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Games for Teaching/Learning Quantum Mechanics: A Pilot Study with High-School Students

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Abstract: The teaching of quantum physics is challenging, not the least because teachers must overcome the traditional narrative approach, students must gain a conceptual understanding of fundamentals, and citizens must become aware of quantum technologies. Quantum games are powerful tools to overcome obstacles and push one's limits without fear of failure. We report on a pilot study involving twenty high-school student volunteers, consisting of a compact intervention module on the concepts of quantum states, properties, measurement, superposition, and entanglement within the framework of the Model of Educational Reconstruction, followed by playing a game, quantum TiqTaqToe. The outcomes of this research-based learning environment are discussed via the qualitative analysis of students' answers to two open questionnaires. We find that students grasped the concepts of superposition and, with special awareness, entanglement, the game proving effective to help students experience their implications in quantum behavior. The informal and stimulating tournament atmosphere favored intertwining of the game with learning goals. Our central message is that the use of quantum game tools fits a teaching/learning environment in manners often not well understood in the literature; it enhances awareness of the nature of new and non-intuitive concepts, increases complementarity with other languages within the process of thinking about physics, boosts student engagement, and improves intervention efficiency and effectiveness.

Keywords: quantum mechanics; secondary school; student learning; quantum games

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1. Introduction

The teaching/learning of Quantum Mechanics (QM) in secondary school is a challenge that requires overcoming the narrative approach to quantum concepts often used in traditional approaches [1]. Younger generations especially need to understand the fundamental concepts and pick up on the coherence in the conceptual construction of quantum theory as a way of thinking [2]. This is crucial to comprehending the foundational role of quantum theory in building knowledge of the world [3,4] and to provide all those conceptual and operational tools that citizens must be aware of with respect to the quantum technologies involved in everyday devices or those will be on the market in the future [5–7].

An increasingly wide literature has addressed this goal, providing very different responses in terms of choice of approach, content, and the weight placed on the analysis of concepts, rather than the analysis of complex contexts such as light–matter interactions, atomic physics, and the role played by formalism [2–4,8–14]. Interpretative choices have determined the way in which concepts are approached and learned and the phenomenological contexts in which they are addressed [15–18]. The advent of new quantum technologies such as quantum computing, quantum cryptography, and teleportation pose new challenges when pushing to identify new approaches to quantum physics [5–7,19,20].

A certain tradition has been established in all high-school school contexts in which elements of quantum physics have been introduced [10,13–15,18]. Based on a historical reconstruction of the milestones in the interpretation of quantum phenomena, this traditional approach often addresses in a descriptive manner the steps that led to the formulation of the 'old physics' of quanta [14,18]. The lack of awareness of the interpretative hypothesis produces semi-classical ways of thinking that contrast with the quantum view of phenomena, which activates distorted ideas as to the nature of science. This emerges clearly from systematic research conducted with high-school students as well as from selected experiments involving university students [21–27]. In order to overcome the descriptive dimension, proposals have been pushed forward looking at how to base educational approaches to QM on a suitable formalism [14,18,25], which we know plays a central and conceptual role in the theory [28,29]. The diffused availability of information and communication technologies provides tools that can help in overcoming formal difficulties by developing simulations or open environments for ideal experiments on quantum phenomena, which can bridge the gap between formal aspects and concepts [30–32]. An interesting way of looking at quantum physics is to directly point to the foundational concepts of the theory and discuss them in specific phenomenological contexts, as with two-state systems (polarization of light, spin, interferometry) [2,3,33–36].

Two central concepts of quantum mechanics, totally absent from everyday life, are quantum superposition and entanglement. These in turn demand the related concepts of quantum states and measurement [17,18,22–26,36]. In quantum mechanics, experimental activities are not easily at hand, and the least required math can be quite challenging even in high schools [37]. The question thus arises whether complementary approaches can be designed to support effective learning processes. Here, we explore the idea that games designed for educational purposes (Games With A Purpose, GWAP) may be tools to stimulate creative thinking and enrich cognitive experiences through the involvement of students in recreational and/or competitive activities.

Games are involved in a wide spectrum of human activities, including in the natural expression of the human behavior [38], in mathematical expression of the human way of thinking [39], and in economic behavior. The textbook by mathematician John von Neumann and economist Oskar Morgenstern is the groundbreaking work creating game theory [40]. Games are connected to human abstraction and formalization. McGonigal [41] discusses at length how the power of GWAP approaches to citizen-science research and education are expressed by several traits. A game is composed of:

- a goal, providing players with a purpose;
- a set of rules, i.e., constraints, which can be opportunities as well, as they can be engineered to unleash creative, visual, and strategic thinking;
- a feedback system reinforcing motivations and guiding players to successfully complete the goal;
- voluntary participation, guaranteeing the freedom to leave or keep up while remaining safe, all essential traits for enjoyable experience and enhanced motivation and creativity.

The playful moment has a significant influence in a person's development [42], with a transitional function between the concreteness of action and thoughts that are totally free from action, i.e., the ability to perform abstraction. The transition from action to abstraction is an internal process which favors the development of logical memory and abstract thought. The playful context of the game offers an opportunity to decontextualize with respect to school activity, motivates active personal learning processes, and produces connections with playful-symbolic skills.

The rules of the game, which cannot be missing, relate to the emotional sphere, becoming a goal (work) and a source of learning [43] while making the game more and more attractive. Perception is the spring that pushes us to act for this transition. Playing increases the degree of awareness related to one own's actions. Playful activity allows us to experiment with various frames and/or living conditions without conditioning us [44]. The person thus expands his/her vision of the world and "experiences the way of structuring thinking towards the universe" [42,45]. Games offer access to knowledge tools and to metacognition, which can help us to understand how physics operates.

In the last decade GWAPs have flourished, being designed and used for both scientific research [46–53] and science education, in particular in the quantum domain [54–56]. Here, they have been boosted by productions made within quantum game jams [57–59] inspired by pioneering work on games on quantum computers [60]. Quantum games offer students the opportunity to explore and experience counterintuitive quantum behavior in a context allowing them to manipulate it in the form of a hands-on/minds-on activity. As peculiar and promising as GWAPs for quantum physics education might be, their diffusion calls for studies assessing their role in learning.

In this work, we report on one compact experiment that we have performed on the role played by games as engaging contexts in which to explore and experience quantum behavior, thereby familiarizing students with basic quantum concepts. We investigate how the game and game strategies have influenced, in an operational manner, the conceptualization process on quantum states, superposition, entanglement, and measurement, introduced by means of a planned research-based approach using a two-state system based on polarization phenomena.

The paper is organized as follows. We illustrate materials and methods in Section 2, including starting research questions, a description of the game, and the adopted research strategy. We then report our results in Section 3, based on the analyzed data from the experiment. Finally, in Section 4, we discuss their significance as well their present and future implications.

2. Materials and Methods

We designed a compact teaching/learning experience on the concepts of quantum states, superposition, entanglement, and measurement based on the use of the TiqTaq-Toe [61] game application (see Figure 1 and Section 2.3) and hosted within the environment of the QPlayLearn platform [55].

Figure 1. The game of Quantum TiqTaqToe. The game allows students to become familiar with several of the fundamental concepts of quantum physics, such as the concepts of state, measurement, superposition, and entanglement. Left grid, superposition: blinking orange X symbols represent one X particle in a superposition state of two different grid positions. Central grid, entanglement: blinking orange Q-ish symbols represent two entangled X and O particles. The snapshot on the right illustrates a case in which one X and one O particle (blue symbols) have entered a state in a single grid position, that needs to be measured, and therefore is not fixed; one O/X particle is in a superposition state of two different grid positions (blinking orange/blue symbols) (accessed on 17 April 2022), while one X and one O particle are entangled (blinking yellow Q-ish symbols). From www.qplaylearn.com (accessed on 17 April 2022). Credits: Evert van Nieuwenburg [61].

The pilot study was carried out as part of the IDIFO (Didactic Innovation in Physics and Orientation) project led by Physics Education Research Unit of the University of Udine, which implements the Scientific Degree Plan in nineteen collaborating Italian Universities [https://urdf.uniud.it/pls (accessed on 17 April 2022)]. The didactic experiment was organized with the collaboration of the Liceo Scientifico "Leonardo da Vinci" in Treviso, a town in Northern Italy. It was carried out in remote mode due to the COVID-19 pandemic, specifically over two afternoon webinars. Twenty students from the last three years who had different math and physics starting backgrounds participated.

Two introductory interactive lectures of 3 h in total were prepared leading up to the 1 h TiqTaqToe tournament. Two open-ended items questionnaires were the instruments used to monitor students' variational learning process and to acquire information on the role played in their learning process by the quantum game. We then analyzed the results of the questionnaires with reference to the main qualitative research methods [62] based on conceptual change [63,64].

2.1. Research Setting

The research setting presented two sides of one same coin. An extensive literature on learning processes in the many different implementations of differentiated teaching/learning approaches [10,13,16–18,22–26,35,36] has highlighted a persistent lack of mastery of key concepts and crucial aspects, such as quantum states, superposition, entanglement, and quantum indeterminism. Therefore, strategies and methods of different nature are needed with respect to cognitive frameworks.

The use of quantum games and interactive tools has been growing, and has been promoted in a cross-disciplinary manner over the last few years as a tool to educate students and citizens about quantum science and technologies. Currently, this field of activity is the subject of attention of the pilot project QUTE4E—Quantum Technology Education for Everyone of the QTEdu-Coordination and Support Action of the European Quantum Flagship [65,66], aimed at creating "the learning ecosystem necessary to inform and educate society about quantum technologies".

The theoretical framework underlying the rationale of the research-based proposal on basic quantum concepts is the Model of Educational Reconstruction [67], which uses previous development of the Dirac approach in the context of optical polarization [3,33,34,36]. In addition, we have used materials, videos and videopills developed by the QPlayLearn platform [55]. The integrated playful proposal is based on a study of the role of games in learning processes [38–42,60], the research of which produced the game TiqTaqToe [55,61]. The consequent research-based planning of the introductory lessons is as follows.

Part I: the concepts of quantum state, property, and measurement. Preparatory to the TiqTaqToe play, an introductory presentation has been focused on the foundational concepts of quantum state, property, and measurement with its stochastic nature, as well as superposition and entanglement in a two-state system. With this aim, the polarization of light has previously been exploited as a toolbox (JQM) [68] designed and analyzed in a previously-developed simulated quantum microworld. JQM offers the opportunity to compare macroscopic phenomenology with ideal single-photon experiments, the former explorable in real labs at high intensity with light detectors (photodiodes) and the latter with polaroids, birefringent crystals, photon beams and detectors [68]. The rationale of the presentation is a simplified version of the didactic proposal developed [3,33,34,69] and tested for several years by our group [29,62–64,67,70], as described in the flow-chart of Table 1.

Table 1. Rationale and flow chart of the research-based educational path.

- Exploration of the interaction of light with ordinary filters and polaroids, looking to the transmitted light intensity;
- Linear polarization of light as a prepared property detected by polaroids in the macroscopic world, and Malus' law;
- The Roger, Grange, and Aspect experiment [71] and confirmation of the photonic nature of light in Einstein's hypothesis concerning the photoelectric effect [72];
- Malus' law is valid while reducing light intensity and polarization, and the property is a single-photon property;
- Exploration of different interactions of polarized photons with polaroids that have different allowed directions in transmission and identification of
    - Mutually exclusive properties, when the result of the interaction is certain
    - Incompatible properties and uncertainty principle, when the result of the interaction is predictable by means of the probability associated with Malus' law
- The state of the polarized photon identified by a vector living in an abstract space, as opposed to the lab, where only the measured properties are present. Introduction of the superposition principle, $\mathbf{w} = \alpha\mathbf{u} + \beta\mathbf{v}$, where **w** is a generic superposition of the base states **u** and **v**;
- Comparison of the results of the interaction of polarized photons with polaroids, and comparison of the expected results with the hypothesis of the polaroid state as a vector with respect to that of a statistical mixture or simultaneous existence of two properties;
- QM measurement as a transition of the polarized photon into a new state: precipitation of the system in the measured states and genuine stochastic nature;
- Phenomenological exploration of light polarization by means of birefringent crystals and polaroids at the macroscopic level, in order to identify ordinary and extraordinary refraction with respect to polarization;
- Interaction of polarized photons with birefringent crystals, helping with understanding of
- Entangled state;
- No trajectory;
- Non locality;
- Formalism: transition probability from state **u** to state **w** as a projector;
- Writing the state of a generic polarized photon and identifying the probabilistic meaning of the coefficients of the two base vectors.

Part II: From the concepts of quantum superposition and entanglement to the game. We now turn to the introduction of the concepts of quantum superposition and entanglement, which are central to the quantum game used in this study. Considering the diverse audience and the need to be as directly as possible for orientation towards the gameplay activity, we used a combination of non-formal and formal resources, which are available at QPlayLearn [55].

As discussed at length in [5], this is a platform aimed at students and teachers of all grades, educators, and the general public. To this end, it was conceived around a dictionary of basic quantum mechanics concepts, listed in the *Quest* menu, each of which is introduced using different approaches in different corresponding sections: playful in the Play section containing quantum games, descriptive in the *Discover* section with quantum pills and interviews with scientists, mathematical in the *Learn* section (presently aimed at high-school or more advanced students), applied in the *Apply* section, which uses the Strangework platform [73] for quantum computing via Qisqit [74], and imaginative in the *Imagine* section through activities such as the treasure hunt *Photonic trail* and the *Quantum Jungle*. In each approach, a different language or their combination is preeminent, allowing users with different backgrounds and instructional degrees to be more easily engaged visually, verbally, formally (i.e., mathematically), and artistically.

