Scale-space budget equations for inhomogeneous (quasi-)periodic turbulent flow

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EFMC14 - 14th European Fluid Mechanics Conference

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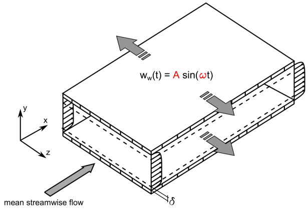
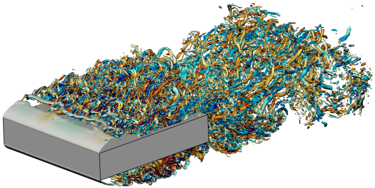
We are interested in studying turbulent flows with periodic features:

- flows subjected to an external periodic forcing



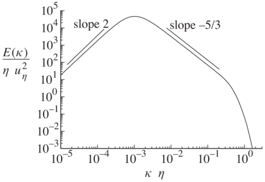
- flows which exhibit quasi-periodic features

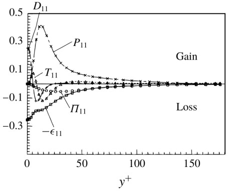


FEATURES OF THE NEW TOOL

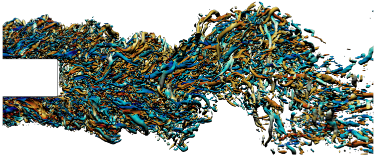
Development of a new statistical tool to study:

- the compound space of scales and positions

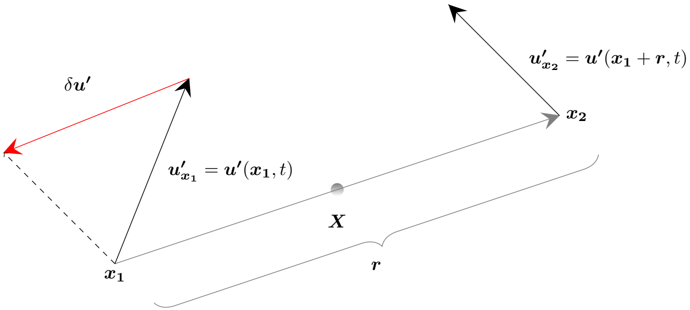




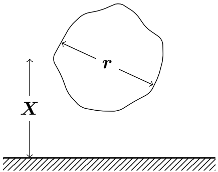
- the interaction between the periodic and stochastic flow fields



$$\left\langle \delta u_i' \delta u_j' \right\rangle = \left\langle \left[u_i'\left(\boldsymbol{X} + \frac{\boldsymbol{r}}{2}, t\right) - u_i'\left(\boldsymbol{X} - \frac{\boldsymbol{r}}{2}, t\right) \right] \left[u_j'\left(\boldsymbol{X} + \frac{\boldsymbol{r}}{2}, t\right) - u_j'\left(\boldsymbol{X} - \frac{\boldsymbol{r}}{2}, t\right) \right] \right\rangle = \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$



$$u = U + u'$$



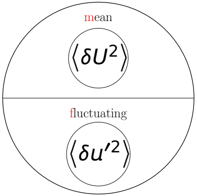
Turbulent stresses at the physical position $\boldsymbol{X}$ and up to the scale $\boldsymbol{r}$.

GKE: Generalised Kolmogorov Equation

(Kolmogorov, JFM 1941; Hill, JFM 2001)

$$\left\langle \delta u_i'^2 \right\rangle = tr \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$

$$1eq: \qquad \frac{\partial \Phi_k^f}{\partial r_k} + \frac{\partial \Psi_k^f}{\partial X_k} = P^f + D^f \qquad k = 1, 2, 3$$



$$u = U + u'$$

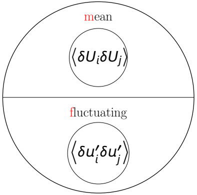
- Budget equation for the trace of $\left\langle \delta u_i' \delta u_j' \right\rangle$: the scale energy $\left\langle \delta u'^2 \right\rangle$.
- Production, transport and dissipation of the kinetic energy in the compound space of scales and positions.

