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A General Stable Approach to Modeling and Coupling Multilayered Systems with Various Types of Layers

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A general stable approach to modeling and coupling multilayered systems with various types of layers

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Presentation Available at Herrick e-Pubs: https://docs.lib.purdue.edu/herrick

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PURDUE MECHANICAL ENGINEERING

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Agenda

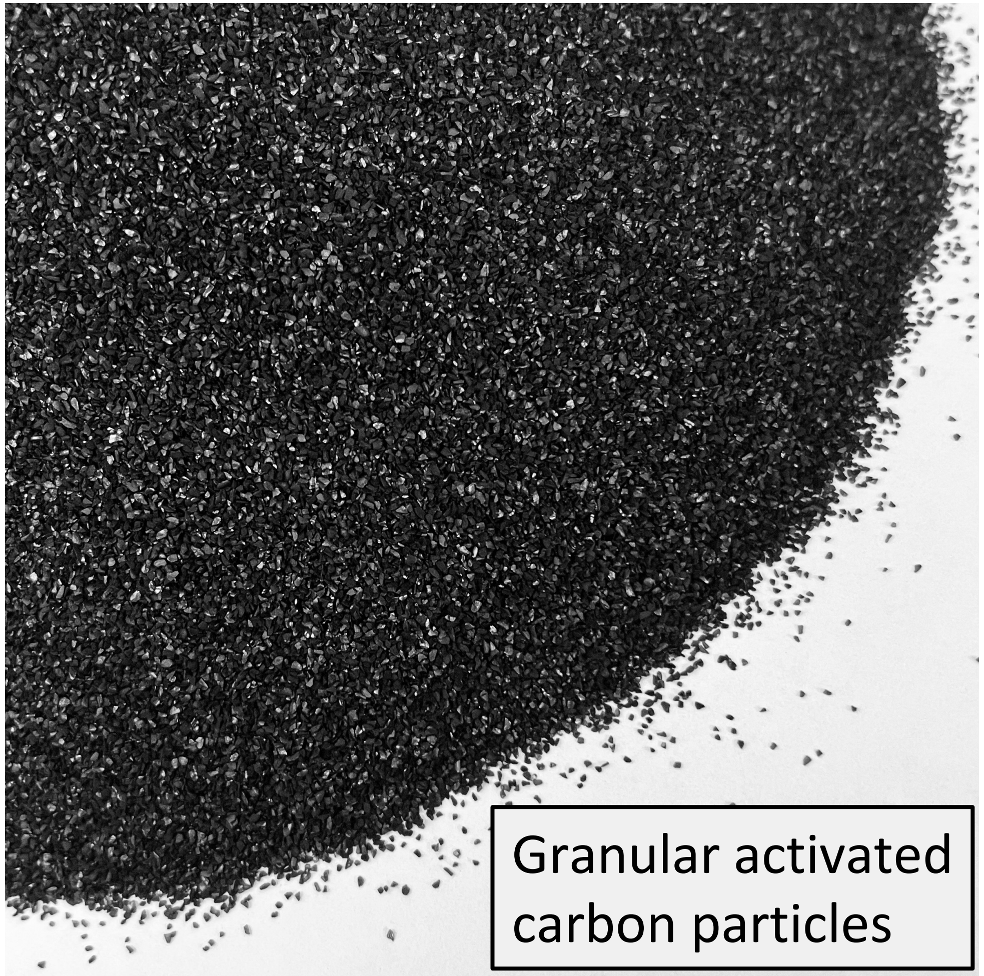
- Motivation & Literature
- Methodology
- Example Results
- Conclusions

Motivation & Literature

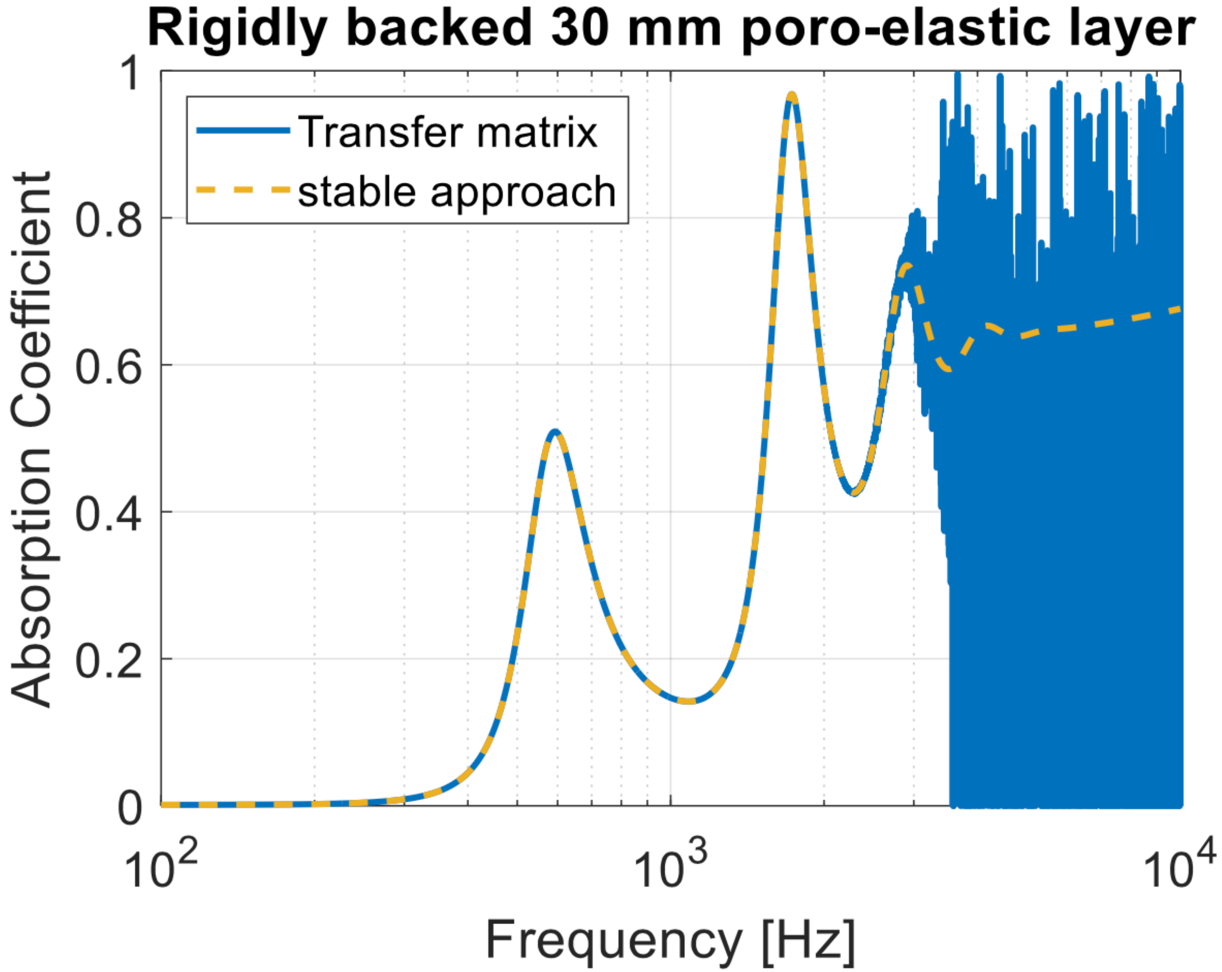
Motivation

σ [Rayls/m]	ϕ	α_∞	ρ_b [kg/m³]
1.5×10^6	0.92	1.3	24
E - Pa	η	ν	θ
6000	0.004	0.27	0°

Stacks of activated carbon are known to be poro-elastic (Mo *et al.,* 2021)



