MASTER

ACTUARIAL SCIENCE

MASTER'S FINAL WORK

INTERNSHIP REPORT

ANALYSIS OF THE SOLVENCY RISK IN DEFINED BENEFIT PENSION SCHEMES (UK)

JOANA TORRÃO PINHO

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ORIENTAÇÃO:

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Abstract

This report was done during an internship in the Willis Towers Watson Lisbon office, where I had the opportunity to perform various actuarial valuations of UK pension schemes.

Roughly speaking, an actuarial valuation assesses if the scheme has enough assets to cover its liabilities. In particular, a solvency valuation assumes the closure of the scheme and allows us to calculate the price charged by an insurance company to assume the responsibility of paying all the benefits accrued to that date. To perform these valuations, some important decisions must be made, such as establishing the economic and demographic assumptions that best represent reality.

By law, the actuarial report of a pension scheme must provide an estimate of its solvency on the date of valuation. Such estimate is naturally calculated in the context of a solvency valuation, and it is an important topic. This was the source of motivation for our work, in particular the idea of testing the impact of changes in the chosen assumptions by performing sensitivity analysis. Furthermore, by calculating the ratio between the amount of the scheme's assets and the value of calculated liabilities, we obtain the solvency level.

In this report we will use this measure to evaluate the scheme's economic position on solvency risk and we will provide some investment advice, according to the level found in the different scenarios produced. Because of confidentiality requirements, a dummy client was used to help us in the analysis performed.

Key words: Pension Schemes, Solvency Level, Actuarial Assumptions, Solvency Risk.

Resumo

Este relatório foi elaborado durante um estágio no escritório de Lisboa da Willis Towers Watson, onde eu tive a oportunidade de realizar várias avaliações atuariais em esquemas de pensões do Reino Unido.

De grosso modo, uma avaliação atuarial estima se um esquema tem ativos suficientes para cobrir o valor dos seus passivos. Em particular, uma avaliação de solvência pressupõe o encerramento do esquema e permite-nos calcular o preço cobrado por uma seguradora para assumir a responsabilidade de pagar todos os benefícios acumulados até essa data. Para realizar estas avaliações, algumas decisões importantes devem ser tomadas, tais como estabelecer pressupostos económicos e demográficos que melhor representem a realidade.

Por lei, o relatório atuarial de um esquema de pensões deve providenciar uma estimativa da sua solvência na data de avaliação. Essa estimativa é naturalmente calculada no contexto de uma avaliação de solvência e é um tópico importante. Esta foi a fonte de motivação para o nosso trabalho, em particular a ideia de testar o impacto das alterações nos pressupostos escolhidos através da realização de análises de sensibilidade. Além disso, calculando o quociente entre o valor dos ativos do esquema e o valor dos passivos estimados, obtém-se o nível de solvência.

Neste relatório usaremos esta medida para avaliar a posição económica do esquema quanto ao risco de solvência e daremos algumas recomendações de investimento, de acordo com o nível encontrado nos diferentes cenários produzidos. Devido a requisitos de confidencialidade, um cliente fictício foi usado para nos ajudar na análise realizada.

Palavras-chave: Esquemas de Pensões, Nível de Solvência, Pressupostos Atuariais, Risco de Solvência.

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Acronyms

ALM – Asset Liability Management

CARE – Career Average Revalued Earnings

CMI – Continuous Mortality Investigation

CPI – Consumer Price Index

DB – Defined Benefit

DC – Defined Contribution

GMP – Guaranteed Minimum Pension

GPA – GMP Payment Age

LSC – Lisbon Service Centre

PPF – Pension Protection Fund

PV – Present Value

RPI – Retail Prices Index

SAPS – Self-Administered Pension Scheme

SERPS – State Earnings Related Pension Scheme

UK – United Kingdom

WTW – Willis Towers Watson

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1. Introduction

This report was developed as the Master's Final Work in the context of my master's degree in Actuarial Science. It was done during a 5-month internship in the Lisbon Service Centre (LSC) of Willis Towers Watson (WTW). WTW is a global advisory, broking and solutions company, operating in many countries all over the world. In Lisbon, the LSC performs different actuarial valuations for UK and Western Europe. In my internship I have worked with UK pension schemes, performing calculations in Defined Benefit Schemes, namely carrying out actuarial valuations.

During this time at the LSC, I had the opportunity to question the importance of the choice of the actuarial assumptions, both economic and demographic, used to perform the different valuations. These assumptions meant to represent in the best possible way all the information of the scheme, such as membership information, market conditions, pension and salary increases and types of retirement. This led to the opportunity of performing sensitivity analysis to test the impact of changes in the chosen assumptions.

At the same time, the detailed work performed at the LSC gave me the opportunity to analyse different risks present in Defined Benefit (DB) schemes. Solvency risk represents the likelihood of the scheme to become insolvent and it can be attenuated with an adequate management of the pension fund. When the LSC attains their results on the valuations performed, the work goes to the consultant team in the UK, who will advise the client regarding the investment strategy to manage the pension fund for the next years. This gave me the thought that I could be the one analysing the riskier scenarios and project ways to invest.

Thus, the objective of this report will be to study solvency risk throughout the different scenarios produced by the sensitivity analysis.

This report is structured in five chapters. In Chapter 2, we will present a brief introduction of UK pension schemes, with focus on state schemes and occupational schemes. The basic concepts on these topics will be introduced as well as the notion of actuarial valuations. Chapter 3 discusses the topic of risk, presenting ways to manage and mitigate risks in Defined Benefit schemes. Furthermore, we will analyse the different types of risks that are present in this kind of schemes. Chapter 4 contains a case study on the sensitivity analysis of various assumptions of an actuarial valuation, considering all the concepts previously addressed. The chapter closes with the main conclusions of these analysis and with some investment advice regarding the different risk scenarios that have been found. Finally, in Chapter 5 we wrap up with a few final thoughts.

2. Pension Funds in the UK

A pension fund is linked to the investment of the contributions made by the members, or on behalf of them, of a pension scheme, so that they are entitled to receive benefits at retirement. Members can belong to various statuses, being active when they are still working and building pensionable service, deferred when they leave the scheme but are still working, retiree when they retire, or dependent in cases where they financially depend on a member who has died, for example spouses or children.

To deeply understand the concept, we will now explore how pension schemes work and of what type they can be in the specific case of the UK.

2.1 State Pension Schemes

The State Pension Scheme was introduced in 1908 aiming to guarantee that all people would be provided with a basic amount of money during retirement. The amount each one receives does not depend on their salaries during the working years, but on National Insurance contributions that they have made while working. Everybody can make contributions to National Insurance between the age of sixteen and State Pension Age. The State Pension Age is currently 66 years old, but it is predicted to rise to 67 years old between 2026 and 2028. When a person reaches this age, they are entitled to claim the State Pension. (Age UK, 2021)

Before 2016, the State Pension was composed of two tiers, namely a flat-rate basic pension and an additional one related with earnings, called State Earnings Related Pension Scheme (SERPS), but from 2016 onwards only one tier exists, the Basic State Pension. The Basic State Pension increases in April every year, the rate of increase being the highest of the following three percentages: the growth in average earnings; 2.5%; the annual variation of the consumer price index (CPI) at previous September. (Willis Towers Watson, 2021)

2.1.1 Guaranteed Minimum Pension

From 6 April 1978 to 5 April 1997, it was possible for Defined Benefit Occupational Pension Schemes to contract out their members from the SERPS, meaning that the scheme would be substituting the government in paying these benefits. As a reward, both employers and employees would have a payment reduction on National Insurance contributions. (Zajmi, A., 2019)

This new minimum pension provided by the scheme is called Guaranteed Minimum Pension (GMP). Since GMP was designed to replicate the state benefits, there are statutory rules that schemes must follow[1] to grant members with a retirement pension that is at least as much as the one they would have received under the SERPS. These rules define the age at which benefits start being paid and the increases each component receives once it is in payment. The age mentioned is the GMP payment age (GPA) which is 65 years old for males and 60 years old for females.

The calculation of GMP is divided in two parts:

(i) Pre 88 GMP is the part accrued from 6 April 1978 to 5 of April 1988 and it is calculated by the following formula:

$$\text{Pre 88 GMP} = \frac{\qquad\qquad}{\qquad \times \qquad} \tag{1}$$

where Earnings represents the members' earnings during the period accrued, the Contribution Rate is the employee's contribution rate and the Accrual Rate = 4 × "Working Life";

(ii) Post 88 GMP is the part accrued from 6 April 1988 to 5 April 1997 and is calculated using the same formula but, this time, Accrual Rate = 5 × "Working Life".

"Working Life" denotes the number of complete tax years between the start and end date. The start date is the latest of 6 April 1978 and the 6 April in the year of the member's 16th birthday. The reference to calculate the end year is the 5 April before member's GPA date. Thus, "Working Life" will be between 20 and 49 years for males and between 20 and 44 years for females (Willis Towers Watson, 2019a,b).

[1] https://www.willistowerswatson.com/en-GB/Insights/2019/02/questions-you-are-too-afraid-to-ask-about-gmp-equalisation#3.What%20is%20GMP?

2.2 Occupational Pension Schemes

In the UK, the most common pension schemes are the non-State Pension Schemes, more precisely, Occupational Pension Schemes. These schemes are set up by employers, as a benefit that is paid on top of the State Pension Scheme, but contributions are usually made by both employers and employees. To calculate the amount of the pension, we need to consider the type of scheme: Defined Contribution (DC), Defined Benefit or Hybrid. (Broadbent, J., Palumbo, M., Woodman, E., 2006)

2.2.1 Defined Contribution Schemes

In a DC scheme, the contribution amount is a fixed value agreed by the two parts involved, this is the sponsor and the employee, at the beginning of the contract. Usually, it is deducted directly from the employee's pay and some portion of it can be matched by the employer. This contribution is made to a 'pot' that needs to be managed, and the money is invested in a fund according to the level of risk that the member is willing to take. When retirement is reached, this 'pot' is used to buy a pension. In this way, the pension at retirement results from the combination of the amount accumulated during the employee's working career, which depends on the contributions made and the investment returns earned, and the choice of retirement product.

Nowadays, this type of scheme is being preferred by the employers since contributions are fixed, in the sense that no additional contributions are required if something goes wrong. However, it can be worst for employees once they are the ones bearing the risk.

2.2.2 Defined Benefit Schemes

In a Defined Benefit scheme, the benefit amount that the member receives at retirement is given by a formula and does not depend on the fund's investment performance. This amount can be calculated in different ways, the following two being the most common ones: one is based on final salary and the other is based on career average revalued earnings (CARE). The first option pays to the member a percentage of the final pensionable salary, based on formula (2) below (Pires F., 2018):

$$= \quad \% \times \qquad \times \qquad , \qquad (2)$$

where % represents the accrual rate, which is the proportion of the employee's earnings he/she will get as a pension for each year in the scheme.

The second option (CARE) is calculated in a similar way, but it uses the average of the earnings of the member's working career as we can see in formula (3) (Pires F., 2018).

$$= \% \times \quad \times \quad ' \quad . \qquad (3)$$

As we can see, in DB schemes the employee does not bear the risk. However, for the employer this type of scheme can be very heavy since it is necessary to make the promised payments at retirement. To guarantee a best performance of the fund, the employer will have the help of a trustee[2] to manage the pension scheme. The role of the trustee is to decide how the pension scheme's assets will be invested, having in mind the best interest of the members and to ensure that there is enough money to pay the members' pensions at retirement.

Another important feature of this type of schemes is that there is the option for commutation, meaning that at retirement the member has the option to take a tax-free lump sum in exchange of smaller pension amounts throughout the rest of the years. This value can be calculated in many ways. The most common is expressed as a fixed percentage of the total value of the pension, usually not more than 20%. Another approach is to use specific table factors that specify how much pension can be commuted at each age. These factors normally decrease as age increases, since commutation will substitute part of the annuity paid to the pensioners.

2.2.3 Hybrid Schemes

A Hybrid Scheme results from the combination of both DC and DB schemes. The biggest example of this type of schemes is the Cash Balance plan, which for tax, accounting and regulation is treated as a DB scheme but, similarly to what happens in DC, the income from scheme's investments does not affect the benefit amounts promised to the members.

