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The renovation of Santa Maria di Collemaggio Basilica: the intervention on the nave's columns

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Abstract
The earthquake of L'Aquila in 2009 caused the partial collapse of the S. Maria di Collemaggio Basilica. The XIII century nave's columns showed serious cracking throughout their height. Therefore, a retrofitting intervention was planned with the purpose of protecting these valuable elements from future additional damages.
The intervention designed involved the complete unloading and reloading of the columns. To avoid additional damages, it was necessary to verify that the procedure would not alter significantly the existing stress state and would not lead to the occurrence of tensile stresses greater than the materials' tensile strength. Therefore, a 3D FE model of the colonnade and the supported wall has been realized and a construction stage non-linear analysis implemented. The geometry was modeled through the acquisition of the results of a laser scanner survey in order to reflect faithfully the Basilica's irregularities (lack of verticality of the wall, eccentricities, different dimensions from one column to the other). Moreover, elastic-brittle constitutive laws were assigned to the model's materials to reproduce the intrinsic non-linearity of the masonry.
The analysis results in terms of stresses and displacements were attentively analyzed. Furthermore, a comparison between the numerical results and the values measured during the intervention has been executed. An acceptable deviation between the values was observed and, therefore, the reliability of the model was assured.

Keywords: retrofitting, masonry columns, numerical analysis, construction stage

1. Introduction
Although they showed a good earthquake resistant capacity and did not collapse, the two lines of octagonal columns that separate the nave from the aisles of the Basilica di Collemaggio have been seriously damaged during the 2009 Earthquake. These columns have great historical and artistic values since they date back to the XIII century. Thus, a retrofitting intervention was designed with the two purposes of, first, enhancing the seismic structural behavior of the columns and, second, protecting these valuable elements from future additional damages.
First, the damages, the geometry, the materials and the behavior of the columns under seismic event have been investigated and analyzed.
The investigation campaign has led to a confident factor CF = 1.5, obtained through the following partial confidence factors:

- deep geometrical survey (C_{F1} = 1);

- extended inspection on details (C_{F2} = 0.06);

- limited investigations on materials' mechanical properties (C_{F3} = 0.06);

- avalaibility of geotechnical data and foundations, limited investigation on soil (C_{F4} = 0.03).

1.1 Geometry and material properties

The geometry has been detected through a laser scanner survey to reflect faithfully the Basilica's irregularities (lack of verticality of the wall, eccentricities, different dimensions from one column to the other). The dimensions of the columns vary irregularly and therefore it was necessary to analyze them one by one. The sections' diameter varies around an average value of 116 cm while the basement's characteristic dimension belongs to a range between 150-170 cm. The columns are approximately 4 m high, not considering the capital and the basement, whose average heights are respectively of 0.53 m and 0.57 m. The pointed arches' rise is approximately 3.95 m.

Regarding the materials' properties, the columns, the basement and the capital are made of a local sedimentary rock called '*breccia aquilana'* that showed a considerable high value of compressive strength equal to f_m = 66.68 MPa. The earthquake-resistant capacity of the columns is likely due to the good mechanical properties of the stone. It is worth mentioning, however, that the columns are not homogenous. Firstly, they are not made of one sole material. In fact, original ashlars are made of *breccia aquilana*, ashlars replaced during restoration interventions are made of Maiella or other local stones and mortar has been used to fill the cracks caused by local material detachments occurred over the past years. Moreover, the columns turned to have an inner core made of a low-quality incoherent material. The discontinuities in the material along with the complete collapse of the transept area are for sure one of the causes that led to the damage of the colonnade.

The masonry of the nave's walls supported by the two lines of column is characterized by poor mechanical property with limited compressive strength f_m = 2 MPa.

Regarding the tensile strength, a value equal to 10 per cent of the compressive strength is taken for both the material considered, that is respectively 6 MPa for the *breccia aquilana* stone constituting the columns and 0.2 MPa for the masonry of the nave's walls.

Finally, it should be mentioned that in the 70s, during the restoration of Superintendent Moretti, the two last octagonal columns next to the transept were completely rebuilt using concrete mortar.

1.2 Survey on the damages

The central columns of both the colonnades were sensibly more damaged (close to collapse) than the external columns. In fact, if under a horizontal force parallel to the nave all the columns exhibit a shear-type behavior and are almost uniformly stressed, in the out-of-plane direction the behaviour of the columns is not uniform. In particular, internal columns happen to be the most stressed. The out-of -plane displacement of the supported wall is, in fact, larger in the middle than at the edges where the wall is constrained by the façade and the transept-frame.

Different types of damages have been identified:

- relative sliding between adjacent ashlars, Fig. 1a;
- local detachment of material and opening of pre-existent cracks, Fig. 1b;
- stone blocks expelled or crushed in the proximity of the basement due to both compression and bending, Fig. 1c;
- vertical cracks due to excessive compression, Fig. 1d.

1.3 The retrofitting intervention

The damages' survey and the analyses implemented have pointed out the necessity of a retrofitting intervention on the nave's columns of the Basilica, even if they did not collapse. Although the excellent quality of the *breccia aquilana*, characterized by a considerable high compressive strength, the presence of cracks and the fact that the material is not homogenous make it difficult to assure an adequate response of the columns to seismic events. Moreover, the characteristics of the columns' inner core are unknown. The most likely hypothesis is that they are made of an incoherent material with low compressive resistance.

