Public Infrastructure Strategically Supplied by Governments
and Trade in a Ricardian Economy*

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Abstract

In a simple two-country Ricardian economy with public infrastructures, we consider a simultaneous and non-cooperate game between governments with respect to public infrastructure supply. Then it is shown that a country with larger(smaller) factor endowment exports a good whose production is more(less) dependent on public infrastructures and both countries will gain from trade as long as factor endowment differs between countries. However, the following special features appear. (i)Any incompletely specializing country produces two goods at an inner point of the production possibility set. (ii) If factor endowment is the same between countries, the trading equilibrium is attained by the pattern of specialization such that each country specializes in one good different with each other and both countries become better off. Which country specializes in which good is indeterminate. The result shows a typical case of symmetric-breaking.

JEL classification: F11; H41

Key words: Public infrastructure; Nash equilibrium; Comparative advantage, Ricardian economy

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1. Introduction

According to the penetration of recent surging globalisation into the world economy, many countries recognize the importance of public infrastructure as one of the key elements to determine a country's comparative advantage in trade. The rapid and robust growth of the Chinese economy is gradually overwhelming the Asian economic region and extending its influence on the world economy. Many trading countries are aware of the impact of the public infrastructure supplied by China on the existing trade patterns in world trade. They are forced to consider the strategic supply of public infrastructure by considering the strategic behavior of China.[1]

The One Belt and One Road Initiative advocated by the Chinese government is regarded as a powerful and effective strategy for China to take a solid economic and political leadership among Asian countries and over some regions of Africa and Eastern Europe. There is no doubt that a drastic change in the patterns of trade will be brought in the world economy by this Chinese strategic behaviour.[2] In order to deal with this foreseen change, the advanced countries, like Japan, U.S., and Australia, became nervous and began to carry out various strategic economic policies, including the public infrastructure supply, to maintain the existing benefits bearing from world trade.[3]

Taking this feature of the recent world trade into account, we propose a simple general equilibrium model to investigate substantial properties concerning a rather complicated real trading economy and show propositions of trade patterns and gains from trade. The existing studies along the line of our study may be those of Connolly (1970), Shimomura (2007), Yanase and Tawada (2019), and Yen, Wu and Miranda (2019), for example, in the sense that the game-theoretic treatment is accommodated into the analysis. Connolly (1970), however, did not examine either the topic of patterns of trade or gains from trade, while Shimomura (2007) and Yanase and Tawada (2019) focused on public infrastructures of the unpaid factor type defined by Meade (1952) where no external economies are generated substantially. Yen, Wu, and Miranda (2019) treated a similar topic to ours in a continuum of goods and one-factor model, but they relied on a numerical simulation in an essential part of the analysis so that their analysis lacks generality.

In the present study, we bring public infrastructures of the creation of atmosphere type defined by Meade (1952) into a simple Ricardian trading economy where only one primary factor exists. Thus the public infrastructures are supposed to be law and education systems, research and development activities, communication infrastructures, managerial skill formation, and so on. Then we examine the patterns of trade and gains from trade under the supposition that each government takes the strategic behavior for the public infrastructure supply to compete with each other in the Cournot fashion.

Our analysis shows that multiple Nash trading equilibria may emerge but, according to the payoff dominance criterion proposed by Harsanyi and Selten (1988), only one of them decisively remains as a trading equilibrium except for two specific cases. Then we obtain the result that a country larger in the factor endowment has a comparative advantage in the good more heavily dependent on public infrastructure and exports that good because the country can supply a larger amount of the public infrastructure than the other country. Both countries will become better off by trade. So, the results of the trade patterns and trade gains seem reasonable in the light of Ricardian trade theory, by which we can confirm the robustness of a law of international comparative advantage of Ricardian trade theory.

But we should notice that this result is not so easy to guess *a priori,* because, in the strategic game setting, once a country with a smaller factor endowment decides to supply a larger amount of public infrastructures in order to take an advantage in a good more heavily dependent on public infrastructures, the best strategic response of the other country is to take an advantage in the other good by a smaller supply of public infrastructures. Then, the trade pattern is reversed and this could be also a Nash. Our analysis proves that the latter case never occurs.

In our analysis, there are two interesting facts newly appear. One is that any incompletely specializing country produces two goods in an inner point of the production possibility set, implying the production is inefficient. We will argue this point in connection to Suga and Tawada (2007), where, if the government supplies the public infrastructure efficiently, an incompletely specializing country will lose by trade.

The other is the result derived in the case where the two countries are perfectly identical with respect to all aspects, including the scale of factor endowment. In this case, two asymmetric Nash equilibria appear and the pure strategic, non-cooperative and normal game between governments leaves these two trading equilibria equally possible, implying that the trading equilibrium is indeterminate. And every country can enjoy a positive gain from trade no matter of which Nash equilibrium brings a trading equilibrium. This is a contrasting result to the traditional one where, if two country are the same with respect to every aspect, both countries stay at the autarkic state even under free trade. Our result gives one example of the symmetry-breaking equilibrium cases pointed out by Matsuyama(2002) and examined by Chatterjee(2017) in the framework of international trade.

Here we should refer to empirical studies, for our analysis is purely theoretical. The existing empirical studies investigating the pattern of trade based on public infrastructure are very few. An exceptional study is Yeaple and Golub (2007), from whose estimation results we cannot confirm a clear relationship between the pattern of trade and the role of public infrastructure.[4] The main obstacle for the advanced empirical studies is the difficulty of getting detailed public infrastructural data across countries and a lack of clear and unified estimation results on the effect of public infrastructure on each production sector. Another important and serious problem for the execution of the empirical studies relating public infrastructure is that the public infrastructures are usually supplied by the government or firms guided by the government. Then, the public infrastructure supply can be made use of as a political implement, which implies that we need to exclude the impact of the political behaviour of the government in order to see the purely economic effect of the public infrastructure supply to international trade. As a typical example, we can see this point in the Chinese One Belt and One Road Strategy. Chinese foreign direct investment for the construction of public infrastructures has not only economic purpose but also political one. This mixing up of these two purposes makes the various investments inefficient or seemingly oversupplied with respect to the risk management and many serious troubles concerning the financial debts and property rights emerge between China and the trading countries receiving Chinese investment.[5] Therefore, for the more elaborated empirical researches, we need to exclude the political factors affecting the public infrastructure supply.

The present paper is organized as follows: The next section introduces a simple model based on the Ricardian economy. The autarkic equilibrium of a country is described in Section 3. Then we proceed with the analysis to seek Nash equilibria between two countries in an open economy from Sections 4 to 6. It is shown in Section 7 that there may be multiple Nash equilibria in the trading economy, but only one of these Nash equilibria can be selected as a trading equilibrium. Section 9 is devoted to two specific cases where careful treatment is required. The last section is for our conclusion.

2. Model

Consider a world economy where two countries called Home and Foreign exist. Each of those countries produces consumption goods 1 and 2 by using labour under constant returns to scale technologies. We suppose that the government of each country supplies public infrastructures to serve for the production of consumption goods 1 and 2 in its own country. Let us call a bundle of public infrastructures simply as a public intermediate good hereafter.

For a moment, we focus on the economy of the Home country. The production function of consumption good i is supposed to be

$$Q_i = R^{\varepsilon_i} L_i \ , \ i = 1,2, \tag{1}$$

where ε_i, $i = 1$ and 2, are parameters satisfying $0 < \varepsilon_2 < \varepsilon_1$, Q_i is the output of good i, L_i is labour used in the production of good i, and R is the amount of a public intermediate good available in both consumption good industries.

We should notice in (1) that the public intermediate good serves for production without any congestion between two industries and works like external economies in production. So an increase in the supply of a public intermediate good gives a rise in labour productivity in both industries. Notice further that, by the assumption that $0 < \varepsilon_2 < \varepsilon_1$, the labour productivity is more sensitive to the level of the public intermediate good supply in good 1 than in good 2.

The government of the Home country can supply the public intermediate good by the production function,

$$R = L_R, \tag{2}$$

where L_R is the labour input for producing the public intermediate good.

The country is endowed with a constant amount of labor, denoted as L. Hence under the full employment of labour, the following equation must hold.

$$L_1 + L_2 + L_R = L. \tag{3}$$

All consumers have identical preferences exhibited by the linearly homogenous Cobb-Douglas utility function so that the aggregate indirect utility is represented by

$$U = \frac{I}{p_1^{\alpha} p_2^{1-\alpha}}, \tag{4}$$

where α is a parameter satisfying $0 < \alpha < 1$, U is the social utility, I is the national income, and p_i is the price of good i, for $i = 1$ and 2.

We assume that all private markets are under perfect competition. Then the profit maximization of good i yields the production equilibrium conditions,

$$p_i R^{\varepsilon_i} = w \quad , \quad i = 1,2, \tag{5}$$

for a given level of R, where w is the wage rate.

Remind that (5) is valid for any positive production, and thus profits never accrue, implying that the source of national income is labour income only. So the government imposes a lump-sum tax on labor income to finance the supply of a public intermediate good. Then the national income should be

$$I = w(L - R). \tag{6}$$

Finally, we see how to determine the level of the public intermediate good supply. We suppose that the government aims to maximize the social utility with respect to the public intermediate good supply. The national welfare is measured by the level of this social utility. Then the supply amount of the public intermediate good is determined by the solution of R for the following national welfare maximization problem:

$$\underset{R}{Max}\frac{w(L-R)}{p_1^{\alpha}p_2^{1-\alpha}}.$$

Concerning the economy of Foreign country is the same to that of Home, except the scale of labour endowment. Hereafter, we use the variables of Foreign country as those of Home with an asterisk, and proceed with the case where $\alpha = 0.5$.

3. Autarkic Equilibrium

Subsequently, we focus on the Home country and its autarkic equilibrium under the assumption that $\alpha = 0.5$. Once R is given, the utility at the autarkic equilibrium is determined as

$$U = R^{\varepsilon_A}(L-R), \tag{7}$$

from (4), (5) and (6), where $\varepsilon_A \equiv (\varepsilon_1 + \varepsilon_2)/2$.

Then the equilibrium level of R is determined by the national welfare maximization so that, at the autarkic equilibrium, we have

$$R = R_A \equiv \frac{\varepsilon_A}{1+\varepsilon_A}L,$$

$$p \equiv \frac{p_1}{p_2} = p^A \equiv R_A^{\varepsilon_1-\varepsilon_2},$$

$$U = U_A \equiv R_A^{\varepsilon_A}(L-R_A).$$

Figure 1 illustrates the Home's autarkic equilibrium. We can deal with Foreign's autarkic equilibrium similarly.