[2] https://www.gov.uk/guidance/pension-trustees-appointment-and-role

During my internship, my focal point was Defined Benefit Schemes. Noting that actuarial risks (related with all the calculations performed by an actuary, as explained in topic 3.3 of this work), are more present in those schemes, from now on, I will focus on them.

2.3 Actuarial Valuation

An actuarial valuation of a scheme is an analysis performed by an actuary, required at least every three years. This analysis aims to study the financial position of the pension scheme, through the estimation of the cost of paying out pensions for all its members, by calculating the value of liabilities and comparing it with the value of assets. In order to conduct a valuation, we need to set some assumptions, such as:

Financial assumptions

- Discount rate
- Pensions increases
- Inflation rate
- Salary increases

Demographic assumptions

- Mortality
- Proportion married and age difference
- Early retirement
- Ill-health retirement
- Normal retirement age
- Withdrawal from the scheme
- Commutation

It is important to mention that the proportion married and age difference are not always provided in data, so we need to add it as an assumption. In the cases we receive data, we use the assumption that proportion married will decrease over the years because it is affected by the spouses' probability of survival.

Using these assumptions, and bearing in mind the fact that the scheme's liabilities can be calculated by different methods, means that the valuation can take different values. So now it is worth to look to the different valuation methods and analyse their purposes. (The Pensions Regulator, 2021b)

Funding / Technical Provision Valuation: The main goal of this valuation is to evaluate if the scheme has enough assets to cover its liabilities and calculate the scheme's costing method for the upcoming years. This is related with the process of calculating technical provisions, to find the amount of money necessary to fully pay members' benefits when they retire. These estimates are subject to the investment strategy used by the scheme and prudent economic and demographic assumptions.

PPF / Section 179 measure: Pension Protection Fund (PPF) valuation is mandatory by the section 179 of the Pensions Act 2004[3]. It was created to pay the benefits to the members of the scheme when the employer gets insolvent and there are not enough assets to cover the compensation level. The assumptions used in this type of valuation are pre-defined by the PPF prudently and the result is generally less generous for participants than that in a funding valuation.

Solvency / Insurance Buy-out: To calculate a solvency estimate, actuaries assume the closure of the scheme at valuation date and treat actives as deferred members. Since solvency measures the price of a buy-out, with this valuation we can know how much money is needed to buy all the benefits accrued from an insurance company. Usually, schemes apply a de-risking strategy while performing a buy-out, to prevent insolvency. Calculating these estimates does not mean applying a buy-out however it let us know what will be needed if schemes want to follow that path. This is the most expensive approach, but for the members and trustees it is considered the safest method to protect the scheme's liabilities.

Self-sufficiency: When a scheme has enough assets to cover liabilities and no more contributions will be made, it reaches a self-sufficiency level. From this point on, the investment strategy of assets should be held at a low-risk basis to minimize the chances of depending on the employer.

Accounting Valuation: Employers use the accounting valuation to present their annual Reports and Accounts, providing a consistent dimension of accounting costs across similar companies and allowing stakeholders to know that pension schemes are considered in liabilities. The assumptions used are defined in the relevant accounting standards of the home country of the company.

[3] https://www.legislation.gov.uk/2004

3. Risk

As in any financial instrument, Defined Benefit schemes are subject to risks and it is relevant to study which ones are they and to find ways to mitigate them.

In general, according to the Corporate Finance Institute[4], risk is the probability that the real results will differ from the anticipated ones and it measures the uncertainty an investor is willing to take to make a profit from an investment. In another perspective it can be a situation involving exposure to danger. Thus, it is important to study the level of risk to achieve balance between the projected amounts and the actual outcomes. Two essential topics in respect to this are risk management and risk mitigation.

Risk management: The risk management process comprises three steps: identifying, analysing and responding. It presupposes the idea of an early control of future outcomes, to mitigate the risk impact and avoid a significant deviation from an anticipated result.

Risk mitigation: The mitigation process is finding a way of attenuating the impact that risk can have on a scheme. It involves planning for major disasters and prioritize what risks will have the bigger impact on the scheme.

3.1 Measurement and mitigation of risk in Defined Benefit Schemes

The best way to measure the risk in defined benefit schemes is to carefully analyse the actuarial valuation. On the financial perspective, risk exposure associated to the funding of benefits can be detected by an actuarial valuation, which can help the trustees to identify those risks and make the appropriate decisions to mitigate them.

3.2 Managing risk in Defined Benefit Schemes

Risk management is becoming an essential and complex function inside financial organizations. In DB schemes, the two main goals should be reducing the pension cost of contributions and minimizing the risk of benefit reductions to the members. To

[4] https://corporatefinanceinstitute.com/resources/knowledge/finance/risk/

achieve this, according to the fund's objectives, the key tasks are to project, to monitor and to revise the investments made by the fund.

Asset Liability Management (ALM) is the name of a process that manages the allocation of assets considering the liabilities, in the direction of having the resources to meet the commitment of not failing to pay the benefits, avoiding illiquidity (Neuhaus, W., 2020). It is composed of a set of tools designed to maximize the likelihood that the aims of profitability and solvency of the company are reached. At an economic level, this is done by comparing the present value of liabilities with the present value of assets, figuring out where to allocate them. ALM implies creating mathematical scenarios of pension fund future liabilities and assets. The conventional approach to do this is by selecting a main scenario and carrying out some tests around it (Blome, S., Fachinger, K., Franzen, D., Scheuenstuhl, G., Yermo, J., 2007). Also, ALM allows the discovery of alternative strategies that will make meeting the plan's objectives easier.

3.3 Risks present in Defined Benefit Schemes

Actuarial risks are related with all the calculations performed by an actuary, namely with the choice of assumptions defined for the valuations. In this section we will focus on five main actuarial risks present in DB schemes that are of interest to us: interest rate risk, inflation rate risk, longevity risk, solvency risk and covenant risk. Besides this, we will talk about general risks, precisely market risk, credit risk and operational risk.

3.3.1 Interest Rate Risk

Scheme's liabilities are very sensible to fluctuations of long-term interest rates, so we can say that pension schemes have a significant connection with interest rates, since they are used to price their liabilities on a solvency basis.

Long-term interest rates in the UK are typically determined by gilts. Gilts[5] are debt issued by the UK government and are considered low risk due to the financial safety of the government.

[5] https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/gilts/

The risk measure used to quantify changes in liabilities due to fluctuations in interest rates is the duration, usually expressed in years. The duration can also be seen as the average payment period of liabilities and the equation is expressed as follows:

$$= \sum_{=1} \frac{\frac{\times}{(1+\ )}}{}, \qquad (4)$$

$$= \sum_{=1} \frac{}{(1+\ )}, \qquad (5)$$

where $\ ,\ = 1, \dots ,\ $, represents cash flow in year *t*, is the discount rate (and is the number of years), see (Broverman, S. A., 2017).

It is easy to understand that borrowers always search for low interest rates, while for lending the most attractive rates are the highest ones. But in pension schemes it is not that straightforward. When in DB schemes companies calculate their liabilities using the present value of future cashflows, they usually use a discount rate linked to long-term interest rates and a reduction in interest rates means an increase in the value of liabilities. The impact of this reduction varies with the duration of assets and liabilities. DB pension funds with long-dated, interest rate-sensitive liabilities will, unless they are hedged, have duration of liabilities greater than the duration of assets. Therefore, a fall in long-term interest rates may cause a significant negative impact on DB schemes, in the sense that the present value of liabilities is very likely to significantly exceed the present value of assets.

This condition can be different when the benefits of the pension plan are linked to salary or inflation (Antolin, P., Schich, S., Yermo, J., 2010). Protracted low interest rates periods anticipate future economic conditions consisting of long periods of low inflation rate on salaries. This reduces the volume of benefits to be paid in the future, and lowers returns on funds, which also makes future pension benefits lower. Thus, the impact of interest rates in pensions linked to salary is attenuated.

Lastly, it is important to notice that different methods of valuation can produce different impacts on liabilities, from low interest rates periods. The reason for this is the fact that the assumptions about the discount rates used vary from method to method.

Summarizing, the two main points to be aware of are: (i) to verify if inflation and future salaries are considered in the calculation of liabilities; (ii) to determine the rate used to discount the value of liabilities to the valuation date.

3.3.2 Inflation Rate Risk

Inflation risk is linked to the prospect of inflation weakening the outcome of an investment. Failing to anticipate changes to inflation rates causes a risk that the future real return on an investment will be less than expected. This happens in the case where inflation rates are higher than anticipated by the scheme, creating a situation where the growth in payments is bigger than predicted, thus producing a loss that results in more contributions to the fund. However, the opposite can also happen.

3.3.3 Longevity Risk

Longevity risk is associated to the possibility of the members' life expectancy being longer than what was predicted, resulting in payment of benefits for a longer time.

We can highlight three reasons that impact longevity (Rodrigues, R. M., 2020):

1. Mortality improvements: as mentioned before, when schemes are estimating the cost of paying out the pensions to their members, they use mortality assumptions, but it may happen that population live more than what was expected. This is becoming more frequent because life expectancy is improving in a faster way due to medical and scientific improvements.
2. Heterogenous population: The second reason is based on the choice of the mortality tables used in actuarial valuations. Traditionally they include two variables, age and sex, and the population is treated linearly. However nowadays other factors can have great impact on these tables too, such as education level, professional career and economic stability.
3. Risk of anti-selection: Adverse selection is the promise of coverage of events with an above average probability of incidence and whose premiums are not enough to cover the risk that is insured. An example of this is health issues that can shorten longevity, such as hypertension, diabetes and smoking.

Since it is hard to anticipate the future, this is a very complex risk to control. To attenuate this, schemes can use de-risking methods to transfer the risk to another party that is willing to have that kind of risk in their portfolio.

3.3.4 Solvency Risk

A pension scheme becomes insolvent when its sponsoring employer can no longer support the scheme and there are not enough assets in the pension fund to cover the liabilities, so the financial obligations are not met. As mentioned before, the PPF was created to protect members in this situation, guaranteeing that they receive compensations.

Another important regulatory framework was created in 2009 by the European Union, called Solvency II. This package of rules aims to promote transparency, competitiveness and comparability between companies, by analysing their governance system and requiring them to produce risk and solvency assessment on a regular basis. Solvency II requires companies to hold capital in relation to their risk profiles, to ensure that they have sufficient resources to overcome financial problems.

After Brexit, the future of Solvency II in the United Kingdom became uncertain. However, the House of Commons Treasury Committee has been trying to understand Solvency II strengths and weaknesses to chart its destiny in UK (Institute and Faculty of Actuaries, 2021).

3.3.5 Covenant Risk

The employer's obligation and financial capability to assure their DB scheme now and in the future is called the employer's covenant[6]. In this type of schemes, as mentioned before, the company promises to ensure the payment of present and future benefits, with the help of a trustee. Alongside the management of the allocation of the assets, the trustees need to judge the strength of the employer's covenant, which can be considered weak or strong. The first case happens when there is a wide gap between the amount needed to pay the benefits and the value obtained by the scheme's

[6] https://www.pensionsadvisoryservice.org.uk/pension-problems/pension-security/the-employer-covenant

investments performance. To bridge this difference, the trustee may ask the employer to make additional contributions.

In this way, covenant risk is about the capability of the company to fulfil its obligations.

3.3.6 Market Risk

The assets that the scheme invests in might not produce the expected outcome and the company will have to bear the difference. When this happens because of changes in market factors, such as interest rates, stock markets or exchange rates, it is due to market risk (Franzen, D. , 2010). This can cause a problem because most of the benefits increase with inflation, both pre-retirement and once in payment. Besides, we can say that market risk increases with exposure to equity risk present in the assets of the scheme. Asset Liability Management is a way to mitigate this type of risk.

3.3.7 Credit Risk

Credit risk is “the potential that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms” (Basel Committee on Banking Supervision, 1999). Furthermore, the effective management of credit risk is a critical part of the complex strategy to control risk and is vital to the success of any banking organization.

In the case of Defined Benefit schemes, as they invest in non-guaranteed assets, either equities or debt, there is an associated risk that the issuer might fail the payments. When the assets owned by the scheme are managed by a different (management) company, asset incomes can be disturbed by the failure of that third party, as well as the access to those assets can be compromised and not be as easy as it should.