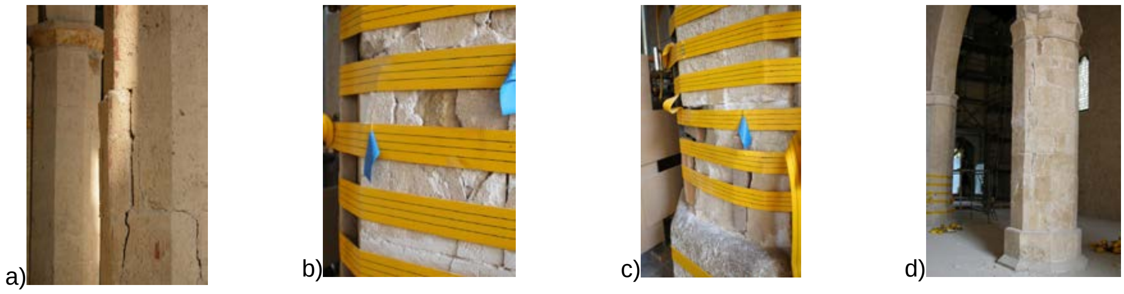


Fig. 1 Damages on the columns – a) relative sliding between ashlars b) local material detachment c) damage in the proximity of the basement d) vertical cracks

All the columns will undergo restoration except for the two columns next to the transept which were rebuilt in the 70s and don't' exhibit serious damages, Fig. 2a.

Two kinds of retrofitting intervention have been designed depending on the state of damage. Concerning the most damaged columns (central columns), the restoration involves the complete disassembly and reassembly substituting the most seriously damaged stone blocks and the columns inner core. The cracks affecting the lateral columns will be instead restored by local substitution of the most damaged stone blocks and injections of mortar, avoiding the complete disassembly of the columns. Both the types of intervention, however, require the complete unloading of the columns and to this aim a suspension system able to support the wall's nave on behalf of the column during its restoration has been designed. The unloading and reloading procedure had to meet a fundamental requirement in order not to cause additional damages: altering as less as possible the pre-existent state of stress avoiding the occurrence of tensile stresses greater than the material tensile strength.

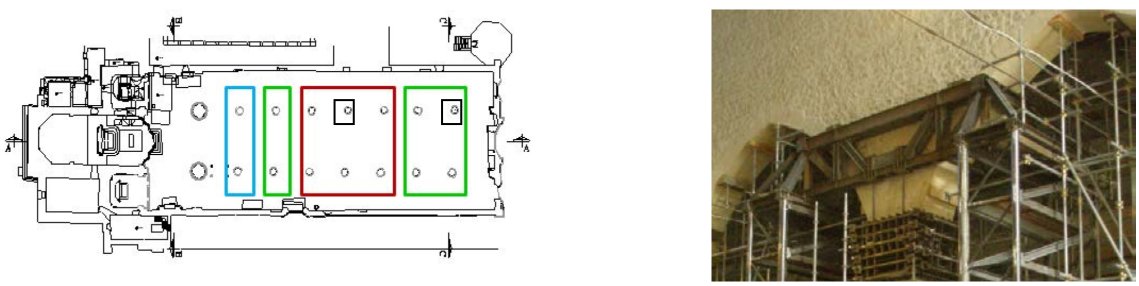
The designed procedure is based on the use of a friction device made of steel ("clamp") which is assembled just above the stone capital in order to clamp the end of the two arches converging in the column, Fig. 2c. The space between the steel plates constituting the clamp and the masonry is filled with mortar to guarantee a uniform distribution of stresses. Through pre-stressing bars, the clamp is tightened around the wall and the compressive force applied makes friction forces arise between the clamp and the masonry. The compression applied must be enough to prevent any sliding between the two surfaces but compatible with the masonry compressive strength to avoid damage. The clamp is then vertically connected through four steel bars to a provisional steel truss portal. The load carried by the column is progressively transferred to the steel portal by applying tension forces to the connecting bars using hydraulic jacks. Once the column is completely unloaded, the joint between the superior edge of the capital and the wall above is cut using diamond wire and the retrofitting intervention can begin. The restored column is finally reloaded by progressively releasing the tension in the vertical bars. The whole procedure of unloading-reloading is attentively monitored: sensors are placed at different points of the column, arches and clamp to control the displacements step by step. It is worth mentioning that the unloading-reloading procedure is developed for one column a time and with load steps corresponding to 100 bars pressure in each of the hydraulic jacks.

This procedure was chosen among other proposals because it does not alter the static scheme of the colonnade. The reaction transferred by the column to the wall is, in fact, punctually substituted by the tensile forces inside the four bars connecting the clamp to the supporting portal. However, to ensure that the unloading-reloading would not affect the stress state exceedingly and to analyse the eventual variations of the stress field, a construction stage non-linear analysis reproducing the entire procedure has been implemented on a 3D FE model of the nave's wall.

