No. 007/2020/MKT

Deceptive Advertising and Consumer Verification

Yi Xiang

Department of Marketing

China Europe International Business School (CEIBS)

Inseong Song*

Seoul National University

March 2020

* Yi Xiang is an Associate Professor of Marketing at China Europe International Business School. email: yixiang@ceibs.edu and Inseong Song is a Professor of Marketing at Seoul National University, Seoul, Korea, +82-2-880-2649, Fax: +82-2-878-3154, e-mail: isong@snu.ac.kr.

Inseong Song acknowledges the research support from Institute of Management Research at Seoul National University.

Deceptive Advertising and Consumer Verification

Abstract

We study the incentive of a firm to provide deceptive information on the value of its product. Consumers take into account the possibility of certain exaggeration or deception in firm's claims. Therefore they may discount such claims and search for extra information to verify the messages. Such reactions from the consumers would in turn influence the firm's incentives to conduct deceptive advertising. Based on a simple model we find that a monopoly firm with a low quality product has a stronger incentive to send out a deceptive message when the consumers' prior belief is favorable to the product. We also investigate the firm's choice on the format of the message when delivering false claims, i.e., explicit claims or subtle messages. Our result indicates that the direct claims are more persuasive than the subtle claims in a wide range of the parameter space. However, when both formats are effective, deceptions with subtle claims are more profitable to the firm because consumers will have less incentive to conduct the verification. Finally, we found that the presence of an independent information source does reduce the firm's incentive to make deceptive claims, but it is most influential when the firm has a moderate reputation.

Keywords: Advertising, persuasion, consumer search, game theory.

1 Introduction

On July 14, 2010, Nestlé Healthcare Nutrition Inc. reached an agreement with the Federal Trade Commission (FTC) to stop advertising that its children's drink "Boost® Kid Essentials" can prevent illness, increase immunity and reduce school absence. The ad was considered deceptive.

In the policy statement on deception, the FTC put forth a working definition: "The commission will find deception if there is a misrepresentation, omission, or practice that is likely to mislead the consumer acting reasonably in the circumstances to the consumer's detriment." An important element in the FTC definition is the "reasonable consumer". When it comes to the judgment of the deceptiveness of an advertisement, the FTC considers the group to which the advertising is targeted and also looks into whether their interpretation of or reaction to the message is reasonable in light of the circumstances. The FTC also takes the position that because consumers expect certain exaggeration or inflated claims in advertising, they recognize puffery and may perceive such claims untrue and discount their truthfulness (FTC 1983).

This paper studies how such reasonable consumers react to a firm's public claims and how that reaction in turn influences the firm's incentive to supply deceptive information. We start with three important observations on deceptive marketing activities.

The first is the increased awareness of deceptive advertisements on the consumer side. Advertising Standard Canada found that from 2000 to 2009, the number of consumer complaints on deceptive advertising has increased significantly (ASC 2010). In a survey across 50 markets worldwide, Nielsen (2009) found that only 8% consumers completely trust TV advertisements. The scenario is even worse for advertisements in other media: 7% for newspaper ads, 6% for ads in magazines and radio, and less than 4% of consumers trust online advertisements. The increasingly skeptical consumers are likely to search for extra information to verify the advertisements. This has become relatively easier with the recent development in information technology and the readily accessible information.

The second observation is the increasingly sophisticated marketing communication skills on the firm side. Advertisers constantly try to stretch their claims within the boundary of law. When outright lies are not feasible, firms may supply indirect, subtle but misleading information about their products. OTC drug manufacturers and dietary supplements producers often exaggerate their test results, and claims such as "9 out 10 clients would recommend Jenny Craig" prevail the weight-loss advertising space.

The third observation relates to the reliability of the easily accessible information. While consumers can search for extra information to evaluate the products, the information may not be fully reliable. The AC Nielsen survey found that only 13% people fully trust online consumer opinions and that number jumps to 34% if the information is from an acquaintance. The huge discrepancy suggests that consumers realize that the information online may not be fully trustworthy and they take into account the source credibility of the information.

Our approach to examining these trends is based on the assumption that consumers, upon receiving the advertisements, may search for extra information and check the truthfulness of the messages. We start from a simple model where the product claim is directly sent from the firm to the consumers. We then move to more sophisticated settings where firms can supply a subtle message to mislead the consumers. We also examine the issue of source credibility of search information. The model examines the strategic interactions among the major players in the communication, as well as the equilibrium outcome of these interactions.

Our results indicate that these simple observations regarding marketing communication have significant impact on the firms' incentive to make deceptive marketing claims. In fact, we show that 1) firms with higher reputation are more likely to engage in deception, 2) subtle and misleading messages are less persuasive but more profitable than explicit claims, and 3) extra independent information could reduce deceptive communication but the impact is rather limited.

2 Related Literature

The issue of deceptive persuasion has been well studied in psychology and consumer behavior areas, where researchers have focused on consumers' responses to the false messages (Darke and Ritchie 2007, Gardner 1975, Johar 1995, Richards 1990, Shimp and Preston 1981). Our paper focuses on the firm's incentive to supply deceptive messages and we assume consumers respond to the firm's claim (possibly false) rationally. Nagler (1993) proposed a model where firms have incentive to lie when consumers are boundedly rational. In our model, consumers are fully rational and they can check the firm's message. In this aspect, our paper is closely related to the pioneering work by Glazer and Rubinstein (2004). While they explore the optimal verification strategies from the perspective of the listener (receiver), our study focuses on the firm's (the sender's) incentive and strategies to cheat. Another related work is by Glaeser (2005), where competing politicians may supply fake stories to create mutual hatreds among ethnical groups. In comparison, our paper examines not only the firm's incentive to cheat but also the feasibility and effectiveness of different deception strategies.

Our model involves a firm trying to deliver information on the value of a product to a group of consumers, and in this regard, this paper is also related to research on advertising coverage (Butters 1977, Grossman and Shapiro 1984, Robert and Stahl II 1993). This stream of research largely focuses on the reach and the coverage of the advertisement, i.e., to how many consumers and to whom to send the message. Instead of studying the advertising reach, we assume that all potential consumers are informed of the existence of the product, but they are yet to be convinced of the true value of a purchase. The firm's decision is then whether and how to provide extra or even false information to persuade the consumers. This relates our paper to the research on advertising content. Anderson and Renault (2006) build a theoretical model studying the firm's choice in its advertising content: should a firm provide price message, the information about the product attributes or both. Mayzlin and Shin (2009) examine the signalling effect of uninformative advertising. While these papers assume partial but truthful information revelation, we explicitly allow the product information to be untrue. Several papers have also examined the issue of false information in the market place. Anderson and Simester (1998) explored the impact of sales sign in the stores where the retailer may actually place the sales signs on more expensive products. Kopalle and Lehmann (2006) studied the management of quality expectation, where firms may overstate the quality of the products. In a more recent study, Kuksov and Xie (2010) analyzed the firm's incentive to offer customers unexpected frills. Our paper differs in that we allow the consumers to verify the firm's claim, thus creating strategic interactions between the firm and the consumers.

Our paper also examines the issue of the credibility of the readily accessible information. In this regard, our model is related to the studies on the third-party product reviews or information intermediaries. Lizzeri (1999) studied how much information would be revealed by certification intermediaries and showed that under some conditions a monopoly intermediary would reveal minimal information, i.e., information only whether a quality is above some minimum level. Albano and Lizzeri (2001) showed that, when quality is a choice variable, seller's incentive to provide high-quality goods increases as certification intermediary improves the information that buyers have about quality. Chen and Xie (2005) studied how a seller should adapt its marketing strategies to product reviews by third parties. Lerner and Tirole (2006) investigated a seller's choice among potential certifiers. Kuksov and Xie (2010) examined the impact of customer ratings on firm's strategies. More relevant to our study is the work by Faulhaber and Yao (1989). In their model, there are two pools of potential firms in the market, one consisting of firms with high-quality service and the other one consisting of "fly-by-night" firms with low-quality services. With an assumption that all reviewers are perfectly accurate in identifying the type of new firms, they found that decreasing the cost of providing information would lead to more high-quality firms and less "fly-by-night" firms. Unlike these studies, our model studies the seller's incentive to provide a deceptive message to the market. The third party review is not necessarily perfect and consumers recognize the possibility of an unreliable review.

In a broad sense, our treatment of information transmission between the firm and the consumers is similar to the research in communication games (Crawford and Sobel 1982, Dewatripont and Tirole 2005, Farrell and Gibbons 1989). There, research mainly focuses on the quality of the signals between the sender and the receiver, the information transmission efficiency of the communication, and sender/receiver's incentive to provide/receive the information. By allowing unfaithful information from the firm, our paper also contributes to the literature on communication theory.

