Mathematical model of the movement of a potato body along the surface of a spiral separator

V. Bulgakov[1] and S. Pascuzzi[2,a]

[1]National University of Life and Environmental Sciences of Ukraine, Kyiv, Ukraine; [2]Department of Agricultural and Environmental Science, University of Bari Aldo Moro, Bari, Italy.

Abstract

The potato is grown in over one hundred countries spread over all latitudes, with the most diverse climatic conditions, from the areas near the Arctic Circle to the southern end of the South American continent. The potato is an irreplaceable component of the food tradition of many countries and is the most cultivated species after cereals (wheat, rice and corn). The authors set up and developed a new clod and rubbish spiral separator for potatoes, which is protected by the patent of Ukraine. The operation modalities of this machine are briefly described in the paper. The aim of this study is the development of a mathematical model concerning the movement of a potato body along the surface the spiral separator. In particular, equations are found for determining the coordinates of the position of a potato tuber on the surface of a spiral separator depending on the parameters of the spiral (spiral line) and the size of the tuber. Furthermore, the velocity of tuber movement in the stream formed by the surfaces of two neighboring spirals in the direction of the horizontal, vertical and longitudinal axis are determined. The results of this study will be used in the future to analytically considering the dynamics of movement of a potato tuber on the surface of a spiral separator, taking into account the conditions that exclude its damage.

Keywords: potatoes, cleaning, separator, mathematical model, optimal parameters

INTRODUCTION

The potato is grown in over one hundred countries spread over all latitudes, with the most diverse climatic conditions, from the areas near the Arctic Circle to the southern end of the South American continent (Scott, 2001). The potato is an irreplaceable component of the food tradition of many countries and is the most cultivated species after cereals (wheat, rice and corn) (Bourget, 2004). For most of the twentieth century, Europe was the largest potato producer in the world and currently, while contributing 38.8% to world production, Europe has been overtaken by Asia (42.5%) (FAOSTAT). Conversely, Ukraine in 2017 with over than 22×10^9 kg, was the fifth largest producer of potatoes in the world (FAOSTAT). The narrowest link in the mechanized technologies of potato production remains his harvesting (Petrov, 2004). In this regards, theoretical and experimental studies, as well as numerous tests of various types of potato harvesting machines, have established that high-quality cleaning of potato tubers from clod and plant impurities is possible when a significant mass of soil and other components of the pile (remains of tops, roots, strong soil formations, stones and others) is immediately separated from the tubers within the process of dig of the potato or immediately after its lifted so that a large mass of heap do not move along with the tubers inside the machine (Keijbets, 2008; Ichiki et al., 2013; Manetto et al., 2017).

The systems of soil and impurities removal used on commercially produced potato harvesters do not always provide a high degree of separation of soil and impurities (Zaltzman and Schmilovitch, 1985). This happens, most often, as a result of intensive sticking of moist soil to the surfaces of the separating working body. Technical systems of more intensive impact on the removal of the soil lead to an increase in undesirable damage to tubers (Esehaghbeygi and Besharati, 2009).

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[a]E-mail: simone.pascuzzi@uniba.it

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Many researchers and designers worked on the problem of creating effective and reliable potato heap separators during digging, as well as at fixed potato clearing stations (Al-Mallahi et al., 2010; Bentini et al., 2006). However, despite the large variety of technological processes of cleaning potato heaps during its harvesting, so far there are relatively few studies on the optimization of spiral separators. The authors set up and developed a new clod and rubbish spiral separator for potatoes, which is protected by the patent of Ukraine (Bulgakov et al., 2017). Taking into account this machine, whose operation modalities are briefly described in the paper, the aim of this study is the development of a mathematical model concerning the movement of a potato body along the surface the spiral separator.

MATERIALS AND METHODS

The developed clod and rubbish spiral separator for potatoes

Figure 1 shows the scheme of the set up and developed spiral potato separator, whose technological process is carried out in such a way that the layer dug out of the soil containing potato tubers is conveyed by the feed conveyor (6 in Figure 1) to the cleaning surface formed by the cleaning rollers (1 in Figure 1). Next, the pile is spread on the cleaning surface of the rollers (1 in Figure 1). The coil springs (2 in Figure 1) are mounted on the hubs (3 in Figure 1), which are connected with the drive shafts (4 in Figure 1). The springs (2 in Figure 1), rotating in the same direction, capture and separate the soil particles and plant impurities pushing them outside the cleaner.