The Discover section contains the Quantum Pills, which are five -minute animations dealing with a central concept; these play the twofold role of engaging students and of globally revising the basic concepts (of mutual exclusive and incompatible properties, quantum state and superposition, measurement, genuine stochastic nature of measured properties, etcetera). This allows students to catch up with the essential meaning and implications of the elementary concepts before they are introduced to a formal description of quantum states in the form of Dirac notation, inspired by the experience in [75]. The latter amounts to a generalization of the concept of vectors and vector properties in two dimensions, and formally represents a quantum state vector in a two-dimensional basis, e.g., a quantum computational or a spin 1/2 basis. Additional activities to reinforce the formalism are described in Section 2.4.1, and the TiqTaqToe quantum game is described [61] in Section 2.3.

As anticipated, the second step of the research plan was the game. Of the quantum games available today, several are available via online platforms [60]. However, there are few systematic studies where the role of quantum games in the learning process has been assessed and the results fed back for optimal design, either for quantum games designed from scratch or for pre-existing game types. In fact, such study should occur prior to game development, in order to allow for (i) identifying the traits that quantum games should possess in order to support the teaching/learning process in engaging students while complementing other tools in the presence of fragile experimental and/or formal literacy; (ii) benchmarking quantum games' effectiveness in relation to teaching/learning activity and context; and (iii) benchmarking the role that quantum games can play in the learning process, along with their added value and any relevant limitations.

2.2. Research Questions

In the present work, we pose the following research questions.

(RQ0) How do students see the main concepts addressed in the introductory lesson? What are their ideas on these concepts?

Our goal was to understand how the concepts addressed in the webinar impressed the students, how they have activated their reasoning using their knowledge of the content and methodological competence, and how the concepts raised their interest. In particular, we wanted to investigate questions about the concepts of states and properties, the differences between a state and a property, the probabilistic nature of measurement and stochastic evolution of systems after interaction with a measuring apparatus (e.g., photons on polaroids and birefringent crystals), the superposition principle, and entanglement and non-locality.

As a marginal aspect, we set the stage in such a manner as to explore whether the students were able to identify and distinguish between mutually exclusive and incompatible properties, on the one hand, and states as abstract vector spaces (Hilbert space) on the other.

We further aimed to identify which the different applications among those proposed (qubits, teleportation, quantum computing and logic) impressed students the most. Our goal was to determine whether any of the illustrated applications were spontaneously referred to by the students.

Regarding the game, which focused on quantum states, superposition, entanglement, and measurement, our working hypothesis was that it might help students to understand the selected concepts.

(RQ1) We wanted to investigate whether, how, and to what extent a quantum game could be useful to raise awareness of the role played in QM by basic concepts such as superposition and entanglement, which are known from the literature [22–26,36] to suffer from conceptual difficulties and mystification. We selected Quantum TiqTaqToe by Evert van Nieuwenburg [61] as the most well-suited quantum game among the existing options, as we found it to possess a clearer setting for identification of the different concepts while allowing for a progressive degree of complexity in which superposition and entanglement-related rules can enter one at a time.

(RQ1) We wanted to explore how a quantum-game tool could contribute to learning goals related to basic concepts in quantum physics and how it fits into the teaching/learning environment when associated with other activities, its role in engaging students and in boosting intervention efficiency (as opposed to effectiveness), and the degree of complementarity of the game vs. other languages used in different steps of the process of thinking about physics. In fact, the primary question (RQ0) above can alternatively be viewed as whether or not the quantum game can be considered a form of experimental environment when used to help create a proper understanding of quantum concepts. Following up on the process of thinking about physics, it is possible to ask whether the quantum game can be used as a form of symbolic tool to test understanding. For example, it might provide a prior form of conceptualization useful for implementing quantum-specific strategies in game-playing.

In the following section, we familiarize readers with the characteristics of quantum TiqTaqToe before diving into a description of the strategy that we adopted to maximize the impact of the game's traits on the teaching/learning activity.

2.3. Description of the Game TiqTaqToe

Quantum TiqTaqToe is a quantum version of the famous game "tic-tac-toe" (known also as "tris" or "noughts and crosses") in which two players alternatively take turns marking the spaces in a three-by-three grid with the symbol *X* or *O.* The winner is the player who manages to first place three of their marks in a vertical, horizontal, or diagonal row. Considering the simplicity of the game, good play from both parties always leads to a draw. However, this usually does not happen in the quantum version because of the quantum moves available to the player.

There are several quantum versions inspired by classical tic-tac-toe; the one we refer to here and used in our pilot study is the game quantum TiqTaqToe, developed by Evert van Nieuwenburg, assistant professor in condensed matter physics at the Niels Bohr International Academy [61]. At its core, quantum TiqTaqToe is very similar to regular classical tic-tac-toe. By adjusting the quantumness slider at the beginning of a play, the players can set which of the quantum moves they will have at their disposal during the game. Overall, there are four moves (see Figure 1 for the main examples):

- Single moves: press a single square. If a box symbol turns red (blue), it can no longer (it can) be used for quantum moves.
- Superposition: press a single empty box, then drag the mouse, hold, and release it on another empty box.
- Entanglement: press a single box, then drag the mouse, hold, and release it on another box. This seems similar to the superposition move, however, in this case one of the two involved boxes must be already filled with the other player's symbol.
- Measurement: completing the grid generates a measurement operation. The resulting single box symbols turn red and remain fixed for the rest of the round. Only these symbols count towards a win!

The level of quantumness identifies the different quantum moves that can be enabled, which have four different overall levels:

- No Quantumness: essentially, the classical game of tic-tac-toe;
- Minimal Quantumness: the superposition move is available;
- Moderate Quantumness: the superposition and entanglement moves are available;
- High Quantumness: the Moderate Quantumness moves are available, as is the option of entangling a square with part of the other player's superposition, creating a three-square entangled state.

In each of the four levels, measurement takes place only when the grid is filled. At the No Quantum level, it only happens once at the end of the game, as there are no probabilistic outcomes resulting from a measurement. In the other levels, it can happen several times in the same round. Indeed, with the additional quantumness, players can obtain probabilistic outcomes from a measurement which makes the *X* and *O* states collapse in a single square. Note that in the No Quantum and Minimal Quantumness levels a single move is definitive, and corresponds to the single symbols immediately turning red. In the Moderate and High Quantumness levels, on the other hand, each move is not definitive, as measurement is needed in order for the symbols to turn red. At all levels, only three red symbols in a row ever count towards a win.

2.4. Research Strategy, Criteria, and Methods

Having introduced our research questions in Sections 2.1 and 2.2 and described the quantum-game tool that best fits our didactic aim in Section 2.3, we now proceed to illustrate in detail the strategy that we adopted to that purpose and how it was implemented in our intervention.

In order to effectively work out the answers to the research questions (RQ0)-(RQ2) posed in Section 2.2, we chose to split the intervention in two sessions, S1 and S2, each conducted in one afternoon and followed by a questionnaire designed to validate the intervention via a conceptual class analysis of the students' answers.

More specifically, during approximately two hours in the first afternoon session (S1), we provided the class with two short introductory lectures (see Section 2.4.1 below). The first lecture was focused on the concepts of quantum states, properties, and measurement, which are crucial accompaniments to the concepts central to the quantum game (1 h). The second lecture was focused on the concepts of quantum superposition and entanglement, which again are central to TiqTaqToe (1 h). After the first session, the students were provided with the first questionnaire (see Section 2.4.3 below).

In order to enhance the visibility of the effect, if any, we conceived the introductory lectures to be compact in order to provide students with the minimal background necessary to more rapidly build awareness about the game rules while at the same time leaving plenty of room to use the quantum game as a tool.

During the second afternoon session (S2), after splitting the class into teams, a TiqTaqToe tournament was played for approximately two hours (see Section 2.4.2 below), after which a second questionnaire (Q2) was submitted to the students (see Section 2.4.3 below).

In order to maximize student engagement, special attention was devoted to the design and conduct of the game session.

The relational strategy between the two sessions, S1 and S2, was conceived to tackle questions posed in (RQ1) and (RQ2). First of all, the introduction of the basic concepts prior to the game-playing was a functional part of answering question (RQ2); in particular, it helped to explore whether students could apply prior conceptualizations to conceive quantum-specific strategies in the gameplay, as if the latter were a symbolic playground. Second, question (RQ1) can be answered by inspecting the answers to Q1 and Q2 and comparing them, in particular, whether and how playing the game reinforced students' understanding of the quantum physics concepts, in essence using the quantum game as an experimental playground.

Last, in order to dig into questions (RQ1) and (RQ2), and in particular to discern how they might be affected by different setting conditions, we introduced two special cases. First, we diversified the learning group of students by forming it from the three last years of instruction in scientific studies in the same high school; thus, the students had different math and physics backgrounds. The number of students in the group was limited to 20, considering the main lab-type traits of the activity (see Section 2.4.4). Second, we diversified the teaching approach and corresponding language when introducing the different basic quantum concepts (see Section 2.4.1). For the foundational concepts of quantum states, property, and measurement, which, while functional, were not central to the game, we used the example of photon polarization, which is well established in the literature [3,33,34,36]. For the concepts of quantum superposition and entanglement, which were central to gameplay, we adopted an explorative approach combining non-formal QPlayLearn resources [55] with the formal introduction of Dirac notation in order to better inspire to the students' learning experience [75]; see as well [6,18,28,29].

2.4.1. The Introductory Lectures

Part I was implemented according to the planned path described in Table 1, starting from simple experiments with polaroids and birefringent crystals, then working with ideal experiments in JQM to build the concepts of mutual exclusive and incompatible properties, quantum states, the superposition principle, the stochastic nature of measurement, the non-trajectory and non-local nature of quantum physics, and entangled states.

In Part II, the lecture flow began with watching the "quantum pills" videos on quantum physics, superposition, and entanglement (10′). Then, the students were introduced to a formal description of quantum states in Dirac notation (duration 30′). This part included activities with Qiskit, introducing the concept of operators and of quantum logic ports along with explicit examples of how to compose superposition states with a Hadamard gate and entangled states with Hadamard + CNOT gates. The next 15′ min of time were devoted to engaging students with the application of superposition and entanglement concepts to a teleportation protocol, partly using the animation "Teleportation explained—How to teleport a Schrödinger's cat" from the OneMinutePhysics youtube channel [76].

Finally, we were ready to provide a formal description of TiqTaqToe [61]. With reference to Figure 1, X and O were formalized as our quantum particles, which can be in a "position" state $|n\rangle_X$ or $|m\rangle_O$. Here, n, m are numbers from 1 to 9 that identify each grid box from left to right and top to bottom: in the top row, the box numbers are 1, 2, 3 from left to right, while in bottom row they are 7, 8, 9. With this in mind, the quantum superposition state of particle X on, e.g., the left panel of Figure 1 can be cast as $\left|Q\right\rangle_X = \frac{1}{\sqrt{2}}(|3\rangle_X + |8\rangle_X)$, while the entangled state between particle X and particle O on the right panel of Figure 1 is $\left|Q\right\rangle_{XO} = \frac{1}{\sqrt{2}}(|9\rangle_X|9\rangle_O + |9\rangle_X|5\rangle_O)$.

2.4.2. Game Conduction: How and Why

In our pilot study, we used the first three levels of the game, e.g., (a) No Quantum, (b) Minimal Quantumness, and (c) Moderate Quantumness. We decided to avoid the last level, (d) High Quantumness, as it introduces quite complicated three-box entangled states the comprehension of which is beyond the scope of this work. On the other hand, we thought that it could represent a nice at-home challenge for students eager to further their exploration of the topic after the guided activity.

After the formal introduction of Quantum TiqTaqToe, as described in the previous section, we were ready to start the Tournament. We first introduced the rules of the game, explaining the different quantum moves available for the players according to the quantumness slider setting. In particular, we created illustrative situations using the superposition and entanglement moves to allow the students to familiarize themselves with the concept of measurement and to visualize the meaning of probabilistic outcomes in the game. Increasing the quantumness to the Moderate level, we focused on the different meaning of having a blue or a red symbol representing the single-box state of the particle, X or O, before and after a measurement. In the initial tutorials, we encouraged the students to identify how the quantum moves introduce new options in developing game strategies. In order to start the tournament, the students were split into pairs randomly formed by their high-school teachers. The first round consisted of eight games:

- two with the No Quantumness setting;
- three with the Minimal Quantumness setting;
- three with the Moderate Quantumness setting.

For each game, the points were assigned as follows: 1, 2, and 3 points for each No Quantumness, Minimal Quantumness, and Moderate Quantumness game, respectively. After all of the initial pairs completed their match of six games, we collected the eight highest scores, who passed towards the second round. In case of draws, we allowed for a tie-breaker consisting of a single game with the Moderate Quantumness setting.

In the second round, we formed new pairs by associating the first with the eighth, the second with the seventh, etc. Each match was made up of two games with the Minimal Quantumness setting and three games with the Moderate Quantumness setting. The points were the same as in the first round, namely, two for each Minimal Quantumness game and three for each Moderate Quantumness game. The two highest scores qualified for the 1st–2nd place final and the next two for the 3rd–4th place final. In case of draws for the qualifications, we allowed for a tie-break using the Moderate Quantumness setting, as in the first round.

The two finals consisted of three games each, one with the Minimal Quantumness setting and two with the Moderate Quantumness setting, scored as in the previous rounds.

Students actively participated throughout the whole tournament, and showed enjoyment during the playful part of the activity. As we discuss in the analysis section, the use of the game seemed effective for promoting the students' understanding of the concepts.

2.4.3. Learning Process Monitoring Tools and Analytical Methods

Our research questions work as path-finders in an almost unexplored land. For this reason, for the present study we started by adopting a conceptual class analysis approach [37] to investigate the students' understanding and awareness of the learning process. To this end, we designed two different questionnaires, Q1 and Q2, delivered after sessions S1 and S2 and containing ten and eight open-ended items, respectively. Questionnaire Q1 (see Table 2) was designed to evaluate, in relationship to RQ0, the concepts that students learned during the introductory compact lectures, which were delivered with the two different approaches described in Section 2.4.1, as well as the concepts learned and the conceptual nodes remaining unresolved. Questionnaire Q2 (see Table 3) was designed to evaluate the role of the game of TiqTaqToe as a motivational and especially a learning tool. In addition, Q2 served to collect information on the whole didactic proposal, including the most and least appreciated highlights as well as proposed changes.

