Improving Student Knowledge Transfer between Mathematics and Engineering Courses Through Structured Cross-Disciplinary Collaboration: A SUMMIT-P Initiative

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Abstract

Student learning across STEM disciplines has been shown to increase with greater integration of applications in mathematics courses. One challenge of this effort is that identical constructs are often presented differently in the partner disciplines than in the mathematics courses. This leads to student confusion and an inability to transfer critical knowledge in their disciplinary courses, even for students who have mastered the mathematical paradigms. An interdisciplinary team at VCU consisting of mathematics and engineering faculty has worked to improve the knowledge transfer required for the integration of applications in the Differential Equations curriculum. This work is part of the multi-institutional SUMMIT-P initiative which aims to transform first- and second-year mathematics through collaboration with partner disciplines. The collaborative efforts have uncovered a variety of differently presented but identical constructs in categories ranging from notation up through higher-level interpretation. We provide some specific examples and analyses of these constructs and the implications for knowledge transfer and pedagogical concerns. Conversations around mathematics and disciplinary imperatives served to create a holistic view of the role mathematics and partner discipline professors have in improving learning outcomes.

Keywords

interdisciplinary, translation, concept mapping, applications, curriculum development

Current research indicates that student learning across STEM disciplines increases as the integration of applications in mathematics courses via interdisciplinary faculty partnerships increases (Filippas et al., 2020). Fostering such a beneficial partnership can be challenging especially in a large, research-intensive urban institution such as Virginia Commonwealth University (VCU). At VCU, mathematics and engineering faculty alike have repeatedly witnessed that students enrolled in traditional STEM curricula often have difficulty transferring knowledge between mathematics classes and their major discipline, ranging from elementary terminology to overarching concepts. Several faculty across these disciplines have taken on the initiative to improve student knowledge transfer as part of the larger project, A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnership (SUMMIT-P).

SUMMIT-P is an extension of work begun in the Curriculum Renewal Across the First Two Years (CRAFTY) project (Ganter & Haver, 2020). The central goal of SUMMIT-P is to develop innovative educational paradigms via collaborative, interdisciplinary partnerships, and thereby improve mathematics instruction for students of all disciplines. Each institution within the consortium developed its own strategies and collaborations based on its perceived needs or preferences. VCU has been an active institution in SUMMIT-P since its launch in the Fall semester of 2016. The VCU team, led by a mathematics PI and an engineering co-PI and including additional faculty from both disciplines, has worked to use interdisciplinary faculty partnerships to effect change in the undergraduate mathematics curriculum with a particular focus on connections to engineering.

Grant activities for beginning structured engagement in productive conversations with faculty across disciplines included site visits (Hofrenning et al., 2020), classroom visits, and “fishbowl” style conversations (Piercey et al., 2020) in which one group of participants observes the discussions of the other group. Given the energy required for an effective fishbowl conversation, our experience indicated that we would not be able to engage mathematics or partner discipline faculty in more than one. Therefore prior to any discussions, the VCU team surveyed all STEM faculty to assess the needs of faculty and students, gather information about their needs for specific course content in a variety of mathematics courses, and develop a sense of the degree to which faculty would be engaged in this process. One of the first strategic choices the team made after the results of the survey were analyzed was to use Differential Equations as our pilot course. This decision was made in part to allow us to develop our learning modules for a smaller cohort of primarily engineering students with similar backgrounds and interests, and later apply them to the more general-interest courses. A second consideration was the high level of interest and sense of urgency imparted by engineering faculty as evidenced by the degree to which they engaged with the survey.

Throughout the conversations about differential equations and engineering, the team has come to recognize that identical mathematical constructs and engineering systems are often presented differently in the partner disciplines when compared to this and other mathematics courses. This leads to confusion and the inability to transfer critical knowledge to disciplinary courses regardless of the level of mastery of mathematical paradigms. What has emerged is a long-term effort to create and maintain a durable infrastructure for discussing terminology, concepts, and applications in the mathematics courses that align with how the students will need to access the content in their subsequent engineering or science courses. This requires regular contact between mathematics and engineering faculty to continuously update and improve our interdisciplinary network of discipline-specific terms, notation, and concepts. Our conversations and collaborations on teaching have evolved from specific content topic concerns, to shared application examples, to the current process we are engaged in of developing a shared mapping. Below, we provide some specific examples of differently presented but identical constructs in categories ranging from notation up through higher-level interpretation and the implications for knowledge transfer and pedagogical concerns. We hope that this process proves to be a useful roadmap for collaborations at other institutions.

Process

The long-term goal of the VCU team is to create and grow an enduring structure to provide a network of connections between the mathematics concepts and methods and the engineering or other partner discipline applications. The foundation for this structure so far has grown into regular conversations, common exam questions, and exemplar problems to bolster curriculum development in differential equations. The current initiative towards discovering and identifying ways to improve knowledge transfer has led to the start of an adaptable document that will provide a mapping of mathematical terms and processes to their engineering counterparts, as well as a comparative mapping of cross-disciplinary engineering terms and methods. This document will provide a latitudinal and longitudinal equivalency matrix that will aid students in making connections between similar concepts across different subject areas with mathematics serving as the common factor and will continue to grow over time as the VCU team maintains interactions with engineering and science faculty.

