Chapter 13
The Harrod Model

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13.1 Introduction

The theoretical foundation of the Keynesian growth theory is the so-called Harrod–Domar model. This is the consolidated opinion we find in the economic literature. Nevertheless, after a careful reading of both the original writings of Harrod and Domar, that "model" stands out as a commingling of two models, which had different aims and different hypotheses. As pointed out by Pugno [15, p. 152], the Harrod model is really a result of many works written over the period of the author's whole intellectual life. The first draft dates back to 1938,[1] where, as Harrod always confirmed until his last book published in 1973, the central and crucial aim was to account for the unstable growth path characterizing capitalistic economies.

Part of this chapter has appeared in [13].

[1] Besomi [3] has edited this draft.

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In spite of this, as clearly shown by Besomi [2], Harrod's readers interpreted, almost unanimously,[2] his contributions as a theory of economic growth [2]. In particular, soon after the publication of Domar's paper in 1946, the similarities between their formulas (firstly noticed by Schelling, pp. 864-866 [17]) became a sufficient condition to unite the two approaches. Therefore, by the early 1960s, it was a common practice to speak of the "Harrod–Domar model."

13.2 Domar's Approach to Economic Dynamics

As Domar himself wrote in the introduction of his 1946 paper [7], his aim was to investigate "the relation between capital accumulation and employment." Defined the productive capacity as the total output produced when all productive factors (labor included) are fully employed, Domar looks for the conditions a growing economy must satisfy to preserve the full employment over time. He pointed out that the growth problem was entirely absent from the Keynesian system, because it was not concerned with changes in the productive capacity. The Keynesian approach dealt with the investment expenditure as an instrument for generating income and disregarded the extremely essential fact that investment also increased the productive capacity [6, pp. 72-73]. The twofold impact of investment in the economic system allowed Domar to identify the tools to derive the conditions under which the economy could grow in a full employment equilibrium. First, the net investment I increases the productive capacity P, and second, the change of I increases the income Y by means of the Keynesian multiplier.

Domar carried out his analysis on a very abstract and simplified level, so that he defines the potential social average investment productivity as[3] $\sigma = \frac{\dot{P}}{I}$, i.e.,

$$\dot{P} = \sigma I. \tag{13.1}$$

Since, by virtue of the Keynesian multiplier,

$$\dot{Y} = \frac{1}{\alpha}\dot{I} \tag{13.2}$$

[2]There are few exceptions. Among others, Boianovsky [4], suggested that the Harrod and Domar growth models faced problems of economic instability, not long-term growth.

[3]From now on, a dot over the variable will indicate the operator d/dt and continuous time assumption.

(α being the marginal propensity to save). The necessary equilibrium condition between productive capacity and aggregate demand leads to

$$\frac{1}{\alpha}\dot{I} = \sigma I \tag{13.3}$$

because $P = Y \Leftrightarrow \dot{P} = \dot{Y}$. Assumed that σ and α are constants, it follows that

$$I(t) = I_0 e^{\alpha\sigma t}. \tag{13.4}$$

Therefore, as long as $\alpha\sigma$ remains constant, "the maintenance of full employment requires investment to grow at a constant rate" [6, p. 75]. As, by assumption, the equilibrium between productive capacity and income has existed since the time $t = 0$, the integration between zero and t of Eq. (13.2) yields

$$Y(t) = \frac{1}{\alpha}\left(I_0 e^{\alpha\sigma t} + B\right) \tag{13.5}$$

because $\frac{1}{\alpha} I_0 = Y_0$ and $B = 0$. The conditions for a steady growth are thus demonstrated.

In the second part of his paper, Domar emphasized the possible disequilibria of the economic system. Probably, this is to account for the dynamic instability of his simple mathematical model. We think that instability was the main element that led to combine Domar's approach with Harrod's dynamic theory. However, as we shall see in the next section, the Harrod instability has nothing to do with the mathematical notion of instability characterizing the Domar model.

