The Effects of Primary and Conditioned Reinforcement on Choice Under Rapid-acquisition Conditions

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Abstract

BACKGROUND: Studies examining preference in concurrent schedules and concurrent-chains procedures have primarily involved steady-state designs with repeated exposure to unchanging reinforcer contingencies. More recently, the acquisition of preference has been studied using rapid-acquisition designs in which reinforcer contingencies change unpredictably across sessions. Grace and colleagues have proposed the cumulative decision model (CDM; Christensen & Grace, 2010; Grace & McLean, 2006; see Grace, 2016 for review) which accounts for acquisition of choice in concurrent chains as well as at steady state. However, the effects of conditioned reinforcement (i.e., terminal-link entries) are not considered in the CDM, and few studies have examined how relative terminal-link entry rate affects acquisition of choice.

AIMS: The first aim was to how preference is affected by rapidly changing reinforcer contingencies in concurrent schedules and concurrent chains. The second aim of this research was to develop a model of choice which could account for preference exhibited in concurrent schedules under rapid-acquisition conditions and use this model to extend the CDM to account for effects of conditioned reinforcement.

METHODS: Experiment 1 was conducted using concurrent schedules and examined preference in terms of sensitivity to reinforcer ratio when the distribution of log reinforcer ratios was uniform or bimodal. Experiment 2 examined sensitivity to terminal-link entry ratio when left and right terminal-link durations were both short (4 s) or long (16 s). Experiment 3 examined sensitivity to terminal-link entry ratio when initial-link durations were both short (4 s) or long (16 s). Experiment 4 examined sensitivity to terminal-link immediacy ratio when terminal links signalled which initial-link alternative resulted in terminal-link entry (cued) and when they did not (uncued). Left and right terminal links were of unequal durations (4 s and 16 s) or (16 s and 4 s). Experiment 5 examined sensitivity to terminal-link entry ratio when terminal links were cued and uncued but left and right terminal-link durations were equal.

RESULTS: In our concurrent schedules experiment, preference was stronger when the distribution of reinforcers was bimodal (approximate matching) and weaker when its distribution was uniform (severe under matching). In Experiment 2, short terminal-link durations resulted in approximate matching while long terminal links produced severe under matching. Bias was strong in pigeons in Experiment 3 but not systematic in terms of order or initial-link duration. Preference between short and long initial links was also approximately equal. However, there was a decrease in preference during the last 12 sessions for short initial links. In Experiment 4, preference was much stronger in cued terminal links (severe over matching) compared to uncued (matching). In Experiment 5, when left and right terminal-link durations were equal, preference was approximately equal between cued and uncued terminal links (severe under matching). However, preference was stronger when birds had prior exposure to uncued terminal links and preference was pooled across cued and uncued conditions (approximate matching). The decision model we developed was able to describe the difference in preference in Experiment 1 but was unable to explain it. This model was also able to account for observed preference in our concurrent-chains experiments, with the exception of Experiment 3. Only experiments using concurrent chains resulted in strong biases. Biases were strongest when terminal links were long and there was no prior exposure to short terminal links. Bias was weakest during short terminal links but only when there was prior exposure to long terminal links. When left and right terminal links were of equal durations, bias was stronger with prior exposure to cued terminal links and weaker with prior exposure to uncued terminal links. Bias was strongest when terminal links were uncued with no prior exposure to cued terminal links. Bias was weakest when terminal links were uncued but with prior exposure to cued terminal links.

CONCLUSION: Reinforcer contingencies such as terminal-link duration and signalled terminal links have been observed in previous research. Although the effect of initial-link duration on preference was not observed, there was a decrease in preference at the end of training for short initial links. This indicates that preference under rapid-acquisition conditions could be similar to that observed in steady-state research if pigeons had repeated exposure to unchanging terminal-link entry ratios. Results indicate that preference acquisition under rapidly changing terminal-link entry and immediacy ratios is similar to that observed in steady state studies. The strong biases observed are inversely related to the strength of preference. Although we offer one plausible explanation for this, further scrutiny of this relationship is required to understand it. In terms of predictive accuracy, the decision model we developed was able to predict observed preference and accounted for an adequate amount of variance in the data, with the exception of Experiment 5. The model's accounts of preference at the individual trial level were largely successful. However, when left and right terminal-link durations were equal and did not change, there was a more noticeable effect of the preceding session's terminal-link entry rates on current responding which indicates a molar component of responding prevalent under these conditions in Experiment 5, More research is required to improve our model's ability to account for this molar component. Moreover, additional empirical work is required to allow the model to explain preference acquisition in concurrent schedules under rapid acquisition conditions while satisfying parameter invariance.

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List of Abbreviations

ANOVA – analysis of variance

CCM – Contextual Choice Model

CDM – Cumulative Decision Model

CI – confidence interval

COD – changeover delay

DRT – Delay-reduction Theory

EWMA – exponentially weighted moving average

FI – fixed interval

GML – Generalized Matching Law

HVA – Hyperbolic Value-added Model

LL – lower limit

Max – maximum

MaxVar – maximal-variation

Min – minimum

MinVar – minimal-variation

PRBS – pseudorandom binary series

RS – response strength

SS – session sixth

UL – upper limit

VAC – variance accounted for

VI – variable interval

Chapter 1

The Effects of Primary Reinforcement and Terminal-link Entry Ratio on the Acquisition of Choice Behavior: A Literature Review

1.1 A Historical Overview

Decision-making is based on the choices which are made across different time frames. How these choices are made has become an important topic of research in the experimental analysis of behaviour. Quantifying how important is far from simple. A search of the PsycInfo database using the keyword "choice" within the Journal of the Experimental Analysis of Behaviour yielded 4041 results, published from 1958 to 2021. However, Mazur (2001) found that 47% of published studies in the *Journal of the Experimental Analysis of Behaviour* from 1997 to 1998 allowed subjects to choose between at least two operant responses. It is likely that the figure obtained from a general search of the PsycInfo database under-estimates the distribution of research which is based on response allocation.

Choice in the current context involves the allocation of time to two or more activities. It has been argued that all behaviour is choice due to the fact that every situation allows for at least two activities (Baum, 2010). Even in a laboratory, it is impossible to create an environment so restricted that the subject is limited to engaging in a single activity. For example, rats bring their own background activities such as licking and biting in terms of how they interact with the response levers. Therefore, if behaviour entails choice then referring to choice is also to refer to behaviour. Following this line of reasoning, the dynamics of choice entail the dynamics of behaviour, and are determined by reinforcement. There are two main types of reinforcement; primary and conditioned.

Primary reinforcers are stimuli such as food, which fulfil the intrinsic, biological needs of an organism, while conditioned reinforcers are stimuli which have acquired the potency of a reinforcer based on individual experiences of each organism (Herrnstein, 1964). A conditioned reinforcer is a neutral stimulus which is repeatedly paired with a primary reinforcer and therefore becomes associated with primary reinforcement. In this context of learning, primary and conditioned reinforcement are functionally equivalent as reinforcers in terms of their effect on choice behaviour.

1.2 Steady State Models of Concurrent Schedules

The first procedure to consider in reviewing operant research on choice is concurrent schedules. During a concurrent schedule of reinforcement, at least two simple schedules of reinforcement are simultaneously made available to a subject (Alsop & Elliffe, 1988). Responding on these schedules commonly results in access to reinforcement in the form of food. The major question is how behaviour is allocated between these alternatives. Studies using concurrent schedules have looked at the influence of reinforcer type, magnitude, probability, delays and immediacy on response allocation. The first published study which used concurrent schedules to examine response allocation was conducted by Findley (1958). Further research on delay to rewards and the size of the rewards have shed more light on how choice behavior is affected by these factors (Grace, Sargisson & White, 2012).

The first types of studies which examine behaviour utilize steady state designs. In steady state designs, subjects receive many sessions of training where the contingencies remain unchanged until choice behaviour has stabilized. The contingencies are then altered and training in the next experimental condition begins (Grace & Hucks, 2012). Consideration of these studies resulted in the development of much steady-state research using concurrent schedules to explore how the behaviour associated with primary reinforcement was subject to greater stimulus control.

Predictive models of choice operate under the assumption that behaviour is a consequence of the obtained reinforcement assigned to each choice. One such model is the Matching Law, which has been shown to be a good predictor of choice behaviour in concurrent schedules of reinforcement. An important factor which affects the relative response ratio is the ratio of reinforcement. Choice behaviour is measured as the log ratio of responding between alternatives (Chung & Herrnstein, 1967). The matching law states that choice behaviour is “matched” to the relative intensity of the received rewards for each choice. Determiners of relative response rate have been extended to immediacy and magnitude ratios of reinforcers.

The difference in the conditions is commonly implemented by varying the delays to reinforcement because delay has been found to be a variable which influences choice. Chung et al. (1967) conducted an experiment where pigeons were required to peck one of two response keys. They found that the relative frequency of reinforcement did not sufficiently account for changes in the relative frequency of responding. The delay was essential for accounting for changes in responding. They also found that changing the delay associated with one response key affected the rate of responding on both keys.

1.2.1 The matching law

One contingency which affects the response rate is the rate of reinforcement. According to Herrnstein (1970), when a pigeon's pecks on two response keys are reinforced on two variable interval (VI) schedules, the pigeon distributes its pecks between the keys as follows:

$$\frac{B_L}{B_L + B_R} = \frac{R_L}{R_L + R_R}, \qquad (1.01)$$

where B_L is the number of pecks on the left key, B_R is the number of pecks on the right key, and R_L is the rate of reinforcement delivered by the left key and R_R is the rate of reinforcement delivered by the right key. This effect is known as the matching law. Baum and Rachlin (1969) provided additional support for this relationship by finding that the time allocated to each alternative and number of reinforcers obtained were also well described by this equation. They proposed that matching occurs depending on the relative value of each activity:

$$\frac{T_1}{T_2} = (\frac{r_1}{r_2})(\frac{a_1}{a_2})(\frac{i_1}{i_2}) = (\frac{V_1}{V_2}) \qquad (1.02)$$

where T refers to the time spent pecking on a response key while V is the value of pecking on that key. The relative value of each behaviour is computed as the product of relative rate, amount and immediacy of reinforcement (r, a, and i). Chung and Herrnstein (1967) found that it didn't matter how one measured behaviour. Regardless of whether it was response rate or time allocation, pigeons matched their relative frequency of their responding to relative reinforcer values. This happened regardless of what property of the reinforcer was varied; rate, immediacy or magnitude with immediacy being defined as the reciprocal of delay.

Aside from the phenomenon of matching in choice behaviour, two other outcomes also occur; under-matching and over-matching (Baum, 1974). Numerous studies of choice eventually gave rise to a more general version of the matching law known as the generalized matching law (GML):

$$\text{Log}\left(\frac{B_L}{B_R}\right) = s.\text{Log}\left(\frac{R_L}{R_R}\right) + \text{Log}\ c \qquad (1.03)$$

where s is the slope of the regression line and *Log c* is the inherent bias for a particular alternative. Matching occurs when the value of s is 1. Undermatching refers to any level of preference which falls short of matching whereby the slope of the regression line is less than 1. This occurs when changes in preference works against changes in reinforcement. Over-matching occurs when changes in preference occur not only in the direction of changes in reinforcement but are more extreme than matching. This form of the matching law makes analysis much simpler because straight lines can be fitted using regression analysis.

In Equation (1.2), the c parameter is called bias because it indicates the direction and magnitude of preference when equal relative magnitudes of reinforcement (R_L=R_R) would predict indifference. According to Baum et al. (1974), bias represents a certain amount of preference which is not accounted for. This occurs primarily through some independent variable affecting preference which has not been taken into account. From this perspective, the occurrence of bias is an indicator of the experimenter's inability to control all the independent variables which affect choice. The occurrence of these biases is more deeply explored via an extension of the matching law called the quantitative law of effect.

1.2.2 The quantitative law of effect

The matching law proposed by Herrnstein describes the relationship between measures of reinforcement and measures of the subsequent desired behaviour (Heyman & Monaghan, 1987). Measures of reinforcement include but are not limited to the delay, immediacy, rate and amount of reinforcement. In a similar way, the matching law has resulted in a much better understanding of the effects of reinforcement on the behaviour of organisms. The equation proposed by Herrnstein (1970) is a typical hyperbola,

$$B = \frac{kr}{r + r_{\mathrm{e}}}, \qquad (1.04)$$

where B denotes the response rate, r refers to the rate of reinforcement of the target behaviour, k and r_e are parameters whose respective values are obtained by fitting Equation (1.3) to the data. In other words, the matching law says that the rate of responses is dependent on reinforcement rate and other variables which are represented by the parameters k and r_e (reinforcement of extraneous behaviour).
The sum of r and r_{e} (the maximum amount of total behaviour) within a given time frame is constant.

While there are two competing interpretations of the matching law, one interpretation of the matching law is that one or both parameters are affected by the reinforcement efficacy and motor performance determinants (Staddon, 1970). Theories based on this interpretation predict that response requirements or different schedules of reinforcement will affect both parameters simultaneously. Studies examining this interpretation have found that varying the response requirement or reinforcement schedules influenced both parameters.

The other interpretation of the matching law states that k refers to the motor component of the reinforced response while r_e refers to the rate of reinforcement for the response that is not reinforced (Heyman et al., 1987). This means that k can be thought of as the response rate for the reinforced response if the organism spent all its time engaged in only the reinforced behaviour from experimental procedures. The magnitude of k is therefore equal to the asymptotic response rate and is measured in the same units as B such as responses per minute. The magnitude of r_e is measured in the same units as r such as reinforcers per hour.

According to the matching law, response rate is a negatively accelerating function of the rate of reinforcement (Heyman et al., 1987). Response rate approaches the maximum asymptote but does not reach it. When r_e is equal to r, the amount of reinforcement for the desired behaviour and other behaviours are equal, which means that response rate should be half of the asymptote. This means that features of an experiment which affect the ease in which reinforcement is obtained should affect k but not r_e. Manipulations which affect the perceived value of the reinforcer such as deprivation should influence r_e but not k. Specifically, as the perceived value of the reinforce decreases, higher reinforcement rates are required to obtain a response rate equal to half of the asymptotic response rate.

Evidence for this interpretation of the matching law was demonstrated by Heyman et al. (1987) where they varied deprivation levels and lever weight separately across two experiments. When they varied deprivation levels in rats, the perceived value of the experimental reinforcer (water) should increase and result in a decrease in r_e since more time should be spent obtaining reinforcement. This effect was observed in the deprivation experiment. When they increased the lever weight causing access to reinforcement to be more difficult, r_e should be unaffected while k should decrease. This effect was observed in their lever-weight varying experiment.

Response topography is affected by r_e and therefore influences response rate. Heyman et al. (1987) varied deprivation levels and found a clear-cut increase in extraneous behaviour. Response topography does not have to rely on the deprivation or satiation level of subjects. It can also be affected by reinforcer magnitude as shown by Heyman and Monaghan (1994).

They varied the levels of sucrose concentration and found that stronger sucrose concentrations resulted in greater increases in response rates at lower reinforcement schedules. The asymptotic response rate remained unaffected.

1.3 Steady State Models of Concurrent-Chains Choice

Autor (1960) introduced a procedure to study choice between conditioned reinforcers. Pigeons pecked at two keys which produced access to one of two conditioned reinforcers where the pairing with primary reinforcement was maintained throughout the study. Controls in the study made it possible to separate the effects of primary and conditioned reinforcement on choice. Autor found that preference for conditioned reinforcement was determined by the frequency of pairing with primary reinforcement. Autor's procedure is known as concurrent chains and has been widely used in choice.

In a typical concurrent-chains procedure with pigeons, the subject responds to two concurrently available keys, each illuminated by the stimulus associated with the initial link of one of the chains. Responses on each key occasionally result in the stimulus for the terminal link of the chain for that particular key (MacEwen, 1972). In such schedules, the independent variable most often involves a difference in the conditions, arranged during the terminal links. Such a difference usually involves varying lengths of the delay between responding in the terminal links and receiving reinforcement. The dependent variable is the distribution of responses in the initial, concurrently presented links in the chain. Alternatively, it is also possible for delays in the initial links to be varied while delays of the terminal links are kept constant. Such a design would investigate the effect of initial link schedules as a predictor of choice.

In concurrent chains, terminal-link stimuli are commonly viewed as conditioned reinforcers. This is because they are paired with reinforcement and their presentation not only affects but also maintains response allocation during the initial links. Therefore, initial link responding is indicative of the relative value of terminal-link stimuli as conditioned reinforcers (Grace and Savastano, 2000). For example, if terminal-link reinforcement schedules associated with the left and right alternatives are fixed –interval (FI) 8 seconds and FI 16 seconds, initial-link response allocation typically favours the alternative associated with the FI 8 second reinforcement schedule. In this way, responding during the initial-link phase tends to favour the alternative leading to the terminal link which is associated with a shorter delay to reinforcement.

Initial results in these studies were moderately well accounted for by the matching law. Results from later studies were better accounted for by models such as delay reduction, contextual choice and hyperbolic value-added models, with the last two being based on the hyperbolic decay model (Mazur et al., 2001). These models share the assumption that choice in the initial links is dependent on the relative value of the terminal links.

1.3.1 Delay-reduction theory

Although the matching law provides a good model of prediction for preference in many concurrent schedules of reinforcement, it fails to predict certain behaviour exhibited in concurrent chains. According to Fantino (1969) matching does not occur when entry into the terminal links is controlled by unequal variable interval (VI) schedules. In order to account for preference in such schedules, he proposed a model known as delay-reduction theory (DRT). This theory states that choice behaviour depends on the amount of reduction in the expected time to primary reinforcement, signified by entry into a terminal link, relative to the amount of reduction in expected time to reinforcement, signified by entry into the other terminal link. According to this model,

$$\frac{B_L}{B_L + B_R} = \frac{T - t_L}{2T - t_L - t_R}, \qquad (1.05)$$

where B_L is the number of responses made on the left key during the initial link, B_R is the number of responses made on the right key during the initial link, T is the average time to reinforcement calculated from the onset of the initial links, t_L is the expected time to reinforcement for the left terminal link and t_R is the expected time to reinforcement for the right terminal link.

One other factor which affects choice is the relative length of the terminal links. MacEwen et al. (1972) conducted an experiment using concurrent chained schedules where the initial links were constantly on a VI 30 second schedule or two VI 60 second schedules. The delays to reinforcement in the terminal links were varied across conditions. The results of his study indicated that preference for the shorter of two terminal links was greater when the terminal links were overall long. This phenomenon is known as the terminal link effect. Williams and Fantino (1978) reproduced this effect in their study while keeping the ratios of the duration of the left and right terminal links equal. They observed that the matching law produced an inaccurate prediction of preference in their study and failed to account for the terminal link effect.

Relative response ratios, which are commonly used to measure preference, are not only dependent on the ratio of the delays but also the absolute values of the delays. By varying the delays in the terminal links and relative reinforcer immediacy, preference for the left or right response key during the initial links could be controlled. The matching law makes an incorrect prediction of response ratios by predicting that relative response ratios would remain unchanged. The matching law makes such a prediction because the ratio of the delays in the left and right response keys of the terminal links are kept constant. This also indicates a limit of the matching law. Specifically, the matching law only takes into account ratios of delay and not the absolute values of delay. It is because of this reason that DRT produces a much more accurate prediction of preference.

Another phenomenon which is not adequately explained by the matching law is the initial link effect. Fantino et al. (1969) conducted a study where the terminal links were always VI 30-seconds and VI 90-seconds. Initial links were varied across conditions, from 600 seconds to 40 seconds. According to the matching law, preference should always be 75%

in favour of the VI 30-second terminal link. However, the results from his study showed that subjects were near indifferent between the terminal links when the initial links were 600s. When the VI schedules were short, subjects preferred the VI 30-second terminal link. DRT was able to predict the initial link effect.

1.3.2 Contextual Choice Model

In the case of the contextual choice model (CCM), terminal link sensitivities to changes in the relative initial-link and terminal link durations are an increasing function of terminal link duration and a decreasing function of initial-link duration (Grace, 1994). In the CCM, sensitivity to terminal-link reinforcement contingencies varies in accordance with the temporal context of reinforcement:

$$\frac{B_{\mathrm{L}}}{B_{\mathrm{R}}} = b\left(\frac{\mu_{1\mathrm{R}}}{\mu_{1\mathrm{L}}}\right)^{a_1}\left[\left(\frac{\mu_{2\mathrm{R}}}{\mu_{2\mathrm{L}}}\right)^{a_2}\left(\frac{\chi_{i\mathrm{L}}}{\chi_{i\mathrm{R}}}\right)^{a_i}\right]^{\left(\frac{Tt}{Ti}\right)^k} , \qquad (1.06)$$

where *R* refer to the rate of conditioned reinforcement (terminal-link entry rate), *k* is the scaling parameter which represents how sensitive terminal link sensitivities (a_2 and a_i) are related to changes in relative initial and terminal-link durations. *L* and *R* refer to terminal-link contingencies of the left and right alternatives and *b* is a bias parameter. T_t/T_i is the ratio of terminal-link to initial-link duration per reinforcement. This ratio decreases with increasing initial-link duration and increases with decreasing terminal-link duration resulting in the CCM being able to predict the initial link and terminal link effect.

Equation 1.05 reduces to become the GML as T_t approaches zero which suggests that the CCM can be viewed as an extension of the GML that can be used in concurrent schedules and concurrent chains. From this perspective, although the GML is unable to account for certain phenomenon in concurrent chains procedures, using the GML as a base for developing new predictive models of choice is still a viable option. Empirical evaluations of CCM have indicated that it accounts for approximately 90% of the variance across multiple studies which used relative initial link responding as measures of exhibited preference (Grace et al., 2012).

Although the CCM is able to account for a substantial amount of exhibited preference in concurrent schedules and concurrent chains procedures, it is limited by measuring terminal-link value as a function of the average delay to reinforcement. If the average delay to reinforcement during terminal links is equal for both alternatives, the CCM predicts that preference for either alternative should be equal since both alternatives have the same terminal link value. Herrnstein (1964) conducted a concurrent chains study whereby the initial links were both VI 60 seconds and each terminal link trial resulted in a maximum of two reinforcers. During the terminal-links, the alternative that led to reinforcement was either a VI 15-second followed by a FI 15-second or a FI 15-second followed by a VI 15-second schedule. Subjects consistently preferred the variable schedule over the fixed schedule despite the average delay to reinforcement being equal. According to the CCM, since it predicts that preference should be equal for both the VI and FI terminal links, the CCM fails to predict greater exhibited preference for variability.

1.3.3 Hyperbolic value-added model

The hyperbolic value-added model (HVA) proposed by Mazur et al. (2001) is another model for choice behaviour that was based on the GML. This model assumes that the relative values of the terminal and initial link stimuli are discounted according to a hyperbolic function:

$$\frac{B_{\mathrm{L}}}{B_{\mathrm{R}}} = b\left(\frac{R_{\mathrm{L}}}{R_{\mathrm{R}}}\right)^{ar}\left(\frac{V_{\mathrm{L}} - a_d V_i}{V_{\mathrm{R}} - a_d V_i}\right), \qquad (1.07)$$

where V_L and V_R are the values of the left and right terminal-link stimuli, V_i being the value of the initial links and a_d denotes a sensitivity parameter. As delays to reinforcement increase, the perceived values of reinforcement decrease according to a parabolic function as described by the hyperbolic decay model (Mazur, 1984). Since terminal-link stimuli are associated with reinforcement, the value of the terminal-links decreases with increasing terminal-link duration. Specifically, terminal-link stimuli are associated with shorter delays to reinforcement therefore their value is greater than initial-link stimuli.

The ability of the CCM and HVA to describe archival data in studies using concurrent chains was compared (Mazur et al., 2001). By fitting both models to sets of data used by Grace et al. (1994), both models accounted for approximately equal percentages of variance in initial-link response allocation (CCM = 90.8&; HVA = 89.6%). Predictions by a version of the DRT with added parameters for sensitivity provided a moderately good description of the data albeit somewhat less than the CCM and HVA, with 83.0% of the variance being accounted for. Although the CCM and HVA provide a good description of the data, which model does so to a better extent could not be distinguished based on overall goodness of fit.

1.4 Reinforcement Contingencies That Affect Choice

1.4.1 Reinforcement probability

One variable which has been examined is the probability of reinforcement during the terminal links. During these studies, terminal links result in either blackout (extinction) or reinforcement, with an assigned probability to each of these two outcomes (Kendall, 1974). Using a concurrent chains procedure, Kendall found that pigeons responded more on the alternative leading to the terminal link which resulted in reinforcement 50% of the time than the one which led to terminal links resulting in reinforcement 100% of the time. These results were problematic because they indicated a preference for the alternative which resulted in lower overall rates of reinforcement (less rich schedule). Greater preference for the 50% alternative when the outcomes are signalled has been due to the conditioned reinforcer value of the stimulus being associated with reinforcement. The reason being that this stimulus signals a relatively greater reduction in delay to reinforcement, compared to the stimulus associated with 100% reinforcement (Dunn & Spetch, 1990). Evidence in favour of this hypothesis was supported by including a 5-second gap on the stimulus associated with 50% reinforcement probability. Including this temporal gap resulted in lower preference for that alternative (McDevitt, Spetch, and Dunn, 1997). When a similar temporal gap was imposed on the alternative associated with 100% reinforcer probability, there was no systematic influence on preference for that alternative.

1.4.2 Terminal-link duration

Multiple studies have examined the effects of terminal-link delays on choice. One consistent result which has been replicated is that preference depends on terminal-link durations relative to initial-link durations. Specifically, as terminal-link delays increase relative to initial-link delays, preference increases (Grace et al., 2012). This terminal-link effect was replicated by Grace (2004) who varied terminal-link schedules but were maintained in a constant ratio while their delays relative to initial-link duration were increased. He found that the increase in preference was a negatively accelerated function of average terminal-link delay relative to initial-link delay, indicating there was limit on how strongly terminal-link delays influence preference relative to initial-link delays.

1.4.3 Initial-link duration

Berg and Grace (2006) examined the effect of initial-link duration on preference in concurrent chains. In their experiment, pigeons responded on a concurrent chains procedure where the terminal links were FI 8 seconds and FI 16 seconds. Across conditions, the initial-link duration was either short (VI 8 seconds) or long (VI 24 seconds). Initial-link durations were switched every 20 sessions. All possible combinations of initial-link durations preceding and following the switch in initial-link schedules were explored (short-short, short-long, long-short, long-long). They found that preference acquisition was faster when durations of initial links following the switch were long. Moreover, initial-link duration had no effect on the rate at which preference was acquired.

1.5 Acquisition of Choice in Steady-state Studies

The matching law explains the relationship between response allocation and reinforcer rate over an extended period of time, after choice behaviour has stabilized. A majority of research studies have used the matching law to examine choice behaviour (Baum, 2003). The GML and other models of behavioural choice such as DRT and the CCM have been able to account for choice behaviour in steady-state studies. During the past 20 years, there has been an increased emphasis on the acquisition of preference. The reason being that understanding how response allocation changes in accordance with different reinforcer contingencies could provide valuable insight into processes underlying steady-state choice. Eventually, these insights should be able to explain why matching occurs as well as account for other results in steady state choice.

1.5.1 Preference acquisition in concurrent schedules

A series of studies in the 1990s examined the acquisition of choice in concurrent schedules and established certain basic preference acquisition processes that occur in concurrent schedules (Bailey and Mazur, 1990; Mazur and Ratti, 1991). These studies involved making a choice between two alternatives of differing reinforcement probabilities but with a constant difference in reinforcer probability between the alternatives. Their aim was to determine whether acquisition of preference was dependent on the ratio or difference in reinforcer probability. They found that preference acquisition occurred more rapidly with greater ratios. For instance, when the probability of reinforcement on the left and right alternatives were .12 and .02 versus .30 and .40, preference acquisition occurred more quickly in the .12 and .02 condition.

When reinforcer ratios were kept constant, the rate of acquisition of preference was higher in conditions that resulted in higher overall probability of reinforcement (Mazur, 1992). For example, the ratio of reinforcer probability was kept constant at 2:1 and reinforcement probabilities of the left versus the right alternatives ranged from .04:.02 to .16:.08. Preference acquisition occurred most rapidly in the .16:.08 conditions. Mazur described a model based on a linear operator rule that could account for these results. This model states that the response strength for a particular alternative increases with the occurrence of reinforcement but decreases when reinforcement does not occur. Using this model, Mazur was able to show that it predicted these results more accurately than several previous models for choice acquisition in concurrent schedules.

Another major area of research in concurrent schedules is within-session preference acquisition. Davison and Baum (2000) exposed pigeons to seven different pairs of concurrent VI-VI schedules during each session. With the overall reinforcer rate kept constant, the reinforcer ratios of the left versus the right alternative were 1:27, 1:9, 1:3, and 1:1, 3:1, 9:1 and 27:1. Each session, all seven ratios were implemented pseudo-randomly such that all seven ratios occurred. They found that preference tended to shift towards the alternative that resulted in the last reinforcer. Moreover, after preference had shifted towards the just-reinforced alternative, preference gradually decreased in such a way that by the time 25 seconds had elapsed after the last reinforcer, preference reflected an indifference towards either alternative. Successive reinforcers from the same alternative produce a smaller effect on preference compared to reinforcement that follows a series of reinforcers from the other alternative.

1.5.2 Preference acquisition in concurrent chains

Mazur, Blake and McManus (2001) examined the effect of terminal and initial link contingencies on preference in pigeons. During certain conditions, the terminal-link durations were kept constant at 1:1 while the response alternative during the initial link phase which resulted in richer terminal-link entry (80% vs 20%) was varied. In the other conditions, either alternative during the initial-link phase resulted in terminal-link entry 50% of the time while the terminal-link delays were varied (2 seconds versus 18 seconds or 5 seconds versus 20 seconds). They found that acquisition of preference was quicker when terminal-link entry arranged during the initial-links was changed instead of varying terminal-link durations.

There have also been other studies which examined whether initial–link responding is dependent on overall terminal-link duration (Mazur, 2002a). In his study, terminal-link ratio was kept constant at 1:1. Terminal link delays were either both 1 second or 20 seconds while the proportion of terminal-link entries from responding on the left alternative ranged from 10% to 90%. He found that percentage of responses on the richer alternative changed when the proportion of terminal link entries assigned to the left alternative of the initial links was switched. However, this switch in preference was the same regardless of terminal link duration. Results of his study suggested that when terminal-link durations were kept equal, preference acquisition occurred independently of absolute terminal link duration.

The results of Mazur et al. (2002a) were challenged by a study conducted by Grace (2002a, experiment 1). In Grace's experiment, a successive reversal design was used in which the position of the richer terminal link schedule was switched every 20 sessions. Terminal link schedules were either both variable interval (VI) or FI and their overall durations were short (8 seconds vs 16 seconds) or long (16 seconds vs 32seconds). All possible combinations of terminal link pairs were examined across conditions. He found that high value terminal link schedule pairs (short over long, VI over FI) resulted in faster changes in preference when they occurred after the reversal. When high value terminal schedule pairs preceded the reversal, changes in preference were slower. This slower change in preference can be attributed to Grace's study investigating how long it took for preference to switch from one asymptote to another under a steady-state rather than rapid acquisition design.

1.6 Preference in Rapid Acquisition Studies

The second type of choice acquisition in concurrent chains involves rapid acquisition designs. This procedure involves changing terminal link schedules unpredictably from session to session. This design is termed “rapid acquisition” because the development of preference reaches its asymptote within individual sessions (Grace, Bragason, & McLean, 2003).

Grace et al. (2003) investigated initial-link preference using a rapid acquisition study. In their study, the duration of the left alternative was always FI 8 seconds. The duration of the right terminal link was FI 4 seconds or FI 16 seconds. Terminal link duration for the right alternative was varied based on a 31-step pseudorandom binary series (PRBS) to ensure assignment of terminal link duration was random (Hunter & Davison, 1985; Schofield & Davison, 1997). After two PRBS blocks (62 sessions) of training, response allocation was determined solely by the immediacy ratio of the terminal links of the current session. Moreover, sensitivity to relative immediacy rapidly rose and reached its asymptote within the first half of each training session.

Grace and McLean (2006) tested whether the range of variation in terminal-link delays across sessions affected the rate of preference acquisition. In their experiment, the minimal variation condition entailed left terminal links that were FI 8 seconds and right terminal links which were either FI 4 seconds or FI 16 seconds. In the maximal variation condition, a unique pair of FI terminal link delays sampled from a potentially infinite sample of delays and immediacy ratios was used per session. Although the range of variability of terminal link delays was greater in the maximal variation condition, both conditions resulted in the left and right alternatives being equally often associated with the shorter-delayed terminal link. Across both conditions, the average delay of the terminal link and the expected delay of the left versus the right terminal links were kept equal. In both conditions, the position of the shorter terminal link was changed daily according to a 31-step PRBS. Although the standard deviation of response accuracy for the shorter delay terminal link was greater in the maximal variation condition, they found no systematic differences in the sensitivity of response rate to immediacy ratio between the minimal and maximal variation groups.

1.7 A Decision Model

Grace et al. (2006) proposed a decision model which could account for their results. According to this model, allocation of responses during initial links is determined by response strengths associated with each alternative:

$$\frac{B_{\mathrm{L}n}}{B_{\mathrm{R}n}} = \frac{RS_{\mathrm{L}n}}{RS_{\mathrm{R}n}}, \qquad (1.08)$$

where B is initial-link response rate, RS is response strength and L, R and n denote left and right alternatives and trial number. Response strength (RS) for a particular alternative changes only after reinforcement has been earned in a terminal link. Therefore, only the RS of the alternative which resulted in terminal-link entry and subsequently, reinforcement, is updated. It is assumed that the subject makes a "decision" as to whether the preceding delay was long or short, relative to the history of experienced delays. If the delay for a terminal link was judged to be short, RS for the corresponding initial link increases. If the delay was judged to be long, RS decreases. The increase and decrease in RS for short and long terminal links occur according to the following linear operator rule:

$$RS_{n+1} = RS_n + (Max_{RS} - RS_n) * \Delta \qquad (1.09a)$$

(for ‘‘short delays”);

$$RS_{n+1} = RS_n - (RS_n - Min_{RS}) * \Delta \qquad (1.09b)$$

(for ‘‘long delays”).

In Equation (1.08a) and Equation (1.08b), RS is response strength, n and n + 1 denote trial numbers, Max_{RS} and Min_{RS} refer to the maximum and minimum asymptotic response strengths, and Δ is a parameter for learning rate. Although a single pair of equations defines the model, separate response strengths for each alternative are maintained. Either Equation (1.08a) or Equation (1.08b) is used to update the RS for each alternative at the end of each trial. In their study, Grace et al. (2006) reset the RS of each alternative to 0.5 at the beginning of each session.

The model is able to make reasonably good predictions for response allocation which are reflective of the obtained data. Since the relationship between the standard deviation of response accuracy and the accuracy of correctly classifying a delay as long or short is an inverse one, with other parameter values held constant, smaller standard deviations result in greater sensitivities to immediacy ratios. Because consistently smaller standard deviations were obtained in the minimal variation condition, this seems to suggest that the model predicts sensitivity to relative immediacy ratios will be greater in the minimal variation. However, sensitivity also depends on the learning rate parameters (Δ_L, Δ_R). Predictions of this model are able to reproduce the obtained pattern of individual differences in sensitivity to relative immediacy ratios.

The model is still incomplete for a number of reasons. In traditional steady-state designs, schedule parameters do not change until response allocation is stable. The resetting of response strengths to intermediate values at the beginning of each session would be erroneous. The model also cannot accurately account for certain well-known effects involving overall terminal-link and initial-link duration such as the initial-link effect (Christensen & Grace, 2008). Moreover, since preference during each session in rapid-acquisition designs are primarily affected by that same session's reinforcement contingencies, values of the learning rate parameters and the carryover changes in within-session response strength must be primarily dependent on the frequency of environmental change. Mazur (1997) found that changes in subsequent reinforcement have a greater impact on subsequent choice behaviour when the overall rate of reinforcement is low and reinforcement contingencies are changed often in the recent past. Despite these limitations, an extension of this basic model which could simulate "real-time" preference in terms of whether the delay associated with each alternative was classified as long or short in every trial, could be used. This version of the model could potentially account for concurrent chains choice behaviour in both steady state and rapid-acquisitions designs.

1.7.1 The cumulative decision model (CDM)

Building on the decision model proposed by Grace et al. (2006), a decision model was developed to account for acquisition of choice within each session. The model proposes that after reinforcement in a terminal link, pigeons make a decision about whether the just-experienced delay was short or long (Grace & McLean, 2015). If the delay was judged to be short, responding in the initial link that results in the terminal link increases. If the delay is judged to be long, responding decreases. In terms of predicting response strength for each alternative at the end of each trial, the CDM combines equations 8a and 8b into a single expression. Changes in response strength for each alternative are described according to a linear operator rule,

$$\Delta RS_{n+1} = (p_s)\,(\Delta)(RS_{Max} - RS_n) + (1\text{-}\ p_s)(\text{-}\ \Delta)(RS_n - RS_{Min}). \qquad (1.10)$$

ΔRS_{n+1} is the change in response strength after reinforcement on trial n, p_s is the probability that the delay was judged to be short, RS_{Max} and RS_{Min} are maximum and minimum response strengths (normally set at 1 and 0.01) and Δ is a learning parameter. The equation states that the change in response strength is a constant proportion of the difference between the current and asymptotic strengths, depending on the probability that the delay was judged to be long or short.

The CDM assumes that delays form a normal distribution and are scaled according to a logarithmic function (Grace et al., 2006). The model calculates the probability of a ‘‘short’’ decision as the total area under the normal distribution to the right of the previous delay, log *D*. With exposure to consecutive trials, subjects compare the delay associated with each alternative to a criterion (log *C*) to decide whether the delay is long or short. This criterion is the mean of the distribution and is obtained by finding the average of the log delays across both alternatives. The accuracy of judging whether delays are long or short is given by the standard deviation (σ) of the distribution. From the beginning to the end of each session, the probability, *p*, that a delay, log *D*, is judged to be short is described in the following manner,

$$1 - \Phi\,(\log D, \log C, \sigma), \tag{1.11}$$

with Φ being the cumulative normal distribution with a mean of log *C* and standard deviation = σ being evaluated at log *D*. When σ was high, delays were less accurately classified and response allocations approximated a linear function of log immediacy ratios, thereby resulting in generalized matching. When σ was low, delays were more accurately classified and response allocations followed a non-linear, sigmoidal function of the log immediacy ratios. Since the CDM is simply the combination of Equations (1.08a) and (1.08b), it does not take into account effects of overall terminal-link and initial-link duration.

By incorporating the delay between the onset of initial links and terminal-link entries and the delay between terminal-link onset and occurrence of reinforcement into the criterion (log *C*), Christensen et al. (2008) proposed this could account for the effects of terminal link and initial link duration. This was because when pigeons decided whether a delay was long or short, they did not clearly discriminate between initial-link or terminal-link intervals. When initial-link duration was included, the criterion increased resulting in the value of *p* increasing for both terminal links. This version of the decision model was thus able to predict the initial-link effect. The model also predicted that preference would decrease when initial-link durations were very short. Moreover, with terminal- and initial links included in the criterion, increasing terminal-link durations resulted in increases in the criterion but this increase was less than proportional (Christensen & Grace, 2009a). This resulted in increases in preference since the ratio of *p* for left and right terminal links increased. This allowed the predictions of the model to account for the terminal-link effect.

In order to account for changes in *RS* across sessions, Christenson and Grace (2009b) proposed a linear-operator term coupled with an exponentially weighted moving average (EWMA; Killeen, 1981) for updating the criterion:

$$\log C_{N+1} = \beta(\log D_N) + (1 - \beta)\log C_N. \tag{1.12}$$

Equation (1.11) assumes that the criterion is updated after every transition (i.e. initial-link to terminal-link, and terminal-link entry to reinforcement or after reinforcement). The criterion values after N and $N + 1$ transitions are denoted by $\log C_N$ and $\log C_{N+1}$ while $\log D_N$ is the Nth stimulus-transition interval, and β is the weight of the most recent interval. Moreover, Christensen and Grace (2010) proposed that the memory of the most-recently experienced delay for each alternative can be computed as a EWMA of the history of delays for that particular alternative. This is because when VI schedules are used, it may be more difficult for subjects to decide whether the delay which was just experienced is long or short. According to Equation (1.9), Since *RS* is updated after the delivery of reinforcement every decision is made retrospectively. Unlike FI schedules, when the delay associated with each alternative is variable within each session, the memory of the just-experienced delay is possibly influenced by previous delays of that session.

The addition of computing a separate EWMA for each terminal link in addition to equation 10 is done so for the purpose of predicting preference for variability (Christensen et al., 2010), preference favouring VI over FI schedules. Specifically, when terminal links are VI x FI x, preference is predicted to be stronger in the VI terminal link which is consistent with this observed phenomenon in the study by Herrnstein et al. (1964). This is because even though the arithmetic mean delay is equal across both FI and VI schedules, the geometric mean (average of the log delays) are lower for VI schedules. Furthermore, the geometric mean of delays that occur according to a FI schedule equals its arithmetic mean but the geometric mean of delays that follow a VI schedule is less than its arithmetic mean. The model will predict weaker preference when both the terminal links are VI schedules (e.g., VI x VI y) compared to corresponding FI schedules (e.g., FI x FI y). This prediction occurs for the same reason it can predict the terminal-link effect: as overall terminal-link duration increases, criterion increases but to an extent which is less than proportional.

An extension of the CDM has been recently proposed to apply to terminal links which differ in magnitude or probability of reinforcement (Kyonka & Grace, 2008). It was suggested that decisions are not made on the basis of delay but rather, the value of the outcome. Value was defined as the ratio of magnitude to delay, $V = M/D$. In logarithmic terms,

$$\text{Log } V = \log M - k(\log D). \tag{1.13}$$

k is a parameter which describes the relative impact of magnitude and delay ratios on choice. This equation is a generalized version of the CDM equation because when magnitudes are equal, decisions are made purely on the basis of delay. This generalization has been shown to be able to predict effects of reward magnitude on delay and probability discounting which is similar to results shown in research with humans.

1.8 Choice in Rapid-acquisitions Conditions - Current Study

1.8.1 Effects of primary reinforcement on choice

Experiment 1/Chapter 2 investigates choice behaviour exhibited under rapid-acquisition conditions in concurrent schedules. In terms of the reinforcement contingencies which affect preference, this experiment examines how the distribution of reinforcers affects choice. Specifically, is choice is primarily affected by the average reinforcer ratio or the distribution of reinforcers? In the case of the latter, how does the distribution of reinforcers affect preference when the distribution is uniform and when it is bimodal?

1.8.2 Effects of terminal-link duration on choice

The Experiment 2/Chapter 3 involves investigating choice under rapid-acquisition conditions in concurrent chains. Reinforcer ratios are either 1:3 or 3:1. Although the durations of the left terminal link always equal that of the right terminal link, terminal link durations are short in one condition and long in the other. With left and right initial-link durations kept equal and constant, this experiment examines the effects of the absolute value of terminal-link durations on choice. Depending on whether terminal-link durations are short or long, the ratio of terminal-link to initial-link durations is either 1:2 or 2:1. This experiment looks at how the absolute value of terminal-link durations affects sensitivity to relative terminal-link entry ratio.

1.8.3 Effects of initial-link duration on choice

Like the second experiment, this experiment examines choice in concurrent chains procedures under rapid acquisitions designs. Reinforcement ratios are either 1:3 or 3:1. Although the durations of terminal links are kept equal and constant, initial links are either short or long. Depending on whether the initial-link duration is short or long, the ratio of terminal-link to initial-link duration is either 1:2 or 2:1. Experiment 3/Chapter 4 examines how the absolute value of initial link duration affects sensitivity to relative terminal-link entry ratio.

1.8.4 Effects of different delayed and signalled terminal-link stimuli

In this concurrent chains procedure, reinforcement ratios are always 1:1. The ratio of terminal link durations is either 1:4 or 4:1. Experiment 4/Chapter 5 investigates the effects of differentially cueing terminal-link stimuli on choice behaviour, specifically, whether initial-link choice between unequal terminal-link delays depends on whether unique stimuli are correlated with the terminal links (cued) or the same stimulus is associated with both terminal links (uncued). Thus, this experiment examines the effects of signalled terminal-link stimuli on the sensitivity to terminal-link durations.

1.8.5 Effects of signalled terminal-link stimuli on choice

In Experiment 5/Chapter 6, Reinforcement ratios are either 1:3 or 3:1. Initial-link durations and terminal-link durations remain unchanged. The left terminal-link duration always equals the duration of the right terminal link. The average durations of the initial links and terminal links are also equal. In this experiment, the colour of terminal-link stimuli will either signal which initial-link alternative resulted in terminal-link entry (cued) or will be independent of initial-link responding (uncued). This experiment examines the effects of signalled terminal-link stimuli on terminal-link entry ratio.

1.9 Specific Aims

The three aims of this study are built upon a review of the extensive literature on choice behaviour. This review is geared towards expanding the knowledge of how choice behaviour occurs under rapid-acquisition conditions. This is because much of the literature is based on steady state conditions while research into rapid acquisition choice behaviour is a lot scarcer. These aims are an initial step to understanding not only what level of preference behaviour is exhibited but also how this level of choice behaviour is eventually reached.

1.9.1 Aim I – rapid acquisition conditions resulting in matching behaviour

As reviewed above, the occurrence of matching behaviour in concurrent schedules is well-established but less so in concurrent chains (Luco, 1990). The first purpose is determining whether matching behaviour occurs in rapid-acquisitions conditions for both concurrent schedules and concurrent chains. This is done to determine the effects of primary reinforcement (in concurrent schedules) and conditioned reinforcement (in concurrent-chains) on choice. Conditioned reinforcement refers to entry into the terminal links. Based on the data gathered under rapid acquisition conditions, evidence for reinforcement contingencies resulting in matching and non-matching behaviour can be obtained.

1.9.2 Aim II – a predictive model of which accounts for matching

The second aim of the study is to develop an extension of the CDM to create a predictive model which accounts for matching and non-matching behaviour under rapid acquisition conditions. If primary and conditioned reinforcement are assumed to be functionally equivalent (Green 1980), then a single predictive model of choice should be able to account for the effects of both types of reinforcement. Thus far, the emphasis of most of the research on reinforcer probability has been on the effects of signalling or whether the choice between risky alternatives depends on deprivation (Caraco, Martindle, and Whitham, 1980). There have been relatively few studies which have manipulated probability with other aspects of reinforcement such as immediacy and magnitude being held constant. The aim is to create a model which can account for preference in rapid acquisition and eventually, also account for preference in steady state choice.

1.9.3 Aim III – underpinnings of matching

The development of an extension of the CDM results in an examination of what gives rise to matching and deviations from matching (severe under-matching and severe over-matching). Described in terms of changes in response strength, this description adds to the knowledge of what gives rise to different levels of preference by examining choice behaviour at the level of individual trials within each session. This analysis of choice behaviour at the molecular level allows for an explanation of choice behaviour in terms of within-session rates of preference acquisition and attained asymptotic preference.

Chapter 2

Experiment 1: The Effects of Reinforcer Distribution on Choice in Concurrent Schedules

2.1 Introduction

This first experiment examines the effects of varying the distribution of primary reinforcer ratios across sessions in a rapid-acquisition procedure. Specifically, are asymptotic preference and the rate of its acquisition different between bimodal and uniform relative reinforcer distributions in concurrent schedules? Moreover, we seek to explain how preference is acquired. This is achieved by fitting the data to a decision model we developed. The GML will be used descriptively when evaluating this decision model. We plan to investigate if predictions of the decision model result in generalised matching.

In this experiment we wanted to determine the effects of varying the range of primary reinforcement ratios on choice behaviour under rapid-acquisition conditions. A single VI 45s schedule started timing and pigeons had to peck on two illuminated keys. There were a total of 12 possible intervals for each cycle which, across each session resulted in an average delay of 45 seconds per cycle. A concurrent-schedules procedure was arranged, in which food reinforcement was scheduled probabilistically to either the left or right key after the interval had elapsed on each cycle.

There were two groups in this experiment, the minimal –variation and the maximal-variation group. In the minimal-variation group, the reinforcer ratios (left/right) were either 4:1 or 1:4. For the maximal-variation group, the probability of left-key reinforcement was determined randomly for each session, and the reinforcement ratios (left/right) varied uniformly (in logarithmic terms) between 1:8 and 8:1, such that the expected reinforcement ratios for sessions in which the left (or right) key was richer was 4:1 (or 1:4). For both groups, whether the left or right key had the richer reinforcement ratio was varied according to a pseudo-random binary series (PRBS). In this way, pigeons had to learn which key was the richer alternative every single session.

2.2 Method

2.2.1 Subjects

Twenty-seven pigeons of mixed breed and sex were maintained at 85% of their free-feeding weight through appropriate post-session feeding. Pigeons were caged individually in a vivarium which ran according to a 12-hour day and night cycle (lights were switched on at 06:00). Home cages were supplied with grit and water which was available at all times to pigeons when they were in their home cages.

2.2.2 Procedure

Each pigeon was exposed to a concurrent schedule of reinforcement. Each session ended after every pigeon had received 72 reinforcers or after 60 minutes had elapsed**.** A single VI 45-s schedule only started timing after the first response of each cycle. Durations of each cycle were sampled from an exponential progression of 12 intervals (Fleshler and Hoffman, 1962). Durations were sampled without replacement such that all 12 intervals were used six times. At the start of each cycle, both side keys were lighted yellow, and a probability gate was sampled to determine if the reinforcer for that cycle would be assigned to the left or right key. Responding on an alternative resulted in reinforcement if (a) it was made on the pre-selected key (b) an interval selected from a VI 45sschedule had elapsed, and (c) a 1.5 second changeover delay (COD) was satisfied (i.e., at least 1.5 seconds had elapsed following a changeover to the alternative which was assigned to result in reinforcement. Each reinforcer consisted of 3 seconds of access to food.

The two groups in the experiment differed based on the degree of variation of the programmed relative reinforcement rate, or more specifically, the programmed relative probability for reinforcement for the left key. In the minimal-variation group (MinVar), the probability that a reinforcer was assigned to the left key was either .8 or .2, so that the programmed reinforcer ratios (left/right) were either 4:1 or 1:4. For the maximal-variation (MaxVar) group, the probability of left-key reinforcement was determined randomly for each session, subject to the following constraints: (a) both the left and right alternatives were equally often associated with a greater probability of reinforcement; (b) the log programmed reinforcement ratios varied uniformly between log (1/8) or log (8), and (c) the expected log reinforcement ratios across sessions were log (1/4) or log (4). In this way, the programmed reinforcement ratio was the same for both groups for sessions in which the left or right key had the higher reinforcement rate, but the MinVar group used two programmed reinforcement ratios whereas for the MaxVar group the programmed ratio was sampled randomly for each session. The setup for the concurrent schedule is shown in Figure 2.01.

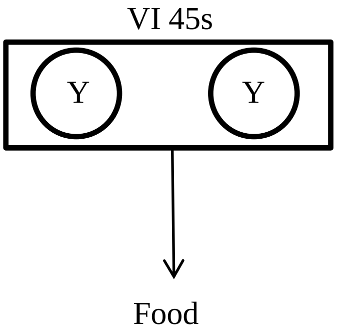


Fig 2.01. The concurrent-schedules procedure. Two keys are lighted yellow. Responses are reinforced according to a single VI 45-second schedule that results in access to food. Y = yellow key.