The students are engaged in the process through activities in both mathematics and engineering classes that ask them to compare the same mathematical construct through the lens of a variety of applications, discover their common factors, and juxtapose the methods, solutions, and outcomes in each case analyzed. Students learn that mathematics provides an “agnostic” solution and engineers provide the application-specific interpretation, aiding in building up the students’ critical thinking and design skills. Additionally, the development of an adaptable document is useful for sustaining the collaboration by using its periodic review to drive systematic collaborative efforts. The specific examples of knowledge transfer described below include some ways in which students have already become part of the process both within and extracurricular to the course assessments.

Technical Examples

As students progress in more advanced mathematics courses, particularly courses such as Differential Equations (DE), they are given opportunities to connect mathematics with practical applications. This often requires translating the description of a physical problem into the corresponding mathematical construct. This can be challenging if different expressions, notations, and/or conventions are used in mathematics vs partner disciplines. We present examples of these collected over the last few years and describe efforts to bridge this gap.

Notation

Several mathematics constructs have multiple types of acceptable notation but there are instances where one is used more in mathematics contexts and another in engineering. One fundamental example of this is the notation used for derivatives. The Leibniz notation dy/dx is ubiquitous and perhaps most useful because it clearly identifies the independent variable. Mathematicians also use the more compact Lagrange or "prime" notation, $y'(x)$ or sometimes abbreviated as simply y'. Less common in mathematics is Newton or "dot" notation $\dot{y}$, often used to indicate derivatives with respect to time and therefore more prevalent in applied disciplines such as engineering and physics. It is reasonable to introduce all types of notations in a mathematics class and indicate the context in which they will be seen. Similarly, Leibniz notation is often used for partial derivatives as is the "subscript" notation, but the order in which derivatives are taken is indicated differently for each style and should be reiterated for novice students.

The imaginary unit or imaginary number, that is the root of the equation $x^2 + 1 = 0$, is conventionally referred to as i in mathematics. This becomes a conflict in disciplines such as electrical and control systems engineering where i is used to denote current. In these cases, j is used instead. It is important then to clarify to students in a mathematics class that when dealing with complex numbers i is the $\sqrt{-1}$, but in a circuit application i is current, and j may be needed to represent $\sqrt{-1}$. Another example of notation conflict is dealing with units. Mathematics majors focused on theoretical mathematics may not need to care about units, and there may not be as much emphasis on them in earlier classes. In application problems, however, they are a necessary component of the quantitative solution. To complicate matters, one common DE textbook used at VCU frequently uses imperial or United States (US) units which would seem logical in a US university course, but recently partner discipline faculty have clarified they work exclusively with SI units. This simple knowledge sharing between faculty has aided in streamlining instruction in recent years.

Terminology

Application-focused courses have several examples of mathematical terms describing processes that have different terms in other related disciplines. It is imperative that students can recognize these and "translate" between them to make the connections required to solve real-world problems. Perhaps the most basic of these in continuous mathematics is describing the *derivative* as both the *tangent slope* and the *rate of change*. Students often come out of calculus having memorized derivative formulas without retaining an understanding of what a derivative is both mathematically and practically. A typical modeling problem will often state something about the rate of change (growth, decay) of a quantity that has certain behavior, which should immediately be a prompt to write down a derivative but has still proven to be a challenge for our typical students in DE and partner discipline courses.

Coordinate systems have multiple naming conventions that can conflate terminology between disciplines. The VCU introductory DE course operates with the standard Cartesian coordinate system, but discipline-specific courses that draw on multi-variable calculus to develop partial differential equations draw upon spatial coordinate knowledge. As an example, cylindrical coordinate systems are often used to represent a 3-D cylindrical domain in fewer spatial dimensions by exploiting symmetry. The height or length down the center of the cylinder, or the axial coordinate, is also called the z coordinate. The distance from the z axis is called the radial distance or radius and can be denoted by r, ρ, or R, and the angular position around the cylinder is the angular coordinate or azimuth and can be denoted as either θ or ϕ. To further complicate coordinate system terminology, the symbols for theta and phi have two forms: θ or ϑ, and ϕ or φ correspondingly. When asked out of context if these are the same letters, over 80% of a junior-level engineering class voted "no".

The cumulative effect of having to navigate through this variety of "standard" symbols and nomenclature can be intimidating to a new learner. It would be very helpful for someone in that situation to have access to a reference where conflicting terminology, symbols, and conventions are linked and defined in context.

Conceptual Examples

We have thus far presented examples of terminology and notation that can differ between mathematics and partner disciplines. With more advanced content, numerous mathematical concepts have a universal interpretation between disciplines but for multiple applications that are usually seen for the first time in DE, and so should be introduced with this in mind.

Translation

DE provides exposure to translating a physical problem into a mathematical model in conjunction with learning the tools for deriving solutions to differential equations. Students then further encounter more real-world modeling problems in their partner discipline courses involving the calculation of quantities based on known information like rates of change. As was done with *derivative* above, key phrases with direct mathematical meaning are discussed with students as part of a procedure for translation. For example, students learn that "the rate of growth of a population is proportional to the current population" translates to $dP/dt = kP.$ A chart of typical "translations" encountered in modeling is shown in Table 1. As part of the team's efforts to understand students' ability to translate, a related set of terms, including some of those in Table 1, were given to DE students as an ungraded "quiz" to see if they could recognize them and give some sort of definition. Results were categorized post hoc by the team as theoretical, conceptual, or incorrect. The percentages of student responses appear in Table 2 in the Solutions section below. Some results were surprising; students had very low response rates for the term "transient", even though they could conceptualize "steady state". In addition, only 5% could provide a contextual example for "rate constant".

