The Development of Electrical and Radio engineering: the Role of M. Krylov and M. Bogolyubov’s Nonlinear Mechanics

Alla Lytvynko
Department of History and Sociology of Science and Technology
SI «G.M. Dobrov Institute for Scientific and Technological Potential and Science History Studies NAS of Ukraine»
Kyiv, Ukraine
litvinko@ukr.net

Olena Voitiuk
Department of History and Sociology of Science and Technology
SI «G.M. Dobrov Institute for Scientific and Technological Potential and Science History Studies NAS of Ukraine»
Kyiv, Ukraine
helen_zt24@ukr.net

Halyna Zvonkova
Department of History and Sociology of Science and Technology
SI «G.M. Dobrov Institute for Scientific and Technological Potential and Science History Studies NAS of Ukraine»
Kyiv, Ukraine
zvonkova@ukr.net

Mariia Stankova
Department of History and Sociology of Science and Technology
SI «G.M. Dobrov Institute for Scientific and Technological Potential and Science History Studies NAS of Ukraine»
Kyiv, Ukraine
0936600528@ukr.net

Olexandr Korniienko
Department of History and Sociology of Science and Technology
SI «G.M. Dobrov Institute for Scientific and Technological Potential and Science History Studies NAS of Ukraine»
Kyiv, Ukraine
korney38@i.ua

Artem Zabuga
Department of General and Theoretical Physics
National Technical University of Ukraine“Igor Sikorsky Kyiv Polytechnic Institute”
Kyiv, Ukraine
azabuga@ukr.net

***Abstract*—The article is devoted to the description of significant asymptotic theory of nonlinear oscillations, created by M. Krylov and M. Bogolyubov in the early 1930s (nonlinear mechanics) for the development of electrical and radio engineering, as well as other branches of science and technology. It is shown that in the second half of the XIX century due to the rapid development of technology, in particular, increasing power of steam engines, turbines and their vibrations, the problems of nonlinear oscillations began to attract the attention of scientists. It is emphasized that the attention to nonlinear oscillations increased after Lee de Forrest’s invention of the triode lamp in 1906, due to awareness of the necessity to generate non-extinguishing electromagnetic oscillations and the subsequent rapid development of radioelectronics. It is shown that the centers for research of nonlinear problems in 1907-1921 were teams led by G. Barkhausen and G. Möller in Germany, and in 1922-1929 – the scientific school of B. Van der Pol in Holland. Since 1925, the team of physicists led by L. I. Mandelstam (Moscow) began active developing the theory of nonlinear oscillations, the advantage of which was the clarity and geometric representation of any dynamical system state. Since 1927 M. M. Krylov and M. M. Bogolyubov in Kyiv developed methods of perturbation theory for the general case, a well-founded mathematical method for the study of general conservative systems with a small parameter, effective ways to obtain approximate formulas (the principle of equivalent linearization, symbolic methods). Methods for considering quasi-periodic modes in an electronic generator, which is periodically affected by an external force, have been developed. In 1932 M. Bogolyubov made a report “Static and dynamic stability of synchronous machines” at the First Electrotechnical Congress in Paris, which attracted considerable attention of the scientific community. Later M. Bogolyubov significantly influenced the development of physical, mathematical and technical sciences and became a leader of scientific teams, in particular at the Institute of Mathematics, Kyiv University, Institute of Theoretical Physics.**

Keywords*—*history of science and technology, electrical engineering, radio engineering, theory of non-linear oscillations

I. INTRODUCTION

The first problems related to nonlinear oscillations arose after the work of L. Euler and J. Lagrange. S. Poisson, M. Ostrogradsky, I. Gulden, A. Lindstedt, A. Poincare and other mathematicians, mechanics and astronomers of the XIX century. They solved some nonlinear problems of the theory of oscillations. In the second half of that century due to the rapid development of technology, in particular, with increasing power of steam engines and turbines, the problems of nonlinear oscillations began to attract the attention of scientists, as growing the power of these units led to a rise in the mass of their regulators.