3 The Base Model

We start from a simple setup where there is direct communication between a firm and the targeted consumers. More specifically, we assume there is a monopolist firm selling one product to the consumers and a purchase of the product brings a return of r to the firm. The quality of the product can be either low or high, bringing a *net* value of V_l or V_h to a consumer, where $V_h > 0 > V_l$. The firm knows the true quality level and the consumers only know that with probability θ, $V = V_h$, and with probability $1 - \theta$, $V = V_l$, where $0 < \theta < 1$. Consumers will buy the product if and only if the expected value is non-negative. To focus on the deceptive message, we assume that $\theta V_h + (1 - \theta)V_l < 0$. This means that consumers will not buy the product without any further information.

Knowing that no consumer will buy the product a priori, the firm can supply a message, denoted by s, to the consumers, claiming that the product brings V_h to them. Notice that the firm knows the true value of V, and when $V = V_l$, this message is a deceptive claim. In other words, s can be either a truthful reflection of the quality (if $V = V_h$) or a overstatement (if $V = V_l$). As pointed out by Kopalle and Lehmann (2006), understatement of product quality is rare in practice and involves managing consumers' surprises during the actual consumption experience. Our focus here is the consumers' purchase decision and we shall assume that the firm does not understate its quality.

We assume that when $V = V_h$, supplying s does not cost the firm anything, and when $V = V_l$, supplying s would cost the firm c. One can think of this cost c as the punishment from regulatory agents, the reputation loss, the cost of being caught deceptive, or the cost of issuing a corrective advertisement. This cost structure basically captures the firm's cost of lying, but does not include the general cost of advertising design and dissemination. The firm knows its cost but the consumers do not know the exact value of c. They only know that $c \sim U[0, 1]$.

It is worth noting that we assume all consumers know the existence of the product but not its quality level. This assumption allows us to focus on the impact of the deceptive message without worrying about the issue of advertising reach as in the literature of advertising coverage. From the consumers' point of view, if the firm does not supply s, then they can correctly infer that $V = V_l$. If the firm does supply s, the consumers believe that s is false with probability β, i.e., $\beta = P(s|V = V_l)$.[1] Understanding that this message could be deceptive, consumers then decide to whether to search for more information to verify the message. However, the verification will cost time and effort, denoted by k. Consumers are assumed to be heterogeneous in their verification cost such that $k \sim U[0, 1]$. Suppose a consumer decides to check the message. If the verification finds out that the message is false then the effort pays off because the consumer can avoid the potential loss of buying a bad product. However, if the verification finds out that the message is true, then the consumer could have bought the product without checking the message. Therefore, when making the verification decisions, consumers will take into account the value of the product V, the *ex ante* probability of $V = V_h$, and the probability that the message is false.

This framework encompasses a wide variety of deceptive communications. For example, the product can be an actual product on the market, a company's reputation for environmental friendliness, or a proposed tax reform from the politician. Here r can be interpreted as monetary return, the reputation gain of the firm, or impact of the politician. To the consumers, the value V can be either the net utility of the product or the benefit from the tax reform. The message s can be an advertisement campaign, a claim in the promotional materials, or the press release by a firm or a politician.

We now analyze the consumers' decisions on verification and purchase. After receiving the message, a consumer makes two sequential decisions: whether to verify the message and whether to purchase the product. If she decides to check the message then she will find out the truth and will act accordingly (i.e., buy only if $V = V_h$). If she decides not to verify the message, she will just trust it and buy the product right away. In the latter case, she is persuaded by the message. We start from the consumers' posterior belief of the product value upon receiving the

[1] This probability β comes from the consumers' uncertainty about the firm's cost of lying c, and shall be further examined in equilibrium.

message ($P(V_h|s)$). The consumers know that when $V = V_h$, the firm will always send s at zero cost ($P(s|V_h) = 1$,) and when $V = V_l$ the firm will send s with probability β. Thus from the consumers' point of view, $P(s) = \theta + (1 - \theta)\beta$. This gives us:

$$P(V_h|s) = \frac{P(s|V_h)P(V_h)}{P(s)} = \frac{\theta}{\theta + (1 - \theta)\beta}. \quad (1)$$

Notice that this probability is always larger than the prior $P(V = V_h) = \theta$. This means that a consumer's belief of the product being a high quality increases when she receives the message, even though the message might be a false claim. Therefore, her posterior expected value of the product becomes:

$$E(V|s) = \frac{\theta}{\theta + (1 - \theta)\beta} V_h + (\frac{(1 - \theta)\beta}{\theta + (1 - \theta)\beta}) V_l. \quad (2)$$

Now the consumer needs to decide whether to search and check the truthfulness of the message. If she does not search, her expected value remains the same as in Equation (2). If she incurs a cost of k and check the truthfulness of the message, she will find out the true value of the product, i.e., whether $V = V_h$ or $V = V_l$. If $V = V_l$, then she will not buy the product and has a payoff of 0. If $V = V_h$, she will buy it and enjoy a value of V_h. Notice that at this stage, consumers' belief of the value of the product is given by $P(V = V_h|s)$ and $P(V = V_l|s)$. Therefore, the expected return with verification is given by

$$E_k(V|s) = \frac{\theta}{\theta + (1 - \theta)\beta} V_h + (\frac{(1 - \theta)\beta}{\theta + (1 - \theta)\beta})0, \quad (3)$$

where the subscript k means that the consumer incurs effort k to search and check the truthfulness of the message s. From the above equation, we can calculate the benefit of conducting the verification, denoted by g:

$$g = E_k(V|s) - E(V|s) = \frac{-(1 - \theta)\beta}{\theta + (1 - \theta)\beta} V_l. \quad (4)$$

If $k < g$, then the consumer will conduct the verification. Otherwise, she will just buy the product right away.[2] Denote by $k^* \equiv g$, the verification cost of the consumer who is indifferent between search and no search.

[2]This requires $E(V|s) > 0$, which will be examined in the equilibrium.

When $V = V_h$, all consumers will buy irrespective of their verification decisions. Thus the firm's demand is 1. We shall focus on the case of $V = V_l$, where firm may have an incentive to supply the deceptive message.

When $V = V_l$, the firm knows that its quality level is low. If it does not supply s, then every consumer in the market correctly infers that $V = V_l$, and no one buys the product, resulting in a profit of zero. If the firm does supply a deceptive message, then only the consumers who do not check will buy the product. This gives the firm's demand: $D = 1 - k^*$. Meanwhile, supplying false claims costs the firm c. So the profit from deceptive communication is $\pi(s) = r(1 - k^*) - c$. When $\pi(s) > 0$, the firm will provide the false message. Denote by $c^* \equiv r(1 - k^*)$ the threshold below which the firm will engage in deceptive persuasion.

Although the firm knows its cost of deception c, the consumers only know that $c \sim U[0, 1]$. In equilibrium, their belief of the firm's deception probability β has to be consistent with their knowledge of the firm's cost: $\beta = F(c^*)$. This gives us the equilibrium:

$$c^* = \frac{1}{2(1-\theta)}(r(1+V_l)(1-\theta) - \theta + \sqrt{4r\theta(1-\theta) + (\theta - r(1+V_l)(1-\theta))^2}). \tag{5}$$

Proposition 1. *When $V = V_l$, the firm will supply deceptive message if $c < c^*$, and its incentive to do so increases with θ and V_l ($\frac{\partial c^*}{\partial \theta} > 0$ and $\frac{\partial c^*}{\partial V_l} > 0$).*

By definition, $c^* \equiv r(1 - k^*) = \pi(s) - c$. Thus given the cost of deceptive communication c, a higher c^* means a higher $\pi(s)$, which means a higher return from deception for the firm. Therefore c^* essentially measures the firm's incentive to supply false claims when $V = V_l$. A higher c^* means that the firm has a stronger incentive to lie. Proposition 1 suggests that when consumers believe that the product quality is high (θ), the firm is more motivated to supply deceptive message in case of a low quality product (i.e., when $V = V_l$). This obtains because when the consumers think very high of the product, the potential gain from verification is small and they will have less incentive to check the message. With fewer consumers checking the message, more consumers will buy the product without search and check. This translates into a larger demand for the firm. Thus the firm has a stronger reason to engage in deception. This finding is also consistent with results from empirical studies. Using data from FTC, Kopalle and Lehmann (2006) found that companies' reputation is positively correlated with quality overstatement.

Similarly, when V_l is low, the consumers face a potentially large loss if they do not check the message and end up buying the low quality product. This means a higher gain for consumers from checking the message, implying fewer consumers will buy without verification. If the worst case scenario is really bad (V_l really low), the firm would have less incentive to send a false message because consumers are very likely to spend extra effort to check its validity.

