TOWARDS RELIABLE AND COST-EFFECTIVE DNS OVER RIBLETS

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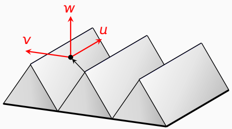
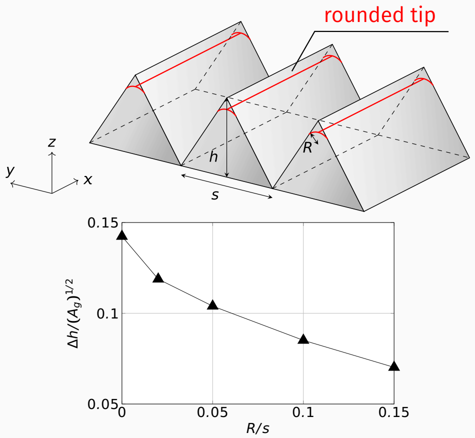


DR PERFORMANCE: IMPORTANCE OF THE SHARP CORNER

The drag reduction performance of the riblets depends on the sharpness of their tip.

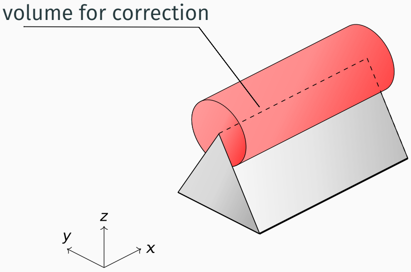
Consequences for DNS:
An extremely fine grid is required near the tip.



Adapted from Garcia–Mayoral & Jimenez, Phil. Trans. R. Soc. A (2011)

Analytical correction of the corner singularity

Luchini, "Higher-order difference approximations of the Navier-Stokes equations", J. Comput. Phys. (1991)



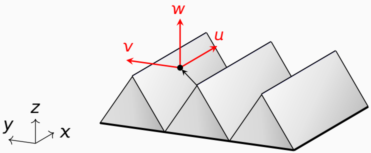
Stokes Problem:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \frac{1}{\rho}\nabla P = \nu\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0$$

$\downarrow$

Analytical solution



Two uncoupled problems

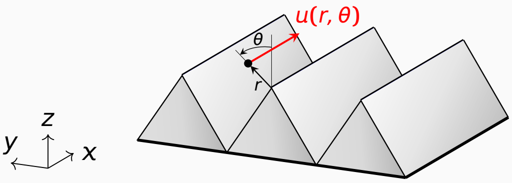
Analytical correction of the corner singularity

- || to the edge

$$0 = \nu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\downarrow$$

1D Laplace problem



$u(r, \theta)$

Analytical correction of the corner singularity

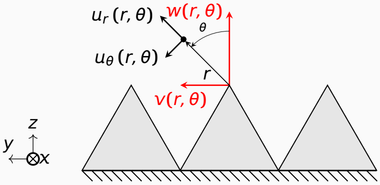
- ⊥ to the edge

$$\frac{\partial P}{\partial y} = \nu\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial P}{\partial z} = \nu\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

↓

2D Stokes problem



$$v(r,\theta) = u_r sin(\theta) + u_\theta cos(\theta)$$

$$w(r,\theta) = u_r cos(\theta) - u_\theta sin(\theta)$$

$$p(r,\theta)$$

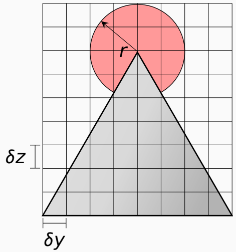
Analytical solution: procedure

- Switch from cartesian to polar coordinates
- Assume variables separation
- Impose the boundary conditions
- Choose in the spectrum of exact solutions the dominant one, uniquely identified by the requirement that it must reduce with continuity to a linear velocity when the surface is flat

Analytical solution: implementation into a DNS solver

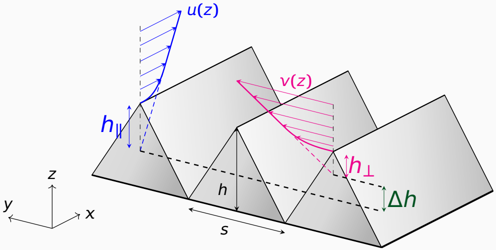
Integration of the analytical corner correction with a IBM solver (Luchini, Eur. J. Mech. B Fluids (2016))

- correction imposed to $\nu\nabla^2\mathbf{u}$ and ∇P
- correction imposed implicitly
- r = $2\delta y$