2. The 3D FE model

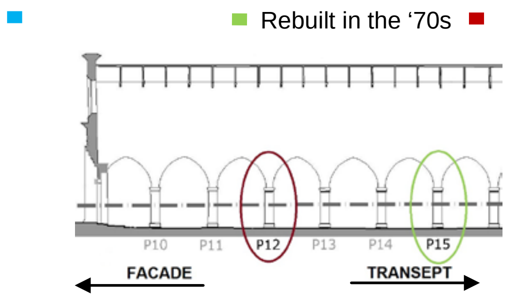
The whole process of unloading-reloading has been reproduced stage by stage, in order to have a complete control on the procedure. Since the columns' substitution procedure affects significantly only the nave's walls, a partial model of the Basilica has been realized, adequately constrained and loaded in order to take into account the effect of the remaining part of the building.

a)



Lateral columns

Central columns



b)

Fig. 2 Retrofitting intervention – a) columns analyzed b) subdivision of the columns depending on the state of damage c) unloading procedure: the clamp and the steel portal

In the following, two representative columns are analyzed: the 12th and the 15th columns, Fig. 2b and Fig. 2a.

The 12th column is sited in the middle of the colonnade and therefore it experiences the largest out of plane displacements under the horizontal forces that necessarily arise as a consequence of the wall's marked out of plumb. Moreover, it was seriously damaged by the earthquake and therefore, it requires to be completely disassembled and reassembled.

The 15th column, instead, is the closest to the collapsed transept area, not considering the very first one rebuilt in concrete during the Moretti's intervention. Because of the collapse of the transept, the adjacent wall is subject to larger in-plane displacement because the last arch's thrust is not equilibrated. Furthermore, as already mentioned, the column will undergo to a different renovation intervention with respect to the 12th column, consisting only in mortar injection and 'sew-unstitched' intervention on the most damaged areas. The 15th column was, in fact, less affected by the earthquake.

2.1 Geometry and boundary conditions

A partial model of the Basilica has been realized through the software Midas FEA, reproducing the left nave's wall and colonnade. It was essential to capture the real shape of the wall, with all its irregularities, in order to have a faithful model able to reproduce the real behaviour. Therefore, the geometry has been imported through the acquisition of a 3D point-cloud, collected through a detailed laser scanner survey. The colonnade, connected to the façade on one side, is completely unrestrained on the right side, where it was connected to the transept frame. The area, in fact, collapsed as a consequence of the large pillars' structural failure.

Two clamp's models have been reproduced in order to suit the dimensions of both the columns 12th and 15th. The smaller clamp corresponds to the 12th column while the bigger one to the 15th one. The clamps have been sited around the wall about 100 mm above the capital superior edge, as prescribed in the intervention design, Fig. 3b.

The mortar layer between the steel clamp and the masonry has been reproduced following the prescriptions on maximum and minimum thicknesses (9.5 mm, 2 mm).

The steel portal has been then positioned, trying both to centre the column and have the bars connecting the portal to the clamp as vertical as possible.

Finally, regarding the meshing operation, a mesh size of 100 mm has been generally adopted. A mesh refinement was necessary, instead, on the clamp, on the mortar and in the adjacent area. The thicknesses of either the clamp's plates or the mortar (14 mm and 20 mm respectively) require a reduction of the mesh size. However, in order to contain the computational cost and to avoid an exceedance slowing down of the analysis, the minimum mesh size has been set to 40 mm.

Concerning the geometrical and static boundary conditions, the ones described below have been assigned, Fig. 3a:

- fix-ends at the bottom of the basements of the columns. The deformability of the Basilica's pavement has been neglected;
- hinges applied on the wall's lateral surface corresponding to the façade to constrain horizontal displacements in both in-plane and out-of-plane directions. The aim is to reproduce the restraining effect of the façade on the wall;

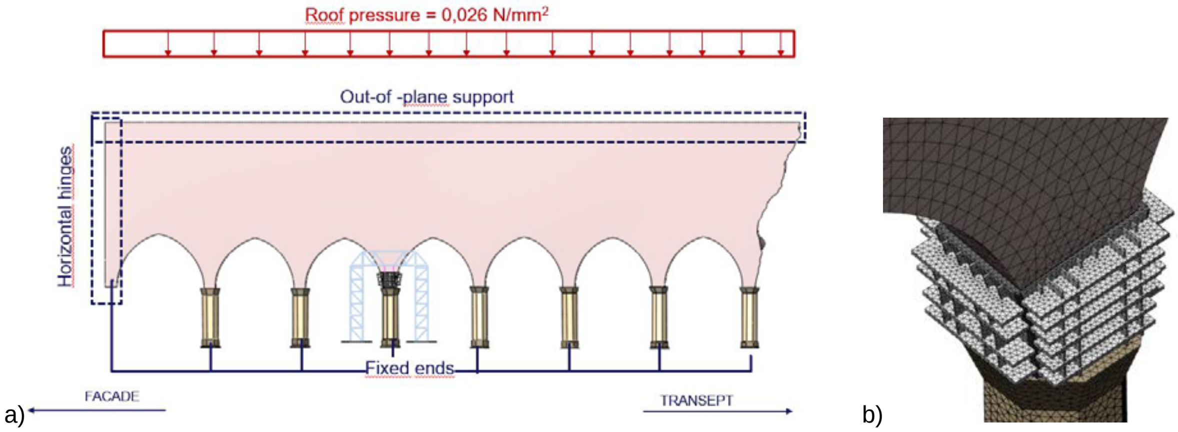


Fig. 3 3D FE model of the colonnade – a) geometry and boundary conditions b) detail of the model of the clamp sited above the capital

- a set of simple supports, applied to the wall's top edge, constraining displacements in the out-of - plane direction. The supports reproduce the effective restraining action of the roof.
- a pressure load reproducing the load transferred by the roof to the nave's wall has been applied to the wall's upper edge.