4 Subtle Deception on Probability

In the previous section, we have assumed that the message from the firm is about the exact value of the product, i.e., an s is a claim that $V = V_h$. Under such a framework, when a consumer finds the message false, she correctly infer that the product is of low quality and $V = V_l$. However, in some cases, the firm's deceptive message is not touting the value of the product such as "Boost Kid Essentials can reduce school absences." Rather, the message is to create an impression that the product is highly *possible* of delivering a good value. For example, such claims as "9 out 10 clients would recommend Jenny Craig" by a weight loss program can create an impression that the program will most likely work, although the firm does not fully guarantee its effectiveness. This kind of claims are also observed in the advertisement and promotional materials for drugs, food, dietary supplements, and experience goods. AstraZeneca claimed that its "Seroquel XR" plus an antidepressant can help antidepressant to achieve 50% greater remission rate[3]. However, the FDA considered this claim an overstatement of efficacy and issued a warning letter to the pharmaceutical firm in 2010 (Safarik 2010). Similarly, Kellogg Co. advertised that attentiveness improved by 20% in children who ate its Frosted Mini-Wheats cereal. The company was subsequently warned by the FTC on this claim. One can think of this type of false claims as deceptive probability, where the firm only claims on the probability of success (or high value) for its product.

[3] 50% greater remission rate means 50% more patients achieved remission.

We therefore modify the base model to accommodate this type of probability claims. In particular, we assume that θ, the probability of the product being of high quality, can take two possible values: $\underline{\theta}$ and $\overline{\theta}$, where $\overline{\theta} > 1/2 > \underline{\theta} = 1 - \overline{\theta}$. The consumers do not know the exact value of θ. Instead, they have a prior belief on the probability, $\alpha \equiv P(\theta = \overline{\theta})$ and their expectation of the probability is $\tilde{\theta} = \alpha\overline{\theta} + (1-\alpha)\underline{\theta}$. With this information structure, the firm's message is modeled as an assertion of $\theta = \overline{\theta}$, denoted by s_θ. If the actual value of θ equals $\overline{\theta}$ then s_θ is a truthful message, and if the actual probability is $\underline{\theta}$ then s_θ is a false claim. Similar to the base model, the consumers know that there is a positive probability β that s_θ is a deceptive claim, i.e., $\beta = P(s_\theta | \theta = \underline{\theta})$.

Upon receiving the message s_θ, the consumers update their beliefs of θ and decide whether to check the truthfulness of the *message*. Notice that consumers do not search for the true value of the product, but just check the message itself. There are two reasons for this: 1) the consumers already know the existence of the product and the firm can use the message to attract attentions to certain aspect of the product; 2) consumer verification may not yield perfect knowledge of the product although they know more about the product quality (Glazer and Rubinstein 2004). If a consumer incur effort to verify the message, she can find out whether $\theta = \overline{\theta}$ or $\theta = \underline{\theta}$, but the exact value of V remains unknown to her. We assume that

$$\overline{\theta}V_h + (1-\overline{\theta})V_l > 0 > \underline{\theta}V_h + (1-\underline{\theta})V_l, \tag{6}$$

which implies that a consumer will not buy the product if she finds that $\theta = \underline{\theta}$. Similar to the pervious section, the firm knows its private cost of supplying a false message and consumers only know that $c \sim U[0,1]$.

The calculation of the equilibrium follows directly as in the base model, and the details are provided in the appendix. Here we provide the equilibrium cost threshold for deception:

$$\begin{aligned} c_\theta^* =& \frac{1}{2(1-\alpha)}\Big(r\big(1+\underline{\theta}V_h + V_l(1-\underline{\theta})\big)(1-\alpha) - \alpha \\ &+ \sqrt{4r\alpha(1-\alpha) + \big(\alpha - r(1+\underline{\theta}V_h + V_l(1-\underline{\theta}))(1-\alpha)\big)^2}\Big). \end{aligned} \tag{7}$$

Proposition 2. *When the message is on the probability, the firm's incentive to lie increases with α, $\underline{\theta}$, and V_l.*

Proposition 2 shares some characteristics with Proposition 1 and the firm's incentive to cheat depends on the consumers' incentive to verify the message. When α is larger, the consumers believe that the product is more likely to be a good one. Consequently, they have less incentive to verify the firm's message. $\underline{\theta}$ and V_l relate to the potential loss of buying a bad product. When these two parameters are larger, the loss becomes smaller and the gain from verification is smaller. Thus the consumers have less incentive to check the firm's claim.

Different from the findings in the base model, when the message is subtle and addresses the probabilities, the firm's incentive to engage in deceptive communication may also be influenced by the consumers' information structure, i.e., $\underline{\theta}$. A higher $\underline{\theta}$ means a tightly distributed prior. This suggests that when the consumers have a more consistent belief about the product, the firm has less incentive to supply deceptive advertising.

An interesting question naturally arises: would the firm be more willing to make outright claim on its product quality or to supply subtle messages that the product is likely to be a good one. We now compare firm's incentive to lie on the value directly and that to lie on the probability.

The consumers do not know what the firm knows about the product. Stated in another way, the consumers are relatively passive and the firm's choice of message format does not have any signalling effect on consumers' posterior beliefs. [4] Denote by s_v the message on V. A first observation in the comparison is that, before verification, the consumers' posterior belief on V_h is higher when the firm's message is on the product value:

$$P(V = V_h|s_v) \geq P(V = V_h|s_\theta). \tag{8}$$

The above inequality can be obtained directly from Equation (1) and (A-1), and it suggests that, a direct claim on the product value is more persuasive than a subtle claim on the likelihood of a high value. This is intuitive. A consumer's purchase decision relies on her estimate of the product value, and the claim on the product value directly addresses her concern in this dimension.

[4] This means that consumers do not assume that the firm has the choice over s_v or s_θ. This avoids the complicated signaling issue and allows us to focus on the interaction of consumer search and firm deception.

While direct claims on value (s_v) is more persuasive than subtle claims on probability (s_θ), they are not necessarily more effective in converting consumer beliefs into actual purchase. This is because consumers understand that the message (either s_v or s_θ) can be a false claim, and they may spend effort to verify the message. A deceptive claim is more effective when more consumers will buy the product without verification, and this decision depends on the consumers' expected gain from this action. In the appendix, we show that when the message is on the probability, consumers' expected gain from verification is

$$g_\theta = (1 - P(\overline{\theta}|s_\theta))(\overline{\theta}(V_h - V_l) - V_h). \tag{9}$$

Let $\tilde{\theta} \equiv \alpha\overline{\theta} + (1-\alpha)\underline{\theta}$, from Equation (4), the expected gain from verifying a value claim (g_v) is :

$$g_v = (1 - P(V_h|s_v)) - V_l = \frac{-(1-\tilde{\theta})\beta}{\tilde{\theta} + (1-\tilde{\theta})\beta} V_l. \tag{10}$$

From g_θ and g_v, we can calculate the equilibrium cost threshold for the firm to supply false message. We denote c_v^* the equilibrium cost threshold of false claim on the value and c_θ^* the equilibrium cost threshold of false claim on the probability θ. Recall that the equilibrium cost threshold reflects the firm's incentive to cheat. If $c_\theta^* > c_v^*$, then we know that the firm has stronger incentive to supply deceptive message on the probability, and *vice versa*. Our key interest in this comparison is to study how the firm's optimal deception strategy changes with consumer priors. Therefore, we focus on the case of $r = V_h = 1$ and $V_l = -1$.

Proposition 3. *When $r = V_h = -V_l = 1$, the firm has more incentive to lie on the probability θ ($c_\theta^* > c_v^*$) only when both α and $\overline{\theta}$ are large.*

Figure 1 shows the parameter space where the firm has a stronger incentive to supply deceptive message on the probability than on the value ($c_\theta^* > c_v^*$).

Proposition 3 can have several implications for deceptive marketing communication. We already know that the probability claims are less persuasive than the value claims. This means that upon receiving the message, consumers are less convinced of the high value if the message asserts that the product is *probably* of good value. When the consumers' prior (θ) is not high,

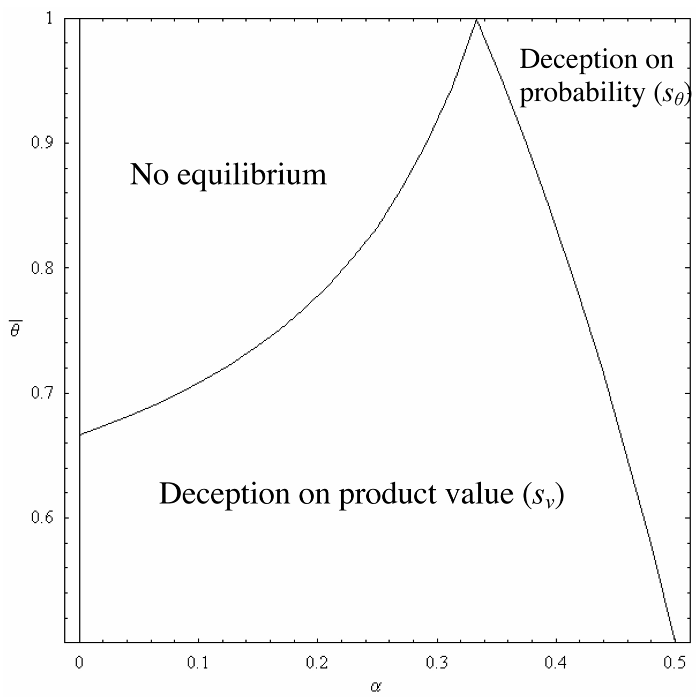

Figure 1: Parameter space where $c_\theta^* > c_v^*$, when $V_h = 1$, $V_l = -1$, and $r = 1$.

the message alone can not persuade the consumers to buy the product. On the other hand, if the message addresses the product value directly, consumers are more convinced and may purchase the product without further verification. This is shown in the central region of Figure 1, where probability claims are not effective but value claims are.

