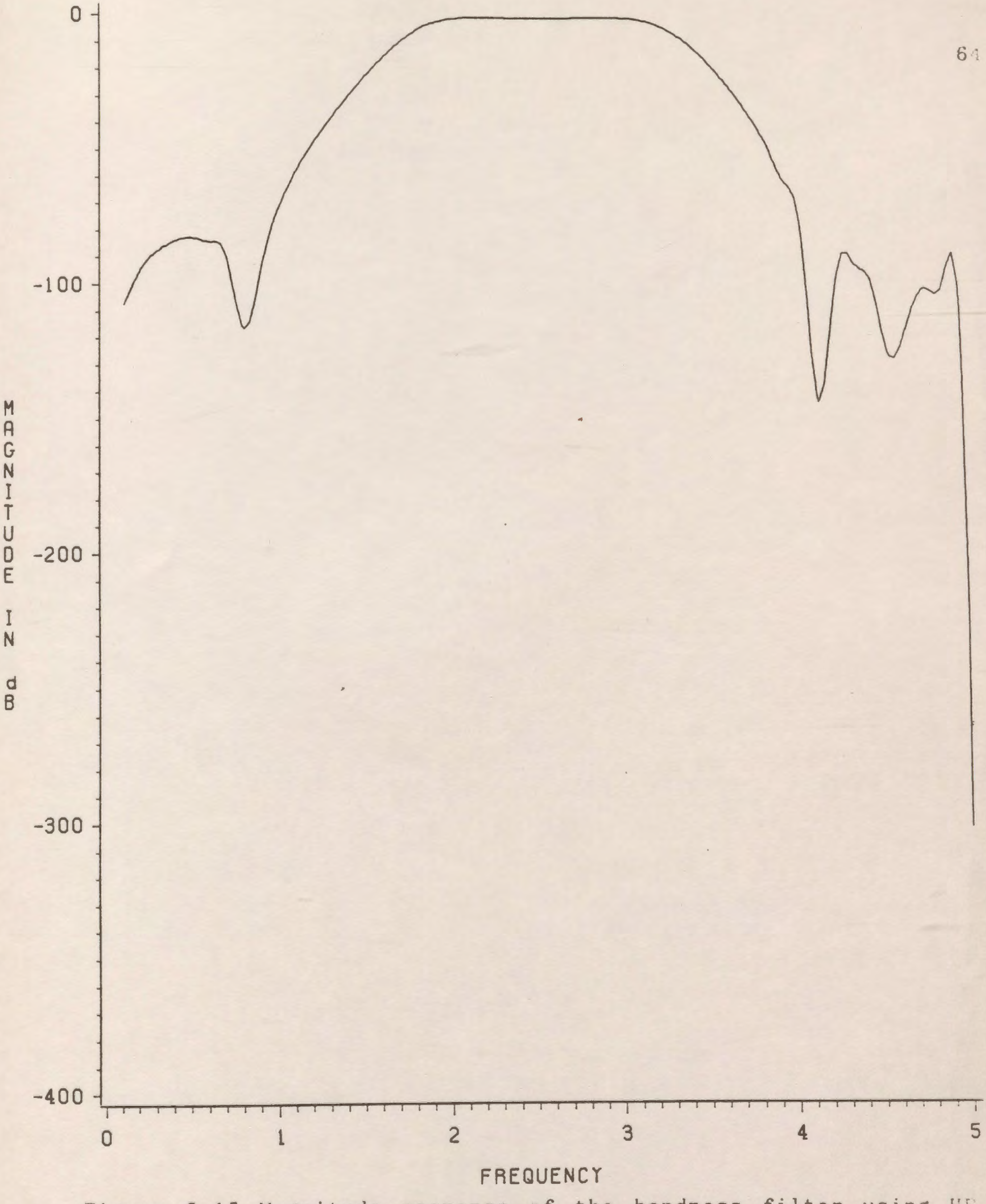


Figure 2.15 Magnitude response of the bandpass filter using HP without linear phase

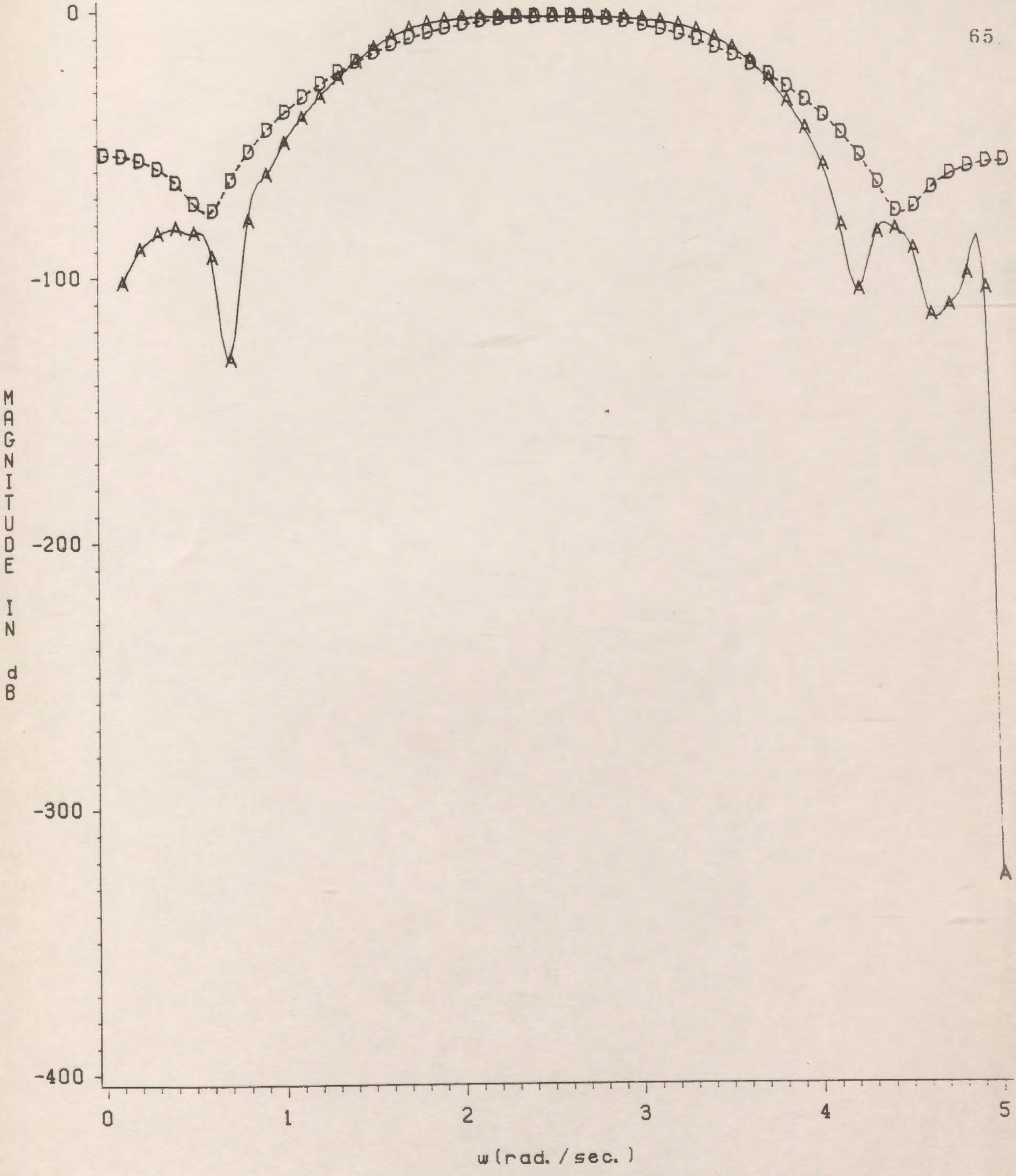


Figure 2.16 a Magnitude response of the bandpass filter using HF with linear phase compared with direct method

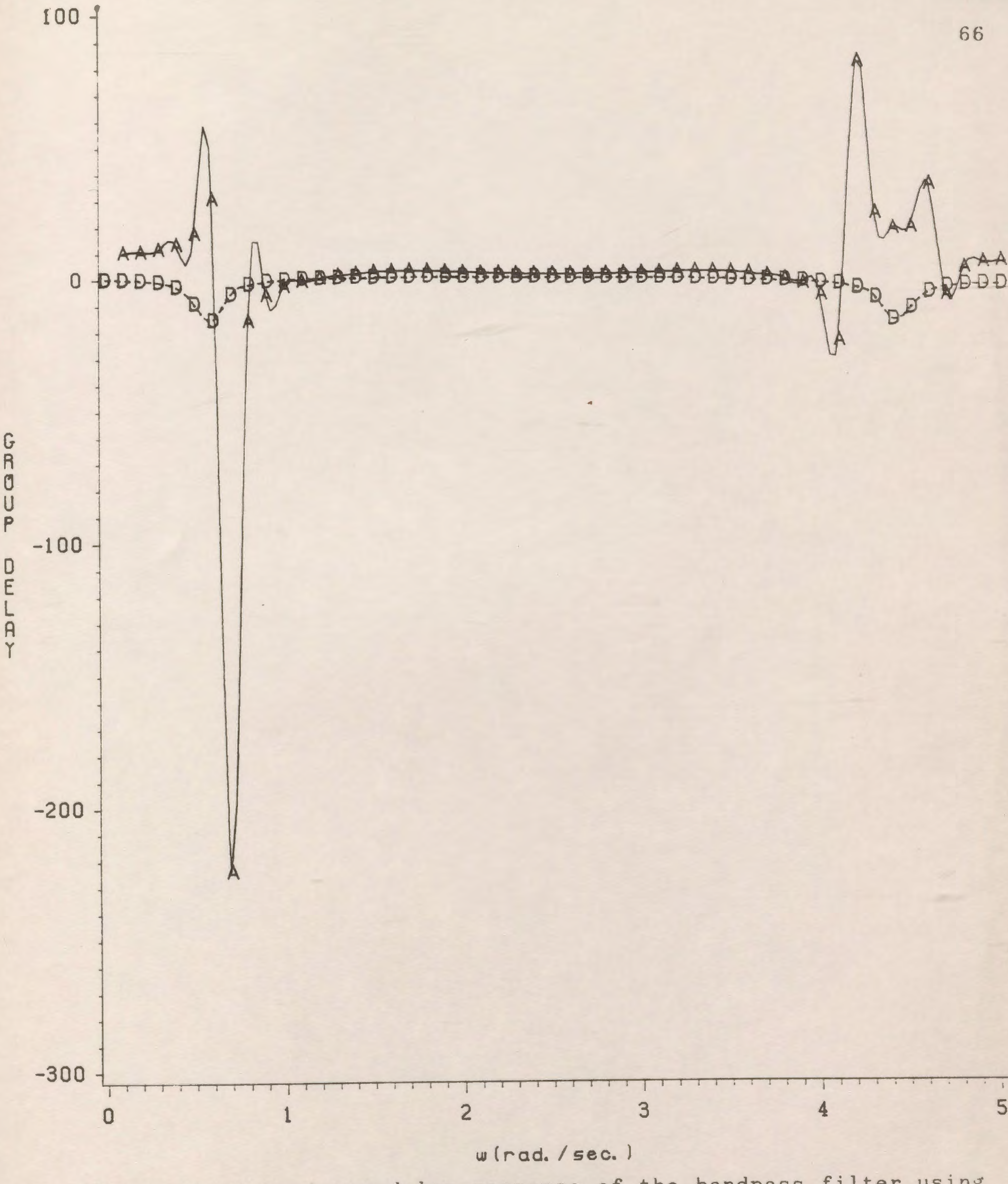


Figure 2.16 b Group delay response of the bandpass filter using HP with linear phase compared with direct method

