FORWARD METHOD FOR VERTICAL AIR MOTION ESTIMATION FROM FREQUENCY MODULATED CONTINUOUS WAVE RADAR RAIN MEASUREMENTS

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ABSTRACT

Abstract — Vertically-pointed Frequency-Modulated Continuous-Wave (FMCW) radar measurements of rain are greatly influenced by strong vertical winds (vertical air motion, VAM) in convective rain scenarios. Particularly, 2nd order products such as rain rate (RR) and drop size distribution (DSD) experience high estimation errors due to VAM. In this work, we consider the estimation of VAM from vertically-pointed FMCW radar measurements in order to correct VAM-corrupted rain 2nd order products. We present preliminary research on a forward method to estimate VAM velocity at a particular height from S-band FMCW radar measurements in convective rain scenarios. The method relies on the parameterization of the DSD as a gamma distribution. It estimates the VAM along with the constitutive parameters of the gamma distribution by means of a parametric solver. The methodology is tested over long-duration, high-resolution measurements by the University of Massachussetts FMCW radar and validated against a ground-based disdrometer in the context of the Verification of the Origins of Rotation in Tornadoes Experiment-Southeast (VORTEX-SE).

1. INTRODUCTION

Radars and disdrometers have been widely used to measure precipitation processes in the atmospheric boundary layer [1]. Ground-based S-band frequency modulated continuous wave (FMCW) radars have been used to assess the atmospheric boundary layer precipitation microphysical processes for more than 40 years [2], as they are unaffected by rain attenuation [3], [4]. Vertically-pointed FMCW radars permit the derivation of key rain 2nd order (integral?) parameters such as DSD and RR, among others [5]. The radar high spatial and temporal resolution permit an accurate monitoring of precipitation vertical profiles. Disdrometers record raindrop counts (at ground level) for different diameters during a measurement interval, and precisely derive rain DSDs from which 2nd order parameters such as RR can be obtained. However, each of these devices has its limitations. On one hand, the disdrometer is not able to give information of the vertical variations of precipitation. Moreover, large errors are common in scenarios with small diameter raindrops and low rain. On the other hand, radars have difficulty precisely measuring precipitation at low heights due to interference from ground clutter, near-field effects, and parallax influence (for FMCW radars employing dual antennas). Additionally, radars estimate the DSD from radar Doppler spectrum by assuming that rain drops are Rayleigh scatterers that fall at their terminal velocities, which are determined by the drop diameter. In practice, the droplets falling velocity are influenced by the vertical air motion (VAM) [6], which arises as a radar-measured spectrum shift in the velocity axis. In the presence of large VAM, such as in convective rain scenarios, radar-derived DSD and 2nd order parameters may be corrupted [4], [6]. The VAM estimation and correction from stand-alone Doppler radar measurements has been of interest since the beggining of radar usage in precipitation measurement [7]. Lhermitte [8] proposed a method to differentiate VAM and raindrops terminal velocity in W-band (λ = 3.2 mm) radars by exploiting Mie scattering. The VAM is determined by comparing the observed spectrum to a predicted spectrum assuming no VAM. However, this is only feasible for very-short wavelengths. Hauser and Amayenc [7] proposed a fitting method in which the DSD was assumed to be with an exponential form characterised by two parameters (Marshall-Palmer distribution). This methodology optimised the best fit between the theoretical spectrum retrieved from the DSD model (shifted by VAM) with respect to the experimental spectrum observations. However, it required exponentially distributed size distributions and it is not suited for convective rain scenarios. More recently, Tridon and Van Baelen [6] proposed a VAM-correction method by shifting the radar-measured spectrum to maximise the correlation with a no-VAM scenario. Rocadenbosch et al. [4] proposed a VAM estimation method based on the correspondence between Z-RR measurements with three different Z-RR models. It consisted on a trial-and-error procedure in which the radar-measured spectrum was shifted until Z-RR relationship matched theoretical models. A similar approach was proposed by Kim and Lee [9], which resorted to radar reflectivity empirical relationships as well to estimate the VAM and then unshift the spectrum. However, they require user expertise in rain radar observations for an accurate correction. In contrast, here, departing from the proposal by Hauser and Amayenc [7], a forward method to estimate the VAM from stand-alone radar measurements is presented. The foundations of the forward method are to parameterise the DSD as a Gamma distribution and to project this parametric DSD through the radar processing chain up to the retrieved radar-measured reflectivity. The methodology is tested over experimental data measured by a vertically-pointed FMCW radar and validated by a disdrometer as a reference during a convective rain event in VORTEX-SE campaign in 2017. The paper is structured as follows: section II describes the VORTEX-SE campaign and presents the OTT Parsivel2 disdrometer and the UMASS S-band radar; section III revisits the disdrometer and rain radar operation procedure as well as the VAM correction method; section IV shows a case study of the VAM correction method; and section V gives concluding remarks.