For both groups, whether the left or right key was assigned a greater probability of reinforcement varied across sessions according to a pseudo-random binary series (PRBS). The PRBS consisted of 31-steps and was identical to that used by Hunter and Davison (1985). For each 31-session block, the position of the more frequently reinforced alternative was Left (L), Right (R), R, R, L, L, R, L, L, L, R, L, R, L, R, R, R, R, L, R, R, L, R, L, L, R, R, L, L, L, L. 13 pigeons were randomly assigned to the minimal-variation group while 14 pigeons were assigned to the maximal-variation group. The pigeons were run in squads at different times, with consecutive numbering indicating that pigeons were in the same squad (e.g., Pigeons 111-118 were in the same squad). All pigeons completed at least 116 sessions of training (minimum: 116, maximum: 274; average = 183). The number of training sessions for each pigeon is shown in Table 2.01.

Table 2.01

Number of Training Sessions by Condition

MinVar		MaxVar	
Subject number	Training sessions	Subject number	Training sessions
111	164	115	164
112	163	116	164
113	163	117	164
114	163	118	164
172	116	173	117
181	274	174	116
182	274	175	154
183	272	176	154
184	274	177	154
225	157	178	154
226	157	185	234
227	157	186	234
228	157	187	234
		188	234

Note. There are 13 birds in the MinVar group and 14 in the MaxVar group. Bird 171 of the MinVar group was excluded due to non-responding.

2.3 Results

2.3.1 Sensitivity Analysis

Because the number of completed sessions across birds varied from squad to squad, a correlation analysis between the number of completed sessions and sensitivity to reinforcement ratio was conducted to determine if sensitivity was dependent on the number of completed sessions. Analysis was conducted on the last 50 sessions of each group. In the MaxVar group, the correlation between the number of completed sessions and sensitivity values was not significant, $r(12) = .29$, $p = .31$. The corresponding correlation for the MinVar group was also not significant, $r(11) = .009$, $p = .976$.

The generalized matching law was used to measure sensitivity to the obtained reinforcement ratio:

$$\mathrm{Log}\left(\frac{B_L}{B_R}\right) = a\mathrm{Log}\left(\frac{R_L}{R_R}\right) + \log b, \qquad (2.01)$$

B is the number of responses made, subscripts $_L$ and $_R$ refer to the left and right alternatives. a is the sensitivity of response allocation to the reinforcer ratio, and Log b is the inherent bias for a particular alternative. Sensitivity and bias were measured as the slope and intercept of regressions of log response ratios on log reinforcer ratios.

2.3.2 Between-session preference acquisition

Sensitivity values for pigeons in the MaxVar condition are shown in Figure 2.02. Data points are given for every block of 25 sessions. The data point at session 25 denotes the sensitivity of relative response ratio to reinforcer ratio as determined by pooling data from individual sessions 1-25, then applying the GML to the data across sessions. This analysis was completed for each block of 25 sessions for all pigeons. In the MaxVar group, sensitivity was 0.35 during the first 25 sessions and 0.39 during the last 25 sessions. The MinVar group exhibited a sensitivity of 0.37 during the first 25 sessions and gradually increased to reach 0.62 by the last 25 sessions. Across both groups peak sensitivity values were reached by the last two 25-session blocks. In addition to peak sensitivity at the end of training being higher in the MinVar group, there was also more individual variation in sensitivity values. Sensitivity values across MaxVar birds showed less variability throughout training to the extent that mean sensitivity values of the first 25 and last 25 sessions were approximately equal.

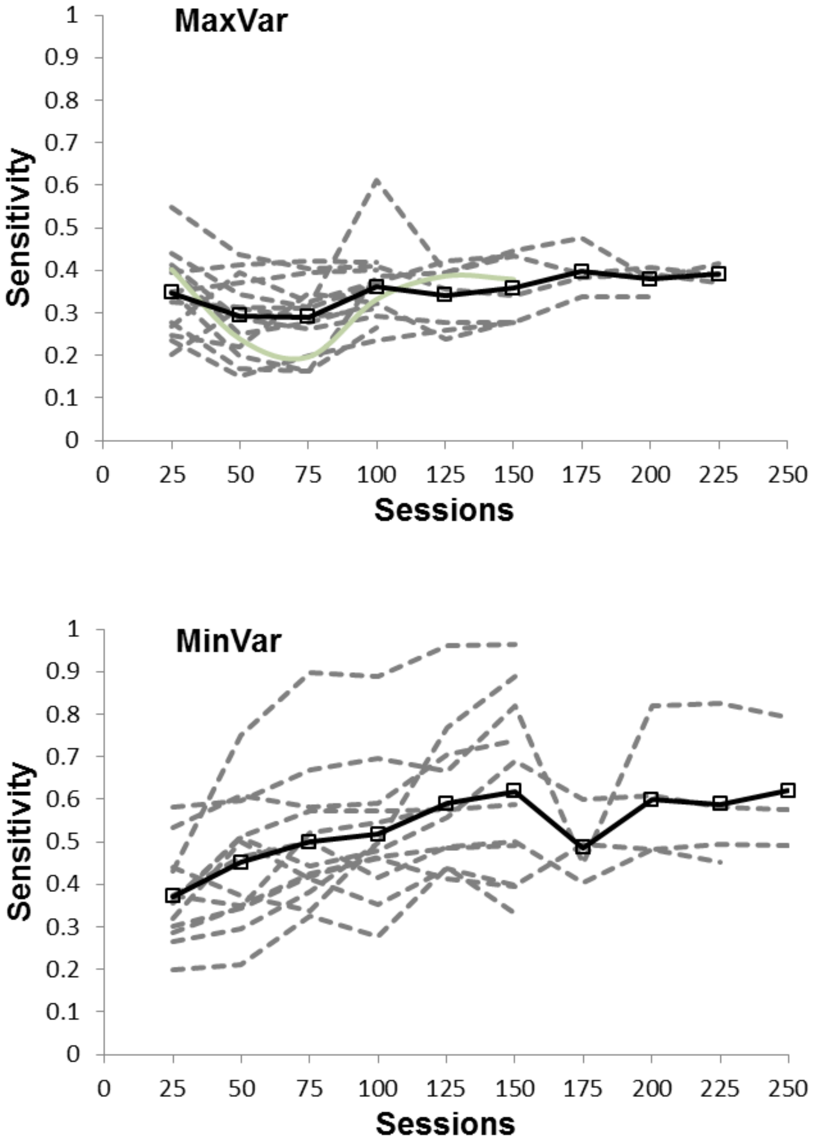


Fig 2.02. Full-session sensitivity values by 25-session blocks in MaxVar (top panel) and MinVar (bottom panel) groups. Dotted lines represent sensitivity values of individual birds. Solid lines are the mean sensitivity values across all birds in each group.

2.3.3 Preference at the beginning and at the end of training

Although Figure 2.02 suggests that sensitivity increased for the MinVar but not MaxVar groups across training sessions, the mean sensitivity by group for later sessions is computed from a decreasing number of birds due to different amounts of completed sessions. In order to determine if preference changed between the beginning and the end of training, we compared sensitivities for both groups for the first and last blocks of 25 sessions. Figure 2.03 shows the mean sensitivity values of each group during the first 25 and last 25 training sessions. Sensitivity values during the first 25 sessions were approximately equal for both MinVar ($M = 0.37$, $SD = 0.11$) and MaxVar ($M = 0.35$, $SD = 0.10$) groups. During the last 25 sessions, MinVar birds exhibited higher sensitivity values ($M = 0.60$, $SD = 0.20$) than MaxVar birds ($M = 0.35$, $SD = 0.06$). Sensitivity values were entered into a 2 × 2 repeated measures analysis of variance (ANOVA) with condition (minimal/maximal) and session block (first 25/last 25) as factors. There was a significant main effect for session block, $F(1, 25) = 28.41$, $p < .001$, $\eta^2 = .53$, and a significant main effect for condition, $F(1, 25) = 10.65$, $p = .003$, $\eta^2 = .30$. Moreover, there was a significant interaction between condition and session block, $F(1, 25) = 24.68$, $p < .001$, $\eta^2 = .50$. These results support the noticeable increase in sensitivity values of the MinVar between the first 25 and last 25 sessions. These results also indicate that this increase in sensitivity between the first and last 25 sessions did not occur in MaxVar birds.

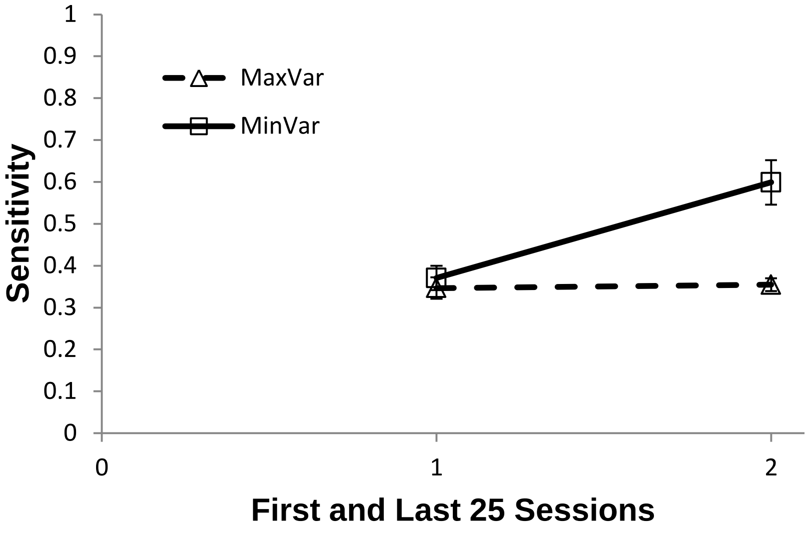


Fig 2.03. Sensitivity of relative response ratio to relative reinforcer ratio for MinVar and MaxVar groups during the first 25 and last 25 sessions. The value of "1" on the horizontal axis refers to the first 25 sessions. The value of "2" refers to the last 25 sessions. Error bars represent standard errors.

2.3.4 Lag sensitivities

To determine how response allocation changed within sessions, and how current and prior-session reinforcer ratios affected responding, we conducted a lag multiple regression analysis (Schofield et al., 1997). For each session, response allocation was computed separately for each set of 12 reinforcer cycles (six per session, 'session sixths'). For each session sixth, log response ratios were predicted by the current (log) session reinforcer ratio ('lag0'), and the log ratios from the prior 5 session ('lag1-lag5') according to the following equation:

$$\text{Log}(\frac{B_{0L}}{B_{0R}}) = a_0\text{Log}(\frac{R_{0L}}{R_{0R}}) + a_1\text{Log}(\frac{R_{1L}}{R_{1R}}) + a_2\text{Log}(\frac{R_{2L}}{R_{2R}}) +$$

$$a_3\text{Log}(\frac{R_{3L}}{R_{3R}}) + a_4\text{Log}(\frac{R_{4L}}{R_{4R}}) \ + a_5\text{Log}(\frac{R_{5L}}{R_{5R}}) + \text{Log}\, b, \qquad (2.02)$$

B represents the number of responses made on each alternative, *R* is the number of reinforcers obtained from responding on each alternative, *a* is the sensitivity to relative reinforcer ratio and subscripts lag 0 through 5 (0 = current session, 1 = previous session) denote the current session up to the preceding five sessions, and Log *b* is a bias parameter.

Figure 2.04 shows mean sensitivity to relative reinforcement ratio for lag 0 through lag 5 for all subjects across both groups during the last 50 sessions. Lag coefficients were entered into a 2 × 6 × 6 repeated measures analysis of variance (ANOVA) with condition (minimal/maximal), session sixth and lag as factors. Mauchly's test of sphericity indicated that the assumption of sphericity had been violated for within-subjects factors lag, χ^2 (14) = 79.95, $p < .001$, and session sixth, χ^2 (14) = 69.60, $p < .001$. Therefore, Greenhouse-Geisser corrections for sphericity were used for lag, ($\varepsilon = .46$) and session sixth, ($\varepsilon = .48$). The main effect of condition was significant, $F(1, 25) = 16.59$, $p < .001$, $\eta^2 = .40$. Moreover, the main effect of lag was also significant, $F(5, 125) = 147.43$, $p < .001$, $\eta^2 = .86$. Although the main effect of session sixth was not significant, $F(5, 125) = 1.61$, $p = .20$, $\eta^2 = .06$, there was a significant interaction between lag and session sixth, $F(25,625) = 71.90$. $p < .001$, $\eta^2 = .74$. The condition x lag x session sixth interaction was also significant $F(25, 625) = 5.30$, $p = .009$, $\eta^2 = .18$. In terms of the interaction between lag and session sixth, this difference persisted from the first to the last session sixth only between lag 0 and lag 1. There were no differences in sensitivity by session sixths between consecutive lags (e.g., lag 1 versus lag 2, lag 2 versus lag 3). These results indicate that the primary differences between groups occurred for the Lag0 coefficients, which were higher for the MinVar group.

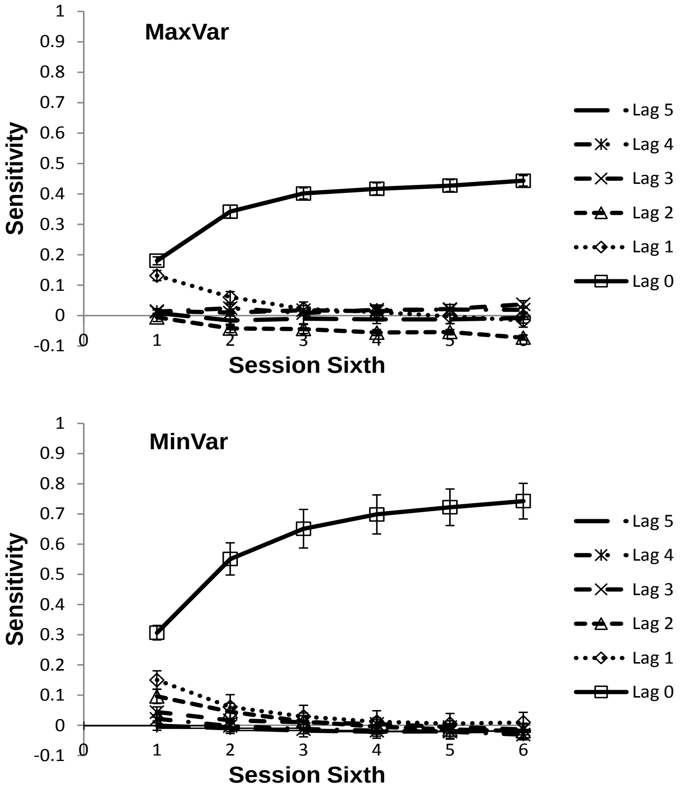


Fig 2.04. Sensitivity of relative response ratio of each session to obtained reinforcer ratio of that same session and preceding five sessions in MaxVar (top panel) and MinVar (bottom panel). Lag 0 denotes sensitivity to reinforcer ratio of that same session. Lag 1 is sensitivity to reinforcer ratio of the preceding session. Results were obtained from the last 50 sessions of each group. Error bars represent standard errors.

2.3.5 Within-session preference acquisition

Figure 2.05 shows the mean sensitivity of relative response ratio to relative reinforcement ratio by session sixth for lag 0 during the last 50 sessions of MaxVar and MinVar groups. Sensitivity during the first session sixth was approximately equal in both MinVar (*M* = 0.31, *SD* = 0.09) and MaxVar (*M* = 0.18, *SD* = 0.05) groups. Sensitivity increased over the course of consecutive session sixths in both groups. By the third session sixth, sensitivity values were substantially higher in MinVar (*M* = 0.65, *SD* = 0.23) compared to the MaxVar (*M* = 0.40, *SD* = 0.07) group and were close to peak sensitivity values. By the end of the last session sixth, sensitivity values were higher in MinVar (*M* = 0.74, *SD* = 0.21) compared to the MaxVar group (*M* = 0.44, *SD* = 0.06). The difference in sensitivity values between MinVar and MaxVar groups is approximately equal for both the third and last session sixth. This result strongly suggests that the greatest increase in sensitivity from the beginning to the end of each session occurred within the first half of each session.

Across both groups, mean sensitivity values of relative response ratio to relative reinforcement ratio gradually increased from the first session sixth to consecutive session sixths. Across both MinVar and MaxVar groups, the greatest increase in sensitivity occurred within the first three session sixths (first half of each session) to the extent that sensitivity values reached asymptotic levels within the first half of each session. This increase in sensitivity was overall greater in the MinVar group compared to the MaxVar group.

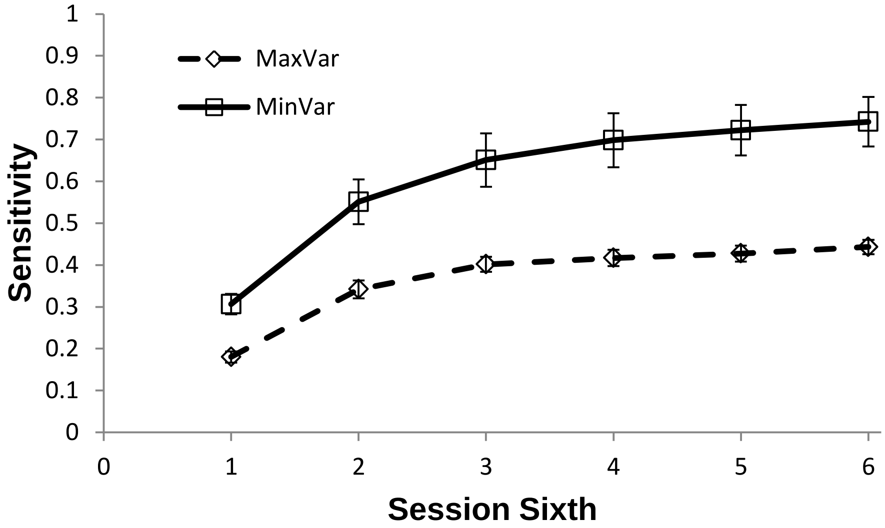


Fig 2.05. Lag 0 sensitivity of relative response ratio to obtained reinforcer ratio in MaxVar and MinVar by session sixth. Results were obtained from the last 50 sessions of each group. Error bars represent standard errors.

2.3.6 A predictive model of choice for concurrent schedules

We attempted to develop a model to account for these results. The model assumes that choice responding depends on the relative response strength of an alternative, with response strength, in turn, determined by reinforcement and extinction processes. Response strength for each alternative is updated at the end of every cycle (i.e., reinforcement delivery) and varies between 0 and 1. If responding on an alternative produces access to a reinforcer, response strength for that alternative increases according to a linear operator rule. The model also assumes that extinction processes operate during the cycle, and that response strength for an alternative might decrease while the pigeon is responding to that alternative, as well as when the pigeon is responding to the other alternative. These decreases in response strength due to extinction are modelled by exponential functions, with separate rate parameters. Specifically, the strengthening of an alternative after reinforcement is computed as follows:

$$RS_{\mathrm{updated}} = RS_{\mathrm{current}} + \alpha(1- RS_{\mathrm{current}}), \qquad (2.03)$$

where RS_{updated} is the response strength of the alternative after reinforcement, RS_{current} is the response strength of that alternative on the current cycle, and α is a learning rate parameter (constant) that determines what proportion of the difference between current and asymptotic (i.e., 1) RS is added as a result of reinforcement. Equation (2.03) only applies to response strength for the reinforced alternative.

Extinction is modelled as an exponential function of time spent responding. Specifically, the decrease in response strength, ΔRS_L, on the left alternative due to extinction is defined according to the following equation:

$$\Delta RS_{\mathrm{L}} = RS_{\mathrm{L}} - RS_{\mathrm{L}}(1-e^{-\beta_{\mathrm{o}} t_{\mathrm{L}} - \beta_{opp} t_R}), \qquad (2.04)$$

where RS_L is the response strength of the left alternative during the current cycle, β_o is the decay rate for response strength based on time spent responding to the left key, t_{L} is time spent responding on the left alternative, t_R is time spent responding on the right alternative and β_{opp} is the decay rate at which left response strength decreases with time spent responding on the right alternative. Similarly, the decrease in response strength for the right alternative occurs according to the following equation:

$$\Delta RS_R = RS_R - RS_R(1-e^{-\beta_o t_R - \beta_{opp} t_L}), \qquad (2.05)$$

RS_R is the response strength of the right alternative during the current cycle, β_o is the decay rate of right response strength based on time spent responding on that alternative, t_L is time spent responding on the left alternative, t_R is time spent responding on the right alternative and β_{opp} is the rate at which right response strength decreases with time spent responding on the left alternative. According to Equation (2.04) and Equation (2.05), response strength on both alternatives decreases during the cycle. Response strength at the beginning of each session (during the first cycle) was reset at 0.50.

The model was applied to data from the last 50 sessions of training for each pigeon. For each pigeon, data from individual cycles were used, including the time spent responding on the left and right alternatives (used to model extinction), and the location of the reinforcer for that cycle. The model was used to calculate simulated responses for each cycle in the following way: The overall average response rate was calculated for each pigeon by pooling the total responses (left and right) and total time spent responding across the 50 sessions. Based on the duration of each cycle, the model then calculated simulated responses for each alternative for that cycle as the obtained cycle duration x overall average response rate x relative response strength. For example, if the pigeons' overall response rate was 1.5 resp/sec, the duration of a cycle was 10 s and the response strengths for the left and right alternative were .75 and .25, the model would simulate (i.e., predict) that 11.25 responses would be made to the left alternative (1.5 x 10 x .75) and 3.75 responses to the right alternative (1.5 x 10 x .25).

At the end of each cycle, changes in response strength were dependent on which alternative resulted in reinforcement and which did not, as determined by Equation (2.03) to Equation (2.05). For every session sixth (every 12 reinforcers), the simulated number of responses on each alternative was computed. Predicted relative response ratios of the model were calculated as the log ratio of the simulated responses on the left and right alternatives. Response strength was reset at the beginning of every session (first cycle of each session). Based on the best-fit parameter values, predictions of relative response ratios by the model were computed for every session sixth (12 reinforcers) for every pigeon using the GML. For both groups, this was done for the last 50 sessions of training. Thus, there were 300 data points for each pigeon (6 session sixths x 50 sessions). Percentage of variance accounted for (VAC), the best-fit parameter values of Log b, α, β_o and β_{opp} as well as the α to β_{opp} ratio are shown for the response allocation data for individual pigeons in Table 2.02.

Because the model uses time spent responding on each cycle to update response strength via the extinction process, we wanted to check that the model was not tautological, that is, it was not able to reproduce perfectly the time allocation data. Thus best-fitting parameter values for the model were also estimated from the log time allocation ratios. These parameter estimates are shown for individual pigeons in Table 2.03. Bird 171 was omitted from the model fits due to multiple non-completed sessions. The average slope and intercept of obtained log response ratios plotted against predicted log response ratios for each group are listed in Table 2.04.

2.3.7 Quality of the model fits

Tables 2.02 and 2.03 show that results of analyses were largely similar, regardless of whether the model was fitted to response allocation or time allocation data. For response allocation, VAC ranged between 0.75 and 0.93 for the MinVar group and 0.51 and 0.84 for the MaxVar group. Average VAC by response allocation was significantly greater for the MinVar group (M = 0.89) compared to MaxVar (M = 0.77), *t*(25) = 3.94, *p* = .001, *d* = 1.53. Similar results were obtained for time allocation. VAC ranged between 0.64 and 0.94 for the MinVar group and 0.58 and 0.88 for the MaxVar group. Average VAC for time allocation was significantly greater for the MinVar group (M = 0.89) compared to MaxVar (M = 0.77), *t*(25) = 4.20, *p* < .001, *d* = 1.61. The greater VAC for the MinVar group is likely due to the increased range of log response and time allocation ratios (i.e., greater sensitivity) for these pigeons compared to the MaxVar group.

Table 2.02

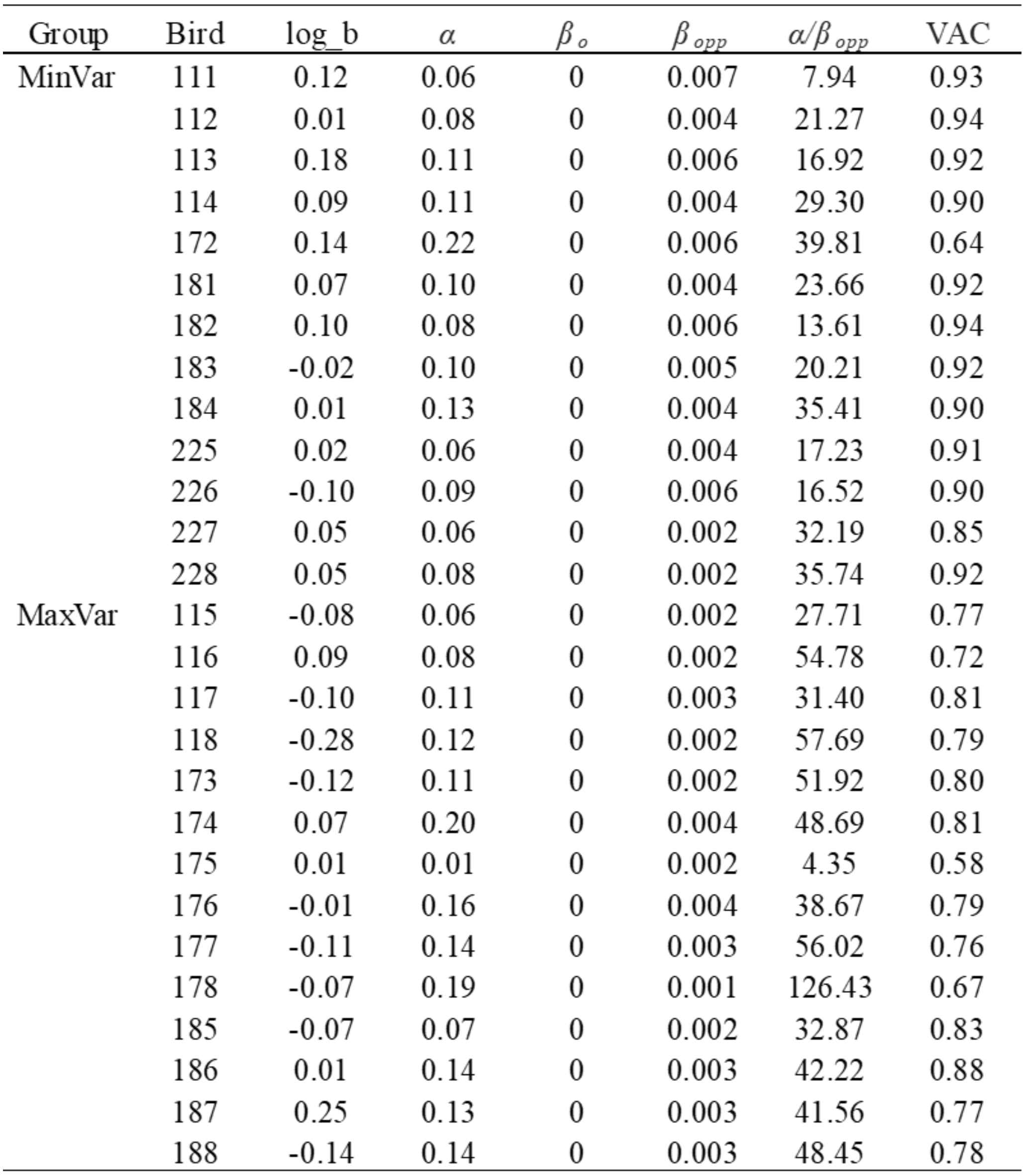
Best-fit Parameter Values and VAC Based on Response Allocation

Group	Bird	log_b	α	β_o	β_{opp}	α/β_{opp}	VAC
MinVar	111	0.12	0.09	0	0.006	14.83	0.93
	112	-0.02	0.09	0	0.003	27.33	0.93
	113	0.10	0.14	0	0.006	22.72	0.90
	114	0.09	0.14	0	0.003	39.39	0.90
	172	0.01	0.24	0	0.006	42.32	0.75
	181	0.09	0.13	0	0.003	49.19	0.90
	182	0.07	0.10	0	0.005	20.92	0.93
	183	-0.04	0.13	0	0.004	33.93	0.92
	184	0.04	0.17	0	0.004	41.23	0.91
	225	-0.03	0.11	0	0.003	31.60	0.91
	226	-0.11	0.11	0	0.006	19.18	0.90
	227	-0.07	0.12	0	0.002	47.82	0.76
	228	0.08	0.10	0	0.002	53.88	0.89
MaxVar	115	-0.03	0.05	0	0.001	49.43	0.76
	116	0.12	0.11	0	0.001	73.74	0.71
	117	-0.09	0.14	0	0.003	42.81	0.82
	118	-0.28	0.14	0	0.003	43.55	0.82
	173	-0.03	0.16	0	0.003	58.03	0.79
	174	0.19	0.25	0	0.005	53.55	0.82
	175	0.01	0.08	0	0.001	58.24	0.67
	176	0.07	0.22	0	0.004	53.63	0.79
	177	-0.13	0.18	0	0.004	51.36	0.83
	178	0.08	0.15	0	0.002	70.08	0.51
	185	-0.10	0.09	0	0.002	46.89	0.82
	186	-0.03	0.19	0	0.004	51.68	0.84
	187	0.16	0.19	0	0.004	48.89	0.78
	188	-0.15	0.18	0	0.003	56.48	0.84

Note. The first half of the table (birds 111 to 228) are for MinVar birds while the second half (birds 115 to 188) are for the MaxVar condition. β_{opp} required at least 3 decimal places for any discernible difference. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. VAC was computed based on session sixth data.

Table 2.03

Best-fit Parameter Values and VAC Based on Time Allocation



Group	Bird	log_b	α	β_o	β_{opp}	α/β_{opp}	VAC
MinVar	111	0.12	0.06	0	0.007	7.94	0.93
	112	0.01	0.08	0	0.004	21.27	0.94
	113	0.18	0.11	0	0.006	16.92	0.92
	114	0.09	0.11	0	0.004	29.30	0.90
	172	0.14	0.22	0	0.006	39.81	0.64
	181	0.07	0.10	0	0.004	23.66	0.92
	182	0.10	0.08	0	0.006	13.61	0.94
	183	-0.02	0.10	0	0.005	20.21	0.92
	184	0.01	0.13	0	0.004	35.41	0.90
	225	0.02	0.06	0	0.004	17.23	0.91
	226	-0.10	0.09	0	0.006	16.52	0.90
	227	0.05	0.06	0	0.002	32.19	0.85
	228	0.05	0.08	0	0.002	35.74	0.92
MaxVar	115	-0.08	0.06	0	0.002	27.71	0.77
	116	0.09	0.08	0	0.002	54.78	0.72
	117	-0.10	0.11	0	0.003	31.40	0.81
	118	-0.28	0.12	0	0.002	57.69	0.79
	173	-0.12	0.11	0	0.002	51.92	0.80
	174	0.07	0.20	0	0.004	48.69	0.81
	175	0.01	0.01	0	0.002	4.35	0.58
	176	-0.01	0.16	0	0.004	38.67	0.79
	177	-0.11	0.14	0	0.003	56.02	0.76
	178	-0.07	0.19	0	0.001	126.43	0.67
	185	-0.07	0.07	0	0.002	32.87	0.83
	186	0.01	0.14	0	0.003	42.22	0.88
	187	0.25	0.13	0	0.003	41.56	0.77
	188	-0.14	0.14	0	0.003	48.45	0.78

Note. The first half of the table (birds 111 to 228) are for MinVar birds while the second half (birds 115 to 188) are for the MaxVar condition. β_{opp} required at least 3 decimal places for any discernible difference. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. VAC was computed based on session sixth data.

To check if there were systematic deviations of the model's predictions from the data, we examined scatterplots of obtained and predicted log response ratios for both response and time allocation measures of choice for each pigeon. In terms of goodness-of-fit, MaxVar pigeons with the best and poorest fits are shown in Figure 2.06. Figure 2.07 shows the best and poorest fits from the MinVar birds. Left panels show plots using response allocation while the right panels were created using time allocation. The slope and y-intercept of each plot are also given. The dashed line in each panel is the line of best fit based on the session sixth data points and the solid line is the line of ideal prediction. In each plot, the closer the dashed line and solid line are to each other, the greater the accuracy of the model's predictions. Plots of the MaxVar group consistently showed lower VAC compared to the MinVar group (see appendix A1 and A2 for plots of the other MaxVar and MinVar birds, respectively). Overall, fits of the model using time allocation were similar (see appendix B1 and B2 for the rest of the MaxVar and MinVar plots, respectively).

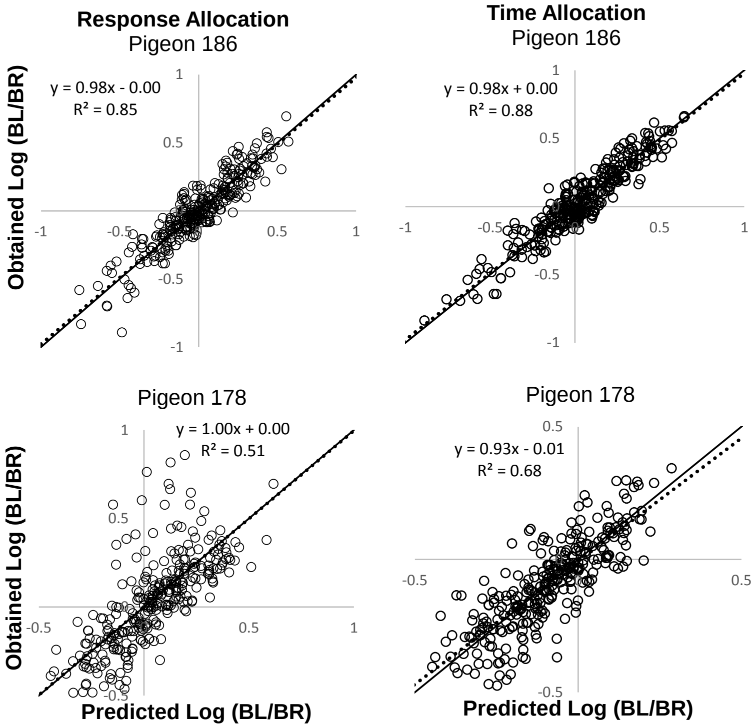


Fig 2.06. Obtained log ratios plotted against predicted log ratios for the best-fit and poorest-fit pigeons in the MaxVar group. Log ratios in the left panels were based on response allocation. Log ratios in the right panels were based on time allocation. Data points are based on session sixth responding. In order to emphasize the scatter pattern of data points, not all axes are of equal limits.

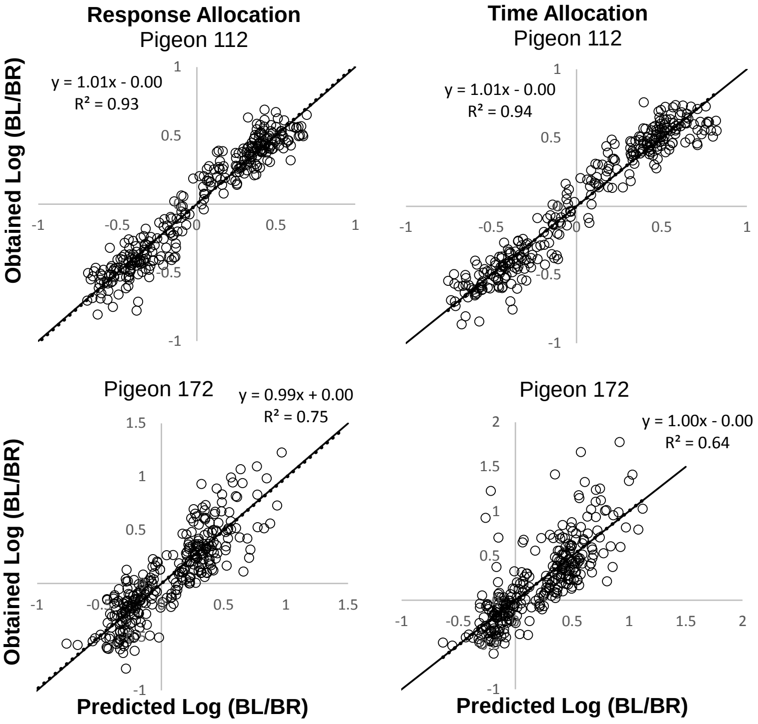


Fig 2.07. Obtained relative response ratios (vertical axis) plotted against predicted relative response ratios for the best-fit and poorest-fit pigeons in the MinVar group. The left panels indicate log ratios based on response allocation. The right panels indicate log ratios based on time allocation. Data points are based on session sixth responding. In order to emphasize the scatter pattern of data points, not all axes are of equal limits.

2.3.7.1 Predictive accuracy of the model. In order to determine whether the predictive accuracy of the model was different between MinVar and MaxVar birds, independent-samples *t*-tests were conducted on the slopes, intercepts and VAC of predicted log ratios plotted against obtained log ratios. The average of these parameter values for each group as well as the corresponding *t*-scores of the difference in parameter values between each group are shown in Table 2.04. This was conducted separately for response allocation and time allocation. In terms of the average slope per group using response allocation data, there was a difference between MinVar (*M* = 1.00, *SD* = 0.01) and MaxVar (*M* = 0.97, *SD* = 0.02) groups, *t*(25) = 5.29, *p* < .001, *d* = 2.01. The average slope per group also yielded a difference when time allocation was used in place of response allocation between MinVar (*M* = 1.00, *SD* = 0.01) and MaxVar (*M* = 0.96, *SD* = 0.02) groups, *t*(25) = 7.58, *p* < .001, *d* = 2.82. For both response and time allocation, there was no difference in average values of intercept between MinVar and MaxVar birds. These two trends persist across both groups for the rest of the pigeons. Across both groups, the y-intercept was consistently zero indicating little evidence of systematic over or under-prediction regardless of whether response allocation or time allocation was used to measure sensitivity. Overall, there were no systematic deviations detected between predicted and obtained data in the best and poorest fit plots of each group. In terms of goodness-of-fit, even though the average slope for MaxVar birds had a gradient that was further from one, this difference was ultimately small for both response and time allocation data.

Table 2.04

Average of Slope, Intercept and VAC for Predicted Versus Obtained Log Ratios

Scale	Group	Slope	Slope 95% CI [LL, UL]	Intercept	VAC
Responses	MinVar	1.00		0.00	0.89
	MaxVar	0.97		0.00	0.77
Time	MinVar	1.00		0.00	0.89
	MaxVar	0.96		0.00	0.77
Responses	*t*	5.29**	[0.02, 0.04]	0.51	3.91**
Time	*t*	7.58**	[0.03, 0.05]	1.35	4.16**

Note. The top two rows indicate data based on response allocation. The following two rows indicate data based on time allocation. The confidence intervals denote the difference between the slopes of the MinVar and MaxVar groups for response and time allocation. UL and LL represent the upper and lower limit of the confidence interval.

* indicates *p* < .05. ** indicates *p* < .01.

2.3.7.2 Determining parameter invariance. In order to determine if the model could account for the greater sensitivity to the reinforcer ratio for the MinVar group with invariant parameter values (e.g., Nevin, 1984), independent-samples *t*-tests were conducted to determine if the values of each parameter as well as the VAC and α to β_{opp} ratio were significantly different between groups. Table 2.05 shows the average of the best-fit parameter values in each group. This was conducted separately using response allocation data and time allocation data. These tests were conducted for parameters estimated from both response and time allocation. β_o was omitted from the *t*-test since its best fit values were zero across all birds. In terms of the α parameter, there was also no significant difference between MinVar (*M* = 0.13, *SD* = 0.04) and MaxVar (*M* = 0.15, *SD* = 0.05) groups, *t*(25) = -1.31, *p* = .202. There was a significant difference in β_{opp} values between MinVar (*M* = 0.004, *SD* = 0.001) and MaxVar (*M* = 0.003, *SD* = 0.001) birds, *t*(25) = 2.50, *p* = .019; *d* = 0.96. . In terms of the α to β_{opp} ratio, it was higher in MaxVar (*M* =54.17, *SD* =8.91) than MinVar (*M* =34.18, *SD* =12.59), *t*(25) = -4.79, *p* < .001, *d* = 1.83. In terms of the Log *b* parameter, there was no significant difference between MinVar (*M* = 0.02, *SD* = 0.07) and MaxVar (*M* = -0.02, *SD* = 0.13) groups, *t*(25) = 0.97, *p* = .341. VAC was significantly different between both groups with more VAC in MinVar (*M* = 0.89, *SD* = 0.06) compared to the MaxVar (*M* = 0.77, *SD* = 0.09) group, *t*(25) = 3.94, *p* = .001, *d* = 1.53.

2.3.7.3 Predictive accuracy between response and time allocation data. In order to determine if the model's predictions of the difference in sensitivity between MinVar and MaxVar birds would change if time spent responding was used in place of response allocation, independent-samples *t*-tests was conducted to determine if the values of each parameter based on time spent responding as well as the resulting VAC and α to β_{opp} ratio were significantly different between the MinVar and MaxVar groups. β_o was omitted from the *t*-test since its best fit values were zero across all birds. Similar results were obtained with one difference; in terms of the Log *b* parameter, there was a significant difference between

MinVar (*M* = 0.06, *SD* = 0.08) and MaxVar (*M* = -0.04, *SD* = 0.13) groups, *t*(25) = 2.34, *p* = .027; *d* = 0.91. These analyses showed that the model is unable to account for the observed difference in sensitivity between groups using invariant parameter values. We found that the model is able to describe differences in sensitivity to reinforcer ratios but is unable to explain this difference.

Table 2.05

Average of Best-fit Parameter Values and VAC

Scale	Group	log_b	α	β_o	β_{opp}	α/β_{opp}	VAC
Responses	MinVar	0.02	0.13	0	0.004	34.18	0.89
	MaxVar	-0.02	0.15	0	0.003	54.17	0.77
Time	MinVar	0.06	0.10	0	0.005	23.83	0.89
	MaxVar	-0.04	0.12	0	0.003	47.34	0.77
Responses	*t*	0.97	-1.31	NA	2.50*	-4.79**	3.94**
Time	*t*	2.34*	-1.06	NA	3.78**	-2.98**	4.20**

Note. The top two rows indicate data based on response allocation. The following two rows indicate data based on time allocation.

* indicates *p* < .05. ** indicates *p* < .01.

2.3.7.4 Summary of results based on session sixth data. How well the model could be fitted to the data in terms of using response allocation and time allocation yielded similar results. The average of the model's parameter values for each group using response allocation and time allocation are shown in Table 2.05. In terms of goodness-of-fit of the model, there was no noticeable difference in terms of average VAC between MinVar plots using response allocation data and MinVar plots using time allocation data. There was also no difference in terms of average VAC in MaxVar birds, regardless of whether we used response or time allocation to measure preference. The difference in the average slopes of MinVar and MaxVar birds although significant was also quite small. In both MinVar and MaxVar birds, average

VAC did not change when time allocation was used in place of response allocation to measure sensitivity. Although the model was able to describe the obtained data reasonably well, it was unable to explain the difference in sensitivity between MinVar and MaxVar birds. This is because in terms of the two primary parameters which were used to explain the difference in sensitivity (α and β_{opp}), there was a difference in β_{opp}). Moreover, the greatest difference occurred only in the α to β_{opp} ratio as its effect size was larger than that of the difference between groups for the β_{opp} parameter. In terms of how well the model accounts for the obtained sensitivity data, the model serves as a good descriptive model but fails to explain the observed difference in sensitivity between MinVar and MaxVar birds. These trends persisted regardless of whether preference was measured in terms of number of responses or time spent responding. Measuring preference in terms of time allocation yielded no noticeable improvements in terms of allowing the model to explain the difference in observed sensitivity between MinVar and MaxVar groups. Further analyses will focus on using response allocation data.

Results presented above (Figures 2.03 and 2.04) show that similar to previous research with rapid-acquisition choice procedures (e.g., Grace et al., 2003), sensitivity increased over the first half of the session but was approximately stable over the second half. Thus, responding during blocks 4-6 could serve as an estimate of the asymptotic preference obtained in each session. We ran several analyses to determine if asymptotic response allocation, both obtained and predicted by the model fits, was consistent with the generalized matching law. Specifically, we wanted to determine if our decision model could predict generalised matching and whether the model's predictions of preference was consistent with our subjects' exhibited preference.

2.3.7.5 The model's predictive accuracy of asymptotic preference. As previous plots of sensitivity have shown, preference stabilizes within the first half of each session and asymptotic levels of preference are exhibited during the second half. The next task was to determine whether the model was able to predict observed asymptotic matching and severe undermatching. Each session consisted of one data point and this analysis was conducted on the last 50 sessions of training resulting in each plot producing 50 data points. Based on the best-fit parameter values, resulting predictions of log response ratios by the model and obtained log response ratios were computed. Obtained log response ratios were then plotted against log reinforcer ratios. The model's predicted log ratios were also plotted against log reinforcer ratios. These plots for the best and worst fit MaxVar birds are shown in Figure 2.08. The plots for the best and worst fit MinVar birds are shown in Figure 2.09. In terms of the slopes of the GML, differences between them based on simulated and obtained data were minor. In terms of the intercepts of the GML, those based on simulated and obtained data were also approximately equal.

In terms of the slopes of MaxVar and MinVar birds based on obtained data, the slopes are steeper in the MinVar birds. The intercepts were approximately equal in both MinVar and MaxVar groups (see appendix C for the rest of the plots of the simulated data and appendix D for plots of the obtained data). These results are consistent with the greater asymptoticpreference exhibited by MinVar birds over the course of training. Plots of the MaxVar group consistently showed lower VAC compared to the MinVar group.

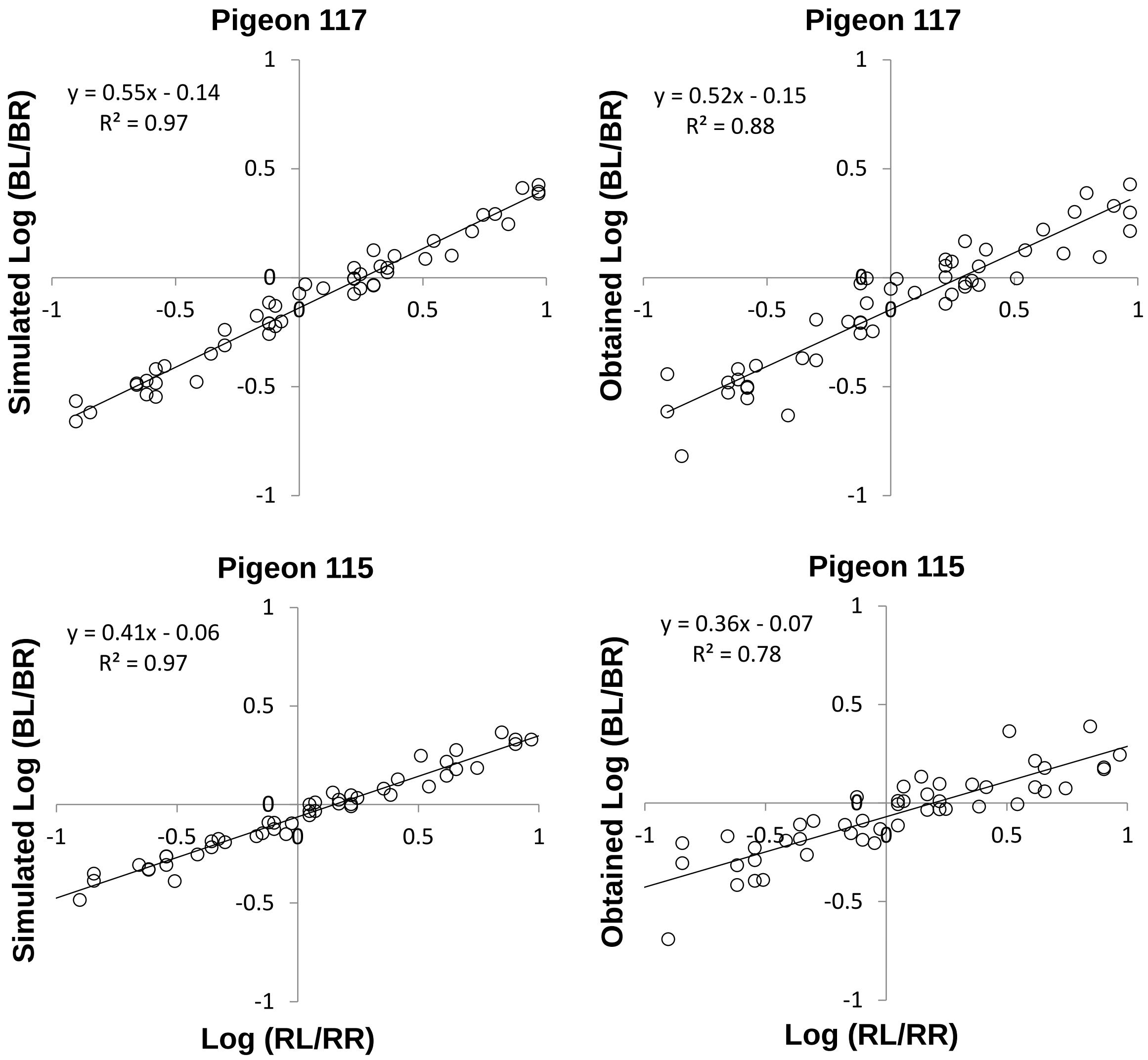


Fig 2.08. Best and worst fit graphs of Log response ratios (vertical axis) plotted against log reinforcer ratios for MaxVar birds. Left panels show plots based on predicted log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

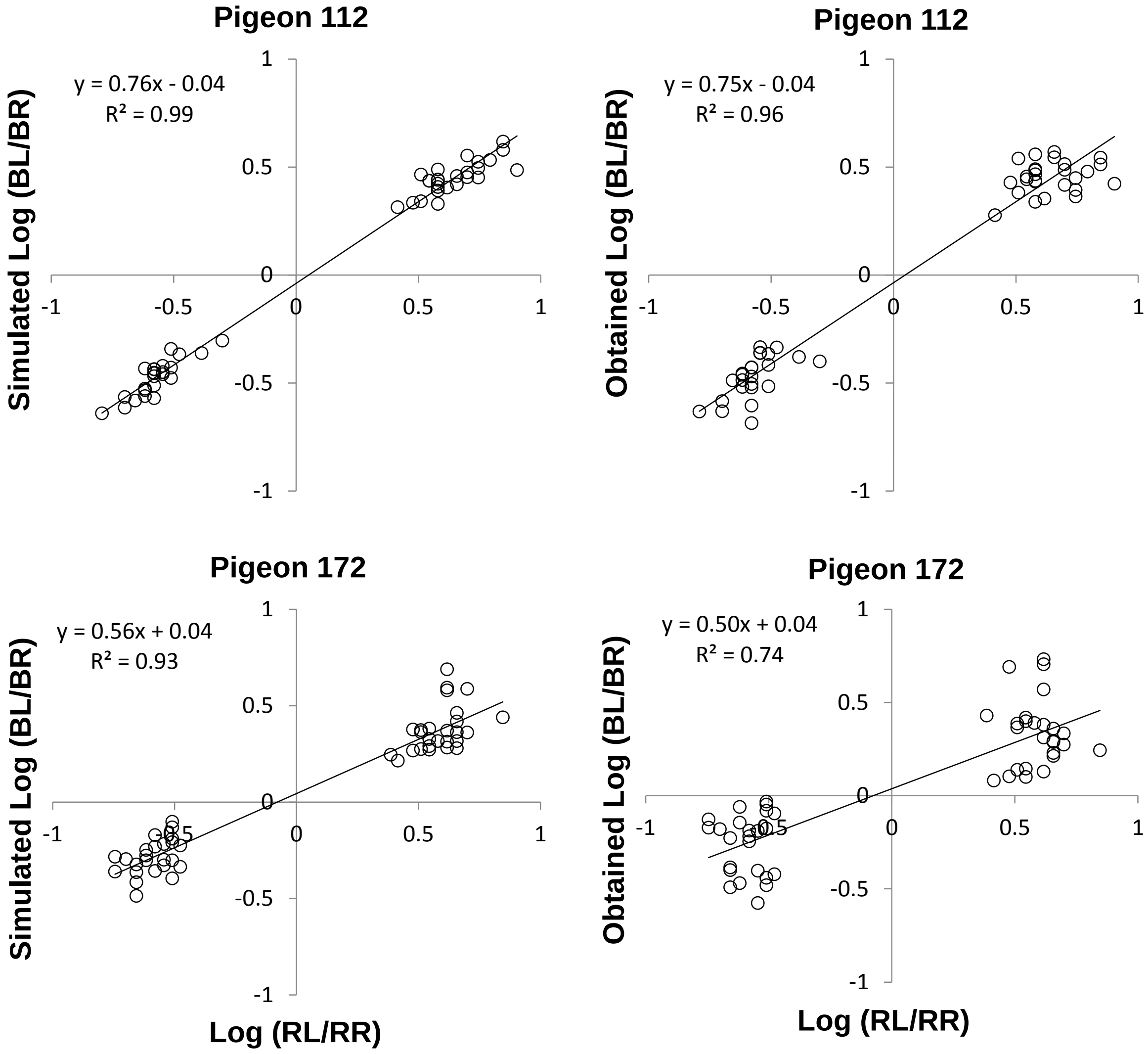


Fig 2.09. Best and worst fit graphs of Log response ratios (vertical axis) plotted against log reinforcer ratios for MinVar birds. Left panels show plots based on predicted log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

In order to assess whether the model was able to accurately predict asymptotic preference, predictions of asymptotic preference levels were compared to exhibited asymptotic preferences across all pigeons. Table 2.06 shows the average slope, intercept and VAC for GML fits to obtained and simulated data. Paired samples *t*-tests were conducted to determine if there was a difference in the average slope, intercept and VAC for each group. In terms of the MaxVar birds, there was a significant difference between obtained (*M* = 0.43, *SD* = 0.07) and predicted (*M* = 0.46, *SD* = 0.06) slopes, *t*(13) = 7.14, *p* < .001, *d* = 1.98. There was no difference between obtained (*M* = -0.03, *SD* = 0.17) and predicted (*M* = -0.04, *SD* = 0.17) intercepts, *t*(13) = -0.13, *p* = .896. There was also a difference between obtained (*M* = 0.83, *SD* = 0.04) and predicted (*M* = 0.96, *SD* = 0.02) VAC, *t*(13) = 10.64, *p* < .001, *d* = 2.95. Similar differences were found in the MinVar birds. There was a significant difference between obtained (*M* = 0.72, *SD* = 0.23) and predicted (*M* = 0.74, *SD* = 0.22) slopes, *t*(12) = 2.24, *p* = .045, *d* = 0.65. There was no difference between obtained (*M* = 0.04, *SD* = 0.11) and predicted (*M* = 0.04, *SD* = 0.11) intercepts, *t*(12) = -0.29, *p* = .773. There was also a difference between obtained (*M* = 0.92, *SD* = 0.06) and predicted (*M* = 0.98, *SD* = 0.02) VAC, *t*(12) = 4.05, *p* = .002, *d* = 1.17.

