Application of Mixed Integer Nonlinear Programming (MINLP) Optimization through GAMS for Component Selection in Vapor Compression Refrigeration

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ABSTRACT

Vapor compression system and component modeling tools are essential for feasibility or design studies of HVAC&R solutions. Such tools frequently rely on scaling factors, for example to estimate the needed heat exchanger surface area or swept volume of the compressor. However, when designing systems using existing components, their capacities or dimensions are not variable and selecting components becomes a mixed integer nonlinear programming (MINLP) optimization. In this type of optimization, not the optimal swept volume of the compressor is sought, but rather whether one or two units of model A, B, or C result in the best value for the objective function. The Generic Algebraic Modeling System (GAMS) is an established and powerful modeling environment for MINLP but is rarely used in the field of vapor compression refrigeration. This paper demonstrates the use of GAMS in making optimal selections of refrigerant, compressor, evaporator and condenser for a chiller from a library totaling 6000 possible combinations. The total computational time for the optimization in GAMS was 11 seconds, a task for which the Engineering Equation Solver (EES) needed 621 seconds. The GAMS language also allows a more convenient implementation using integer variables and set representations.

Keywords: Vapor compression cycle, component selection, optimization, MINLP, GAMS

1. Introduction

Mixed integer nonlinear programming is a branch of optimization where models include integer variables. Powerful solvers exist to solve those optimization problems. The Generic Algebraic Modeling System (GAMS, 2022) is a modeling environment which interfaces conveniently with a variety of such solvers. The commercial software started as a World Bank project in the 1970s and is now a widely applied tool to solve optimization problems. It is frequently used for economic decisions, energy or commodity distribution and transportation. López-Flores et al. (2021) showed the applicability of GAMS to thermal systems for an industrial process comprising several hot and cold streams which needed to be cooled or heated. The optimization problem was to design a heat exchanger network and add chillers, heat pumps, boilers and Organic Rankine Cycles (ORC) to minimize an objective variable, either cost or energy consumption. Martinelli et al. (2022) used GAMS in a different study to optimize a system composed of a heat exchanger network, refrigeration cycle and ORC by allowing different architectures and determining ideal high and low side operating pressures all simultaneously. GAMS was also used for absorption chillers (Chávez-Islas and Heard, 2009), thermoacoustic refrigeration (Tartibu et al., 2015) or ORC working fluid selection (Schilling et al., 2021). In general, however, GAMS is still scarcely used in the field of refrigeration and air-conditioning. The present study demonstrates the use of GAMS in three ways that are rarely shown in the literature:

- An optimal chiller configuration is sought given predefined component libraries
- The chiller is optimized considering multiple different ambient temperatures
- The chiller can be designed with multiple compressors in parallel of which some may be turned off depending on the ambient temperature.

Results and practicality of implementation are directly compared to EES (Klein and Alvarado, 2002), a modeling environment not designed for integer programming. Section four is written as a small tutorial to coding in GAMS using set representations.

2. Optimization Problem

2.1 Problem Statement

A warehouse is to be kept at an air temperature of 8 °C. Seasonal variations in ambient temperature are lumped into three bins with a weighting factor w relative to the total yearly operating time of 6000 hours as shown in Table 1 . The ambient temperature dictates the cooling load and acts as the temperature of the heat sink for the chiller. The life cycle cost (LCC) for an eight-year period is to be minimized choosing either of the refrigerants Ammonia, R134a, R32, R404A, R407C or R410A as the refrigerant and by selecting an evaporator, condenser and up to three compressors from component libraries. Compressors are of variable-speed type but limited to $f \leq 3600$ RPM. Any on-off cycling to avoid operation at very low compressor speeds is not modeled. A variable number of compressors may be turned on for the different seasons, but all compressors running must operate at the same frequency. The problem statement is hypothetical and only serves to demonstrate the utility and performance of GAMS.

Table 1 : Considered ambient temperatures and their weighting for the 6000 operating hours per year.