2. INSTRUMENTS

The Verification of the ORigins of Rotation in Tornadoes Experiment-Southeast (VORTEX-SE) measurement campaign aimed at studying how different environmental scenarios affected the formation, characteristics, and evolution of tornadoes in the Southeast United States. The second measurement campaign of VORTEX-SE took place between 8 March and 8 May 2017 in northern Alabama. It involved multiple fixed and mobile instruments assessing the spatial and temporal evolution of storm events. During the experiment, the UMass S-band FMCW radar was deployed at the Scottsboro, Alabama airport along with an OTT Parsivel2 disdrometer, part of the Portable In-situ Precipitation Sensor (PIPS) package deployed by Purdue University. The S-band FMCW radar was developed by the Microwave Remote Sensing Laboratory from the University of Massachusetts (UMASS). It is a transportable radar which uses two parabolic dish antennas of 2.4 m diameter with 34 dB gain, with a transmitter of 250 W [10]. It is able to vertically profile the volume reflectivity spectral density as a function of velocity (η(v)) with temporal and spatial resolutions of 16 s and 5 m, respectively. The radar bandwidth permits to measure drop falling velocities up to 14 m/s. Thanks to the radar signal wavelength, the atmospheric boundary layer can be studied both in clear-air and precipitation scenarios. The OTT Parsivel2 is a laser-based disdrometer able to measure the ground-level rain droplets distribution as a function of diameter and falling velocity [11]. Its operation is based on the shadowing effect that drops generate when passing through a light band. From the hydrometeors distribution, 2nd order parameters such as RR and DSD can be derived. It has been widely used in measurement campaigns, and here, it will be used as a reference.

3. METHOD

A. Radar Data Products

Following [4], the DSD is obtained as the ratio of the volume reflectivity density with respect to the drop diameter, η(D) [m−1/mm], to the single-particle backscattering cross section of a drop of diameter D, σ(D) [m2/drop]. It can be formulated as

$$N(D) = \frac{\eta(D)}{\sigma(D)}. \quad (1)$$

FMCW radars are able to measure the volume spectral reflectivity, η(v), which is the volume reflectivity (or radar cross section per unit volume) per unit Doppler velocity. In order to retrieve the DSD from radar measurements (see Equation 1 above), we need to express the spectral reflectivity as a function of drop diameter. To do so, the relationship η(D)∂D = η(v)∂v is used. This relationship was found empirically by [12], and was put expressed analytically by [13] as

$$v_n(D)[\text{m/s}] = (9.65 - 10.3e^{-0.6 \cdot D[\text{mm}]})\delta v(h), \quad (2)$$

where δv(h) is the height-dependent density correction for the terminal fall velocity, which is formulated as

$$\delta v(h) = 1 + 3.68 \cdot 10^{-5} h + 1.71 \cdot 10^{-9} h^2, \quad (3)$$

where h is the measurement height. From the DSD, second order products such as the reflectivity factor Z and the RR can be derived. The radar reflectivity factor can be obtained as the sixth power of the DSDas

