Towards a comprehensive framework for V2G optimal operation in presence of uncertainty

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***Abstract*— As the global fleet of Electric Vehicles keeps increasing in number, the Vehicle To Grid (V2G) paradigm is gaining more and more attention. From the grid point of view an aggregate of electric vehicles can act as a flexible load, thus able to provide balancing services. The problem of computing the optimal day-ahead charging schedule for all vehicles in the fleet is a challenging one, especially because it is affected by many sources of uncertainty. In this paper we consider the uncertainty deriving from arrival and departure times, arrival energy and services market outcomes. We propose a general optimization framework to deal with the day ahead planning that encompasses different kind of use-cases. We adopt a robust paradigm to enforce the constraints and an expectation paradigm for the cost function. For all constraints and cost terms we propose an exact formulation or a very tight approximation, even in the case of piece-wise linear battery dynamics. Numerical results corroborates the theoretical findings.**

***Index Terms*— Vehicle to grid; Ancillary services; Uncertain optimization**

I. Introduction

THE global fleet of Electric Vehicles (EV) has expanded exponentially during the last decade, arriving to more than 7.2 millions by 2019, [1]. For the electric grid, EVs are flexible loads that can be exploited to improve the efficiency of network operation. However, the impact of a single EV on the grid is marginal and a large population is required to offer adequate services to the system operator, [2]. Aggregators are new market participants, whose aim is to properly coordinate the actions of costumers to participate in electricity markets, [3]. An aggregation of energy users can support the grid operation, for example alleviating technical constraints, reducing peak-load or acting as a virtual power plant. Within the electrical market, the integration of vehicle charging systems offers new opportunities but also new issues [4]–[8]. It will be possible to plan vehicle recharges so that production overloads can be managed or vehicles can be used as synthetic inertia, but at the same time there are instabilities due to simultaneous and unpredictable access to infrastructure in supply points that may already be critical. The use of vehicle aggregators can be a solution for many of these issues. A central coordinator that gathers the needs of vehicles and the network and schedules both charging and discharging in accordance with pricing policies. Two aggregation approaches can be distinguished in literature. Incentive-based methods use price or incentive signals to influence the consumers behavior. These are unidirectional solutions with reduced infrastructure requirements. However, they can suffer from reliability issues because the consumption decision is taken by final users, that can be influenced by external phenomena [9], [10]. On the other hand, direct control approaches, assume that the aggregator can directly manage the loads operation, guaranteeing robustness in the actuation of consumption plans, while requiring bidirectional communication infrastructure [11].

One of the main challenges when designing an aggregation strategy coping with EV needs is handling uncertainty in the fleet behavior. Arrival and departure times of each vehicle, and also the initial State Of Charge (SOC) are fundamental parameters that the aggregator requires from each vehicle to properly schedule the power flows. When participating in wholesale markets, the aggregator must also deal with uncertainty in the outcomes of ancillary services, [12].

In the context of electrified transportation, different approaches for the aggregation of EV fleets using direct control have been proposed. They differ in the objective function to be optimized, the timeline of the operation and the approach to deal with uncertainty. For example, in [13] the authors describe a model for the day-ahead optimization of energy provision for a fleet of EVs in a deterministic setting, considering also regulation services. The study concludes that the optimal bidding strategies exhibit high sensitivity to input parameters that are uncertain during the actual operation of the system.

Most of the existing solutions to face uncertainty in the aggregation operation use stochastic optimization to maximize the expected profit of the aggregator, imposing some limit to the probability of noncompliance of the imposed constraints. In [11], the authors use stochastic programming to solve the scheduling of a fleet of EVs maximizing the profit of the aggregator by charging the vehicles and participating in ancillary services markets. The model considers uncertainty in market prices, vehicles availability and reserves activation. A proper scenario generation and a conditional value at risk formulation is followed. [14] formulates a stochastic linear program to minimize the recharging cost of the fleet, facing uncertainty in energy prices, renewable sources generation and inflexible loads served by the aggregator. No uncertainty is considered

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about availability of EVs. [12] formulates a stochastic optimization problem, considering market prices uncertainty only. The authors use stochastic p-robust optimization to trade-off expected profit and maximum related regret. [15] applies game theory to represent the decision process of EV owners and use the resulting models to derive an optimal strategy to participate in energy markets for the aggregator. Recharging level constraints are imposed in probability, limiting the noncompliance risk. [16] presents a model to schedule the over-night charging of EV in residential context. Uncertainty in arrival time and SOC are considered. Detailed information about the probability distribution of uncertain information is required to minimize the risk of constraint violation, including network overloading, voltage deviation, and low recharging level. [17] uses a scenario approach to solve a stochastic day-ahead dispatch problem for a EVs aggregator, considering also participation in the reserves market. The model considers uncertainty in market prices only and deals with it using a risk-constrained formulation. [18] proposes a scenario approach to solve a two-stage stochastic EV charging scheduling problem. The model handles uncertainty in EVs availability (arrival and departure time) and arrival SOC. A receding horizon approach is applied for real-time operation and EVs with similar availability patterns are aggregated to reduce the number of scenarios considered in the optimization.

A second research line to solve the EVs recharging scheduling is the use of robust optimization, where the uncertainty is formulated in a worst-case scenario. [19] combines robust and stochastic programming techniques to solve the day-ahead bidding problem of an EV fleet aggregator. The model considers scenarios to represent market prices, while aggregated power and energy limits, described by confidence bounds, are employed instead of arrival and departure times, to represent vehicles behavior. [2] formulates a multi-objective optimization problem minimizing the operational costs and maximizing the flexibility of the fleet. The model considers the uncertainty in the arrival time and SOC for each vehicle, while the departure time is assumed known. A receding horizon strategy is employed to compensate the deviation caused by the uncertainty, guaranteeing feasibility in the worst-case. [20] uses a hierarchical model to solve the day-ahead energy provision problem of an aggregator of EVs with V2G capability. The model uses a robust formulation considering the uncertainty in vehicles availability and energy requirements. The upper level problem maximizes the profit of the operation while two low level problems determine the worst-case scenarios of battery draining and power exchange deviations with the grid. [21] proposes a robust solution to the day-ahead scheduling of an aggregator that integrates wind power and manages a fleet of EVs with V2G capabilities. The framework considers the uncertainty in wind power generation and arrival SOC of the vehicles. EVs availability is assumed known.

From the previous review it can be noticed that existing formulations based on stochastic optimization require detailed information about the probability distributions of uncertain variables and in most cases result in computationally demanding solutions, while the obtained strategies cannot guarantee to satisfy the requests from the TSO for all the possible scenarios. On the other hand, robust formulations mostly focus in the day ahead planning of the charging/discharging profiles, without incorporating ancillary services in the aggregator operation.

In this work we propose a general framework to formulate a day-ahead EVs recharging scheduling problem for an aggregator that operates a parking lot of a company, coping with most of the limitation of existing solutions. The aggregator participates in the energy and ancillary services markets. In the formulation we consider different types of cost and constraints which can be selected depending on the specific application. The model uses a robust formulation to deal with uncertainties in vehicles availability, arrival SOC and TSO service activation signal in the constraints. The formulation guarantees that the optimal schedule is feasible for the worst-case realization of the uncertain variables (which in turn guarantees that the TSO requests can always be satisfied), while the aggregator maximizes its expected revenue for energy provision and participation in ancillary services provision. We propose a detailed derivation of the robust counterpart of all constraints and show how they can be approximated to reduce the computational complexity in obtaining the solution. We also provide the analytic expression of the expected value of the cost function (or an approximation of it) for simple distributions of the uncertainty parameters. We pay particular attention to deriving tight approximations so as not to make the formulation overly conservative. Numerical simulations show that the proposed robust scheduling framework allows to maximize the profit of the aggregator while satisfying all the requests from the TSO and the energy demand of the EV fleet, in front of a large amount of realizations of the uncertain variables. Moreover, the computational cost of the algorithm is low, allowing to handle a large amount of EVs in the operation of the V2G service.

Notation: For any scalar $v \in \mathbb{R}$, we denote its positive part with $[v]^+ = \max\{v, 0\}$ and its negative part with $[v]^- = \max\{-v, 0\}$, so that we can always express $v = [v]^+ - [v]^-$ with $[v]^+$ and $[v]^-$ both non-negative. The indicator function over a set $V \subset \mathbb{R}^n$ is denoted as $\mathbf{1}_V(v)$ and for any $v \in \mathbb{R}^n$ we have $\mathbf{1}_V(v) = 1$ if $v \in V$ and $\mathbf{1}_V(v) = 0$ otherwise. For a generic random variable X, we denote as $\mathbb{P}_X$ its associated probability measure, with $\mathbb{P}_X\{X \in \mathcal{X}\}$ the probability of the event $X \in \mathcal{X}$, and with $\mathbb{E}_X[\,\cdot\,]$ the expectation (when clear from the context we will use $\mathbb{P}$ and $\mathbb{E}$ without subscript).

II. Proposed Framework

Consider the electric vehicle (EV) parking lot of a company. Assume the company has N employee with an EV, each of which has one charging station assigned[1]. Since for most of the day the vehicles are parked, the company would like to leverage their internal batteries to provide ancillary services to the main grid. To this end, the company has to estimate the power exchange profile and the maximum amount of upward and downward power variations it will be able to provide for the next day and communicate them to the main grid. In this section we formulate a mathematical program to optimally

[1] This assumption admittedly restrict the applicability of the framework, but it enable us to simplify some derivations.

plan such profiles, the company's goal being to minimize the EVs charging costs and maximize the revenues associated with the ancillary service provision. To formulate the finite horizon optimal control problem we consider a one-day time horizon discretized into T time intervals (referred to as time-slots) indexed by $k = 0, \dots, T-1$.

A. Electric Vehicles Modeling

Let us denote by a_i (arrival) and d_i (departure) the first and last *full* time-slots in which EV i is plugged into its charging station, meaning that vehicle i arrives during time-slot $a_i - 1$ and departs during time-slot $d_i + 1$. Even though employees and their EVs are the same from one day to the next, their time of arrival/departure may change. Quantities a_i and d_i are thus considered to be uncertain, but limited to some intervals $a_i \in [\underline{a}_i, \overline{a}_i]$ and $d_i \in [\underline{d}_i, \overline{d}_i]$, with $a_i, \underline{a}_i, \overline{a}_i, d_i, \underline{d}_i, \overline{d}_i \in \{0, \dots, T-1\}$ and $a_i \leq d_i$.

Each EV is equipped with a battery, whose State Of Charge (SOC) at the beginning of time-slot k is denoted by $e_{k,i}$ and, for $k \in [a_i, d_i]$, obeys the following dynamics

$$e_{k+1,i} = \alpha_i \, e_{k,i} + \tau \, \eta_{k,i} \, p_{k,i}, \tag{1}$$

where τ represents the time-slot duration, $p_{k,i}$ denotes the average power used to charge ($p_{k,i} > 0$) or discharge ($p_{k,i} < 0$) the battery during time-slot k, $\alpha_i \in (0, 1]$ models self-discharging losses, and

$$\eta_{k,i} = \begin{cases} \eta_i^+ & p_{k,i} \geq 0 \\ \frac{1}{\eta_i^-} & p_{k,i} < 0 \end{cases} \tag{2}$$

models charging/discharging losses, $\eta_i^+, \eta_i^- \in (0, 1]$ being the charging/discharging efficiencies. Note that, since $p_{k,i}$ cannot be positive and negative at the same time, charging and discharging are mutually exclusive.

The battery SOC always stays within a minimum $e_i^{\min} > 0$ and a maximum $e_i^{\max} > 0$ value and therefore

$$e_i^{\min} \leq e_{k,i} \leq e_i^{\max} \qquad k \in [a_i, d_i] \tag{3}$$

must hold for any EV i. By noticing that (1), with $\eta_{k,i}$ as in (2), can be equivalently expressed as

$$e_{k+1,i} = \alpha_i \, e_{k,i} + \tau \min \left\{ \eta_i^+ p_{k,i}, \frac{1}{\eta_i^-} p_{k,i} \right\}, \tag{4}$$

we can explicitly compute $e_{k,i}$ in (3) as

$$e_{k,i} = \alpha_i^{k-a_i} \, e_{a_i,i} + \tau \sum_{t=a_i}^{k-1} \alpha_i^{k-1-t} \min \left\{ \eta_i^+ p_{t,i}, \frac{1}{\eta_i^-} p_{t,i} \right\}, \tag{5}$$

as a function of $p_{t,i}$, $t \in [a_i, k-1]$, for all $k \in [a_i, d_i + 1]$.

To ease the notation, we use the symbol e_i^0 in place of $e_{a_i,i}$ (i.e., $e_i^0 \equiv e_{a_i,i}$) to denote the SOC at arrival. Note that if $a_i > 0$, then $e_i^0 \neq e_{0,i}$. The arrival SOC e_i^0 is also considered to be uncertain, with $e_i^0 \in [e_i^\mu - e_i^\Delta, e_i^\mu + e_i^\Delta] \subset [e_i^{\min}, e_i^{\max}]$, $e_i^\mu, e_i^\Delta > 0$ being the midpoint and the half-width of the uncertainty interval, respectively.

The power exchange $p_{k,i}$ between any charging station and its connected EV is limited by a maximum amount $p_i^{\max} > 0$ when the vehicle is present and is zero otherwise. Thus it must hold

$$p_{k,i} \in [-p_i^{\max}, p_i^{\max}] \qquad k \in [a_i, d_i] \tag{6a}$$

$$p_{k,i} = 0 \qquad k \notin [a_i, d_i] \tag{6b}$$

for any EV i. Moreover, the overall power that the parking lot can exchange with the grid is also limited by $p^{\max} > 0$, and hence

$$-p^{\max} \leq \sum_{i=1}^{N} p_{k,i} \leq p^{\max}. \tag{7}$$

Power limits are assumed to be symmetric, but all derivations can be readily extended to asymmetric power upper and lower bounds as, e.g., in V1G scenarios.