13.3 Harrod's Approach to Economic Dynamics

Preliminarily, we have to point out that Harrod never formalized his ideas in terms of difference or differential equations. Nevertheless, Harrod (1959, p. 451) [10] acknowledges that there is a similarity between Domar's work and his own contribution to the theory of a growing economy. This similarity only concerns a potential increase of output (productive capacity) per unit of new investment designed by Domar as σ. Harrod wrote (1959, p. 452) that he considered "how many units of new investment are required ... to produce an extra unit of output." In other words, Domar's σ is equivalent to his capital coefficient C_r, because C_r is "valued on the basis that the new investment is no more nor less than that required to produce additional output."[4] As C_r deals with a steady rate of growth of income

[4] Let us mention that C_r was denoted by Harrod as C in his 1939 paper. Specifically, the equivalence is such that $\sigma = 1/C_r$.

denoted by Harrod $G_w = s/C$, where s is Domar's α, the formal identity of the two equilibrium conditions seems to be evident.

Now, we have to point out that this identical result exclusively concerns the equilibrium condition, while it does not entail the equality $\alpha\sigma = \frac{\dot{I}}{I} = G_w = \frac{\dot{Y}}{Y}$, which follows from Domar's assumptions. In fact, Harrod (1959, pp. 452-453) clearly wrote, "I make no such assumption .... In my equilibrium equation there is no reference, explicit or implicit, to $\dot{I}$ or I." To understand Harrod's viewpoint, we have to recall the process leading to his "fundamental equation" (Harrod, 1939, p. 17).

As said by Harrod, capital goods include both equipment and stock-in-trade, the actual saving in a period is always equal to the increment of the capital stock,

$$S = I = C\dot{Y}, \tag{13.6}$$

where S is the aggregate saving and C "the increase in the volume of goods of all kinds (I) outstanding at the end over that outstanding at the beginning of the period divided by the increment of production in that same period" (Harrod, 1948, p. 78). Dividing both sides of Eq. (13.6) by Y, we have

$$\frac{S}{Y} = C\frac{\dot{Y}}{Y} \Rightarrow \frac{S/Y}{C} = \frac{\dot{Y}}{Y} = G, \tag{13.7}$$

i.e., the actual (effective) rate of growth of income. According to the Keynes proposition, saving is necessarily (ex post) equal to investment, but this does not mean that saving will be "equal to ex ante investment ..., since unwanted accretion or depletions of stocks may occur, or equipment may be found to have been produced in excess of, or short of, requirements" (Harrod, 1939, p. 19). To express the equilibrium of a steady advance, Harrod deduced his fundamental equation

$$G_w = \frac{\widehat{S}/Y}{C_r}, \tag{13.8}$$

where G_w is the warranted rate of growth (i.e., the rate of growth of production equating ex ante saving and investment), C_r is the desired capital coefficient (in Harrod's sense), and the expected fraction of income saved $\widehat{S}/Y$.[5]

Let us note that both Eqs. (13.7) and (13.8) refer to the average propensity to save that Harrod denotes by s. This implies that it is not entirely true that Domar's α is Harrod's s, unless s is explicitly assumed to be constant. If this is not the case, the marginal propensity to save may differ from the average. To see the consequence on the steady growth, we can derive the Domar equation from the equilibrium $S(Y) = I$. In fact, differentiation of both sides with respect to time yields $S'\dot{Y} = \dot{I}$.

[5]Harrod refers to the expected saving in his 1973 book. The symbol $\widehat{S}$ is introduced by us.

As from Eq. (13.6), $\dot{Y} = I/C$, by substitution we get $\dfrac{S'}{C} = \dfrac{\dot{I}}{I}$, i.e., Eq. (13.2), and from Eq. (13.7), we have $\dfrac{S/Y}{C} = \dfrac{\dot{Y}}{Y}$. Therefore, if $S/Y \neq S'$, then $\dot{Y}/Y \neq \dot{I}/I$. To confirm this result, we can define a link between the two rates of growth. Dividing $\dot{Y}/Y$ by $\dot{I}/I$, we get $\dot{Y}/Y = (1/E_s)\dot{I}/I$, where $E_s = S'/(S/Y)$ is the elasticity of saving with respect to income. This coefficient is typically greater than one, so that $\dfrac{\dot{Y}}{Y} < \dfrac{\dot{I}}{I}$. This happens because, being residual between earnings and consumptions, the aggregate saving has the tendency to vary quicker than income, either in the case the business activity is rising or declining (see [5, 14]). Although the variability of the average propensity to save questions the possibility of a steady growth, we cannot say that this result reflects Harrod's thought. In fact, Harrod prevalently founds his reasoning on the disequilibrium between ex ante and ex post investments. The disequilibrium is the main ingredient of his "instability principle." To understand this principle better, it may be useful to list some specific Harrod's assumptions often neglected by the growth theorists:

(1) It is crucial to avoid the mistake of considering C (or C_r) as the traditional capital/output ratio or as the technical accelerator coefficient. Harrod (1948, p. 84) explicitly points out that "C_r may not be equal to the capital coefficient in the economy as a whole." Specifically, C is the ratio of additional goods "of all kinds" (i.e., new equipment and additional stocks) to the production increase carried out at a given period. The mean of C_r is similar, but unlike C, C_r is the ratio of desired additional goods to the expected production increase based on entrepreneurs' previous expectations. Neither C nor C_r may be assumed as constant over time.

(2) The quantity C defined by Harrod is measurable and consistent with a stylized fact described by Kaldor [12]: in the long run, the capital/output ratio K/Y has the tendency to remain constant. Incidentally, let us notice that empirical data suggested by Romer [16] confirm this statement. If we admit that tendency to remain constant means that the capital/output ratio may change inside a bounded interval, so that its average is constant over time, the consistency between C and $k = K/Y$ can be easily proved. The differentiation of k with respect to time yields (after some rearrangement)

$$\dot{k} = \frac{\dot{Y}}{Y}\left(\frac{I}{\dot{Y}} - \frac{K}{Y}\right) = G(C - k). \tag{13.9}$$

From the analytical point of view, this result does not require any particular comment. Given $G \neq 0$, the sign of $\dot{k}$ still depends on the difference $(C - k)$ that may change over time.

(3) Equation (13.9) allows us to realize that there is a difference between Harrod's own time scale and the usual notion of long run. Harrod always refers to the "long period" pertaining to the typical industrial trade cycle. The long period is

much less than the long run, and, according to the particular phase of the cycle, meaningful differences between C and k are possible. This is because sudden increases or cuts in inventories with respect to a slower change in the productive capacity affect these differences.

Equipped with these assumptions, we can now explain the instability principle. Looking at Eq. (13.9) and following Harrod (1948, p. 81), we can say that it expresses “the conditions in which producers will be content with what they are doing.” This equilibrium condition must be compared with what actually happens to be confirmed. Therefore, Harrod considered Eq. (13.7) and wrote, “the greater G is, the lower C will be.” Consequently, if G has a value above G_w, “C will have a value below C_r.” This implies that “there will be insufficient goods in the pipe-line and/or insufficient equipment.” Therefore, orders will be increased and the production rises. In other words, if the actual growth is above the line of growth consistent with a steady advance, the actual growth rate will further increase. This leads to a new C that will be further below C_r. If $G < G_w$, the reasoning needs to be reversed. Harrod (1948, p. 86) affirms that this is “an extraordinarily simple and notable demonstration of the instability of an advancing system. Around the line of advance, ..., centrifugal forces are at work, causing the system to depart further and further from the required line of advance.”

As Harrod did not attempt to build the instability principle in mathematical terms, we think this is the reason why a contradiction emerges in his reasoning, because C_r seems to be constant. This implies that the gap between C and C_r will be always increasing. Really, C and C_r are interconnected variables, and the difference between them cannot become explosive.

13.4 A Mathematical Foundation of Harrod’s Instability

To give a mathematical foundation to Harrod’s instability, a slight shifting from his definition of the warranted rate of growth is necessary. Since in Harrod’s G_w there are several ambiguities (see Besomi (1998, pp. 51-53)) [2], we assign to G_w a practical meaning. In other terms, we interpret this rate of growth as the expected rate of growth founded on the firms’ business forecasts. Following Sportelli [20], we set $G_w = \frac{\dot{Y}_e}{Y}$, where $\dot{Y}_e$ is an expected change of income. Furthermore, according to Harrod’s definition of C_r, we assume that in every period the firms decide the investment looking at an expected change of demand:

$$I_j = C_r \dot{Y}_e. \tag{13.10}$$

If ex post it turns that the effective change of demand $\dot{Y}$ is greater than $\dot{Y}_e$, then the effective investment will be less than ex ante I_j, because stocks are below the desired level. This implies that the actual desired coefficient C_r has become greater than the actual C. It follows that, if capacity utilization is near full levels, each firm will decide investments, either to restore stocks levels or to increase (if profitable) its actual productive capacity, to make it consistent with the level of production.

