CHIRP BASED ADAPTIVE OPTIMAL KERNAL FOR LIMITED NONSTATIONARY SIGNALS

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Time varying spectra are ubiquitous in practise, and manifest themselves in speech, radar/sonar returns and biomedical signals, etc. Joint time-frequency distributions (TFDs) is a powerful tool to analyze these signals. Nevertheless, the conventional methods do not deliver good performance in missing sample cases due to noise-like artifacts spreading in the time-frequency (TF) domain as well as the ambiguity domain. Therefore, this paper introduces a novel TFD which can effectively suppress the effects of limited data, namely the chirp-based adaptive optimal kernel TFD. The method uses a two-dimension mask to attenuate the cross-terms between different components as well as those between the same components appearing in WignerVille distribution (WVD). However, in contrast with traditional RIDs, which belong to the Cohen's class, the proposed kernel is signal-dependent, being applied for a windowed signal, not the whole signal, and it can partially combat missing samples. The first crucial feature of the proposed method is that its kernel is designed based on chirps' properties. According to [1], any non-stationary windowed signal can be approximated as a sum of chirps. Moreover, chirp's auto-terms always reside in only a half of the ambiguity domain, which does not cover the Doppler axis [1]. Therefore, the mask for any windowed nonstationary signal just needs to cover the half not containing the Doppler axis. The advantageous point is that the artifact distribution caused by missing samples always appears along the Doppler axis [1]. Thus, by filtering out the region along the Doppler axis, the chirp-based kernel gives improved time frequency representation (TFR) in the case of incomplete data. The second important feature of the new approach is that the kernel is signal-dependent. In contrast with the radiant Gaussian kernel (RGK) [2], our method actually solves an optimization problem just in the useful half of the ambiguity domain, not the whole domain. This not only reduces the complexity but also improves the TFD estimation results because almost half of interference and noise-like artifacts caused by missing samples are mitigated.

The optimization process is conducted in polar coordinate. The original kernel is Gaussian, which is expressed following,

$$C(g,q) = e^{-\frac{(g\Delta r)^2}{2\sigma(q\Delta\phi)^2}} \qquad (1)$$

$$g = 0,...,G-1; q = 0,...,Q-1,$$

where g and q are the radius and angle indices, Δr and $\Delta\phi$ are the radius and angle step sizes, $\sigma(q\Delta\phi)$ is the spread parameter. At the start, σ is equal for all angles. Then, the spread parameter is updated by the gradient ascent method to match with signal in the ambiguity plane in such a way that it is large at a certain angle if the magnitude of the AF is large. The optimization problem is written as follows,

$$\max_{\substack{C(n;g,q),\forall g,\\ |q\Delta\phi|\le\pi/4,3\pi/4\le q\Delta\phi\le 5\pi/4}} \sum_{g=0}^{G-1}\sum_{q=0}^{Q-1} r\left|A_{ss}(n;g,q)C(n;g,q)\right|^2$$

$$\text{Subject to } C(n;r,q)=e^{-\frac{(g\Delta r)^2}{2\sigma^2(q)}} \tag{2}$$

$$\sum_{q=0}^{N-1}\sigma^2(q)\le a,$$

where a is the kernel volume ($1\le a\le 5$). The current-time slice of the TFR is computed as one slice (at time n only) of the two-dimensional Fourier transform of the short time ambiguity function -kernel product, expressed as follows,

$$TFR(n,k)=\sum_p\sum_b A(n;p,b)C(n;p,b)e^{-j2\pi np/N_w}e^{-j2\pi bk/N_w}, \tag{3}$$

