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Mathematics Education, Western and Eastern Teaching Approaches Combined With Arts

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Abstract

The paper describes the experimentation results of the application in the classroom of a novel learning and teaching approach for studying mathematics. The method results from a combination of art and technology to stimulate and motivate secondary school students in mathematics often considered tedious and challenging to understand. This helped students revive the art perception displaying its hidden science base and understanding that most artists' reasoning is reducible to mathematical concepts.

Keywords: Mathematics Education, Active Learning, Teaching Approaches
ACM 2012 CCS Concepts: Applied computing → Education
Mathematics Subject Classification 2020: 97U50, 68U01

1 Introduction

Nowadays, when Europe is coping with a period of change and the economic crisis, in particular, which is slowing down the development and the social progress highlighting the structural weaknesses within the European Countries, the role of R&D is difficult to overestimate.

On the other hand, there are other international challenges, such as globalisation, resource exploitation, and human ageing. As stated in the European Communication EUROPE 2020 – *A strategy for smart, sustainable and inclusive growth* of 2010 [1], the Commission focuses on a stronger strategy that can transform the European Union into a smart, sustainable and inclusive growth for a high level of employment, productivity and social cohesion.

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This paper presents the principal results of the doctoral thesis "Mathematics Education, Western and Eastern Teaching Approaches Combined With Arts" by Michela Tramonti, Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences, successfully defended before a Scientific Jury on January 21, 2021.

According to EUROSTAT [2], published in *Science, Technology, and Innovation in Europe*, 1.68% of people in Europe work in the Research & Development (R&D) field, but the situation is changing completely at a National level for some countries as Italy where the percentage of people working as "scientists and engineers" is very low, around 0.61% [2].

Although Research & Development are the keywords for Europe of the 21st century, the obvious national and institutional barriers limit the strengthening of European Research.

These data are still confirmed by the following figure representing the intensity of R&D activities in 2016 in Europe, where Italy is placed in a low ranking of the average of 2.03% reached by some European countries.

Schreiner and Sjǿberg suggest that *it might be that we have now passed the era in which the work of physicists, technicians, and engineers is seen as crucial to people's lives and well-being* [3]. Today's youths are more interested in who they will be rather than what they will do. Negative stereotypes of scientists, engineers, researchers, and other STEM (Science, Technology, Engineering and Mathematics) experts' careers can be found amongst youth in most of the western world, even the United States. The only exception is Japan, where the Government makes wide investments in the educational field [4].

School, television, and newspapers feed the people's imagination with the idea that scientists' work is a hard and demanding activity and unknown and mysterious. It is enough to think about emblematic figures of scientists like Frankenstein or Doc, the character of the movie *Back to the Future*, or cartoons, which continue to represent *science* in a caricatured and imaginative way. As a result, the scientist's figure appears isolated, immersed in a special work – strange and incomprehensible to everyone at the same time.

Further studies show that women remain under-represented in R&D in every region worldwide. Some of the latest UNESCO data shows that, in North America and Western Europe, the average representation for women in R&D is 32% (is the lowest average found in Luxemburg with only 21% and in the Netherlands with 24%). A different situation is found in Central and Eastern Europe, where the average rises to 40%. In fact, in Latvia, Lithuania and Northern Macedonia, woman represents more than 50%, and in Albania, Bulgaria, Croatia, Estonia, Moldova and Romania, the percentage exceeds 40%. Despite the positive trend occurring in Eastern Europe, many studies indicate a disparity between the number of women studying science and those who go working as scientists professionally. Overall, women account for a minority of the world's researchers *(UNESCO Institute for Statistics, June 2018)* [5].

In this context, the European Commission's High-Level Expert Group on Science Education Renewal has made the points that *Teachers are key players ... being part of a network allowing them to improve the quality of their teaching and support their motivation and that the articulation between national activities and those funded at the European level must be improved* [6].

In this framework, it is necessary to develop a high-quality teaching and learning environment where students can approach the study of scientific subjects with more interest and motivation adequately supported by teachers in their learning, which will be more effective if it is *meaningful*, i.e. active, constructive, intentional, authentic and collaborative [7].

The research intended to exploit the potentiality emerging from a creative integration between Western and Eastern learning approaches combined with the use of the arts. The potential effect would allow students to develop and improve their creativity-based learning skills and problem-solving skills.

Moreover, it aimed at approaching the students to the study of STEM *(Science, Technology, Engineering and Mathematics)* subjects through both meaningful and mastery learning. This allowed the development of a more effective Educational and Training environment for teachers and their students who have benefited from the use of more attractive and fun pedagogical tools in the study of mathematics.

In detail, the research objectives can be synthesised as follows:

- Exploiting the possibility to find a combination between Western and Eastern approaches (mainly Singapore's method) in mathematics teaching.
- Finding the result of this combination in the arts.

Demonstrating the objectives above encourages meaningful learning in developing the research skills to improve the school performance (short-term objective) and increased interest towards a future perspective in the mathematics field (long-term objective).

2 Study of the problem

The PISA survey is promoted by the Organization for Economic Cooperation and Development (OECD) [8]. It aims at assessing the performances of 15 years-old students in reading, mathematics, and science every three years. It focuses on how far students near the end of compulsory education have acquired knowledge and skills essential for full participation in society. This survey contributes to the assessment of mathematical and science literacy through several proposed questions always set in a real context.