Attention to nonlinear oscillations increased after the invention by Lee de Forest of a triode lamp in 1906 and the subsequent rapid development of radio electronics. At that time the task was not to overcome the harmful oscillatory process, but in using it to generate undamped electromagnetic oscillations. The tasks in electrical engineering, radio physics, acoustics, and solid state physics have become a priority. However, the existing theory of linear differential equations did not describe nonlinear processes.

Initially, there was an attempt to keep linear terminology and linear mathematical methods when interpreting new nonlinear problems, correcting solutions. Supposing system as nonlinear, we need to consider it in each case individually. D’Alembert, J. Lagrange, J. Maxwell, I.Vyshnegradsky, and M. E. Zhukovsky tried to act in this way. In 1907-1921 the centers for research of nonlinear problems were teams led by G. Barkhausen and G. Möller in Germany, in 1922-1929 – the school of B. Van der Pol in Holland. Active development theories of nonlinear oscillations began by a team of physicists led by L. I. Mandelstam (Moscow) starting from 1925, and by M. M. Krylov and M. M. Bogolyubov in Kyiv since 1927.

In 1907 G. Barkhausen made an important contribution to the theory of nonlinear switches, in 1920 he built an electronic tube to create high-frequency oscillations and formulated equations that connect the coefficients of the tube. G. Möller considered the case when the period of the exciting force is equal to the natural period of oscillations of the system [1]. G. Barkhausen and G. Möller carried out linearization of nonlinear problems, rejecting nonlinear terms in differential equations and introducing linear ones instead. These "quasilinear methods" perfectly describe problems close to sinusoidal. In 1933 A. I. Berg and Y. B. Kobzarev also developed a "quasilinear method" to solve the problems of radio engineering.

Among the representatives of another direction, which was to consider each nonlinear problem outside the connection with other nonlinear problems, it is necessary to mention the works of 1922-1929 performed by the Dutch radio-physicist B. Van der Pol [2]. He was the first to show the impossibility of describing nonlinear systems by linear theory. The oscillations excited by the tube generator have a finite, and, as a rule, large amplitude and do not depend on the initial conditions. Linear non-conservative systems generate only fading or only increasing oscillations. B. Van der Paul drew attention to this. His method of "slowly changing coefficients" was used in the 1920s for creating the theory for generator of quasi-harmonic radio oscillations on a three-electrode lamp. It was a significant step in developing the theory of nonlinear oscillations by "improving" the principle of superposition. At that nonlinearity is taken into account basing on slow change of the linear factor. In 1934 Van der Paul method was strictly mathematically grounded by L. I. Mandelstam and M. D. Papaleksi. Later its independent substantiation was given by M. M. Krylov and M. M. Bogolyubov [3].

Among the first significant works in which nonlinear problems were studied are the works of E. Appleton (England), devoted to the study of propagation, attenuation and reflection of radio waves, Kogi (Japan), who observed that at certain settings the period of self-excitation self-oscillating system becomes a multiple acting force [4], A. Lénard and A. Cartan and E. Cartan (France) [5], as well as R. Peierls. The latter scientist created the quantum theory of thermal conductivity of crystalline bodies. It takes into account the anharmonicity of atoms in the crystal lattice due to small corrections of the linearly based solution [6-8].

Starting from the beginning of XX century the method of fitting began to use for solving nonlinear problems. It was applied by M. D. Papaleksi, who used it to solve the problem of electric current rectifier. The method was in using function represented by a broken line in place of nonlinear equation part, and instead of a nonlinear equation, a system of linear ones was considered. At the junctions, the integration constants are adjusted based on the requirements of the solution continuity, and often also of its derivative.