3.3.8 Operational Risk

The Basel Committee on Banking Supervision describes operational risk as “the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events” (Basel Committee on Banking Supervision, 2016). These relate to human errors, failed processes and information systems, fraud, accidents, natural disasters and problems linked to private management. Sometimes the risk can be seen as the failure of the recovery plan, such as having inadequate plans to recoup from a fire, for example not having insurance, or holding insufficient data to allow benefits to be precisely calculated.

The definition of operational risk also comprises external risk as a type of operational risk. It happens when government impose a regulatory adjustment such as changes on taxes or in solvency requirements, that may lead to additional problems on the firm trough, for example, matching new requirements or new compensation schemes.

Even though operational risk is harder to accurately identify (than market or credit risk, for instance), it is usually the one bringing more losses to the schemes[7].

3.4 De-Risking strategies

DB de-risking refers to the implementation of investment strategies to improve the chance of a pension scheme fully paying its benefits on time, reducing its own level of risk, more precisely solvency risk (Pension and Lifetime Savings Association, 2019). It can be a very complex topic and there is not a strategy that fits all schemes, since the decision will depend on the target and objective of each one.

Buy-in and buy-out contracts are the most common methods used by schemes (Ruilin, T., & Jeffrey C., 2020). Under a buy-out, the scheme allocates all its liabilities and assets to an insurance company that will carry the legal responsibility of paying the scheme members' pensions. By doing this, the scheme trustees handover all their obligations to the insurance firm, and the scheme members become policyholders of the insurer. Consequently, all risks are transferred to the insurer. In a buy-in contract, the administration of the scheme will not change hands, and assets enough to cover the present value of its liabilities are allocated to an insurance company. In return, the insurer makes the commitment of paying future cashflows equal to the pension amounts of the scheme's members. Essentially, the insurance company makes periodical payments to the scheme and the scheme makes the payments to pensioners. The buy-in works as an asset of the scheme. In terms of risk this implies that the scheme

[7] https://www.mckinsey.com/business-functions/risk-and-resilience/our-insights/the-future-of-operational-risk-management-in-financial-services

shifts most of the risk to the insurer company. Nevertheless, the scheme is taking another risk, the external risk, once it becomes dependant on the third party of the contract.

The final type of de-risking strategy we will discuss is longevity swaps. Longevity swaps are similar to buy-in contracts in the sense that the scheme holds its members and the obligation of payments. In this type of contracts, the other party (an insurer, for instance) takes the longevity risk. The pension plan will make frequent payments to the insurer based on expected mortality rates, whereas the insurer will pay to the scheme amounts according to actual mortality.

Longevity swaps are very attractive because they allow trustees to manage separately longevity risk and investment risk, they reduce external risk present in buy-out and they hedge funding needs, since cash flows are traded throughout the life of the deal[8]. Furthermore, the demand for longevity swaps is expected to increase during the next years because of the constant increase in life expectancy.

These strategies can be expensive. For an updated overview of these subjects see (Blake, D. P., & Cairns, A. J. G., 2021).

[8] https://www.swisslife.com/en/home/media/media-releases/news-archiv/longevity_swaps__a.html

4. Cash Flow Analysis

A cash flow stream represents the entry and out-go of cash from a business or project, roughly working like a bank account. Through the analysis of cash flows, we can know if we are in a deficit or surplus situation and predict how much will be needed to keep the business on track (Institute and Faculty of Actuaries, 2018). Hence, in this project the goal will be to analyse liability cash flows of a pension scheme valuation process and conclude what measures to take according to its financial position.

As mentioned in Chapter 2, a solvency valuation assumes that the scheme closes at valuation date, and we will evaluate all members at that date as if no more future salary will be paid. Thus, actives will be treated as deferred pensioners.

In this part of the work, the target is to perform a sensitivity analysis of the assumptions that may impact a Solvency run. In this way, we will be able to see, through the cash flows produced, the changes in the values of liabilities and conclude which assumptions lead to riskier scenarios, mostly translating cash flows into financial positions and then into investment suggestions.

To help us accomplishing our goal, it is important to present the concept of solvency level. The solvency level measures the present value of liabilities as a percentage of the value of scheme's assets, pointing the level of deficit or surplus identified. A level of 100% implies that the scheme has enough resources to support all the payments promised to employees. Furthermore, we will see what is recommended to do in cases where the level is lower or higher than this. The objective of our analysis, as already mentioned, is to isolate the effect of the choice of valuation assumptions and, therefore, the value of assets will remain the same throughout the different analysis. This might have an impact on solvency level, but the main goal is to study only liabilities. The value assumed for assets is then £ 181 110 820, defined in the beginning of this work to represent a starting point of 90% solvency level, compared with the initial solvency valuation scenario. This is consistent with the average funding level observed on UK pension schemes over 2021 (The Pensions Regulator, 2021a).

4.1 Data

To produce the analysis mentioned, we will use a dummy client subject to a funding valuation approach. For this reason, there is not a real source to the tables and figures. The rules that define this client are shown below.

Table 1 - Scheme information

General information	
Open/closed to new entrants	Closed at 04/01/2011
Open/closed to future accrual	Open
Retirement Benefits	
Normal retirement age	65 years
Pension calculation	1/60 × final pensionable salary
Spouse's pension	50% × pre commuted member's pension

For the economic assumptions, we are assuming a discount rate of 1% and an inflation rate RPI of 3.7% and CPI of 2.5%. Furthermore, the pensions in payment for deferreds, retirees and dependants are subject to increases according to the following rules:

- The amount correspondent to the Guaranteed Minimum Pension accrued before 6 April 1988 does not increase once in payment.
- The amount correspondent to the Guaranteed Minimum Pension accrued after 6 April 1988 increases in line with CPI, with a floor of 0% and a cap of 3% (for example, if CPI is 2.5%, we will use 2.5%; however, if it is -0.2%, we will use 0% or if it is 5%, we will use 3%).

Other than these two amounts, we have the excess pension divided into three tranches:

- The amount accrued until 5 April 1997 does not receive increases.
- The amount accrued between 6 April 1997 and 5 April 2005 increases annually in line with RPI with a floor of 0% and a cap of 5%.
- The amount accrued from 6 April 2005 onwards increases annually in line with RPI, subject to a floor of 0% and a cap of 2.5%.

Finally, the actives' salary increases on 6 April are linked to RPI, which means that we will use the real rates tabulated. Thus, in this analysis, salary will be more sensitive to changes in inflation as it has no minimum or maximum defined.

The demographic assumptions used were:

	Actives	Non-actives
Mortality tables	Males: S2NMA*1.05; CMI_2019_M_(1_50%) Females: S2NFA*1.03; CMI_2019_F_(1_50%)[9]	
Proportion married	Males: 90% Females: 90%	Males: 90% Females: 80%
Age difference	Females are on average 3 years younger than males	
Early retirement	No allowance	
Ill health retirement	Based on accrued pension	Not applicable
Commutation	No allowance	

Regarding the scheme's membership information, we present a condensed table.

Table 2 – Scheme membership

Active members	
Number	212
Total net Salary (annual)	£ 7 402 269
Average age (weighted by salary)	55.1 years
Deferred members	
Number	519
Total deferred pension p.a.	£ 1 580 855
Average age (weighted by pension)	55.5 years
Retirees	
Number	383

[9] A more detailed explanation is given in Section 4.4.

Total pension payable p.a.	£ 1 591 978
Average age (weighted by pension)	71.6 years
Dependants	
Number	53
Total pension payable p.a.	£ 120 356
Average age (weighted by pension)	70.9 years

Applying all these assumptions to data and performing a technical provisions valuation, we will obtain the following blue cash flow, based on predicted payments throughout the years. In the same graphic, we will see the orange cash flow produced under a solvency valuation. Values of payments are lower in the solvency valuation, since we are using a salary escalation rate higher than the inflation rate. In a solvency valuation, actives are considered deferred pensioners, which means that their pension will grow with inflation instead of salary escalation.

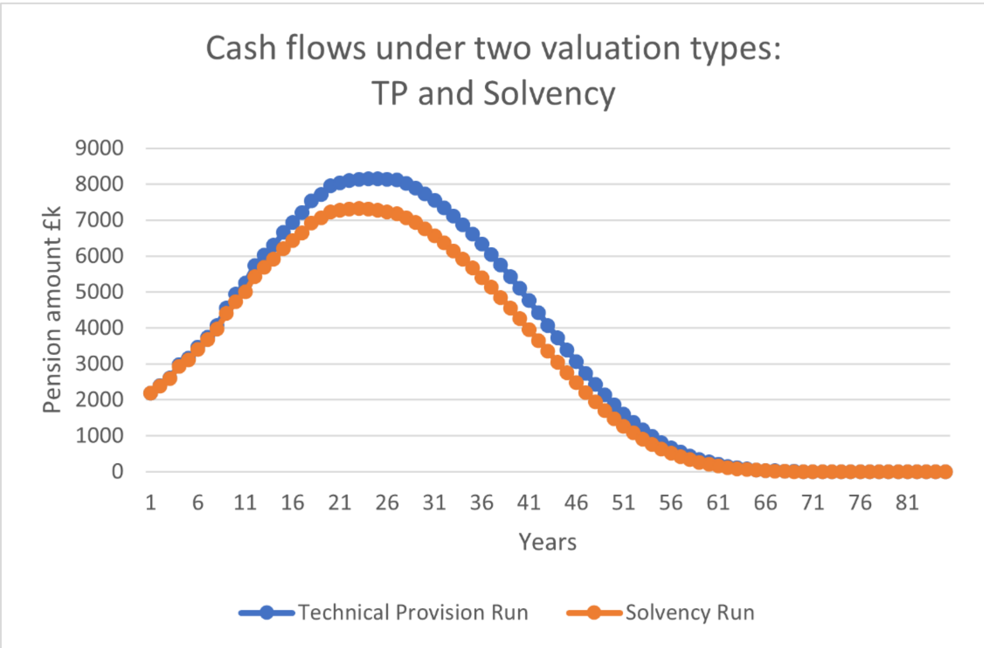

Figure 1 – Cash Flows under two valuation types (TP and Solvency)

We will now progress with three sensitivity analyses made using solvency valuation assumptions, specifically discount rate, inflation rate and mortality tables.

4.2 Discount rate

On the first sensitivity analysis we will study the impact of changes in discount rate. As we have seen in Chapter 3, the discount rate allows us to calculate the present value of our liabilities. The value of the individual yearly cashflows is nominal, thus the only item that will change will be the present value of liabilities, which will be calculated using the different rates selected for the test. As mentioned before, the original discount rate was 1%, so the analysis goes through rates between 0.5% and 1.5%.

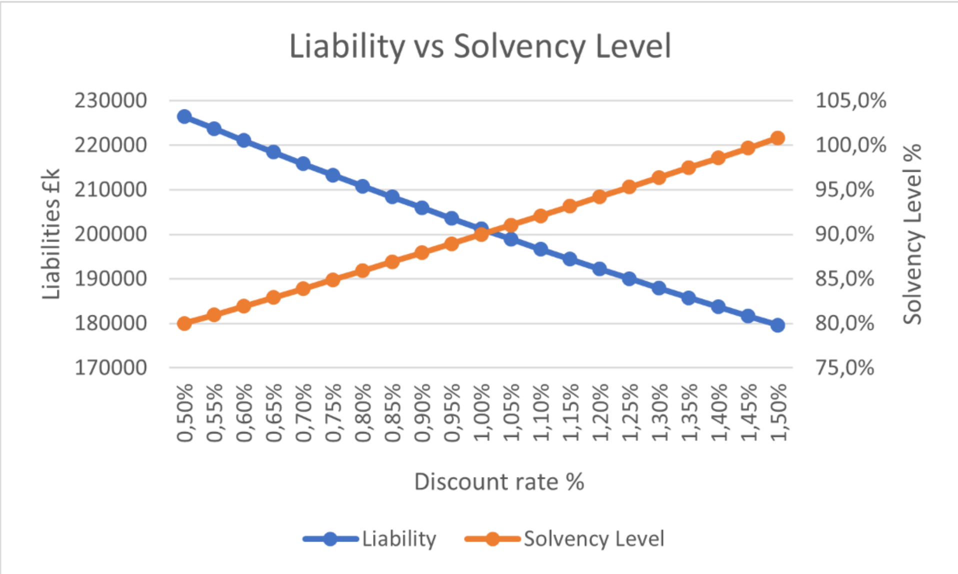

Figure 2 – Liability vs solvency level: discount rate scenario

In this graphic, we can see that discount rate and solvency level are inversely proportional and this will happen in all scenarios because of the definition of solvency level we have seen before, stating that solvency level equals assets divided by liabilities. Thus, since assets are kept the same, this will always be an inverse relation.