(Figure 1)

4. Preliminary Analysis for Trading Equilibrium

We are ready for the analysis of international trade. Suppose two consumption goods to be tradeable, the public intermediate good to be non-tradeable, and labour to be internationally immobile. Free trade is supposed to prevail between two countries. In each country, the government supplies a certain amount of the public intermediate good to maximize the country's welfare under a given level of the public intermediate good of the other country. Hence the equilibrium of the public intermediate good supply in each country is a Nash equilibrium of a non-cooperative normal game between countries. Once the level of a public intermediate good supply is determined in each country, any competitive firms face the production possibility frontier (PPF) of their own country under a given level of the intermediate good supply. The PPF under a given level of the intermediate good supply becomes a straight line, denoted as *PPF(R)* for Home and *PPF(R*)* for Foreign and called as a restricted PPF. Then, according to the Ricardian trade theory, any trading equilibrium must be such that at least one country specializes completely in a commodity in which the country has a comparative advantage.

So, any trading equilibrium must belong to one of the following six specialization patterns in production.

[I] Home incompletely specializes, and Foreign specializes in good 2.
[II] Home specializes in good 1, and Foreign specializes in good 2.
[III] Home specializes in good 1, and Foreign incompletely specializes.
[IV] Home specializes in good 2, and Foreign incompletely specializes.
[V] Home specializes in good 2, and Foreign specializes in good 1.
[VI] Home incompletely specializes, and Foreign specializes in good 1.

In which good a country has a comparative advantage is determined by difference in the slope of the Ricardian straight line of *PPF(R)* between countries, or equivalently difference in labour productivity ratio of two goods between countries under a given level of the public intermediate good of each country. In order to characterize a trading equilibrium by specialization patterns, we define various sets of (R, R^*) as follows;

$$E \equiv \{((R, R^*)|0 < R < L \;\; and \;\; 0 < R^* < L^*\}$$
$$E^+ \equiv \{(R, R^*) \in E | R \geq R^*\}$$
$$E^- \equiv \{(R, R^*) \in E | R < R^*\}$$
$$E^I \equiv \{(R, R^*) \in E^+ | R^{*\varepsilon_2}(L^* - R^*) < R^{\varepsilon_2}(L - R)\}$$
$$E^{II} \equiv \{(R, R^*) \in E^+ | R^{\varepsilon_2}(L - R) \leq R^{*\varepsilon_2}(L^* - R^*) \;\; and \;\; R^{*\varepsilon_1}(L^* - R^*) \leq R^{\varepsilon_1}(L - R)\}$$
$$E^{III} \equiv \{(R, R^*) \in E^+ | R^{\varepsilon_1}(L - R) < R^{*\varepsilon_1}(L^* - R^*)\}$$
$$E^{IV} \equiv \{(R, R^*) \in E^- | R^{\varepsilon_2}(L - R) < R^{*\varepsilon_2}(L^* - R^*)\}$$
$$E^V \equiv \{(R, R^*) \in E^- | R^{*\varepsilon_2}(L^* - R^*) \leq R^{\varepsilon_2}(L - R) \;\; and \;\; R^{\varepsilon_1}(L - R) \leq R^{*\varepsilon_1}(L^* - R^*)\}$$
$$E^{VI} \equiv \{(R, R^*) \in E^- | R^{*\varepsilon_1}(L^* - R^*) < R^{\varepsilon_1}(L - R)\}$$

Concerning the characterization of a trading equilibrium by specialization patterns, we can establish

Theorem 1

The trading equilibrium under a given pair (R, R^*) *follows specialization pattern [i] if and only if* $(R, R^*) \in E^i$, *for* $i = I, \cdots, VI$.

Proof. It is sufficient to prove the necessary part of each case since all patterns are covered by these cases from [I] to [VI]. We prove the case of [I] only because the rest of the cases can be proved in a similar manner. Suppose that the trading equilibrium follows a specialization pattern [I]. Then, Home incompletely specializes. So, in Home, the followings are true at the equilibrium: The budget constraint is

$$p_1 D_1 + p_2 D_2 = p_1 Q_1 + p_2 Q_2,$$

where D_i is demand in good i. The production equilibrium condition is

$$p_1 R^{\varepsilon_1} = w = p_2 R^{\varepsilon_2},$$

and the national income is

$$I = w(L - L_R).$$

Then the demand and supply of good 2 are,

$$D_2 = R^{\varepsilon_2}(L - R)/2 \quad \textit{and} \quad Q_2 = R^{\varepsilon_2} L_2,$$

respectively.

In Foreign where production is specialized in good 2, the followings are true at the equilibrium: The budget constraint is,

$$p_1 D_1^* + p_2 D_2^* = p_2 Q_2^*.$$

The production equilibrium condition is,

$$p_2 R^{*\varepsilon_2} = w^* \geq p_1 R^{*\varepsilon_1},$$

and the national income is

$$I^* = w^*(L^* - R^*).$$

So the demand and supply of good 2 are, respectively,

$$D_2^* = R^{*\varepsilon_2}(L^* - R^*)/2 \quad \text{and} \quad Q_2^* = R^{*\varepsilon_2}(L^* - R^*).$$

By this specialization pattern, Home must import good 2 and Foreign must export it. Let Home's import and Foreign's export of good 2 be M_2 and E_2^*, respectively. Then we have

$$M_2 = D_2 - Q_2 = \frac{1}{2} R^{\varepsilon_2}(L - R) - R^{\varepsilon_2} L_2,$$

$$E_2^* = Q_2^* - D_2^* = \frac{1}{2} R^{*\varepsilon_2}(L^* - R^*),$$

from which we obtain

$$R^{\varepsilon_2}(L - R) > R^{*\varepsilon_2}(L^* - R^*),$$

since $M_2 = E_2^*$ and $L_2 > 0$ at the trading equilibrium.

The fact that $R > R^*$ is evident since the production equilibrium conditions of two countries assure

$$R^{\varepsilon_1 - \varepsilon_2} = \frac{p_2}{p_1} \geq R^{*\varepsilon_1 - \varepsilon_2}$$

where $\varepsilon_1 - \varepsilon_2 > 0$. Q.E.D.

We should notice in the proof that how labour endowment differs between countries does not matter to Theorem 1.

For the later analysis, we calculate the social utilities of two countries and the equilibrium price ratio under a given pair of R and R^* in each E^i, for $i = I, \ldots, IV$. We, however, omit the calculation since it is tedious. We simply give the calculation results in Table 1.

(Table 1)

5. Restricted Nash Equilibria

Our next step is to seek Nash equilibrium pairs (R, R^*), which yield a trading equilibrium when the strategy space is confined in region $E^i, i = I, \dots, VI$. Let (R^i, R^{*i}) stand for this kind of an equilibrium strategy pair, for $i = I, \dots, VI$. Then (R^i, R^{*i}) must satisfy

$$U(R^i, R^{*i}) \geq U(R, R^{*i}) \quad for\ all\ (R, R^{*i}) \in E^i$$

and

$$U^*(R^i, R^{*i}) \geq U(R^i, R^*) \quad for\ all\ (R^i, R^*) \in E^i,$$

where $U(R, R^*)$ and $U^*(R, R^*)$ are, respectively, the social utilities of Home and Foreign attained at the trading equilibrium under given (R, R^*).

Now we consider a game where Home and Foreign governments determine the levels of the public intermediate good supply to maximize the social utilities of their own countries under the supposition that the amount of the public intermediate good supply of the other country is given. In this game, we assume that the strategy set (R, R^*) faced by two governments is E^i, for $i = I, \dots, VI$. Therefore, there are six kinds of games.

Consider, for example, the game confined in E^I. According to Table 1, for a given R^*, Home government maximizes $U = R^{\varepsilon_A}(L - R)$ with respect to R under given R^*, where $(R, R^*) \in E^I$. The optimal solution is then $R_A \equiv \varepsilon_A L/(1 + \varepsilon_A)$, which is independent of the value of R^*. On the other hand, for a given R, the Foreign government maximizes $U^* = R^{*\varepsilon_2}(L^* - R^*)/R^{-(\varepsilon_1 - \varepsilon_2)/2}$ with respect to R^*. Then the optimal R^* is given as $R_2^* \equiv \varepsilon_2 L^*/(1 + \varepsilon_2)$, which is independent of R. Therefore, the Nash equilibrium of the two-country game in E^I is that Home's optimal strategy is R_A while that of Foreign is R_2^*. Thus, we have $(R^I, R^{*I}) = (R_A, R_2^*)$.

Similarly, for the game confined in E^{II}, the Home government maximizes

$$U = U^* = [R^{\varepsilon_1}(L - R)]^{1/2}[R^{*\varepsilon_2}(L^* - R^*)]^{1/2},$$

with respect to R, so that the optimal R of Home is R_1, which is independent of R^*. Likewise, the Foreign government maximizes the above utility with respect to R^* and then obtains R_2^* as the optimal solution, which is independent of R. Thus, the Nash equilibrium of the game restricted the strategy set to E^{II}. is $(R^{II}, R^{*II}) = (R_1, R_2^*)$.

By the use of a similar method, we can deal with all other cases, and we obtain the following results for the rest of the cases: $(R^{III}, R^{*III}) = (R_1, R_A^*)$, $(R^{IV}, R^{*IV}) = (R_2, R_A^*)$, $(R^V, R^{*V}) = (R_2, R_1^*)$, $(R^{VI}, R^{*VI}) = (R_A, R_2^*)$. We call these Nash equilibrium strategy pairs as restricted Nash equilibrium strategy ones since the strategy set of the governments is confined in E^i, which is a subset of the whole strategy set E.

By the above discussion, we establish

Theorem 2

The restricted Nash equilibrium strategy pair for the government game restricted in $E^i, for\ i = I, \dots, VI$, *becomes* (R_A, R_2^*) *for* E^I, (R_1, R_2^*) *for* E^{II}, (R_1, R_A^*) *for* E^{III}, (R_2, R_A^*) *for* E^{IV}, (R_2, R_1^*) *for* E^V, *and* (R_A, R_1^*) *for* E^{VI}, *where* $R_i \equiv \varepsilon_i L/(1+\varepsilon_i)$ *and* $R_i^* \equiv \varepsilon_i L^*/(1+\varepsilon_i)$, *for i=1, 2, A.*

Therefore, whatever is the region of the game, the restricted Nash equilibrium strategy is $R_1(R_1^*)$, $R_2(R_2^*)$, and $R_A(R_A^*)$ respectively, if Home (Foreign) specializes in good 1, good 2, and specializes incompletely. Moreover, the amount of $R_i(R_i^*)$ corresponds to the level of the public intermediate good maximizing the output of good i when Home (Foreign) specializes in good i. The amount of $R_A(R_A^*)$ corresponds to that of the Home (Foreign) autarkic equilibrium level of the public intermediate good. The results of Theorem 2 are shown in Table 1.