AGKE: Anisotropic Generalised Kolmogorov Equations

(Gatti et al., JFM 2020)

$$\left\langle \delta u_i' \delta u_j' \right\rangle = \begin{bmatrix} \langle \delta u' \delta u' \rangle & \langle \delta u' \delta v' \rangle & \langle \delta u' \delta w' \rangle \\ \langle \delta u' \delta v' \rangle & \langle \delta v' \delta v' \rangle & \langle \delta v' \delta w' \rangle \\ \langle \delta u' \delta w' \rangle & \langle \delta v' \delta w' \rangle & \langle \delta w' \delta w' \rangle \end{bmatrix}$$

$$6 \;\; eqs: \qquad \frac{\partial \Phi^f_{ij,k}}{\partial r_k} + \frac{\partial \Psi^f_{ij,k}}{\partial X_k} = P^f_{ij} + \Pi^f_{ij} + D^f_{ij} \qquad k = 1, 2, 3$$



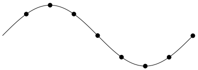
- Budget equations for each component of $\left\langle \delta u_i' \delta u_j' \right\rangle$.
- Production, transport, redistribution and dissipation of the turbulent stresses in the compound space of scales and positions.

$$u = U + u'$$

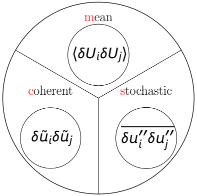
φAGKE: Phase-aware Anisotropic Generalised Kolmogorov Equations

$$6 \cdot n_{ph} \;\; eqs: \quad \frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

$$6 \cdot n_{ph} \;\; eqs: \quad \frac{2\pi}{T}\frac{\partial \overline{\delta u''_i\delta u''_j}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



- Budget equations for each component of $\overline{\delta\tilde{u}_i\delta\tilde{u}_j}$, $\overline{\delta u''_i\delta u''_j}$.
- Production, transport, redistribution, dissipation and inter-phase interaction of the turbulent stresses in the compound space of scales and positions.

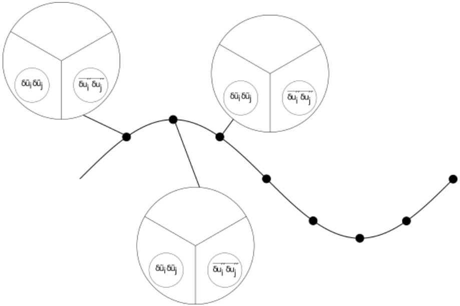


$$u = U + \underbrace{\tilde{u} + u''}_{u'}$$

φAGKE: EVOLUTION ACROSS PHASES

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

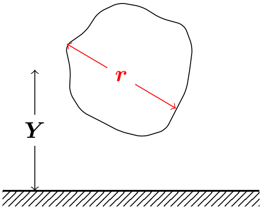
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i\delta u''_j}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: TRANSPORT AMONG SCALES

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

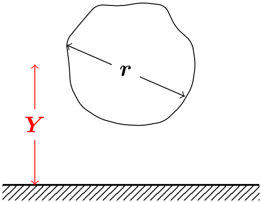
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i\delta u''_j}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: TRANSPORT IN THE PHYSICAL SPACE

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

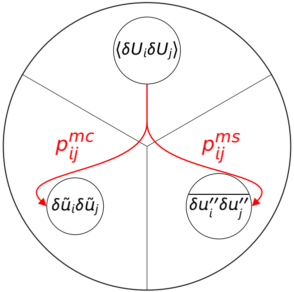
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u_i''\delta u_j''}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: MEAN PRODUCTION

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial \varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

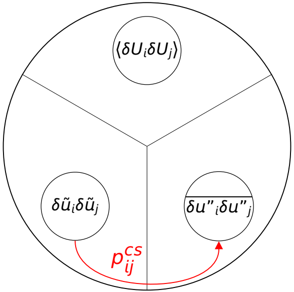
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i\delta u''_j}}{\partial \varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: COHERENT-STOCHASTIC PRODUCTION

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

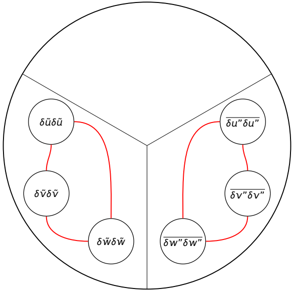
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i \delta u''_j}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: PRESSURE STRAIN

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

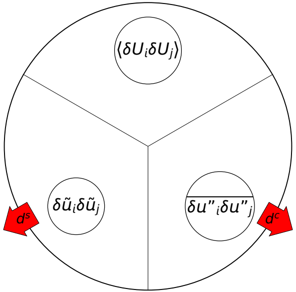
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i\delta u''_j}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: DISSIPATION