Season (S)	T_{amb} [°C]	w
S1	20	0.5
S2	30	0.3
S3	40	0.2

2.2 Component Libraries

The component libraries comprise 10 evaporators (Ev1…Ev10), 10 condensers (Cd1…Cd10) and 10 compressors (Cp1…Cp10). Heat exchangers are described with two specifications: UA value and cost. To create the database, two evaporators and two condensers were selected from a design software with price information (Guentner, 2022). From the specification sheets, UA values were derived and linear fits for cost versus UA were created for the two evaporators and the two condensers. The fits are shown with solid lines in Figure 1. The 10 evaporators and 10 condensers for the case study are shown as dots in Figure 1 and were created with a random function in proximity to the linear fits. Compressors are modeled with five specifications: swept volume V_{swept}, cost, overall isentropic efficiency $\eta_{i,o}$, volumetric efficiency η_v and a heat loss efficiency $\eta_{h,l}$ relating the actual outlet state with an adiabatic one. A compressor with $V_{swept} = 70\ cm^3$ for \$3000 was used as a reference and 10 compressors were artificially generated by varying V_{swept} in a range of +/- 30% and the cost in a range of +/- 40%. Small random values were added to avoid a perfectly linear distribution. $\eta_{i,o}$ was randomized between 0.55 and 0.8 for each compressor. η_v and $\eta_{h,l}$ were also randomized but always forced to be greater than $\eta_{i,o}$. The characteristics of the 10 compressors are shown in Figure 2.

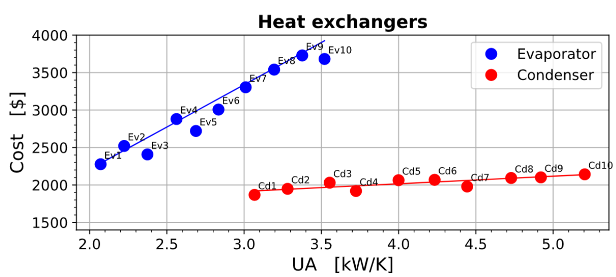

Figure 1: Cost and UA values of evaporators and condensers in component library.

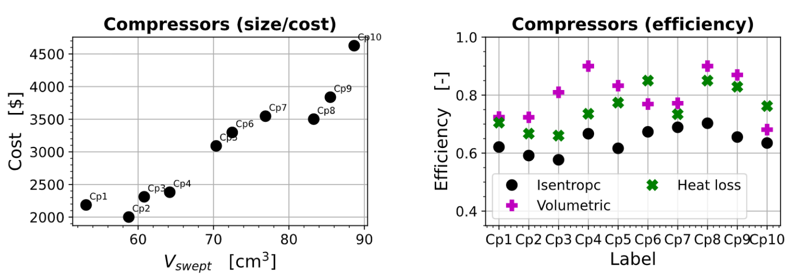

Figure 2: Cost, swept volume and efficiencies of compressors in component library.

3. Thermodynamic Model

3.1 Basic Equation System

The model without any integer variables is called the "basic equation system". Heat exchangers are modeled with a lumped UA value that is constant even for different air flow rates and isobaric flows are assumed for both the air and refrigerant side. Condensers are dry cooled and no moisture removal or frost formation is modeled for the evaporators. Overall isentropic efficiency, volumetric efficiency and heat loss ratio of each compressor are constant. The expansion valve is modeled as isenthalpic. State points are labeled as shown in Figure 3 and equations and parameters of the basic model are listed in Table 2 .

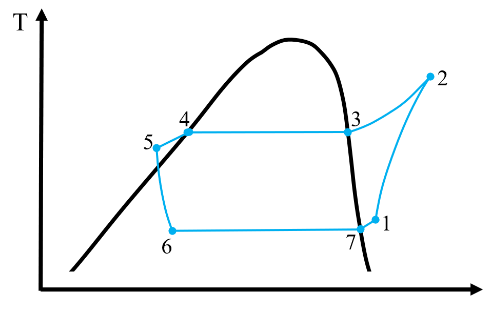

Figure 3: Definition of VCC-state points in T-s diagram.

Table 2 : Basic equation system (without discrete (integer) variables).