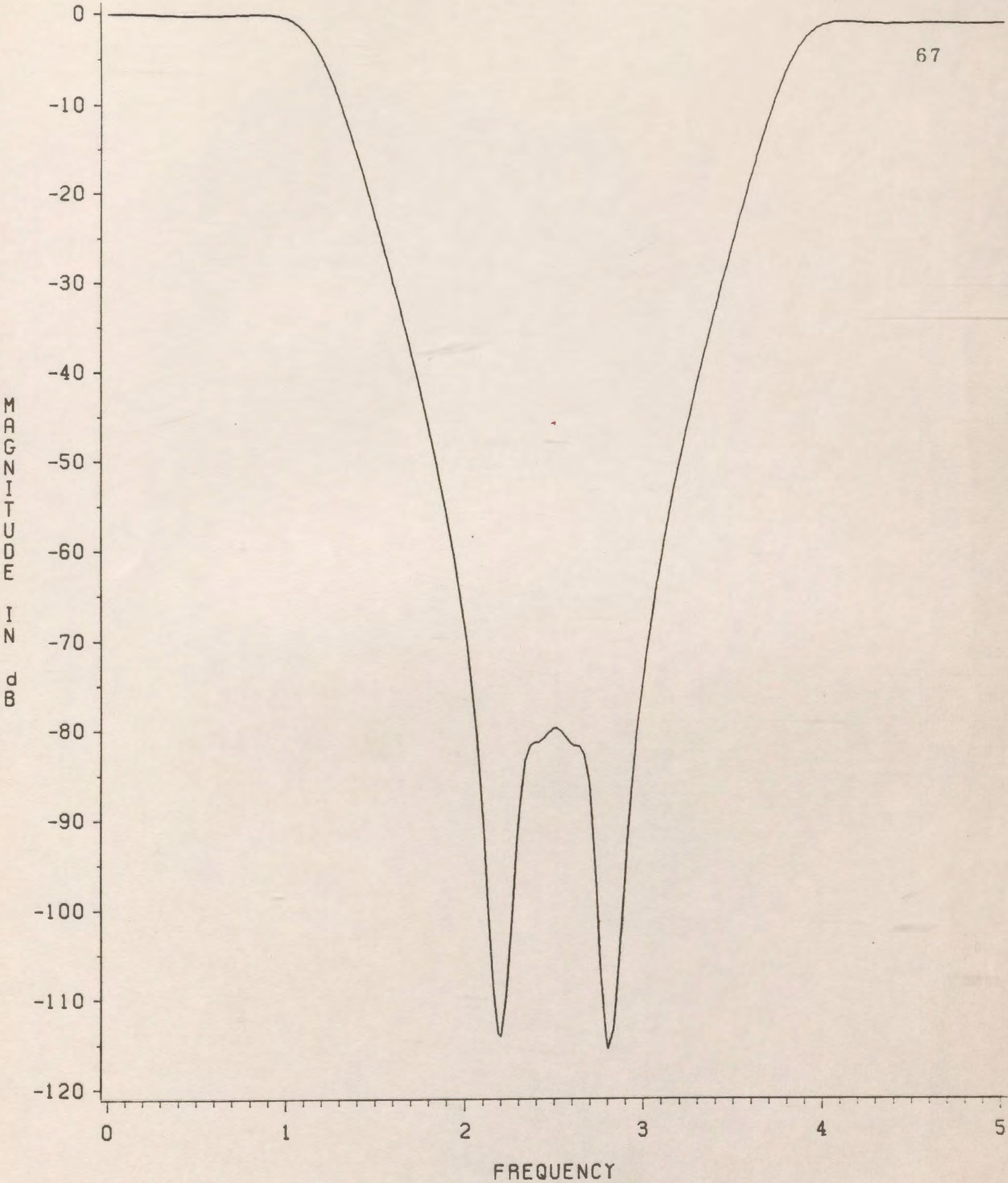


Figure 2.17 Magnitude response of the bandstop filter using HP without linear phase

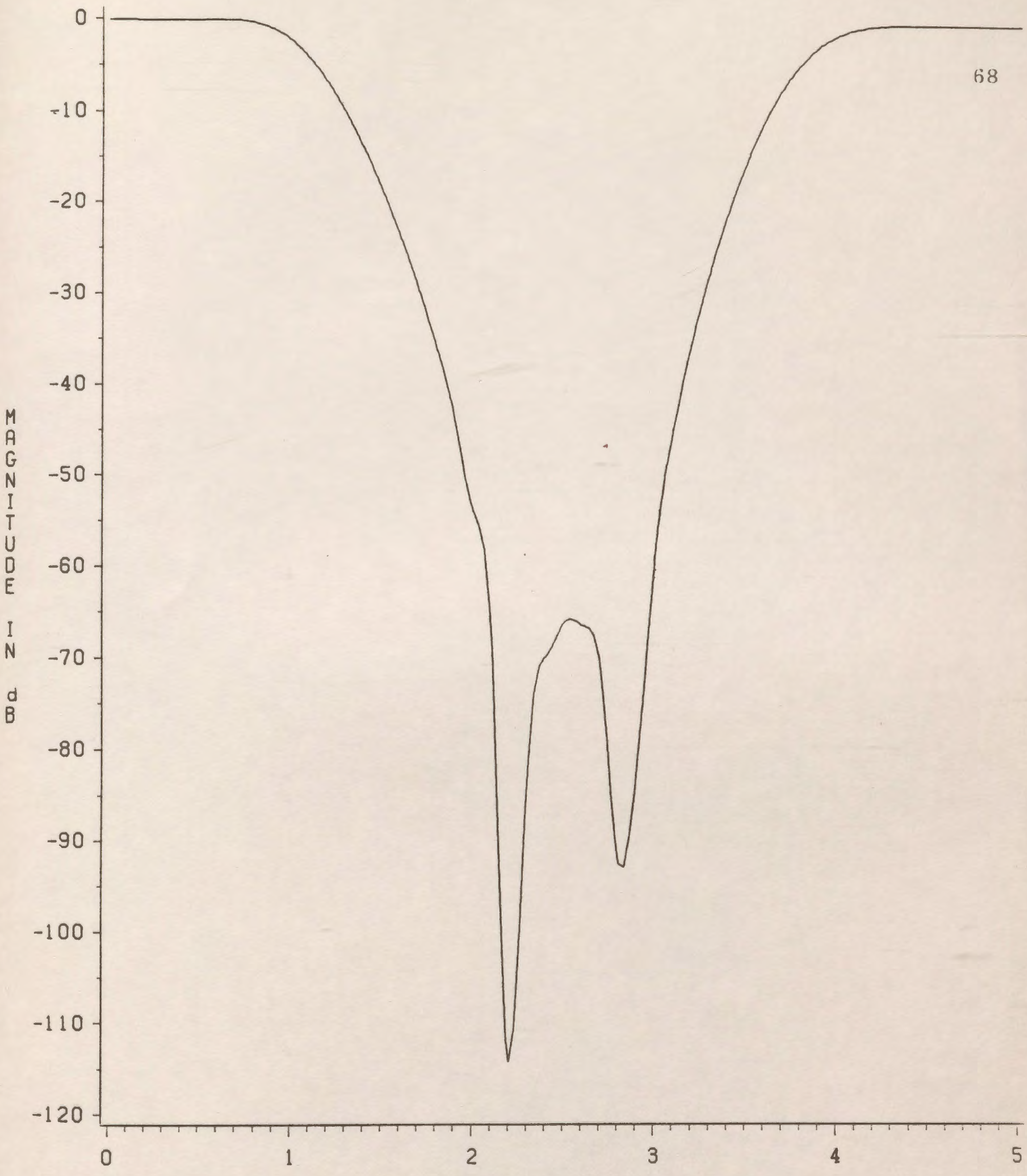


Figure 2.18 a Magnitude response of the bandstop filter using H₂ with linear phase

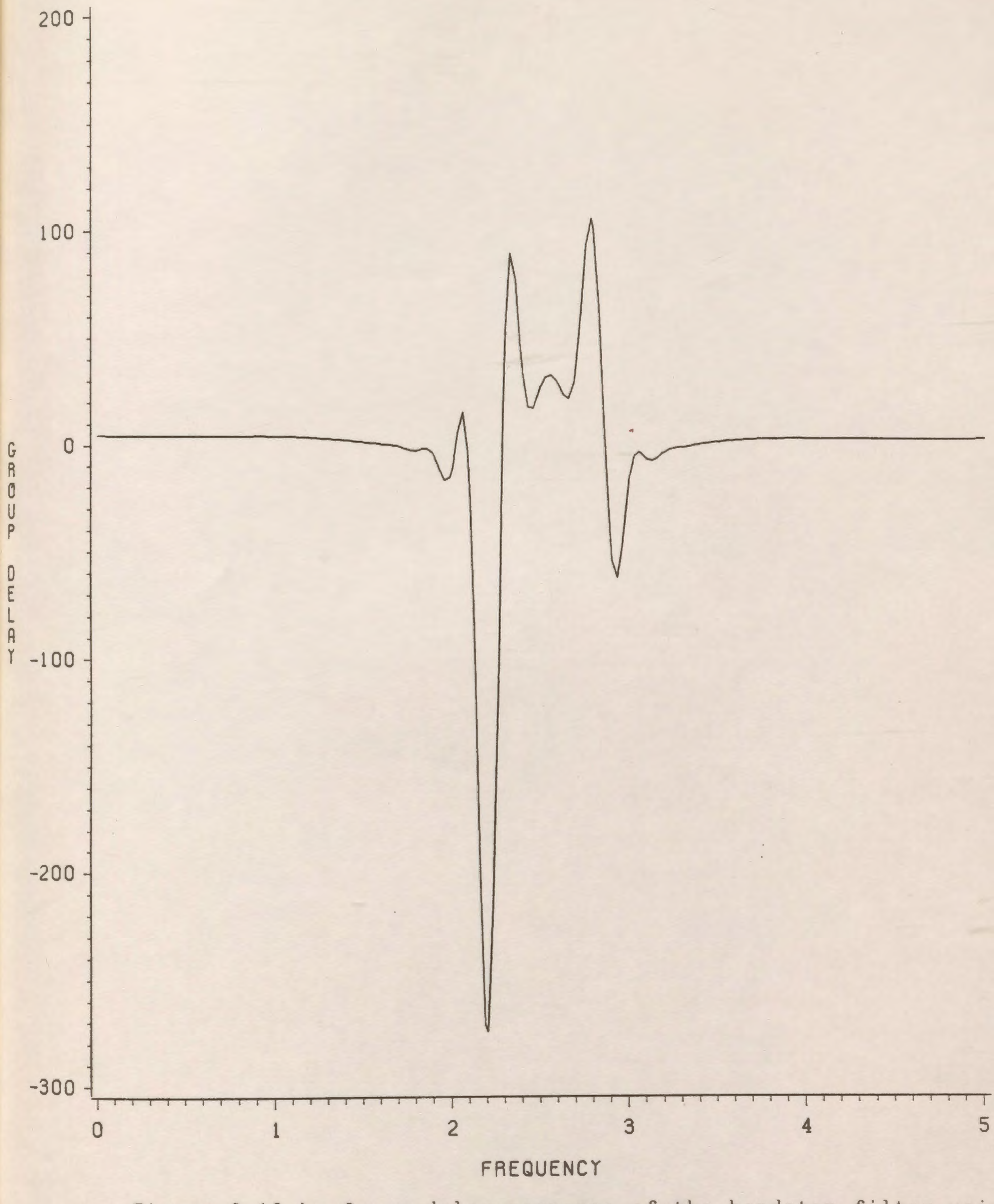


Figure 2.18 b Group delay response of the bandstop filter using HP with linear phase