Table 2.06

Average of Slope, Intercept and VAC for GML fits to Obtained and Simulated Data

Data	Group	Slope	Intercept	VAC
Obtained	MaxVar	0.43	-0.03	0.83
	MinVar	0.72	0.04	0.92
Simulated	MaxVar	0.46	-0.04	0.96
	MinVar	0.74	0.04	0.98
MinVar Diff	*t*	2.24*	-0.29	4.05**
MaxVar Diff	*t*	7.14**	-0.13	10.64**

Note. Data was based on responding during the second half of each session.

Diff denotes the difference between obtained and simulated data.

* indicates *p* < .05. ** indicates *p* < .01.

2.3.7.6 Residual analysis. In order to determine if predictions of the model were able to be incorporated into a version of the GML which could predict approximate matching and under matching, a residual analysis similar to that used by Sutton, Grace, McLean and Baum (2008) was conducted on the difference in predicted log response ratios of the model and those of the GML. Residuals were plotted against predicted log response ratios of the GML. In terms of the simulated data, residuals were calculated as the difference between predicted log response ratios of the GML and predicted log response ratios of the model during the second half of each session. In terms of the obtained data, residuals were calculated as the difference between the GML's predictions of log response ratios and log response ratios calculated from data collection (the pigeons' responding). This was conducted separately for MinVar and MaxVar birds using simulated and obtained data. These plots are shown in Figure 2.10 where the top panels show plots of the MaxVar birds and the lower panels show those of the MinVar group. As expected, there is a greater scatter of residuals in the obtained data. Moreover, there is a slight systematic trend in the residuals of the simulated data for the MinVar group.

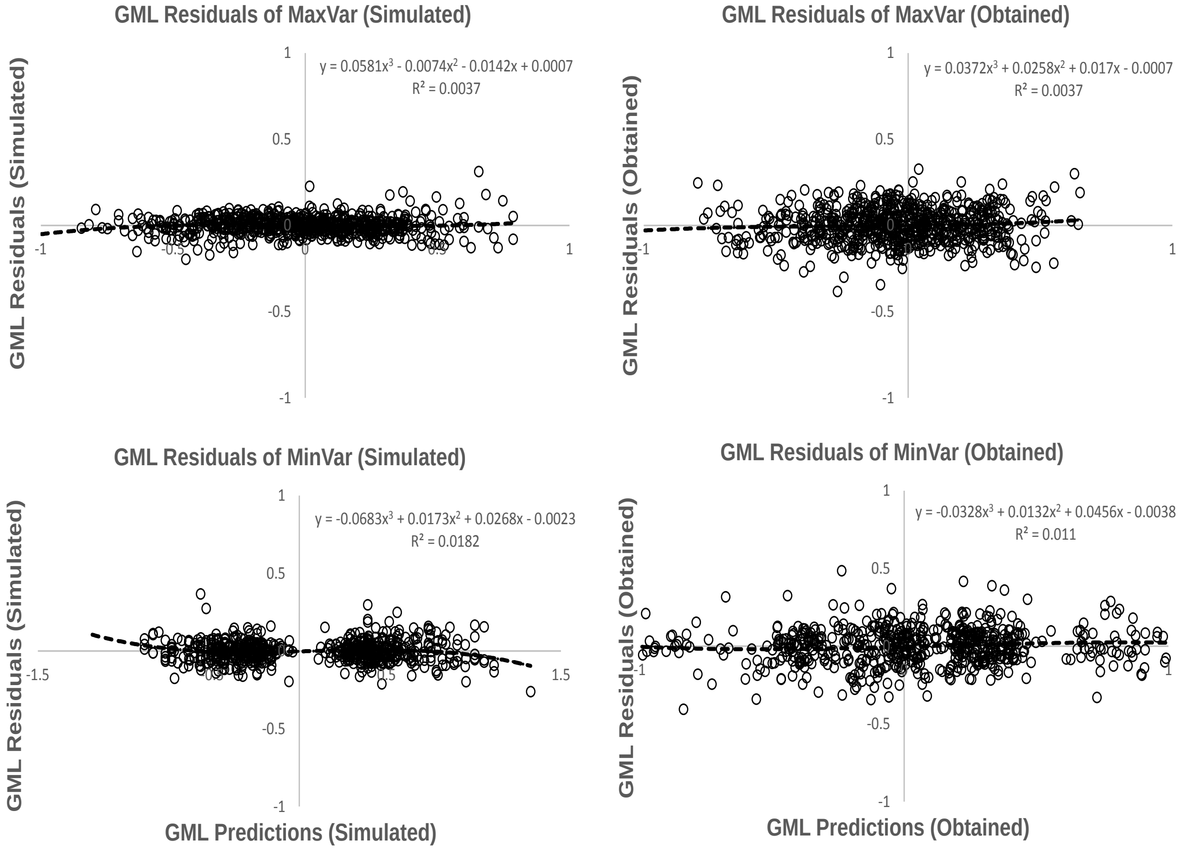


Fig 2.10. All panels show response allocation predicted by the GML for a range of reinforcer ratios of 1:8 to 8:1 in MaxVar birds and 1:4 or 4:1 in MinVar birds. Upper panels show residuals of the MaxVar group while the lower panels show residuals of the MinVar birds. Left panels show residuals based on simulated data while the right panels show residuals based on obtained data.

In order to test for systematic deviations in the GML's predictions, polynomial regressions were conducted in which the residuals were regressed on the predicted values. Polynomial regressions were then conducted in which each model's residuals were regressed on the GML's predicted log response ratios. Linear, quadratic, and cubic terms were included in all regressions. Table 2.07 shows the results of this analysis. In terms of the simulated data the linear and cubic coefficients were significant predictors of the log response ratios for the simulated data. Even then, these systematic deviations occurred only in the MaxVar group and these deviations were relatively minor. In terms of the obtained data, the linear coefficient was a significant predictor even though the regression model was not a significant predictor of the pigeons' exhibited log response ratios. This indicates there were no systematic deviations from the GML's predictions across obtained data.

Table 2.07

Results of Polynomial Regressions Performed on GML Residuals

Dependent variable	Group	Predictor variables	*B*	S.E. of *B*	R^2	*F*
GML residuals Simulated	MaxVar	Linear	-0.014	0.011		
		Quadratic	-0.007	0.017		
		Cubic	0.058	0.039		
					0.004	0.872
GML residuals Obtained	MaxVar	Linear	0.017	0.025		
		Quadratic	0.026	0.047		
		Cubic	0.037	0.105		
					0.004	0.858
GML residuals Simulated	MinVar	Linear	0.027**	0.010		
		Quadratic	0.017	0.015		
		Cubic	0.068**	0.020		
					0.018**	4.000
GML residuals Obtained	MinVar	Linear	0.046*	0.018		
		Quadratic	0.013	0.020		
		Cubic	-0.033	0.021		
					0.011	2.391

Note. Dependent variables were residuals from the GML, computed using log response ratios. Independent variables were log predicted values, plus squares and cubes of log predicted values ('quadratic' and 'cubic'). *B* is the unstandardised regression coefficient, and R^2 is the proportion of variance accounted for.

* indicates $p < 0.05$, ** indicates $p < 0.01$, *** indicates $p < 0.001$.

Mean sensitivity to log reinforcer ratio remained relatively unchanged during the maximal variation condition from the beginning to the end of training. During the minimal variation condition, sensitivity to log reinforcer ratios increased from the beginning of training to reach asymptotic levels by the end of training. Across both conditions, sensitivity had stabilized by the last 30 sessions of training. Over the last 30 sessions, mean sensitivity reached asymptotic levels by the second half of each session. Asymptotic sensitivity levels were lower in MaxVar birds indicating severe under-matching. MinVar birds exhibited approximate matching during the second half of each session.

We computed exhibited preference based on response and time allocation. Fitting the model to session sixth data indicated little difference in predictive accuracy of the model between data based on response allocation and time allocation. The model was able to account for more of the variance in the data in MinVar birds. In terms of asymptotic sensitivity values, although there was a significant difference between predicted and exhibited preference for obtained data based on response and time allocation, these differences were ultimately small. Analyses of preference in later experiments were all based on response allocation. Residual analyses of sensitivity over the second half of each session indicated little evidence of systematic deviations from predictions of the GML. The model is therefore able to describe the changes in preference but is ultimately unable to explain them.

2.4 Discussion

These results show that preference in concurrent schedules with unpredictable changes in reinforcer ratios across sessions depends on the distribution of those ratios. When programmed reinforcer ratios were either 4:1 or 1:4 (MinVar group), sensitivity was greater and reached approximate matching during the last 50 sessions of training. By contrast, when programmed reinforcer ratios had the same expected value but were sampled from a uniform distribution (from 8:1 to 1:8; MaxVar group), preference showed considerable undermatching, with asymptotic sensitivity of .50 or less.

We proposed a model to account for acquisition of preference. The model states that reinforcement and extinction processes occur during each trial. Specifically, responding that produces reinforcement increases response strength for that alternative, but response strength is also assumed to decrease continuously in time according to an extinction process. The best-fitting model results were obtained when extinction only occurred to the alternative that was not currently responded to – that is, while a pigeon responded to the left key, response strength to right key was decreasing, and vice versa. Residual analyses of sensitivity over the second half of each session indicated little evidence of systematic deviations from predictions of the GML. The model is therefore able to describe the differences in preference between the MinVar and MaxVar groups but cannot fully explain them. The problem with the model is shown by the significant differences in the α parameter and α to β_{opp} ratio. The largest effect size occurred in the α to β_{opp} ratio. Thus parameter invariance was not satisfied.

The α to β_{opp} ratio describes the relationship between the increase in response strength and the loss in response strength prior to reinforcement. A lower ratio indicates a more stable net change in response strength due to these two processes. This is evident in the MinVar group and results in stronger preference and therefore approximate matching behaviour. In the MaxVar group, the ratio is higher indicating response strength reaching its asymptote more quickly. Due to relatively much weaker losses in response strength, once asymptotic response strength is reached, the process of losing response strength translates to response strength remaining close to its asymptote. This means increases due to reinforcement affect response strength to a smaller extent and become limited by the asymptote. Specifically, asymptotic response strength creates a ceiling effect and results in preference for the MaxVar group being weaker. Due to the violation of parameter invariance, we consider an alternative model of choice to resolve this issue. We also consider broader issues relevant to acquisition of choice under concurrent schedules.

2.4.1 Bayesian Model

We explored if an alternative model might be able to account for these results. The rationale for the model was to regard reinforcement history, in terms of variation in reinforcer ratios across sessions, as analogous to a Bayesian prior distribution. As reinforcers are obtained during a session, the distribution is updated, and response allocation depends on the updated distribution.

Specifically, the prior relative reinforcement distribution was modelled as 13 reinforcer ratio bins, equally spaced in logarithmic terms from 8:1 (log = .903) to 1:8 (log = -.903), with a density for each bin (d_i, with i = 1 to 13). In terms of reinforcement probabilities for the left alternative, these bins corresponded to p_{Li} = .889, .850, .800, .739, .667, .586, .500, .414, .333, .261, .200, .150 and .111. For the MinVar group, the prior distribution at the beginning of each session had the densities for p_L = .800 and .200, d_3 and d_{11} = .50, with all remaining d_i = 0. For the MaxVar group, the prior distribution was uniform, that is all bins had d_i = .077 (1/13). Response allocation was assumed to be determined by the expected value of p_L:

$$\widehat{p_L} = \sum_{i=1}^{13} d_i p_i \tag{2.06}$$

The effect of reinforcement was to update the densities so that $\widehat{p_L}$ would shift toward the reinforced alternative (i.e., increase with reinforcement to left-key responding, and decrease with reinforcement to right-key responding). A linear operator rule was used, with parameter α. Specifically, the updated densities were:

$$d_{i,n+1} = d_{i,n} + \alpha * \left[\frac{p_{Li} * d_{i,n}}{\widehat{p_{Ln}}} - p_{Li}\right] \text{ (after left-key reinforcement)} \tag{2.07a}$$

$$d_{i,n+1} = d_{i,n} + \alpha * \left[\frac{(1-p_{Li})*d_{i,n}}{(1-\widehat{p_{Ln}})} - p_{Li}\right] \text{ (after right-key reinforcement)} \qquad (2.07b)$$

where *n*, *n*+1 indicates the trial number and $\widehat{p_{Ln}}$ is the expected probability of left-key reinforcement for trial *n*.

To illustrate how the model described can potentially account for the difference in preference for the MinVar and MaxVar groups, Figure 2.10 below shows predictions for a single session (72 trials) with $p_L = 0.80$ and $\alpha = 1.0$. The expected value for probability of left-key reinforcement ($\widehat{p_L}$) increased more rapidly for the prior distribution based on the MinVar condition.

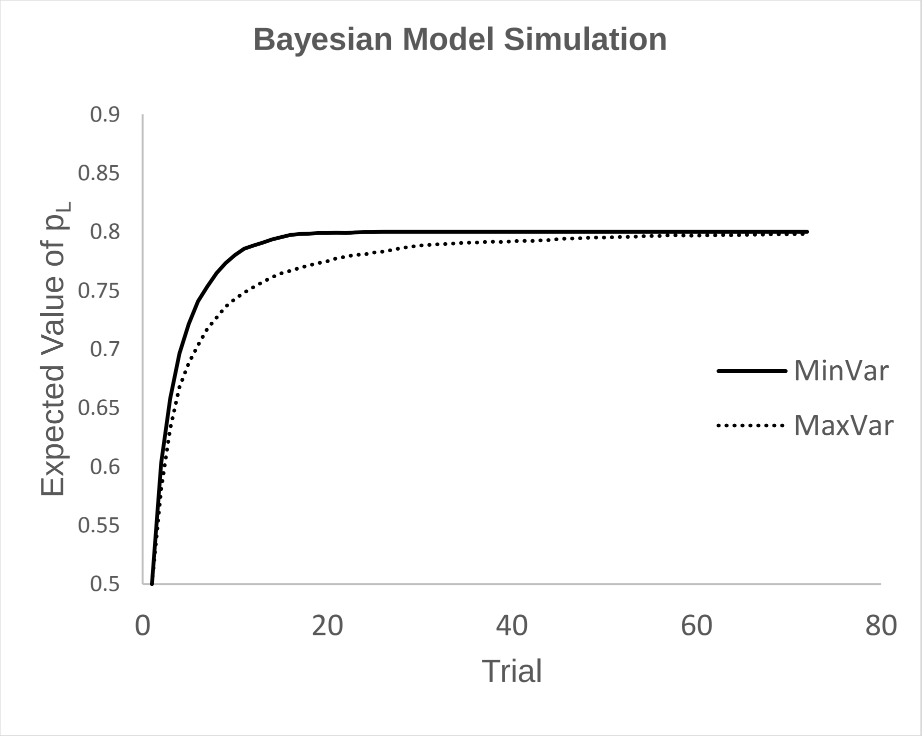


Fig 2.11. Simulations of expected probability of reinforcement from the left alternative in the MinVar and MaxVar group. Simulations are based on consecutive trials across one session.

The Bayesian model was fitted to data for individual pigeons from Experiment 1. Predicted log response ratios were calculated for each session sixth (12 reinforcers) across the last 50 sessions of training, based on the log ratio of predicted probabilities and with a bias term (Log *b*) included. Thus, there were two parameters estimated for each pigeon (α and Log *b*) and 300 data points for each pigeon (6 session sixths x 50 sessions). The corresponding VAC using the estimated values for these two parameters as well as the slope and intercept of log response ratios plotted against predicted log response ratios are shown in Table 2.08.

We assessed the Bayesian model's goodness-of-fit for each group. The Bayesian model had lower average VAC in the MaxVar group compared to the MinVar group. In the MinVar group, the Bayesian model accounted for an average of 84% of the variance but only 69% in the MaxVar group, $t(25) = 4.97$, $p < .01$, $d = 1.91$.

We also compared the goodness-of-fit of both models;theBayesian model and the decision model we introduced prior to it . The Bayesian model had lower average VAC compared to the other decision model. In the MinVar group, the earlier decision model accounted for 89% of the variance compared to 84% for the Bayesian model, $t(12) = 3.59$, $p < .01$, $d = 1.00$. In the MaxVar group, the prior decision model accounted for 77% of the variance compared to 65% of the Bayesian model, $t(13) = 19.43$, $p < .001$, $d = 5.20$.

We also assessed parameter invariance and predictive accuracy of the model by comparing the best-fit parameter estimates for *a*, Log b and the slopes and intercepts of obtained response allocation plotted against predicted response allocation of the Bayesian model. In terms of the *α* parameter, the average value was greater in the MinVar group (*M* = 0.17) compared to the MaxVar group (*M* = 0.07), *t*(25) = 2.50, *p* = .027, *d* = 0.98. Thus the Bayesian model did not demonstrate parameter invariance. There was no significant difference between groups in terms of the Log *b* parameter, *t*(25) = 1.37, *p* = .184. Slopes of obtained vs. predicted plots were significantly greater in the MinVar group (*M* = 0.94) compared to MaxVar (*M* = 0.86), *t*(25) = 2.62, *p* = .015, *d* = 1.00, while the intercepts did not differ systematically, *t*(25) = 0.80, *p* = .429.

Table 2.08

Best-fit Parameter Values and VAC for the Bayesian model

Group	Bird	Log *b*	*α*	Slope	Intercept	VAC
MinVar	111	0.16	0.56	1.26	-0.02	0.90
	112	-0.02	0.13	0.96	0.00	0.93
	113	0.20	0.24	0.95	-0.02	0.88
	114	0.11	0.08	0.92	0.00	0.84
	172	0.05	0.08	0.76	0.00	0.54
	181	0.12	0.07	0.88	0.03	0.84
	182	0.12	0.31	1.00	0.02	0.92
	183	-0.05	0.13	0.92	0.00	0.88
	184	0.03	0.08	0.85	0.06	0.82
	225	-0.03	0.12	0.94	0.00	0.88
	226	-0.16	0.33	1.01	0.03	0.88
	227	-0.04	0.05	0.91	-0.01	0.71
	228	0.10	0.05	0.90	0.02	0.84
MaxVar	115	-0.05	0.05	0.90	-0.02	0.65
	116	0.14	0.04	0.82	0.09	0.59
	117	-0.12	0.09	0.88	-0.03	0.74
	118	-0.37	0.07	0.85	-0.08	0.68
	173	-0.06	0.06	0.87	0.00	0.63

	174	0.22	0.08	0.81	0.05	0.65
	175	0.00	0.04	0.91	0.05	0.56
	176	0.06	0.09	0.77	0.04	0.66
	177	-0.16	0.07	0.84	-0.02	0.72
	178	0.06	0.05	0.77	-0.04	0.38
	185	-0.12	0.06	0.93	-0.02	0.74
	186	-0.03	0.08	0.89	-0.01	0.71
	187	0.24	0.09	0.85	0.00	0.65
	188	-0.18	0.07	0.88	-0.05	0.70
MinVar	*M*	0.05	0.17	0.94	0.01	0.84
MaxVar	*M*	-0.03	0.07	0.86	0.00	0.65
	t	1.37	2.50*	2.62*	0.80	4.97**

Note. The first half of the table (birds 111 to 228) are for MinVar birds while the second half (birds 115 to 188) are for the MaxVar condition. The slope and intercept are obtained from plotting obtained log response ratios against predicted log ratios of session sixth data. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. VAC was computed based on session sixth data.

* indicates $p < .05$. ** indicates $p < .01$.

We conducted polynomial regressions in which the residuals of the Bayesian model were regressed on predicted values (not reported here). In contrast to the earlier decision model, there was systematic variation in the residuals of the predicted log ratios. The linear, quadratic and cubic components were all significant for the MinVar group, $R^2 = 0.115$, $F(3, 646) = 27.97$, $p < .001$.The polynomial regression was also significant with linear, quadratic and cubic terms being also significant for the MaxVar group, $R^2 = 0.045$, $F(3, 696) = 10.86$, $p < .001$.

Overall, the Bayesian model fails to provide an adequate account of the Experiment 1 results. It accounts for less variance in the data compared to the other decision model, but this is to be expected because the Bayesian model has fewer parameters. However it does not fully account for the difference in sensitivity between the MinVar and MaxVar groups, as shown by the lack of invariance in estimated values of the α parameter. Another limitation of the Bayesian model is that it does not provide a principled basis for how the prior distribution corresponding to reinforcement probability is established.

Although the earlier model also has similar problems (e.g., the lack of parameter invariance), its predictions do result in generalized matching. Residual analysis of the earlier model's predictions showed very little systematic deviation, in contrast to the Bayesian model. For these reasons, in the subsequent experiments with the concurrent-chains procedure we used the earlier model as a basis for modelling choice.

2.4.2 Effects of Reinforcer Ratio Distributions in Rapid-Acquisition Concurrent Schedules

In terms of modelling choice behavior in concurrent schedules, the assumption that response strength decreases due to 'background' extinction processes is, to our knowledge, a novel contribution. Previous studies have emphasized the need for strengthening of responding due to reinforcement (Schofield et al., 1997; Mazur et al., 1992; Davison et al., 2000). The model proposed here is based on strengthening by reinforcement but also adds extinction dynamics to predict choice. The model does a good job at describing the data but is not adequate in terms of explaining the observed preference between MinVar and MaxVar groups.

The best- fit values for the extinction processes proposed in our decision model, denoted by the β parameters, are consistent with previous literature on extinction processes in concurrent schedules. Davison and Hunter (1976) arranged a series of two- and three-alternative choice where at least one of the alternatives resulted in extinction. They found that some responding on the alternative which led to extinction was still maintained. This is consistent with our best-fit parameter values being non-zero only for the alternative not being responded on. This is because if extinction occurs on the alternative that is being responded on, responding on that alternative that results in extinction should reach cessation. Moreover, if an alternative that is being responded on loses response strength, response strength for that alternative would reach zero quite rapidly on schedules that arrange multiple successive reinforcers. This was not the result we found in the MaxVar group where reinforcer ratios for some sessions were greater than 5:1 (more than 60 reinforcers on one alternative out of a maximum obtainable 72 for the whole session).

The primary empirical contribution of the present results is to show that choice in a concurrent-schedules procedure in which reinforcer ratios vary unpredictably across sessions depends on the distribution of those ratios. Experiment 1 of Schofield et al. (1997) exposed pigeons to rapidly changing reinforcement ratios. Similar to the MinVar group, the reinforcer ratio distributions were bimodal. They found no significant differences between response allocation and obtained reinforcer ratio. Given that they had three sets of reinforcer ratios (2:1 vs 1:2, 4:1 vs 1:4 and 8:1 vs 1:8), their results strongly suggest that the range of reinforcer ratios did not affect sensitivity. When the distribution of reinforcement was bimodal (MinVar), preference reached approximate matching. However, the present study showed that when the distribution of reinforcer ratios was uniform in the MaxVar condition, preference only reached severe undermatching.

Rapid adaptation of response allocation to reinforcement schedules which changed unpredictably across sessions is consistent with obtained data in other concurrent schedules of reinforcement under rapid-acquisition conditions (Maguire et al., 2007; Hunter et al., 1985; Schofield et al., 1997). Similar to these studies, our results showed that after sufficient training had been provided, preference for the richer alternative of each session was primarily influenced by the reinforcement ratios that occurred during that session. There was no indication that the reinforcement ratios of previous sessions influenced preference of each current session. In both MinVar and MaxVar conditions, preference increased during the first half of each session and stabilized thereafter. Moreover, we showed that the asymptotic preference predicted by our decision model was consistent with the GML. Sutton et al.(2008)

compared the GML and the contingency discriminability model (Davison & Jenkins, 1985) when using them to fit response allocation and time allocation data obtained from concurrent schedules. The contingency discriminability model describes the relationship between log reinforcer ratio and log response ratio via a sigmoidal relationship in contrast to the GML, although the predictions of the contingency discriminability model can appear to be very close to linear. Sutton reported a meta-analysis of residuals pooled across studies and showed that the residuals of the contingency discriminability model had a systematic pattern predicted if the GML was the 'true' model. These results indicate that the relationship between log response ratio and log reinforcer ratio follows a linear trend which we have obtained in Experiment 1. Residual analysis of our decision model's predictions showed little systematic trends in polynomial regressions of the residuals indicating that the decision model is a viable framework for modelling dynamic processes underlying choice in concurrent schedules.

However there are two unresolved questions associated with the model and results of Experiment 1. First, the difference in sensitivity between the MinVar and MaxVar groups suggests a molar effect of prior reinforcer ratios on preference. Specifically, the distribution of reinforcers in preceding sessions influences response allocation of each current session in a way that cannot be explained as an effect of hysteresis. Since the distribution of reinforcers is different in MinVar and MaxVar groups, the impact of this molar effect differs for each group, thereby resulting in higher asymptotic preference in the MinVar group.

This result violates the 'locality' assumption common to linear-operator models of choice – that responding is determined by current state variables (e.g., response strength) and recent reinforcement history (Davis, Staddon, Machado & Palmer, 1993). Davison et al. (2000) proposed a "cumulative effects" model which takes into account the learning of subjects from the beginning of each experimental condition and is 'nonlocal' because it depends on the entire reinforcement history. This model was able to account for a range of preference from matching to near indifference and was sensitive to the order of experimental conditions. Given that this model is able to also accurately account for preference observed in experimental procedures of daily reversal and extinction in concurrent schedules suggest a persistent non-locality effect in the acquisition of preference.

The results of the present study also suggest that the difference between the MinVar and MaxVar groups – the effect of the distribution of reinforcer ratios – is a nonlocal, molar effect. This is different from hysteresis which takes only into account the recent history of reinforcer distributions. In experiment 1, this is shown by the decreasing regression coefficients of increasingly preceding sessions. Specifically, the coefficients of the first session sixth were always non-zero and only turned towards zero across the span of that session.

A second, related question is whether the difference in responding between MinVar and MaxVar is a quantitative or qualitative one. A 'quantitative' difference would be distinct patterns of response allocation produced by change in the value of a model parameter. By contrast, a 'qualitative' difference would correspond to distinct topographies of choice responding that could not be accommodated by changing the value of a parameter. Grace et al. (2006) provides an example of a 'quantitative' difference in responding. They studied rapid acquisition in a concurrent-chains procedure in which variation in terminal-link delays across session was similar to the MinVar and MaxVar conditions. In their equivalent 'MinVar' condition, the left terminal link was FI 8 s and right terminal links were either FI 4 s or FI 16 s, whereas in their 'MaxVar' condition, a unique pair of FI terminal link delays (sampled from a pseudorandom distribution) was used each session. They found that pigeons showed both linear and nonlinear patterns of responding in the MaxVar condition, which were produced by a 'decision model' that was the precursor of the CDM. When the sigma parameter in the model was large, corresponding to relatively 'inaccurate' decisions, log response ratios were linearly related to the log immediacy ratio, whereas when the sigma parameter was small a nonlinear, sigmoidal response pattern was obtained. For the present study, the question is whether the difference in responding between the MinVar and MaxVar conditions can be fully explained by varying the parameters of the decision model (quantitative) or is there an underlying process which affects response topography (qualitative). Further research is needed to resolve this question. A possible study would be similar to that of Experiment 1 but with every pigeon being trained under MinVar and MaxVar conditions. This would allow an investigation of whether preference under MinVar conditions is affected by prior exposure to MaxVar and whether preference during MaxVar was affected by prior exposure to MinVar.

Although the decision model provides a viable descriptive framework for modelling choice in concurrent schedules, it is unable to explain the difference in sensitivity between MinVar and MaxVar groups via parameter invariance. This is due to the possibility of a molar component of reinforcer rate which affects preference.

This molar effect of the preceding session's reinforcement rate on choice is consistent with previous results from a steady-state study conducted by Landon, Davison and Elliffe (2002). They arranged seven reinforcer ratios ranging from 27:1 to 1:27 under a single concurrent VI 30s schedule. They found that although the effects of the most recently obtained reinforcer had a much larger effect on choice compared to the preceding reinforcers, previous reinforcers still had a small effect on preference. This effect of current and preceding reinforcers on responding during each session was greatest when reinforcer ratios were the most extreme (27:1 and 1:27). Davison and McCarthy (1988) re-analysed data from the study by Hunter et al. (1985). They computed the cumulative sum of average sensitivity to previous sessions' reinforcer ratios found that the average sum of this sensitivity was 0.62 when taking into account the preceding 3 sessions but increased to 0.70 when the preceding 10 sessions were included (Davison & McCarthy, 1988). In order to account for this molar effect, we introduced the Bayesian model which failed to do so.

This effect of preceding sessions' reinforcement rates on current responding also extends to rapid acquisition studies in concurrent schedules. Hunter et al. (1985) used a 31-step PRBS in concurrent VI 60s VI 240s or concurrent VI 240s VI 60s schedules and found that the preceding two to three sessions' reinforcement rates still affected current responding. Given that the use of the PRBS rendered it impossible to know which reinforcer ratio was going to be used for each session, this indicates some form of prior exposure to reinforcement was carried over to each session. This effect is also evident in a study by Staddon and Davis (1990) who found that although responding in pigeons was most strongly affected by the alternative that was most recently rewarded, pigeons regressed to their earlier preferences overnight. This was attributed to the effect of the most-recently rewarded alternative weakening as time passed.

The decision model takes into account the local effects of reinforcement on choice as documented by Davison et al. (2000). They found that when reinforcement contingencies changed within each session, individual reinforcers resulted in predictable changes in responding. Specifically, successive reinforcers from the same alternative resulted in weaker effects on responding while a single reinforcer obtained from the opposite alternative following a succession of reinforcers from that same alternative had a much stronger effect on responding. The model describes this phenomenon in terms of the difference between the current RS of each alternative and asymptotic RS of that same alternative.

Modelling behaviour in concurrent schedules is more difficult as the independent variables of choice, namely reinforcement contingencies, are not as easily separated as concurrent chains. The goal of studying choice behaviour lies in the level of analysis used to examine and predict it. The results indicate that molar and molecular changes in responding take place contingently. This is stated by Landon et al. (2000) whereby molar or molecular analyses alone are insufficient to model concurrent VI behaviour. This is because molecular analyses can describe long-term changes in choice behaviour to some extent but molar analyses fail to explain the more local effects of reinforcers. The decision model needs to incorporate a separate component to take into account for the effects of prior reinforcement rate on current responding in concurrent VI schedules. The issue of parameter invariance may then be resolved.

Chapter 3

Experiment 2: The Effects of Terminal-link Entry Ratio and Terminal-link Duration on Choice

3.1 Introduction

One main goal of this thesis is to understand how choice behaviour is affected by different reinforcer contingencies in a rapidly changing environment in terms of primary and conditioned reinforcement. The previous chapter shows results indicating that the distribution of primary reinforcer ratios affects the acquisition of asymptotic preference. The next step involves investigating the effects of reinforcer contingencies on choice in regard to conditioned reinforcement. In this chapter, preference is measured in a manner similar to the concurrent schedules experiment in the context of sensitivity to reinforcer ratio. Specifically, how sensitivity to the conditioned reinforcer ratio (i.e., terminal-link entry ratio) is affected by the absolute value of terminal-link duration when left and right terminal links are equal duration. This question has not been investigated in prior research.

Based on the results from the experiment in this chapter, a subsequent aim is to create a model which can account for the observed results in terms of preference acquisition. It was shown that the decision model used in the previous chapter is problematic in terms of explaining the difference in preference. However, the consistently high accuracy of predicted relative response ratios and adequate VAC indicate that it serves as a viable descriptive foundation which can be used to develop a decision model for concurrent chains. With certain additions to the decision model used in the previous chapter, a decision model for concurrent chains can hopefully be developed that is not only descriptive but also explanatory for the purposes of accounting for preference between short and long terminal-link durations.

For this first study of concurrent chains in this thesis, we wanted to determine how sensitivity to conditioned reinforcement of choice (terminal-link entry ratio) was affected by terminal-link duration under rapid-acquisition conditions. Each cycle consisted of completing an initial link and terminal link phase. Each initial link consisted of a single VI 8s schedule and pigeons had to peck on two lighted keys. For each initial link, there was 12 possible intervals that, across each session resulted in an average initial-link duration of eight seconds.

A concurrent-chains procedure was arranged, in which terminal-link entry was scheduled probabilistically to either the left or right key after the interval had elapsed at the end of each initial link. As each interval in the schedule timed out, pecking on the key which had been randomly assigned an outcome (left or right key) resulted in access to the corresponding terminal link. For half of the sessions, the assignment (left/right) was 1:3. In the remaining sessions, the assignments were reversed (3:1). Left and right terminal-link durations were always equal. Short terminal-link durations were FI 4s FI 4s while long terminal-links were FI 16s FI 16s. A 1s changeover delay COD was used. Reinforcement consisted of 3.5s of access to food. For both groups, whether the left or right key had the richer reinforcement ratio was varied according to a PRBS. In this way, pigeons had to learn which key was the richer alternative every single session. Every pigeon was exposed to both conditions. Half of the pigeons were started on short terminal links while the other half of the pigeons were started on long terminal links. In this way, we were able to test for the effects of terminal-link duration on sensitivity to terminal link entry ratio.

3.2 Method

3.2.1 Subjects

7 pigeons of mixed breed and sex were maintained at 85% of their free-feeding weight through appropriate post-session feeding. Pigeons were caged individually in a vivarium which ran according to a 12-hour day and night cycle (lights were switched on at 06:00). Home cages were supplied with grit and water which was available at all times to pigeons when they were in their home cages.

3.2.2 Procedure

One complete trial of the concurrent chains procedure occurred when the pigeons had responded during an initial link phase and a terminal link phase. A 72-cycle program of concurrent chains was be used for this experiment. Completing each session consisted of pigeons completing 72 trials of concurrent chains. At the start of each trial, side keys were lighted yellow to signal the beginning of the initial links. Terminal link entry was signalled by lighting the side keys either red or green. During each terminal link, the color of the side key depended on whether a left or right initial link response produced the terminal link (red-left, right-green). Terminal link responses were reinforced by access to grain according to FI schedules.

In the initial link, a single VI 8s schedule operated. As each interval in the schedule timed out, the outcome was assigned randomly to the left or right key and the next peck on the assigned key produced access to the corresponding terminal link. In one half of all sessions, the assignment was 75% -left, 25% right. In the remaining sessions, the assignments were reversed. Terminal links were always the same for left and right – FI 4s and FI 4s in one condition, and FI 16s and FI 16s in the second condition. The order in which schedules favoured the left and right keys across sessions was determined by a PRBS, as used by Hunter et al. (1985). The use of PRBS ensured the order in which terminal link entries occurred across sessions was completely randomized. Each presentation of the PRBS required 31 sessions of concurrent chains. In one condition, terminal-link durations were FI 4s seconds (short). In the other condition, terminal-link durations were FI 16s seconds (long). The concurrent chains procedure for this experiment is shown in Figure 3.01. The short terminal-link condition is shown on the left while the long condition is shown on the right.

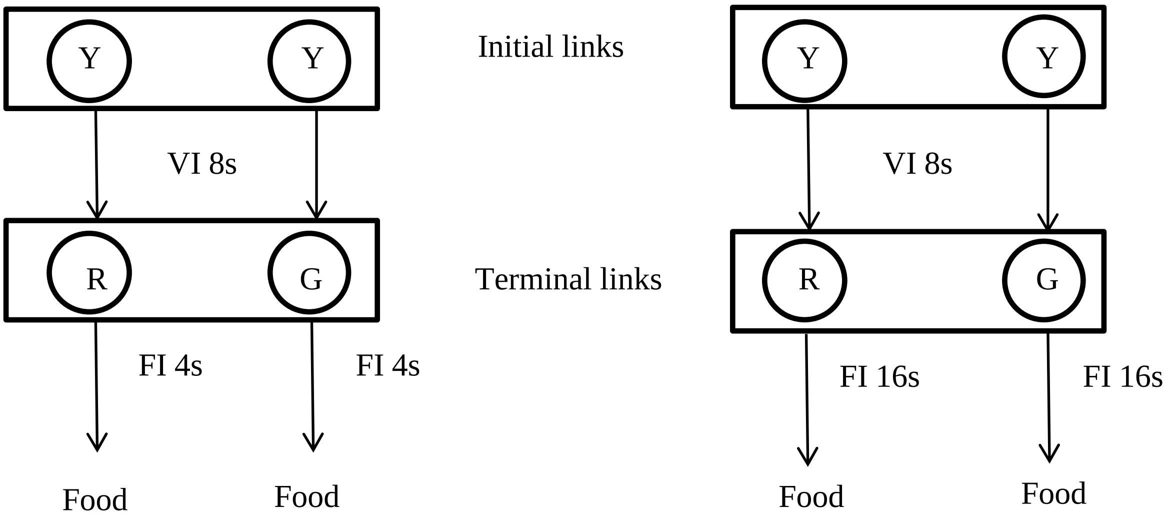


Fig 3.01. Procedure for each bird in Experiment 2. Side keys are lighted yellow. Initial links are a single VI 8-second schedule that results in terminal-link entry. Terminal links are either both FI 4 seconds or FI 16 seconds. Left terminal links are light red while right terminal links are lighted green. The end of each terminal link results in access to food. Y = yellow key, R = red key, G = green key. Left versus right terminal-link entry ratios are 1:3 or 3:1.

Every pigeon was exposed to both conditions. Half of the pigeons were started on short terminal links while the other half of the pigeons were started on long terminal links. Every pigeon was subject to both short and long terminal-link conditions. For each 31-session block, the position of the more frequently reinforced alternative was Left (L), Right (R), R, R, L, L, R, L, L, L, R, L, R, L, R, R, R, R, L, R, R, L, R, L, L, R, R, L, L, L, L. Due to non-responding issues, the number of completed sessions varied between pigeons. With the exception of 2 pigeons during the long terminal condition, pigeons completed at least 75 sessions of training (minimum: 42, maximum: 124; average = 101). The number of training sessions for each pigeon is shown in Table 3.01.

Table 3.01

Number of Completed Sessions by Pigeon and Condition

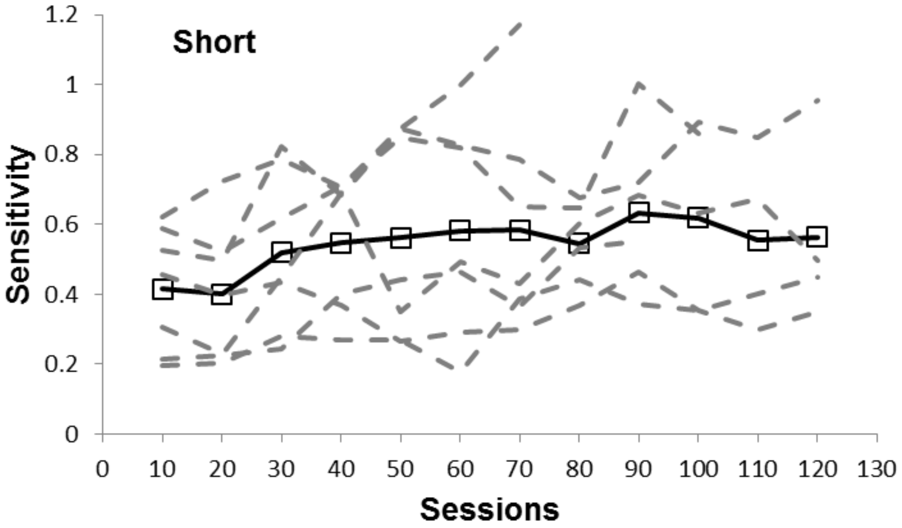
Bird	Short	Long
171	123	119
172	86	50
173	75	42
174	123	114
176	106	89
177	124	123
178	124	123
Average	108	94

Note. Experimental conditions are denoted by “short” for short terminal-link durations and “long” for long terminal-link durations. Birds 171 to 174 were started on short terminal links. Birds 176 to 178 were started on long terminal links.

3.3 Results

3.3.1 Between-session preference acquisition

Since the number of completed sessions varied across birds and conditions, a correlation analysis between the number of completed sessions and sensitivity to relative terminal-link entry ratio was conducted to determine if asymptotic sensitivity levels were dependent on the number of completed sessions. Sensitivity to the relative terminal-link entry ratio was calculated over the last 30 sessions of training for each pigeon and condition**.** Across both short and long terminal-link durations, there was no significant correlation between the number of completed sessions and sensitivity values, $r(12) = -.03$, $p = .913$. Sensitivities to relative terminal-link entry ratio for blocks of 10 sessions are shown in Figure 3.02**.** The data point at session 10 denotes the sensitivity of relative response ratio to relative terminal-link entry ratio as determined by pooling data from individual sessions 1-10, then applying the GML to the data across sessions. This analysis was completed for each block of 10 sessions for all pigeons. Across sessions, there appeared to be an increase in sensitivity in the short condition but not the long condition. In the short terminal link condition, mean sensitivity was 0.42 during the first 10 sessions and 0.56 during the last 10 sessions. In the long terminal link condition, mean sensitivity was 0.45 during the first 10 sessions and decreased to 0.37 by the last 12 sessions. Across both conditions, peak sensitivity values were reached by the last three 10-session blocks.



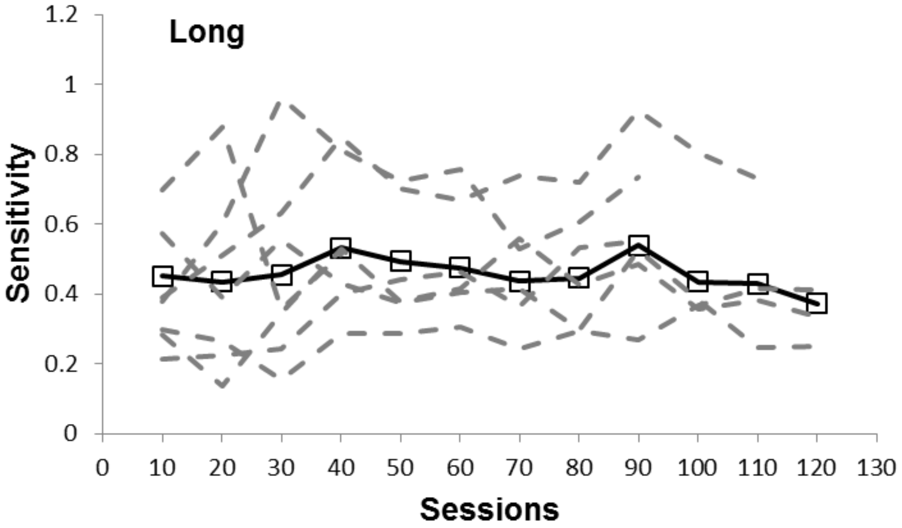


Fig 3.02. Full-session sensitivity values by 10-session blocks in short (top panel) and long (bottom panel) terminal-link durations. Dotted lines represent sensitivity values of individual birds. Solid lines are the mean sensitivity values across all birds in each condition. The last session block consists of 12 sessions.

3.3.2 Preference at the beginning and at the end of training

Because Figure 3.02 suggests that sensitivity increased during short terminal links but not long terminal links across consecutive training sessions, we compared sensitivities for both conditions for the first and last blocks of 20 sessions. Figure 3.03 shows the mean sensitivity values of each condition during the first 20 and last 20 training sessions. Sensitivity values during the first 20 sessions were approximately equal for both short (M = 0.41, SD = 0.17) and long (M = 0.48, SD = 0.16) conditions. During the last 20 sessions, short terminal links resulted in higher sensitivity values (M = 0.74, SD = 0.27) compared to long terminal links (M = 0.51, SD = 0.18). A repeated-measures ANOVA with condition (short/long) and session block (first 20/last 20) as factors found a significant effect for session block, $F(1, 12) = 15.25$, $p = .002$, $\eta^2 = .56$, and a significant interaction between condition and session block, $F(1, 12) = 11.49$, $p = .005$, $\eta^2 = .489$, but no significant effect for condition, $F(1, 12) = 0.68$, $p = .426$, $\eta^2 = .054$. These results confirm that sensitivity increased from the first 20 to the last 20 sessions in the short but not long terminal-link condition.

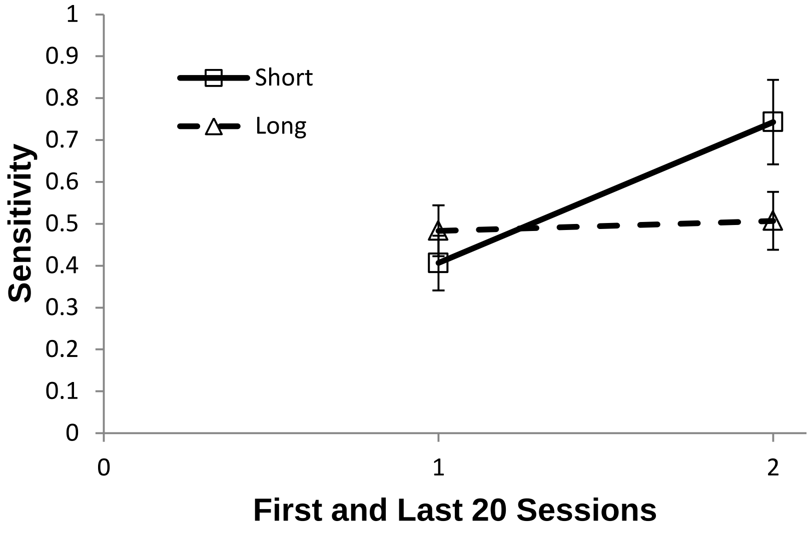


Fig 3.03. Sensitivity of relative response ratio to relative terminal-link entry ratio across short and long terminal-link durations during the first 20 and last 20 sessions (as indicated by “1” and “2” on the horizontal axis). Error bars represent standard errors.

3.3.3 Lag 0 and lag 1 analysis

We wanted to determine how sensitivity to terminal-link entry ratio changed within sessions and was affected by terminal-link duration, the order in which pigeons completed the conditions (short terminal-link duration then long, or vice versa), and the terminal-link entry ratio for the current and prior sessions. For this we conducted a lag multiple regression analysis to obtain estimates of sensitivity for each session sixth to current and prior terminal-link entry ratios, and bias:

$$\text{Log}(\frac{B_{0L}}{B_{0R}}) = \text{Log } b + a_0\text{Log}(\frac{R_{0L}}{R_{0R}}) + a_1\text{Log}(\frac{R_{1L}}{R_{1R}}) , \qquad (3.01)$$

where *B* represents the number of responses made on each alternative, *R* is the number of reinforcers (terminal-link entries) obtained from responding on each alternative, *a* is the sensitivity to relative entry ratio, subscripts lag 0 and 1 denote the current session and previous session, and Log *b* is a bias parameter.

Figure 3.04 shows mean sensitivity to relative terminal-link entry ratio for lag 0 and lag 1 (lag 0 = current session, lag 1 = previous session) for all subjects across both groups during the last 30 sessions. Lag coefficients were entered into a 2 × 2 × 2 ×6 repeated measures analysis of variance (ANOVA) with order (short followed by long/long followed by short)/condition (short/long), lag and session sixth as factors. Sensitivity to terminal link entry ratio increased across session sixths as evidenced by the significant main effect for session sixth, $F(5, 25) = 12.98$, $p < .001$, $\eta^2 = .72$. The main effect of lag was significant, $F(1, 5) = 83.19$, $p < .001$, $\eta^2 = .94$. The effect of condition was significant, $F(1, 5) = 7.11$, $p = .045$, $\eta^2 = .59$, as sensitivity was greater with short terminal links. For lag 0, mean sensitivity was slightly higher during the first session sixth in short ($M = 0.36$) compared to long ($M = 0.27$) terminal links. By the last session sixth, sensitivity was much higher in short ($M = 0.87$) in contrast to long ($M = 0.65$). There was a significant interaction between lag and session sixth, $F(25, 125) = 45.21$, $p < .001$, $\eta^2 = .90$. These results indicate that the primary differences between groups occurred for the Lag0 sensitivities, which were higher for the short terminal-link condition. There was no significant main effect for order, $F(1, 5) = 0.002$, $p = .965$, and all interactions involving order also failed to reach significance, although the two-way interactions between order and session sixth and terminal-link duration approached significance (*p*s = .06 and .13, respectively).

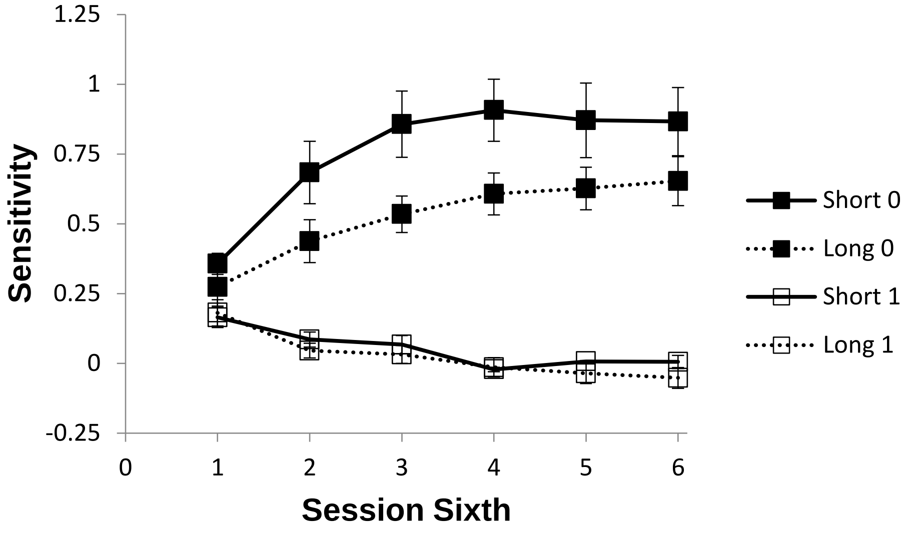


Fig 3.04. Lag analysis of previous session terminal-link entry ratios as predictors of preference in the current session for short and long terminal-link durations. "0" refers to the sensitivity of each current session's log response ratio to log terminal-link entry ratio of that same session. "1" refers to the sensitivity of each current session's log response ratio to log terminal-link entry ratio of the previous session. Error bars represent standard errors.