Granular activated carbon particles

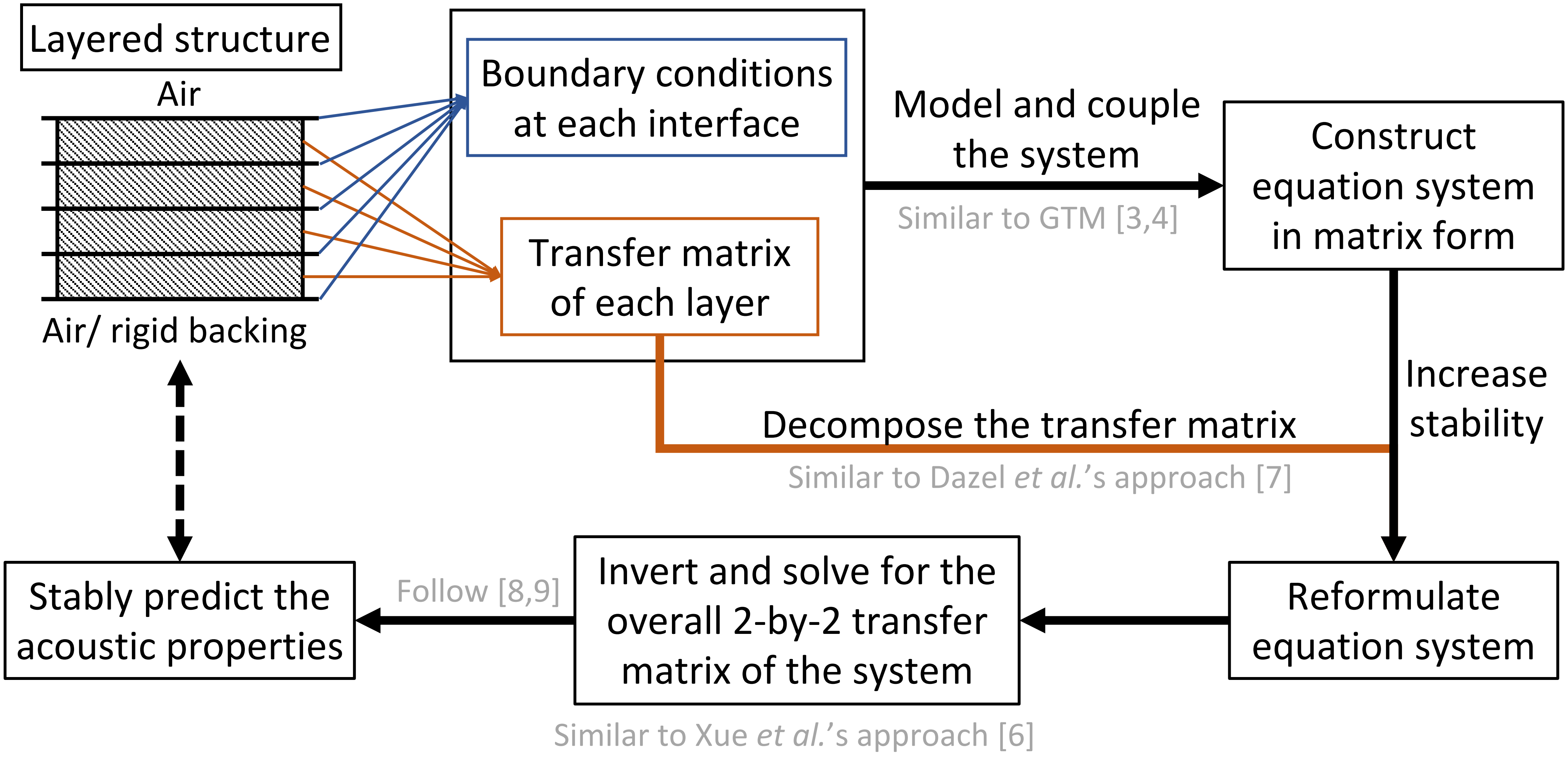


Previously-proposed methods

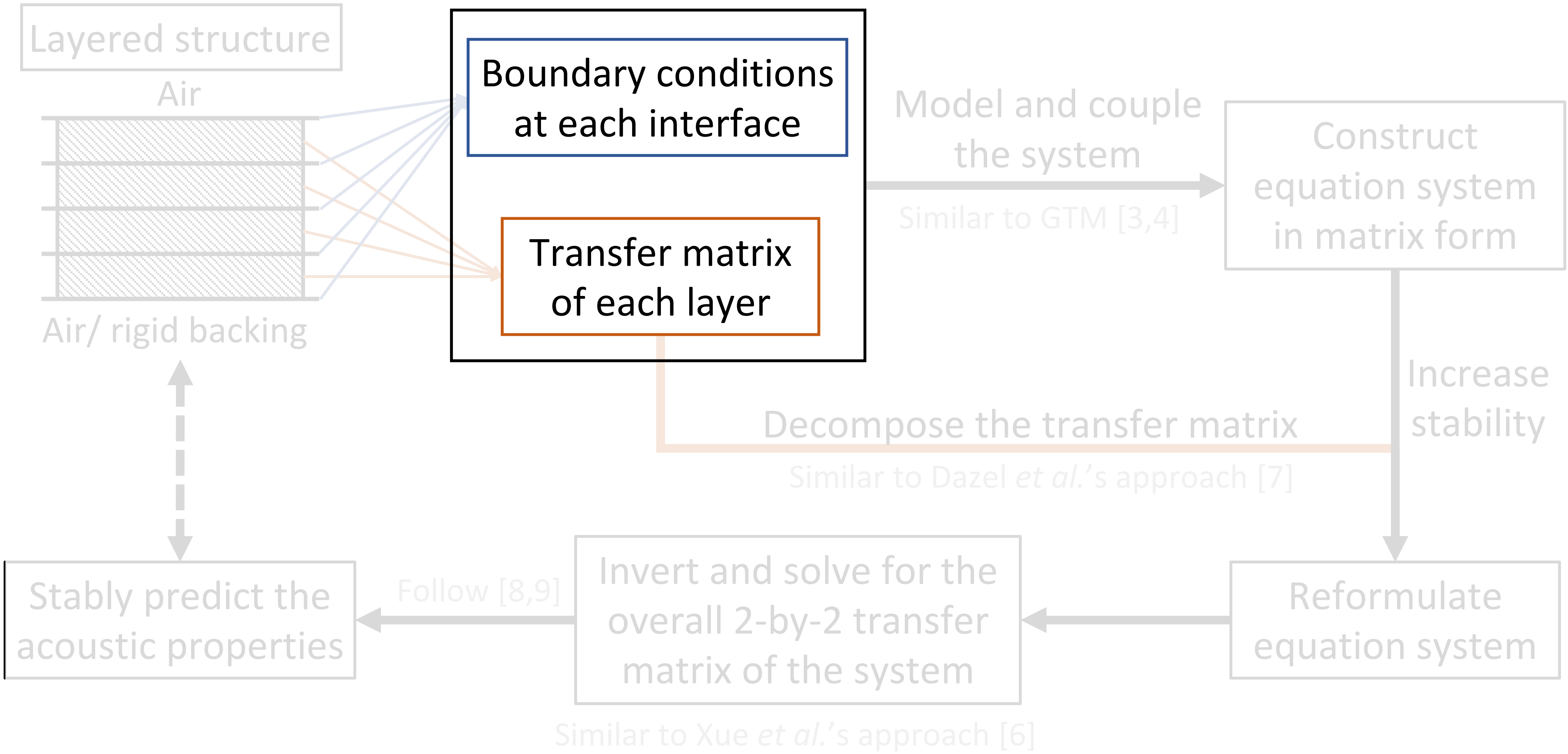
	Variables	General method?	Effort to redesign the system	Stability
Arbitrary coefficient method (ACM) [2,3]	Amplitude of waves	√	Time-consuming	Unstable
Global transfer matrix method (GTM) [4,5]	State vector	√	Easy	Unstable
Xue *et al.*'s method [6]	State vector	x	Easy	Unstable
Dazel *et al.*'s method [7]	Information vector	√	Easy	Stable

Methodology

Overview of the stabilized TMM



Overview of the stabilized TMM



Matrix representation – transfer matrix

Layer Type	Waves	Transfer matrix	State vector
Fluid	1 dilatational	$[\boldsymbol{T}^{\boldsymbol{f}}]_{2\times2}$	$\boldsymbol{V}^{\boldsymbol{f}} = \left[p \quad v_z^f\right]^T$
Elastic-solid	1 dilatational + 1 rotational	$[\boldsymbol{T}^{\boldsymbol{s}}]_{4\times4}$	$\boldsymbol{V}^{\boldsymbol{s}} = [v_x^s \quad v_z^s \quad \sigma_{zz}^s \quad \tau_{xz}^s]^T$
Poro-elastic	2 dilatational + 1 rotational	$[\boldsymbol{T}^{\boldsymbol{p}}]_{6\times6}$	$\boldsymbol{V}^{\boldsymbol{p}} = \left[v_x^s \quad v_z^s \quad v_z^f \quad \sigma_{zz}^s \quad \tau_{xz}^s \quad \sigma_{zz}^f\right]^T$

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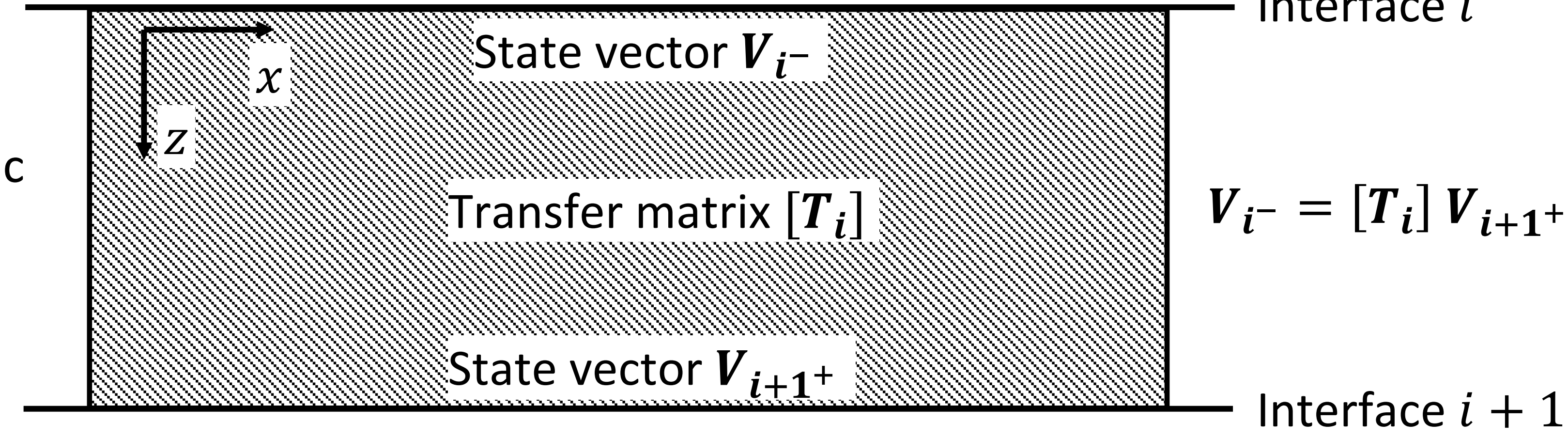
Poro-elastic	2 dilatational + 1 rotational	$[\boldsymbol{T^p}]_{6\times6}$	$\boldsymbol{V^p} = \left[v_x^s \quad v_z^s \quad v_z^f \quad \sigma_{zz}^s \quad \tau_{xz}^s \quad \sigma_{zz}^f\right]^T$

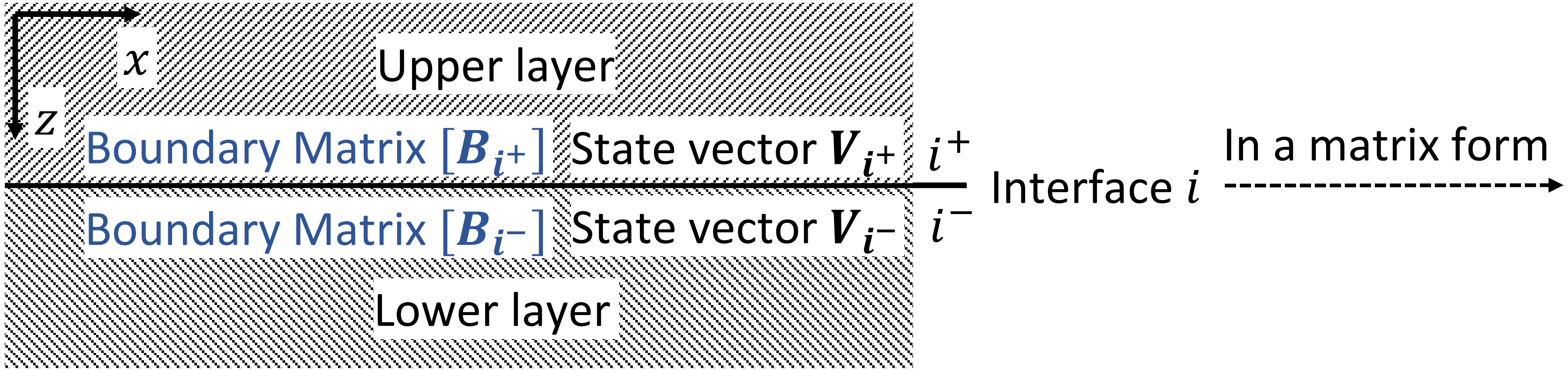
A layer of acoustic material



$$\boldsymbol{V_{i^-}} = [\boldsymbol{T_i}]\,\boldsymbol{V_{i+1^+}}$$

Matrix representation – boundary conditions

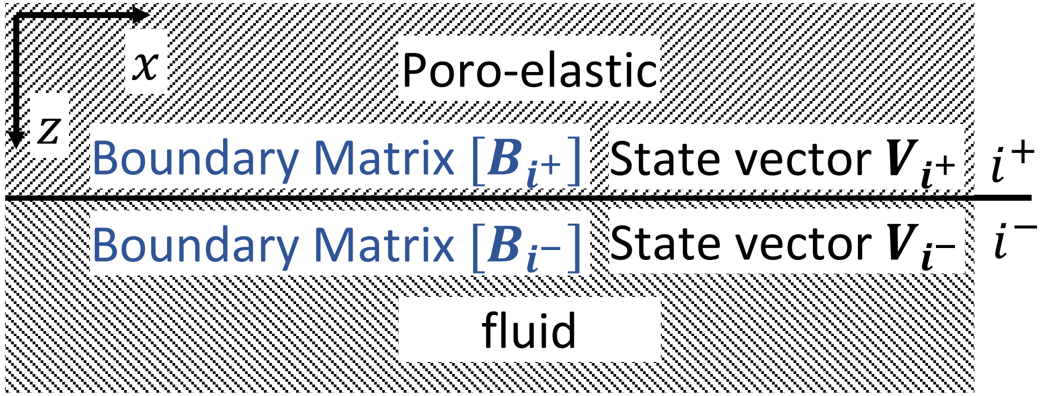
General expression:



$$[\boldsymbol{B}_{i^+}]\boldsymbol{V}_{i^+} = [\boldsymbol{B}_{i^-}]\boldsymbol{V}_{i^-}$$