At any time-slot k, if an EV is charging its owner pays c_k^{v+} per energy unit to the company, otherwise if the vehicle is discharging the company pays $c_k^{v-} > c_k^{v+}$ per energy unit to the owner. The cost incurred by the company for vehicle i is thus given by

$$c_{k,i}^{\text{veh}} = c_k^{v-} \left[\tau p_{k,i} \right]^- - c_k^{v+} \left[\tau p_{k,i} \right]^+, \tag{8}$$

and the overall cost for vehicles charging/discharging over the entire horizon is

$$c^{\text{veh}} = \sum_{k=0}^{T-1} \sum_{i=1}^{N} c_{k,i}^{\text{veh}}. \tag{9}$$

B. Vehicles Residual Energy

Depending on the considered application, one may want to handle the residual SOC $e_{d_i+1,i}$ of EV i at departure differently. If we know that vehicle i will return to the parking lot the next day (like in the company parking lot considered here), then part of the energy $e_{d_i+1,i}$ stored in EV i will be available the next day. To monetize such residual energy we can consider a (negative) cost given by

$$c^{\text{soc}} = -c^{\text{avg}} \sum_{i=1}^{N} e_{d_i+1,i}, \tag{10}$$

where c^{avg} is an average energy unit price over the considered time horizon.

Another aspect that can be easily incorporated into the framework is a minimum requirement on the SOC at departure, which can be taken into account enforcing the constraint

$$e_{d_i+1,i} \geq e_i^\circ, \tag{11}$$

where $e_i^\circ \in [e_i^{\min}, e_i^{\max}]$ is EV i desired SOC at departure, which can be an absolute value, or it can be expressed as a desired increase γ_i with respect to the SOC at arrival e_i^0, i.e., $e_i^\circ = e_i^0 + \gamma_i$.

C. Day Ahead and Ancillary Services Markets

Since the company has to establish a baseline power exchange profile with the main grid and the offered capacity for the ancillary services, it is convenient to express $p_{k,i}$ as

$$p_{k,i} = p_{k,i}^{\text{dam}} + p_{k,i}^{\text{asm}}, \tag{12}$$

where $p_{k,i}^{\rm dam}$ is the portion of $p_{k,i}$ purchased on the day-ahead market, while $p_{k,i}^{\rm asm}$ is the portion of $p_{k,i}$ used to dispatch ancillary services.

At any time-slot k, buying an energy unit on the day-ahead market costs $c_k^{e+} < c_k^{v+}$, while the grid pays $c_k^{e-} < c_k^{e+}$ for energy unit sold. The company buys energy whenever the net power requested by all EVs is positive, and sells energy otherwise. The cost incurred by the company for the day-ahead market over the entire horizon is thus given by

$$c^{\rm dam} = \sum_{k=0}^{T-1} c_k^{e+} \left[\sum_{i=1}^{N} \tau p_{k,i}^{\rm dam}\right]^+ - c_k^{e-} \left[\sum_{i=1}^{N} \tau p_{k,i}^{\rm dam}\right]^- . \quad (13)$$

As for the ancillary services, since they are typically divided into upward and downward services (see Remark 1) , it is convenient to further express $p_{k,i}^{\rm asm}$ as

$$p_{k,i}^{\rm asm} = s_{k,i}^+ [\omega_k]^+ - s_{k,i}^- [\omega_k]^- , \quad (14)$$

where $s_{k,i}^+ \geq 0$ and $s_{k,i}^- \geq 0$ is the maximum power variation offered by EV i in time-slot k for the downward and upward services respectively, while $\omega_k \in [-1, 1]$ is the actual service signal provided by the Transmission System Operator (TSO) for time-slot k: $\omega_k = 1$ if the TSO requests all the offered downward power variation; $\omega_k = -1$ if the TSO requests all the offered upward power variation; $\omega_k = 0$ if the TSO does not request any variation with respect to the baseline power profile; or any fractional value in case a fraction of the offered flexibility is requested. The overall capacity offered by the company for time-slot k is simply given by $\sum_{i=1}^N s_{k,i}^+$ for upward variations and $\sum_{i=1}^N s_{k,i}^-$ for downward variations.

An energy unit bought on the ancillary service market costs $c_k^{s+} < c_k^{e+}$, while an energy unit sold pays $c_k^{s-} \in (c_k^{e-}, c_k^{v-})$. The company buys energy on the ancillary service market whenever $\omega_k > 0$ and sells it when $\omega_k < 0$. The total cost incurred by the company for the ancillary service market over the entire horizon is thus given by

$$c^{\rm asm} = \sum_{k=0}^{T-1} c_k^{s+} \sum_{i=1}^{N} \tau s_{k,i}^+ [\omega_k]^+ - c_k^{s-} \sum_{i=1}^{N} \tau s_{k,i}^- [\omega_k]^- . \quad (15)$$

Clearly, the signal ω_k is not known at the time when the company has to define the baseline profile and the offered service capacity, as it refers to the next day, and therefore ω_k is also considered to be uncertain.

Finally, the ancillary service market may require to provide upwards and downwards services for a certain number of consecutive time slots $T^{\rm asm}$ (e.g., for 2 hours in Italy). In this cases we need to impose the additional constraints

$$\sum_{i=1}^{N} s_{(r-1)T^{\rm asm},i}^+ = \cdots = \sum_{i=1}^{N} s_{rT^{\rm asm}-1,i}^+ \qquad r \in [1, \tfrac{T}{T^{\rm asm}}], \quad (16a)$$

$$\sum_{i=1}^{N} s_{(r-1)T^{\rm asm},i}^- = \cdots = \sum_{i=1}^{N} s_{rT^{\rm asm}-1,i}^- \qquad r \in [1, \tfrac{T}{T^{\rm asm}}]. \quad (16b)$$

Remark 1: Note that we here assumed the presence of a single service market with costs c_k^{s+} and c_k^{s-} and service signal ω_k. However, if multiple bidding markets are considered, then different costs $c_k^{s_j+}$ and $c_k^{s_j-}$, different signals ω_k^j, and different allotted capacities $s_{k,i}^{j+}$ and $s_{k,i}^{j-}$ for each ancillary service j could be easily introduced in the proposed framework without any conceptual leap. The same formulation applies also to capacity services, where the aggregator offers a given capacity and is remunerated independently on how much of the total capacity is actually requested. In this case the uncertain signal is either 0 or 1 with a distribution given by the probability of acceptance. ■

D. Optimal Planning

We are now in a position to formulate the mathematical program the company has to solve to optimally plan its baseline profile and the amount of ancillary services offered to the grid, which reads as

$$\begin{array}{lll} \displaystyle\min_{p_{k,i}^{\rm dam}, s_{k,i}^+, s_{k,i}^-} & (9) + (10) + (13) + (15) & (\mathcal{P}_\delta) \\ \text{subject to:} & (7) & \forall k \\ & (3), (6), (11) & \forall i \\ & (16) & \\ & s_{k,i}^+, s_{k,i}^- \geq 0 & \forall i, \ \forall k \end{array}$$

where $p_{k,i}^{\rm dam}$, $s_{k,i}^+$, $s_{k,i}^-$, for all $i = 1, \ldots, N$ and $k = 0, \ldots, T-1$ are the decision variables, $e_{k,i}$ and $e_{d_i,i}$ in (3), (10), and (11) are given by (4), (9) can be computed using (8), and $p_{k,i}$ in (7), (6), (4), and (8) is expressed as $p_{k,i} = p_{k,i}^{\rm dam} + s_{k,i}^+ [\omega_k]^+ - s_{k,i}^- [\omega_k]^-$ combining (12) and (14).

Unfortunately, the cost and the constraints of $\mathcal{P}_\delta$ depend on the uncertain parameters a_i, d_i, e_i^0, for all $i = 1, \ldots, N$ and ω_k for all $k = 0, \ldots, T-1$, which we will collectively refer to as the uncertainty δ taking values in a set Δ. This renders problem $\mathcal{P}_\delta$ ill-posed as δ is not known at the time a solution to $\mathcal{P}_\delta$ has to be computed. This issue is tackled in the next section, where we show how to handle δ and compute an uncertainty-aware solution to the optimal planning problem.

III. Taming Uncertainty

We propose to handle the uncertainty differently based on whether it appears in the cost function or in the constraints and based on the type of constraint. Since constraints (3) and (6) represents physical limitations on EVs battery operations, we propose a robust paradigm, enforcing those constraints for all possible values of $\delta \in \Delta$. As for constraint (11), which is not a hard constraint, we can either pursue a robust approach, or impose the weaker requirement that

$$\mathbb{E}[e_{d_i+1,i}] \geq \mathbb{E}[e_i^\circ], \quad (17)$$

for all $i = 1, \ldots, N$, so that the employees are "on average" satisfied with their SOC at departure. Finally, for the cost function, we propose to minimize the expected cost.

To reduce the conservatism resulting from adopting a robust paradigm, we also propose to parametrize component $p_{k,i}^{\rm dam}$ of the power exchanged by EV i during time-slot k in the day-ahead market as

$$p_{k,i}^{\rm dam} = \bar{p}_{k,i} - \vartheta_{k,i}(e_i^0 - e_i^\mu), \quad (18)$$

where $\bar{p}_{k,i}$ is the baseline control policy for EV i while $\vartheta_{k,i}(e_i^0 - e_i^\mu)$, with $\vartheta_{k,i} \geq 0$, is introduced to adapt the optimal control policy given the actual arrival SOC EV i will have the next day. Given the uncertainty interval of e_i^0, we have $(e_i^0 - e_i^\mu) \in [-e_i^\Delta, e_i^\Delta]$.

Finally, the resulting (deterministic) optimal control problem reads as

$$\begin{aligned} \min_{\bar{p}_{k,i},\vartheta_{k,i},s^+_{k,i},s^-_{k,i}} \quad & \mathbb{E}[(9)+(10)+(13)+(15)] && (\mathcal{P}) \\ \text{subject to:} \quad & (7)\ \delta \in \Delta && \forall k \\ & (3),(6)\ \delta \in \Delta && \forall i \\ & (11)\ \delta \in \Delta,\ \text{ or } (17) && \forall i \\ & (16) && \\ & s^+_{k,i}, s^-_{k,i}, \vartheta_{k,i} \geq 0 && \forall i,\ \forall k \end{aligned}$$

where the decision variables are now $\bar{p}_{k,i}$, $\vartheta_{k,i}$, $s^+_{k,i}$, $s^-_{k,i}$, for all $i = 1,\dots,N$ and $k = 0,\dots,T-1$ and the cost and constraints depends on those variables similarly to $\mathcal{P}_\delta$ given that now $p_{k,i} = \bar{p}_{k,i} - \vartheta_{k,i}(e_i^0 - e_i^\mu) + s^+_{k,i}\,[\omega_k]^+ - s^-_{k,i}\,[\omega_k]^-$ owing to (18).

Next, we shall show how to compute the robust version of the constraints and the expected values appearing in $\mathcal{P}$.

A. Assumptions on Uncertainty

For the subsequent derivations, we make the following assumptions.

Assumption 1 (Independence): The random variables e_i^0, a_i, d_i for all $i = 1,\dots,N$, and ω_k for all $k = 0,\dots,T-1$, are all independent from each other. ■

This assumption ensures that most of the subsequent estimates for the constraints in $\mathcal{P}$ are tight and eases the computation of the expected values in the cost function. In case Assumption 1 is not satisfied, constraints estimates are still valid, but are not tight anymore. Unfortunately, the expected values in the cost function has to be recomputed.

Assumption 2 (Non-zero Stopover): Arrival and departure time windows overlap is at most equal to a single time-slot, i.e., $\overline{a}_i \leq \underline{d}_i$. ■

Assumption 2 ensures that we are certain that each EV stays attached to its charging station for at least one (full) time-slot. Relaxing this assumption requires a modification of the approach. In those time-slots when the vehicle is possibly not plugged, the company can still buy energy from the DAM but it will not be guaranteed that that power will be absorbed. In such a case the company will incur in an unbalancing cost as it will absorb less quantity than the quantity expected. These additional costs have to be properly minimized together with the rest. This extension is left as a future improvement of the proposed framework.

Assumption 3 (Minimum SOC at Arrival): We assume that $e_i^\mu - e_i^\Delta \geq e_i^{\min}/\alpha_i^{\overline{a}_i - \underline{a}_i}$ to ensure that EV i does not arrive with a critically low SOC. ■

Note that when α_i is close to 1, as is typically the case, $e_i^{\min}/\alpha_i^{\overline{a}_i - \underline{a}_i}$ in Assumption 3 is slightly above $e_i^{\min}$ meaning that, in practice, we are only requiring the vehicle to arrive with a non-empty battery.

Assumption 4 (Distribution): The random variables e_i^0, a_i, d_i for all $i = 1,\dots,N$, all have a uniform distribution over their support. The random variable ω_k, for all $k = 0,\dots,T-1$, is defined as follows: with probability π^+, ω_k is drawn from a uniform distribution over the interval $[0,1]$, with probability π^-, ω_k is drawn from a uniform distribution over the interval $[-1,0]$, and with probability $\pi^0 = 1 - \pi^+ - \pi^-$, we have that $\omega_k = 0$. ■

Assumption 4 is only required to compute the expected values in $\mathcal{P}$ analytically. In the data-based solution strategy proposed in Section III-E this assumption is not necessary.

