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Vincenzo Merella, Stephen E. Satchell

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Asset pricing with utility from external anticipation*

Vincenzo Merella† Stephen E. Satchell‡

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Abstract

We show that augmenting household's preferences with utility from anticipation of external factors significantly improves the performance of the consumption-based asset pricing model. Specifically, our predictions match the realized returns on equity and on risk-free assets, and helps in explaining the observed equity premium volatility. This is due to the novel forward-looking component of preferences exerting an effect on households' decision that countervails the standard market incentives to invest. Our findings stem from simulating the model with different data frequencies and confidence indicators as proxies for external anticipation. The model rationalizes the conventional wisdom that confidence makes households feel richer, hence willing to consume more. Our results also suggest that the observed predictive power of confidence on consumption growth might be justified by anticipatory utility.

JEL Classification: G12, E21

Keywords: Asset Pricing, Utility from Anticipation, Equity Premium Puzzle

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†University of Cagliari, University of Economics Prague and Birkbeck Centre for Applied Macroeconomics, University of London. Email: merella@unica.it.

‡Trinity College, University of Cambridge, and Finance Department, University of Sydney. Email: ses11@econ.cam.ac.uk.

1 Introduction

The media, financial analysts and regulators closely watch the moods of households, investors and managers seeking signals regarding possible evolution patterns of the economic conditions. Why is that so? According to conventional wisdom, agents with positive attitudes tend to feel wealthier: this may influence their investment decisions and, in particular, induce them to consume more. Though not always sharing this popular view, economists have produced a variety of empirical contributions on the subject. Notwithstanding, no established theory exists on the links between agents' moods and financial outcomes.

This paper attempts to narrow this gap in the literature. Specifically, we study asset returns in an economy in which investment decisions are influenced by a forward-looking component capturing *external* anticipatory utility. The term 'external' reflects our choice of referring to future occurrences that are beyond the household's control, such as general business conditions and labor market outlook. Utility from anticipation arises from the current pleasant or distressing feelings about future positive or negative outcomes.

With households' preferences represented by a function of consumption and external anticipation, our consumption-based asset pricing model can rationalize two stylized facts: the low average return on risk-free assets and the large equity premium. Furthermore, it helps in justifying a third one: the large standard deviation of the equity return. We argue that these facts are observed because the market incentive to invest is tempered by the forward-looking component of households' preferences reflecting utility from anticipation.

Two mechanisms are at play in our model. On the one hand, larger future payoffs encourage households to increase saving, as always. On the other hand, positive anticipation makes households feel richer, hence willing to consume more. Investment decisions thus depend on the expectation regarding whether anticipatory feelings eventually match the realized future performance of the economy. If the correlation between these two variables is positive, then in our framework external anticipation tends to be positive when risky assets payoffs are large, and vice versa. As a result, consumption is less responsive to financial incentives than in the standard consumption-based asset pricing theories: our model is able to replicate the key moments of asset returns, even if the observed consumption growth volatility is low.

Classical asset pricing theories require implausibly large values of risk aversion for consumption-based models to match the observed risk-free and excess returns.[1] One way to improve these models' predictions is to rationalize the observed low volatility of consumption growth by enriching within-period utility with habit formation. With *internal* habit, utility of current consumption may depend on past individual consumption (Constantinides, 1990); with exter-

[1] Comprehensive surveys of the asset pricing literature can be found in Mehra (2012), Ludvigson (2013) and Campbell (2015).

nal habit, on aggregate consumption (Abel, 1990). Campbell and Cochrane (1999, p. 208) eloquently state that habit formation "captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it." This paper takes the symmetric stance in a temporal perspective: anticipation of a future stimulus alters the perception of current stimuli and responses to them. On the one hand, we may envisage habit formation as having an impact on investment decisions through the way agents portray the *current* situation affecting their *future* consumption. On the other hand, we may envision anticipation as exerting its influence through the way agents feel that *future* conditions affect their *current* consumption.

In a seminal paper, Loewenstein (1987) explicitly links anticipation to *internal* factors such as the "pleasurable deferral of a vacation, the speeding up of a dental appointment, the prolonged storage of a bottle of expensive champagne" (p. 666).[2] The author defines utility from anticipation as proportional to the future stream of utility from personal consumption, a formalization later borrowed by the few contributions providing asset pricing applications: Caplin and Leahy (2001) investigate the role of anxiety in determining the risk-free rate of return and the equity premium; Kuznitz, Kandel and Fos (2008) study the effect of anticipatory utility on the mean allocation to stocks. We depart from this approach by focusing on *external* factors. We do so for two reasons.

First, we choose to investigate the role of utility from external anticipation because referring to occurrences like business and labor market conditions has the advantage to minimize potential countervailing effects due to idiosyncrasies. When considering internal factors, heterogeneous tastes over the same consumption activities may generate diversified, possibly contradictory feelings across individuals; furthermore, households' bundles of consumption activities may have different compositions, resulting in varied combinations of savoring and dreading emotions. Conversely, external factors might exert different degrees of influence on distinct individuals, yet they are unlikely to induce conflicting feelings.

Second, resorting to external anticipation has the advantage that it bears no effect on the time separability property of intertemporal utility. Anticipatory utility influences current investment decisions, which in turn affect future consumption. Since *internal* anticipation is defined by the stream of future consumption, a time dependency arises in an analogous way as in the models featuring internal habit formation. In contrast, the household cannot alter *external* occurrences, hence no mechanism of this sort emerges. In this paper, abandoning the time separability assumption is exclusively devoted to disentangle intertemporal elasticity of substitution from the coefficient of relative risk aversion, as initially suggested by Epstein and Zin (1989) and Weil (1989). This is key to our analysis since, as we discuss later, the intertemporal elasticity

[2] For a discussion on the origin and the relevance of anticipatory utility, see Frederick, Loewenstein and O'Donoghue (2002).

of substitution dictates whether positive anticipation leads, everything else held constant, to a rise or a fall in current consumption.[3]

We investigate the quantitative performance of our model using consumer confidence indicators as proxies for the external factors; namely, the Conference Board's Consumer Confidence Index (CC), and the University of Michigan's Consumer Sentiment Index (CS). Both indices are based on five-question surveys, which include queries about current and future general market conditions and job availability. The financial markets, the media and the business community interpret the indices of consumer confidence as indicators of changes in household income or wealth. Higher confidence, the typical story goes, signals better economic conditions; this, in line with conventional wisdom, makes agents feel richer and, accordingly, more prone to consume. Both the interpretation given to the indicators and the type of questions asked to construct them suggest that external anticipation and consumer confidence are a fitting match.[4]

Interestingly, a number of empirical studies find that consumer confidence predicts consumption growth, over and above other commonly used economic indicators. Economists rationalize this finding in a number of different ways: news about exogenous future productivity shocks; animal spirits on economic activity; precautionary savings motives owing to changes in the forecast variance of consumption; habit formation.[5] Once again, no attention is paid to anticipatory utility, although it might be argued that the mechanism discussed above linking investment decisions to external anticipation would in fact be able to generate the correlation underlying the predictive power of consumer confidence over consumption growth.

Due to the choice of the proxy for external anticipation, the quantitative section of this paper relates to the literature that investigates the potential connection between confidence indicators and asset pricing. Economists have produced a string of empirical contributions regarding this issue. Examples include Lemmon and Portniaguina (2006), who investigate the time-series relationship between consumer confidence and the returns of small stocks; Ho and Hung (2009) and Bathia and Bredin (2018), who include investor sentiment in conditional models to study the relevance of the size, value, liquidity and momentum effects on individual stocks returns; Chung, Hung and Yeh (2012), who study the potential asymmetry of the predictive power of

[3] See, in particular, Footnote 7 on page 6 and Appendix A.1 for further details.