	Mean score in PISA 2018			Long-term trend[1]			Short-term change[2]			Top/low students[3]	
	Reading	Math	Science	Reading	Math	Science	Reading	Math	Science	[4]	[5]
	Mean	Mean	Mean	Score dif.	Score dif.	Score dif.	Score dif.	Score dif.	Score dif.	%	%
OECD average	487	489	489	0	−1	−2	−3	2	−2	15,7	13,4
Italy	476	487	468	0	5	−2	−8	−3	−13	12,1	13,8

Table 1: Extracted from the snapshot of performance in mathematics, reading and science – PISA 2018 results. Source: OECD, PISA 2018 Database, [9].
[1]Long-term trend: Average rate of change in performance, per three-year-period
[2]Short-term change in performance (PISA 2015 to PISA 2018)
[3]Top-performing and low-achieving students
[4]Share of top performers in at least one subject (Level 5 or 6)
[5]Share of low achievers in all three subjects (below Level 2)

Concerning the PISA last survey realised in 2018, the mean scores achieved for mathematics have registered a negative tendency in countries considered to be at the forefront of education, like Finland (where the score has decreased by −12 in the last three years) one hand. While on the other hand, the results have confirmed the negative ranking (below the OECD average) in other countries, like Greece.

In Italy, if the survey realised in 2015 have shown that the pupils' performance reached in mathematics a score on the OECD average (490), the last results collected in 2018 present again a negative situation (487, −2) under the OECD average (489) (Table 1). Probably this inverse tendency is still due to the use of traditional learning-teaching methodologies in mathematics study [10].

If the total score achieved in Italy (487) is examined taking into account the gender difference, the final result represents a different scenario: the boys have reached the main score of 494 corresponding to +5 with respect to the OECD Average 489, but the girls – only a score of 479 with a decreasing of −10 below the OECD average. This factor underlines the complexity of the Italian situation where gender differences strongly affect the students' performance in mathematics [10].

Another important international survey is the TIMSS, developed by the International Association for the Evaluation of Educational Achievement (IEA) to allow participating nations to compare students' educational achievement. It is an international assessment of fourth-grade and eighth-grade students' mathematics and science knowledge. This includes achievement in each content and cognitive domain and overall mathematics and science achievement.

The data, collected by the last TIMSS survey (2015) on mathematics, show that East Asian countries, particularly Singapore, are top achievers in fourth and eighth grades in Mathematics. Italy has achieved an invariable average (507 and 494) in both at fourth and eighth grades in mathematics, considering trends from 2011 to 2015. While countries like Finland and Germany show a lower average achievement at fourth grade in the same period [11].

The data demonstrate that for fourth-grade students, Italy has achieved +2.6 from the TIMSS Scale Centerpoint (500), while for eighth-grade students, the results show still a negative situation −2.5 from TIMSS Scale Centerpoint (500). At the same levels, Singapore is placed at the top, reaching 618 (+3.8) for fourth grade and 621 (+3.2) for eighth grade.

The gap between Asian countries and the following highest countries was 23 points in 2015, unchanged from 2011 at fourth grade, while at eighth grade, the gap between Asian countries and the next highest countries was 48 points in 2015, increasing from 31 in 2011. However, with a few exceptions, like Norway, Belgium (at fourth grade) and Slovenia, Hungary (at eighth grade), European students often lack key basic competencies in mathematics, science and technology [11].

In this context, most Ministries of Education share and agree on the educational objectives and results that are the base of these surveys by underlining the importance to encourage the development of the mathematics skills and their assessment through the national exams at the end of the compulsory education. In fact, based on these last results reached by the students' performances, the European Ministers aim to reduce the average of students with difficulties in literacy, mathematics and science getting a percentage lower than 15% within 2020 (from the section News in EU Commission web site) [12].

2.1 Substantial differences in mathematics performances between Western and Eastern countries

In 2018, students participated in the OECD-PISA international survey on mathematics, science and reading by providing an opportunity to compare student performance over time from 2009. Based on the results achieved, the students from the following Asian countries were top-ranked: Singapore reached a score of 569 above the OECD average 489, followed by Hong Kong – China (551), Macau-China (558), Beijing-Shanghai-Jiangsu-Zhejiang (Regions of China participating in testing) (591), Fig. 1, [10].

Comparing the previous PISA results, these countries confirmed their higher positions by reporting no differences between boys and girls. In comparison, Ital-

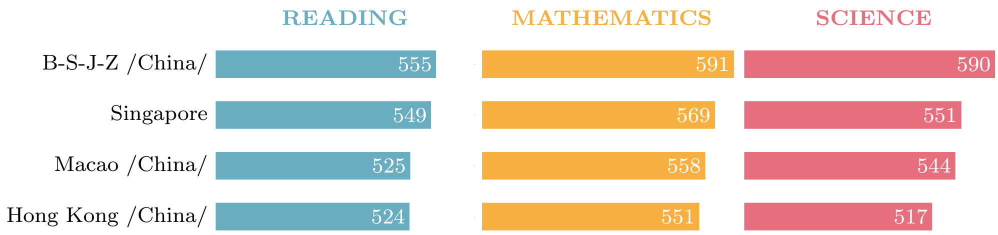

Figure 1: Extracted from the snapshot of students' performance in reading, mathematics and science. Source: OECD Education Statistics: PISA Results – 2018, [9].

ian students' last performances in mathematics demonstrate a negative trend not overcoming the OECD average score by ranking at 31st place. Also, in other countries, like Finland, even if still placed over the OECD average, the final ranking list underlines a negative trend concerning the previous results. In particular, in Finland, PISA data in 2006 indicates a negative relation between the performance mean score and the students' interest score: highest in science score, lowest in interest. This shows that the negative score in the students' interest has, over the years, led to a lowering of the performance score, losing in PISA survey of 2018 points [10].

An interesting factor underlining the difference between Western and Asian students, emerging from these administered surveys, is the quality of relationships between teacher and student analysed in the survey framework Do teacher-student relation affect students' well-being at school. As a result, students in Belgium, Hong Kong, China, Japan, Liechtenstein, Shanghai-China, Singapore, Switzerland, and Chinese Taipei have achieved a high score (above-average OECD) and above-average performances in mathematics [13].

