On the Accuracy of Geoid Heights Derived from Discrete GNSS/Levelling Data Using Kriging Interpolation

Emanuele Alcaras, Pier Paolo Amoroso, Ugo Falchi, and Claudio Parente

Abstract

Local geoid models presenting higher resolution than global ones are generally derived by a combination of different datasets, integrating individual pure astrogeodetic, gravimetric and GNSS/levelling solutions. To define local geoid, different interpolators may be applied starting from dataset of geoid height values. It is well known that the accuracy of the resulting models depends not only by interpolation method, but also by points numerosity and distribution. This article aims to analyse the performance of Kriging approaches in dependence of the density of the dataset. The experiments are carried out on geoid heights extracted in random way from an already existing local geoid model: different subsets are organized containing an increasing number of points in the same area and each of them is submitted to Kriging interpolations (Universal Kriging and Ordinary Kriging). The resulting models are compared with the original one and residuals are calculated to evaluate the accuracy in dependence of point density. The results demonstrate the efficiency of the Kriging methods, highlighting the possibility to achieve higher accuracy (a few centimetres) using a point density of 1 point/100 sqkm, in absence of gravity anomalies. Ordinary Kriging provides better results than Universal Kriging but the undulations between the resulting models are minimal (a few millimetres) when a high number of points is involved. Furthermore, the results highlight the limit of the leave one out Cross validation since it supplies higher residuals than direct comparison for both Universal Kriging and Ordinary Kriging, when few points are used.

Keywords

Accuracy · Geoid height · Interpolation · Kriging · Local geoid

1 Introduction

The determination of the geoid, the equipotential surface of the earth's gravitational field that is closest to an average ocean surface (Barzaghi et al. 2002), is essential to measure the heights above the sea level. In fact, it represents the reference surface for orthometric heights, i.e., levelled heights corrected for gravity effects. It is known that the orthometric height of a point is nothing more than the distance from the point to the geoid, measured along a plumb line. The information on the geoid height (or geoid undulation), approximately defined as difference between the orthometric altitude and the ellipsoidal altitude, is fundamental in many

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application fields, e.g., for geophysical studies relating to crustal structures (Rapp 1974) and oceanographic studies relating to the topography of the sea surface (Blinken and Koch 1999). Different techniques can be adopted for geoid modelling and detailed descriptions of them are available in literature (Erol and Çelik 2004a; Eteje and Oduyebo 2018). We can distinguish at least five different approaches: the GNSS/levelling technique, the Gravimetric technique, the Astrogeodetic technique, the Satellite technique, the hybrid approach (including and integrating two or more techniques). Those approaches differ for used data; particularly the satellite technique incorporates orbit perturbations (ranging to satellites), gradiometry, satellite-to-satellite tracking, etc.

There are global geoid models (GGMs) such as EGM 1996 (Smith and Milbert 1997) and EGM 2008 (Pavlis et al. 2008; Barzaghi et al. 2016; Maglione et al. 2018): they represent correctly only the external gravity potential. The geoid must be derived by considering topography and its mass density variations. A GGM supports the conversion of ellipsoidal heights to orthometric heights with accuracies varying between few centimetres to even a metre (Denker et al. 2009; Pavlis et al. 2012; Alcaras et al. 2022). There are also local geoid models that present higher level of accuracy: they are generally developed using local (surface or aerial) gravity data compared with the GNSS/levelling measurements (Sideris and She 1995; Huang et al. 2007). In fact, to determine an accurate local geoid, it is necessary to take full advantage of all types of data/information in an integrated solution (Chen and Luo 2004). In other terms, the accuracy of a local geoid model can be improved by integrating an existing gravimetric geoid model with the ellipsoidal height and orthometric height derived from GNSS/levelling (You 2006).