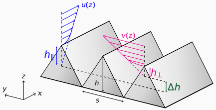
Validation: protrusion heights

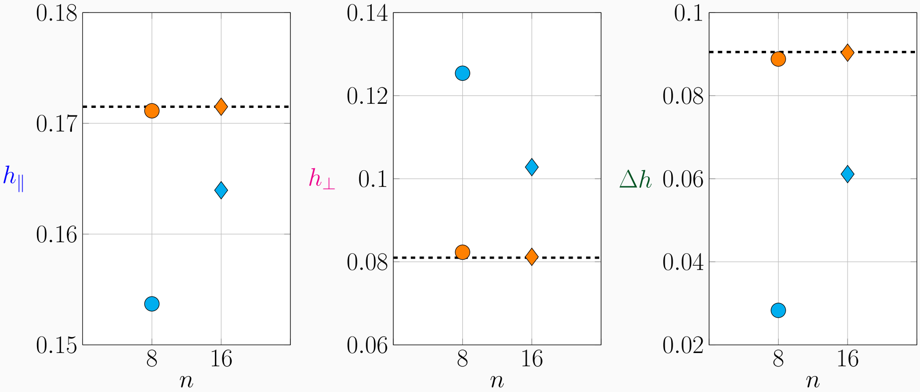
- For laminar flows, the protrusion height $\Delta h = h_{\parallel} - h_{\perp}$ can be computed exactly
- For turbulence flows, drag reduction performance is proportional to Δh



Validation: protrusion heights

Protrusion heights without and with corner correction with 8 (●) and 16(◆) points per riblet (n):



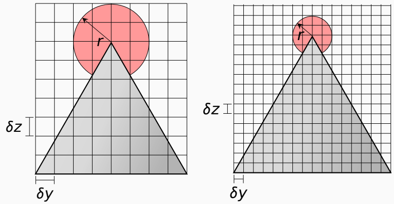


Simulation parameters

We performed two sets of DNS of a turbulent half channel flow with the wall covered by riblets at CPG by an IBM code written in CPL language.

Re_τ	L_x^+	L_y^+	δx^+	n	δy^+	$\delta z_{max}/\delta z_{min}$	h/s	s^+	r
200	1500	416	6.3	16 (8)	1 (2)	1.3	$\sqrt{3}/2$	16	2

	n_x	n_y	n_z
$n = 8$	240	208	94
$n = 16$	240	416	186



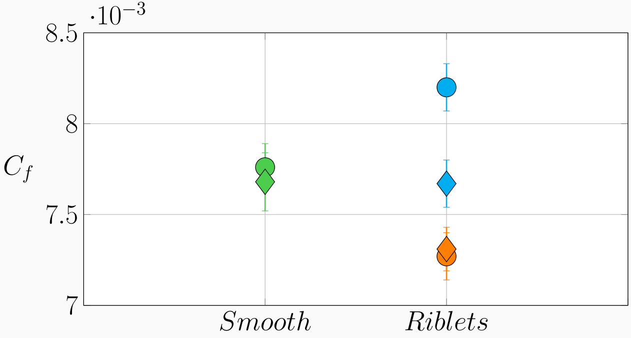
Experimental result: $DR = 5\%$ (Bechert et al., J. Fluid Mech. (1997))

Turbulent results: drag reduction performances

Friction coefficient for the cases

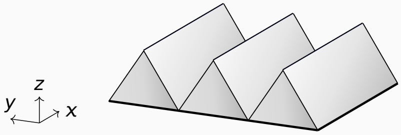
- smooth
- with riblets
  - without corner correction
  - with corner correction

with 8 (●) and 16(♦) points per riblet (n).



DNS with analytical correction: $DR = 4.8\%$

Preliminary extension to 3D sinusoidal riblets



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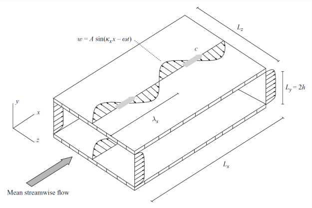

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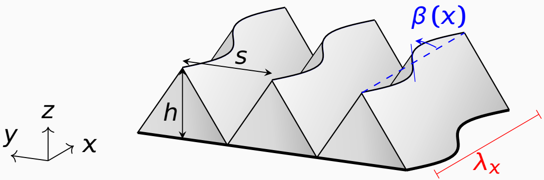
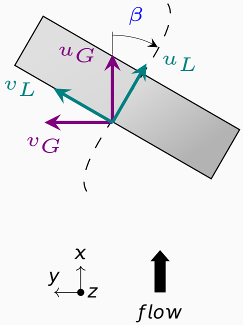
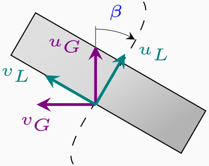