2.2 Materials

Both the masonry of the nave's wall and the *breccia aquilana* stone constituting the columns are characterized by a strongly different behaviour in tension and compression. To take into account the non-linearity, elastic-brittle constitutive laws have been assigned to the materials.
The columns are modeled with two different materials: *breccia aquilana* in the outer band and masonry in the inner core. Since the characteristics of the columns' inner cores were unknown before the restoration intervention took place, similar properties to the ones assigned to the wall's masonry have been adopted.
Two different materials are also assigned to the supported wall. The wall is, in fact, made of chaotic masonry with almost no tensile strength while the arches are made of regular stone blocks of *breccia aquilana*. The model reproduces the material variation as displayed in Fig. 4.
Finally, the respective classes of steel have been assigned to the clamp, the bars and to the portal.

2.3 Construction stage

The purpose of the analysis is to reproduce the stress and displacement field of the Basilica's elements throughout the whole columns' unloading-reloading procedure. Therefore, a construction stage analysis has been performed, taking into account all the main phases historically involved in the retrofitting intervention. It was fundamental, in fact, to study the evolution of stresses and strains with respect to the existing configuration of the structure. The sequence of the construction phases is summarized in the list below.

1. Existing configuration: the first stage is representative of the structure's state after the earthquake, before the retrofitting intervention. The structure is, at this stage, loaded only by its self-weight.
2. Assembly of the clamp: the second stage represents the positioning and assembling of the clamp. The clamp and the mortar layer are placed above the column's capital. No additional load or boundary condition are assigned. The software computes automatically the new total weight, comprehensive of the weight of the clamp and the mortar.

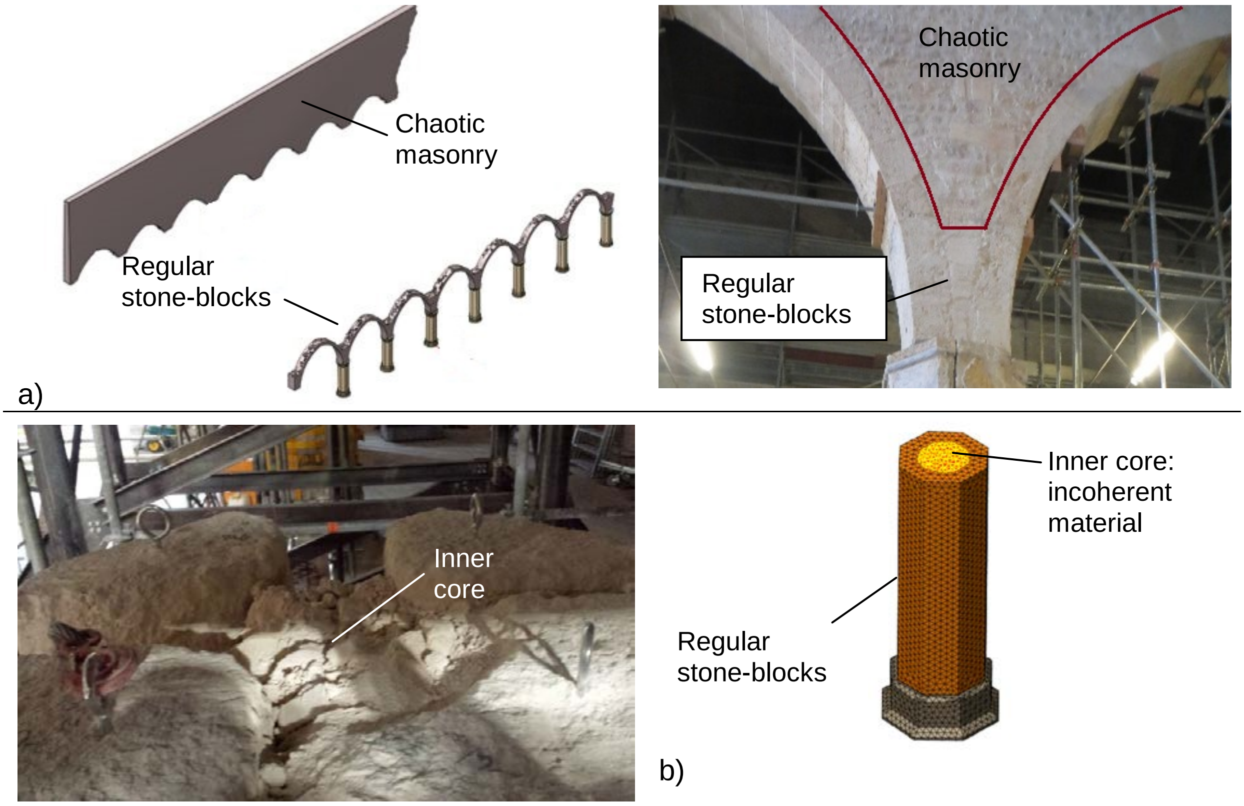


Fig. 4 Materials - a) different kinds of masonry b) presence of an inner core of incoherent material inside the column