B. Power Constraints

Let us consider the robust counterpart of constraint (6). From (6b), $p_{k,i} = 0$ whenever $k \notin [a_i, d_i]$. Since this has to be true for every possible combination of $a_i \in [\underline{a}_i, \overline{a}_i]$ and $d_i \in [\underline{d}_i, \overline{d}_i]$ we need to enforce $p_{k,i} = 0$ whenever $k \notin [\overline{a}_i, \underline{d}_i]$, which is to say that in all those time-slots in which we are not certain that EV i is attached to the charging station, we do not plan charge/discharge the vehicle. Instead, by (6a), for any $k \in [\overline{a}_i, \underline{d}_i]$ we need to enforce $-p_i^{\max} \leq p_{k,i} \leq p_i^{\max}$. Clearly

$$\begin{aligned} p_{k,i} &= \bar{p}_{k,i} - \vartheta_{k,i}(e_i^0 - e_i^\mu) + s^+_{k,i}\,[\omega_k]^+ - s^-_{k,i}\,[\omega_k]^- \\ &\leq \bar{p}_{k,i} - \vartheta_{k,i}(e_i^0 - e_i^\mu) + s^+_{k,i} \\ &\leq \bar{p}_{k,i} + \vartheta_{k,i} e_i^\Delta + s^+_{k,i}, \end{aligned}$$

where the first inequality is due to $\omega_k \in [-1,1]$ together with $s^+_{k,i}$, $s^-_{k,i}$, $[\omega_k]^+$, and $[\omega_k]^-$ all being non-negative, and the second inequality is due to $(e_i^0 - e_i^\mu) \in [-e_i^\Delta, e_i^\Delta]$ together with $\vartheta_{k,i} \geq 0$. Note that, under Assumption 1, the bound is tight. Similarly,

$$\begin{aligned} p_{k,i} &= \bar{p}_{k,i} - \vartheta_{k,i}(e_i^0 - e_i^\mu) + s^+_{k,i}\,[\omega_k]^+ - s^-_{k,i}\,[\omega_k]^- \\ &\geq \bar{p}_{k,i} - \vartheta_{k,i}(e_i^0 - e_i^\mu) - s^-_{k,i} \\ &\geq \bar{p}_{k,i} - \vartheta_{k,i} e_i^\Delta - s^-_{k,i}. \end{aligned}$$

The robust counterpart of constraint (6) is thus equivalent to

$$\begin{aligned} &\bar{p}_{k,i} + \vartheta_{k,i} e_i^\Delta + s^+_{k,i} \leq p_i^{\max} && k \in [\overline{a}_i, \underline{d}_i], && \text{(19a)} \\ &\bar{p}_{k,i} - \vartheta_{k,i} e_i^\Delta - s^-_{k,i} \geq -p_i^{\max} && k \in [\overline{a}_i, \underline{d}_i], && \text{(19b)} \\ &p_{k,i} = 0 && k \notin [\overline{a}_i, \underline{d}_i], && \text{(19c)} \end{aligned}$$

where the interval $[\overline{a}_i, \underline{d}_i]$ is non-empty under Assumption 2, meaning that we allow $p_{k,i} \neq 0$ for at least one time-slot.

Given the (tight) upper and lower bounds found for $p_{k,i}$, we can also construct upper and lower bounds for $\sum_{i=1}^N p_{k,i}$ (which are also tight due to independence across vehicles) and derive the robust counterpart of constraint (7) as

$$\sum_{i=1}^N \bar{p}_{k,i} + \vartheta_{k,i} e_i^\Delta + s^+_{k,i} \leq p^{\max}, \tag{20a}$$

$$\sum_{i=1}^N \bar{p}_{k,i} - \vartheta_{k,i} e_i^\Delta - s^-_{k,i} \geq -p^{\max}. \tag{20b}$$

C. State of Charge Constraints

Let us now consider constraint (3) with $e_{k,i}$ expressed as in (5). We have the following result

Theorem 1: Under Assumptions 1, 2, and 3 the robust counterpart of constraint (3) for EV i is given by

$$\alpha_i^{k-\overline{a}_i} e_i^{\mu} + \min_{\{\eta_{t,i}\in H_i\}_t} \overline{g}_{k,i}(\{\eta_{t,i}\}_t) \le e_i^{\max} \tag{21}$$

and

$$\alpha_i^{k-\underline{a}_i} e_i^{\mu} + h_{i,k} \ge e_i^{\min}(k) \tag{22a}$$

$$h_{k,i} \le \underline{g}_{k,i}(\{\eta_{t,i}\}_t) \qquad \left\{\eta_{t,i} \in \{\eta_i^+, \tfrac{1}{\eta_i^-}\}\right\}_{t=\overline{a}_i}^{k-1} \tag{22b}$$

for each $k \in [\overline{a}_i + 1, \underline{d}_i + 1]$, with $H_i = [\eta_i^+, \frac{1}{\eta_i^-}]$,

$$e_i^{\min}(k) = \begin{cases} e_i^{\min} & k \in [\overline{a}_i + 1, \underline{d}_i] \\ \frac{e_i^{\min}}{\alpha_i^{\overline{d}_i - \underline{d}_i}} & k = \underline{d}_i + 1 \end{cases}$$

$$\overline{g}_{k,i}(\{\eta_{t,i}\}_t) = \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} + s_{t,i}^+) + \left|\alpha_i^{k-\overline{a}_i} - \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}\vartheta_{t,i}\right| e_i^{\Delta}$$

$$\underline{g}_{k,i}(\{\eta_{t,i}\}_t) = \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} - s_{t,i}^-) - \left|\alpha_i^{k-\underline{a}_i} - \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}\vartheta_{t,i}\right| e_i^{\Delta},$$

and $h_{k,i}$ being auxiliary continuous decision variables. ■

While constraint (22) is convex, constraint (21) is non-convex and can be either dealt with a dedicated solver, or approximated. Moreover, even though (22) is convex, the number of inequalities required to define it grows exponentially with k. We next discuss an interesting case in which constraints (22) and (21) can be greatly simplified.

Let us consider the case in which the compensator parameters $\vartheta_{k,i}$ satisfy the following linear constraint

$$\frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\underline{d}_i - t} \vartheta_{t,i} \le \alpha_i^{\underline{d}_i - \underline{a}_i + 1}, \tag{23}$$

then we have the following result.

Proposition 1: Under Assumptions 1, 2, and 3, if we enforce the additional constraint (23), then constraints (21) and (22) can be equivalently enforced as

$$\alpha_i^{k-\overline{a}_i}(e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \overline{h}_{t,i} \le e_i^{\max} \tag{24a}$$

$$\overline{h}_{t,i} \ge \eta_i^+(\bar{p}_{t,i} + s_{t,i}^+ - \vartheta_{t,i} e_i^{\Delta}) - p_i^{\max}(1 - b_{t,i}) \tag{24b}$$

$$\overline{h}_{t,i} \ge \tfrac{1}{\eta_i^-}(\bar{p}_{t,i} + s_{t,i}^+ - \vartheta_{t,i} e_i^{\Delta}) - p_i^{\max} b_{t,i} \tag{24c}$$

and

$$\alpha_i^{k-\underline{a}_i}(e_i^{\mu} - e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \underline{h}_{t,i} \ge e_i^{\min}(k) \tag{25a}$$

$$\underline{h}_{t,i} \le \eta_i^+(\bar{p}_{t,i} - s_{t,i}^- + \vartheta_{t,i} e_i^{\Delta}) \qquad t \in [\overline{a}_i, \underline{d}_i] \tag{25b}$$

$$\underline{h}_{t,i} \le \tfrac{1}{\eta_i^-}(\bar{p}_{t,i} - s_{t,i}^- + \vartheta_{t,i} e_i^{\Delta}) \qquad t \in [\overline{a}_i, \underline{d}_i] \tag{25c}$$

for each $k \in [\overline{a}_i + 1, \underline{d}_i + 1]$, with $e_i^{\min}(k)$ defined as in Theorem 1, $\overline{h}_{t,i}$ and $\underline{h}_{t,i}$ additional continuous decision variables, and $b_{t,i} \in \{0, 1\}$ additional binary decision variables. ■

Remark 2: Constraint in (23) has the following intuitive explanation. For the sake of simplicity let us assume $\alpha_i = \eta_i^+ = \eta_i^- = 1$. Then, (23) becomes $\tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \vartheta_{t,i} \le 1$, implying

$$\left|\tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \vartheta_{t,i}(e_i^0 - e_i^{\mu})\right| \le \left|e_i^0 - e_i^{\mu}\right|$$

for any e_i^0. This means that the amount of energy provided to EV i over the time frame $[\overline{a}_i, \underline{d}_i]$ to compensate the uncertainty in e_i^0 cannot exceed $|e_i^0 - e_i^{\mu}|$. But this is not limiting at all as it would not make sense to compensate for more than $|e_i^0 - e_i^{\mu}|$. ■

Even though the result given by Proposition 1 greatly simplifies imposing the robust counterpart of constraint (3), constraint (24) requires us to solve a Mixed-Integer program, whose complexity grows exponentially with the number of binary decision variables. For those cases in which computing resources are limited, we propose the following convex approximation of (24).

Corollary 1: Constraint (24) can be approximated as

$$\alpha_i^{k-\overline{a}_i}(e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \times \eta_i^+(\bar{p}_{t,i} + s_{t,i}^+ - \vartheta_{t,i} e_i^{\Delta}) \le e_i^{\max}. \tag{26}$$

■

If one is interested in enforcing the robust counterpart of constraint (11), then, if the desired SOC at departure e_i° is given in absolute terms, this would simply entail changing the definition of $e_i^{\min}(k)$ in (22a) or (25a) to

$$e_i^{\min}(k) = \begin{cases} e_i^{\min} & k \in [\overline{a}_i + 1, \underline{d}_i] \\ \frac{e_i^{\circ}}{\alpha_i^{\overline{d}_i - \underline{d}_i}} & k = \underline{d}_i + 1 \end{cases}. \tag{27}$$

Otherwise, if the desired SOC at departure is given as $e_i^{\circ} = e_i^0 + \gamma_i$, we have the following robust counterpart of (11), which holds irrespectively of whether (23) is enforced or not.

Proposition 2: Under Assumptions 1, and 2 the robust counterpart of constraint (11) for EV i when $e_i^{\circ} = e_i^0 + \gamma_i$ is given by

$$\left(\alpha_i^{\overline{d}_i - \underline{a}_i + 1} - 1\right)(e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{d_i} \alpha_i^{\overline{d}_i - t} \underline{\hbar}_{t,i} \ge \gamma_i \tag{28a}$$

$$\underline{\hbar}_{t,i} \le \eta_i^+(\bar{p}_{t,i} - s_{t,i}^- - \vartheta_{t,i} e_i^{\Delta}) \qquad t \in [\overline{a}_i, \underline{d}_i] \tag{28b}$$

$$\underline{\hbar}_{t,i} \le \tfrac{1}{\eta_i^-}(\bar{p}_{t,i} - s_{t,i}^- - \vartheta_{t,i} e_i^{\Delta}) \qquad t \in [\overline{a}_i, \underline{d}_i] \tag{28c}$$

where $\bar{h}_{t,i}$ are additional continuous decision variables. ■

Finally, note that the alternative constraint (17) need not to be robustified, as it is already deterministic. Since the term $\mathbb{E}[e_{d_i+1,i}]$ on the left hand side of (17) also appears in the cost function, its exact computation is deferred to the next section. As for the right hand side of (11), $\mathbb{E}[e_i^\circ]$ is either equal to e_i°, if e_i° is given in absolute terms, or is equal to $\mathbb{E}[e_i^0] + \gamma_i$ if the desired SOC is given as $e_i^\circ = e_i^0 + \gamma_i$.

D. Cost Function

Lastly, we focus on the cost function of $\mathcal{P}$. Since it is composed by four terms and, by linearity of the expected value operator it holds

$$\mathbb{E}[(9)+(10)+(13)+(15)] = \mathbb{E}[(9)]+\mathbb{E}[(10)]+\mathbb{E}[(13)]+\mathbb{E}[(15)],$$

we can focus on each term separately. In the following, we will compute all expected values under Assumption 4 and we will make use of the following result.

Lemma 1: Let $J(x,\delta)$ be a convex function of x for any value of an uncertain parameter $\delta \in \Delta$ with probability measure $\mathbb{P}_\delta$. Then

$$\tilde{J}(x) = \mathbb{E}[J(x,\delta)] = \int_\Delta J(x,\delta)\,\mathrm{d}\mathbb{P}_\delta$$

is a convex function of x. ■

1) Vehicles Charging/Discharging: Let us start by observing that, since $c_k^{v+} < c_k^{v-}$, $c_{k,i}^{\mathrm{veh}}$ in (8) can equivalently be expressed as $c_{k,i}^{\mathrm{veh}} = \max\{-c_k^{v+}\tau p_{k,i}, -c_k^{v-}\tau p_{k,i}\}$, which is a convex function of the decision variables, since $p_{k,i}$ is linear in the decision variables, for any fixed value of the uncertainty parameters e_i^0 and ω_k. By (9), also c^{veh} is convex and, as a consequence of Lemma 1, $\mathbb{E}[c^{\mathrm{veh}}]$ will also be a convex function of the decision variables. By linearity of expectation, from (9) and (8), we have

$$\mathbb{E}[c^{\mathrm{veh}}] = \sum_{k=0}^{T-1}\sum_{i=1}^{N}\mathbb{E}[c_{k,i}^{\mathrm{veh}}],$$

and we can thus focus on the computation of $\mathbb{E}[c_{k,i}^{\mathrm{veh}}]$.

