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Abstract. A desirable aspect of financial time series analysis is that of successfully detecting (in real time) market phases. In this paper we implement HMMs and HSMMs with normal state-dependent distributions to Bitcoin/USD price dynamics, and also compare this with S&P 500 price dynamics, the latter being a benchmark in traditional stock market behaviour which most literature resorts to. Furthermore, we test our models' adequacy at detecting bullish and bearish regimes by devising mock investment strategies on our models and assessing how profitable they are with unseen data in comparison to a buy-and-hold approach. We ultimately show that while our modelling approach yields positive results in both Bitcoin/USD and S&P 500, and both are best modelled by four-state HSMMs, Bitcoin/USD so far shows different regime volatility and persistence patterns to the one we are used to seeing in traditional stock markets.

Keywords: Hidden Markov Models · Hidden Semi-Markov Models · Cryptocurrencies · Filtering · Nowcasting

1 Introduction

The relatively short history of Bitcoin and other cryptocurrencies is filled with numerous events that have drastically affected its value. The following three reasons justify the intense volatility experienced by cryptocurrencies: (i) cryptocurrency wealth distribution is more disproportionate than that of traditional financial assets, (ii) public understanding is subjective and highly divided, and lastly (iii) regulation from governments, for and against cryptocrurrencies, has greatly impacted their value. Some important events are mentioned hereafter. In 2015, the U.S. Commodity Futures Commission declared that cryptocurrencies essentially are not considered as currencies, but more as a commodity, and hence could not be regulated. 2017 saw Japan pass a law to accept Bitcoin as a legal form of payment, Bitcoin was split into two derivative digital currencies (the Bitcoin chain BTC and the Bitcoin cash chain BCH), and China's government ceased domestic exchanges. In 2018, South Korea prohibited anonymous cryptocurrency trading, social media platforms such as Facebook and Twitter banned cryptocurrency advertisements, and the UK's Financial Conduct Authority (FCA) issued advice on the high risks of investing in the unregulated market of cryptocurrencies. While 2017 was, generally, a bull year for cryptocurrencies, 2018 has seen much decline in their value and some cryptocurrencies have even been wiped out. Presently, however, the value of one Bitcoin is on the rise again and worth more than $9000.

The aim of this paper is that of identifying market regimes - mainly bull and bear market phases - of cryptocurrencies through the use of hidden Markov models (HMMs) and hidden semi-Markov models (HSMMs). We shall implement mock investment strategies on test data, and compare to a buy-and-hold approach, to determine how well these regimes are identified. When prices are on the rise for a relatively long period of time, the market condition is said to be a bull market, and when prices fall steeply with respect to recent highs, the market condition is referred to as a bear market. Two other phases which may be detected in the process are corrections and rallies, with the former being a period of steady decrease amid a bull market, and the latter being a period of slow increase within a bull or bear market. It is possible that HMMs and HSMMs may struggle to distinguish between these two states due to the fact that neither is associated with a steep change. Our research allows for the mean, and not just the volatility, to depend on the states - this is at times ignored in the literature. Due to high correlation between cryptocurrency dynamics, we consider the daily closing prices of Bitcoin/USD (BTC/USD), for the dates ranging from 01/01/2016 to 28/01/2019 for a total of 1124 trading days. Bitcoin is around 50% of the crypto market. Since traditionally, positive trends with low volatility and negative trends with high volatility have respectively been labelled as bull and bear markets, we shall compare and contrast our findings with a de facto standard stock market - the 'S&P 500' where the dates considered are 01/01/2000 - 28/01/2019. This can be invested in collectively via the S&P 500 Index Fund.

The following is a review of existent literature related to cryptocurrencies and the use of HMMs and HSMMs to model financial assets. Starting with the former, [8] fit various parametric distributions on cryptocurrency returns. Furthermore [2, 4, 9, 14, 19] fit generalised autoregressive conditional heteroscedastic (GARCH) models and its variants in their single-regime form. [15] look at the application of Markov switching autoregressive models to Bitcoin. Recent publications which involve the modelling of Bitcoin volatility dynamics at multiple regimes are [1, 3, 7] - though different approaches were used for modelling in these papers with slightly varying results, in all cases, multi-regime dynamics within a heteroscedastic framework was detected. The following, on the other hand, are examples of literature using HMMs and HSMMs to model different phases of financial asset price movements. [18] show that a normal-HMM is capable of reproducing most of the stylised facts for daily S&P 500 return series established by [10, 11]. However, they only allow the standard deviations to vary by the state, while the means are fixed at zero. Recently, [17] applied a four-state HMM for stock trading by predicting monthly closing prices of the

