Collective Excitations and Nonequilibrium Phase Transition in Dissipative Fermionic Superfluids

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We predict a new mechanism to induce collective excitations and a nonequilibrium phase transition of fermionic superfluids via a sudden switch on of two-body loss, for which we extend the BCS theory to fully incorporate a change in particle number. We find that a sudden switch on of dissipation induces an amplitude oscillation of the superfluid order parameter accompanied by a chirped phase rotation as a consequence of particle loss. We demonstrate that when dissipation is introduced to one of the two superfluids coupled via a Josephson junction, it gives rise to a nonequilibrium dynamical phase transition characterized by the vanishing dc Josephson current. The dissipation-induced collective modes and nonequilibrium phase transition can be realized with ultracold fermionic atoms subject to inelastic collisions.

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Introduction.—Collective excitations of superconductors and superfluids have been widely studied in condensed matter physics [1–20]. Recent experimental progress in ultracold atoms has enabled studies of out-of-equilibrium dynamics of superfluids [21–24]. For example, a periodic modulation of the amplitude of the order parameter excites the Higgs amplitude mode, which has been observed with ultracold fermions [23] and in solid-state systems by light illumination on BCS superconductors [25–33]. As for collective phase modes, the Nambu-Goldstone mode exists in neutral superfluids, and the relative-phase Leggett mode has been predicted for multiband superfluids [2,33–38]. In particular, ultracold atoms allow for a dynamical control of various system parameters, offering an ideal playground to investigate collective modes. However, they suffer from atom loss due to inelastic scattering, which has received little attention in literature.

In dissipative open quantum systems, the dynamics, after environmental degrees of freedom are traced out, is nonunitary and described by a completely positive and trace-preserving map [39,40]. Such nonunitary dynamics is relevant for atomic, molecular, and optical systems, drastically changing various aspects of physics such as quantum critical phenomena [41,42], quantum phase transitions [43–45], quantum transport [46,47], and superfluidity [48,49]. In particular, high controllability of parameters in ultracold atoms has enabled investigations of nonequilibrium quantum dynamics induced by dissipation [49–63], and studies of fermionic superfluidity in ultracold atoms undergoing inelastic collisions have achieved remarkable progress [48,49,64–71]. The effect of particle loss in fermionic superfluids has been studied in the framework of the non-Hermitian BCS theory [49]; however, it ignores a significant change in particle number due to quantum jumps. It is crucially important to go beyond the non-Hermitian framework to describe the long-time dynamics of a superfluid and associated collective modes of the order parameter.

In this Letter, we theoretically investigate collective excitations and a nonequilibrium phase transition of fermionic superfluids driven by a sudden switch on of two-particle loss due to inelastic collisions between atoms. By formulating a dissipative BCS theory that fully incorporates a change in particle number, we find that dissipation fundamentally alters the superfluid order parameter and induces collective oscillations in its amplitude and phase. In particular, we elucidate that a coupling between the order parameter and dissipation leads to a chirped phase rotation, in sharp contrast to the case of an interaction quench in closed systems [see Fig. 1(a)].

To experimentally observe the collective phenomena induced by dissipation, we propose introducing a particle loss in one of two coupled superfluids to induce a relative-phase oscillation analogous to the Leggett mode [2,33–38] [see Fig. 1(b)]. The phase mode causes an oscillation of a

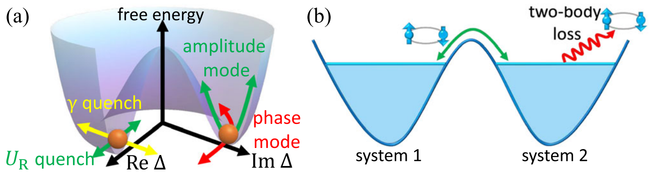

FIG. 1. (a) Schematic illustration of the amplitude and phase modes in a Mexican-hat free-energy potential as a function of the complex order parameter Δ, when either the interaction U_R or the dissipation γ is suddenly switched on. A sudden quench of the interaction U_R and that of the dissipation γ kick Δ in a direction parallel and perpendicular to the radial direction, respectively. Note that a finite change of γ excites both the phase and amplitude modes. (b) Two superfluids coupled via a Josephson junction, where one superfluid (system 2) is subject to two-body loss.