Matrix representation – boundary conditions

General expression:



In a matrix form

$$[\boldsymbol{B}_{\boldsymbol{i}^+}]\boldsymbol{V}_{\boldsymbol{i}^+} = [\boldsymbol{B}_{\boldsymbol{i}^-}]\boldsymbol{V}_{\boldsymbol{i}^-}$$

More specifically, interface between a poro-elastic layer ($\boldsymbol{i}^+$) and a fluid layer ($\boldsymbol{i}^-$):

Boundary conditions:

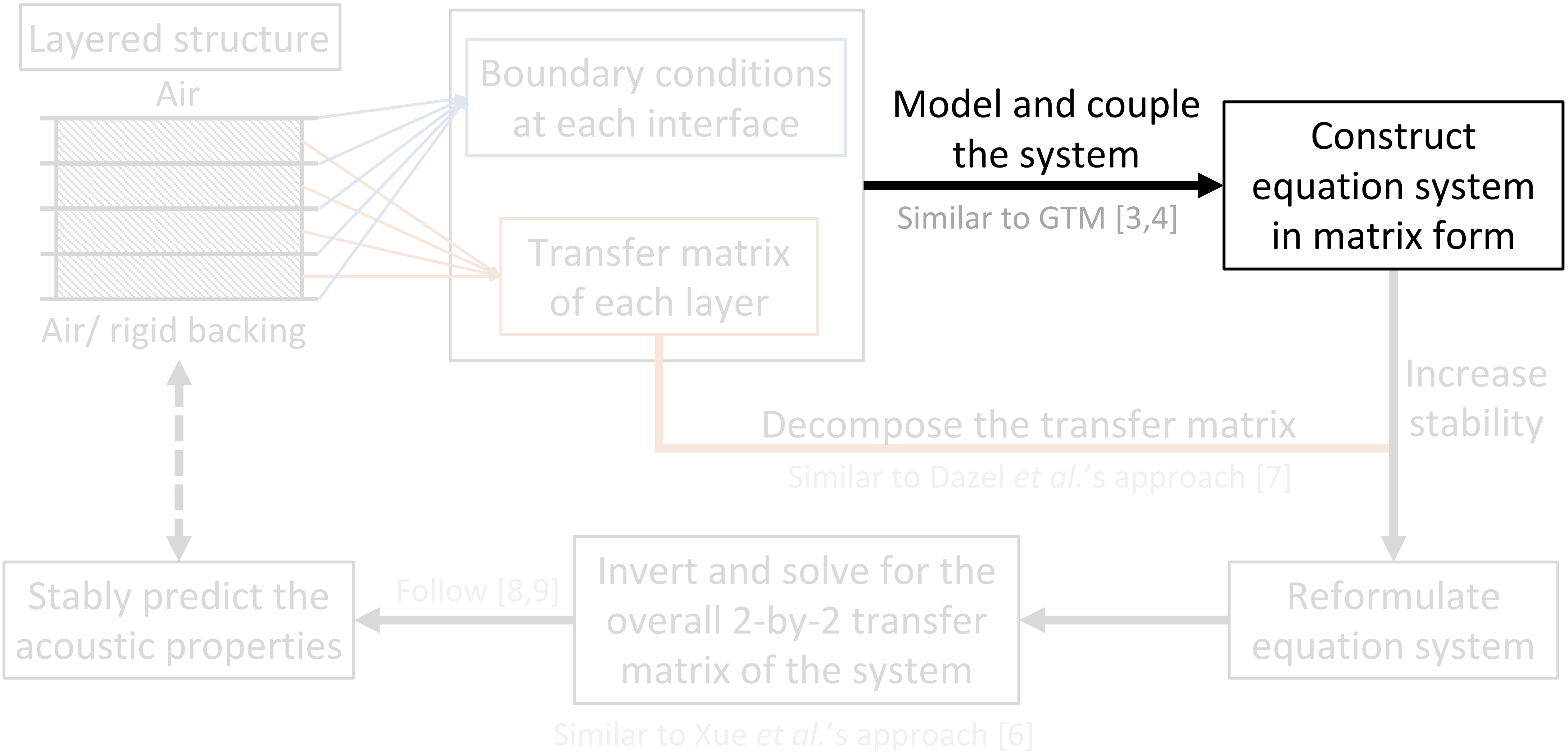
- $[\sigma_{zz}^s]^p = -(1-\phi)[p]^f$
- $\left[\sigma_{zz}^f\right]^p = -\phi[p]^f$
- $(1-\phi)[v_z^s]^p + \phi\left[v_z^f\right]^p = \left[v_z^f\right]^f$
- $[\tau_{xz}^s]^p = 0$

ϕ: porosity

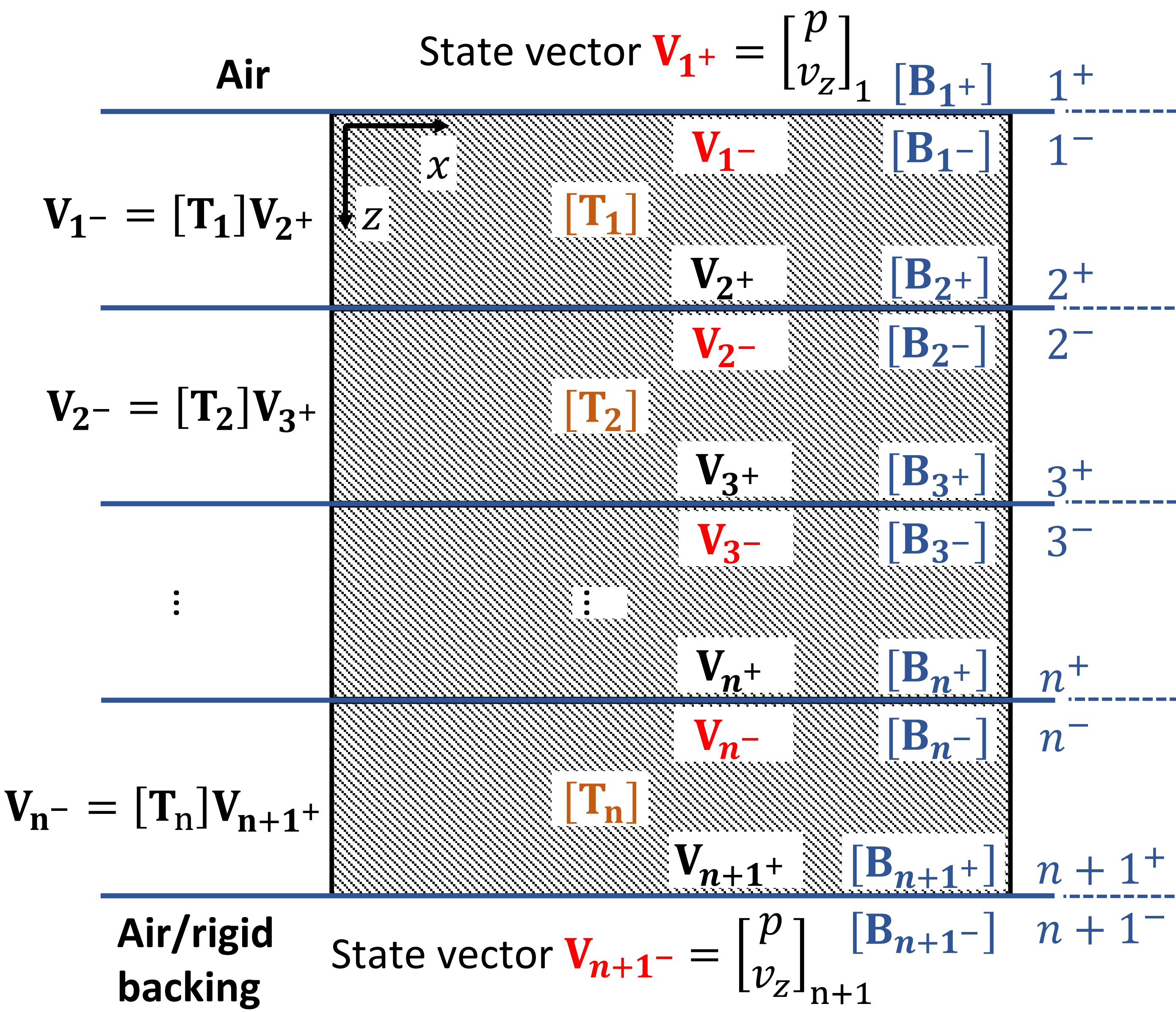
Matrix form: $[\boldsymbol{B}_{\boldsymbol{i}^+}]$ $\boldsymbol{V}_{\boldsymbol{i}^+}$ = $[\boldsymbol{B}_{i^-}]$ $\boldsymbol{V}_{\boldsymbol{i}^-}$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1-\phi & \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_x^s \\ v_z^s \\ v_z^f \\ \sigma_{zz}^s \\ \tau_{xz}^s \\ \sigma_{zz}^f \end{bmatrix} = \begin{bmatrix} -(1-\phi) & 0 \\ -\phi & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ v_z^f \end{bmatrix}$$

Overview of the stabilized TMM



Model and couple the layered system



Boundary conditions at interfaces:

$$[\boldsymbol{B}_{1^+}]\boldsymbol{V}_{1^+} = [\boldsymbol{B}_{1^-}]\boldsymbol{V}_{1^-},$$

$$[\boldsymbol{B}_{2^+}]\underbrace{[\boldsymbol{T}_1]^{-1}\boldsymbol{V}_{1^-}}_{\boldsymbol{V}_{2^+}} = [\boldsymbol{B}_{2^-}]\boldsymbol{V}_{2^-},$$

$$[\boldsymbol{B}_{3^+}]\underbrace{[\boldsymbol{T}_2]^{-1}\boldsymbol{V}_{2^-}}_{\boldsymbol{V}_{3^+}} = [\boldsymbol{B}_{3^-}]\boldsymbol{V}_{3^-},$$

…

$$[\boldsymbol{B}_{n^+}]\underbrace{[\boldsymbol{T}_{n-1}]^{-1}\boldsymbol{V}_{n-1^-}}_{\boldsymbol{V}_{n+1^+}} = [\boldsymbol{B}_{n^-}]\boldsymbol{V}_{n^-},$$

$$[\boldsymbol{B}_{n+1^+}]\underbrace{[\boldsymbol{T}_n]^{-1}\boldsymbol{V}_{n^-}}_{\boldsymbol{V}_{n^+}} = [\boldsymbol{B}_{n+1^-}]\boldsymbol{V}_{n+1^-}.$$