S&P 500, showing that the HMM is superior to the buy-and-hold strategy as it yields larger percentage profits under different training and testing periods. Modelling literature on financial time series using HSMMs is, on the other hand, quite limited. [16] implemented a three-state HSMM to describe the dynamics of the Chinese stock market index (CSI 300) returns. The authors assumed normal state-dependent distributions with logarithmic dwell-time distributions, and also implemented a profitable trading strategy. In the next section, we discuss the modelling approach implemented in this paper.

2 General Methodology

The daily adjusted close prices of BTC/USD and S&P 500 were obtained for suitably chosen time periods, not equal in length, which encapsulate the swings the financial instrument goes through. Log returns of the daily adjusted close prices were taken, and the HMM and HSMM models were then fitted on the log returns. Mathematically, an m-state HMM consists of two processes: (i) an unobserved (hidden) discrete-time m-state Markov chain, $(Z_n)_{n\in\mathbb{N}}$, taking values in a finite state-space, $\mathcal{S} = 1, 2, \ldots, m$, and (ii) a state-dependent process, $(Y_n)_{n\in\mathbb{N}}$, whose outcomes (observations) are assumed to be generated by one of m distributions corresponding to the current state of the underlying discrete-time Markov chain (DTMC). The distribution of Y_n is assumed to be conditionally independent of previous observations and states, given the current state Z_n. For a thorough review of HMMs, refer to [21]. One drawback of basic HMMs is due to the one time lag memory of the underlying first order DTMC which is inherently geometric. One possible way to circumvent this problem is to consider general state (possibly not geometric) dwell-time distributions, $d_i(r)$, leading to the HSMM framework. Thus, HSMMs generalise HMMs by explicitly modelling state persistence and state switches separately. This is achieved by considering a discrete-time semi-Markov chain (DTSMC), $(S_n)_{n\in\mathbb{N}}$ with state-space $\mathcal{S}$. For a thorough account of HSMMs, refer to [11] and references therein.

Since the log returns take values in the real space $\mathbb{R}$, we assume the HMM specification $a_{ij} = \mathrm{P}(Z_n = j|Z_{n-1} = i)$ and $Y_n|Z_n = i \sim N(\mu_i, \sigma_i)$ where a_{ij} are the transition probabilities, and the state-dependent distributions are assumed to be normal with mean μ_i and standard deviation σ_i, for each hidden state i. Similarly, the HSMM specification assumes $q_{ij} = \mathrm{P}(S_n = j|S_{n-1} = i, S_n \neq i)$, $q_{ii} = 0$, $d_i(r) \sim NBinom(v_i, p_i)$ and $Y_n|S_n = i \sim N(\mu_i, \sigma_i)$ - here model state switches are denoted by q_{ij}, $d_i(r)$ models state persistence via negative binomial dwell-time distributions (of which the geometric distribution is a special case) with parameters v_i and p_i, while we once again assume normal state-dependent distributions as for HMMs. Parameter estimation of HMMs can be carried out by either direct numerical maximisation (DNM) of the likelihood via Newton-type methods or by the Expectation-Maximisation (EM) algorithm. Both methods are described in [21]. HSMMs are usually fitted via the EM algorithm as described in [12]. For state inference, the Viterbi algorithm in [20] can be applied for both HMMs and HSMMs to obtain a sequence of most likely hidden states.

The daily log return series are then analysed as follows: (i) suitable HMMs and HSMMs on the complete time series are fitted by varying the number of assumed states; (ii) the optimal model based on the Akaike information criterion (AIC), Bayesian information criterion (BIC), and Hannan-Quinn information criterion (HQC) is chosen; (iii) the chosen time period is split into mutually exclusive training and testing periods; (iv) an expanding window method is implemented by first fitting the optimal model on the training set, and then iteratively adding one time point from the test set (until testing period is exhausted) to the training period and applying the Viterbi algorithm as a filtering procedure to nowcast the current most likely hidden state after parameter re-estimation; and (v) finally, investment strategies based on the model features arising from the Viterbi algorithm are applied to determine models' success at determining market phases. The data analysis presented next is carried out in RStudio by using the packages *HiddenMarkov* of [13] and *hsmm* of [6]. In the next section, we look at the modelling of the different market phases of both BTC/USD and S&P 500, and also draw comparisons.