On the other hand, in the upper right corner of Figure 1, where both probability claims and value claims are effective, the firm will take into account consumers' verification strategy. If there are more consumers incurring effort to verify the message, then the deception is less successful and less attractive to the firm. Here we find that more consumers will verify the value message. This comes from fact that, searching and checking a probability claim only gives the consumer imperfect knowledge of the product, resulting in a smaller expected gain from verification. Therefore, less consumers will check the probability message, and the firm is more willing to supply probability deceptions.

In summary, probability claims are less persuasive than value claims and the deceptions on probability have more constraints (smaller feasible parameter space). However, when both types of claims are feasible, deceptions on probability are more powerful and profitable.

5 The Reliability of Search Information

When receiving the firm's message (potentially deceptive), a rational consumer may search for extra information and check the message in depth. In this section, we examine the consumers' search and verification strategies in more details.

Consider the following scenario: Canon claims that its lenses with the latest "Image Stabilization" technology could reduce hand-shake by up to 4 stops.[5] A consumer interested in the Canon IS lenses may search on the Internet for related reviews of the IS function. However, she is not sure the quality of the online reviews: did the review agent employ human subjects to study the hand-shake reduction or the report just borrowed the data from Canon?

Today, the widely available information has dramatically decreased consumers' search cost and made their verification easier. On the other hand, this free information may not be fully reliable. After all, it is costly to investigate the firm's claim and even the review agents may not conduct a comprehensive investigation. We consider the review agent as an independent information provider who care about its reputation. On the consumer side, we assume that this review information is free, and consumers understand that the information may not be fully correct and they could launch their own investigation at a cost. For example, while the lenses review sites such as www.slrGear.com are one click away, a consumers may find its information less reliable and decide to visit the Canon showroom to test the lenses herself.

Compared to the base model, we now have three players in the game: a firm deciding whether to cheat, a review agent deciding whether to investigate, and the consumers deciding whether to trust the firm, or the review, or neither. If a consumer chooses not to trust the firm and the review, she can launch her own investigation. We shall focus on the case where the firm's

[5]Hand shake leads to blurred image, and the blur becomes more severe with slower camera shutter speed. 4 stop IS means clear images can be obtained at the shutter speed 4 times slower than a regular speed.

message is about the value of the product, i.e., s_v. Consistent with the base model, we assume that the review agent has the same prior information as the consumers. That is, the review agent believes that $P(V = V_h) = \theta$ and it knows $c \sim U[0, 1]$. This implies that the review agent shares with the consumers the same belief that with probability β, the firm will supply a deceptive message when $V = V_l$. If the review agent decides to investigate the message, it incurs an effort e. We assume that only the review agent knows its investigation cost e. Neither the firm nor the consumers know this, and all they know is that $e \sim U[0, 1]$. This suggests that both the firm and the consumers believe that the review agent will investigate with a probability, denoted by ϕ. As in the base model, the value of ϕ will be determined in the equilibrium together with β.

Upon receiving the message s_v and before publishing its report to the public, the review agent decides whether to investigate. If the review agent does not investigate the message, it could just borrow the data from the firm and issue a report of t_h, stating that $V = V_h$. If the review agent investigates and finds that the message is true, it will also report t_h. However, if it investigates and finds that the message is false, then it will report t_l, asserting that $V = V_l$. Similar to the consumer verification, we assume that if the review agent investigates, it will find out the truth, i.e., the exact value of the product.

Consumers then receive the report t and decide whether to trust the report. If a consumer trusts the report then she will act according to the report's recommendation, i.e., buy the product if $t = t_h$ and does not buy if $t = t_l$. If a consumer chooses not to trust the report and decides to conduct her own investigation, then she incurs a cost k as in the base model and her purchase decision depends on the outcome of her investigation.

5.1 Consumers' belief after receiving the report

If the firm does not supply a signal s_v, then the review agent correctly infers that $V = V_l$. If the firm supplies an s_v but the review agent finds it false through investigation, the review agent also knows that $V = V_l$. Under either of the two circumstances, the review agent will issue a report of $t = t_l$. Therefore, when a consumer sees t_l, she knows for sure that $V = V_l$.

However, when a consumer sees a report of t_h, she is not sure whether $V = V_h$ or $V = V_l$. On the one hand, it is possible that the product is of high quality (V_h). On the other hand, it is also possible that the firm supplied a false claim but the review agent just copied the firm's message into its report t_h without any investigation. Therefore, the consumer's *ex post* probability of $V = V_h$ is:

$$P(V_h|t_h) = \frac{P(t_h|V_h)P(V_h)}{P(t_h)}. \tag{11}$$

We now calculate each item in the above equation. When $V = V_h$, the firm sends out a message s_v at zero cost. If the review agent does not investigate, then it will report according to the message thus $t = t_h$. If the review agent investigates the message, it will find $V = V_h$ and the report is still $t = t_h$. Thus $P(t_h|V_h) = 1$. Obviously $P(V_h) = \theta$. For $P(t_h)$, we have:

$$P(t_h) = P(t_h|s_v)P(s_v) + P(t_h|\text{no } s_v)P(\text{no } s_v). \tag{12}$$

Notice that this $P(t = t_h)$ measures the consumers' *ex ante* belief that the review agent will issue a report of t_h, and it comprises three types of uncertainty: ϕ, θ, β. The first uncertainty is that the consumers do not know whether the report is based on a solid investigation or a mere copy of the firm's statement. In particular, they believe that, given the message s_v from the firm, with probability ϕ the review agent will investigate and with probability $1 - \phi$ there will be no investigation. In the latter case, the report will be t_h since the review agent will just repeat the firm's claim (s_v). If the review agent does investigate, then the report depends on the outcome of the investigation, which is related to the other two types of uncertainty. From Equation 1, we have: $P(t_h|s_v) = \phi\frac{\theta}{\theta + (1-\theta)\beta} + (1-\phi)$. From the base model we know that $P(s_v) = \theta + (1-\theta)\beta$. Obviously, $P(t_h|\text{no } s_v) = 0$ since the review agent correctly infers that $V = V_l$ and will report t_l.

Combining the above analysis, we have:

$$P(V_h|t_h) = \frac{\theta}{\phi\theta + (1-\phi)(\theta + (1-\theta)\beta)} = \frac{\theta}{\theta + (1-\phi)(1-\theta)\beta)}. \tag{13}$$

Equation 13 is intuitively appealing. When the review agent will surely investigate ($\phi = 1$), its report speaks of the truth ($P(V_h|t_h) = 1$). When the review agent does not investigate at all, its report is considered uninformative and equivalent to the firm's claim ($P(V_h|t_h) = \dfrac{\theta}{\theta + (1-\theta)\beta} = P(V_h|s_v)$).

Proposition 4. *Consumers consider the review report more trustworthy than the firm's claim.*

Proof: When $t = t_l$, consumers know for sure that $V = V_l$. When $t = t_h$, it is easily checked that $P(V_h|t_h) \geq P(V_h|s_v)$. $\square$

This is intuitive. Since consumers know that there is a positive probability that the review agent will investigate the truthfulness of the message, obviously a report of t_h is more trustworthy than the message from the firm s_v. Although the report is more trustworthy than the firm's message, it does not mean that consumers will *fully* trust it. Because it is possible that the review agent did not investigate, after reading the report, a consumer still needs to decide whether to check the message by herself. This decision depends on the content of the report (i.e., $t = t_l$ or $t = t_h$) and the consumer's search cost k.

If the report says that the product is of low value ($t = t_l$), then the consumers know that either the review agent has investigated and found $V = V_l$ or the firm did not supply any message. Thus the consumers correctly infer that $V = V_l$. In this case, no further check is necessary.

If the report says that the value of the product is high ($t = t_h$), then a consumer's decision to search depends on her search cost k and the expected gain from the search. Denoted by g_t the expected gain from further check, we have:

$$g_t \equiv E_k(V|t_h) - E_n(V|t_h) = -V_l\left(1 - P(V_h|t_h)\right), \tag{14}$$

where the subscript t means the presence of a report. If $k < g_t$, then the consumer will search. Otherwise, she will trust the report and just buy the product right away.