L. I. Mandelstam and his school managed to cover oscillatory processes in radio engineering, automation, acoustics, aerodynamics, to discover new types of resonance, in particular, n-th resonance and create its theory, to build a theory of stationary oscillations at n-th resonance, to discover the phenomenon of asynchronous excitation, to develop a new method of generating electromagnetic energy using parametric generators [9-12]. Their advantage was the extraordinary clarity, the geometric representation of any state of a dynamic system. It has been theoretically established that an unexcited generator can be excited under the influence of an electromotive force of twice its frequency; if the frequency of the external force differs significantly from twice the frequency of the system, the system almost does not deviate, behaves as a normal linear model.

These works by L. Mandelstam and M. Papaleksi [13, 14], as well as a report on their study near the n-th resonance, reported by M. Papaleksi at the I All-Union Congress of Physicists in Odessa in August 1930, made the foundations theory of nonlinear oscillations and drew the attention of scientists to the method of small parameter. The works of M. Krylov and M. Bogolyubov on nonlinear mechanics became important. This research is devoted to their analysis.

II. Methods of Investigations

The study is based on general scientific principles, as well as historical and interdisciplinary research methods. The application of the methodological principle of historicism made possible to determine the priority design developments in the intellectual heritage of M. Krylov and M. Bogolyubov. Using of bibliographic, archival, source methods and content analysis contributed to the search and systematization of primary information. Historical-chronological and subject-logical methods allowed to draw conclusions about the content and nature of the scientific achievements of the scientist on the creation and development of native electrical and radio engineering.

III. Results and Discussion

In 1927 M. Krylov and M. Bogolyubov began to develop another branch of the theory of nonlinear oscillations, based on the construction of asymptotic expansion in powers of a small parameter, called by the authors as "nonlinear mechanics." M. Bogolyubov created methods of perturbation theory for the general case, a strictly substantiated mathematical approach for studying general conservative systems with a small parameter, methods for studying the existence and stability of quasiperiodic solutions, effective methods for obtaining approximate formulas (equivalent linearization principle, symbolic methods). The method of integral manifolds and the strict theory of the averaging method also became classics.

In 1931 the First All-Union Conference on Oscillations was held which pointed out the most important theory for the rapid development industry. It was also noted that periodic regimes are more or less developed, and quasi-periodic and almost periodic ones require the fastest development.

M. Krylov became interested in the problem of oscillations and especially in the problem of nonlinear equations in 1908. Attending in 1908 A. Picard's lectures at the Sorbonne, he drew attention to nonlinear equations and became acquainted with the ideas of the new Ritz method. In 1909-1911 he published works on the theory of nonlinear oscillations [15-17].

In 1924 M. Krylov's student M. Bogolyubov wrote the first scientific work "On the behavior of solutions of linear equations in infinity". On July 1, 1925 the Presidium of the USSR Academy of Sciences made a special decision: "Taking into account the phenomenal abilities in mathematics, to consider Mykola Bogolyubov a graduate student of the Research Department of Mathematics in Kyiv since June 18, 1925." In his postgraduate card even then M. Bogolyubov wrote that he spoke French, German, English and Italian [18]. It proves a high educational level of the young scientist.

In 1927 the work of M. M. Bogolyubov "On the calculation of forced oscillations that validate certain nonlinear differential equations" was published [19]. On July 8, 1928 at a meeting of the Department of Agricultural Mechanics of the Institute of Technical Mechanics, M. M. Bogolyubov defended his work on the topic: "On some new methods in variational calculus."

In 1932 M. M. Krylov and M. M. Bogolyubov sent three messages to the Paris Academy of Sciences, which were published. They contained developed methods for considering quasi-periodic modes in an electronic generator, periodically affected by an external force [20-22]. The method of obtaining an approximate solution of the nonlinear equation used in radio engineering was firstly described here. This work also studied the complex phenomena of resonance in the nonlinear case. In addition to the main resonance, which is only possible in linear theory, resonances differential tones (battements) in the middle of individual zone were considered, their width and location were determined. These studies, as the authors noted, "laid the foundation of nonlinear mechanics ... a new branch of mathematical physics, which is immediately applied to all those branches of science where nonlinear oscillations are studied" [23, p.7].