2.6.4 COMPARISONS

Table 2.18 presents a relative comparison of the errors obtained in the passband and stop band between direct and all-pass method. This includes the magnitude error as well as group-delay error. A careful scrutiny of Table 2.18 clearly reveals that the all-pass method outperforms the other greatly for the magnitude response while direct method is relatively superior to the all-pass method for the group delay response. For example, in the lowpass filter design the stopband magnitude error is approximately 15 times smaller using the all-pass method than using the direct method and the group delay error is approximately 1.5 times bigger. It must be also noticed that the all-pass method is computationally more efficient than the direct method. It needs only 8 multiplications while the direct method meeds 9 multiplications. As explained in chapter 2.4, the complementary filter, which can be obtained automatically, is another merit over the direct design method.

type \ error		passband mag.	stopband mag.	group delay
LP	Direct Method	$0.901 \; 10^{-2}$	$0.809 \; 10^{-1}$	$0.140 \; 10^{-1}$
	All-Pass Meth.	$0.508 \; 10^{-2}$	$0.626 \; 10^{-2}$	$0.175 \; 10^{-1}$
	All-Pass Meth. (Mag. only)	$0.419 \; 10^{-4}$	$0.141 \; 10^{-3}$	
HP	All-Pass Meth.	$0.374 \; 10^{-2}$	$0.482 \; 10^{-2}$	$0.705 \; 10^{-2}$
	All-Pass Meth. (Mag. only)	$0.517 \; 10^{-6}$	$0.234 \; 10^{-5}$	
BP	Direct Method	$0.336 \; 10^{-1}$	0.135	$0.100 \; 10^{-1}$
	All-Pass Meth.	$0.121 \; 10^{-1}$	$0.158 \; 10^{-1}$	$0.746 \; 10^{-1}$
	All-Pass Meth. (Mag. only)	$0.147 \; 10^{-2}$	0.270 10-2	
BS	All-Pass Meth.	$0.173 \; 10^{-1}$	$0.138 \; 10^{-1}$	$0.332 \; 10^{-1}$
	All-Pass Meth. (Mag. only)	$0.146 \; 10^{-2}$	$0.321 \; 10^{-2}$	

Table 2.18

Chapter III
DESIGN OF 1-D DIGITAL FILTERS WITH INTEGER COEFFICIENTS

3.1 INTRODUCTION

Digital signal processing algorithms are realized either with special-purpose digital hardware or as programs for a general-purpose digital computer. In both cases the filter coefficients can only be expressed using a finite number of bits. As we have seen from the previous section, the digital filter is characterized by a set of real numbers, namely its coefficients. Obviously, altering these numbers will alter the characteristics of the filter. By quantizing the coefficients we actually obtain a different filter from the one we have originally designed. In fact, the quantized filter may fail to meet specifications even though the unquantized filter does. In the floating point arithmetics, one simple way to analyze the performance of a filter is to consider rounding the mantissa of the numbers. But, it would be preferable to realize the filter with its designed finite wordlength coefficients rather than just truncating or rounding the infinite wordlength coefficients and use them in the implementation.

3.2 ALGORITHM FOR INTEGER PROGRAMMING

To reduce the hardware cost and complexity of implementing a filter, it is desirable to have filters with integer coefficients. Of the many integer optimization techniques which exist the technique based on the discretization and reoptimization techniques[40] is chosen to minimize the performance error caused by truncating the coefficients because the cost functions which are highly non-linear do not lend themselves to linear integer programming routine and the tree search method is lengthy and inefficient. The proposed program is simple, easy to be manipulated, and gives good results although it is suboptimal in nature. It is an improved form of an algorithm proposed by Y.Wan and M.M.Fahmy[61,63]. The algorithm starts by varying the first most sensitive element of the parameter vector in a direction such that to decrease the performance index. This sensitivity is measured by the following formula.

$$Sn(i) = \left| \frac{\{H(x_1,x_2,\ldots,x_i,\ldots,x_N) - H(x_1,x_2,\ldots,x_i+1,\ldots,x_N)\}}{H(x_1,x_2,\ldots,x_i,\ldots,x_N)} \right| + \left| \frac{\{H(x_1,x_2,\ldots,x_i,\ldots,x_N) - H(x_1,x_2,\ldots,x_i-1,\ldots,x_N)\}}{H(x_1,x_2,\ldots,x_i,\ldots,x_N)} \right| \quad (3.1)$$

The variation is carried out by integer quantization step of $q=2^{-b}$, where b is the number of bits used in representing the coefficients. In the program proposed here, b was chosen to be 0 to quantize the parameters to integer values. The same procedure is repeated until all the elements of the parameter vector have been changed. This gives a complete cycle which is repeated many times until the improvement in the performance index is the minimum. The algorithm is described by the flowchart in Fig.3.2 and is summarized in the following steps.

Step 1 : get the most sensitive parameter (among the rest), and truncate the chosen coefficients of the filter to integer. Evaluate the performance index with the new coefficients.

Step 2 : Change the chosen parameter by a quantization step, i.e., 1, in the positive or the negative direction in such a way that to decrease the objective function E(x). This process is repeated in the same direction until a minimum value of E(x) is reached. Fix the chosen parameter.

Step 3 : Repeat step 1 and 2 for x_i, i = 1,2, ...,N where N is the dimension of **x** in their sensitivity order for the remaining parameters. Let E_1 be the performance index after this complete cycle.

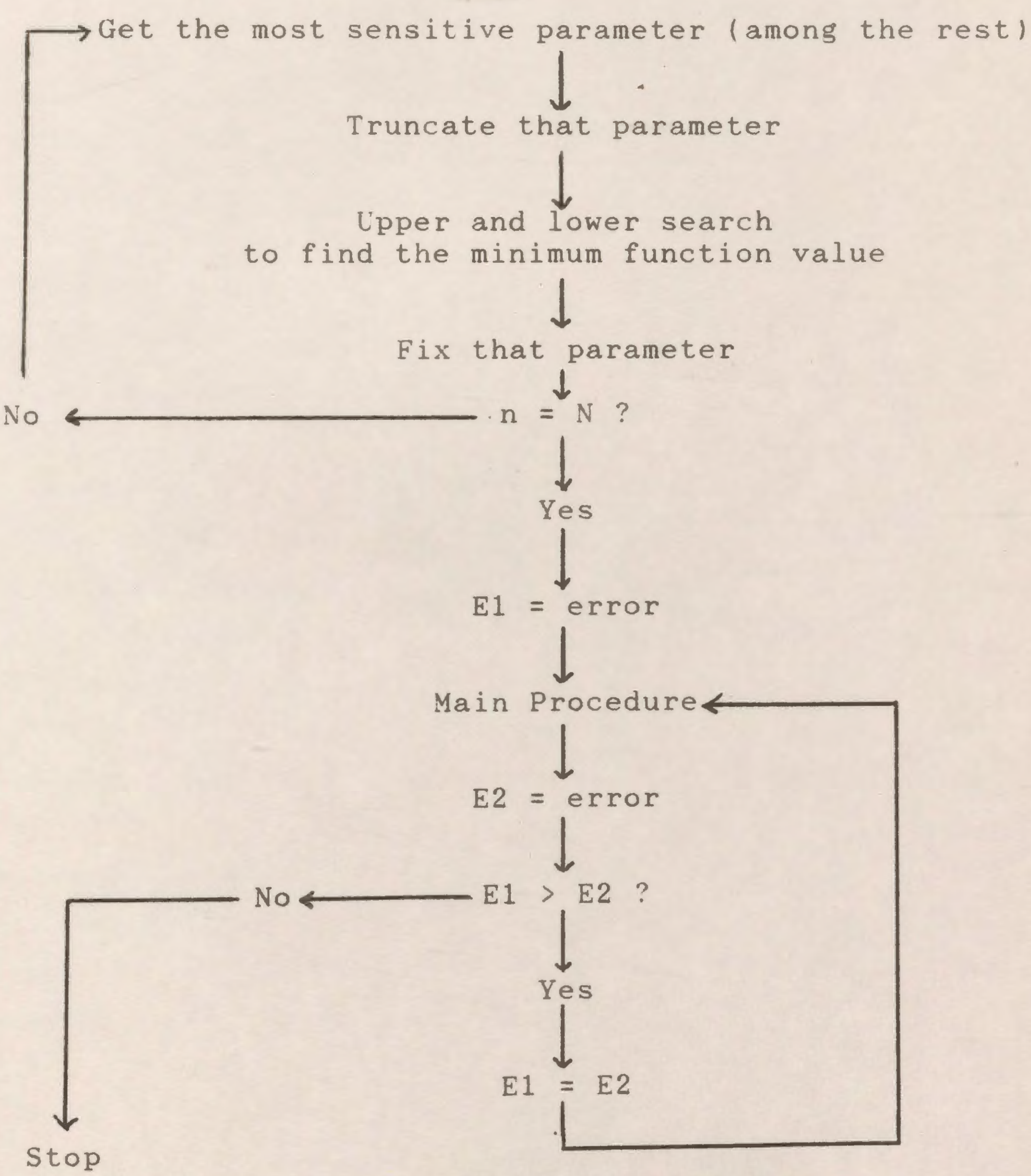
Step 4 : Repeat the complete cycle(step 1 to step 3) again with the integer parameters chosen in the previous steps and let the performance index be called E_2.

Step 5 : Evaluate the difference between the two errors.