The collected data show a strict relation between student well-being and school performance. Indeed, the countries, such as China, at the top of the ranking, are still those where young students get the best scores in mathematics and language understanding tests. The results of the PISA survey show that positive and constructive teacher-student relations are a good ground for better performance in mathematics. Therefore, this could be a key factor for fostering students' social and emotional development at school.

Comparing countries and economies with two indicators, teacher-student relation and mathematics performance, shows that developing a competitive, knowledge-based global economy is crucial in some countries, like Shangai-China, Singapore, Hong Kong-China, and Chinese Taipei students show high performance.

On the contrary, countries like Italy, France, Hungary, Bulgaria, Greece are still below the OECD average [13].

Looking through the PISA results in 2015 [11], another relevant factor comes out. In particular, the results related to students' science beliefs, engagement and motivation are interesting if the achievements in Asian countries are compared with some Western countries, such as Italy. High performances in Asian countries, e.g. Singapore, express a more positive and inclusive image of science in students. This encourages a positive attitude towards scientific careers and learning motivation enhanced.

While low performances, as in Italy, are strictly related to the creation of a negative mental image, stereotyped on who are the scientists and the engineers and on who are the people choosing this kind of career. Since science knowledge and understanding are useful even beyond the scientists' work and are necessary for full participation in a science-based technology world, school scientific topics should be promoted more positively by enhancing new resources and methodologies to increase student interest and enjoyment [14].

2.2 Student motivation in mathematics study

About 79% of students define mathematics as a big obstacle to their learning process. The difficulties revealed are often related to its being considered more abstract than the others. About 83% of students work using visual memory [15]. This means that if we can imagine a history lesson as a film or a cartoon, it is impossible to do the same with a mathematics lesson. For instance, a lesson on inequalities hardly can activate visual memory as a literature subject does. It can most stimulate the photographic memory, which is only 7% of the visual one [16].

Besides, the theory should be learned through doing exercises to understand the solution process by activating the procedural memory. This allows students to get skills through learning by doing.

Students often do not perceive a practical utility in mathematics study unless they study specific subjects, such as economics, where the utility is directly evident. In short, most students don't have enough stimuli of being interested in mathematics.

Mathematics is not complicated by itself but must be studied differently from other disciplines. The suggested learning approaches usually include full immersion, an association between image and concept, or using memory techniques [16].

In the case of memory techniques, for example, mnemonics can help students remember formulas or demonstrations by looking at them once because these stimulate the innate abilities of the student's mind.

However, these learning methods can help students in a short-term study, but they don't seem effective if we think in long-term learning.

Another issue, which can impact student motivation in mathematics study, is directly related to the teaching-learning process.

The traditional one is focused more on content mastery than on the learning abilities development and enhancement of research aptitude. The mainstream education system is oriented towards the teacher: the teacher gives, and students receive [17]. Therefore the student evaluation is related to a correct or incorrect answer. This education system focuses on school performance and results and doesn't develop their lifelong learning capacity.

The didactic activities in the classroom can be realised in different ways by teachers. For example, by presenting, in the beginning, general principles on the topic to be studied, and then letting students deduce particular behaviours or, on the contrary, by starting from phenomena, partial information, well-defined known behaviours up to building laws, processes, considerations, and general behaviour. This way to proceed is a part of the traditional teaching methodology, focused on the discipline and knowledge and the completeness of the message communicated¸ exploiting the deductive learning mode.

On the other hand, in a context of an active school, deductive teaching is replaced by the inductive modality, centred on the student's educational process involving his/her bio-psychological, socio-cultural and value structure.

Consequently, the factors which can make inductive teaching interesting, regard the increased attention given to students in terms of effective learning aiming to exploit and develop their motivation.

According to the motivational theories, motivation is a mainspring for the individual to act. If a strong reason drives the individuals, they will be able to: reach own objectives; develop a positive vision on their work/study; produce new energies needed to change; increase their self-esteem and their abilities; realise their personal and professional development helping others in this process. One factor that increases motivation is interest: motivating students means arousing their interest in the study by searching and discovering information that will promote student learning [18].

Meaningful learning aims at connecting a learning process to relevant concepts already owned, existing in the cognitive structure of the subject [19]. This concept comes from the constructivist theory of knowledge, in other words, there is no knowledge without a process of meaning construction by the learner.

The learners construct their knowledge in meaningful contexts through the object's manipulation, tools, and observation and interpretation of their actions' results. In these terms, meaningful learning becomes contextualised and complex. The students learn more and better if they cope with authentic tasks strictly connected to the real world, where they meet their everyday life's real problems.

The same thing occurs when a student approaches math studies. People often forget that reality, as well as all disciplines, as known by man, is based on mathematical concepts: math could be found in human beings, in architecture, plants, and animals, and as a part of them, it also regulates their characteristics.

Despite this strong presence of mathematics concepts, their connection doesn't appear so evident during the learning process. Often, teachers offer students a too theoretical approach, causing the perception that mathematics is abstract and far away from everyday life. This influences the way to learn mathematics privileging more storage capacity than problem-solving skills.

3 The research approach developed

The reference theoretical framework of the proposal had a heuristic function and practice orientation by allowing guiding the tasks to be carried out up to the achievement of the results.

The main research model was adapted from the *Theory of Didactical Situations in Mathematics*, elaborated by Guy Brousseau [20, 21], aiming to define the conditions under which "a learner is led to do mathematics, how to use it and how to invent it".

Based on the constructivist approach, Brousseau defines three situations in the teaching process:

1. A-didactical situation is a context containing all the conditions which allow students to establish a relationship with a specific knowledge regardless of their teacher. This means that students' actions and answers depend on such relationship with the problem or the difficulty to be solved or overcome, even if this relation is no so explicit.
2. Non-didactical situation is the context that is not organised to allow students to learn specific knowledge. For example, all operations with natural numbers may be considered as a non-didactical situation.
3. Didactical situation is the one in which the situation is designed and well-organised to favour knowledge. It is enough to consider all the activities done in the classrooms where the teacher intentionally "teaches" and students are forced to learn [20].