3.3.4 Strength of bias

Inspection of the individual results suggested that bias values in the lag multiple regression analysis may have differed systematically between short and long conditions. For most of the pigeons, strength of bias (log *b*) for a particular key during the initial links were often very different in the short and long condition, although all pigeons showed a consistent direction of bias across conditions (i.e., sign of log *b*). To investigate this, we calculated the absolute value of the bias (log *b*) estimates from the multiple regressions. Absolute bias values were analysed to allow us to calculate averages to determine if position preference (left or right) was overall stronger in the short or long condition.

Results are shown in Figure 3.05 below. A repeated-measures ANOVA (2 × 2 × 6 short followed by long/long followed by short)/condition (short/long) and session sixth as factors found a significant main effect for condition, *F*(1, 5) = 11.60, *p* = .019, η^2 = .70. Absolute bias with short terminal links was weaker overall (*M* = 0.34) compared to long (*M* = 0.57) terminal links.

Although there was no main effect for session sixth, *F*(5, 25) = 0.29, *p* = .682, there was a significant three-way interaction for condition, session sixth and order, *F*(5,25) = 3.02, *p* = .029, η^2 = .38, and a two-way interaction for session sixth and order, *F*(5,25) = 3.36, *p* = .019, η^2 = .40. Pigeons that were started on short terminal links exhibited decreasing bias across consecutive session sixths when terminal links were long, whereas pigeons that were started on long terminal links exhibited increasing bias across consecutive session sixths with long terminal links. Although the effect for order was not significant, *F*(1, 5) = 0.29, *p* = .614, there was a significant interaction between order and condition, *F*(1, 5) = 16.44, *p* = .010, η^2 = .77. Pigeons which were started on short terminal links exhibited approximately equal strengths of bias during short (*M* = 0.45) and long (*M* = 0.40) terminal links. Pigeons which were started on long terminal links exhibited much stronger bias in long (*M* = 0.75) compared to short (*M* = 0.24) terminal links. Post-hoc tests (Fisher LSD) showed that none of the other pairwise comparisons were significant.

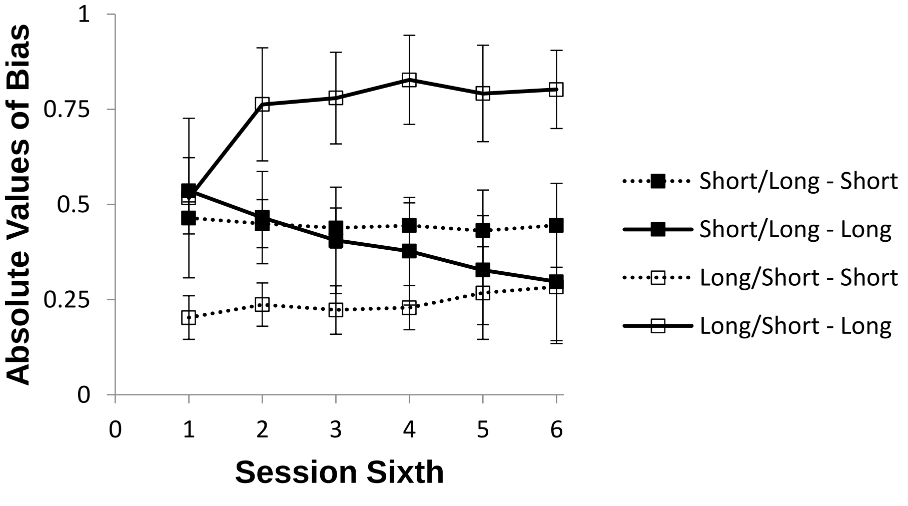


Fig 3.05. Mean absolute values of bias by session sixth over the last 30 sessions for short and long terminal links based on the order which pigeons began the study. Filled squares represent pigeons that were started on short terminal links. Unfilled squares represent pigeons that were started on long terminal links. Dashed lines represent mean bias during short terminal links. Solid lines represent bias exhibited during long terminal links. Error bars represent standard errors.

These results show that when pigeons were initially exposed to the long terminal-link condition, bias (i.e., left/right position preference) strongly determined response allocation even though relative terminal-link entry rate was changing across sessions. When these pigeons were subsequently exposed to short terminal links, the bias decreased. However, pigeons that were first trained with short terminal-link durations had overall weaker levels of bias, which did not increase when shifted to long terminal links. Thus, the order of training with short and long terminal-link durations affected the degree to which position preference controlled response allocation.

3.3.5 Extending the cumulative decision model

A major goal of the present research was to test if the cumulative decision model (CDM; et al., 2010; Grace, 2016) could be extended to predict acquisition of choice when the location of the richer alternative (in terms of terminal-link entry ratio) changed unpredictably across sessions. As noted in the Introduction, the CDM was originally developed to explain the effects of terminal-link variables on choice. When the CDM was applied to steady-state data from archival studies, Christensen et al. (2010) concatenated a terminal-link value ratio (computed by the CDM) to a generalized-matching model based on terminal-link entry ratios.

Here, we assumed that a similar process as that described by the model for Experiment 1 for choice in concurrent schedules operated during the initial links, to replace the generalized-matching component of the CDM.

Specifically, initial-link responding was determined by response strength, which could decrease according to an extinction process. As in Chapter 2, the RS of the opposite alternative was assumed to decrease during visits by a continuous exponential decay process. One change to the process of changing RS during initial links was that RS at the beginning of each session was not reset to 0.50. Instead, the RS at the beginning of each session was carried over from the last trial of the previous session. Therefore, this model is called the “carryover” model. On terminal-link entry, the response strength of the initial link that produced the entry increased by an increment determined by a linear operator. However, because this increase could only affect initial-link responding after the terminal link was finished, we assumed that its effect would be reduced, depending on the length of the terminal link. This was accomplished by assuming that the probability that the entry would be assigned to the correct initial link decreased, according to an exponential function, during the terminal link, analogous to a diffusion process (with rate parameter β_{diff}):

$$Diff = e^{(-\beta_{\text{diff}}\,T)}, \qquad (3.02)$$

where *Diff* denotes the probability that the pigeon will accurately discern which key during the initial links resulted in terminal-link entry and T is terminal-link duration. All other aspects of the model remained the same as the original CDM. At the end of the terminal link, the outcome was compared against a criterion, and initial-link response strength was updated depending on the probability that the outcome was judged ‘good’ relative to the criterion. Analyses showed that varying the β_{diff} parameter did not substantially improve VAC of model fits and was therefore fixed at 0.02.

The model was applied to data from the last 30 sessions of training for each pigeon. For each pigeon, the total duration of individual cycles was used to approximate the time spent responding on the left and right alternatives (used to model extinction), as well as the location of the reinforcer for that cycle. The model was used to calculate simulated responses for each cycle in the following way: The overall average response rate was calculated for each pigeon by pooling the total responses (left and right) and total time spent responding across the 30 sessions. Based on the duration of each cycle, the model then calculated simulated responses for each alternative for that cycle as the obtained cycle duration x overall average response rate x relative response strength.

During each cycle, decreases in response strength were determined by Equation (2.04) and Equation (2.05). At the end of each cycle, changes in response strength were dependent on which alternatives resulted in terminal-link entry and delivery of reinforcement, as determined by Equation (2.03). For every session sixth (every 12 reinforcers), the simulated number of responses on each alternative was computed. Predicted relative response ratios of the model were calculated as the log ratio of the simulated responses on the left and right alternatives.

Based on the best-fitting parameter values, predictions of relative response ratios by the model were computed for every session sixth (12 reinforcers) for every pigeon. For both groups, this was done for the last 30 sessions of training. Thus, there were 180 data points for each pigeon per condition (6 session sixths x 30 sessions), and three parameters were estimated (log b, α , and β_{opp}). Several parameters from the CDM were fixed rather than freely estimated, including α_{TL}, α_{Cr} and σ. α_{TL} is a learning parameter (constant) that denotes that proportion of the difference between asymptotic (i.e., 1) response strength and current response strength is added as a result of the terminal link outcome (delivery of reinforcement). Because the updating of response strength every cycle is determined by whether an experienced delay is judged to be long or short relative to a criterion, an EWMA is used to update the criterion based on the intervals between stimuli (Christensen et al., 2010).

The reinforcement history in terms of log initial-link and terminal-link delays is a distribution of delays that follows an inverse cumulative normal distribution whereby its mean is the criterion (α_{Cr}) and standard deviation (σ) as a parameter. σ determines the accuracy of decisions whereby when its values are small, decisions are accurate but when its values are large, decisions are not accurate. Because the left and right and right terminal-link delays were always equal, the values of α_{TL}, α_{Cr} and σ were fixed at typical values used in previous studies (0.2, 0.1 and 0.3, respectively). Table 3.02 shows the percentage of VAC, the best-fit parameter values of Log *b*, α, β_{opp} as well as the slopes and intercepts of obtained log response ratios plotted against the model's predicted log response ratios . β_o was initially allowed to vary but VAC of the model fits were very marginal (less than 3%). Therefore, β_o was omitted since it was fixed at a value of zero across all birds. Moreover, β_o will be fixed at this value for all successive experiments.

3.4.1 Quality of the extended CDM's model fits

3.4.1.1 Parameter invariance. In order to determine if the model could account for greater sensitivity to terminal link entry ratio during short terminal links, dependent-samples t-tests were conducted to determine if the values of each parameter as well as the VAC were significantly different between short and long terminal links. In terms of the α parameter, the difference between short ($M = 0.036$, $SD = 0.38$) and long ($M = 0.003$, $SD = 0.005$) terminal links approached significance, $t(6) = 2.45$, $p = .050$, $d = 0.92$, consistent with the increased sensitivity in the short condition. There was no significant difference in β_{opp} values between short ($M = 0.020$, $SD = 0.015$) and long ($M = 0.017$, $SD = 0.009$) terminal links, $t(6) = 0.42$, $p = .691$. In terms of the log *b* parameter, there was no significant difference between short ($M = 0.03$, $SD = 0.42$) and long ($M = 0.05$, $SD = 0.68$). VAC was significantly higher in short

(*M* = 0.69, *SD* = 0.12) compared to long (*M* = 0.46, *SD* = 0.14) terminal links, $t(6) = 2.68$, $p = .037$, $d = 1.04$.

These results show that the model accounted for more variance overall in the short terminal-link condition. This is likely due to the greater range of log response ratios in the short condition (and hence greater variance). The data were relatively variable across pigeons, and estimated bias values were significantly greater in the long terminal links. This may have contributed to the relatively poorer fits of the model to the long condition. Results also suggested that α parameter was greater in the short condition, although the difference did not quite reach significance.

Table 3.02

Best-fit Parameter Values and VAC

Group	Bird	Log b	α	β_{opp}	Slope	Intercept	VAC
Short	171	0.49	0.000	0.008	1.00	0.00	0.67
	172	-0.32	0.089	0.000	1.02	0.01	0.47
	173	0.36	0.020	0.045	1.00	0.00	0.83
	174	-0.64	0.029	0.011	1.00	0.00	0.70
	176	0.39	0.091	0.021	1.02	0.00	0.67
	177	0.14	0.020	0.032	1.01	0.00	0.80
	178	-0.19	0.006	0.023	1.00	0.00	0.73
Long	171	0.63	0.002	0.012	0.97	0.02	0.45
	172	-0.18	0.000	0.018	0.98	0.00	0.51
	173	0.24	0.001	0.012	0.94	0.01	0.20
	174	-0.65	0.008	0.035	0.98	-0.01	0.69
	176	0.82	0.013	0.014	0.97	0.04	0.49

	177	0.51	0.000	0.019	0.97	0.01	0.47
	178	-0.99	0.000	0.008	0.95	-0.05	0.42
Short	*Mean*	0.03	0.037	0.020	1.01	0.00	0.69
Long	*Mean*	0.05	0.003	0.017	0.97	0.00	0.46
	t	-0.14	2.45	0.42	8.15**	-0.13	2.68*

Note. The first half of the table are for short terminal links while the second half are for long terminal links. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. VAC was computed based on session sixth data.

* indicates $p < .05$. ** indicates $p < .01$.

3.4.1.2 Predictive accuracy of the extended CDM. To check if there were systematic deviations of the model's predictions from the data, we examined scatterplots of obtained and predicted log response ratios of each pigeon. In terms of the goodness-of-fit and predictive accuracy of the model, Figure 3.06a shows the plots for Pigeons 171 to 174 while Figure 3.06b shows plots for Pigeons 176 to 178. Left panels show plots of short terminal links while the right panels show long terminal links. The slope and y-intercept of each plot are also given. The dashed line in each panel is the regression of obtained on predicted values, and the solid line indicates ideal prediction. In each plot, slopes and intercepts of the dashed line that are close to 1.0 and 0.0, respectively, indicate that there are no systematic errors in prediction.

Comparisons of the slopes and intercepts of short and long terminal-link durations confirmed that the model was able to predict data in short terminal links more accurately. The mean sensitivity of the slope for short terminal links was 1.01, 95% CI [1.00, 1.01], while the corresponding mean for the long condition was 0.97, 95% CI [0.95, 0.98]. For the intercepts, the mean for both conditions was 0.00 (short 95% CI [0.00, 0.00], long 95% CI [-0.02, 0.02]. Thus, predictions did not deviate systematically from obtained data in the short condition, whereas with long terminal links there was a significant tendency for the model to overpredict (i.e., obtained values were slightly less extreme than predicted), as evidenced by the slope < 1.

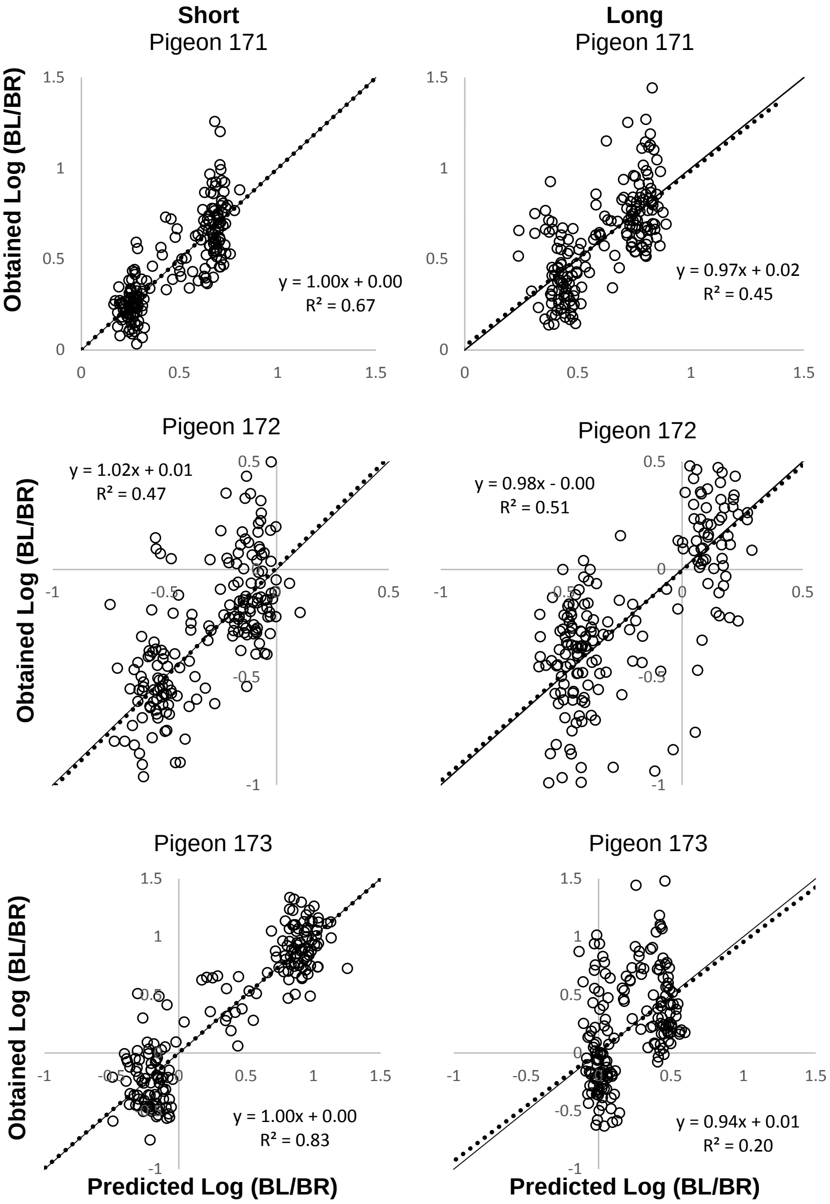


Fig 3.06a. Obtained log ratios plotted against predicted log ratios for short and long terminal-link durations for Pigeons 171 to 173. Left panels show short terminal links while right panels show long terminal links. Data points are based on session sixth responding. In order to display better the scatter of individual data points, not all axes have identical scales.

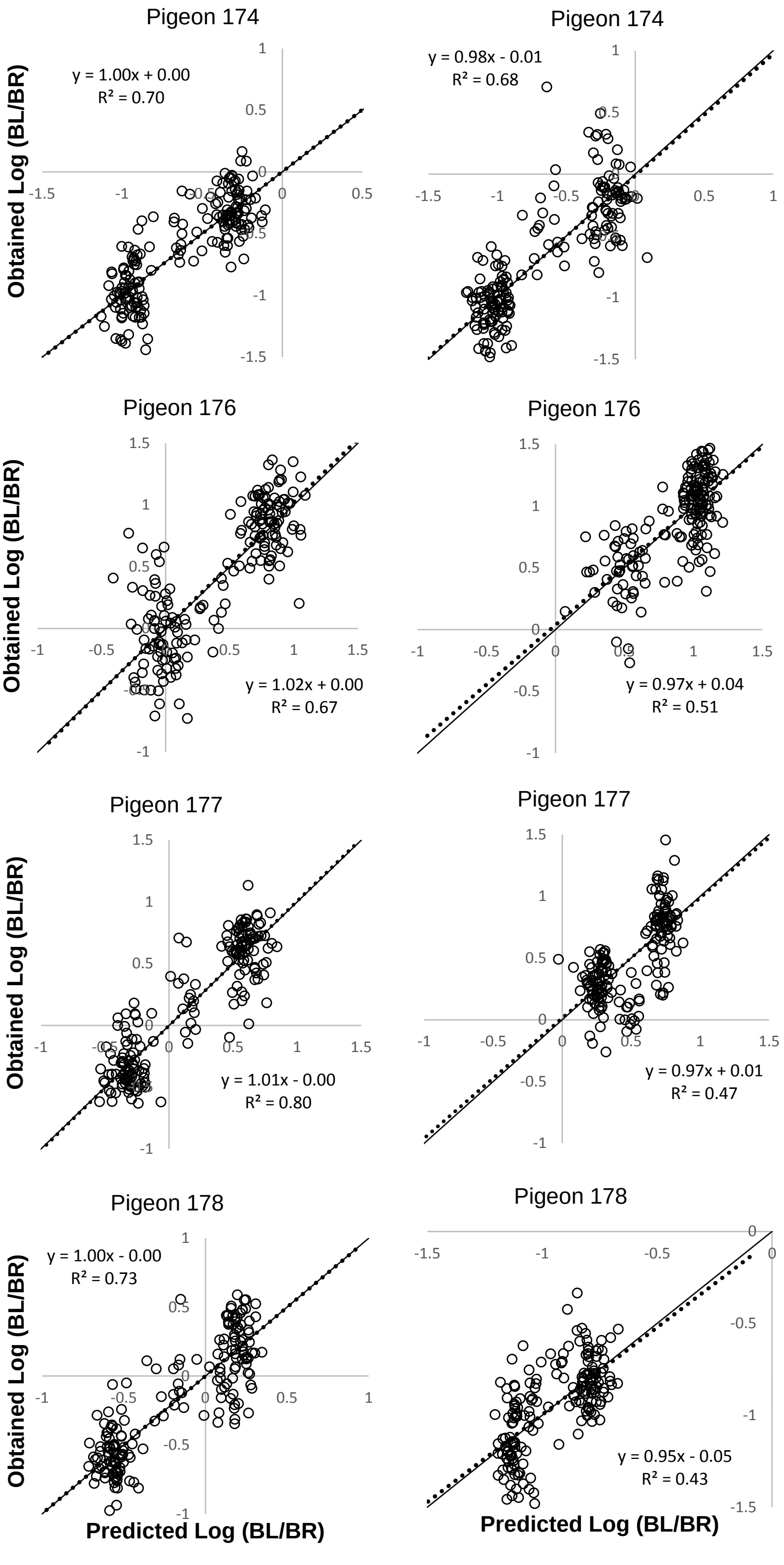


Fig 3.06b. Obtained log ratios plotted against predicted log ratios for short and long terminal-link durations for Pigeons 174 to 178. Left panels show short terminal links while right panels show long terminal links. Data points are based on session sixth responding. In order to display better the scatter of individual data points, not all axes have identical scales.

3.4.1.3 The extended CDM's predictive accuracy of asymptotic preference.

Results presented above (Figure 3.04) show that similar to previous research with rapid-acquisition choice procedures (e.g., Kyonka & Grace, 2007), sensitivity increased over the first half of the session but was approximately stable over the second half. Therefore, responding during each block of the fourth to sixth session sixth could serve as an approximation of attained asymptotic preference during each session. Several analyses were conducted to determine if asymptotic response allocation, both obtained and predicted by the model fits, was consistent with the generalized matching law.

As previous plots of sensitivity have shown, preference stabilizes within the first half of each session and asymptotic levels of preference are reached by the second half. In terms of asymptotic preference levels, the next task was to determine whether this model was able to predict the observed trend of approximate matching and under matching. Each session consisted of one data point and this analysis was conducted on the last 30 sessions of training resulting in each plot producing 30 data points. Based on the best-fit parameter values, the model's resulting predictions of log response ratios and obtained log response ratios were computed. Obtained and predicted log response ratios were separately plotted against log terminal-link entry ratios. For short terminal links, Figure 3.07a shows the plots for Pigeons 171 to 173 while Figure 3.07b shows the plots for Pigeons 174 to 178. For long terminal links, Figure 3.08a shows the plots for Pigeons 171 to 174 while Figure 3.08b shows plots for Pigeons 176 to 178. In terms of the slopes of the GML, differences between them based on simulated and obtained data were minor. In terms of the intercepts of the GML, those based on simulated and obtained data were also approximately equal.

In terms of the slopes of short and long terminal links based on obtained data, the slopes are steeper in short terminal links. The intercepts were noticeably distanced from the point of origin in both short and long terminal links. These results are consistent with the greater peak preference exhibited during short terminal links over the course of training. VAC during short terminal links was also consistently higher than long terminal links.

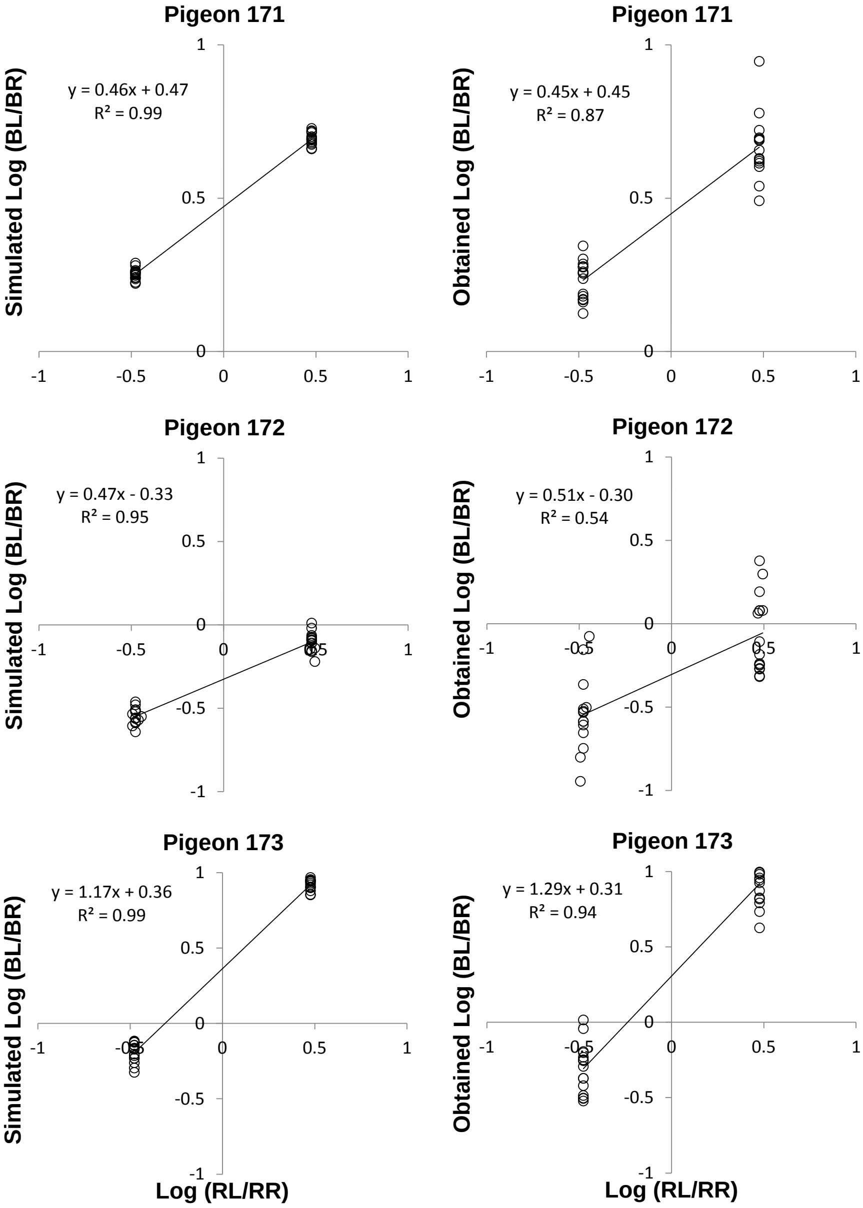


Fig 3.07a. Log response ratios (vertical axis) plotted against log reinforcer ratios for short terminal-link durations for Pigeons 171 to 173. Left panels show plots based on predicted log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

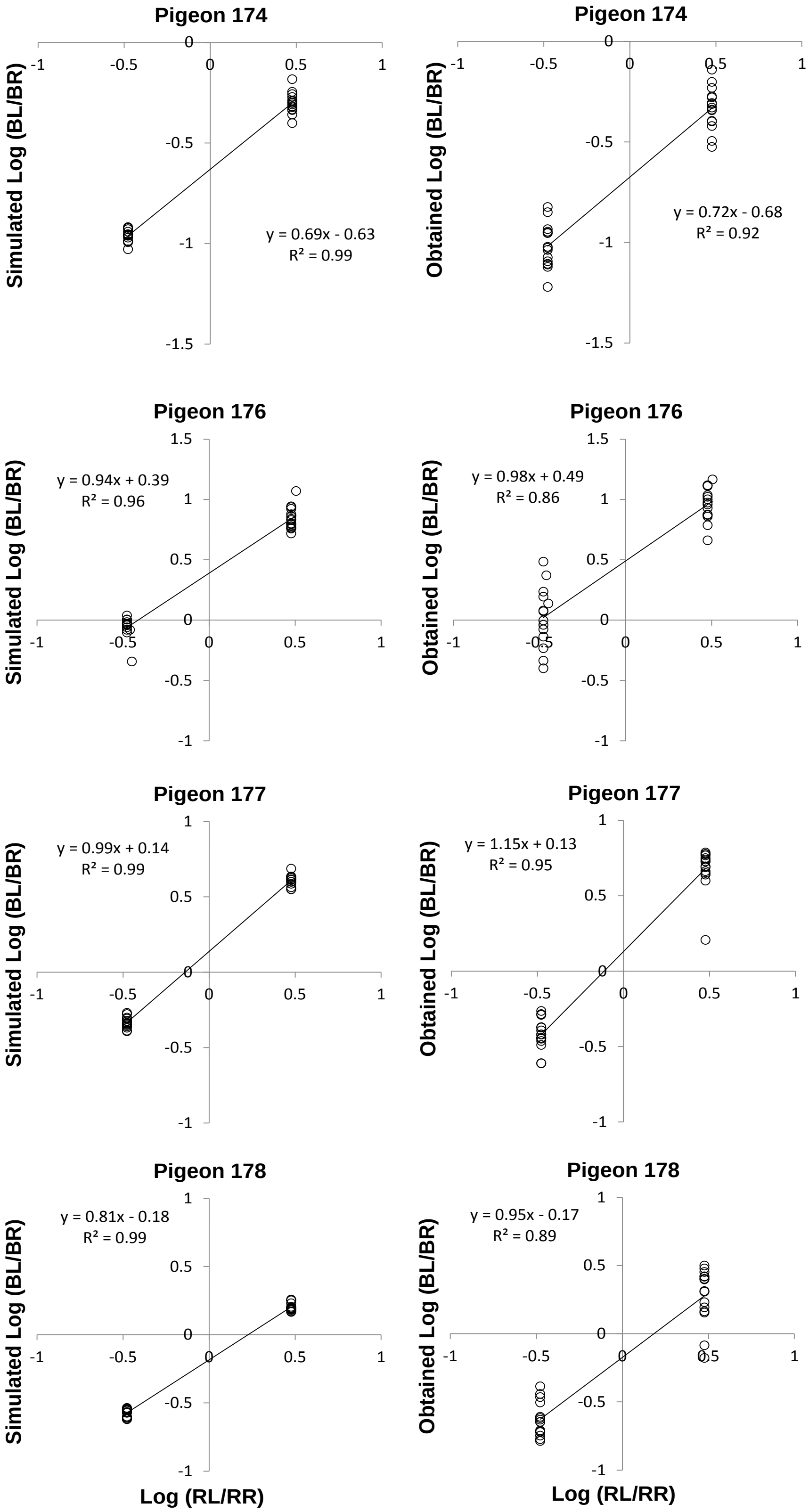


Fig 3.07b. Log response ratios (vertical axis) plotted against log reinforcer ratios for short terminal-link durations for Pigeons 174 to 178. Left panels show plots based on predicted log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

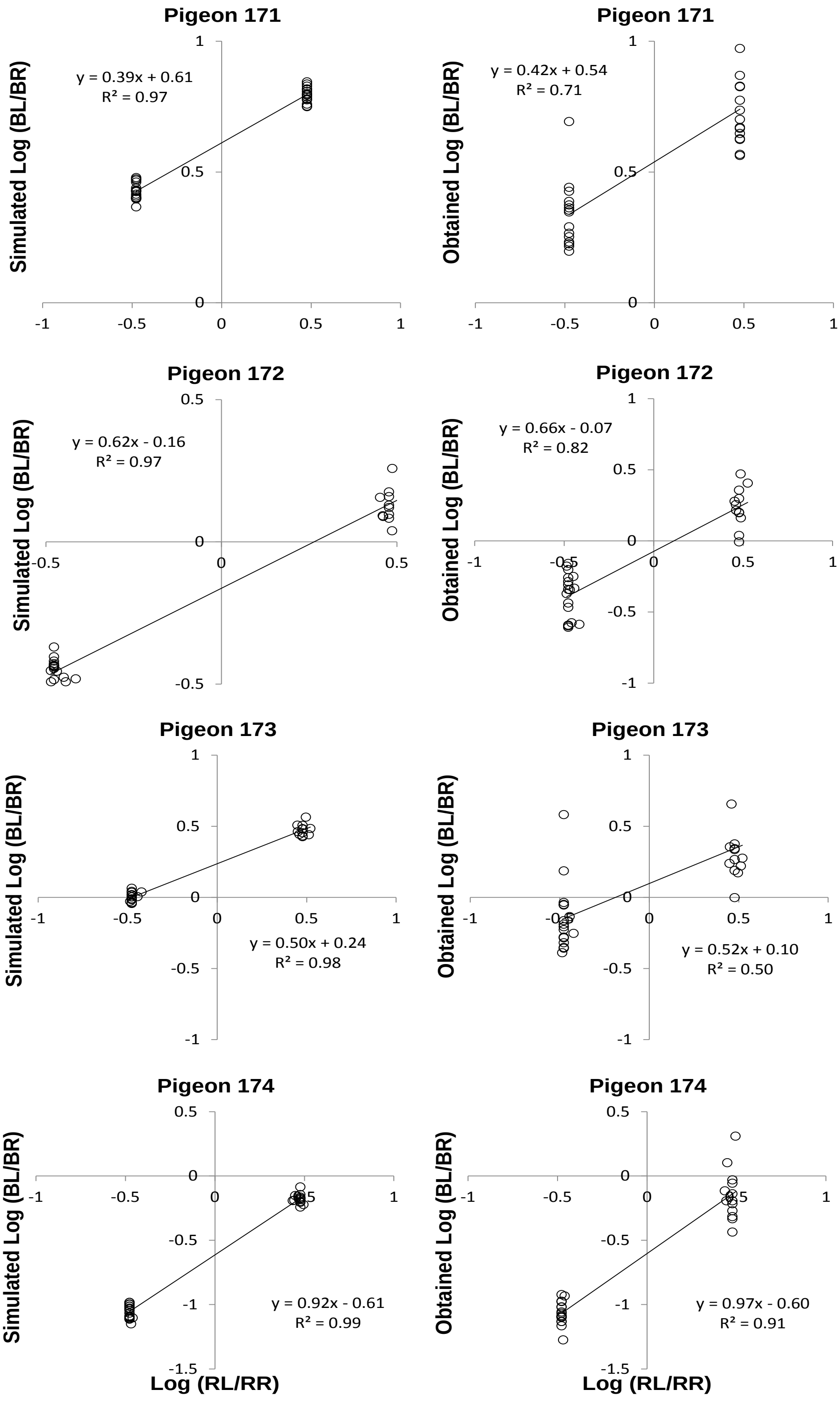


Fig 3.08a. Log response ratios (vertical axis) plotted against log reinforcer ratios for long terminal-link durations in Pigeons 171 to 174. Left panels show plots based on predicted log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

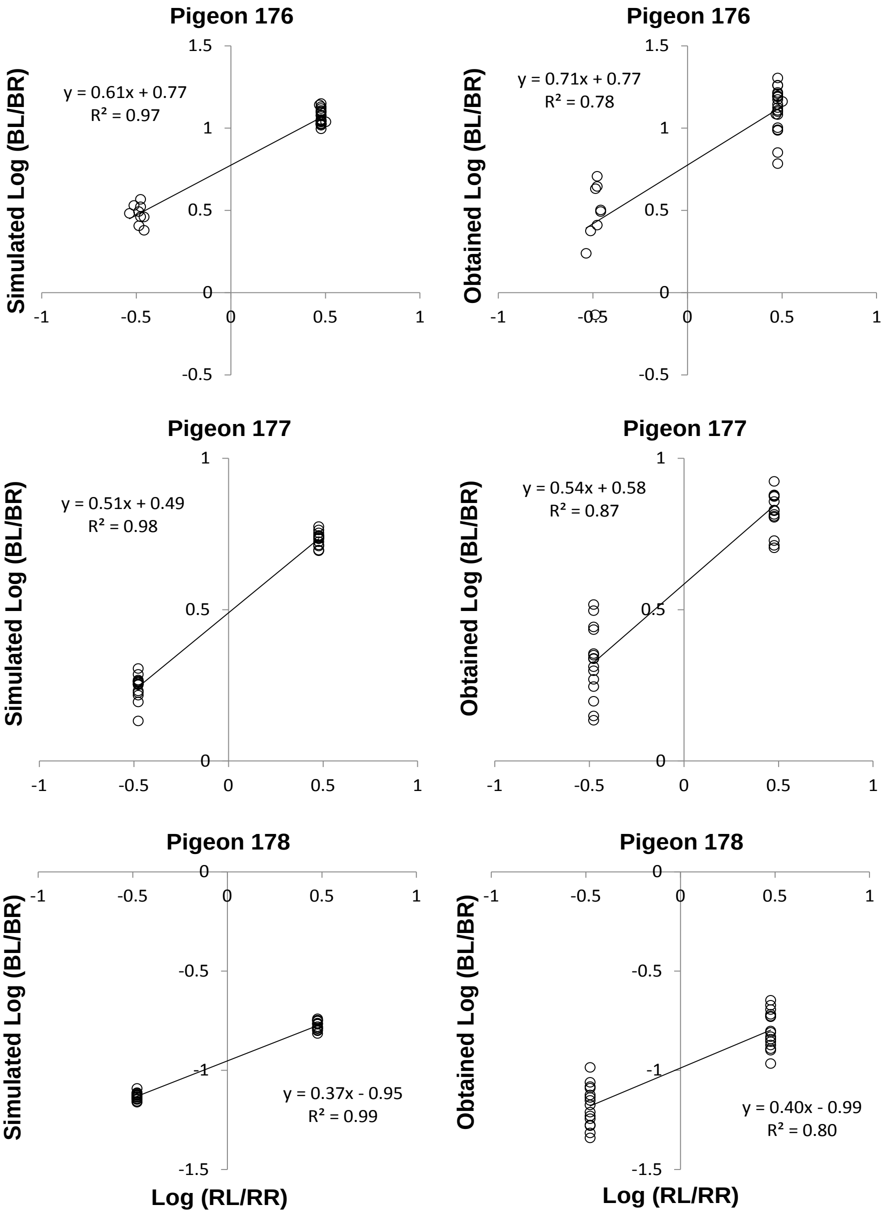


Fig 3.08b. Log response ratios (vertical axis) plotted against log reinforcer ratios for long terminal-link durations in Pigeons 176 to 178. Left panels show plots based on predicted log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

Finally, we assessed the extent to which the asymptotic data (i.e., second half of sessions) and the model's predictions were consistent with generalized matching. Table 3.03

shows the average slope, intercept and VAC for GML fits to obtained and simulated data. Paired-sample *t* tests were conducted to determine if there was a difference in the average slope, intercept and VAC between short and long terminal links. In terms of VAC in short terminal links, there was a significant difference between predicted (*M* = 0.98, *SD* = 0.02) and obtained (*M* = 0.85, *SD* = 0.14) VAC, *t*(6) = 2.63, *p* = .039, *d* = 0.99. In Long terminal links, there was a significant difference between predicted (*M* = 0.98, *SD* = 0.01) and obtained (*M* = 0.77, *SD* = 0.14) VAC, *t*(6) = 4.14, *p* = .006, *d* = 1.56. During short terminal links, there was a significant difference between predicted (*M* = 0.79, *SD* = 0.27) and obtained (*M* = 0.86, *SD* = 0.32) slopes, *t*(6) = -3.03, *p* = .023, *d* = 1.16. There was no significant difference between predicted (*M* = 0.03, *SD* = 0.42) and obtained (*M* = 0.03, *SD* = 0.43) intercepts, *t*(6) = -0.07, *p* = .945. Similar differences were found in long terminal links. There was a significant difference between predicted (*M* = 0.56, *SD* = 0.19) and obtained (*M* = 0.60, *SD* = 0.20) slopes, *t*(6) = -4.22, *p* = .006, *d* = 1.58. There was no significant difference between predicted (*M* = 0.06, *SD* = 0.65) and obtained (*M* = 0.05, *SD* = 0.65) intercepts, *t*(6) = 0.27, *p* = .795.

Table 3.03

Average of Slope, Intercept and VAC for GML fits to Obtained and Simulated Data

Data	Condition	Slope	Intercept	VAC
Obtained	Short	0.86	0.03	0.85
	Long	0.60	0.05	0.77
Simulated	Short	0.79	0.03	0.98
	Long	0.56	0.06	0.98
Short Diff	*t*	-3.03*	-0.07	2.63*
Long Diff	*t*	-4.22*	0.27	4.14**

Note. Diff denotes the difference between obtained and simulated data. Values are computed using responding during the second half of each session, over the last 30 sessions.

* indicates *p* < .05. ** indicates *p* < .01.

3.4.2 Residual analysis

A residual analysis was conducted to test if there were systematic deviations in the obtained and simulated data from generalized matching, similar to Experiment 1. Polynomial regressions were conducted in which the residuals were regressed on the predicted and obtained values. The results of the regressions are shown in Table 3.04. In terms of the simulated data, the linear, quadratic and cubic coefficients were all non-significant predictors of the log response ratios for the simulated data. A similar trend was observed in the obtained data where the linear, quadratic and cubic coefficients were all not significant predictors of log response ratios for the obtained data. These results strongly suggest that there were no systematic deviations from the GML's predictions for both simulated and obtained data. Although statistical tests of systematic trends are not significant, we wanted to determine if there were any weak systematic trends that did not reach significance. Figure 3.09 shows the scatter of residual data points for short and long terminal links. The scatter of data points indicates little evidence of linear, quadratic or cubic trends.

Table 3.04

Results of Polynomial Regressions Performed on GML Residuals

Dependent variable	Group	Predictor variables	*B*	S.E. of *B*	R^2	*F*
GML residuals	Short	Linear	-0.002	0.016		
Simulated		Quadratic	0.001	0.012		
		Cubic	0.003	0.023		
					0.000	0.008
GML residuals	Short	Linear	-0.008	0.029		
Obtained		Quadratic	0.001	0.016		
		Cubic	0.005	0.019		
					0.000	0.029
GML residuals	Long	Linear	0.000	0.011		
Simulated		Quadratic	0.000	0.006		

		Cubic	0.000	0.011		
					0.000	0.000
GML residuals Obtained	Long	Linear	-0.005	0.029		
		Quadratic	0.000	0.012		
		Cubic	0.003	0.015		
					0.000	0.012

Note. Dependent variables were residuals from the GML, computed using log response ratios. Independent variables were log predicted values, plus squares and cubes of log predicted values (‘quadratic’ and ‘cubic’). *B* is the unstandardised regression coefficient, and R^2 is the proportion of variance accounted for.

*indicates $p < 0.05$, ** indicates $p < 0.01$, *** indicates $p < 0.001$.

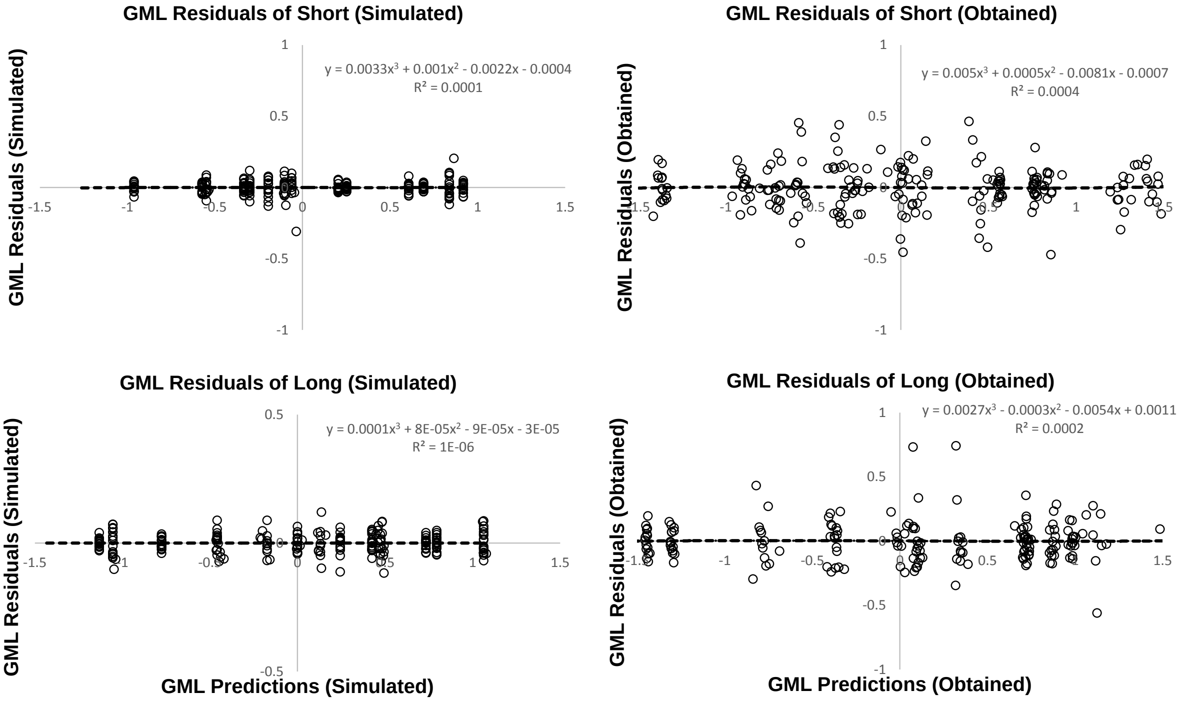


Fig 3.09. All panels show response allocation predicted by the GML for short or long terminal links. Upper panels show residuals of short terminal links while the lower panels show residuals of long terminal links. Left panels show residuals based on the carryover model’s simulated data while the right panels show residuals based on obtained data. Each data point is computed from the second half of each session over the last 30 sessions.

3.5 Summary

The major goal of Experiment 2 was to test if the duration of equal terminal links affected acquisition of preference in concurrent chains when their relative frequency varied unpredictably across sessions. Results showed that sensitivity to the relative terminal-link entry ratio was greater when the terminal-link duration was short (4 s) than when it was long (16 s). An extension of the CDM was developed which included the major features of the model from Experiment 1. The model provided a reasonably good description of the data, with relatively more variance accounted for in the short terminal-link duration condition. This may have been due to overall greater bias effects on preference for pigeons that were first trained in the long duration condition. Importantly, there were no systematic differences in the estimated values of the model parameters (α and β_{opp}), suggesting that the model was able to predict the overall greater sensitivity to the terminal-link entry ratio in the short condition.

Chapter 4

Experiment 3: The Effects of Terminal-link Entry Ratio and Initial-link Duration on Choice

4.1 Introduction

The previous chapter shows an effect of terminal-link duration on preference for the more frequently occurring terminal link in the form of increasing preference with shorter terminal links. In the context of the effects of conditioned reinforcement on choice behaviour, the next step involves investigating the effects of initial-link duration on sensitivity to terminal-link entry ratio. In this chapter, we also measured preference in terms of sensitivity to terminal-link entry ratio. We wanted to determine if the absolute values of initial-link duration affect the acquisition of choice between terminal links with the same delay. This question has not been addressed in prior research.

The carryover model has proven to be able to account for changes in preference due to terminal-link duration while maintaining parameter invariance. However, we wanted to determine if the carryover model could account for changes in preference due to initial-link duration. However, the consistently high accuracy of predicted relative response ratios and adequate VAC for short terminal links indicate that it serves as a viable explanatory decision model for concurrent chains. With lower sensitivity values of severe undermatching, we expected mean VAC to be lower than for the group with short terminal-link durations.

In this study, we wanted to determine the effects of initial-link duration on sensitivity to terminal-link entry ratio under rapid-acquisition conditions. Similar to the previous study, each trial consisted of an initial link phase and terminal link phase. Pigeons were made to peck two lighted keys in the initial links and the single lighted key in the terminal links. In this study, initial-link durations were either long or short. When initial links were short, a single VI 4s schedule timed out, while long initial links consisted of a single VI 16s schedule of reinforcement. For each initial link, there were a total of 12 possible intervals which, across each session resulted in an average initial-link duration of either 4 seconds or 16 seconds. A concurrent-chains procedure was arranged, in which terminal-link entry was scheduled probabilistically to either the left or right key after the interval had elapsed at the end of each initial link. In the initial links, when the interval in the schedule timed out, terminal-link entry was assigned randomly to the left or right key, and the next peck on the assigned key produced access to the corresponding terminal link. For half of the sessions, the ratio of terminal-link entries assigned to the left and the right key was 3:1. In the remaining sessions, it was 1:3. Terminal link durations were always the same for left and right keys – FI 8s FI 8s. A 1s COD was used. Reinforcement consisted of 3.5s of access to food. For both groups, whether the left or right key had the richer reinforcement ratio was varied according to a PRBS. All pigeons were exposed to both short and long initial links. Half of the pigeons were started on short initial links while the other half of the pigeons was started on long initial links. In this way, pigeons had to learn which initial-link key resulted in the richer alternative every single session.

4.2 Method

4.2.1 Subjects

Eight pigeons of mixed breed and sex were maintained at 85% of their free-feeding weight through appropriate post-session feeding. Pigeons were caged individually in a vivarium which ran according to a 12-hour day and night cycle (lights were switched on at 06:00). Home cages were supplied with grit and water which was available at all times to pigeons when they were in their home cages.

4.2.2 Procedure

One complete trial of the concurrent chains procedure occurred when pigeon responded during an initial link phase and a terminal link phase. A 72-cycle program of concurrent chains will be used for this experiment. Each session consisted of pigeons completing 72 trials of concurrent chains. At the start of each trial, side keys were lighted yellow to signal the beginning of the initial links. Terminal link entry was signalled by extinguishing the side keys and lighting the side keys either red or green. During the terminal links, the color of the side key was dependent on whether a left or right initial link response resulted in entry into the terminal link (red-left, right-green). Terminal link responses were reinforced by access to grain according to FI schedules.

In the initial links, short initial-link durations were operated using a single VI 4s schedule and long durations operated using a single VI 16s schedule. As each interval in the schedule timed out, the outcome was assigned randomly to the left or right key, and the next peck on the assigned key produced access to the corresponding terminal link. In one half of all sessions, the assignment was 75% on the left key and 25% on the right key. In the remaining sessions, the assignments were reversed. Terminal links were always the same for left and right keys – FI 8s and FI 8s. The order, in which schedules favour the left and right keys, across sessions, was determined by a PRBS, which was used by Hunter et al. (1985). The use of PRBS ensured the order in which terminal link entries occurred across sessions was completely random. Each presentation of the PRBS required 31 sessions of concurrent chains. Every pigeon was exposed to both conditions. Half of the pigeons were started on short initial links while the other half of the pigeons started on long initial links.

This experiment examined the effect of varying initial link durations on sensitivity to terminal-link entry ratios. For each 31-session block, the position of the more frequently reinforced alternative was Left (L), Right (R), R, R, L, L, R, L, L, L, R, L, R, L, R, R, R, R, L, R, R, L, R, L, L, R, R, L, L, L, L. Due to non-responding issues, data was only available from 5 pigeons. These pigeons each completed 62 sessions of training in each condition. The concurrent chains procedure for this experiment is shown in Figure 4.01. The short initial-link condition is shown on the left while the long condition is shown on the right.