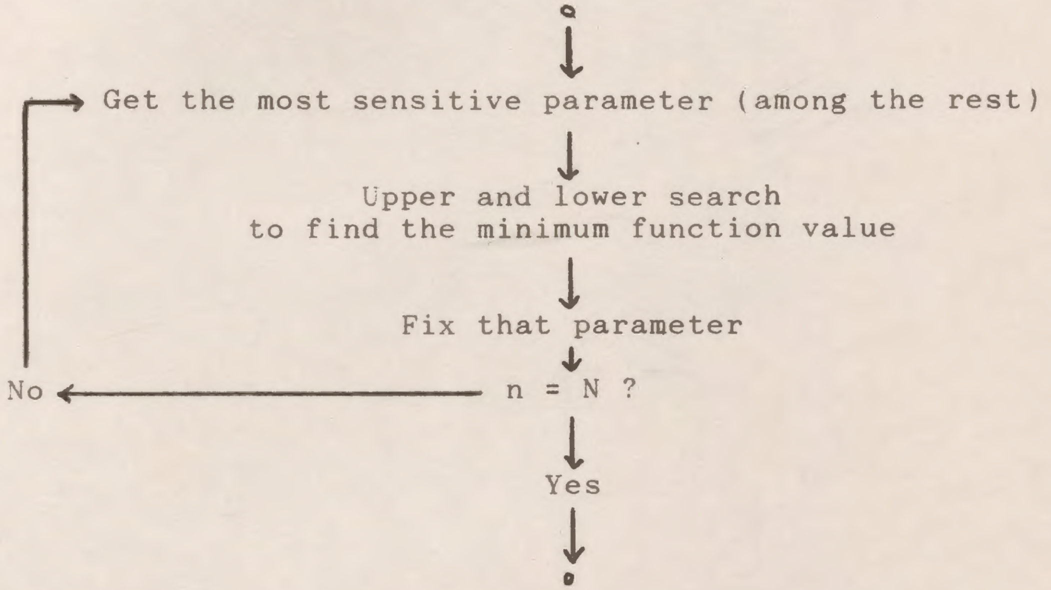
$$E = E_1 - E_2 \qquad (3.2)$$

If $E \leq 0$, then terminate the algorithm, otherwise set $E_1 = E_2$ and go to step 4.

*. Algorithm



*. The main procedure in the algorithm is explained below.



3.3 EXAMPLES

The lowpass and bandpass filters with integer coefficients are designed to show the usefulness of the algorithm proposed. The filters with infinite precision coefficients designed in chapter II are used as original filters with same specifications.

Table 3.1 shows the integer coefficients of the low-pass filter transfer function without linear phase and Table 3.2, that with linear phase. And the integer coefficients of the band-pass filter transfer function without linear phase is in Table 3.3 and those with linear phase is in Table 3.4. Figure 3.1 illustrates the magnitude response of integer coefficients low-pass filter without linear phase and figure 3.2 a,b shows the magnitude and group delay responses of the desired integer coefficients filter. In Figure 3.3 a,b, the plots of lowpass integer coefficient filter obtained by simply rounding the parameters are shown which does not seem to be acceptable as expected. Figure 3.4 shows the plot of fourth order bandpass filter without linear phase with integer coefficients and figure 3.5 a,b shows those with linear phase. In the plots, the solid line with the character 'C' represents filters with infinite coefficients and dotted line with 'I' represents the filters with integer coefficients.

u_{11} = 2	u_{13} = 26	u_{21} = 2	u_{23} = 14
u_{12} = 4	u_{14} = 2	u_{22} = 4	u_{24} = 11
σ_{11} = 3	σ_{13} = 12	σ_{21} = 12	σ_{23} = 13
σ_{12} = 28	σ_{14} = 0	σ_{22} = -13	σ_{24} = 4
a_{11} = 5	a_{12} = 0	a_{21} = 2	a_{22} = 9
r_{11} = 2	r_{12} = 4	r_{21} = 9	r_{22} = 10
g_{11} = 43	g_{12} = 20	g_{21} = 60	g_{22} = 30

Table 3.1 Integer coefficients of lowpass filter without linear phase

u_{11} = 1	u_{13} = 24	u_{21} = 3	u_{23} = 14
u_{12} = 4	u_{14} = 2	u_{22} = 3	u_{24} = 11
σ_{11} = 4	σ_{13} = 10	σ_{21} = 12	σ_{23} = 12
σ_{12} = 28	σ_{14} = -1	σ_{22} = -24	σ_{24} = 5
a_{11} = 4	a_{12} = 2	a_{21} = 4	a_{22} = 8
r_{11} = 2	r_{12} = 2	r_{21} = 9	r_{22} = 9
g_{11} = 43	g_{12} = 20	g_{21} = 83	g_{22} = 48

Table 3.2 Integer coefficients of lowpass filter with linear phase

u_{11}	=	6	u_{13}	=	26	u_{21}	=	-7	u_{21}	=	13
u_{12}	=	22	u_{14}	=	1	u_{22}	=	5	u_{22}	=	2
σ_{11}	=	10	σ_{13}	=	14	σ_{21}	=	12	σ_{23}	=	5
σ_{12}	=	33	σ_{14}	=	0	σ_{22}	=	-5	σ_{24}	=	1
a_{11}	=	3	a_{12}	=	0	a_{21}	=	6	a_{22}	=	5
r_{11}	=	3	r_{12}	=	4	r_{21}	=	6	r_{22}	=	15
g_{11}	=	23	g_{12}	=	17	g_{21}	=	25	g_{22}	=	39

Table 3.3 Integer coefficients of bandpass filter without linear phase

u_{11}	=	4	u_{13}	=	27	u_{21}	=	-6	u_{23}	=	12
u_{12}	=	22	u_{14}	=	1	u_{22}	=	6	u_{24}	=	2
σ_{11}	=	9	σ_{13}	=	17	σ_{21}	=	14	σ_{23}	=	7
σ_{12}	=	37	σ_{14}	=	0	σ_{22}	=	-5	σ_{24}	=	1
a_{11}	=	3	a_{12}	=	2	a_{21}	=	7	a_{22}	=	2
r_{11}	=	3	r_{12}	=	2	r_{21}	=	6	r_{22}	=	14
g_{11}	=	91	g_{12}	=	15	g_{21}	=	28	g_{22}	=	44

Table 3.4 Integer coefficients of bandpass filter with linear **phase**

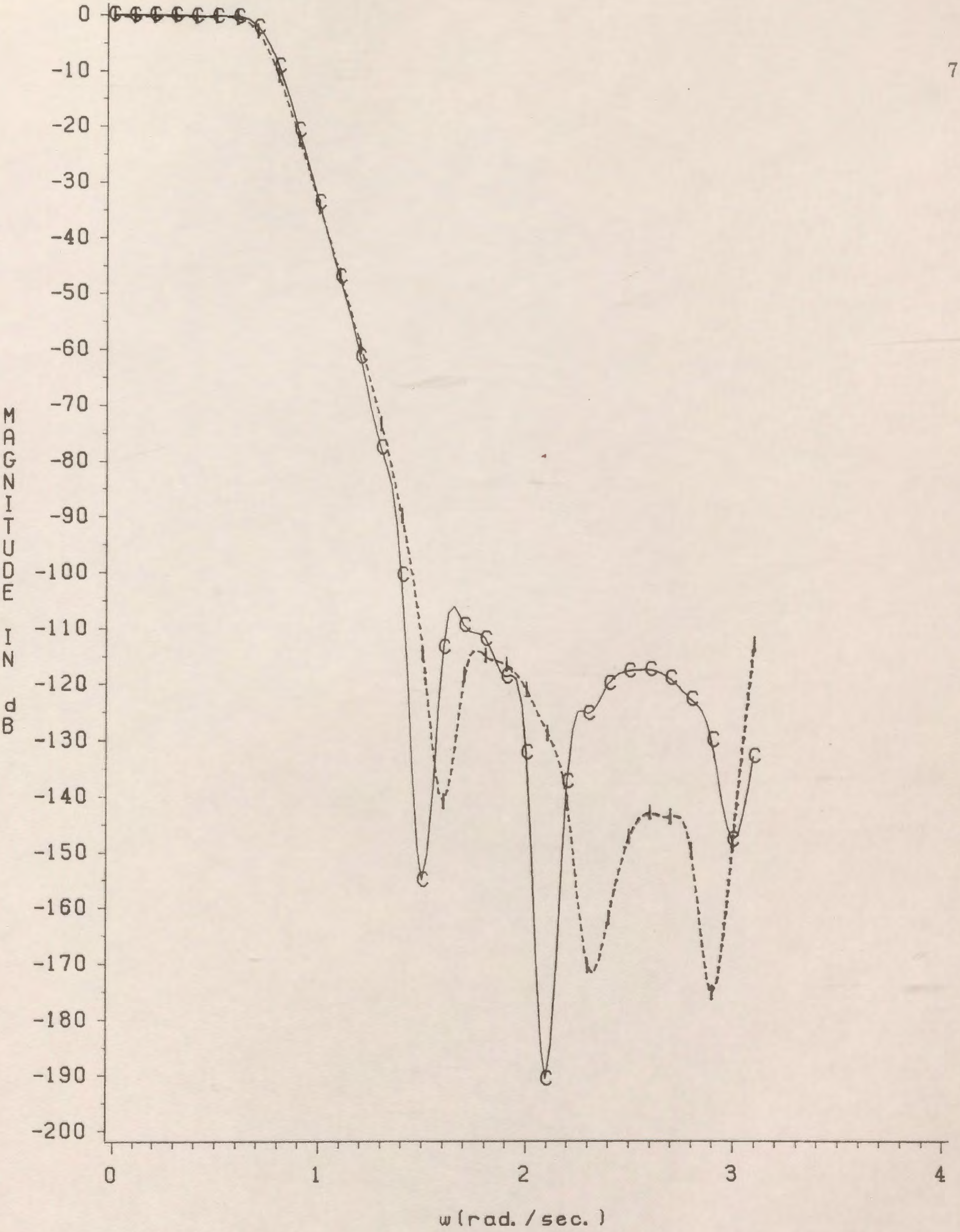


Figure 3.1 Magnitude response of integer coefficient Lowpass filter(I) without linear phase compared with infinite precision coefficient filter(C)