Model and couple the layered system

Boundary conditions at interfaces:

$$[\boldsymbol{B}_{1^+}]\boldsymbol{V}_{1^+} = [\boldsymbol{B}_{1^-}]\boldsymbol{V}_{1^-},$$

$$[\boldsymbol{B}_{2^+}][\boldsymbol{T}_1]^{-1}\boldsymbol{V}_{1^-} = [\boldsymbol{B}_{2^-}]\boldsymbol{V}_{2^-},$$

$$[\boldsymbol{B}_{3^+}][\boldsymbol{T}_2]^{-1}\boldsymbol{V}_{2^-} = [\boldsymbol{B}_{3^-}]\boldsymbol{V}_{3^-},$$

$$\cdots$$

$$[\boldsymbol{B}_{n^+}][\boldsymbol{T}_{n-1}]^{-1}\boldsymbol{V}_{n-1^-} = [\boldsymbol{B}_{n^-}]\boldsymbol{V}_{n^-},$$

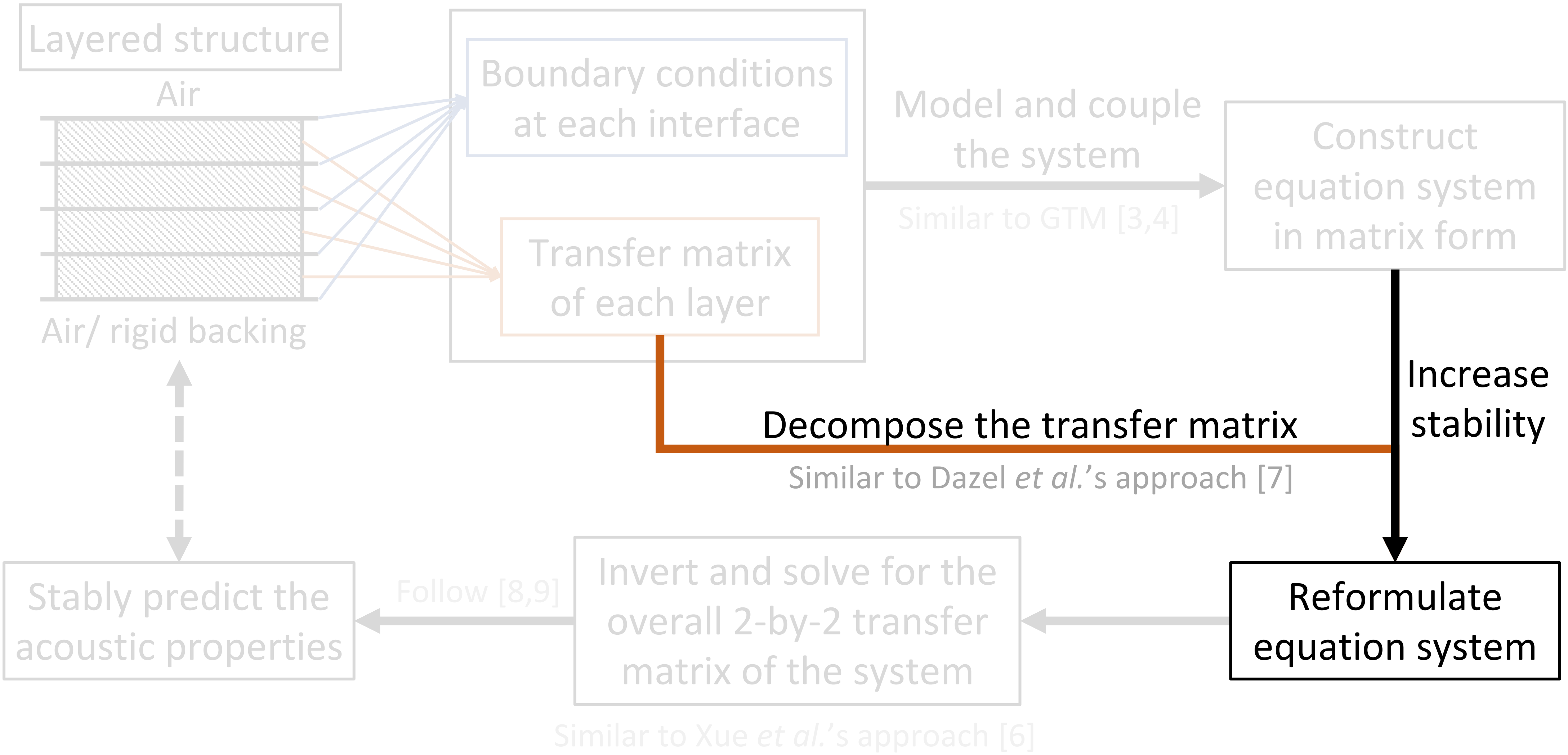
$$[\boldsymbol{B}_{n+1^+}][\boldsymbol{T}_n]^{-1}\boldsymbol{V}_{n^-} = [\boldsymbol{B}_{n+1^-}]\boldsymbol{V}_{n+1^-}.$$

Matrix form equation system:

Global Matrix $[\boldsymbol{A}]$

$$\begin{bmatrix} [\boldsymbol{B}_{1^+}] & -[\boldsymbol{B}_{1^-}] & \cdots & [0] & [0] \\ [0] & [\boldsymbol{B}_{2^+}][\boldsymbol{T}_1]^{-1} & \cdots & [0] & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & [0] & \cdots & [\boldsymbol{B}_{n^+}][\boldsymbol{T}_{n-1}]^{-1} & -[\boldsymbol{B}_{n^-}] \\ [0] & [0] & \cdots & [0] & [\boldsymbol{B}_{n+1^+}][\boldsymbol{T}_n]^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{1^+} \\ \boldsymbol{V}_{1^-} \\ \cdots \\ \boldsymbol{V}_{n-1^-} \\ \boldsymbol{V}_{n^-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ [\boldsymbol{B}_{n+1^-}] \end{bmatrix} \boldsymbol{V}_{n+1^-}.$$

Overview of the stabilized TMM



Decompose of the transfer matrix

In the traditional TMM:

When there is a significant disparity between the magnitudes of the waves: i.e.,

- at higher frequencies
- for a thick layer
- for extreme parameter values

The most attenuated wave's contribution can be masked by numerical errors.

→

Instability occurs when inverting the global matrix

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When there is a significant disparity between the magnitudes of the waves: i.e.,

- at higher frequencies
- for a thick layer
- for extreme parameter values

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→ Instability occurs when inverting the global matrix

Decomposition – extract wave attenuation terms

$$[\boldsymbol{T}] = [\boldsymbol{\Phi}][\boldsymbol{\Lambda}][\boldsymbol{\Phi}]^{-1}$$

With wave attenuation terms

E.g., for a solid layer:

$$[\Lambda^s] = \begin{bmatrix} e^{jk_{13}d} & 0 & 0 & 0 \\ 0 & e^{-jk_{13}d} & 0 & 0 \\ 0 & 0 & e^{jk_{33}d} & 0 \\ 0 & 0 & 0 & e^{-jk_{33}d} \end{bmatrix}$$

Reformulate the equation

Equation system:

$$\begin{bmatrix} [\boldsymbol{B}_{1^+}] & -[\boldsymbol{B}_{1^-}] & \cdots & [0] & [0] \\ [0] & [\boldsymbol{B}_{2^+}][\boldsymbol{T}_1]^{-1} & \cdots & [0] & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & [0] & \cdots & [\boldsymbol{B}_{n^+}][\boldsymbol{T}_{n-1}]^{-1} & -[\boldsymbol{B}_{n^-}] \\ [0] & [0] & \cdots & [0] & [\boldsymbol{B}_{n+1^+}][\boldsymbol{T}_n]^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{1^+} \\ \boldsymbol{V}_{1^-} \\ \cdots \\ \boldsymbol{V}_{n-1^-} \\ \boldsymbol{V}_{n^-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ [\boldsymbol{B}_{n+1^-}] \end{bmatrix} \boldsymbol{V}_{n+1^-}.$$

Reformulate the equation

Equation system:

$$\begin{bmatrix} [\boldsymbol{B}_{1^+}] & -[\boldsymbol{B}_{1^-}] & \cdots & [0] & [0] \\ [0] & [\boldsymbol{B}_{2^+}][\boldsymbol{T}_1]^{-1} & \cdots & [0] & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & [0] & \cdots & [\boldsymbol{B}_{n^+}][\boldsymbol{T}_{n-1}]^{-1} & -[\boldsymbol{B}_{n^-}] \\ [0] & [0] & \cdots & [0] & [\boldsymbol{B}_{n+1^+}][\boldsymbol{T}_n]^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{1^+} \\ \boldsymbol{V}_{1^-} \\ \cdots \\ \boldsymbol{V}_{n-1^-} \\ \boldsymbol{V}_{n^-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ [\boldsymbol{B}_{n+1^-}] \end{bmatrix} \boldsymbol{V}_{n+1^-}.$$