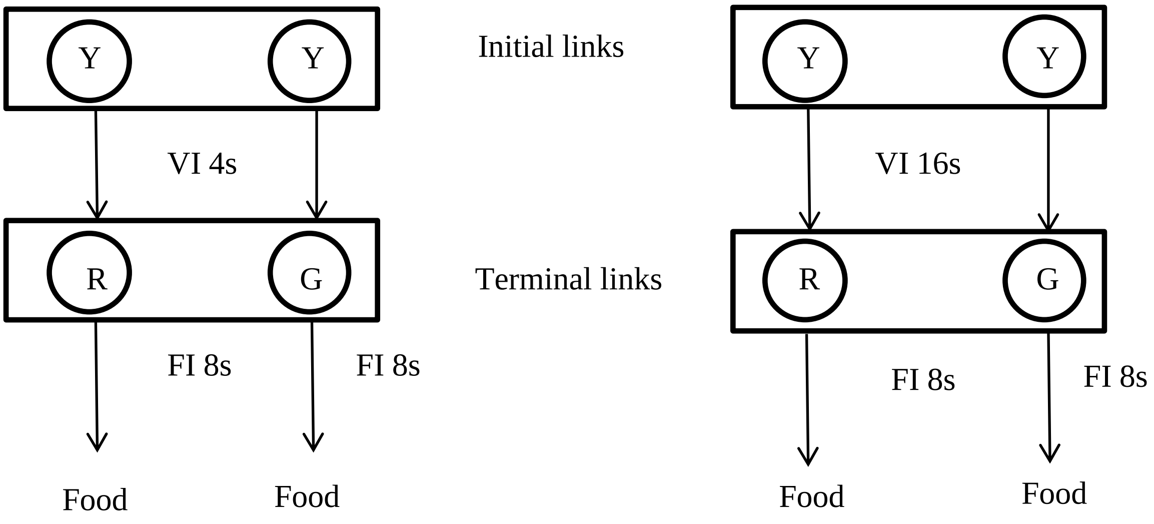


Fig 4.01. Procedure for each bird in Experiment 3. Side keys are lighted yellow. Initial links are a single VI 4-second or VI 16 second schedule that results in terminal-link entry. Terminal links are both FI 8. Left terminal links are light red while right terminal links are lighted green. The end of each terminal link results in access to food. Y = yellow key, R = red key, G = green key. Left versus right terminal-link entry ratios are 1:3 or 3:1.

4.3 Results

4.3.1 Between-session preference acquisition

Values for sensitivity to relative terminal-link entry ratio from the beginning to the end of each condition are shown in Figure 4.02. Data points are given for every block of 10 sessions. The data point at session 10 denotes the sensitivity of relative response ratio to relative terminal-link entry ratio as determined by pooling data from individual sessions 1-10, then applying the GML to the data across sessions. This analysis was completed for each block of 10 sessions for all pigeons. When initial links were short, mean sensitivity was 0.51 during the first 10 sessions and 0.39 during the last 10 sessions. When initial links were long, mean sensitivity was 0.39 during the first 10 sessions and increased to 0.48 by the last 12 sessions. Mean sensitivity values were similar and approximately stable for both conditions during the last 30 sessions.

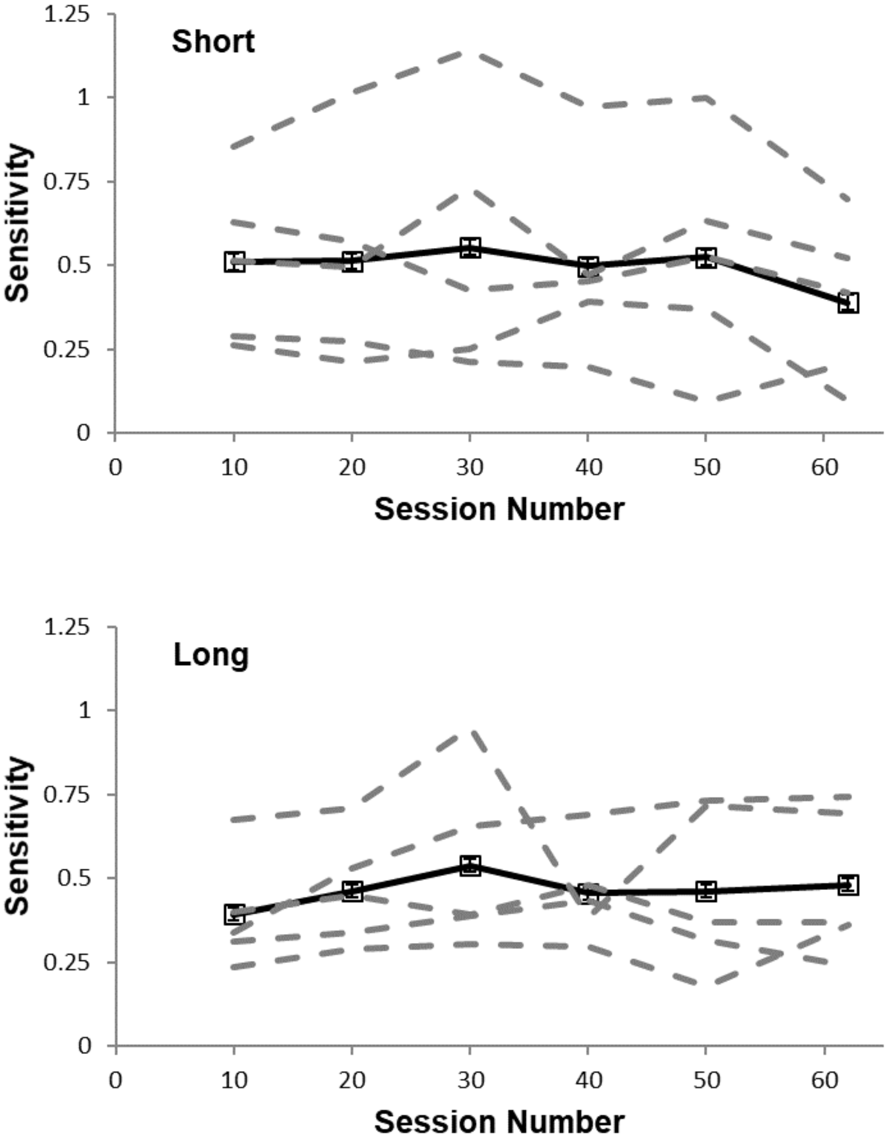


Fig 4.02. Full-session sensitivity values by 10-session blocks in short (top panel) and long (bottom panel) initial-link durations. Dotted lines represent sensitivity values of individual birds. Solid lines are the mean sensitivity values across all birds in each condition. The last session block consists of 12 sessions.

4.3.2 Preference at the beginning and at the end of training

Figure 4.03 shows the mean sensitivity values of each condition during the first 20 and last 20 training sessions. Sensitivity values during the first 20 sessions were approximately equal for both short (*M* = 0.50, *SD* = 0.26) and long (*M* = 0.54, *SD* = 0.15) initial-link durations. During the last 20 sessions, sensitivity values with short initial links were slightly lower (*M* = 0.44, *SD* = 0.25) compared to long initial links (*M* = 0.52, *SD* = 0.23). A repeated-measures ANOVA found no significant effects for condition, block, or their interactions (all *ps* > .20). Sensitivity values did not differ across training or between initial-link conditions.

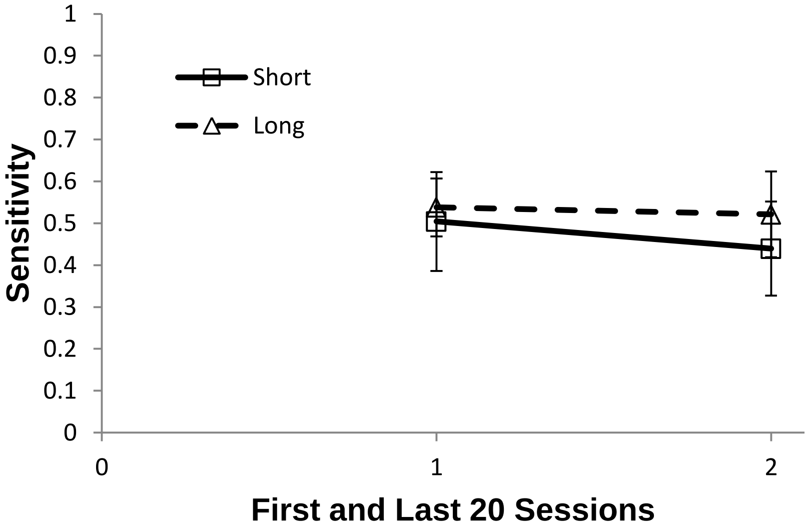


Fig 4.03. Sensitivity of relative response ratio to relative terminal-link entry ratio across short and long initial-link durations during the first 20 and last 20 sessions. The value of "1" on the horizontal axis refers to the first 20 sessions. The value of "2" refers to the last 20 sessions. Error bars represent standard errors.

4.3.3 Lag 0 and lag 1 analysis

A lag multiple regression analysis (using Equation 3.01) was carried out similar to Experiment 2 to provide estimates of sensitivity to current and prior session terminal-link entry ratios for each session sixth, and bias. This was based on the last 30 sessions of each condition. Average sensitivity coefficients are shown in Figure 4.04. Lag coefficients were entered into a 2 × 2 × 2 × 6 repeated measures ANOVA with order (short followed by long/long followed by short)/condition (short/long), lag and session sixth as factors. Because there were no significant main effects or interactions involving order (*p*s > .17), the analysis was re-run omitting order as a factor. There was no significant main effect for condition, $F(1, 3) = 0.04$, $p = .844$. The main effect for session sixth was significant, $F(5, 15) = 2.81$, $p = .044$, $\eta^2 = .41$. The main effect of lag was significant, $F(1, 3) = 12.34$, $p = .025$, $\eta^2 = .76$. There was a significant interaction between lag and session sixth, $F(5, 15) = 11.09$, $p < .001$, $\eta^2 = .74$, showing that difference between lag coefficients increased over session sixths. No other interactions were significant (all *p*s > .20).

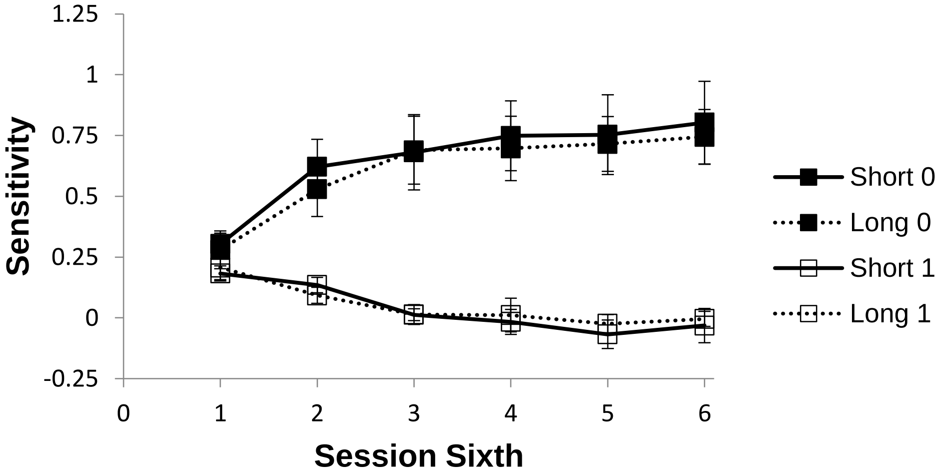


Fig 4.04. Lag analysis of previous session terminal-link entry ratios as predictors of preference in the current session for short (left panel) and long (right panel) initial-link durations. “0” refers to the sensitivity of each current session’s log response ratio to log terminal-link entry ratio of that same session. “1” refers to the sensitivity of each current session’s log response ratio to log terminal-link entry ratio of the previous session. Error bars represent standard errors.

4.3.4 Strength of bias

Relatively large estimates of bias were found, similar to Experiment 2. We analysed the absolute value of the bias (log *b*) estimates from lag multiple regressions, to determine if the strength of the bias was stronger in the short or long condition. Average absolute bias values are shown in Figure 4.05 below. A repeated-measures ANOVA (2 × 2 × 6 short followed by long/long followed by short)/condition (short/long) and session sixth as factors found no significant main effect for condition, *F*(1, 3) = 1.71, *p* = .282. There was also no main effect for session sixth, *F*(5, 15) = 1.68, *p* = .200. Bias stayed approximately equal across consecutive session sixths. The effect for order was also not significant, *F*(1, 3) = 0.02, *p* = .896. There were no other significant effects or interactions. Position preferences were strong and idiosyncratic across pigeons but were consistent for individual pigeons. Whether pigeons were initially exposed to long or short initial-link condition did not affect strength of bias between conditions. Moreover, although the strength of bias was slightly different between short and long initial links, these differences were not significant. Thus bias, while strong, did appear to be affected by initial-link duration.

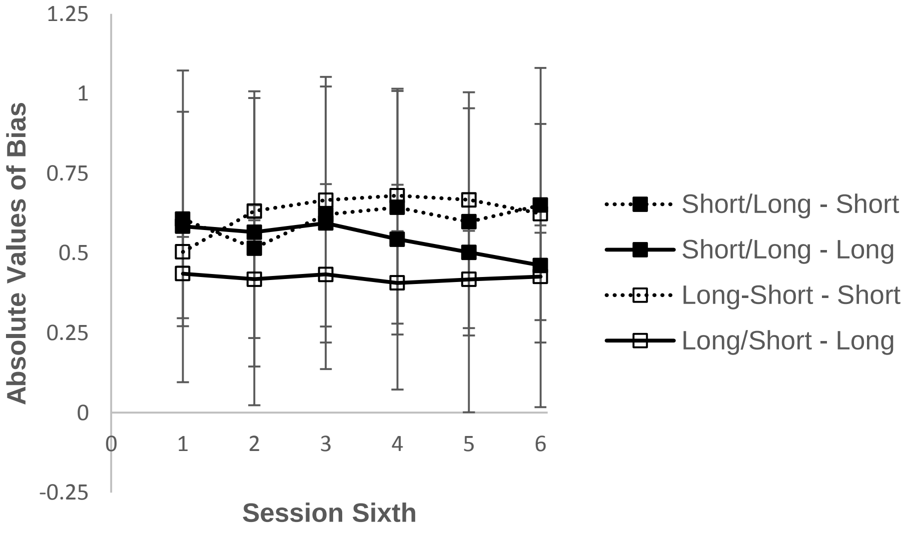


Fig 4.05. Mean absolute values of bias by session sixth over the last 30 sessions for short and long initial links based on the order which pigeons began the study. Filled squares represent pigeons that were started on short initial links. Unfilled squares represent pigeons that were started on long initial links. Dashed lines represent mean bias during short initial links. Solid lines represent bias exhibited during long initial links. Error bars represent standard errors.

4.3.5 Model fitting analyses

We applied the carryover model (as in Experiment 2) to data from the last 30 sessions of training for each pigeon. Analyses showed that varying the β_{diff} parameter did not substantially improve VAC of model fits and was therefore fixed at 0.02. For each pigeon, the total duration of individual cycles was used to approximate the time spent responding on the left and right alternatives (used to model extinction), as well as the location of the reinforcer for that cycle. The model calculates simulated responses for each cycle in the following way: The overall average response rate was calculated for each pigeon by pooling the total responses (left and right) and total time spent responding across the 30 sessions. Based on the duration of each cycle, the model then calculated simulated responses for each alternative for that cycle as the obtained cycle duration x overall average response rate x relative response strength.

During each cycle, decreases in response strength were determined by Equation (2.04) and Equation (2.05). At the end of each cycle, changes in response strength were dependent on which alternatives resulted in terminal-link entry and delivery of reinforcement, as determined by Equation (2.03). For every session sixth (every 12 reinforcers), the simulated number of responses on each alternative was computed. Predicted relative response ratios of the model were calculated as the log ratio of the simulated responses on the left and right alternatives.

Based on the best-fit parameter values, predictions of relative response ratios by the model were computed for every session sixth (12 reinforcers) for every pigeon. Thus, there were 180 data points for each pigeon (6 session sixths x 30 sessions) and the values of three parameters, α, β_{opp} and log *b* were estimated. The values of these parameters and percentage of VAC are shown for individual pigeons in Table 4.01. β_o was omitted since it was fixed at a value of zero across all birds. The other parameters α_{TL}, α_{Cr} and σ were fixed at typical values used in previous studies (0.2, 0.1 and 0.3, respectively), as for Experiment 2.

We wanted to determine whether the predictive accuracy of the model was different between short and long initial links. Dependent-samples t-tests were conducted on the slopes, intercepts and VAC of predicted log response ratios plotted against obtained log response ratios. The average of the slope, intercept, and VAC during initial-link durations as well as the corresponding *t*-scores of the difference in these parameter values between short and long initial links are also shown in Table 4.01.

4.3.5.1 Goodness-of-fit and parameter invariance. Paired-samples *t*-tests were conducted to determine if the mean values of each parameter and VAC of the model were significantly different between short and long initial links. Since there was no significant difference in sensitivity based on initial-link duration, we did not anticipate a difference in α and β_{opp} values. The difference in VAC between the short (*M* = 0.46, *SD* = 0.20) and long (*M* = 0.61, *SD* = 0.16) initial links was not significant, $t(4) = -2.46$, $p = .070$. In terms of the α parameter, there was also no significant difference between short (*M* = 0.092, *SD* = 0.157) and long (*M* = 0.013, *SD* = 0.011) initial links, $t(4) = 1.09$, $p = .339$. There was no significant difference in β_{opp} values between short (*M* = 0.009, *SD* = 0.010) and long (*M* = 0.009, *SD* = 0.009) initial links, $t(4) = 0.01$, $p = .993$. In terms of the log *b* parameter, initial-link duration did not result in a significant difference between short (*M* = 0.12, *SD* = 0.77) and long (*M* = -0.08, *SD* = 0.71), $t(4) = 1.12$, $p = .324$.

Table 4.01

Best-fit Parameter Values and VAC

Group	Bird	Log b	α	β_{opp}	Slope	Intercept	VAC
Short	193	0.10	0.372	0.000	1.02	0.00	0.35
	194	1.07	0.000	0.006	1.01	-0.01	0.38
	195	0.69	0.026	0.017	0.99	0.01	0.63
	197	-0.65	0.041	0.000	0.91	-0.05	0.23
	198	-0.61	0.020	0.021	0.99	-0.01	0.69
Long	193	-0.62	0.006	0.023	0.99	-0.01	0.69
	194	1.00	0.028	0.001	1.04	-0.03	0.42
	195	0.25	0.012	0.012	0.98	0.00	0.78
	197	-0.72	0.000	0.003	0.99	0.00	0.47
	198	-0.30	0.019	0.004	1.00	0.00	0.69
Short	*M*	0.12	0.09	0.01	0.98	-0.01	0.46
Long	*M*	-0.08	0.01	0.01	1.00	-0.01	0.61
	t	1.12	1.09	0.01	-0.70	-0.38	-2.46

Note. The first half of the table is for short initial links while the second half is for long initial links. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. All parameter values and VAC were computed using session sixth data.

We plotted log response ratios predicted by the model for each session sixth against the log response ratios obtained from the pigeons' data. This was conducted to determine if there were systematic deviations of the model's predictions from the data. In terms of the goodness-of-fit and predictive accuracy of the model, Figure 4.06a shows the plots for Pigeons 193 and 194 while Figure 4.06b shows the plots for Pigeons 195 to 198. Left panels show plots of short initial links while the right panels show long initial links. The slopes and y-intercepts are also given. The dashed line in each panel was obtained by regressing the obtained on the predicted values and the solid line indicates ideal prediction.

4.3.5.2 Predictive accuracy of the extended CDM. Comparisons of the slopes and intercepts of short and long initial links indicated that the model's predictive accuracy was slightly higher in long initial links. In terms of model fits to short initial links, the mean value of the slope was 0.98, 95% CI [0.95, 1.02]. The mean value of the slope during long initial links was 1.00, 95% CI [0.98, 1.02]. The mean intercept of short initial links was -0.01, 95% CI [-0.03, 0.01]. The mean intercept of long initial links was -0.01, 95% CI [-0.02, 0.00]. Across both short and long initial links, the y-intercept was consistently close to zero indicating little evidence of systematic over or under-prediction.

In order to determine if there were differences in predictive accuracy of the model between short and long initial links, paired *t*-tests were also conducted on the slopes and intercepts of obtained log response ratios plotted against predicted log response ratios of the model. In terms of the slopes, there was no significant difference between short (*M* = 0.98, *SD* = 0.04) and long (*M* = 1.00, *SD* = 0.02) initial links, *t*(4) = -0.70, *p* = .521. In terms of the intercepts, there was also no significant difference between short (*M* = -0.01, *SD* = 0.02) and long (*M* = -0.00, *SD* = 0.01) initial links, *t*(4) = -0.38, *p* = .726.

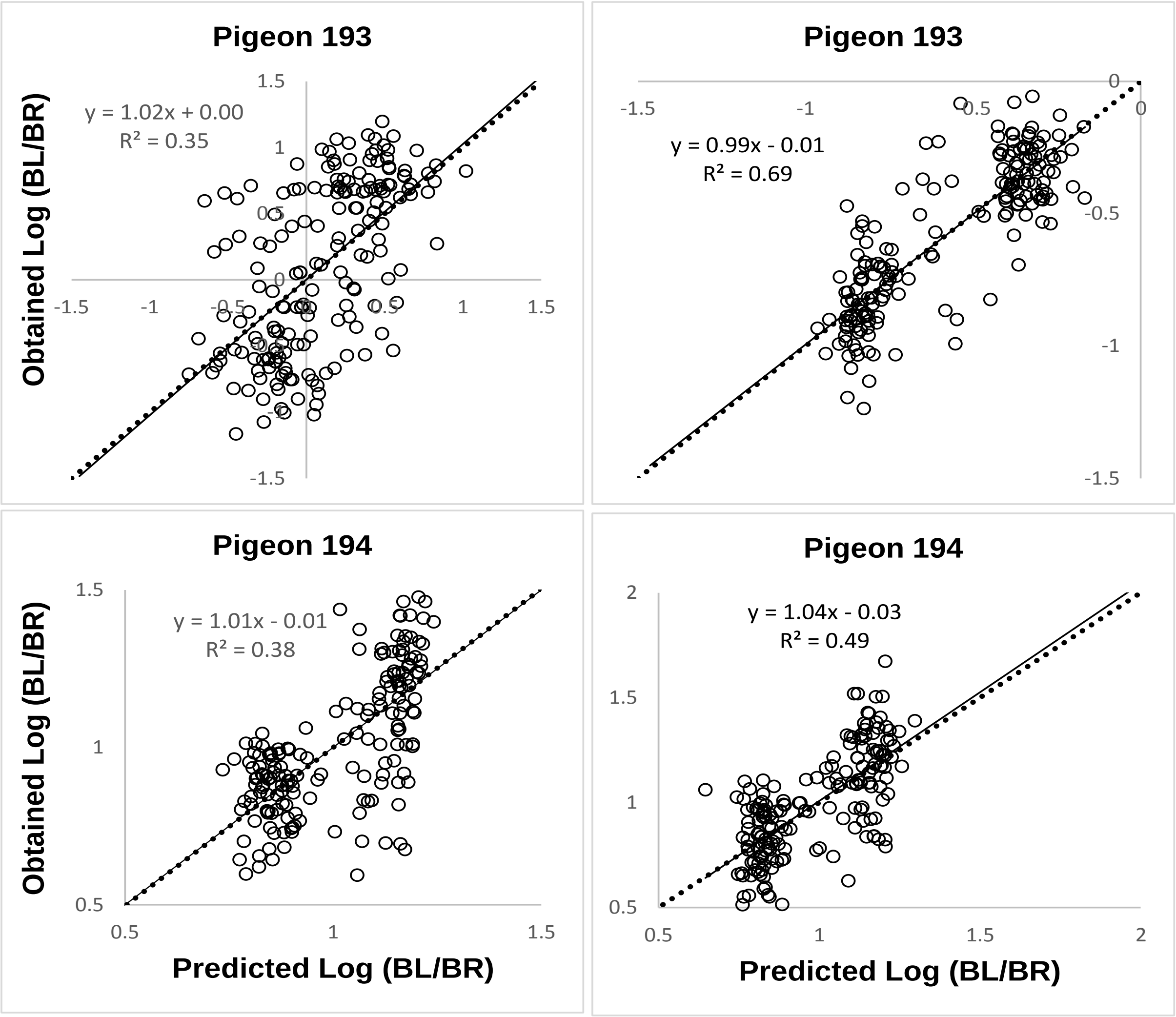


Fig 4.06a. Obtained log ratios plotted against predicted log ratios for short and long initial-link durations in Pigeons 193 and 194. Left panels show short initial links while right panels show long initial links. Data points are based on session sixth responding. In order to display better the scatter of individual data points, not all axes have identical scales.

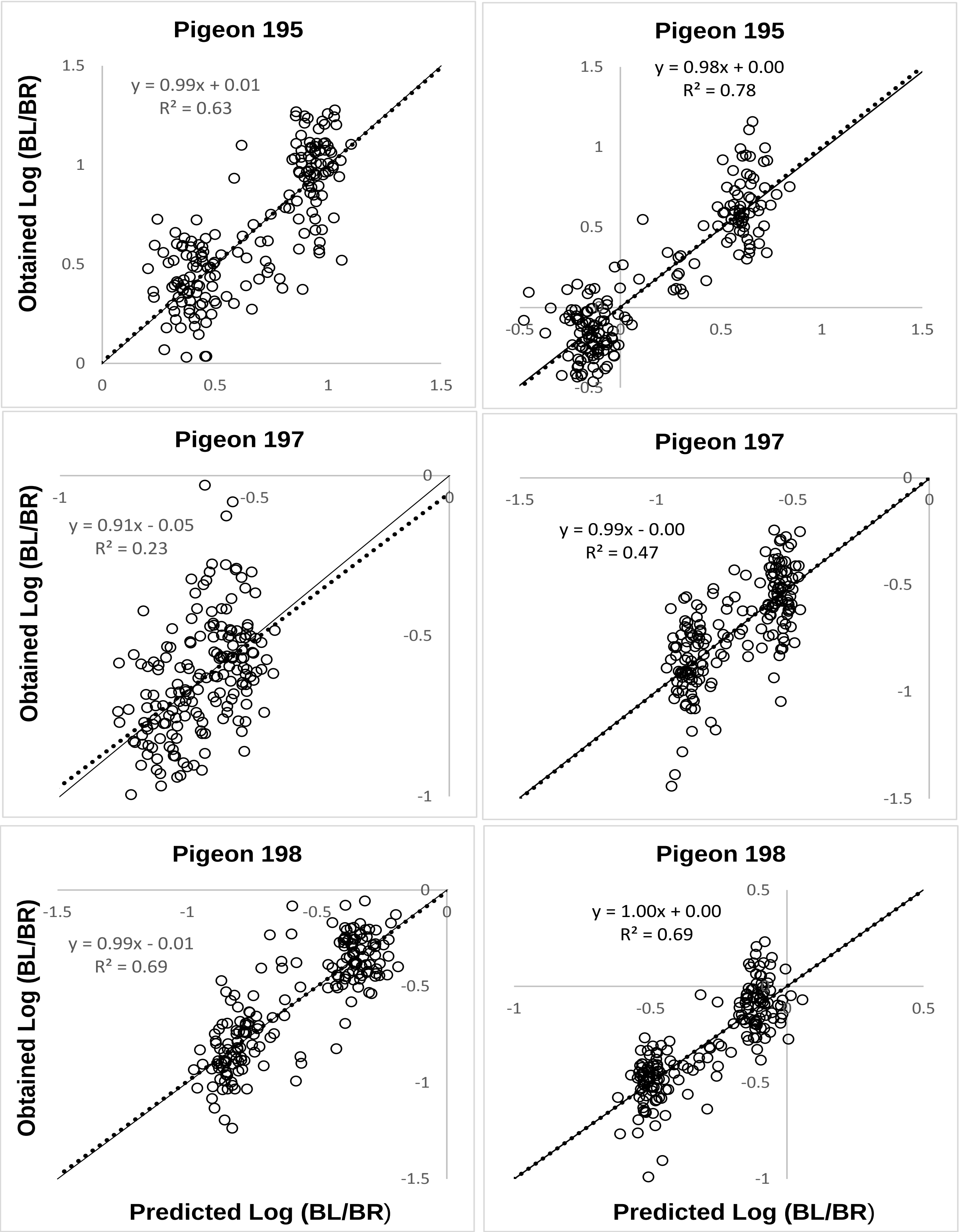


Fig 4.06b. Obtained log ratios plotted against predicted log ratios for short and long initial-link durations in Pigeons 195 to 198. Left panels show short initial links while right panels show long initial links. Data points are based on session sixth responding. In order to display better the scatter of individual data points, not all axes have identical scales.

4.3.6 Model's predictions of asymptotic preference

Results presented above (Figure 4.04) show that similar to previous research with rapid-acquisition choice procedures (e.g., Maguire et al., 2007), sensitivity to terminal-link entry ratio increased over the first half of the session but was approximately stable over the second half. Therefore, responding during each block of the fourth to sixth session sixth could serve as an approximation of attained asymptotic preference. Several analyses were conducted to determine if asymptotic response allocation, both obtained and predicted by the model fits, was consistent with the GML.

As previous plots of sensitivity have shown, preference stabilizes within the first half of each session and asymptotic preference levels are reached by the second half. In terms of asymptotic preference levels, the next task was to determine whether the CDM fitted to the session sixth data was able to make accurate predictions of asymptotic preference levels. Each session consisted of one data point and this analysis was conducted on the last 30 sessions of training. Based on the best-fit parameter values, the model's resulting predictions of log response ratios and obtained log response ratios were computed. Obtained and predicted log response ratios were separately plotted against log terminal-link entry ratios. During short initial links, plots for Pigeons 193 to 197 are shown in Figure 4.07a while plots for Pigeon 198 are shown in Figure 4.07b. During long initial links, plots for Pigeons 193 and 194 are shown in Figure 4.08a while plots for Pigeons 195 to 198 are shown in Figure 4.08b.

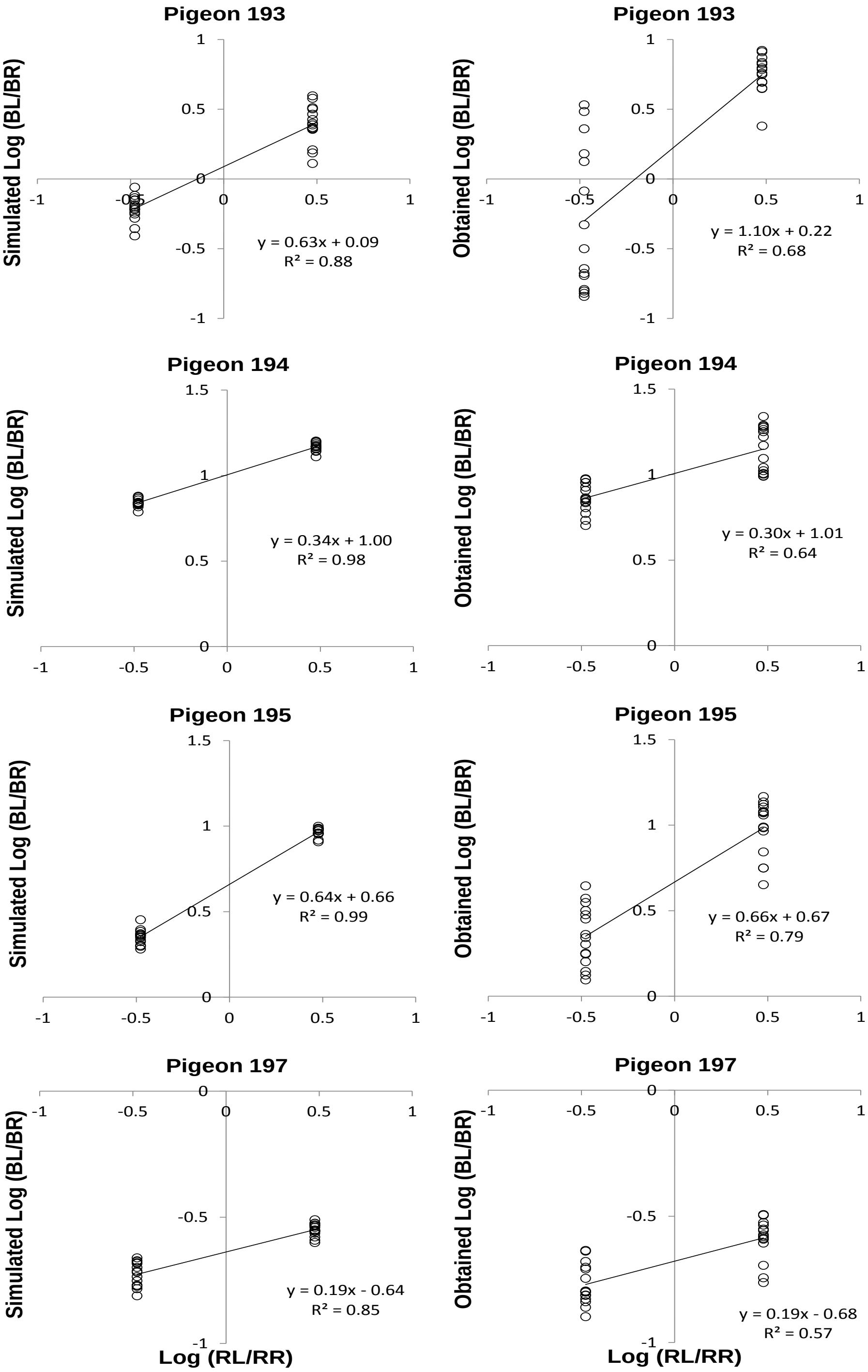


Fig 4.07a. Log response ratios (vertical axis) plotted against log terminal-link entry ratios for short initial-link durations for Pigeons 193 to 197. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

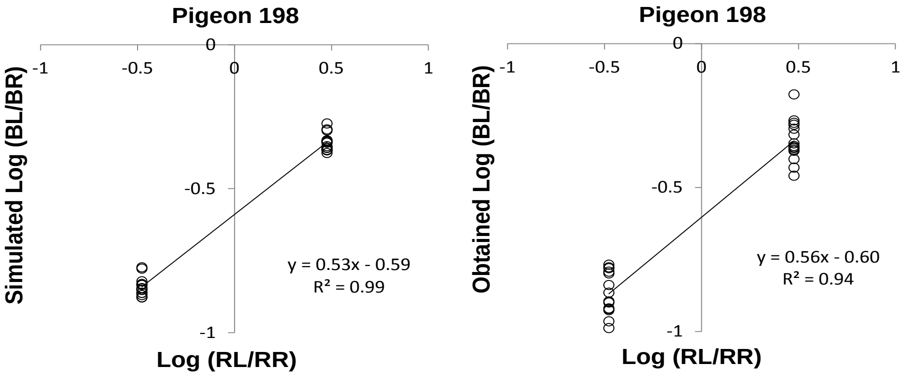


Fig 4.07b. Log response ratios (vertical axis) plotted against log terminal-link entry ratios for short initial-link durations for Pigeon 198. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

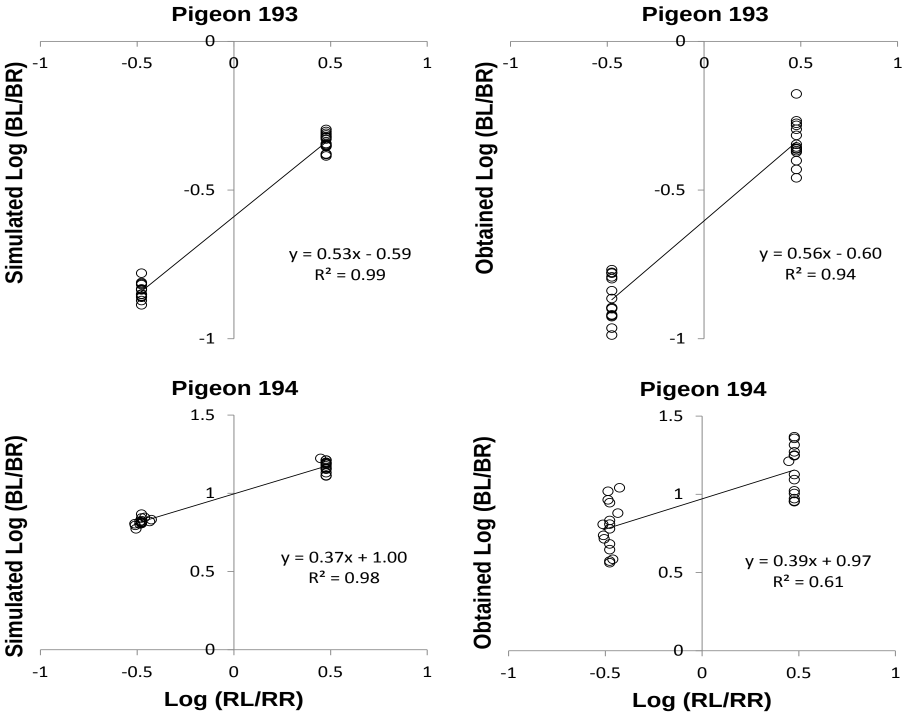


Fig 4.08a. Log response ratios (vertical axis) plotted against log terminal-link entry ratios for long initial-link durations for Pigeons 193 and 194. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

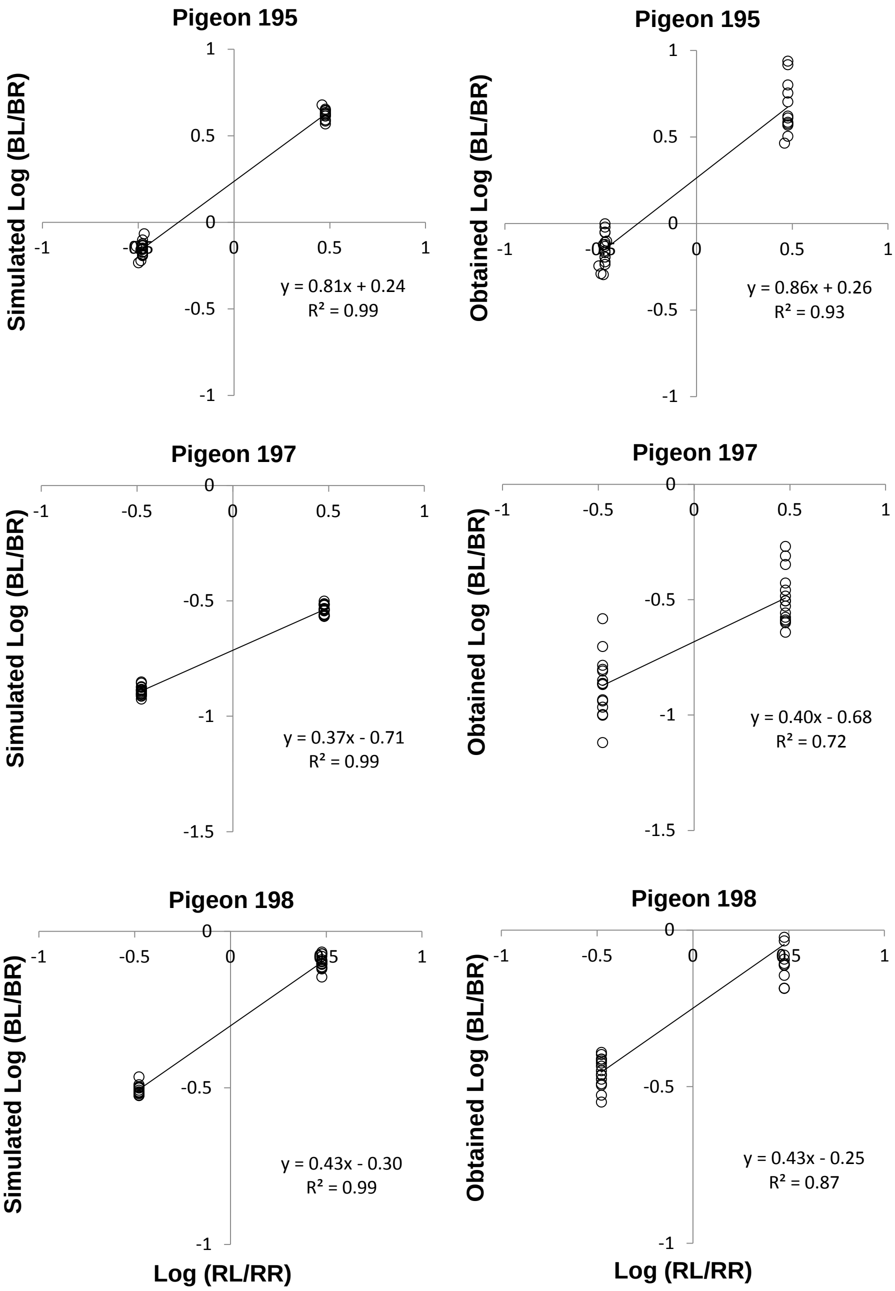


Fig 4.08b. Log response ratios (vertical axis) plotted against log terminal-link entry ratios for long initial-link durations for Pigeons 195 to 198. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

4.3.7 Goodness-of-fit to asymptotic responding

Average parameters for GML fits to the data in Figures 4.07 and 4.08 are shown in Table 4.02. Paired samples t-tests were conducted to determine if there were systematic differences in the average VAC, slope, and intercept between short and long initial links. During short initial links, there was a significant difference between predicted (*M* = 0.94, *SD* = 0.07) and obtained (*M* = 0.72, *SD* = 0.14) VAC, *t*(4) = 4.39, *p* = .012, *d* = 1.96. There was no significant difference between predicted (*M* = 0.47, *SD* = 0.20) and obtained (*M* = 0.56, *SD* = 0.36) slopes, *t*(4) = -1.02, *p* = .366. There was also no significant difference between predicted (*M* = 0.10, *SD* = 0.73) and obtained (*M* = 0.12, *SD* = 0.75) intercepts, *t*(4) = -0.69, *p* = .528. Results were different for long initial links. There was no significant difference between predicted (*M* = 0.99, *SD* = 0.004) and obtained (*M* = 0.81, *SD* = 0.14) VAC, *t*(4) = 2.77, *p* = .050. The difference between predicted (*M* = 0.50, *SD* = 0.18) and obtained (*M* = 0.53, *SD* = 0.20) slopes was in the same direction as the short condition, but reached significance, *t*(4) = -3.20, *p* = .033, *d* = 1.44. There was no significant difference between predicted (*M* = -0.07, *SD* = 0.70) and obtained (*M* = -0.06, *SD* = 0.68) intercepts, *t*(4) = -0.84, *p* = .448. Thus, results indicated that the sensitivity to the relative entry ratio in the predicted data was somewhat less extreme than obtained, but only was significant with long initial links.

Table 4.02

Average of Slope, Intercept and VAC for GML fits to Obtained and Simulated Data

Data	Condition	Slope	Intercept	VAC
Obtained	Short	0.56	0.12	0.72
	Long	0.53	-0.06	0.81
Simulated	Short	0.46	0.10	0.94
	Long	0.50	-0.08	0.99
Short Diff	*t*	-1.02	-0.69	4.39*
Long Diff	*t*	-3.20*	-0.84	2.77

Note. Diff denotes the difference between obtained and simulated data. Values are computed using responding during the second half of each session, over the last 30 sessions.

* indicates $p < .05$. ** indicates $p < .01$.

4.3.8 Residual analysis

Finally, we conducted a residual analysis to test if there were systematic deviations from the GML in the obtained and simulated data. Residuals from GML fits in Figures 4.06 and 4.07 were plotted against predicted log response ratios of the GML. In order to test for systematic deviations in the GML's predictions, polynomial regressions were conducted whereby the residuals were regressed on the predicted values. This polynomial regression was the same as the one conducted in Experiments 1 and 2. Table 4.03 shows the results. In terms of the simulated data, the linear, quadratic and cubic coefficients were all non-significant predictors of the log response ratios for the simulated data. Similar results were observed in the obtained data where the linear, quadratic and cubic coefficients did not reach significance for the obtained data. These results strongly suggest that there were no systematic deviations from the GML's predictions for the simulated and obtained data. Although statistical tests of systematic trends are not significant, we wanted to determine if there were any indications of weak systematic trends. Figure 4.09 shows the scatter of residual data points for short and long initial links. The scatter of data points indicates little evidence of linear, quadratic or cubic trends.

Table 4.03

Results of Polynomial Regressions Performed on GML Residuals

Dependent variable	Group	Predictor variables	*B*	S.E. of *B*	R^2	*F*

GML residuals Simulated	Short	Linear	0.000	0.018		
		Quadratic	0.000	0.018		
		Cubic	0.000	0.025		
					0.000	0.000
GML residuals Obtained	Short	Linear	-0.014	0.057		
		Quadratic	-0.008	0.046		
		Cubic	0.013	0.052		
					0.000	0.021
GML residuals Simulated	Long	Linear	0.000	0.009		
		Quadratic	0.000	0.006		
		Cubic	0.000	0.010		
					0.000	0.000
GML residuals Obtained	Long	Linear	-0.005	0.036		
		Quadratic	-0.002	0.019		
		Cubic	0.004	0.028		
					0.000	0.008

Note. Dependent variables were residuals from the GML, computed using log response ratios. Independent variables were log predicted values, plus squares and cubes of log predicted values (‘quadratic’ and ‘cubic’). B is the unstandardized regression coefficient, and R^2 is the proportion of variance accounted for.

*indicates $p < 0.05$, ** indicates $p < 0.01$, *** indicates $p < 0.001$.

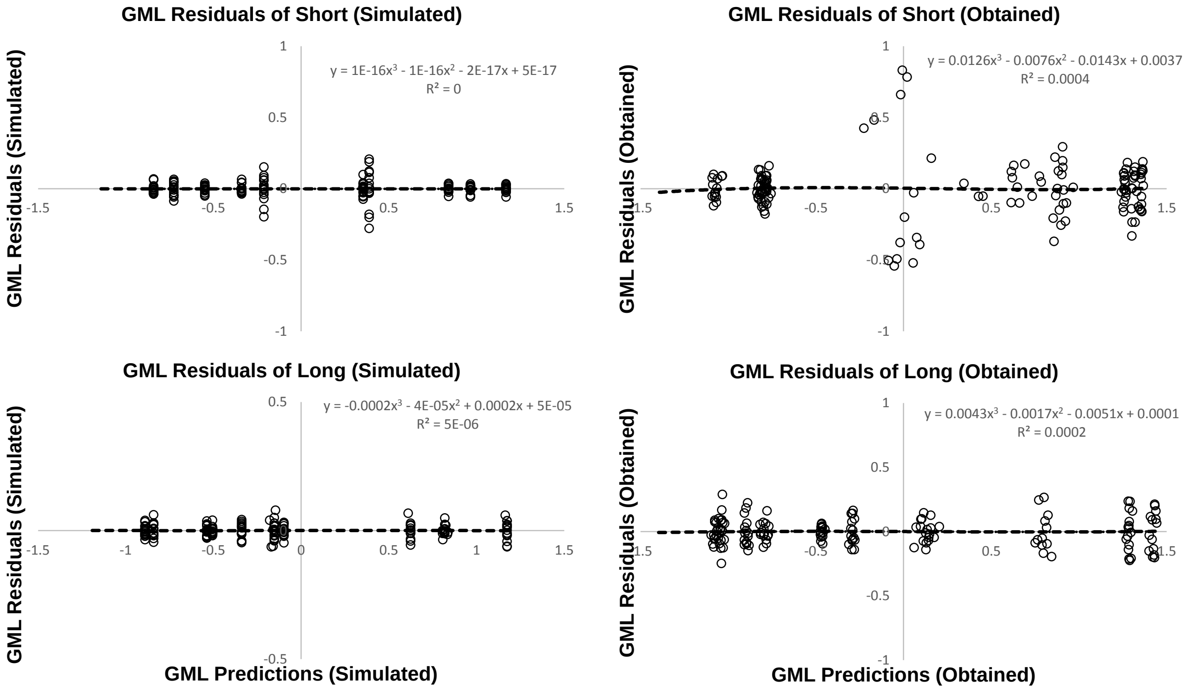


Fig 4.09. All panels show response allocation predicted by the GML for short or long initial links. Upper panels show residuals of short initial links while the lower panels show residuals of long initial links. Left panels show residuals based on the carryover model's simulated data while the right panels show residuals based on obtained data. Each data point was computed from the second half each session over the last 30 sessions.

4.4 Summary

The main goal of Experiment 3 was to test if the duration of equal initial links affected acquisition of preference in concurrent chains when relative frequency of terminal links varied unpredictably across sessions. Results showed that sensitivity to the relative terminal-link entry ratio was approximately equal regardless of whether initial-link duration was short (4s) or long (16s). An extension of the CDM applied to the data from Experiment 2 was also applied to Experiment 3 data. The model provided a reasonably good description of the data, with relatively equal variance accounted for between short and long initial links. Similar to Experiment 2, there were no systematic differences in the estimated values of the model parameters (α and β_{opp}).

Chapter 5

Experiment 4: The Effects of Terminal-link Immediacy Ratio and Signalled Terminal-link Stimuli on Choice

5.1 Introduction

So far, the effects of initial and terminal-link duration on conditioned reinforcement have been examined. The previous chapter showed no significant effect of initial-link duration on preference for the more frequently occurring terminal link regardless of initial-link duration. In the previous two experiments, terminal-link stimuli have always signalled which initial-link alternative resulted in terminal-link entry. In this way, the colors of terminal-link stimuli provide information on correlations between initial-link responding and terminal-link entry. The next step was determining if this information affects response allocation for the richer and poorer alternatives. In this chapter, we also measured preference in terms of sensitivity to terminal-link immediacy ratio. Terminal-link key colors were either correlated (cued) with terminal-link entry or uncorrelated (uncued).

In light of the goal of creating a decision model that can explain choice behaviour when varying contingencies of conditioned reinforcement, the carryover model has shown to be able to explain preference based on the collected data while maintaining parameter invariance. Thus far, the criterion of parameter invariance is satisfied when applied to varied terminal-link and initial-link durations. The next step is extending the CDM via the carryover model to satisfy this criterion while taking into account signalling effects of terminal-link stimuli on preference for the shorter terminal link. With higher mean sensitivity values of approximate matching, we expected mean VAC to be higher than Experiments 2 and 3.

In this study, we wanted to determine the effects of cued and uncued terminal links on sensitivity to unequal left and right terminal-link delay ratios under rapid-acquisition conditions. Each trial consisted of an initial link phase and a terminal link phase. Pigeons were made to peck two lighted keys in the initial links and the single lighted key in the terminal links. This study had two conditions; cued and uncued terminal links. When terminal links were cued, the color of the key during the terminal-link phase was depended on whether a left or right initial link response produced the terminal link. When terminal links were uncued, the color of the lighted key during the terminal links was not dependent upon which initial link response resulted in terminal-link entry. Every pigeon was exposed to both cued and uncued terminal links. Each initial link consisted of a single VI 8s. For each initial link, there was a total of 12 possible intervals which, across each session resulted in an average initial-link duration of eight seconds. For all sessions, the assignment to the left and right keys was always 1:1. However, terminal- link durations were always different for the left and right keys. In half of the sessions, durations of the left and right terminal links were FI 4s and FI 16s. In the remaining sessions, left and right terminal links were FI 16s and FI 4s. A 1s COD was used. Reinforcement consisted of 3.5s of access to food. For both groups, whether the left or right initial-link key was associated with the terminal link with the shorter delay varied according to a PRBS. Although, every pigeon was exposed to both conditions, half of the pigeons were started on cued terminal links while the other half of the pigeons were started on uncued terminal links. In this way, pigeons had to learn which initial link key was the richer alternative every single session.

5.2 Method

5.2.1 Subjects

Eight pigeons of mixed breed and sex were maintained at 85% of their free-feeding weight through appropriate post-session feeding. Pigeons were caged individually in a vivarium which ran according to a 12-hour day and night cycle (lights were switched on at 06:00). Home cages were supplied with grit and water which was available at all times to pigeons when they were in their home cages.

5.2.2 Procedure

One complete trial of the concurrent chains procedure occurred when the pigeons had responded during an initial link phase and a terminal link phase. A 72-cycle program of concurrent chains was used for this experiment. Completing each session consisted of pigeons completing 72 trials of concurrent chains. At the start of each trial, side keys were lighted yellow to signal the beginning of the initial links. Terminal link entry was signalled by extinguishing the side keys and lighting the centre key either red or green. When terminal links were cued, the color of the centre key depended on whether a left or right initial link response produced the terminal link (left-red, right-green). When terminal links were uncued, the centre key was lighted red or green with equal probability but was not correlated with the location of the prior initial-link response. Every pigeon was exposed to both cued and uncued conditions, and the order of conditions was counterbalanced across pigeons.