6. Restricted Nash Equilibrium and Country Size

In this section, we examine which restricted Nash equilibrium exists in what size of the country. We assume that $L \geq L^*$, implying that Home is not smaller than Foreign. Let L^* be fixed and consider a change in L to see how different is the size of labour endowment between two countries. Before proceeding further, we prove the following lemma.

Lemma 1

Let $x(L) \equiv R(L)^\delta \big(L - R(L)\big)$, *where* $R(L) \equiv \varepsilon L/(1+\varepsilon)$ *and* δ *and* ε *are positive parameters. Then it holds that*

$$\frac{dx}{dL} > 0.$$

Proof.

$$\frac{dx}{dL} = \delta R^{\delta-1}\left(\frac{\varepsilon}{1+\varepsilon}\right)(L-R) + R^{\delta} - R^{\delta}\left(\frac{\varepsilon}{1+\varepsilon}\right)$$

$$= \delta R^{\delta-1}\left(\frac{\varepsilon}{1+\varepsilon}L\right) - \delta R^{\delta}\left(\frac{\varepsilon}{1+\varepsilon}\right) + R^{\delta} - R^{\delta}\left(\frac{\varepsilon}{1+\varepsilon}\right)$$

$$= R^{\delta}\left(\delta + 1 - \frac{\varepsilon}{1+\varepsilon}(\delta+1)\right) > 0.$$

Q.E.D.

First, we consider the case where $R^* \leq R$, so that any restricted Nash equilibrium falls into one of E^I, E^{II}, and E^{III}. For our analysis, we employ the following definitions:

Definition

L^2_{2-A} *is the level of* L *satisfying* $R_A^{\varepsilon_2}(L - R_A) = R_2^{*\varepsilon_2}(L^* - R_2^*)$.
L^2_{2-1} *is the level of* L *satisfying* $R_1^{\varepsilon_2}(L - R_1) = R_2^{*\varepsilon_2}(L^* - R_2^*)$.
L^1_{2-1} *is the level of* L *satisfying* $R_1^{\varepsilon_1}(L - R_1) = R_2^{*\varepsilon_1}(L^* - R_2^*)$.
L^1_{A-1} *is the level of* L *satisfying* $R_1^{\varepsilon_1}(L - R_1) = R_A^{*\varepsilon_1}(L^* - R_A^*)$.

Then, by Theorem 2 and the definitions of E^i, for $i = I, II, and\ III$, it is clear that

$$(R_A, R_2^*) \in E^I \quad if\ and\ only\ if \quad L^2_{2-A} < L.$$

$$(R_1, R_2^*) \in E^{II} \quad if\ and\ only\ if \quad L^1_{2-1} \leq L \leq L^2_{2-1}.$$

$$(R_1, R_A^*) \in E^{III} \quad if\ and\ only\ if \quad L < L^1_{A-1}.$$

Define $x_i(R) \equiv R^{\varepsilon_i}(L - R)$ in $[0, L]$ and $x_i^*(R^*) \equiv R^{*\varepsilon_i}(L^* - R^*)$ in $[0, L^*]$. Then we can depict the graphs of $x_i(R)$ and $x_i^*(R^*)$ as in Figure 2. $x_i(R)$ and $x_i^*(R^*)$ are increasing, respectively, in $(0, R_i)$ and $(0, R_i^*)$ and decreasing, respectively, in (R_i, L) and (R_i^*, L^*). So $x_i(R)$ and $x_i^*(R^*)$ take a maximum value at R_i and R_i^*, respectively. Moreover, the whole curve of $x_i^*(R)$ lies below that of $x_i(R)$ for $L^* < L$.

(Figure 2)

Now we can show

Lemma 2

$$L_{2-1}^{1} < L_{A-1}^{1} < L^{*} < L_{2-A}^{2} < L_{2-1}^{2}.$$

Proof. We show that $L_{2-1}^{1} < L_{A-1}^{1}$. By the graph of $x_1^*(R^*)$ in Figure 2 and the fact that $R_2^* < R_A^* < R_1$, it is obvious that $R_2^{*\varepsilon_1}(L^* - R_2^*) < R_A^{*\varepsilon_1}(L^* - R_A^*)$. Thus, by the definitions of L_{2-1}^{1} and L_{A-1}^{1}, we have $R_1^{\varepsilon_1}(L_{2-1}^{1} - R_1) < R_1^{\varepsilon_1}(L_{A-1}^{1} - R_1)$. This, together with Lemma 1, assures that $L_{2-1}^{1} < L_{A-1}^{1}$.

We show that $L_{2-A}^{2} < L_{2-1}^{2}$. By the definitions of L_{2-1}^{1} and L_{A-1}^{1}, we have $R_A^{\varepsilon_2}(L_{2-A}^{2} - R_A) = R_1^{\varepsilon_2}(L_{2-1}^{1} - R_1)$. The graph of $x_2(R)$ in Figure 2, and the fact that $R_2 < R_A < R_1$ assures that $R_1^{\varepsilon_2}(L - R_1) < R_A^{\varepsilon_2}(L - R_A)$ for any L. So we have $R_1^{\varepsilon_2}(L_{2-A}^{2} - R_1) < R_A^{\varepsilon_2}(L_{2-A}^{2} - R_A)$ and $R_1^{\varepsilon_2}(L_{2-A}^{2} - R_1) < R_1^{\varepsilon_2}(L_{2-1}^{2} - R_1)$. Thus, because of Lemma 1, we obtain $L_{2-A}^{2} < L_{2-1}^{2}$.

We show that $L^* < L_{2-A}^{2}$. By the definition of L_{2-A}^{2}, it is clear that $R_A^{\varepsilon_2}(L_{2-A}^{2} - R_A) = R_2^{*\varepsilon_2}(L^* - R_2^*)$. Since $x_2^*(R^*)$ is maximized at $R^* = R_2^*$, $R_A^{*\varepsilon_2}(L^* - R_A^*) < R_2^{*\varepsilon_2}(L^* - R_2^*)$. Hence $R_A^{\varepsilon_2}(L_{2-A}^{2} - R_A) > R_A^{*\varepsilon_2}(L^* - R_A^*)$, which implies $L_{2-A}^{2} > L^*$ by Lemma 1.

Finally, we show that $L_{A-1}^{1} < L^*$. By the definition of L_{A-1}^{1}, we have $R_1^{\varepsilon_1}(L_{A-1}^{1} - R_1) = R_A^{*\varepsilon_1}(L^* - R_A^*)$, so that we have $R_1^{*\varepsilon_1}(L^* - R_1^*) > R_1^{\varepsilon_1}(L_{A-1}^{1} - R_1)$ by $R_1^{*\varepsilon_1}(L^* - R_1^*) > R_A^{*\varepsilon_1}(L^* - R_A^*)$. Thus, in view of Lemma 1, it is clear that $L^* > L_{A-1}^{1}$. Q. E. D.

Based on Lemma 2, we obtain

Lemma 3

Let the strategy set of the governments be E^+. Then the strategic pair of a restricted Nash equilibrium exists as $(R_1, R_2^*) \in E^{II}$ for any $L \in [L^*, L_{2-1}^{2}]$, and $(R_A, R_2^*) \in E^{I}$ for any $L \in (L_{2-A}^{2}, \infty)$.

Proof. It is clear from the definitions of R_1, R_1^*, and R_A that $R_2^* < R_1$ and $R_2^* < R_A$ for any L no smaller than L^*. This, together with Lemma 2, assures that $(R_1, R_2^*) \in E^{II}$. and $(R_A, R_2^*) \in E^{I}$. So, by Theorem 2, the assertion of this lemma holds. Q. E. D.

Lemma 3 gives a diagram displaying the relationship between the specialization pattern of equilibrium and the country size, which is shown in Figure 3.

(Figure 3)

We turn our attention to the case where the strategic space is confined to E^-. Then the possible specialization patterns are [IV], [V], and [VI]. An analysis can be made of these in a similar method as [I], [II], and [III] in E^+. We first present the definitions of particular levels of L for dealing with

[IV], [V], and [VI].

Definition

L_{A-2}^{2} *is the level of* L *satisfying* $R_2^{\varepsilon_2}(L-R_2) = R_A^{*\varepsilon_2}(L^*-R_A^*)$.

L_{1-2}^{1} *is the level of* L *satisfying* $R_2^{\varepsilon_1}(L-R_2) = R_1^{*\varepsilon_1}(L^*-R_1^*)$.

L_{1-2}^{2} *is the level of* L *satisfying* $R_2^{\varepsilon_2}(L-R_2) = R_1^{*\varepsilon_2}(L^*-R_1^*)$.

L_{1-A}^{1} *is the level of* L *satisfying* $R_A^{\varepsilon_1}(L-L_A) = R_1^{*\varepsilon_1}(L^*-R_1^*)$.

Then we have

$$(R_2, R_A^*) \in E^{IV} \quad if\ and\ only\ if \quad L < L_{A-2}^{2}.$$

$$(R_2, R_1^*) \in E^{V} \quad if\ and\ only\ if \quad L_{1-2}^{2} \le L \le L_{1-2}^{1}.$$

$$(R_A, R_1^*) \in E^{VI} \quad if\ and\ only\ if \quad L_{1-A}^{1} < L.$$

Employing a similar method to prove Lemma 2, we obtain

Lemma 4

$L_{1-2}^{2} < L_{A-2}^{2} < L^* < L_{1-A}^{1} < L_{1-2}^{1}$.

Due to Theorem 2 and Lemma 4, we can assert

Lemma 5

Let the strategy set of the governments be E^{-}. *Then, if a restricted Nash equilibrium exists, the strategic pair of the equilibrium is* $(R_2, R_1^*) \in E^{V}$ *for any* $L \in (L^*, L_{1-2}^{1}]$ *and* $(R_A, R_1^*) \in E^{VI}$ *for any* $L \in (L_{1-A}^{1}, \infty)$.

Based on Lemma 5, we can illustrate a diagram similar to Figure 3, which is shown in Figure 4.

(Figure 4)

Lemma 5 is slightly different from Lemma 3 though both diagrams are look-alike. Lemma 3 asserts the existence of a restricted Nash equilibrium characterized by the specialization pattern, while Lemma 5 asserts only what is the specialized pattern at a restricted Nash equilibrium for each L if the equilibrium exists. We need to show $R < R^*$ at the equilibrium for the existence, which will be discussed later.