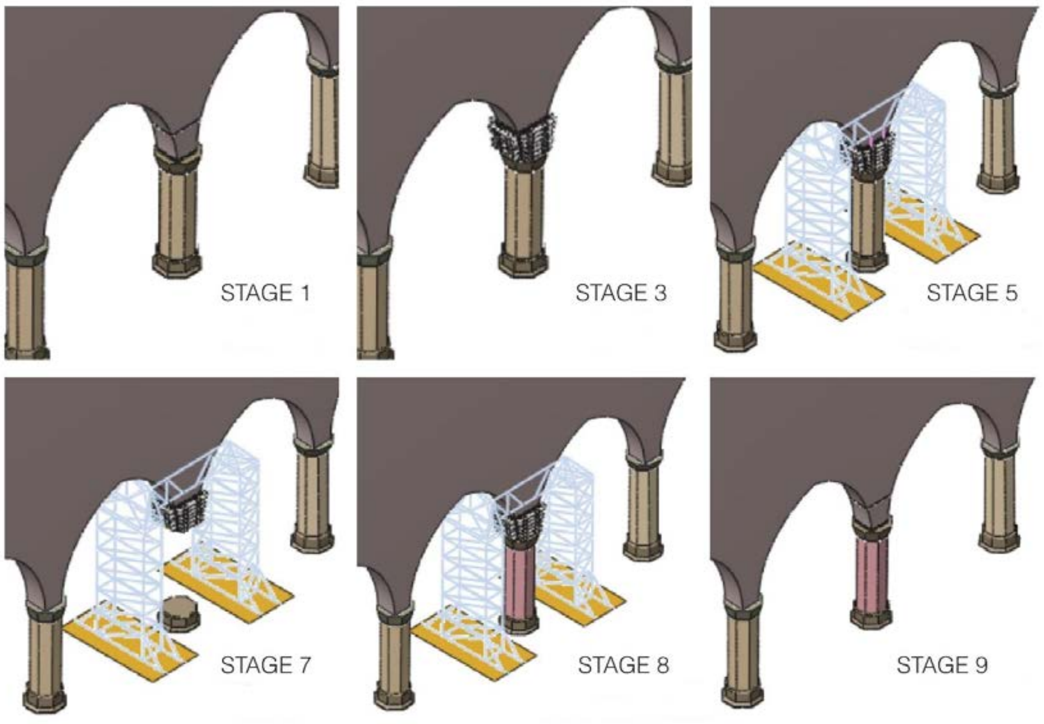


Fig. 5 Most significant stages of the unloading-reloading of the 12th column

3. Clamping: compressive forces are applied to the clamp to simulate the tensioning of the pre-stressing bars.
4. Assembly of the support portal: the steel portal at this step is not connected to the clamp and therefore bears only its self-weight. To avoid damages to the Basilica's pavement, a layer of sand will be interposed between the portal's foundation and the pavement with the aim of redistributing the stresses. Therefore, a spring bed with adequate stiffness has been assigned to the portal foundations to reproduce the sand behaviour and deformability.

5. Column's unloading: the stage reproduces the column's unloading procedure. As explained in the previous chapter, the load is transferred from the column to the portal by tensioning the bars connecting the portal to the clamp using hydraulic jacks. In the model, the column is completely unloaded when the vertical reactions of its basement equal the column's weight.
6. Detachment of the column from the above wall: once the column is completely unloaded the joint between the capital and the wall above is cut using diamond wire and the disassembly may begin. To reproduce this step in the model, the solid representing the wall has been trimmed just under the clamp. This way, a new solid shaped like a parallelepiped with a thickness equal to the distance between the clamp's inferior edge and the capital's superior edge (100 mm) has been created. The cut will be simulated by inactivating the mesh set corresponding to this solid and thus, the column will be detached from the wall above.
7. Retrofitting intervention: regarding column n°12 the model reproduces the complete disassembly. Moreover, the 12^{th} column restoration is reproduced in the model by modifying the mechanical properties of the inner core, in particular, the compressive strength changes from 2 MPa to 40 MPa. The retrofitting intervention of column n° 15, instead, does not involve the complete disassembly but only mortar injections and local 'sew-unstitched' intervention and therefore, it has been decided not to modify the initially modelled column. For the sake of completeness, it should be mentioned that the Italian code allows considering an increment of the mechanical properties of the material due to this kind of intervention, §Table C8A.2.2 Circolare n°617. However, in the present analysis, it has been decided to neglect this beneficial effect.
8. Reloading of the restore column: once the retrofitting intervention is completed the column is reloaded. In construction site, this phase is realized by the simultaneous unloading of the vertical bars connecting the clamp to the portal and the loading of four flat jacks placed in the space between the capital and the wall. Once the load is completely transferred to the column, the cut is filled with mortar, the flat jacks are removed and the supported wall is again connected to the capital.
9. Removal of the suspension system (clamp, mortar and steel portal): the last stage consists of the removal of the clamp, the mortar layer and the whole supporting steel structure. The model is constituted by the same elements of Stage 1 (except for the restore 12th column) but the stress and displacement field will be different because of the load history and the rehabilitation intervention it has undergone.

3. Results

The results of the analyses are highly satisfying and confirm the feasibility of the retrofitting intervention proposed. First of all, the stresses occur during the whole unloading-reloading phase are far below the material strengths for both the columns analysed and therefore no cracks are developed. The maximum tensile force recorded occurred inside the *breccia aquilana* masonry and is about 1.3 MPa for both the columns. As already mentioned the value is compatible with the mechanical property of the material (f_t = 6 MPa).