In the initial link, a single VI 8s schedule operated. As each interval in the schedule timed out, the outcome was assigned randomly to the left or right key and the next peck on the assigned key produced access to the corresponding terminal link. For all sessions, the assignment was 50% -left, 50% right. Terminal links durations were always different for the left and right keys. In half of the sessions, durations of the left and right terminal links were FI 4s and FI 16s. In the remaining sessions, the assignments were reversed – left and right terminal links were FI 16s and FI 4s. The order in which terminal link durations were assigned daily was determined by a PRBS, as used by Hunter et al. (1985). The use of PRBS ensured the order in which terminal link entries occurred across sessions was completely randomized. Each presentation of the PRBS required 31 sessions of concurrent chains. Every pigeon was exposed to both sets of terminal-link durations. Figure 5.01 shows the concurrent-chains schedule we used for cued terminal links.

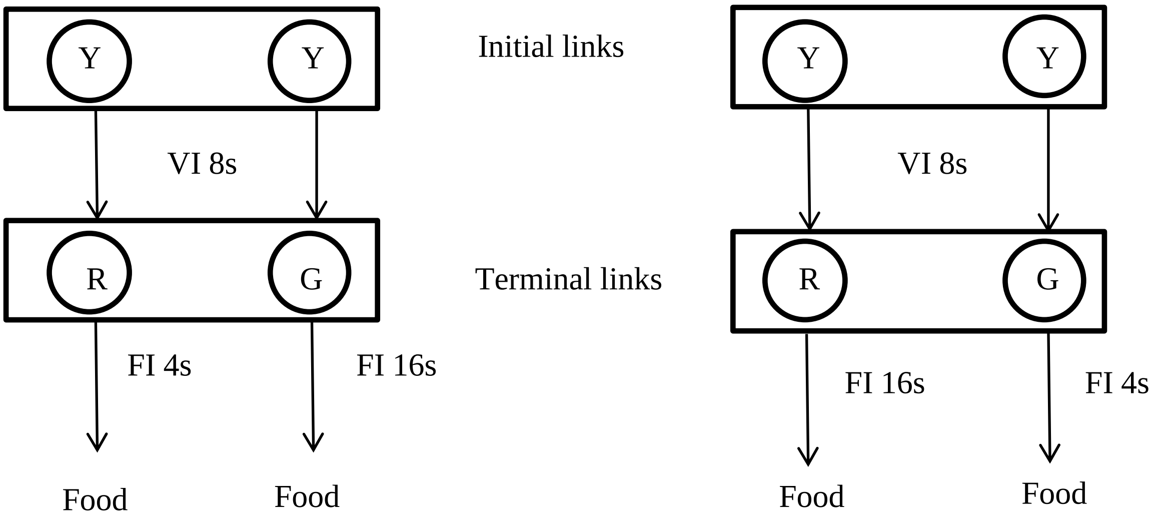


Fig 5.01. Concurrent-chains schedule for the cued condition. Side keys are lighted yellow. Initial links consist of a single VI 8s schedule that results in terminal-link entry. Terminal links are FI 4s and FI 16s or FI 16s and FI 4s. Left terminal links are light red while right terminal links are lighted green in the cued condition. In the uncued condition, terminal-link keys are randomly lighted red or green. The end of each terminal link results in access to food. Y = yellow key, R = red key, G = green key.

5.3 Results

5.3.1 Between-session preference acquisition

Sensitivities to relative immediacy ratio for blocks of 10 sessions are shown in Figure 5.02**.** The data point at session 10 denotes the sensitivity of relative response ratio to relative immediacy ratio as determined by pooling data from individual sessions 1-10, then applying the GML to the data across sessions. This analysis was completed for each successive block of 10 sessions for all pigeons. Across sessions, there appeared to be an increase in sensitivity in both cued and uncued terminal links. In the cued condition, mean sensitivity was 1.02 during the first 10 sessions and 1.33 during the last 10 sessions. In the uncued condition, mean sensitivity was 0.30 during the first 10 sessions and increased to 0.77 by the last 12 sessions. Across both conditions, peak sensitivity values were reached by the last three 10-session blocks.

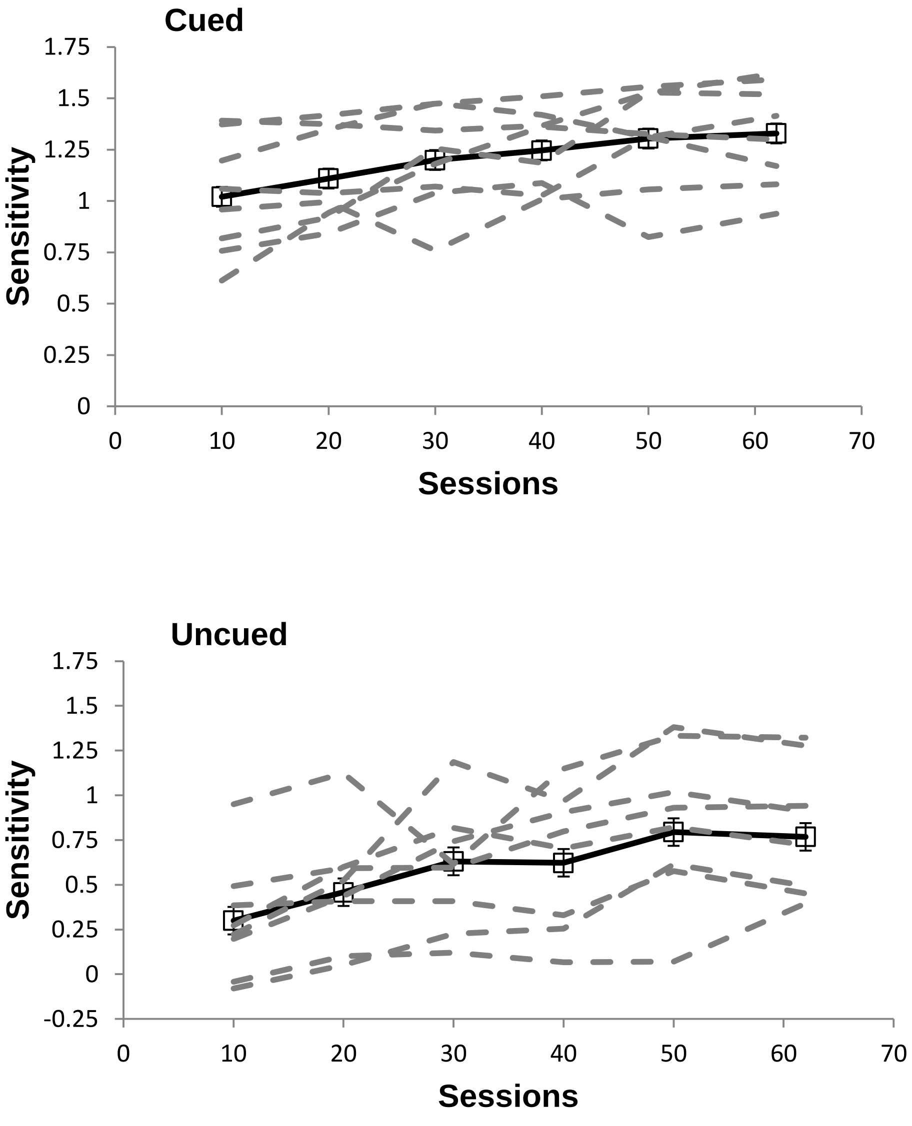


Fig 5.02. Full-session sensitivity values by 10-session blocks in cued (top panel) and uncued (bottom panel) terminal links. Dotted lines represent sensitivity values of individual birds. Solid lines are the mean sensitivity values across all birds in each condition. The last session block consists of 12 sessions.

5.3.2 Preference between the beginning and at the end of training

Figure 5.03 shows the mean sensitivity values of each condition during the first 20 and last 20 training sessions. Sensitivity values during the first 20 sessions were higher in cued (M = 1.07, SD = 0.24) compared to uncued (M = 0.42, SD = 0.30) terminal links. During the last 20 sessions, sensitivity values were greater with cued terminal links (M = 1.32, SD = 0.25) compared to uncued (M = 0.84, SD = 0.36). Sensitivity values were entered into a 2 × 2 × 2 repeated measures ANOVA with order (cued followed by uncued/uncued followed by cued), condition (cued/uncued) and session block (first 20/last 20) as factors. There was a significant main effect for session block, $F(1, 6) = 42.03$, $p = .001$, $\eta^2 = .88$, as well as a significant main effect for condition, $F(1, 6) = 84.07$, $p < .001$, $\eta^2 = .93$. These results confirm that sensitivity increased from the first to the last 20 sessions and was greater in the cued condition. Between the beginning and the end of training, there was a greater increase in sensitivity with uncued terminal links, but the interaction was not significant. There was no significant main effect for order, $F(1, 6) = 0.45$, $p = .528$. All remaining interactions were not significant ($ps > .380$).

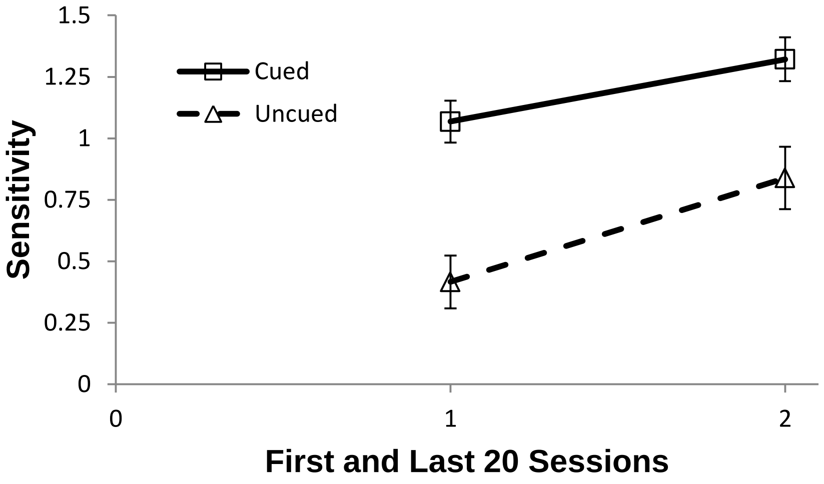


Fig 5.03. Sensitivity of relative response ratio to relative immediacy entry ratio across cued and uncued terminal links during the first 20 and last 20 sessions. The value of "1" on the

horizontal axis refers to the first 20 sessions. The value of “2” refers to the last 20 sessions. Error bars represent standard errors.

5.3.3 Lag 0 and lag 1 analysis

As in previous experiments, we conducted a lag regression analysis to determine how sensitivity to relative terminal-link duration changed within sessions. We examined how initial-link response allocation depended on the terminal-link immediacy ratio from the current and prior session using a generalized matching model:

$$\text{Log}\left(\frac{B_{0L}}{B_{0R}}\right) = \text{Log } b + a_0 \text{Log}\left(\frac{I_{0L}}{I_{0R}}\right) + a_1 \text{Log}\left(\frac{I_{1L}}{I_{1R}}\right), \qquad (5.01)$$

where *B* represents the number of responses made on each alternative, *I* is the immediacy of each terminal link (i.e., reciprocal of delay), *a* is the sensitivity to relative immediacy ratio, subscripts 0 and 1 denote the current session and previous session, and Log *b* is a bias parameter.

For both cued and uncued conditions, a lag multiple regression analysis was conducted to obtain estimates of sensitivity for each session sixth to current and prior terminal-link immediacy ratios, and bias according to Equation (5.01). Figure 5.04 shows mean sensitivity to relative terminal-link immediacy ratio for lag 0 and lag 1 for all subjects across both groups during the last 30 sessions. Lag 0 sensitivity increased more rapidly and reached a greater level in the cued condition.

This observation was confirmed by a repeated-measures ANOVA with order (cued followed by uncued/uncued followed by cued), condition (cued/uncued), lag and session sixth as factors. Because the main effect of order and all its interactions were not significant, we re-ran the analysis pooled across order. The main effect of condition was significant, $F(1, 7) = 19.11$, $p = .003$, $\eta^2 = .73$. The main effect for session sixth was also significant, $F(5, 35)$

= 25.76, $p < .001$, $\eta^2 = .79$. There was also a main effect for lag, $F(1, 7) = 47.50$, $p < .001$, $\eta^2 = .87$. There was a significant interaction between condition and session sixth, $F(5, 35) = 5.09$, $p = .001$, $\eta^2 = .42$. There was an interaction between lag and condition, $F(1, 7) = 28.22$, $p = .001$, $\eta^2 = .80$. There was also an interaction between lag and session sixth, $F(5, 35) = 38.95$, $p < .001$, $\eta^2 = .85$. The condition x lag x session sixth interaction was also significant, $F(5, 35) = 7.19$, $p < .001$, $\eta^2 = .51$. These results confirm that sensitivity to the terminal link immediacy ratio in the current session increased more rapidly and to a greater overall level in the cued condition. Averaged across the second half of sessions, sensitivity to relative immediacy was $M = 1.36$ and $M = 0.98$ in the cued and uncued conditions, respectively.

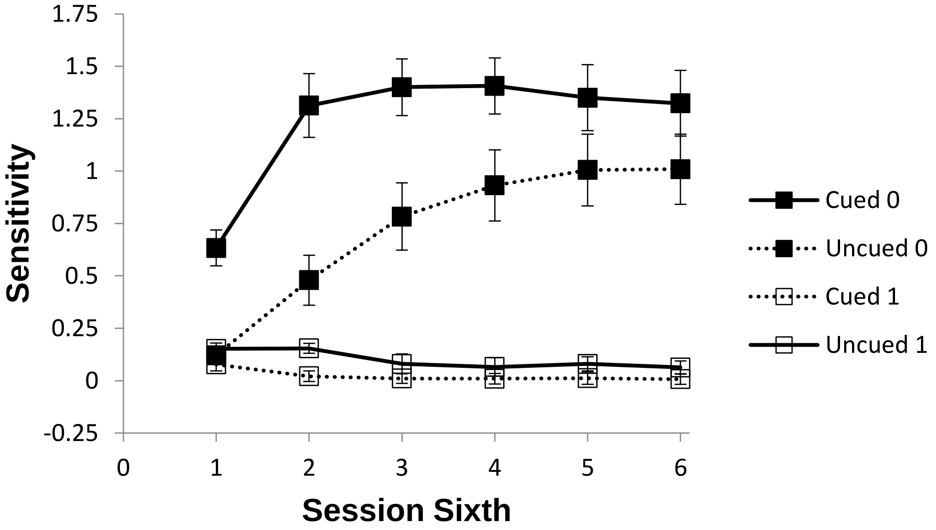


Fig 5.04. Lag analysis of previous session immediacy ratios as predictors of preference in the current session for cued and uncued terminal links. "0" refers to the sensitivity of each current session's log response ratio to log immediacy ratio of that same session. "1" refers to the sensitivity of each current session's log response ratio to log immediacy ratio of the previous session. This was conducted on the last 30 training sessions. Error bars represent standard errors.

5.3.4 Strength of bias

We checked if the strength of bias differed depending on whether the terminal links were cued and the order in which conditions were completed. As in Experiment 3, we calculated the absolute value of the bias (log *b*) estimates from the multiple regressions in Figure 5.04. Absolute bias values were analysed to allow us to calculate averages to determine if the strength of bias varied regardless of the left/right preference that each pigeon might show.

Average absolute bias values are shown in Figure 5.05 and appeared to decrease over session sixths. Because a repeated-measures ANOVA (2 × 2 × 6 cued followed by uncued/uncued followed by cued)/condition (cued/uncued) and session sixth as factors found no significant main effect for order, $F(1, 6) = 0.87$, $p = .386$, or interactions involving order (all $ps > .236$), we re-ran the analysis collapsing across order. There was a significant main effect for session sixth, $F(5, 35) = 2.85$, $p = .029$, $\eta^2 = .29$. Average absolute bias values were $M = 0.19$ and $M = 0.12$ in the first and last session sixths, respectively. None of the other effects or interactions were significant (ps > .42). Thus the strength of bias decreased gradually within sessions for both cued and uncued conditions.

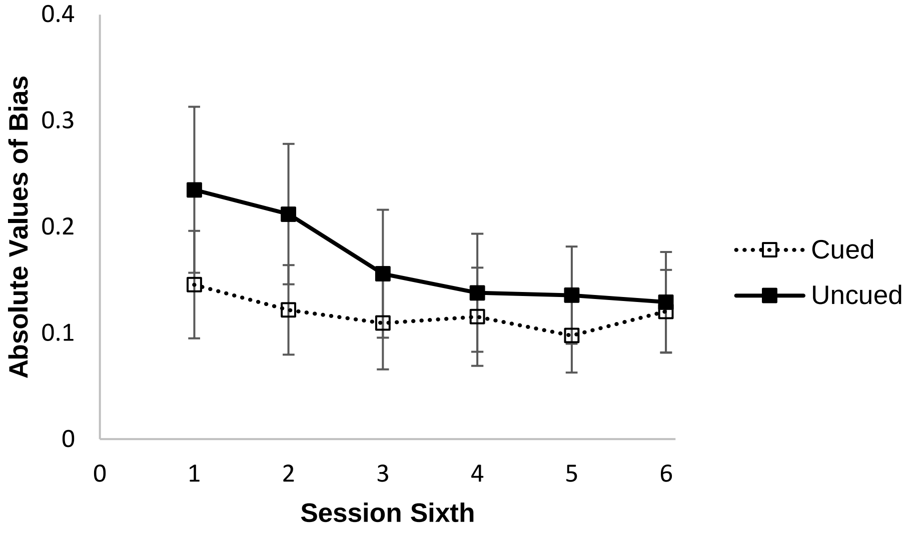


Fig 5.05. Absolute values of bias by session sixth over the last 30 sessions for cued and uncued terminal links. Unfilled squares represent data points during cued terminal links. Filled squares represent data points of uncued terminal links. Error bars represent standard errors.

5.3.5 Extending the CDM to accommodate signalling effects

As previously noted, a major goal of the present research was to test if the CDM could be extended to predict acquisition of choice in situations different from previous studies which have varied terminal-link reinforcement conditions or initial-link duration (e.g., Grace et al., 2006; Christensen et al., 2008, 2009). An extended version of the CDM was applied to the results from Experiments 2 and 3, in which the terminal-link delays were equal but the relative frequency of terminal-link entry was varied unpredictably across sessions. In Experiment 4, the signalling of unequal terminal-link delays (cued vs uncued) was varied across conditions in a rapid-acquisition design.

The effect of cueing can be accommodated by the CDM without modification. When applying the CDM to archival steady-state data, Christensen et al. (2010) proposed that decisions about terminal-link delays were made on the basis of an EWMA of delays associated with the particular terminal link. Although they did not apply the model to results from studies with cued vs uncued terminal links, their approach is applicable here, provided we assume that a EWMA is calculated for each terminal-link stimulus. The EWMA is defined as:

$$\text{Log } D_n = \beta_{\text{TLstim}} (\text{Log } D) + (1 - \beta_{\text{TLstim}})(\text{Log } D_{\text{n-1}}), \qquad (5.02)$$

where Log D_n is the result of the EWMA used to make the decision on the current trial, Log D is the delay experienced on a terminal-link trial, and Log $D_{\text{n-1}}$ is the value of the EWMA carried over from the previous trial. β_{TLstim} is a parameter that determines the weight given to the most recently experienced terminal link delay; when $\beta_{\text{TLstim}} = 1$, Log D_n is determined solely by the most recent delay, whereas for lower values of β_{TLstim} the previous value of the EWMA is relatively more influential.

In Equation (5.02), the value of β_{TLstim} is unimportant for cued terminal links, because separate EWMAs are calculated for each stimulus, so Log D_n will always equal Log D (for FI terminal links, as used here). However, in the uncued condition the same stimulus is associated with both terminal-link delays, so Log D_n will be closer to the average of the terminal-link delays provided that $\beta_{\text{TLstim}} < 1$. We applied the CDM used in Experiment 3 with Equation (5.02) to data from the last 30 sessions of training for each pigeon. Analyses in Experiments 2 and 3 showed that varying the β_{diff} parameter did not substantially improve VAC of model fits and was therefore fixed at 0.02. For each pigeon, we recorded reinforcer location of each cycle, time spent on each initial-link alternative (used to model extinction), average response rate per experimental condition as well as the model's simulated responses in the same way as Experiments 2 and 3.

5.3.6 Model fitting

During each cycle, decreases in response strength were determined by Equation (2.04) and Equation (2.05). At the end of each cycle, changes in response strength were dependent on which alternatives resulted in terminal-link entry and delivery of reinforcement, as determined by Equation (2.03). For every session sixth (every 12 reinforcers), the simulated number of responses on each alternative was computed. Predicted relative response ratios of the model were calculated as the log ratio of the simulated responses on the left and right alternatives.

Based on the best-fit parameter values, predictions of relative response ratios by the model were computed for every session sixth (12 reinforcers) for every pigeon. Thus, there were 180 data points for each pigeon (6 session sixths x 30 sessions) and two parameters, β_{opp} and log b were estimated. Unlike Experiments 2 and 3, α was not estimated since terminal-link entry ratio of the left and right initial-link alternatives was always 1:1. α was instead set at 0.02 (the mean value of α in Experiment 2, averaged across both conditions). The values of these parameters and percentage of VAC are shown for individual pigeons in Table 5.01. β_o was omitted since it was fixed at a value of zero across all birds. The other parameters α_{TL} and α_{Cr} were fixed at typical values used in previous studies (0.2, and 0.1 respectively), as for Experiment 3. σ was allowed to vary. In contrast to Experiments 2 and 3, one set of parameter values was fitted across both conditions, with the exception of Pigeon 175 where it was found that estimating separate bias values for the cued and uncued conditions resulted in a substantially improved fit.

5.3.6.1 Goodness of fit. Considering that one set of parameter values was used to fit the data to both conditions (with the exception of an additional bias parameter for Pigeon 175), the model fitted the data very well. On average, the model accounted for 80% of the variance in the session sixth log initial-link response ratios. The values of β_{TLstim} were all less than 1 ($M = 0.58$), indicating that terminal-link delays from previous cycles influenced responding when delays were not differentially cued. The values of σ ranged from 0.14 to 0.27 ($M = 0.22$) which was consistent with values obtained in previous experiments.

Table 5.01

Best-fit Parameter Values

Bird	Log *b*	β_{opp}	σ	β_{TLstim}	VAC
171	-0.03	0.007	0.24	0.37	0.75
172	0.08	0.005	0.20	0.15	0.72
173	-0.04	0.010	0.20	0.88	0.88
174	0.01	0.009	0.27	0.77	0.84
175	-0.39, -0.09	0.000	0.24	0.51	0.66
176	-0.35	0.004	0.14	0.44	0.84
177	-0.10	0.018	0.24	0.64	0.85
178	0.08	0.019	0.22	0.85	0.89
M	-0.09	0.010	0.22	0.58	0.80

Note. Pigeon 175 had two log *b* values estimated when fitting the data because the direction of bias greatly differed between cued and uncued conditions. The first value is for cued terminal links while the second value is for uncued terminal links. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. VAC was computed based on session sixth data. Parameter values were fitted using session sixth data.

5.3.6.2 Predictive accuracy of the model. To determine if there were systematic deviations of the model's predictions from the data, we examined scatterplots of obtained and predicted log response ratios for individual pigeons. Figure 5.06a shows these scatterplots for Pigeons 171 to 174 while Figure 5.06b shows them for Pigeons 175 to 178. Left and right panels show plots of cued and uncued conditions, respectively. The slope and y-intercept for regressions (dashed line) in each plot are also shown. In each plot, slopes and intercepts of the dashed line that are close to 1.0 and 0.0, respectively, indicate that there are no systematic errors in prediction. Comparisons of the slopes and intercepts of cued and uncued terminal links confirmed that the model was able to predict data in cued terminal links more accurately. The mean sensitivity of the slope for cued terminal links was 1.01, 95% CI [1.01, 1.02], while the corresponding mean for the uncued condition was 0.97, 95% CI [0.94, 1.00]. In terms of the intercepts, the mean for cued terminal links was 0.00, 95% CI [-0.03, 0.03]. The mean of uncued terminal links was also 0.00, 95% CI [-0.07, 0.03]. Overall, slopes and intercepts (shown in Table 5.02) were close to 1 and 0 in both conditions, respectively, so predictions of the model did not systematically differ from the obtained data in cued terminal links. The regression R^2 values were significantly greater in the cued condition, $t(7) = 2.60$, $p = .036$, $d = 0.92$, likely due to the greater range of log response ratios in the cued condition (and hence greater variance).

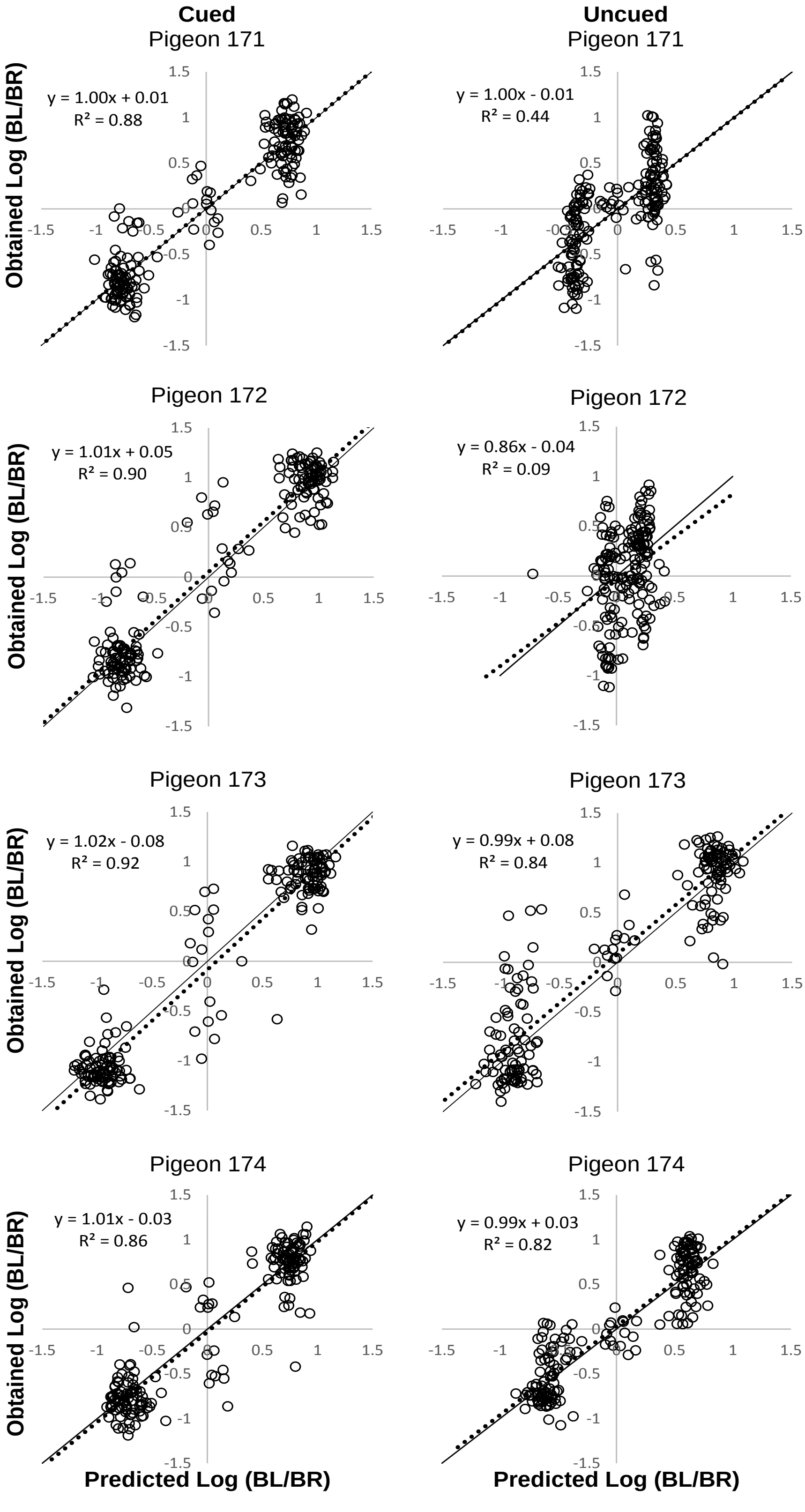


Fig 5.06a. Obtained log ratios plotted against predicted log ratios for cued and uncued terminal links for Pigeons 171 to 174. Left panels show cued terminal links while right panels show uncued terminal links. Data points are based on session sixth responding.

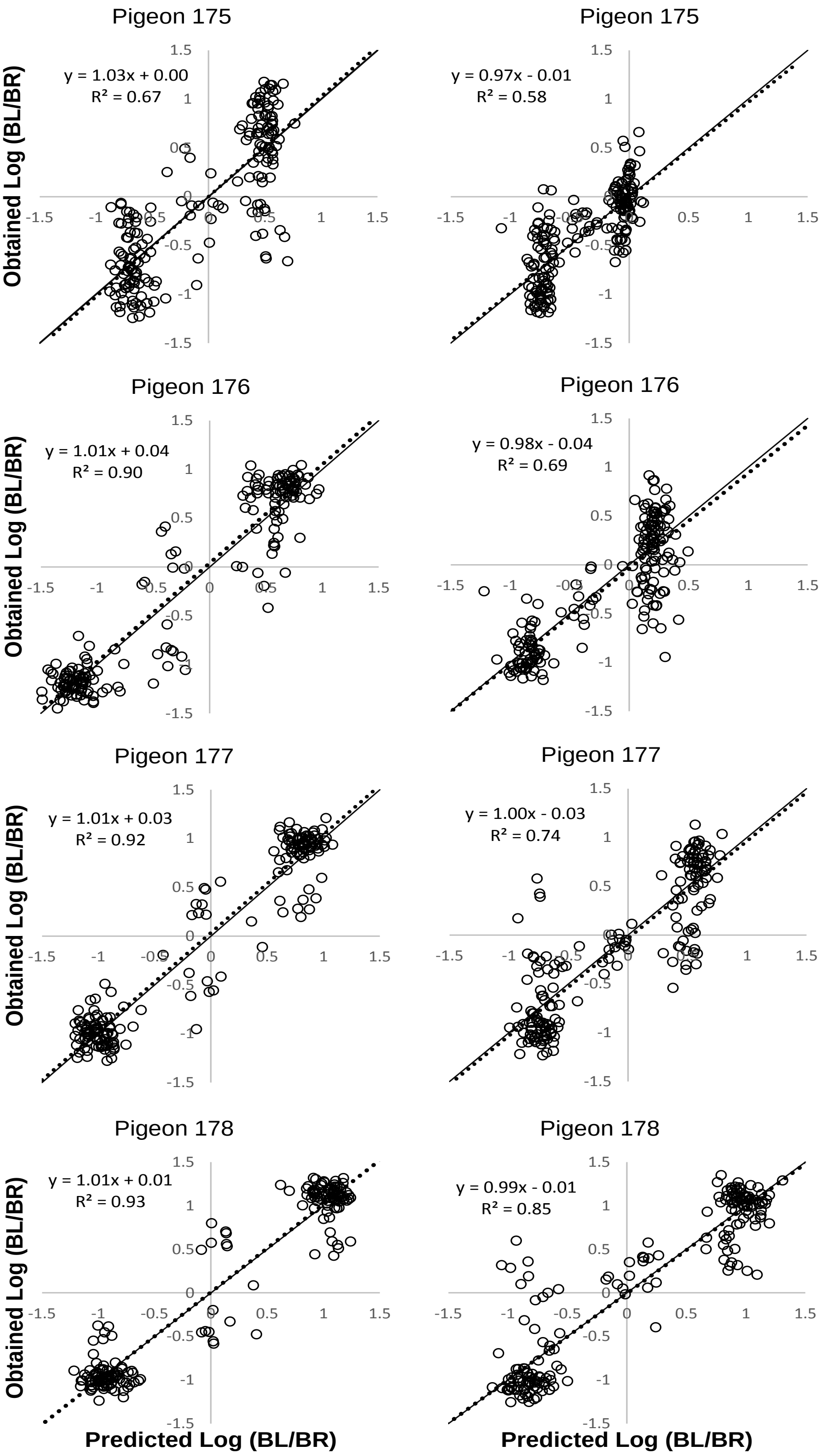


Fig 5.06b. Obtained log ratios plotted against predicted log ratios for cued and uncued terminal links for Pigeons 175 to 178. Left panels show cued terminal links while right panels show uncued terminal links. Data points are based on session sixth responding.

Table 5.02

Slope, Intercept and R^2 of Predicted Plotted Against Obtained Log Ratios

Group	Bird	Slope	Intercept	R^2
Cued	171	1.00	0.01	0.88
	172	1.01	0.05	0.90
	173	1.02	-0.08	0.92
	174	1.01	-0.03	0.86
	175	1.03	0.00	0.67
	176	1.01	0.04	0.90
	177	1.01	0.03	0.92
	178	1.01	0.01	0.93
Uncued	171	1.00	-0.01	0.44
	172	0.86	-0.04	0.09
	173	0.99	0.08	0.84
	174	0.99	0.04	0.82
	175	0.97	-0.01	0.58
	176	0.98	-0.04	0.69
	177	1.00	-0.03	0.74
	178	0.99	-0.01	0.85
Cued	*M*	1.01	0.00	0.87
Uncued	*M*	0.97	0.00	0.63
	t	2.37	0.26	2.60*

Note. R^2 values range from 0 to 1. 1 being 100% or all variability in data is accounted for. R^2 was computed based on session sixth data.

* indicates $p < .05$. ** indicates $p < .01$.

5.3.6.3 Asymptotic preference. Results presented above (in Figure 5.04) show that sensitivity increased over the first half of the session but was approximately stable during the second half. This result is similar to previous experiments, so we conducted several analyses using responding pooled over the second half of each session to determine if asymptotic response allocation was consistent with the GML. This analysis was conducted both for obtained data and responding predicted by the model fits to the full-session data over the last 30 sessions of training.

Figures 5.07a and 5.07b show scatterplots of asymptotic responding in the cued condition as a function of log terminal-link immediacy ratios for Pigeons 171 to 174 and

Pigeons 175-178, respectively. Responding predicted by the model fits (i.e., simulated data) are shown in the left columns and obtained data in the right columns. Corresponding results for the uncued condition are shown in Figures 5.08a and 5.08b. Regression lines indicate the GML fits in these scatterplots, and slopes, intercepts and R^2 values are also shown.

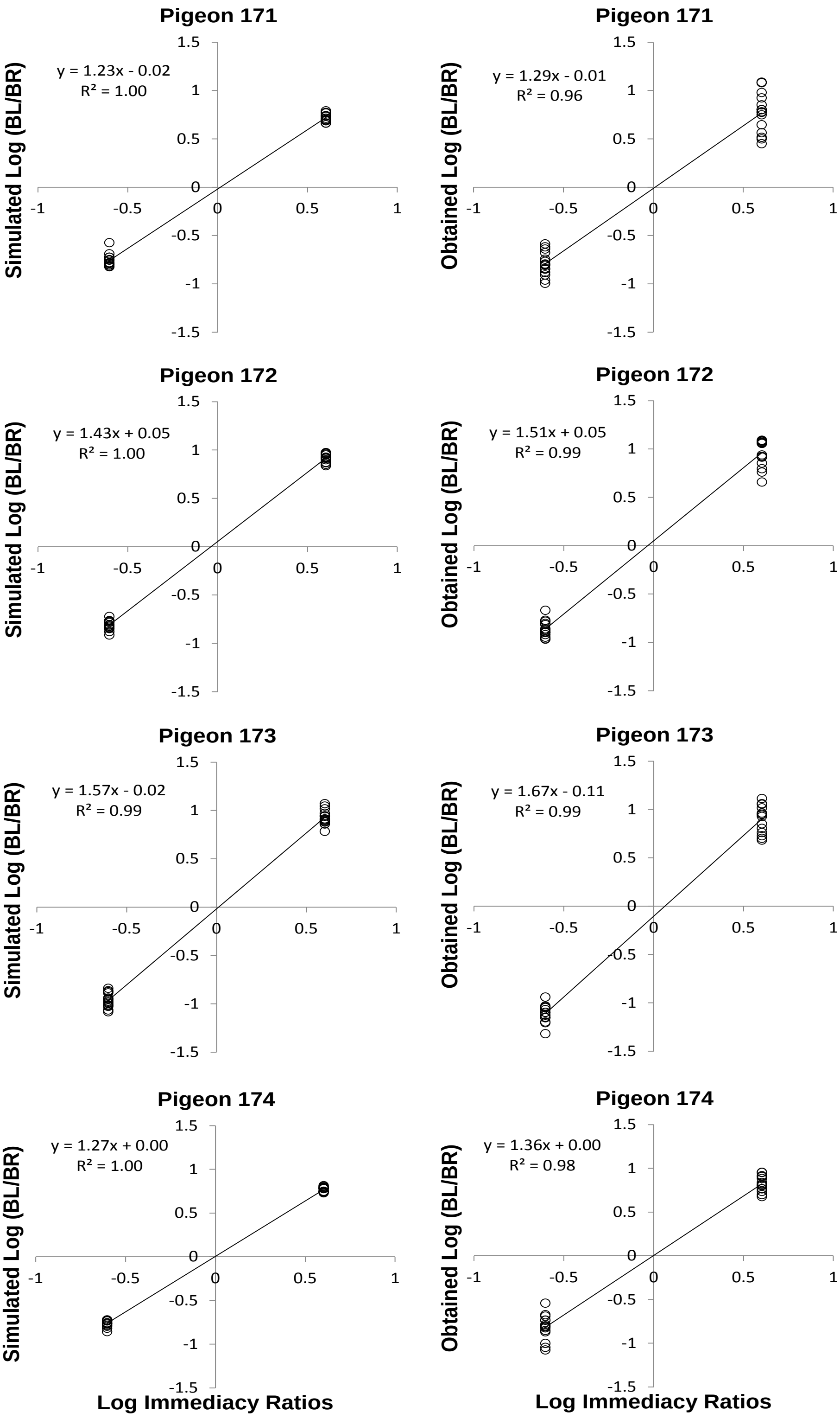


Fig 5.07a. Log response ratios (vertical axis) plotted against log immediacy ratios when terminal links were cued for Pigeons 171 to 174. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

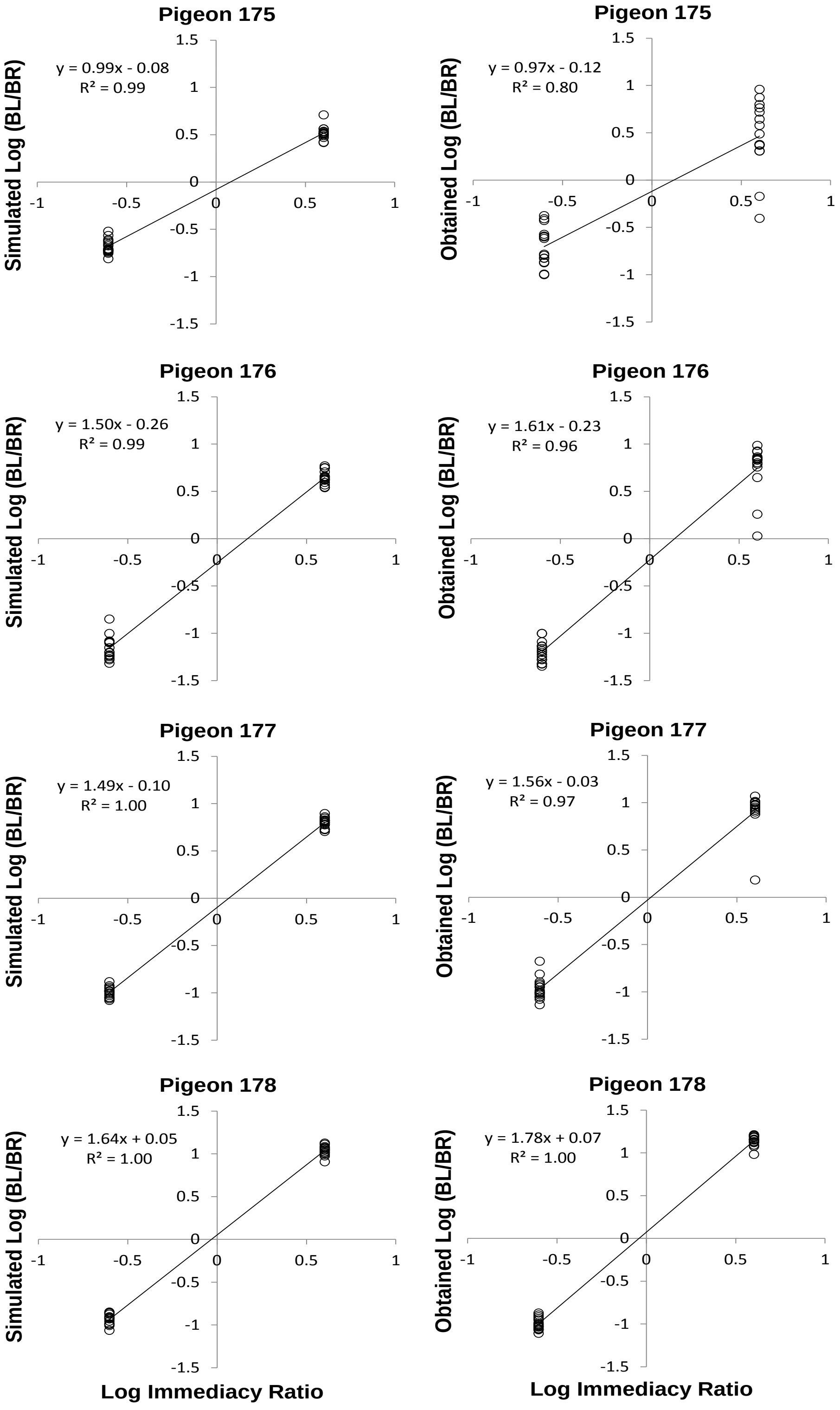


Fig 5.07b. Log response ratios (vertical axis) plotted against log immediacy ratios when terminal links were cued for Pigeons 175 to 178. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

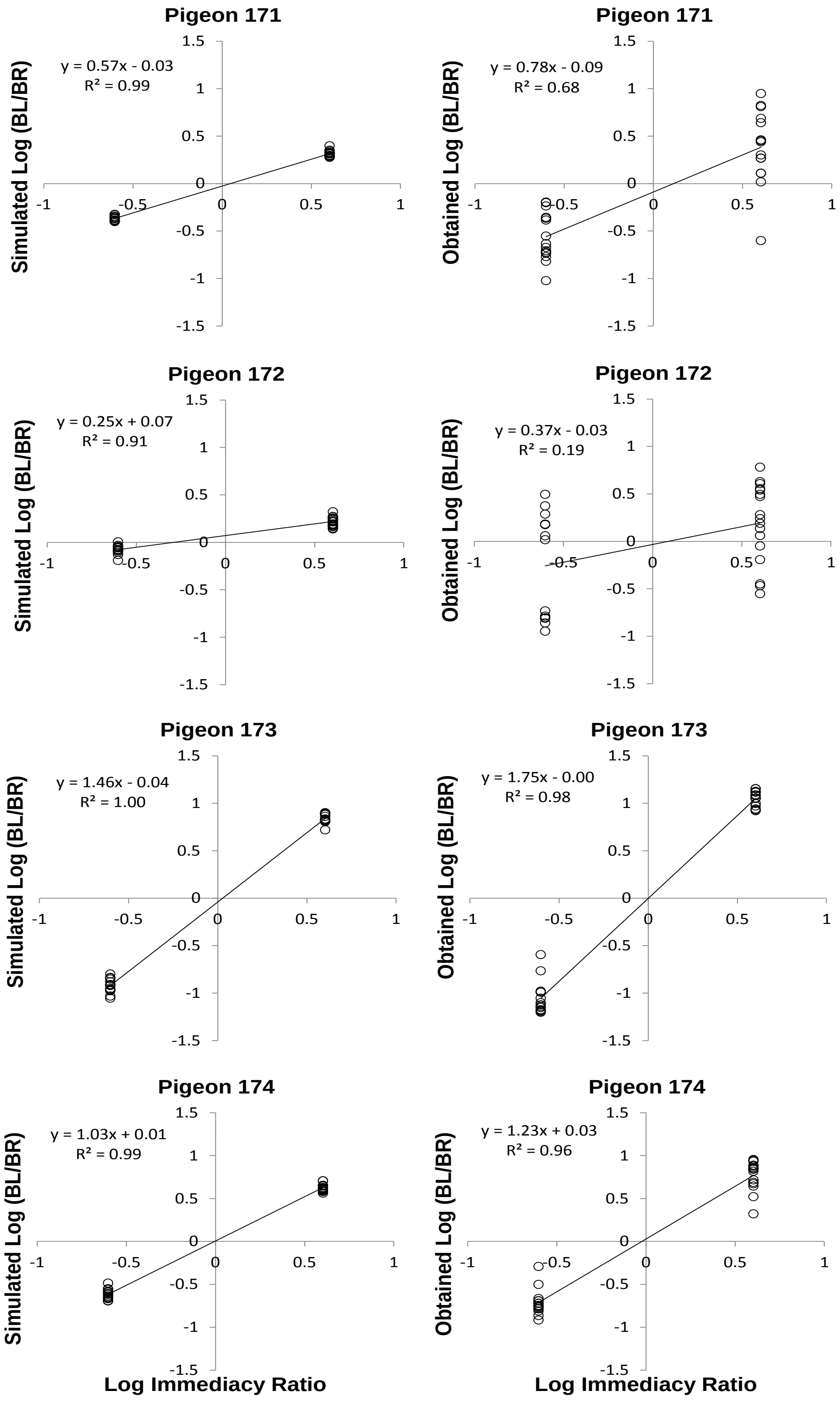


Fig 5.08a. Log response ratios (vertical axis) plotted against log immediacy ratios for uncued terminal links in Pigeons 171 to 174. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

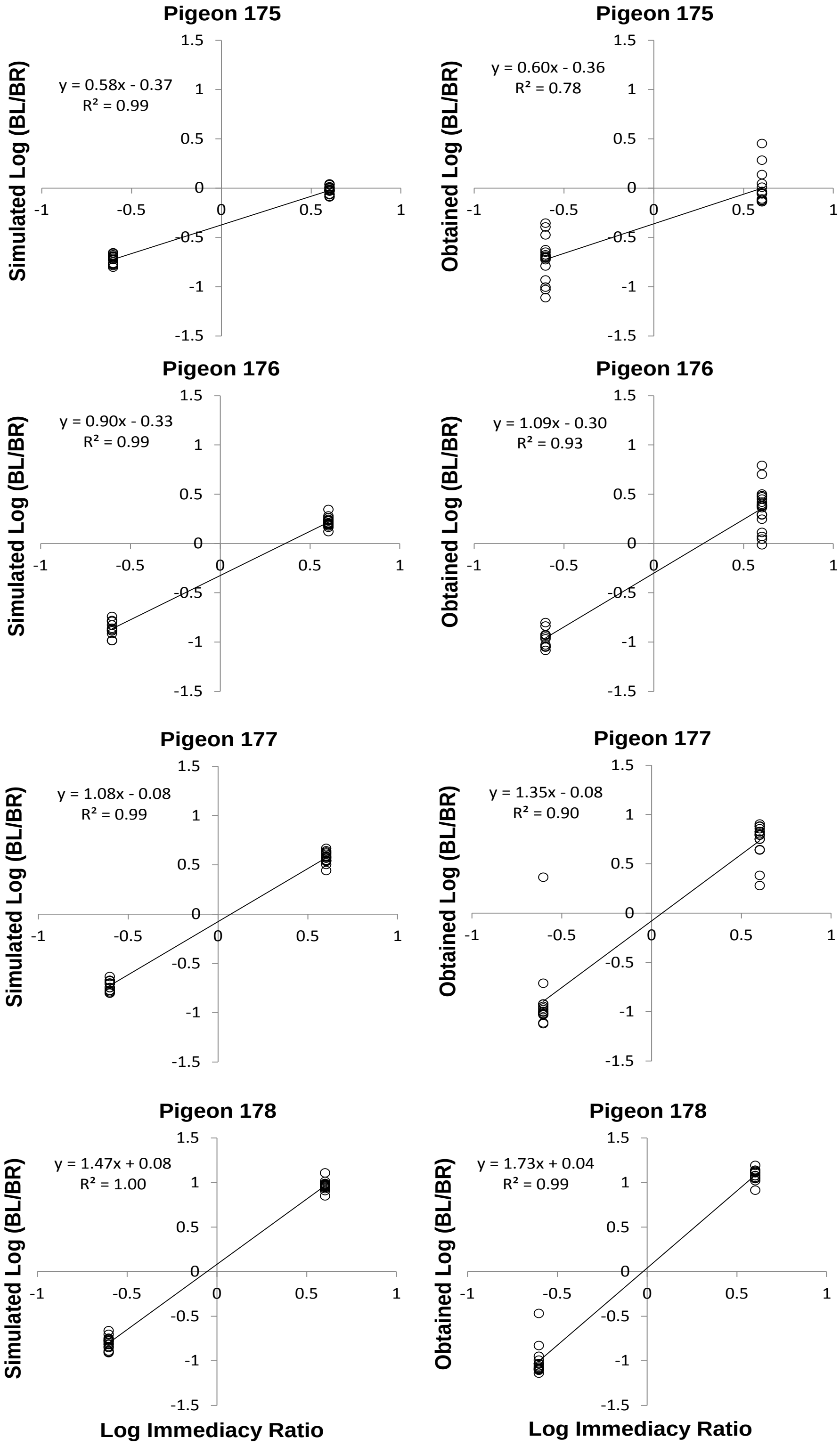


Fig 5.08b. Log response ratios (vertical axis) plotted against log immediacy ratios for uncued terminal links in Pigeons 175 to 178. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

Table 5.03 shows the average slope, intercept and VAC for GML fits to obtained and simulated data. Paired-sample *t* tests were conducted to determine if there was a difference in the average slope, intercept and VAC between cued and uncued terminal links. Specifically, was there a difference between the model's simulations and the obtained data in terms of asymptotic preference. During cued terminal links, there was a significant difference between simulated (*M* = 1.39, *SD* = 0.21) and obtained (*M* = 1.47, *SD* = 0.26) slopes, *t*(7) = -4.74, *p* = .002, *d* = 1.69. There was no significant difference between simulated (*M* = -0.05, *SD* = 0.10) and obtained (*M* = -0.05, *SD* = 0.10) intercepts, *t*(7) = 0.00, *p* > .999. In terms of VAC in cued terminal links, there was no significant difference between simulated (*M* = 0.99, *SD* = 0.01) and obtained (*M* = 0.96, *SD* = 0.06) VAC, *t*(7) = 1.82, *p* = .112. Similar differences were found in uncued terminal links. There was a significant difference between simulated (*M* = 0.92, *SD* = 0.43) and obtained (*M* = 1.11, *SD* = 0.50) slopes, *t*(7) = -6.20, *p* < .001, *d* = 2.17. There was no significant difference between simulated (*M* = -0.09, *SD* = 0.17) and obtained (*M* = -0.10, *SD* = 0.15) intercepts, *t*(7) = 0.72, *p* = .496. In terms of VAC, there was no significant difference between simulated (*M* = 0.98, *SD* = 0.03) and obtained (*M* = 0.80, *SD* = 0.27) VAC, *t*(7) = 2.12, *p* = .072.