Clearly, $c_{k,i}^{\mathrm{veh}}$ is a function of $p_{k,i}$ which, in turn, is a function of the uncertain quantities ω_k and e_i^0. By definition of expectation

$$\begin{aligned}
\mathbb{E}[c_{k,i}^{\mathrm{veh}}] &= \int_{\omega_k\in[-1,1]}\int_{e_i^0\in[e_i^\mu - e_i^\Delta, e_i^\mu + e_i^\Delta]} c_{k,i}^{\mathrm{veh}}\,\mathrm{d}\mathbb{P}_{e_i^0}\mathrm{d}\mathbb{P}_{\omega_k}\\
&= \int_{\omega_k\in(0,1]}\int_{e_i^0\in[e_i^\mu - e_i^\Delta, e_i^\mu + e_i^\Delta]} c_{k,i}^{\mathrm{veh}}\,\mathrm{d}\mathbb{P}_{e_i^0}\mathrm{d}\mathbb{P}_{\omega_k}\\
&\quad + \int_{\omega_k=0}\int_{e_i^0\in[e_i^\mu - e_i^\Delta, e_i^\mu + e_i^\Delta]} c_{k,i}^{\mathrm{veh}}\,\mathrm{d}\mathbb{P}_{e_i^0}\mathrm{d}\mathbb{P}_{\omega_k}\\
&\quad + \int_{\omega_k\in[-1,0)}\int_{e_i^0\in[e_i^\mu - e_i^\Delta, e_i^\mu + e_i^\Delta]} c_{k,i}^{\mathrm{veh}}\,\mathrm{d}\mathbb{P}_{e_i^0}\mathrm{d}\mathbb{P}_{\omega_k},
\end{aligned}$$

where in the second equality we split the integral over the ω_k domain $[-1,1]$ in the three intervals $[-1,0)$, $[0,0]$, and $(0,1]$, to deal with the distinct probability measure in each case (cf. Assumption 4). Using Mathematica [22], we can analytically compute the three integrals and, from the result[2], we can also see how each term is a piece-wise convex function defined over a polyhedral partition of the $\bar{p}_{k,i}$, $\vartheta_{k,i}$, $s_{k,i}^+$, and $s_{k,i}^-$ domain. Following a reasoning similar to the one in [24], we can construct a piece-wise affine upper bound of each term and then recombine them together. The resulting expression is reported in (29). Note that the upper bound is tight because it ensures, by construction, that equality holds at any two region border of the polyhedral partition.

2) Vehicles Residual Energy: Taking the expectation of (10) and using the linearity property of expectation we obtain

$$\mathbb{E}[c^{\mathrm{soc}}] = \mathbb{E}\left[-c^{\mathrm{avg}}\sum_{i=1}^{N} e_{d_i+1,i}\right] = -c^{\mathrm{avg}}\sum_{i=1}^{N}\mathbb{E}[e_{d_i+1,i}],$$

and we only need to compute $\mathbb{E}[e_{d_i+1,i}]$, whose expression is given in the following proposition and can also be used for the constraint in (17).

Proposition 3: Under Assumptions 1 and 2 we have

$$\begin{aligned}
\mathbb{E}[e_{d_i+1,i}] &= \alpha_i^{\tilde{d}_i-\tilde{a}_i+1}\mathbb{E}[e_i^0]\\
&\quad + \tau\sum_{t=\overline{a}_i}^{\underline{d}_i}\alpha_i^{\tilde{d}_i-t}\mathbb{E}\left[\min\left\{\eta_i^+ p_{t,i}, \tfrac{1}{\eta_i^-}p_{t,i}\right\}\right]
\end{aligned}$$

where $\tilde{a}_i = -\log_{\alpha_i}\mathbb{E}[\alpha_i^{-a_i}]$ and $\tilde{d}_i = \log_{\alpha_i}\mathbb{E}[\alpha_i^{d_i}]$. ■

Note that $\min\{\eta_i^+ p_{t,i}, \frac{1}{\eta_i^-}p_{t,i}\}$ is a concave function of the optimization variables (as $p_{t,i}$ is linear in the decision variables for any fixed value of e_i^0 and ω_t) and thus, owing to Lemma 1, also $\mathbb{E}[\min\{\eta_i^+ p_{t,i}, \frac{1}{\eta_i^-}p_{t,i}\}]$ is concave. Since $\tau, \alpha_i > 0$, also $\mathbb{E}[e_{d_i+1,i}]$ is concave, implying that constraint (17) is convex and, since we have a minus sign inside the expression of c^{soc}, than also $\mathbb{E}[c^{\mathrm{soc}}]$ is convex in the decision variables. Moreover,

$$\mathbb{E}\left[\min\left\{\eta_i^+ p_{t,i}, \tfrac{1}{\eta_i^-}p_{t,i}\right\}\right] = -\mathbb{E}\left[\max\left\{-\eta_i^+ p_{t,i}, -\tfrac{1}{\eta_i^-}p_{t,i}\right\}\right],$$

which is similar to $\mathbb{E}[c_{k,i}^{\mathrm{veh}}]$ except that there is no τ, c_k^{v+} and c_k^{v-} are replaced by η_i^+ and $\frac{1}{\eta_i^-}$, and $p_{k,i}$ by $p_{t,i}$. Therefore, under the additional Assumption 4, we can compute the right hand side of the previous relation using (29) with said substitutions.

3) Day-Ahead Market: First, let us notice that since $\tau > 0$ and $c_k^{e+} > c_k^{e-}$, (13) can be equivalently expressed as

$$c^{\mathrm{dam}} = \tau\sum_{k=0}^{T-1}\max\left\{c_k^{e+}\sum_{i=1}^{N}p_{k,i}^{\mathrm{dam}}, c_k^{e-}\sum_{i=1}^{N}p_{k,i}^{\mathrm{dam}}\right\},$$

which is a convex function of the decision variable for any fixed value of the uncertainty, as an effect of $p_{k,i}$ being linear in the decision variables for any fixed value of the uncertainty. Owing to Lemma 1, we know that also $\mathbb{E}[c^{\mathrm{dam}}]$ is convex in the decision variables. Unfortunately, even though $\mathbb{E}[c^{\mathrm{dam}}]$ is convex, computing its analytic expression under Assumption 4 is too involved and is left as a future research effort.

[2]In the interest of space, we do not report here the resulting expression, but the Mathematica notebook used is available in [23].

$$
\begin{aligned}
\mathbb{E}[c_{k,i}^{\mathrm{veh}}] \leq \tau \tfrac{\pi^0}{2} \max \Big\{ & -2c_k^{v+} p_{k,i},\ -2c_k^{v-} p_{k,i},\ (c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} - (c_k^{v-}+c_k^{v+}) p_{k,i} \Big\} + \tau \tfrac{\pi^+}{6} \max \Big\{ -6c_k^{v+} p_{k,i} - 3c_k^{v+} s_{k,i}^+,\ -6c_k^{v-} p_{k,i} \\
& -3c_k^{v-} s_{k,i}^+,\ (c_k^{v+}-c_k^{v-})\vartheta_{k,i} e_i^{\Delta} - 3(c_k^{v+}+c_k^{v-}) p_{k,i} - 3c_k^{v+} s_{k,i}^+,\ (c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} - (5c_k^{v+}+c_k^{v-}) p_{k,i} - 3c_k^{v+} s_{k,i}^+,\ (c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} \\
& -(c_k^{v+}+5c_k^{v-}) p_{k,i} - (c_k^{v+}+2c_k^{v-}) s_{k,i}^+,\ 3(c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} - 3(c_k^{v+}+c_k^{v-}) p_{k,i} - (c_k^{v+}+2c_k^{v-}) s_{k,i}^+ \Big\} + \tau \tfrac{\pi^-}{6} \max \Big\{ 3c_k^{v+} s_{k,i}^- \\
& -6c_k^{v+} p_{k,i},\ 3c_k^{v-} s_{k,i}^- - 6c_k^{v-} p_{k,i},\ (c_k^{v+}-c_k^{v-})\vartheta_{k,i} e_i^{\Delta} - 3(c_k^{v+}+c_k^{v-}) p_{k,i} + 3c_k^{v-} s_{k,i}^-,\ (c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} - (c_k^{v+}+5c_k^{v-}) p_{k,i} \\
& +3c_k^{v-} s_{k,i}^-,\ (c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} - (5c_k^{v+}+c_k^{v-}) p_{k,i} + (c_k^{v-}+2c_k^{v+}) s_{k,i}^-,\ 3(c_k^{v-}-c_k^{v+})\vartheta_{k,i} e_i^{\Delta} - 3(c_k^{v+}+c_k^{v-}) p_{k,i} + (c_k^{v-}+2c_k^{v+}) s_{k,i}^- \Big\}
\end{aligned} \tag{29}
$$

In this work, we propose to upper bound it as follows. Taking the expectation on both sides of the previous expression and using the linearity property yields

$$
\begin{aligned}
\mathbb{E}[c^{\mathrm{dam}}] &= \tau \sum_{k=0}^{T-1} \mathbb{E}\left[\max \left\{ c_k^{e+} \sum_{i=1}^{N} p_{k,i}^{\mathrm{dam}}, c_k^{e-} \sum_{i=1}^{N} p_{k,i}^{\mathrm{dam}} \right\} \right] \\
&\leq \tau \sum_{k=0}^{T-1} \sum_{i=1}^{N} \mathbb{E}\big[\max \{ c_k^{e+} p_{k,i}^{\mathrm{dam}}, c_k^{e-} p_{k,i}^{\mathrm{dam}} \} \big],
\end{aligned}
$$

where the inequality is due to the monotonicity property of expectation and we recall that $p_{k,i}^{\mathrm{dam}}$ is defined in (18). Similarly to Section III-D.1, we can compute the expected value of the (k,i)-th term exactly using Mathematica [22] and then upper bound it with a piece-wise affine function, yielding

$$
\begin{aligned}
&\mathbb{E}\big[\max \{ c_k^{e+} p_{k,i}^{\mathrm{dam}}, c_k^{e-} p_{k,i}^{\mathrm{dam}} \} \big] \\
&\leq \max \left\{ c_k^{e+} \bar{p}_{k,i}, c_k^{e-} \bar{p}_{k,i}, \tfrac{(c_k^{e+}+c_k^{e-})\bar{p}_{k,i} + (c_k^{e+}-c_k^{e-})\vartheta_{k,i} e_i^{\Delta}}{2} \right\}.
\end{aligned}
$$

4) Ancillary Services Market: Taking the expectation of (15) and using the linearity property of expectation yields

$$
\begin{aligned}
\mathbb{E}[c^{\mathrm{asm}}] &= \mathbb{E}\left[\sum_{k=0}^{T-1} c_k^{s+} \sum_{i=1}^{N} \tau s_{k,i}^{+} [\omega_k]^{+} - c_k^{s-} \sum_{i=1}^{N} \tau s_{k,i}^{-} [\omega_k]^{-} \right] \\
&= \sum_{k=0}^{T-1} c_k^{s+} \sum_{i=1}^{N} \tau s_{k,i}^{+} \mathbb{E}[[\omega_k]^{+}] - c_k^{s-} \sum_{i=1}^{N} \tau s_{k,i}^{-} \mathbb{E}[[\omega_k]^{-}],
\end{aligned}
$$

and we thus only need to compute $\mathbb{E}[[\omega_k]^+]$ and $\mathbb{E}[[\omega_k]^-]$. By definition of expectation,

$$
\begin{aligned}
\mathbb{E}[[\omega_k]^{+}] &= \mathbb{E}[\max\{\omega_k, 0\}] = \int_{\omega_k \in [-1,1]} \max\{\omega_k, 0\} \, \mathrm{d}\mathbb{P}_{\omega_k} \\
&= \int_{\omega_k \in (0,1]} \omega_k \, \mathrm{d}\mathbb{P}_{\omega_k} = \int_{\omega_k \in (0,1]} \omega_k \tfrac{1}{4} \, \mathrm{d}\omega_k = \tfrac{1}{8}
\end{aligned}
$$

and, with similar derivations, $\mathbb{E}[[\omega_k]^-] = \frac{1}{8}$. Thus the ancillary services part of the expected cost is given by

$$
\mathbb{E}[c^{\mathrm{asm}}] = \tfrac{1}{8} \sum_{k=0}^{T-1} \left(c_k^{s+} \sum_{i=1}^{N} \tau s_{k,i}^{+} - c_k^{s-} \sum_{i=1}^{N} \tau s_{k,i}^{-} \right) \tag{30}
$$

and is linear in $s_{k,i}^+$ and $s_{k,i}^-$.

E. Data-based Solution

Suppose now that the (possibly joint) probability measure of a_i, d_i, e_i^0, and ω_k is unknown, and thus Assumption 4 is not necessarily satisfied. If we have access to historical data we can approximate the expression of the cost function of $\mathcal{P}$ using those data. While computing the expected values for discrete random variables like a_i and d_i is easy, computing expectations with respect to e_i^0 and ω_k is not. However, owing to the independence granted by Assumption 1, we can turn e_i^0 and ω_k into discrete random variables by gridding. This way, we can approximate the expectation of any function $\varphi(\xi, e_i^0)$ as

$$
\mathbb{E}_{e_i^0}[\varphi(\xi, e_i^0)] \approx \sum_{j} \mathbb{P}\{e_i^0 \in \mathcal{I}_i^j\} \varphi(\xi, \mathcal{E}_i^j),
$$

where $\mathcal{I}_i^j$ is the j-th bin of the interval $[e_i^{\mu} - e_i^{\Delta}, e_i^{\mu} + e_i^{\Delta}]$, $\mathcal{E}_i^j$ is the center of $\mathcal{I}_i^j$, and $\mathbb{P}\{e_i^0 \in \mathcal{I}_i^j\}$ is estimated from the available data. Clearly, such approximation preserves convexity, thus the resulting data-based problem still has a convex cost function.