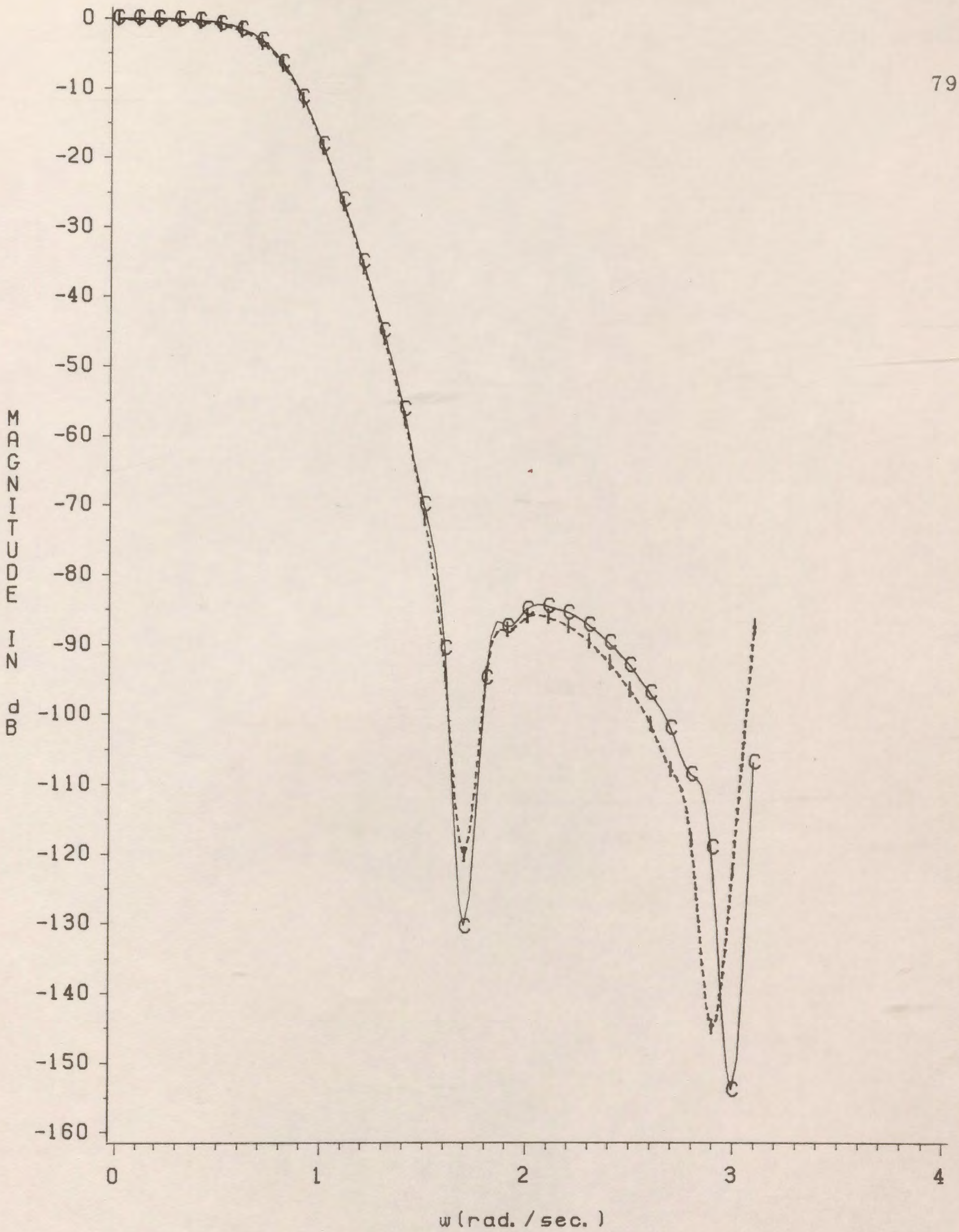


Figure 3.2 a Magnitude response of integer coefficient Lowpass filter(I) with linear phase compared with infinite precision coefficient filter(C)

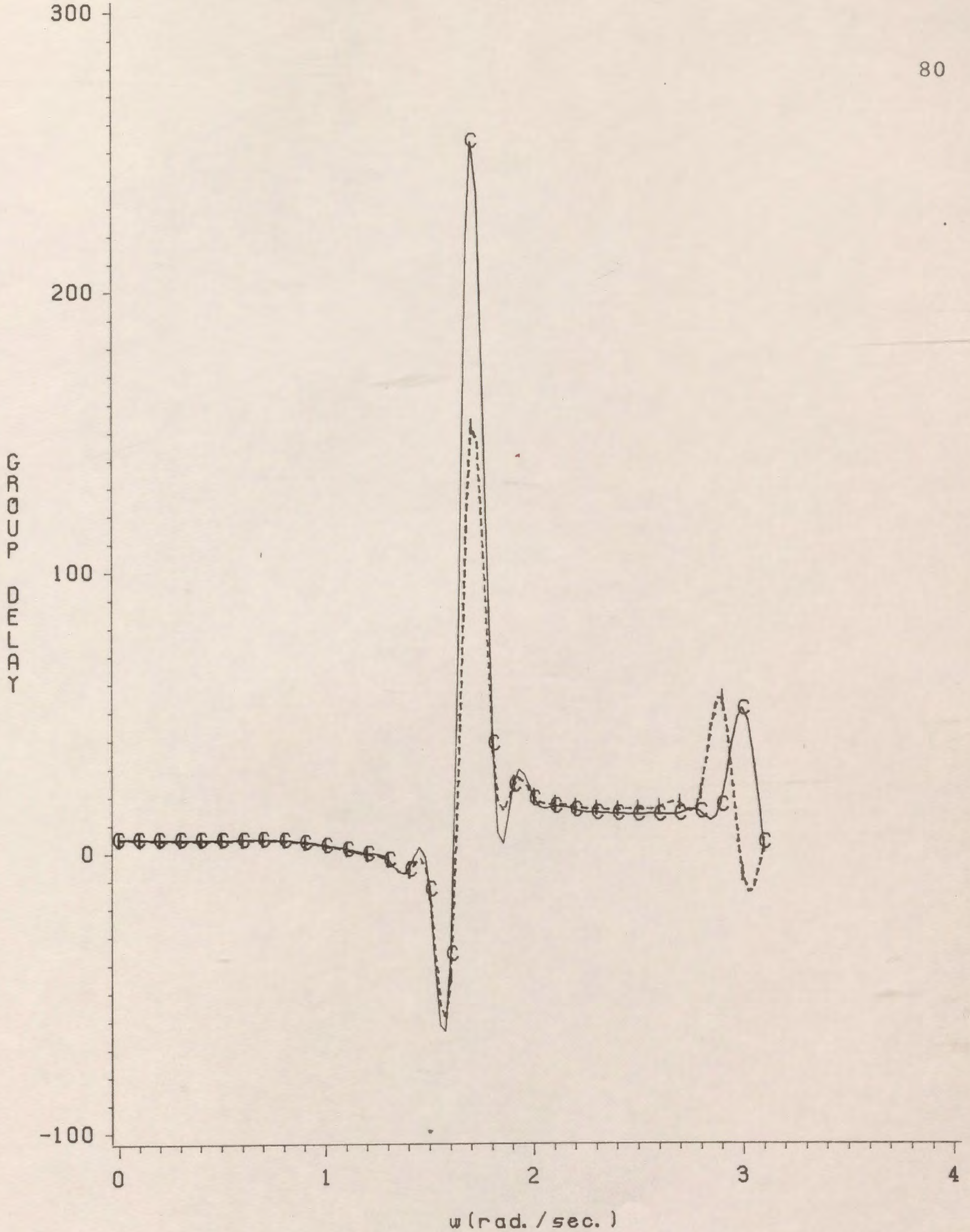


Figure 3.2 b Group delay response of integer coefficient Lowpass filter(I) with linear phase compared with infinite precision coefficient filter(C)

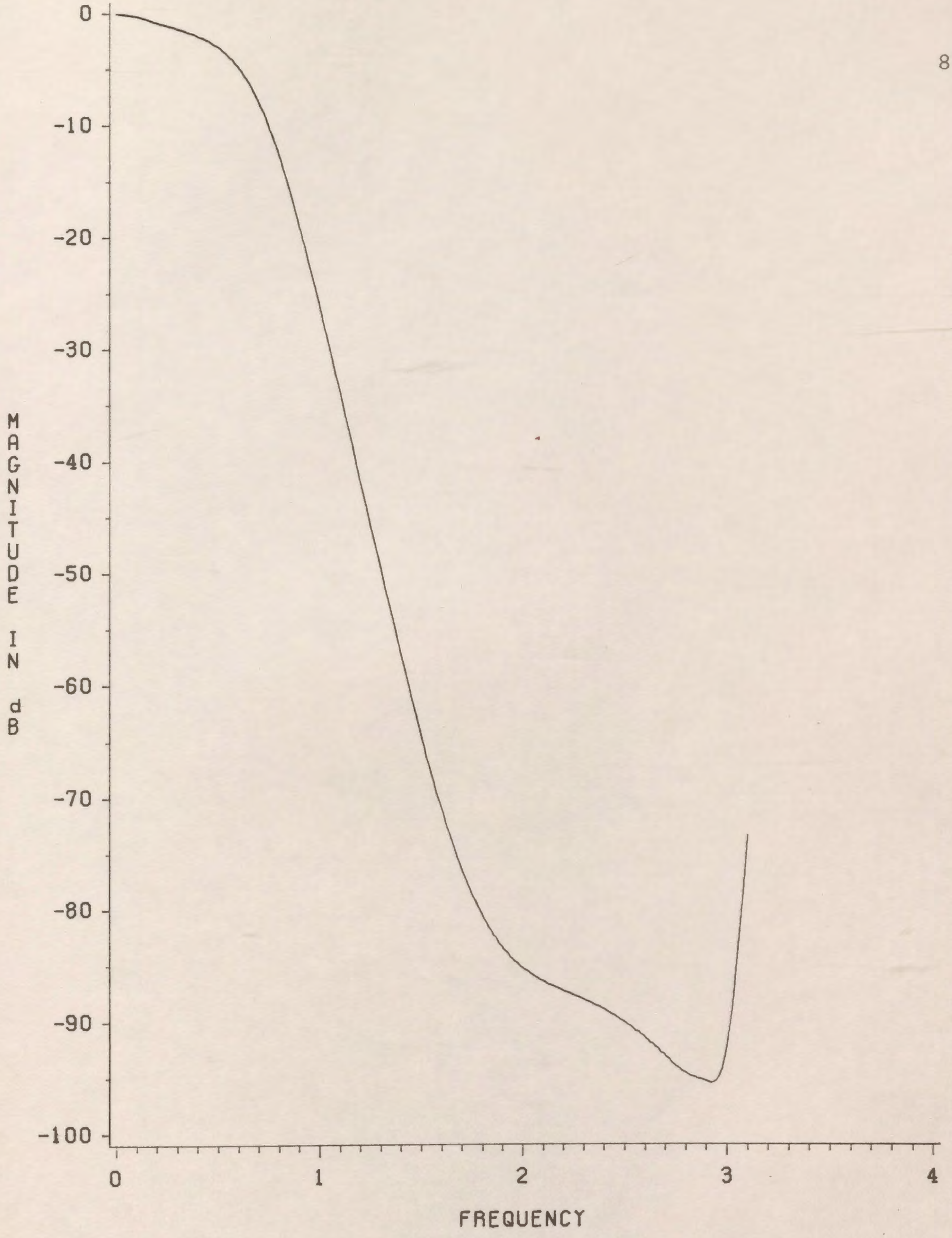


Figure 3.3 a Magnitude response of lowpass filter with linear phase obtained by simply rounding the parameters