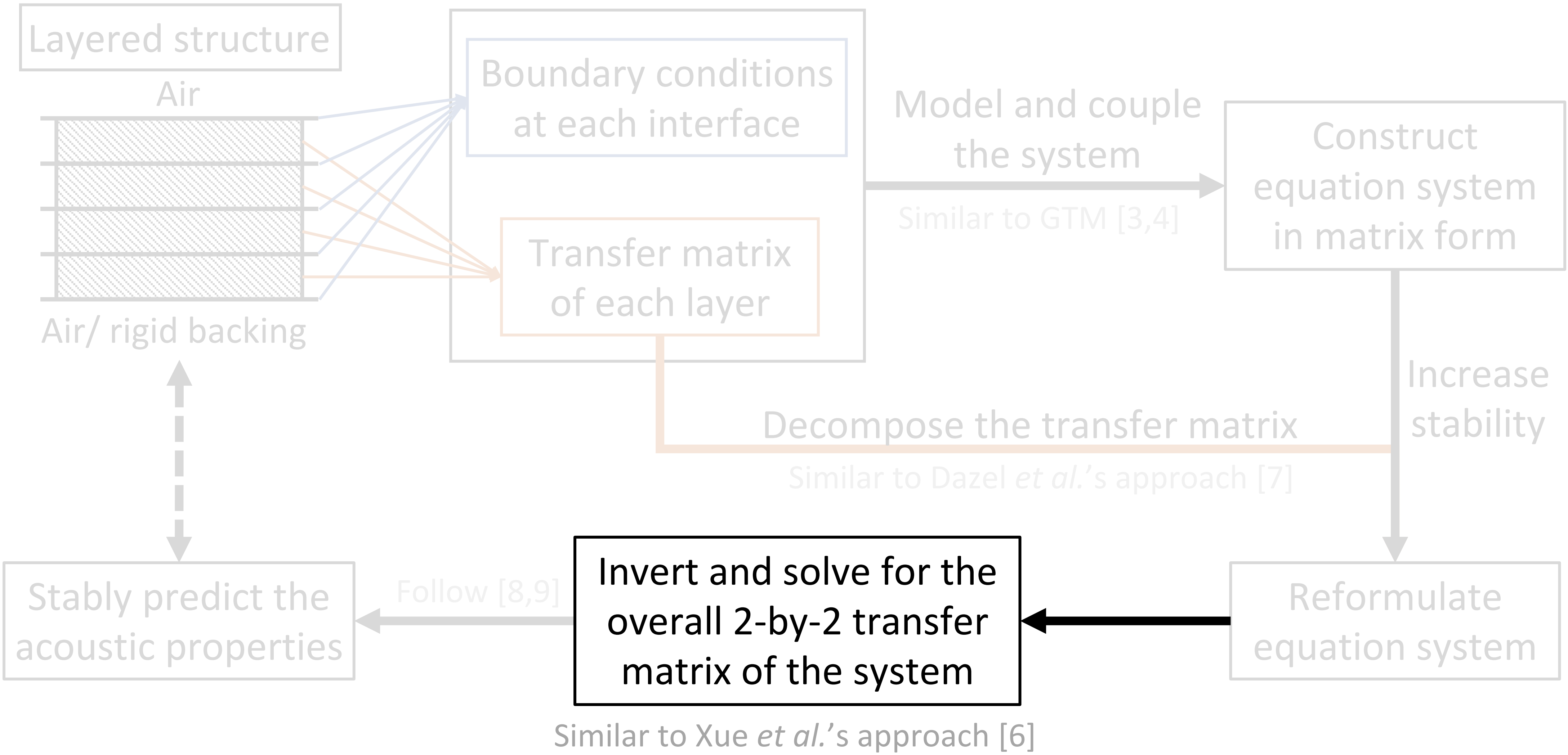
Global Matrix $[\boldsymbol{A}]$

Reformulate

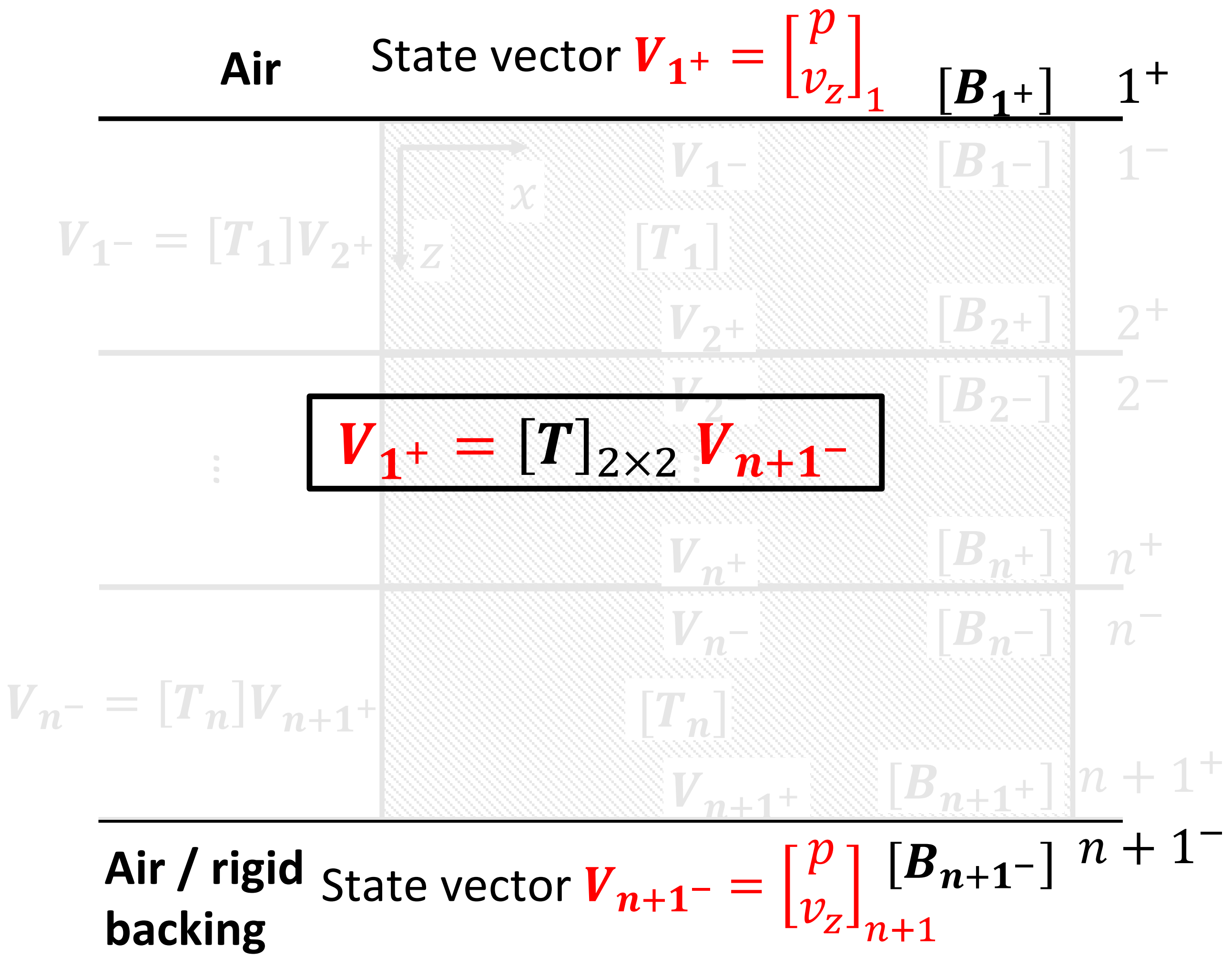
No significant numerical errors

$$\underbrace{\begin{bmatrix} [\boldsymbol{B}_{1^+}] & -[\boldsymbol{B}_{1^-}][\boldsymbol{\Phi}_1] & \cdots & [0] & [0] \\ [0] & [\boldsymbol{B}_{2^+}][\boldsymbol{\Phi}_1][\boldsymbol{\Lambda}_1]^{-1} & \cdots & [0] & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & [0] & \cdots & [\boldsymbol{B}_{n^+}][\boldsymbol{\Phi}_{n-1}][\boldsymbol{\Lambda}_{n-1}]^{-1} & -[\boldsymbol{B}_{n^-}][\boldsymbol{\Phi}_n] \\ [0] & [0] & \cdots & [0] & [\boldsymbol{B}_{n+1^+}][\boldsymbol{\Phi}_n][\boldsymbol{\Lambda}_n]^{-1} \end{bmatrix}}_{[\boldsymbol{A}_1]} \begin{bmatrix} [\boldsymbol{I}] & [\boldsymbol{0}] & \cdots & [\boldsymbol{0}] & [\boldsymbol{0}] \\ [\boldsymbol{0}] & [\boldsymbol{\Phi}_1]^{-1} & \cdots & [\boldsymbol{0}] & [\boldsymbol{0}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [\boldsymbol{0}] & [\boldsymbol{0}] & \cdots & [\boldsymbol{\Phi}_{n-1}]^{-1} & [\boldsymbol{0}] \\ [\boldsymbol{0}] & [\boldsymbol{0}] & \cdots & [\boldsymbol{0}] & [\boldsymbol{\Phi}_n]^{-1} \end{bmatrix}$$

Overview of the stabilized TMM



2-by-2 transfer matrix that relates $\boldsymbol{V}_{1^+}$ and $\boldsymbol{V}_{n+1^-}$

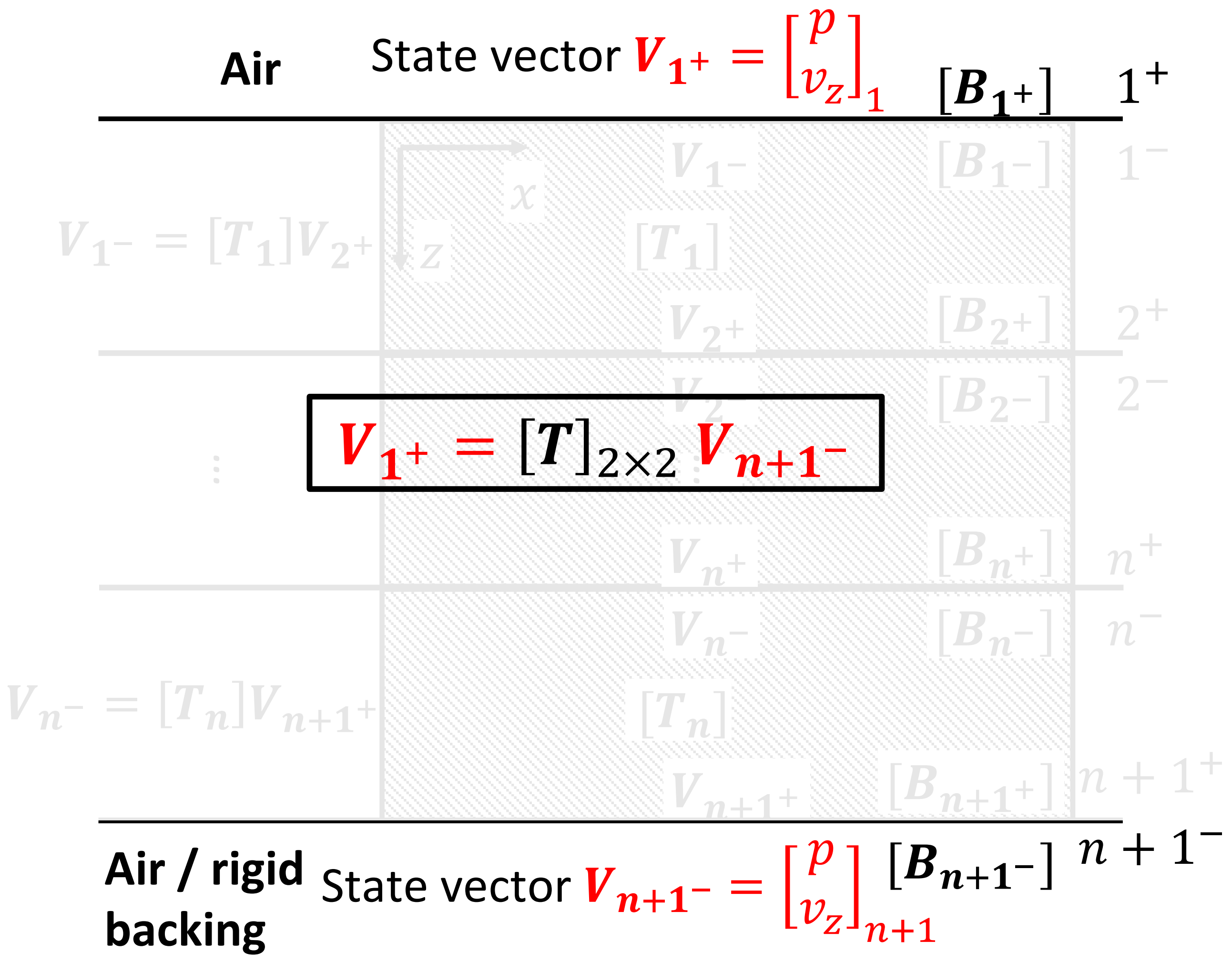


$$\boldsymbol{V}_{1^+} = [\boldsymbol{T}]_{2\times 2}\, \boldsymbol{V}_{n+1^-}$$

Given equation system:

$$[A]\begin{bmatrix} \boldsymbol{V}_{1^+} \\ \boldsymbol{V}_{1^-} \\ \cdots \\ \boldsymbol{V}_{n-1^-} \\ \boldsymbol{V}_{n^-} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ [\boldsymbol{B}_{n+1^-}] \end{bmatrix} \boldsymbol{V}_{n+1^-}.$$

2-by-2 transfer matrix that relates $\boldsymbol{V}_{1^+}$ and $\boldsymbol{V}_{n+1^-}$