The present value was calculated using (5) and when we raise the value of discount rate, we are increasing the value of the denominator, so we are expecting liabilities to decrease. When liabilities are decreasing, the solvency level is increasing. Additionally, we can see that this is the scenario that produces the bigger gap between lower and higher solvency levels found, which will have impact on the conclusions.

4.3 Inflation rate - CPI

There are two indices used to measure inflation rate in the UK, namely Retail Prices Index (RPI) and Consumer Prices Index (CPI). The main difference between them is that RPI includes the costs of housing such as mortgage interest payments and council tax, while CPI does not. In this way, RPI is always higher than CPI.

In our study case, the calculation of liabilities is linked to CPI, thus the target of the analysis is this rate. It is common practice to derive CPI assumption from the RPI, specifically CPI = RPI - margin. The target of our analysis is to change only CPI, so we will change the margin. The base CPI is 2.5% and, as said, we will assume that RPI keeps its original value of 3.7% which means that, because of the rule of RPI being higher than CPI, we will study CPI rates in the range 0.5% to 3.5%.

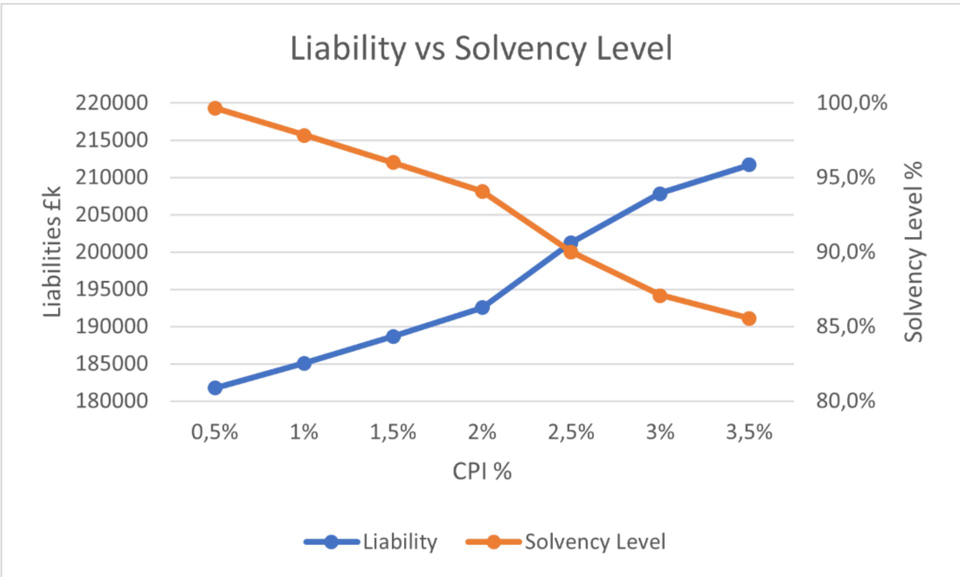

Figure 3 – Liability vs solvency level: inflation rate scenario

Conversely to what occurred with the discount rate, in this scenario when we raise the value of CPI, the value of liabilities also grows. When we change the CPI from 2.5% to 3.5%, we are testing what happens when the CPI-RPI margin converges to zero and our conclusion can be that it causes a riskier scenario, since the solvency level drops. However, in this scenario, the lines produced are not as linear as in the analysis made before, because some tranches of the pension are subject to caps in CPI, such as post 88 GMP.

In Figure 4, we can see the impact of varying CPI. The lower CPI rate produces the lower cash flow and the higher one, in dark blue, is the one with maximum CPI. Nevertheless, the curves found are close to each other, which leads us to conclude that liabilities are not very sensitive to changes in CPI.

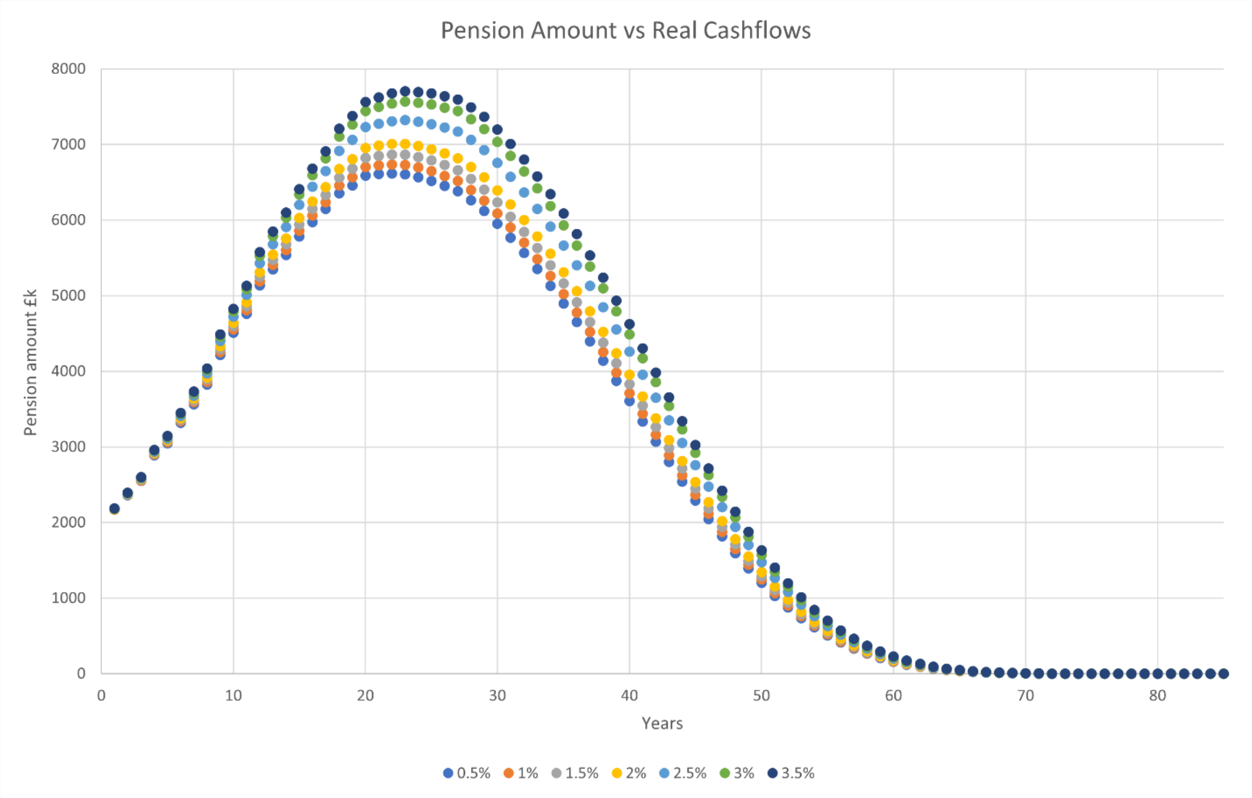

Figure 4 – Pension Amount vs Real Cash Flows: inflation rate scenario

4.4 Mortality tables

On this next sensitivity test we will study the impact of changing mortality assumptions, through the tables used.

The Continuous Mortality Investigation[10] (CMI) is the department of the Institute and Faculty of Actuaries responsible for the research into mortality data investigation and for the disclosure of the results through practical tools that are helpful for actuaries, namely mortality tables. In February 2014, the CMI published the second version of the Self-Administered Pension Scheme (SAPS) mortality tables, known as 'S2' series. To produce these tables, they have collected pension scheme mortality experience information between 2004 and 2011. This series contains 18 tables split by gender,

[10] https://www.actuaries.org.uk/learn-and-develop/continuous-mortality-investigation

pension amounts bands, health status and dependant status. In our study case the tables used were S2NMA and S2NFA, which represent normal health tables for males and females, respectively. In addition, we can define a base multiplier to adjust these tables into our scheme, for example if the multiplier is 0.95 then all the original mortality rates q_x will decrease by 5%. In our study case, for males we use one base multiplier of 1.05 and for females one base multiplier of 1.03, which are realistic values consistent with overall known experience.

On top of that, we can apply improvements to the base tables (Deloitte, March 2021). In a general way, such improvements are a function of the elements shown in (6) below

_ _ _(− %_ _) (6)

year denotes the year of use, *G* is the gender, the − % represents the reduction in mortality rates from one year to the next, for example due to medicine or economic constraints, is the smoothing parameter and reflects how we balance the past known information with future forecasts and is the initial additional parameter, allowing for improvements in the near future. In our study we have used CMI_2019_M_(1_50%) for males and CMI_2019_F_(1_50%) for females, which means that our − is 1.50%. This is an illustrative value, based on common practice in the UK (Institute and Faculty of Actuaries, n.d.) To simplify, we have opted to disregard smoothing or initial additional parameters.

In this analysis the main goal is to evaluate how the value of liabilities changes when we vary the base multiplier, precisely between 0.80 and 1.30 for males and 0.78 and 1.28 for females. We will do the same changes to males and females' base multiplier simultaneously and that the ranges of the multiplier were chosen randomly.

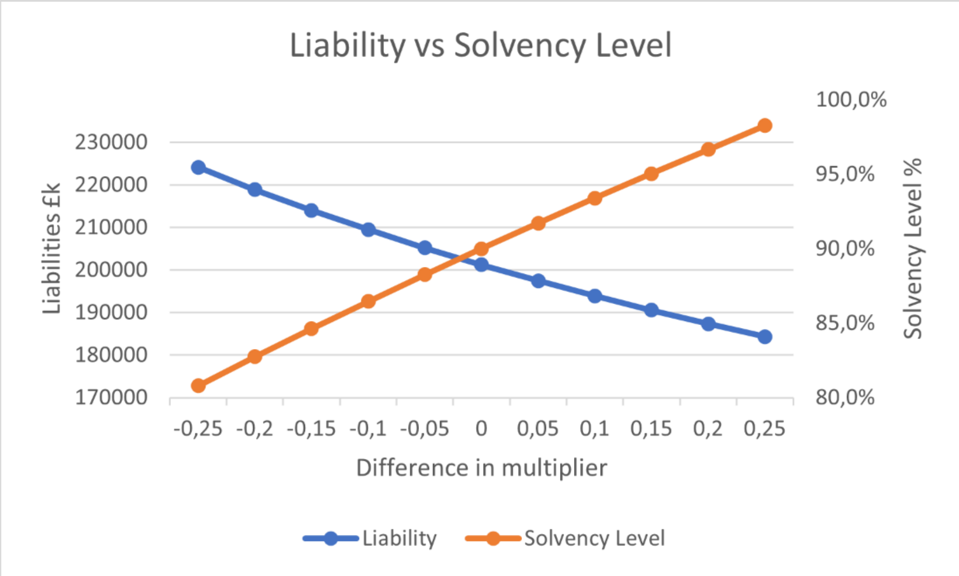

Figure 5 – Liability vs solvency level: mortality tables scenario

By decreasing the value of the multiplier, we are assuming that people live longer, so we are assuming that the company will have to pay pension for longer periods of time. In this way, the value of liabilities is expected to increase as we can confirm by looking into Figure 5.

Finally, we will present all the cash flows produced under the different scenarios as they all change in each iteration of the test. As we can see in Figure 6, the curves produced with lower q_x multipliers are more distant to each other than the ones using the higher multiplier amounts. This leads us to conclude that liabilities calculations are more sensitive to the lower rates. Combining what we found in Figure 5 and Figure 6, we can state that the higher solvency level found was 98.3%, which represents a difference of +8.3% than the original solvency rate of 90%, while the lowest one was 80.8%, representing a decrease of 9.2%, making this a very risky scenario.