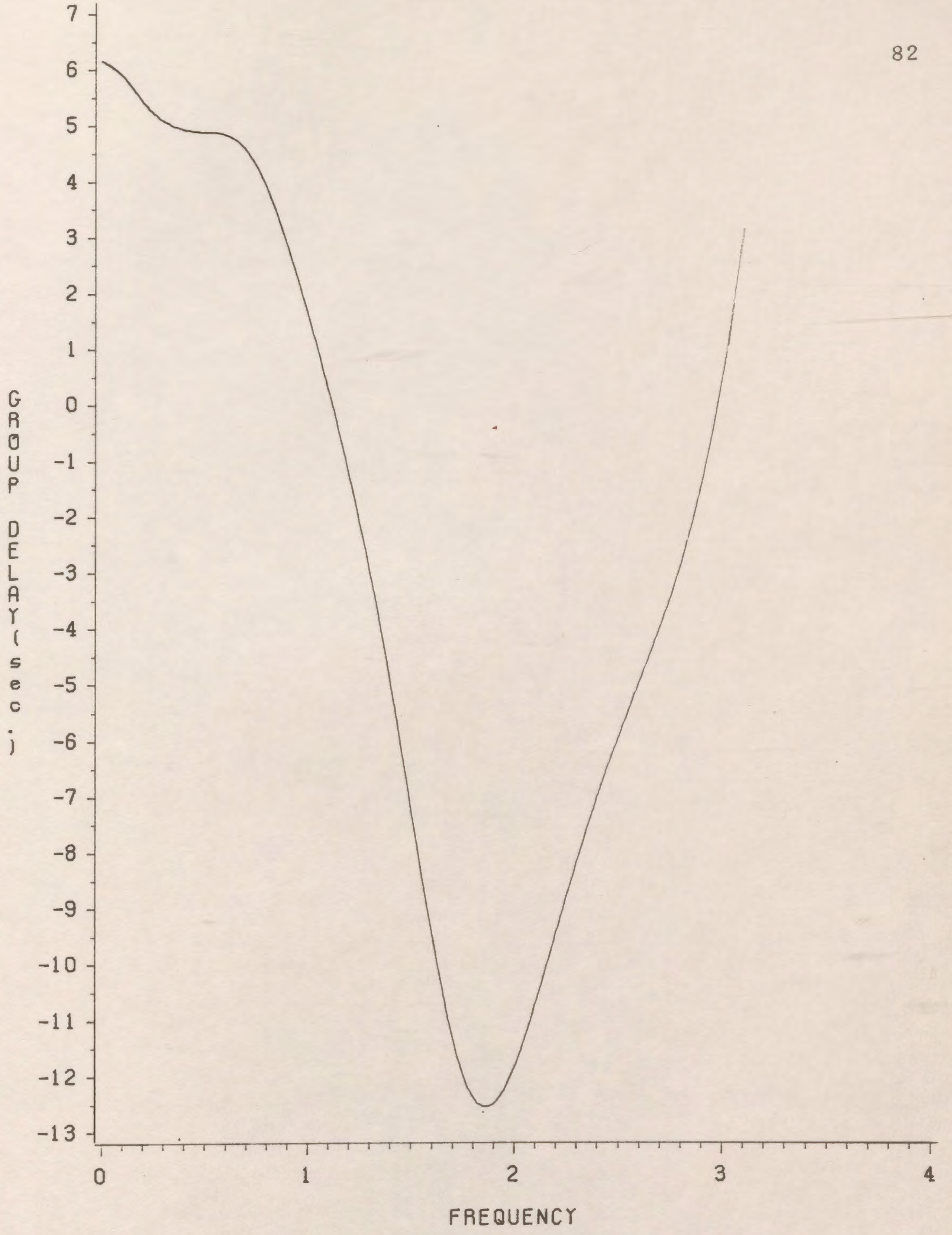


Figure 3.3 b Group delay response of lowpass filter with linear phase obtained by simply rounding the parameters

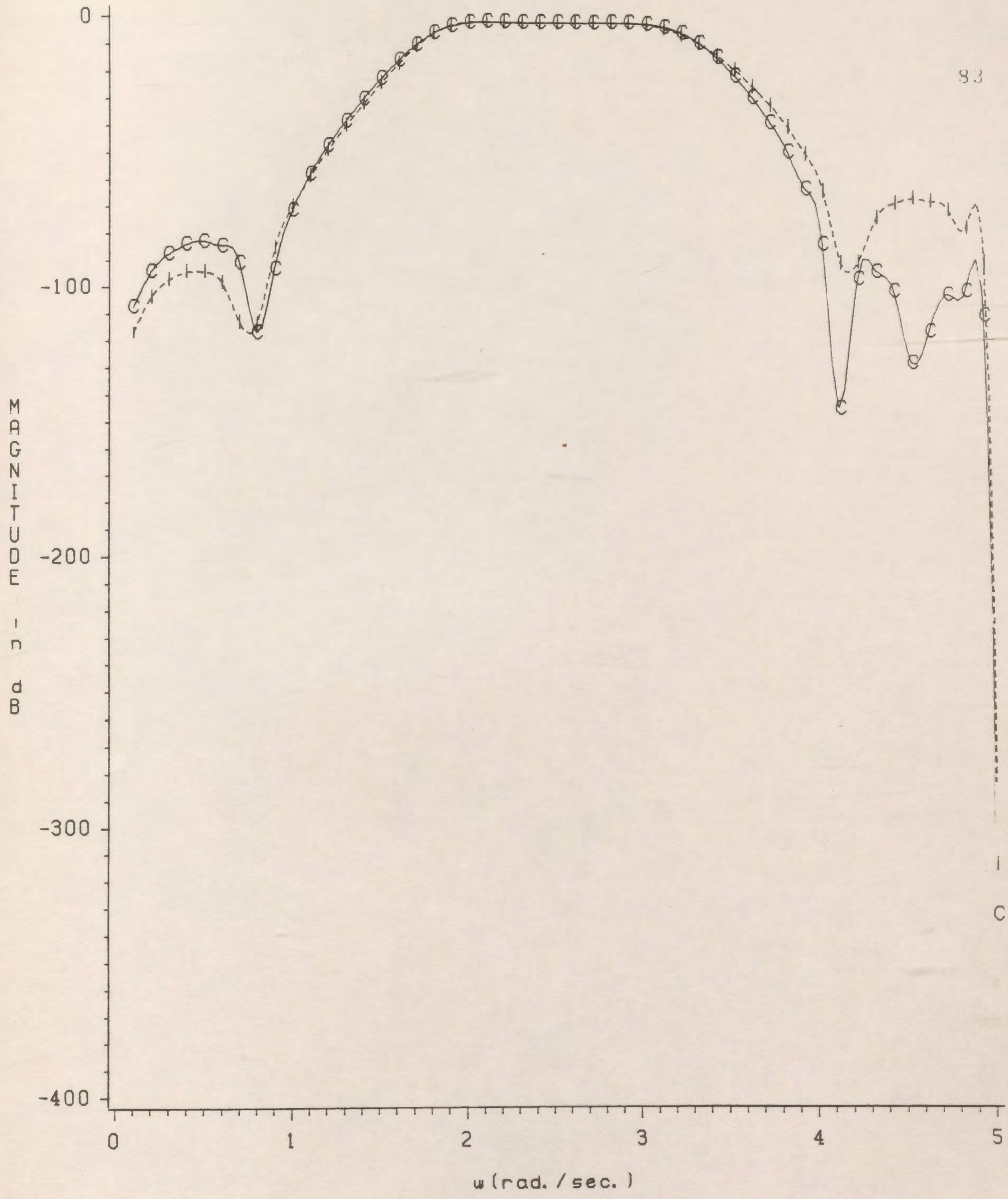


Figure 3.4 Magnitude response of integer coefficient bandpass filter(I) without linear phase compared with infinite precision coefficient filter(C)

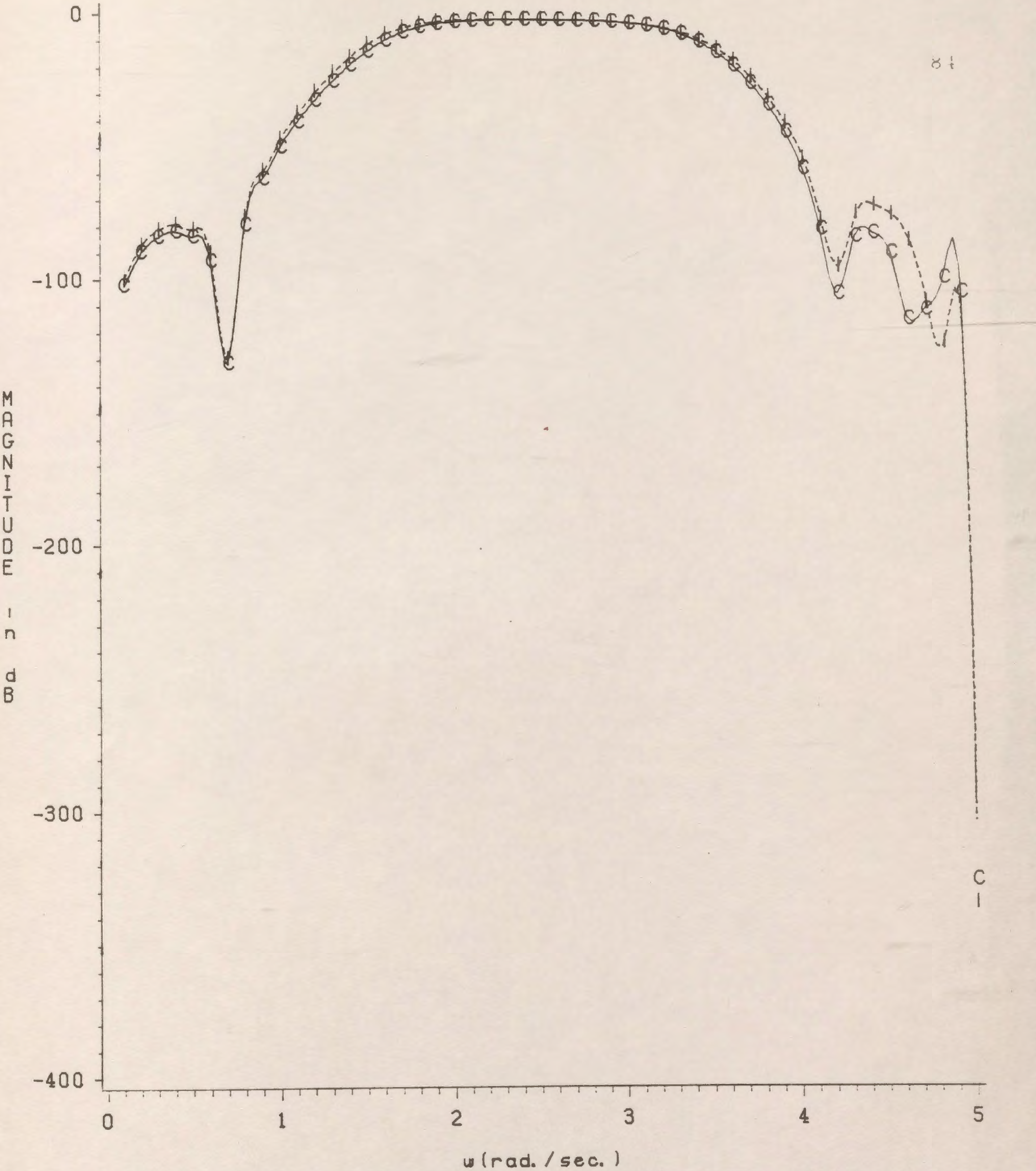


Figure 3.5 a Magnitude response of integer coefficient bandpass filter(I) with linear phase compared with infinite precision coefficient filter(C)