3 Estimation and State Inference for HMM and HSMM models

This section is divided into two parts, where first we present the model fit on the complete series and state inference outputs using the Viterbi algorithm for BTC/USD, and this is followed by the same for S&P 500. A comparison of the properties of the two series will ensue. Not more than four states were considered as the algorithms experienced numerical issues for five states or more.

3.1 BTC/USD

In Table 1, we see the relevant goodness-of-fit criteria for 2-, 3- and 4-state HMMs and HSMMs. It can be seen that the homogeneous 4-state normal-HSMM provides the best fit throughout for all information criteria.

Table 1. Goodness-of-fit of stationary normal-HMMs and homogeneous normal-HSMMs for 2,3 and 4 states based on the entire series of daily log returns of BTC/USD.

	Likelihood	AIC	BIC	HQC
2-state HMM	2924.454	5860.980	5891.056	5872.302
3-state HMM	2872.646	5769.292	5829.587	5792.078
4-state HMM	2846.546	5737.093	5837.586	5775.070
2-state HSMM	2887.872	5779.743	5789.793	5783.541
3-state HSMM	2857.086	5720.171	5735.245	5725.868
4-state HSMM	**2837.926**	**5683.852**	**5703.950**	**5691.447**

For brevity, we present the parameter estimates for the best model (4-state HSMM) only. The parameter estimates (ordered by increasing volatility), obtained via the EM algorithm, are given by,

$$\hat{\mathbb{Q}} = \begin{pmatrix} 0 & 0.831 & 0.000 & 0.169 \\ 0.901 & 0 & 0.000 & 0.099 \\ 0.000 & 0.270 & 0 & 0.730 \\ 0.006 & 0.769 & 0.225 & 0 \end{pmatrix}, \ \hat{\boldsymbol{\delta}}(1) = (1, 0, 0, 0)),$$

$$\hat{\boldsymbol{v}} = (0.351, 0.204, 10.573, 0.143), \ \hat{\boldsymbol{p}} = (0.180, 0.126, 0.170, 0.031),$$

$$\hat{\boldsymbol{\mu}} = (0.099, 0.603, 0.376, -0.702), \ \hat{\boldsymbol{\sigma}} = (0.617, 2.135, 4.162, 7.376),$$

where $\hat{\mathbb{Q}}$ contains estimates of the state switches q_{ij}, $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{p}}$ contain estimates of the negative binomial parameters for the dwell-times of the different states, while $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\sigma}}$ contain estimates of the normal distribution parameters for the different states. Finally $\hat{\boldsymbol{\delta}}(1)$ is the initial distribution of the DTSMC, which suggests that the series starts from state 1. The normal state-dependent parameters allow us to attach the following interpretations. State 3 can be associated with a bull market due to the moderately high mean, common occurrence and strong persistence. State 4 can be associated with a bear market due to the large (and only) negative mean with relatively weak persistence. Both states exhibit very high volatility, though the bear state exhibits a stronger drift and volatility. Attaching interpretations to state 1 and 2 can be a bit more tricky, as both have weak persistence. State 1 appears to be a market correction/rally state due to its low drift and volatility, while state 2 appears to be an additional bull state with stronger drift, smaller volatility and weak persistence.