Table 5.03

Average of Slope, Intercept and VAC for GML fits to Obtained and Simulated Data

Data	Condition	Slope	Intercept	VAC
Obtained	Cued	1.47	-0.05	0.96
	Uncued	1.11	-0.10	0.80
Simulated	Cued	1.39	-0.05	0.99
	Uncued	0.92	-0.09	0.98
Cued Diff	*t*	-4.74**	0.00	1.82
Uncued Diff	*t*	-6.20**	0.72	2.12

Note. Diff denotes the difference between obtained and simulated data. Values are computed using responding during the second half of each session, over the last 30 sessions.

* indicates *p* < .05. ** indicates *p* < .01.

5.3.7 Residual analysis

Finally, we conducted a residual analysis to test if there were systematic deviations in the obtained and simulated data from generalized matching, similar to Experiments 1 through 3. Polynomial regressions were conducted in which the residuals were regressed on the simulated and obtained values. Results are shown in Table 5.04. In terms of the simulated data, the linear, quadratic and cubic coefficients were all non-significant predictors of the log response ratios for the simulated data. A similar trend was observed in the obtained data where the linear, quadratic and cubic coefficients were all not significant predictors of log response ratios for the obtained data. These results strongly suggest that there were no systematic deviations from the GML's predictions for both simulated and obtained data. Figure 5.09 shows the scatter of residual data points for the second half of the last 30 sessions. The scatter of data points indicates no discernible signs of linear, quadratic or cubic trends.

Table 5.04

Results of Polynomial Regressions Performed on GML Residuals

Dependent variable	Group	Predictor variables	*B*	S.E. of *B*	R^2	*F*
GML residuals Simulated	Cued	Linear	0.000	0.016		
		Quadratic	0.000	0.016		
		Cubic	0.000	0.018		
					0.000	0.000
GML residuals Obtained	Cued	Linear	-0.005	0.018		
		Quadratic	0.002	0.010		
		Cubic	0.002	0.007		
					0.001	0.053
GML residuals Simulated	Uncued	Linear	0.000	0.016		
		Quadratic	0.000	0.011		
		Cubic	0.000	0.024		
					0.000	0.000
GML	Uncued	Linear	0.003	0.044		

residuals					
Obtained	Quadratic	0.001	0.017		
	Cubic	-0.003	0.020		
				0.000	0.011

Note. Dependent variables were residuals from the GML, computed using log response ratios. Independent variables were log predicted values, plus squares and cubes of log predicted values (‘quadratic’ and ‘cubic’). *B* is the unstandardized regression coefficient, and R^2 is the proportion of variance accounted for.

*indicates $p < 0.05$, ** indicates $p < 0.01$, *** indicates $p < 0.001$.

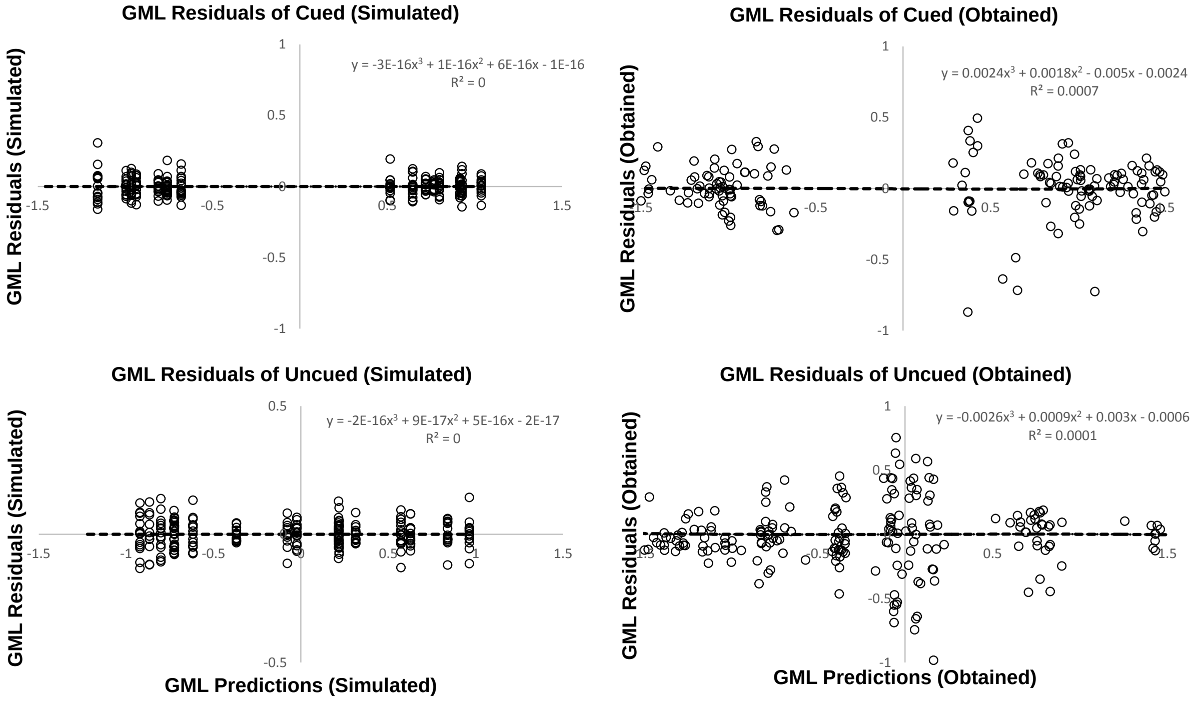


Fig 5.09. All panels show response allocation predicted by the GML for cued or uncued terminal links. Upper panels show residuals of cued terminal links while the lower panels show residuals of uncued terminal links. Left panels show residuals based on the carryover model’s simulated data while the right panels show residuals based on obtained data. Each data point was computed from the second half each session over the last 30 sessions.

5.4 Summary

Experiment 4 attempted to replicate a well-known finding from steady-state choice with a rapid-acquisition task: Providing differential terminal-link cues increases sensitivity to delay (Alsop et al., 1994; Williams et al., 1978). Results showed that sensitivity to immediacy ratios was higher when terminal links were signaled with differential cues than when the same center-key stimulus was associated with each alternative. A modification of the CDM applied to Experiment 2 and 3 data was developed to account for the results. This model provided a very good description of the data, with relatively equal variance accounted for between cued and uncued conditions and no systematic differences in the estimated values of the model parameters (α and β_{opp}). During the last 30 sessions, bias for a particular alternative was weaker compared to Experiment 3, and decreased over the course of each session in both cued and uncued conditions.

Chapter 6

Experiment 5: The Effects of Terminal-link Entry Ratio and Signalled Terminal-link Stimuli on Choice

6.1 Introduction

The previous chapter shows strong evidence for signalling effects of terminal links in terms of preference for the shorter terminal link. Specifically, preference for the shorter terminal link is stronger when terminal-link key colors were correlated with terminal-link entry. In terms of examining signalling effects of terminal-link stimuli on choice behaviour, the next step was determining if signalling effects of terminal-link stimuli affect preference for the more frequently occurring terminal link. In this chapter, we measured preference in terms of sensitivity to terminal-link entry ratio. Terminal-link key colors were either correlated with terminal-link entry or random. We know that signalled terminal links affect preference towards conditioned reinforcement in terms of terminal link delays. The question is whether this information conveyed by terminal links also extends to terminal-link entry ratio. This was conducted because if signalling effects of terminal-link stimuli affect choice behaviour, then the results of chapters 3 and 4 that measured preference in terms of terminal-link entry ratio could be a combination of duration and signalling effects.

The carryover model can explain the acquisition of preference and therefore asymptotic preference across multiple varied contingencies of conditioned reinforcement. In addition to being able to explain choice behaviour for varied initial and terminal-link durations, the carryover model has is able to explain signalling effects of terminal links on preference for the shorter terminal link while maintaining parameter invariance. The next step is extending the CDM through the carryover model to satisfy this criterion while taking into account signalling effects of terminal-link stimuli on preference for the more frequent conditioned reinforcer (terminal-link entry). With mean sensitivity values of severe undermatching, we expected mean VAC of the carryover model to approximate VAC in Experiments 2.

In this study, we wanted to determine the effects of cued and uncued terminal links on sensitivity to terminal-link entry ratio under rapid-acquisition conditions. Pigeons were made to peck two lighted keys in the initial links and the single lighted key in the terminal links. Each trial consisted of an initial link phase and a terminal link phase. This study had two conditions; cued and uncued terminal links. When terminal links were cued, the color of the centre key depended on whether a left or right initial link response produced the terminal link. When terminal links were uncued, the color of the centre key was not dependent upon which initial link response resulted in terminal-link entry. Every pigeon was exposed to both cued and uncued terminal links. In each initial link, a single VI 8s schedule operated. For each initial link, there were a total of 12 possible intervals which, across each session resulted in an average initial-link duration of eight seconds. A concurrent-chains procedure was arranged, in which terminal-link entry was scheduled probabilistically to either the left or right key after the interval had elapsed at the end of each initial link. As each interval in the schedule timed out, pecking on the key which had been randomly assigned an outcome (left or right key) resulted in access to the corresponding terminal link. For half of the sessions, the ratio of the assignment (left/right) was 1:3. In the remaining half of the sessions, it was 3:1. Terminal links durations were always FI 8s FI 8s for the left and right keys. A 1s changeover delay COD was used. Reinforcement consisted of 3.5s of access to food. For both groups, whether the left or right key had the richer reinforcement ratio was varied according to a PRBS. In this way, pigeons had to learn which key was the richer alternative every single session.

6.2 Method

6.2.1 Subjects

Eight pigeons of mixed breed and sex were maintained at 85% of their free-feeding weight through appropriate post-session feeding. Pigeons were caged individually in a vivarium which ran according to a 12-hour day and night cycle (lights were switched on at 06:00). Home cages were supplied with grit and water which was available at all times to pigeons when they were in their home cages.

6.2.2 Procedure

Each trial of the concurrent chains procedure was completed when the pigeons had responded during an initial link phase and a terminal link phase. A 72-cycle program of concurrent chains was used for this experiment. Completing each session consisted of pigeons completing 72 trials of concurrent chains. At the start of each trial, side keys were lighted yellow to signal the beginning of the initial links. Terminal link entry was signalled by extinguishing the side keys and lighting the centre key either red or green. When terminal links were cued, the color of the centre key depended on whether a left or right initial link response produced the terminal link (red-left, right-green). When terminal links were uncued, the color of the centre key was not correlated which initial link response resulted in terminal-link entry (red and green centre keys were equally likely for left and right initial links). Every pigeon was exposed to both cued and uncued terminal links.

For half of the sessions, half of the pigeons were started cued terminal links while the remaining half of the pigeons was started on uncued terminal links. For the remaining sessions, the order was then reversed. In the initial link, a single VI 8s schedule operated. As each interval in the schedule timed out, the outcome was assigned randomly to the left or right key and the next peck on the assigned key produced access to the corresponding terminal link. For half of the sessions, the assignment was 75% -left, 25% right. In the remaining half of the sessions, the order was reversed. The order in which terminal link entry ratios were assigned occurred daily and was determined by a PRBS, as used by Hunter et al. (1985).Terminal links durations were always FI 8s FI 8s for the left and right keys The use of the PRBS ensured the order in which terminal link entries occurred across sessions was completely randomized. Each presentation of the PRBS required 31 sessions of concurrent chains. Every pigeon was exposed to both sets of terminal-link durations. Figure 6.01 shows the concurrent-chains procedure we used for cued terminal links. The uncued condition used the same setup but terminal-link key colors were randomised.

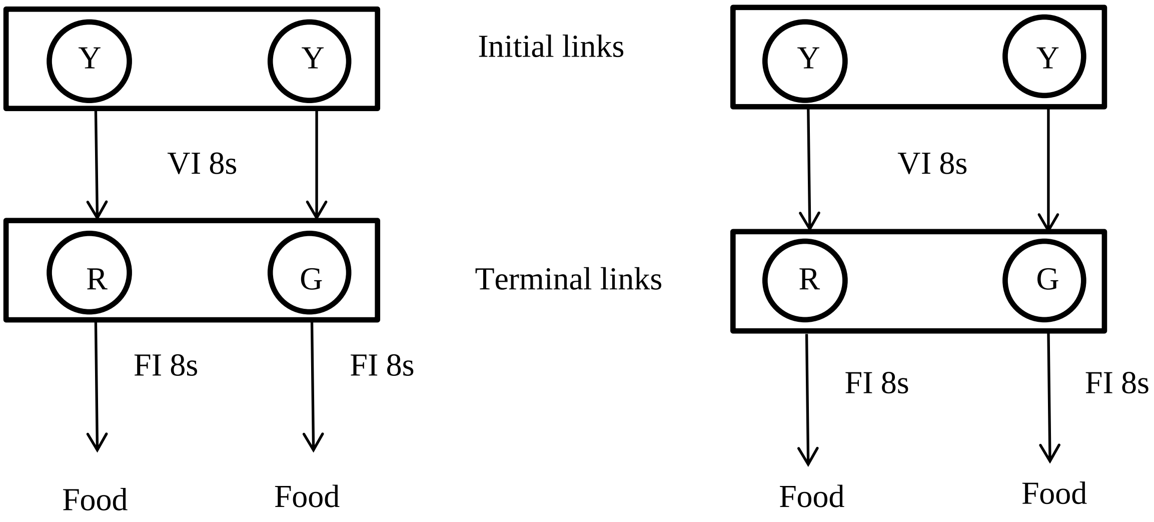
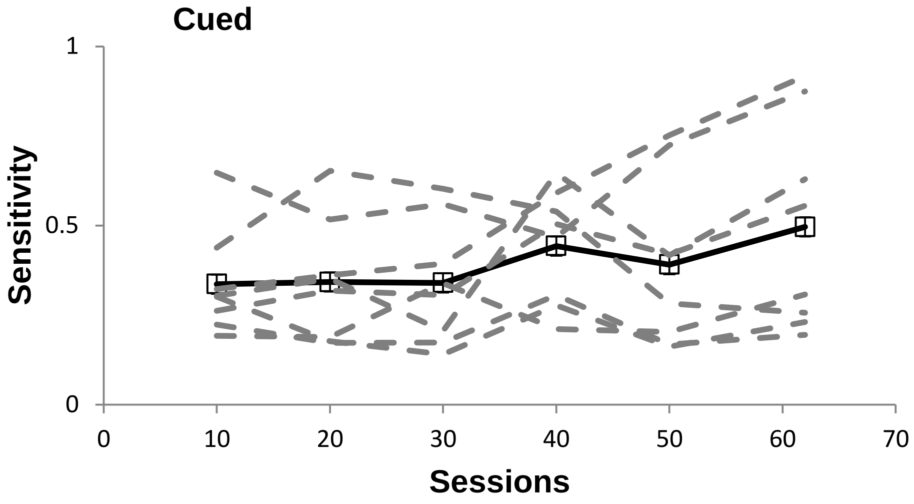


Fig 6.01. Concurrent-chains schedule for the cued condition. Side keys are lighted yellow. Initial links consist of a single VI 8s schedule that results in terminal-link entry. Both terminal links are FI 8s. Left terminal links are light red while right terminal links are lighted green in the cued condition. In the uncued condition, terminal-link keys are randomly lighted red or green. The end of each terminal link results in access to food. Y = yellow key, R = red key, G = green key.

6.3 Results

6.3.1 Between-session preference acquisition

Sensitivities to relative terminal-link entry ratio for blocks of 10 sessions are shown in Figure 6.02. The data point at session 10 shows the sensitivity of relative response ratio to relative terminal-link entry ratio based on applying the GML to data pooled from sessions 1-10. This analysis was completed for each successive block of 10 sessions for all pigeons. Across sessions, sensitivity to terminal link entry ratio was similar for both cued and uncued terminal links. In the cued condition, mean sensitivity was 0.34 during the first 10 sessions and 0.50 during the last 10 sessions. In the uncued condition, mean sensitivity was 0.38 during the first 10 sessions and increased to 0.44 by the last 12 sessions.



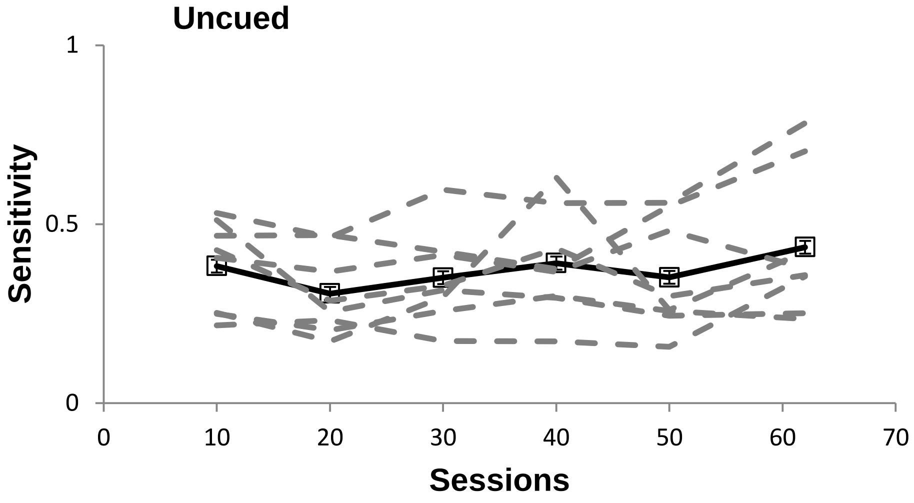


Fig 6.02. Full-session sensitivity values by 10-session blocks in cued (top panel) and uncued (bottom panel) terminal links. Dotted lines represent sensitivity values of individual birds. Solid lines are the mean sensitivity values across all birds in each condition. The last session block consists of 12 sessions.

6.3.2 Preference at the beginning and at the end of training

In order to determine if sensitivity to terminal-link entry ratio was different after training, we examined mean preference levels at the beginning and at the end of training. Figure 6.03 shows the mean sensitivity values of each condition during the first 20 and last 20 training sessions. Sensitivity values during the first 20 sessions were approximately equal in cued (*M* = 0.36, *SD* = 0.14) and uncued (*M* = 0.34, *SD* = 0.07) terminal links. During the last 20 sessions, sensitivity was still approximately equal in cued (*M* = 0.47, *SD* = 0.28) and uncued (*M* = 0.40, *SD* = 0.16) terminal links. A repeated-measures ANOVA found no effects of session block, condition or condition order on sensitivity values (all *p*s > .10). Between the beginning and the end of training, there was no systematic difference in mean sensitivity for cued and uncued terminal links.

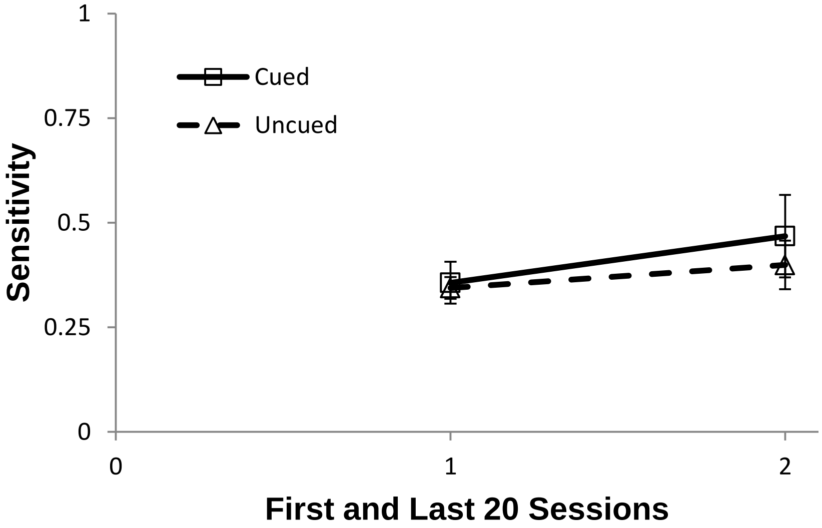


Fig 6.03. Sensitivity of relative response ratio to relative terminal-link entry ratio across cued and uncued terminal links for the first 20 and last 20 sessions. The value of “1” on the horizontal axis refers to the first 20 sessions. The value of “2” refers to the last 20 sessions. Error bars represent standard errors.

6.3.3 Lag 0 and lag 1 analysis

A lag regression analysis was conducted on the session-sixth data pooled over the last 30 sessions, using Equation (3.01). Figure 6.04 shows mean sensitivity to relative terminal-link entry ratio for lag 0 and lag 1 (lag 0 = current session, lag 1 = previous session) for all pigeons and both conditions. Figure 6.3 suggests that sensitivity did not differ systematically between conditions. This was confirmed by a 2 × 2 × 2 × 6 repeated measures ANOVA with order (cued followed by uncued/uncued followed by cued)/condition (cued/uncued), lag and session sixth as factors. The main effect of condition was not significant, $F(1, 6) = 0.08$, $p = .789$. There were significant main effects for session sixth, $F(5, 30) = 9.11$, $p < .001$, $\eta^2 = .60$, and lag, $F(1, 6) = 31.19$, $p = .001$, $\eta^2 = .84$, as well as their interaction, $F(5, 30) = 31.82$, $p < .001$, $\eta^2 = .84$. Thus, control by the current and prior session entry ratios increased and decreased within sessions, respectively, similarly in both cued and uncued conditions.

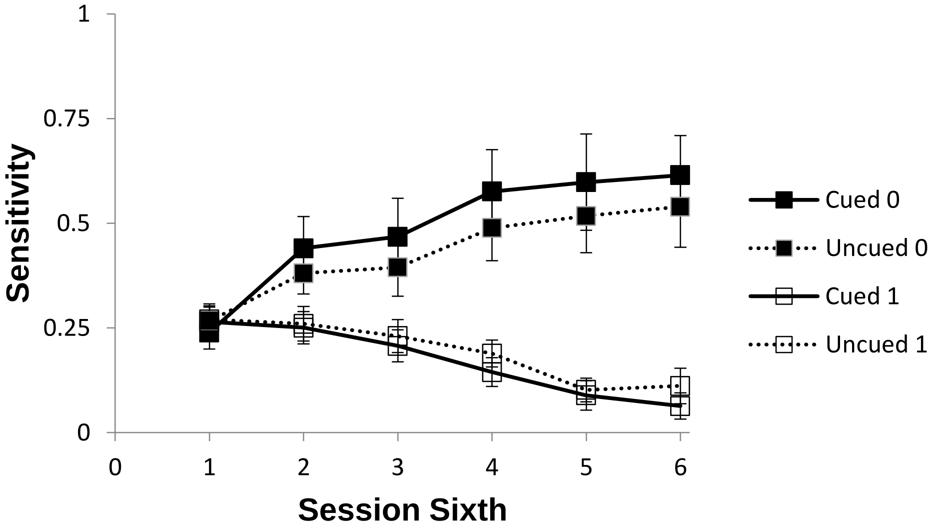


Fig 6.04. Lag analysis of previous session’s terminal-link entry ratios as predictors of preference in the current session for cued and uncued terminal links. “0” refers to the

sensitivity of each current session's log response ratio to log terminal-link entry ratio of that same session. "1" refers to the sensitivity of each current session's log response ratio to log terminal-link entry ratio of the previous session. This was conducted on the last 30 training sessions. Error bars represent standard errors.

However there were also significant effects related to the order in which the conditions were completed that suggests that whether terminal links were differentially cued affected responding. The main effect of order was significant, $F(1, 6) = 8.10$, $p = .029$, $\eta^2 = .57$, as was the interaction between session sixth and order, $F(5, 30) = 4.31$, $p = .005$, $\eta^2 = .42$, and the order x lag x session sixth interaction, $F(5, 30) = 4.13$, $p < .006$, $\eta^2 = .41$. All other interactions were not significant (*ps* > .234).

To understand these effects, we examined a plot of the three-way interaction between order, lag and session sixth. As Figure 6.05 shows, sensitivity values were overall greater when the uncued condition was completed first (*M*s = .38 and .26 for uncued-cued and cued-uncued, respectively). More specifically, lag 1 coefficients were greater early the session, while lag 0 coefficients were greater later in the session. This result suggests that sensitivity to unpredictable changes in the entry ratio was overall greater when pigeons were trained first with uncorrelated terminal-link stimuli.

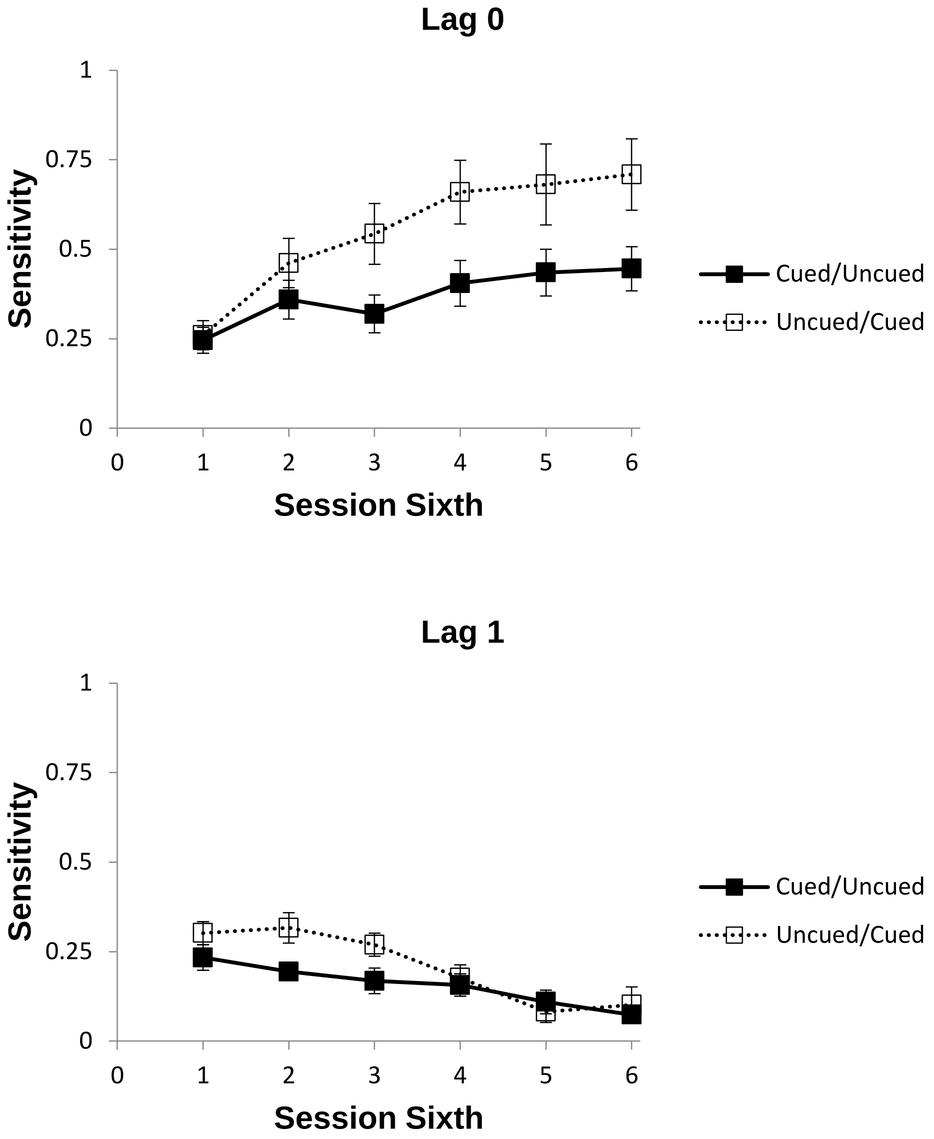


Fig 6.05. Lag 0 (top panel) and lag 1 (bottom panel) coefficients by order, pooled across cued and uncued conditions. “0” refers to the sensitivity of each current session’s log response ratio to log terminal-link entry ratio of that same session. “1” refers to the sensitivity of each current session’s log response ratio to log terminal-link entry ratio of the previous session. The series “cued/uncued” refers to birds that first completed the cued terminal link condition. The series “uncued/cued” refers to birds that first completed the uncued condition. Analysis was based on data from the last 30 sessions. Error bars represent standard errors.

One possible interpretation of these results is that pigeons which began training in the cued condition attended to the differential terminal-link stimuli, whereas those pigeons that were trained first on the uncued condition did not. Because the terminal-link stimuli were not predictive of differential reinforcement (and thus were irrelevant to the discrimination that pigeons were learning), these stimuli may have competed with initial-link cues correlated with terminal-link entry, thus reducing sensitivity to the entry ratio in the cued condition. When these birds were later exposed to the uncued condition, attending to the differential terminal-link stimuli would have interfered to a greater extent with learning the initial-link discrimination, further reducing sensitivity. By contrast, pigeons that were first trained with uncued terminal links may have learned not to attend to the colour of the centre key, so the terminal-link stimuli would not compete with initial-link cues for control over responding.

6.3.4 Strength of bias

Estimates of bias from the regressions yielded some relatively large values. Most of the birds exhibited bias for the right key with the strongest bias being -0.23. Bird 193 exhibited a particularly strong bias for the left key with the strongest bias value of 0.71 in the uncued condition. In order to determine the strength of these biases, we analysed the absolute value of the bias (log *b*) estimates from lag multiple regressions, to determine if the strength of the bias was stronger in the cued or uncued condition. A repeated-measures ANOVA (2 × 2 × 6 cued followed by uncued/uncued followed by cued)/condition (cued/uncued) and session sixth as factors. Average absolute bias values are shown in Figure 6.06 below. The interactions between condition and order and between condition, session sixth and order were significant, $F(1, 6) = 6.29$, $p = .046$, $\eta^2 = .51$, and, $F(5, 30) = 3.21$, $p = .019$, $\eta^2 = .35$, respectively. As Figure 6.05 shows, bias was stronger in the uncued condition for birds exposed first to the cued condition, and stronger in the cued condition for birds exposed first to the uncued condition, suggesting that the strength of bias generally increased across the experiment. Differences in bias were more pronounced after the first session sixth. No other main effects or interactions were significant (*p*s > .13).

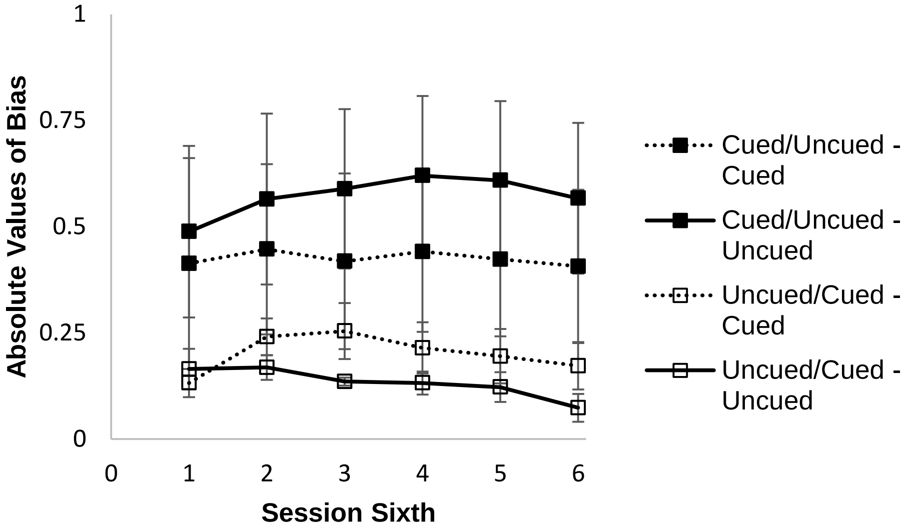


Fig 6.06. Mean absolute values of bias by session sixth over the last 30 sessions for cued and uncued terminal links based on the order which pigeons began the study. Filled squares represent pigeons that were started on cued terminal links. Unfilled squares represent pigeons that were started on uncued terminal links. Dashed lines represent mean bias during cued terminal links. Solid lines represent bias exhibited during uncued terminal links. Error bars represent standard errors.

6.3.5 Model fitting

Because terminal-link delays were always equal for the left and right alternatives, we modelled the data using the CDM from Experiment 2 ((Equation (2.04) and Equation (2.05)). Analyses in Experiments 2 and 3 showed that varying the β_{diff} parameter only resulted in small improvements in the VAC of model fits and was therefore fixed at 0.02. We recorded reinforcer location of each cycle and time spent on each initial-link alternative (used to model extinction) for each pigeon similar to Experiment 2. Calculations of average response rates and the model's simulated responses were similar to Experiments 2 and 3. The model was fitted to data for individual pigeons from the last 30 sessions of each condition.

Parameter values and percentages of VAC are shown for individual pigeons in Table 6.01. β_o was omitted since it was fixed at a value of zero across all birds. The other parameters α_{TL}, α_{Cr} and σ were fixed at typical values used in previous studies (0.2, 0.1 and 0.3, respectively), as in Experiment 3. The model was fitted separately to data from the cued and uncued conditions. Overall, the quality of the model fits was mediocre, with average VAC of .49 and .44 for the cued and uncued conditions, respectively.

6.3.5.1 Parameter invariance and goodness-of-fit. We assessed if the model's predictive accuracy and parameter estimates were different between conditions. Results of paired-samples *t*-tests shown in Table 6.01 indicated that the mean values of each parameter and VAC of the model were not significantly different between cued and uncued conditions. The difference in VAC between the cued (*M* = 0.49, *SD* = 0.16) and uncued (*M* = 0.44, *SD* = 0.14) terminal links was not significant, *t*(7) = 1.68, *p* = .137. In terms of the α parameter, there was also no significant difference between cued (*M* = 0.004, *SD* = 0.005) and uncued (*M* = 0.023, *SD* = 0.046) terminal links, *t*(7) = -1.11, *p* = .305. In terms of β_{opp} values, there was no significant difference between cued (*M* = 0.013, *SD* = 0.011) and uncued (*M* = 0.007, *SD* = 0.008) terminal links, *t*(7) = 1.48, *p* = .183. In terms of the log *b* parameter, cueing terminal links did not result in a significant difference between cued (*M* = -0.12, *SD* = 0.44) and uncued (*M* = -0.16, *SD* = 0.50), *t*(7) = 0.59, *p* = .571.

Table 6.01

Best-fit Parameter Values

Group	Bird	Log b	α	β_{opp}	Slope	Intercept	VAC
cued	191	-0.11	0.006	0.019	1.00	0.00	0.44
	192	-0.10	0.000	0.006	0.98	0.00	0.37

	193	0.61	0.000	0.005	0.98	0.01	0.41
	194	-0.94	0.012	0.008	0.99	-0.01	0.55
	195	-0.26	0.006	0.007	1.00	0.00	0.45
	196	-0.30	0.004	0.023	1.00	0.00	0.68
	197	0.22	0.000	0.005	0.97	0.01	0.28
	198	-0.07	0.000	0.034	1.02	0.00	0.77
uncued	191	-0.49	0.000	0.005	0.97	-0.01	0.49
	192	-0.10	0.040	0.001	1.05	0.01	0.29
	193	0.71	0.014	0.003	0.99	0.01	0.35
	194	-1.06	0.000	0.005	1.00	0.00	0.53
	195	-0.11	0.000	0.023	1.01	0.00	0.49
	196	-0.18	0.000	0.012	0.98	0.00	0.46
	197	0.12	0.134	0.000	1.08	-0.01	0.26
	198	-0.13	0.000	0.010	1.00	0.00	0.67
cued	*M*	-0.12	0.004	0.013	0.99	0.00	0.49
uncued	*M*	-0.15	0.02	0.01	1.01	0.00	0.44
	t	0.59	-1.11	1.48	-1.02	0.36	1.68

Note. The first half of the table refers to cued terminal links while the second half refers to uncued terminal links. VAC values range from 0 to 1. 1 being 100% or all variability in data being accounted for. All parameter values and VAC were computed using session sixth data.

6.3.5.2 Predictive accuracy of the extended CDM. Scatterplots of obtained and predicted log response ratios for each pigeon were examined to determine if there were systematic deviations of the model's predictions from the data. Scatterplots for Pigeons 191 to 194 are shown in Figure 6.07a while those for Pigeons 195 to 198 are shown in Figure 6.07b. Plots for cued and uncued conditions are shown in the left and right panels. Comparisons of the regression slopes and intercepts of cued and uncued terminal links indicated that the model was able to accurately predict data in cued and uncued conditions, with slopes and intercepts close to 1.0 and 0.0, respectively. The mean sensitivity of the slope for cued terminal links was 0.99, 95% CI [0.98, 1.00], while the corresponding mean for the uncued condition was 1.01, 95% CI [0.98, 1.03]. In terms of the intercepts, the mean for cued terminal links was 0.00, 95% CI [0.00, 0.01]. The mean of uncued terminal links was also 0.00, 95% CI [-0.01, 0.00]. *t* tests comparing the slopes and intercepts between the cued and uncued conditions were not significant.

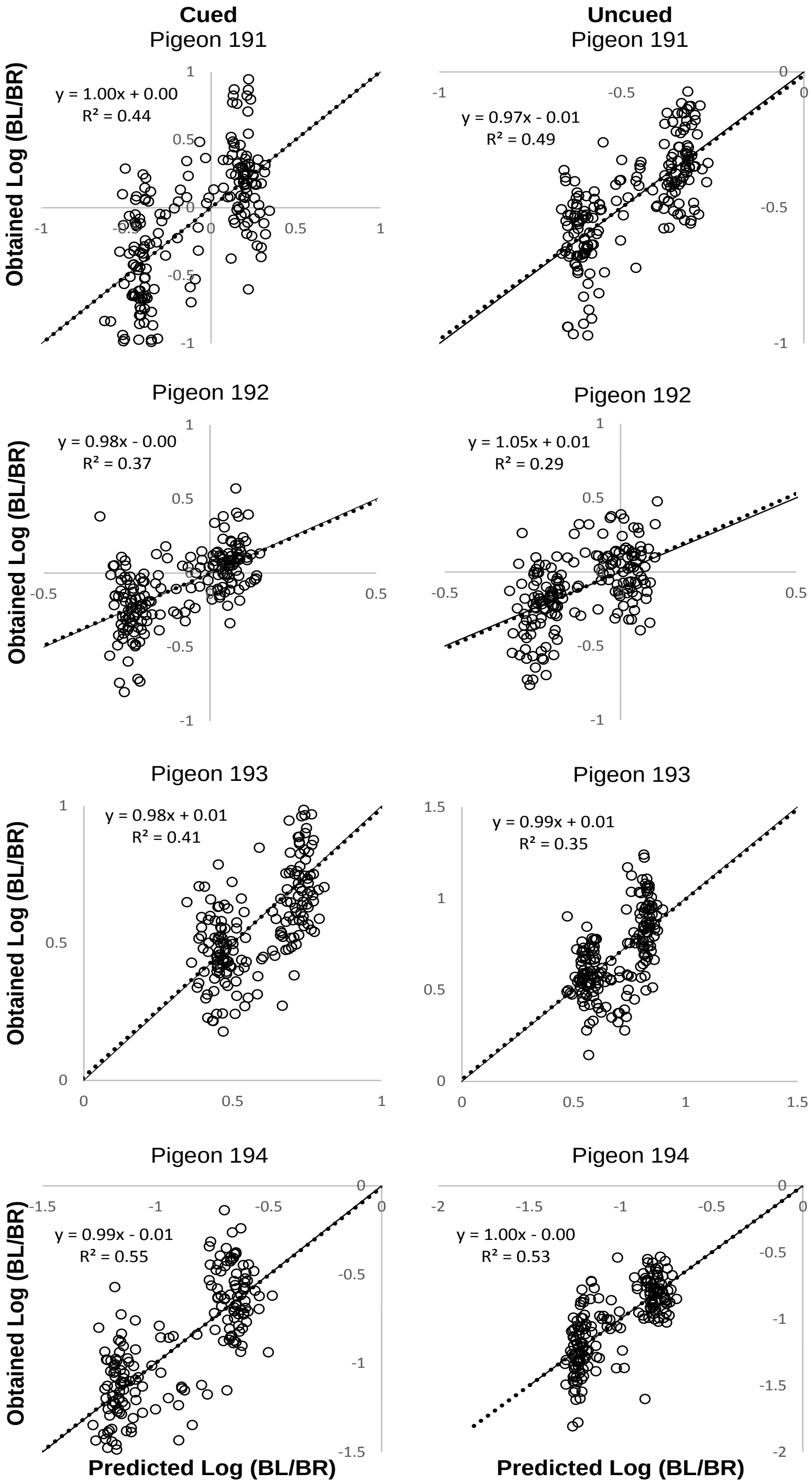


Fig 6.07a. Obtained log ratios plotted against predicted log ratios for cued and uncued terminal-link for Pigeons 191 to 194. Left panels show cued terminal links while right panels

show uncued terminal links. Data points are based on session sixth responding. In order to better display the scatter of individual data points, not all axes are of identical scales.

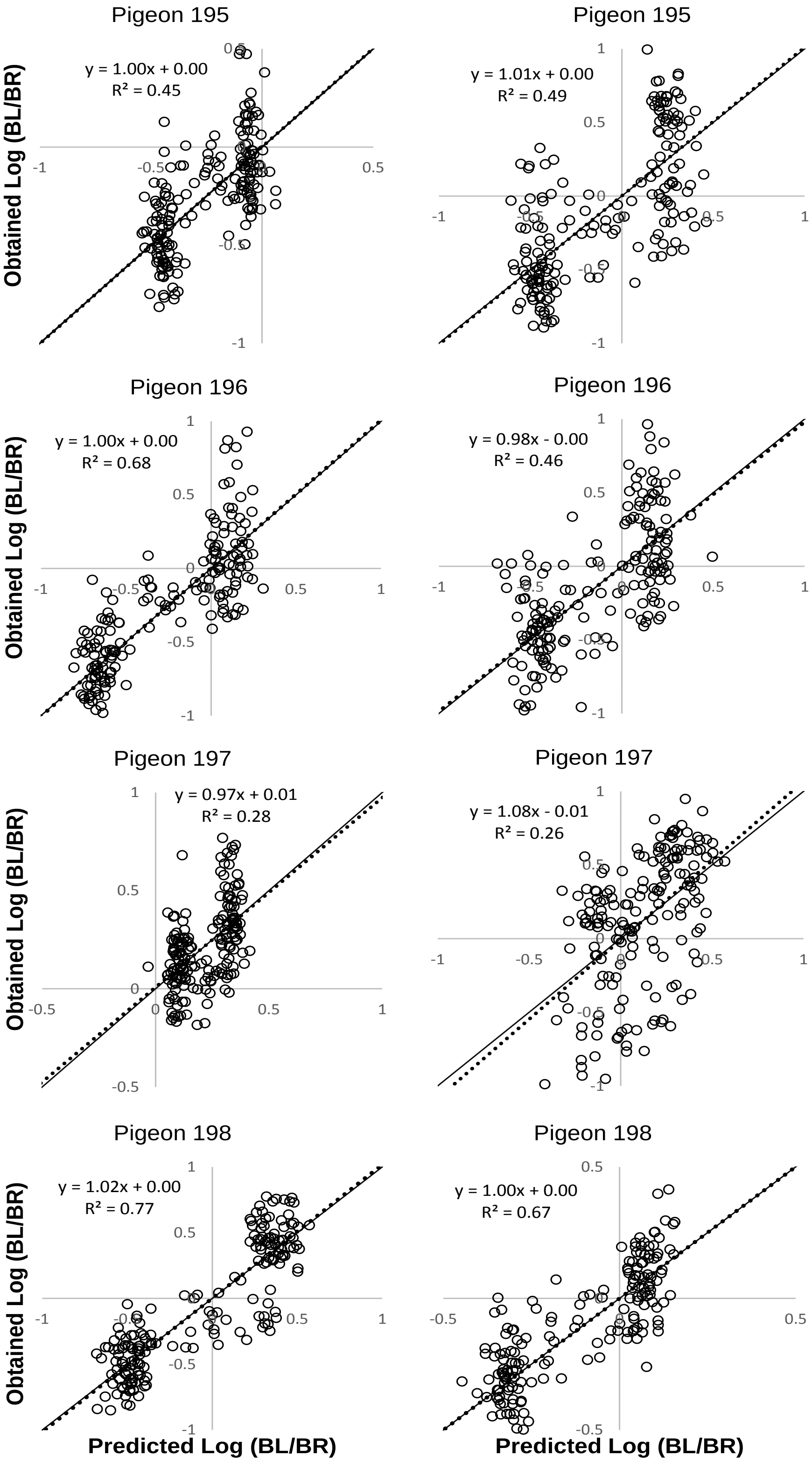


Fig 6.07b. Obtained log ratios plotted against predicted log ratios for cued and uncued terminal-link for Pigeons 195 to 198. Left panels show cued terminal links while right panels

show uncued terminal links. Data points are based on session sixth responding. In order to better display the scatter of individual data points, not all axes are of identical scales.

6.3.5.3 Asymptotic preference. Results presented above (in Figure 6.04) show that sensitivity increased over the first half of the session but was approximately stable during the second half. This result is similar to previous experiments, so we conducted several analyses using responding pooled over the second half of each session to determine if asymptotic response allocation was consistent with the GML. This analysis was conducted both for obtained data and responding predicted by the model fits to the full-session data over the last 30 sessions of training.

Figure 6.08a and Figure 6.08b show scatterplots of asymptotic responding in the cued condition as a function of log terminal-link entry ratios for Pigeons 191 to 194 and Pigeons 195 to 198, respectively. Responding predicted by the model fits (i.e., simulated data) are shown in the left columns and obtained data in the right columns. Corresponding results for the uncued condition are shown in Figure 6.09a and Figure 6.09b. Regression lines indicate the GML fits in these scatterplots. Slopes, intercepts and VAC values are also shown.