IV. Numerical Example

We consider the case of a company parking lot composed of $N = 100$ slots, each assigned to a single user indexed with i. The 24 hours time horizon is discretized into $T = 96$ time slots of $\tau = 15$ minutes each. Vehicle i arrives uniformly at random between 6:00 AM and 7:45 AM and leaves uniformly at random between 4:00 PM and 8:00 PM. For each vehicle i, we set $\eta_i^+ = \eta_i^- = 0.97$, $p_i^{\max} = -p_i^{\min} = 22$ kW, $e_i^{\max} \in [40,\ 70]$ kWh, $e_i^{\min} = 0$ kWh, $e_i^0 \in [0.1,\ 0.5]e_i^{\max}$ kWh is extracted according to a uniform distribution, and $e_i^{\circ} = 0.7e_i^{\max}$. The maximum power that can be exchanged with the grid is set to $p^{\max} = -p^{\min} = 600$ kW. The energy prices are shown in Figure 1. As for the ancillary service market, we set the acceptance probabilities to $\pi_k^+ = 0.3$ and $\pi_k^- = 0.1$ based on real Italian market data (see [25]), and we set a time constraint of 2 hours (i.e., $T^{\mathrm{asm}} = 8$ time slots) in (16).

A. Analytic Solution

We solve problem $\mathcal{P}$ with the expected value of the recharge cost term (8) expressed analytically as in (29), and the expected values of (13) and (15) expressed analytically as described in Sections III-D.3 and III-D.4, respectively. Maxima appearing in analytic expressions are dealt with by means of epigraphic reformulation so as to make the overall problem linear. The robust counterpart of (6) and (7) are imposed using (19) and (20), respectively. After imposing the additional constraint (23), we use (25) and (26), from Corollary 1 and Proposition 1 respectively, to approximate the robust counterpart of (3). Similarly, we use (25) with the modification in (27) to impose the robust counterpart of (11). We do not impose constraint (17) and we ignore term (10) in

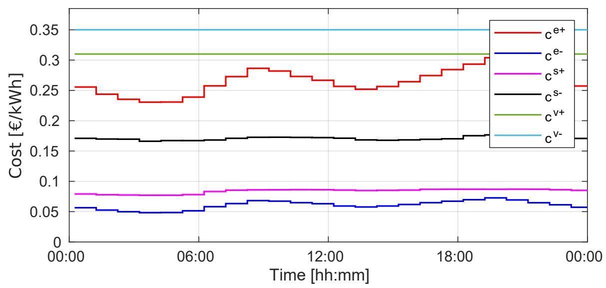

Fig. 1. Day-ahead market, ancillary service market, and vehicle charging/discharging prices.

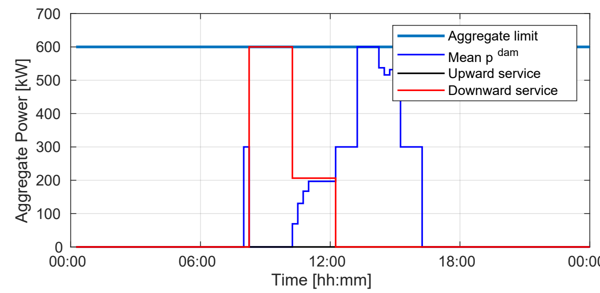

Fig. 2. Power exchange profile with the main grid. Analytical solution.

the cost function. The resulting optimization problem

$$
\begin{aligned}
\min_{\bar{p}_{k,i},\vartheta_{k,i},s^+_{k,i},s^-_{k,i}} \quad & \mathbb{E}[(9)+(13)+(15)] & & (31)\\
\text{subject to:} \quad & (20) & \forall k &\\
& (19), (26) & \forall i &\\
& (25) \text{ with } e_i^{\min}(k) \text{ as in } (27) & \forall i &\\
& (16) & &\\
& s^+_{k,i}, s^-_{k,i}, \vartheta_{k,i} \geq 0 & \forall i,\ \forall k &
\end{aligned}
$$

is modeled via YALMIP [26] in MATLAB 2021a and solved using CPLEX [27] on a laptop with i5-9400H processor and 32GB of RAM. The computation time is less than 4 seconds.

In Figure 2 the resulting power profile at the point of connection with the main grid is shown. The optimal strategy consists in offering two downward services at 8:00 AM and at 10:00 AM, essentially to buy a non deterministic amount of energy at a discounted price (cf. Figure 1 where $c_k^{s+} < c_k^{e+}$). The market bids are all concentrated between 8:00 AM and 4:00 PM because in those time intervals the vehicles availability is guaranteed.

Figure 3 shows the results of 1000 different scenarios (extracted according to the distributions described at the beginning of this section) for the behavior of vehicle 1, where each scenario is obtained by sampling the uncertain variables. In the top plot the profile of the power bought on the energy and ancillary services markets is shown. The vehicle contributes to the aggregate downward service with the red profile. The second plot from above shows 1000 profiles of the percentage state of charge of the vehicle (different colors) along with their worst cases (dashed lines) and state of charge constraints (magenta). It can be noted that different profiles may have different lengths: this is due to the fact that besides the SOC at arrival, also arrival and departure times are random. Note also that the largest portion of realizations is close to the minimum value because the probability of having high requests of downward services is low. This plot highlights the conservativity of the robust solutions that is limited by worst-case realizations even if they are very unlikely. Possible countermeasures to reduce conservativism are briefly discussed in the conclusions. The third plot shows the value of the compensator parameter along the considered time horizon: as can be noted by comparing it with the second plot, a non-zero value of the compensator reduces the span of possible values for the vehicle's energy. On the contrary, offering a service increases the span of possible values for the vehicle's energy because the energy exchanged with the grid is uncertain. In fact, on the one hand the optimal

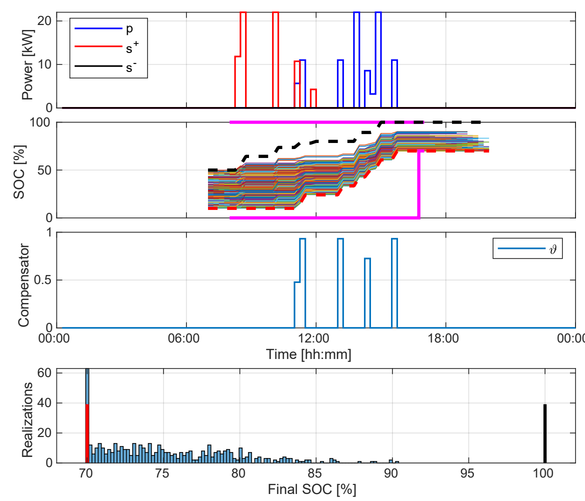

Fig. 3. Simulation results for a vehicle (capacity $e_i^{\max} = 59$ kWh) with 1000 uncertainty realizations. Top to bottom: 1) market bids, 2) SOC percentage profiles, 3) compensator action, 4) final SOC percentage histogram. Top three plots share the same x-axis.

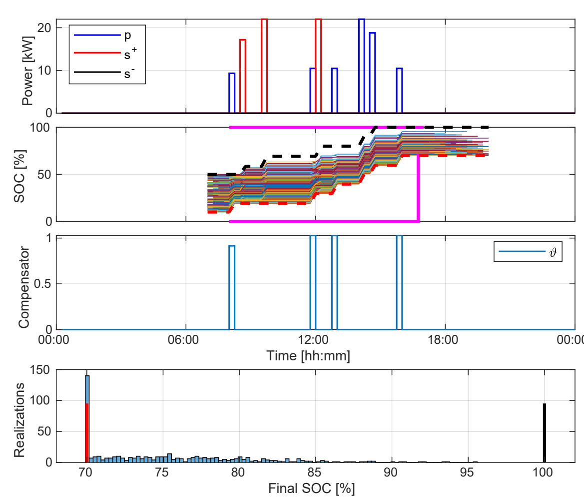

Fig. 4. Simulation results for another vehicle (capacity $e_i^{\max} = 51$ kWh) with 1000 uncertainty realizations. Top to bottom: 1) market bids, 2) SOC percentage profiles, 3) compensator action, 4) final SOC percentage histogram. Top three plots share the same x-axis.

solution aims at maximizing downward services so as to buy more energy at a cheaper price, but this leads to an increase in the span of possible values for the vehicle's energy content, which has to be counteracted using the compensator action, which reduces the span of possible values for the vehicle's energy so as it can fit into the gap between the minimum and maximum energy constraints before departure. Without such compensating action, the problem might even be unfeasible if the energy span at arrival is bigger than the allowed energy gap before departure. Finally, the last plot shows the distribution of the SOC percentage at the time of departure: in all cases the energy is contained in the limits (red and black vertical lines), as enforced by the robust paradigm.

Figure 4 shows the results for a different car: as can be noted, these are very similar to the ones in Figure 3, as the car is charged with a downward bid saturating the maximum power in some time intervals. The compensator term is necessary also for this second car to ensure feasibility: the span of possible energy realizations is indeed reduced from an initial SOC variability of 40% (from 10% to 50% at 7:00 AM) to a final SOC variability of 30% (from 70% to 100% at 5:00 PM), cf. second panel of Figure 4.

B. Data-based Solution

In order to test the efficacy of the data-based solution explained in Section III-E, we also solve the sampled counterpart of (31). The expected value in the cost function are approximated gridding each e_i^0 and ω_k into 5 bins and estimating their probability extracting 1000 realization of the uncertain parameters, drawn from the same uniform distributions described at the beginning of Section IV, so as to produce comparable results with the analytical solution presented before.

Figure 5 shows the result obtained for the power exchange profile with the grid. As can be noted, the power absorption profile is very similar to the analytical one shown in Figure 2 and the corresponding optimal costs differ by 4% only. The approximation quality of the data-based solution clearly depends on the number of samples: the higher the number of samples, the better the accuracy, but the higher the computation cost. In general, the data-based solution should be used in those cases when the distribution of the uncertainty is not known and only samples are available, or when the distribution is known but the analytical formulation of the expected value is too complicated to derive. If the support of the distributions is known, the data-based formulation only affects the cost function, and not the robust formulation of the constraints. In Figure 6 we report the solution for one vehicle, which shows that constraints on power and energy are satisfied. Please note that the 1000 uncertainty realizations in Figure 6 are different from those used to estimate the bin probabilities.

V. Conclusion

In this paper we proposed a framework for optimizing the operation of a fleet of electric vehicles while providing ancillary services to the electric grid. We discussed different types of constraint and cost functions to make the approach versatile and suitable for different contexts. We considered different sources of uncertainties and provided either a robust formulation for the constraints or an explicit formulation for expectations of the different terms composing the cost function. We derived very tight approximations, if not exact formulations, of each constraint and cost terms even in presence of a piece-wise linear model of the vehicles battery. In future works we will further enrich the framework allowing for imbalances in the power bought from the energy market through a disturbance feedback-like scheme: this will allow to reduce the conservativism of the robust approach, to guarantee a more flexible and profitable operation of the fleet, and to consider scenarios other than the company parking lot. We will also investigate the extension of the approach to the aggregation of domestic users, each equipped with a battery, and an uncertain photovoltaic generation and load.

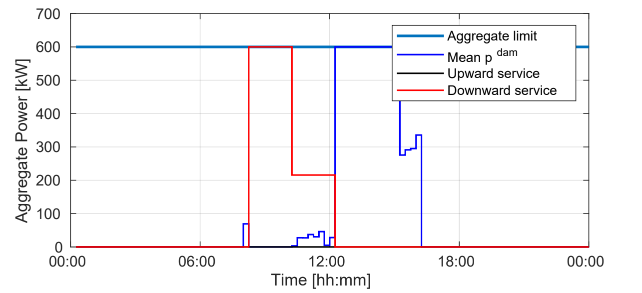

Fig. 5. Power exchange profile with the main grid. Data-based solution.

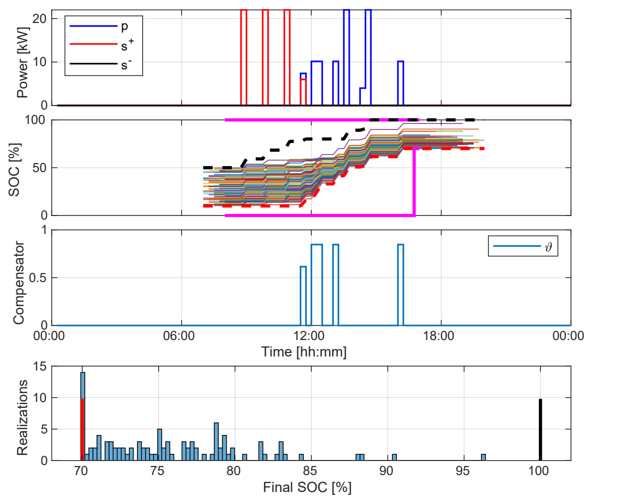

Fig. 6. Simulation results for a vehicle (capacity $e_i^{\max} = 60$ kWh) with 1000 uncertainty realizations. Top to bottom: 1) market bids, 2) SOC percentage profiles, 3) compensator action, 4) final SOC percentage histogram. Top three plots share the same x-axis.