This leads to an increase in I (at least in inventories only), which will work in its turn for a new $\dot{Y}$ according to the monotonic multiplier effect. In the course of the period, a new C will be progressively attained, and, at the same time, as firms are careful to acquire any new information generated by the system to forecast future demand, a new Y_e will arise. The comparison of this new Y_e with the perceived current level of demand allows firms to define the actual $\dot{Y}_e$. In a period of rising business activity, this value is positive, so that a further amount of investment will be justified. This leads to a new value of C_r, which will differ from its past value. By virtue of their definition, both C and C_r change along the given period. Therefore, if we assume a sequence of periods with $\dot{Y} > \dot{Y}_e$, then I_j will be pushed forward to I, while C_r will be pushed ahead of C. This conclusion allows us to infer that there is a path dependence of C_r on the difference $\dot{Y} - \dot{Y}_e$. The greater this difference is, the more violent the thrust forward of C_r will be.

It is clear that $\dot{Y} < \dot{Y}_e$ leads to contrary conclusions. In any case, over a given period, the difference between C_r and C never becomes explosive, because the justified investment evolves according to the following derivative:

$$\dot{I}_j = \dot{C}_r \dot{Y}_e + C_r \ddot{Y}_e, \tag{13.11}$$

where the first term on the right-hand side can be interpreted as stock investments filling existing storehouse gap and the latter term as additional goods aimed at restoring actual stock levels to sustain the new $\dot{Y}_e$ and, eventually, the new equipment required by the change $\ddot{Y}_e$. Along the cycle, the sign of $\ddot{Y}_e$ may be reversed. Sooner or later, this happens, and so the sign of $\dot{I}_j$ will change. Consequently, the growth of C_r will slow down initially, and its value will decrease as soon effective investments I exceed I_j. Hence, changes in the value of C (which follows C_r) will be bounded over time.

We think that the C_r path dependence on the difference $\dot{Y} - \dot{Y}_e$ is the basic component of Harrod's instability principle. Looking at the wide variety of literature inspired by Harrod's dynamic theory, we found only one approach able to give a mathematical foundation to the instability. This is the work by Alexander [1], which received an explicit approval by Harrod (1951, p. 263). The Harrod model described later encloses Alexander's intuitions and takes into account the dynamic link between C and C_r. Furthermore, it stresses the interaction between Harrod's three rates of growth (i.e., the actual, warranted, and natural). Discrepancies between these three growth rates are cause and consequence of economic cycles. All this is in accordance with many of Harrod's theses.

13.5 Cycles

As already mentioned, Kaldor and Harrod laid down the basis for the modern theory on growth and cycles. In particular, Kaldor suggested that growth depends on income distribution and that the shifts between wages and profits determine the savings ratio. Therefore, an equilibrium is achieved when G_n (the rate of growth required for a full employment) equates G_w (the warranted rate of growth).

Keynes argued that in the short run, through the multiplier, more demand (e.g., investments, public spending, and exports) translates into an increase in output. Harrod shares the same opinion regarding the short term, but agrees with Domar about the twofold impact of investment in the economic system. In fact, "he notes that investment not only induces production through the multiplier, but also simultaneously expands capacity. On this basis he shows that investment is sustainable only if it is self-consistent, and for this to hold it must follow a particular growth path which he calls the warranted path" [18]. In other terms, in Harrod's view, it is the discrepancy between the natural rate of growth (G_n), the warranted rate of growth (G_w), and the actual one (G) that generates instability. This instability could be lessened when the economy is open to foreign trades.