Equations	**Comments and constants**
$\dot{Q}_e = (T_{amb} - T_b)UA_{house}$	Cooling demand; $T_b = 5\,°C, UA_{house} = 0.5\ kW/K$
$\epsilon_e = 1 - \exp(-UA_e/(\dot{m}_{a,e}\, c_p))$	Evap. effectiveness; $c_p = 1\ kJ/(kg \cdot K), \dot{m}_{a,e} = \{1,2,3\} kg/s$ for season 1, 2 and 3, respectively.
$\epsilon_c = 1 - \exp(-UA_c/(\dot{m}_{a,c}\, c_p))$	Cond. effectiveness, $\dot{m}_{a,c} = \dot{m}_{a,c}$ for each season
$\dot{Q}_e = \epsilon_e \dot{m}_{a,e}\, c_p(T_{a,in,e} - T_e)$	Evap. heat transfer rate; $T_{a,in,e} = 8\,°C$
$\dot{Q}_c = \epsilon_c \dot{m}_{a,c}\, c_p(T_c - T_{a,in,c})$	Cond. heat transfer rate
$T_1 = T_e + \Delta T_{sh}; T_5 = T_c - \Delta T_{sc}$	State points 1 and 5; $\Delta T_{sh} = 5\ K;\ \Delta T_{sc} = 5\ K$
$\dot{Q}_e = \dot{m}_r(h_1 - h_6)$	Dictates refrigerant mass flow rate
$\dot{Q}_c = \dot{m}_r(h_2 - h_5)$	Closes energy balance
$h_5 = h_6$	Isenthalpic expansion
$\eta_{i,o} = (h_{2s} - h_1)/(h_{2a} - h_1)$	h_{2a}: Adiabatic discharge state, h_{2s}: Isentropic discharge state
$\eta_{h,l} = (h_2 - h_1)/(h_{2a} - h_1)$	h_2: Actual discharge state
$\dot{m}_r = \eta_v \cdot f \cdot V_{swept} \cdot \rho_1$	Relationship of compressor frequency and mass flow rate
$\dot{W}_{comp} = \dot{m}_r(h_{2a} - h_1)$	Compressor power draw
$\dot{W}_e = a_0 \dot{m}_{a,e} + a_1$	Evaporator fan power draw; $a_0 = 0.4, a_1 = -0.34$
$\dot{W}_c = a_0 \dot{m}_{a,c} + a_1$	Condenser fan power draw
$\dot{W}_{tot} = \dot{W}_{comp} + \dot{W}_{evap} + \dot{W}_{cond}$	Total power draw
$COP = \dot{Q}_e/\dot{W}_{tot}$	Coefficient of performance
$EC = (w^{S1}\dot{W}_{tot}^{S1} + w^{S2}\dot{W}_{tot}^{S2} + w^{S3}\dot{W}_{tot}^{S3}) \cdot runtime \cdot c_{el}$	Energy cost; $c_{el} = 0.12\ \$/kWh$; $runtime = 6000h/year.$ Superscripts indicate seasons.
$CC = \left(NOC \cdot c_{comp} + c_{evap} + c_{cond}\right) f_{install}$	Capital cost; $f_{install} = 2$, NOC is the number of needed compressors.
$LCC = P_1 EC + P_2 CC$	Life cycle cost. P_1 and P_2 as in Duffie and Beckman (2013) or in the supplemental material of Brendel et al. (2020); $d = 0.01, dp = 0.25, i_{inf} = 0.02, m = 0.085, M_s = 0.03, N_l = 5, N_D = 10, R_v = 0, tax_{in} = 0.25, tax_{pr} = 0.02, V_r = 0.8$