Appendix I
Proof of Theorem 1

Given that, by (19c), $p_{k,i} = 0$ for all $k \notin [\overline{a}_i, \underline{d}_i]$ (which is non-empty under Assumption 2) and recalling that $e_{a_i,i} = e_i^0$, the SOC $e_{k,i}$ in (5) can be equivalently expressed as

$$\begin{aligned} e_{k,i} &= \alpha_i^{k-a_i} e_i^0 + \tau \sum_{t=\overline{a}_i}^{\min\{k-1,\underline{d}_i\}} \alpha_i^{k-1-t} \min\left\{\eta_i^+ p_{t,i}, \tfrac{1}{\eta_i^-} p_{t,i}\right\} \\ &= \begin{cases} \alpha_i^{k-a_i} e_i^0 & k \in [a_i, \overline{a}_i] \\ \tilde{e}_{k,i} & k \in [\overline{a}_i + 1, \underline{d}_i + 1] \\ \alpha_i^{k-(\underline{d}_i+1)} e_{\underline{d}_i+1,i} & k \in [\underline{d}_i + 2, d_i + 1] \end{cases} \end{aligned} \tag{32}$$

with

$$\tilde{e}_{k,i} = \alpha_i^{k-a_i} e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min\left\{\eta_i^+ p_{t,i}, \tfrac{1}{\eta_i^-} p_{t,i}\right\}$$

since $k-1 \leq \underline{d}_i$ for all $k \in [\overline{a}_i + 1, \underline{d}_i + 1]$. We now consider the three cases in (32) separately.

A. Case $k \in [a_i, \overline{a}_i]$

By (32), if $k \in [a_i, \overline{a}_i]$, then

$$e_{k,i} = \alpha_i^{k-a_i} e_i^0 \leq e_i^0 \leq e_i^{\max},$$

where the first inequality is due to $\alpha_i \in (0,1]$ and $k \geq a_i$ and the second inequality is due to $e_i^0 \in [e_i^{\min}, e_i^{\max}]$. Similarly,

$$e_{k,i} = \alpha_i^{k-a_i} e_i^0 \geq \alpha_i^{\overline{a}_i - a_i} e_i^0 \geq \alpha_i^{\overline{a}_i - \underline{a}_i} (e_i^\mu - e_i^\Delta) \geq e_i^{\min},$$

where the first inequality is due to $\alpha_i \in (0,1]$ and $k \leq \overline{a}_i$, the second inequality is due to $a_i \geq \underline{a}_i$ and $e_i^0 \geq e_i^\mu - e_i^\Delta$, and the last inequality is due to Assumption 3.

For $k \in [a_i, \overline{a}_i]$, constraint (3) is thus automatically satisfied for all possible values of the uncertain parameters without imposing any constraint.

B. Case $k \in [\overline{a}_i + 1, \underline{d}_i + 1]$

Let us first focus on the upper bound in (3). Since we want $e_{k,i} \leq e_i^{\max}$ for all possible values of a_i, e_i^0, and ω_t, with $t = \underline{a}_i, \ldots, k-1$, it is sufficient to enforce that

$$\max_{a_i, e_i^0, \{\omega_t\}_t} e_{k,i} \leq e_i^{\max}.$$

To this end, we shall derive the left hand side as a function of the decision variables. To start, we notice that, for any fixed $p_{t,i}$,

$$\min_{\eta_{t,i} \in H_i} \eta_i\, p_{t,i} = \min\left\{\eta_i^+ p_{t,i}, \tfrac{1}{\eta_i^-} p_{t,i}\right\} \tag{33}$$

where H_i is the continuous interval $[\eta_i^+, \frac{1}{\eta_i^-}]$. Indeed the left hand side is a linear program and its optimal solution is obtained setting $\eta_{t,i} = \eta_i^+$ or $\eta_{t,i} = \frac{1}{\eta_i^-}$. Then, if $k \in [\overline{a}_i + 1, \underline{d}_i + 1]$, by (32) and (33),

$$\begin{aligned} \max_{a_i} e_{k,i} &= \max_{a_i} \left\{\alpha_i^{k-a_i} e_i^0\right\} + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min_{\eta_{t,i} \in H_i} \eta_{t,i}\, p_{t,i} \\ &= \alpha_i^{k-\overline{a}_i} e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min_{\eta_{t,i} \in H_i} \eta_{t,i}\, p_{t,i}, \end{aligned}$$

where the first equality is due to a_i appearing only in the term outside the summation and the second equality is due to $e_i^0 \geq 0$, $\alpha_i \in (0,1]$, and $a_i \leq \overline{a}_i$. Then,

$$\begin{aligned} \max_{a_i} e_{k,i} &= \alpha_i^{k-\overline{a}_i} e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min_{\eta_{t,i} \in H_i} \eta_{t,i}\, p_{t,i} \\ &= \alpha_i^{k-\overline{a}_i} e_i^0 + \min_{\{\eta_{t,i} \in H_i\}_t} \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}\, p_{t,i} \\ &= \min_{\{\eta_{t,i} \in H_i\}_t} \left\{\alpha_i^{k-\overline{a}_i} e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}\, p_{t,i}\right\}, \end{aligned} \tag{34}$$

where in the second equality we brought the minimum over $\eta_{t,i}$ for each t outside the summation using the fact that τ and α_i are both positive and the minima are independent across t under Assumption 1, and in the third equality we brought the e_i^0 term inside the minimum since it does not depend on any $\eta_{t,i}$. If we now expand $p_{t,i}$ as

$$p_{t,i} = \underbrace{\overline{p}_{t,i} + \vartheta_{t,i} e_i^\mu + s_{t,i}^+ [\omega_t]^+ - s_{t,i}^- [\omega_t]^-}_{q_{t,i}(\omega_t)} - \vartheta_{t,i} e_i^0, \tag{35}$$

we get

$$\begin{aligned} \max_{a_i} e_{k,i} &= \min_{\{\eta_{t,i} \in H_i\}_t} \Bigg\{ \underbrace{\left(\alpha_i^{k-\overline{a}_i} - \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i} \vartheta_{t,i}\right)}_{\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)} e_i^0 \\ &\qquad + \underbrace{\tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}\, q_{t,i}(\omega_t)}_{f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t)} \Bigg\} \\ &= \min_{\{\eta_{t,i} \in H_i\}_t} \underbrace{\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t) e_i^0 + f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t)}_{\overline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t)} \end{aligned} \tag{36}$$

and consequently

$$\begin{aligned} \max_{a_i, e_i^0, \{\omega_t\}_t} e_{k,i} &= \max_{e_i^0, \{\omega_t\}_t} \max_{a_i} e_{k,i} \\ &= \max_{e_i^0, \{\omega_t\}_t} \min_{\{\eta_{t,i} \in H_i\}_t} \overline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t) \\ &= \min_{\{\eta_{t,i} \in H_i\}_t} \max_{e_i^0, \{\omega_t\}_t} \overline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t) \end{aligned} \tag{37}$$

where the second equality is due to (34) with (36), and the last equality is due to Sion's minimax theorem, [28], which holds since e_i^0, $\{\omega_t\}_t$, and $\{\eta_{t,i}\}_t$ all lies in convex compact sets, and for any fixed e_i^0 and $\{\omega_t\}_t$ the function $\overline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \cdot)$ is linear and for any fixed $\{\eta_{t,i}\}_t$ the function $\overline{f}_{k,i}(\cdot, \cdot, \{\eta_{t,i}\}_t)$ is continuous and quasi-concave, as a consequence of being linear in e_i^0 and $q_{t,i}(\omega_t)$, with $q_{t,i}(\omega_t)$ monotonically non-decreasing in ω_t. Then,

$$\begin{aligned} &\max_{e_i^0, \{\omega_t\}_t} \overline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t) = \\ &\qquad \max_{e_i^0} \overline{f}^0_{k,i}(\{\eta_{t,i}\}_t) e_i^0 + \max_{\{\omega_t\}_t} f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t) \end{aligned} \tag{38}$$

and we can focus on the two maxima separately. As for the first term,

$$\begin{aligned}\max_{e_i^0} \overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^0 &= \overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^\mu + \max_{e_i^0-e_i^\mu} \overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)(e_i^0 - e_i^\mu)\\ &= \overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^\mu + |\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)|e_i^\Delta, \end{aligned} \tag{39}$$

where the first equality we added and subtracted the term in e_i^μ and the second equality is due to $(e_i^0 - e_i^\mu) \in [-e_i^\Delta, e_i^\Delta]$. As for the second term, since by Assumption 1 all ω_t are independent, and since $\tau, \alpha_i, \eta_{t,i} > 0$, then

$$\begin{aligned}\max_{\{\omega_t\}_t} f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t) &= \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i} \max_{\omega_t} q_{t,i}(\omega_t)\\ &= \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i}(\bar{p}_{t,i} + \vartheta_{t,i}e_i^\mu + s^+_{t,i}), \end{aligned} \tag{40}$$

where the latter equality holds by (35) together with the fact that $\omega_t \in [-1, 1]$ and both $s^+_{t,i}$ and $s^-_{t,i}$ are non-negative. Using (39) and (40) in (38) and recalling the definition of $\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)$, we obtain

$$\max_{e_i^0,\{\omega_t\}_t} \overline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t) = \alpha_i^{k-\overline{a}_i}e_i^\mu + |\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)|e_i^\Delta + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i}(\bar{p}_{t,i} + s^+_{t,i}),$$

which can be used in (37) to get

$$\max_{a_i,e_i^0,\{\omega_t\}_t} e_{k,i} = \alpha_i^{k-\overline{a}_i}e_i^\mu + \min_{\{\eta_{t,i}\in H_i\}_t} \left\{ |\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)|e_i^\Delta + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i}(\bar{p}_{t,i} + s^+_{t,i}) \right\}.$$

The robust counterpart of the upper bound in (3) in case $k \in [\overline{a}_i + 1, \underline{d}_i + 1]$ can be obtained requiring the right hand side of the previous relation to be no-greater than $e_i^{\max}$, which is exactly (21).

As for the lower bound in (3). Since we want $e_{k,i} \geq e_i^{\min}$ for all possible values of a_i, e_i^0, and ω_t, with $t = \underline{a}_i, \ldots, k-1$, it is sufficient to enforce that

$$\min_{a_i,\{\omega_t\}_t,e_i^0} e_{k,i} \geq e_i^{\min}.$$

Similarly to the upper limit above, we shall express the left hand side as a function of the decision variables. Using equivalence (33), we have

$$\begin{aligned}\min_{a_i} e_{k,i} &= \min_{a_i} \left\{\alpha_i^{k-a_i}e_i^0\right\} + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min_{\eta_{t,i}\in H_i} \eta_{t,i}\, p_{t,i}\\ &= \alpha_i^{k-\underline{a}_i}e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min_{\eta_{t,i}\in H_i} \eta_{t,i}\, p_{t,i}, \end{aligned}$$

where the first equality is due to a_i appearing only in the term outside the summation and the second equality is due to $e_i^0 \geq 0$, $\alpha_i \in (0, 1]$, and $a_i \geq \underline{a}_i$. Then, similarly to (34) and (36),

$$\begin{aligned}\min_{a_i} e_{k,i} &= \alpha_i^{k-\underline{a}_i}e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \min_{\eta_{t,i}\in H_i} \eta_{t,i}\, p_{t,i}\\ &= \alpha_i^{k-\underline{a}_i}e_i^0 + \min_{\{\eta_{t,i}\in H_i\}_t} \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i}\, p_{t,i}\\ &= \min_{\{\eta_{t,i}\in H_i\}_t} \left\{\alpha_i^{k-\underline{a}_i}e_i^0 + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i}\, p_{t,i}\right\}\\ &= \min_{\{\eta_{t,i}\in H_i\}_t} \underbrace{\underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^0 + f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t)}_{\underline{f}_{k,i}(e_i^0,\{\omega_t\}_t,\{\eta_{t,i}\}_t)}, \end{aligned} \tag{41}$$

where in the second equality we brought the minimum over $\eta_{t,i}$ for each t outside the summation using the fact that τ and α_i are both positive and the minima are independent across t, in the third equality we brought the e_i^0 term inside the minimum since it does not depend on any $\eta_{t,i}$, and in the last equality we used (35) together with the definition of $f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t)$ and $\underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)$, the latter being identical to $\overline{f}^0_{k,i}(\{\eta_{t,i}\}_t)$ except for $\alpha_i^{k-\underline{a}_i}$ in place of $\alpha_i^{k-\overline{a}_i}$. Then,

$$\begin{aligned}\min_{a_i,e_i^0,\{\omega_t\}_t} e_{k,i} &= \min_{e_i^0,\{\omega_t\}_t} \min_{a_i} e_{k,i}\\ &= \min_{e_i^0,\{\omega_t\}_t} \min_{\{\eta_{t,i}\in H_i\}_t} \underline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t)\\ &= \min_{\{\eta_{t,i}\in H_i\}_t} \min_{e_i^0,\{\omega_t\}_t} \underline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t), \end{aligned} \tag{42}$$

where the second equality is due to (41) and in the second equality we swapped the two minima. Similarly to the upper limit case,

$$\min_{e_i^0,\{\omega_t\}_t} \underline{f}_{k,i}(e_i^0, \{\omega_t\}_t, \{\eta_{t,i}\}_t) = \min_{e_i^0} \underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^0 + \min_{\{\omega_t\}_t} f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t) \tag{43}$$

and we can focus on the two minima separately. As for the first term,

$$\begin{aligned}\min_{e_i^0} \underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^0 &= \underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^\mu + \min_{e_i^0-e_i^\mu} \underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)(e_i^0 - e_i^\mu)\\ &= \underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)e_i^\mu - |\underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)|e_i^\Delta, \end{aligned} \tag{44}$$

where the first equality we added and subtracted the term in e_i^μ and the second equality is due to $(e_i^0 - e_i^\mu) \in [-e_i^\Delta, e_i^\Delta]$. As for the second term, since by Assumption 1 all ω_t are independent, and since $\tau, \alpha_i, \eta_{t,i} > 0$, then

$$\begin{aligned}\min_{\{\omega_t\}_t} f^\omega_{k,i}(\{\omega_t\}_t, \{\eta_{t,i}\}_t) &= \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i} \min_{\omega_t} q_{t,i}(\omega_t)\\ &= \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t}\eta_{t,i}(\bar{p}_{t,i} + \vartheta_{t,i}e_i^\mu - s^-_{t,i}), \end{aligned} \tag{45}$$

where the latter equality holds by (35) together with the fact that $\omega_t \in [-1,1]$ and both $s^+_{t,i}$ and $s^-_{t,i}$ are non-negative. Using (44) and (45) in (43) and recalling the definition of $\underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)$, we obtain

$$\min_{e_i^0,\{\omega_t\}_t} \underline{f}_{k,i}(e_i^0,\{\omega_t\}_t,\{\eta_{t,i}\}_t) =$$
$$\alpha_i^{k-\underline{a}_i} e_i^\mu - |\underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)| e_i^\Delta + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} - s^-_{t,i}),$$

which can be used in (42) to get

$$\min_{a_i,e_i^0,\{\omega_t\}_t} e_{k,i} = \alpha_i^{k-\underline{a}_i} e_i^\mu + \min_{\{\eta_{t,i}\in H_i\}_t} \Bigg\{ -|\underline{f}^0_{k,i}(\{\eta_{t,i}\}_t)| e_i^\Delta + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} - s^-_{t,i}) \Bigg\}.$$