Geoid height values (GHVs) already known in specific points (Geoid Height Points, GHPs) can be interpolated to define a local Geoid model (GM) (Erol and Çelik 2004b; Das et al. 2018; Falchi et al. 2018). Since different interpolation methods can be adopted (Erol and Erol 2021; Erol and Erol 2013), different results are expected (Ferrara and Parente 2021): there is no absolutely best interpolation method but only the optimal choice under certain circumstance (Yang et al. 2004). Nevertheless, some studies show the high level of performance of Kriging interpolators (Erol and Çelik 2004b; Falchi et al. 2018). For consequence, we decide to consider these algorithms for our study.

This article aims to analyse the relationship between the density of GHPs and the accuracy of each local geoid derived from those points using Universal (UK) and Ordinary Kriging (OK) interpolators. Since the spatial complexity of the function to be interpolated, the results are related to the roughness of the considered surface. To have a valid reference for calculating the accuracy of the resulting models, an already existing local geoid model concerning Corsica Isle (France) is chosen and assumed as source for extracting different subsets containing different number of points in the same area.

The article is organised as follows. Section 2 describes the materials and methods: 12 different datasets are selected including an increasing number of GHPs from 24 to 960; OK and UK interpolators are applied to each dataset. Section 3 presents and discusses the results comparing the levels of accuracy of 24 GMs, 12 for each interpolation algorithm in dependence of the number of the GHPs including in each dataset; particularly the accuracy is tested using the starting GM as reference. Section 4 draws out our conclusions.

2 Data and Methods

The experiments are carried out on geoid heights extracted in random way from an already existing local geoid model concerning Corsica Isle (France) and covering an area located between the following ellipsoidal WGS84 coordinates: lon min = 8° 24′ 00″, lon max = 9° 44′ 00″, lat min = 41° 11′ 15″, lat max = 43° 12′ 45″. The Geoid model includes 81 rows × 40 columns, presents a grid spacing of 1.5′ in latitude and 2′ in longitude and covers an area of about 24,767.74 sqkm (Institut Géographique National - IGN 2010). It is an adaptation of the QGC02 model, the gravimetric quasi-geoid model for the Corsica region (Duquenne et al. 2004), to 60 GNSS/levelling points: it has been assessed by using 15 independent GNSS/levelling points, showing differences with a RMSE of 3.4 cm (L'Ecu 2009). The geoid heights range between 44.947 m and 50.592 m; roughness, i.e. the degree of the surface irregularity that is calculated by the largest intercell difference of a central pixel and its surrounding cell, ranges between 0.007 m and 1.302 m. Figures 1 and 2 show respectively: the study area with the geolocalization of the dataset and the 3D visualization of the geoid model.

The Geoid model is converted in grid vector points and 12 different subsets are extracted in random way from them including an increasing number of elements in the study area from 24 to 960. Each extracted point coincides with the respective grid node: no interpolation algorithm is applied in this phase and the value provided by the initial grid is preserved in any case. In order to ensure a sufficiently homogeneous distribution of the points over the whole considered area, a grid presenting cell size 19.9995′ (long.) × 20.25′ (lat.) is introduced. For consequence the geoid area is subdivided in 24 cells (mean area: 1031.99 sqkm) and an equal number of GHPs (minimum 1, maximum 40) is maintained in each cell for each subset. Figure 3 shows two subsets including respectively 240 GHPs (0.010 point/sqkm) and 960 GHPs (0.039 point/sqkm).