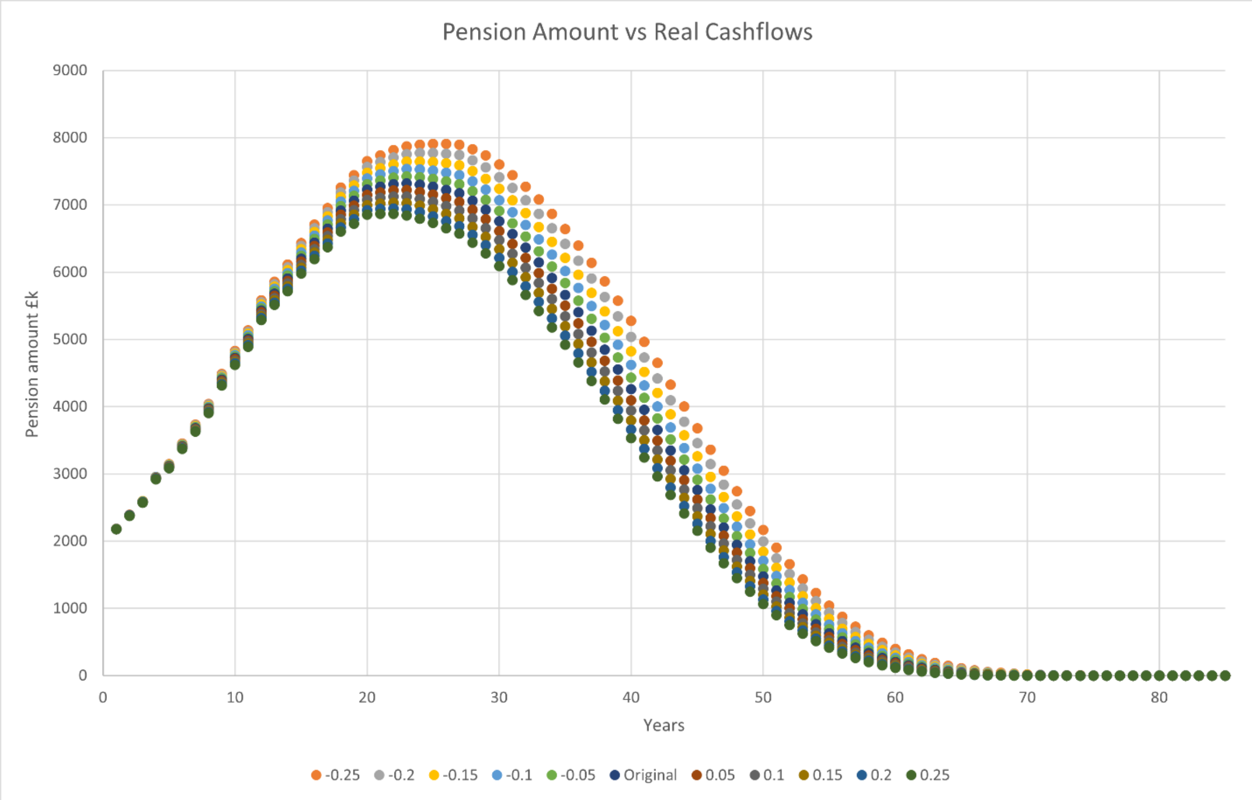

Figure 6 – Pension Amount vs Real Cash Flows: mortality tables scenario

4.5 Comparison of the different assumptions

As we saw, all the assumptions modify the value of liabilities, but this does not materialize in the same way for all of them. Hence, in this next part we will compare the three assumptions previously studied. To do this, we will now compare the present value of cash flows.

The first difference, as expected, is that when we increase CPI, the value of liabilities also grows. The conclusion is the opposite for the discount rate i and the q_x multiplier, this is also as expected. When we raise the values of these variables, the value of liabilities decreases. These results are shown in Figures 2, 3 and 5.

The second difference (of a greater interest) lies in the fact that the lines representing the individual yearly cash flows are closer in some scenarios than others. We can observe this fact in the next graphs, representing a closer look of the ones with the original cash flows, specifically the amounts of the cash flows between ages 20 and 25. For an easier understanding, the scales used in the three graphics are the same.

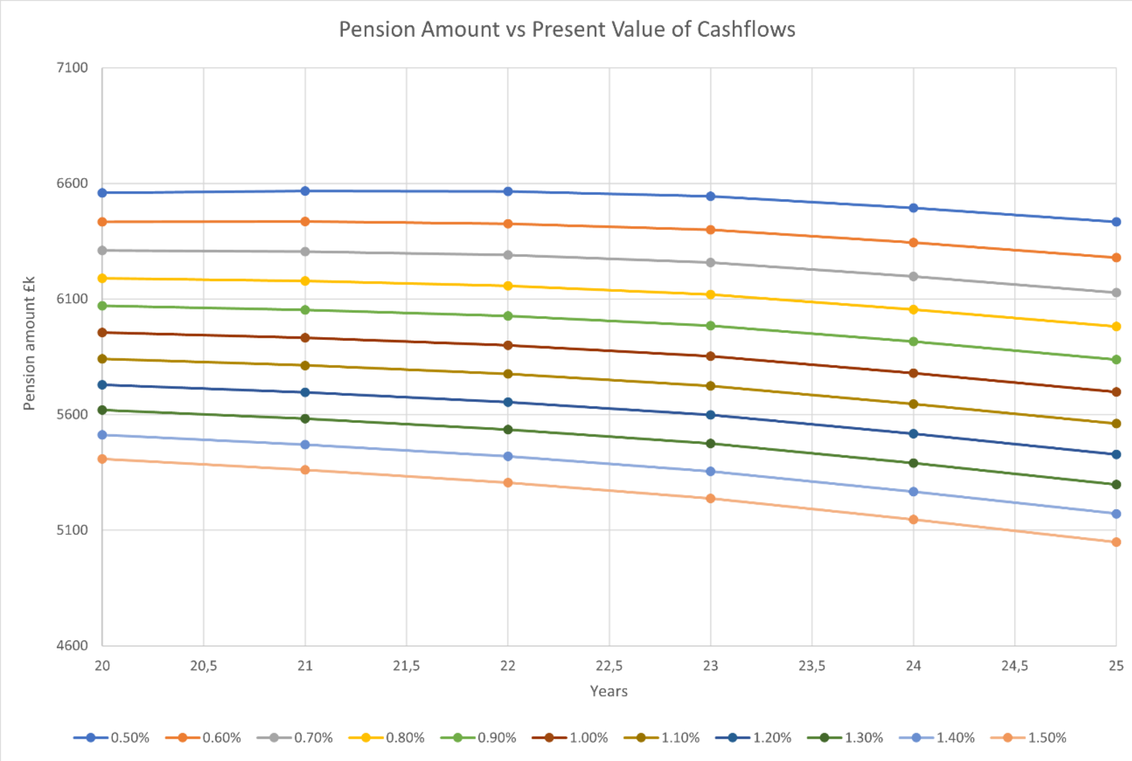

Figure 7 – Pension Amount vs Present Value of Cash Flows: discount rate scenario

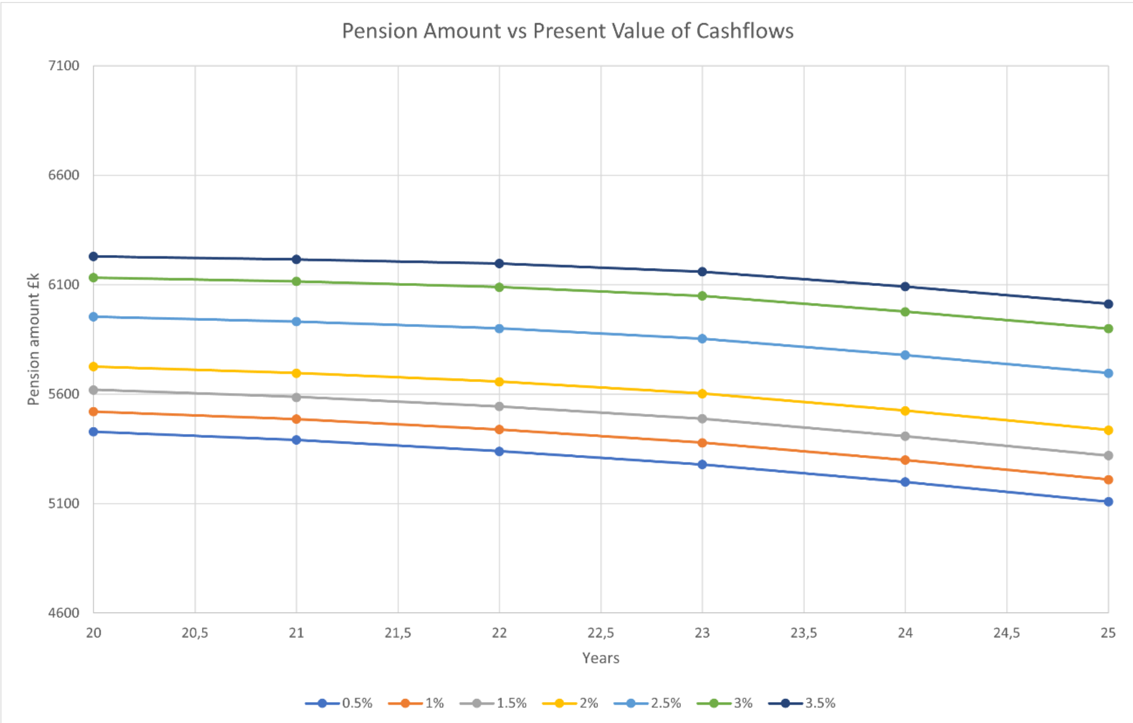

Figure 8 – Pension Amount vs Present Value of Cash Flows: inflation rate scenario

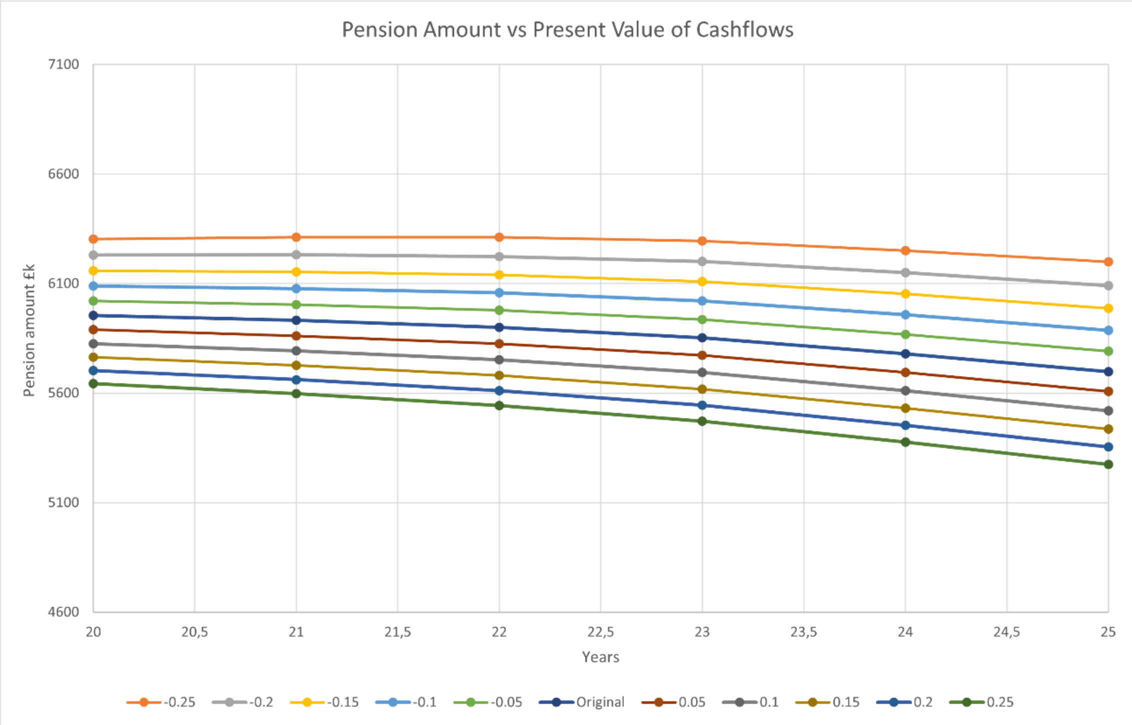

Figure 9 – Pension Amount vs Present Value of Cash Flows: mortality tables scenario

We can see that changing the assumption on q_x multiplier produces the closer cash flows lines, meaning that the impact of these alterations is smaller than the effects from other changes. On the other hand, as already mentioned, the assumption with respect to the discount rate is the one that produces larger variability. To conclude, we can say that the value of liabilities is more sensible to changes of the discount rate than to changes of the CPI rate - or to changes of the q_x multiplier.