[4] Among the several alternatives that one may use to proxy anticipation, tools that analyze the popularity of top search queries and keywords such as Google Trends or Twitter API have recently attracted a great deal of attention in quantitative and empirical studies. While these tools represent a great resource when working with high frequency data, in the context of our paper an overall index of anticipation obtained by aggregating the information they supply might be too sensitive to the arbitrary choices that one must make in order to construct it, both in terms of selection of queries and aggregation weights.

[5] For a list of early contributions to the literature addressing the empirical question of the link betwen consumer confidence and consumption growth, see Bram and Ludvigson (1998, footnote 3, p. 76). Among the most relevant work on the interpretations of these empirical findings, see Acemoglu and Scott (1994), Carroll, Fuhrer and Wilcox (1994), Ludvigson (2004) and Barsky and Sims (2012). More recent contributions can be found in Dees and Brinca (2013), Dreger and Kholodilin (2013), and Lahiri, Monokroussos and Zhao (2015).

investor sentiment on stock returns during economic expansions and recessions. We complement these authors' work by offering a possible theoretical rationale for the object of their empirical studies.

The paper is organized as follows. Section 2 illustrates the consumption-based asset pricing model with preferences augmented with utility from anticipatory feelings on external factors. Section 3 describes the data that we use for our quantitative exercise, details the procedure that we adopt to simulate the model, and discusses the resulting findings. Section 4 concludes. The appendix contains some mathematical derivations.

2 The model

We begin our analysis by developing a consumption-based asset pricing model in which households' preferences include utility from anticipation. Specifically, we assume that the external factors, from which anticipatory feelings arise, influence within-period utility of consumption in a multiplicative way. The rest of our framework is analogous to the Epstein-Zin-Weil (EZW) model: households' preferences are represented by a utility function à la Kreps and Porteus (1978); two assets, one risk-free and the other state-contingent, are traded; free portfolio formation and the law of one price hold.

Formally, we let the representative household's lifetime utility U_t from date t onward be represented by the function

$$U_t = \left[(1-\beta)\left(\psi_t c_t\right)^{\rho} + \beta \mu_t \left\{U_{t+1}\right\}^{\rho}\right]^{\frac{1}{\rho}} \tag{1}$$

where: c is consumption and ψ is external anticipation.[6] The most obvious example of factor generating households' anticipatory feelings is the stream of future *aggregate* consumption. Accordingly, one may think of external anticipation as formalized by the expression $\psi_t \equiv E_t\left\{\sum_{\tau} c_{t+\tau}\right\}$. However, we can afford to be agnostic regarding which factors actually generate those feelings, since we opt for the use of proxies to calibrate our model. The term $\mu_t\left\{\cdot\right\}$ is a 'certainty equivalent' operator, conditional on information at date t, specified as the function of the expected value of future lifetime utility

$$\mu_t\left\{U_{t+1}\right\} = \left(E_t\left\{\left(U_{t+1}\right)^{1-\alpha}\right\}\right)^{\frac{1}{1-\alpha}}$$

The preference parameters $\beta > 0$ and $\alpha > 0$ represent the subjective discount factor and the

[6] Typically, one would be tempted to introduce anticipation as something else than a linear function of the external factors upon which it is specified; for instance, the power function ψ^{γ}. By refraining from doing so, we are able to use the property of homogeneity of degree one of the utility function to simplify matters before simulating the model, at the cost of imposing an additional constraint ($\gamma = 1$) to the calibration of our model (which admittedly may reduce the generality of our quantitative results).

relative risk aversion coefficient, respectively; $\rho \in (0,1)$ governs the intertemporal elasticity of substitution $1/(1-\rho)$.[7]

In line with the EZW model, and in contrast with the (MP) model by Mehra and Prescott (1985), here the timing of the resolution of uncertainty matters to households. Consider two consumption streams. Stream A includes c_0 today and tomorrow, then either a series of c_0 or c_1 with some positive probabilities from the day after tomorrow. Stream B includes c_0 today, then either a series of c_0 or c_1 with the same probabilities as in stream A from tomorrow. Stream A has thus a later resolution of uncertainty than B. A household prefers an early resolution if $1-\alpha-\rho<0$; a late resolution if $1-\alpha-\rho>0$. The coefficient of relative risk aversion is commonly estimated in the range of 1 to 2, though Mehra and Prescott (1985) suggest values up to 10 might be justified since the relevant empirical exercises can be challenged on several grounds.[8] Coupled with the range of admitted values for ρ, in our framework these considerations correspond to an implicit assumption of preference for early resolution.

Households maximize lifetime utility subject to the budget constraint

$$(p_{t+1}+y_{t+1})\,z_t+b_t \geq c_t+p_{t+1}z_{t+1}+q_{t+1}b_{t+1}$$

where b is the bond holding, q is the bond price, z is the stock holding, y is the stock dividend and p is the stock price. To ease notation, we denote the aggregate state with $s=(\psi,\theta,y,x)$. The variables involved in the determination of the state are levels and growth factors of external anticipation and dividends, respectively related by the two equalities

$$\psi_{t+1}=\theta_{t+1}\psi_t \text{ and } y_{t+1}=x_{t+1}y_t$$

with the pair (θ,x) following a Markov chain. Keeping this in mind, the representative household's dynamic program can be formalized as

$$v\{z_t,b_t,s_t\}=\max_{c_t,z_{t+1},b_{t+1}}\left[(1-\beta)(\psi_t c_t)^{\rho}+\beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho}\right]^{\frac{1}{\rho}}$$

[7] The values in the admitted range for the intertemporal elasticity of substitution correspond to those generating the effect on investment decisions that we target. Namely, positive anticipation should make households willing to consume more, hence it should raise marginal utility of consumption. We refer to Appendix A.1 for a formalized version of this argument. It might be argued that the choice of excluding negative values for ρ is counterfactual, since the intertemporal substitution is estimated to be quite low: for example, Hall's (1988) estimate is about 0.1. Those findings, however, follow from considering consumption as the only factor generating households' utility, hence they are unlikely to represent the appropriate values to be considered when utility is a function of some other factor as well.

[8] Classical instances of such empirical findings include Friend and Blume (1975), Kydland and Prescott (1982) and Epstein and Zin (1991). See Campbell (1996) for an example of contribution challenging these studies and concluding that the relative risk aversion is of substantially higher magnitude than one.

subject to

$$(p\{s_t\}+y_t)\,z_t+b_t \geq c_t+p\{s_t\}\,z_{t+1}+q\{s_t\}\,b_{t+1}$$

where $\mu_s\{\cdot\}$ is the certainty equivalent conditional on the state s; likewise, the stock and bond prices, $p\{s\}$ and $q\{s\}$, are also conditional on the state s; $v\{\cdot\}$ is the household's value function conditional on the asset holdings z and b as well as on the state s.[9]

The first-order condition for holdings of the stock is

$$\begin{aligned}(\psi_t)^{\rho}(c_t)^{\rho-1}p\{s_t\} &= \beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho-1+\alpha}\cdot \\ &\quad E_t\left\{(v\{z_{t+1},b_{t+1},s_{t+1}\})^{1-\rho-\alpha}(\psi_{t+1})^{\rho}(c_{t+1})^{\rho-1}(p\{s_{t+1}\}+y_{t+1})\Big|\,s_t\right\} \end{aligned}\tag{2}$$