5.2 The review agent's payoff

In the presence of a potential deceptive message, the value of the report to the consumers depends on 1) how it changes the consumers' verification decision, and 2) how it changes the consumers' purchase decision. A consumer, who would have incurred the effort to check the firm's message, may not do so after reading the report. In this case, the benefit of the report is the saved verification effort. Another consumer, who would have bought the product without verifying the firm's message, may forgo the purchase after reading the report. In this case, the benefit of the report is the avoidance of a wrong purchase. We assume that the consumers will attribute these benefits to the review agent and consider it responsible. This constitutes the review agent's reputation. Sometime the report could also mislead the consumers to purchase a low quality product. We assume that a consumer will find out the true value of the product after purchase. When the report recommends a low value product, the review agent is considered irresponsible and it suffers a reputation loss. The review agent maximizes its reputation gain while taking into account the cost of investigations. Obviously, the reputation gain from launching an investigation depends on the content/recommendation of the report and the consumers' own verification cost k. Denote by $k_t^* \equiv g_t$ the verification cost of the consumer who is indifferent between search and no search when there is a report t, and by k_s^* for the case when there is the firm's message s only.

If the report is t_l, then consumers know for sure that $V = V_l$ and no one further checks the message and no one buys the product. Compared to the case without the report, consumers whose verification cost is higher than k_s^* (here k_s^* equals k^* in the base model and the subscript s means there is only the firm's message s) will buy the product without check and end up with V_l. That is, the report t_l saves these consumers $-V_l$. Therefore, the reputation gain, denoted by b, from these consumers equals the saved loss ($b = -V_l$). For other consumers whose verification cost is lower than k_s^*, if there is no report, they would check the message s_v by themselves and will find out $V = V_l$. Therefore, the review agent's reputation gain from these consumers is the saved verification effort ($b = k$).

If the report is t_h, there are three segments of consumers: consumers with low verification cost ($k < k_t^*$), with intermediate verification cost ($k_t^* \leq k < k_s^*$), and those with high verification cost ($k \geq k_s^*$). The consumers with low verification cost will always check the message by themselves, and for these consumers the report does not bring any benefit. Thus the review agent's reputation gain from these consumers is zero ($b = 0$). Consumers with high verification cost will never check the message with or without the report. In the case of $t = t_h$, the report does not bring any gain or loss to these high cost consumers ($b = 0$).[6] For those consumers with intermediate cost, the report changes their verification decisions: without the report, they would check the message themselves since $k < k_s^*$; with the report, they choose to trust the report because $k \geq k_t^*$. If $V = V_h$, then the report saves these consumers' verification effort and the review agent's reputation gain is $b = k$. If $V = V_l$, these consumers think the review agent is irresponsible and the report leads to a loss for them. So the review agent's reputation gain is $b = V_l + k$ (since $V_l < 0$, this actually means a reputation loss to the review agent).

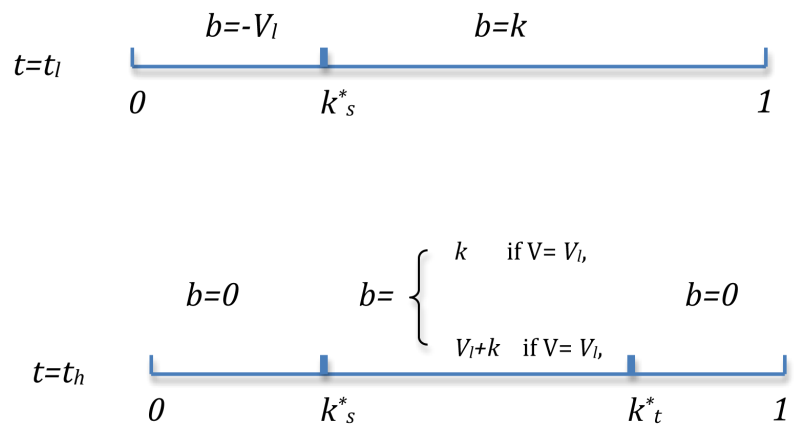

Figure 2: Reputation gain from different consumer segments.

Figure 2 summarizes the reputation gains from different consumer segments with different reports when the firm supplies a message s_v. It suggests that a report of $t = t_l$ can bring in more reputation. However, such a report is feasible only after an investigation. When the review agent receives the message s_v from the firm, it does not know whether the message is true or false and its belief on the probability of $V = V_h$ changes from θ to $P(V_h|s_v) = \dfrac{\theta}{\theta + (1-\theta)\beta}$. If the review agent investigates the message, it faces two possible outcomes: 1) $V = V_l$, and the

[6]If $V = V_h$, then the consumers think that they would have bought the product anyway even without the report, thus there is no benefit from the report. If $V = V_l$, the consumers will also think that they would have been cheated by the firm and bought the product anyway, even without the report, thus they do not blame the review agent.

review agent will issue a report of $t = t_l$, which impresses all consumers; or 2) $V = V_h$, a report of t_h is published and only those consumers with intermediate verification cost are impressed. The second possibility means a smaller reputation gain and may not cover the investigation cost e. Therefore, when deciding whether to conduct an investigation, the review agent calculates its overall expected reputation gain, denoted by B_e, where the subscript e refers to the decision of launching an investigation at the effort e:

$$B_e = P(V_h|s_v) \int_{k_t^*}^{k_s^*} kdk + (1 - P(V_h|s_v)) \left(\int_0^{k_s^*} kdk + \int_{k_s^*}^{1} (-V_l)dk \right). \tag{15}$$

If the review agent does not investigate, its report will be $t = t_h$. The consumers will either follow the report and buy the product, or exert an effort to verify the message themselves. In either case, the consumers will know the true value of the product. If $V = V_l$, then the review agent suffers a reputation loss. If $V = V_h$, those consumers with intermediate verification cost will thank the review agent, which means a reputation gain without effort. The expected reputation gain(loss) is:

$$B_0 = \int_{k_t^*}^{k_s^*} \left(P(V_h|s_v)k + (1 - P(V_h|s_v))(V_l + k) \right) dk, \tag{16}$$

where the superscript 0 refers to the decision of "no investigation" and zero effort is incurred.

5.3 Equilibrium

Because of the investigation cost e, the review agent will investigate if and only if $B_e - B_0 \geq e$. Notice that the review agent does not know the firm's cost of lying c, and this uncertainty is the basis for the belief of a message being deceptive (β). Similarly, the firm does not know the review agent's investigation cost e, and this uncertainty at the firm side is the basis for the firm's belief that review agent will investigate (ϕ). In the equilibrium, these uncertainties influence each other and jointly determine the equilibrium-consistent beliefs β^* and ϕ^*. Therefore, the equilibrium is calculated by jointly solving the following equations:

$$\begin{cases} B_e - B_0 & = \phi \\ \pi(s_v) & = \beta. \end{cases} \tag{17}$$

Similar to the section 4, we focus on the case of $r = V_h = -V_l = 1$. In the appendix, we show that the equilibrium exists and is unique. It is worth noticing that even with a review agent, the firm's incentive to supply deception still increases with consumers' prior θ.[7] Our major interest is how the presence of an objective review agent changes the firm's incentive to make deceptive claims. For this purpose, we need to compare the equilibrium cost threshold of deceptive advertising when there is the firm alone (denoted by c_s^*) and that when there is an independent review agent (denoted by c_t^*).

Proposition 5. *The presence of an independent review agent decreases the firm's incentive to be deceptive ($c_t^* \leq c_s^*$), but the impact diminishes as consumers' prior beliefs become more extreme (i.e., when θ is close to 0 or 1).*

Our treatment of the independent review agent resembles an extra information source to the consumers. Many information providers fall into this functional category: the Consumer Report publishes product reviews, critics comment on forthcoming movies/shows, the media report stories about government policies and company news. With the capacity of launching independent investigations, these information providers make deceptive communication more difficult and is supported in our equilibrium.

One would expect that the review agent is more willing to investigate when the firm has a bad reputation (lower θ) or when the firm has a strong incentive to lie (higher β). However, when we take into account the consumers' verification behaviors, this may not hold any more. In fact, it is easily checked that when $\theta \longrightarrow 0$, the review agent will not investigate at all ($\phi^* \longrightarrow 0$).

This is because consumers' evaluation of the size of the uncertainty is small under such a prior belief. Consumers' perceived uncertainty, measured as the perceived variance of the product value, $\theta(1-\theta)$, is maximized at $\theta = 0.5$. When consumers perceive a small level of uncertainty, the review agent would not investigate to save investigation cost. If consumers believe the product is almost surely of a low quality, most consumers will verify the claim regardless of the review. The review agent then rationally expect that there is little room to gain by doing the investigation

[7]See Appendix for proof.

for the consumers. In the other extreme case where consumers believe the product is surely of a high quality, the firm will almost surely cheat ($\beta_t^* \longrightarrow 1$). Interestingly, the review agent has little incentive to investigate the firm's message ($\phi^* \longrightarrow 0$ and $e^* \longrightarrow 1$). This obtains because the firm's reputation is so good ($\theta \longrightarrow 1$) and the review agent is willing to take the chances and save the investigation cost.