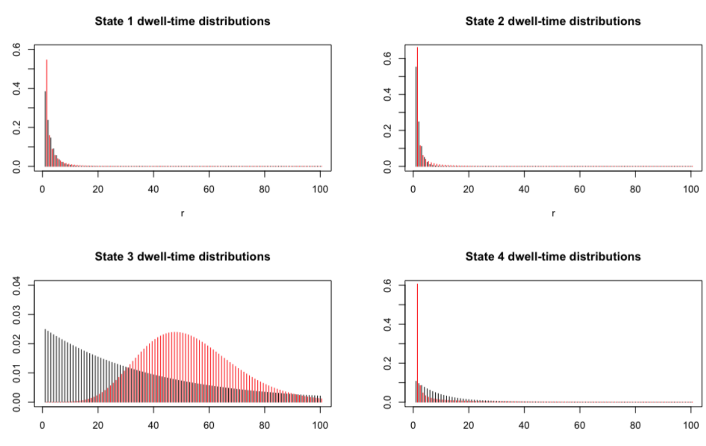
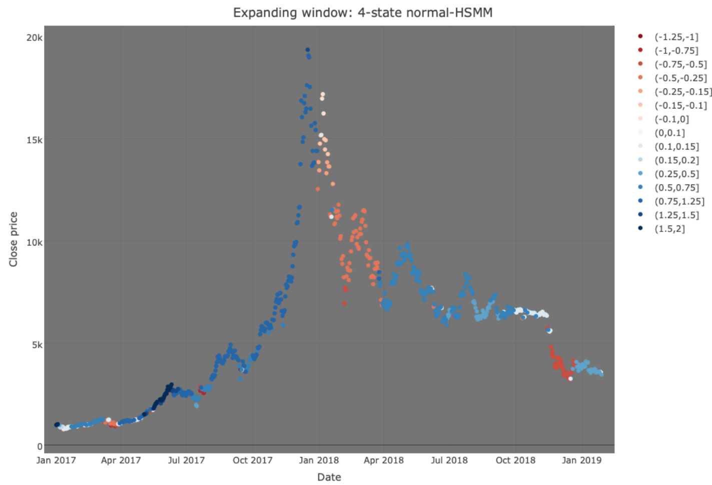
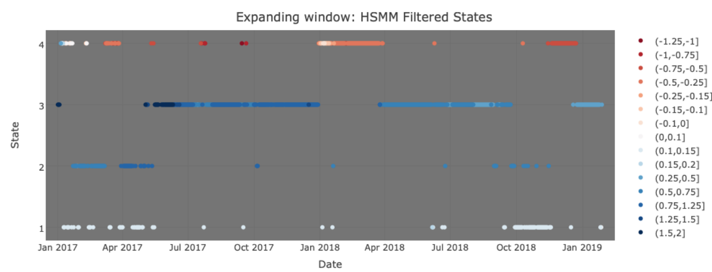


Fig. 1. BTC/USD: state dwell-time distributions for the homogeneous 4-state normal-HSMM (red) and for the stationary 4-state normal-HMM (black).

The dwell-time distributions for the 4-state HSMM are compared with the equivalent geometric dwell-time distribution of the 4-state HMM in Fig. 1. For states 1 and 2, the geometric and negative binomial distributions closely resemble each other and show a lack of persistence in these states. The HSMM dwell-time distribution for state 3, however, is clearly non-geometric as it shows an extremely



(a) BTC/USD close price.



(b) Hidden state sequence.

Fig. 2. Expanding window: 4-state normal-HSMM filtering via the Viterbi algorithm on BTC/USD. The colours vary by the mean, while the sizes vary by the volatility.

high persistence with a modal run length of 47 time steps until a state-switch. For state 4, the HMM geometric distribution shows a higher persistence than the negative binomial dwell-time distribution of the HSMM.

We next employ the expanding window procedure for BTC/USD, where we take the training period to be 01/01/2016 - 31/12/2016 and the testing period to be 01/01/2017 - 28/01/2019. Fig. 2 shows that the 4-state HSMM, based on a filtering method, can capture the hidden economic regimes pertaining to bull and bear market phases quite well, since upward (positive) trends are generally a shade of blue while sharp downward (negative) trends are generally orange to red in colour. Observe that the Viterbi algorithm assigns most of the test period in the third state - the bull state. Then, at the start of 2018 the value of one Bitcoin starts plummeting, which is identified early by the Viterbi algorithm as state 4 - the bear state. Moreover, the last days of the testing period switch between states 1 and 2. Ultimately, the 4-state normal-HSMM seems to perform fairly well in detecting the changing market conditions.

3.2 S&P 500

We shall now fit the same models to S&P 500. It is typically more common to see HMM-type models implemented on S&P 500, due to the fact that the features most commonly associated with financial time series can be found here. We can thus also use this stock market index as a benchmark for comparison. Also for this case, a homogeneous 4-state normal-HSMM with negative binomial dwell-time distributions was found to be the best, and the following parameter estimates were obtained,

$$\hat{\mathbb{Q}} = \begin{pmatrix} 0 & 0.998 & 0.002 & 0.000 \\ 0.973 & 0 & 0.023 & 0.004 \\ 0.000 & 0.767 & 0 & 0.233 \\ 0.000 & 0.000 & 1.000 & 0 \end{pmatrix}, \ \hat{\boldsymbol{\delta}}(1) = (0,0,1,0)),$$