The stress state inside the column during the whole procedure has also been analysed. Again, both the columns analysed gave comparable results, therefore just the results corresponding to column n°12 will be illustrated in the figures below. Fig. 8 shows the variations of the stress distribution from the existing configuration to the unloading phase and after the complete detachment of the column from the nave's wall. As it was expected, the maximum compressive stress is recorded in correspondence of the stiffer outer band (where the stone blocks are placed). A decompression of the column is observed in correspondence to the unloading phase, Stage 5.

Concerning the masonry enclosed inside the clamp, it has been verified that the compressive forces applied were not exceeding the material resistance. As already mentioned, the force applied by the clamp to the masonry must guarantee the occurrence of an adequate friction force but must not overcome the material compressive strength. Fig. 7 displays the principal compressive stress distribution at the existing configuration, after the application of the forces and at the final configuration after the removal of the clamp. The maximum compressive stress recorded is around 11.8 MPa, largely below the maximum value allowed f_m = 66.68 MPa.

The values of the tensile force to be applied to the four bars connecting the clamp to the portal to reach the complete unloading of the two columns, "unloading load", have also been computed. This load corresponds to the sum of the tension forces that are in the bars when the vertical reactions at the basement of the column equal the column's weight. In fact, in such a case, the column is bearing only itself and does not carry the nave's wall anymore. The "unloading load" of column n°12 is 2066 kN while the value assessed for the 15^{th} column is 2228 kN.

Moreover, to study in detail the unloading phase, the vertical base reactions of both the columns and of the support portals have been plotted as a function of the tension force in the vertical bars. As it is shown in Fig. 9, it emerged that the trends of the base reactions are linear: while the column's base

reactions decrease the ones of the steel portal increase. However, it appears that the portal's total vertical base reaction overcomes the decrease in the column's base reaction of 473 kN in the case of column n°12 and 502 kN for column n°15. This apparent contradiction is due to the fact that also the other columns are partially unloaded by the procedure. This is evident in the graphs reported in Fig. 11: the vertical base reactions of the columns immediately next to the one involved in the retrofitting intervention decrease significantly. The procedure has instead negligible effects on the columns sited far away from the column unloaded.
Finally, the displacement field has been studied. In fact, in addition to assuring that the stress distribution does not vary exceedingly, it is important to verify that no remarkable displacements occur during the unloading-reloading intervention. The trends of the displacement during the unloading phase for different control points have been plotted, in particular:

- vertical displacement of the keystones of the arches adjacent to the column;
- vertical and horizontal displacements of the masonry enclosed in the clamp;

Fig. 12 and Fig. 13 show the displacements' trends as a function of the tension in the vertical bars. It can be seen that the maximum displacements are of the order of one-tenth of a millimeter, perfectly compatible with the requirement.

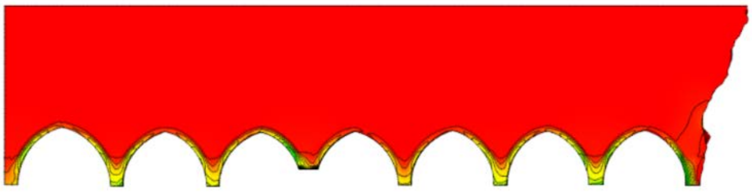


Fig. 6 Compressive stress distribution in the nave's wall at Stage 5. The maximum value is equal to 0.7 MPa

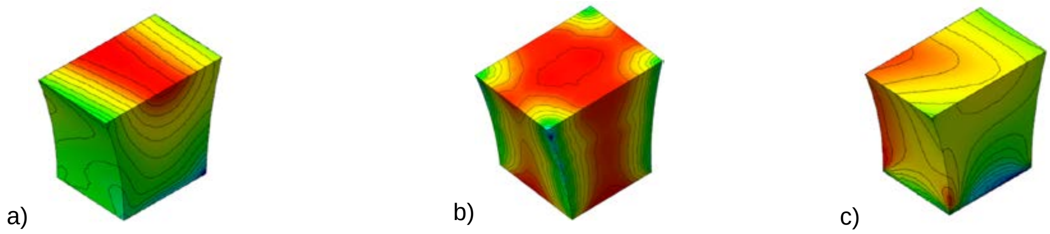
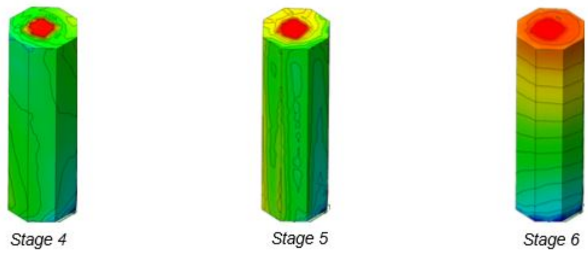


Fig. 7 Principal compressive stresses inside the masonry enclosed in the clamp at existing configuration (a), under the compression of the clamp (b) and at the final stage (c)



Stage	Average compressive stress
4	-1,9 MPa
5	- 0,4 MPa
6	-0,12 MPa

Fig. 8 Stress distribution inside the column n° 12 during the retrofitting at Stages n° 4,5 and 6 and relative average values of compressive stresses