The robust counterpart of the lower bound in (3) in case $k \in [\overline{a}_i+1, \underline{d}_i+1]$ can be obtained requiring the right hand side of the previous relation to be no-smaller than $e_i^{\min}$. Since the argument of the minimization over $\{\eta_{t,i} \in H_i\}_t$ is concave, the minimum is achieved at $\{\eta_{t,i} \in \{\eta_i^+, \frac{1}{\eta_i^-}\}\}_t$ and therefore the desired robust constraint can be expressed as in (22) by means of an epigraphic reformulation.

C. Case $k \in [\underline{d}_i+2, d_i+1]$

If $d_i = \underline{d}_i$, the interval $[\underline{d}_i+2, d_i+1]$ is empty and we can safely neglect this case, so let us consider $d_i \geq \underline{d}_i + 1$. Similarly to the first case, by (32), if $k \in [\underline{d}_i+2, d_i+1]$, then

$$e_{k,i} = \alpha_i^{k-(\underline{d}_i+1)} e_{\underline{d}_i+1,i} \leq e_{\underline{d}_i+1,i} \leq e_i^{\max},$$

where the first inequality is due to $\alpha_i \in (0,1]$ and $k \geq \underline{d}_i + 1$ and the second inequality is due to (21). As for the minimum SOC constraint,

$$e_{k,i} = \alpha_i^{k-(\underline{d}_i+1)} e_{\underline{d}_i+1,i} \geq \alpha_i^{\overline{d}_i-\underline{d}_i} e_{\underline{d}_i+1,i},$$

where the inequality is due to $\alpha_i \in (0,1]$ and $k \leq \overline{d}_i+1$. Since the right hand side of the previous relation is not guaranteed to be greater than $e_i^{\min}$, we need to tweak the right hand side of (22a) when $k = \underline{d}_i + 1$ to make sure

$$e_{\underline{d}_i+1,i} \geq \frac{e_i^{\min}}{\alpha_i^{\overline{d}_i-\underline{d}_i}}.$$

This last consideration concludes the proof. ■

Appendix II
Proof of Proposition 1

Let us start by noticing that, for all $k \in [\overline{a}_i+1, \underline{d}_i+1]$,

$$\tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i} \vartheta_{t,i} \leq \frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \vartheta_{t,i}, \tag{46}$$

since $\tau, \alpha_i > 0$, $\vartheta_{t,i} \geq 0$ for all t, and $\eta_{t,i} \leq \frac{1}{\eta_i^-}$ for all $\eta_{t,i} \in H_i$. Moreover, for any $\bar{k} \in [\overline{a}_i, \underline{d}_i]$,

$$\frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\bar{k}} \alpha_i^{\bar{k}-t} \vartheta_{t,i} \leq \alpha_i^{\bar{k}+1-\underline{a}_i} \implies \frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\bar{k}-1} \alpha_i^{\bar{k}-1-t} \vartheta_{t,i} \leq \alpha_i^{\bar{k}-\underline{a}_i}$$

since

$$\begin{aligned}
\frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\bar{k}-1} \alpha_i^{\bar{k}-1-t} \vartheta_{t,i} &= \frac{1}{\alpha_i} \frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\bar{k}-1} \alpha_i^{\bar{k}-t} \vartheta_{t,i} \\
&\leq \frac{1}{\alpha_i} \frac{\tau}{\eta_i^-} \left[\sum_{t=\overline{a}_i}^{\bar{k}-1} \alpha_i^{\bar{k}-t} \vartheta_{t,i} + \vartheta_{\bar{k},i} \right] \\
&= \frac{1}{\alpha_i} \frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\bar{k}} \alpha_i^{\bar{k}-t} \vartheta_{t,i} \\
&\leq \frac{1}{\alpha_i} \alpha_i^{\bar{k}+1-\underline{a}_i} \\
&= \alpha_i^{\bar{k}-\underline{a}_i},
\end{aligned}$$

where the first equality is obtained multiplying and dividing by α_i, the first inequality is due to $\frac{1}{\alpha_i} \frac{\tau}{\eta_i^-} \vartheta_{\bar{k},i} \geq 0$, in the second equality we included $\vartheta_{\bar{k},i}$ inside the summation, the second inequality is due to the left hand side of the implication above, and the last equality is trivial. Iterating the above implication we have that

$$\begin{aligned}
&\frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\underline{d}_i-t} \vartheta_{t,i} \leq \alpha_i^{\underline{d}_i-\underline{a}_i+1} \\
&\implies \frac{\tau}{\eta_i^-} \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \vartheta_{t,i} \leq \alpha_i^{k-\underline{a}_i} \\
&\implies \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i} \vartheta_{t,i} \leq \alpha_i^{k-\underline{a}_i} &\text{(47a)} \\
&\implies \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i} \vartheta_{t,i} \leq \alpha_i^{k-\overline{a}_i}, &\text{(47b)}
\end{aligned}$$

for all $k \in [\overline{a}_i+1, \underline{d}_i+1]$ and all $\eta_{t,i} \in \{\eta_i^+, \frac{1}{\eta_i^-}\}$, where the second implication is due to (46) and the latter implication is due to $\alpha_i^{k-\underline{a}_i} \leq \alpha_i^{k-\overline{a}_i}$ as a consequence of $\alpha_i \in (0,1]$ and $\underline{a}_i \leq \overline{a}_i$.

Under Assumptions 1-3, Theorem 1 holds. Moreover, by (47a) and (47b), the argument of the absolute values in the expressions of $\underline{g}_{k,i}(\{\eta_{t,i}\}_t)$ and $\overline{g}_{k,i}(\{\eta_{t,i}\}_t)$ respectively are always non-negative. We can thus remove the absolute values and simplify the expressions to

$$\overline{g}_{k,i}(\cdot) = \alpha_i^{k-\overline{a}_i} e_i^\Delta + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} + s^+_{t,i} - \vartheta_{t,i} e_i^\Delta)$$

$$\underline{g}_{k,i}(\cdot) = -\alpha_i^{k-\underline{a}_i} e_i^\Delta + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} - s^-_{t,i} + \vartheta_{t,i} e_i^\Delta).$$

Recalling that (22) is equivalent to

$$\alpha_i^{k-\underline{a}_i} e_i^\mu + \min_{\{\eta_{t,i}\in H_i\}_t} \underline{g}_{k,i}(\{\eta_{t,i}\}_t) \geq e_i^{\min}(k) \tag{48}$$

and substituting the simplified expressions for $\overline{g}_{k,i}(\{\eta_{t,i}\}_t)$

and $\underline{g}_{k,i}(\{\eta_{t,i}\}_t)$ in (21) and (48) respectively, we have

$$\alpha_i^{k-\overline{a}_i} e_i^{\mu} + \min_{\{\eta_{t,i}\in H_i\}_t} \Bigg\{ \alpha_i^{k-\overline{a}_i} e_i^{\Delta} + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) \Bigg\} \le e_i^{\max}$$

$$\alpha_i^{k-\underline{a}_i} e_i^{\mu} + \min_{\{\eta_{t,i}\in H_i\}_t} \Bigg\{ -\alpha_i^{k-\underline{a}_i} e_i^{\Delta} + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \eta_{t,i}(\bar{p}_{t,i} - s_{t,i}^{-} + \vartheta_{t,i} e_i^{\Delta}) \Bigg\} \ge e_i^{\min}(k),$$

which can be further simplified to

$$\alpha_i^{k-\overline{a}_i}(e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \times \min_{\eta_{t,i}\in H_i} \eta_{t,i}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) \le e_i^{\max}$$

$$\alpha_i^{k-\underline{a}_i}(e_i^{\mu} - e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \times \min_{\eta_{t,i}\in H_i} \eta_{t,i}(\bar{p}_{t,i} - s_{t,i}^{-} + \vartheta_{t,i} e_i^{\Delta}) \ge e_i^{\min}(k),$$

since the minima over $\eta_{t,i}$ are independent across different t's. For both cases, note that the minimum over each $\eta_{t,i} \in H_i$ is achieved either at $\eta_{t,i} = \eta_i^{+}$ or $\frac{1}{\eta_i^{-}}$. The bound involving $e_i^{\min}(k)$ is equivalent to its epigraphic reformulation in (25a), while handling the bound involving $e_i^{\max}$ requires further efforts.

Since α_i and τ are both positive, we can rewrite the bound involving $e_i^{\max}$ as

$$\alpha_i^{k-\overline{a}_i}(e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \overline{h}_{t,i} \le e_i^{\max}$$

$$\overline{h}_{t,i} \ge \min_{\eta_{t,i}\in\{\eta_i^{+}, \frac{1}{\eta_i^{-}}\}} \eta_{t,i}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) = \begin{cases} \eta_i^{+}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) & \bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta} \ge 0 \\ \frac{1}{\eta_i^{-}}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) & \text{otherwise} \end{cases},$$

where the first constraint is already (24a). The constraint on $\overline{h}_{t,i}$ can instead be interpreted as two alternative constraints, with the sign of $\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}$ determining which one is active. Following the procedure described in [29], the inequality constraint involving $\overline{h}_{t,i}$ can be equivalently enforced using linear inequalities by introducing the additional binary variable $b_{t,i} \in \{0, 1\}$ representing which of the two constraints is enforced. If we choose $b_{t,i} = 1$ to represent the positive case, the constraint on $\overline{h}_{t,i}$ can be imposed using the following linear inequalities

$$\overline{h}_{t,i} \ge \eta_i^{+}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) - M(1 - b_{t,i}) \tag{49a}$$

$$\overline{h}_{t,i} \ge \tfrac{1}{\eta_i^{-}}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) - M\, b_{t,i} \tag{49b}$$

where M is a large constant which renders (49b) ineffective when $b_{t,i} = 1$ and renders (49a) ineffective when $b_{t,i} = 0$. Since imposing (24a) implicitly requires $\overline{h}_{t,i}$, the solver will automatically set $b_{t,i}$ to enforce the loosest constraints between (49a) and (49b). When $b_{t,i} = 1$, (49) simplifies to

$$\overline{h}_{t,i} \ge \eta_i^{+}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) \tag{50a}$$

$$\overline{h}_{t,i} \ge \tfrac{1}{\eta_i^{-}}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) - M \tag{50b}$$

and to make (50b) ineffective it is sufficient to impose

$$\eta_i^{+}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) \ge \tfrac{1}{\eta_i^{-}}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) - M,$$

which is equivalent to

$$M \ge (\tfrac{1}{\eta_i^{-}} - \eta_i^{+})(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}).$$

It is easy to see that $M = (\frac{1}{\eta_i^{-}} - \eta_i^{+}) p_i^{\max}$ satisfies this requirement since

$$\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta} \le \bar{p}_{t,i} + s_{t,i}^{+} + \vartheta_{t,i} e_i^{\Delta} \le p_i^{\max},$$

where the first inequality is due to $\vartheta_{t,i} e_i^{\Delta} \ge 0$, and the second inequality is due to (19a). A similar reasoning shows that the same value of M is valid also the case $b_{t,i} = 0$. Since, in all practical cases, $\frac{1}{\eta_i^{-}} - \eta_i^{+} < 1$, we can set $M = p_i^{\max}$ in (49a) and (49b) to obtain (24b) and (24c), respectively, thus concluding the proof. ∎