Given equation system:

$$\begin{bmatrix} \boldsymbol{V}_{1^+} \\ \boldsymbol{V}_{1^-} \\ \cdots \\ \boldsymbol{V}_{n-1^-} \\ \boldsymbol{V}_{n^-} \end{bmatrix} = [\boldsymbol{A}]^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ [\boldsymbol{B}_{n+1^-}] \end{bmatrix} \boldsymbol{V}_{n+1^-}.$$

$$[\boldsymbol{A}]^{-1} = \begin{bmatrix} [\boldsymbol{I}] & [0] & \cdots & [0] & [0] \\ [0] & [\boldsymbol{\Phi}_1] & \cdots & [0] & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & [0] & \cdots & [\boldsymbol{\Phi}_{n-1}] & [0] \\ [0] & [0] & \cdots & [0] & [\boldsymbol{\Phi}_n] \end{bmatrix} [\boldsymbol{A}_1]^{-1},$$

$$= \begin{bmatrix} [\boldsymbol{A}^*_{1,1}] & [\boldsymbol{A}^*_{1,2}] & \cdots & [\boldsymbol{A}^*_{1,n-1}] & [\boldsymbol{A}^*_{1,n}] \\ [\boldsymbol{A}^*_{2,1}] & [\boldsymbol{A}^*_{2,2}] & \cdots & [\boldsymbol{A}^*_{2,n-1}] & [\boldsymbol{A}^*_{2,n}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [\boldsymbol{A}^*_{n-1,1}] & [\boldsymbol{A}^*_{n-1,2}] & \cdots & [\boldsymbol{A}^*_{n-1,n-1}] & [\boldsymbol{A}^*_{n-1,n}] \\ [\boldsymbol{A}^*_{n,1}] & [\boldsymbol{A}^*_{n,2}] & \cdots & [\boldsymbol{A}^*_{n,n-1}] & [\boldsymbol{A}^*_{n,n}] \end{bmatrix}.$$

2-by-2 transfer matrix that relates $\boldsymbol{V}_{1^+}$ and $\boldsymbol{V}_{\boldsymbol{n}+1^-}$

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Given equation system:

$$\begin{bmatrix} \boldsymbol{V}_{1^+} \\ \boldsymbol{V}_{1^-} \\ \cdots \\ \boldsymbol{V}_{\boldsymbol{n}-1^-} \\ \boldsymbol{V}_{\boldsymbol{n}^-} \end{bmatrix} = [\boldsymbol{A}]^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ [\boldsymbol{B}_{\boldsymbol{n}+1^-}] \end{bmatrix} \boldsymbol{V}_{\boldsymbol{n}+1^-}.$$

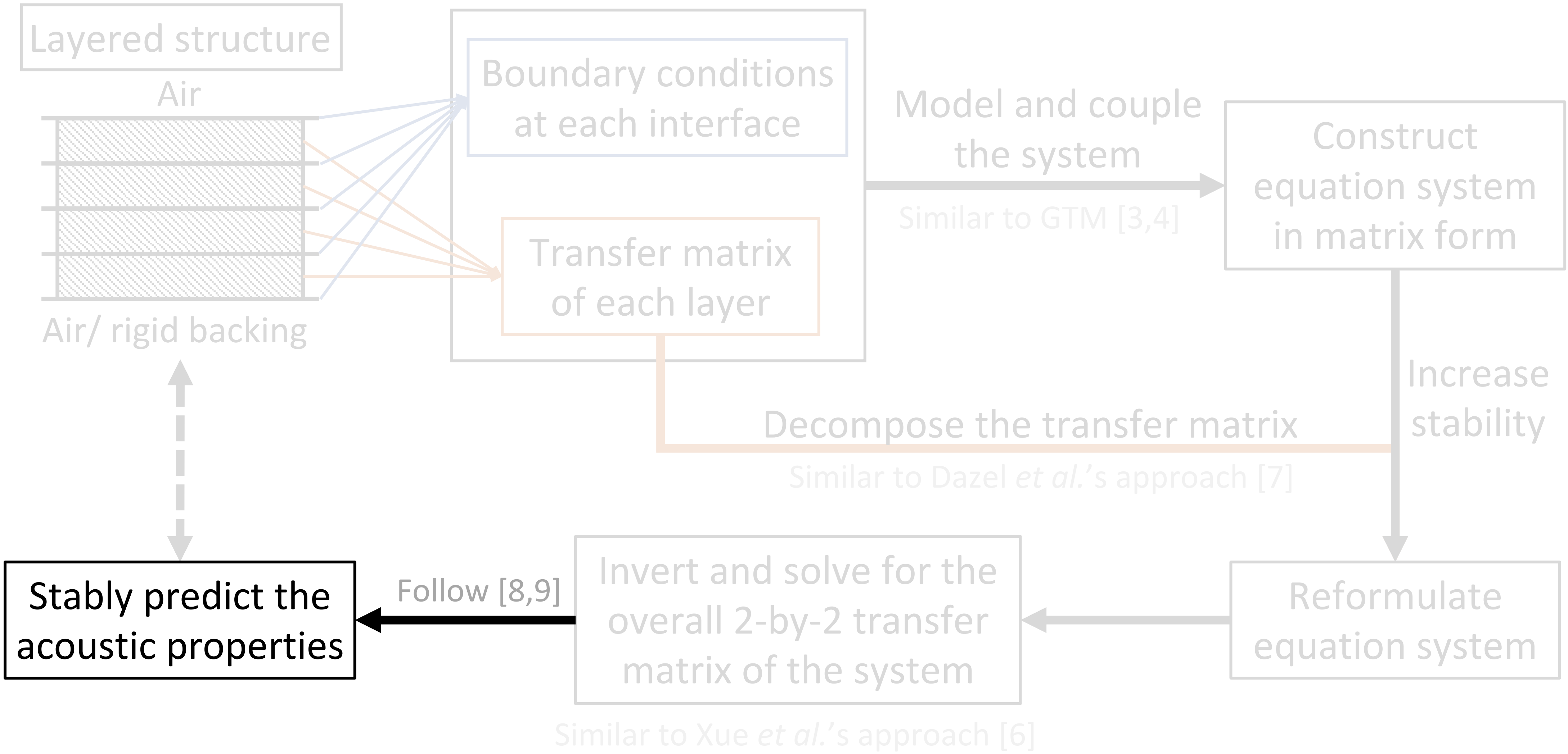
$$[\boldsymbol{A}]^{-1} = \begin{bmatrix} [\boldsymbol{I}] & [\boldsymbol{0}] & \cdots & [\boldsymbol{0}] & [\boldsymbol{0}] \\ [\boldsymbol{0}] & [\boldsymbol{\Phi}_1] & \cdots & [\boldsymbol{0}] & [\boldsymbol{0}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [\boldsymbol{0}] & [\boldsymbol{0}] & \cdots & [\boldsymbol{\Phi}_{\boldsymbol{n}-1}] & [\boldsymbol{0}] \\ [\boldsymbol{0}] & [\boldsymbol{0}] & \cdots & [\boldsymbol{0}] & [\boldsymbol{\Phi}_{\boldsymbol{n}}] \end{bmatrix} [\boldsymbol{A}_1]^{-1},$$

$$= \begin{bmatrix} [\boldsymbol{A}^*_{1,1}] & [\boldsymbol{A}^*_{1,2}] & \cdots & [\boldsymbol{A}^*_{1,\boldsymbol{n}-1}] & [\boldsymbol{A}^*_{1,\boldsymbol{n}}] \\ [\boldsymbol{A}^*_{2,1}] & [\boldsymbol{A}^*_{2,2}] & \cdots & [\boldsymbol{A}^*_{2,\boldsymbol{n}-1}] & [\boldsymbol{A}^*_{2,\boldsymbol{n}}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [\boldsymbol{A}^*_{\boldsymbol{n}-1,1}] & [\boldsymbol{A}^*_{\boldsymbol{n}-1,2}] & \cdots & [\boldsymbol{A}^*_{\boldsymbol{n}-1,\boldsymbol{n}-1}] & [\boldsymbol{A}^*_{\boldsymbol{n}-1,\boldsymbol{n}}] \\ [\boldsymbol{A}^*_{\boldsymbol{n},1}] & [\boldsymbol{A}^*_{\boldsymbol{n},2}] & \cdots & [\boldsymbol{A}^*_{\boldsymbol{n},\boldsymbol{n}-1}] & [\boldsymbol{A}^*_{\boldsymbol{n},\boldsymbol{n}}] \end{bmatrix}.$$

$$\boldsymbol{V}_{1^+} = [\boldsymbol{A}^*_{1,\boldsymbol{n}}][\boldsymbol{B}_{\boldsymbol{n}+1^-}]\boldsymbol{V}_{\boldsymbol{n}+1^-},$$

$$[\boldsymbol{T}]_{2\times2} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = [\boldsymbol{A}^*_{1,\boldsymbol{n}}][\boldsymbol{B}_{\boldsymbol{n}+1^-}].$$

Overview of the stabilized TMM



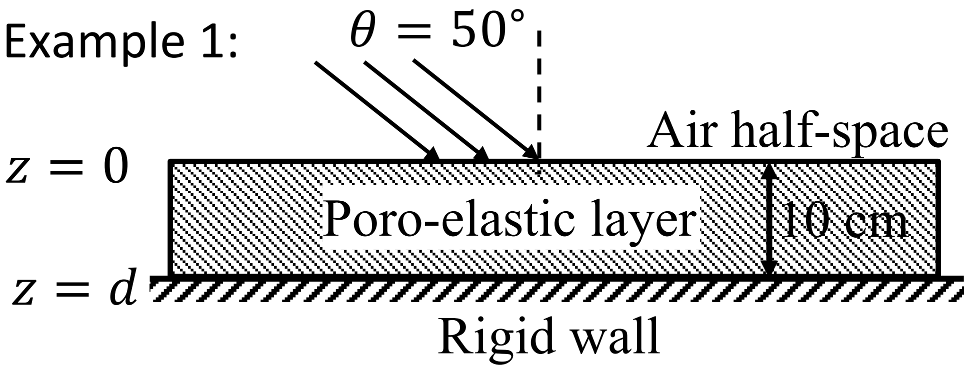
Solve for acoustic properties [6,8,9]

With $\boldsymbol{V}_{\mathbf{1}^+} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \boldsymbol{V}_{\boldsymbol{n}+\mathbf{1}^-}$:

Layered system fixed on a rigid wall:

- $R = \frac{T_{11} \cos\theta/(T_{21}\,\rho_0 c) - 1}{T_{11} \cos\theta/(T_{21}\,\rho_0 c) + 1}$.