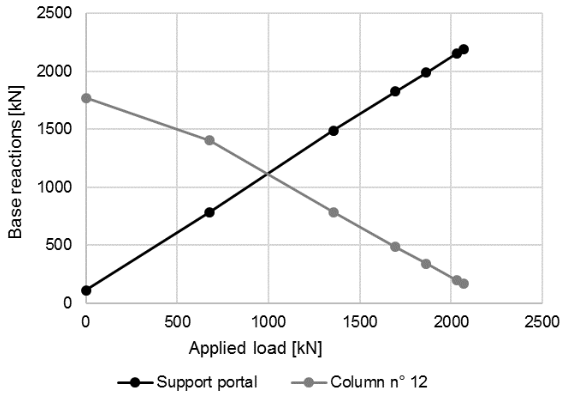


Fig. 9 Trends of the base reactions of both the column n° 12 and the portal

Pressure [bar]	N_{tot} [kN]	Column n° 12 Base reactions [kN]	Support portal Base reactions [kN]
610	2066.91	165.00	2186.63
600	2033.03	194.60	2153.13
550	1863.61	340.41	1988.13
500	1694.19	485.13	1824.37
400	1355.35	783.34	1486.93
200	677.68	1402.79	785.99
0	0	1767.91	109

Fig. 10 Values of the base reactions and the corresponding applied load. The trends are plotted in Fig. 9

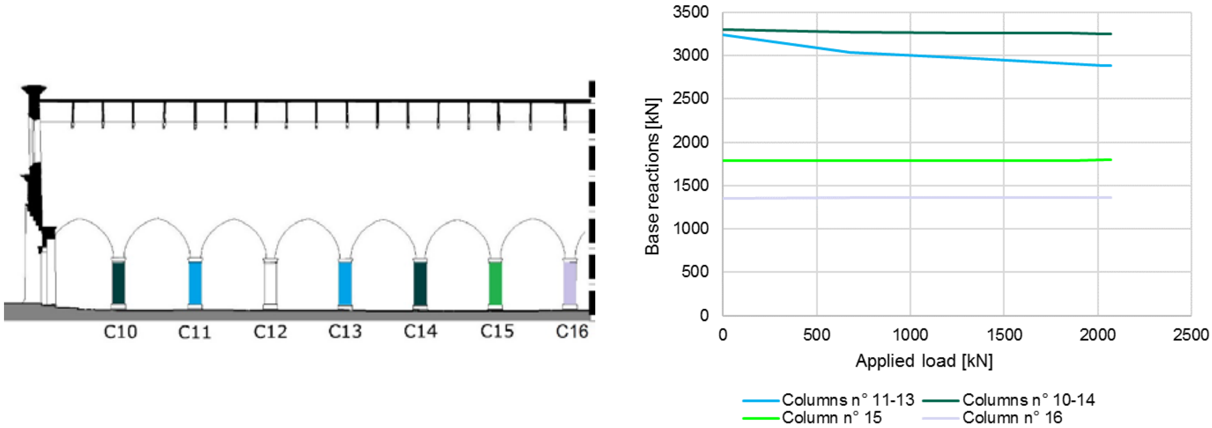


Fig. 11 Vertical reactions at the basements of the columns adjacent to column 12th. It can be observed that the columns immediately next to the 12th column are significantly unloaded (light blue decreasing trend) while the other are less affected (horizontal trends)

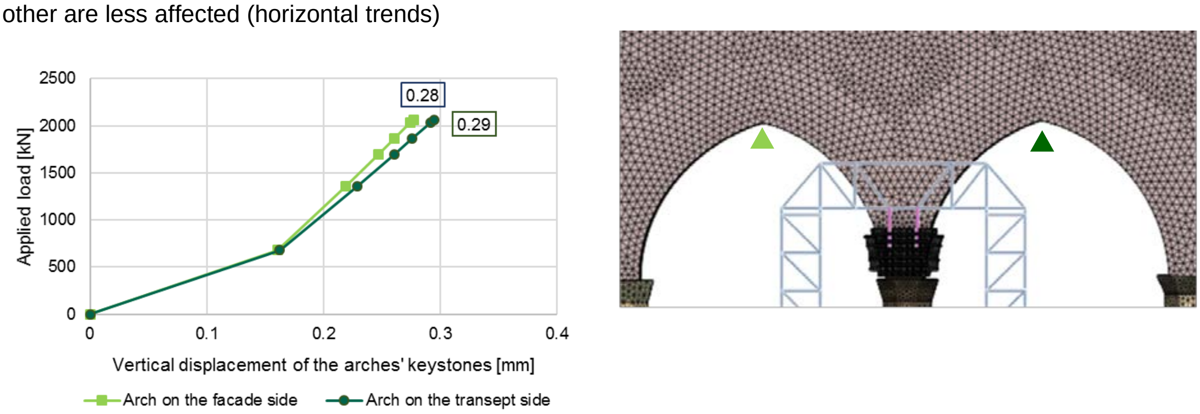


Fig. 12 Vertical displacements of the arches' keystones during the development of the intervention. The behaviour of the arches is perfectly symmetrical and the maximum displacement is 0.29 mm