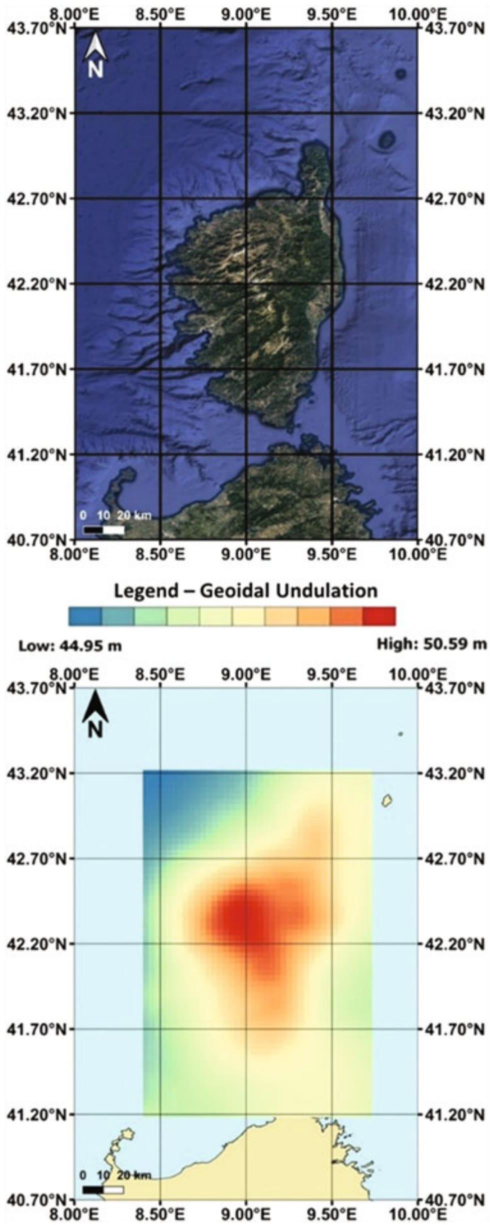

Fig. 1 The study area referred to WGS84 ellipsoidal coordinates: territorial framework of Corsica edited from Google Earth data (Upper); Initial dataset: geoid of Corsica (grid spacing: 1.5′ in latitude and 2′ in longitude) (Lower)

Each subset is submitted to Kriging interpolators, namely OK and UK, both based on the geo-statistical model which uses the spatial correlation between sampled points to estimate the value at an unknown point (Krivoruchko 2012).

Kriging interpolation methods assume that the spatial variation of any continuous attribute is often too irregular to be modelled by a simple mathematical function, so a stochastic surface is more suitable to represent it (Oliver and Webster 1990).

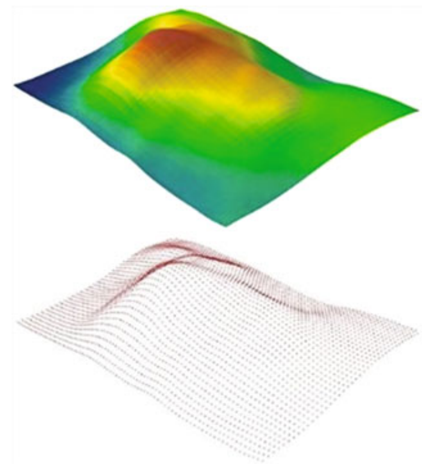

Fig. 2 Initial geoid model in 3D visualization as continuous surface (upper) and as grid points (lower)

For consequence Kriging methods can supply models that better represent and describe the geoid heights since it allows a more consistent prediction of the values in the non-sampled points. To understand the difference between the OK and the UK, a very wide range of sources is available in the literature and can be consulted (Martin and Simpson 2003; Kiš 2016).

OK assumes the model:

$$z(x_0) = \sum_{i=1}^{n} \lambda_i z(x_i) \tag{1}$$

where λ_i are the kriging weights. The function $z(x_i)$ is composed of a deterministic component μ and a random function $\varepsilon(x_i)$ (ESRI 2016).

$$z(x_i) = \mu + \varepsilon(x_i) \tag{2}$$

The deterministic component is a constant value for each x_i location in each area.

UK assumes the model (ESRI 2016):

$$z(x_i) = \mu(x_i) + \varepsilon(x_i) \tag{3}$$

where, $z(x_i)$ is the variable of interest, $\mu(x_i)$ is some deterministic function and $\varepsilon(x_i)$ is random variation (Gundogdu and Guney 2007).

Unlike OK, where the mean μ is assumed constant over the entire region of study, UK assumes that the mean $\mu(x_i)$ is dependent on the spatial location (Mesić Kiš 2016).