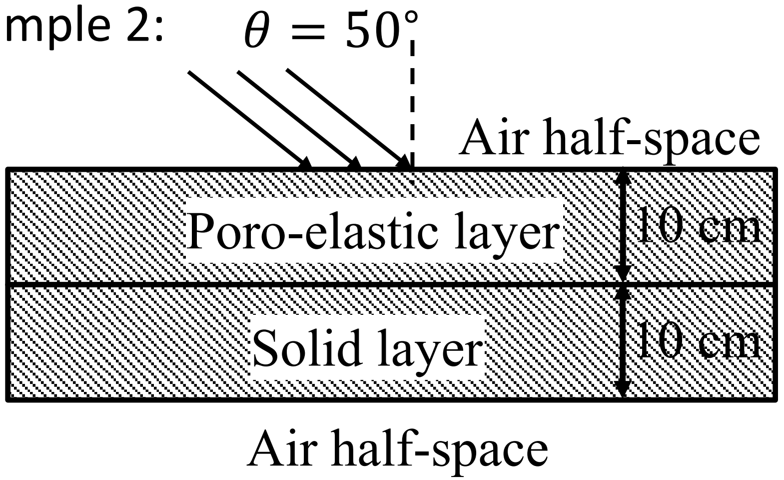
Example 1:



Layered systems with fluid on both sides:

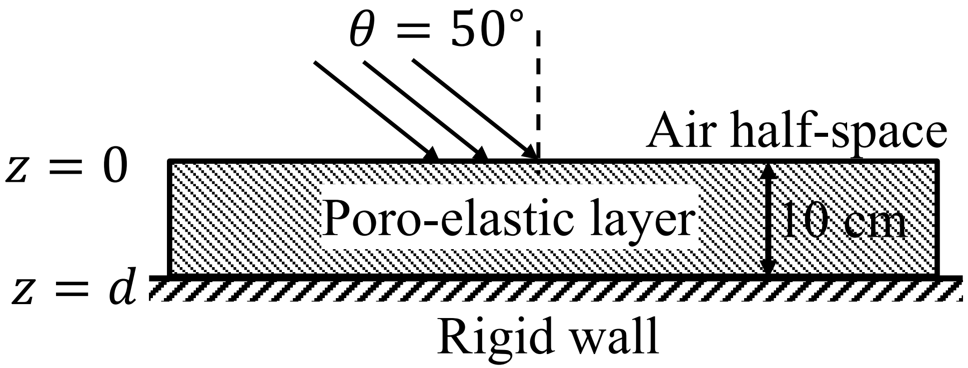
- $T = \frac{2e^{jk_z d}}{T_{11} + T_{12} \cos\theta/\rho_0 c + T_{21}\,\rho_0 c/\cos\theta + T_{22}}$,
- $R = \frac{T_{11} + T_{12} \cos\theta/\rho_0 c - T_{21}\,\rho_0 c/\cos\theta - T_{22}}{T_{11} + T_{12} \cos\theta/\rho_0 c + T_{21}\,\rho_0 c/\cos\theta + T_{22}}$,
- $\alpha = 1 - |R|^2$,
- $TL = 20 \log_{10} \frac{1}{|T|}$.

Example 2:



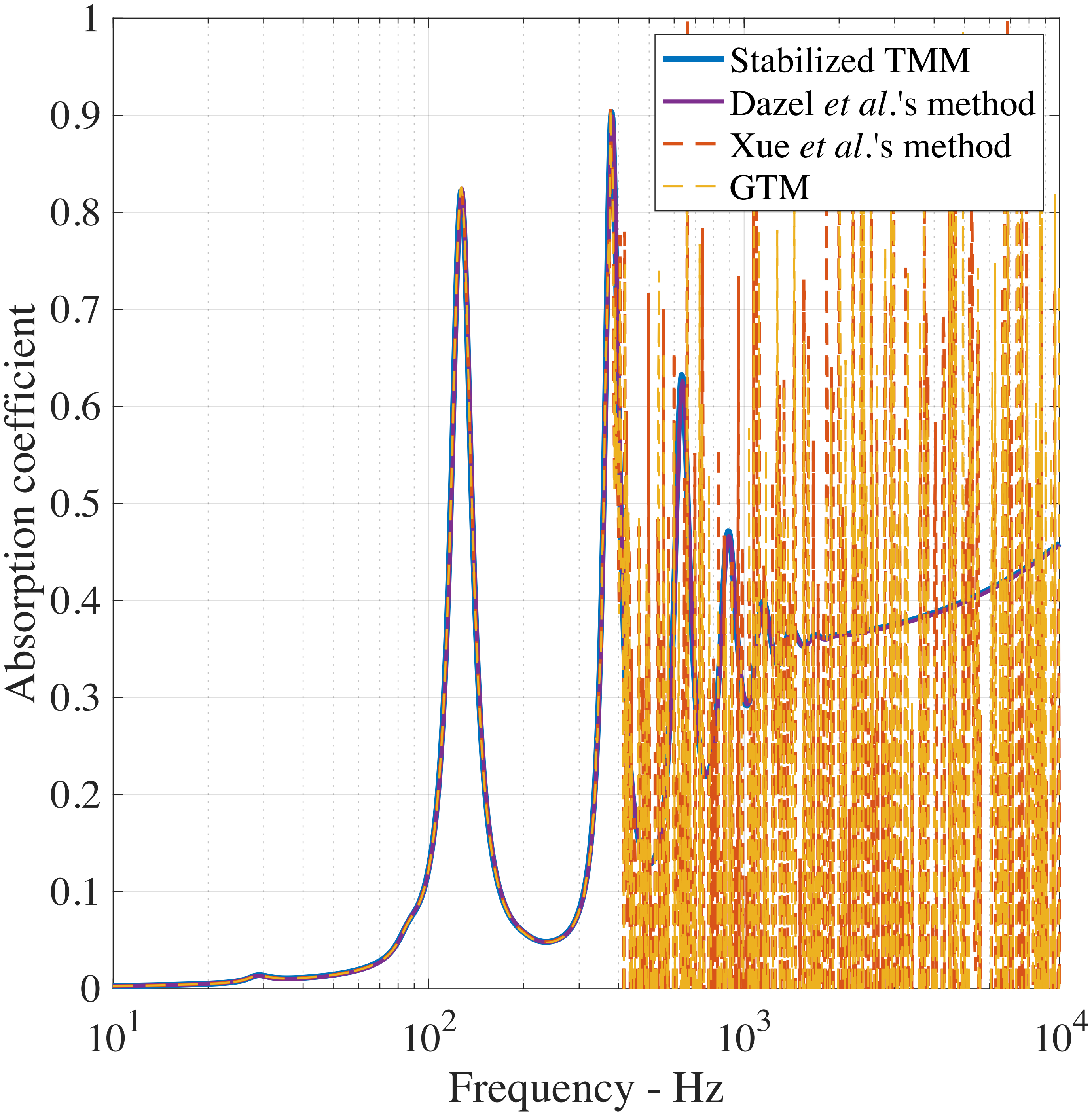
Example results

Example 1

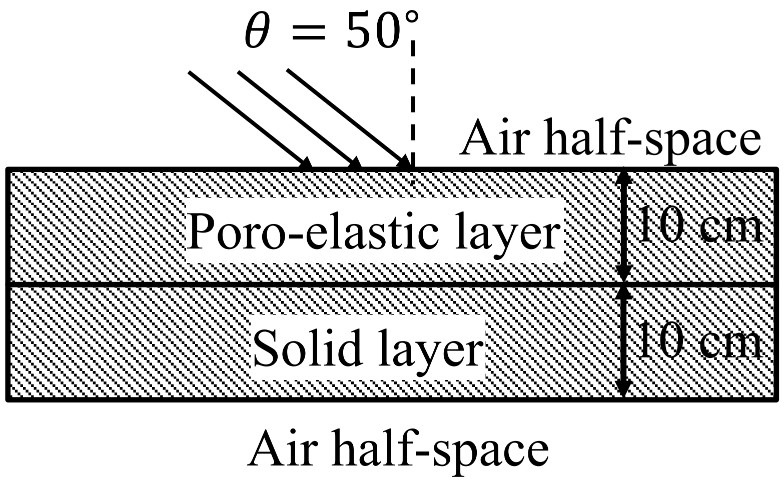


Parameters for poro-elastic layer

σ – Rayls/m	4×10^6
ϕ	0.4
α_∞	1.75
Λ – m	9.3×10^{-6}
Λ' – m	2.0×10^{-5}
ρ_1 – kg/m^3	120
E - Pa	4×10^4
η	0.2
ν	0.3