The first-order condition concerning the riskless asset is analogous and obtained simply by plugging in $q\{s_t\}$ for $p\{s_t\}$ and 1 for the payoff $p\{s_{t+1}\}+y_{t+1}$, obtaining

$$\begin{aligned}&(\psi_t)^{\rho}(c_t)^{\rho-1}q\{s_t\} = \\ &\quad \beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho-1+\alpha}E_t\left\{(v\{z_{t+1},b_{t+1},s_{t+1}\})^{1-\rho-\alpha}(\psi_{t+1})^{\rho}(c_{t+1})^{\rho-1}\Big|\,s_t\right\}\end{aligned}$$

Imposing equilibrium (consumption must equal dividends, i.e. $c=y$; the representative household constantly holds all the stock, i.e. $z=1$; but no bond, i.e. $b=0$) and rearranging, the model's asset pricing formulas for the equity price becomes

$$p\{s_t\}=E_t\left\{\beta\left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}}\right)^{1-\rho-\alpha}(\theta_{t+1})^{\rho}(x_{t+1})^{\rho-1}(p\{s_{t+1}\}+y_{t+1})\Bigg|\,s_t\right\}\tag{3}$$

where to simplify notation we let $V\{s\}=v\{1,0,s\}$, representing the household's value function in equilibrium.

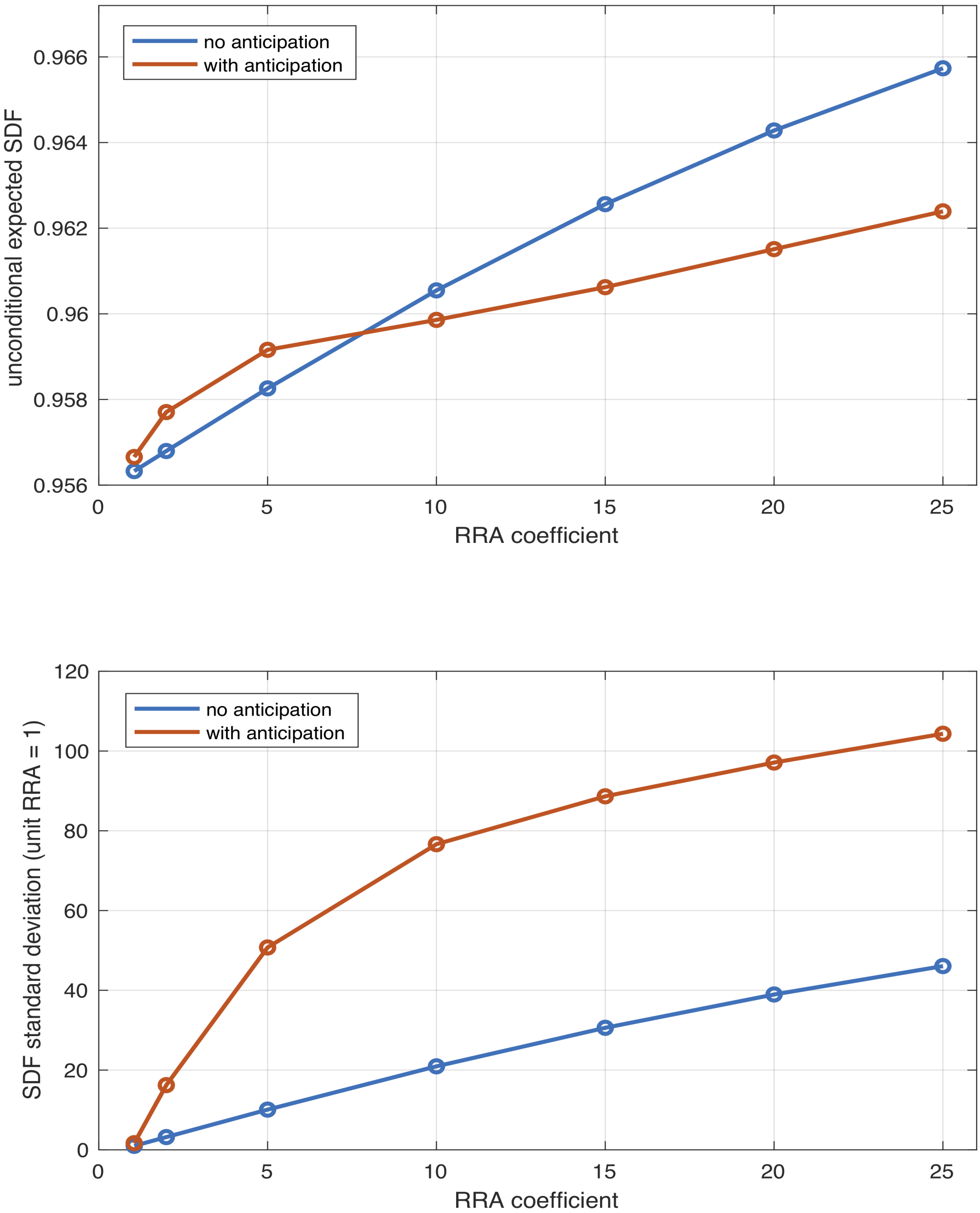
The stochastic discount factor (SDF), given by

$$m\{s_t,s_{t+1}\}=\beta(x_{t+1})^{\rho-1}(\theta_{t+1})^{\rho}\left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}}\right)^{1-\rho-\alpha}\tag{4}$$

incorporates three terms. The first term, $\beta(x_{t+1})^{\rho-1}$, is the product between the subjective discount factor and a non-increasing power function of consumption growth. It represents the SDF in the MP model, and is one of the two terms comprising the SDF in the EZW model. The second term, $(\theta_{t+1})^{\rho}$, is a concave function of the innovation in external anticipation. Taken in isolation, it reflects the impact of anticipatory utility on household's choice abstracting from uncertainty. The third term, $\left(V\{s_{t+1}\}/\mu_{s_t}\{V\{s_{t+1}\}\}\right)^{1-\rho-\alpha}$, involves the household's value function and reflects the household's preferences for the timing of resolution of uncertainty. If

[9] For a derivation of the solution of the household's problem, see Appendix A.2.

Figure 1. Stochastic discount factor's mean and standard deviation as relative risk aversion varies.



Source. Authors' calculations.

Notes. The subjective discount factor is set to $\beta = 0.975$, with unit intertemporal elasticity of substitution ($\rho = 0$). The SDF standard deviations reported in the bottom panel are expressed as variations from the value obtained by setting RRA = 1, in percentage terms.

early resolution is preferred, i.e. $1-\rho-\alpha<0$, then asset payoffs in states where realized lifetime utility is lower than the conditional certainty equivalent will have a greater impact on the asset price than payoffs in states where the opposite occurs, just like in the EZW model. The difference, of course, is that here the value function also depends on external anticipation: that is, anticipatory utility affects the magnitude of the potential rise in the volatility of the SDF relative to that generated by the first, standard term.[10]

Figure 1 illustrates the evolution of mean and standard deviation of the SDF as the coefficient of relative risk aversion varies. We consider the case of unit intertemporal elasticity of substitution, i.e. the most conservative value in the admitted range ($\rho=0$). Moments of consumption growth are calibrated on the quarterly dataset described in Section 3. External anticipation is given a 0.8 mean, a 12.4 standard deviation and a 0.4 correlation with consumption growth.[11] The top panel deals with the average values of the SDF. We note that the values delivered by the model with anticipation are larger than those by the EZW model at low levels of RRA. The bottom panel concerns the volatility of the SDF. There, the values delivered by our model are substantially larger than the ones by the EZW model already at low levels of RRA; the gap keeps rising up to a RRA coefficient equal to 10, then maintains the same proportion for larger values of it.

