# JOURNAL OF ACADEMY OF BUSINESS AND ECONOMICS TM

**EDITOR-In-CHIEF** 

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A Publication of the International Academy of Business and Economics®

### A SURVEY ON MACROECONOMIC DATA IN THE EUROZONE AND A CONTROL DASHBOARD MODEL BASED ON THE KAM AND NEKHOROSHEV THEOREMS AND THE HÉNON ATTRACTOR

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dx.doi.org/10.18374/JABE-21-3.6

### ABSTRACT

Starting from the examination of the main macroeconomic parameters that have characterized the structure of the Eurozone in the last decade – and their systemization – our aim was to apply a model suitable for describing its dynamics. In particular, the Kolmogorov-Arnold-Moser theorem was adapted to the question, up to low level perturbations caused by negative economic conditions, the first symptoms of financial or exogenous crises, and other turbulence affecting the economy. We then applied Nekhoroshev's theorem to represent the phenomena characterized by the occurrence of stronger resonance as well as the reactions of the system to the control and recovery measures implemented by the ECB Governing Council.

The goal of the paper is to propose the adoption of a systemic stability planning and control dashboard – also suitable for the support and stimulation of growth cycles – with attention to optimal performance, which can be identified in compliance with (or restoration of) the macroeconomic trajectories determined in the model by the Hénon Attractor.

The proposed scheme may find useful application – both for evaluation and operational purposes – in the current period, characterized by the complex and compromised scenario brought about by the SARS-COVID2 pandemic emergency, which has obviously imposed structured measures to support the economy.

*Keywords:* Macroeconomic Cycles, Monetary Policy Analysis, Optimizing Dynamic Models, Eurozone, Hamiltonian, Quasi-periodic Solution, Covid19 Crisis

# **1. INTRODUCTION**

This paper focuses on the Euro System. It starts from the collection and systematization of macroeconomic and monetary policy data, beginning before the Covid19 crisis up to now. The aim of the research is to observe the dynamics of the system in the perturbations of the scenario, and to identify a mathematical model describing the phenomena that are triggered, also capable of suggesting a response forecast, in particular – currently – concerning any recovery measures adopted and in particular the allocations of the Recovery Fund (Luo, 2021).

As is known, in 2020, the world economy – the Eurozone being no exception – was hit by a severe contraction, perhaps even a recession, caused by the rapid emergence of the aforementioned SARS-Covid2 pandemic and the consequent and reactive measures to contain its diffusion, concretized in the limitation of the social activities on the one hand, and of productivity and business on the other. However, since this condition is exogenous to the economy – unlike the global financial crisis of 2008 – with the reduction of restrictions, as during the summer of 2020, positive signs of a rebound in the indicators were noted (Wasserfallen *et al.*, 2018), (Genschel and Jachtenfuchs, 2018).

The role of public finance, in this context, has been crucial for mitigating severe effects of the crisis: the European Union has suspended the budgetary constraints for Member States, enforcing, for the first time, the general escape clause of the Stability and Growth Pact. Important interventions have been made in the EMU countries to sustain businesses and support credit through forms of public guarantees, generating large-scale budget deficits. In all the EMU countries, in 2020 the deficit reached 7.2 per cent of GDP (compared to 0.6 per cent in 2019), interrupting a ten-year trend towards improvement (ISTAT, Rapporto Annuale 2021). These results reflect the significant worsening of the primary balance (from +1.0 percent in 2019 to -5.7 per cent), at least partially offset by the reduction in financial expenses.

# 2. GRAPHICAL REPRESENTATIONS OF EMPIRICAL OBSERVATIONS

The graphs elaborated from the data-sets (see 'Appendix B') are shown below. For methodological consistency, the Member States with a population – as of December 2019 – of less than 5 million inhabitants were eliminated (Data center Eurostat, 2021). The following Member States were therefore selected: Belgium, Germany, Greece, Spain, France, Italy, Netherlands, Austria, Portugal, Slovakia, Finland (consequently, were excluded: Estonia, Ireland, Cyprus, Latvia, Lithuania, Luxembourg, Malta, Slovenia. See 'Appendix A').

# FIGURE 1 – REAL GDP GROWTH RATE (CHAIN LINKED VOLUMES, PERCENTAGE CHANGE ON PREVIOUS PERIOD) (= $\delta$ )



Source: Eurostat (Data center Eurostat, 2021)







FIGURE 3 – INFLATION RATE (ANNUAL AVERAGE RATE OF CHANGE)  $(= \eta)$ 

Source: Eurostat (Data center Eurostat, 2021)

FIGURE 4 – TOTAL GENERAL GOVERNMENT EXPENDITURE (PERCENTAGE OF GROSS DOMESTIC PRODUCT - GDP) (=  $\theta$ )





FIGURE 5 – FINAL CONSUMPTION EXPENDITURE OF GENERAL GOVERNMENT, CURRENT PRICES IN MILLION EURO (=  $\mu$ )

Source: Eurostat (Data center Eurostat, 2021) FIGURE 6 – QUARTERLY FINANCIAL ACCOUNTS FOR GENERAL GOVERNMENT, IN MILLION UNITS OF NATIONAL CURRENCY (=  $\nu$ )



Source: Eurostat (Data center Eurostat, 2021)



FIGURE 7 – CURRENT ACCOUNT, MAIN COMPONENTS, NET BALANCE - ANNUAL DATA, % OF GDP  $(=\xi)$ 

Source: Eurostat (Data center Eurostat, 2021) FIGURE 8 – IMPORTS OF GOODS AND SERVICES, IN PERCENTAGE OF GROSS DOMESTIC PRODUCT (GDP) (= 0)