Figure 3 shows the equilibrium cost threshold of deception at the firm side. It has two implications. First, the presence of a review agent does effectively decrease the firm's incentive to lie. Second, this effect is stronger when consumers are more uncertain about the value of the product (θ in the middle of (0, 1) means the consumers' prior has a large variance).

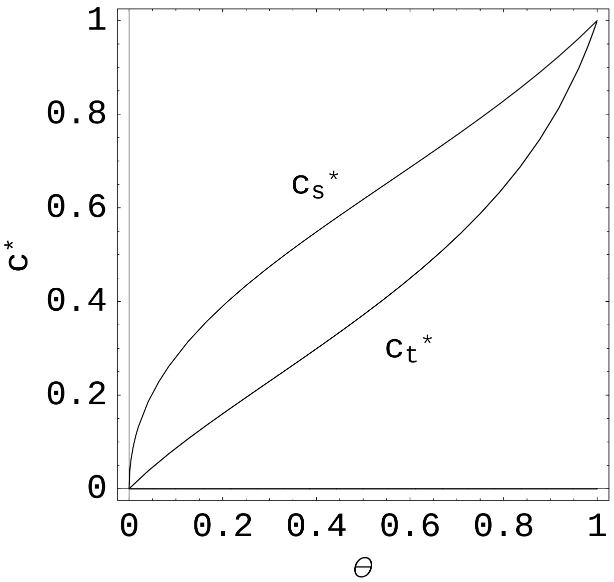

Figure 3: Equilibrium cost threshold of deception with and without review agent.

6 Discussion

We develop a simple framework to study the firm's incentive to provide deceptive information when consumers have imperfect knowledge on the true value of the product. We explicitly recognize reasonably rational consumers' reactions to the firm's messages. That is, the consumers may take into account the possibility of certain exaggeration or deception in the firm's claims and therefore they may discount its claims and search for extra information to verify the truthfulness of such claims. Such reactions from reasonable consumers would in turn influence the firm's incentives to supply deceptive advertising. We summarize our findings along three points. First, firms with a low quality product are more likely to engage in deception when they have higher reputation. That is, a firm has a stronger incentive to send out a deceptive message when the consumers have more favorable prior belief toward its product. For the consumers, obtaining extra information is costly. So they will engage in verification only when the benefit from it exceeds the cost of information acquisition. When the consumers have a favorable prior belief, the net expected gain from obtaining extra information is low. This result suggests that firms with higher reputation are more likely to deny product failure.

Second, we also investigate the firm's choice of message format when delivering persuasive claims. Firms may provide a message directly asserting the value of the product or a message on the probability that the product is of a high quality. Our result indicates that, when left alone, product claims are more persuasive than probability claims in a wide range of the parameter space. However, when both formats are effective, deceptions on probability are more profitable to the firm because fewer consumers will verify the claim when they receive probability-formatted messages as the expected gain from verification is smaller. This kind of outcome is likely to happen when consumers have a strongly favorable prior belief on the quality of product, i.e., consumers believe that the probability of a good quality is very high. Combined together with the first result, consumers' favorable belief increases firms' incentive to send deceptive signals, and as such belief becomes stronger firms are likely to choose less persuasive but subtle message format.

Finally, we study the impact of an independent information provider on the firm's incentive to deceive. As expected, the existence of an independent information source reduces the firm's incentive to provide deceptive claims but such effect decreases as consumers' prior belief becomes extreme. In other words, when the consumers' belief on the probability of a high quality product is very large or very small, the deception-deterrence introduced by the presence of an independent information source becomes minimum.

Our stylized model does have its own limitations. We assume a single-period interaction between players. Our model can be further extended mainly along several directions. One is the incorporation of multi-period interactions. We have assumed simultaneous moves of consumer verification and the firm's decision on false claims. Although equilibrium under this assumption implicitly can be interpreted as the evolutionary outcome of repeated interactions between the firm and the consumers, explicitly modeling and endogenizing the reputation building would be an interesting direction for future research. Another interesting venue for further research would be to study the firm's choice between deception and costly quality improvement. It would be interesting to study whether and how firms would choose to improve quality by investing in R&D or to send deceptive claims.

References

2010. *Annual Report 2009-2010*. Advertising Standard Canada.

Albano, Gian Luigi, Alessandro Lizzeri. 2001. Strategic certification and provision of quality. *International Economics Review* **42**(1) 267–283.

Anderson, Eric T., Duncan I. Simester. 1998. The role of sale signs. *Marketing Science* **17**(2) 139–155.

Anderson, Simon P., Régis Renault. 2006. Advertising content. *The American Economic Review* **96**(1) 93–112.

Butters, Gerard R. 1977. Equilibrium distributions of sales and advertising prices. *Review of Economics and Statistics* **44** 465–491.

Chen, Yubo, Jinhong Xie. 2005. Third-party product review and firm marketing strategy. *Marketing Science* **24**(2) 218–240.

Crawford, Vincent P., Joel Sobel. 1982. Strategic information transmission. *Econometrica* **50**(6) 1431–1451.

Darke, Peter R., Robin J.B. Ritchie. 2007. The defensive consumer: Advertising deception, defensive processing, and distrust. *Journal of Marketing Research* **44** 114–127.

Dewatripont, Mathias, Jean Tirole. 2005. Modes of communication. *Journal of Political Economy* **113**(6) 1217–1238.

Farrell, Joe, Robert Gibbons. 1989. Cheap talk with two audiences. *American Economic Review* **79**(5) 1214–1223.

Faulhaber, Gerald R., Dennis A. Yao. 1989. Fly-by-night firms and the market for product reviews. *Journal of Industrial Economics* **38**(1) 65–77.

FTC. 1983. FTC policy statement on deception. Federal Trade Commission.

Gardner, David M. 1975. Deception in advertising: A conceptual approach. *Journal of Marketing* **39** 40–46.

Glaeser, Edward L. 2005. The political economy of hatred. *The Quaterly Journal of Economics* **120**(1) 45–86.

Glazer, Jacob, Ariel Rubinstein. 2004. On optimal rules of persuasion. *Econometrica* **72**(6) 1715–1736.

Grossman, Gene M., Carl Shapiro. 1984. Informative advertising with differentiated products. *Review of Economic Studies* **51**(1) 63–81.

Johar, Gita V. 1995. Consumer involvement and deception from implied advertising claims. *Journal of Marketing Research* **32** 267–279.

Kopalle, Praveen K., Donald Lehmann. 2006. Setting quality expectations when entering a market: What should the promise be? *Marketing Science* **25**(1) 8–24.

Kuksov, Dmitri, Ying Xie. 2010. Pricing, frills, and customer ratings. *Marketing Science* **29**(5) 925–943.

Lerner, Josh, Jean Tirole. 2006. A model of forum shopping. *American Economic Review* **96**(4) 1091–1113.

Lizzeri, Alessandro. 1999. Information revelation and certification intermediary. *Rand Journal of Economics* **30**(2) 214–231.

Mayzlin, Dina, Jiwoong Shin. 2009. Uninformative advertising as an invitation to search. Working Paper, Yale University.

Nagler, Matthew G. 1993. Rather bait than switch: Deceptive advertising with bounded consumer rationality. *Journal of Public Economics* **51** 359–378.

Richards, Jef I. 1990. *Deceptive Advertising: Behavioral Study of a Legal Concept*. Lawrence Earlbaum Associates, Inc., Hillsdale, NJ.

Robert, Jacques, Dale O. Stahl II. 1993. Informative price advertising in a sequential search model. *Econometrica* **61**(3) 657–686.

Safarik, Michelle. 2010. Warning letter: Seroquel XR extended-release tablets. Division of Drug Marketing, Advertising, and Communication, Food and Drug Administration.

Shimp, Terence A., Ivan L. Preston. 1981. Deceptive and nondeceptive consequences of evaluative advertising. *Journal of Marketing* **45** 22–32.