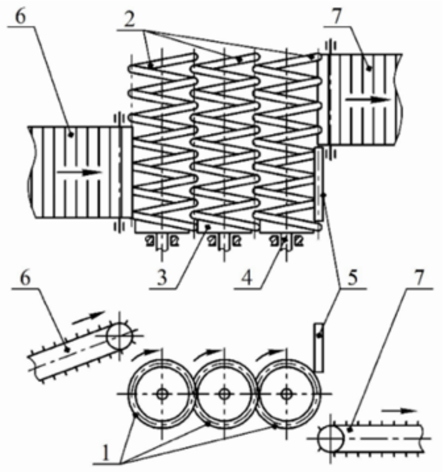


Figure 1. Scheme of the spiral potato separator: 1 – cleaning rollers; 2 – spiral springs; 3 – hubs; 4 – drive shafts; 5 – protective screen; 6 – loading conveyor; 7 – unloading conveyor.

As the spiral springs (2 in Figure 1) are mounted on the hubs (3 in Figure 1) of the cantilever, their ends are able of oscillating movements in longitudinal-vertical planes, so activating the separation of soil and plant impurities through the coils themselves of the springs (2 in Figure 1) and the gaps between the individual springs (2 in Figure 1). Potatoes tubers as solid bodies being on top of the active wave-form surface formed by the coil springs (2 in Figure 1) of the rollers (1 in Figure 1) are involved in a complex movement in both the radial and axial directions. Furthermore, as the coil springs (2 in Figure 1) are mounted with eccentricity on the hubs (3 in Figure 1) and with mutual overlap, the surface on which the potato tubers are located is forced to make oscillating movements of its entire surface with a small amplitude, which contributes to intensive rotations of the potato tubers and consequently to an efficient sifting of soil particles downwards beyond the separator. The protective screen (5 in Figure 1) avoids loss of potato tubers, which are conveyed in the direction of the cantilever end and to the unloading conveyor (7 in Figure 1).

Preliminary aspect for the development of the mathematical model

The movement of a single potato tuber is analyzed, considering its position on the surface of the springs separator. The equivalent scheme of this operative condition is reported in Figure 2. Two spiral cleaning rollers (springs) 1 and 2, installed on the same level, with mutual overlap, whose centers are denoted by O_1 and O_2 are located in the transverse plane. The cleaning springs have the following radii, respectively: spring 1 – R_1, spring 2 – R_2. The distance between the axis centers of the springs O_1 and O_2 is a_W. At the same time, the potato tuber is approximated by a body, whose shape is close to that of a ball, with radius R_k and center, indicated by point C. Consider the case of a potato tuber placed between two spiral cleaning rollers (springs), positioned so that the left side of the potato is in the space between the neighbor coils of the spring 1 (i.e. it is in contact with its two coils) and at the same time, its right side leans on one coil of the spring 2. Springs 1 and 2 rotate around their own rotation axes at the same angular speed ω . The arrows show the rotation directions of springs 1 and 2 (Figure 2). The diameters d_n of the coil of both the springs 1 and 2 is $2r$.

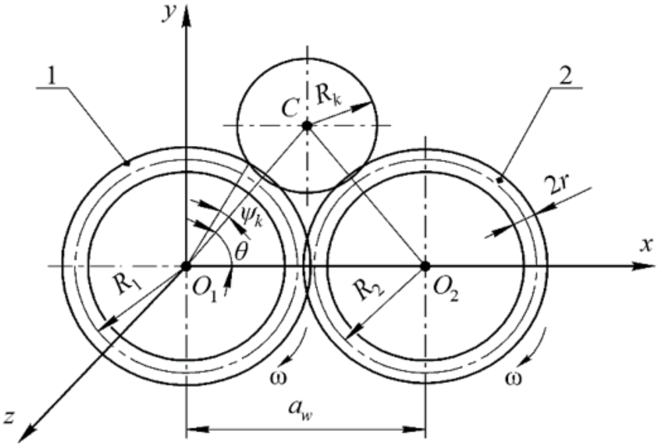


Figure 2. The equivalent scheme concerning the location of the potato tuber on the surface of the spring separator: 1 – the first spring; 2 – the following spring.

A spatial Cartesian coordinate system xO_1yz (Figure 2) is considered, in which: *i*) the axis O_1x is directed along the line connecting the centers of the springs, starting at point O_1; *ii*) the axis O_1y coincides with the longitudinal axis of the spring 1; *iii*) the axis O_1z is perpendicular to the plane formed by the two previous axes.