Source: Eurostat (Data center Eurostat, 2021)



FIGURE 9 - EXPORTS OF GOODS AND SERVICES, IN PERCENTAGE OF GROSS DOMESTIC **PRODUCT (GDP)**  $(= \pi)$ 

FIGURE 10 – DOMESTIC CREDIT TO PRIVATE SECTOR BY BANKS (% OF GDP) (=  $\rho$ )



Source: World Bank (Data center World Bank, 2021)

Source: Eurostat (Data center Eurostat, 2021)



FIGURE 11 – DIRECT INVESTMENT IN THE REPORTING ECONOMY - FOURTH QUARTER DATA, % OF GDP (=  $\sigma$ )

The following considerations will take into account and move from the trajectories and cycles observable in the dynamics of the variables represented above.

### 3. MODELING

The conjugated action of economic and monetary policy, at the Eurozone level, expresses a macroeconomic dynamic that can be represented by quasi-periodic motions, at least for limited levels of perturbations. The aim of this contribution is to model this phenomenon, first by borrowing the theoretical KAM emanations (Kolmogorov-Arnold-Moser) (Wang *et al.*, 2018), (Palmero and Diaz, 2020).

Starting from the general problem of Poincaré dynamics, the reasoning of the canonical system can be conducted through the Hamiltonian:

$$H(x, y) = h(x) + \chi f(x, y; \chi)$$
(1)

where:

- $y \in \mathbb{T}^n$ : represents the angular variables [with regard to the variables considered in the survey, as reported in the caption of figures 1-11:  $y = f(\epsilon, \theta, \mu, \nu, o, \pi)$ ];
- x ∈ K ⊂ R<sup>n</sup>: are the action variables [with regard to the variables considered in the survey, as reported in the caption of figures 1-11: x = f(δ, η, ξ, ρ, σ)], where K is the open in which they are defined;
- χ is the parameter that indicates both the eventual perturbation event and the control maneuver. This
  is the case of impulses coming from negative conjunctures, from financial crises, as well as from
  exogenous and unexpected events that destabilize the economy, such as the current Covid19 crisis,
  as well as the responses from the central policy *control panel*, for example the Quantitative Easing
  of 2014-2015 or the Recovery Fund of 2021;

Source: Eurostat (Data center Eurostat, 2021)

- in all variables, the function is analytic;
- $f(x, y; \chi)$ , developed in power series of  $\chi$ , converges around  $\chi = 0$ .

When the system is unperturbed – and no particular control stimuli are observed – we can see, through Hamilton's equations:

$$\dot{y} = \frac{\partial h}{\partial x} := \omega(x)$$
$$\dot{x} = 0$$

that the phase space  $\mathbb{K} \subset \mathbb{R}^n$  has a process of invariant tori, parameterized by *x*, with quasi-periodic motion with frequencies  $\omega(x) = \frac{\partial h}{\partial x}$  (Giorgilli, 2007).

# FIGURE 12 – TRAJECTORY ON A TORUS AND SECTION OF QUASI-PERIODIC MOTION ON INVARIANT TORI (Franceschini, 2006-2007), (Benettin, 2001)



There may be heterocline connections: it is interesting and important to study and be able to intercept – that is, to direct, through governmental economic policy interventions – the trends between the different points of equilibrium in the path through the phase space, which can give indications on the outlook for economic growth or decline.

Assuming h(x) non-degenerate (that is, the Hessian of the unperturbed Hamiltonian is never null), if we put  $\chi \neq 0$  in H(x, y) we can observe the persistence in the system of invariant tori characterized by strongly non-resonant frequencies only as long as  $\chi$  remains at sufficiently small levels. In this circumstance – therefore, conserving the low values of the perturbations and control parameters – the frequencies, by resonance factor k, satisfy the Diophantine condition:

$$|\langle k, \omega \rangle| \ge \frac{\gamma}{|k|^{\tau}}, \ \gamma > 0, \ \tau \ge n-1, \ 0 \neq k \in \mathbb{Z}^n$$
(2)

We can then proceed with the analysis by introducing the vector  $\lambda \in \mathbb{R}^n$  of the frequencies of the trajectories (Giorgilli, 2007), where we will have:

$$H(x,y) = \sum_{i} \lambda_{i} x_{i} + \mathsf{P}(x,y) \quad (3)$$

With:

 $P(x, y) = O(x^2)$ and Hamilton's equations:

$$\dot{y}_i = \omega + \frac{\partial \Gamma}{\partial x_i}$$
$$\dot{x}_i = -\frac{\partial P}{\partial y_i}$$

For x(0) = 0 and any y(0) it is easy to see that the torus x = 0 is invariant and the relative dynamics of motion are quasi-periodic with frequencies  $\lambda_i$ . However, let's now try to qualify the reasoning towards defining outcomes of a general nature, with the aim of characterizing the macroeconomic (systemic) cycles of the countries of the Euro Area, by substituting the normal form of (3) for the Birkoff normal form, that is, constructed by canonical transformation close to the identity of the variables. Starting again from a condition  $\chi = 0$ , we can identify an invariant torus  $x^*$  – with Diophantine-like frequencies  $\lambda = \omega(x^*)$  – and proceed with the Taylor series development around it. We arrive at:

$$H(x,y) = \sum_{i} \lambda_{i} x_{i} + \frac{1}{2} \sum_{i,j} S_{i,j}(y) x_{i} x_{j} + U(y) + \sum_{i} W_{i}(y) x_{i} + O(x^{3})$$
(4)

Where:

- $S_{i,j}(y)$  is a symmetric matrix;
- U(y) and  $W_i(y)$  are the contributions obtained from the development of the perturbation / control parameter  $\chi f(x, y)$ , and they follow it in dimension. Mathematically they can be eliminated through a procedure of successive approximations, thanks to the rapidity of convergence of the method which reduces and even avoids the accumulation of small dividers;
- the final form of (4) is supported by the translation  $x' = x x^*$ , with the elision of the quotes for simplification of the expression.