7. Selection between Restricted Nash Equilibria for a Nash Equilibrium

Lemma 3 reveals that two restricted Nash equilibrium strategy pairs exist for each $L \in (L_{2-A}^2, L_{2-1}^2]$ in the strategy set E^+. They are (R_1, R_2^*) and (R_A, R_2^*). Foreign strategy is the same as R_2^* between these strategy pairs. Thus, if an equilibrium strategy pair at which Home utility is greater than that of the other equilibrium strategy pair, that equilibrium strategy pair remains as a candidate for a Nash equilibrium pair and the other drops. The same argument to this can be applied to Lemma 5.

So our aim of this section is to seek which restricted Nash equilibrium would drop out in each of Lemmas 3 and 5. We begin with the case of Lemma 3. We will compare the Home utility $U(R_1(L), R_2^*(L^*))$ with that of $U(R_A(L), R_2^*(L^*))$ for any $L \in [L_{2-A}^2, L_{2-1}^2]$, where $R_1(L) \equiv \varepsilon_1 L/(1+\varepsilon_1)$, $R_A(L) \equiv \varepsilon_A L/(1+\varepsilon_A)$, $R_2^*(L^*) \equiv \varepsilon_2 L^*/(1+\varepsilon_2)$, $U(R_1(L), R_2^*(L^*)) \equiv [R_1(L)^{\varepsilon_1}(L - R_1(L))]^{1/2}[R_2^*(L)^{\varepsilon_1}(L^* - R_2^*(L^*))]^{1/2}$, and $U(R_A(L), R_2^*(L^*)) \equiv R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)$.

We can easily observe

$$\begin{aligned}
&U(R_1(L_{2-A}^2), R_2^*(L^*)) \\
&\qquad = [R_A(L_{2-A}^2)^{\varepsilon_2}(L_{2-A}^2 - R_A(L_{2-A}^2))]^{1/2}[R_1(L_{2-A}^2)^{\varepsilon_1}(L_{2-A}^2 - R_1(L_{2-A}^2))]^{1/2} \\
&\qquad > [R_A(L_{2-A}^2)^{\varepsilon_2}(L_{2-A}^2 - R_A(L_{2-A}^2))]^{1/2}[R_A(L_{2-A}^2)^{\varepsilon_1}(L_{2-A}^2 - R_A(L_{2-A}^2))]^{1/2} \\
&\qquad = R_A(L_{2-A}^2)^{\varepsilon_A}[L_{2-A}^2 - R_A(L_{2-A}^2)] = U(R_A(L_{2-A}^2), R_2^*(L^*)) \qquad (8)
\end{aligned}$$

and

$$\begin{aligned}
&U\big(R_1(L_{2-1}^2), R_2^*(L^*)\big) \\
&\qquad = \big[R_1(L_{2-1}^2)^{\varepsilon_1}\big(L_{2-1}^2 - R_1(L_{2-1}^2)\big)\big]^{1/2}\big[R_1(L_{2-1}^2)^{\varepsilon_2}\big(L_{2-1}^2 - R_1(L_{2-1}^2)\big)\big]^{1/2} \\
&\qquad = R_1(L_{2-1}^2)^{\varepsilon_A}\big(L_{2-1}^2 - R_1(L_{2-1}^2)\big) < R_A(L_{2-1}^2)^{\varepsilon_A}\big(L_{2-1}^2 - R_A(L_{2-1}^2)\big) \\
&\qquad = U\big(R_A(L_{2-1}^2), R_2^*(L^*)\big). \qquad (9)
\end{aligned}$$

Moreover, since $U\big(R_1(L), R_2^*(L^*)\big)$, and $U\big(R_A(L), R_2^*(L^*)\big)$ are both continuous with respect to L, (8) and (9) assure that their graphs intersect in the interval $[L_{2-A}^2, L_{2-1}^2]$. We show that the intersection point is unique. Recalling Lemma 1, it is clear that $dU\big(R_A(L), R_2^*(L^*)\big)/dL > 0$ and $dU\big(R_1(L), R_2^*(L^*)\big)/dL > 0$. Hence, to see that two curves intersect only once, it suffices to prove $dU\big(R_A(L), R_2^*(L^*)\big)/dL < dU\big(R_1(L), R_2^*(L^*)\big)/dL$ at any intersection point.

Simple calculation yields

$$\frac{dU\big(R_A(L),R_2^*(L^*)\big)}{dL}=R_A(L)^{\varepsilon_A}>0$$

and

$$\frac{dU\big(R_1(L),R_2^*(L^*)\big)}{dL}=\frac{1}{2}\big[R_1(L)^{\varepsilon_1}\big(L-R_1(L)\big)\big]^{-1/2}\big[R_2^*(R^*)^{\varepsilon_2}\big(L^*-R_2^*(L^*)\big)\big]^{1/2}R_1(L)^{\varepsilon_1}>0.$$

Let L_{II-I} be the level of L such that $U\big(R_A(L),R_2^*(L^*)\big)=U\big(R_1(L),R_2^*(L^*)\big)$. Then, we have

$$R_2^*(L^*)^{\varepsilon_2}(L^*-R_2^*(L^*))^{1/2}=\big[R_1(L_{II-I})^{\varepsilon_1}\big(L-R_1(L_{II-I})\big)\big]^{-1/2}R_A(L_{II-I})^{\varepsilon_A}(L_{II-I}-R_A(L_{II-I})),$$

from which it follows that

$$\frac{dU\big(R_1(L_{II-I}),R_2^*(L^*)\big)}{dL}$$

$$=\frac{1}{2}\big[R_1(L_{II-I})^{\varepsilon_1}\big(L_{II-I}-R_1(L_{II-I})\big)\big]^{-1}R_A(L_{II-I})^{\varepsilon_A}\big(L_{II-I}-R_A(L_{II-I})\big)R_1(L_{II-I})^{\varepsilon_1}$$

$$=R_A(L_{II-I})^{\varepsilon_A}\frac{L_{II-I}-R_A(L_{II-I})}{2(L_{II-I}-R_1(L_{II-I}))}<R_A(L_{II-I})^{\varepsilon_A}=\frac{dU\big(R_A(L_{II-I}),R_2^*(L^*)\big)}{dL},$$

by virtue of

$$0<\frac{L_{II-I}-R_A(L_{II-I})}{2(L_{II-I}-R_1(L_{II-I}))}=\frac{1+\varepsilon_1}{2(1+\varepsilon_A)}=\frac{1+\varepsilon_1}{2+\varepsilon_1+\varepsilon_2}<1.$$

Thus we can conclude that

$$0<\frac{dU\big(R_1(L),R_2^*(L^*)\big)}{dL}<\frac{dU\big(R_A(L),R_2^*(L^*)\big)}{dL}$$

at $L=L_{II-I}$, from which the following lemma is obtained:

Lemma 6

There is a unique L_{II-I} *such that* L_{II-I} *is included in the open interval* (L_{2-A}^2,L_{2-1}^2) *and* *satisfies*

$$U\big(L_1(L_{II-I}), L_2^*(L^*)\big) = U(L_A(L_{II-I}), L_2^*(L^*)).$$

Moreover, it holds that

$$U\big(L_1(L), R_2^*(L^*)\big) > U\big(L_A(L), R_2^*(L^*)\big) \textit{ for any } L < L_{II-I}$$

and

$$U\big(L_A(L), R_2^*(L^*)\big) > U\big(L_1(L), R_2^*(L^*)\big) \textit{ for any } L > L_{II-I}.$$

Next, we deal with the case of Lemma 5, where two restricted Nash equilibrium strategic pairs exist in E^- for any$(L^1_{1-A}, L^1_{1-2}]$. The strategy pairs are (R_2, R_1^*) and (R_A, R_1^*). So we examine which pair remains as a strategy pair possible to be a Nash equilibrium strategy pair. This could be analysed along the same line to obtain Lemma 6. We have to compare the Home utilities under these strategy pairs.

We omit a detailed analysis and simply display a similar lemma of this case as follows:

Lemma 7

In any open interval (L^1_{1-A}, L^1_{1-2})*, there is a unique* L_{V-VI} *such that*

$$U\big(R_2(L_{V-VI}), R_1^*(L^*)\big) = U\big(R_A(L_{V-VI}), R_1^*(L^*)\big).$$

Moreover, it holds that

$$U\big(R_A(L), R_1^*(L^*)\big) < U\big(R_2(L), R_1^*(L^*)\big) \textit{ for any } L < L_{V-VI}$$

and

$$U\big(R_2(L), R_1^*(L^*)\big) < U\big(R_A(L), R_1^*(L^*)\big) \textit{ for any } L > L_{V-VI}.$$

In view of Lemmas 6 and 7, we can draw Figure 5, which gives a diagram of the relationship between the type of possible Nash equilibrium strategy pairs and the scale of Home labour endowment.

(Figure 5)

As seen in Figure 5, two types of restricted Nash equilibrium pairs in E^- or E^+ cannot coexist for any level of L. There is, however, a possibility that one in E^+ and the other in E^- coexist for some interval of L. However, the coexisting strategy pairs are completely different for any L. Hence each of these four strategy pairs in Figure 5 may become a Nash equilibrium pair. All Nash equilibrium pairs, if they exist, must be one of these four pairs.

Now we establish

Theorem 3

Suppose free trade prevails between Home and Foreign and each government strategically supplies the public intermediate good to maximize its own country's welfare. Then, the following hold:

(R_1, R_2^*) *is a candidate of the Nash equilibrium strategy pair for any* $L \in [L^*, L_{II-I}]$.

(R_A, R_2^*) *is a candidate of the Nash equilibrium strategy pair for any* $L \in [L_{II-I}, \infty)$.

(R_2, R_1^*) *satisfying* $R_2 < R_1^*$ *is a candidate of the Nash equilibrium strategy pair for any* $L \in [L^*, L_{V-VI}]$.

(R_A, R_1^*) *satisfying* $R_A < R_1^*$ *is a candidate of the Nash equilibrium strategy pair for any* $L \in [L_{V-VI}, \infty)$.

We do not have any other possible strategic pairs to attain a trading equilibrium.

8. Nash Equilibria, Patterns of Trade and Gains from Trade

In view of Theorem 3 in the previous section, there are four different couples as candidates of Nash equilibrium strategy pairs as follows:

(a) (R_1, R_2^*) and (R_2, R_1^*) for $L \in [L^*, Min[L_{V-VI}, L_{II-I}]]$.
(b) (R_A, R_2^*) and (R_2, R_1^*) for $L \in [L_{II-I}, L_{V-VI}]$ if $L_{II-I} < L_{V-VI}$.
(c) (R_1, R_2^*) and (R_A, R_1^*) for $L \in [L_{V-VI}, L_{II-I}]$ if $L_{V-VI} < L_{II-I}$.
(d) (R_A, R_2^*) and (R_A, R_1^*) for $L \in [Max[L_{V-VI}, L_{II-I}], \infty)$.