In order to determine the position of the potato tuber in the adopted coordinate system, the center of the potato tuber (point C) is connected with the centers of the springs O_1 and O_2. The resulting segments have inclinations to the axis O_1x at angles θ. The segment connecting the center O_1 of the spring 1 and the point of contact of the potato tuber with the coils of the spring 1 has a slope to the line O_1C at angle ψ_k.

RESULTS AND DISCUSSION

First, the motion of an arbitrary point on the surface of a spring at an arbitrary time is analytically considered.

In this coordinate system, the parametric equations of a spiral spring in general form, referred to the i-th spring that rotates, for any moment in time, have the following form (Bulgakov et al., 2018a):

$$\begin{cases} x_i = x_{0i} + R_i \cos(\psi_{0i} + \psi_i) \\ y_i = y_{0i} + R_i \sin(\psi_{0i} + \psi_i) \\ \qquad z_i = z_{0i} \pm \frac{S_i \psi_i}{2\pi} \end{cases} \tag{1}$$

where: x_{0i}, y_{0i}, z_{0i} – coordinates of the center of the spring i-th; $\psi_i = \omega t$ – independent angular parameters of the spirals; ψ_{0i} – the initial value of the independent angular parameter ψ_i, which determines the placement of the cross section of the i -th spiral spring at the initial moment of time, i.e. at t=0. S_i - spiral pitch of the spring i-th.

The sign in the third equation of the system (1) determines the direction of the spiral winding: if the spiral has the clockwise winding, then the “plus” sign; otherwise, a “minus”.

A formalized description of the surface of spiral elements is simplified using a special helical coordinate system $\rho O \varphi \psi$, in which the placement of an arbitrary point is determined by the parameters ρ, φ and ψ, (where ρ and φ – respectively the radial and angular parameters of the polar coordinate system in the cross sectional plane; ψ – the angular parameter that specifies the placement of the cross section along the length of the spiral bar) (Bulgakov et al., 2018b; Pascuzzi et al., 2017).

Therefore, for a spiral that rotates at an angular velocity ω around a fixed own longitudinal axis $O_i z$, the following system of equations is obtained after some transformations from the system of Equation 1:

$$\begin{cases} x_i = (R + \rho_i \cos\varphi_i) \cdot \cos(\psi_0 + \omega t) + \rho_i \sin\varphi_i \cdot \sin\gamma \cdot \sin(\psi_0 + \omega t) \\ y_i = (R + \rho_i \cos\varphi_i) \cdot \sin(\psi_0 + \omega t) - \rho_i \sin\varphi_i \cdot \sin\gamma \cdot \sin(\psi_0 + \omega t) \\ \qquad z_i = \frac{S \omega t}{2\pi} + \rho_i \cos\gamma \cdot \sin\varphi_i \end{cases} \tag{2}$$

i=1,2.

The resulting system of Equation 2 describes the movement of an arbitrary point along the surface of one spiral, which rotates around its own axis. These equations and their further transformations make it possible to obtain kinematic parameters that will characterize the motion of the components of the heap, which will be fed to the surface of separator. In this regard, the system of Equation 2 is used to determine the kinematic characteristics in the case of potato tuber on the surface of the spiral separator. In the first approximation the potato is considered having a form close to the shape of a sphere (Figure 3). The radial parameter of its center C is determined with the equation (Figure 2):

$$\rho = R_k + \frac{d_n}{2} \tag{3}$$

where R_k – potato tuber radius; d_n – winding diameter of the coil.

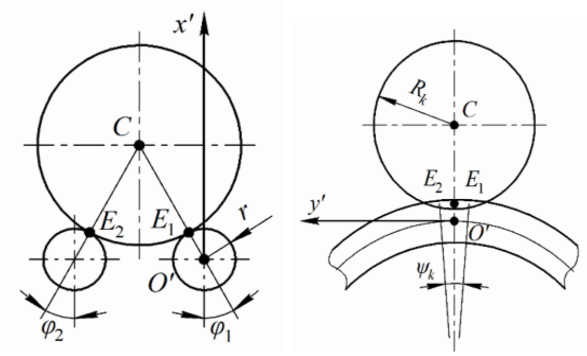
Furthermore, at the point of contact E_1 and E_2 (Figure 3), the normals to the surface of the potato and the surfaces of the coils coincide, and the reactions from the coils come through the center of the potato. After all, the coordinates x_C, y_C, z_C of the center of the potato with a radius R_k, using the parameters ρ, φ and ψ, can be represented both through the parameters of the contact point E_1, and through the parameters of the contact point E_2.