Josephson current around a nonvanishing dc component. Remarkably, when dissipation becomes strong, the coupled system undergoes a nonequilibrium phase transition characterized by the vanishing dc Josephson current, which can be regarded as a generalization of a dynamical phase transition [11,12,72,73] to dissipative quantum systems. Our findings can experimentally be tested with ultracold atoms through introduction of dissipation via a photoassociation process [50,59].

Dissipative BCS theory.—We consider ultracold fermionic atoms described by the three-dimensional attractive Hubbard model

$$H = \sum_{k\sigma} \epsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - U_R \sum_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} c_{i\uparrow}, \quad (1)$$

where $U_R > 0$, $\epsilon_{\boldsymbol{k}}$ is the single-particle energy dispersion, and $c_{\boldsymbol{k}\sigma}$ ($c_{i\sigma}$) denotes the annihilation operator of a spin-σ fermion with momentum $\boldsymbol{k}$ (at site i). When the system is subject to inelastic collisions, scattered atoms are lost to a surrounding environment, resulting in dissipative dynamics as observed experimentally [50,55,56,58]. Here, we study the time evolution of the density matrix ρ which is described by the Lindblad equation [39,40]

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i[H,\rho] - \frac{\gamma}{2}\sum_i (\{L_i^\dagger L_i, \rho\} - 2L_i \rho L_i^\dagger), \quad (2)$$

where $L_i = c_{i\downarrow} c_{i\uparrow}$ is a Lindblad operator that describes two-body loss with loss rate $\gamma > 0$. We note that the kinetic energy of lost atoms is large because of large internal energy of atoms before inelastic collisions. Under such situations, atoms after inelastic collisions are quickly lost into the surrounding environment and the Born-Markov approximation is justified [74–76].

We first study how the standard BCS theory is generalized in open dissipative systems by formulating a time-dependent mean-field theory in terms of a closed-time-contour path integral [77,78]. We start with a generating functional defined as

$$Z = \mathrm{tr}\rho = \int \mathcal{D}[c_-, \bar{c}_-, c_+, \bar{c}_+] e^{iS} = 1, \quad (3)$$

with an action

$$S = \int_{-\infty}^{\infty} dt \Big[\sum_{\boldsymbol{k}\sigma} (\bar{c}_{\boldsymbol{k}\sigma+} i\partial_t c_{\boldsymbol{k}\sigma+} - \bar{c}_{\boldsymbol{k}\sigma-} i\partial_t c_{\boldsymbol{k}\sigma-}) - H_+ + H_- + \frac{i\gamma}{2}\sum_i (\bar{L}_{i+} L_{i+} + \bar{L}_{i-} L_{i-} - 2L_{i+}\bar{L}_{i-}) \Big], \quad (4)$$

where the subscripts $+$ and $-$ denote forward and backward paths, $H_\alpha = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} \bar{c}_{\boldsymbol{k}\sigma\alpha} c_{\boldsymbol{k}\sigma\alpha} - U_R \sum_i \bar{c}_{i\uparrow\alpha} \bar{c}_{i\downarrow\alpha} c_{i\downarrow\alpha} c_{i\uparrow\alpha}$, $L_{i\alpha} = c_{i\downarrow\alpha} c_{i\uparrow\alpha}$, and $\bar{L}_{i\alpha} = \bar{c}_{i\uparrow\alpha} \bar{c}_{i\downarrow\alpha}$ ($\alpha = +, -$). Note that the action has U(1) symmetry under $c_{i\sigma\alpha} \to e^{i\theta} c_{i\sigma\alpha}$ though the particle number of the system is not conserved [79,80]. By introducing auxiliary fields via the Hubbard-Stratonovich transformation, we rewrite the action in a quadratic form of fermionic Grassmann fields as [49,81]

$$S = \int dt \Big\{ \sum_{\boldsymbol{k}} \Big[\bar{\psi}^t_{\boldsymbol{k}+} \begin{pmatrix} i\partial_t - \epsilon_{\boldsymbol{k}} & -\Delta \\ -\Delta^* & -i\partial_t + \epsilon_{\boldsymbol{k}} \end{pmatrix} \psi_{\boldsymbol{k}+} - \bar{\psi}^t_{\boldsymbol{k}-} \begin{pmatrix} i\partial_t - \epsilon_{\boldsymbol{k}} & -\Delta \\ -\Delta^* & -i\partial_t + \epsilon_{\boldsymbol{k}} \end{pmatrix} \psi_{\boldsymbol{k}-} \Big] \Big\}, \quad (5)$$