A classic example of a translatable differential equation is the "mixing problem", describing the change in the amount of salt in a saltwater tank as built up from a mass balance paradigm, i.e. "rate of change of quantity = rate in - rate out." While this paradigm is a concrete way of understanding a change in an actual mass of something, it can also translate to living organisms in a closed environment or to non-STEM quantities, such as money. Thus, when presenting the theory of building a differential equation, it is advantageous to present the translation in multiple forms and describe multiple contexts in which it is used without necessarily solving the problem or going into detail about the application.

Solutions

The VCU team has observed that even the seemingly fundamental phrases "solution to differential equation" or "solve the differential equation" carry imprecise meanings for students, most of whom in earlier courses have only had to solve for variables that represent scalars. A solution to a differential equation is mathematically defined as a function defined on an interval

Table 1
Examples of Problem Statements, their Equivalent Mathematical Term, and Symbol Options

Problem Statements	Mathematical Term	Symbol Options
Is	Is equal to	$=$
Rate of change, Growth / decay	First-order derivative	$y', \dot{y}, \frac{dy}{dx}, Dy$
Rate of acceleration, deceleration	Second-order derivative	$y'', \ddot{y}, \frac{d^2y}{dx^2}, D^2y$
Is proportional to	Varies directly with	$y \propto x, y = kx$
Has an n −order dependence		$y = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$
Has an exponential dependence		$y = Ae^{kx}$
Difference, sum	Minus, plus	$-, +$
Interaction, ratio	Multiplied (times), Divided by	$\times$ or $\cdot$; $\div$ or $/$
Half-life	The time at which the value of y is equal to 0.5 its original value	$y(t_{0.5}) = 0.5y(t = 0) = 0.5y_0$
Double, triple		$y(t) = 2y_0, y(t) = 3y_0$

that reduces a differential equation to an identity, but practically it describes the time or spatially dependent behavior of a quantity under study. Since students traditionally begin solving differential equations in the DE course before having physical context this connection can be missing early on. Later in the course in an introductory modeling scenario, students may be asked to find the “equation of motion”, meaning the solution to a differential equation describing the motion of a system as a function of time. It is critical for an instructor to explicitly lead students to a comprehensive understanding of what a solution is.

While most introductory DE courses focus on applications and behavior of solutions, they also allocate time to explore when a unique solution to an initial value problem exists. The first step is understanding that an *initial value problem* comprises a differential equation and the initial condition together. Applying a procedure to solve a differential equation does not demonstrate mastery, but instead, a practitioner must also be concerned with if and how the solution dynamics relate to a given data point. Most students have rarely had to consider if there is an answer to the problem they are asked to solve, or if there may be more than one valid answer. The closest analogy they may have encountered is solving a 2×2 system of linear equations where it’s clear that a unique solution represents a point of intersection, no solution is parallel lines, and the existence of infinitely many solutions means the lines are the same. This analogy put forth in DE classes in recent years at VCU has helped students transfer their understanding to real engineering problems that might offer more than one possible mathematical solution; the role of the engineer is to apply further knowledge of the system dynamics to limit the solutions to one physically possible solution. The mathematics majors will find this theoretical treatment to be the foundation for future work. Meanwhile, the engineering majors will use their physical understanding of a system, including approximations and limitations, to ensure that a solution is possible or to discover that the model is inadequate for the case under consideration.

Table 2
Responses to Differential Equations Term Identification Quiz

Term	Student Response: Theoretical/Symbolic	Student Response: Conceptual	Student Response: Missing/Wrong
Derivative	81%	15%	3%
Rate of change	29%	40%	20%
Variable	63%	34%	1%
Parameter	5%	17%	77%
Transient	3%	12%	84%
Proportional	27%	25%	46%
Linear	41%	51%	6%
Steady state	5%	39%	55%
Homogeneous	20%	32%	46%
Forcing function	20%	12%	67%
Tangent	12%	63%	24%
Source	0%	31%	68%
Area under a curve	34%	53%	12%
Integral	60%	25%	13%
Equilibrium	1%	51%	46%
Mass balance	1%	10%	87%
Rate constant	25%	5%	68%

Note. The set of terms was provided to students as an ungraded DE class quiz to assess their ability to define them theoretically (symbolically) or conceptually. The table provides % that responded correctly to each term as well as % wrong or missing.

DE is one of the first courses where students gain experience translating the description of a solution from a mathematical expression to a graphical or visual representation. This is particularly useful for autonomous, or time-independent, first order differential equations for which representative solutions are easy to sketch with a direction field but may be more cumbersome to solve explicitly. Students explore qualitatively how solutions are related, how they are impacted by initial conditions and parameter values, and what long-term behavior looks like. Solutions to second-order constant coefficient initial conditions that represent the equation of motion similarly describe the observed behavior of the state quantity. For example, an underdamped system is pictured as the name describes, with the underdamped variable typically overshooting its steady state, establishing damped oscillations, and eventually settling into its steady state response. Students can change the physical coefficients in the problem (i.e., increase a damping coefficient to increase damping) and observe the change in behavior of a solution, made even more accessible by the appropriate use of technology. The skills for representation and interpretation of results are more easily transferred to partner discipline-specific courses with the foundation given in DE.