4.6 Projecting Extreme Cases

In this section, we will start by projecting cash flows for the next five years, using the original assumptions and further assuming that the value of assets will be equal to the value of projected liabilities, to guarantee a solvency level of 100%. After that, we will study what will happen to the same cash flows in the extreme cases seen in the previous sections. More specifically, in the riskier scenarios of lower discount rates, lower q_x multipliers and higher CPI rates. Tables 3, 4, 5 and 6 below display the results. Table 3 shows the projections of future cash flows under the original assumptions. Tables 4, 5 and 6 contain the future cash flows under the three extreme scenarios characterized before. For more details see Appendixes A, B, C and D.

Table 3 – Solvency level for future years using discount rate of 1%, male qx of 1.05, female qx of 1.03 and CPI of 2.5%

Year	0	1	2	3	4	5
Assets	201,234,245	201,055,071	200,669,970	200,078,235	199,125,399	197,986,612
Liabilities	201,234,245	201,055,071	200,669,970	200,078,235	199,125,399	197,986,612
Solvency level	100%	100%	100%	100%	100%	100%

Table 4 – Solvency level for future years using discount rate of 0.5%

Year	0	1	2	3	4	5
Assets	201,234,245	201,055,071	200,669,970	200,078,235	199,125,399	197,986,612
Liabilities	226,408,113	225,354,068	224,091,124	222,619,585	220,786,384	218,768,032
Solvency level	88.9%	89.2%	89.5%	89.9%	90.2%	90.5%

Table 5 – Solvency level for future years using CPI of 3.5%

Year	0	1	2	3	4	5
Assets	201,234,245	201,055,071	200,669,970	200,078,235	199,125,399	197,986,612
Liabilities	216,227,271	216,196,524	215,960,177	215,516,358	214,705,085	213,704,450
Solvency level	95.1%	95.0%	95.0%	94.9%	94.8%	94.8%

Table 6 – Solvency level for future years using qx multiplier of 0.8 for males and 0.78 for females

Year	0	1	2	3	4	5
Assets	201,234,245	201,055,071	200,669,970	200,078,235	199,125,399	197,986,612
Liabilities	224,162,758	224,211,234	224,051,468	223,681,509	222,944,642	222,015,384
Solvency level	89.8%	89.7%	89.6%	89.4%	89.3%	89.2%

Comparing the three extreme cases, we can conclude that the scenario expected to cause more risk in the future is when q_x multiplier decrease, because the solvency level dropped the most, from 100% to 89.2%, a drop of 10.8%.

Given that sometimes these scenarios can occur at the same time, it is very important to always keep track of the solvency level of the scheme, so that the adequate actions to avoid insolvency can be taken at the proper time. At this point, the goal is to examine the solvency level of the different scenarios and advise the investors of the fund's assets, in the best way possible, considering their solvency position.

In the cases where the solvency level is higher than 100%, it means that the scheme has enough assets to cover the value of its liabilities. Thus, being in a safe spot, investors can avoid market risk and invest in more conservative investments, essentially treasury bills, corporate bonds or cash deposits.

On the other hand, for levels below 100%, it means that the risk of becoming insolvent is higher. Hence, the strategy used must be to choose riskier investments, such as venture capital or stocks, to achieve a higher return and reduce the deficit. At a first sight, this methodology seems to add another risk to the portfolio, but, since we are in a deficit situation, we need to bet all our possibilities, or we will fail anyway. Even in these bold scenarios, we should keep an adequate level of prudence, always present when we are dealing with pension funds' investments. Another approach is to invite the employees and sponsors to make additional contributions or use both methods. In the end, the decision will be entirely of the fund manager, having in mind the risk profile of the scheme's participants.

Our study shows that when inflation increases, the scheme will need more resources to preserve the adequate solvency level. In this scenario, since the risk of insolvency is higher, we suggest investing in riskier financial goods. On the contrary, when the discount rate or the mortality rate increases, as the solvency level improves, is preferable to follow the conservative approach.

5. Conclusions

My internship at WTW was a very rewarding opportunity as my first professional experience. I had the chance to learn more about UK pension funds and develop my skills in Excel. I realised that performing an actuarial valuation is a demanding job and I am honoured to be part of it.

With this report we have tried to provide an overview of how actuarial assumptions and market conditions affect, or will affect in the future, the solvency level of a scheme and of the actions to keep it at a safe position. The subject I have worked on this internship report was not only challenging but also a great opportunity to learn by discussing this topic with more experienced colleagues.

To achieve this report goal, I have performed three sensitivity analysis that led to various conclusions. As we saw, the most serious scenarios found, where the solvency risk decreases, are low mortality multiplier, high inflation and low discount rate. In these cases, scheme's investors must take riskier investments to obtain a higher return. In contrast, in the cases where solvency level is higher than 100%, investors can follow a more conservative investment strategy, since the payment of the pensions is already guaranteed.

Thus, it is essential to highlight the importance of continuously tracing solvency level in pension schemes. The safest way to do this is keeping the fund well managed to prevent unexpected riskier situations that can happen in the future and to know how to invest in the presence of these scenarios, to hedge the outcome.

For future research, we propose doing the same analysis for different schemes, to have a better picture of the sensibility issues. Another idea is to develop a more robust model, using stochastic variables instead of the deterministic ones used in this study, which is a much more complex challenge.

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Appendix A – Future provisions cash flows under original scenario

	Scenario: Normal					
	Cashflow year 0	Cashflow year 1	Cashflow year 2	Cashflow year 3	Cashflow year 4	Cashflow year 5
Assets	201234245	201055071	200669970	200078235	199125399	197986612
Liabilities	201234245	201055071	200669970	200078235	199125399	197986612
Solvency level	100%	100%	100%	100%	100%	100%
1	2169818.69	0	0	0	0	0
2	2348447.66	2371932.14	0	0	0	0
3	2522015.16	2547235.31	2572707.66	0	0	0
4	2838369.18	2866752.87	2895420.40	2924374.60	0	0
5	2978126.71	3007907.97	3037987.05	3068366.92	3099050.59	0
6	3228730.37	3261017.67	3293627.85	3326564.13	3359829.77	3393428.07
7	3448245.25	3482727.70	3517554.98	3552730.53	3588257.84	3624140.41
8	3685420.77	3722274.98	3759497.73	3797092.71	3835063.63	3873414.27
9	4043301.44	4083734.45	4124571.80	4165817.52	4207475.69	4249550.45
10	4298099.78	4341080.77	4384491.58	4428336.50	4472619.86	4517346.06
11	4512404.89	4557528.94	4603104.23	4649135.28	4695626.63	4742582.89
12	4844429.94	4892874.24	4941802.98	4991221.01	5041133.22	5091544.55
13	5017218.67	5067390.86	5118064.77	5169245.41	5220937.87	5273147.25
14	5166205.98	5217868.04	5270046.72	5322747.19	5375974.66	5429734.40
15	5367196.06	5420868.02	5475076.70	5529827.47	5585125.74	5640977.00
16	5519898.72	5575097.71	5630848.69	5687157.17	5744028.74	5801469.03
17	5642549.23	5698974.72	5755964.47	5813524.11	5871659.35	5930375.95
18	5811171.05	5869282.76	5927975.58	5987255.34	6047127.89	6107599.17
19	5875411.82	5934165.94	5993507.60	6053442.67	6113977.10	6175116.87
20	5954625.77	6014172.03	6074313.75	6135056.89	6196407.46	6258371.53
21	5931889.40	5991208.29	6051120.38	6111631.58	6172747.90	6234475.38
22	5900485.59	5959490.45	6019085.35	6079276.21	6140068.97	6201469.66
23	5853080.80	5911611.61	5970727.73	6030435.00	6090739.35	6151646.75
24	5779308.09	5837101.17	5895472.19	5954426.91	6013971.18	6074110.89
25	5697493.69	5754468.63	5812013.32	5870133.45	5928834.79	5988123.13
26	5604465.57	5660510.23	5717115.33	5774286.48	5832029.35	5890349.64
27	5509306.88	5564399.95	5620043.95	5676244.39	5733006.84	5790336.91
28	5371187.16	5424899.03	5479148.02	5533939.50	5589278.89	5645171.68
29	5217893.97	5270072.91	5322773.64	5376001.38	5429761.39	5484059.00
30	5039094.90	5089485.85	5140380.71	5191784.52	5243702.36	5296139.39
31	4850254.57	4898757.11	4947744.69	4997222.13	5047194.35	5097666.30
32	4653253.77	4699786.31	4746784.17	4794252.01	4842194.53	4890616.48
33	4448154.69	4492636.24	4537562.60	4582938.22	4628767.61	4675055.28
34	4236131.20	4278492.51	4321277.44	4364490.21	4408135.12	4452216.47
35	4018080.80	4058261.61	4098844.22	4139832.67	4181230.99	4223043.30
36	3794966.54	3832916.21	3871245.37	3909957.83	3949057.40	3988547.98
37	3567813.87	3603492.01	3639526.93	3675922.20	3712681.42	3749808.23
38	3337786.41	3371164.28	3404875.92	3438924.68	3473313.92	3508047.06
39	3106199.91	3137261.91	3168634.53	3200320.87	3232324.08	3264647.32
40	2874495.90	2903240.86	2932273.27	2961596.00	2991211.96	3021124.08
41	2644242.05	2670684.47	2697391.31	2724365.23	2751608.88	2779124.97
42	2417134.58	2441305.92	2465718.98	2490376.17	2515279.93	2540432.73
43	2194914.45	2216863.60	2239032.24	2261422.56	2284036.78	2306877.15
44	1979258.62	1999051.21	2019041.72	2039232.13	2059624.46	2080220.70

45	1771806.98	1789525.05	1807420.30	1825494.50	1843749.45	1862186.94
46	1574029.80	1589770.09	1605667.80	1621724.47	1637941.72	1654321.13
47	1387165.30	1401036.95	1415047.32	1429197.79	1443489.77	1457924.67
48	1212335.87	1224459.23	1236703.82	1249070.86	1261561.57	1274177.19
49	1050436.34	1060940.70	1071550.11	1082265.61	1093088.27	1104019.15
50	902079.94	911100.74	920211.74	929413.86	938708.00	948095.08
51	767665.66	775342.31	783095.74	790926.69	798835.96	806824.32
52	647291.44	653764.36	660302.00	666905.02	673574.07	680309.81
53	540699.62	546106.61	551567.68	557083.35	562654.19	568280.73
54	447349.21	451822.70	456340.93	460904.34	465513.38	470168.51
55	366442.33	370106.75	373807.82	377545.90	381321.35	385134.57
56	296986.49	299956.35	302955.92	305985.48	309045.33	312135.78
57	237907.90	240286.98	242689.85	245116.75	247567.92	250043.60
58	188120.64	190001.85	191901.87	193820.88	195759.09	197716.68
59	146607.61	148073.69	149554.42	151049.97	152560.47	154086.07
60	112417.97	113542.15	114677.57	115824.34	116982.59	118152.41
61	84661.51	85508.13	86363.21	87226.84	88099.11	88980.10
62	62516.03	63141.19	63772.60	64410.32	65054.43	65704.97
63	45201.57	45653.58	46110.12	46571.22	47036.93	47507.30
64	31959.67	32279.27	32602.06	32928.08	33257.37	33589.94
65	22066.80	22287.46	22510.34	22735.44	22962.80	23192.42
66	14859.43	15008.02	15158.10	15309.68	15462.78	15617.41
67	9743.52	9840.95	9939.36	10038.76	10139.14	10240.53
68	6212.70	6274.82	6337.57	6400.95	6464.96	6529.61
69	3848.09	3886.57	3925.43	3964.69	4004.34	4044.38
70	2311.94	2335.06	2358.41	2382.00	2405.82	2429.88
71	1345.42	1358.87	1372.46	1386.18	1400.05	1414.05
72	757.46	765.03	772.68	780.41	788.22	796.10
73	412.14	416.26	420.42	424.63	428.87	433.16
74	216.57	218.73	220.92	223.13	225.36	227.61
75	109.78	110.88	111.99	113.11	114.24	115.38
76	53.64	54.18	54.72	55.27	55.82	56.38
77	25.27	25.52	25.78	26.04	26.30	26.56
78	11.47	11.58	11.70	11.82	11.94	12.05
79	5.04	5.09	5.14	5.19	5.24	5.29
80	2.13	2.15	2.17	2.20	2.22	2.24
81	0.90	0.91	0.92	0.92	0.93	0.94
82	0.36	0.36	0.36	0.37	0.37	0.37
83	0.13	0.13	0.13	0.14	0.14	0.14
84	0.04	0.04	0.04	0.04	0.05	0.05
85	0.00	0.00	0.00	0.00	0.00	0.00
86	0.00	0.00	0.00	0.00	0.00	0.00
87	0.00	0.00	0.00	0.00	0.00	0.00
88	0.00	0.00	0.00	0.00	0.00	0.00
89	0.00	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00