13.5.1 Harrod's Knife-Edge

According to Harrod, "for a country in which G_w is tending to exceed G_n, there is by consequence a chronic tendency to depression (because G cannot exceed G_n), a positive value of the balance of trade expressed as a fraction of income (i.e., the net export rate) may be beneficial" [9]. Therefore, Harrod "predicts that incompatibilities between long-term saving and investment opportunity are all but certain to cause prolonged unemployment (which will be structural where G_n exceeds G_w and demand deficient where G_w exceeds G_n) with persistent inflation in addition wherever long-term saving is inadequate for the natural rate of growth" [8]. In terms of public policy, "the difficulties may be too great to be dealt with by a mere anti-cycle policy" [11], and hence the government should increase public investment when $G_w > G_n$ or, conversely, seek to generate more long-term savings when $G_w < G_n$ (see Figs. 13.1 and 13.2).

13.5.2 Discussion

The model we are testing (Sportelli et al. [19]) claims that Harrod's speculation holds true only for a specific set of parameters and with positive net exports coupled with competitiveness in foreign markets. In those specific conditions, regular cycles in the long period can be achieved. In the following, we list some variables/equations

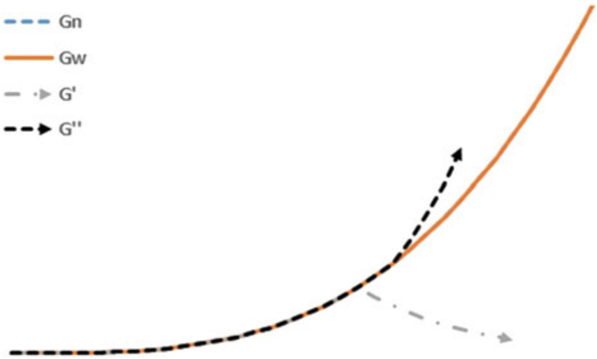

Fig. 13.1 The Harrod knife-edge denoting an unstable equilibrium. When $G = G_n = G_w$, there is sustainable full employment. A departure from this condition may lead to recession (G') or booming periods (G')

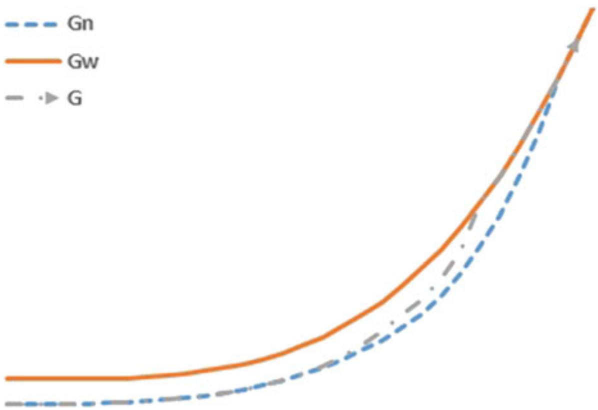

Fig. 13.2 Supply-side policy to raise the natural growth path. When $G = G_w > G_n$, there is a permanent unemployment equilibrium. Policy-makers may employ supply-side policies in order to increase both the actual growth G and the natural growth G_n

that will be used in the ensuing part where some assumptions will be made and new variables will be identified (Table 13.1).

As in [19], we assume that

(A) The desired capital is an increasing function Φ of the difference between the current and the expected changes of demand, i.e.,

$$C_r = \Phi\left(\frac{\dot{Y} - \dot{Y}_e}{Y}\right) = \Phi\left(G - G_w\right) \tag{13.12}$$

such that $\Phi' > 0$ and $\Phi(0) = C^* > 1$, because $\dot{Y} = \dot{Y}_e$ implies $I = I_j$.

Table 13.1 List of variables in the Harrod model

Variable	Description
I_j	Ex-ante investment including both equipment and desired inventory stocks
I	Ex-post investment including both equipment and effective inventory stocks
S	Ex-post saving
E	Exports
M	Imports
$X = E - M$	Balance of trade
Y	Effective demand
$S/Y = \Sigma$	Share of income saved
$x = X/Y$	Ratio of balance of trade to income (or simply the net export rate)
$I/Y = \Sigma - x$	Share of income invested
$G = \dot{Y}/Y$	Actual rate of growth of domestic income
Y_e	Expected demand
$C_r = I_j/\dot{Y}_e$	Desired capital coefficienttnote:DesCap
$C = I_j/\dot{Y}$	Actual capital coefficienttnote:AcCap
$G_w = \dot{Y}_e/Y$	Warranted expected rate of growth of aggregate demand
G_n	Technical progress (rate of growth)
G_f	Rate of growth of foreign demand
ϕ	Sensitivity of the difference between actual and warranted relative changes of demand