$$Z = \int_0^\infty N(D) D^6 dD, \quad (4)$$

and the RR can be estimated from the DSD third moment as a function of drop terminal fall velocity

$$RR = \frac{\pi}{6} \int_0^\infty N(D) D^3 v(D) dD. \quad (5)$$

B. VAM influence.
Without VAM, the radar-measured Doppler velocities match the raindrops terminal falling velocities (vDoppler = v(D)). In presence of VAM, the hydrometeors falling velocities are determined by both the drop terminal velocity as a function of diameter (v(D) Equation 2) and the VAM velocity (vV AM). Then, the radar-measured Doppler velocity is given by

Therefore, drop diameter retrieval from velocity measurements and subsequent derivation of η(D), DSD, and RR need to include the correction v(D) = vDoppler − vV AM . As vV AM could not be measured with the available instrumentation, we face a non-linear inverse problem in which we need to find the best vV AM estimation given radar volume reflectivity density η(v) measurement [14].

$$v_{Doppler} = v(D) + v_{VAM}. \tag{6}$$

C. Forward model for VAM estimation
We propose a forward model approach to solve the inverse problem of radar RR retrieval as a function of the VAM correction by constraining the DSD as a gamma distribution:

$$N(D) = N_0 D^{\mu} e^{-\Lambda D}, \tag{7}$$

in which N0 , μ, and Λ are the intercept, shape, and slope constitutive parameters of the distribution, respectively [15]. In an inverse problem, the parameters to be estimated are represented by the state vector x, and the measurements actually made to estimate x can be represented by the measurement vector z [14]. In our inverse problem, the state vector x to be estimated is formed by the DSD gamma distribution parameters and the VAM. It can be formulated as

$$\boldsymbol{x} = [N_0, \mu, \Lambda, v_{VAM}]^T. \tag{8}$$

The measurement vector z is defined as the radar-measured volume reflectivity density η(v). It is formulated as

$$\boldsymbol{z} = \eta(v). \tag{9}$$

z is a N × 1 dimension vector, being N the number of velocity bins measured by the radar. We depart from Equation 1 in order to obtain the volume reflectivity density as a function of diameter from the DSD as

$$\eta(D) = N(D)\sigma(D). \tag{10}$$

Then, we make use of the relationship η(v)∂v = η(D)∂D to obtain the volume reflectivity density as a function of velocity as

$$\eta(v) = N(D)\sigma(D)\frac{\partial D}{\partial v}. \tag{11}$$

For each state vector x there is an ideal measurement vector z related by a forward function f(·). The radar DSD-to-reflectivity forward function is defined as the expanded form of Equation 11 above as

$$f(\boldsymbol{x}) = N_0 D(v_{VAM})^{\mu} e^{-\Lambda D(v_{VAM})} \sigma(D(v_{VAM})) \frac{\partial D(v_{VAM})}{\partial v}, \tag{12}$$

where D(vV AM) is the velocity-to-diameter relationship (see Equation 2) and vVAM is VAM velocity correction (see Equation 6). Then, the forward model can be defined as

$$\boldsymbol{z} = f(\boldsymbol{x}) + \epsilon, \tag{13}$$

where ε is a residual error term. In order to solve the inverse problem, and thus, to estimate the state vector x, we resort to a constrained non-linear least-squares (LSQ) method, finding the optimal x that minimizes the squared error ε2 between the model observation f(x) and the actual observation z. It can be formulated as an optimization problem as

$$\boldsymbol{x} = \arg\min_{\boldsymbol{x}} ||\eta(v) - \hat{\eta}(v, \boldsymbol{x})||^2 \tag{14}$$

where η ˆ(v,x) is the estimated radar volume reflectivity, output of the radar DSD-to-reflectivity forward function f(x). The block diagram in Figure 1 represents the optimization problem. The optimization problem in Equation 14 is solved by means of the trust-region-reflective algorithm [16].