3.2 Implementation of Thermodynamic Properties

The following thermodynamic properties are needed for the model: $h_1(T_1, P_1), s_1(T_1, P_1), h_{2s}(s_1, P_2), h_5(T_5, P_5)$. Moreover, $x_6(h_6, P_6)$ and $T_2(h_2, P_2)$ are desired for debugging and comparisons with EES results (subscripts refer to Figure 3). It is possible to include properties in GAMS through dynamically linked libraries (Manassaldi et al., 2021, 2019). In this study, properties were included through polynomials that were fitted for the expected range of temperatures and pressures. This was found to be computationally fast and sufficiently accurate. For example, given the outdoor temperature range of $20\,°C < T_{amb} < 40\,°C$, the condensation temperature may be bounded to $25\,°C < T_c < 55\,°C$. The saturation pressure was fitted with a first-order polynomial ($P(T) = c_0 + c_1 T$) to 5 evenly distributed samples from this range. This is shown in Figure 4 for R32 and led to $R^2 = 0.993$. The range of expected evaporation temperatures is narrower and resulted in an even better fit. Figure 5 shows fits for $\mathrm{h_1, T_2, h_5, x_6}$, all of which are dependent on two properties. Expected pressures and temperatures (enthalpies) were defined and the area within was fitted to 15 samples (three sets of 5 at three different pressure levels). Samples either evenly fill the area between the bounds or hug the vapor dome. This fitting was automated in Python and the coefficients were copy-pasted into GAMS. Refitting to adjusted bounds can be accomplished within minutes, if necessary. A second-order polynomial was only used to fit h_{2s} since it was found to have a relatively large potential to improve the accuracy of the overall model ($h_{2s}(s_1, P_2) = c_0 + c_1 s_1 + c_2 s_1^2 + c_3 P_2 + c_4 P_2^2 + c_5 s_1 P_2$). All other properties that were dependent on two other properties were fitted with first-order polynomials (for example $h_1(T_1, P_2) = c_0 + c_1 T_1 + c_2 P_1$). The process was repeated for all refrigerants. The fitted polynomials were also imported into EES to allow a performance comparison between EES solving the built-in property functions and EES retrieving properties from the polynomials.

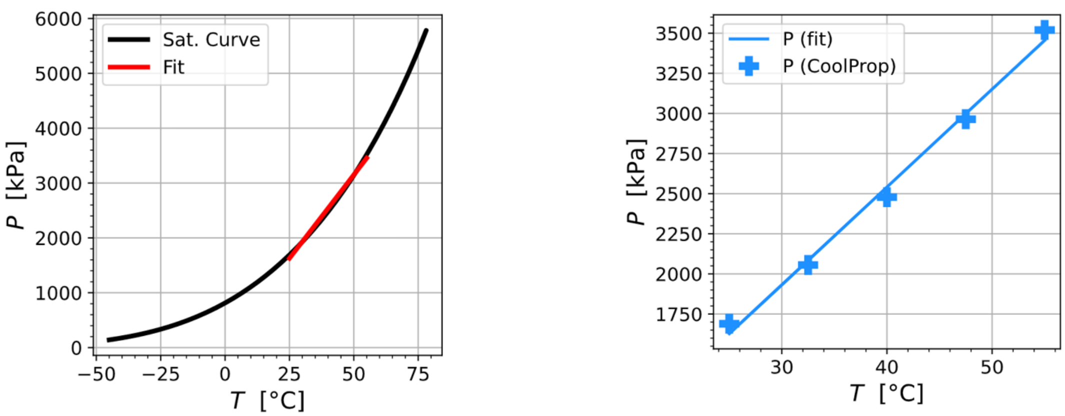

Figure 4: Linear fit of saturation curve with bounds relevant for problem statement.

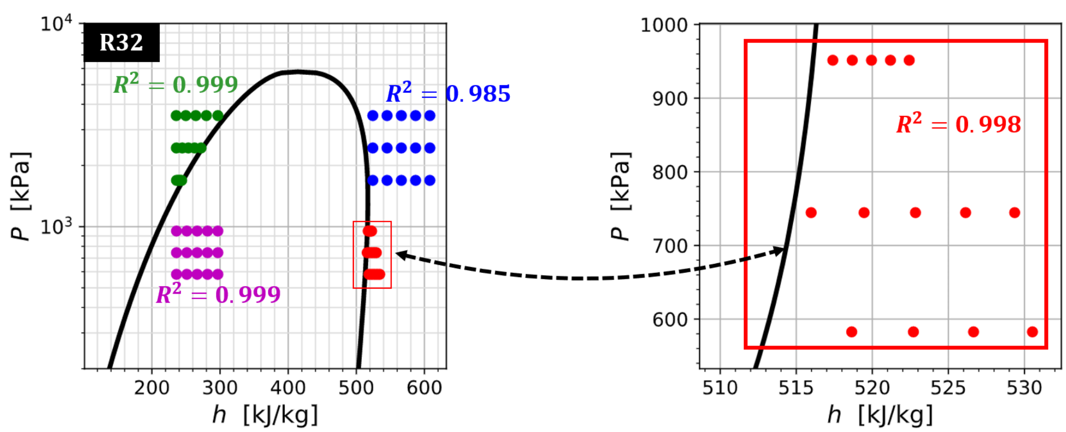

Figure 5: Areas for linear fits in P-h diagram for h_1, T_2, h_5, x_6.