"The requirement for new capital divided by the increment of output to sustain which the new capital is required" [9]
"The increase in the volume of goods of all kinds outstanding at the end over that outstanding at the beginning of the period divided by the increment of production in the same period" [9]

So that, ex-post, at the equilibrium, $C_r = C^*$, $I_j = I$, $\Phi(0) = C^* > 1$, and $G = G_w$ (or equivalently, $\dot{Y} = \dot{Y}_e$). Denoted $\varphi > 1$ as a reaction parameter representing how sensitive are firms to discrepancies between actual and warranted relative changes of demand, the linearization of (13.12) in $G - G_w$ can be expressed as

$$C_r = \Phi\left(G - G_w\right) = \left[C^* + \varphi(G - G_w)\right]. \tag{13.13}$$

(B) According to Alexander [1], changes in the growth rate of income depend on the difference between ex-ante and ex-post investments, that is,

$$U = I_j - I = C_r \dot{Y}_e - I, \tag{13.14}$$

so that dividing by Y and considering that $I/Y = (S - X)/Y = \Sigma - x$, the relative gap $u = U/Y$ can be written as

$$\begin{aligned} u = U/Y &= I_j/Y - I/Y = C_r G_w - (\Sigma - x) \\ &= \Phi\left(G - G_w\right) G_w - \Sigma + x. \end{aligned} \tag{13.15}$$

Therefore, $\dot{G}$ can be expressed as a function F of u with F increasing (resp., decreasing) with u, and if we assume F to be linear, we obtain

$$\begin{aligned}\dot{G} &= F(u) = F\left(\Phi\left(G - G_w\right) G_w + \Sigma - x\right) \\ &= \alpha\left\{\left[C^* + \varphi(G - G_w)\right] G_w - \Sigma + x\right\}\end{aligned} \tag{13.16}$$

with $0 < \alpha < 1$, because investment changes in the productive capacity make investment sticky.

(C) The saving rate varies over time depending on unforeseen differences between technical progress and the rate of growth and on income fluctuations:

$$\dot{\Sigma} = \varepsilon\left(G_n - G_w\right) + \delta\dot{G}_w, \tag{13.17}$$

where ε and δ are sensitivity parameters, and the variable $\dot{G}_w$ describes the economic cycle.

(D) We set the following Eq. (13.18), where changes in the ratio of the trade balance depend on G_f, G_n, and G as follows:

$$\frac{\dot{x}}{x} = \Psi(G_f, G_n, G) \quad \text{with} \quad \frac{\partial\Psi}{\partial G_f} > 0,\ \frac{\partial\Psi}{\partial G_n} > 0 \text{ and } \frac{\partial\Psi}{\partial G} < 0. \tag{13.18}$$

We assume that Eq. (13.18) can be rewritten as

$$\frac{\dot{x}}{x} = \Psi(G_f, G_n, G) = \left(\zeta G_f + \sigma G_n - \mu G - m\right) \tag{13.19}$$

with $\zeta, \sigma, \mu > 0$ denoting the sensitivities of the balance of trade to foreign rate of growth, technical progress, and domestic growth rate respectively. We set $m > 0$ because $Y(G_f, 0, 0) < 0$, i.e., a constant domestic production without technical progress has a negative effect on the balance of trade or, equivalently, $\zeta G_f - m < 0$.

(E) The expected rate of change of aggregate demand is defined as an adaptive expectation, i.e.,

$$\dot{G}_w = \gamma(G - G_w), \tag{13.20}$$

where $\gamma \geq 1$ denotes how quick the expected rate of growth adjusts to the actual growth.