Appendix

Proof of Proposition 2:

Similar to the base model, the probability that the firm will supply a message is:

$$P(s_\theta) = \alpha + (1-\alpha)\beta.$$

Let $P_{\overline{\theta}} \equiv P(\theta = \overline{\theta}|s_\theta) = \dfrac{\alpha}{\alpha + (1-\alpha)\beta}$, obviously,

$$P(V_h|s_\theta) = P_{\overline{\theta}}\overline{\theta} + (1 - P_{\overline{\theta}})\underline{\theta}. \tag{A-1}$$

If a consumer does not search, her expected value of the product is:

$$E_n(V|s_\theta) = P(V_h|s_\theta)V_h + (1 - P(V_h|s_\theta))V_l, \tag{A-2}$$

where the subscript n indicates no search and check. If she searches and finds out $\theta = \underline{\theta}$, then she does not buy the product and have zero value. If she searches and finds out $\theta = \overline{\theta}$, then she will buy the product with a return of $\overline{\theta}V_h + (1-\overline{\theta})V_l$. Therefore, her expected value of the product is

$$E_k(V|s_\theta) = P_{\overline{\theta}}(\overline{\theta}V_h + (1-\overline{\theta})V_l), \tag{A-3}$$

where the subscript k indicates that the consumer incurs effort k to search and check the truthfulness of the message s_θ. Now we can calculate a consumer's gain from searching and checking the message:

$$g_\theta = (1 - P_{\overline{\theta}})(\overline{\theta}(V_h - V_l) - V_h). \tag{A-4}$$

Following the logic in the base model, we can calculate the equilibrium cost threshold for the firm to cheat on θ:

$$\begin{aligned} c^*_\theta =& \frac{1}{2(1-\alpha)}\Big(r\big(1 + \underline{\theta}V_h + V_l(1-\underline{\theta})\big)(1-\alpha) - \alpha \\ &+ \sqrt{4r\alpha(1-\alpha) + \big(\alpha - r(1 + \underline{\theta}V_h + V_l(1-\underline{\theta}))(1-\alpha)\big)^2}\Big). \end{aligned} \tag{A-5}$$

It can be checked that $\dfrac{\partial c^*_\theta}{\partial \alpha} \geq 0$, $\dfrac{\partial c^*_\theta}{\partial \underline{\theta}} \geq 0$, $\dfrac{\partial c^*_\theta}{\partial V_l} \geq 0$. **Q.E.D.**

Proof of Proposition 3:

For any deceptive claim to be effective, there must exist some consumers who will buy the product without searching and checking the message. This requires that in equilibrium $E_n(V|s_\theta) > 0$ and $E_n(V|s_v) > 0$. Solving these two inequalities yields:

$$\begin{cases} \overline{\theta} < \dfrac{2-3\alpha}{3-6\alpha} \Longleftrightarrow E_n(V|s_v) > 0 \\ \overline{\theta} > \dfrac{3-5\alpha}{2-2\alpha} \Longleftrightarrow E_n(V|s_\theta) > 0. \end{cases} \tag{A-6}$$

It can be easily checked that, when $\overline{\theta} \in (1/2, 1)$ and $\alpha \in (0, 1/2)$, $\overline{\theta} > \frac{3-5\alpha}{2-2\alpha} \Longrightarrow \overline{\theta} < \frac{2-3\alpha}{3-6\alpha}$. This means that if the firm cheats on the probability, the equilibrium space is smaller than cheating on the value. Further more, one can show that when $\overline{\theta} > \frac{3-5\alpha}{2-2\alpha}$, $c^*_\theta > c^*_v$. **Q.E.D.**

Proof of equilibrium existence on information reliability

With some simplification, we have the following:

$$\begin{cases} B_e = \frac{k_s^{*2}}{2} - \frac{yk_t^{*2}}{2} - V_l(1-y)(1-k_s^*) \\ B_0 = \frac{(k_s^{*2} - k_t^{*2})}{2} + V_l(1-y)(k_s^* - k_t^*) \\ k_s^* = \frac{-V_l\beta(1-\theta)}{\theta+\beta-\theta\beta} = \frac{-V_l\beta(1-\theta)}{\theta+\beta-\theta\beta} \\ k_t^* = \frac{-V_l\beta(1-\theta)(1-\phi)}{\theta+\beta(1-\theta)(1-\phi)} = -V_l(1 - \frac{\theta}{\theta+\beta(1-\theta)(1-\phi)}) \\ y = P(V_H|S_H) = \frac{\theta}{\theta+\beta-\theta\beta} \end{cases} \tag{A-7}$$

The firm's profit depends on how many consumers will buy without search and check the truthfulness of its message s_v, so we have: $\pi = r * (1-\phi) * (1-k_t^*)$. We need to solve the following equations simultaneously:

$$\begin{cases} B_e - B_0 = \phi & \text{(A-8)} \\ \pi(s_v) = \beta. & \text{(A-9)} \end{cases}$$

We focus on the case of $V_l = -1$ and $r = 1$. From the equation (A-8)

$$\begin{aligned} \phi &= B_e - B_0 \\ &= \frac{k_s^{*2}}{2} - \frac{yk_t^{*2}}{2} - V_l(1-y)(1-k_s^*) - [\frac{(k_s^{*2} - k_t^{*2})}{2} + V_l(1-y)(k_s^* - k_t^*)] \\ &= (1-y)(\frac{k_t^{*2}}{2} - V_l(1-k_t^*)) \\ &= (1-y)(-V_l - \frac{V_l^2}{2} + \frac{1}{2}(\frac{\theta}{\theta+\beta(1-\theta)(1-\phi)})^2) \qquad \text{(let } V_l = -1) \\ &= \frac{1}{2}\frac{\beta(1-\theta)}{\theta+\beta-\theta\beta}[1 + (\frac{\theta}{\theta+\beta(1-\theta)(1-\phi)})^2]. \end{aligned} \tag{A-10}$$

From the equation (A-9), we have:

$$\beta = \pi(s_v) = r * (1-\phi) * (1-k_t^*) = (1-\phi)\frac{\theta}{\theta+\beta(1-\theta)(1-\phi)}, \tag{A-11}$$

and this leads to:

$$\phi = 1 - \frac{\beta\theta}{\theta - \beta^2(1-\theta)}. \tag{A-12}$$

From (A-12), we have:

$$\begin{aligned} &\theta + \beta(1-\theta)(1-\phi) = \theta + \beta(1-\theta)\frac{\beta\theta}{\theta - \beta^2(1-\theta)} = \frac{\theta^2}{\theta - \beta^2(1-\theta)} \\ &\Rightarrow \frac{\theta}{\theta + \beta(1-\theta)(1-\phi)} = \frac{\theta - \beta^2(1-\theta)}{\theta} = 1 - \frac{1-\theta}{\theta}\beta^2 \end{aligned} \tag{A-13}$$

Inserting the above two equations into (A-10), we have:

$$\begin{aligned} &1 - \frac{\beta\theta}{\theta - \beta^2(1-\theta)} = \frac{1}{2}\frac{\beta(1-\theta)}{\theta + \beta(1-\theta)}[1 + (1 - \frac{1-\theta}{\theta}\beta^2)^2] \\ &\Longleftrightarrow \frac{\theta - \beta^2(1-\theta) - \beta\theta}{\theta - \beta^2(1-\theta)} = \frac{\beta(1-\theta)}{\theta + \beta(1-\theta)}[1 - \frac{1-\theta}{\theta}\beta^2 + \frac{1}{2}(\frac{1-\theta}{\theta})^2\beta^4] \\ &\Longleftrightarrow (\theta - \beta^2(1-\theta) - \beta\theta)(\theta + \beta(1-\theta)) = \beta(1-\theta)(\theta - \beta^2(1-\theta))[1 - \frac{1-\theta}{\theta}\beta^2 + \frac{1}{2}(\frac{1-\theta}{\theta})^2\beta^4] \end{aligned} \tag{A-14}$$

The left hand side equals:

$$\begin{aligned} &\theta^2 + \beta\theta(1-\theta) - \beta^2\theta(1-\theta) - \beta^3(1-\theta)^2 - \beta\theta^2 - \beta^2\theta(1-\theta) \\ &= \theta^2 + \beta(\theta - 2\theta^2) - \beta^2 2\theta(1-\theta) - \beta^3(1-\theta)^2 \end{aligned} \tag{A-15}$$

And the right hand side equals:

$$\begin{aligned} &(\beta\theta(1-\theta) - \beta^3(1-\theta)^2)(1 - \frac{1-\theta}{\theta}\beta^2 + \frac{1}{2}(\frac{1-\theta}{\theta})^2\beta^4) \\ &= \beta\theta(1-\theta) - \beta^3(1-\theta)^2 + \frac{1}{2}\frac{(1-\theta)^3}{\theta}\beta^5 - \beta^3(1-\theta)^2 + \frac{(1-\theta)^3}{\theta}\beta^5 - \frac{1}{2}\frac{(1-\theta)^4}{\theta^2}\beta^7 \end{aligned} \tag{A-16}$$

Then we get:

$$\frac{1}{2}\frac{(1-\theta)^4}{\theta^2}\beta^7 - \frac{3}{2}\frac{(1-\theta)^3}{\theta}\beta^5 + (1-\theta)^2\beta^3 - 2\theta(1-\theta)\beta^2 - \theta^2\beta + \theta^2 = 0 \tag{A-17}$$

Divided by $\frac{\theta^2}{2}$:

$$(\frac{1-\theta}{\theta})^4\beta^7 - 3(\frac{1-\theta}{\theta})^3\beta^5 + 2(\frac{1-\theta}{\theta})^2\beta^3 - 4(\frac{1-\theta}{\theta})\beta^2 - 2\beta + 2 = 0 \tag{A-18}$$

Let $\lambda \equiv \frac{1-\theta}{\theta}$, we have:

$$\lambda^4\beta^7 - 3\lambda^3\beta^5 + 2\lambda^2\beta^3 - 4\lambda\beta^2 - 2\beta + 2 = 0 \tag{A-19}$$

Any β that satisfies Equation (A-19) will be the equilibrium belief β^*. From this we can easily derive ϕ^*, thus c^* and e^* respectively. We now show the existence of such equilibrium β^*.