Before turning to the description of our quantitative findings, let us have a brief look at how the SDF generates asset prices and the resulting returns. Recall that $cov\left\{m,R\right\}=E\left\{m\,R\right\}-E\left\{m\right\}E\left\{R\right\}$ and $\rho_{m,R}=cov\left\{m,R\right\}/\left(\sigma\left\{m\right\}\sigma\left\{R\right\}\right)$; furthermore, consider that for any asset on the efficient mean-variance frontier it holds that $R=a+b\,m$.[12] Then, from the central asset pricing formula, $E\left\{m\,R\right\}=1$, we may obtain the following three equations that our simulation exercises obey in determining the risk-free return

$$R^f = 1/E\left\{m\right\} \tag{5}$$

$$EP = \frac{b^2\sigma^2\left\{m\right\}}{E\left\{m\right\}} \tag{6}$$

$$\sigma\left(R\right) = b^2\sigma\left\{m\right\} \tag{7}$$

where R^f is the risk-free return, EP is the equity premium, $\sigma\left(R\right)$ is the equity return volatility and b is a value governed by the preference parameters. From (5) we learn that the risk-free return is merely the reciprocal of the SDF expected value. Our exercise thus suggests that

[10] If the household is indifferent to the timing of resolution of uncertainty, i.e. $1-\rho-\alpha=0$, then the SDF is ordinally equivalent to $m\left\{s_t,s_{t+1}\right\}=\beta\left(x_{t+1}\right)^{-\alpha}\left(\theta_{t+1}\right)^{1-\alpha}$. In this case, the term $\left(\theta_{t+1}\right)^{1-\alpha}$ captures the response of investment decisions to uncertainty: payoffs in states where anticipatory utility is higher have a smaller impact than payoffs in states where the opposite occurs if the coefficient of relative risk aversion is larger than one, and vice versa.

[11] These values correspond to those obtained by using the Consumer Sentiment Index as a proxy for anticipation.

[12] See e.g. Cochrane (2005, Chapter 1) for further details.

including anticipation can predict lower riskless rates than the standard framework for modest levels of risk aversion. From (6) we establish that the equity premium is proportional to the ratio between the SDF's variance and mean. In light of our simulation results, we expect the model to be able to predict larger equity premia at virtually any level of relative risk aversion. Finally, (7) indicates that the equity return volatility is proportional to the SDF's standard deviation. Our simulations are then suggestive of the predictions on $\sigma(R)$ following a similar pattern as those on EP. Each of these three predictions has the potential to represent an improvement over those delivered by the standard consumption-based asset pricing model.

3 Quantitative results

The preference parameters defining the SDF are β, ρ and α; the technological parameters are the elements defining the stochastic processes for x and θ, including the transition probability matrix, denoted by π, that characterizes the Markov chain process governing the pair (θ, x). We refer to the previous section for the reasons that drive our choice of the ranges of admitted values for the parameters governing the intertemporal elasticity of substitution, $\rho \in (0, 1)$, and the relative risk aversion coefficient, $\alpha \in [1, 10]$. The subjective discount factor takes the value $\beta = 0.975$. Both the values taken by the pair (θ, x) across the states and the matrix governing the pair's transition process are calibrated on the observed confidence indicators (for external anticipation) and consumption growth data.

Data

We use two sets of data to perform our quantitative exercises. The first one is the original dataset set up by Mehra and Prescott (1985). We refer to it as the historical or yearly dataset. The second one, which we refer to as our or quarterly dataset, is constructed as follows. The existing observations of our proxies for external anticipation dictate the time span. The Conference Board's Consumer Confidence Index (CC) has the shortest time series, available from the second quarter of 1967. The University of Michigan's Consumer Sentiment Index (CS) dates back to the fourth quarter of 1958, but for a better comparison we align all the time series to the shortest. The dataset therefore spans from the second quarter of 1967 to the first quarter of 2019. The consumption growth time series is calculated using the Bureau of Economic Analysis' United States personal consumption expenditures on non-durable goods and services, at constant prices and seasonally adjusted. For robustness, our exercises are run using the two confidence indicators as a proxy for external anticipation in turn.

The financial figures that we target are computed as follows. The equity returns are derived from the price and dividend time series of the Standard & Poor's 500 composite index. As a series for the bond returns, we use the quarterly and yearly data of annual based nominal yield

Table 1. Descriptive statistics, quarterly data.

	consumption growth	risk-free return	excess return	consumer confidence	consumer sentiment
mean	2.08	1.19	6.18	2.52	0.79
std. dev.	1.82		15.4	22.7	12.4

Source. Authors' calculations based on the dataset describe in Section 3 (204 observations).

on three-month U.S. government treasury bills. As always, these rates are reported using the bank discount convention.[13] The nominal returns are then converted in real terms by using the price index provided by the Bureau of Economic Analysis of the U.S. Department of Commerce (for homogeneity, the same source as the one we choose for consumption data). All time-series in our dataset are annualized.

Tables 1 and 2 respectively report the descriptive statistics and the correlations of the variables involved in the quarterly dataset. We may draw a comparison between these figures and those gathered from the yearly dataset only with reference to consumption growth and asset returns, since historical data on confidence indicators do not exist. In the yearly dataset, the average consumption growth rate is 1.8%, with a standard deviation of 3.6; the risk-free rate is 0.8%; the average excess return is 6.2%, with a standard deviation of 16.7; the correlation between consumption growth and excess return is 0.4.

Compared to the historical data reported in the classical asset pricing papers, therefore, the consumption growth rate and the return on the risk-free asset are larger in our dataset (about +0.3% and +0.4%, respectively); the average excess return and its correlation with consumption growth are about the same; the standard deviations of consumption growth and excess return are lower (about half and −1.3, respectively). Regarding the confidence indicators, we note that CC has both a larger average and volatility than CS (+1.3% and +10.3, respectively). The correlations with consumption growth are positive and, again, stronger for CC than CS (+0.17 larger). The correlations with the excess return are positive and comparable (with only a difference of +0.03 in favor of CC). As expected, the correlation between the two confidence indicators is positive and quite large (just above 0.8).

Methodology

We investigate the quantitative effects of the introduction of external anticipatory utility in the household's preference specification by exploring whether this helps in improving the model's

[13] See, *e.g.*, Mayle (2007).

Table 2. Correlations spreadsheet, quarterly data.

	excess return	consumer confidence	consumer sentiment
consumption growth	0.409	0.564	0.394
excess return		0.585	0.550
consumer confidence			0.809

Source. Authors' calculations based on the dataset describe in Section 3 (204 observations).

predictions on a number of central financial measures; namely, the average return on risk-free assets; the equity premium; the standard deviation of the equity return. In order to do so, we need to implement and solve an iterative routine to calculate the SDF and, thereby, price dividend ratios and asset returns. Before illustrating our findings, we briefly explain how the iterative process works and what it depends upon.