where $\bar{\psi}_{\boldsymbol{k}\alpha} = (\bar{c}_{\boldsymbol{k}\uparrow\alpha}, c_{-\boldsymbol{k}\downarrow\alpha})^t$ and $\psi_{\boldsymbol{k}\alpha} = (c_{\boldsymbol{k}\uparrow\alpha}, \bar{c}_{-\boldsymbol{k}\downarrow\alpha})^t$ ($\alpha = +, -$), with t denoting transposition. Here Δ is the superfluid order parameter which can be determined from the requirement that the action be extremal as [81]

$$\Delta = -\frac{U}{N_0}\sum_{\boldsymbol{k}} \mathrm{tr}(c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow} \rho) \equiv -\frac{U}{N_0}\sum_{\boldsymbol{k}} \langle c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow} \rangle, \quad (6)$$

where $U = U_R + i\gamma/2$ is an effective complex coupling constant including a contribution from the atom loss [49], and N_0 is the number of sites. Importantly, the order parameter includes the loss rate γ, which leads to dissipation-induced collective modes as discussed below. The action (5) describes the mean-field time-evolution equation of the density matrix as

$$\frac{d\rho}{dt} = -i[H_{\mathrm{eff}}, \rho], \quad (7)$$

$$H_{\mathrm{eff}} = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \begin{pmatrix} \epsilon_{\boldsymbol{k}} & \Delta \\ \Delta^* & -\epsilon_{\boldsymbol{k}} \end{pmatrix} \Psi_{\boldsymbol{k}}, \quad (8)$$

where $\Psi_{\boldsymbol{k}} = (c_{\boldsymbol{k}\uparrow}, c^\dagger_{-\boldsymbol{k}\downarrow})^t$ is the Nambu spinor. In the Supplemental Material [81], we show that Eq. (7) can be derived from two different methods, i.e., the mean-field theory for the Lindblad equation and the time-dependent Bogoliubov–de Gennes analysis. While Eq. (7) describes unitary evolution, it is consistent with the original Lindblad equation (2) as a consequence of the time-dependent BCS ansatz [81].

We use Anderson's pseudospin representation [1,7–12,14,32] defined by $\boldsymbol{\sigma}_{\boldsymbol{k}} = \frac{1}{2}\Psi_{\boldsymbol{k}}^{\dagger} \cdot \boldsymbol{\tau} \cdot \Psi_{\boldsymbol{k}}$ and $H_{\rm eff} = 2\sum_{\boldsymbol{k}} \boldsymbol{b}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}_{\boldsymbol{k}}$, where $\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)$ is the vector of the Pauli matrices. The pseudospins satisfy the commutation relations $[\sigma_{\boldsymbol{k}}^j, \sigma_{\boldsymbol{k}}^k] = i\epsilon_{jkl}\sigma_{\boldsymbol{k}}^l$. For simplicity of notation, we omit the bracket and regard $\boldsymbol{\sigma}_{\boldsymbol{k}}$ as the expectation value of the pseudospin operator. By using the commutation relation of the pseudospins, Eq. (7) is mapped to the Bloch equation:

$$\frac{d\boldsymbol{\sigma}_{\boldsymbol{k}}}{dt} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}}, \tag{9}$$

$$\boldsymbol{b}_{\boldsymbol{k}} = (\mathrm{Re}\Delta, -\mathrm{Im}\Delta, \epsilon_{\boldsymbol{k}}). \tag{10}$$

Equation (9) shows that the superfluid dynamics is characterized by precession of a pseudospin in an effective magnetic field $\boldsymbol{b}_k$. Here, the order parameter is determined self-consistently from the pseudospin expectation value as

$$\Delta = |\Delta|e^{i\theta} = -\frac{U}{N_0}\sum_{\boldsymbol{k}}(\sigma_{\boldsymbol{k}}^x - i\sigma_{\boldsymbol{k}}^y). \tag{11}$$

It is noteworthy that the norm of the pseudospin is conserved by the Bloch equation (9). The time evolution of the particle number due to particle loss is obtained from Eq. (7) as

$$\frac{1}{N_0}\frac{dN}{dt} = -\frac{2\gamma|\Delta|^2}{|U|^2}, \tag{12}$$

which reflects the dynamics of the order parameter.