However, the use of this model for this issue – also on the basis of the empirical evidence presented in the previous paragraph, and the related trajectories – reassures the existence of a stability of the system up to moderate perturbative events, which can in any case be controlled by the action of monetary policy. Moreover, manifestations of more marked resonances in the dynamics of the macroeconomic indicators of the countries due to supra-systemic critical events, but also to the interactions among the same Member States, were neglected, such as for example, imbalances in trade and unexpected gaps in workforce flows among the countries of the Area (Gräbner *et al.*, 2020).

The application of the Kolmogorov-Arnold-Moser theorem is in fact sufficiently representative in periodic intervals characterized by weak (non-linear) perturbations – in the data-sets presented, for example, from 2015 to 2019 – in which the conservative capacity of the system is apparent: the invariant tori *deformed* by weak perturbations and small corrective maneuvers, maintain pairs of fixed points in common with the undeformed tori, whose points can be elliptical, with the same dynamics of the general system and originating in a fractal or hyperbolic structure, where a form similar to saddle points develops as well as principles of chaotic behavior of the system Under these conditions, both the points *in entry* (stable manifold) and the outward points (unstable manifold) with respect to the fixed point, remain, however, invariant sets.

Nevertheless, since the dynamics of the system also go through periods of stronger resonances and perturbations, let's go back to (1) and apply Nekhoroshev's theorem (Cong *et al.*, 2020), (Benettin, 2001):

$$H(x,y) = h(x) + \chi f(x,y)$$
 (5)

where:

- $(x, y) \in K \times \mathbb{T}^n$  and *H* is analytic in a complex neighborhood of the real domain  $D = K \times \mathbb{T}^n$ ;
- *h* let *steepness*, or simplifying quasi-convex, i.e. with equations:

$$\frac{\partial h}{\partial x} \cdot u = 0$$
,  $\frac{\partial^2 h}{\partial x \partial x} u \cdot u = 0$ 

which only admit a trivial solution u = 0

then there are the positive constants  $\zeta$ ,  $\chi_*$ ,  $\alpha$ ,  $\beta \ [\alpha = \beta = \frac{1}{2n}]$ , so that, if  $\chi < \chi_*$ :

$$\|x(t) - x(0)\| < \zeta \left(\frac{\chi}{\chi_*}\right)^{\alpha}, \quad |t| \le \zeta e^{(\chi_*/\chi)^{\beta}}$$
(6)

Geometrically, by restricting *h* on an orthogonal plane  $\Phi$  to  $\varphi(x^*)$ , we can observe that for a  $\forall x^* \in K$ , the dynamic with departure close to  $x^*$  is *squashed* on  $\Phi$  or its subspaces: the elliptical trajectory is replaced with a hyperbolic structure, in which the asymptotes allow an *escape route* for the diffusion of the actions.

# FIGURE 13 – THE ELLIPTICAL TRAJECTORY IS REPLACED WITH A HYPERBOLIC STRUCTURE (QUASI-CONVEXITY HYPOTHESIS) (Benettin, 2001)



# 4. ANALYSIS

In the period following the 2008-2011 economic-financial crises, there was a continuation of perturbed cycles and strong resonances, with evidence of chaotic trajectories and risks of subcritical bifurcations with respect to stable control orbits (Guiso et al., 2019). In fact, as can be seen in the figures (1-11) presented in the second paragraph, the initial control response of the ECB was weak and in particular focused on purchases of financial assets, through liquidity auctions, without new monetary issues. In December 2011 and February 2012, as the dynamics of the system continued towards a hyperbolic structure, long-term refinancing operations (LTROs) (Andrade et al., 2019) were carried out in two auctions (with the participation also of commercial banks and for a total of about €10<sup>12</sup>): however, these were interventions with maturity (a maximum of three years ), since the resources were formed in loan ratios and their recasting progressively reduced the balance sheet total of the Central Bank. The angular effects of the LTROs were however limited to the financial sphere, failing to stimulate the real economy and having very little impact on the (re) growth of production and employment levels (in any case, it is believed that there have been some positive effects on GDP, albeit indirect, caused by the holding of the stock exchanges). In 2014 (June), considering the persistence of stagnation and the risk of a further disturbance with deflationary outcomes, a TLTRO (Targeted LTRO) plan was carried out: an injection of net liquidity to the banking sector, through long-term loans, aimed at greater disbursements of credit to productive companies. However, even this operation did not allow the recovery of the toroidal trajectories, notably due to the continuation of the credit crunch (Ben Bouheni and Hasnaoui, 2017).