3D riblets: issues

- Global reference frame:
  decoupling into 1D Laplace and 2D Stokes problems fails
- Local reference frame: decoupling is possible, but velocity components are intermixed
  - discretization becomes explicit
  - discretization becomes challenging due to staggered grid



3D RIBLETS: PROVISIONAL SOLUTION



Assumption: local misalignment of the riblets section is small ($\beta(x)_{max} = 2°, \lambda_x^+ = 1500$)

$$\begin{Bmatrix} u_G \\ v_G \end{Bmatrix} = \begin{bmatrix} f(\beta, c_{lap}, c_{st}) & g(\beta, c_{lap}, c_{st}) \\ p(\beta, c_{lap}, c_{st}) & q(\beta, c_{lap}, c_{st}) \end{bmatrix} \begin{Bmatrix} u_L \\ v_L \end{Bmatrix}$$

Solution: limitation to the diagonal components of the correction matrix

3D riblets: preliminary results

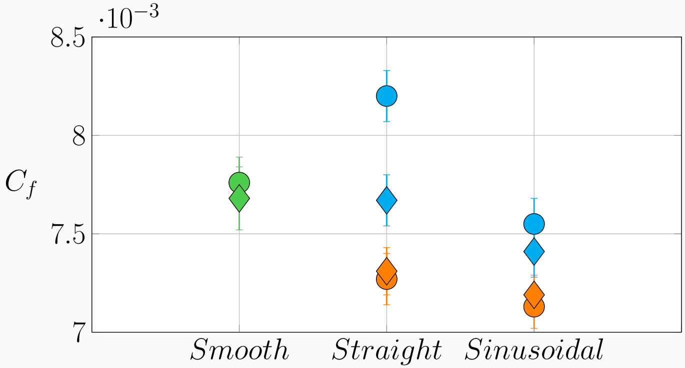
Friction coefficient for the cases

- smooth
- with riblets
  - without corner correction
  - with corner correction

with

- 8 (●)
- 16(◆)

points per riblet



Conclusions

An analytical correction for the corner singularity was applied to the turbulent flow over ribelts

- reliable: increased accuracy in computing Δh
- effective: much fewer points per riblets are needed for a given accuracy

obtaining

- DR of 4.8% for the reference configuration
- DR of $+30\%$ for sinusoidal riblets compared to the reference configuration.

Analytical Corner Correction: Stokes problem with streamfunction-vorticity formulation

$$\begin{cases} \nabla \cdot \mathbf{u} = 0 \\ \nabla^2 \mathbf{u} - \nu^{-1} \nabla p = 0 \end{cases} \implies \begin{cases} \nabla^2 \boldsymbol{\psi} = \boldsymbol{\omega} \\ \nabla^2 \boldsymbol{\omega} = 0. \end{cases}$$

The steady $\psi - \omega$ Stokes system in polar coordinates is

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \omega$$
$$\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r}\frac{\partial \omega}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2} = 0.$$

By imposing a variable separation for $\psi(r,\theta) = \mathrm{P}(r)\mathrm{F}(\theta)$ and $\omega(r,\theta) = \mathrm{R}(r)\mathrm{G}(\theta)$, calling $\chi = \mathrm{G}''/\mathrm{G}$ and $k = -\sqrt{\chi} < 0$:

$$\begin{aligned} r^2\mathrm{R}'' + r\mathrm{R}' - \chi\mathrm{R} = 0 \\ \mathrm{G}'' + \chi\mathrm{G} = 0 \end{aligned} \quad \Longrightarrow \quad \mathrm{R} = ar^{-\sqrt{\chi}} + br^{\sqrt{\chi}} = ar^{k}$$

since $r \ll 1$, we obtain:

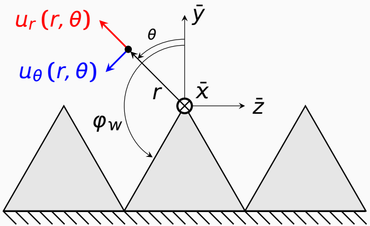
$$\omega(r,\theta) = r^{k}\left[C_1 \cos(k\theta) + C_2 \sin(k\theta)\right].$$

$$\psi(r,\theta) = r^{k+2}\left[D_1 \cos((k+2)\,\theta) + D_2 \sin((k+2)\,\theta) + D_3 \cos(k\theta) + D_4 \sin(k\theta)\right].$$