Collective excitations: Phase and amplitude modes.—We numerically solve the Bloch equation (9) self-consistently under the condition (11). As an initial state, we prepare a BCS ground state with $\gamma = 0$, whose pseudospin representation is given by $\sigma_{\boldsymbol{k}}^x(0) = -\Delta_0/\sqrt{\epsilon_{\boldsymbol{k}}{}^2 + \Delta_0^2}$, $\sigma_{\boldsymbol{k}}^y(0) = 0$ and $\sigma_{\boldsymbol{k}}^z(0) = -\epsilon_{\boldsymbol{k}}/\sqrt{\epsilon_{\boldsymbol{k}}{}^2 + \Delta_0^2}$ with $\Delta_0 \in \mathbb{R}$. The single-particle energy $\epsilon_{\boldsymbol{k}}$ is measured from the Fermi energy of the initial state. The bandwidth W is defined by the energy difference between the upper and lower edges of the energy spectrum with a constant density of states. We then switch on the atom loss γ at $t = 0$. The ensuing dynamics shown in Fig. 2 are obtained by the second-order Runge-Kutta method. In the long-time limit, the amplitude of the superfluid order parameter Δ is suppressed due to dissipation, indicating a decay of superfluidity [see Fig. 2(a)]. We note that the order parameter decays in the long-time limit due to a decrease of the particle number [see Fig. 2(b)], and such behavior has no counterpart in the quench in isolated systems [10,11]. Remarkably, after the dissipation γ is introduced, the U(1) phase of the order parameter rotates and shows chirping, i.e., its angular velocity increases with time [see Figs. 2(a) and 2(b)] as a consequence of the dynamical shift of the Fermi level [81]. This property is unique to the dissipative superfluid and distinct from the usual dynamics in isolated systems where the U(1) phase stays constant [10–12]. The phase rotation is understood from an initial-state free energy as a function of Δ [see Fig. 1(a)]. When dissipation is introduced, the sudden quench of the imaginary part of U in Eq. (11) pushes the order parameter towards the direction perpendicular to the radial direction irrespective of the initial choice of the gauge. Another way to understand the phase rotation is to introduce an effective chemical potential as $\Delta(t) = \exp[-2i\int_0^t \mu_{\rm eff}(t)dt]\Omega(t)$ $(\Omega \in \mathbb{R})$. By performing a global gauge transformation from Δ to Ω, the Bloch equation is written in the Larmor frame on which the energy dispersion is given by $\xi_{\boldsymbol{k}}(t) = \epsilon_{\boldsymbol{k}} - \mu_{\rm eff}(t)$. This gauge transformation indicates that the phase rotation corresponds to a decrease of the effective chemical potential, which is consistent with the behaviors of $\dot{\theta}$ and N in Fig. 2(b). This result can naturally be understood from the fact that the phase and the particle number are conjugate variables.

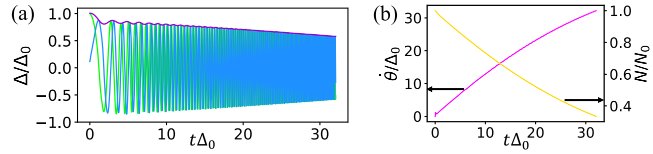

FIG. 2. Dynamics of a superfluid after the atom loss with $\gamma = 2.81\Delta_0$ is switched on for the initial state with $U_R = 12.2\Delta_0$ and bandwidth $W = 46.8\Delta_0$, where Δ_0 is the superfluid order parameter in the absence of the atom loss. (a) Real parts (light green), imaginary parts (blue), and the amplitude (violet) of the order parameter. (b) Angular velocity (pink) and particle number (yellow) plotted against time. The figures indicate a chirped phase rotation and an amplitude oscillation of Δ.

We also find amplitude oscillations in $|\Delta|$ as shown in Fig. 2(a). The amplitude oscillations are more pronounced when the interaction and the dissipation are simultaneously quenched [81]. The mechanism behind the oscillations is that the quench of the imaginary part of U changes the absolute value of Δ [see Fig. 1(a)]. The frequency of the amplitude oscillation is close to $2\Delta_0$ at an early stage, and increases as time evolves. This behavior is distinct from that of an isolated system, where the amplitude mode is characterized by the constant frequency. Such behavior can be observed from the measurement of the time-dependent particle number via Eq. (12).