In January 2015 – to provide impulses to restore the inflation rate to around 2% and to re-stabilize the macroeconomic cycles of the system – we then proceeded with Quantitative Easing (in December of the same year, phase two, QE2, was launched, extending monetary stimulus until 2017 and subsequently, until the end of 2018), (Ralph *et al.*, 2017), (Dedola *et al.*, 2021) mainly based on: purchase of public and private debt securities (60-80 billion euros of monthly purchase); reduction in the yields of countries' bonds; additional interest rate for banks and financial intermediaries who opted to keep debt securities on deposit with themselves; negative deposit rate with the ECB (-0.2%; -0.4% since 2016); extension of the liquidity on loan to banks that increase lending to the productive sector by at least 2.5% (Centro studi e ricerche Intesa Sanpaolo Group, 2021).

These ultra-expansionary monetary policy measures progressively re-stabilized the system. Their overall impact reduced the real value of household financial debt, with a consequent net increase in consumption and even economic growth in the medium term (Curcuru *et al.*, 2018). However, the further effect of the

general revaluation of assets led to an increase in inequalities within each Member State (Desogus and Casu, 2020).

As pointed out in the introduction, the effects of the SARS-COVID2 pandemic have disrupted the economic system, as can still be seen in the graphs presented above. The first control maneuver that is being implemented is centered on the Next Generation EU fund (Picek, 2020). Ecofin – following the positive results of the assessments of the European Commission – approved National Recovery and Resilience Plans: the countries will receive pre-financing equal to 13% of the total amount of each plan. The funds are raised on the markets through debt issues. The program is scheduled to last until 2026, with six-month reviews on compliance with the set objectives and the adequacy of the resources implemented.

On the side of the ECB, the change in forward guidance is expected to ensure greater room for monetary maneuver. On inflation, the transition to a symmetrical target of 2% is being implemented; temporary and limited deviations will be allowed, but without automatic compensation mechanisms (inflation averaging). Tools from Expanded Asset Purchase Program (APP) and TLTRO will also be used. The dynamics of the system will be monitored, albeit with no forecast – at the moment – of changes in the monetary policy (Crescenzi *et al.*, 2021).

In the control configuration proposed here, for the neutralization of the perturbations, the optimization of the action variables and the stability of the system – which can even offer conditions for growth – we refer back to the Hénon System (Dubeibe *et al.*, 2018), (Asai *et al.*, 2022), defined by a pair of equations:

 $\begin{aligned} x_{t+1} &= y_t + 1 - ax_t^2 \\ y_{t+1} &= bx_t \end{aligned}$ 

with parameters a > 1 and b > 0.



FIGURE 14 – ILLUSTRATION OF THE HÉNON ATTRACTOR (Franceschini, 2006-2007)

This geometric model allows the permanence of regular trajectories (fractals) to be visualized in a limited space, conditioned by a strange attractor that keeps them stable. Strong perturbations or inadequate control measures that are excessively expansive or unbalanced among the countries of the system, could lead to chaotic orbits and compromise the overall dynamics.

### 5. CONCLUSIONS

Through the joint use of the models presented and the applications with calibration of the variables with respect to the empirical data collected, it is believed that an ex-post evaluation tool of the effects of the crises that perturb the system and of the effectiveness of the reaction measures implemented by the central

government institution can be obtained. Furthermore, through a historicization of the cycles, it is possible to extract probabilistic forecasts or operate stress-testing.

The stability to which it is possible to strive or preserve, however, has a macroscopic nature: the variations on each country – albeit with regular aggregate trajectories – can probably show under-imbalances, such as the emergence or growth of inequalities (mentioned above) – also caused or aggravated by the same control and correction maneuvers that affect the general dynamics of the system (Frieden and Walter, 2017), (Lenza and Slacalek, 2018).

The same resonances that are formed – for example – in the balance of payments among countries or in the international labor markets, although not necessarily negative in the economy of trade between the nations involved, can suffer interference and partial interdictions due to the manifestation of the response mechanisms to the management stimuli of systemic cycles.

However, the migratory phenomena observed in Europe – both of external origin and between the countries considered – are *second level* perturbations, being characterized by greater intensity: they therefore require a more structured intervention and a process of correction that at the same time does not cause forced blocks to these flows. The same rationale applies to the reaction, both corrective and conservative, to the implications of socio-commercial relations with countries of global economic importance, particularly active and aggressive, such as China. Managing responses to these types of perturbative forces represents a significant challenge for the presented model, and probably an intrinsic operational limitation.

Furthermore, the substantial permanence of the (relative) macroeconomic values of each Member State is expected: the predetermined hegemonies, that is the richest states, will remain such, as it will be difficult to observe significant developments – or *overtaking* – by countries of the multi-country system with low productivity (Gräbner *et al.*, 2020).

The adoption of a dashboard based on this model, would mitigate alterations of the economic system – especially in the medium term – and lower the perception of uncertainty in the economic and (micro) managerial action, reducing *ex-ante* the impacts individual adaptive choices, not easily predictable.