$$\hat{\boldsymbol{v}} = (0.079, 0.112, 7.755, 0.455), \ \hat{\boldsymbol{p}} = (0.028, 0.044, 0.119, 0.015),$$

$$\hat{\boldsymbol{\mu}} = (0.107, 0.000, -0.068, -0.270), \ \hat{\boldsymbol{\sigma}} = (0.449, 0.970, 1.540, 3.385),$$

where the initial distribution suggests that the series starts from state 3. Note that the parameter estimates for the negative binomial parameters show clear deviations from geometric distributions. Hence, the following interpretations were considered: (i) state 1 can be associated with a bull market due to a large positive mean with low volatility and an eventual highly likely switch to state 2, (ii) state 4 can be associated with a bear market due to the large negative mean and high volatility with an eventual and almost certain switch to state 3, and (iii) states 2 and 3 can both be interpreted as market correction/rally phases, where the former is characterised by an almost zero mean with low volatility, while the latter has a negative mean with larger volatility. Note that the less volatile correction state, i.e. state 2, is likely to transition to the bull state or to the more volatile correction state, i.e. state 3, while the latter can transition to the bear state or to the other correction state.

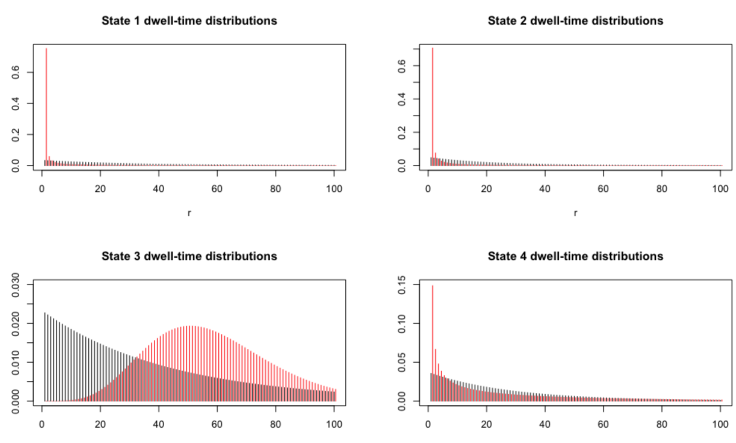
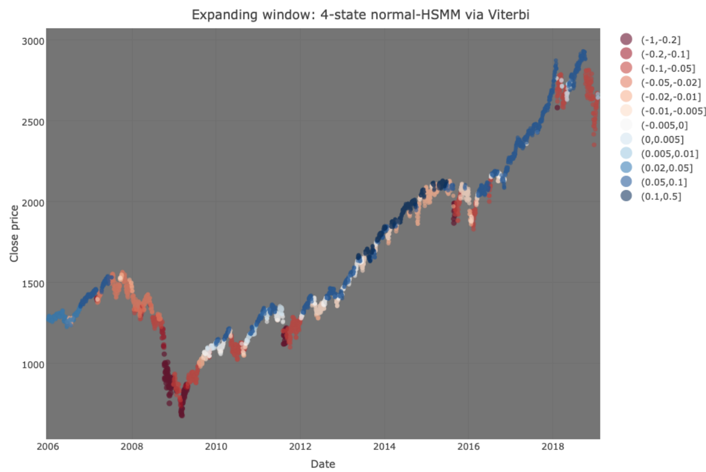
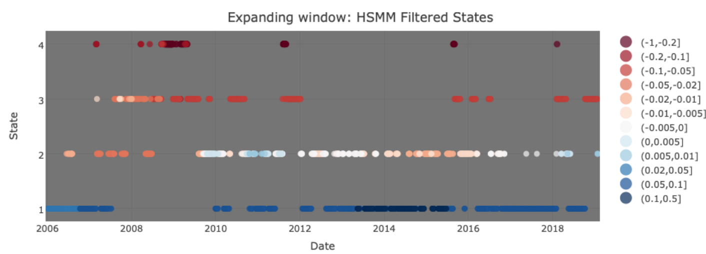


Fig. 3. S&P 500: state dwell-time distributions for the homogeneous 4-state normal-HSMM (red) and for the stationary 4-state normal-HMM (black).



(a) S&P 500 close price.



(b) Hidden state sequence.

Fig. 4. Expanding window: 4-state normal-HSMM filtering via the Viterbi algorithm on S&P 500. The colours vary by the mean, while the sizes vary by the volatility.