Since the pair (θ, x) is assumed to follow a Markov chain and lifetime utility is homogeneous of degree one in ψc, the SDF depends only on (θ_t, x_t) and (θ_{t+1}, x_{t+1}), with θ_t and x_t appearing just in the conditioning of the certainty equivalent. For some function ϕ, the equilibrium value function can be written as

$$V\{s_t\} = \phi\{\theta_t, x_t\}\,\psi_t y_t \tag{8}$$

After some simple algebra, the third term in (4) becomes

$$\left(\frac{V\{s_{t+1}\}}{\mu_{s_t}\{V\{s_{t+1}\}\}}\right)^{1-\rho-\alpha} = \left(\frac{\phi\{\theta_{t+1}, x_{t+1}\}\,\theta_{t+1}x_{t+1}}{\mu_{\theta_t,x_t}\{\phi\{\theta_{t+1}, x_{t+1}\}\,\theta_{t+1}x_{t+1}\}}\right)^{1-\rho-\alpha}$$

which in turn implies that the SDF reads

$$m\{\theta_t, \theta_{t+1}, x_t, x_{t+1}\} = \beta\,(x_{t+1})^{\rho-1}\,(\theta_{t+1})^{\rho}\left(\frac{\phi\{\theta_{t+1}, x_{t+1}\}\,\theta_{t+1}x_{t+1}}{\mu_{\theta_t,x_t}\{\phi\{\theta_{t+1}, x_{t+1}\}\,\theta_{t+1}x_{t+1}\}}\right)^{1-\rho-\alpha}$$

In this expression, x_{t+1} and ϕ are vectors, while m is the matrix

$$m\{i,j\} = \beta\,(x\{j\})^{\rho-1}\,(\theta\{j\})^{\rho}\left(\frac{\phi\{j\}\,\theta\{j\}\,x\{j\}}{\mu\{i\}}\right)^{1-\rho-\alpha} \tag{9}$$

where

$$\mu\{i\} = \left(\sum_{j=1}^{S}\pi\{i,j\}\,(\phi\{j\}\,\theta\{j\}\,x\{j\})^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{10}$$

Together with (3), (9) and (10) entail that the equity price is homogeneous in ψc. Using (3), the price/dividend ratio, defined as $\omega\{s\} = p\{s\}/y$, can be written as

$$\omega\{i\} \equiv \frac{p\{i\}}{y\{i\}} = \sum_{j=1}^{S} \pi\{i,j\}\, m\{i,j\}\, x\{j\}\,(1+\omega\{j\})$$

The bond price is simply

$$q\{i\} = \sum_{j=1}^{S} \pi\{i,j\}\, m\{i,j\}$$

We need to compute the matrix $m = m\{i,j\}$ to solve these equations. This task requires the calculation of the function ϕ. Here is where it becomes necessary to resort to the iterative procedure. Recall that $\phi\{\theta, x\}\,\psi y$ is the value function in equilibrium, which in turn represents the household's maximized lifetime utility. We can therefore write

$$\phi\{\theta_t, x_t\}\,\psi_t y_t = \left[(1-\beta)\,(\psi_t y_t)^{\rho} + \beta\left(\mu_{\theta_t,x_t}\left\{\phi\{\theta_{t+1}, x_{t+1}\}\,\psi_{t+1} y_{t+1}\right\}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

which, dividing both sides by $\psi_t y_t$, using $\psi_{t+1} = \theta_{t+1}\psi_t$, $y_{t+1} = x_{t+1} y_t$ and the homogeneity of μ, becomes

$$\phi\{\theta_t, x_t\} = \left[(1-\beta) + \beta\left(\mu_{\theta_t,x_t}\left\{\phi\{\theta_{t+1}, x_{t+1}\}\,\theta_{t+1} x_{t+1}\right\}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

This expression corresponds to the vector

$$\phi\{i\} = \left[1-\beta+\beta\,(\mu\{i\})^{\rho}\right]^{\frac{1}{\rho}} \tag{11}$$

with $\mu\{i\}$ as in (10). To solve for ϕ, we treat the last two equations as a mapping that, at the k-th iteration, takes the vector ϕ^{k-1} into a new vector ϕ^{k}. Specifically: given ϕ^{k-1}, we first use (10) to generate a vector of certainty equivalents μ^{k}; we then use μ^{k} to obtain ϕ^{k} using (11). This two-step procedure is repeated iteratively until the change in ϕ produced by successive iterations is sufficiently small to be considered negligible.

Results

We now present a brief summary of the various attempts that we have made to simulate our model. Our first exercises concern the model without anticipation (the EZW model). We begin by running a set of simulations in which the model is calibrated using the historical data. This gives us a base calculation to compare with the existing literature. We then run simulations of the EZW model calibrated on quarterly data. The resulting figures serve as a benchmark for the findings obtained from the exercises that immediately follow. Namely the simulations of the model with anticipation proxied by two indicators, respectively the Consumer Confidence Index

Table 3. Simulation of the model without external anticipation, yearly data.

IES	RRA 1	5	10	∞	
1	4.49	4.02	3.44	**0.84**	risk-free
	0.13	0.61	1.20	3.97	EP, mean
	0.00	0.00	0.01	0.22	EP, st. dev.
5	2.92	2.65	2.31	2.14	risk-free
	0.12	0.57	1.13	4.01	EP, mean
	0.00	0.00	0.01	**0.23**	EP, st. dev.
10	2.71	2.46	2.16	2.31	risk-free
	0.12	0.57	1.12	**4.02**	EP, mean
	0.00	0.00	0.01	**0.23**	EP, st. dev.
∞	2.51	2.28	2.01	2.48	risk-free
	0.12	0.56	1.11	**4.02**	EP, mean
	0.00	0.00	0.01	**0.23**	EP, st. dev.
U.S. data (1889-1978)		0.80	risk-free return		
		6.18	excess return, mean		
		16.7	excess return, st. dev.		

Source. Authors' calculations based on the dataset described in Section 3 (90 observations).

Notes. The subjective discount factor is set to $\beta = 0.975$. All values are in percentage terms.

(CC) and the Consumer Sentiment Index (CS), calibrated on quarterly data.

Table 3 summarizes the results of our first set of simulations. The EZW model is calibrated on yearly data from 1889 to 1978. The table reports the simulation outcomes for each pair of values (α, ρ) from the sets $\alpha = \{1, 5, 10, \infty\}$ and $\rho = \{0, 0.8, 0.9, 1\}$. Excess return volatility (third target) rises with both the relative risk aversion coefficient (RRA) and the intertemporal elasticity of substitution (IES). The risk-free return (first target) does the opposite, with the exception of large values of both RRA and IES for which the evolution reverts. The excess return (second target) steadily rises with RRA and (barely) declines with IES. In line with the typical findings in the literature, the model's simulation performs well in terms of the first target only for implausibly large values of RRA. When IES is also large, the simulations also offer the best performance regarding the other two targets, though these remain well off the respective observed figures.

Table 4 offers a snapshot of the findings obtained from our second set of simulations. The EZW model is calibrated on quarterly data from the second quarter of 1967 to the last quarter of 2018. The table reports the simulation outcomes for each pair of values (α, ρ) already considered in our first set of simulations. The trends of the three variables of interest as each of the parameter's values varies are confirmed, though the exception regarding the risk-free return

Table 4. Simulation of the model without external anticipation, quarterly data.

IES	RRA 1	5	10	∞	
1	4.58	4.37	4.12	2.81	risk-free
	0.02	0.25	0.52	1.88	EP, mean
	0.00	0.03	0.14	1.24	EP, st. dev.
5	2.92	2.64	2.34	2.54	risk-free
	0.13	0.65	1.20	1.94	EP, mean
	0.00	0.09	0.32	1.28	EP, st. dev.
10	2.70	2.41	2.10	2.50	risk-free
	0.15	0.71	1.29	**1.95**	EP, mean
	0.00	0.09	0.35	**1.29**	EP, st. dev.
∞	2.48	2.18	**1.87**	2.46	risk-free
	0.17	0.77	1.39	**1.95**	EP, mean
	0.00	0.10	0.37	**1.29**	EP, st. dev.
U.S. data (1967q2-2018q4)		1.19		risk-free return	
		6.18		excess return, mean	
		15.4		excess return, st. dev.	