Further understanding and communication about the long-term solution of a dynamic system again leads to the use of multiple terms, meaning identical concepts from different perspectives. Specific to a time-dependent system, the *limit as time goes to infinity* and the *asymptotic* behavior both mathematically describe long-term behavior. Students fresh out of calculus may still be challenged by the notion of a limit and even an asymptote and therefore may not understand their practical implications, but should be able to transfer the idea of short-term vs long-term in time for greater understanding of the mathematical description. This distinction in an applied discipline however is often called *transient* vs *steady-state* with reference to a time-dependent system. The common DE text used at VCU introduces the latter terms as a foreshadowing for context that will be encountered by engineering and science majors. Furthermore, engineering students continue to conflate "transient" with "time-domain". This can be explained by two common practices in the engineering community: (i) the quickness with which an engineering curriculum proceeds, spending relatively little time discussing and demonstrating transient response and its importance, and (ii) the practice, in engineering simulators such as MultiSim, of using "transient" as a catch-all term to describe any time-domain solution.

Applications

As part of the SUMMIT-P partnership with engineering, the VCU DE course has recently transitioned to a greater focus on modeling, applications, and behavior of solutions as related to parameters. One beneficial consequence is familiarizing students with a variety of letters describing independent and dependent variables. For example, population growth, amount of radioactive decay, and attenuation of light intensity can all be described by the same form of a first order separable differential equation but typically describe the dependent variable with $\mathrm{P}, \mathrm{A}, \mathrm{I}$ respectively. Similarly, students learn that a typical mixing problem with constant inflow and proportional outflow has the same mathematical formulation as a description of drug metabolism with constant drug delivery and proportional metabolism, and so can be solved with the same tools though they would usually have different symbolic representations. Because a differential equation is usually solved symbolically to generalize for all possible values of variables and parameters, it is common to represent scalars and constants with representative letters such as "k" for a proportionality constant. Since students in differential equations are still novices with symbolic mathematics, the practice of modeling in this course is beneficial to understanding the meaning and purpose of a differential equation.

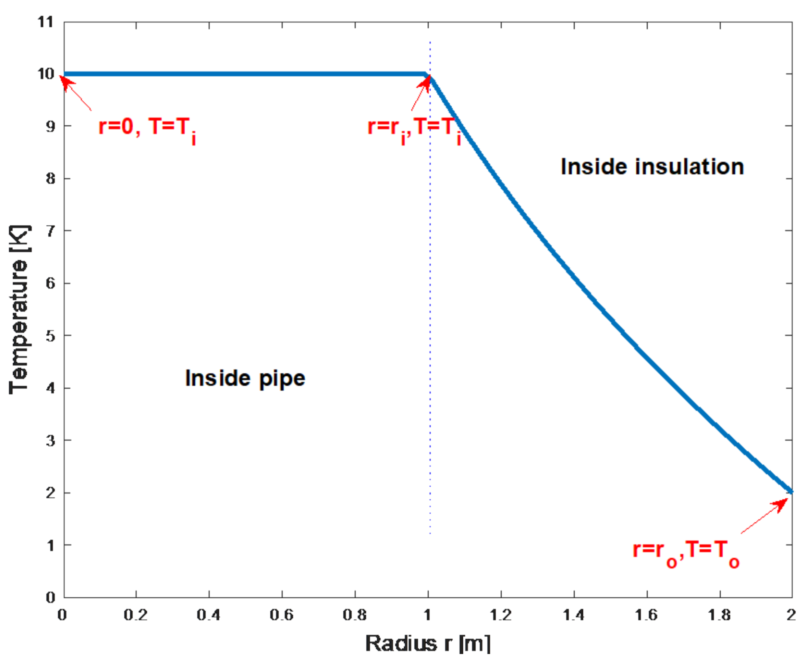
One classic example of a differential equation with analogs in multiple sciences that is given extensive attention in the VCU course is the second order spring-mass-dashpot (-damper) equation. This equation describing the displacement x of an attached mass is often derived in the class from balancing the forces of acceleration on the mass, the behavior of the spring, and any damping medium that resists motion. This equation has the advantage that its solution describes physical spring dynamics that are observable to the naked eye, and thus easily demonstrated in the classroom with accessible physics equipment. Concepts such as the equation of motion, damping, and forcing function then have real physical meaning that is immediately connected to the mathematical notation. In DE at VCU with many electrical and computer engineering majors, this second order differential equation is also presented as its circuit analog, a voltage balance on a Resistor-Inductor-Capacitor (R-L-C) circuit determined by Kirchhoff's voltage law (KVL). In this differential equation, the state variable is charge q, the mathematical equivalents of acceleration, damping, and spring forces are analogous to the voltage relationships for the inductance L, resistance R, and (the reciprocal of) the capacitance C correspondingly, and system dynamics are forced by an input voltage. The third hydraulic analog using the same differential equation applies to fluid flow induced by a pressure potential, including flows in physiology and biology, so it is of interest to our biomedical and chemical engineering students. It is not treated as extensively in the VCU course as in the prior two but even its brief mention in the course shows students the ubiquitous and transferable nature of this differential equation and its potential uses in future applications in partner disciplines.