Collective excitations: Leggett mode.—To observe the chirped phase rotation of the superfluid order parameter that is a unique feature of dissipative superfluids, we propose that the phase rotation induced by dissipation can be detected when two superfluids are connected via a Josephson junction, which has been realized in ultracold atoms [85–91]. As the phase difference in the two superfluid order parameters is gauge-invariant, it leads to an observable Josephson current. We introduce dissipation to one of the two superfluids as schematically illustrated in Fig. 1(b) and assume that they are coupled via a tunneling Hamiltonian [2,38]

$$H_{\rm tun} = -\frac{V}{N_0}\sum_{kk'}(c^\dagger_{1k\uparrow}c^\dagger_{1-k\downarrow}c_{2-k'\downarrow}c_{2k'\uparrow} + \text{H.c.}), \qquad (13)$$

where $V > 0$ is the amplitude of Cooper-pair tunneling between system 1 without dissipation and system 2 with two-particle loss. By performing a mean-field analysis, we can write the system Hamiltonian as $H_{\rm syst} = H_1 + H_2 + H_{\rm tun} = H'_1 + H'_2$, where $H_i \equiv \sum_{k\sigma}\epsilon_k c^\dagger_{ik\sigma}c_{ik\sigma} + \sum_k(\Delta_i c^\dagger_{ik\uparrow}c^\dagger_{i-k\downarrow} + \text{H.c.})$ $(i = 1, 2)$ is the mean-field Hamiltonian of system i and $H'_i \equiv H_i - (V/N_0)\sum_{kk'}(\langle c_{j-k'\downarrow}c_{jk'\uparrow}\rangle c^\dagger_{ik\uparrow}c^\dagger_{i-k\downarrow} + \text{H.c.})$ $[(i, j) = (1,2)$ or $(2,1)]$. In the pseudospin respresentation, the Hamiltonian is written as $H'_i = 2\sum_k \boldsymbol{b}_{ik}\cdot\boldsymbol{\sigma}_{ik}$ with an effective magnetic field $\boldsymbol{b}_{ik} = (\text{Re}\Delta'_i, -\text{Im}\Delta'_i, \epsilon_{ik})$, which yields the Bloch equation $d\boldsymbol{\sigma}_{ik}/dt = 2\boldsymbol{b}_{ik}\times\boldsymbol{\sigma}_{ik}$. The self-consistent conditions for the order parameters read $\Delta_1 = |\Delta_1|e^{i\theta_1} = -(U_R/N_0)\sum_k(\sigma^x_{1k} - i\sigma^y_{1k})$ and $\Delta_2 = |\Delta_2|e^{i\theta_2} = -(U/N_0)\sum_k(\sigma^x_{2k} - i\sigma^y_{2k})$, where N_0 is the number of sites of each system. Here, the relations $\Delta'_i = \Delta_i - (V/N_0)\sum_k(\sigma^x_{jk} - i\sigma^y_{jk})$ $[(i, j) = (1, 2)$ or $(2,1)]$ are satisfied. Then, the Josephson current between the two superfluids is given by the rate of change in the particle number of system 1:

$$\frac{1}{N_0}\frac{dN_1}{dt} = -\frac{4V|\Delta_1||\Delta_2|}{U_R|U|}\sin(\theta_2 - \theta_1 + \delta), \qquad (14)$$

where $\delta = \tan^{-1}(-\gamma/2U_R)$ is the phase shift due to the sudden switch on of the atom loss.