Concerning each of these couples, we investigate which strategy pair is selected as a trading equilibrium pair. To see this, we introduce the following criterion, which is the payoff dominance of Harsanyi and Selten (1988). Suppose that there are two Nash equilibrium strategy pairs $(R_i, R_i^*), i = A, B$. Let U_i and U_i^* be the social utilities of Home and Foreign, respectively, under $(R_i, R_i^*), i = A, B$. Then, if $U_A > U_B$ and $U_A^* > U_B^*$, the strategy pair (R_A, R_A^*) is selected as a trading equilibrium pair. Subject to this criterion, we examine each couple of two strategy pairs.

Consider (a). Our purpose is to compare $U\big(R_1(L),\ R_2^*(L^*)\big)$ with $U\big(R_2(L), R_1^*(L^*)\big)$, where

$$U\big(R_1(L), R_2^*(L^*)\big) = \big[R_1(L)^{\varepsilon_1}\big(L - R_1(L)\big)\big]^{\frac{1}{2}}\big[R_2^*(L^*)^{\varepsilon_2}\big(L^* - R_2^*(L^*)\big)\big]^{\frac{1}{2}}$$
$$= U^*\big(R_1(L), R_2^*(L^*)\big) \tag{12}$$

and

$$U\big(R_2(L), R_1^*(L^*)\big) = \big[R_2(L)^{\varepsilon_2}\big(L - R_2(L)\big)\big]^{\frac{1}{2}}\big[R_1^*(L^*)^{\varepsilon_1}\big(L^* - R_1^*(L^*)\big)\big]^{\frac{1}{2}}$$
$$= U^*\big(R_2(L), R_1^*(L^*)\big). \tag{13}$$

Since

$$\frac{dx_i\big(L, R(L)\big)}{dL} = \left(\frac{\varepsilon_i}{1+\varepsilon_i}\right)^{\varepsilon_i} L^{\varepsilon_i} = R_i^{\varepsilon_i},$$

where $x_i\big(L, R(L)\big) \equiv R_i(L)^{\varepsilon_i}\big(L - R_i(L)\big)$, we have

$$\frac{dU\big(R_1(L), R_2^*(L^*)\big)}{dL} = \big[R_1(L)^{\varepsilon_1} R_2^*(L^*)^{\varepsilon_2}\big(L^* - R_2^*(L^*)\big)\big]^{1/2}$$

$$= \left[R_1^*(L^*)^{\varepsilon_1} R_2^*(L^*)^{\varepsilon_2} \left(\frac{L}{L^*}\right)^{\varepsilon_1} \big(L^* - R_2^*(L^*)\big)\right]^{1/2} > 0 \qquad (14)$$

and similarly

$$\frac{dU\big(R_2(L), R_1^*(L^*)\big)}{dL} = \left[R_1^*(L^*)^{\varepsilon_1} R_2^*(L^*)^{\varepsilon_2} \left(\frac{L}{L^*}\right)^{\varepsilon_2} \big(L^* - R_1^*(L^*)\big)\right]^{1/2} > 0. \qquad (15)$$

Hence, under $L^* \leq L$, it is clear that

$$0 < \frac{dU\big(R_2(L), R_1^*(L^*)\big)}{dL} < \frac{dU\big(R_1(L), R_2^*(L^*)\big)}{dL}$$

by (14), (15), and the fact that $R_2^* < R_1^* < L^*$. This, together with (12) and (13), implies

$$U\big(R_2(L), R_1^*(L^*)\big) = U^*\big(R_2(L), R_1^*(L^*)\big)$$
$$< U\big(R_1(L), R_2^*(L^*)\big) = U^*\big(R_1(L), R_2^*(L^*)\big),\ \ for\ any\ \ L \in [L^*, \infty), \qquad (16)$$

because

$$U\big(R_1(L^*), R_2^*(L^*)\big) = U\big(R_2(L^*), R_1^*(L^*)\big).$$

Therefore, (R_1, R_2^*) becomes a trading equilibrium strategy in (a).

We treat (b). As for this case, if $L_{V-VI} < L_{II-I}$, (b) does not appear. We show that $L_{V-VI} < L_{II-I}$. Making use of Lemmas 6 and 7 and inequality (16), we can draw Figure 6 where the graph of the continuous and monotonously increasing function $x_A(L)$ must have a unique intersection with each increasing curve of $U\big(R_1(L), R_2^*(L^*)\big)$ and $U\big(R_2(L), R_1^*(L^*)\big)$, the slope of $x_A(L)$ is steeper than the slope of these two curves at their intersections and $U(R_1(L), R_2^*(L^*)) > U(R_2(L), R_1^*(L^*))$ for all $L > L^*$. By this figure, we have $L_{V-VI} < L_{II-I}$.

(Figure 6)

As for (c), for any $L \in [L_{V-VI}, L_{II-I}]$, we can observe

$$\begin{aligned} U(R_1(L), R_2^*(L^*)) &\equiv \left[R_1(L)^{\varepsilon_1}(L - R_1(L))\right]^{1/2}\left[R_2^*(L^*)^{\varepsilon_2}(L^* - R_2^*(L^*))\right]^{1/2} \\ &> R_A(L)^{\varepsilon_A}(L - R_A(L)) \equiv U(R_A(L), R_1^*(L^*)). \end{aligned} \tag{17}$$

Therefore, (R_1, R_2^*) is preferable to (R_A, R_1^*) for Home. The fact that $L \in [L_{V-VI}, L_{II-I}]$ implies that $L_{1-A}^1 \leq L$. So $R_A(L)^{\varepsilon_1}(L - R_A(L)) \geq R_1^*(L^*)^{\varepsilon_1}(L - R_1^*(L^*))$, which yields

$$\begin{aligned} U^*(R_A(L), R_1^*(L^*)) &\equiv R_A(L)^{\varepsilon_2-\varepsilon_1} R_1^*(L^*)^{\varepsilon_1}(L^* - R_1^*(L^*)) \\ &\leq R_A(L)^{\varepsilon_2-\varepsilon_1} R_A(L)^{\varepsilon_1}(L - R_A(L)) < R_A(L)^{\varepsilon_A}(L - R_A(L)) \\ &\equiv x_A(L) < U^*(R_1(L), R_2^*(L^*)) \end{aligned}$$

by the fact that $L \in [L_{V-VI}, L_{II-I}]$ and Figure 6. Therefore, Foreign prefers (R_1, R_2^*) to (R_A, R_1^*). Eventually, (R_1, R_2^*) remains and (R_A, R_1^*) drops out.

Finally, we are concerned with (d). In this case, Home always selects R_A as a strategy. So we compare $U^*(R_A(L), R_2^*(L^*))$ with $U^*(R_A(L), R_1^*(L^*))$, where

$$U^*(R_A(L), R_2^*(L^*)) \equiv R_A(L)^{(\varepsilon_1-\varepsilon_2)/2} R_2^{*\varepsilon_2}(L^*)(L^* - R_2^*(L^*))$$

and

$$U^*(R_A(L), R_1^*(L^*)) \equiv R_A(L)^{(\varepsilon_2-\varepsilon_1)/2} R_1^{*\varepsilon_1}(L^*)(L^* - R_1^*(L^*)).$$

Given $L_{II-I} \leq L$, it is verified that

$$\begin{aligned} &\left[R_2(L)^{\varepsilon_2}(L - R_2(L))\right]^{1/2}\left[R_1^*(L^*)^{\varepsilon_1}(L^* - R_1^*(L^*))\right]^{1/2} \\ &\quad\leq \left[R_1(L)^{\varepsilon_1}(L - R_1(L))\right]^{1/2}\left[R_2^*(L^*)^{\varepsilon_2}(L^* - R_2^*(L^*))\right]^{1/2} \\ &\quad< R_A(L)^{\varepsilon_A}(L - R_A(L)). \end{aligned}$$

This assures

$$R_1^*(L^*)^{\varepsilon_1}\big(L^* - R_1^*(L^*)\big) < \big[R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)\big]^2\big[R_2(L)^{\varepsilon_2}\big(L - R_2(L)\big)\big]^{-1},$$

from which we have

$$U^*\big(R_A(L), R_1^*(L^*)\big) < R_A(L)^{(\varepsilon_1-\varepsilon_2)/2}\big[R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)\big]^2\big[R_2(L)^{\varepsilon_2}\big(L - R_2(L)\big)\big]^{-1}$$
$$= R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)^2/\big(L - R_2(L)\big).$$

Similarly, since

$$R_2^*(L^*)^{\varepsilon_2}\big(L^* - R_2^*(L^*)\big) > \big[R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)\big]^2\big[R_1(L)^{\varepsilon_1}\big(L - R_1(L)\big)\big]^{-1},$$

we have

$$U^*\big(R_A(L), R_2^*(L^*)\big) > R_A(L)^{(\varepsilon_1-\varepsilon_2)/2}\big[R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)\big]^2\big[R_1(L)^{\varepsilon_1}\big(L - R_1(L)\big)\big]^{-1}$$
$$= R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)^2/\big(L - R_1(L)\big). \quad (18)$$

Hence we obtain

$$U^*\big(R_A(L), R_2^*(L^*)\big) > U^*\big(R_A(L), R_1^*(L^*)\big), \quad (19)$$

for $L - R_1(L) < L - R_2(L)$. So (R_A, R_2^*) is superior to (R_A, R_1^*) for Foreign. As a result, (R_A, R_2^*) is selected in (d).

Consequently, the following lemma is established by Theorem 3:

Lemma 8

(i) For any $L \in (L^*, L_{II-I})$*, the strategic pair* (R_1, R_2^*) *is a sole candidate to attain a trading equilibrium.*

(ii) For any $L \in (L_{II-I}, \infty)$*, the strategic pair* (R_A, R_2^*) *is a sole candidate to attain a trading equilibrium.*

(iii) For $L = L^*$*, two strategic pairs* (R_1, R_2^*) *and* (R_2, R_1^*) *are sole candidates to attain a trading equilibrium.*

(iv) For $L = L_{II-I}$*, two strategic pairs* (R_A, R_2^*) *and* (R_1, R_2^*) *are sole candidates to attain a trading equilibrium.*

Now we are in a position to show that all strategic pairs appearing in Lemma 8 are a Nash equilibrium in E.