From the third equation of system (2), the axial coordinate z_i, which takes into account the position of the potato tuber on the surface of the separator can be determined with some transformations and the equation to evaluate the axial velocity of the potato tuber is:

$$\frac{dz_i}{dt} = \frac{S}{2\pi}\left(\frac{d\theta_i}{dt} - \omega\right) + \left(R_k + \frac{d_n}{2}\right) \cos\gamma \cdot \cos\vartheta_i \cdot \frac{d\vartheta_i}{dt} \tag{4}$$

The velocity of movement of the potato tuber in the stream formed by the surfaces of adjacent spirals in the direction of the vertical x and horizontal axes y will be determined by differentiating Equation 2 (Bulgakov et al., 2018c). Therefore, after some transformations, the following system of equations allows to assess the speed of movement of the potato along the axes:

$$\begin{cases} \frac{dx_i}{dt} = -(R + \rho_i cos\varphi_i) \cdot sin(\psi_0 + \omega t) \cdot \omega + \rho_i sin\,\varphi_i \cdot sin\,\gamma \cdot cos(\psi_0 + \omega t) \cdot \omega \\ \frac{dy_i}{dt} = (R + \rho_i cos\varphi_i) \cdot sin(\psi_0 + \omega t) \cdot \omega + \rho_i sin\,\varphi_i \cdot sin\,\gamma \cdot sin(\psi_0 + \omega t) \cdot \omega \\ \frac{dz_i}{dt} = \frac{S \cdot \omega}{2\pi} + \rho_i \cdot sin\,\varphi_i \cdot cos\,\gamma \end{cases} \tag{5}$$



a) b)

Figure 3. The equivalent scheme of contact of potato tuber with the surface of the spiral separator: a) view in the transverse plane; b) view in the longitudinal plane.

We further define the effect on the angular coordinates in a special spiral coordinate system of certain geometrical parameters. To do this, equate the second equation of system (2) to zero:

$$(R + \rho \cdot cos\,\varphi)sin\,\psi = \left(R_k + \frac{d_n}{2}\right) sin\,\gamma \cdot sin\varphi \cdot cos\psi \tag{6}$$

After certain transformations you can obtain:

$$tan\,\psi = \frac{R \cdot tan^2\,\gamma(\pi - \psi)}{R + \frac{1}{cos\,\gamma}\sqrt{\rho^2 \cdot cos^2\gamma - R^2(\pi - \psi)^2}} \tag{7}$$

Equation 6 highlights that the angle ψ depends on the geometrical parameters of the coil spring and varies depending on the radial parameter ρ, the radius of the coil *R* and the winding parameter γ. In this case, the angular parameters of the points of contact and the center of the tuber in the polar coordinate system are:

$$\varphi_{1,2} = \pm arcsin\frac{S(\pi - \psi_k)}{2\pi \cdot \rho_c \cdot cos\,\gamma} \tag{8}$$

Equations 7 and 8 allow the exact definition of parameters ψ and φ accordingly.

You can also get simpler, but approximate expressions for determining the specified parameters. To do this, select the arc δl on the coil, which corresponds to the angle ψ on the projecting plane. Its projection on the plane of the cross section is $\delta l \cdot cos\gamma$. On the other hand, the projection of the arc on the plane can also be expressed as $(\psi \cdot R)$. Then $\delta l \cdot cos\gamma = \psi R$. For small values of angles, the equality is $sin\,\psi = \psi$, therefore, from the last relation you find the sine of the angular parameter ψ:

$$sin\,\psi = \frac{\delta l \cdot cos\,\gamma}{R} \tag{9}$$

Furthermore, taking into account the inclination of the winding, it is not difficult to conclude that $\delta l = S \cdot sin\gamma$, therefore:

$$sin\,\psi = \frac{S \cdot sin\gamma \cdot cos\,\gamma}{R} = \frac{S \cdot sin\,2\gamma}{2R} \tag{10}$$

On the other hand, on the basis of geometric considerations, it should be noted that:

$$\begin{cases} sin\,\varphi = \frac{S}{d_n + 2R_k} \\ cos\,\varphi = \frac{\sqrt{d_n^2 + 4d_n R_k + 4R_k^2 - S^2}}{d_n + 2R_k} \end{cases} \tag{11}$$

We investigate the dependences of the angles ψ and φ accordingly, on the radius of the coil elements with a pitch of winding *S*=40 mm and a radius of potato tuber R_k=25 mm, calculated from approximate dependencies and exact ones, by plotting for this graphical dependencies. The graph of the dependence of the angular parameter φ from the radius of the coil (Figure 4) shows that its value can be calculated either by Equation 8 for precise determination, or by Equation 11 for approximate determination. This is especially evident for the case when the radius of the coil is greater than 60 mm. For calculation, it is more convenient to use, with sufficient accuracy for practical determination (Equation 11).