We numerically solve the coupled Bloch equations for $\boldsymbol{\sigma}_{ik}$. We assume that dissipation γ and tunneling V are turned on at $t = 0$ for the BCS ground state. The numerical results for weak dissipation are shown in Figs. 3(a1)–3(d1). In Figs. 3(a1) and 3(b1), the dynamics of two superfluids almost synchronize with each other because the timescale of particle loss is comparable with the inverse tunneling rate. In the pseudospin picture, the dynamics of particle numbers shown in Fig. 3(c1) can be interpreted as the nutation of pseudospins. Importantly, we see that, although the particle number of the system decreases in time, the corresponding amplitude of the order parameter stays almost constant. This implies that the condensate fraction against the total particle number becomes larger than that of the initial state. As inferred from Fig. 3(d1), the Josephson current oscillates around its dc component. Such behavior is reminiscent of Shapiro steps in a Josephson junction under irradiation of a microwave [92]; however, in the present case, the Josephson current oscillates *spontaneously without any external field*. Moreover, from Fig. 3(d1), the frequency of the oscillation of the phase difference between the two systems is close to that of the relative-phase mode known as the Leggett mode [2,38] whose dispersion relation is given by $\omega_L = 2\sqrt{(\lambda_{12} + \lambda_{21})|\Delta_1||\Delta_2|/\det\lambda}$, where $\lambda_{11} = \lambda_{22} = U_R/W$, $\lambda_{12} = \lambda_{21} = V/W$, and $\det\lambda = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}$. We note that ω_L includes the effect of loss through the order parameters. The Leggett mode with frequency ω_L has been discussed in the context of a collective mode in a multiband superconductor irradiated by light [38]. The agreement between the frequencies of the relative-phase modes in very different situations can be understood as follows. When dissipation is weak, the time evolution of an order parameter is given by $\Delta_i(t) = \exp[-2i\int_0^t dt\mu_{i\rm eff}(t)]|\Delta_i(t)|$ with an effective chemical potentials $\mu_{i\rm eff}$. Then, by performing a global gauge transformation from $c_{ik\sigma}$ to $c_{ik\sigma}\exp(i\int_0^t\sum_i\mu_{i\rm eff}dt/2)$, we can linearize the Bloch equation with respect to the relative phase difference between Δ_i's by following Ref. [38].

Nonequilibrium phase transition.—In the presence of strong dissipation, the order parameter of system 2 oscillates faster than that of system 1 [see Figs. 3(a2) and 3(b2)] and the phase difference monotonically increases in time [see Fig. 3(d2)]. This is because the dissipation rate larger than the tunneling rate makes system 1 fail to follow the

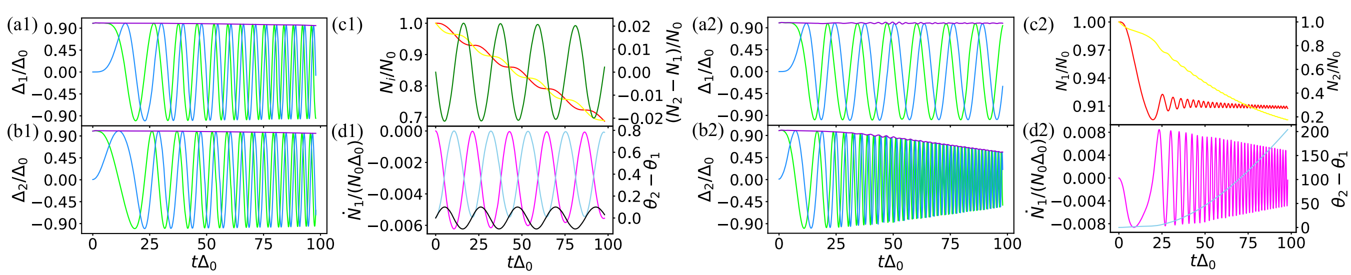

FIG. 3. Dynamics of two fermionic superfluids after the switch on of the atom loss γ and the tunnel coupling $V = 0.02\Delta_0$ with $U_R = 3.06\Delta_0$ and bandwidth $W = 5.11\Delta_0$, where $\gamma = 0.03\Delta_0$ for (a1)–(d1) and $\gamma = 0.06\Delta_0$ for (a2)–(d2). (a),(b) Real parts (light green), imaginary parts (blue), and amplitudes (violet) of the order parameter for systems 1 and 2. (c) Particle numbers of system 1 (red) and system 2 (yellow), and their difference [green, in (c1)]. (d) Josephson current (pink) and phase difference (light blue) between the two systems. The black curve in (d1) shows an oscillation at frequency ω_L for comparison.