Source. Authors' calculations based on the dataset described in Section 3 (204 observations).

Notes. The subjective discount factor is set to $\beta = 0.975$. All values are annualized and in percentage terms.

here occurs also at milder levels of IES. The model performs generally worse when calibrated on our dataset. Both the risk-free return and the excess return are farther from their observed figures, even when we consider the best performing simulation. There is, however, a general rise in the values concerning the third target, although it is too mild even at relatively high level of RRA to be considered a substantial improvement.

The model with utility from anticipation yields more accurate predictions. Table 5 reports the figures obtained by simulating the model with external anticipation, calibrated on quarterly data using CC as a proxy for anticipation. The simulations are run for each pair of values (α, ρ) from the sets $\alpha = \{1, 3, 5, 10\}$ and $\rho = \{0, 0.68, 0.9, 1\}$. All targets exhibit evolutions as one of the parameters vary that may depend on the value of the (other) parameter being kept constant. Specifically, the risk-free return declines as RRA (resp., IES) increases for sufficiently low levels of IES (resp., RRA), while rises otherwise. Both the equity premium and its volatility rise monotonically with RRA for sufficiently low levels of IES, but first increase then decline otherwise; in contrast, they monotonically go up with IES, irrespective of the level of RRA.

Figure 2 helps in clarifying the rationale for our findings. Like in Figure 1, the top panel illustrates the evolution of the SDF's unconditional mean as the RRA coefficient varies, this time for different levels of IES; the bottom panel, the SDF standard deviation's. In isolation, the top

Table 5. Simulation of the model with CC as a proxy for external anticipation, quarterly data.

	RRA				
IES	1	3	5	10	
1	4.67	4.25	4.16	4.03	risk-free
	0.03	0.59	0.73	0.88	EP, mean
	0.01	0.23	0.31	0.42	EP, st. dev.
3.1	**1.20**	2.93	6.31	11.76	risk-free
	6.20	11.41	10.27	6.98	EP, mean
	1.88	4.43	4.94	3.69	EP, st. dev.
10	-1.02	2.80	7.34	14.6	risk-free
	11.02	14.85	13.10	8.83	EP, mean
	3.45	5.51	6.12	4.65	EP, st. dev.
∞	-2.18	2.79	7.85	15.9	risk-free
	13.67	16.36	14.31	9.63	EP, mean
	4.34	5.96	**6.59**	5.05	EP, st. dev.

U.S. data (1967q2-2018q4)	1.19	risk-free return
	6.18	excess return, mean
	15.4	excess return, st. dev.

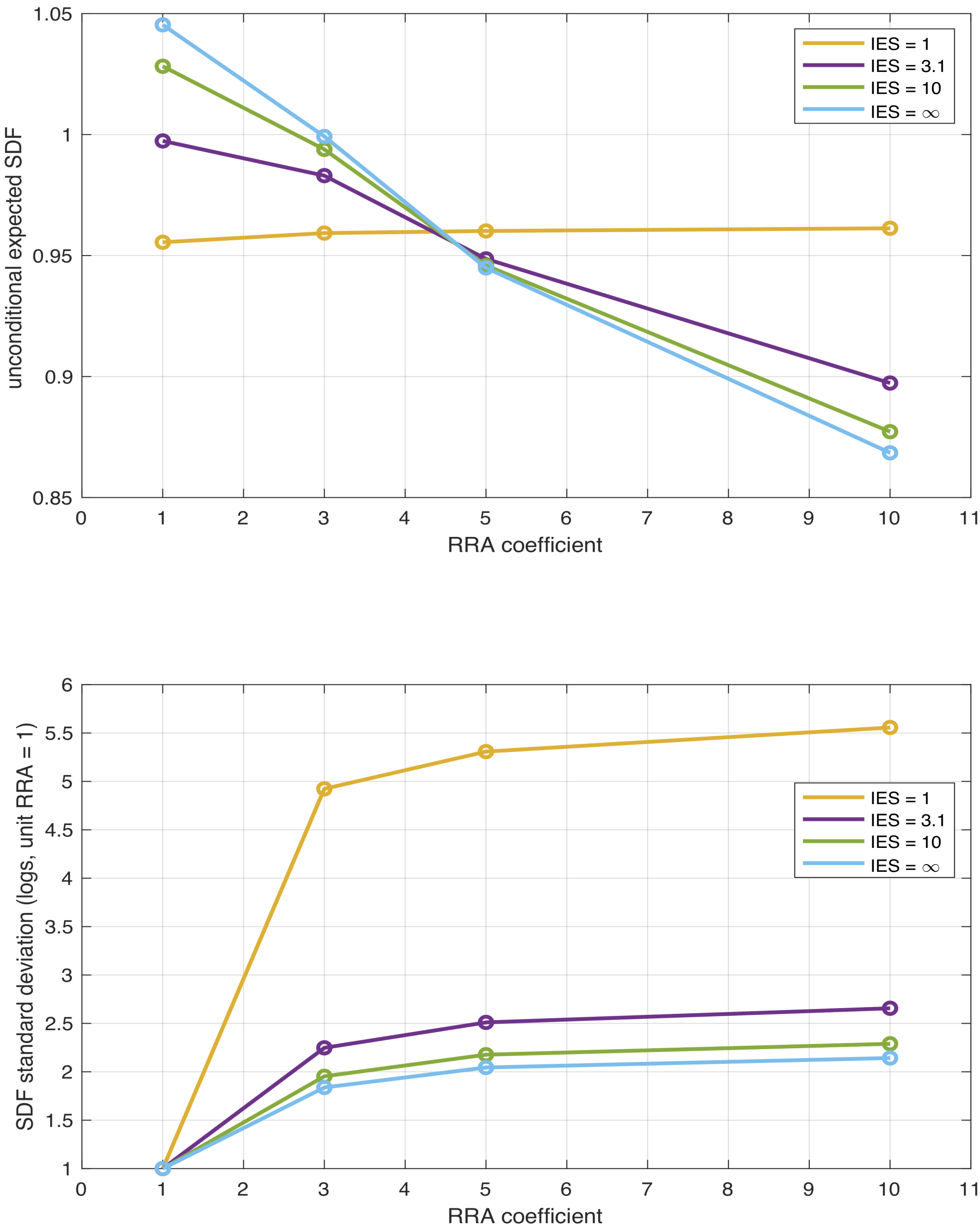
Source. Authors' calculations based on the dataset described in Section 3 (204 observations).

Notes. The subjective discount factor is set to $\beta = 0.975$. CC stands for the consumer confidence index. All values are annualized and in percentage terms.

panel explains the evolution of the risk-free return, which according to (5) equals the reciprocal of the SDF's unconditional mean. At unit IES, the average SDF monotonically increases with RRA, hence the riskless rate steadily declines. The opposite occurs at the higher levels of IES. The equity premium volatility (7) is proportional to the SDF's standard deviation $\sigma\{m\}$. However, the proportionality factor b^2 is affected by the variations in the preference parameters (β, ρ, α). Since $\sigma\{m\}$ is monotonically increasing with the RRA coefficient, we learn from (7) that b^2 is a non-increasing function of α, at least for the larger values of the latter. Finally, (6) indicates that the equity premium EP is proportional to the ratio between the SDF's variance $\sigma^2\{m\}$ and mean $E\{m\}$. As $\sigma^2\{m\}$ rises and $E\{m\}$ declines with α, we would expect a monotonically increasing evolution for EP. But, once again, the variations in the proportionality factor b^2 revert this prediction for large levels of risk aversion. In other words, at the higher levels of IES, the monotonic decline in the SDF mean induce agents to hold more risky portfolios and, when the rise in the SDF volatility slows down, both the variable of interest begin falling too due to the varying b^2, notwithstanding the larger risk aversion that causes those variations in the first place.