Appendix B – Future provisions cash flows under low discount rate scenario

	Scenario: Extremely low discount rate					
	Cashflow year 0	Cashflow year 1	Cashflow year 2	Cashflow year 3	Cashflow year 4	Cashflow year 5
Assets	201234245	201055071	200669970	200078235	199125399	197986612
Liabilities	226408113	225354068	224091124	222619585	220786384	218768032
Solvency level	88.9%	89.2%	89.5%	89.9%	90.2%	90.5%
1	2175209.56	0	0	0	0	0
2	2365995.17	2377825.15	0	0	0	0
3	2553500.65	2566268.15	2579099.49	0	0	0
4	2888101.64	2902542.15	2917054.86	2931640.14	0	0
5	3045384.10	3060611.02	3075914.07	3091293.64	3106750.11	0
6	3318073.44	3334663.81	3351337.13	3368093.82	3384934.29	3401858.96
7	3561292.74	3579099.21	3596994.70	3614979.68	3633054.58	3651219.85
8	3825180.38	3844306.28	3863527.81	3882845.45	3902259.68	3921770.98
9	4217511.47	4238599.03	4259792.03	4281090.99	4302496.44	4324008.92
10	4505592.99	4528120.95	4550761.56	4573515.37	4596382.94	4619364.86
11	4753777.36	4777546.25	4801433.98	4825441.15	4849568.35	4873816.19
12	5128953.57	5154598.34	5180371.33	5206273.19	5232304.55	5258466.07
13	5338317.87	5365009.46	5391834.50	5418793.68	5445887.65	5473117.08
14	5524187.74	5551808.68	5579567.73	5607465.56	5635502.89	5663680.41
15	5767657.79	5796496.08	5825478.56	5854605.95	5883878.98	5913298.37
16	5961265.24	5991071.57	6021026.93	6051132.06	6081387.72	6111794.66
17	6124039.81	6154660.01	6185433.31	6216360.48	6247442.28	6278679.49
18	6338428.84	6370120.99	6401971.59	6433981.45	6466151.36	6498482.11
19	6440381.37	6472583.28	6504946.19	6537470.92	6570158.28	6603009.07
20	6559686.10	6592484.53	6625446.95	6658574.18	6691867.05	6725326.39
21	6567150.08	6599985.83	6632985.76	6666150.69	6699481.44	6732978.85
22	6564882.58	6597707.00	6630695.53	6663849.01	6697168.25	6730654.10
23	6544538.70	6577261.39	6610147.70	6643198.44	6676414.43	6709796.50
24	6494200.31	6526671.31	6559304.66	6592101.19	6625061.69	6658187.00
25	6434117.65	6466288.24	6498619.68	6531112.78	6563768.34	6596587.18
26	6360549.87	6392352.62	6424314.39	6456435.96	6488718.14	6521161.73
27	6283660.80	6315079.10	6346654.50	6378387.77	6410279.71	6442331.11
28	6156606.07	6187389.10	6218326.04	6249417.67	6280664.76	6312068.09
29	6010652.81	6040706.07	6070909.60	6101264.15	6131770.47	6162429.32
30	5833567.70	5862735.54	5892049.22	5921509.46	5951117.01	5980872.60
31	5642889.56	5671104.00	5699459.52	5727956.82	5756596.61	5785379.59
32	5440628.43	5467831.58	5495170.73	5522646.59	5550259.82	5578011.12
33	5226699.39	5252832.89	5279097.05	5305492.54	5332020.00	5358680.10
34	5002330.20	5027341.85	5052478.56	5077740.95	5103129.66	5128645.30
35	4768446.68	4792288.91	4816250.36	4840331.61	4864533.27	4888855.93
36	4526072.73	4548703.09	4571446.61	4594303.84	4617275.36	4640361.74
37	4276328.68	4297710.33	4319198.88	4340794.87	4362498.85	4384311.34
38	4020524.78	4040627.40	4060830.54	4081134.69	4101540.37	4122048.07
39	3760182.43	3778983.34	3797878.26	3816867.65	3835951.99	3855131.75
40	3497007.14	3514492.17	3532064.63	3549724.96	3567473.58	3585310.95
41	3232893.10	3249057.56	3265302.85	3281629.36	3298037.51	3314527.70
42	2969930.46	2984780.11	2999704.01	3014702.53	3029776.04	3044924.92

43	2710306.21	2723857.74	2737477.03	2751164.41	2764920.23	2778744.84
44	2456171.11	2468451.96	2480794.22	2493198.19	2505664.18	2518192.50
45	2209671.90	2220720.26	2231823.86	2242982.98	2254197.90	2265468.89
46	1972784.50	1982648.42	1992561.66	2002524.47	2012537.09	2022599.78
47	1747230.59	1755966.75	1764746.58	1773570.31	1782438.16	1791350.35
48	1534617.96	1542291.05	1550002.51	1557752.52	1565541.28	1573368.99
49	1336295.08	1342976.56	1349691.44	1356439.90	1363222.10	1370038.21
50	1153275.24	1159041.62	1164836.83	1170661.01	1176514.32	1182396.89
51	986314.39	991245.96	996202.19	1001183.20	1006189.12	1011220.06
52	835792.43	839971.39	844171.25	848392.11	852634.07	856897.24
53	701632.89	705141.06	708666.76	712210.10	715771.15	719350.00
54	583385.82	586302.75	589234.26	592180.43	595141.33	598117.04
55	480253.06	482654.32	485067.59	487492.93	489930.40	492380.05
56	391161.87	393117.67	395083.26	397058.68	399043.97	401039.19
57	314908.22	316482.76	318065.18	319655.50	321253.78	322860.05
58	250245.85	251497.08	252754.57	254018.34	255288.43	256564.88
59	195993.77	196973.74	197958.61	198948.40	199943.14	200942.86
60	151034.72	151789.89	152548.84	153311.59	154078.15	154848.54
61	114309.52	114881.07	115455.47	116032.75	116612.91	117195.98
62	84828.75	85252.89	85679.16	86107.55	86538.09	86970.78
63	61639.70	61947.90	62257.64	62568.92	62881.77	63196.18
64	43799.05	44018.04	44238.13	44459.32	44681.62	44905.03
65	30391.83	30543.79	30696.51	30849.99	31004.24	31159.27
66	20567.19	20670.03	20773.38	20877.25	20981.63	21086.54
67	13553.27	13621.03	13689.14	13757.58	13826.37	13895.50
68	8684.88	8728.30	8771.94	8815.80	8859.88	8904.18
69	5406.10	5433.13	5460.29	5487.59	5515.03	5542.61
70	3264.16	3280.48	3296.88	3313.37	3329.94	3346.58
71	1909.00	1918.55	1928.14	1937.78	1947.47	1957.21
72	1080.10	1085.50	1090.93	1096.38	1101.87	1107.37
73	590.62	593.57	596.54	599.52	602.52	605.53
74	311.89	313.45	315.02	316.60	318.18	319.77
75	158.90	159.69	160.49	161.29	162.10	162.91
76	78.02	78.41	78.81	79.20	79.60	79.99
77	36.94	37.12	37.31	37.50	37.68	37.87
78	16.85	16.93	17.02	17.10	17.19	17.27
79	7.44	7.47	7.51	7.55	7.59	7.62
80	3.16	3.18	3.19	3.21	3.23	3.24
81	1.34	1.35	1.35	1.36	1.37	1.37
82	0.53	0.54	0.54	0.54	0.54	0.55
83	0.20	0.20	0.20	0.20	0.20	0.20
84	0.07	0.07	0.07	0.07	0.07	0.07
85	0.00	0.00	0.00	0.00	0.00	0.00
86	0.00	0.00	0.00	0.00	0.00	0.00
87	0.00	0.00	0.00	0.00	0.00	0.00
88	0.00	0.00	0.00	0.00	0.00	0.00
89	0.00	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00

Appendix C – Future provisions cash flows under high inflation rate scenario

Scenario: Extremely high CPI

	Cashflow year 0	Cashflow year 1	Cashflow year 2	Cashflow year 3	Cashflow year 4	Cashflow year 5
Assets	201234245	201055071	200669970	200078235	199125399	197986612
Liabilities	211704462	211625909	211338011	210839034	209970991	208910967
Solvency level	95.1%	95.0%	95.0%	94.9%	94.8%	94.8%
1	2173858.84	0	0	0	0	0
2	2356785.78	2380353.64	0	0	0	0
3	2535528.39	2560883.67	2586492.51	0	0	0
4	2860293.87	2888896.81	2917785.78	2946963.64	0	0
5	3006377.87	3036441.65	3066806.07	3097474.13	3128448.87	0
6	3266592.54	3299258.47	3332251.05	3365573.57	3399229.30	3433221.59
7	3496930.38	3531899.68	3567218.68	3602890.86	3638919.77	3675308.97
8	3747729.34	3785206.63	3823058.70	3861289.28	3899902.17	3938901.20
9	4122675.30	4163902.05	4205541.07	4247596.48	4290072.45	4332973.17
10	4389917.19	4433816.36	4478154.52	4522936.07	4568165.43	4613847.08
11	4620183.39	4666385.22	4713049.07	4760179.56	4807781.36	4855859.17
12	4973735.46	5023472.82	5073707.55	5124444.62	5175689.07	5227445.96
13	5163817.91	5215456.09	5267610.65	5320286.76	5373489.63	5427224.52
14	5330274.07	5383576.82	5437412.58	5491786.71	5546704.58	5602171.62
15	5550422.22	5605926.45	5661985.71	5718605.57	5775791.62	5833549.54
16	5726591.90	5783857.82	5841696.40	5900113.36	5959114.49	6018705.64
17	5864317.41	5922960.58	5982190.19	6042012.09	6102432.21	6163456.53
18	6055885.34	6116444.19	6177608.64	6239384.72	6301778.57	6364796.36
19	6137279.47	6198652.27	6260638.79	6323245.18	6386477.63	6450342.41
20	6230198.54	6292500.53	6355425.53	6418979.79	6483169.59	6548001.28
21	6216554.07	6278719.61	6341506.81	6404921.88	6468971.09	6533660.80
22	6197641.22	6259617.63	6322213.80	6385435.94	6449290.30	6513783.21
23	6159517.25	6221112.43	6283323.55	6346156.79	6409618.35	6473714.54
24	6090959.24	6151868.83	6213387.52	6275521.39	6338276.61	6401659.37
25	6014188.22	6074330.10	6135073.40	6196424.14	6258388.38	6320972.26
26	5926861.92	5986130.54	6045991.84	6106451.76	6167516.28	6229191.44
27	5835646.23	5894002.69	5952942.71	6012472.14	6072596.86	6133322.83
28	5698881.36	5755870.18	5813428.88	5871563.17	5930278.80	5989581.59
29	5546297.05	5601760.02	5657777.62	5714355.39	5771498.95	5829213.94
30	5365410.23	5419064.33	5473254.98	5527987.53	5583267.40	5639100.08
31	5173361.72	5225095.33	5277346.29	5330119.75	5383420.95	5437255.16
32	4972345.83	5022069.29	5072289.98	5123012.88	5174243.01	5225985.44
33	4762091.68	4809712.60	4857809.73	4906387.82	4955451.70	5005006.22
34	4543847.02	4589285.49	4635178.35	4681530.13	4728345.43	4775628.88
35	4318456.64	4361641.20	4405257.62	4449310.19	4493803.29	4538741.33
36	4086843.83	4127712.27	4168989.39	4210679.29	4252786.08	4295313.94
37	3850011.54	3888511.66	3927396.77	3966670.74	4006337.45	4046400.82
38	3609128.20	3645219.48	3681671.67	3718488.39	3755673.27	3793230.01
39	3365546.76	3399202.23	3433194.25	3467526.19	3502201.45	3537223.47