Theorem 4

(i) If $L \in (L^*, L_{II-I}]$*, then the strategic pair* (R_1, R_2^*) *is a Nash equilibrium.*

(ii) If $L \in [\,L_{II-I},\ \infty)$*, then the strategic pair* (R_A, R_2^*) *is a Nash equilibrium.*

(iii) If $L = L^*$*, then two strategic pairs* (R_1, R_2^*) *and* (R_2, R_1^*) *are Nash equilibria.*

(iv) If $L = L_{II-I}$*, then two strategic pairs* (R_A, R_2^*) *and* (R_1, R_2^*) *are Nash equilibria.*

Proof. We show that $(R_1, R_2^*)$is a Nash for any $L \in [L^*, L_{II-I}]$. First of all, we should notice that for any R satisfying $R \in [0,\ L]$, $PPF(R)$ is below $PPF(R_1)$ ($PPF(R_2)$) if $R > R_1$ ($R < R_2$). Thus the effective strategy set for Home is the interval $[R_2, R_1]$. Similarly, the effective strategy set of Foreign is the interval $[R_2^*, R_1^*]$. Since (R_1, R_2^*) is a Nash equilibrium in E^+, we have

$$U(R_1, R_2^*) > U(R, R_2^*) \ \text{ for all } \ R \geq R_2^*,$$

from which we have

$$U(R_1, R_2^*) > U(R, R_2^*) \ \text{ for all } \ R \in [R_2, R_1],$$

by the fact that $R_2 \geq R_2^*$. Similarly, we also have

$$U^*(R_1, R_2^*) > U^*(R_1, R^*) \ \text{ for all } \ R^* \in [R_2^*, R_1^*],$$

since $R_1 \geq R_1^*$. Therefore, we proved that (i') The strategic pair (R_1, R_2^*) is a Nash equilibrium for all $L \in [L^*, L_{II-I}]$.

Next, we show that (R_A, R_2^*) is a Nash for all $L \in [L_{II-I},\ \infty)$. Applying the same technique used to prove (i'), we have

$$U(R_A, R_2^*) > U(R, R_2^*) \ \text{ for all } \ R \in [R_2, R_1],$$

since (R_A, R_2^*) is a Nash in E^+ and $R_2 \geq R_2^*$. Moreover, by a similar reason, we have

$$U^*(R_A, R_2^*) > U^*(R_A, R^*), \text{ for all } \ R^* \ \text{ such that } \ R^* \in [R_2^*, R_1^*] \ \text{ and } \ R^* \geq R_A.$$

We have also

$U^*(R_A, R_2^*) > U^*(R_A, R^*)$, for all R^* such that $R^* \in [R_2^*, R_1^*]$ and $R^* > R_A$.

This is because $L_{II-I} \leq L$ implies (R_A, R_2^*) is a Nash in E^- and this, together with (19), yields

$$U^*(R_A, R_2^*) > U^*(R_A, R_1^*) > U^*(R_A, R^*) \text{ for all } R^* > R_A.$$

So we have (ii') the strategic pair (R_A, R_2^*) is a Nash for any $L \in [\, L_{II-I}, \infty)$. These (i') and (ii'), together with Lemma 8, assure the theorem. Q.E.D.

The following theorem concerning the production and trade patterns are obvious due to Lemma 8 and Theorem 4.

Theorem 5

Suppose that $L^* < L$. *Then, for any* $L \in (L^*, L_{II-I})$, *Home and Foreign specialize in goods 1 and 2, respectively, and for any* $L \in (L_{II-I}, \infty)$, *Home diversifies and Foreign specializes in good 2, at the trading equilibrium, where* L_{II-I} *is the level of* L *such that Home utility at the trading equilibrium is the same between* (R_A, R_2^*) *and* (R_1, R_2^*). Thus, *at any trading equilibrium, Home (Foreign) exports good 1 (2) and imports god 2 (1) for any* $L \in (L^*, L_{II-I}) \cup (L_{II-I}, \infty)$.

Proceeding further, the theorem of the gains from trade is also established as follows:

Theorem 6

At a trading equilibrium for any $L \in (L^*, L_{II-I}) \cup (L_{II-I}, \infty)$, *each country can enjoy a positive gain from trade when the country specializes but stays at the same level of utility as that of autarky when the country diversifies. If both countries specialize at the trading equilibrium, the level of utility is the same between countries.*

Proof. Taking into account that $x_A(L) = R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)$ and $U\big(R_1(L), R_2^*(L^*)\big) = U^*\big(R_1(L), R_2^*(L^*)\big)$, the assertion of the theorem clearly holds from Table 1 and Figure 6, where $x_A^*(L^*) \equiv R_A^*(L^*)^{\varepsilon_A}\big(L^* - R_A(L^*)\big)$ in the figure. In particular, it can be shown by the use of (18) that $U^*\big(R_A(L), R_2^*(L^*)\big) > R_A^*(L^*)^{\varepsilon_A}\big(L^* - R_A(L^*)\big)$ for any $\in (L_{II-I}, \infty)$, because $R_A(L)^{\varepsilon_A}\big(L - R_A(L)\big)^2 / \big(L - R_1(L)\big) > R_A^*(L^*)^{\varepsilon_A}\big(L^* - R_A(L^*)\big)$. Q. E. D.

So far, we treated the strategy pairs (R_A, R_1^*) and (R_2, R_1^*) as if they are Nash equilibria. As seen in Theorem 4, however, whether they are Nash equilibria or not does not matter for the determination of the trading equilibrium. Even the existence of those strategic pairs for any level of L no smaller than L^* does not matter. Thus, we do not discuss this problem.[6] Furthermore, we excluded two cases where $L = L^*$ and where $L = L_{II-I}$. In both of these cases, there are two Nash equilibria, so that we face the selection problem between two Nash equilibria. Next section deals with this problem.

9. Two Specific Cases

This section deals with two specific cases which are left untouched. One is that $L = L^*$ and the other is that $L = L_{II-I}$.

(i) The case that $L = L^*$.

In this case, there are two Nash equilibrium pairs, which are (R_1, R_2^*) and (R_2, R_1^*). Whichever is the strategy pair (R_1, R_2^*) or (R_2, R_1^*), the utility at the trading equilibrium is the same, and furthermore, it is also the same between countries. In addition to this, Figure 7 assures the level of this utility is higher than the autarkic utility attained by (R_A, R_A^*) for both countries. Therefore, (R_1, R_2^*) and (R_2, R_1^*) are preferable to (R_A, R_A^*). However, in the present framework, which pair between (R_1, R_2^*) and (R_2, R_1^*) is selected as an equilibrium pair is indeterminate because (R_1, R_2^*) and (R_2, R_1^*) are indifferent to each country from the country welfare point of view. This result exhibits a typical example of the symmetry-breaking in Ricardian type of trading economies, while Chatterjee (2017) discussed it in Heckscher-Ohlin type of trade economies in a more comprehensive fashion.

(ii) The case where $L = L_{II-I}$.

In this case, there exist two Nash equilibrium pairs (R_1, R_2^*) and (R_A, R_2^*). For Home, the welfare is the same between these two pairs, while, Foreign's utilitiy levels at (R_1, R_2^*) and (R_A, R_2^*) are, respectively,

$$\begin{aligned} U^*\big(R_1(L_{II-I}), R_2^*(L_{II-I})\big) &\equiv \big[R_1(L_{II-I})^{\varepsilon_1}\big(L_{II-I} - R_1(L_{II-I})\big)\big]^{1/2}\big[R_2^*(L^*)^{\varepsilon_2}\big(L^* - R_2^*(L^*)\big)\big]^{1/2} \\ &> R_A^*(L)^{\varepsilon_A}\big(L^* - R_A^*(L^*)\big) \end{aligned}$$

and

$$\begin{aligned} U^*\big(R_A(L_{II-I}), R_2^*(L_{II-I})\big) &\equiv R_A(L_{II-I})^{(\varepsilon_1-\varepsilon_2)/2}R_2^*(L^*)^{\varepsilon_2}\big(L^* - R_A^*(L^*)\big) \\ &> R_A^*(L)^{\varepsilon_A}\big(L^* - R_A^*(L^*)\big). \end{aligned}$$

Therefore, we can conclude that, under trade, Foreign necessarily specializes in good 2 while Home may specialize in good 2 or diversifies in production alternatively. Moreover, whichever the Nash equilibrium under trade, Home's welfare level is the same to that of autarky and Foreign becomes better off. Thus, which Nash equilibrium is selected is indeterminate under the pure strategies.

The following theorem summarizes the results obtained in this section.

Theorem 7

(i) In the case where $L = L^{*}$, *there are two possibilities of the pattern of specialization. These are expressed in the way that Home specializes in one good and exports it and Foreign specialized in the other good and export it. Which country specializes in which good is indeterminate. At a trading equilibrium, both countries have a positive gain from trade and the welfare level is the same between two countries.*

(ii) In the case where $L = L_{II-I}$, *at the trading equilibrium, the public intermediate good supply of Foreign is* R_2^{*} *and that of Home is indecisive between* R_1 *and* R_A *under the pure strategic game. At the trading equilibrium, Foreign always specializes in good 2 while Home specializes in good 1 or alternatively diversities production. Once a trading equilibrium is attained, the level of Home welfare stays at the same as that of autarky and that of Foreign becomes higher than that of autarky.*

10. Conclusion

In this paper, we introduced a public intermediate good into a simple Ricardian economy. We analysed the patterns of trade and gains from trade under the supposition that each country's government determines the supply level of the public intermediate good strategically to maximize the welfare of the country under a non-cooperative normal game.

By the analysis, we obtained the result that, as far as factor endowment differs between countries, the patterns of specialization follow a similar law to that of Ricardian comparative advantage, that is, a country endowed with a larger factor endowment supplies a larger amount of the public intermediate good and thus has a comparative advantage in a good which enjoys higher productivity by the public intermediate good than the other good. Thus, the country endowed with a larger factor endowment exports the good with higher productivity. Once trade is opened, any country never loses by trade. Therefore, we confirmed the robustness of the comparative advantage theory of Ricardian economy even in our setting where the public infrastructures of atmosphere creation type are accommodated and two governments interact with each other in the public infrastructure supplies.

In our present setting, however, we have two interesting aspects newly emerge.

1. Concerning an incompletely specializing country, production occurs at an inner point of the production possibility set, as shown in Figure 7. In the figure, point P is the production point, and C is

the consumption point under trade where C exactly coincides with the autarkic equilibrium point, and the social indifference curve is tangent to the PPF there. The figure implies that inefficiency occurs in production under trade. Nevertheless, the welfare of this incompletely specializing country is protected from getting worse off once after the trade. Suga and Tawada (2007) investigated the gains from trade in the same framework but under the supposition that each government supplies the public intermediate good at each production point by the Samuelson-Kaizuka condition. So, the production carries out at the surface of the PPF and, because of this, the country necessarily loses by trade since the PPF is bowed in to the origin.