Fig. 3 shows that the geometric distributions corresponding to the 4-state HMM are all persistent. However, the negative binomial distributions assume different shapes, showing high persistence in states 3 and 4, while a lack of persistence in states 1 and 2.

The expanding window results using the Viterbi algorithm for the S&P 500 series for the 4-state HSMM can be seen in Fig. 4. In this case, our method can more accurately identify bull and bear markets since upward trends are generally blue while downward trends are generally red. Note how during the period 2010 - 2015, the S&P 500 Index is on the rise with few short periods of drops in price highlighting market corrections. These instances are captured by the 4-state HSMM as very pale (sometimes white) colours, implying a mean which is very close to zero with moderate volatility (state 2). In conclusion, it seems that the 4-state HSMM is better at determining market phases for the S&P 500 than it is for the more volatile BTC/USD.

3.3 Comparison of Results for BTC/USD and S&P 500

Given the previous outputs, we now compare and contrast the features for both BTC/USD and S&P 500. The 4-state HSMMs in both cases reveal two states indicative of bull and bear behaviour, with two "in-between" states. However, while Bitcoin has had strong and persistent bull phases with savage and weakly persistent bear phases, for most part S&P 500 tended to switch between bull and stable/bull correction phases with rare but persistent bear phases. Also, BTC/USD can exhibit higher volatility in comparison to S&P 500, as it is more novel and prone to external events. Indeed, both bull and bear markets for BTC/USD are volatile, while for S&P 500 only bear markets are volatile. For this reason, cryptocurrencies have often been remarked to be excessively volatile and subject to speculation and hence not, as yet, currency-like in their behaviour.

Secondly, BTC/USD states are less interpretable in terms of market phases than S&P 500. While our models seem to perform well in detecting the bear states, for BTC/USD it is harder to distinguish between bull phases and more stable ones. For S&P 500, on the other hand, steep upward trends are associated with the lowest volatility while steep downward trends tend to be the most volatile. Ultimately, the 4-state HSMM appears to be an effective modelling framework for both BTC/USD and S&P 500. Therefore, we shall implement two mock model-based investment strategies using filtered states on both the 4-state HSMM and 4-state HMM equivalent, with the aim of assessing the suitability of HSMMs, and whether they are an improvement of HMMs for determining market phases.

4 Using Investment Strategies to Assess Model Adequacy

In order to analyse the success of HMMs and HSMMs in determining bull and bear features, we devise two mock investment strategies and apply them with the expanding window procedure on both BTC/USD and S&P 500, using the buy-and-hold as a benchmark. For simplicity the following assumptions were made for each strategy: (i) the actions (buy or sell) are not subject to transaction costs; (ii) the testing period is entered with an initial capital of $20,000; (iii) the first action is to buy on the first day of the testing period; (iv) if a buy signal is given, financial assets are bought only if enough capital is at hand, in which case, the maximum possible amount of capital is invested (v) if a sell signal is given, financial assets are sold in their entirety if and only if they are owned. The investment strategies are defined hereafter.

Strategy 1 - Buy-and-Hold: This is a naive investment strategy which is used for comparative purposes only. It is defined by the following two actions: (i) buy on the first day of the testing phase and, (ii) sell on the last day of the testing phase.

Strategy 2 - Regime: This strategy is based on the way we arbitrarily associate the states obtained via the Viterbi algorithm, under the expanding window procedure (see earlier explanations for more detail). At each state change, apply the following actions: (i) if state i^*_{n-1} is associated with a bear market and state i_n^* is associated with a bull market then buy as many financial positions as possible at time n; (ii) if state i_{n-1}^* is associated with a bull market state and state i_n^* is associated with a bear market then sell all financial positions at the close price of time n; otherwise (iii) do nothing. For Bitcoin we shall consider state 3 as a bull state and state 4 as a bear state, while other states will not be labelled since they are ambiguous and infrequent. For S&P 500, on the other hand, we shall consider states 1 and 2 as bull states and states 3 and 4 as bear states, based on the probabilities in $\hat{\mathbb{Q}}$ connecting them.

Strategy 3 - Drift: This strategy is based on the drift of the states obtained via the Viterbi algorithm, under the expanding window. Given arbitrary $\epsilon \geq 0$, at each state change, apply the following actions: (i) if $\hat{\mu}_{n-1} < 0$ and $\hat{\mu}_n > \epsilon$ then buy as many financial positions as possible at time n; (ii) if $\hat{\mu}_{n-1} > 0$ and $\hat{\mu}_n < -\epsilon$ then sell all financial positions at the close price of time n; otherwise (iii) do nothing.