Both OK and UK analyse the variability of the points with increasing distances (variance) and adopt a mathematical model to describe it. Usually, a software for Kriging method application provides the user with different types of

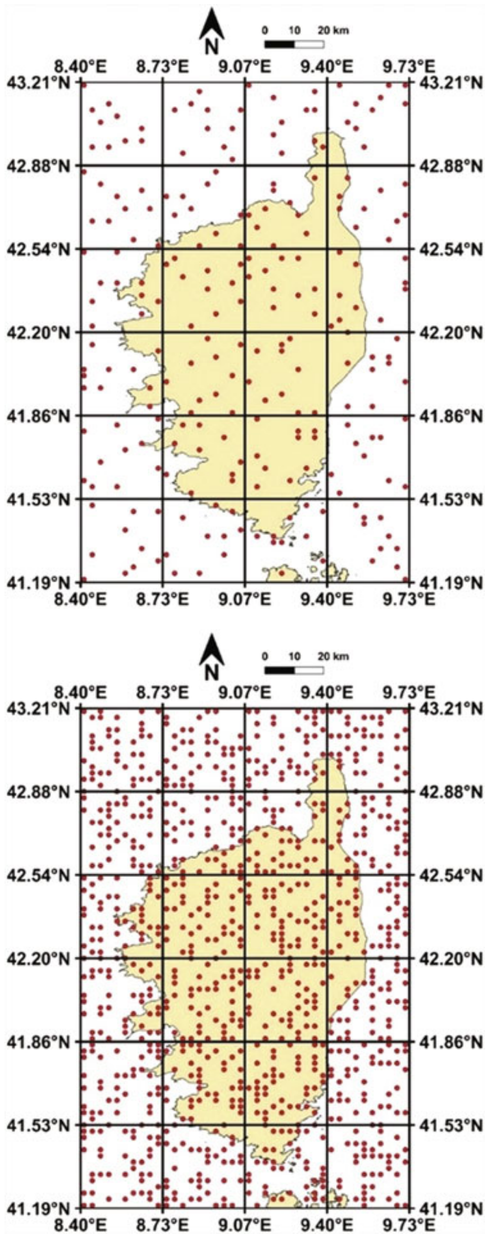

Fig. 3 Examples of subsets extracted from the initial models: the subset including 240 points (upper) and the subset including 960 points (lower) used for Kriging interpolations

semi-variogram, the mathematical function that graphically represents the spatial correlation between the input point values (Jian et al. 1996). In this study the choice of the mathematical model to fit the experimental data is carried out using the best performing one that results Stable model (ESRI 2016). We fix Lag = 12; Minimum neighbours = 2; Maximum neighbours = 5; 4 sectors with 45° offset. We also apply the optimization option supplied by the software that allows to increase the result accuracy. For consequence specific parameters are automatically determined, e.g. lag size and research radius.

The resulting GMs are tested by means of leave one out cross validation (Fasshauer and Zhang 2007) as well as using direct comparison with the original geoid. The subsequent residuals between initial undulation values and corresponding interpolated values are used to analyse and evaluate the accuracy in dependence of point density.

3 Results and Discussion

Significant statistical parameters (minimum, maximum and root mean square error) of all residuals for each dataset are shown in Table 1 for OK applications analysed by Cross validation, and in Table 2 for the same applications analysed by direct comparison.

In a similar way, significant statistical parameters of all residuals for each dataset are shown in Table 3 for UK applications analysed by Cross validation, and in Table 4 for the same applications analysed by direct comparison.

Table 1 Statistics of the residuals produced by cross validation for the Ordinary Kriging

Count	Min (m)	Max (m)	Mean (m)	RMSE (m)
24	−0.65	1.66	0.237	0.609
48	−0.41	0.45	0.024	0.192
72	−0.41	0.48	0.004	0.142
96	−0.30	0.23	0.007	0.104
120	−0.20	0.26	0.003	0.085
144	−0.19	0.21	0.000	0.069
168	−0.15	0.18	0.000	0.058
192	−0.17	0.21	0.005	0.049
216	−0.11	0.16	0.004	0.041
240	−0.10	0.15	0.003	0.039
480	−0.13	0.09	0.001	0.022
960	−0.09	0.07	0.000	0.012