The performance of the model with external anticipation resulting from Table 5 is striking.

Figure 2. SDF's mean and standard deviation as RRA varies, ψ proxied by CC.



Source. Authors' calculations.

Notes. The subjective discount factor is set to $\beta = 0.975$. The SDF standard deviations reported in the bottom panel are expressed as variations from the value obtained by setting RRA = 1, in log terms.

Table 6. Simulation of the model with CS as a proxy for external anticipation, quarterly data.

	RRA				
IES	1	2.7	5	10	
	4.65	4.51	4.38	4.15	risk-free
1	0.03	0.22	0.38	0.64	EP, mean
	0.00	0.08	0.18	0.31	EP, st. dev.
	1.57	**1.19**	2.22	5.19	risk-free
5.3	2.36	**6.18**	7.47	6.62	EP, mean
	0.31	1.92	3.50	3.71	EP, st. dev.
	1.12	0.81	2.02	5.33	risk-free
10	2.90	6.94	8.24	7.25	EP, mean
	0.37	2.11	3.80	4.04	EP, st. dev.
	0.59	0.37	1.79	5.50	risk-free
∞	3.56	7.81	9.10	7.95	EP, mean
	0.46	2.30	4.12	**4.40**	EP, st. dev.
U.S. data (1967q2-2018q4)			1.19	risk-free return	
			6.18	excess return, mean	
			15.4	excess return, st. dev.	

Source. Authors' calculations based on the dataset described in Section 3 (204 observations).

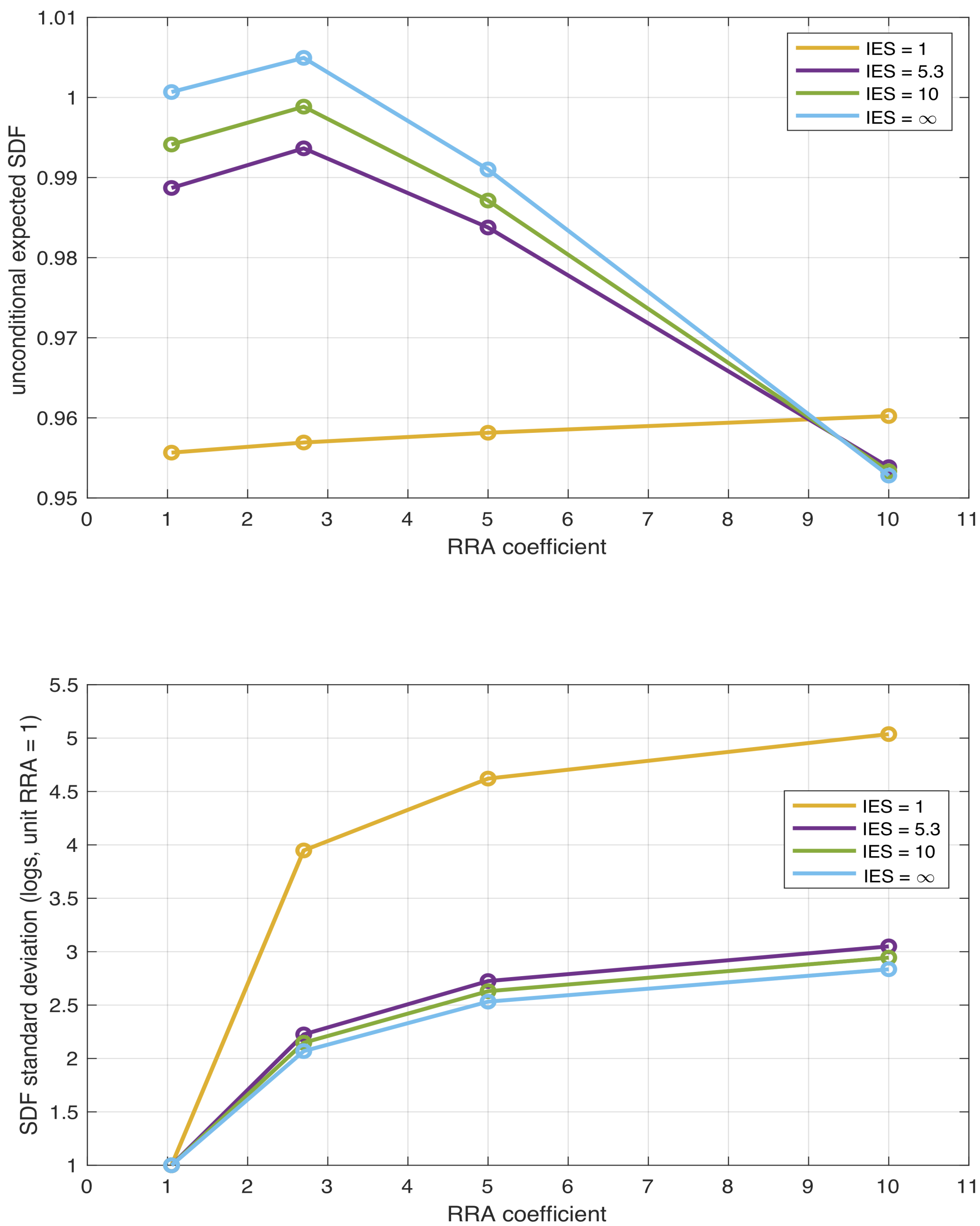
Notes. The subjective discount factor is set to $\beta = 0.975$. CS stands for the consumer sentiment index. All values are annualized and in percentage terms.

With a fairly reasonable level of IES ($= 3.1$) and the unit value for the RRA coefficient indicated in the literature as the actual level of relative risk aversion, our simulation hits the first two targets (risk-free return and equity premium) with less that a 0.02% error margin! The other exercises suggest that the model also represents an improvement over the benchmark with regard to explaining the equity premium volatility, though even the best performing simulation in this respect, run with reasonable RRA ($= 5$) but implausibly large IES ($\to \infty$), falls short of getting near the targeted figure (6.59 vs. 15.4).

The model with external anticipation maintains a fine performance also when the proxy for anticipatory utility is the Consumer Sentiment Index. Table 6 collects the simulation's outcomes of the model with external anticipation, once again calibrated on quarterly data. The pair of values (α, ρ) used here are from the sets $\alpha = \{1, 2.7, 5, 10\}$ and $\rho = \{0, 0.81, 0.9, 1\}$. The evolutions of the variables of interest as the parameters vary are analogous to those discussed above concerning the case of external anticipation proxied by CC. The only exception is that the equity premium volatility now rises monotonically throughout. The reason is more easily understood by resorting again to an illustration of the evolution of the SDF's unconditional mean and standard deviation as the RRA coefficient varies.

Figure 3 portrays these evolutions for different levels of IES in the top panel for the level

Figure 3. SDF's mean and standard deviation as RRA varies, ψ proxied by CS.



Source. Authors' calculations.

Notes. The subjective discount factor is set to $\beta = 0.975$. The SDF standard deviations reported in the bottom panel are expressed as variations from the value obtained by setting RRA = 1, in log terms.

and in the bottom panel for the volatility of the SDF, respectively. The decline in the SDF's mean and the rise in the SDF's standard deviation are less pronounced than those represented in Figure 2 for the case of CC being used as a proxy for external anticipation: in this case, from (5)-(7) we learn that these variations suffice to revert the trend of the equity premium for sufficiently high values of both parameters, but they are not sufficient to exert the same effect on the equity premium volatility.