3.3 Bounds

Bounds on operational variables were defined for both GAMS and EES as shown in Table 3. The evaporation and condensation pressure needed relatively wide bounds to accommodate all refrigerants. Additionally, lower bounds of 0 were imposed in both GAMS and EES for all variables that should always be positive. EES required additional lower bounds for $\Delta h_{61}, h_2, h_{2a}, P_1, P_2, P_5, P_6, \rho_1, s_1, T_1, T_2$ and T_5 for reliable convergence across all 6000 system configurations. Using the previous solution as the initial guess value was enabled in EES.

Table 3 : Bounds imposed in EES and GAMS.

Variable	**Lower**	**Upper**	**Variable**	**Lower**	**Upper**
T_e [°C]	-5	5	P_e [kPa]	200	950
T_c [°C]	25	55	P_c [kPa]	665	3519
COP [-]	0.5	50			

3.4 Validation of GAMS model against EES results

The model was executed in both environments with compressor Cp5, evaporator Ev5 and condenser Cd5 across all seasons in Table 1 for each refrigerant to validate the EES and GAMS codes against each other. The air flow rates were set as $\dot{m}_{a,e} = \dot{m}_{a,c} = \{1,2,3\}\ kg/s$ for S1, S2 and S3, respectively. No degree of freedom was left, such that the results shown in Table 4 represent the accuracy of the GAMS model given the thermophysical property approximations. The life cycle cost deviation is ≤2.6% for the six refrigerants. Errors result mainly from Δh_{12} affecting the modeled compressor power draw directly and Δh_{61}, affecting the modeled mass flow rate and thereby indirectly the compressor work. Other major contributions to the LCC (installation cost, cooling demand, fan power, P1, P2) were identical for the two approaches because no approximations were necessary in the GAMS model. When executing EES with the polynomials used in GAMS, the LCC is identical for GAMS and EES up to $1.

Table 4 : Validation of GAMS solution against EES results by comparing life cycle cost. Cp5, Ev5 and Cd5 were imposed as compressor, evaporator and condenser.

Refrigerant	**LCC [$] (EES)**	**LCC [$] (GAMS)**	**Deviation [%]**
Ammonia	54266	54641	-0.7
R134a	63693	64051	-0.6
R32	57880	58498	-1.1
R404A	56629	57103	-0.8
R407C	55193	56627	-2.6
R410A	57845	58415	-1.0

4. Equations for Integer Constraints

4.1 GAMS

The GAMS language allows the use of sets. For example, all compressors form the set X (upper case) and its separate elements are generically denoted as x (lower case). Similarly, the evaporators y are contained in the set Y, the condensers z are contained in the set Z, the three seasons s are contained in S (compare with Table 1) and the six refrigerants r form the set R. The element symbols are written as superscripts to variables and should not be confused with exponents. For example, U^x is a binary variable defining whether a compressor model is used, thus it exists for each compressor in the component library. Equations that limit the algorithm to choose only one model each for the compressor, evaporator and condenser are

$$\sum_{x \in X} U^x = 1; \sum_{y \in Y} U^y = 1; \sum_{z \in Z} U^z = 1.$$

Some equations from Table 2 have to be adjusted when implementing integer decisions in GAMS. For example, the evaporator effectiveness equation is written with the sum of all products $U^y UA_e^y$ instead of a single UA_e variable. Because only one U^y can be non-zero, the equation will eventually be evaluated with only one UA value but the algorithm can choose freely which one. The air flow rate and therefore the effectiveness are season dependent and therefore have the superscript s. The following equation is therefore duplicated for each season s in S.

$$\epsilon_e^s = 1 - \exp\left(-\left(\sum_{y\in Y} U^y\, UA_e^y\right) / \left(\dot{m}_{a,e}^s\, c_p\right)\right), \qquad \forall s \in S$$