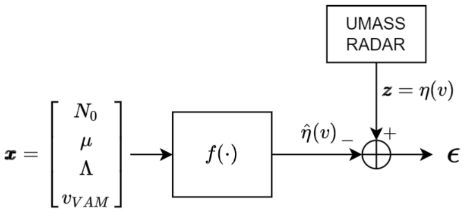

Fig. 1. Block diagram of the forward model algorithm. f(.) is the radar DSD-to-reflectivity forward function. The substractor is used to compute the residual error, ε, between the measured and the estimated radar volume reflectivity density.

4. RESULTS AND DISCUSSION

The algorithm was tested over experimental data measured by the UMASS S-band FMCW radar and the OTT Parsivel2 during VORTEX-SE 2017 campaign (see section II). The VAM was estimated by means of the forward model method from radar volume reflectivity density measurements at 500 m. Reflectivity density measurements were 1-min averaged in order to reduce their uncertainty [7]. Then, radar measurements were corrected considering VAM estimations, and RR and DSD radar products at 500 m were recomputed. The VAM-corrected RR estimations were compared against the disdrometer RR as a reference, assuming vertical correlation between 500 m and 0 m measurement heights considering 10-min average ensembles. Although collision and coalescence processes are present in the precipitation process, correlation coefficients of ρ ≃ 0.75 were found for the RRs between the radar measurements at 500 m and

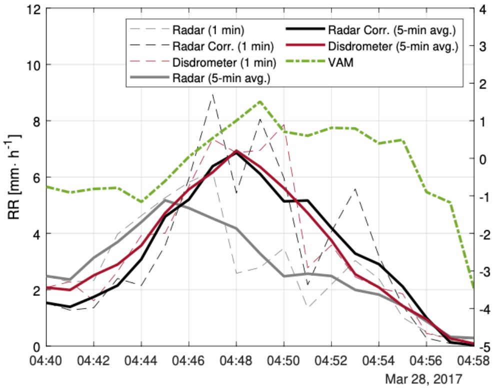

Figure 2
Time series representing the radar-measured RR (1 min), with (dashed black) and without (dashed gray) VAM correction, the disdrometer-measured RR (1 min, dashed red) and the VAM estimated by the forward method (green). Solid traces are the 5-min averaged versions of the respective dashed plots.

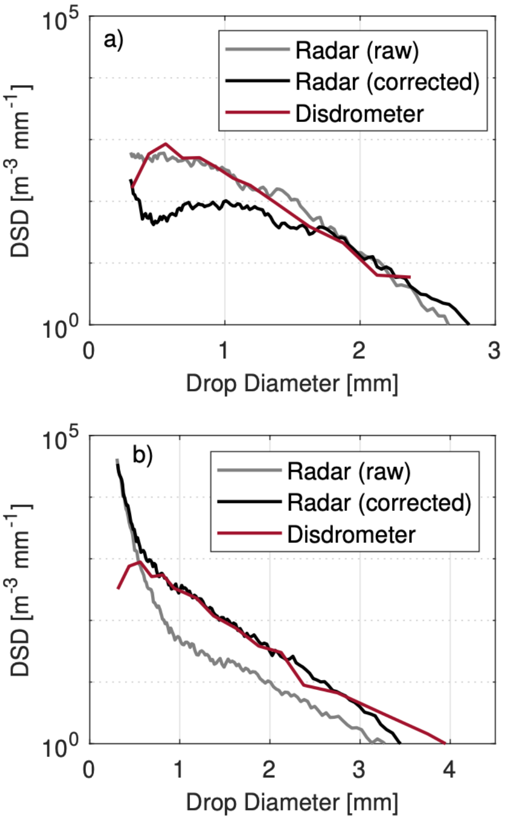

Figure 3
Comparison between DSDs measured by the radar, with (black) and without (gray) VAM correction, and the disdrometer (red). Two case examples are shown: (a)) biased VAM estimation; (b)) good VAM estimation.