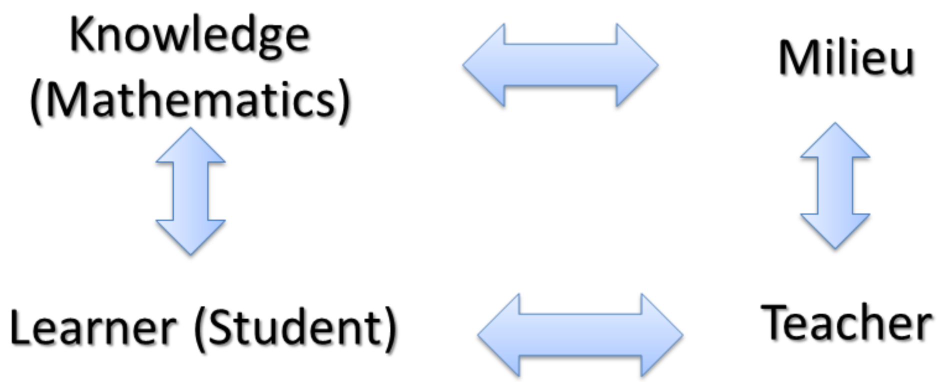

Figure 2: The didactical quadrangle of Guy Brousseau.

The three situations identified can be imagined like interaction systems between one or more subjects with a milieu that is a context or means. According to the theory of didactical situations, teachers have to encourage their students a behaviour, which they should take independently to demonstrate their knowledge. For this reason, an important element is a milieu that teachers know well or have prepared by themselves for students.

To synthesise the different situations occurring, Brousseau refers to the didactical quadrangle where the four apices are constituted by Knowledge (in this case Mathematics), Teacher and Learner/Student and the milieu, Fig. 2, [22].

In this constructivist theoretical framework, problem-solving is a fundamental tool to learn mathematics. This means that students' need to solve a problematic situation gives rise to a reflection action, which becomes knowledge accordingly. Therefore, in mathematics teaching, the encouragement of student's problem-solving ability can be a good means to favour learner's reflection and motivation.

The model proposed in this research work results from the integration of the three types of didactical situations identified by Brousseau. Therefore, the learning/teaching situation developed according to the three phases of the Eastern learning/teaching approach (concrete, pictorial and abstract), was *A-didactical, Non-didactical, Didactical* as follows:

1. A-didactical, because the students learned mathematical topics by discovering that different relationships exist among things or mathematical concepts (even if they cannot be so explicit) and by developing, accordingly, problem-solving skills avoiding memorising the solution procedure only.

2. Didactical because several worksheets were prepared for the students before starting the experimentation phase. These instructions led the student from the concrete phase to pictorial up to the abstract one.
3. Non-didactical, because the teachers had the function to mediate and support the learning process through their students' creativity and imagination. The use of creativity from students was let free, especially when they produced their art-works on the base of mathematics concepts studied.

Therefore, re-considering the figure of Didactical Quadriangle of Guy Brousseau (Fig. 2) from the point of view of the proposed model, the art was identified as the context or better the *milieu* to be used to reach knowledge.

3.1 Arts as learning means in mathematics study

In the proposed approach, the introduction of arts helps students connect mathematics to reality better and understand its actual application in their everyday lives.

As *Steiner and Schwarz* [23] claim in education, it is necessary to have tools and methodologies that can guarantee learners' active involvement and creative inclusion to let them test the interconnection of different languages, such as visual and sensory, verbal and nonverbal. This favours the development of both cognitive and emotional dimensions.

Artists of all ages have tried to create works that represent human beings and dimensions. But to do it in the best way, they had also to devote themselves to the studies of geometry and architecture. In arts, we find some mathematics concepts such as perspective and proportion. One of the well-known artists, who used mathematics as one of the most important components in his art-works, was the artist *Mauritius Cornelius Escher* (1898-1972).

In this context, art is a way for students to express their creativity and find harmony in developing both cognitive and emotional dimensions. At the same time, it obeys mathematics rules and propositions. Similarly, mathematics reconstructs reality according to its laws, getting an idealised replica of the subject. Consequently, art and mathematics are closely related.

Exploiting the potentialities of the interconnections between mathematics and art, students can discover how mathematics and scientific rules impact various aspects of reality.

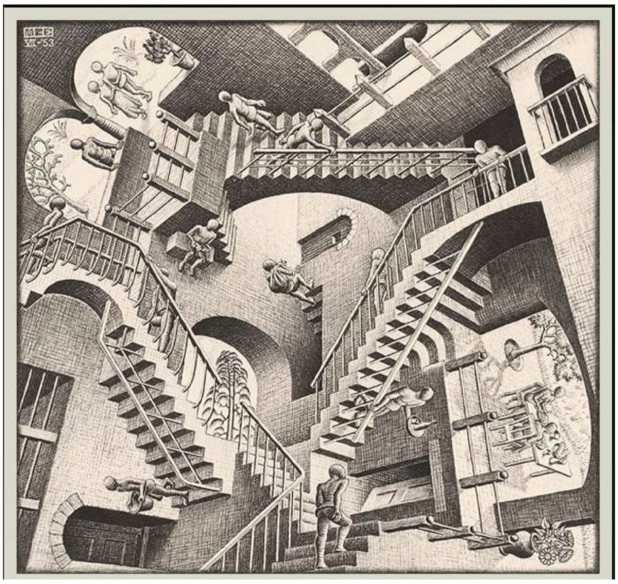

Figure 3: Relativity. Source: https://mcescher.com/.