Table 2. Questionnaire Q1.

Item Number	Question
I1	What did I learn about Quantum Mechanics with this experience
I2	Tell your classmate what is meant by mutually exclusive properties and what is meant by incompatible properties (and what is different)
I3	Illustrate to your classmate what is meant by state of a quantum system
I4	How would you explain to your classmate the difference between property and state of a quantum system?
I5	Illustrate to your classmate the characteristics of measurement in quantum mechanics
I6	Illustrate to your classmate the principle of superposition of quantum states
I7	Tell a classmate of yours what an entangled state is
I8	Illustrate to your classmate the difference between a state of quantum superposition and an entangled state (you can also help yourself, if you feel like, with the example of the Tiq-Taq-Toe game)
I9	Illustrate to your classmate, why is it impossible to attribute a trajectory to a quantum system
I10	What would you like to be changed to improve your mastery of some of the concepts addressed?

Table 3. Questionnaire Q2.

Item Number	Question
I1	What game strategies did I use in the different game stages, in order to win
I2	What aspects of Quantum Mechanics did I recognize in the game. Choose from the following aspects and for each of them explain how did you recognize it: 1. Property 2. State 3. Distinction between property and state (explain) 4. Measurement 5. Superposition principle 6. Entanglement
I3	Was the game helpful in clarifying the following concepts? Explain the answer in detail for the concepts of: 1. Property 2. State 3. Distinction between property and state (explain) 4. Measurement 5. Superposition principle 6. Entanglement
I4	Were the concepts outlined in the seminar useful for gaming strategies? Answer by explaining in detail which concepts and how they were useful
I5	What would you like to be explained further?
I6	Indicate the three aspects you liked the most in the whole activity and why
I7	Indicate the three aspects you liked least in the whole business and why
I8	Advices and suggestions for when we will repeat the activity with students like you

As Table 2 shows, Q1 questions were formulated in such a way as to ask students to explain to a classmate the different foundational QM concepts enucleated in the introductory presentations. With this choice, we wanted to ease the students into using a colloquial/daily language without feeling the need to provide definitions, which are otherwise recoverable on the internet. Items D1-I9 provided information towards answering RQ0. Questions I1 and I8 provided guidance in answering RQ2. The last item, I10, allowed us to gain feedback about the concepts from the webinar.

The first part of the Q2 questionnaire was aimed at understanding the role of the game and the strategies used, in order to both grasp the students' perceptions and provide feedback for the researchers. In particular, we wanted to investigate how competition strategies and learned concepts were integrated, or, vice versa, how the concepts learned in the lectures guided the students' competition strategies.

In the analysis process, after reading the 20 answers to each of the 18 questions, we created a shared classification of the answers, including a conceptual independent alternative or interpretative vision of the question faced [62]. These represented the different possible conceptions of the subject, and were defined operationally by typical student responses. For the different categories, we therefore evaluated the frequency of occurrence to identify those answers that were more frequent. We were then able to perform comparisons between related issues.

2.4.4. Context and Experimental Protocol

Having detailed the different parts of our research strategy, in this section we now summarize the experimental conditions. The context was a high school for scientific studies, "Leonardo da Vinci" (TV, Italy), in remote mode due to the COVID-19 pandemic. The learning group consisted of 20 students, 5 females and 15 males, of different ages, from 16 to 19 years old: by degree, there were thirteen K12, five K11, and two K10 students. The experimental protocol is reported in Table 4.

Table 4. Experimental protocol.

Item	Description
1. Session S1	Introduction to concepts of quantum states, properties, and measurement using photon polarization (1 h) [3,29,33,34,36,69] Introduction to concepts of superposition and entanglement with the QPlayLearn resources [53], i.e., using Quantum Pill videos on quantum physics, superposition, and entanglement (10′) (Section 2.4.1) Formal description of quantum superposed and/or entangled states as represented in TiqTaqToe using Dirac notation (30′) Application of superposition and entanglement concept to a teleportation protocol using a One-minute physics video (15′) [76]
2. Q1 delivery	Ten questions (see Table 2)
3. Session S2	Team tournament with the TiqTaqToe game [61] (see Section 2.4.2)
4. Q2 delivery	Eight questions (see Table 3)
5. Q1–Q2 analysis	Conceptual class analysis of students' answers (Sections 2.4.3 and 3)

3. Results: Data and Data Analysis

We now discuss the analysis performed on the two questionnaires, Q1 and Q2, proceeding with each individual in Q1 from I1 to I10 and in Q2 from I1 to D8, respectively. In the following, considering the small number of students, we provide the results in terms of absolute numbers. However, in order to ease reading and comparisons, we occasionally provide results as percentages as well.

3.1. Q1-I1: What Did I Learn from This Experience?

In Q1-I1 item, 75% of the answers concern specific issues, 22% general and epistemic issues, and 3% surprising and/or peculiar issues (see Figure 2). Answers are most often rich and well articulated (16/20), including several aspects. While referring to Appendix A for details, two examples are discussed below.

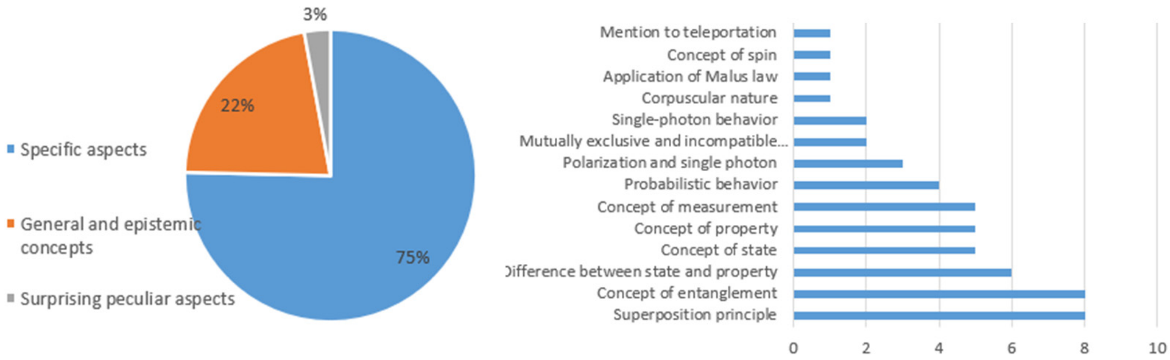

Figure 2. Distribution of answers to item Q1-I1: *What I did learn about Quantum Mechanics from this experience?* Left panel: Circle diagram of typologies of answers. Right panel: zoom on specific aspects.

Davide's sentence exemplifies the most populated category of those who indicated specific matters:

Davide: "*I think that the TiqTaqToe tournament is a very effective method to establish well the principles of superposition and entanglement. Personally, they weren't very clear to me with the theoretical explanation but thanks to the game I was able to understand them better. In addition to this, I learned another conception of terms such as state, properties and measurement precisely following the reasoning of the teachers*".

Matching this sentence with the repeated and correct application of the concepts performed by Davide in the game leads us to infer that his understanding benefitted from playing the game and from the sequence of activities he was exposed to. Note that the game was recalled first, even though it was the last activity. The gameplay is indicated as the activity allowing for comprehension of superposition and entanglement. Presumably, this might be due to the exemplification of concepts operationally which were not sufficiently clear after the original solely theoretical explanation. In addition, the game may have allowed this student to grasp the foundational concepts of quantum states, properties, and measurement.

Paolo's sentence exemplifies the class of those students whose questions indicated general and epistemic aspects:

Paolo "*I understood how much the world in which we live is very complex and can be investigated from different points of view (in the case of QM from a microscopic point of view). Despite the complexity of the world, governed by particles motion, I have understood on how the work of twentieth-century physicists managed to describe most situations through simple formulas with a universal character. I also confirmed the fact that to do physics you need a lot of imagination: imagination leads to surprising discoveries. I really admire those physicists who, thinking about the corpuscular nature of matter, brake through the senses barrier to navigate "quantum" seas, precisely*".

In fact, this sentence highlights important aspects of the nature of physics and the role played by QM, which the student has identified and re-elaborated based on his knowledge, albeit expressed with excessive emphasis or naivety. We can notice the different ways of looking at science, in which he grasps the following: the crucial node of the underlying microscopic vision in contrast to the macroscopic one of classical physics, leaving aside the clarification of the limits of the applicability of QM (which emerged in two more answers), namely, the universality and relative simplicity of physical laws despite the complexity of natural phenomena, the role of imagination in scientific discovery, which recalls the vision fostered by Einstein and Feynman, and the need to go beyond our senses, attributed to humans having a corpuscular nature, an expression that seems to imply a reference to Parmenian philosophy.

Among the specific aspects, there are notable references to specific context-related aspects (7/20) concerning single-photon polarization (3) and behavior (2), the application of Malus' law (1), and the concept of spin (1). Almost 40% of the students identified specific and more innovative aspects such as superposition (8/20) and entanglement (8/20), more than 30% mentioned the distinction between states and properties (6/20), and 25% mentioned the individual concepts of quantum states (5/20), properties (5/20), and measurement (5/20). Only 20% cited probabilistic quantum behavior (4/20) and a few (2/20) quoted the existence of mutually exclusive and incompatible properties, while individual students indicated corpuscular nature and hints about teleportation. Importantly, the spontaneous recollection of all these founding elements of QM by the students highlights the effectiveness of the activity.

Among the general aspects, a little more than one third of the students stated that they understood the basic concepts of QM (6/20) without specifying which ones, one fifth noted the microscopic approach to the investigation of reality (4/20), and individual students noted the complexity of the world, the existence of formulas with universal character, the importance of imagination, study, and research, and the way in which QM solves the flaws of classical mechanics.

Even if the individual statements of the students are partial and partly generic, their whole can offer the opportunity for a larger group discussion digging into the aspects which the literature has highlighted to be important [2–4,14–18,22–26,35,77].

Two sentences are included here among the surprising aspects. The first: "for quantum mechanics, on a scientific level, the descriptive dimension is not satisfactory" highlights with simple words the need to capture the role of formalism in quantum theory. The second reads: "the impossibility of accurately determining the final state starting from the initial conditions: it is possible, for example, to provide only a probabilistic evaluation of the outcome". From the full answer, we can infer that this student had a clear idea that the probabilistic evaluation is related to the measurement action, though with an as yet unclear distinction between the state (evidently defined) and the result of a measurement (stochastically indeterminate, in general).

Here, we note that only two students explicitly mentioned TiqTaqToe, and only a single student mentioned an application among the aspects of interest (teleportation). Finally, individual students highlighted the following: understanding of aspects previously seen only in popular books in a discursive manner; topics not covered in school, or clarifying concepts learned in chemistry class but not understood, such as spin, and a new phenomenology, polarization.

3.2. Q1-I2: Illustrate to a Classmate the Concept of Mutually Exclusive and Incompatible Properties

The answers on the understanding of mutually exclusive properties evidence four classes (A–D) of perspective, displayed in the left panel of Figure 3:

(A) They produce a certain result (11/20), following a measurement or in interaction with polaroid (3/11);
(B) Possession in exclusive terms, as if it were a state (an application of the Aristotelean principle of the excluded third) (5/20);
(C) Events that cannot occur simultaneously (1/20);
(D) Events corresponding to probability 1 (1/20).

About three quarters of the students; (14/20) sentences included examples offered during the webinar. The context of optical polarization plays (5/20) a fundamental role (1/5) in completing conceptual identifications (4/5), such as for spin (1/20). The outcomes reveal the need to clarify that a property value is the outcome of a measurement event, not the event itself, an aspect often too subtle to be caught.

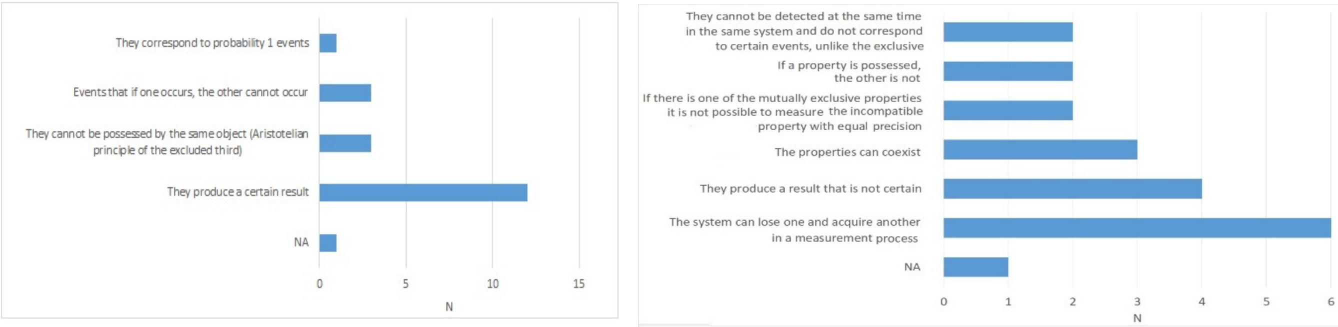

Figure 3. Distribution of categories of answers to item Q1-I2: *Tell your classmate what is meant by mutually exclusive* (**left**) *and by incompatible* (**right**) *properties*.

The scenario involving the identification of incompatible properties is, not surprisingly, more fragmented (see right panel in Figure 3), as this is known to be a more problematic topic for students [36,77,78].

The most frequent identification of incompatible properties is that, in a measurement process, the system can lose one property and acquire another (6/20). Other characterizations are (i) they produce a result that is not certain, in contrast to the certain outcome for mutually exclusive properties; (ii) it is not possible to measure the incompatible property with equal precision, an aspect hiding the identification of uncertainty with indeterminacy; (iii) if a property is possessed, the other does not possess it, leaving incompatibility and mutual exclusivity undistinguished; (iv) properties can coexist, revealing a classical vision of superposition. The first category in the right panel of Figure 3 is the richest, and it has therefore been separated from others: "*They cannot be detected at the same time in the same system, however, unlike the mutually exclusive ones, the incompatible properties do not correspond to certain events*".