Analytical Corner Correction: boundary conditions

The coefficients D_i are given after the following boundary and symmetry conditions are provided:

$$u_r(r, \pm\varphi_w) = 0 \qquad \text{no penetration}$$
$$u_\theta(r, \pm\varphi_w) = 0 \qquad \text{no-slip}$$
$$u_r(r, \theta) = -u_r(r, -\theta) \qquad u_r \text{ odd in } \theta$$
$$u_\theta(r, \theta) = u_\theta(r, -\theta) \qquad u_\theta \text{ even in } \theta.$$



The symmetry conditions lead to $D_2 = D_4 = 0$, and the definition of the stream-function gives u_r and u_θ depending on $\gamma = k + 1$ as

$$u_r(r,\theta) = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = -r^\gamma \left[D_1(\gamma+1)\sin\left((\gamma+1)\theta\right) + D_3(\gamma-1)\sin\left((\gamma-1)\theta\right)\right]$$
$$u_\theta(r,\theta) = -\frac{\partial\psi}{\partial r} = -(\gamma+1)r^\gamma \left[D_1\cos((\gamma+1)\theta) + D_3\cos((\gamma-1)\theta)\right].$$

The boundary conditions are used to find the ratio between the coefficients D_3 and D_1, that is

$$\frac{D_3}{D_1} = \frac{\cos\left((\gamma+1)\varphi_w\right)}{\cos\left((\gamma-1)\varphi_w\right)}. \tag{1}$$

We set $D_1 = 1$.

Analytical Corner Correction: boundary conditions

The last constant to find is γ, whose value is given solving numerically $det(\mathbf{Q}(\gamma)) = 0$.

$$\underbrace{\begin{bmatrix} (\gamma+1)\sin\left((\gamma+1)\varphi_w\right) & (\gamma-1)\sin\left((\gamma-1)\varphi_w\right) \\ \cos\left((\gamma+1)\varphi_w\right) & \cos\left((\gamma-1)\varphi_w\right) \end{bmatrix}}_{\mathbf{Q}(\gamma)} \begin{bmatrix} D_1 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The solution depends on the geometry considered: for the problem at hand, with $\varphi_w = \pi/6$, the result is $\gamma \approx 0.51222$.

The last unknown for the Stokes problem is the pressure:

$$\nu\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{1}{r^2}\left(\frac{\partial^2 u_r}{\partial \theta^2}-2\frac{\partial u_\theta}{\partial \theta}-u_r\right)\right]-\frac{1}{\rho}\frac{\partial p}{\partial r}=0$$

$$\nu\left[\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}+\frac{1}{r^2}\left(\frac{\partial^2 u_\theta}{\partial \theta^2}+2\frac{\partial u_r}{\partial \theta}-u_\theta\right)\right]-\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta}=0$$

$$\frac{1}{\nu}p\left(r,\theta\right)=-4\gamma D_3 r^{\gamma-1}\sin\left(\left(\gamma-1\right)\theta\right).$$

The expression for p can not be used itself, because it is not guaranteed that p is symmetric and continuous inside the body. A correction can be implemented to choose a continuous branch for the solution, considering $\tilde{\theta} = \theta f(\theta)$ where $f(\theta) \neq 1$ only if $|\theta| > \varphi_w$, so that p is given by

$$\frac{1}{\nu} p\left(r, \theta\right) = -4\gamma D_3 r^{\gamma-1} \sin\left(\left(\gamma - 1\right)\theta f\left(\theta\right)\right)$$

$$f(\theta) = \begin{cases} 1 + \dfrac{|\theta| - \pi}{\pi - \varphi_w}\left(\dfrac{1}{\gamma - 1} - 1\right) & \text{if } |\theta| > \varphi_w \\ 1 & \text{otherwise.} \end{cases}$$

Analitical correction: Laplace Problem

The Laplace problem reads:

$$\nabla^2 u = 0 \Longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0,$$

and a variable separation leads to the general solution

$$u(r,\theta) = r^m\big[C\cos(m\theta) + D\sin(m\theta)\big].$$

No-slip boundary conditions, namely $u\,(r, \pm\varphi_w) = 0$, lead to $\cos(m\varphi_w) = 0$ and so $m\varphi_w = \pi/2$. The symmetry condition, $u\,(r,\theta) = u\,(r,-\theta)$, gives $D = 0$ and the final expression for u, namely

$$u = Cr^m \cos(m\theta)\,.$$

C here is a free constant that can be set to 1 to have a unique solution.

$$u^{(t+\Delta t)} = u^{(t)} + (lapl + \mathrm{NL} + \nabla p)\,\Delta t - u^{(t+\Delta t)} imbc \Delta t \Longrightarrow u^{(t+\Delta t)} = \frac{u^{(t)} + RHS\Delta t}{1 + imbc\Delta t}$$