Figure 4: Gravity. Source: https://mcescher.com/.

3.2 An Eastern learning approach: the Singapore method

The research approach was focused on combining the mathematics approaches set in West and East, between Western analytical methods and Eastern holistic approaches, namely Singapore's method.

The theoretical reference system under consideration could be schematised by referring to Aristoteles with the organisation of bivalent logic characterising the way of arguing in Western culture and therefore of our students, and to Confucius with Confucian method transmission of the Tao, and the book I Ching [24].

As Leung stated: *Aristoteles, more than any other thinker, determined Western intellectual history's orientation and content. He was the author of a philosophical and scientific system that, through the centuries, became the support and vehicle for both medieval Christian and Islamic scholastic thought: until the end of the 17th century, western culture was Aristotelian. And, even after the intellectual revolutions of centuries to follow, Aristotelian concepts and ideas remained embedded in Western thinking* [24]. For example, Euclidean geometry, the first structured language in mathematics history, is a model of Aristotelian bivalent logic.

In fact, Eastern researchers prefer, in general, to determine the relationships among things, such as among phenomena or objects, following a different approach from the Aristotelian hypothetical deductive one [25].

In other words, the aim is to favour a relational thought connected to a concrete arithmetic concept and develop skills for recognising relationships among variables to be able to work on them dynamically. This approach, mediated by the teacher, can promote in the student a stronger recognition of the relationship between syntax and semantics of arithmetic writing. For example, the elements' theory is defined based on their function in a situation. This is analogous to the method mostly used in mathematics, according to which an object is determined from the relationships established with other objects.

Therefore, the mathematical reasoning results can be assimilated to a set of analysis and synthesis processes that evolve in the development of both abstraction and generalisation phases through so-called variations as well as the observation and recognition of invariant elements can give rise to generalisations through the search of algorithmic procedures unifying and contextualised in more fields. This process has been called by some mathematics teachers the *trilogy of problem-solving* [26].

In this meta-cognition *independent* process, the definitions, patterns of hypothetical-deductive reasoning demonstrations are not used. However, this is focused on the ability to rationally review the information and data provided by the problematic situation met, learn how to categorise them concerning knowledge previously achieved, choose the best ways for their representation, and draw up concrete conclusions [27].

Generally, the Singapore method is defined as the *Concrete-Pictorial-Abstract method* (CPA) applying the so-called *Concrete-Representational-Abstract* (CRA) strategies [28]. The *concrete* phase refers to a manipulative experience with real objects to understand how they work. Students learn how to transfer their knowledge acquired through the real objects into a mental image, into a diagram or drawing during the pictorial phase. Finally, in the abstract, they learn to use mathematics symbols, such as X for multiplication.

Thanks to this transition, starting from concrete objects to pictures arriving then to symbols, the Singapore method offers many and various opportunities to learn mathematical concepts, especially for those students who have difficulties. In this way, students have time to face mathematics concepts by testing many different approaches to solving similar problems [29].

The main feature of this approach is visual and model-drawing strategies that underline mathematics and word problems and leave out the memorisation occurring through repetitive exercises. In effect, Singaporean textbooks often use visual aids like strip models, which are pictorial representations of mathematical problems requiring a basic arithmetic operation to be solved. These tools are used to help the young student learn to choose more appropriate arithmetic operations to solve a specific problem.

The approach aims to visually teach the student how to represent mathematics concepts before applying abstract symbols like numbers and equations. This helps students make their connections and draw generalisations about the concepts learnt and not simply memorise disconnected and isolated facts.

In this context, students experience different types of representations of the same concept by finding correspondences and relationships among them. This step conducts students to consolidate the conceptual understanding.

Inspired by the Singapore method, we combined the three phases (concrete, pictorial and abstract) with the Aristotelian method of logic deductive by using a particular tool: the art (Fig. 5).

The introduction of the art to combine and join Eastern and Western learning/teaching approaches turns Fig. 5 into the following diagram, Fig. 6.

This procedure lets students face variations in mathematics problems, a fundamental element used systematically in Chinese mathematics teaching. In these types of problems, we have a constant element that Arabic numbers can represent as usually they are specified in the textbook or by the general mathematics

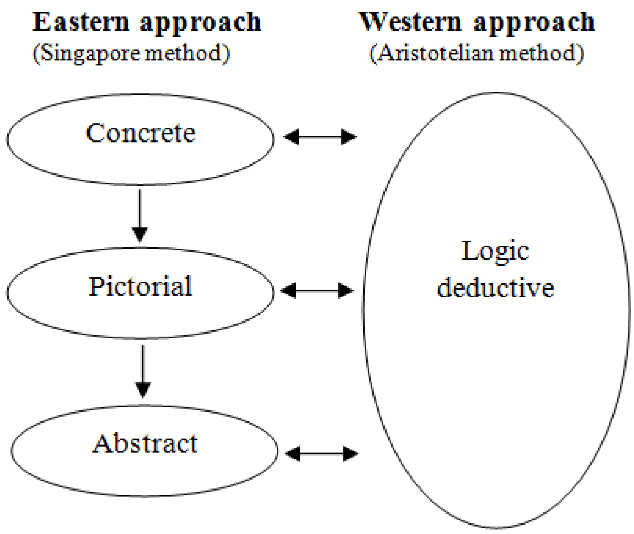

Figure 5: Combination between the Singapore method and the Aristotelian method.