At a conference on the study of nonlinear oscillations held in Paris in late January 1933, Van der Paul highly appreciated the work of Ukrainian scientists. New asymptotic methods were first published in 1932 in the monograph of M. Krylov and M. Bogolyubov "Investigation of the longitudinal stability of the airplane" [24]. Soon the monograph "On the vibrations of synchronous machines. On the stability of parallel operation of n synchronous machines", which contained one of the first solutions of nonlinear problems in the theory of regulation was published [25]. These results were reported in 1932 at the First Electrotechnical Congress in Paris. M. Krylov himself could not take part, but presented joint report with M. Bogolyubov "Static and dynamic stability of synchronous machines", which caused a lot of attention and lively discussion.

Soon the monograph of scientists "Application of methods of nonlinear mechanics to the theory of stationary oscillations" [26] was published. It studied self-oscillating systems in which stationary quasi-periodic oscillations were considered, quasi-periodic solutions were found, and it was established that the properties of quasi-periodicity of the first approximations are similar to those of exact solutions. In the same year, a total of 8 works by M. Krylov and M. Bogolyubov were published. A special place among them belongs to the monograph "Introduction to nonlinear mechanics" (1937), in which they continued to spread asymptotic methods in the case of non-conservative systems. A great attention here was paid to the practical computational implementation of asymptotic methods. For this purpose the method of equivalent linearization was developed. It allowed obtaining the equations of the first approximation for amplitude and phase according to the problem without using exact equations of motion. Later, they introduced a broader principle of generalized virtual work.

Thus, as a result of intensive development of numerous ideas from mathematical physics M. Krylov and M. Bogolyubov in 1932 laid the foundations of a new approach to the problems of nonlinear oscillations for a new branch in the theory of nonlinear oscillations, called nonlinear oscillation mechanics, or nonlinear mechanics. The developed asymptotic methods turned out to be applicable for many phenomena of physics and technology. Rather simple formulas for their calculation allowed achieving practical results at once. Already in 1932, with the help of this mathematical approach according to the order from the Kharkiv Aviation Institute, the longitudinal stability of the aircraft was studied. The stability of parallel operation of electric machines, oscillations of building structures, oscillations and stability of rods were studied at various plants and research institutes, and in 1935 the oscillations of frame structures were calculated.

The resulting formulas of the second approximation for determining the frequency of stationary oscillations in electronic generators allowed defining the influence of overtones on frequency stability. Frequency division resonances, internal resonances in systems with many degrees of freedom were studied. The questions of using nonlinear elements to prevention of resonance in mechanical engineering were developed.

Along with the development of new asymptotic methods, M. Krylov and M. Bogolyubov paid great attention to their strict substantiation. In this way, an approximation method of averaging was developed, its accuracy was established, and an error estimate was given. The first grounding of the averaging principle was given by L. Mandelstam and M. Papaleksi in 1934. M. Krylov and M. Bogolyubov also showed that the first approximation obtained by the averaging principle coincides with the approximate solution by the method of Van der Paul.

Among the first applications of developments in the field of asymptotic decompositions it is necessary to note the study of internal friction influence during oscillations of rods, turbine blades, etc. (G. Pisarenko), the influence of nonlinear couplings on rotational oscillations of crankshafts (V. Natanzon), the mutual influence of relaxation and weak oscillations, self-oscillating processes and stability issues in higher-order control systems (E. Chopov), etc.