(F) The dynamics of technological progress is described by a continuous, increasing nonlinear function of share of income saved and devoted to investments:

$$G_n = G_n(\Sigma) = \beta(\xi - \Sigma)\Sigma, \quad \text{with} \quad \beta > 1 \quad \text{and} \quad 0 < \xi < 1. \tag{13.21}$$

Therefore, Harrod's dynamics [19] can be written as

$$\begin{aligned}\dot{G} &= \alpha\left\{\left[C^* + \varphi(G - G_w)\right]G_w - \Sigma + x\right\}\\ \dot{\Sigma} &= \varepsilon\,(G_n - G_w) + \delta\dot{G}_w\\ \dot{x} &= \left(\zeta G_f + \sigma G_n - \mu G - m\right)x.\end{aligned} \tag{13.22}$$

By replacing on it Eqs. (13.20) and (13.21), we obtain the following specification we want to test:

$$\begin{aligned}\dot{G} &= \alpha\left\{\left[C^* + \varphi(G - G_w)\right]G_w - \Sigma + x\right\}\\ \dot{G}_w &= \gamma(G - G_w)\\ \dot{\Sigma} &= \varepsilon\,[\beta(\xi - \Sigma)\Sigma - G_w] + \delta\gamma(G - G_w)\\ \dot{x} &= \left[\zeta G_f + \sigma\beta(\xi - \Sigma)\Sigma - \mu G - m\right]x,\end{aligned} \tag{13.23}$$

where α, γ, ε, β, δ, ζ, σ, and μ are the parameters that will be calibrated in Chap. 18.

References

1. Alexander, S.S.: Mr. Harrod's dynamic model. Econ. J. **60**(240), 724–739 (1950)
2. Besomi, D.: Failing to Win Consent: Harrod's Dynamics in the Eyes of His Readers, pp. 38–88. Palgrave Macmillan UK, London (1998)
3. Besomi, D., et al.: Introduction to 'an essay in dynamic theory': 1938 draft by Roy F. Harrod. Hist. Polit. Econ. **28**, 245–252 (1996)
4. Boianovsky, M.: Beyond capital fundamentalism: Harrod, domar and the history of development economics. Camb. J. Econ. **42**(2), 477–504 (2018)
5. Deaton, A.: Financial policies and saving. In: Klaus Schmidt-Hebbel, J.S., Luis Serven (eds.) The Economics of Saving and Growth, chap. 4. Cambridge University Press, Cambridge (1999)
6. Domar, E.: Essays in the Theory of Economic Growth. Cambridge University Press, New York (1957)
7. Domar, E.D.: Capital expansion, rate of growth, and employment. Econometrica **14**, 137–147 (1946)
8. Eltis, W.: Harrod–Domar Growth Model, pp. 1–5. Palgrave Macmillan, London (2016). https://doi.org/10.1057/978-1-349-95121-5_1267-1
9. Harrod, R.F.: Towards a Dynamic Economics: Some Recent Developments of Economic Theory and Their Application to Policy. MacMillan and Company, London (1948)
10. Harrod, R.F.: Domar and dynamic economics. Eco. J. **69**(275), 451–464 (1959)
11. Harrod, R.F.S.: Economic Essays, 2nd edn. Macmillan, London (1972)
12. Kaldor, N.: Capital accumulation and economic growth. In: The Theory of Capital, pp. 177–222. Springer, Berlin (1961)
13. Orlando, G., Della Rossa, F.: An empirical test on Harrod's open economy dynamics. Mathematics **7**(6), 524 (2019). https://doi.org/10.3390/math7060524
14. Piscitelli, L., Sportelli, M.: A simple growth-cycle model dysplaying sil'nikov chaos. Econ. Complexity Non-linear Dyn. Multi-Agents Econom. Learn. **14**, 3–30 (2004)

15. Pugno, M.: Harrod's economic dynamics as a persistent and regime-changing adjustment process. In: Economic Dynamics, Trade and Growth, pp. 152–178. Springer, Berlin (1998)
16. Romer, P.: Capital Accumulation and Long-Term Growth. Modern Business Cycles Theory. Blackwell, Oxford (1989)
17. Schelling, T.C.: Capital growth and equilibrium. Am. Econ. Rev. **37**(5), 864–876 (1947)
18. Shaikh, A.: Economic policy in a growth context: a classical synthesis of Keynes and Harrod. Metroeconomica **60**(3), 455–494 (2009). https://doi.org/10.1111/j.1467-999X.2008.00347.x
19. Sportelli, M., Celi, G.: A mathematical approach to Harrod's open economy dynamics. Metroeconomica **62**(3), 459–493 (2011)
20. Sportelli, M.C.: Dynamic complexity in a Keynesian growth-cycle model involving Harrod's instability. J. Econ. **71**(2), 167–198 (2000)