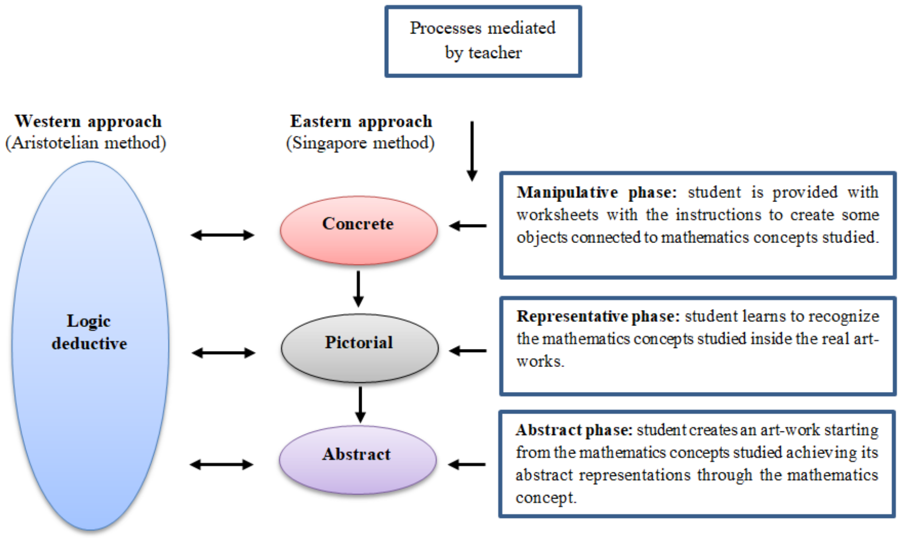

Figure 6: Representation of Western and Eastern approaches (Singapore's method) combined with art.

concept to be studied. In contrast, the context or other surrounded elements change. For instance, a student can learn to recognise the different symmetries working on several friezes and plan ornaments. In this example, the symmetry is the constant element, and the ornament is the context changing.

This allows students to learn, analyse, and see the problem from different points of view. Moreover, identifying the relationships among the various mathematics problems, they don't only memorise the resolution procedure but also develop and reinforce problem-solving skills.

3.3 Experimentation and evaluation

The preliminary phase of the experimentation included the following phases:

(i) Definition of the mathematics concepts, on the base of the school curriculum, and the selection of the art-works to be investigated by the students. Teachers and researchers worked together during this task.
(ii) Construction of the teaching/learning materials to be used during the experimentation activity. In particular, three worksheets, as guidelines, were constructed to provide the teachers with the instructions to proceed through the three phases concrete, pictorial and abstract. The instructions described the materials (such as paper with specific shape, pencils, and colours) and/or software applications (e.g. GeoGebra) to be used, the objective, the vocabulary, activity sequence structured on the base of the three phases (concrete, pictorial and abstract).

The experimentation phase divided into concrete, pictorial and abstract were mediated by teachers and/or the researcher who supported the students' study by introducing every phase and by stimulating them with questions and observations. In detail, these three phases were implemented as follows:

(i) Concrete phase (Fig. 7) – students were invited, for example, to build some objects (e.g. pentagram to study geometrical figures), figures (e.g. friezes, ornaments to study the symmetry) with instructions (provided by the teacher or the researcher) and the equipment available. The aim was to let students have the experience and become familiar with the mathematical concepts studied through object manipulation.

    As agreed with teachers, the GeoGebra application was used for this purpose for several reasons.

    First of all, Italian teachers consider it (GeoGebra) user-friendly when used with low ICT skilled students on PC, Mobile, and Tablet.

Secondly, due to students' previous experiences with the software, they were able to concentrate directly on the subject to be explored without first being worried about a new tool to be learnt.

(ii) *In the pictorial (or representative) phase, students started their research by finding* the art-works or the real-life application containing the mathematical concept or formula studied of the previous phase both individually and in team working. This helped students recognise the same concept in different contexts (and then in different art-works – Fig. 8). This put their attention on the different existing representations of mathematics concepts.

During this phase, they used what is strictly known as the art-works and found the mathematical concept in other contexts as well, e.g. in nature (Fig. 8).

(iii) *Abstract phase* – students were invited to create their art-work starting from the mathematics concept studied in the two previous phases by achieving the abstraction representation of the mathematical concept (Fig. 9).

(iv) Finally, a part of these works was uploaded in a 3D virtual museum, developed by the *Institute for Computer Science and Control, Hungarian Academy of Science*, in collaboration with the *Institute of Mathematics and Informatics, Bulgarian Academy of Sciences.*

The virtual museum, Mathematics and Arts, was realised to show some of the final outputs of the research work, in particular, the objectives and model proposed by establishing the structure and the tasks followed during the three phases (concrete-pictorial-abstract).

Moreover, by clicking on the art-works exposed, users can get some information about the author(s), the object created and its relation to the real world, as well as the mathematics concept represented (Fig. 10) and they can launch a video showing the key moments of the experimentation phase realised.

The evaluation phase included different stages:

(i) Research sample formation – 3 mathematics teachers of lower and upper secondary school and 3 art teachers, and 130 secondary school students of different ages, 11-13 for the first cycle and 14-16 – for the second one.

(ii) Qualitative and quantitative data collection – different tools were prepared and used to verify the method's effectiveness and usability – semi-structured questionnaires and grid for the observation participatory method and group discussion.

(iii) Coding and data processing – classification and management of the quantitative and qualitative data collected to analyse them based on relations among variables found.

Figure 7: Experimentation time – Concrete phase with students at Istituto Tecnico Superiore Bianchini in Terracina (Italy).