Again, it can be observed that each of the students' sentences does not allow for a clear and univocal identification of when one should speak of incompatible properties (except the first). However, integrating the different positions provides a sufficiently coherent and complete identification of what is meant in QM by incompatible properties. The webinar that we designed was evidently able to activate this wealth of meanings, which is deserving of a further in-depth discussion about underlying ontologies [17]. In addition, a further moment of synthesis, collaborative construction, and negotiation of meanings would be required, which are known to be a constitutive element of science and learning [79–82]. Considering the briefness of the webinar, however, this was left to the students' teachers.

3.3. Q1-I3: Illustrate to a Classmate the Concept of Quantum State

More than half of the students identified the concept of quantum state with a vector (11/20), albeit with very different meanings (see Figure 4).

In seven cases, the vector is identified either with the system itself or the description of its characteristics. In the other three cases, it is instead identified with its mathematical properties, i.e., a vector for which the superposition principle applies or can be cast in different manners. Two students identify the state with a formal description of the set of possible properties that a system can acquire after a measurement, that is, a mathematical representation "*which takes into account every possible property that a system may have*". In the other cases, the state is variously identified with a probability distribution, which recalls the well-known identification between probability amplitude and probability from the literature [25,29,78], and an intrinsic system characteristic identifying the state and property [25,36].

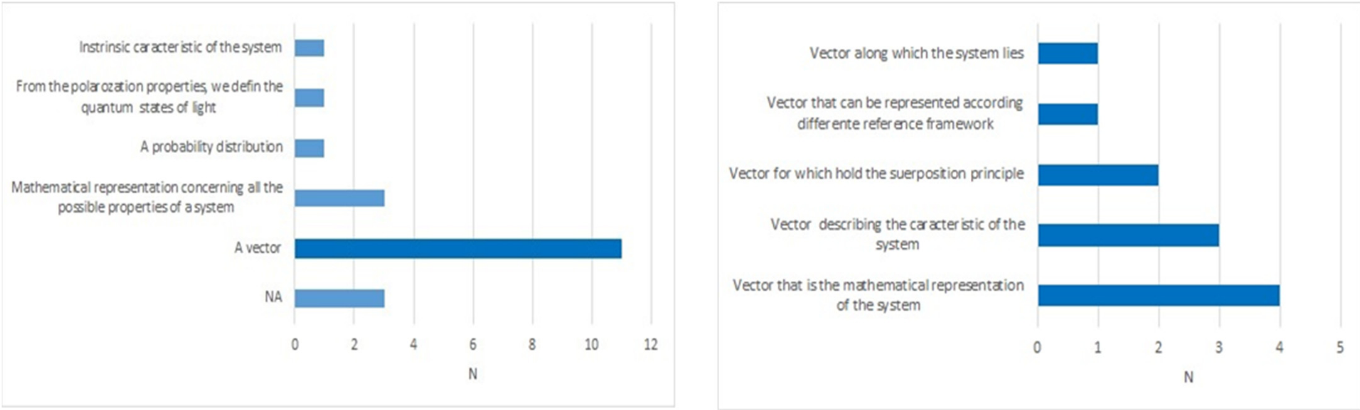

Figure 4. Distribution of answers to item Q1-I3: *Tell your classmate what is meant by state of a quantum system.*

3.4. Q1-I4: Illustrate to a Classmate the Distinction between Quantum State and Property

Among the eighteen respondents, the property is defined in two ways:

P1: "The property is a precise/intrinsic characteristic" (10/18);

P2: "The property is a measurable expression of the state or the value of the state (8/18).

The state, on the other hand, is defined in four ways:

S1: "*More general*" (of the property) (7/18);

S2: "*A vector*" (7/18), which, for someone, "*falls into a property at the moment of measurement and allows to determine the probability of the outcomes*";

S3: "*Considering the condition in which the system is found*" (3/18);

S4: "*The set of all measurable properties*" (1/18).

Figure 5 shows how the answers to the two aspects of the question are related. It is clear that P1 is related only to S1 and S2. P2 is instead related to all four S1–S4, with S3 prevailing. This item was particularly challenging for the students, for whom the very concept of what a state (whether classical or quantum) is in physics was not for [23,24,26]. This can probably be traced back the answer of one student: "*I am the classmate they should explain this to*".

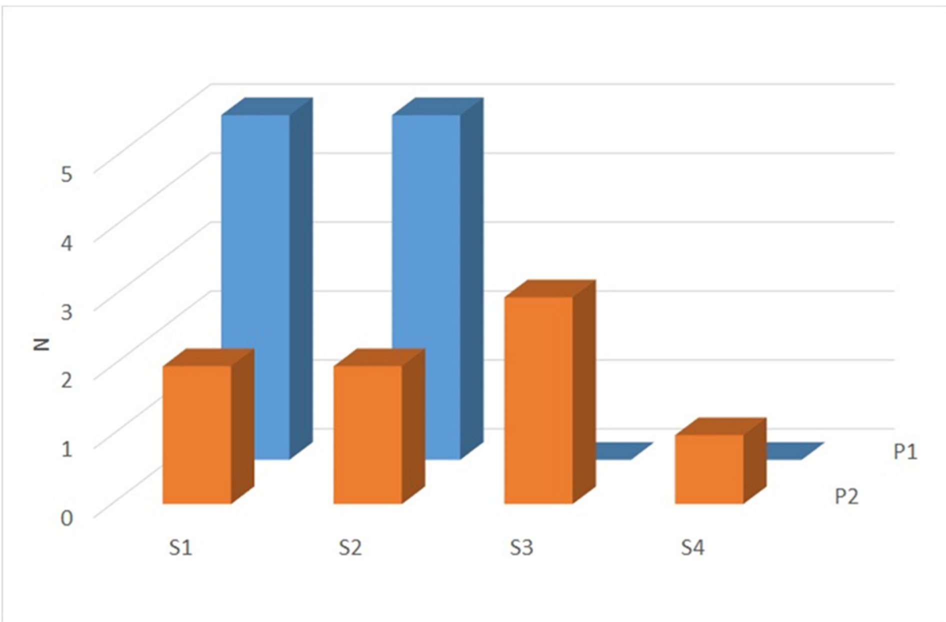

Figure 5. Distribution of the categories of students' answers to item Q1-I4*: Tell your classmate the distinction between quantum state and property*.

3.5. Q1-I5: Illustrate to a Classmate the Characteristics of Quantum Measurement

Two indicators show that the concept of quantum measurement was particularly problematic for almost one third of the students. Four students left the answer completely blank, and the variety of answers was low (see Figure 6 and example answers in Appendix B). Undoubtedly, the concept of measurement is not easy to summarize in a few words. Students had difficulty spontaneously providing a unique and complete identification, as often occurs with concepts requiring caution and thoroughness. In any case, the expressed conceptual aspects were correctly identified, even if incomplete.

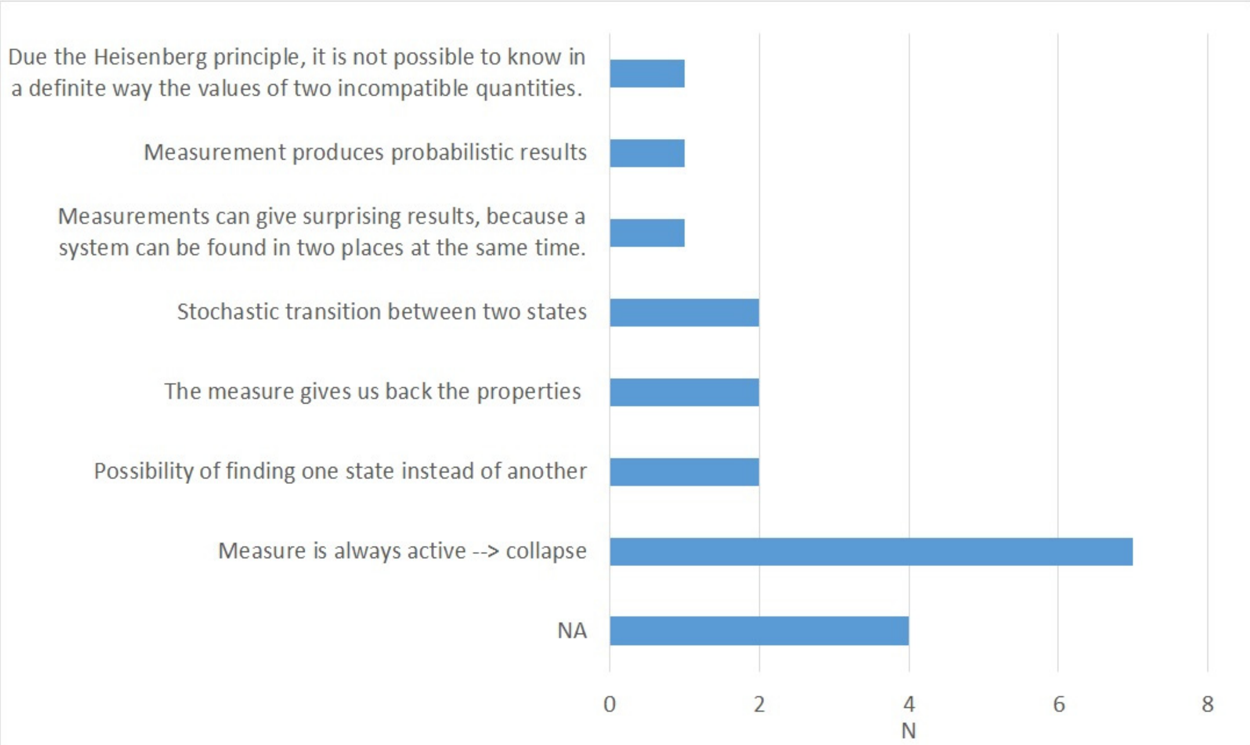

Figure 6. Distribution of answers to item Q1-I5: *Illustrate to your classmate the concept of quantum measurement*.

The prevailing category indicates that quantum measurement is always active, involving the concept of state collapse. Only one student mentioned that the system collapses in a measurement. This is not correct in general, however, the idea might have been activated by the used of selective measurement in the exercise involving a Polaroid. The other aspects again highlight a rich panorama, which, however, needs to be recomposed. Statements such as "*Stochastic transition between two states*" and "*Due the Heisenberg principle, it is not possible to know in a definite way the values of two incompatible quantities*" must be connected, as they represent two orthogonal modes of looking at the measurement process in QM.

3.6. Q1-I6: Illustrate to a Classmate the Concept of Quantum Superposition

The superposition principle was mainly illustrated as the sum of states (or sum of state vectors), which we consider an adequate answer (Figure 7).

In the didactic path, not enough weight was given to normalization or to the fact that a linear combination should be referenced rather than a sum of vectors. Problems with understanding were only notable in the three answers in which superposition was identified as the coexistence of incompatible properties or with the sum of properties.

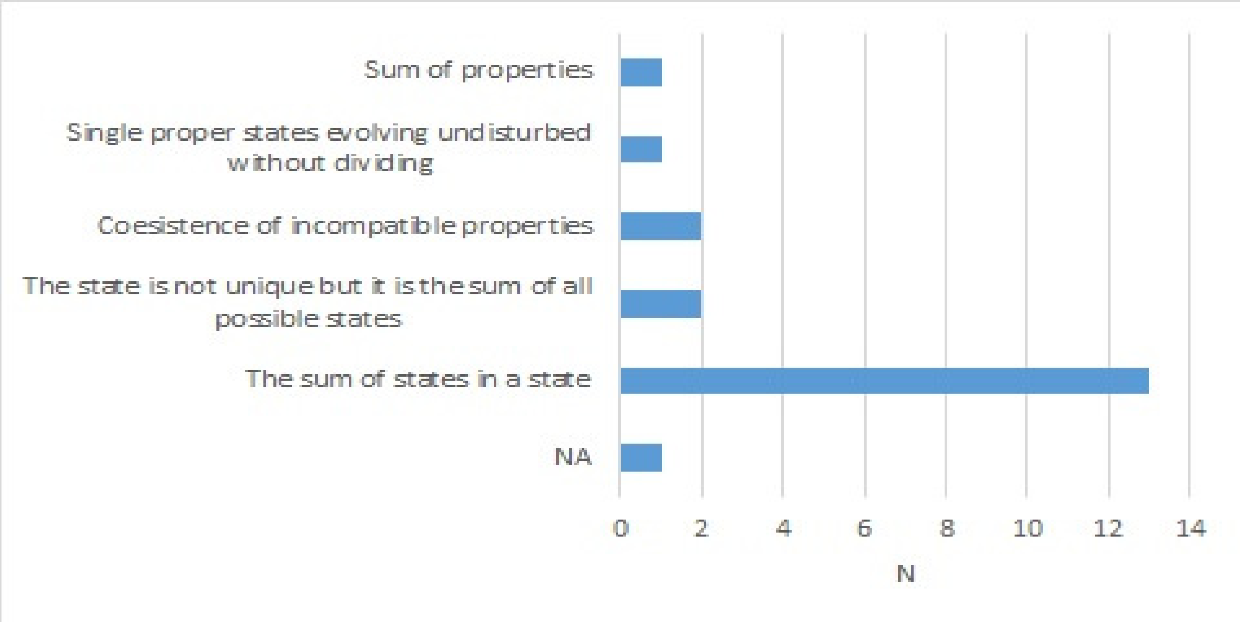

Figure 7. Distribution of answers to item Q1-I6: *Illustrate to your classmate the concept of superposition of quantum states.*

3.7. Q1-I7: Illustrate to a Classmate the Concept of Entangled State

A large majority of students (almost 80%) recognize the entangled-state concept as a non-local correlation among system states (Figure 8). Surprisingly, the concept of entanglement is adequately characterized by most of the students as distant correlation states in which the properties of two entangled systems cannot be attributed individually unless a measurement is performed that causes the collapse of both parties. This is a very good result of the present experiment. Only two cases emerge, in which an entangled state is incorrectly identified with a produced state or as a synonym for superposition.