Example 2



Poro-elastic layer	
σ – Rayls/m	4×10^{6}
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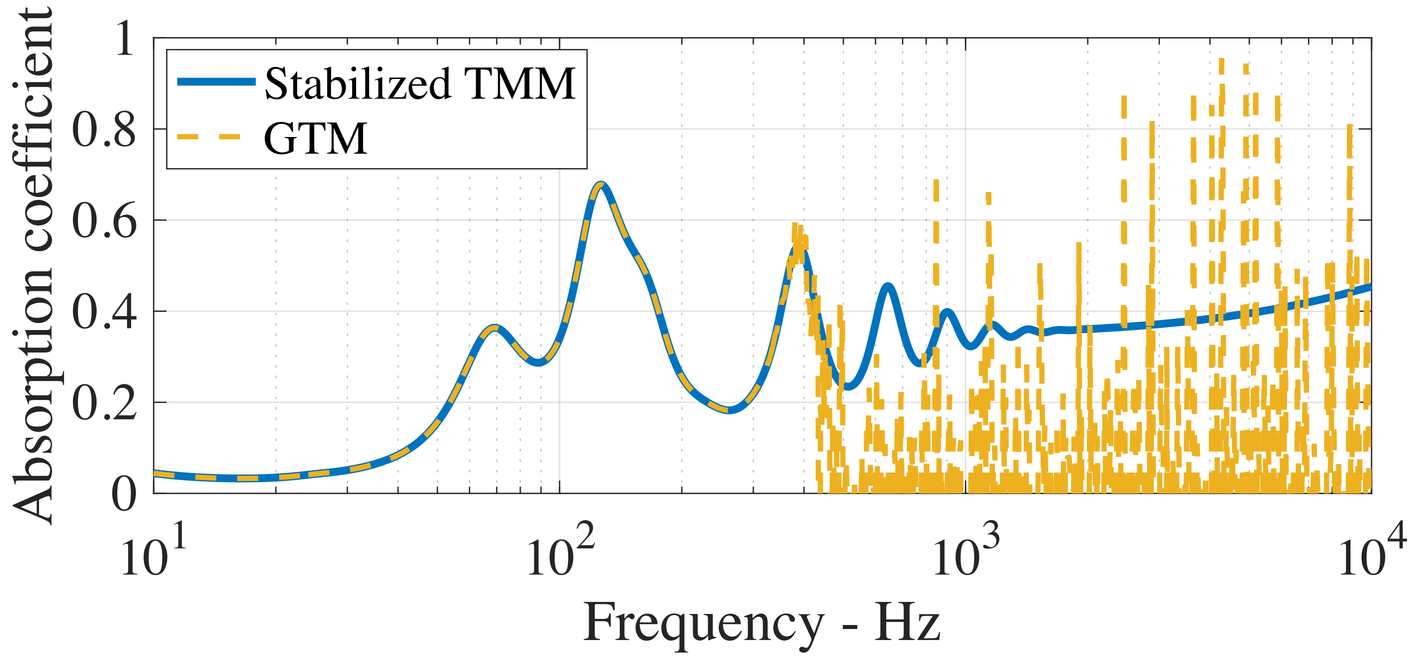
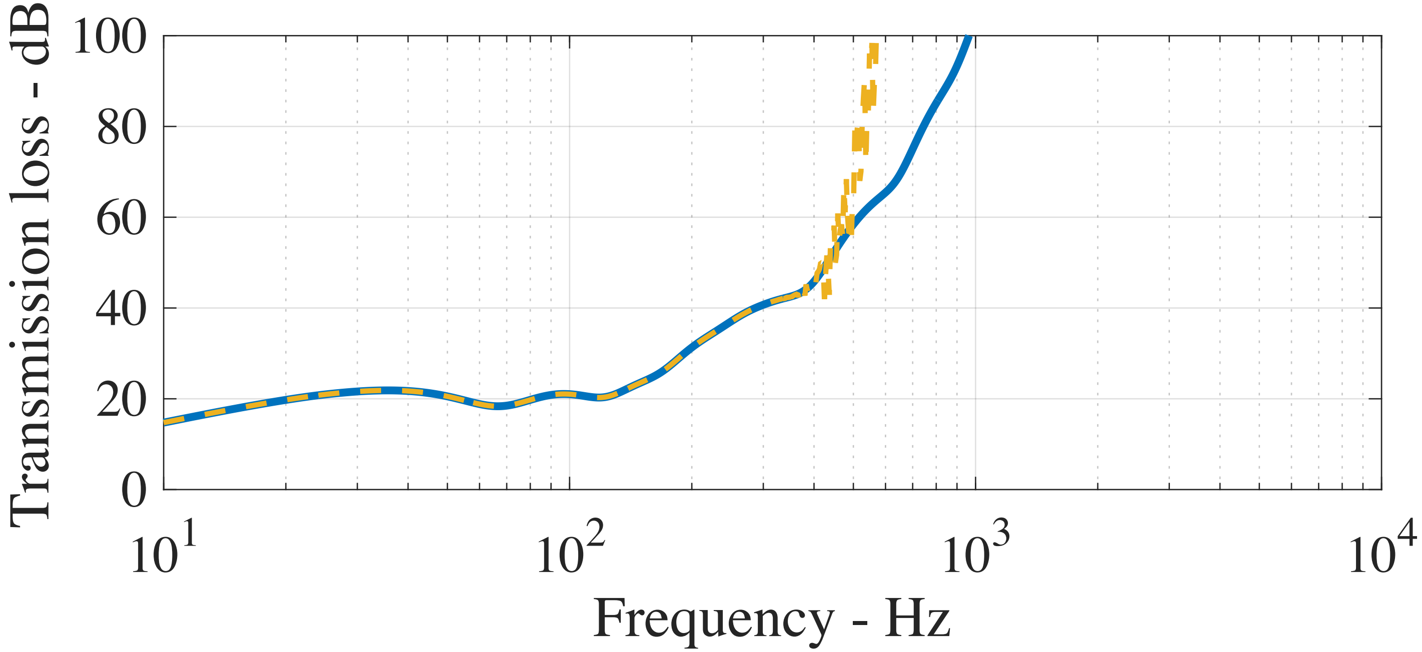
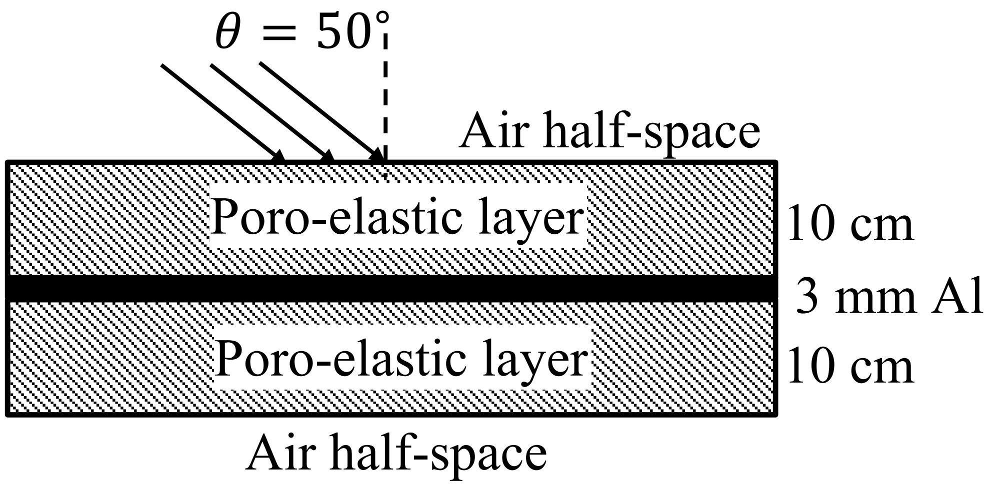
Λ' – m	2.0×10^{-5}
ρ_1 – kg/m^3	120
E - Pa	4×10^{4}
η	0.2
ν	0.3

Solid layer	
ρ_b – kg/m^3	1000
E - Pa	1×10^{5}
η	0.5
ν	0.4



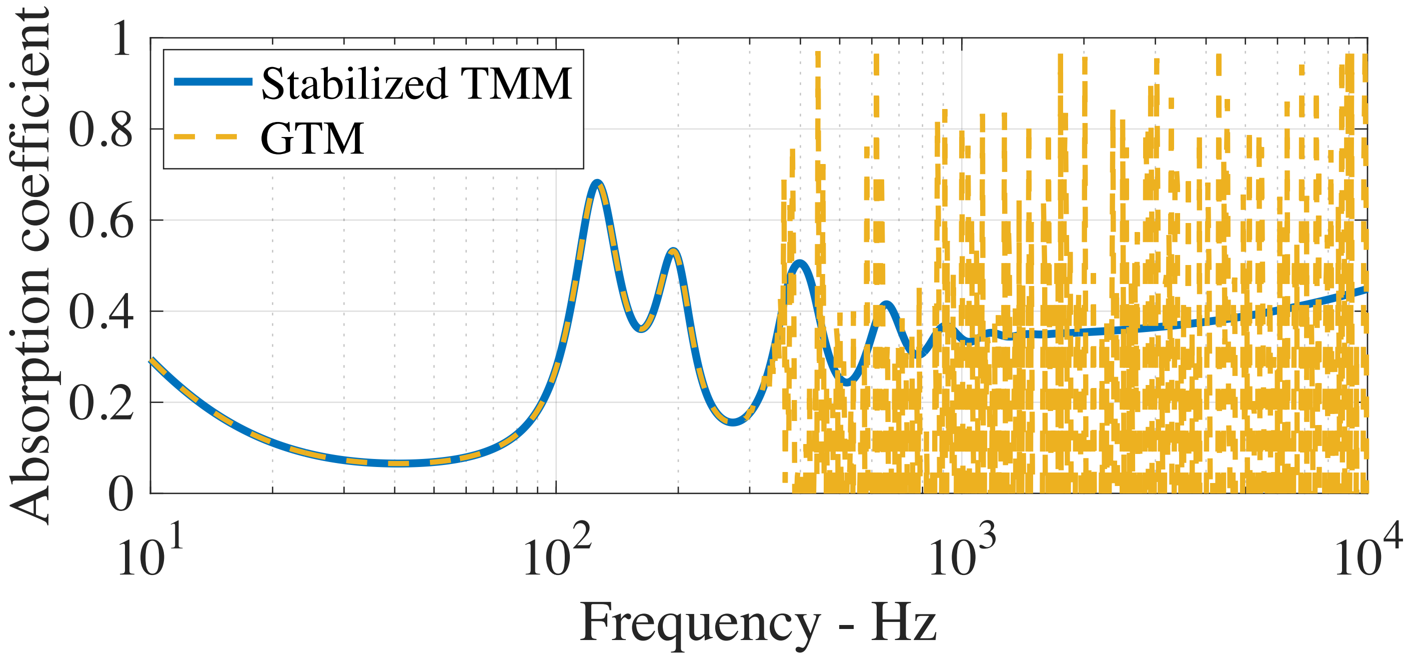


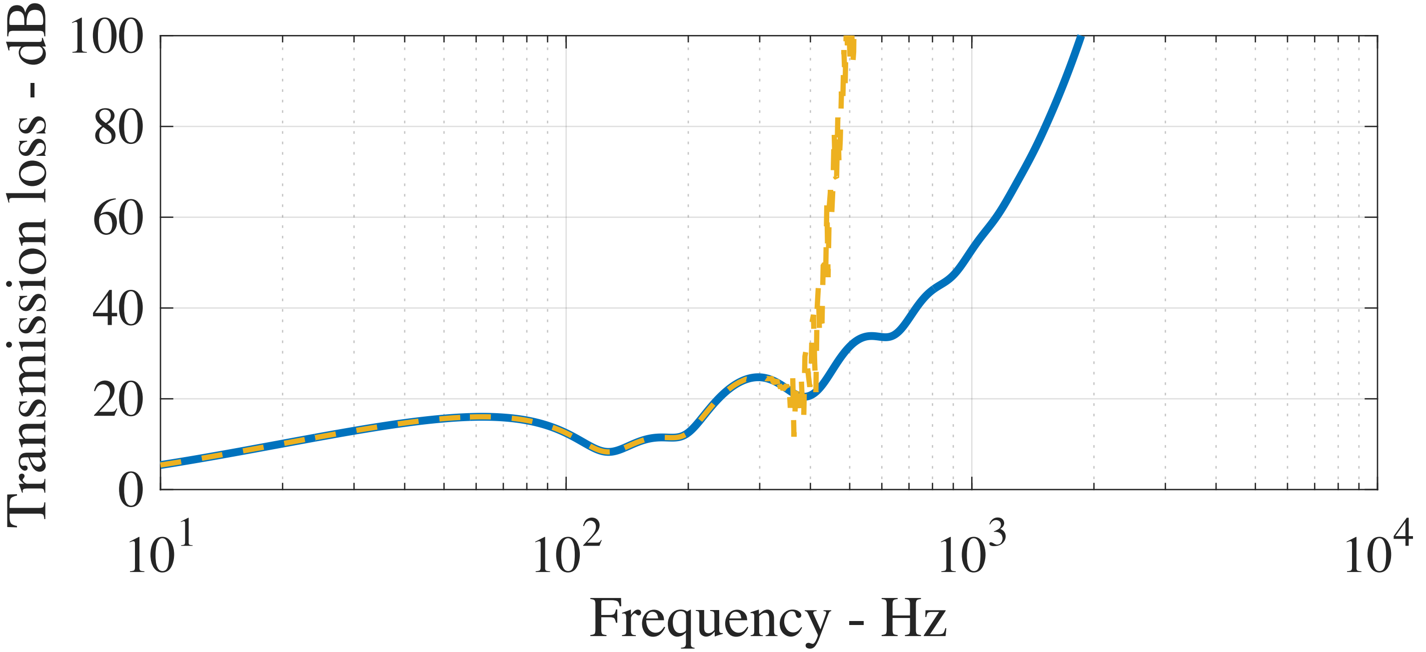
Example 3



Poro-elastic layer	
σ – Rayls/m	4×10^{6}
ϕ	0.4
α_∞	1.75
Λ – m	9.3×10^{-6}
Λ' – m	2.0×10^{-5}
ρ_1 – kg/m^3	120
E - Pa	4×10^{4}
η	0.2
ν	0.3

Stiff panel	
ρ_b – kg/m^3	2700
E_0 - Pa	7.0×10^{10}
η_p	0.003
ν_p	0.33





Conclusions

Conclusions

- A stable, general, robust, and straightforward approach (stabilized TMM) is proposed to model and couple of multi-layered systems consisting of various layer types.
- This approach models the layered system as a two-by-two transfer matrix. Therefore, it can be conveniently connected to other systems with the same dimension and makes the redesign of complicated systems much easier.
- As a modelling tool, this approach makes up for the deficiency of the traditional methods and makes it possible to model and couple thick layers of materials (e.g., granular materials) in a layered system over a wide frequency range.

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