Vignette – Virginia Commonwealth University

As part of the earlier collaborative work, the VCU team asked the broader engineering faculty to identify and submit typical course assignments that use introductory differential equations. These were compiled into a catalog of differential equations-focused application problems for the mathematics faculty to be able to introduce in DE to aid in knowledge transfer. The example shared here from mechanical engineering describes heat loss from a cylindrical pipe under steady state conditions (Filippas et al., in press) (see Figure 1 for a typical solution). In its original engineering-focused format, it was assumed that students already understood the heat transfer context and had passed the pre-requisite DE course being discussed here. Attention is given to using symmetry to transform a 3-D problem into a 1-D problem with dependence only on radius. The VCU team expanded on the details of heat transfer, set up the appropriate differential equation for the students, and made the mathematical questions more explicit in the first adaptation of this problem specifically for the DE course itself. For a particular implementation in Fall 2021, one mathematics professor modified it further to guide the students towards creating the differential equation themselves, plus added reasonable values for calculating and graphing a typical solution. In this version given to students in DE as classwork, only 2 of 7 questions directly used DE content – solving symbolically with separation of variables and finding the particular solution with a general initial condition. However, providing context around the DE content helps address issues discussed earlier such as like notation, terminology, and translation which smooths the knowledge transfer to future engineering courses. In fact, engineering students are often excited when this problem appears because they recognize the engineering style of the problem and have, on occasion, pulled out an engineering textbook to show the professor a similar problem!

Figure 1

Graph of Temperature for Steady-state Heat Loss through a Cylindrical Pipe Problem



Note. This problem is a 1-D modification of a 3-D physical scenario simplified by using radial symmetry.

Vignette – Norfolk State University

Another member of the SUMMIT-P consortium, Norfolk State University, also set up a teaching collaboration between their mathematics and engineering departments. Their team created several teaching modules on various engineering applications of differential equations. One of the most successful projects explores the application of a system of two second-order differential equations in the context of wireless power transfer (WPT) such as that which might be used for a smartphone charging pad. In this system, the two equations represent the transmitter circuit and the receiver circuit in the WPT. With R_1, L_1, C_1 *as the RLC components of the transmitting circuit of the WPT and* R_2, L_2, C_2 *as the RLC components of the receiving circuit of the WPT,* M *as the mutual inductance coupling the transmitting and the receiving circuits, and* $V(t)$ *as the input sinusoidal voltage source, a simplified system of equations that govern a WPT is given by*

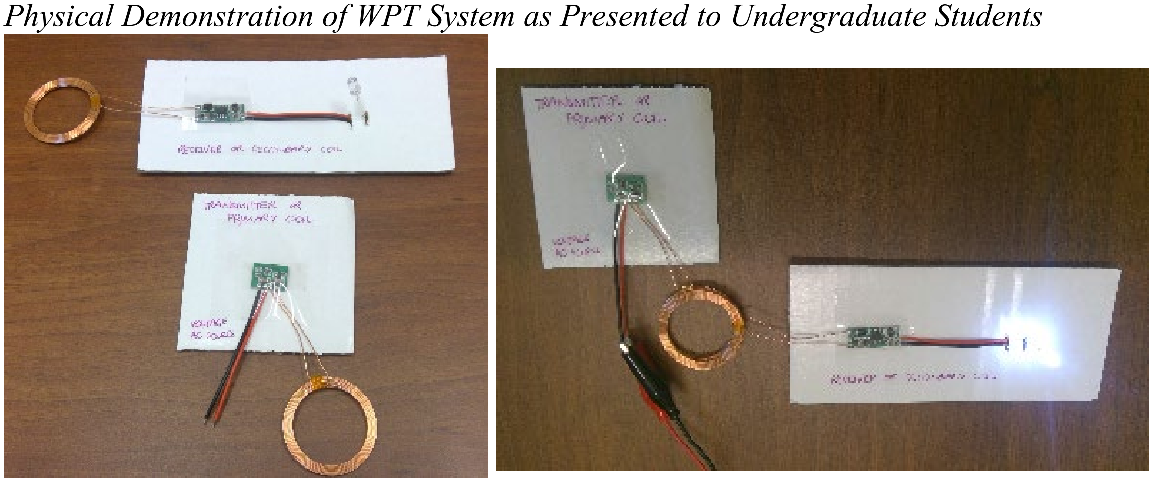
$$\begin{cases} L_1 i_1'' + M i_2'' + R_1 i_1' + \frac{1}{C_1} i_1 = V'(t) \\ L_2 i_2'' + M i_1'' + R_2 i_2' + \frac{1}{C_2} i_2 = V'(t) \end{cases}.$$

Here, the dependent variables are the currents $i_1(t)$ *and* $i_2(t)$ *of the transmitting and the receiving circuits, respectively.*

This teaching module is currently embedded in the latter part of a first course in differential equations. The module includes the physical demonstration pictured in Figure 2. Panel (a) shows the receiver circuit (secondary coil) has a LED that is initially off. When the transmitter circuit (or primary coil) is coupled to the receiver circuit, electrical power is wirelessly transferred so that the LED lights up as seen in panel (b). Students see that an electrical circuit can be viewed from a mathematical perspective so that they can transfer their mathematical skills to the engineering classroom. Faculty emphasize that each component in the circuit represents a term in the differential equation. In this way, students see a tangible example of an engineering application outside an engineering laboratory and learn that analyzing a circuit is tantamount to investigating the solutions to a differential equation.

Figure 2

Physical Demonstration of WPT System as Presented to Undergraduate Students



(a) (b)

Note. (a) Receiver's LED is off. (b) Primary coil (transmitter circuit) is coupled to the secondary coil, which turns the LED on.