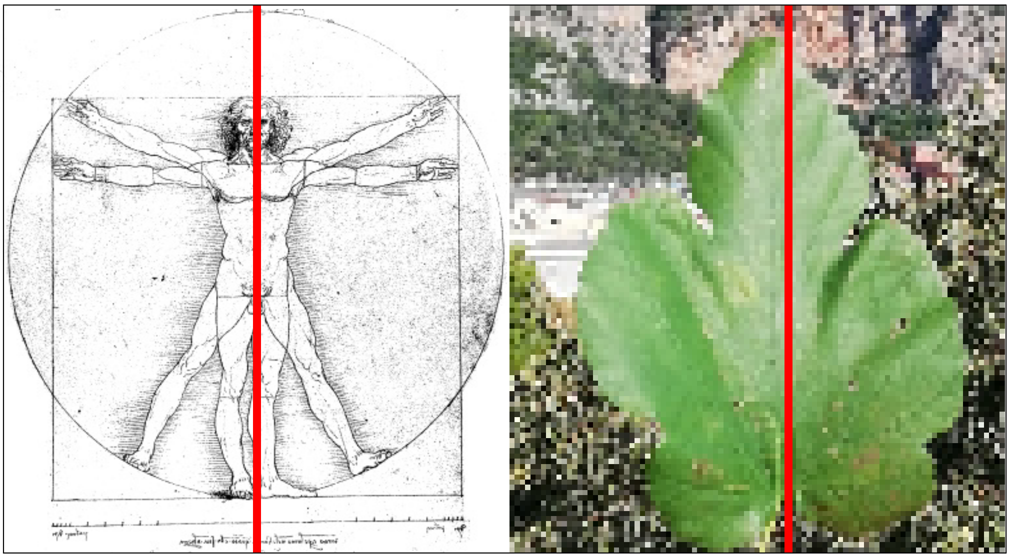

Figure 8: Experimentation time – Pictorial phase – art-work and object from nature selected for the symmetry study by students at Istituto Tecnico Superiore Bianchini in Terracina (Italy).

Figure 9: Experimentation time – Abstract phase – art-work realised by a group of students from Istituto Tecnico Superiore Bianchini in Terracina (Italy).

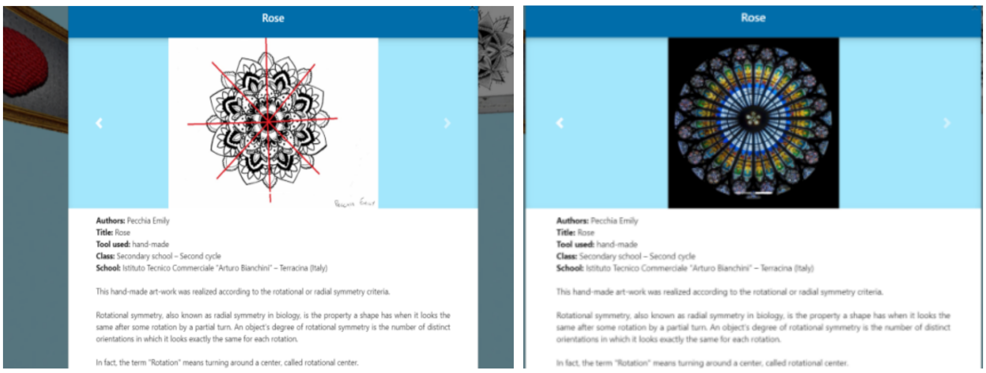

Figure 10: An example of the information card prepared per each art-work created by students and published in the virtual 3D Museum.

4 Presentation of the research data

The piloting phase was implemented with about 130 secondary school students of different ages, 11-13 years old for the first cycle (44,62%) and 14-16 years old for the second one (55,38%) with 43,1% male and 56,9% female.

4.1 Student initial attitude towards Mathematics

Usually, students define mathematics as a big obstacle for their study path because the difficulties often revealed are related to its being considered more abstract than the other disciplines.

In particular, 57,70% of students stated that their major difficulty in mathematics study is that this discipline is too abstract than the other subjects.

This factor derives from the different nature of the topics to be studied in the mathematics curricula and/or from the teaching method applied in the classroom. Usually, in secondary school – first-grade teachers often use visual tools to explain and introduce scientific connections to their students. However, these methods do not always work effectively if we consider the diversity of subjects students should study at school. The students are expected to use a different kind of ability and capability to learn the lesson of histories, mathematics or other subjects.

Students, growing up, start to perceive mathematics as an abstract subject, and for this reason, most of them see this discipline as something far from reality and not easily applicable. Students often do not perceive a practical future utility in mathematics study unless they study specific subjects, such as economics, where the utility is directly evident. The reason is that they often don't have a stimulating overview of being interested in mathematics.

4.2 Students' initial attitude towards the combination of Mathematics and Art

Another relevant element investigated with the first part of the questionnaire is the initial capability to perceive the connections between mathematics concepts and art, seen as art-works and as real objects in nature.

The data show that most students (82,23%) never thought that mathematics could be included in this unusual aspect, "art". Besides, a quite high percentage of students (23,80%) held a neutral position by underlining the difficulty of seeing and comprehending the relation between mathematics and art.

However, 61,60% of students, intrigued by this new experience, consider this means art, a way to support and make easier mathematics learning (Fig. 11).

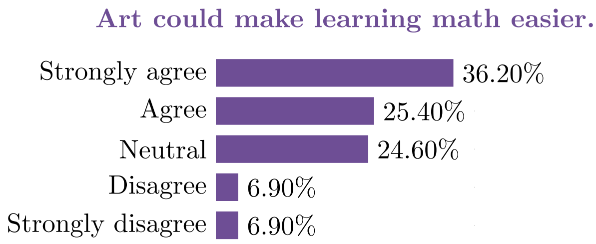

Figure 11: Art and its power to facilitate mathematics learning.

4.3 Students' attitude and perception of the experience

After the piloting phase, a second part of the questionnaire was submitted to the students. It was focused on understanding the impact of the carried out experience on students in terms of initial motivation, technology application in mathematics study and art as a supporting tool in mathematics study.

The 43,85% of the students were very motivated and interested in participating in this new experience because, according to the qualitative feedback, they discovered something unexpected, or, in other words, something they have never thought about.

For this reason, their curiosity pressed them during all the experience collaborating with other classmates by working in small groups.