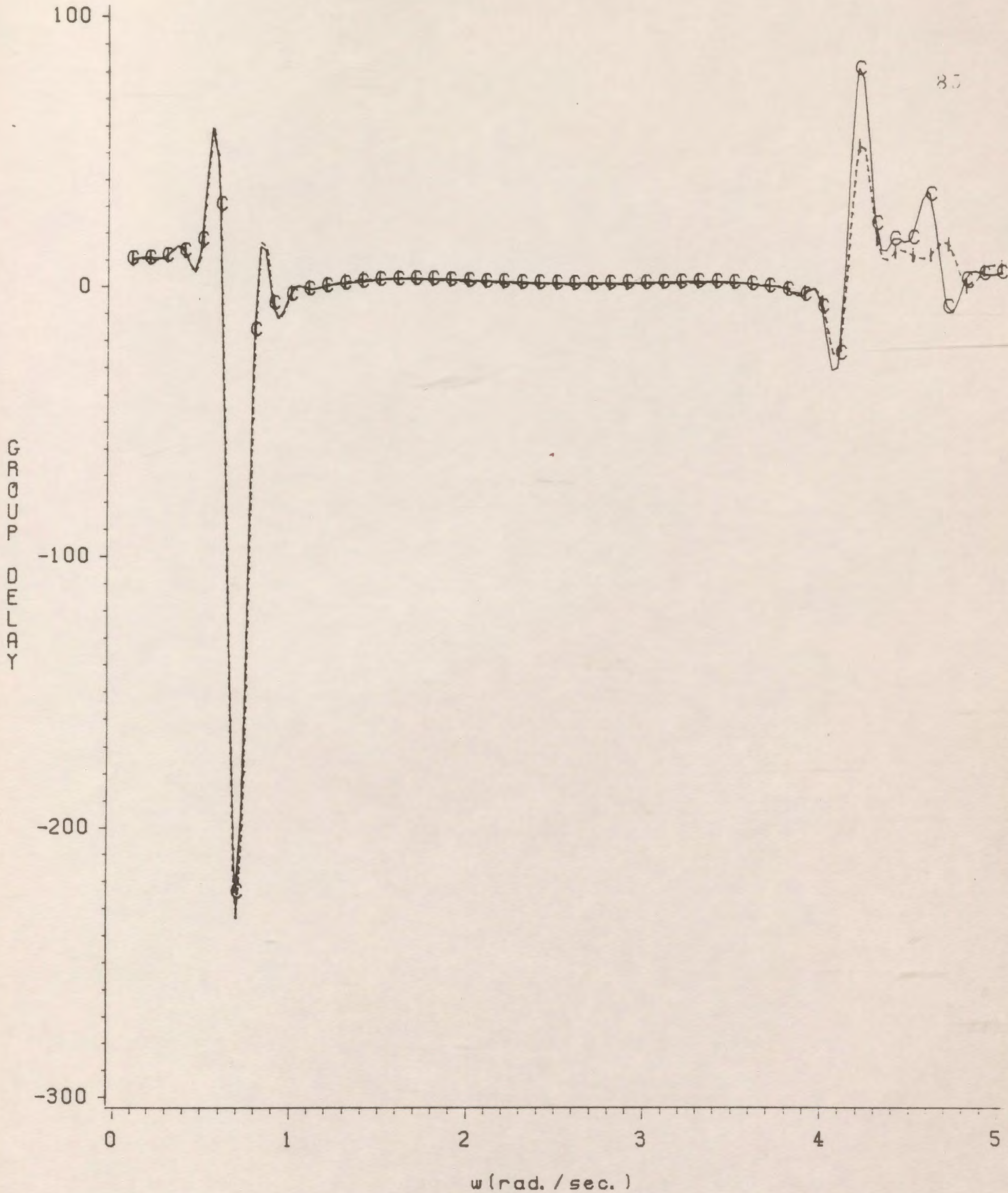


Figure 3.5 b Group delay response of integer coefficient bandpass filter(I) with linear phase compared with infinite precision coefficient filter(C)

3.5.3 COMPARISONS

Table 3.6 presents a relative comparison between integer and infinite precision coefficients filters errors. This includes the magnitude error as well as group-delay error. As we can see in the table, the differences between these two are small enough to ignore in most cases. And in some parts, the error with integer coefficient filter is smaller than that with the infinite precision coefficient filter even the overall error is bigger. If the infinite precision filter is designed to have the optimal value, the error of the integer coefficient filter cannot be smaller and if it is smaller, it means that the original filter is not globally optimal. The errors with simple rounding of the variables are shown for lowpass filter which is too big compared to the original filter error. The reason we could get the good results is partly due to the insensitive structure of the filter as we explained before.

error type	passband mag.	stopband mag.	group delay	overall
LP using two all pass filters with linear phase				
Infinite Prec.	$0.508\ 10^{-2}$	$0.626\ 10^{-2}$	$0.175\ 10^{-1}$	$0.288\ 10^{-1}$
Integer	$0.661\ 10^{-2}$	$0.547\ 10^{-2}$	$0.172\ 10^{-1}$	$0.293\ 10^{-1}$
Rounding	$0.871\ 10^{-1}$	$0.881\ 10^{-1}$	0.117	0.204
LP using two all pass filters without linear phase				
Infinite Prec.	$0.419\ 10^{-4}$	$0.141\ 10^{-3}$		$0.183\ 10^{-3}$
Integer	$0.169\ 10^{-3}$	$0.190\ 10^{-3}$		$0.359\ 10^{-3}$
BP using two all pass filters with linear phase				
Infinite Prec.	$0.121\ 10^{-1}$	$0.159\ 10^{-1}$	$0.746\ 10^{-1}$	0.103
Integer	$0.828\ 10^{-2}$	$0.232\ 10^{-1}$	$0.723\ 10^{-1}$	0.104
BP using two all pass filters without linear phase				
Infinite Prec.	$0.147\ 10^{-2}$	$0.270\ 10^{-2}$		$0.417\ 10^{-2}$
Integer	$0.274\ 10^{-2}$	$0.107\ 10^{-1}$		$0.134\ 10^{-1}$

Table 3.6 Errors between integer and infinite precision coefficient filters

Chapter IV
CONCLUSION

In this thesis, the design of 1-D IIR filter with linear phase using two all-pass filters with or without integer coefficients is studied. The design of 1-D IIR filter using two all-pass filters has been focused on recently by many researchers[32,36,51,52,55,54] for its computational efficiency. The new filtering scheme is found to have other benefits. The complementary filters are obtained by just changing the sign of one of the all-pass sections. Also the new scheme is proven to have extremely low pass-band sensitivity.[54] But most of the research is done without linear phase characteristics which are important in many applications.

In this thesis, the unconstrained optimization procedures are used to search for the coefficients such that the filter approximates the ideal magnitude and group delay responses. So it lends itself to the flexibility of designing virtually any type of filters besides standard filters such as lowpass, highpass or bandpass filters. Also stable polynomials are generated to ensure the stability of the filter structurally. That means, whatever the value we select for the parameter of the polynomials, the poles of the filter lie inside the unit circle in z domain which enable us to use the unconstrained optimization technique for the design of stable 1-D IIR digital filters satisfying a prescribed magnitude and constant group delay specifications.

The New Stability Test reported by V.Ramachandran and C.S.Gargour[50] is used to generate a stable polynomial which gave generally good results. Also, the Hurwitz polynomial is generated in two different methods. The first method involves the use of partial derivatives as a procedure in deriving the desired polynomial while the second method makes use of an additional resistance matrix and avoids the need for partial derivatives hence simplifying the filter design procedure. It is worth mentioning that the second method needs more parameters than the first one.

This may be regarded as a disadvantage since larger number of coefficients means more computational complexity which is clearly undesirable. But, through many filter designs, it has been observed that the large number of coefficients actually provides higher degree of freedom and flexibility in the process of optimization. As expected, the second method gave excellent results while the first did not.

The designing procedure using HP is divided by two steps. The first step is to approximate the magnitude response using least mean square error criteria with a powerful optimization package MINOS[29] which needs provided gradients, while the second is to approximate the group delay response using the min-max criteria with the direct search optimization package[2]. It should be noticed that new cost function for constant group delay characteristics proposed with the design using HP is much more efficient than the conventional one which is used in the filter design using the new stability test. It enables us to use the single step optimization procedure to design a optimum filter while the conventional one does not. Critical comparisons made between the proposed design and a direct design using a general 1-D IIR filter transfer function[71] show that the new techniques have overall better performances. It should be also noted that the proposed method requires one less multiplication than the general 1-D IIR filter we used for the comparison.

After infinite precision coefficient filters are designed, a discretization method is given to design integer coefficients 1-D filters. The algorithm is based on the optimization method proposed by Wan and Fahmy[61]. By this algorithm, the error caused by truncating the filter coefficients is minimized. The improvement is made by using the sensitivity measure to find the most sensitive parameter for discretization and optimization. Also, the with the proposed algorithm, we can find the optimal discrete parameter values by comparing and substituting the error obtained while the previous one ca not. Comparison made between the filters with infinite precision coefficients and integer coefficients proves the usefulness of the proposed algorithm.

Finally, the contributions of this thesis can be summarized as follows.

1. Introduction of a new function by combining a lms error for the magnitude response and a rational minimax error criteria for the phase response of the filter.

2. Introduction of a 2 step optimization procedure in order to speed up the optimization process,

3. Introduction of two stability approach to the design of 1-D IIR filter using two all-pass sections having a prescribed magnitude with or without constant group delay response and integer coefficients.

REFERENCES

1. E.Guillemin, "The Mathmatics of Circuit Analysis", John Wiley and Sons, New York, 1949

2. R.Hooke and T.A.Jeeves, "Direct search solution of numerical and statistical problems", J.Ass.Comput.Mach., Vol.8, pp.212-229, Aug. 1961

3. F.Kuo, "Network Analysis and Synthesis", Bell Telephone Laboratories, Inc., 1962

4. H.J.Orchard, "Indoctorless filters", Electron. Lett., vol.2, pp. 224-225, Sept. 1966

5. R.J.Dakin, "A tree-search algorithm for mixed integer programming problems", Computer J., vol.8, (1966) pp. 250-255

6. E.A.Robinson, "Statistical Communication and Detection", Hafner, New York, 1967, Chapters 6 and 7

7. J.L.Shanks, "Recursion Filters for Digital Processing", Geophysics 32,33 , 1967

8. A.G.Deczky, "General expression for the group delay of digital filters", Electron. Lett., vol.5, pp 663-665, Dec. 1969

9. B.C.Kuo, "Discrete-Data Control Systems", Engle-Wood Cliffs, Prentice-Hall, 1970, pp.132-134

10. K. Steiglitz, "Computer-Aided Design of Recursive Digital Filters", IEEE Trans. Audio Electroacoust., Vol. AU-18, June 1970.