Students' responses both from class and extracurricular demonstrations have been positive, even among students who have not formally studied differential equations. The tangible and portable demonstration facilitates the knowledge transfer between the differential equations and engineering concepts. At the end of each presentation, students leave with a version of the following mantra, "Every RLC (resistance-impedance-compliance) circuit is a higher-order differential equation with constant coefficients."

Cognitive Load Analysis

Having identified knowledge transfer as a nontrivial component of student learning of differential equations concepts, the VCU team questioned whether the instructors of discipline-specific courses account for the added cognitive load placed on students when solving a problem. For many engineering students, such courses will be their first extensive exposure to context requiring more than just being provided with a differential equation and the corresponding initial, final, or boundary conditions. As students then proceed through an engineering curriculum beyond differential equations, it is often the case that collateral requirements are needed to derive the appropriate differential equation for the application, use the problem description to derive the initial or final conditions, and, finally, to solve the equation and interpret the answer in the context of the specific problem. While the mathematics has not changed, students are required to perform a different set of cognitive tasks in the partner discipline than they are required to do in a mathematics course.

Cognitive Load Map

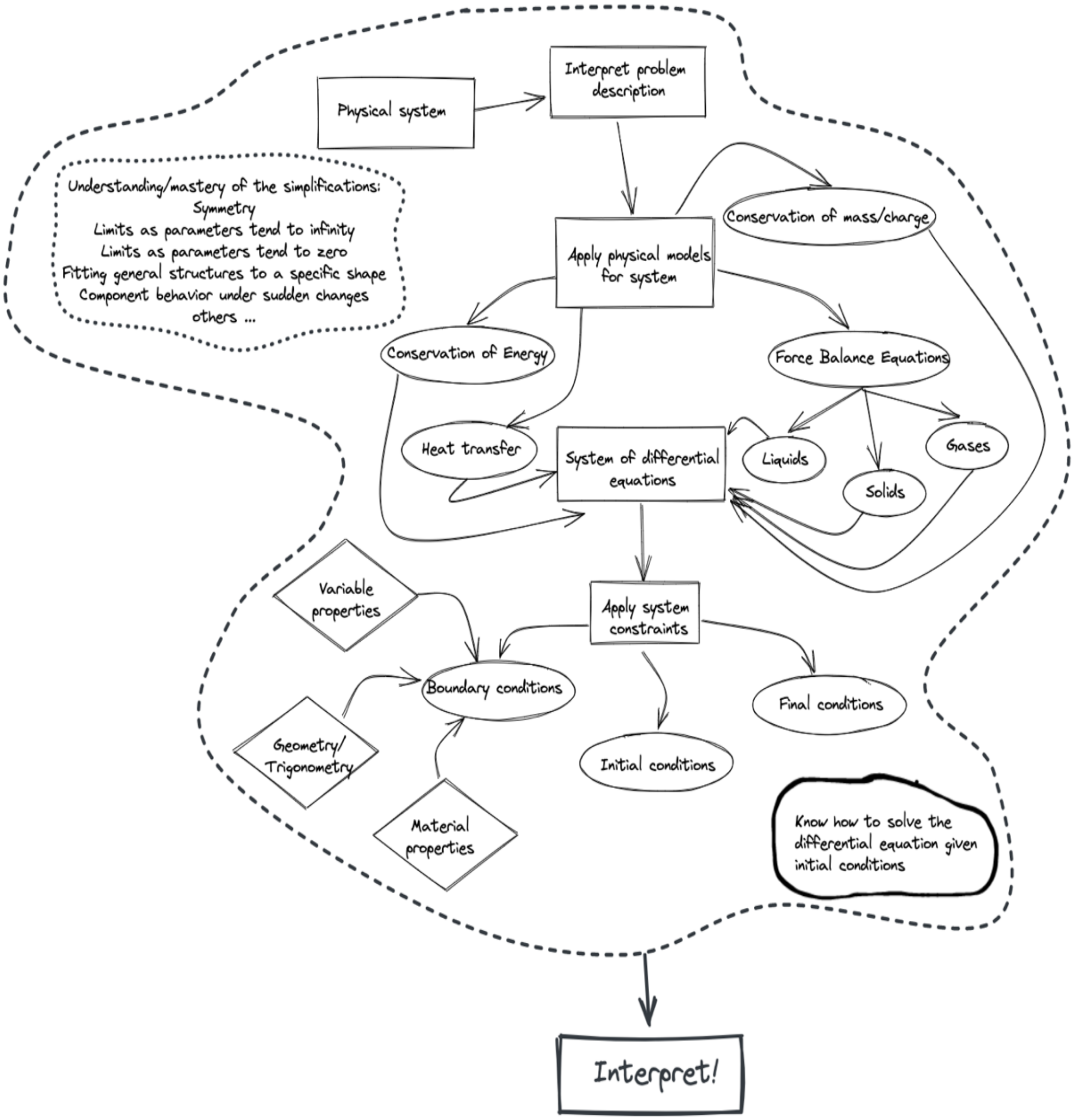
Figure 3 is an example of the knowledge, skills, and intuition an engineering student needs to develop when they are faced with a problem requiring the derivation and solution to a differential equation. The variety of naming conventions and increased complexity of the solution (e.g., being provided initial or final conditions vs having to derive them) is challenging for students who already have trouble seeing all the connections between their mathematics and engineering courses, making it even more important to show how the underlying mathematics is consistent between courses and disciplines. Therefore, it is important to maintain a reasonable cognitive load as the students move from the introductory level, agnostic mathematics course to the science and engineering courses that address problems of increasing complexity. For example, the modeling of a solution through the $e^{\alpha t}$ function is applied in introductory circuits, signals, and systems to introduce students to the transient response, but the $e^{j\omega t}$ function is used to solve more complex problems by implementing phasors in the frequency domain and is typically used to define the steady-state response of a system. Students need to move through these concepts in stages, with a strategic review of fundamental material and intentional linking of prior knowledge to current learning objectives. The weight of the accumulated cognitive load further motivates the creation of an adaptable document that not only links the mathematics to the science and engineering courses, but also the science and engineering courses to each other.