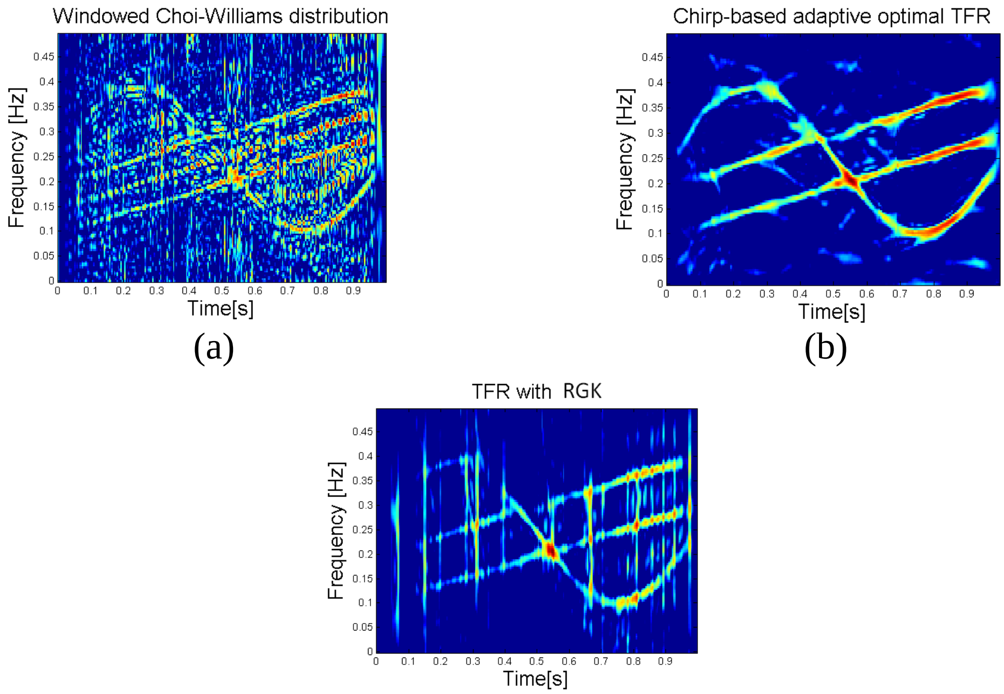
where n, k are discrete time and frequency variables, $n=0,1,2,...,N_w-1$, $k=0,1,2,...,N_w-1$, N_w is the length of the windowed signal, p and b denote the frequency shift (Doppler frequency) and the time lag. To evaluate the performance of the proposed optimal chirp-based kernel, the following signal is used,

$$s(n)=\exp\left\{j(0.15F_s)\cos(2\pi\frac{n}{F_s}+\pi)+j2\pi(0.25F_s)\frac{n}{F_s}\right\}+$$
$$\exp\left\{j2\pi\left[(0.1F_s)\frac{n}{F_s}+(0.2F_s)\frac{n^2}{F_s^2}\right]\right\}+\exp\left\{j2\pi\left[(0.2F_s)\frac{n}{F_s}+(0.2F_s)\frac{n^2}{F_s^2}\right]\right\} \tag{4}$$

The signals are sampled at the Nyquist rate, and then randomly shortened 50% to create the incomplete data to be processed. The two representatives of signal-independent and signal-dependent kernels, Choi-Williams and the RGK, are simulated with the same signals to get a visual comparison with our methods. The TF estimation result is shown in Figure 1. A parameter of concentration level ζ is used to access the accuracy of the resulting TFR [1]. The higher ζ, the more accurate the TF approximation. The windowed Choi-William kernel gives the worst performance as it lets much of cross-terms and missing sample artifacts go through (ζ =0.5). The RGK achieves concentration level of 1.64. The chirp-base adaptive optimal kernel give the best TFRs with the highest concentration level, ζ =5.2.

CONCLUSION

The paper has introduced a novel chirp-based adaptive optimal kernel method. The method obtains superior results when compared with traditional fixed kernels belonging to Cohen‘s class, and with previous signal-dependent kernel method, RGK, in the case of missing samples.



(a) (b)

(c)

Figure 1.TFD obtained by (a) Windowed Choi-Williams distribution; (b) Chirp-based adaptive optimal kernel; (c) RGK of the signal (4) when 50% data missing.

References

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Анотація

Відсутні вибірки та випадкові вибірки нестаціонарних сигналів породжують артефакти, що поширюються як по частоті часу, так і по областях неоднозначності, що призводить до неточної оцінки час-частота. Скориставшись властивістю чірпіння та процесом оптимізації, у статті запропоновано метод, який дає хороший результат частотно-часові розподіли, коли ми маємо обмежений сигнал.

Ключові слова: щебетання, залежне від сигналу ядро, відсутній зразок.

Abstract

Missing samples and randomly sampled non-stationary signals give rise to artifacts that spread over both the time-frequency and the ambiguity domains, which results in inaccurate TF estimation. By taking advantage of chirp property and optimization process, the paper has proposed a method that gives good result of TFD when we have limited signal.

Keywords: chirp, signal- dependent kernel, missing sample.