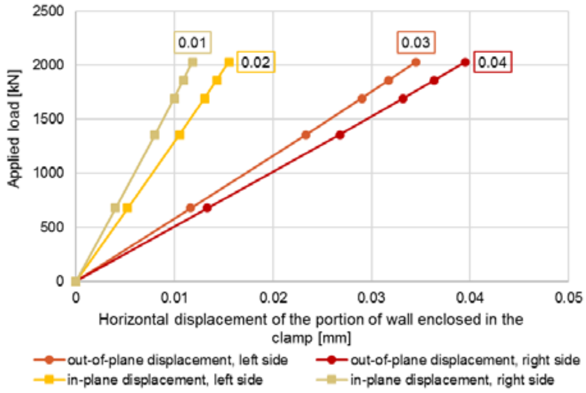


Fig. 13 Horizontal displacements in both in-plane and out-of-plane directions of the portion of wall enclosed in the clamp. The maximum value reached is 0.04 mm

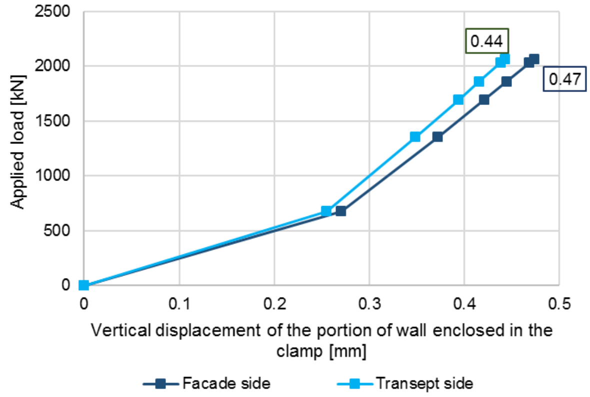


Fig. 14 Vertical displacement of the portion of wall enclosed in the clamp. The maximum value is equal to 0.47 mm

4. Comparison between numerical results and measured values

A comparison with the measurements recorded during the unloading procedure of the 12th column has been performed. The trend of the vertical displacement of the portal's crossbeam during the unloading of the column has been chosen as the parameter to be compared. The load steps (corresponding to 100 bars pressure in each hydraulic jack) have been reproduced in the model by incrementing progressively the axial forces in the bars. As it can be seen in Fig. 15, the results obtained are satisfying. In fact, the two curves almost coincide and the mean error is around 3%. Furthermore, the trend is approximately linear, as it was expected.

It is worth mentioning that the values of displacements are not due only to the deflection of the crossbeams but include the vertical displacement of the portal's foundation on the sand layer below.

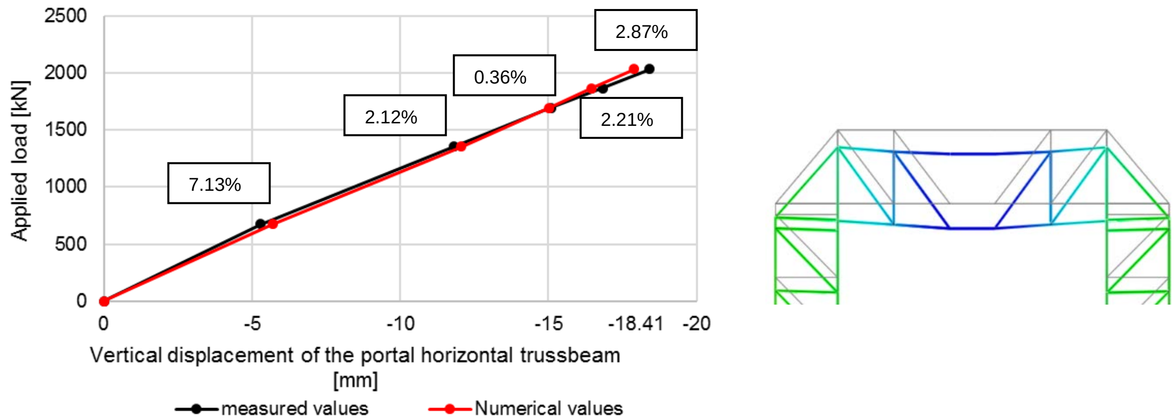


Fig. 15 Comparison between vertical displacements of the portal's horizontal truss beam. The errors are acceptable and the maximum value of displacement recorded is 18.41 mm

Finally, also the comparison of the "unloading load" provided an interesting result. In fact, the value estimated through the model was 2066 kN and the effective value measured in construction site was attested around 2000 kN.

5. Conclusions

The complexity of the intervention consists of working with a historical building already seriously damaged by the recent strong seismic events. The procedure, in fact, should both preserve the artistic value of the structure and concurrently improve the seismic behavior. Particular attention has to be drawn to the variation of the stress field of the masonry wall, during the whole intervention. In fact, masonry is characterized by almost no tensile strength and therefore, the occurrence of eventual tensile stresses could easily lead to the development of additional cracking. It follows that the restoring intervention has to be performed altering as less as possible the existent static scheme. To verify that the intervention designed met the requirements, a 3D FE model, with a geometry acquired from a laser scanner survey, has been implemented. The material non-linear behaviour has been included through the assignment of elastic-brittle constitutive laws. The results of the construction stage analysis executed, returned satisfying results in terms of both stresses and displacements. In fact, the stress values resulted to be always smaller than the ultimate strengths of the materials and the displacements of the supported wall negligible throughout the whole procedure. Moreover, a comparison with the values measured during the unloading procedure of the 12th column has been performed and the average error found is of the 3.5%.

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