Example of Accumulated Cognitive Load

The challenge of transference of knowledge amplified by the accumulated cognitive load is illustrated here with a set of problems that are mathematically simple but require greater

Figure 3

Mind Map of Differential Equations in an Engineering Course



Note. The mind map demonstrates the actual complexity a student faces when solving a differential equation in an engineering course. Solving the equation itself comprises only a portion of the degree of mastery required.

engineering-specific sophisticated thinking by the students. In this example, a Resistor-Conductor (R-C) circuit and a Resistor-Inductor (R-L) circuit as shown in Figure 4 are described by the same form of a differential equation but differ significantly in the engineering concepts the students need to master in order to set up and solve these equations.

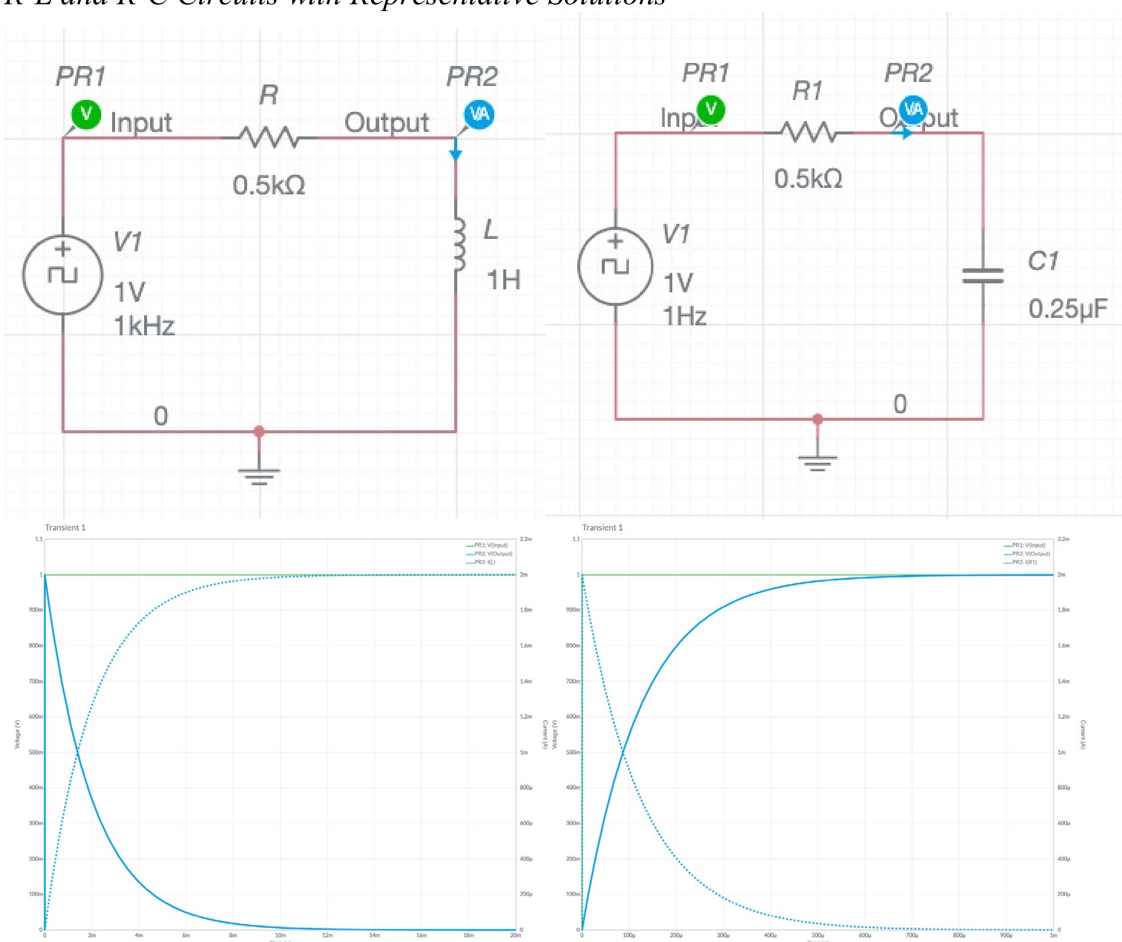
Both circuits in Figure 4 are described by a differential equation of the form $dx/dt + \alpha i = x_s(t)$ with $x(t)$ as current i, $x_s(t)$ relating to a constant or time-varying voltage source, and parameter α relating to a time constant characterizing the circuit's response. Students need to apply KVL to derive this equation using the same components discussed previously in the Applications section of this paper. In the case of the R-L circuit, $\alpha = \alpha_{RL} = -R/L$, while in the

R-C circuit, $\alpha = \alpha_{RC} = -1/RC$. Furthermore, in the above circuits, the square wave is simulating the action of a switch; i.e., at time $t = 0^{-}$ $(t < 0)$, the switch is open, and all initial conditions are zero. At $t = 0^{+}$ $(t > 0)$, the switch closes, connecting the source to R and L or R and C, respectively.