Table 2 Statistics of residuals produced by direct comparison for Ordinary Kriging

Count	Min (m)	Max (m)	Mean (m)	RMSE (m)
24	−0.57	1.45	0.041	0.281
48	−0.54	0.42	−0.027	0.130
72	−0.43	0.27	−0.008	0.089
96	−0.28	0.40	−0.003	0.075
120	−0.25	0.22	−0.005	0.057
144	−0.30	0.23	−0.011	0.055
168	−0.26	0.21	−0.010	0.049
192	−0.24	0.24	−0.008	0.048
216	−0.24	0.15	−0.008	0.045
240	−0.24	0.16	−0.007	0.045
480	−0.13	0.12	−0.005	0.034
960	−0.14	0.09	−0.006	0.032

Table 3 Statistics of the residuals produced by cross validation for the Universal Kriging

Count	Min (m)	Max (m)	Mean (m)	RMSE (m)
24	−4.48	2.21	−0.124	1.402
48	−0.78	1.07	0.056	0.371
72	−0.60	0.64	0.059	0.227
96	−0.50	0.47	0.024	0.168
120	−0.42	0.53	0.006	0.151
144	−0.42	0.41	−0.003	0.129
168	−0.28	0.30	0.005	0.105
192	−0.28	0.26	0.007	0.090
216	−0.26	0.25	0.008	0.080
240	−0.19	0.26	0.005	0.069
480	−0.15	0.12	0.001	0.032
960	−0.11	0.09	0.001	0.018

Table 4 Statistics of residuals produced by direct comparison for Universal Kriging

Count	Min (m)	Max (m)	Mean (m)	RMSE (m)
24	−1.51	1.45	0.162	0.475
48	−1.16	0.77	−0.002	0.270
72	−0.75	0.72	0.008	0.198
96	−0.70	0.64	0.006	0.154
120	−0.56	0.51	0.000	0.123
144	−0.42	0.49	−0.003	0.106
168	−0.34	0.38	−0.004	0.090
192	−0.28	0.35	−0.004	0.078
216	−0.25	0.35	−0.001	0.069
240	−0.25	0.32	−0.003	0.064
480	−0.18	0.15	−0.006	0.039
960	−0.16	0.11	−0.005	0.034

The results demonstrate the efficiency of the Kriging methods, highlighting the possibility to achieve higher accuracy in dependence of an adequate density of GHPs. For OK the RMSE value rapidly decreases from the first to the fifth subset (from 0.609 m to 0.085 m using Cross validation, from 0.281 m to 0.057 m using direct comparison), while the variation slows down in subsequent groups. The trend of RMSE values for UK from the first to the fifth subset is similar, even if higher values are found (from 1.402 m to 0.151 m using Cross validation, from 0.475 m to 0.123 m using direct comparison).

The trend is clearly shown in Fig. 4 which plots the value of the RMSE in the case of OK and UK products directly compared with the initial geoid model.

Both methods of cross validation and direct comparison show a better performance of OK compared to UK. In consideration of the formulas (2) and (3), this seems to remark that it is correct to consider the deterministic component constant over the entire region of study rather than dependent on the spatial location. However, in the presence of a high number of points (480 or 960), the differences between the results of the two interpolators tend to become minimal (e.g. 0.006 m for 960 GHPs using Cross validation, 0.002 m for the same subset using direct comparison). In other terms, the higher number of points reduces the differences because it allows to better define the deterministic component assumed as dependent on the spatial location. Furthermore, the results highlight the limit of the leave one out Cross validation since it supplies higher residuals than direct comparison for both UK and OK, when a few points are used.

For example, Erol and Çelik (2004b) achieved an accuracy of about 0.03 m using UK as an interpolation method, and 1 GHP/3 km. Abdulrahman (2021) achieved an accuracy of about 0.243 m using OK and 1 GHP/0.350 km, but in this case measurements are carried out by means Total Station (Trigonometric Levelling).