Later M. Bogolyubov significantly influenced the development of physical, mathematical and technical sciences and became a leader of scientific teams. Mykola Mykolayovych Bogolyubov got deep results in calculus of variations, functional analysis, and theory of differential equations, probability theory and theory of almost periodic functions. He is one of the founders of asymptotic methods of nonlinear oscillation theory, the author of new approaches to statistical physics relaxation in non-equilibrium processes and the method of obtaining kinetic equations based on the mechanics of the set of particles, created in quantum statistical mechanics microscopic theories of superfluidity and superconductivity. M. M. Bogolyubov also obtained important results in quantum field theory and the theory of elementary particles. Thanks to his works, a new axiomatic approach to quantum field theory was developed, the theory of the scattering matrix was constructed, and dispersion relations were strictly proved for the first time. It became the basis for creating a new language in the theory of strong interactions. In the theory of elementary particles, he independently introduced a new quantum number, which later became known as "color", and built a scheme of strong interactions based on three triplets of quarks with integer charges. All this was a significant step towards the creation of quantum chromodynamics.

Peculiarities of M. M. Bogolyubov's work were a wide creative range, depth and comprehensive coverage of phenomena, unity of issues. The scientist was characterized by talent, pedagogical ability, intelligence, democracy, scientific integrity. Such qualities contributed to the formation of a students group around him as a scientific leader from the 1940s and 1950s in Kyiv, Moscow and Dubna. There are three stages in Kyiv related to the shift of emphasis in his scientific activity. In the first period (late 1930s – early 1940s) the interests of M. Bogolyubov were concentrated in the field of mathematical physics and the theory of nonlinear oscillations, in the second (1940s – about 1965) the scientist aimed to mathematically accurate solving problems of statistical physics and quantum field theory, the third period (since 1966) is characterized by research on the most pressing issues of high-energy physics. This stage was organizationally connected with the establishing in 1966 on his initiative the Institute of Theoretical Physics of the Academy of Sciences of Ukraine. In 1928-1973 M. Bogolyubov worked at the Academy of Sciences of Ukraine, from 1936 to 1949 he was a professor, head of the Department of Theory of Functions, in 1946-1949 – dean of the Faculty of Mechanics and Mathematics of Kyiv University, from 1945 to 1956 he headed Department of the Institute of Mathematics of the Academy of Sciences of Ukraine. In 1948 he was elected an academician of the Academy of Sciences of Ukraine, and from 1966 to 1973 he headed the Institute of Theoretical Physics of the Academy of Sciences of Ukraine. From 1948 M. Bogolyubov worked at the V. Steklov's Mathematical Institute, in 1950-1953 moved to "Arzamas-16", where he participated in the developing the basics of fusion and the creation of nuclear weapons, developed a magnetic thermonuclear reactor.

Until 1959, the scientist remained a professor at the University of Kyiv, from 1944 to 1949 he worked at the Kyiv Institute of Food Industry. In 1957, at his suggestion, a laboratory of the atomic nucleus and elementary particles was established at the Institute of Physics of the Academy of Sciences of Ukraine.

Conclusions

M. M. Bogolyubov's scientific authority was very high all over the world. His works have been published in many languages. He was elected a member of the Academies of Sciences of Bulgaria, Germany, Poland and the United States, and an honorary doctor of the universities of Allahabad (India), Berlin and Chicago, a member of many scientific societies, awarded prizes and medals, including the M. V. Lomonosov gold medal, the M. M. Krylov Prize (1949, 1964), medals of M. Planck (1973), B. Franklin (1974) and others.

One of the first M. Bogolyubov's students, Academician Y. Mitropolsky, said that M. Bogolyubov is a scientist who on scale of scientific thought is equal with such luminaries as A. Poincare, A. M. Lyapunov, A. Kolmogorov. M. Malyuta, a student of M. Bogolyubov, believed that the scientist's knowledge volume was at the level of L. Landau.

Krylov – Bogoliubov methods of nonlinear mechanics became the working tool of a science and have penetrated into the numerous areas - electrical and radio engineering, theoretical physics, astrophysics, theory of accelerating devices, theory of jet engines, theory of a structure of crystals, theory of gyroscopes, nonlinear optics.

At the same time, the using methods developed by scientists at the present stage require further analysis.

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