Similarly, the overall isentropic efficiency is written as the sum of all $U^x\eta_{i,o}^x$ where the binary variable U^x nullifies all $\eta_{i,o}^x$ except the one for the selected compressor. Again, the equation is duplicated for all seasons.

$$\sum_{x\in X} U^x\, \eta_{i,o}^x = \frac{h_{2s}^s - h_1^s}{h_{2a}^s - h_1^s}, \qquad \forall s \in S$$

The computation of the mass flow rate from compressor specifications is usually written as $\dot{m}_r = \eta_v \cdot RPM \cdot V_{swept} \cdot \rho$, multiplied with the number of compressors in parallel. Two changes are introduced to account for the discrete number of compressors in parallel, which may vary with the seasons. The volumetric efficiency can be described like the isentropic efficiency as $\sum_{x\in X} U^x\, \eta_v^x$ because it depends only on the chosen compressor model and not on the number of compressors. An additional integer variable is used for the number of operational compressors to compute the total mass flow rate. It is denoted as $On^{x,s}$, where the superscripts x and s denote that this variable exists for all 30 combinations of the 10 compressor types and the 3 seasons. By substituting η_v, inserting $On^{x,s}$ and rearranging, the following equation is obtained:

$$\sum_{x\in X} \frac{U^x \eta_v^x}{On^{x,s} \cdot V_{swept}^x} = \dot{m}_r^s \cdot \rho_1^s \cdot RPM^s, \qquad \forall s \in S$$

The number of compressors of any type that must be purchased, N^x, is equal to the largest $On^{x,s}$, written as an inequality constraint:

$$N^x \geq On^{x,S}, \qquad \forall x \in X \text{ and } \forall s \in S$$

The formulation of the selection problem for the heat exchangers is simpler because at most one evaporator and condenser may be purchased. N^y and N^z is therefore defined as

$$N^y = U^y, \qquad \forall y \in Y,$$
$$N^z = U^z, \qquad \forall z \in Z.$$

The capital cost is then written in one equation using the set of all components C (comprising the three component sets X, Y, Z). It is multiplied with a constant factor $f_{install}$ to account for assembly and installation cost.

$$CC = f_{install} \sum_{c\in C} N^c c^c, \qquad \forall c \in C$$

The refrigerant choice was implemented into GAMS similarly to the component specifications. For example, to find the evaporation pressure at a given evaporation temperature, the polynomials for all refrigerants were added together but each was multiplied with a binary decision variable U^r, where only one was allowed to be 1.

$$P_e^s = \sum_{r\in R} U^r \left(c_0^r + c_1^r T_e^s\right), \qquad \forall s \in S$$

4.2 EES

In EES, every variable is continuous, but procedures may be used to enforce variables to be discrete as shown in the following. The cooling demand $\dot{Q}_{dem}$ and evaporator enthalpy difference Δh_{61} define a mass flow rate that satisfies the cooling demand at the evaporation temperature of the current iteration.

$$\dot{m}_r = \dot{Q}_e / \Delta h_{61}$$

The minimum number of compressors expressed as a continuous number (NOC_{exact}) given a swept volume and a maximum frequency can be calculated as

$$NOC_{exact} = \frac{\dot{m}_r}{\eta_v \cdot f_{max} \cdot V_{swept} \cdot \rho}.$$

Since the number of compressors must be an integer value, NOC_{exact} is rounded up with a ceiling function.

$$NOC = ceil(NOC_{exact})$$

With NOC, the frequency can be calculated.

$$f = \dot{m}_r / (\eta_v \cdot NOC \cdot V_{swept} \cdot \rho)$$

Since Δh_{61} is a function of the evaporation temperature, the procedure must be part of the overall iteration in the EES solver.