11. A.G.Deczky, "Systhesis of recursive digital filters using the minimum p-error criterion", IEEE Trans. Audio Electroacoust., vol.AU-20, pp. 257-263, Oct. 1972

12. M.Suk and S.K.Mitra, "Computer-aided design of digital filters with finite word lengths", IEEE Trans. Audio Electroacoust., vol.AU-20(Dec. 1972) pp.356-363

13. P.Thajchayapong and P.J.W.Rayner, "Recursive digital filter design by linear programming", IEEE Trans. Audio Electroacoust., vol.AU-21, No.2, pp.107-112, April 1973

14. A.H.Gray,Jr., and J.D.Markel, "Digital lattice and ladder filter synthesis", IEEE Trans. Audio Electroacoust., vol.AU-21, pp.491-500, 1973

15. E.I.Jury, "Inners and Stability of Dynamic Systems", John Wiley & Sons, Inc., 1974, pp.55-60

16. L.R.Rabiner, J.F.Kaiser, O.Herrmann and M.T.Dolan, "Some comparisons between FIR and IIR digital filters", Bell Syst. Tech.J., pp 305-331, Feb. 1974

17. S.K.Mitra and K.Hirano, "Digital allpass networks", IEEE Trans., Circuits Syst., vol. CAS-21, pp. 688-700, Sept. 1974

18. A.V.Oppenheim and R.W.Schafer, "Digital signal processing", Englewood Cliffs, NJ: Prentice-Hall, 1975

19. A.V.Oppenheim, J.S.Lim,G.Kopec and S.C.Pohig, "Phase in Speech an dPictures", IEEE Conf. on Acoustics, Speech and Signal Processing.

20. L.R. Rabiner and B. Gold, "Theory and Application of Digital Signal Processing", Prentice-Hall, Englewood Cliffs, N.J. 1975.

21. T.S. Huang, J.W. Burnett, A.G. Deczky. "Importance of Phase in Image Processing Filters", IEEE Trans. on Acoustics Speech and Signal Processing, Vol. 23, No.6, pp. 529-542, Dec. 1975.

22. G. Daryanani, "Principle of Active Network Synthesis and Design", John Wiley and Sons, 1976.

23. H.W. Schussler, "A Stability Theorem for Discrete Systems", Speech and Signal Processing, Vol. ASSP-24, No.1, pp.87-89, Feb. 1976.

24. B.A.Murtagh and M.A.Saunders, "MINOS user's guide", Stanford University, Feb. 1977

25. J.Szsczupak, S.K.Mitra and E.I.Jury, "Some New Results on

Discrete-System Stability", IEEE Trans. on Acoustics, Speech and Signal Processing, Vol.ASSP-25, No.5, pp.101-102, Feb. 1977

26. C.F.Chen and Y.T.Tsay, "A Stability Theorem for Discrete Systems", Speech and Signal Processing, Vol.ASSP-25, No.1, pp.1200-1202, Aug. 1977

27. P.Steffan, "An Algorithm for Testing Stability of Discrete-Systems", IEEE Trans. on Acoust., Speech and Signal Processing, Vol.ASSP-25, No.5, pp.454-456, Oct. 1977

28. A. Antoniou, "Digital Filters Analysis and Design", McGraw-Hill, New York, 1979.

29. B.A.Murtagh and M.A. Saunders, "MINOS user's manual", Stanford University, June, 1980

30. T.Inukai, "A unified approach to optimal recursive digital filter design", IEEE Trans. on Circuits and Systems, vol.CAS-27, No. 7, July 1980

31. C.Charalambous and A.Antoniou, "Equalization of recursive digital filters", IEEE Proc., vol.127, pt.G, No.5, pp. 219-225, Oct. 1980

32. R.Ansari and B.Liu, "A class of low noise computationally efficient recursive digital filters", Proc. IEEE Int. Symp. Circuits Syst., Chicago, IL, pp. 550-553, April 1981

33. N.Kanaya and S.Yuta, "A digital all-pass transfer function with all possible coefficient combinations", IEEE Trans. Circuits Syst., vol.CAS-28, pp. 1171-1174, Dec. 1981

34. R.Gnanasekaran, "A note on the new 1-D and 2-Dstabilitytheorems for discrete systems", IEEE Trans. Acoustic Speech Signal Process., vol.ASSP-29, No.6, pp.1211-1212, Dec. 1981

35. A.Chottera and G.A.Jullien, "A linear programming approach to recursive digital filter design with linear phase", IEEE Trans. on Circuits and Systems, Vol. CAS-29, pp. 139-149, Mar. 1982

36. B.Liu and R.Ansari, "Quantization effects in computationally efficient realizations of recursive filters", Proc. Int.Symp.Circuits Syst., pp. 716-720, May 1982 IEEE

37. A.M.Davies, "A New z-Domain Continued Franction Expansion", IEEE Trans. on Circuits and Systems, Vol.CAS-29, No.10, pp.658-662, Oct. 1982

38. P.P.Vaidyanathan and S.K.Mitra, "A new approach for sysnthesis of low sensitivity digital filter structures based on losless building blocks", Proc. IEEE Int. Conf. on Acoust., Speech, Signal Processing, Boston, MA, pp. 615-618, April 1983

39. P.P.Vaidyanathan and S.K.Mitra, "A general theory for low sensitivity digital filter structures", Proc. IEEE Int. Symp.

on Circuits and Syst., Newport Beach, California, pp.266-269 May 1983

40. Z.Jing and A.T.Fam, " Design of finite wordlength IIR filters by successive discretization and reoptimization", Proc. of IEEE Intern. Conf. on Electrical, Electronics, pp. 516-519, Sep. 1983

41. Y.C.Lim and S.R.Parker, "Discrete coefficient FIR digital filter design based upon an LMS criteria", IEEE Trans., Circuits Syst., vol. CAS-30, No. 10, pp. 723-739, Oct. 1983

42. S.P.Golikeri, "Design of 2-D Stable Recursive Digital Filter Satisfying Prescribed Magnitude and Group Delay Responses", M.A.Sc. Thesis, University of Windsor, Windsor, Canada, 1984

43. M.T.Boraie, "Design of 1-D and 2-D Recursive Digital FIlters Based on a New Stability Test", M.A.Sc. Thesis, University of Windsor, Windsor, Canada, 1984

44. Chi-Tsong Chen, "Linear System Theory and Design", CBC COLLEGE PUBLISHING, HRW, 1984

45. P.P. Vaidyanathan and S.K.Mitra, "Low passband sensitivity digital filters: A generalized viewpoint and synthesis procedures", Proc.IEEE, vol.72, pp. 404-423, Apr. 1984.

46. V.Ramachandran and C.S.Gargour, "An implementation of a stability test of 1-D discrete system based on Schussler's theorem and some conseuent coefficient conditions",

J.Franklin Inst., vol.317, No.5, pp. 341-358, May 1984

47. Y.Neuvo and S.K.Mitra, "Complementary IIR digital filters", Proc. IEEE Int. Symp. Circuits Syst., Montreal, Canada, pp. 234-237, May 1984

48. M.Ahmadi and V.Ramachandran, "New method for generating two-variable VSHPs and its application in the design of two-dimensional recursive digital filters with prescribed magnitude and constant group delay responses", IEE Proc. vol.131, Pt.G, No.4, Aug. 1984

49. V.Ramachandran, C.S.Gargour, M.Ahmadi and M.T.Boraie, "Direct design of recursive digital filters based on a new stability test", J.Franklin Inst., vol.318, No.6, pp. 407-413, Dec., 1984

50. A. Peled and Bede Liu, "Digital Signal Processing: Theory, Design and Implementation", Robert E. Krieger Publishing Co., Malabar, Florida 1985.

51. B.Liu and R.Ansari, "A class of low-noise computationally efficient recursive digital filters, with applications to sampling rate alterations", IEEE Trans. Acoust., Speech, Signal Processing, vol.ASSP-33, pp. 90-97, Feb. 1985

52. T.Saramaki, "On the design of digital filters as the sum of two all-pass filters:, IEEE Trans. Circuits Syst., vol.CAS-32, pp. 1191-1193, Nov. 1985

53. Lonnie C. Ludeman, "Fundamentals of Digital Signal Processing", Harper and Row Publ. Co., 1986.

54. P.P.Vaidyanathan, S.K.Mitra and Y.Neuvo, "A new approach to the realization of low sensitivity IIR digital filters", IEEE Trans. Acoust., Speech, Signal Processing, vol.ASSP-34, pp. 350-361, Apr. 1986

55. M.Refors and T.Saramaki, "A class of approximately linear phase digital filters composed of two allpass subfilters", in Proc. 1986 Int. Symp. on Circuits and Systems (San Jose, CA, May 1986), pp. 678-681

56. M.Ahmadi, M.T.Boraie and V.Ramachandran, "An iterative method for the design of 1-D stable recursive digital filters", IEEE CH2255-8, Aug. 1986 pp. 669-673

57. Z.Jing and A.T.Fam, "A new scheme for designing IIR filters with finite wordlength coefficients", IEEE Trans., Acoust., Speech, Signal Processing, vol.ASSP-34, No.5, Oct. 1986

58. P.P.Vaidyanathan and S.K.Mitra, "A unified structural intepretation of some well-known stability tests for linear systems", Proc.IEEE, vol.75, pp.478-497, Apr. 1987

59. J.Szczupak, S.K.Mitra and J.Fadavi, "Realization of structurally LBR digital allpass filters", in proc. IEEE Int. Symp. on Circuits and Systems (Philadelphia, PA, May 1987), pp. 633-636.

60. P.A.Regalia, S.K.Mitra and P.P.Vaidyanathan, "The digital all-pass filter: a versatile signal processing building block", in Proc. IEEE, vol.76, No.1, Jan. 1988

61. Y.Wan and M.M.Fahmy, "Design of 2-D digital filters with finite wordlength coefficients", Proc. IEEE INt. Conf. on Acoustics, Speech and Signal Processing, CH2561-9,pp. 844-847 1988 IEEE

62. M.Ahmadi, M.Shridar, H.Lee and V.Ramachandran, "A method of 1-D recursive digital filters satisfying a given magnitude and constant group-delay response", The Franklin Inst. J., vol 326, No.3, pp.381-393, 1989

63. K.A.El-Baher and A.I.Abu-El-Haija, "Design of two-dimensional recursive digital filters using separable denominator transfer function and finite wordlength coefficients", to be published

64. H.J.J.Lee, "Generation of 1-D and 2-D stable polynomials and its application in the 1-D and 2-D recursive filter design", Master Thesis, University of Windsor, Windsor, Canada, 1989

65. J.L.Kueston and J.H.Mize, "Optimization Techniques with Fortran", McGrow-Hill, pp.309-319

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