For each strategy we shall record: (i) the number of actions (NOA), (ii) the last sell date (LSD), (iii) the final cumulative amount (FCA), and (iv) the return on investment (ROI) which is the profit/loss made as a percentage of initial capital. With regards to strategy 3, only that ϵ which returned the highest ROI out of the possible grid values is shown. The following grid values for ϵ, based on empirical evidence, were taken: $\epsilon = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ and $\epsilon = 0, 0.005, 0.01, 0.02, 0.03, 0.1$, for BTC/USD and S&P 500, respectively.

Table 2 compares the investment strategies considered for the BTC/USD exchange rate for testing period 01/01/2017 - 28/01/2019. As can be observed, the buy-and-hold strategy is the most inferior of all strategies considered over the given period. Strategy 2 works marginally better for the 4-state HMM model, but for 4-state HSMM model and taking $\epsilon = 0$, Strategy 3 by far outperforms Strategy 2. Furthermore, the 4-state HSMM is superior to the 4-state HMM under all model-based investment strategies, and also yields less actions which, as mentioned, would incur more transaction costs.

Table 2. Investment strategies during the testing period 01/01/2017 - 28/01/2019 for BTC/USD.

Strategy	ϵ	NOA	LSD	FCA ($)	ROI (%)
1 (Buy-and-Hold)	n/a	2	28/01/2019	69,384.79	245.92
2 (Regime/HMM)	n/a	60	10/10/2018	71,967.73	259.84
3 (Drift/HMM)	0	82	19/11/2018	71,299.09	256.50
2 (Regime/HSMM)	n/a	22	20/12/2018	113,540.75	467.70
3 (Drift/HSMM)	**0**	**36**	**20/12/2018**	**147,203.48**	**636.02**

On the other hand, the results for S&P 500 are summarised in Table 3 for testing period 01/01/2006 - 28/01/2019. As can be observed, the buy-and-hold strategy works fairly well for the long testing period, outperforming Strategy 3 for the 4-state HMM. Strategy 2 for the 4-state HMM, however, works better. For the 4-state HSMM, both Strategy 2 and Strategy 3 work better, with Strategy 3 taking $\epsilon = 0.02$ being the most profitable under the assumed circumstances for the S&P 500 Index.

Table 3. Investment strategies during the testing period 01/01/2006 - 28/01/2019 for S&P 500.

Strategy	ϵ	NOA	LSD	FCA ($)	ROI (%)
1 (Buy-and-Hold)	n/a	2	28/01/2019	40,625.75	103.13
2 (Regime/HMM)	n/a	38	11/10/2018	45,275.13	126.38
3 (Drift/HMM)	0.005	46	11/10/2018	32,789.96	63.95
2 (Regime/HSMM)	n/a	46	10/10/2018	41,505.15	107.53
3 (Drift/HSMM)	**0.02**	**16**	**22/03/2018**	**47,898.68**	**139.49**

Upon comparing, Tables 2 and 3 yield some noteworthy revelations. Firstly, the naive buy-and-hold strategy for the considered testing periods works fairly well for S&P 500 while it is the least profitable for BTC/USD. Secondly, the HSMM framework provides a clear improvement over the standard HMM methodology in both cases. Thirdly, it must be noted that Strategy 3 surpassed Strategy 2 for the better performing 4-state HSMM model, indicating that allowing the interpretations of the states to adjust at each step according to the mean of the state can have its advantages. Also, despite high return in the best of strategies, we note that there were periods of huge gains and periods of considerable losses for BTC/USD. However, for S&P500, market phases were more appropriately identified. Finally, had we considered transaction costs, it is very likely that the HSMM framework using Strategy 3 could still result in being the most profitable due to its superior performance with relatively smaller number of transaction costs.

5 Conclusion

In this paper we propose that a more desirable approach for modelling both BTC/ USD and S&P 500, and capturing effectively the dynamics of bull and bear market regimes, is a 4-state normal-HSMM with negative binomial dwell-time distributions. When implementing investment strategies, it has proven to be considerably superior to a buy-and-hold approach for our data, while this was not always the case for HMMs, which constrained dwell-times to be geometric. Indeed, by allowing dwell-time distributions on the states with larger modes, the number of buy/sell actions is greatly reduced in comparison. Although in the case of BTC/USD, the states of the 4-state HSMM model are not as interpretable as in the case of S&P 500, it still provides a good basis for further improvement and future research. On a concluding note, one must pinpoint that S&P 500 is a much older financial instrument with consistent long-term behaviour. On the other hand, inference on BTC/USD behaviour is based on a much shorter history and, as the asset matures, consistent long-term features may also develop.