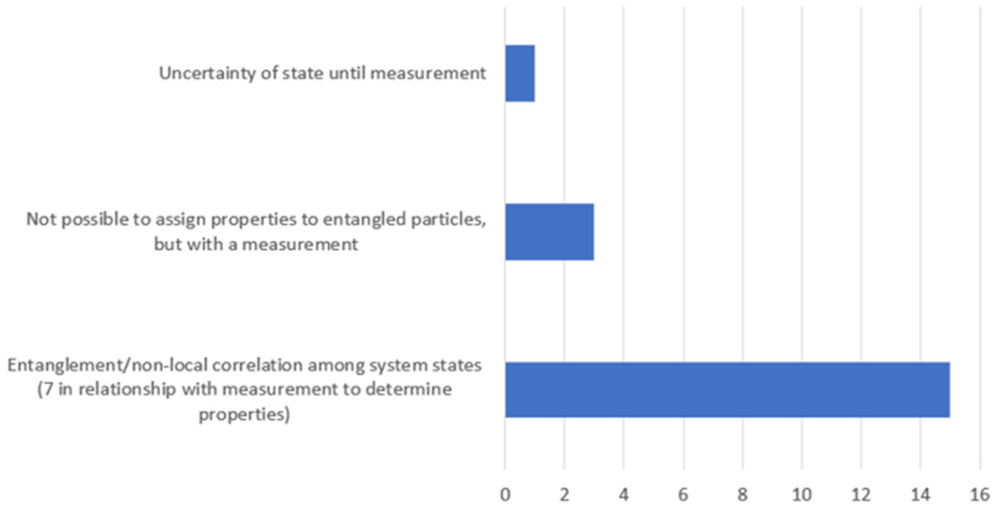

Figure 8. Distribution of answers to item Q1-I7: *Illustrate to your classmate what an entangled state is.*

3.8. Q1-I8: Illustrate to a Classmate the Distinction between Quantum Superposition and Entangled States

Figure 9 shows the types of answers concerning the distinction between superposition and entanglement. Superposition was almost never expressed as a sum. Entanglement was expressed in several manners, including as a correlation and as the impossibility of singling out the properties of single parts, as seen in 3.7 Q1-I7 above. Although encouraged to recall TiqTaqToe, only three students included examples from the game, and these are not particularly illuminating. Overall, however, 50% of the students were able to make a distinction, after noting that the former is related to one particle while the latter is related to two or more, in certain cases even noting this in relationship to the measurement process. On the other hand, 30% of the students confused the two concepts, instead performing an identification.

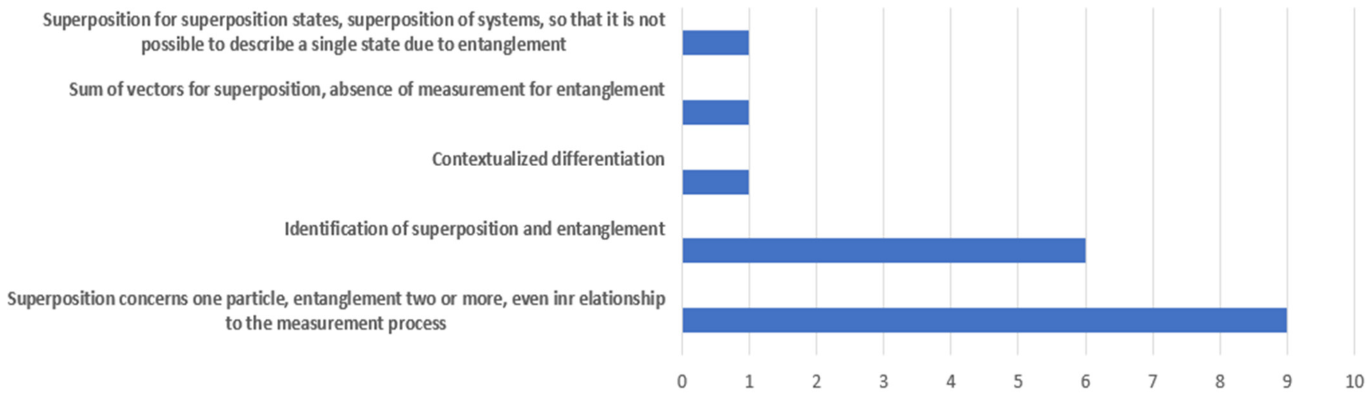

Figure 9. Distribution of answers to item Q1-I8: *Illustrate to your classmate the distinction between quantum superposition and entangled states.*

3.9. Q1-I9: Illustrate to a Classmate the Impossibility of Attributing a Trajectory to a Quantum System

Students found it rather problematic to point out why a trajectory cannot be attributed to a quantum particle. Figure 10 illustrates the six different types of answers identified. The prevailing category, relating to the Heisenberg principle, specified that "*trying to measure and calculate the trajectory of the system would compromise it*" or "*because position and momentum are correlated variables. By accurately measuring the position, for example, the momentum is indeterminate and therefore it is not possible to know in which direction will go and then build the trajectory*".

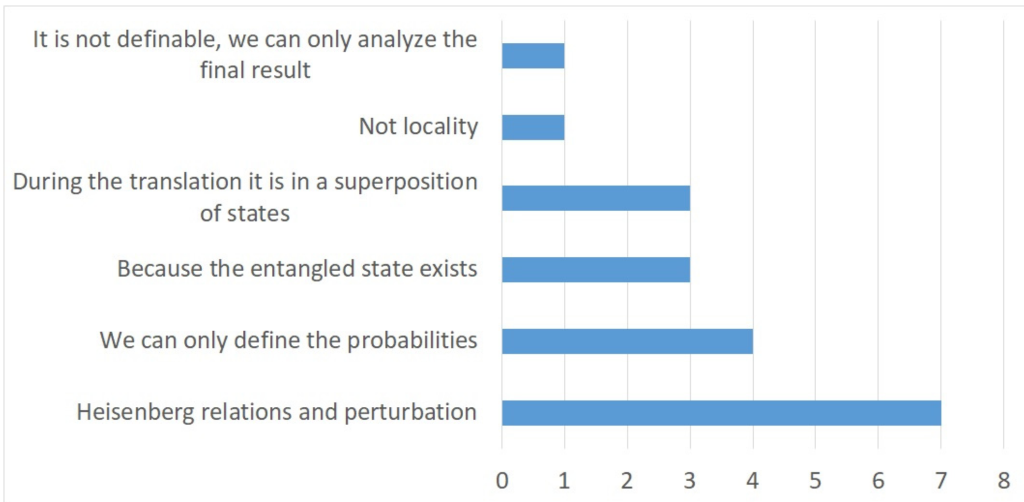

Figure 10. Distribution of answers to item Q1-I9: *Illustrate to your classmate why it is impossible to attribute a trajectory to a quantum system.*

The trajectory concern is then linked to the probability concept: "*Because we cannot be sure that the particles that make up the system move in a certain way. We can only find a probability of the particles to follow a given one trajectory*".

Overall, these assertions are adequate to explain why a trajectory is not attributable to quantum systems, demonstrating that the messages proposed to students on this topic were absorbed. However, most of the students' sentences are rather assertive. Therefore, it is difficult to establish whether they are reproducing well-read expressions from the webinar presentations or whether they are expressing real understanding. In fact, it remains unclear whether the Heisenberg principle refers to the idea that exploring the trajectory of microscopic systems necessarily implies unavoidable perturbations or whether this is impossible in principle.

The three students recalling entanglement, for example, stated: "*It is impossible to attribute a trajectory to the quantum state, because the entangled state is present*". This correlation between entanglement and the impossibility of associating a trajectory presumably emerges in part from the approach used in the first half of session S1. There, we introduced entanglement via the interaction of photons with birefringent crystals; the propagation of photons in crystals actually occurs in a superposition in which polarization and translational states are entangled. However, the students seem to have grasped the correlation in the specific case as opposed to its general meaning. A different approach to entanglement was used in the second part of session S1, although never referring to trajectories.

3.10. Q1-I10: What You Would Like to Be Explained Again?

With the last question of the Q1 questionnaire, we enter a new dimension of analysis, re-examining both the content and evaluation aspects of the webinar (albeit directly). First, 80% of the students asked about specific aspects and only 20% about generic ones.

Figure 11 shows the aspects that were indicated by the students as deserving more in-depth study to allow for better mastery. Entanglement and related aspects (how do you create entangled particles, why is there an intertwining of states) is the most mentioned. It is interesting to note that six of these seven students provided the clearest identifications of the concept of entanglement in Q1-I7, that is, either as a state that foresees a correlation at a distance or as a state in which it is not possible to attribute properties to single parts (the single particles). Evidently, their increased competence provides a critical self-evaluation tool. While these students grasped one aspect of entanglement, they further demonstrate awareness of that they do not fully know how to handle the concept.

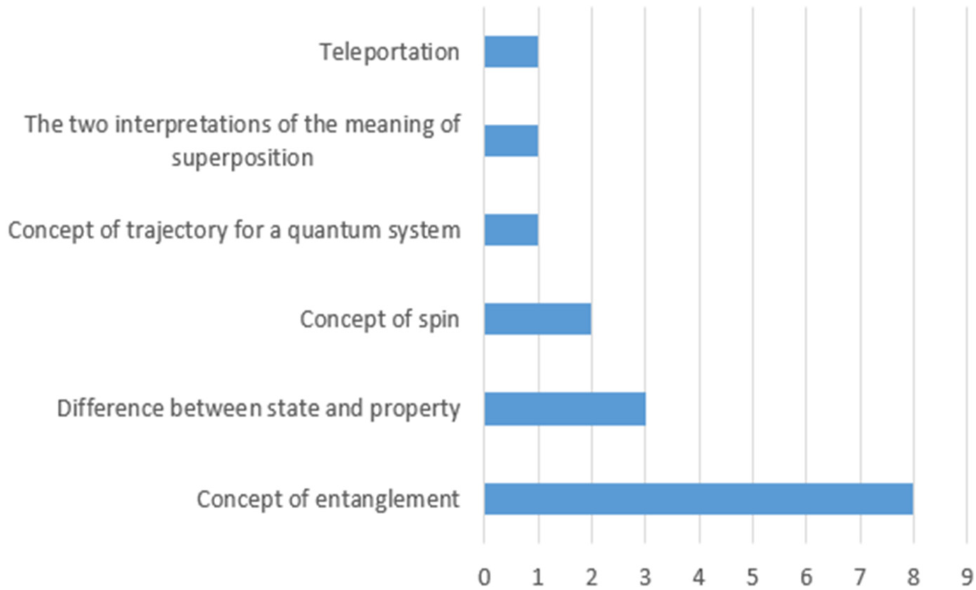

Figure 11. Distribution of answers to item Q1-I10*: What would you like be explained again?*

Those who indicated the need to restate the difference between state and property did not manifest as much awareness. Evidently, this report highlights the need to deepen students' knowledge of basic concepts such as state and property, which were not particularly clear in the answers to specific questions (see for example Q1-I4).

Regarding the apparently generic (all/nothing) answers, it is useful to point out the respective non-trivial motivations: " … *all because, since each aspect is connected/derived from other*"; "*taking into consideration the timing and the remote modality, all the concepts have been explained in a simple and concise way, despite the fact that the subject is extremely complicated*".

3.11. Q2-I1: What Game Strategies Were Used to Win?

As seen from the analysis of the Q1 questionnaire, while the students demonstrated a grasp of the basic concepts, only a few students used the game of TiqTaqToe to exemplify or contextualize their answers. To answer RQ2 and RQ3, it is therefore crucial to consider items Q2 I1-I4, which investigated how the students lived the game experience. Figure 12 illustrates the mutually exclusive macro-categories into which the responses were classified. As to the first (*predict and anticipate opponent's moves*), we can say that it simply extends to the quantum game a typical strategy valid for a game with classical rules. The second begins to include the probabilistic element, which is inherent to the quantum logic with which quantumness levels 2 and 3 of the game were implemented. The third provides an additional quantum level defined by the concept of superposition, which in the game corresponds to being able to occupy, at least virtually, two grid-boxes at the same time, thus preventing the opponent from occupying them as well. The fourth, mentioned by over half of students, highlights the highest quantum level, which in the game can, e.g., be used to ensure that squares virtually occupied by the opponent can become one's own box.

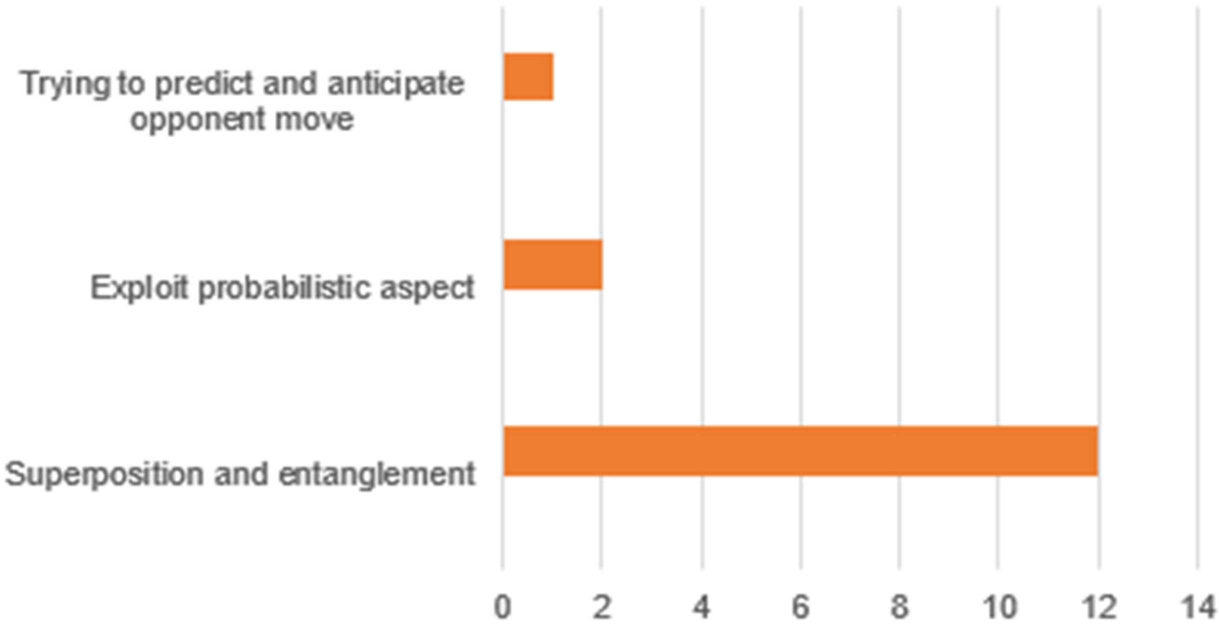

Figure 12. Distribution of answers to item Q2-I11: *What game strategies were used to win?*

This work highlights that quantum rules, hardly accepted as possible in reality, were usefully assimilated by the students in a very short time. Although with different awareness and competence (see Appendix C), the students were able to quickly master them and turn them in their own favor to conceive of suitable strategies to win the game.