Mathematically this is a simple first order non-homogenous differential equation, for which there are standard solution techniques learned in an introductory DE course. Multiple challenges arise beyond the mathematics, however, including identifying equation components in the context of the engineering application and having a piecewise forcing function as the source. To solve this differential equation, students need to first understand that the square wave voltage source is acting as a direct current (constant) at all times except for at time $t = 0$. Solving the equation as a "zero-input response" is analogous to finding its homogeneous solution, giving the characteristic equation and roots λ. Thus, the zero-input response is found to be $i(t) = Ae^{\lambda t} + B$, and the students obtain $\lambda = -R/L$ for the R-L circuit and $\lambda = -1/RC$ for the R-C circuit.

Figure 4

R-L and R-C Circuits with Representative Solutions



Note. Both circuits are described by a differential equation of the form $\frac{dx}{dt} + \alpha x(t) = b x_s(t)$.

Left: R-L circuit (top) and solution $i_L(t)$ (bottom) to the DE $\frac{di}{dt} + \left(\frac{R}{L}\right) i = \frac{v_s}{L}$. In the solution for $i_L(t)$, $\lim_{t\to\infty}(i(t)) = \frac{v_s}{R}$ and $\lim_{t\to\infty}(v_L(t)) = 0$. Note the sharp increase of $v_L = v_s(0^{+})$ at time $t = 0^{+}$and the asymptotic decay to zero as $t \to \infty$.

Right: R-C circuit (top) and solution $i_c(t)$ (bottom) to the DE $\frac{di}{dt} + \frac{1}{RC} i = \frac{1}{R}\frac{dv_s}{dt}$. Similar to the R-L circuit, the solution graph shows $i_C(0^{+})$ increasing sharply and going asymptotically to zero.

Students need to subsequently develop the initial conditions from the engineering context, again using KVL as well as their knowledge of the physical limitations of the devices in the circuit. For example, the current through the inductor and the voltage across the capacitor cannot change instantaneously, but KVL has to be satisfied every time, t. Thus, for the R-L circuit, $v_L(0^+) = v_s(0^+) = Ldi/dt|_{t=0^+}$; so, $di/dt|_{0^+} = v_s(0^+)/L$. Similarly, for the R-C circuit, $v_C(0^+) = 0$, so $v_R = i(0^+)R = v_s(0^+) \Leftrightarrow i(0^+) = v_s(0^+)/R$. Using these context-specific initial conditions students derive the final solutions: for the R-L circuit, $i_L(t) = (v_s/R)(1 - e^{-(L/R)t})$ and for the R-C circuit, $i_C(t) = (v_s/R)e^{-t/(RC)}$.

Fully completing the problem requires that students interpret the solution to understand how each component will respond. The terms L/R for the inductor and RC for the capacitor are considered the "time constants" with units of time. This makes the terms $(R/L)t$ and $t/(RC)$ unitless and therefore consistent with the exponential function. Students observe how units are derived from the nature of the components, and the solution, derived from the laws governing these components, naturally leads to the correct units. Furthermore, the R-L circuit solution $i_L(t)$ in Figure 4 (left) shows a sharp increase of $v_L = v_s(0^+)$ at time $t = 0^+$and an asymptotic decay to zero as $t \to \infty$, whereas the R-C circuit solution $i_c(t)$ (right) shows $i_C(0^+)$ increasing sharply and going asymptotically to zero.

These types of problems use the natural, unforced, or autonomous response, and refer to systems that are not subjected to a continuously changing signal. As such, they might become activated by a sudden change – modeled as initial conditions – but will revert asymptotically to their natural state. Also, this problem deals exclusively with discrete components, so students need to track only one independent variable. More typical and realistic problems deal with changes in space and time, and do not generally have closed-form solutions. This means that students must understand the physical laws governing the system, start from an accurate model for its behavior, but then use symmetry or specific problem simplifications (solution along an axis of symmetry, etc.) to be able to solve the differential equation for a specific case.

Conclusions

We have presented several examples of disparate ways terminology, concepts, and applications are presented in mathematics and engineering courses as well as the added cognitive load placed on new learners when they are required to apply mathematics concepts to complex problems. These challenges possibly serve to disengage students' understanding of the role mathematics plays in their chosen field and the power it has to aid in the systematic and effective design of innovative solutions to engineering problems. It is important, therefore, to continue the intentional links between mathematics and engineering in the engineering courses, through the review of concepts but also through the continuous application of these concepts in problems of gradually increasing complexity.

It behooves faculty, therefore, to address classes that would benefit from the merging of mathematics and discipline-specific terminology, not with the question of whether the course can be taught without this knowledge, but rather as an opportunity for further student engagement. Other successful models include review sessions either on an as-needed basis or a "mathematics bootcamp" at the beginning of the course to review and align terminology. This, however, cannot replace the systematic application of increasingly complex mathematics skills in all coursework and an in-depth discussion of the information these equations provide about the behavior of the systems under review. Regardless of the approach, clearly the only way forward is with good communication between the faculty.

Finally, each mathematics course and partner discipline pairing will have specific concerns but providing the time and opportunity for students to study the beauty and utility of mathematics in the solution to a multitude of everyday challenges that impact the human condition, and to build intentional links between mathematics and their specific field of interest will enhance student learning and improve both student and instructor engagement.

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