Acknowledgements. The research work carried out, was partially funded by the European Social Fund/ENDEAVOUR Scholarships Scheme.

References

1. Ardia,D., Bluteau, K., Reude, M.: Regime changes in Bitcoin GARCH Volatility dynamics. Financ. Res. Lett., In Press (2018).
2. Baur, D.G., Dimpfl, T. and Kuck, K.: Bitcoin, gold and the dollar - a replication and extension. Financ. Res. Lett., **25**, 103-110 (2018).
3. Bonello, A., Suda, D.: Volatility Regime Analysis of Bitcoin Price Dynamics Using Markov Switching GARCH Models. Proceedings of the European Simulation and Modelling Conference, 213-218 (2018).
4. Bouri, E., Azzi, G. and Dyhrberg, A.H.: On the return-volatility relationship in the Bitcoin market around the price crash of 2013. Econstor Economics Discussion Papers, 2016-41 (2018).
5. Bulla, J., Bulla, I.: Stylized facts of financial time series and hidden semi-Markov models. Computat. Statist. & Data Analysis, **51**, 2192 - 2209 (2007).
6. Bulla, J., Bulla, I.: hsmm: Hidden Semi-Markov Models. R package version 0.4, http://CRAN.R-project.org/package=hsmm. Last accessed 12 Mar 2019 (2013).
7. Chappell, D.-R.: Regime heteroskedasticity in Bitcoin: A comparison of Markov switching models. MPRA paper 90682, University Library of Munich, Germany (2018).
8. Chan, S., Chu, J., Nadarajah, S. amd Osterrieder, J.: A statistical analysis of cryptocurrencies. Journal of Risk and Financial Management, **10(12)**, 1-23 (2017).
9. Chu, J., Chan, S., Nadarajah, S. and Osterrieder, J.: GARCH modelling of cyprtocurrencies. Journal of Risk and Financial Management, **10(17)**, 1-15 (2017).
10. Granger, C.-W, Ding,Z.: Some properties of absolute return: An alternative measure of risk. Annales d'Economic et de Statistique, **40**, 67 – 91 (1995a).
11. Granger, C.-W, Ding,Z.: Stylized facts on the temporal and distributional properties of daily data from speculative markets. Department of Economics, University of California, San Diego, unpublished paper (1995b).
12. Guédon, Y.: Estimating hidden semi-Markov chains from discrete sequences. J. Comp. Graphical Statist, **12**, 604 – 639 (2003).
13. Harte, D.: R package 'HiddenMarkov', version 1.8–1, http://homepages.maxnet.co.nz/davidharte/SSLIB/. Last accessed 12 Mar 2019 (2014).
14. Katsiampa, P.: Volatility estimation for Bitcoin, a comparison of GARCH models. Economic Letters **158**, 3-6 (2017).
15. Kodama, O., Pichl, L. and Kaizoji, T.: Regime change and trend prediction for Bitcoin time series data.Proceedings of the CBU International Conference on Innovations in Science and Education **5**, 384-388 (2017).
16. Liu, Z., Wang, S.: Decoding Chinese stock market returns: Three-state hidden semi-Markov model. Pacific-Basic Finance Journal, **44**, 127–149 (2017).
17. Nguyen, N.: Hidden Markov Model for Stock Trading. Int. J. Financial Stud., **6**, 1–17 (2018).
18. Rydén, T., Teräsvirta, T., Åsbrink, S.: Stylized facts of daily return series and the hidden Markov model. J. Appl. Econometr., **13**, 217–244 (1998).
19. Stavroyiannis, S.: Value-at-risk and related measures for the Bitcoin. Journal of Risk Finance **19(2)**, 127-136 (2018).
20. Viterbi, A.J.: Error bounds for convolutional codes and an asymptotically optimal decoding algorithm. IEEE Trans. Inform. Th., **13**, 260–269 (1967).
21. Zucchini, W., MacDonald, I.L., Langrock, R.: Hidden Markov Models for Time Series: An Introduction Using R. 2nd edn. Chapman & Hall/CRC, FL, Boca Raton (2016).