At least three main game strategies exploiting the quantum rules can be identified. The first is based on using the combination of superposition and entanglement to increase the probability to win: "*thanks to the superposition I tried to create one if not two rows by placing in each component of the trio chosen, a 50% chance of having my symbol in one box. … in the negative case I still had the certainty of having two boxes in line occupied by my symbol and that the third was free. At this point, the opponent occupied the missing box but I created an entangled state so the victory could return to my side*". The second focuses on the importance of being the first to move after the collapse, in fact it "*seems that there is important advantage to the player with the first move after the measurement: he could in fact see the outcome of the moves of both players.*" The third strategy includes entanglement, which makes it possible to change a box sign; using entanglement, "*I tried to use it to have the opportunity to exchange my position with that of the opponent*".

Obviously, not all students were able to master the opportunities offered by the quantum laws of the game in such a short time, as expressed in the answers of two students: "*In the last two phases my opponent and I have often relied more on luck than on our intellect, because much (if not all) depended on how the 'particles' were positioned at the time of measurement.*"

3.12. Q2-I2: QM Aspects Recognized in the Game

Figure 13 shows the number of students (out of seventeen respondents) who indicated each of six concepts involved (see legend).

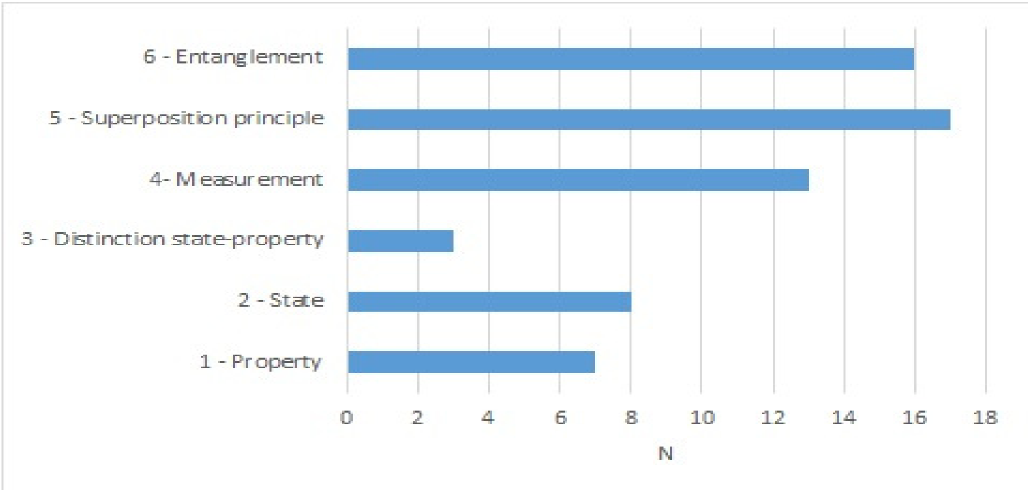

Figure 13. Distribution of answers to item Q2-I2: *Which aspects did I recognize in the game, among: 1. Property 2. State 3. Distinction between property and state (explain) 4. Measurement 5. Principle of superposition 6. Entanglement?*

Fifteen of seventeen students identified three or more aspects, while two identified all six of them. All students provided an explanation of how they recognize them, either limiting themselves to only two examples, or else indicating the game phases in which they recognized superposition and entanglement or indicating superposition, "*because it is more intuitive to understand by playing*". These explanations allow us to appreciate the degree of appropriate understanding reached by the students, starting with the superposition principle, which was cited by all respondents, and entanglement, which was cited by all but one. These two peculiar aspects of quantum mechanics were explicitly referred to as levels of the game, and therefore it is no surprise to find them in the students' quotations. What is significant is that they almost always seem to have recognized them.

Note that the concepts most discussed at later stages of the activity are the very same which collected the most indications from students. The concepts of entanglement, superposition, and measurement, introduced later and at the core of TiqTaqToe, were generally identified in the correct manner. This emerges in the following answer:

Niccolò: "*During the game I was able to recognize the superposition principle, which was one of the possible moves in which two symbols of the same type, not definitive, are formed, which with a subsequent measurement will be determined in one or the other cell. The second aspect that I have recognized is the entanglement represented by the uncertainty of the two symbols in two different cells, which consequently to the measurement will be one symbol or the other in the respective cells. The third, on the other hand, is the measurement, which cannot be absolute for both quantities found in the superposition. I also recognized the property which, being a specific value, was represented by the definitive sign*". *In some cases, superposition and entanglement have been recognized also* *in reference to mutual exclusive properties, like in Michela's statement "In case of superposition or entanglement of cells A and B, the properties Xa and Xb are mutually exclusive just like Oa and Ob".*

The identification of state and property, as well as their distinction, proves to be more challenging. The concept of state is often confused with the symbols themselves, as in Ivan's sentence "*The state was represented by symbols*". It is noticeable, however, that, when correctly identified the process is clearly operational. For example, for Giulia, "*I have identified 3 types of properties: [definitely] empty cell (v), cell with cross (X), cell with circle (O) [. . .] Since the state must contain within itself all the possible properties of a cell before the properties even begin to be defined (i.e., before the game), I recognized in the cell position within the grid the state S, as a superposition of states corresponding to all the properties described above (Sv, Sx, So).*" A similar consideration applies to the difference between state and properties, as in "*The state describes the possible scenarios that a particular cell might run into before it can even take on a definite property, that is, before a player selects it or before revealing the result of a superposition or an entanglement*" (Michela) and "*The difference between state and property can be seen in the fact that only states were involved in the superposition*" (Giulia). The operational environment was not helpful for everyone, as in the case of Andrea, for whom, "*I did not recognize the property, nor the state, but not even the distinction between the two. I found again the concept of measurement when the collapse of the game grid occurred. The superposition principle, entanglement and their mechanisms were found between the second and third phase of the game, where certain symbols could be randomly eliminated*". *See* Appendix D *for other examples of students answers.*

3.13. Q2-I3: Was the Game Useful to Clarify Concepts in the Webinar?

The answers to Q2-I3 (Figure 14) reveal that the game indeed served to clarify the concepts discussed in the webinar, in particular for the three key concepts of measurement, superposition, and entanglement. In fact, a total of 57 concepts were discussed, with 30% of the answers (17/57) declaring the use of the game for the concepts of quantum superposition and of entanglement to be beneficial, 21% (12/57) only for measurement, and as many for quantum states, properties, and their distinction (in total).

0 concepts		2
1 concept		1
2 concepts		3
3 concepts		8
4 concepts		2
5 concepts		0
6 concepts		2

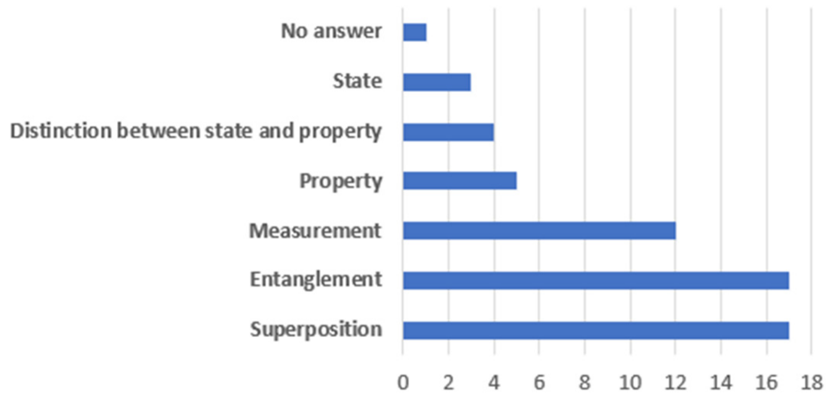

Figure 14. Distribution of answers to item Q2-I3: Was the game helpful in clarifying the following concepts? Explain the answer in detail to the concepts of: 1. Property 2. State 3. Distinction between property and state (explain) 4. Measurement 5. Principle of superposition 6. Entanglement. Right panel: table of how many students (right column) indicated the number of concepts listed in the left column.

In particular, 44% of the responding students (8/18) could recognize superposition as either two states associated with a single move or via the visual representation of one symbol in two boxes (one specifying that this is true until quantum measurement is performed), although 55% could not specify how they recognized the concept in the game. On the other hand, 40% could recognize entanglement either as X and O united in two boxes (one specifying until measurement is performed) or via the visual representation of a symbol, while 50% could not provide the answer for 10% it was simply a game option. Measurement was recognized by 40% of the students as performed at the grid filling stage, with almost as many attributing one single state to superpositions in the process (in one case, to entangled states as well), while 25% were not able to provide either answer. Barely 10% (2/9) were able to identify the quantum state as X or O in a particular position, while almost 50% identified it with the "particle" X or O, 10% with the "property" of the position, and almost 30% could not perform the identification or provide a motivation. Similarly, barely 10% (1/11) could recognize the concept of as the position of X or O on the grid (with one specifying after measurement), 20% linked the property to the symbol's redness, and more than 50% were not able to do either. Not surprisingly, the answers were equally split (25% each) among identifying state and property, identifying neither, not answering, or assigning a different visualization to the state before measurement and of the property afterwards.

3.14. Q2-I4: Were the Concepts in the Webinar Useful to Identify Game Strategies?

Conversely, the answers to question Q2-I4 (Figure 15) show how the introductory webinar was actually used by students in the game phase. In 84% of the answers (left panel in Figure 15), the concepts explained in the introductory part were evaluated as useful for the game, with a preeminent role for the concepts of superposition (45%) and entanglement (38%) and only 17% for quantum measurement (right panel in Figure 15). In addition, the students' game strategies were elaborated on the basis of the concepts, as explicitly mentioned by 11% of students. Only 5% of the students stopped at a lower quantumness level in the game.

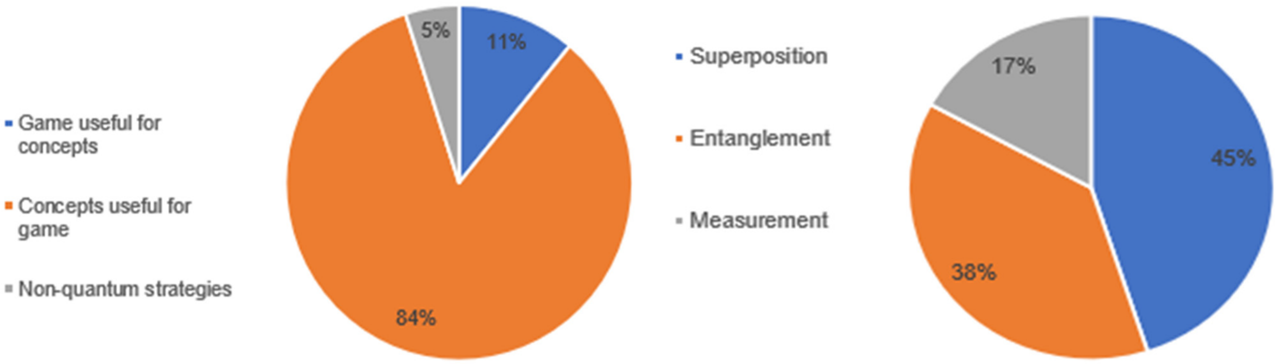

Figure 15. Distribution of answers to item Q2-I4. Left panel: Were the concepts outlined in the webinar useful for gaming strategies? Right panel: Which concepts were useful?

3.15. Q2-I5: How Students Ask for Further Explanations

Despite the short intervention time, as many as 20% of the students grasped the main concepts in QM, except for superposition (Figure 16, Q2-I5). In particular, 21 answers were about epistemic aspects such as the concepts of entanglement (4/29), property (4/20), quantum state (4/20), quantum measurement (3/20), teleportation (3/20), and gameplay strategies (3/20). Note that the indication of gameplay strategy suggests reduced autonomy, as this is the aspect on which we asked for feedback. Individual students asked for the difference between state and property, practical applications of QM, and the concept of spin, while 25% provided generic answers. Evidently, a global perspective cannot be easily provided in such a short and operational intervention, and is further hidden by the specificity of the content and of the individual didactic activities. Significantly, students indicated the foundational concepts of state, property, entanglement and measurement. Teleportation (as already mentioned) is the application that led to the strongest interest.