(Figure 7)

2. As was stated, we also derived the interesting result in the case where two countries are identical even in the size of labor endowment. The traditional result of this case is that there is no trade even after opening trade, but once the public intermediate good is supplied under strategic interaction between governments, trade necessarily occurs between entirely identical countries. The reason is that, because of the increasing returns to scale effect of the public intermediate good, trade by specialization is better than autarky by diversification in production. In this case, however, under the non-cooperate normal game between countries, as long as each government is confined to use pure strategies, one asymmetric Nash equilibrium strategy pair assures that the opposite strategic pair is also a Nash equilibrium. These two strategy pairs are equally possible to be a trading equilibrium, implying that the pattern of trade is indecisive. This is one example of symmetry-breaking in Ricardian economy of international trade, the original problem of which is raised by Matsuyama (2002) and investigated by Chatterjee (2017) in a general setting of the Heckscher-Ohlin type of trading economies.

In this paper, our analysis is limited to the case where $\alpha = 1/2$, so that a natural extension is to allow for any value of α in between 0 and 1. In this general case, we can expect the following: As for the pattern of specialization, we have the same result for α around 1/2. For α sufficiently close to zero, which means that demand in good 1 is fairly weak, the pattern of specialization is [I] for all $L \in (L^*, \infty)$. Moreover, for α sufficiently close to 1, which means a demand in good 1 is extremely strong, the pattern of specialization shifts from [III] to [II] and finally to [I], according to an increase in L starting from L^* to ∞. As for the gains from trade, our proposition seems to carry over as long as the country size is different between countries.

We did not discuss whether the equilibrium supply of public infrastructures is efficient from the world welfare point of view. Suga (2021) tackled this topic in the same framework to ours but under the assumption that the parameter α of the utility function is free but the labour endowment is the same between countries and showed that the public infrastructures are under(over)-supplied in the country diversifying and exporting good 1(2). The reason is that the diversifying country tries to defend from the welfare reduction caused by the strategic game and to keep the welfare at the autarkic level at least by shifting the public infrastructures from optimal level so as to make the terms of trade preferable to the country. Taking this result into consideration, it is plausible in our case that public infrastructures are undersupplied in Home when it is incompletely specializing.

We assumed that all workers are identical in every respect over both countries and showed the gains from the trade theorem in the aggregate demand version. Thus, a natural extension is to relax this assumption and consider multiple individuals who are not necessarily identical. Then, following the propositions proved by Grandmont and McFadden (1972) and Kemp and Wan (1972), it may be shown that, once we could suitably formalize the strategic supply of public infrastructures by the government, there would be a transfer scheme for everyone to be better off under trade in each country. Since late Professor Murray C. Kemp generalized the Kemp and Wan proposition based on the Arrow and Debreu framework and further investigated how far it is applicable in various economies, but not in the game setting economy.[7] So, another interesting topic to consider is to seek the delimitation of the applicability of the Kemp and Wan proposition in the game setting framework.

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[1] See, for example, Asian Development Bank and Asian Development Bank Institute (2009) for various strategies on the public infrastructure supply in Asian region.

[2] See, for example, Miller (2017), who explains the political and economic purpose and impact of this strategy.

[3] For example, the U.S., Japan, Australia, and India are advancing the Free and Open Indo-Pacific Initiative to compete with China's One Belt and One Road Initiative. In this respect, see, for example, Hirakawa et al. (2019).

[4] In the relation between trade and public infrastructure, there is also a study by Bougheas et al. (1999, 2003), which focuses on public infrastructure specialized in transportation.

[5] For these points, see Hirakawa et al. (2019) and Miller (2017), for example.

[6] By the aid of Table 1, we can illustrate the diagram of the response curves of the two countries for each case of the endowment pair (L, L^{*}). Based on these diagrams, we can confirm that the strategy pairs (R_A, R_1^{*}) and (R_2, R_1^{*}) are also the Nsah equilibria wherever they exist. We do not show this fact in order to save the space.

[7] See Essays 5 to 10 in Kemp (1995) for those related works.

Reference

Asian Development Bank and Asian Development Bank Institute, 2009, *Infrastructure for a Seamless Asia*, Asian Development Bank and Asian Development Bank Institute.

Bougheas, S., Demetriades, P. O., and W. Morgenroth, E. L., 1999, Infrastructure, transport costs and trade, *Journal of International Economics* 27, 169-189.

Bougheas, S., Demetriades, P. O., and W. Morgenroth, E. L., 2003, International aspects of public infrastructure investment, *Canadian Journal of Economics* 36(4), 884-910.

Chatterjee, A., 2017, Endogenous comparative advantage, gains from trade and symmetry-breaking, *Journal of International Economics* 109, 102-115.

Connolly, M., 1970, Public Goods, externalities, and international relations, *Journal of Political Economy* 78(2), 279-290.

Grandmont, J. M., McFadden, D., 1972, A technical note on classical gains from trade, *Journal of International Economics* 2, 109-125.

Harsanyi, J. C. and Serten, R., 1988, *A General Theory of Equilibrium Selection in Games*, M.I.T. Press.

Hirakawa, H., Machida, I., Maie, Y., and Ishikawa, K., eds., 2019, *Political Economics of One Belt and One Road: Does China Create A New Frontier? (Ittai-Ichiro no Seiji-Keizaigaku: Chugoku ha Aratana Frontier wo Soshutsu Suruka ?)*, Bunshindo. (in Japanese)

Kemp, M. C., 1995, *The Gains from Trade and the Gains from Aid: Essays in International Trade Theory*, Routledge.

Kemp, M. C. and Wan Jr. H., 1972, The gains from trade, *International Economic Review* 13, 509-522.

Matsuyama, K., 2002, Explaining diversity: Symmetry-breaking in complementarity games, *American Economic Review* 92, 241-246.

Millar, T., 2017, *China's Asian Dream: Empire Building Along the New Silk Road*, ZED Books.

Shimomura, K., 2007, Trade Gains and Public Goods, *Review of International Economics* 15(5), 948-954.

Suga, N., 2021, A two-country model of international trade with a public intermediate good: Inter-governmental competition and comparative advantage, *mimeographed*.

Suga, N. and Tawada, M., 2007, International trade with a public intermediate good and the gains from trade, *Review of International Economics* 15, 284-293.

Suga, N., Tawada, M., and Yanase, A., 2021, Public intermediate good and endogenous Ricardian comparative advantage, *mimeographed*.

Yanase, A. and Tawada, M., 2019, Public infrastructure and trade in a dynamic two-country model, *Review of International Economics* 28(2), 447-465.

Yeaple, S. R. and Golub, S. S., 2007, International productivity differences, infrastructure, and comparative advantage, *Review of International Economics* 15(2), 223-242.

Yen, M-F, Wu, R., and Miranda, M. J., 2019, A general equilibrium model of bilateral trade with strategic investment in commercial infrastructure, *Journal of International Trade and Economic Development* 28(6), 712-731.