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APPENDIX A
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Eurozone states	Polulation (in million)	Eurozone states	Polulation (in million)
Belgium	11.46	Estonia	1.33
Germany	83.02	Ireland	4.9
Greece	10.72	Cyprus	0.88
Spain	46.94	Latvia	1.92
France	67.06	Lithuania	2.79
Italy	60.36	Luxembourg	0.61
Netherlands	17.28	Malta	0.50
Austria	8.86	Slovenia	2.08
Portugal	10.28		
Slovakia	5.45		
Finland	5.52		

Eurostat: 2019 data

# **APPENDIX B**

# DATASET 1 (RELATIVE TO FIGURE 1): REAL GDP GROWTH RATE (CHAIN LINKED VOLUMES, PERCENTAGE CHANGE ON PREVIOUS PERIOD)

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	-2.0	2.9	1.7	0.7	0.5	1.6	2.0	1.3	1.6	1.8	1.8	-6.3
Germany	-5.7	4.2	3.9	0.4	0.4	2.2	1.5	2.2	2.6	1.3	0.6	-4.8
Greece	-4.3	-5.5	-10.1	-7.1	-2.7	0.7	-0.4	-0.5	1.3	1.6	1.9	-8.2
Spain	-3.8	0.2	-0.8	-3.0	-1.4	1.4	3.8	3.0	3.0	2.4	2.0	-10.8
France	-2.9	1.9	2.2	0.3	0.6	1.0	1.1	1.1	2.3	1.9	1.8	-7.9
Italy	-5.3	1.7	0.7	-3.0	-1.8	0.0	0.8	1.3	1.7	0.9	0.3	-8.9
Netherlands	-3.7	1.3	1.6	-1.0	-0.1	1.4	2.0	2.2	2.9	2.4	2.0	-3.8
Austria	-3.8	1.8	2.9	0.7	0.0	0.7	1.0	2.0	2.4	2.6	1.4	-6.3
Portugal	-3.1	1.7	-1.7	-4.1	-0.9	0.8	1.8	2.0	3.5	2.8	2.5	-7.6
Slovakia	-5.5	5.9	2.8	1.9	0.7	2.6	4.8	2.1	3.0	3.7	2.5	-4.8
Finland	-8.1	3.2	2.5	-1.4	-0.9	-0.4	0.5	2.8	3.2	1.3	1.3	-2.8

# DATASET 2 (RELATIVE TO FIGURE 2): GDP AND MAIN COMPONENTS (OUTPUT, EXPENDITURE AND INCOME, CURRENT PRICES), IN MILLION EURO – TIME 2018-Q4 TO 2021-Q1

	2018-Q4	2019-Q1	2019-Q2	2019-Q3	2019-Q4	2020-Q1	2020-Q2	2020-Q3	2020-Q4	2021-Q1
Belgium	122,549.0	114,386.0	119,799.0	115,764.0	126,394.0	113,930.0	103,525.0	111,871.0	121,851.0	116,021.0
Germany	864,390.0	845,820.0	846,940.0	870,150.0	886,140.0	851,060.0	768,870.0	843,870.0	872,380.0	841,310.0
Greece	45,379.2	41,551.4	46,069.8	50,051.0	45,741.2	40,561.1	38,128.6	44,593.1	42,547.0	39,199.4
Spain	315,221.0	298,529.0	315,695.0	305,647.0	324,901.0	289,961.0	250,838.0	281,956.0	298,943.0	280,524.0
France	612,595.0	600,479.0	610,098.0	601,359.0	625,700.0	580,881.0	523,451.0	582,878.0	615,649.0	599,349.0
Italy	469,387.9	427,400.2	444,587.4	443,885.3	475,068.6	402,285.7	372,435.7	423,993.3	452,880.1	408,024.4
Netherlands	199,005.0	197,442.0	207,286.0	199,768.0	208,559.0	201,819.0	191,724.0	198,696.0	207,856.0	201,249.0
Austria	101,725.2	96,074.4	98,844.8	98,974.2	103,682.0	94,104.2	87,503.3	96,696.0	98,993.7	90,733.5
Portugal	52,708.2	51,027.7	53,513.1	54,391.5	55,017.0	50,853.6	46,498.1	52,201.8	52,887.0	49,130.6
Slovakia	23,019.9	21,597.7	23,596.6	24,519.8	24,186.4	21,492.4	21,441.7	24,410.2	24,211.0	21,672.8
Finland	61,079.0	57,355.0	60,777.0	59,706.0	62,423.0	58,161.0	58,185.0	58,898.0	62,300.0	57,769.0

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	0.0	2.3	3.4	2.6	1.2	0.5	0.6	1.8	2.2	2.3	1.2	0.4
Germany	0.2	1.1	2.5	2.2	1.6	0.8	0.7	0.4	1.7	1.9	1.4	0.4
Greece	1.3	4.7	3.1	1.0	-0.9	-1.4	-1.1	0.0	1.1	0.8	0.5	-1.3
Spain	-0.2	2.0	3.0	2.4	1.5	-0.2	-0.6	-0.3	2.0	1.7	0.8	-0.3
France	0.1	1.7	2.3	2.2	1.0	0.6	0.1	0.3	1.2	2.1	1.3	0.5
Italy	0.8	1.6	2.9	3.3	1.2	0.2	0.1	-0.1	1.3	1.2	0.6	-0.1
Netherlands	1.0	0.9	2.5	2.8	2.6	0.3	0.2	0.1	1.3	1.6	2.7	1.1
Austria	0.4	1.7	3.6	2.6	2.1	1.5	0.8	1.0	2.2	2.1	1.5	1.4
Portugal	-0.9	1.4	3.6	2.8	0.4	-0.2	0.5	0.6	1.6	1.2	0.3	-0.1
Slovakia	0.9	0.7	4.1	3.7	1.5	-0.1	-0.3	-0.5	1.4	2.5	2.8	2.0
Finland	1.6	1.7	3.3	3.2	2.2	1.2	-0.2	0.4	0.8	1.2	1.1	0.4