Define: $f(\beta) = \lambda^4\beta^7 - 3\lambda^3\beta^5 + 2\lambda^2\beta^3 - 4\lambda\beta^2 - 2\beta + 2$. Because $0 < \phi < 1$, from Equation (A-12), we have:

$$\begin{cases} \beta^2(1-\theta) - \theta < 0 \\ \beta^2(1-\theta) + \theta\beta - \theta < 0 \end{cases} \Longrightarrow \beta^2(1-\theta) + \theta\beta - \theta < 0 \tag{A-20}$$

This gives us:

$$0 < \beta < \overline{\beta} \equiv \frac{-\theta + \sqrt{4\theta - 3\theta^2}}{2(1-\theta)}.$$

It is easily checked that

$$\begin{cases} f(0) &= 2 > 0 \\ f(\overline{\beta}) &= (\lambda\beta^2 + \beta - 1)(\lambda^3\overline{\beta}^5 - \lambda^2\overline{\beta}^4 + \lambda\overline{\beta}^3 - 2\lambda^2\overline{\beta}^3 + \lambda\overline{\beta}^2 - 2) \\ & \quad -\lambda\overline{\beta}^2(1 + \overline{\beta}^2) < 0 \end{cases} \tag{A-21}$$

Since the function $f(\beta)$ is continuous, we can conclude that there exist at least one β^* such that $f(\beta^*) = 0$, thus the existence of the equilibrium.

Next we show that this equilibrium is unique. The proof of the uniqueness is equivalent to the proof that $f(\beta) = 0$ has only one solution in $(0, \overline{\beta})$.

Following the Mean Value Theorem, if there are at least two roots for $f(\beta)$, then there must exist an $\beta \in (0, \overline{\beta})$ such that $f'(\beta) \geq 0$ and $f(\beta) \leq 0$. Therefore, if we can show that for any $\beta \in (0, \beta_m)$ satisfies $f'(\beta) \geq 0$, $f(\beta) > 0$ holds, then we know that $f(\beta) = 0$ has only one root in $(0, \beta_m)$.

First, we know that

$$\forall \beta \in (0, \overline{\beta}), \quad \beta^2(1-\theta) + \beta\theta - \theta < 0 \Longleftrightarrow \lambda\beta^2 + \beta - 1 < 0 \qquad (\lambda = \frac{1}{\theta} - 1 \in (0, +\infty)). \tag{A-22}$$

Then we have:

$$\begin{aligned} f'(\beta) &= 7\lambda^4\beta^6 - 15\lambda^3\beta^4 + 6\lambda^2\beta^2 - 8\lambda\beta - 2 \\ &= 7\lambda^3\beta^4(\lambda\beta^2 + \beta - 1) - 7\lambda^3\beta^5 - 8\lambda^3\beta^4 + 6\lambda^2\beta^2 - 8\lambda\beta - 2 \\ &< 6\lambda^2\beta^2 - 7\lambda^3\beta^5 - 8\lambda^3\beta^4 - 8\lambda\beta - 2 \end{aligned} \tag{A-23}$$

If $f'(\beta) \geq 0$, we can get

$$\begin{cases} 6\lambda^2\beta^2 - 7\lambda^3\beta^5 - 8\lambda^3\beta^4 - 8\lambda\beta - 2 > 0 \Longrightarrow 6\lambda^2\beta^3 > 7\lambda^3\beta^6 + 8\lambda^3\beta^5 + 8\lambda\beta^2 + 2\beta & \text{(A-24)} \\ 6\lambda^2\beta^2 - 8\lambda\beta - 2 > 0 \Longrightarrow \lambda\beta < \frac{3}{2} & \text{(A-25)} \\ 6\lambda^2\beta^2 - 8\lambda^3\beta^4 > 0 \Longrightarrow \lambda\beta^2 < \frac{3}{4}. & \text{(A-26)} \end{cases}$$

$$\beta f'(\beta) = 7\lambda^4\beta^7 - 15\lambda^3\beta^5 + 6\lambda^2\beta^3 - 8\lambda\beta^2 - 2\beta \tag{A-27}$$

Then:

$$\begin{aligned}
&7f(\beta) = 7\lambda^4\beta^7 - 21\lambda^3\beta^5 + 14\lambda^2\beta^3 - 28\lambda\beta^2 - 14\beta + 14 \\
&= \beta f'(\beta) - 6\lambda^3\beta^5 + 8\lambda^2\beta^3 - 20\lambda\beta^2 - 12\beta + 14 \\
&\geq 2\lambda^2\beta^3 + 6\lambda^2\beta^3 + 14 - 6\lambda^3\beta^5 - 20\lambda\beta^2 - 12\beta \qquad (\because \beta f'(\beta) \geq 0) \\
&> 2\lambda^2\beta^3 + 14 + 7\lambda^3\beta^6 + 8\lambda^3\beta^5 + 8\lambda\beta^2 + 2\beta - 6\lambda^3\beta^5 - 20\lambda\beta^2 - 12\beta \qquad (A-24) \\
&= 2 * \lambda\beta * \lambda\beta^2 + 14 + 7\lambda^3\beta^6 + 2 * (\lambda\beta)^3 * \beta^2 - 12\lambda\beta^2 - 10\beta \qquad \text{(A-28)} \\
&> 14 + 7\lambda^3\beta^6 + \frac{27}{4}\beta^2 - 9\lambda\beta^2 - 10\beta \qquad (A-25) \\
&> 7\lambda^3\beta^6 + \frac{27}{4}\beta^2 - 10\beta + 14 - 9 * \frac{3}{4} \qquad (A-26) \\
&> \frac{27}{4}\beta^2 - 10\beta + \frac{29}{4} \\
&> 0
\end{aligned}$$

The last inequality sign $>$ is concluded by

$$\begin{cases} \frac{27}{4} > 0 \\ \Delta = 100 - 4 * \frac{27}{4} * \frac{29}{4} < 0 \end{cases} \Longrightarrow \forall\beta \in R, \frac{27}{4}\beta^2 - 10\beta + \frac{29}{4} > 0. \tag{A-29}$$

Q.E.D.

Proof of $\frac{\partial c_t^*}{\partial\theta} > 0$: Since $c_t^* = \beta_t^*$ in equilibrium, we just need to show $\frac{\partial\beta_t^*}{\partial\theta} > 0$. β_t^* is the solution for A-19. Consider this as a function of λ and β, denoted by $f(\lambda, \beta)$, we have:

$$\frac{\partial\beta_t^*}{\partial\theta} = -\frac{\partial f/\partial\lambda}{\partial f/\partial\beta_t^*}\frac{d\lambda}{d\theta}. \tag{A-30}$$

From the proof of the equilibrium uniqueness, we know that $\partial f/\partial\beta_t^* \leq 0$. We now examine the sign of $\partial f/\partial\lambda$.

$$\begin{aligned}\frac{\lambda\partial f}{\partial\lambda} - 2f(\lambda,\beta) &= 2\lambda^4\beta^7 - 3\lambda^3\beta^5 + 4(\lambda\beta^2 + \beta - 1) \\ &< 4(\lambda\beta^2 + \beta - 1) - \lambda^3\beta^5 \qquad (\text{from } A-26) \\ &< 0 \qquad (\text{from } A-22).\end{aligned} \tag{A-31}$$

Obviously, $\dfrac{d\lambda}{d\theta} < 0$. We have $\dfrac{\partial\beta_t^*}{\partial\theta} > 0$. **Q.E.D.**

Proof of Proposition 5: Because in equilibrium, $\beta^* = F(c^*)$. Given the uniform distribution of c, we have $c^* = \beta^*$. This holds for the case without the medium and the case with the medium. It is easily check that when $V_l = -1$ and $r = 1$, $\beta_s^* = \overline{\beta} \geq \beta_t^*$. Therefore, we have $c_t^* \leq c_s^*$.

From (A-11), we know that when $\theta \to 0$, $\beta_t^* \to 0$ thus $c_t^* \to 0$. From (A-19), we know that $\theta \to 1$, $\beta_t^* \to 1$ and thus $c_t^* \to 1$. On another note, it can be checked that

$$\begin{cases} \lim\limits_{\theta\to 0} c_s^* & = 0 \\ \lim\limits_{\theta\to 1} c_s^* & = 1. \end{cases} \tag{A-32}$$

Given that $c_t^* \leq c_s^*$, we know that $c_s^* - c_t^*$ reaches its minimum when $\theta \to 0$ or 1. **Q.E.D.**