Being u_{loc} and p_{loc} the analytical solutions for the velocity and the pressure respectively, considering the problem for the x-direction one gets

$$d_u = \underbrace{\left(\frac{lapl\,(u_{loc}\,(x,\cdot))}{Re} - \frac{p_{loc}\,(x+\Delta x,\cdot) - p_{loc}\,(x,\cdot)}{\Delta x}\right)\frac{1}{u_{loc}\,(x,\cdot)}}_{\mathrm{corr}_{stokes}} u\,(x,\cdot)\,,$$

where *lapl*() is the laplacian corrected with the true distance from the body. The Navier-Stokes problem here is not so different: the terms to add inside *imbc* are a contribution from the Laplace problem in u, corr_{lapl}, and from the Stokes problem in v and w, corr_{stokes}.

Considering (u', v') in the local reference frame and (u, v) in the global one, the following additional rotation should be performed:

$$u' = \cos(\beta)\, u + \sin(\beta)\, v, \qquad v' = \cos(\beta)\, v - \sin(\beta)\, u.$$

The *imbc* coefficients in the local reference frame were already found for the straight riblets as

$$d_{u'} = \mathrm{corr}_{lapl}\, u', \qquad d_{v'} = \mathrm{corr}_{stokes}\, v',$$

but to define the corrections in the cartesian global reference frame the two components get mixed into the 2×2 non-diagonal system.

Analytical Corner Correction: rotation

$$\begin{bmatrix} d_u \\ d_v \end{bmatrix} = \begin{bmatrix} \cos^2(\beta)\,\mathrm{corr}_{lapl} + \sin^2(\beta)\,\mathrm{corr}_{stokes} & \left(\mathrm{corr}_{lapl} - \mathrm{corr}_{stokes}\right)\sin(2\beta)/2 \\ \left(\mathrm{corr}_{stokes} - \mathrm{corr}_{lapl}\right)\sin(2\beta)/2 & \cos^2(\beta)\,\mathrm{corr}_{stokes} + \sin^2(\beta)\,\mathrm{corr}_{lapl} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}.$$

$$\begin{cases} d_u = \left(\cos^2(\beta)\,\mathrm{corr}_{lapl} + \sin^2(\beta)\,\mathrm{corr}_{stokes}\right) u \\ d_v = \left(\cos^2(\beta)\,\mathrm{corr}_{stokes} + \sin^2(\beta)\,\mathrm{corr}_{lapl}\right) v. \end{cases}$$

	nppr	$h_{\parallel}$	(err%)	$h_{\perp}$	(err%)	Δh	(err%)
Standard	8	0.1537	(-10.4)	0.1254	(+54.8)	0.02831	(-68.7)
+ Correction	8	0.1683	(-1.9)	0.0811	(+0.2)	0.0872	(-3.7)
Standard	16	0.1639	(-4.4)	0.1028	(+26.9)	0.06111	(-32.5)
+ Correction	16	0.1702	(-0.7)	0.0812	(+0.3)	0.0890	(-1.7)

Table 1: Results of the validation for straight riblets with the immersed boundary correction only (Standard) and with the addition of the corner correction (+ Correction). Errors are estimated as $(h - \bar{h})/\bar{h}$.

$\bar{h}_{\parallel}$	$\bar{h}_{\perp}$	$\Delta\bar{h}$
0.17150	0.08099	0.09051

Table 2: Protrusion heights reference values for $h/s = \sqrt{3}/2$.

Skin Friction Coefficient and U_b - Straight

	n	U_b	$(\Delta U_b^+ \%)$	$C_f \times 10^3$	$(\Delta C_f / C_{f,0} \%)$
Standard	8	15.62	(-2.7)	8.20	(+5.7)
+ Correction	8	16.58	(+3.3)	7.27	(-6.3)
Standard	16	16.14	(+0.1)	7.67	(-0.1)
+ Correction	16	16.54	(+2.6)	7.31	(-4.8)

Table 3: U_b^+ and C_f for the straight case. ΔU_b^+ and ΔC_f are evaluated considering the smooth channel simulation with the same δy^+ of the case considered.

	n	U_b	$(\Delta U_b^+ \%)$	$C_f \times 10^3$	$(\Delta C_f / C_{f,0} \%)$
L Standard	8	16.28	(+1.4)	7.55	(-2.7)
L + Correction	8	16.75	(+4.4)	7.13	(-8.1)
L Standard	16	16.43	(+1.9)	7.41	(-3.5)
L + Correction	16	16.67	(+3.4)	7.19	(-6.4)

Table 4: U_b^+ and C_f for the sinusoidal cases. ΔU_b^+ and ΔC_f are evaluated considering the smooth channel simulation with the same δy^+ of the case considered.