# DATASET 3 (RELATIVE TO FIGURE 3): INFLATION RATE (ANNUAL AVERAGE RATE OF CHANGE)

# DATASET 4 (RELATIVE TO FIGURE 4): TOTAL GENERAL GOVERNMENT EXPENDITURE (PERCENTAGE OF GROSS DOMESTIC PRODUCT - GDP)

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	54.7	53.9	55.3	56.5	56.1	55.6	53.7	53.1	52.0	52.2	52.1	60.0
Germany	48.2	48.1	45.2	44.9	44.9	44.3	44.1	44.4	44.2	44.5	45.2	51.1
Greece	54.1	53.0	55.1	56.7	62.9	50.7	54.1	50.0	48.5	48.5	47.9	60.7
Spain	46.2	46.0	46.2	48.7	45.8	45.1	43.9	42.4	41.2	41.7	42.1	52.3
France	57.2	56.9	56.3	57.1	57.2	57.2	56.8	56.7	56.5	55.6	55.4	62.1
Italy	51.1	49.9	49.2	50.6	51.0	50.9	50.3	49.1	48.8	48.4	48.6	57.3
Netherlands	47.7	47.9	46.8	46.9	46.6	45.9	44.7	43.6	42.4	42.2	42.0	48.1
Austria	54.1	52.8	50.9	51.2	51.6	52.4	51.1	50.1	49.3	48.7	48.6	57.9
Portugal	50.2	51.9	50.0	48.9	49.9	51.7	48.2	44.8	45.4	43.2	42.5	48.4
Slovakia	44.4	42.5	41.6	41.1	42.5	43.3	45.8	42.7	41.3	41.7	42.7	48.0
Finland	54.1	53.9	53.7	55.4	56.8	57.3	56.5	55.6	53.6	53.4	53.2	56.7

# DATASET 5 (RELATIVE TO FIGURE 5): FINAL CONSUMPTION EXPENDITURE OF GENERAL GOVERNMENT, CURRENT PRICES IN MILLION EURO

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	83,818.4	86,182.9	90,418.1	93,853.4	95,520.0	97,663.4	98,404.4	100,046.6	102,506.0	106,005.9	109,670.1	112,558.0
Germany	488,943.0	501,679.0	513,660.0	529,210.0	551,894.0	573,455.0	595,908.0	623,851.0	648,167.0	670,346.0	704,536.0	754,383.0
Greece	55,366.9	50,107.2	44,923.8	41,759.8	37,026.9	36,465.6	36,010.7	35,517.0	36,157.2	35,139.6	36,149.9	37,113.4
Spain	220,705.0	221,331.0	219,898.0	205,982.0	202,852.0	202,678.0	209,910.0	212,278.0	216,332.0	224,689.0	234,937.0	247,295.0
France	466,306.0	478,655.0	488,750.0	500,279.0	510,490.0	518,650.0	523,400.0	530,212.0	543,160.0	549,979.0	560,255.0	577,322.0
Italy	326,155.0	331,166.0	326,718.0	321,754.0	319,441.0	317,979.0	316,344.0	322,650.0	327,002.0	334,637.0	335,049.0	345,009.0
Netherlands	162,444.0	167,744.0	167,706.0	169,945.0	170,326.0	172,465.0	172,354.0	174,842.0	179,491.0	188,611.0	200,134.0	207,633.0
Austria	59,579.6	60,636.8	61,705.0	63,286.7	64,509.8	65,972.4	68,033.4	70,273.9	71,985.9	74,506.0	77,321.2	80,519.7
Portugal	37,374.0	36,987.5	34,700.4	30,857.8	32,134.5	31,839.2	32,080.0	32,799.6	33,673.0	34,834.4	36,007.8	37,890.7
Slovakia	12,814.8	13,200.0	13,148.4	13,125.9	13,465.2	14,017.2	14,862.9	15,343.1	16,035.5	16,731.0	18,477.3	19,729.8
Finland	43,742.0	44,564.0	46,255.0	48,444.0	50,133.0	50,705.0	51,545.0	51,489.0	51,568.0	53,484.0	55,757.0	58,110.0

# DATASET 6 (RELATIVE TO FIGURE 6): QUARTERLY FINANCIAL ACCOUNTS FOR GENERAL GOVERNMENT, IN MILLION UNITS

	2018-Q3	2018-Q4	2019-Q1	2019-Q2	2019-Q3	2019-Q4	2020-Q1	2020-Q2	2020-Q3	2020-Q4
Belgium	-384,733.0	-384,692.0	-402,035.0	-408,900.0	-427,037.0	-402,644.0	-431,561.0	-449,815.0	-457,770.0	-462,556.0
Germany	-1,032,115.0	-1,044,248.0	-1,044,487.0	-1,009,801.0	-1,009,537.0	-960,993.0	-988,559.0	-1,021,558.0	-1,061,172.0	-1,086,166.0
Greece	-260,271.5	-260,406.0	-265,269.6	-268,546.9	-272,751.7	-269,237.3	-277,937.1	-284,025.7	-292,629.3	-297,798.9
Spain	-931,123.0	-946,270.0	-973,243.0	-1,034,364.0	-1,038,959.0	-1,030,118.0	-1,019,615.0	-1,097,258.0	-1,115,715.0	-1,193,073.0
France	-1,811,605.0	-1,845,152.0	-1,907,926.0	-1,953,513.0	-1,976,009.0	-1,919,338.0	-2,017,719.0	-2,102,202.0	-2,145,721.0	-2,177,934.0
Italy	-2,070,573.0	-2,108,078.0	-2,146,603.0	-2,200,642.0	-2,340,780.0	-2,269,325.0	-2,277,237.0	-2,358,168.0	-2,454,081.0	-2,520,468.0
Netherlands	-264,784.0	-269,188.0	-257,551.0	-257,128.0	-260,741.0	-251,561.0	-244,250.0	-261,662.0	-273,472.0	-285,562.0
Austria	-193,356.6	-198,410.8	-202,237.3	-205,852.4	-205,955.0	-195,444.6	-202,436.6	-215,674.5	-222,979.0	-234,990.8
Portugal	-207,464.1	-208,819.4	-211,838.6	-217,607.4	-214,657.9	-212,643.5	-209,127.2	-215,881.2	-219,383.7	-225,147.3
Slovakia	-36,464.8	-36,905.7	-37,989.5	-39,137.4	-40,146.4	-39,535.6	-39,804.3	-42,907.2	-44,459.9	-47,439.7
Finland	144,305.0	127,128.0	138,966.0	142,213.0	142,656.0	150,741.0	120,859.0	131,052.0	136,388.0	152,349.0