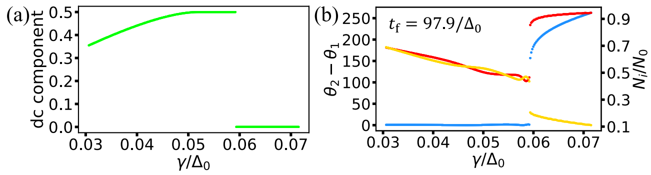

FIG. 4. (a) dc component of the Josephson oscillation defined by $[\max_{0\le t\le t_f}\{\sin(\theta_2(t)-\theta_1(t)+\delta)\}+\min_{0\le t\le t_f}\{\sin(\theta_2(t)-\theta_1(t)+\delta)\}]/2$ with $t_f = 97.9/\Delta_0$. (b) Phase difference between the two systems (blue) and particle numbers of system 1 (red) and system 2 (yellow) after a sufficiently long time ($t_f = 97.9/\Delta_0$). The parameters used are $U_R = 3.06\Delta_0$, $V = 0.02\Delta_0$, and $W = 5.11\Delta_0$.

decay of system 2, resulting in the dynamics similar to that of a single superfluid shown in Fig. 2. In particular, the chirped phase rotation of the superfluid order parameter of system 2 can be detected from the Josephson current [Fig. 3(d2)]. As the superfluidity of system 2 is suppressed, the Josephson current also decays, and the particle number of system 1 settles to a constant after some transient time [see Fig. 3(c2)]. The latter behavior is attributed to the continuous quantum Zeno effect [49,74,75,93–95], which states that strong dissipation prevents tunneling and inhibits loss in system 1. In fact, an effective decay rate of system 1 is given by $\gamma_{\rm eff} \equiv |V_{\rm eff}|^2/\gamma$ with an effective tunneling rate $V_{\rm eff} = V\Delta_2/U_R$ from Eq. (13), leading to suppression of decay $\gamma_{\rm eff} \to 0$ for $|\Delta_2|^2/\gamma \to 0$.

The two dynamically distinct regimes of superfluid behaviors suggest the existence of dynamical phases of matter [72,73] in dissipative superfluids. The qualitative change in the superfluid behaviors with respect to the dissipation strength highlights a dynamical phase transition characterized by the vanishing dc Josephson current [Fig. 4(a)], where the dc component of the Josephson oscillation is defined by $[\max_{0\le t\le t_f}\{\sin(\theta_2(t)-\theta_1(t)+\delta)\}+\min_{0\le t\le t_f}\{\sin(\theta_2(t)-\theta_1(t)+\delta)\}]/2$ [see Eq. (14)] after a sufficiently long time evolution with $t_f = 97.9/\Delta_0$. We emphasize that the dynamical phase transition in dissipative superfluids is essentially distinct from the phase transition between ground states in a non-Hermitian BCS superfluid [49]. The former is caused by a change in particle number in the long-time dynamics, whereas the latter is caused by an exceptional point of a non-Hermitian BCS Hamiltonian, which is relevant to the short-time dynamics during which the number of particles does not change [96]. From Fig. 4(b), we see that the phase difference $\theta_2-\theta_1$ starts to increase monotonically at the critical point and that the difference in particle number $(N_2-N_1)/N_0$ becomes much larger. The behavior of the phase difference is reminiscent of the localization-diffusion transition of a quantum-mechanical particle moving in a washboard potential in the presence of frictional force [97–99]. However, the origin of the transition shown in Fig. 4 is essentially different from frictional force, since it cannot change the particle number. In fact, as shown in the Supplemental Material [81], the dynamical phase transition in Fig. 4 is triggered by the competition between the Josephson coupling and particle loss. Moreover, as the steady state is a vacuum due to the particle loss, the dynamical phase transition is observed only in the transient dynamics, and thus distinct from steady-state transitions.

Conclusions.—We have investigated the loss-quench dynamics of fermionic superfluids, and have demonstrated that the dynamics exhibits amplitude and phase modes with chirped oscillations, the latter of which is a salient feature of a dissipative superfluid. To observe the chirped phase rotation, we have proposed a Josephson junction comprised of dissipative and nondissipative superfluids. We have shown that the relative-phase Leggett mode can be detected from the Josephson current for weak dissipation. Remarkably, when dissipation becomes strong, the superfluids exhibit the unique nonequilibrium phase transition triggered by particle loss. Our prediction can be tested with ultracold atomic systems of ^{6}Li [86–90], for example, by introducing dissipation using photoassociation processes [50,59,100]. It is of interest to explore how the dimensionality or confinement by a trap potential affects the dynamics and associated collective modes [15–18].

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