Turning back to Table 6, we note that the model exactly replicates the observed risk-free return and equity premium with reasonable levels of IES (= 5.3) and RRA (= 2.7). In general, however, the improvement over the benchmark with regard to explaining the equity premium volatility is more modest than that offered by the model with CC as a proxy for anticipatory utility, with the best performing exercise in this respect, with parameters set to the implausibly large values of $\alpha = 10$ and $\rho \to 1$, delivering a figure well off the targeted magnitude (4.4 vs. 15.4).

4 Concluding remarks

This paper investigates the effects of utility from anticipation of external factors on investment decisions and, thereby, on asset prices. Simulations of the model using confidence indicators as proxies for external anticipation indicate a substantial improvement in consumption-based asset pricing relative to the classical frameworks found in the literature. Specifically, the model is able to match the risk-free return and the equity premium observed in the data, and is able to explain over 30% of the equity premium volatility under some parameter specifications. Our framework thus offers a rationale for the role that the media, financial analysts and regulators seemingly assign to the moods of households, investors and managers in shaping financial market outcomes.

In order to have a more transparent understanding of the mechanism through which external anticipation influences asset prices, our model only features anticipatory feelings as an additional element relative to the benchmark Epstein-Zin-Weil framework. Nevertheless, it seems more than plausible to entertain the thought that exploiting the complementarities between external habit formation and external anticipation within a model featuring Kreps-Porteus preferences may not only be sensible, but might also produce further improvements about the rationale for the large observed equity premium volatility. This is beyond this paper's objective, therefore we leave this issue for future research with a sharper focus on the performance of the consumption-based asset pricing model.

The mechanism described in the paper lends some support to the conventional wisdom that publicly available information plays a key role in shaping economic outcomes, especially those of a financial nature. This argument has a straightforward policy implication: regulators might need to understand the fundamental determinants of agents' mood, and offer to the public strong signals to moderate potential swings in those attitudes. In light of our findings, those signals may help in reducing the possibly disruptive large fluctuations that cyclically characterize the financial markets. Furthermore, our findings are suggestive of a potential role for anticipatory utility in explaining the observed predictive power of confidence on consumption growth, since they give grounds for confidence indicators having an impact on agents' current investment decisions within a framework abiding by the rational expectations permanent income hypothesis.

A Appendix

A.1 Admitted range of values for the intertemporal elasticity of substitution

We first compute the marginal utility of current consumption by partially differentiating lifetime utility (1) with respect to c_t

$$\frac{\partial U_t}{\partial c_t} = (1-\beta)\,\psi_t^{\rho} c_t^{\rho-1} U_t^{1-\rho}$$

The effect of a change in the current level of external anticipation on marginal utility of consumption is obtained by differentiating the last expression with respect to ψ_t

$$\frac{\partial^2 U_t}{\partial c_t \partial \psi_t} = (1-\beta)\,(\psi_t c_t)^{\rho-1}\, U_t^{1-\rho} \left[\rho + (1-\rho)\,(1-\beta)\,(\psi_t c_t)^{\rho}\, U_t^{-\rho}\right]$$

In order to have positive anticipation inducing, everything else held constant, larger current consumption, the last expression must be positive, which requires

$$\rho + (1-\rho)\,(1-\beta)\,(\psi_t c_t)^{\rho}\, U_t^{-\rho} > 0 \tag{12}$$

Denote $\Gamma_t \equiv (1-\beta)\,(\psi_t c_t)^{\rho}\, U_t^{-\rho}$. We can manipulate this expression by plugging in the explicit equation defining U_t and recalling from (8) that we can express the certainty equivalent as $\mu_t\{U_{t+1}\} = \mu_t\{\phi\{\theta_{t+1}, x_{t+1}\}\}\,\psi_t c_t$ for some function ϕ. Then we obtain

$$\Gamma_t = \left(1 + \frac{\beta}{1-\beta}\mu_t\{\phi\{\theta_{t+1}, x_{t+1}\}\}^{\rho}\right)^{-1}$$

With $\mu_t\{\cdot\} > 0$, it is straightforward to notice that we must have $\Gamma_t \in (0,1)$. Returning to condition (12) above, we then have

$$\rho\,(1-\Gamma_t) > -\Gamma_t$$

Therefore, $\Gamma_t < 1$ implies

$$\rho > -\frac{\Gamma_t}{1-\Gamma_t} = \frac{\Gamma_t}{\Gamma_t - 1}$$

A sufficient condition for this to hold is that $\rho \geq 0$.

A.2 Derivation of the first-order condition

Denote $W\{c,\mu\} = [(1-\beta)(\psi c)^{\rho} + \beta\mu^{\rho}]^{\frac{1}{\rho}}$, and note that the partial derivatives are:

$$\begin{aligned} W_c\{c,\mu\} &= \frac{1}{\rho}[(1-\beta)(\psi c)^{\rho} + \beta\mu^{\rho}]^{\frac{1}{\rho}-1}(1-\beta)\rho\psi^{\rho}c^{\rho-1} = (1-\beta)(W\{c,\mu\})^{1-\rho}\psi^{\rho}c^{\rho-1} \\ W_{\mu}\{c,\mu\} &= \frac{1}{\rho}[(1-\beta)(\psi c)^{\rho} + \beta\mu^{\rho}]^{\frac{1}{\rho}-1}\beta\rho\mu^{\rho-1} = \beta(W\{c,\mu\})^{1-\rho}\mu^{\rho-1} \end{aligned}$$

The partial derivatives of μ_{s_t} with respect to z_{t+1} is

$$\begin{aligned} &\frac{\partial}{\partial z_{t+1}}\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\} = \\ &\qquad \left(\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right)^{\alpha} E_t\left\{[v\{z_{t+1},b_{t+1},s_{t+1}\}]^{-\alpha}\frac{\partial}{\partial z_{t+1}}v\{z_{t+1},b_{t+1},s_{t+1}\}\middle| s_t\right\} \end{aligned}$$

The first-order condition (FOC) for the choice of z_{t+1} is

$$\begin{aligned} &W_c\left\{c_t,\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right\}p\{s_t\} = \\ &\qquad W_{\mu}\left\{c_t,\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right\}\frac{\partial}{\partial z_{t+1}}\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\} \end{aligned}$$

which we can write as

$$\begin{aligned} &(1-\beta)(\psi_t)^{\rho}(c_t)^{\rho-1}p\{s_t\} = \\ &\quad \beta\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}^{\rho-1+\alpha}E_t\left\{(v\{z_{t+1},b_{t+1},s_{t+1}\})^{-\alpha}\frac{\partial}{\partial z_{t+1}}v\{z_{t+1},b_{t+1},s_{t+1}\}\middle| s_t\right\} \end{aligned}$$

We can use an envelope argument to get an expression for the derivative of v with respect to z. From the budget constraint we have

$$\frac{\partial}{\partial z}c\{z,\cdot\} = p\{s\} + y$$

At state (z_t, b_t, s_t), the derivative is given by

$$\begin{aligned} \frac{\partial}{\partial z_t}v\{z_t,b_t,s_t\} &= (1-\beta)\left(W\left\{c_t,\mu_{s_t}\{v\{z_{t+1},b_{t+1},s_{t+1}\}\}\right\}\right)^{1-\rho}(\psi_t)^{\rho}(c_t)^{\rho-1}(p\{s_t\}+y_t) \\ &= (1-\beta)(v\{z_t,b_t,s_t\})^{1-\rho}(\psi_t)^{\rho}(c_t)^{\rho-1}(p\{s_t\}+y_t) \end{aligned}$$

We now advance this expression one period and plug it into the right hand side of the FOC to get (2).

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