5. Results

5.1 Optimal Configuration

GAMS and EES should pick the optimal refrigerant, compressor, evaporator and condenser from the component library, given the chiller operates in three seasons with the weighting factors as in Table 1 and air flow rates set to 1, 2 and 3 kg/s for S1, S2 and S3, respectively. In EES, this requires an enumerated search containing all possible combinations of refrigerants, compressors, evaporators and condensers, since EES does not include MINLP solvers. The three seasons are implemented using array notation and the determination of the number of compressors as explained in section 4.2. Sorting the 6000 results by the life cycle cost reveals the optimal solution $LCC = \$44470$ to be 1 unit of compressor Cp8 together with Ev1, Cd7 and R32 as the refrigerant as shown in Table 5 . The computation took more than 10 minutes when using the built-in property functions. The same optimal configuration was found using the polynomials from GAMS in EES. The computation time increased, presumably because the property function evaluation contains lookup statements for the coefficients that are evaluated in every iteration. Using the solver SCIP (Bestuzheva et al., 2021), GAMS found the same optimal configuration despite some deviation in the LCC due to the approximations as explained in section 3.4. The computation time was approximately 11 seconds, almost 60 times faster than in EES. A benefit of the parametric table in EES is that all solutions are available. This allows plotting all 6000 life cycle costs in a histogram with 60 bins as shown in Figure 6 to gain insight into the distribution of solutions. The gap between the optimal and second-best solution for the life cycle cost in EES was only \$32, much less than the deviations between GAMS and EES shown in Table 5.

Table 5 : Performance and results of GAMS and EES in configuration study.

Code	Computation time [s]	Configuration [-]	LCC [$]
EES (built-in props)	621	R32 + 1xCp8+Ev3+Cd7	44470
EES (polynomials)	934	R32 + 1xCp8+Ev3+Cd7	44811
GAMS	11	R32 + 1xCp8+Ev3+Cd7	44811

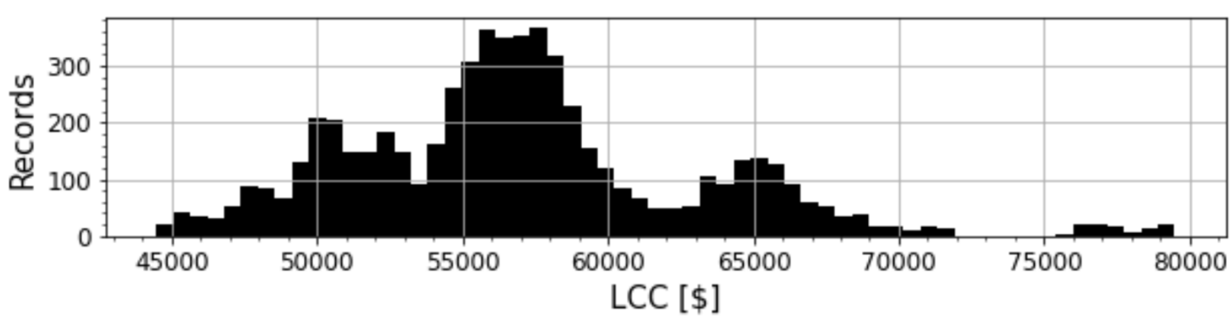

Figure 6: Histogram of all 6000 solutions from EES in 60 bins.

5.2 Parametric Study on UA_{house}

With the fast computation time of GAMS and a convenient interface to Python, parametric studies may be run on certain inputs, as for example the conductance of the house UA_{house}. One execution of a for-loop in Python was used to generate the results shown in Table 6. As UA_{house} increases, the optimal configuration is forced to change to satisfy the higher cooling demand obeying bounds on saturation temperatures and compressor frequency. In EES, this would require separate execution and post-processing to find the optimal solution for each row of the table with an overall significantly longer computation time.

Table 6: Optimal component choice for varying project UA_{house}.

UA_{house} [kW/K]	Refrigerant	Compressor	Evaporator	Condenser
0.3	Ammonia	1xCp4	Ev1	Cd1
0.4	R32	1xCp4	Ev1	Cd4
0.5	R32	1xCp8	Ev3	Cd7
0.6	R32	1xCp8	Ev10	Cd7
0.7	R32	2xCp4	Ev7	Cd7

6. Discussion

The GAMS solution clearly had better computational performance with the same end results and was easier to implement compared to EES for the presented problem statement. However, there are caveats to be aware of, as for example "performance variability", which is essentially an unexpected change in performance prone to occur when solving complex mixed integer optimization problems. According to Koch et al. (2011), even simply changing the order of constraints in the code or adding or removing redundant constraints can impact the performance of solvers, often due to imperfect tie-breaking in the branch and cut optimization method. The coding environment itself and differences in rounding errors due to differently arranged equations can impact the performance, too. Danna (2008) describes performance variability informally as "*a change in performance we do not understand*". This means for example, that the theoretically intuitive addition of tighter (but realistic) bounds does not necessarily improve the performance and may even worsen it, introducing trial and error in the model tuning which is not existent in directly solving the model for all possible combinations of integer variables.