4 Conclusion

The study demonstrates the efficiency of the Kriging methods for local Geoid determination, highlighting the relationship between the density of GHPs and the accuracy of the resulting model. OK provides better results than UK but the undulations between the resulting models are minimal (a few millimetres) when a high number of GHPs is involved. In fact, the limited extension of the considered area advises to take the deterministic component as a constant (OK): vice versa, if considered variable (UK), a higher number of points is necessary to determine its value more accurately. Using a few points, leave one out cross validation supplies higher residuals than direct comparison for both UK and OK, remarking the opportunity to consider this effect when testing GMs. For the analysed study area, using a density of 1 point/100 sqkm, direct comparison highlights that RMSE is less than 5 cm for OK application and less than 7 cm for UK. The current experiments testify that both the interpolation algorithms can be applied to determine accurate local geoid using 3.9 points/100 sqkm. The influence of the analysed region topography on the accuracy results needs further investigation.

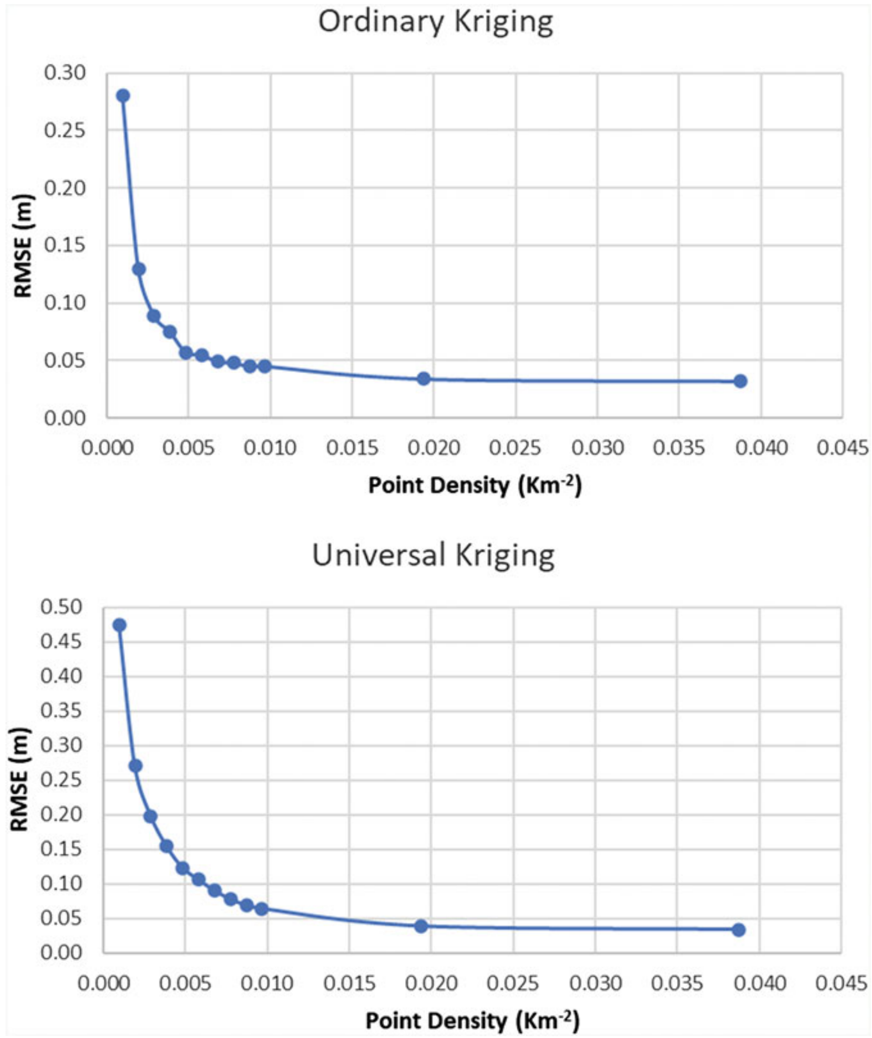

Fig. 4 Ordinary Kriging RMSE and Universal Kriging RMSE values (in meters) plotting for direct comparison

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