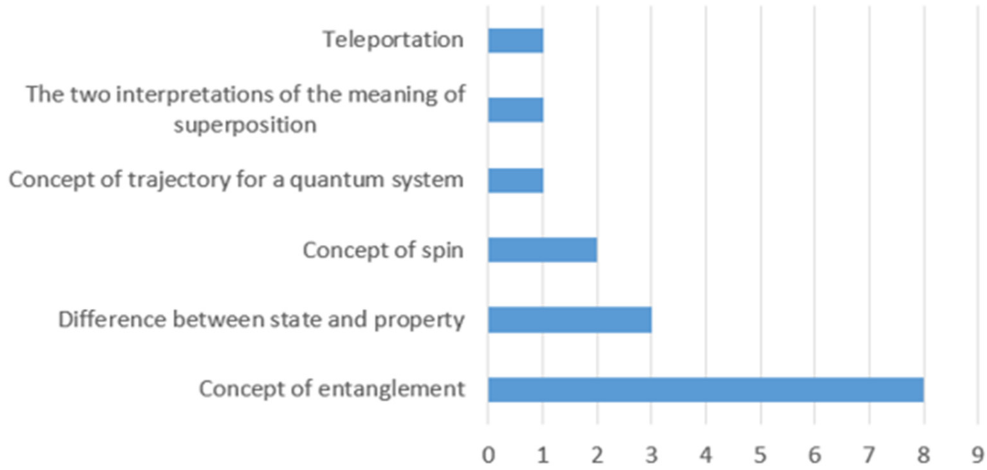

Figure 16. Distribution of answers to item Q2-I5: What would you like be explained further.

3.16. Q2-I6: What Students Liked the Most

Over 67% of the students liked single different aspects, 19% methodological and didactic ones, and 13% the clarity and simplicity of the intervention (Q2-I6, Figure 17). In detail, two main results emerge: over 70% (14/20) appreciated the use of the game to clarify concepts, and more than one third liked the playful engagement provided by the tournament. Almost 20% of the answers about single different aspects appreciated the hints involving teleportation, and the same number appreciated the use of audio-visual recordings.

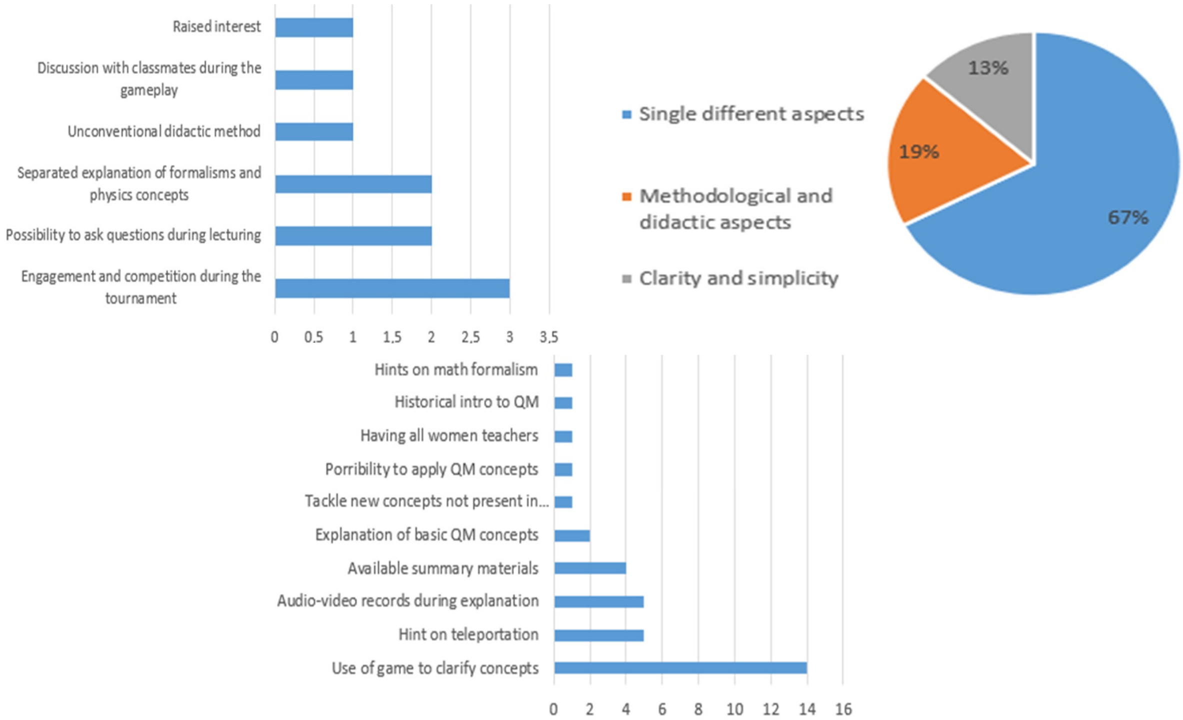

Figure 17. Distribution of answers to item Q2-I6. Top-left panel: Indicate the three aspects you liked the most in the whole activity and why. The top right and bottom panels focus on methodological/didactic aspects and on single different aspects, respectively.

In summary, the use of the game for the purpose of clarifying the concepts was appreciated by most of the students. Even within the reduced percentage of methodological answers, the students showed an appreciation for the playful engagement provided by the tournament.

3.17. Q2-I7: What Students Liked the Least

Least-liked aspects were expressed by fifteen of seventeen students, with eleven indicating three aspects, three indicating two, and four indicating only one aspect (Q2-I7, Figure 18), which were, respectively, as follows: the way of treating selected topics, the organization of the whole activity, and the specific organization of the tournament.

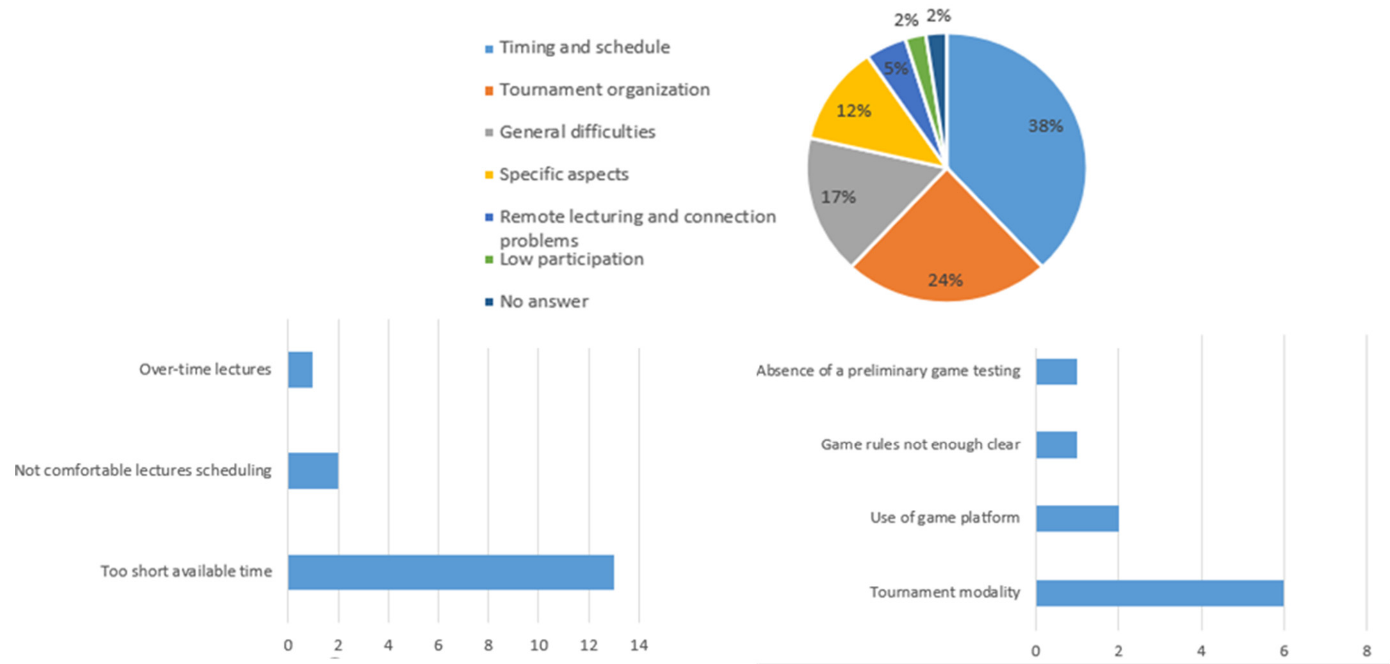

Figure 18. Distribution of answers to item Q2-I7. (**Top**) panel: Indicate the three aspects you liked the least in the whole activity and why. (**Bottom**) panels focus on answers about timing and schedule (**left**) and tournament organization (**right**), respectively.

Regarding the first area, the students responded that certain explanations were too concise or unclear (7) and that complex concepts were not thoroughly investigated (specifically, superposition, entanglement, and the difference between states and properties). Finally, one student found the quantum formalism not relevant, and felt that it could therefore be avoided (1).

The prevailing indication with respect to organization was about timing, either too tight (7) or too short (1). Other aspects were remote activity (3) during afternoons (2), limited involvement of students (2), or exceeding scheduled time (1).

The prevailing indications with respect to the tournament included the modalities, in particular, the use of a single smartphone at a distance (6), the compressed time (4), unclear match rules (1), the need for a training session (1), and the tournament format (1).

3.18. Q2-I8: Suggestions for Future Activities

On Q2-I8, 67% of the consulted students responded about didactic aspects, 21% about organization, and 13% provided generic answers or none at all.

The didactic suggestions were mostly about deepening more complex topics (3/16) and additional interactions with students during class time (2/16), while individual students proposed a variety of very specific actions, such as providing slower explanations, examples, and metaphors, additional focus on experiments, more time to play the game, logical explanations of formalism, lecture summaries, additional QM applications, a different order for the presented topics, playing the game before the conceptual explanations, and separating the explanations from the practical activities.

Finally, four out of five answers about organization involved suggestions to split the group into more classes, while the fifth involved improving the timing of the lectures.

4. Discussion

Teaching/learning Quantum Mechanics in secondary school while providing young generations with the opportunity to experience fundamental quantum concepts and understand the coherence of the theory is a challenge. However, it is crucial both to comprehend the paradigmatic role of quantum theory in building knowledge of the world and to provide citizens with the conceptual and operational tools necessary to face the challenges of a society increasingly pervaded by quantum technologies. To this purpose, it is necessary to explore new perspectives for QM teaching/learning processes in high schools while responding to three crucial questions: what concepts to address and how to most suitably propose them; what phenomenological contexts should be offered to students in order to allow them to construct quantum concepts and acquire experience with quantum behavior; and how to achieve effective engagement with students in order for them to build their own learning process.

In order to unite these goals, we developed a short educational proposal of 4 h length, divided into two 2-h parts (S1 and S2), which integrates the presentation of the fundamental nucleus of quantum theory and its basic formalism with a quantum game tournament.

The first part was planned using the theoretical framework of the Model of Educational Reconstruction by identifying founding nuclei and conceptual difficulties for an educational path activity able to engage students' reasoning. It was divided into two sections, focused first on discussing the basic concepts of quantum state, property, measurement with its stochastic nature, and superposition and entanglement in a two-state system. The phenomenological context offered in the first part relied on the polarization of light, explored using real polaroids at high intensity which can then be re-analyzed in simulated ideal experiments involving single photons interacting with polaroids and birefringent crystals. The rationale of this first presentation is to present a revised and more compact version of the research-based educational proposal developed and tested in previous research [3,29,33,34,36,69,70]. The subsequent half involves in-depth study of the concepts of quantum superposition and entanglement, which are central to the quantum game. We drew the needed resources from the QPlayLearn platform [5,55], including the quantum game TiqTaqToe [61] and the 5 min "quantum pills" on quantum physics and entanglement, i.e., animations dealing with one central concept in an engaging manner. The formal description of quantum states in Dirac notation was then introduced in the case of two-state systems and applied to engage students with basic elements of quantum logic first and teleportation afterwards. Finally, the same Dirac notation was used to provide a formal description of the concepts involved in TiqTaqToe [61].

The second part of the activity was dedicated to the TiqTacToe game tournament, dividing the students into pairs. The students tackled the different levels of quantumness offered by the game, starting from the classic setting, then introducing superposition, and finally entanglement (30 min in total). Due to the mainly game-based nature and the length of the intervention, we chose to focus on the internal coherence of the new way of thinking in terms of QM, rather than discussing classical and quantum interpretations of the game.

The learning process was monitored by means of two different purpose designed open-ended questionnaires, Q1 and Q2, delivered after sessions S1 and S2, respectively. The first questionnaire collected information about how concepts were learned and about the conceptual nodes which remained unresolved. Q2 was designed to evaluate the role of the TiqTaqToe game in engaging students and in their learning of single concepts, as well as to collect information on the whole didactic proposal. We analyzed the students' answers by using qualitative research methods to construct operative response categories and collecting the occurrence frequencies of each category.

The analysis of Q1, which asked about students' beliefs with respect to the main concepts they learned, provided answers to our first research question (RQ0). In fact, it emerged that the majority of students believed that they had a better understanding of the concepts of superposition and entanglement than those of state, property, and their difference. This was certainly favored by the use of the game, which centered on superpositions and entanglement, as again the majority of the students affirmed it in their answers the questions posed in both Q1 and Q2 (see for example the discussion of the responses to Q1-I1, Q1-I10, and Q2-I1–3). At the same time, it is interesting to note that the students showed a significant degree of awareness of being not fully able to master complex concepts such as entanglement; the significance of this is further witnessed by the fact that students indicated entanglement as the major aspect on which they wished for further instruction (see answers to Q1-I10). The game played a new and important role, one not found in previous research; that is, it produced operational appropriation of the concepts of quantum superposition and entanglement, at the same time offering awareness about the nature and role of the new concepts, which are otherwise completely counter-intuitive.

On the other hand, we should remark that the second issue the students indicated involved the distinction between state and property (see items Q1-I4). This is related to the fact that the concepts of quantum state and property, fundamental everywhere in physics, are not stressed in the tradition of classical physics teaching/learning practice, the related ideas being often left vague and substantially indistinct. Although we know that in classical physics the concepts of state and property can in fact be made to coincide without encountering contradictions of principle, The assumption of such an identification in QM, even in principle, leads to irremediable contradictions with phenomenal reality. On the other hand, distinguishing between state and property is a crucial aspect of QM, regardless of the interpretative frame that is assumed, and one on which students' known learning difficulties are centered [14,81]. If the basic vector formalism, introduced in S1 of the activity, mainly allows students to identify the state with a vector (see for example Q1-I3), it was not sufficient in this case to make students confident enough to master the concept of property, which they essentially identified with an intrinsic property of the system or the measurable expression of a state (see Q1-I4).