$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial\varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + \zeta^c_{ij}$$

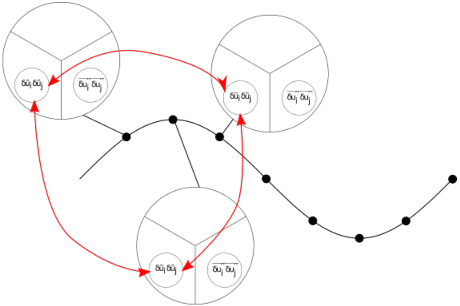
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i \delta u''_j}}{\partial\varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



φAGKE: Interaction across phases

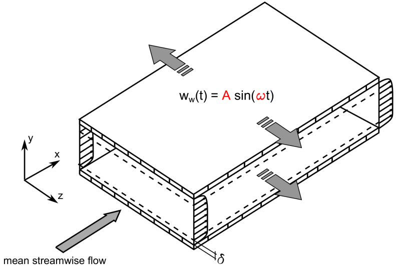
$$\frac{2\pi}{T}\frac{\partial \delta\tilde{u}_i\delta\tilde{u}_j}{\partial \varphi} + \frac{\partial \phi^c_{k,ij}}{\partial r_k} + \frac{\partial \psi^c_{k,ij}}{\partial X_k} = p^{mc}_{ij} - p^{cs}_{ij} + \pi^c_{ij} + d^c_{ij} + {\color{red}\zeta^c_{ij}}$$

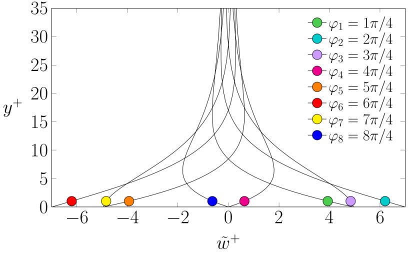
$$\frac{2\pi}{T}\frac{\partial \overline{\delta u''_i\delta u''_j}}{\partial \varphi} + \frac{\partial \phi^s_{k,ij}}{\partial r_k} + \frac{\partial \psi^s_{k,ij}}{\partial X_k} = p^{ms}_{ij} + p^{cs}_{ij} + \pi^s_{ij} + d^s_{ij}$$



$$w_w(t) = A\sin\left(\frac{2\pi}{T}t\right)$$

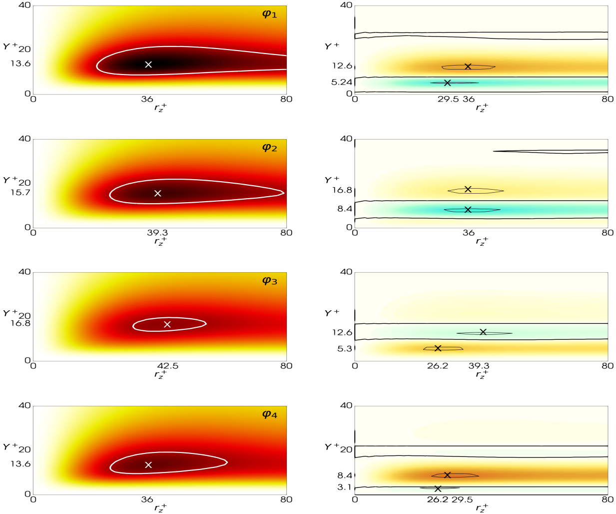
- external periodic forcing
- independent variables: $(r_x, r_y, r_z, Y, \varphi)$
- $\boldsymbol{U}(y) = (U(y), 0, 0)$
- $\tilde{v}(y, t) = 0$





RESULTS: THE PRODUCTION TERMS

$$P_{11}^{ms} = -2\overline{\delta u'' \delta v''}\left(\frac{dU}{dy}\right) \qquad P_{33}^{cs} = -2\overline{\delta v'' \delta w''}\left(\frac{\partial \tilde{w}}{\partial y}\right)$$



We derived phase-aware φAGKE expoiting a triple decomposition to study periodic and quasi-periodic turbulent flows.

Compared to the classical AGKE, φAGKE add new features:

- the scale-space energy exchange among mean, coherent, and stochastic fields
- the mutual interaction of the coherent motions at different phases
- no average over phases