Their participation was self-evaluated as very good at 55,38%, because they were actively involved in all the three phases, concrete-pictorial- abstract, by guaranteeing the finalisation of the whole tasks without raising any relevant critical situation. This is confirmed by the Q3.3, which provides an overall judgment of their final experience with the 64,62% *Excellent*, 29,23% *Very good*, and 6,15% *Good* without receiving any negative feedback.

However, some of them, in particular, 4.1% (2.7% – Disagree and 1.4% – strongly Disagree) of girls, had had some difficulties, mainly during the third phase, when students were expected to create their art-works. On the base of the results achieved, boys seem to be more comfortable using their creativity and being original in creating their art-works (Table 2).

The data described above also show that most of the students felt free to use their creativity and be original in their art-works production during the third phase (abstract). The characteristic trend revealed is in the positive increase from the first target group to the second one.

Another interesting factor to be underlined is the different attitudes of stu-

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
I don't feel comfortable using this method.	1 0,77%	1 0,77%	10 7,69%	31 23,85%	87 66,92%
It was easy to create my artwork using the mathematical concept studied.	61 46,92%	42 32,31%	15 11,54%	8 6,15%	4 3,08%

Table 2: Discovering mathematics in art. 130 valid cases. The percentage calculation is made on the number of units in the single sub-samples. The first sub-sample (11-13 years old students) includes 29 girls and 29 boys. The second sub-sample (14-16 years old students) comprehends 45 girls and 27 boys.

dents towards the misconception that young people have towards mathematics, considered as something abstract. At the end of the experimentation, students discovered that mathematics is not something abstract but related to a real-life application. Also, there is no relevant difference between girls and boys.

Comparing the answers gathered, despite an insignificant percentage of the *strongly disagree* option in girls' sample, the majority of respondents demonstrate a strong opinion that the method has enabled them to consider the mathematics concepts as more concrete and applicable to reality.

Moreover, the main perception revealed is that the use of the art-works in the mathematics study favours the development of an enhanced learning setting enabling students to enjoy the learning process more to the traditional frontal lessons thanks to the exploitation of different languages, such as visual, graphical, verbal, non-verbal, representational and pictorial.

Concerning the contents learned, the survey and the exercise submission have demonstrated that students developed their knowledge in mathematics and the reasoning process based on applicability, imagination, creativity, and problem-solving skills. They learned to deal with the mathematical problem from different points of view thanks to the problem variations. In fact, using the different art-works to study the same mathematical concept allowed them to analyse it from a different perspective and learn logical reasoning bound to the problem studied. This means that even if the art-works can change the context and the background, the mathematical concept behind these works is always recognised and becomes applicable more easily in everyday life, as shown in the Table 3 below.

The qualitative survey showed that students are usually not used to working together, but in most cases, they study individually.

	Strongly agree	Agree	Neutral	Disagree	Strongly disagree	I don't know
The content was easily understood.	111 85,38%	10 7,69%	6 4,62%	0 0,00%	3 2,31%	0 0,00%
The contents learned seem to me more concrete and practical than before.	112 86,15%	12 9,23%	4 3,08%	0 0,00%	2 1,54%	0 0,00%
The content learned can support my study outside the classroom.	79 60,77%	28 21,54%	20 15,38%	0 0,00%	3 2,31%	0 0,00%

Table 3: Content learned. 130 valid cases.

Therefore, this experience showed that teamwork among students could be more motivating and interesting by making the performances more positive in terms of achieved results.

However, a small percentage of 13,85% preferred the first phase, where they could use applications, like *GeoGebra*, to manipulate math concepts and objects. The reason was that they could use technological tools differently from simple entertainment by making animation and playing around.

Looking at the results achieved to the question *Which phase of the activities did you like least?*, the 15,50% (against 3,10% for the second phase and the 7,75% for the third one) of students indicated the first phase (concrete) addressed to the object and concept manipulation related to mathematics study by using the technological applications. In the first step of the method, they didn't feel free to use their creativity and their work the way they did in the third phase.

Finally, not all the students (7,75%) feel comfortable creating something and using their creativity because they are often not used to proceeding independently without being told exactly what to do.

4.4 Comparison of the results of pre and after the experimentation

Comparing the results of pre and after the piloting experience, the students' attitude (82,31%) towards mathematics has changed. However, a percentage of 12,31% remains "neutral" towards this discipline. Students discovered that different things could be connected with art as well as the beauty of the art can be derived from the mathematical concepts.

However, 91,54% of the sample agree with the desire to use the arts in the study of mathematics, mainly with the proposed method (91,5%) because it could be a good incentive to improve and promote the study of mathematics.

In fact, most of the students (79,23% against the 16,92% with a neutral position) agree that the teaching method, generally used for scientific topics, should be changed and diversified by using both the technology and arts to underline the real applicability of the scientific concepts.

5 Conclusions

This research aimed to analyse and exploit the possibility to find a combination between Western and Eastern approaches in mathematics teaching using the arts. Starting from a study of the strengths and weaknesses of both Eastern and Western learning and teaching approaches, we designed a model exploiting the potentialities of both without leaving cultural differences out.

We adopted Singapore's method defined through three phases (concrete, pictorial, abstract) combined with the Western mathematics teaching implemented mainly in Italian schools.

The complex combination was created by using art.

The major result obtained is that students improved their understanding of the subject and developed their creativity through their art-works. This increased the motivation and interest in the study of mathematics, on the one hand, and meaningful improvement of final student performances, on the other hand. Besides, the use of the art-works in the proposed learning and teaching approach offered students a way to go beyond the pure theory and apply their knowledge in the surrounding world.

Moreover, the research work, through the collaboration of teachers from different subjects like art and mathematics, showed the importance of a novel interdisciplinary and multidisciplinary approach in the school curriculum needed to improve and develop, in this case, mathematics skills.

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