the disdrometer in the VORTEX-SE campaign [4]. Figure 2 shows the VAM estimation and correction results in terms of RR during a 20-minute observation period. By comparing the radar-retrieved RR without correction (dashed gray) to the disdrometer (dashed red), it can be observed how from 04:46 to 04:55 they largely disagree, being the first up to 3 mm · h−1 lower than the latter. The VAM estimated by the inverse method presented in subsection III-C (dashed-dot green) shows constant -1 m/s values from 04:40 to 04:44, then it rises with a constant slope up to values around 1 m/s, and finally, at 04:55, it shows negative values down to -3 m/s. These VAM values seem to be in accordance with the RR, as a positive VAM implies a reduction of the radar-retrieved RR [17], corresponding to the 04:46-04:55 period in which the radar measures a lower RR as compared to the reference. After VAM correction, the radar-retrieved RR (dashed black) shows values of the same order of magnitude as the ones for the disdrometer, proving the validity of the VAM estimations. These results are further corroborated when comparing the 5-min window-averaged versions of the RR measurements (solid traces). The radar-measured RR (solid gray) shows significantly lower values in the 04:46-04:55 period with respect to the disdrometer (solid red). After correction, the radar-measured RR (solid black) matches almost ideally the reference. Note that when RR is very low, e.g., at 04:56-04:58 period, the VAM correction does not have a noticeable effect on the radar measurements, as reflectivity density values are too low. During the 04:40-04:44 period, it can be observed that the VAM-corrected radar RR shows lower values than the disdrometer reference. This may be due to an overestimation of negative VAMs. As a result, the algorithm estimates biased DSD parameters in order to match the measurements, i.e., to minimize the squared error between the measurement vector and the output of the forward function (see Equation 14). Figure 3 compares two DSDs measured by the radar, with and without VAM correction, against the reference DSDs measured by the disdrometer. Figure 3 a) plots the radar and the disdrometer DSD measurements at 04:43 UTC. Surprisingly, the VAM-corrected radar RR becomes underestimated (see Figure 2). As it can be observed, the raw DSD radar measurement (gray) virtually overlaps the reference (red). Therefore, the estimated VAM should be close to 0 m/s and no correction should be required. However, the forward method presented overestimates the VAM (VAM≃ −0.8 m/s), compensating the poorer functioning of the forward method with a miss-estimation of the DSD constitutive parameters (N0, μ, and Λ), which leads to wrong DSD retrievals (black). On the other hand, Figure 3 b) depicts the comparison of DSD measurements at 04:49, where a VAM of 1.5 m/s is estimated. Here, a factor 10 difference can be observed between the radar-measured and reference DSDs. After VAM correction, the radar-measured DSD matches almost perfectly the reference. However, as previously reported in the literature [18], optical disdrometers underestimate DSDs at low diameters (see D < 0.5mm at Figure 3 b)).

5. CONCLUSIONS

An inverse method to estimate VAM velocity from stand-alone radar measurements was presented. The method consists on fitting a modelled volume reflectivity density, as a function of the VAM and DSD gamma distribution constitutive parameters, to the radar-measured reflectivity. The method was tested over experimental data captured during a 20-minute period by an S-band FMCW radar and validated with an OTT Parsivel2 disdrometer in the context of VORTEX-SE 2017 measurement campaign. The estimation results found VAM values mainly ranging from -1 m/s up to 1.5 m/s during the period under analysis. After VAM correction, the radar-measured RRs were found to match almost ideally the disdrometer-measured RR for positive VAM values. However, overestimation of VAM negative values was found in no-VAM scenarios, leading to miss-estimation of the DSD constitutive parameters. These results were corroborated by comparing radar-retrieved DSD (with and without VAM correction) to disdrometer measurements. Although promising, the algorithm still needs to be further tested over different stratiform and convective rain scenarios in order to see if overestimation of negative VAM is also found, and how to improve these estimations. The VAM estimations could also be further validated by direct measurements of vertical wind by wind profilers.

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