Appendix III
Proof of Corollary 1

Since (24) is equivalent to

$$\alpha_i^{k-\overline{a}_i}(e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{k-1} \alpha_i^{k-1-t} \overline{h}_{t,i} \le e_i^{\max}$$

$$\overline{h}_{t,i} = \min_{\eta_{t,i}\in\{\eta_i^{+}, \frac{1}{\eta_i^{-}}\}} \eta_{t,i}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta}) \le \eta_i^{+}(\bar{p}_{t,i} + s_{t,i}^{+} - \vartheta_{t,i} e_i^{\Delta})$$

the result trivially follows. ∎

Appendix IV
Proof of Proposition 2

Under Assumption 2, from (32) and (33) we can compute

$$e_{\underline{d}_i+1,i} = \alpha_i^{\underline{d}_i - a_i + 1} e_i^0 + \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\underline{d}_i - t} \min_{\eta_{t,i}\in H_i} \eta_{t,i} p_{t,i}$$

and $e_{d_i+1,i} = \alpha_i^{d_i - \underline{d}_i} e_{\underline{d}_i+1,i}$. To ensure $e_{d_i+1,i} \ge e_i^0 + \gamma_i$ for all possible values of the uncertain parameters, it is therefore sufficient to require that

$$\min_{a_i, d_i, e_i^0, \{\omega_t\}_t} \left\{ \alpha_i^{d_i - \underline{d}_i} e_{\underline{d}_i+1,i} - e_i^0 \right\} \ge \gamma_i.$$

Since the minimum over $d_i \in [\underline{d}_i, \overline{d}_i]$ is achieved at $d_i = \overline{d}_i$ due to $e_{\underline{d}_i+1,i} > 0$ and $\alpha_i \in (0, 1]$ and $e_{\underline{d}_i+1,i}$ and e_i^0 being independent from d_i, the left hand side of the previous relation is equal to

$$\min_{a_i, e_i^0, \{\omega_t\}_t} \left\{ \alpha_i^{\overline{d}_i - \underline{d}_i} e_{\underline{d}_i+1,i} - e_i^0 \right\}.$$

Similarly to the proof of Theorem 1, under Assumption 1, the left hand side of the previous relation is equivalent to

$$\min_{\substack{a_i, e_i^0, \{\omega_t\}_t, \\ \{\eta_{t,i}\in H_i\}_t}} \left\{ \left(\alpha_i^{\overline{d}_i - a_i + 1} - 1 - \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\overline{d}_i - t} \eta_{t,i} \vartheta_{t,i} \right) e_i^0 + \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\overline{d}_i - t} \eta_{t,i} (\bar{p}_{t,i} + \vartheta_{t,i} e_i^{\mu} + s_{t,i}^{+} [\omega_t]^{+} - s_{t,i}^{-} [\omega_t]^{-}) \right\}. \tag{51}$$

Since $\alpha_i \in (0,1]$, $\overline{d}_i - a_i + 1 > 0$ as a consequence of Assumption 2, and $\tau, \eta_{t,i} > 0$, and $\vartheta_{t,i} \geq 0$, the coefficient multiplying e_i^0 is always non-positive and hence the minimum over e_i^0 is achieved at $e_i^0 = e_i^{\mu} + e_i^{\Delta}$. Similarly, since $\tau, \eta_{t,i}, \alpha_i > 0$ and $s_{t,i}^{+}, s_{t,i}^{-} \geq 0$, the minimum over $\{\omega_t\}_t$ is achieved at $\omega_t = -1$ for all t. Moreover, the minimum over a_i is achieved at $\underline{a}_i$ since $e_i^0 > 0$ and $\alpha_i \in (0,1]$. Substituting the values of these minimizers in (51) yields

$$\min_{\{\eta_{t,i}\in H_i\}_t} \left\{ \left(\alpha_i^{\overline{d}_i - \underline{a}_i + 1} - 1 - \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\overline{d}_i - t} \eta_{t,i} \vartheta_{t,i} \right) (e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\overline{d}_i - t} \eta_{t,i} (\bar{p}_{t,i} + \vartheta_{t,i} e_i^{\mu} - s_{t,i}^{-}) \right\},$$

which is equivalent to

$$\left(\alpha_i^{\overline{d}_i - \underline{a}_i + 1} - 1 \right) (e_i^{\mu} + e_i^{\Delta}) + \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\overline{d}_i - t} \times \min_{\eta_{t,i}\in H_i} \eta_{t,i} (\bar{p}_{t,i} - \vartheta_{t,i} e_i^{\Delta} - s_{t,i}^{-}), \tag{52}$$

owing to independence of the minima across t. Since the minimum over $\eta_{t,i}$ is achieved at η_i^{+} or $\frac{1}{\eta_i^{-}}$, then (28) is the epigraphic reformulation of (52) $\geq \gamma_i$ and this observation concludes the proof. ∎

Appendix V
Proof of Lemma 1

The function $\tilde{J}(x)$ can be shown to be a convex function of x as follows:

$$\begin{aligned}
&\tilde{J}(\alpha x_1 + (1-\alpha) x_2) \\
&\quad = \mathbb{E}[J(\alpha x_1 + (1-\alpha) x_2, \delta)] \\
&\quad = \int_{\Delta} J(\alpha x_1 + (1-\alpha) x_2, \delta)\, \mathrm{d}\mathbb{P}_{\delta} \\
&\quad \leq \int_{\Delta} (\alpha J(x_1, \delta) + (1-\alpha) J(x_2, \delta))\, \mathrm{d}\mathbb{P}_{\delta} \\
&\quad = \alpha \int_{\Delta} J(x_1, \delta) \mathrm{d}\mathbb{P}_{\delta} + (1-\alpha) \int_{\Delta} J(x_2, \delta) \mathrm{d}\mathbb{P}_{\delta} \\
&\quad = \alpha \mathbb{E}[J(x_1, \delta)] + (1-\alpha) \mathbb{E}[J(x_2, \delta)] \\
&\quad = \alpha \tilde{J}(x_1) + (1-\alpha) \tilde{J}(x_2),
\end{aligned}$$

where the inequality is due to $J(x, \delta)$ being convex and the monotonicity property of the integral with respect to the measure $\mathbb{P}_{\delta}$. ∎

Appendix VI
Proof of Proposition 3

Given that, by (19c), $p_{k,i} = 0$ for all $k \notin [\overline{a}_i, \underline{d}_i]$ (which is non-empty under Assumption 2), recalling that $e_{a_i,i} = e_i^0$, and using (5), we can express $e_{d_i+1,i}$ as

$$e_{d_i+1,i} = \alpha_i^{d_i - a_i + 1} e_i^0 + \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{d_i - t} \min\left\{ \eta_i^{+} p_{t,i}, \frac{1}{\eta_i^{-}} p_{t,i} \right\}.$$

Taking the expectation of the previous expression, using its linearity property, and the independence between the random variables granted by Assumption 1, we can compute

$$\mathbb{E}[e_{d_i+1,i}] = \mathbb{E}_{d_i}[\alpha_i^{d_i}] \mathbb{E}_{a_i}[\alpha_i^{-a_i}] \alpha \mathbb{E}_{e_i^0}[e_i^0] + \tau \sum_{t=\overline{a}_i}^{\underline{d}_i} \alpha_i^{\underline{d}_i - t} \mathbb{E}_{e_0, \omega_t}\left[\min\left\{ \eta_i^{+} p_{t,i}, \frac{1}{\eta_i^{-}} p_{t,i} \right\} \right].$$

Defining $\tilde{a}_i = -\log_{\alpha_i} \mathbb{E}[\alpha_i^{-a_i}]$ and $\tilde{d}_i = \log_{\alpha_i} \mathbb{E}[\alpha_i^{d_i}]$, we can substitute $\mathbb{E}[\alpha_i^{-a_i}] = \alpha_i^{-\tilde{a}_i}$ and $\mathbb{E}[\alpha_i^{d_i}] = \alpha_i^{\tilde{d}_i}$ into the previous expression and the desired result directly follows. ∎

References

[1] IEA, "Global ev outlook 2020," IEA, Paris, Tech. Rep., 2020.

[2] C. Diaz-Londono, L. Colangelo, F. Ruiz, D. Patino, C. Novara, and G. Chicco, "Optimal strategy to exploit the flexibility of an electric vehicle charging station," *Energies*, vol. 12, no. 20, 2019.

[3] T. E. PARLIAMENT and T. C. O. T. E. UNION, "Directive (eu) 2019/944," EU, Tech. Rep., 2019.

[4] Z. Guo, Z. Zhou, and Y. Zhou, "Impacts of integrating topology reconfiguration and vehicle-to-grid technologies on distribution system operation," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 2, pp. 1023–1032, 2020.

[5] A. S. Al-Ogaili, T. J. Tengku Hashim, N. A. Rahmat, A. K. Ramasamy, M. B. Marsadek, M. Faisal, and M. A. Hannan, "Review on scheduling, clustering, and forecasting strategies for controlling electric vehicle charging: Challenges and recommendations," *IEEE Access*, vol. 7, pp. 128 353–128 371, 2019.

[6] E. Veldman and R. A. Verzijlbergh, "Distribution grid impacts of smart electric vehicle charging from different perspectives," *IEEE Transactions on Smart Grid*, vol. 6, no. 1, pp. 333–342, 2015.

[7] D. Liu, W. Wang, L. Wang, H. Jia, and M. Shi, "Dynamic pricing strategy of electric vehicle aggregators based on ddpg reinforcement learning algorithm," *IEEE Access*, vol. 9, pp. 21 556–21 566, 2021.

[8] D. Qiu, Y. Ye, D. Papadaskalopoulos, and G. Strbac, "A deep reinforcement learning method for pricing electric vehicles with discrete charging levels," *IEEE Transactions on Industry Applications*, vol. 56, no. 5, pp. 5901–5912, 2020.

[9] J. Vuelvas, F. Ruiz, and G. Gruosso, "Limiting gaming opportunities on incentive-based demand response programs," *Applied Energy*, vol. 225, pp. 668–681, 2018.

[10] ——, "A time-of-use pricing strategy for managing electric vehicle clusters," *Sustainable Energy, Grids and Networks*, vol. 25, p. 100411, 2021.

[11] M. Alipour, B. Mohammadi-Ivatloo, M. Moradi-Dalvand, and K. Zare, "Stochastic scheduling of aggregators of plug-in electric vehicles for participation in energy and ancillary service markets," *Energy*, vol. 118, pp. 1168–1179, 2017.

[12] T. Sriyakul and K. Jermsittiparsert, "Optimal economic management of an electric vehicles aggregator by using a stochastic p-robust optimization technique," *Journal of Energy Storage*, vol. 32, p. 102006, 2020.

[13] N. DeForest, J. S. MacDonald, and D. R. Black, "Day ahead optimization of an electric vehicle fleet providing ancillary services in the los angeles air force base vehicle-to-grid demonstration," *Applied Energy*, vol. 210, pp. 987–1001, 2018.

[14] S. Liu and A. H. Etemadi, "A dynamic stochastic optimization for recharging plug-in electric vehicles," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 4154–4161, 2018.

[15] R. Bernal, D. Olivares, M. Negrete-Pincetic, and lvaro Lorca, "Management of ev charging stations under advance reservations schemes in electricity markets," *Sustainable Energy, Grids and Networks*, vol. 24, p. 100388, 2020.

[16] W. Sun, F. Neumann, and G. P. Harrison, "Robust scheduling of electric vehicle charging in lv distribution networks under uncertainty," *IEEE Transactions on Industry Applications*, vol. 56, no. 5, pp. 5785–5795, 2020.

[17] M.-W. Tian, S.-R. Yan, X.-X. Tian, M. Kazemi, S. Nojavan, and K. Jermsittiparsert, "Risk-involved stochastic scheduling of plug-in electric vehicles aggregator in day-ahead and reserve markets using downside risk constraints method," *Sustainable Cities and Society*, vol. 55, p. 102051, 2020.

[18] Z. Wang, P. Jochem, and W. Fichtner, "A scenario-based stochastic optimization model for charging scheduling of electric vehicles under uncertainties of vehicle availability and charging demand," *Journal of Cleaner Production*, vol. 254, p. 119886, 2020.

[19] L. Baringo and R. Snchez Amaro, "A stochastic robust optimization approach for the bidding strategy of an electric vehicle aggregator," *Electric Power Systems Research*, vol. 146, pp. 362–370, 2017.

[20] A. Porras, R. Fernández-Blanco, J. M. Morales, and S. Pineda, "An efficient robust approach to the day-ahead operation of an aggregator of electric vehicles," *IEEE Transactions on Smart Grid*, vol. 11, no. 6, pp. 4960–4970, 2020.

[21] R. Shi, S. Li, P. Zhang, and K. Y. Lee, "Integration of renewable energy sources and electric vehicles in v2g network with adjustable robust optimization," *Renewable Energy*, vol. 153, pp. 1067–1080, 2020.

[22] W. R. Inc., "Mathematica, Version 12.1," champaign, IL, 2020. [Online]. Available: https://www.wolfram.com/mathematica

[23] A. Falsone and R. Vignali, "Mathematica notebook for computing $c_{k,i}^{\mathrm{veh}}$." [Online]. Available: https://falsone.faculty.polimi.it/files/CostVEH.nb

[24] M. Rossini, C. Sandroni, and R. Vignali, "A stochastic optimization approach to the aggregation of electric vehicles for the provision of ancillary services," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 7431–7438, 2020.

[25] "Gestore mercati energetici." [Online]. Available: https://www.mercatoelettrico.org/it/

[26] J. Lofberg, "Yalmip: A toolbox for modeling and optimization in matlab," in *2004 IEEE international conference on robotics and automation (IEEE Cat. No. 04CH37508)*. IEEE, 2004, pp. 284–289.

[27] I. I. Cplex, "V12. 1: Users manual for cplex," *International Business Machines Corporation*, vol. 46, no. 53, p. 157, 2009.

[28] M. Sion, "On general minimax theorems," *Pacific Journal of mathematics*, vol. 8, no. 1, pp. 171–176, 1958.

[29] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

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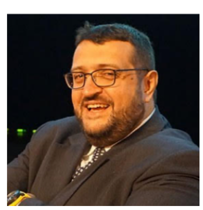

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