40	3120780.28	3151988.08	3183507.97	3215343.04	3247496.48	3279971.44
41	2876507.26	2905272.33	2934325.05	2963668.30	2993304.98	3023238.03
42	2634572.57	2660918.30	2687527.48	2714402.76	2741546.78	2768962.25
43	2396894.00	2420862.94	2445071.57	2469522.29	2494217.51	2519159.68
44	2165348.51	2187002.00	2208872.02	2230960.74	2253270.34	2275803.05
45	1941797.39	1961215.36	1980827.52	2000635.79	2020642.15	2040848.57
46	1727938.32	1745217.70	1762669.88	1780296.58	1798099.54	1816080.54
47	1525234.60	1540486.95	1555891.82	1571450.73	1587165.24	1603036.89
48	1335029.77	1348380.06	1361863.86	1375482.50	1389237.33	1403129.70
49	1158419.03	1170003.22	1181703.25	1193520.28	1205455.48	1217510.04
50	996184.88	1006146.73	1016208.20	1026370.28	1036633.98	1047000.32
51	848864.59	857353.23	865926.77	874586.03	883331.90	892165.21
52	716656.05	723822.61	731060.84	738371.44	745755.16	753212.71
53	599354.47	605348.02	611401.50	617515.51	623690.67	629927.57
54	496432.63	501396.96	506410.93	511475.04	516589.79	521755.69
55	407075.81	411146.56	415258.03	419410.61	423604.72	427840.76
56	330243.80	333546.24	336881.70	340250.52	343653.02	347089.56
57	264795.59	267443.54	270117.98	272819.16	275547.35	278302.82
58	209570.56	211666.27	213782.93	215920.76	218079.97	220260.77
59	163473.62	165108.36	166759.44	168427.04	170111.31	171812.42
60	125470.60	126725.31	127992.56	129272.49	130565.21	131870.87
61	94588.63	95534.51	96489.86	97454.76	98429.30	99413.60
62	69925.89	70625.15	71331.40	72044.71	72765.16	73492.81
63	50623.17	51129.41	51640.70	52157.11	52678.68	53205.46
64	35842.92	36201.35	36563.37	36929.00	37298.29	37671.27
65	24786.32	25034.18	25284.53	25537.37	25792.75	26050.67
66	16719.46	16886.66	17055.52	17226.08	17398.34	17572.32
67	10984.08	11093.92	11204.86	11316.90	11430.07	11544.37
68	7018.64	7088.83	7159.71	7231.31	7303.62	7376.66
69	4357.64	4401.22	4445.23	4489.68	4534.58	4579.92
70	2624.99	2651.24	2677.76	2704.53	2731.58	2758.90
71	1532.00	1547.32	1562.80	1578.42	1594.21	1610.15
72	865.22	873.87	882.61	891.44	900.35	909.35
73	472.41	477.14	481.91	486.73	491.60	496.51
74	249.10	251.59	254.11	256.65	259.21	261.81
75	126.75	128.01	129.30	130.59	131.89	133.21
76	62.18	62.80	63.43	64.06	64.70	65.35
77	29.38	29.67	29.97	30.27	30.57	30.88
78	13.41	13.55	13.68	13.82	13.96	14.10
79	5.91	5.97	6.03	6.09	6.15	6.21
80	2.54	2.56	2.59	2.62	2.64	2.67
81	1.03	1.04	1.05	1.06	1.07	1.09
82	0.40	0.40	0.41	0.41	0.42	0.42
83	0.13	0.13	0.13	0.14	0.14	0.14
84	0.04	0.04	0.04	0.04	0.05	0.05
85	0.00	0.00	0.00	0.00	0.00	0.00
86	0.00	0.00	0.00	0.00	0.00	0.00
87	0.00	0.00	0.00	0.00	0.00	0.00
88	0.00	0.00	0.00	0.00	0.00	0.00
89	0.00	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00

Appendix D – Future provisions cash flows under low mortality scenario

	Scenario: Extremely low q_x multiplier					
	Cashflow year 0	Cashflow year 1	Cashflow year 2	Cashflow year 3	Cashflow year 4	Cashflow year 5
Assets	201234245	201055071	200669970	200078235	199125399	197986612
Liabilities	224162758	224211234	224051468	223681509	222944642	222015384
Solvency Level	89.8%	89.7%	89.6%	89.4%	89.3%	89.2%
1	2171437.02	0	0	0	0	0
2	2354551.68	2378097.19	0	0	0	0
3	2533700.43	2559037.43	2584627.81	0	0	0
4	2857650.36	2886226.86	2915089.13	2944240.02	0	0
5	3005398.23	3035452.21	3065806.74	3096464.80	3127429.45	0
6	3266129.11	3298790.40	3331778.31	3365096.09	3398747.05	3432734.52
7	3496985.96	3531955.82	3567275.38	3602948.13	3638977.62	3675367.39
8	3747227.61	3784699.89	3822546.89	3860772.35	3899380.08	3938373.88
9	4121141.20	4162352.61	4203976.13	4246015.89	4288476.05	4331360.81
10	4392709.92	4436637.02	4481003.39	4525813.42	4571071.55	4616782.27
11	4625092.72	4671343.64	4718057.08	4765237.65	4812890.03	4861018.93
12	4978534.65	5028319.99	5078603.19	5129389.22	5180683.12	5232489.95
13	5172631.23	5224357.54	5276601.12	5329367.13	5382660.80	5436487.41
14	5344366.25	5397809.91	5451788.01	5506305.89	5561368.95	5616982.64
15	5570682.48	5626389.31	5682653.20	5739479.73	5796874.53	5854843.28
16	5749774.83	5807272.58	5865345.30	5923998.76	5983238.74	6043071.13
17	5900040.54	5959040.94	6018631.35	6078817.67	6139605.84	6201001.90
18	6098721.44	6159708.65	6221305.74	6283518.79	6346353.98	6409817.52
19	6192841.71	6254770.12	6317317.82	6380491.00	6444295.91	6508738.87
20	6303439.20	6366473.59	6430138.32	6494439.71	6559384.10	6624977.94
21	6311309.75	6374422.85	6438167.08	6502548.75	6567574.24	6633249.98
22	6310949.27	6374058.77	6437799.35	6502177.35	6567199.12	6632871.11
23	6294582.78	6357528.61	6421103.89	6485314.93	6550168.08	6615669.76
24	6251274.08	6313786.82	6376924.69	6440693.94	6505100.88	6570151.89
25	6199278.03	6261270.81	6323883.52	6387122.36	6450993.58	6515503.52
26	6135043.42	6196393.86	6258357.80	6320941.38	6384150.79	6447992.30
27	6067458.76	6128133.35	6189414.68	6251308.83	6313821.92	6376960.14
28	5954614.91	6014161.06	6074302.67	6135045.69	6196396.15	6258360.11
29	5824535.23	5882780.59	5941608.39	6001024.48	6061034.72	6121645.07
30	5666442.74	5723107.17	5780338.24	5838141.62	5896523.04	5955488.27
31	5495911.57	5550870.68	5606379.39	5662443.18	5719067.61	5776258.29
32	5314720.98	5367868.19	5421546.87	5475762.34	5530519.96	5585825.16
33	5122840.87	5174069.27	5225809.97	5278068.07	5330848.75	5384157.24
34	4921377.44	4970591.22	5020297.13	5070500.10	5121205.10	5172417.15
35	4711142.38	4758253.81	4805836.35	4853894.71	4902433.66	4951457.99
36	4492997.40	4537927.38	4583306.65	4629139.72	4675431.11	4722185.42
37	4267823.20	4310501.43	4353606.45	4397142.51	4441113.94	4485525.07
38	4036611.01	4076977.12	4117746.89	4158924.36	4200513.60	4242518.74
39	3800503.32	3838508.35	3876893.43	3915662.37	3954818.99	3994367.18
40	3560779.33	3596387.13	3632351.00	3668674.51	3705361.25	3742414.86
41	3318857.74	3352046.32	3385566.78	3419422.45	3453616.67	3488152.84
42	3076342.98	3107106.41	3138177.47	3169559.25	3201254.84	3233267.39
43	2834986.28	2863336.14	2891969.50	2920889.20	2950098.09	2979599.07
44	2596566.05	2622531.72	2648757.03	2675244.60	2701997.05	2729017.02

45	2362942.73	2386572.16	2410437.88	2434542.26	2458887.68	2483476.56
46	2135898.66	2157257.65	2178830.23	2200618.53	2222624.72	2244850.96
47	1917037.00	1936207.37	1955569.44	1975125.13	1994876.38	2014825.15
48	1707932.47	1725011.79	1742261.91	1759684.53	1777281.37	1795054.19
49	1509997.65	1525097.63	1540348.60	1555752.09	1571309.61	1587022.71
50	1324392.46	1337636.38	1351012.75	1364522.88	1378168.10	1391949.79
51	1152122.29	1163643.51	1175279.95	1187032.75	1198903.08	1210892.11
52	993929.68	1003868.97	1013907.66	1024046.74	1034287.21	1044630.08
53	850196.54	858698.50	867285.49	875958.34	884717.93	893565.11
54	720989.72	728199.62	735481.62	742836.43	750264.80	757767.44
55	606043.65	612104.09	618225.13	624407.38	630651.46	636957.97
56	504780.40	509828.20	514926.49	520075.75	525276.51	530529.27
57	416405.16	420569.21	424774.91	429022.65	433312.88	437646.01
58	339971.96	343371.68	346805.40	350273.45	353776.18	357313.95
59	274505.13	277250.18	280022.68	282822.91	285651.14	288507.65
60	219000.45	221190.45	223402.36	225636.38	227892.74	230171.67
61	172466.54	174191.21	175933.12	177692.45	179469.38	181264.07
62	133936.32	135275.68	136628.44	137994.72	139374.67	140768.42
63	102491.59	103516.50	104551.67	105597.18	106653.16	107719.69
64	77216.02	77988.18	78768.06	79555.74	80351.30	81154.81
65	57222.03	57794.25	58372.19	58955.91	59545.47	60140.93
66	41676.45	42093.21	42514.14	42939.29	43368.68	43802.36
67	29801.06	30099.07	30400.07	30704.07	31011.11	31321.22
68	20897.55	21106.52	21317.59	21530.77	21746.07	21963.53
69	14365.11	14508.76	14653.84	14800.38	14948.39	15097.87
70	9669.75	9766.45	9864.11	9962.75	10062.38	10163.01
71	6364.13	6427.77	6492.05	6556.97	6622.54	6688.76
72	4099.92	4140.92	4182.33	4224.16	4266.40	4309.06
73	2581.53	2607.34	2633.42	2659.75	2686.35	2713.21
74	1587.82	1603.70	1619.73	1635.93	1652.29	1668.81
75	957.23	966.80	976.47	986.23	996.10	1006.06
76	562.40	568.03	573.71	579.45	585.24	591.09
77	325.06	328.31	331.59	334.91	338.26	341.64
78	183.97	185.81	187.67	189.55	191.44	193.36
79	100.78	101.79	102.81	103.84	104.88	105.93
80	53.32	53.85	54.39	54.93	55.48	56.04
81	27.43	27.70	27.98	28.26	28.54	28.83
82	12.40	12.52	12.65	12.78	12.90	13.03
83	5.06	5.11	5.16	5.21	5.27	5.32
84	2.09	2.11	2.13	2.15	2.18	2.20
85	0.22	0.22	0.22	0.22	0.22	0.23
86	0.09	0.09	0.09	0.09	0.09	0.09
87	0.04	0.04	0.04	0.04	0.04	0.04
88	0.00	0.00	0.00	0.00	0.00	0.00
89	0.00	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00