# DATASET 7 (RELATIVE TO FIGURE 7): CURRENT ACCOUNT, MAIN COMPONENTS, NET BALANCE - ANNUAL DATA, % OF GDP

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	2.0	-1.0	1.7	1.6	-1.9	-0.1	1.0	0.8	1.4	0.6	0.7	-0.8	0.3	-0.2
Germany	6.9	5.7	5.8	5.7	6.2	7.1	6.6	7.2	8.6	8.5	7.8	7.9	7.5	7.0
Greece	-15.2	-15.1	-12.3	-10.1	-8.8	-3.5	-1.4	-0.7	-0.8	-1.7	-1.9	-2.9	-1.5	-6.7
Spain	-9.4	-8.9	-4.1	-3.7	-2.7	0.1	2.0	1.7	2.0	3.2	2.8	1.9	2.1	0.7
France	-0.1	-0.7	-0.6	-0.6	-0.9	-1.0	-0.5	-1.0	-0.4	-0.5	-0.8	-0.8	-0.3	-1.9
Italy	-1.4	-2.8	-1.9	-3.3	-2.8	-0.2	1.1	1.9	1.4	2.6	2.6	2.5	3.2	3.6
Netherlands	6.9	5.0	5.4	7.0	8.6	10.2	9.8	8.5	6.3	8.1	10.8	10.8	9.4	7.0
Austria	3.8	4.5	2.6	2.9	1.6	1.5	1.9	2.5	1.7	2.7	1.4	1.3	2.8	2.5
Portugal	-9.6	-11.8	-10.3	-10.3	-6.0	-1.6	1.6	0.2	0.2	1.2	1.3	0.6	0.4	-1.2
Slovakia	-5.9	-6.4	-3.4	-4.7	-4.9	0.9	1.9	1.1	-2.1	-2.7	-1.9	-2.2	-2.7	-0.4
Finland	4.0	2.5	2.0	1.5	-1.4	-2.1	-1.8	-1.3	-0.9	-2.0	-0.8	-1.8	-0.3	0.8

# DATASET 8 (RELATIVE TO FIGURE 8): IMPORTS OF GOODS AND SERVICES, IN PERCENTAGE OF GROSS DOMESTIC PRODUCT (GDP)

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	66.6	74.1	80.8	80.3	78.5	79.0	76.4	78.2	82.1	83.3	81.2	80.1
Germany	33.1	37.3	40.1	40.2	39.7	39.0	39.3	38.7	40.2	41.2	41.1	38.0
Greece	28.8	29.4	31.3	33.3	32.7	34.0	33.2	32.8	36.5	41.1	41.7	39.4
Spain	23.9	27.0	29.3	29.4	29.0	30.4	30.6	29.9	31.5	32.4	31.9	29.1
France	25.6	28.1	30.4	30.5	30.4	30.8	31.2	30.9	32.0	32.7	32.5	29.9
Italy	23.0	26.9	28.3	27.3	26.2	26.2	26.7	26.0	27.9	28.9	28.4	25.8
Netherlands	54.7	61.7	67.0	69.8	69.7	69.5	75.2	69.3	72.6	74.1	72.7	67.4
Austria	41.9	47.8	51.2	51.2	50.6	50.1	49.3	48.6	50.9	52.4	52.2	49.0
Portugal	34.2	37.7	38.6	38.3	38.5	40.1	39.9	39.1	41.7	43.0	43.2	38.7
Slovakia	68.2	77.5	84.2	85.4	87.9	86.7	88.9	90.8	93.0	94.5	92.0	84.9
Finland	34.0	37.1	39.7	40.4	39.1	37.6	36.0	36.1	37.5	39.7	39.6	35.5