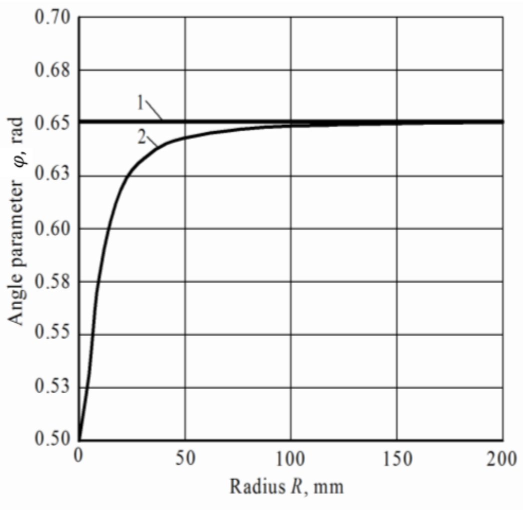


Figure 4. The dependence of the angle *φ* from the radius R of the coil elements with the step of winding S=40 mm and the radius of the tuber R_k=25 mm, calculated through: 1 – the exact Equation 8; 2 – the approximate dependencies (Equation 11).

Analysis of the graphs of the dependence of the parameter ψ from the radius *R* of the spiral (Figure 5) indicates that its determination is possible for small radius only by Equation 7 for accurate determination. With a radius of coils greater than 120 mm, it is possible to use the approximate Equation 10.

So, as a result of considering the kinematics of the body movement of a potato tuber over the surface of a spiral (spiral roller), equations of trajectory, the values of the speed of oscillating motion were obtained, was conducted study of the influence of design parameters (primarily the spiral radius *R*) on the angular parameters, which determine the placement of the potato tuber in the spiral coordinate system.

The movement of the potato tuber along the surface of the spiral roller also occurs due to the backing of the mass, which is constantly fed to the separating working organ. To ensure a continuous separation process and avoid jamming, the layer of potato tubers along the length of the entire spiral separating working body must correspond to the thickness of one tuber.

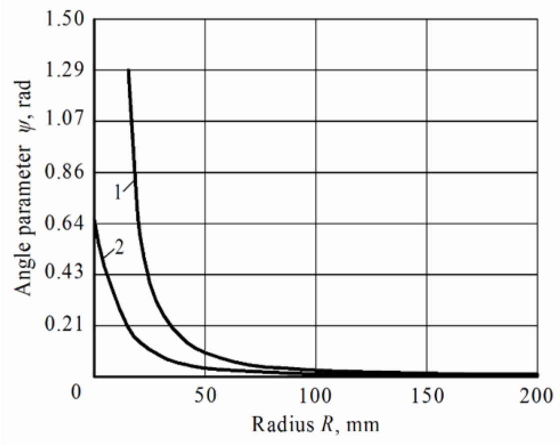


Figure 5. The dependence of the angle ψ from the radius of the spiral elements with the step of winding S=40 mm and the radius of the tuber R_k=25 mm, calculated through: 1 – the exact Equation 7; 2 – the approximate dependencies (Equation 10).

CONCLUSIONS

The following conclusions can be drawn from the study:

- A mathematical model has been developed for the movement of a single potato tuber, as a result of which new kinematic characteristics have been obtained in the event of its falling on the surface of a spiral separator.
- Expressions are found for determining the coordinates of the position of a potato tuber on the surface of a spiral separator depending on the parameters of the spiral (spiral line) and the size of the tuber.
- The velocity of tuber movement in the stream formed by the surfaces of two neighboring spirals in the direction of the horizontal, vertical and longitudinal axis are determined.
- The effect on the angular coordinates in a special helical coordinate system of some geometrical parameters of the spiral and the size of the tuber was investigated. In particular, graphical dependences of the angles ψ and φ accordingly, on the radius of the coil elements R, the pitch of winding S and the radius R_k of the potato tuber according to exact and approximate formulas, are obtained.
- The results of this study will be used by us in the future to analytically considering the dynamics of movement of a potato tuber on the surface of a spiral separator, taking into account the conditions that exclude its damage.

ACKNOWLEDGMENTS

The authors equally contributed to the present study.

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