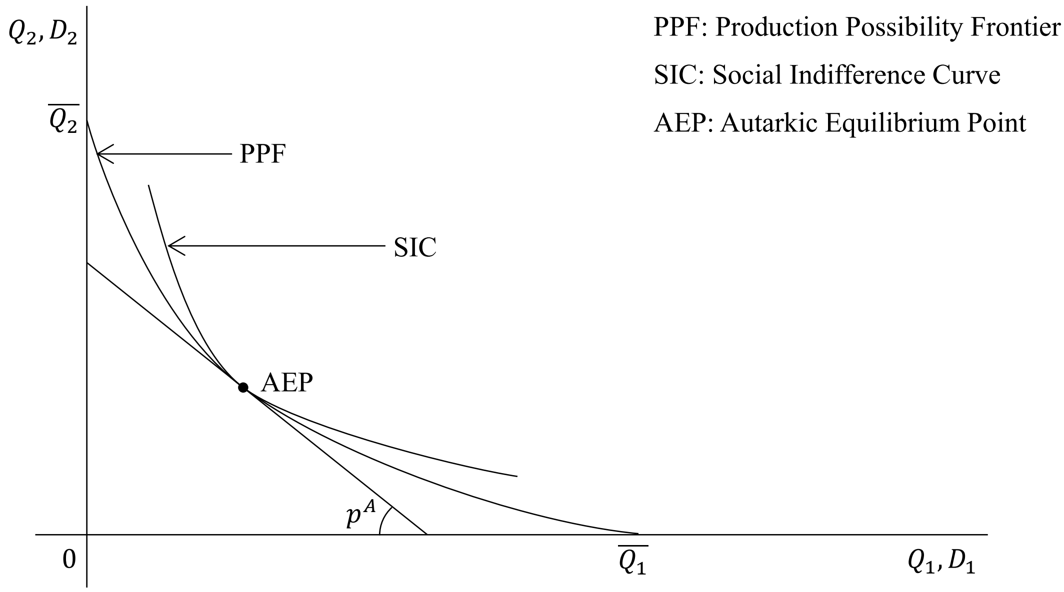


Figure 1. Home Autarkic Equilibrium

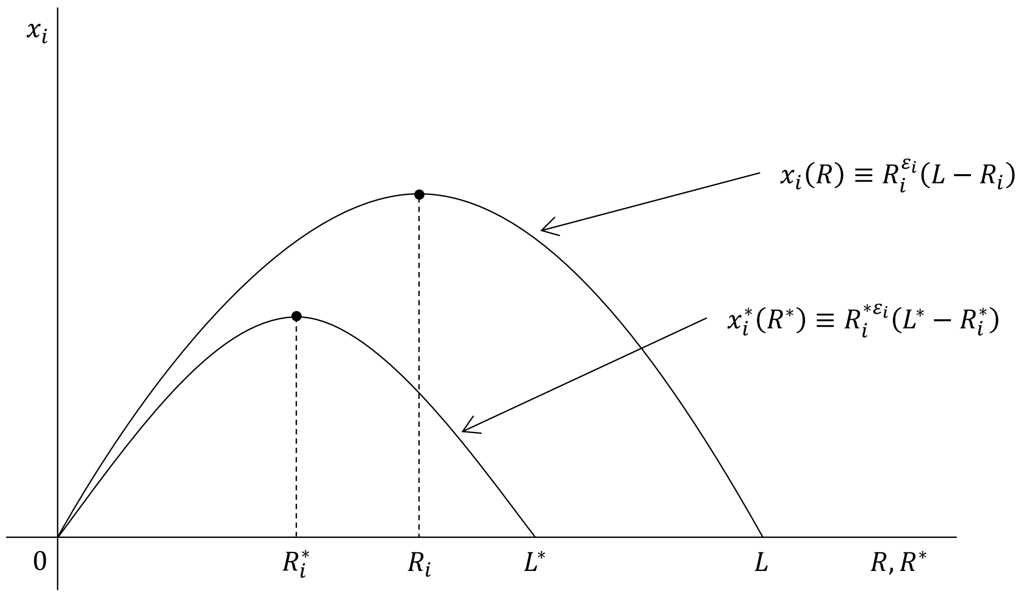


Figure 2. The Graphs of $x_i(R)$ and $R_i^*(R^*)$

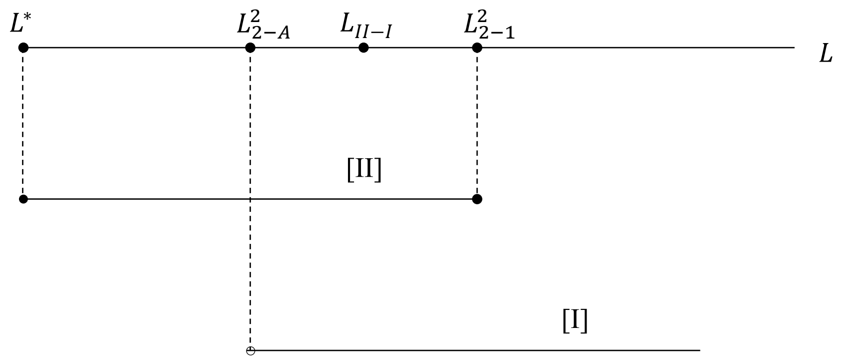


Figure 3. Relationship between Specialization Pattern in E^+ and Country Size

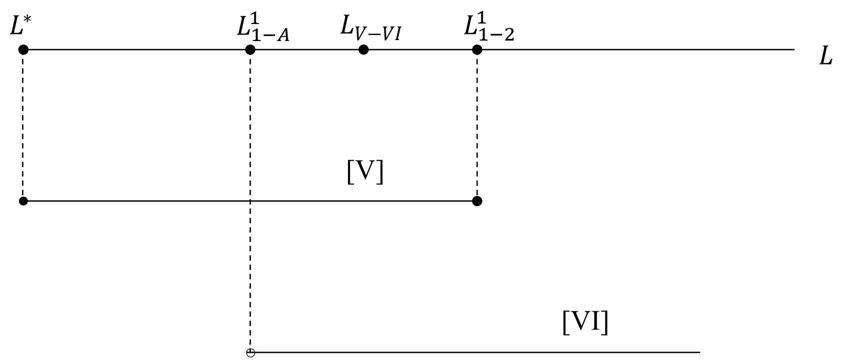
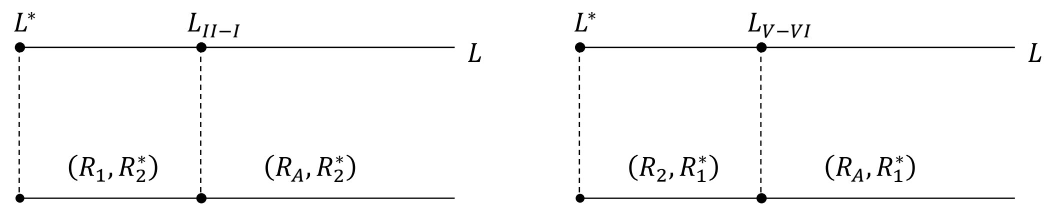


Figure 4. Relationship between Specialization Pattern in E^- and Country Size



5-a. $R \geq R^*$ 5-b. $R < R^*$

Figure 5. Relationship between Nash Equilibrium Strategy Pair and Home Labor Endowment

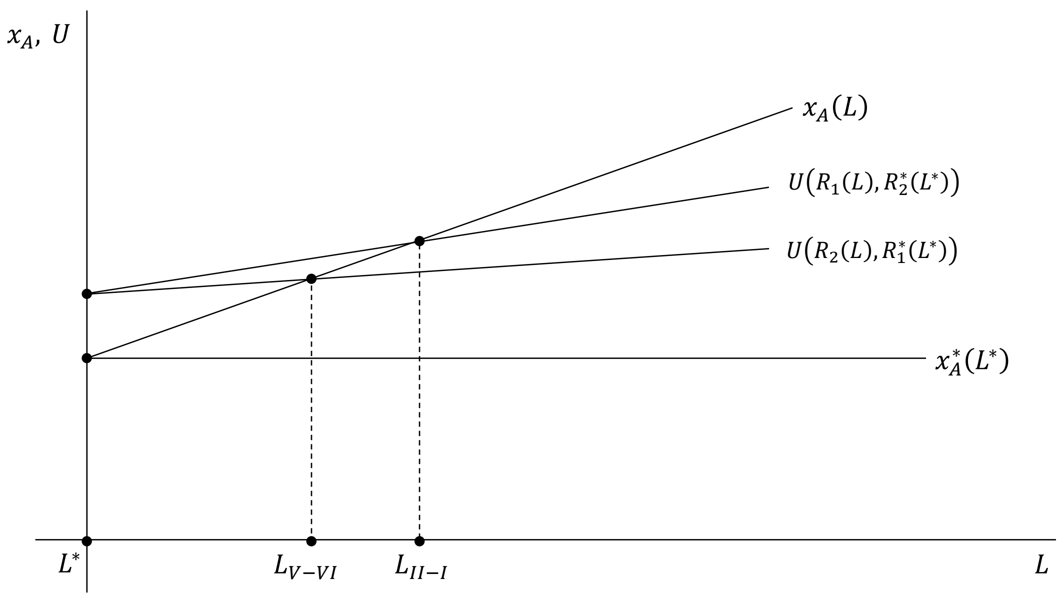


Figure 6. $L_{V-VI} < L_{II-I}$

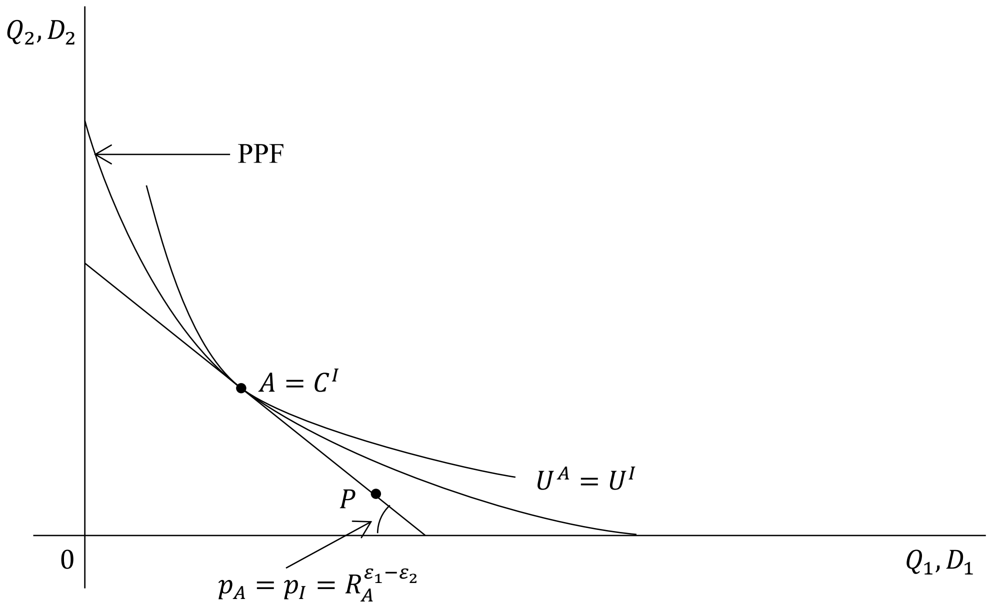


Figure 7. The Production and Consumption in the Case of Incomplete Specialization

Table 1: Specialization Pattern of Trading Equilibrium

	[I]	[II]	[III]
The condition for a Trading Equilibrium	$R \geq R^*,\ 1 < \frac{R^{\varepsilon_2}(L-R)}{R^{*\varepsilon_2}(L^*-R^*)}$	$R \geq R^*,\ \frac{R^{\varepsilon_2}(L-R)}{R^{*\varepsilon_2}(L^*-R^*)} \leq 1 \leq \frac{R^{\varepsilon_1}(L-R)}{R^{*\varepsilon_1}(L^*-R^*)}$	$R \geq R^*,\ \frac{R^{\varepsilon_1}(L-R)}{R^{*\varepsilon_1}(L^*-R^*)} < 1$
Pattern of Specialization	Home: both goods Foreign: good 2	Home: good 1 Foreign: good 2	Home: good 1 Foreign: both goods
Restricted Nash Equilibrium Strategy Pair	(R_A, R_2^*)	(R_1, R_2^*)	(R_1, R_A^*)
Utility	$U = R^{\varepsilon_A}(L-R)$ $U^* = R^{(\varepsilon_1-\varepsilon_2)/2}R^{*\varepsilon_2}(L^*-R^*)$	$U = U^*$ $= [R^{\varepsilon_1}(L-R)]^{1/2}[R^{*\varepsilon_2}(L^*-R^*)]^{1/2}$	$U = R^{*(\varepsilon_2-\varepsilon_1)/2}R^{\varepsilon_1}(L-R)$ $U^* = R^{*\varepsilon_A}(L^*-R^*)$
	[VI]	[V]	[VI]
The condition for a Trading Equilibrium	$R < R^*,\ 1 > \frac{R^{\varepsilon_2}(L-R)}{R^{*\varepsilon_2}(L^*-R^*)}$	$R < R^*,\ \frac{R^{\varepsilon_2}(L-R)}{R^{*\varepsilon_2}(L^*-R^*)} \geq 1 \geq \frac{R^{\varepsilon_1}(L-R)}{R^{*\varepsilon_1}(L^*-R^*)}$	$R < R^*,\ \frac{R^{\varepsilon_1}(L-R)}{R^{*\varepsilon_1}(L^*-R^*)} > 1$
Pattern of Specialization	Home: good 2 Foreign: both goods	Home: good 2 Foreign: good 1	Home: both goods Foreign: good 1
Restricted Nash Equilibrium Strategy Pair	(R_2, R_A^*)	(R_2, R_1^*)	(R_A, R_1^*)
Utility	$U = R^{*(\varepsilon_1-\varepsilon_2)/2}R^{\varepsilon_2}(L-R)$ $U^* = R^{*\varepsilon_A}(L^*-R^*)$	$U = U^*$ $= [R^{*\varepsilon_1}(L^*-R^*)]^{1/2}[R^{\varepsilon_2}(L-R)]^{1/2}$	$U = R^{\varepsilon_A}(L-R)$ $U^* = R^{(\varepsilon_2-\varepsilon_1)/2}R^{*\varepsilon_1}(L^*-R^*)$