# DATASET 9 (RELATIVE TO FIGURE 9): EXPORTS OF GOODS AND SERVICES IN % OF GDP

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	68.8	75.9	80.7	80.4	79.3	79.8	77.8	79.4	83.2	83.0	81.8	80.6
Germany	38.1	42.6	45.1	46.3	45.4	45.6	46.9	46.1	47.2	47.4	46.9	43.8
Greece	19.0	21.8	25.5	28.7	30.3	32.5	32.2	31.3	35.0	39.0	40.1	31.9
Spain	23.1	26.0	29.5	31.5	33.0	33.5	33.6	33.9	35.1	35.1	34.9	30.6
France	24.8	26.8	28.4	29.2	29.4	29.7	30.6	30.2	30.9	31.7	31.6	27.9
Italy	22.4	25.1	26.9	28.4	28.6	29.1	29.7	29.3	30.7	31.4	31.7	29.5
Netherlands	62.2	69.8	75.5	79.5	79.9	80.6	82.7	79.5	83.4	84.7	82.5	77.9
Austria	45.2	51.3	53.9	54.0	53.4	53.4	53.1	52.4	54.1	55.7	55.6	52.6
Portugal	27.3	30.1	34.5	37.8	39.6	40.2	40.6	40.2	42.7	43.4	43.5	36.7
Slovakia	68.0	77.2	84.9	90.9	93.5	91.5	92.0	93.7	95.2	96.4	92.4	85.7
Finland	36.1	38.4	38.9	38.8	38.0	36.5	35.4	34.8	37.5	38.4	39.8	35.9

# DATASET 10 (RELATIVE TO FIGURE 10): DOMESTIC CREDIT TO PRIVATE SECTOR BY BANKS (% OF GDP)

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Belgium	68.443	62.758	58.678	56.801	55.420	54.738	56.407	57.890	60.633	62.786	63.724	66.212	68.363	74.867
Germany	97.129	97.005	98.770	88.491	84.898	83.807	82.091	79.301	78.115	77.302	77.317	77.920	79.719	86.149
Greece	84.404	89.150	87.868	112.380	119.108	118.352	118.564	116.819	112.717	108.523	100.547	91.432	80.711	81.841
Spain	167.948	171.188	173.977	172.016	167.783	158.159	146.500	130.599	119.245	111.507	105.147	98.504	93.699	107.385
France	88.780	92.494	95.036	95.851	96.793	96.541	96.007	94.076	95.077	96.661	99.519	101.930	105.254	121.190
Italy	81.516	83.540	87.377	92.884	93.922	93.606	90.704	88.488	87.197	84.530	79.918	75.698	73.282	82.495
Netherlands	113.374	110.917	117.233	113.666	114.498	116.945	113.839	116.418	111.601	114.564	111.143	105.485	100.029	101.064
Austria	92.603	95.499	97.402	98.189	95.722	93.863	92.157	87.178	85.449	82.846	83.744	84.044	85.553	93.906
Portugal	142.160	151.415	159.793	155.536	156.186	152.871	143.008	129.668	119.753	111.161	102.525	96.913	90.364	99.951
Slovakia	37.355	40.657	44.869	44.672	46.195	46.234	47.791	49.674	52.718	57.056	60.199	62.069	62.868	67.588
Finland	76.274	80.099	86.182	87.939	89.285	91.708	92.861	92.835	94.392	93.901	93.090	93.753	94.847	100.022

	2007- Q4	2008- Q4	2009- Q4	2010- Q4	2011- Q4	2012- Q4	2013- Q4	2014- Q4	2015- Q4	2016- Q4	2017- Q4	2018- Q4	2019- Q4	2020- Q4	2021- Q1
Belgium	52.5	84.9	44.2	39.0	45.1	-18.1	-11.2	2.9	-17.4	7.8	-15.6	-18.4	4.8	2.3	11.7
Germany	-0.5	0.2	3.7	4.1	3.0	4.1	2.9	-1.1	0.9	0.1	2.2	7.8	-0.4	4.7	1.7
Greece	1.2	1.0	-0.1	0.3	3.1	0.5	3.9	1.3	0.8	2.3	1.5	1.8	3.5	1.9	2.3
Spain	11.2	3.4	-0.9	4.5	2.8	4.7	3.8	-3.6	-1.5	0.4	0.2	-3.1	-2.0	1.3	3.6
France	0.7	2.0	1.8	2.9	3.4	0.2	1.0	0.7	2.2	1.2	2.3	5.9	2.0	3.0	0.9
Italy	3.2	-1.2	-0.3	0.7	1.4	0.9	1.5	1.2	0.8	2.0	-0.6	4.2	0.9	-2.3	1.4
Netherlands	102.7	-33.6	25.1	-14.9	47.2	-9.4	34.8	-5.0	25.0	31.1	-5.4	-125.1	-24.8	-19.0	9.6
Austria	36.1	2.2	7.3	-28.9	8.8	3.6	-0.3	3.4	-5.2	-10.5	5.1	-26.2	-23.6	-6.4	0.6
Portugal	1.4	4.2	1.6	5.4	9.9	18.9	14.0	6.9	3.3	5.5	4.8	2.6	3.2	5.2	1.6
Slovakia	9.1	7.8	-4.1	-0.8	0.	2.7	-1.3	-4.0	1.7	8.0	6.9	2.8	4.0	-0.7	4.8
Finland	14.9	11.3	-1.4	4.2	-2.6	-7.0	2.7	6.0	4.2	-1.8	4.7	-10.8	6.6	-4.7	5.2

# DATASET 11 (RELATIVE TO FIGURE 11): DIRECT INVESTMENT IN THE REPORTING ECONOMY -FOURTH QUARTER DATA, % OF GDP

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For COI Statement. Compliance with Ethical Standards.

Funding. The authors declare that their research has been entirely self-funded.

Conflict of Interest. The authors declare that they have no conflict of interest.

# **CREDIT AUTHOR STATEMENT:**

**Marco Desogus:** Conceptualization; Methodology; Mathematical Modelling; Writing, Reviewing and Editing; Supervision.

Elisa Casu: Data Collection, Curation, Systematization, and Investigation; Software and Graphics.