Although GAMS outperformed EES in terms of computational speed, the performance benefit was reduced for a smaller number of decisions variables and an increased complexity of the nonlinear programming problem. For example, the benefit was reduced when choosing components from smaller libraries but making the air flow rates a continuous optimization variable or when introducing a performance penalty to the compressors linearly changing with the compressor frequency.

7. Conclusions

An artificial mixed integer nonlinear programming problem was solved with GAMS and EES. Except for the isentropic discharge enthalpy, first-order polynomials fitted for defined property regions were sufficiently accurate and decreased the computation time compared to using second-order approximations. Integer variables were introduced in EES using rounding functions. Generally, GAMS showed superior performance and practicability for the component selection from available libraries, solving the problem with 6000 discrete options in 11 seconds where EES needed over 10 minutes using a brute force approach. The benefit was amplified for parametric studies of the optimization process. The superiority of GAMS increased more clearly with the number of discrete decisions than with the complexity of the nonlinear programming problem. Performance variability was encountered but insignificant for the given problem size. The need for trial-and-error iterations on bounds and best performing solvers is an expected inconvenience for larger problems.

NOMENCLATURE

Symbols and acronyms		
a_0, a_1	Constants in polynomial	
c_0, c_1	Constants in polynomial	
C	Set of all components	
$c_{comp}, c_{cond}, c_{evap}$	Capital cost of compressor, condenser and evaporator	$

CC	Capital cost	\$
c_{el}	Electricity cost	\$/kWh
COP	Coefficient of performance	-
c_p	Specific heat	kJ/(kg·K)
d	discount rate	-
dp	down payment	-
EC	Energy Cost	\$/year
EES	Engineering Equation Solver	
f	Compressor frequency	1/min
$GAMS$	Generic Algebraic Modeling System	
$f_{install}$	Factor to account for installation and assembly cost	-
h	Enthalpy	kJ/kg
i_{inf}	Inflation	-
LCC	Life cycle cost	
m	Interest rate on loan	
$MINLP$	Mixed integer nonlinear programming	
M_s	Maintenance cost as ratio to capital cost	-
N	Integer variable (number of components to purchase)	
NOC	Number of components	
N_D	Period of depreciation	years
N_l	Period to pay back loan	years
N_p	Project duration considered for economic model	years
On	Integer variable (number of operational compressors in a given season for a given compressor model)	
ORC	Organic Rankine Cycle	
P	Pressure	kPa
P_1, P_2	Factors in computing life cycle cost	
$\dot{Q}$	Heat transfer rate	kW
R_v	Resale value	-
S	Set of all seasons	
T	Temperature	°C
tax_{in}	Income tax	-
tax_{pr}	Property tax	-
U	Binary variable, 1 if component is used	
UA	Conductance	kW/K
V_{swept}	Swept volume of compressor	cm^3
V_r	Ratio of additional property value resulting from system installation	-
w	Weighting factor	-
$\dot{W}$	Electric power	kW

Subscripts		**Greek symbols**	
1..7	state points	$\Delta h_{61}, \Delta h_{12}$	Enthalpy difference between two state points
a	air or adiabatic	ϵ	Effectiveness
amb	ambient	η	Efficiency
b	balance	**Superscripts**	
c	condensation/condenser	c	components
comp	compressor	r	refrigerant
e	evaporation/evaporator	s	season
exact	Continuous value of a variable that is defined as an integer variable	x	compressor
i,o	overall isentropic	y	evaporator
in	inlet	z	condenser

h,l	heat loss	
out	outlet	
r	refrigerant	
s	isentropic	
tot	total	
sc	subcooling	
sh	superheat	
v	volumetric	

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