The concept of measurement was a surprisingly complex one for the students to acquire. Our analysis of their responses to this concept highlighted the large number of students who avoided providing an answer, which led to a great dispersion of answers. In short, among the respondents, one third of the students identified measurement with the collapse of the state, one in seven with a probabilistic process, one in seven with the sum or coexistence of properties, and one in ten with a transition between states; only one student connected measurement to Heisenberg's uncertainty principle. The students' answers capture important aspects of the concept of measurement. In fact, we believe that, within the activity. this concept required time to collaboratively construct a shared meaning for this quite counterintuitive outcome of QM and recompose the spread of the different visions, for instance by pointing out that while in QM a system can be detected in two very distant places, this is simply related to the nonlocal nature of QM.

Reverting to the concepts of superposition and entanglement, we should comment that they are particularly far from our sense-based perception and understanding as well as from the deterministic/causal way in which classical physics describes the macroscopic world. Although they emerge from formalism, understanding and mastering the conceptual implications of this formalism is one of the aspects that characterizes expert knowledge, certainly not the knowledge of students who are approaching the quantum world for the first time. Nonetheless, though far from believing that this profound understanding might have been achieved by students, from their assertions it is evident that the game favored their experience of the implications of those phenomena involving superpositions and entangled states. In other words, because the game operationally exemplifies the realization of a superposition or entangled state and the entailed phenomena, it is capable of activating conceptual understanding of the same. Students' perceptions of the role by the conceptual introduction vs. the formal exemplification offered during the first part of the webinar remains to be clarified. In particular, it is not clear whether the two were perceived as disconnected (one student stated that the formalism part was useless) or integrated into gradually richer knowledge (as other students seemed to express). We believe that in a future intervention it would be welcome to implement the proposal offered by the students themselves of devoting a longer period of time to the study of these concepts.

5. Conclusions

The central message of the present work is that quantum game tools can fit into a teaching/learning environment that, when associated with other very short compacted activities, is engaging for students, boosts intervention efficiency and effectiveness, and enhances complementarity with other languages used in different steps of the physics thinking process. It is surprising how the results of this study showed a priority in terms of overcoming the conceptual difficulties thanks to an operational approach. This provides a privileged attack angle [63,83] for conceptual ownership that is eased by the operational approach while applying the rules in a context that is exemplified by the game itself.

The students in this study were able to grasp typically difficult concepts such as entanglement and superposition [14], identifying them at a high level for operational use in gameplay. The deep nature of the concepts' meaning is gained beforehand as well as by playing. In the literature, it is often stated that it is necessary to deepen students' understanding of conceptual aspects, as these can be counterintuitive. However, in the same literature it appears that there is no identification of concepts on an operational context, only in the context of ritual exercises. The game demonstrates extraordinary potential to provide operational ownership of the intuitive and formal meaning of these concepts, which assume a conceptual role in identifying the specific meaning of the new QM way of thinking, as the distance between the abstract and the operational framework is shortened.

Even more exciting is the fact that the game implies a clear goal (to win), meaning that the game challenge in fact becomes a learning goal. This is a powerful tool for the students, who favor identifying their own learning goals and grasping the perspectives via learning-related action. We note that this is consistent with the challenge posed by the complexity of society to the idea of future, where learning capacities risk being reduced if learning goals are not clear.

Finally, the students in this study demonstrated an ability to discuss superposition and entanglement better than quantum state and property. The students who could better explain the basic concepts were the same students who wanted these concepts to be re-explained, this may be due to increased awareness of what one has understood, and thus of what one is missing, which again is consistent with the opportunity offered by having clear learning goals in the gameplay environment.

The results emerging from this study extend beyond the identification of modalities to overcoming conceptual nodes, and can open a perspective on the new roles games can play in studying the new concepts in physics, many of which are increasingly far from the perspective of everyday realism. This paper opens up a new line of study in the direction described here, which we believe is necessary to dedicate further research efforts to.

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Institutional Review Board Statement: The study was conducted in accordance with the Declaration of Helsinki. All participants were fully informed that anonymity is assured.

Informed Consent Statement: The participating students voluntarily applied, by replying to a free call describing the activity, launched by the Liceo "Leonardo Da Vinci" in Treviso (Italy).

Data Availability Statement: The data presented in this study are available on request from the corresponding authors.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Examples of Student Answers to Item Q1-I1

Bastiano. "*I consolidated the basic concepts of superposition, state, properties and entanglement that I had found in popularization books by Carlo Rovelli and Brian Greene*"

Bernardo. "*I learned that for quantum mechanics, on a scientific level, the descriptive dimension is not satisfactory. Furthermore, it is based on the principles of state and property*".

Silvia: "*I got new concepts, such as state, property and measurement in the field of quantum mechanics, and I experimented with the principles of superposition and entanglement thanks to the tiq-taq-toe tournament. I found the mention of teleportation particularly interesting and understood the importance of study and research in the field of quantum mechanics*".

Michele: "*I was able to discover and understand the fundamental concepts underlying quantum mechanics, such as the difference between property and state, the meaning and physical repercussions of measurement, the principles of entanglement and superposition of states. I also deepened my knowledge of the phenomenon of polarization with the introduction of a new perspective to the phenomenon of a probabilistic nature*".

Silvia: "*I learned that QM deals with the microscopic level by solving problems, which emerged previously, as it treated the microscopic level as if it were macroscopic. For example, the QM deals with the problems of single photons*".

Chiara: "*I had the opportunity to approach a topic that, generally, is not included in our school program. I especially appreciated the fact to be able to give a more exhaustive definition of spin, a topic studied in chemistry for that which concerns atomic orbitals, but leaving out various aspects. I then got to understand the definitions, and the respective differences, between property and state*".

Nicolò: "*QM is that branch of physics that deals with studying the interactions between particles and systems at the microscopic level. Its development was necessary due to the various difficulties presented in studying and describing light phenomena using classical mechanics alone. Postulate fundamental of QM is the impossibility of determining exactly the final state of departure from the initial conditions: for example, it is possible to provide only a probabilistic evaluation of the outcome that it concerns the single particle, but do not predict a priori which path it will take*".

Appendix B. Examples of Student Answers to Item Q1-I5

Davide: "*QM is that branch of physics that deals with studying the interactions between particles and systems at the microscopic level. Its development was necessary due to the various difficulties presented in studying and describing light phenomena using classical mechanics alone. Postulate fundamental of quantum mechanics is the impossibility of determining exactly the final state from the initial conditions: for example, it is possible to provide only a probabilistic evaluation of the outcome that it concerns the single particle, but do not predict a priori which path it will take*".

Bastiano: "*The measurements make states unimaginable for usto be understood because with the restitution of values we have something tangible like property and not something extremely and* *deeply abstract, more complex. The measurement is therefore able to give back some characteristics of what is analyzed to try and understand it better*".

Bianca: "*Quantum measurements are not like those in classical mechanics. This is because classical mechanics studies the motion of macroscopic bodies. In QM however, particles behave in a strange way: sometimes the calculations show that an electron, for example, is in two different positions at the same time. Quantum measurements, therefore, can reveal surprising and impossible results in real life (macroscopic world)*".

Riccardo: "*The measurement causes a quantum system to collapse into one state rather than another, thereby determining the properties of the particle*".

Arianna: "*The measurement in QM is a transition into a state: the plummeting of the system into that measured. The measurement in MQ has a stochastic nature*".

Edoardo: "*It breaks the indeterminate quantum state by collapsing the system.*"

Silvia: "*In QM, the indeterministic and probabilistic nature of the measurement is the fundamental element. The measurement process takes part in the phenomenon that must manifest itself, disturbing it, to the point that it also changes its phenomenology (problem of the collapse of the wave function). Physical quantities that can be determined through a measurement process lead to a result predicted by a probability function and are called observable or predictable*".

Michele: "*In QM, the assumption of classical physics that measurement has no influence on the system is eliminated, as if persons were only spectators of the physical phenomena that arise in front of them by means of the experiment. In fact, in QM it is known that a measurement disturbs the physical system and causes the fact that only one of the properties attributable to the state of a body, that is the one detected*".

Niccolò: "*Measurement in QM is not absolute due to the uncertainty Heisenberg principle, according to which it is not possible to know in a definite way the values of two incompatible quantities*".

Appendix C. Examples of Student Answers to Item Q2-I1

Bastiano: "*My aim was to position the Xs in order to have more chances of hitting tic-tac-toe at the same time mainly with one in the middle and two complementary states. I used entanglement mainly for defensive purposes or to create an additional possibility*".

Bianca: "*For the game, I used different strategies: first of all I always started from the corners and never from the center, to have more chances of winning by superposition or entanglement. It took me one/two games to get it carried away. I found that the superposition and entanglement were a leap into the void, but with the right technique you could predict the opponent's moves*".

Davide: "*In all phases of the game I tried to create two sets of three boxes at the same time in such a way to make it impossible for the opponent to block them both. Leaving aside the first phase where the strategies are already known, in the second and third phase I tried to reproduce them by adapting them to the new means I had available: thanks to the superposition I tried to create one if not two rows by placing in each component of the trio chosen, a 50% chance of having my symbol in one npx. In this way my chances of having a complete row increased, in the negative case I still had the certainty of having two boxes in line occupied by my symbol and that the third was free. At this point the opponent occupied the missing box but I created an entangled state so the victory could return to my side. And so it was in most cases*".

Edoardo: "*In the first phase the classic tricks of the three of a kind, that is to build two pairs that aim to build two triples at the same time, which I am an expert in and in fact I won both rounds easily. In the second phase, since the superposition state variable made things more unpredictable, I initially tried to immediately occupy the central box with a particle, and then surround it with quantum states. It didn't work so I lost twice and decided to occupy the corners and the middle one with the overlap state and won one. The last phase was too complex to manage and I was tired, I played totally random and I lost all three*".

Sonia: "*Some general techniques (occupation of the central box, opposite corners …). The second modality, also offering the superposition, seemed instead to give an important advantage to the player with the first move after the measurement: he/she could in fact see the outcome of the moves of both players, and therefore make their own having the scoreboard available. Therefore,* *where possible, I have tried to arrange the first move after the measurement. The third mode, by presenting the entanglement, made it possible to have the possibility to modify one of the two boxes, by inverting the present symbol. Therefore, finding myself in a situation in which my opponent would have had the opportunity to complete a row, I resorted to entanglement to try to place my symbol on the third box*".

Giulia: "*[While using] entanglement, I tried to use it to have the opportunity to exchange my position with that of the opponent*".

Appendix D. Examples of Student Answers to Item Q2-I2

Michela: "*1. Properties- I have identified 3 types of properties: [definitely] empty cell (v), cell with cross (X), cell with circle (O). In a cell A these 3 properties are two by two mutually exclusive. In case of superposition or entanglement of cells A and B, the properties Xa and Xb are mutually exclusive just like Oa and Ob.*

2. State—Since the state must contain within itself all the possible properties of a cell before the properties even begin to be defined (i.e., before the game), I recognized in the cell position within the grid the state S 'last, as a superposition of the states corresponding to all the properties described above (Sv, Sx, So).

3. Distinction between property and state (explain—The state describes the possible scenarios that a particular cell might run into before it can even take on a definite property, that is, before a player selects it or before revealing the result of an overlap or an entanglement.

4. Measurement- The definition of a property regarding a cell can be done in 4 ways:—the cell is chosen by a player with a classic move outlining the property X (or O depending on the player)—the cell, if it goes into a superposition, once all the other cells have been filled in, it can take on the properties X (or O according to the player), or it can return to state S—the box, if it goes against entanglement, once all the other cells have been filled in, can take property X, or O—the box remained empty until the end of the game and took property v.

5. Superposition principle—The superposition tates SXa and SXb (or SOa and SOb), therefore up to the moment of the "measurement" it is not possible to establish which cell will take the property X (or O) and which will return to state S.

6. Entanglement- If two cells become entangled until the time of "measurement" it is not possible to establish which cell will take ownership of X and which of property O."

Giulia: "*I recognized the state in the symbols (O and X) that had to be placed in the cell that was identifiable in the fact that you could have a 50% chance that your symbol could be in one cell and another. Subsequently, however, I re-recognized the entanglement in the "bond" that could be established between two different states, the measurement was recognized when the entangled and superposed states could occur with a probability of 50% in one of the selected cells. Finally, the difference between state and property can be seen in the fact that only states were involved in the superposition*".

Niccolò: "*During the game I was able to recognize the superposition principle, which was one of the possible moves in which two symbols of the same type, not definitive, are formed, which with a subsequent measurement will be determined in one or the other cell. The second aspect that I have recognized is the entanglement represented by the uncertainty of the two symbols in two different cells, which consequently to the measurement will be one symbol or the other in the respective cells. The third, on the other hand, is the measurement, which cannot be absolute for both quantities found in the superposition. I also recognized the property which, being a specific value, was represented by the definitive sign*".

Andrea: "*I did not recognize the property, nor the state, but not even the distinction between the two. I found again the concept of measurement when the collapse of the game grid occurred. The superposition principle, entanglement and their mechanisms were found between the second and third phase of the game, where certain symbols could be randomly eliminated*".

Ivan: "*2. State The state was represented by symbols; 4. Measurement. I recognized the concept of measurement when the result was shown only when all the cells were filled in. 5. Superposition principle. I recognized the superposition principle in the second type of game, in which you could choose two cells where to place your symbol and in the end both had a 50% chance* *that it would end up in one cell rather than the other. 6. Entanglement. I recognized entanglement in the last type of game, in which different symbols could be coupled"*.

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