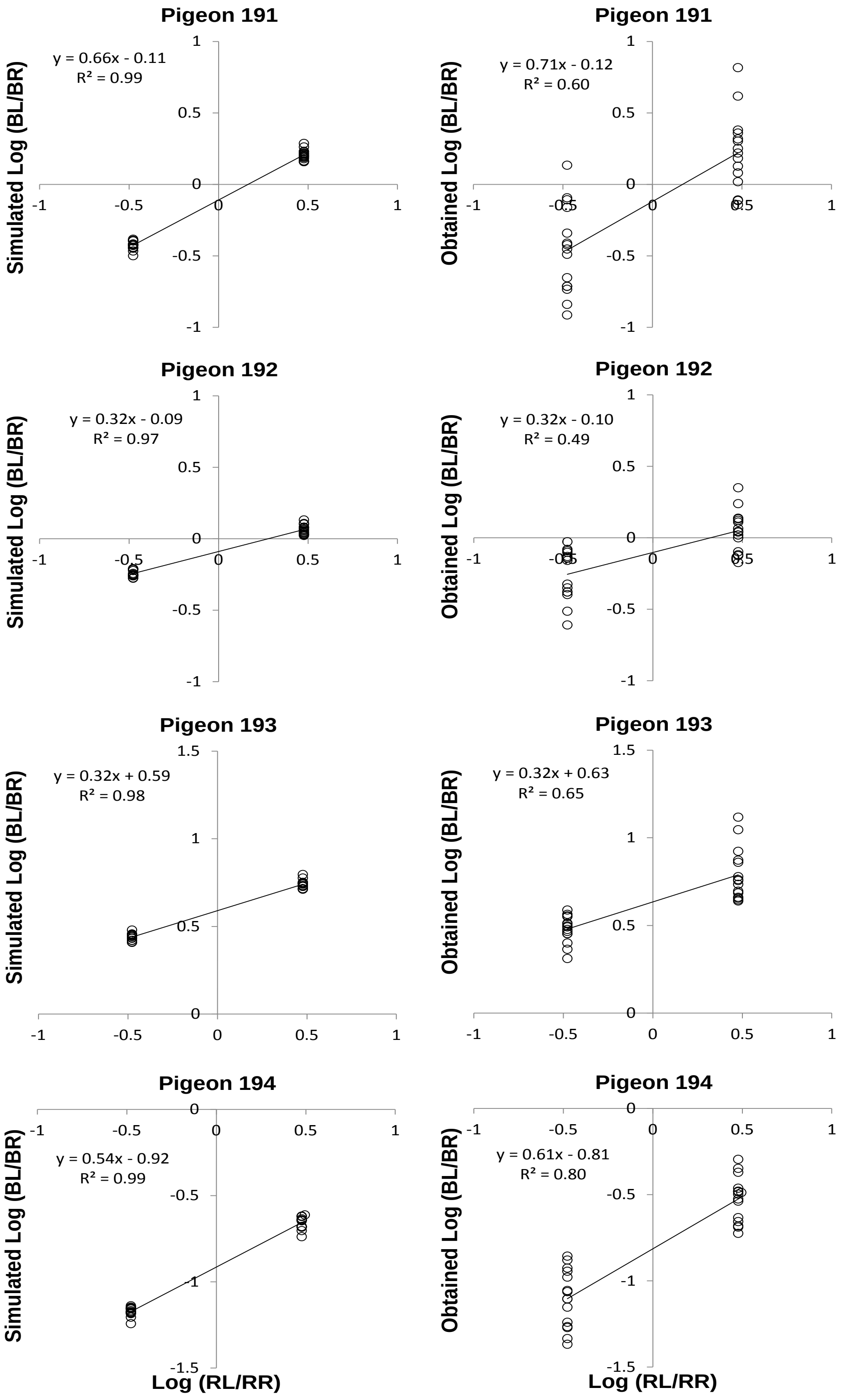


Fig 6.08a. Log response ratios (vertical axis) plotted against log terminal-link entry ratios during cued terminal links for Pigeons 191 to 194. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

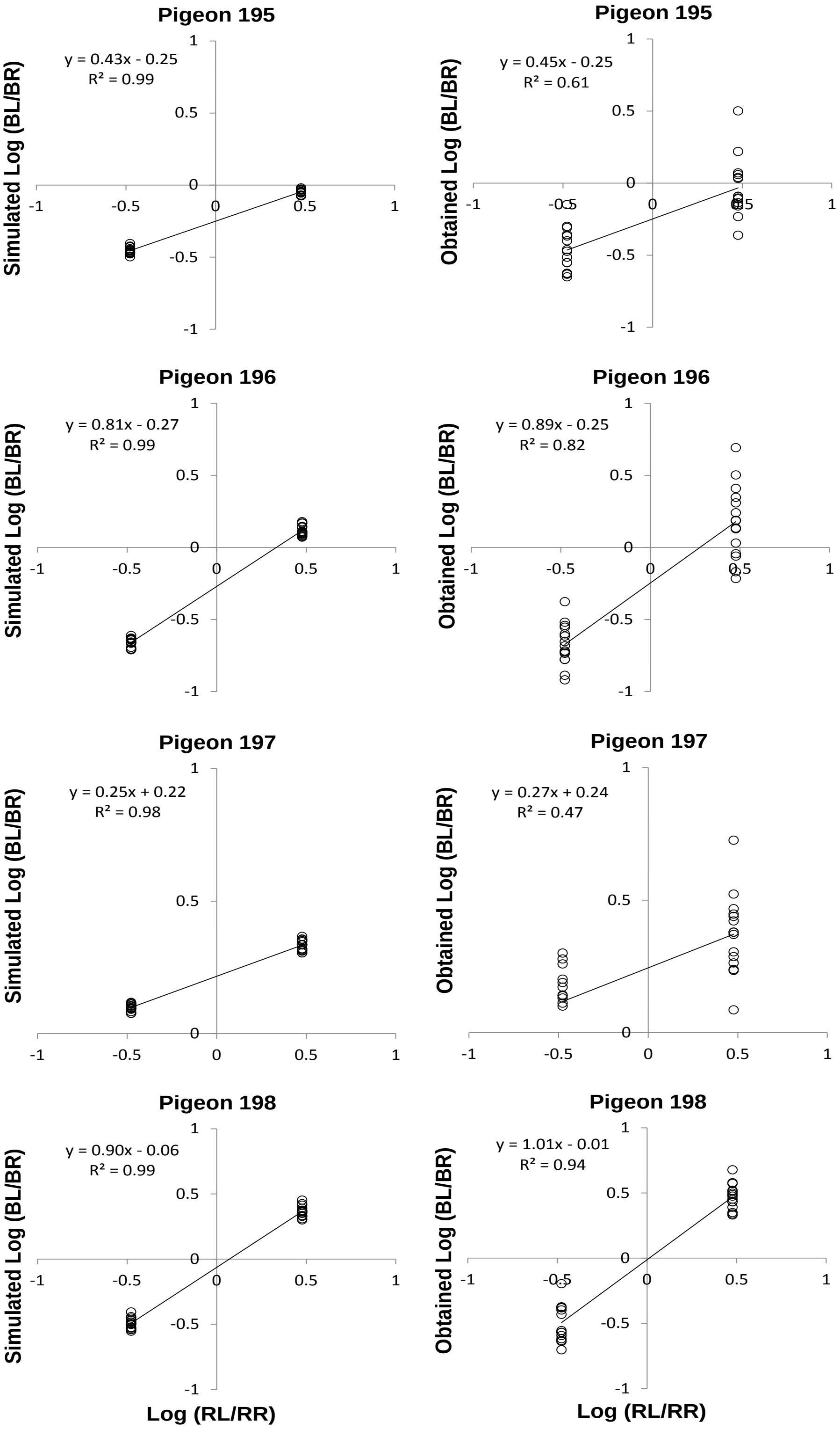


Fig 6.08b. Log response ratios (vertical axis) plotted against log terminal-link entry ratios during cued terminal links for Pigeons 195 to 198. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

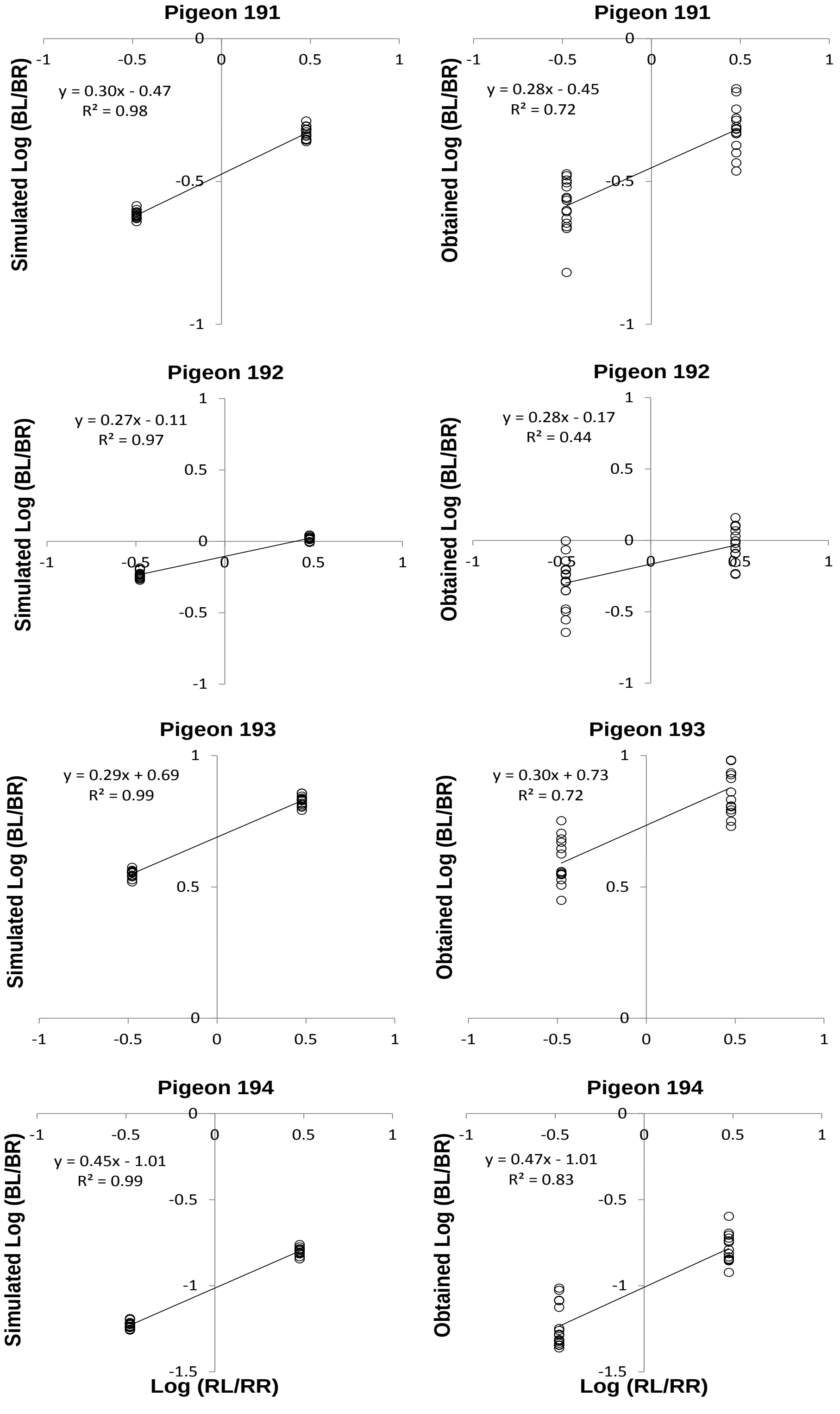


Fig 6.09a. Log response ratios (vertical axis) plotted against log terminal-link entry ratios during uncued terminal links for Pigeons 191 to 194. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

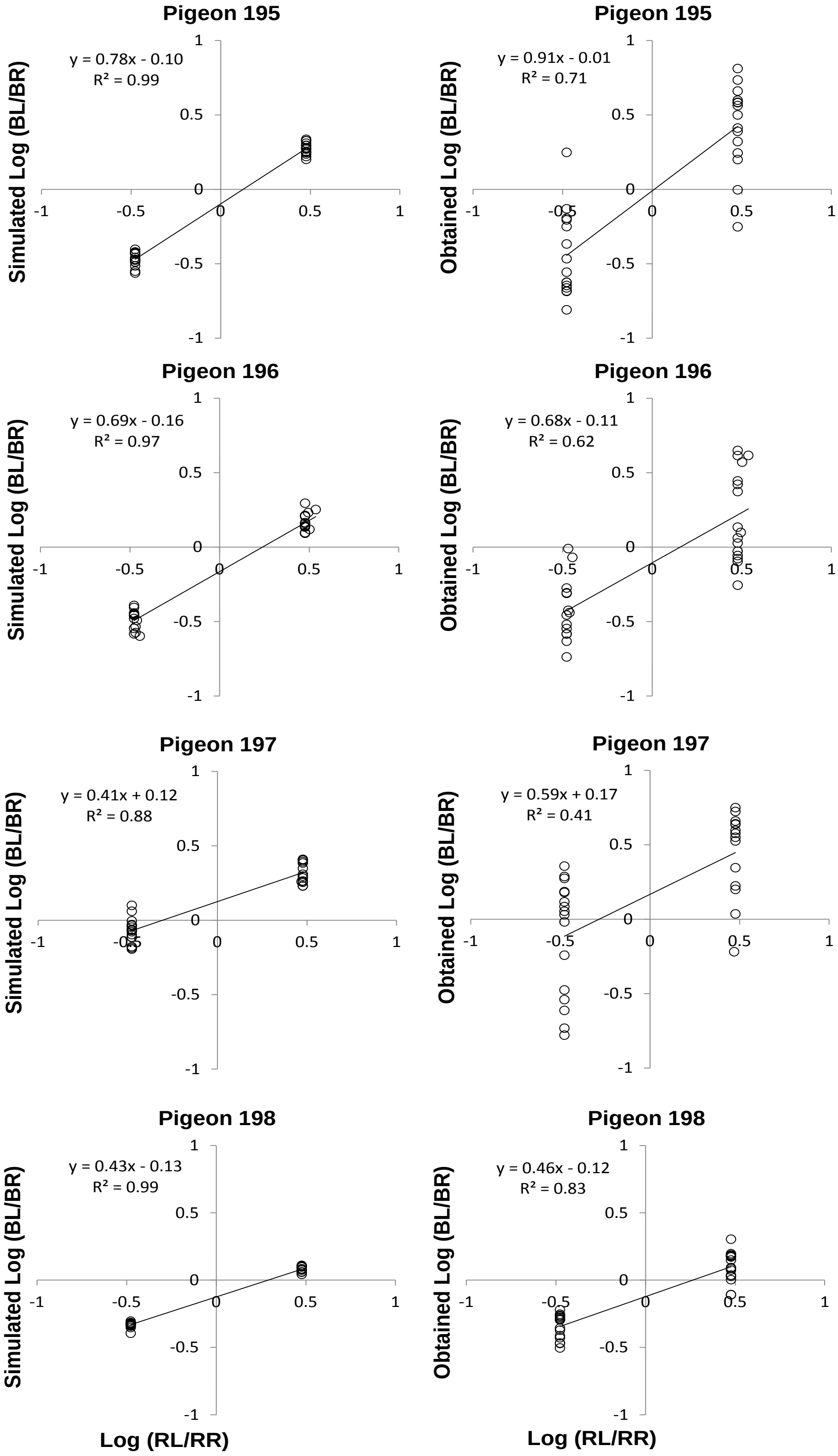


Fig 6.09b. Log response ratios (vertical axis) plotted against log terminal-link entry ratios during uncued terminal links for Pigeons 195 to 198. Left panels show plots based on simulated log response ratios while right panels show plots based on obtained log ratios. Data points are based on responding during the second half of each session.

The average slope, intercept and VAC for GML fits to obtained and simulated data are shown in Table 6.02. We wanted to determine whether there was a difference between the model's simulations and the obtained data in terms of asymptotic preference. Paired-sample *t* tests were conducted to determine if there was a difference in the average slope, intercept and VAC between conditions. When terminal links were cued, there was a significant difference between simulated (*M* = 0.53, *SD* = 0.24) and obtained (*M* = 0.57, *SD* = 0.28) slopes, *t*(7) = -3.07, *p* = .018, *d* = 1.11. There was no significant difference between simulated (*M* = -0.11, *SD* = 0.43) and obtained (*M* = -0.08, *SD* = 0.41) intercepts, *t*(7) = -1.95, *p* = .092. In terms of VAC during cued terminal links, there was a significant difference between simulated (*M* = 0.99, *SD* = 0.01) and obtained (*M* = 0.67, *SD* = 0.17) data, *t*(7) = 5.48, *p* = .001, *d* = 1.94. Similar differences were not found between simulated and obtained data when terminal links were uncued. There was no significant difference between simulated (*M* = 0.45, *SD* = 0.19) and obtained (*M* = 0.50, *SD* = 0.22) slopes, *t*(7) = -1.73, *p* = .128. There was also no significant difference between simulated (*M* = -0.15, *SD* = 0.48) and obtained (*M* = -0.12, *SD* = 0.50) intercepts, *t*(7) = -1.59, *p* = .155. However in terms of VAC, there was a significant difference between simulated (*M* = 0.97, *SD* = 0.04) and obtained (*M* = 0.66, *SD* = 0.16) data, *t*(7) = 6.55, *p* < .001, *d* = 2.31.

Table 6.02

Average of Slope, Intercept and VAC for GML fits to Obtained and Simulated Data

Data	Condition	Slope	Intercept	VAC
Obtained	Cued	0.57	-0.08	0.67
	Uncued	0.50	-0.12	0.66
Simulated	Cued	0.53	-0.11	0.99
	Uncued	0.45	-0.15	0.97
Cued Diff	*t*	-3.07*	-1.95	5.48**

Uncued Diff	*t*	-1.73	-1.59	6.55**

Note. Diff denotes the difference between obtained and simulated data. Values are computed using responding during the second half of each session, over the last 30 sessions.

* indicates $p < .05$. ** indicates $p < .01$.

6.3.6 Residual analysis

Similar to Experiment 4, we conducted a residual analysis to determine if there were systematic deviations in the obtained and simulated data from fits of the generalized matching law. Polynomial regressions were conducted in which the pooled residuals were regressed on the simulated and obtained values. Results are shown in Table 6.03. For the simulated data, the linear, quadratic and cubic coefficients were all non-significant predictors of the log response ratios for the simulated data. A similar result was observed in the obtained data where the linear, quadratic and cubic coefficients were all not significant predictors of log response ratios for the obtained data. These results strongly suggest that there were no systematic deviations from the predictions of generalized matching for both simulated and obtained data. Figure 6.10 shows the scatter of residual data points for the second half of the last 30 sessions. No noticeable traces of linear, quadratic or cubic trends were evident.

Table 6.03

Results of Polynomial Regressions Performed on GML Residuals

Dependent variable	Group	Predictor variables	*B*	S.E. of *B*	R^2	*F*
GML residuals Simulated	Cued	Linear	0.000	0.006		
		Quadratic	0.000	0.010		
		Cubic	0.000	0.011		
					0.000	0.000
GML	Cued	Linear	-0.006	0.037		

residuals						
Obtained		Quadratic	0.004	0.040		
		Cubic	0.006	0.038		
					0.000	0.010
GML						
residuals	Uncued	Linear	0.000	0.009		
Simulated		Quadratic	0.000	0.011		
		Cubic	0.000	0.013		
					0.000	0.000
GML						
residuals	Uncued	Linear	-0.009	0.047		
Obtained		Quadratic	0.006	0.038		
		Cubic	0.007	0.039		
					0.000	0.014

Note. Dependent variables were residuals from the GML, computed using log response ratios. Independent variables were log predicted values, plus squares and cubes of log predicted values ('quadratic' and 'cubic'). *B* is the unstandardised regression coefficient, and R^2 is the proportion of variance accounted for.

*indicates $p < 0.05$, ** indicates $p < 0.01$, *** indicates $p < 0.001$.

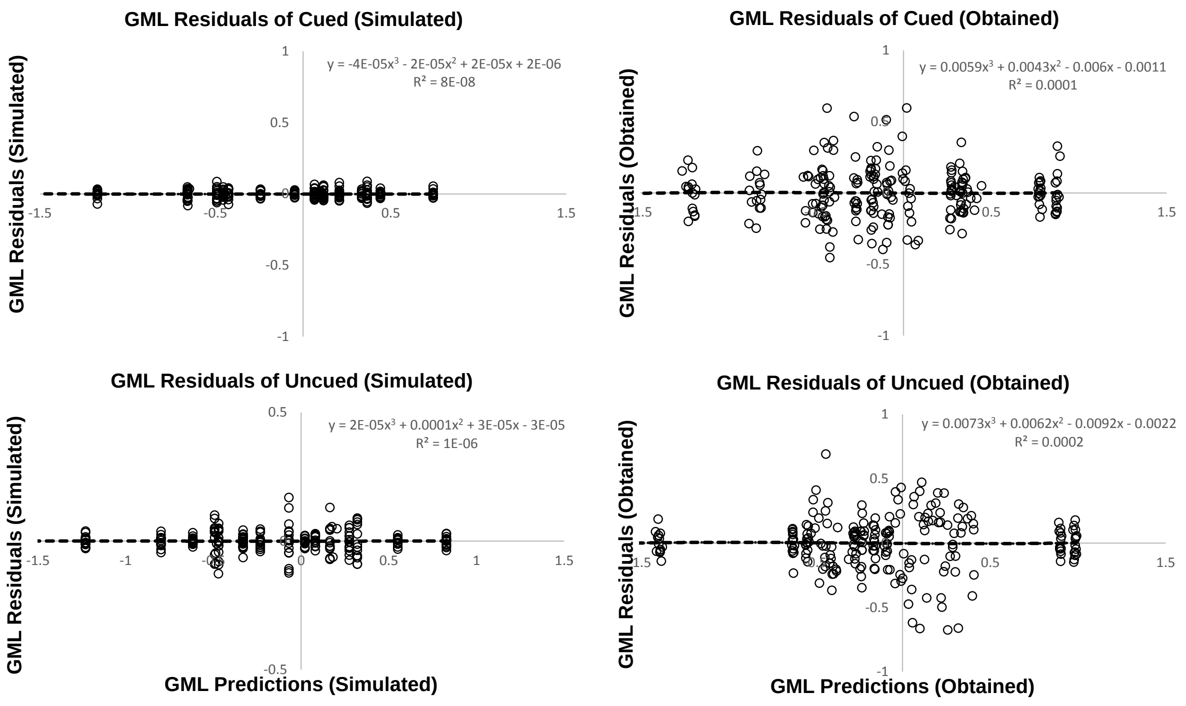


Fig 6.10. All panels show response allocation predicted by the GML for cued or uncued terminal links. Upper panels show residuals of cued terminal links while the lower panels show residuals of uncued terminal links. Left panels show residuals based on the carryover model's simulated data while the right panels show residuals based on obtained data. Each data point was computed from the second half each session over the last 30 sessions.

6.4 Summary

The major goal of Experiment 5 was to test if providing differential cues affected sensitivity to relative terminal-link entry when delays were equal in a rapid-acquisition task (Alsop et al., 1994). Results showed that sensitivity to terminal-link entry ratio was similar regardless of whether terminal links were cued or uncued. However, sensitivity to terminal-link entry ratio was higher when the uncued condition was completed first. The extended CDM used for Experiment 2 and 3 data was used to fit the data. This model provided a mediocre description of the data, with relatively equal variance accounted for between cued and uncued conditions. There were no systematic differences in the estimated values of the model parameters (α and β_{opp}). During the last 30 sessions, bias for a particular key during the initial links was stronger during the second condition for each bird.

Chapter 7

General Discussion

7.1 Overview and Evaluation of Key Results

The primary goal of the research in this thesis was to study how variation in the rates of terminal-link entries in concurrent chains – that is, relative conditioned reinforcement rate – affected choice under rapid acquisition conditions, and to explore how these effects might be included in the cumulative decision model (CDM; Grace et al., 2016). To accomplish this we conducted a series of experiments with concurrent schedules and concurrent chains procedures in which the location (left/right) of the richer alternative varied unpredictably from session to session. Experiment 1 (concurrent schedules) was intended to provide data for developing a model for the effects of varying primary reinforcement rates that could be adapted for conditioned reinforcement with the CDM; the remaining experiments all used concurrent-chains procedures in which the terminal link duration (Experiment 2), initial link duration (Experiment 3) and signal conditions (cued versus uncued) for the terminal links (Experiments 4-5) were varied. Our plan was to extend the CDM to be able to account for the experimental results.

In terms of empirical results, we found that with concurrent schedules (Experiment 1), sensitivity to the primary reinforcer ratio was greater when the distribution of relative reinforcement across sessions was bimodal than when it was uniform. With concurrent chains, sensitivity to the terminal-link entry ratio (i.e., relative conditioned reinforcement rate) was greater with short- than long-duration terminal links (Experiment 2), but sensitivity was not systematically different when initial links were short or long duration (Experiment 3). When left and right terminal links were of different durations, sensitivity was greater when terminal links were cued (Experiment 4). When left and right terminal links were equal duration and their entry ratio was varied, sensitivity was not systematically different when terminal links were cued or uncued (Experiment 5).

In terms of being able to account for observed preference in Experiment 1, the effect of severe undermatching was unexpected as it is not typically observed in concurrent schedules. To our knowledge, no prior model of choice acquisition can account for differences in preference associated with the distribution of reinforcer ratios.

Using the data from Experiment 1, we developed a simple model for acquisition of choice in concurrent schedules. The model had two assumptions: Reinforcers strengthened responding using a linear-operator rule, and response strength to each alternative decreased with time spent responding. This model provided a good account of the data, with an average VAC of .83 for acquisition data (i.e., session sixths), although estimates of the two main parameters (α and β_{opp}) were not invariant across bimodal and uniform relative reinforcement distributions (i.e., MinVar and MaxVar groups). Although the decision model we developed for Experiment 1 fails to properly explain asymptotic preference while maintaining parameter invariance, it is able to provide a good quantitative description of the results. Moreover, residual analyses indicate little systematic trends. Both of these points indicate that although the decision model is incomplete, it serves as a viable framework to be built upon when developing future molecular models of choice.

In light of this, we proposed an extended CDM (carryover) in which the model from Experiment 1 was used to model the effects of conditioned reinforcement. Versions of this extended model were then applied to data from Experiments 2 through 5. How well does the extended CDM account for the data overall? Table 7.01 shows a summary of the parameters which were estimated for fits to individual data in each experiment, and which parameters were fixed or not applicable, and average VAC for session sixth and asymptotic (i.e., second half of sessions) data. Across experiments, there were 3 parameters estimated with the exception of Experiment 4, uncued condition, where 4 parameters were estimated. Table 7.01 also lists the equations which were used to fit the data to the corresponding model of each experiment. Overall, the model provided an adequate account of the acquisition data, with VAC ranging from .47 (Experiment 5) to .80 (Experiment 4). VAC improved for asymptotic data, and ranged from .65 (Experiment 5) to .87 (Experiment 4). These results suggest that the extended CDM was able to describe the data across experiments with a reasonable degree of accuracy. The model is also parsimonious, with 3 or 4 parameters estimated from 180 data points (i.e., 30 sessions x 6 data points per session).

Table 7.01

Summary of Equations Used as well as Estimated and Fixed Parameters Values by Experiment Number and corresponding VACs

Expt No	Log b	α	β_{o}	β_{opp}	α_{TL}	α_{Cr}	σ	β_{diff}	β_{TLstim}	Equations	VAC (SS)	VAC (Asymp)
Expt 1	✓	✓	✓	✓	N/A	N/A	N/A	N/A	N/A	2.03, 2.04, 2.05	0.83, 0.83	0.92
Expt 2	✓	✓	0	✓	0.20	0.10	0.30	0.02	N/A	2.03, 2.04, 2.05, 3.02	0.58	0.79
Expt 3	✓	✓	0	✓	0.20	0.10	0.30	0.02	N/A	2.03, 2.04, 2.05, 3.02	0.53	0.75
Expt 4	✓	0.02	0	✓	0.20	0.10	✓	0.02	✓	2.03, 2.04, 2.05, 3.02, 5.02	0.80	0.87
Expt 5	✓	✓	0	✓	0.20	0.10	0.30	0.02	N/A	2.03, 2.04, 2.05, 3.02	0.47	0.65

Note. Fixed and Estimated Parameters values as well as mean VAC by experiment number. Model fits for Experiment 1 were based on the last 50 sessions. For Experiments 2 to 5, model fits were based on the last 30 sessions of training. Parameter values which were fixed are listed above for each experiment. Fixed values were pre-determined based on values obtained from previous studies. Tick marks denote best-fit parameter estimates from model-fitting. N/A = not applicable because these parameters were not used in the model fits of certain experiments. Expt No = experiment number. VAC (SS) refers to model fits based on session sixth data. VAC (Asymp) refers to fits of the model using the second half of each session. In Experiment 1, the first value for VAC (SS) refers to model fits using response allocation while the second value is based on time allocation. We used response allocation data for analyses of model fits from Experiment 2 onwards so the value listed in VAC (Asymp) for Experiment 1 is based on response allocation. VAC values range from 0 to 1. 1 being 100% or all variability in data is accounted for. In Experiment 1, Equation 2.03 applies after the delivery of food. In Experiments 2 to 5, equation 2.03 applies only after terminal-link entry.

The purpose of developing a new decision model which extends to concurrent chains was to build on existing models of choice. For example, in Experiment 2, left and right terminal links are always of equal durations meaning DRT isn’t applicable since it is predicts choice based on preference for the shorter terminal link. Moreover it is unable to predict preference exhibited by nonhumans towards VI over FI terminal links when the terminal links are equated (Moore, 1984). The CCM faces the same limitation of being unable to account for variability because it calculates the value of terminal links based on average delay to reinforcement. The HVA has been extended to account for differences in preference based on reinforcer contingencies such as delay, magnitude and signalled versus non-signalled terminal links. However, the HVA and other decision models have not been modelled to predict preference based on terminal-link entry ratio. The CDM can potentially account for terminal-link entry ratio but requires certain additions to the core equation. Therefore, we aimed to extend the CDM by creating the carryover model to account for variations in different reinforcer contingencies based on relative frequency of conditioned reinforcement.

7.2 Monte Carlo Simulations and Parameter Invariance

A core criterion of any decision model is whether it can account for observed preference when different reinforcer contingencies are varied. Specifically, whether the model is able to predict the results between conditions while satisfying parameter invariance. For each experiment, we ran dependent *t*-tests to compare the best-fitting α and β_{opp} values between conditions. There were no significant differences found in terms of the α and β_{opp} parameters for fits to individual data. These parameters determine sensitivity to the terminal-link entry ratio, and so the lack of significant differences suggests that the model can account for the differences in preference observed in Experiments 2-5 while satisfying parameter invariance.

To provide a stronger test of the model's ability to predict the major empirical results across experiments, we conducted a Monte Carlo analysis. These simulations used the same set of parameter values, obtained by averaging those from fits to individual data across conditions and experiments. The parameter values were: $\alpha = 0.029$ and $\beta_{opp} = 0.012$. Every simulation of each experimental condition involved randomizing the number of responses made on each alternative during the initial links as well as time spent in the initial links. For each condition, the simulation consisted of 1000 iterations of 10 sessions of simulated data, with sensitivity computed by aggregating simulated responses over the last 5 sessions.

Figure 7.01 shows the mean sensitivity values for each condition and the obtained results. The extended model made accurate predictions for Experiment 2 (greater sensitivity with short terminal links), Experiment 4 (greater sensitivity with cued terminal links) and Experiment 5 (no difference between cued and uncued conditions), but not for Experiment 3, Simulations predicted stronger preference when initial-links were long but the obtained data indicated approximately equal preference regardless of initial-link duration.

Figure 7.02 shows a scatterplot of obtained versus simulated sensitivity. Across all experiments, there was a significant positive correlation between simulated and obtained sensitivity values, $r(6) = .91$, $p = .002$. Although under prediction is evident, this suggests that with the exception of Experiment 3, the model predicted the ordinal results across experiments with a fixed set of parameter values.

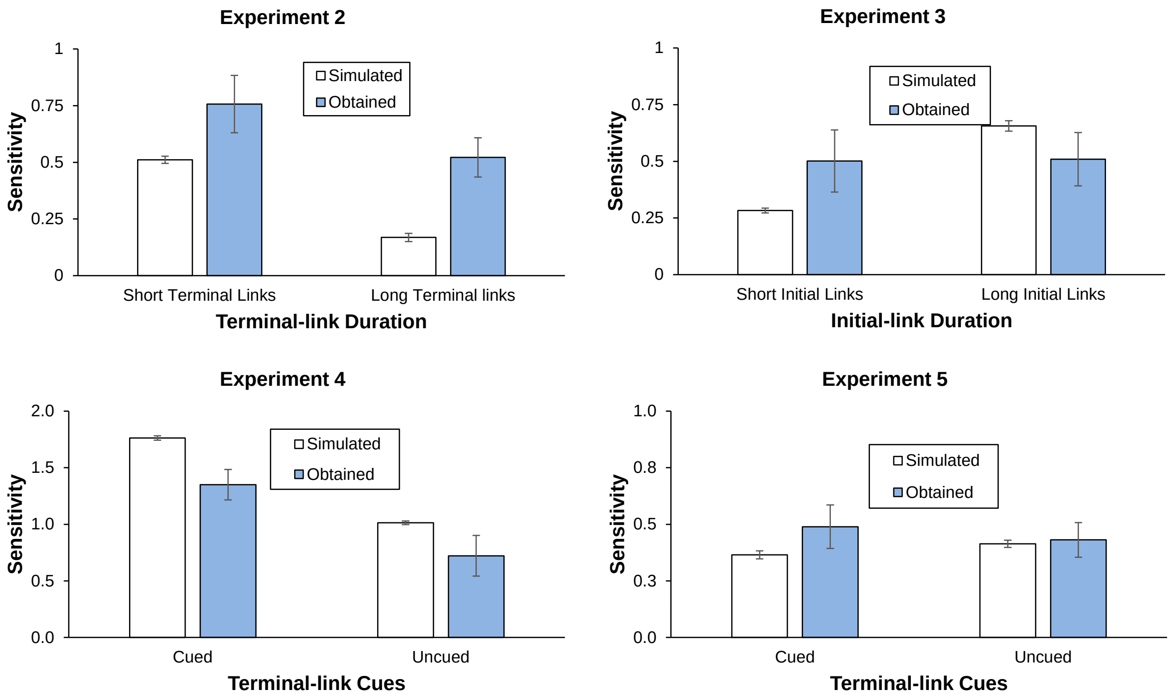


Fig 7.01. Average sensitivity to terminal-link entry ratios for Experiment 2 to Experiment 5 from the obtained data and Monte Carlo simulations. Obtained data was based on the last 30 sessions of training. The same parameter values were used in Monte Carlo simulations in both conditions for each experiment, $\alpha = 0.029$ and $\beta_{opp} = 0.012$. Error bars indicate standard errors.

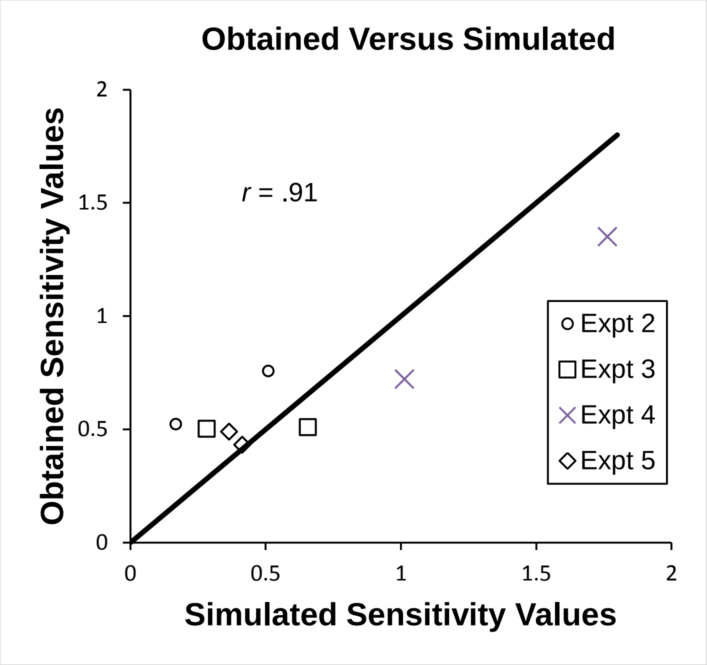


Fig 7.02. Obtained sensitivity plotted against simulated sensitivity for Experiment 2 to Experiment 5. Results were computed using the last 30 sessions of training. The solid line indicates an ideal 1:1 ratio between obtained and simulated.

7.3 The Effects of Reinforcer Contingencies on Bias

Although the carryover model has shown that it can account for asymptotic preference and position preferences (bias) in concurrent schedules and concurrent chains, it is important to examine the effects of reinforcer contingencies on bias. In terms of the GML, the bias term refers to response allocation not accounted for by changes in reinforcer contingencies. Since the allocation of responses is independent of manipulated reinforcer contingencies, the variance in choice associated with changes in bias can be viewed as changes due to factors independent of experimental manipulation. The effect of varying each reinforcer contingency on choice behaviour can therefore be viewed as the sum total of effects from experimental manipulation and factors that are independent of these manipulations. Although we expect a certain level of bias from each pigeon, strong biases potentially indicate a weaker effect of varied reinforcer contingencies on choice behavior, relative to factors which increase the strength of biases (reinforcement of extraneous behaviour). This potentially calls into question results which do not result in significant differences between experimental conditions since the strength of empirical manipulation is too weak to result in these differences and is therefore obscured by extraneous factors.

We observed relatively large values of bias, particularly in Experiments 2, 3 and 5. Because the strength of bias cannot be measured independently of its left/right location, we used absolute values of log *b* which allowed us to aggregate values over pigeons and to compare across experiments. Figure 7.03 shows the average absolute bias for each experiment. In our concurrent chains studies, with the exception of Experiment 4, the strength of mean position preference was quite high. There are certain relationships between reinforcer contingencies and position preference. Figure 7.03 shows the absolute bias values, averaged across pigeons and conditions for Experiments 2-5. Results for the MinVar and MaxVar groups of Experiment 1 are also shown.

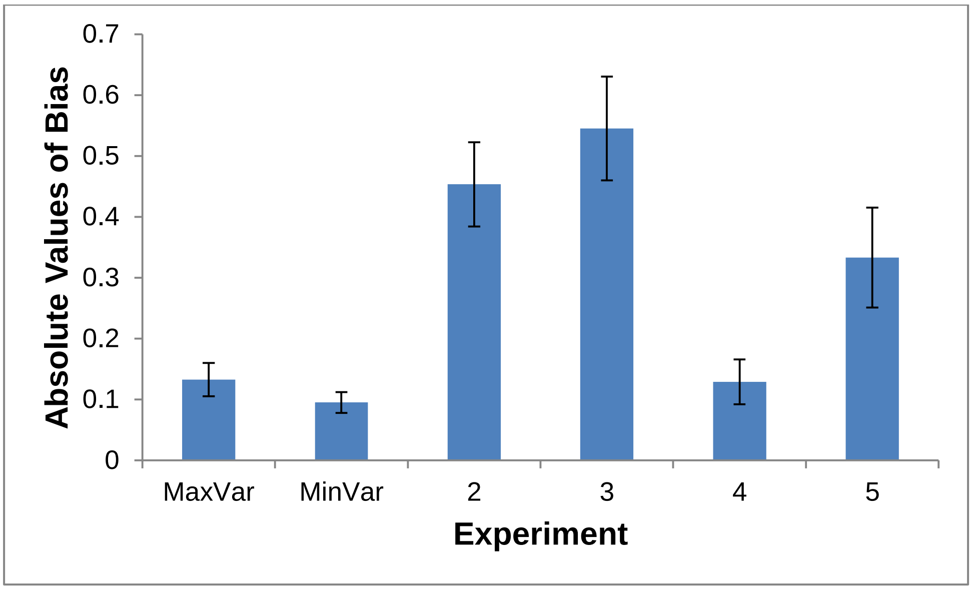


Fig 7.03. Mean strength of bias by experiment. “MaxVar” and “MinVar” data are for Experiment 1. MaxVar and MinVar analyses were conducted using the last 50 sessions of training. Mean Bias for Experiments 2 to 5 were obtained using the last 30 sessions. Error bars show standard errors.

We conducted a correlation analysis between the average absolute preference and the strength of position bias in the concurrent-chains experiments (2 – 5). The average absolute preference was computed in the following manner:

$$\text{avg pref} = 0.5.[Log(\frac{B_L}{B_R})_{\text{richleft}} - \text{Log}(\frac{B_L}{B_R})_{\text{richright}}], \qquad (7.01)$$

where B_L is the number of responses made on the left key, and B_R is the number of responses on the right key. This log response ratio was computed separately for sessions where the left alternative was richer (richleft) and sessions where the right alternative was richer (richright). This was conducted over the last 30 sessions. This allows us to examine the average preference while negating terminal-link entry ratio in each experiment since terminal-link entry ratio was not the same across our concurrent chains experiments. There was a significant negative relation between average absolute preference and bias, $r = -.79$, $p = .019$. A scatterplot is shown in Figure 7.04.

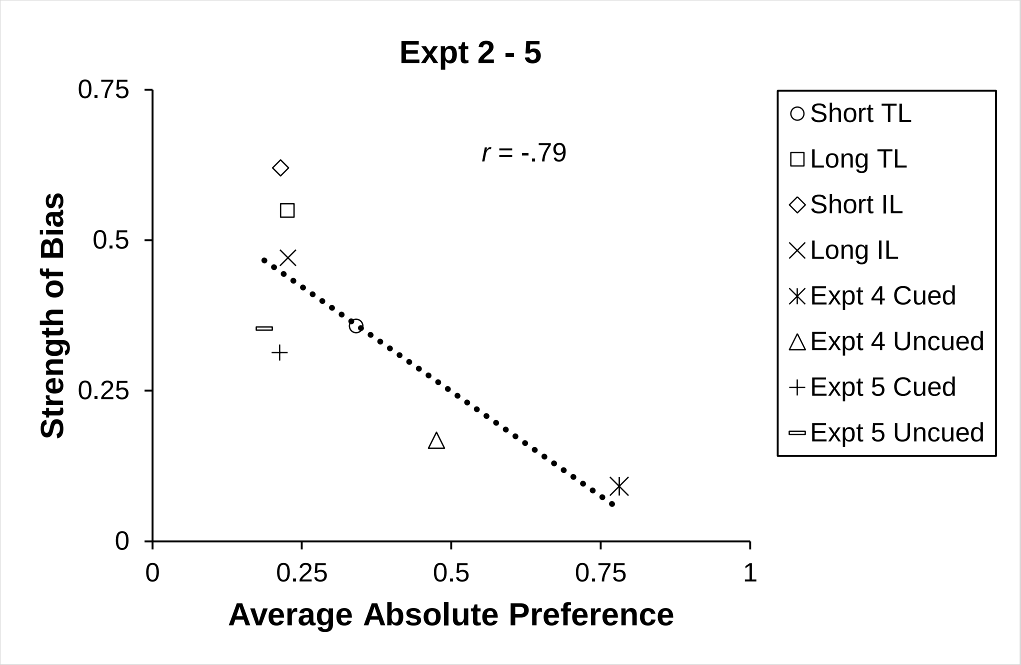


Fig 7.04. Average preference and bias by experiment condition. The dashed line indicates the line of best fit from regressing strength of bias on average preference. Results were computed using the last 30 sessions of training.

Why might there be an inverse relation between the strength of preference and bias? This result was unexpected and because sources of inherent bias have received relatively little attention in the literature compared to determiners of sensitivity, it is worth examining how bias was related to the reinforcer contingencies in our experiments. In order to do this, we isolate the effects of varying each reinforcer contingency on bias for each experiment.

In Experiment 2, terminal-link durations were either short or long. Results suggested that strength of bias was affected by both terminal-link duration and the order in which birds were exposed to short or long terminal links. Bias was strongest when birds were started on long terminal links and had no prior exposure to short terminal links. Bias was weakest during short terminal links if they had prior exposure to long terminal links. If birds were first started on short terminal links, the strength of bias gradually decreased when they were switched to long terminal links. When terminal links were short, strength of bias did not change during each session, regardless of whether they had prior exposure to long terminal links.

In Experiment 3, initial link durations were short or long. Position preference was very strong with both short and long initial links. However, initial-link duration and the order in which birds completed each condition were not associated with changes in position preference. Although position preference was idiosyncratic across pigeons, they were consistent for individual pigeons across conditions. The strength of position preference also did not change between the beginning and the end of each session.

In Experiment 4, the left and right terminal link delays were unequal, and terminal-link stimuli were differential (cued) or non-differential (uncued). Bias was stronger when terminal links were uncued, but was weaker than Experiments 2, 3 and 5.

In Experiment 5, terminal links were cued or uncued, similar to Experiment 4, but delays were equal and the relative terminal-link entry rate was varied. Bias was substantially stronger in pigeons that were started on cued terminal links and weaker in pigeons that were started on uncued terminal links. For pigeons that were started on cued terminal links, the strength of bias increased when they switched to uncued terminal links. For birds that were started on uncued terminal-links, position preference became stronger when they switched to the cued condition. Regardless of which condition birds had prior exposure to, position preference was always stronger in the second condition. Although bias remained relatively unchanged during each each session, position preference gradually weakened over the course of each session during the uncued condition for pigeons that were started on uncued terminal links.

These results suggest that the effects of the experimental manipulation compete with other factors which influence bias. The stronger the effects of the manipulation on preference – such as cueing the terminal-link with the shorter delay in Experiment 4 – the weaker the effect of bias on response allocation. When the manipulation is irrelevant, as in the cued condition of Experiment 5, strong biases can emerge. The strong biases exhibited in Experiments 2, 3 and 5 suggest that manipulation of the relative entry frequency (i.e., conditioned reinforcement rate) of equal-valued terminal links had relatively weaker effects than either when terminal-link values were unequal (Experiment 4) or when relative primary reinforcement rate was varied (Experiment 1).

Regardless of the extent to which biases reduced the effect of experimental manipulation, results of analyses indicate that primary reinforcement in the form of food and conditioned reinforcement in terms of terminal link entries are functionally equivalent. This is specifically in the context of reinforcement in the form of food following each terminal-link entry. Our results are consistent with previous research conducted by Dunn, Williams and Royalty (1987) which support the view that terminal links are associated with primary reinforcement and therefore become conditioned reinforcers. In their study using concurrent chains, they had additional terminal-link entries which were not followed by food. They found the initial link associated with additional non-reinforced terminal links resulted in weaker preference. It is therefore important to review our experiments' effects of reinforcer contingencies on choice behaviour.

7.4 Reinforcement Contingencies that Affect Choice

7.4.1 The effects of terminal-link duration

Experiment 2 showed that sensitivity to the terminal-link entry ratio was greater when the duration of terminal links was short rather than long. Similar to findings in previous research (Hunter et al., 1985; Schofield et al., 1997; Grace et al., 2003), preference increased over the first half of the session and reached an approximate asymptote in the second half of the session. This result was consistent with the Monte Carlo simulations using the extended

CDM (Figure 7.02), confirming that the model predicts the result. The reason is that the criterion in CDM, which is used to judge whether each terminal-link delay is short, is lower with short terminal links. The probability that each terminal-link delay is judged to be short is higher when terminal-link durations are short, relative to the overall history of experienced delays. During short terminal links, changes in RS due to primary reinforcement occur more rapidly. This leads to a greater difference in RS between the richer and poorer alternatives during each session when terminal links are short. This results in approximate matching during short terminal links and undermatching when terminal-link durations are long. Overall, the extension of the CDM serves as a viable model of preference acquisition in concurrent chains by taking into account terminal-link entry ratio as well as the absolute values of terminal-link duration, and is able to account for effects of terminal-link duration while satisfying parameter invariance.

7.4.2 The effects of initial-link duration

Rapid adaptation of response allocation initial-link responding to reinforcement schedules which changed unpredictably across sessions is consistent with obtained data in other rapid-acquisition studies which varied initial-link duration (Kyonka et al., 2009). Over the last 30 sessions, sensitivity of log initial-link response ratios of each session was most sensitive to terminal-link entry ratios of that same session. There was no indication that terminal-link entry ratios of the preceding session influenced initial-link response allocation of each current session.

Results of Experiment 2 found no systematic difference in sensitivity to the terminal-link entry ratio between short and long initial-link conditions. This was not consistent with the Monte Carlo simulations using the extended CDM, which showed that the model predicted greater sensitivity with longer initial links. The model proposes that the criterion which is used to judge whether each terminal-link delay is short is higher during long initial links. The cut-off value of the terminal link which is 0.90 lies to the left of the mean distribution of terminal-link delays. Despite the duration of the terminal links being the same in both conditions, the extended CDM predicts the probability that a terminal-link delay will be judged as “short” is higher during long initial links. During long initial links, changes in RS due to primary reinforcement occur more rapidly. The model predicts that preference should be stronger during long initial links even though this difference between short and long initial links is smaller than that which is predicted when terminal link durations are varied between short or long.

However, the increased sensitivity with longer initial links predicted by the extended CDM has been obtained in previous steady-state studies. Davison (1983) examined the sensitivity to terminal-link entry ratio in concurrent chains schedules of reinforcement. In Experiment 2 of his study, there was always a longer and shorter initial link whereby the duration of the shorter initial link was either 0 s, 30 s or 60 s. Davison found that sensitivity to terminal link entry ratio increased as the duration of the shorter initial link increased (lower in the 0 s condition than the 30 s and 60 s condition). Although this difference was noticeable it was not significant. This result was replicated by Alsop and Davison (1988). They found a similar effect of sensitivity to terminal-link entry ratio increasing with longer initial-link durations. They also found that sensitivity to terminal-link entry ratio reached an asymptote when initial-link durations were longer than 32 seconds.

Why did we fail to find a similar result in Experiment 3? One possibility is that bias and position preferences – which were overall strongest in Experiment 3 – might have been associated with increased variability across pigeons. Although bias is formally independent of the effects of reinforcement contingencies in the generalized matching law (Baum et al., 1974), as Figure 7.04 shows there was a systematic relationship between the strength of preference and bias across our experiments. If our suggestion that extraneous influences on preference compete with reinforcement contingencies and appear as bias, then these influences would likely have been greatest in Experiment 3. The other possibility is that the number of training sessions might have been insufficient; results in Figure 4.01 suggest that sensitivity was decreasing over the last 12 sessions for the short initial link condition. If this trend continued with additional training, the results would be consistent with the difference in preference predicted by the extended CDM.

7.4.3 Effects of signalling terminal links and terminal-link duration

The relationship between conditioned reinforcement and choice was visibly affected by two factors; different durations of left and right terminal links as well as whether terminal links signalled which initial-link alternative resulted in food. Subjects consistently allocated the majority of their initial-link responses to the alternative associated with the shorter terminal-link delay. Preference during terminal-links which did not signal the richer initial-link alternative resulted in severe under matching at the beginning of training and reached matching by the end of training. Signalled terminal links resulted in approximate matching at the beginning of training and reached severe over matching by the end of matching. This strongly suggests that sensitivity to relative terminal-link immediacy ratio is strongly influenced by whether terminal links signalled which initial-link alternative resulted in more frequent terminal-link entry.

Between-session preference acquisition for the shorter-delayed terminal link occurred from the beginning of each condition, regardless of whether terminal links signalled the richer initial-link alternative. Exhibited preference increased at roughly the same rate across consecutive 10-session blocks to stabilize during the last 30 sessions of training. The difference in preference between the beginning and the end of training when terminal links were signalled was approximately equal to the difference observed in non-signalled terminal links. The main difference being that exhibited preference at the beginning of training for signalled terminal links was substantially higher than non-signalled terminal links. This indicates that the most rapid increase in preference occurred during the first 10 sessions of training. Consecutive sessions of training after the first 10 sessions resulted in more incremental changes in preference. This result is consistent with previous studies which showed that preference for the richer alternative changes rapidly with rapidly changing terminal-link delays (Kyonka et al., 2007).

During the last 30 sessions of training, within-session asymptotic preference and rates of preference acquisition varied substantially depending on whether terminal links signalled which initial-link alternative resulted in terminal-link entry. At the beginning of each session, signalled terminal –links resulted in approximate matching while non-signalled terminal links resulted in near indifference. Signalled terminal links resulted in more rapid preference acquisition with asymptotic preference occurring within the first half of each session (severe over matching). Non-signalled terminal links resulted in more gradual preference acquisition with asymptotic preference being reached only during the second half of each session (matching).

The extended CDM explains this difference in preference acquisition in terms of the relative value of the most recently occurring conditioned reinforcer in the uncued condition. The value of the most recently experienced terminal-link relative to the preceding terminal links of each session is denoted by the parameter β_{TLstim}. The best-fit value is noticeably far from 1 (0.57) indicating terminal-link entries preceding the most recent terminal-link entry still influence the response allocation during each new cycle. This indicates that the value of terminal-link stimuli substantially differs depending on whether they are cued with cued terminal links being attributed a higher value. Therefore, changes in RS toward the shorter terminal link are more incremental when terminal links are uncued. This difference in value of the conditioned reinforcer results in overall weaker preference compared to when conditioned reinforcement takes the form of cued terminal-link stimuli.

Exhibited preference in Experiment 4 was consistent with Monte Carlo simulations of the predictions of the CDM. The model proposes that the criterion which is used to judge whether each terminal-link delay is short is more variable in the uncued condition. Specifically, there are separate criteria for the left and right terminal links and this increases the difficulty of discriminating between richer and poorer alternatives. This results in the probability of the shorter terminal link being judged as short being smaller when terminal links do not signal terminal-link duration. The more incremental changes in RS for each initial-link alternative result in a slower process of favouring the shorter terminal links during each cycle, The difference in RS between the left and right alternatives during the initial links is smaller resulting in a smaller difference in response allocation between the initial-link alternatives resulting in lower sensitivity to changing terminal-link durations across sessions. This also explains why sensitivity to terminal-link immediacy ratio of the preceding session during the beginning of each session is higher in the uncued condition.

These results of signalled versus non-signalled terminal-link stimuli are consistent with previous studies varying the amount of information conveyed by terminal links. Williams et al. (1978) arranged different left and right terminal-link durations. In one condition, the terminal links were both lighted white and were therefore similar to our uncued condition. The other condition had a blue side key during the terminal links being associated with the shorter terminal link while the other key (lighted white) was associated with the longer terminal link. They found that preference for the shorter terminal link was substantially weaker in the uncued condition.

By extending the CDM to take into account the effects of signalling during the terminal links, we were able to develop a model which provides a good account of preference acquisition in concurrent chains when the amount of information provided by the terminal links is varied. This information varies directly with whether the terminal links are correlated with the corresponding delays. Although predictions of exhibited response allocation are more accurate when terminal links convey more information (cued), this difference was ultimately small. Similar to Experiment 2, the higher VAC of the model's predictions for cued terminal links can be associated with the higher mean sensitivity which results in greater variability in the data. There is also little evidence that the model over-predicts or under-predicts preference as shown by the intercept of the best-fit regression line being zero.

This modified version of the CDM satisfies parameter invariance as evidenced by no significant differences between cued and uncued conditions in terms of the α and β_{opp} parameters. The best fit parameter values strongly suggested certain persistent trends which were previously observed in Experiment 2 and Experiment 3. There was no loss in RS for that alternative when responding on a particular alternative during the initial links resulted in reinforcement. Entry into each terminal-link results in an increase in RS on the initial-link alternative resulting in terminal-link entry.

This version of the extended CDM has shown that it can account for the effects of terminal-link immediacy ratio on choice. Moreover, the model is equally parsimonious with the version of the model used in Experiment 2 and Experiment 3 as the numbers of parameters which are estimated remain the same. One main difference in terms of the effects of conditioned and primary reinforcement on choice is that the increase in RS from the presentation of food remains equal across both cued and uncued conditions. The increase in RS from primary reinforcement also remains constant across conditions. Therefore, preference is primarily influenced by the subjective value of the most recently occurring terminal-link delay.

As previously illustrated, the extended CDM can predict the phenomenon of severe undermatching in concurrent chains and by extension, the ability to simulate asymptotic preference by also taking into account the effects of terminal-link entry on choice. By using the simulated log response ratios of the extended CDM as a precursor to predictions of the GML this allows the GML to predict the phenomenon of severe overmatching. This is important as this overcomes the major limitation of the GML being unable to account for severe overmatching which was observed in Experiment 4.

7.4.4 Signalling and sensitivity to the terminal-link entry ratio

Log initial-link response ratios of each session showed a noticeable trend in sensitivity to terminal-link entry ratio of the preceding session. Once preference had stabilized, sensitivity of log initial-link response ratios of each session was most sensitive to terminal-link entry ratios of that same session. However, sensitivity to terminal-link entry ratio of the preceding session was approximately equal to terminal-link entry ratio at the beginning of the current session. Moreover, sensitivity to terminal-link entry ratios of the preceding session decreased towards indifference more gradually, compared to Experiment 1 to Experiment 4. This indicates a weakening but noticeable influence of preceding-session terminal-link entry ratios on response allocation during each current session.

Based on the order of exposure to terminal-link stimuli not correlated with terminal-link entry, signalling effects have a discernible effect on preference. Prior exposure to terminal-link stimuli not correlated with terminal-link entry resulted in asymptotic preference reaching approximate matching. Prior exposure to terminal-link stimuli correlated with terminal-link entry resulted in asymptotic preferences of severe undermatching. One plausible explanation is that the signalling of terminal-link stimuli serves to confuse the pigeons in terms of the amount of time which has passed between consecutive presentations of primary reinforcement. This is because the average amount of time between consecutive presentations of food is approximately equal across training sessions. This indicates a molar component of the signalling effect of terminal-link stimuli which serves to weaken preference.

This result was consistent with the Monte Carlo simulations of preference using the extended CDM. The model proposes that preference would be similar between cued and uncued terminal links. We did not include the β_{TLstim} parameter because the terminal link delays remained the same in cued and uncued conditions. Therefore, regardless of which alternative resulted in terminal-link entry, comparisons are made between identical durations when determining whether each terminal-link delay was judged to be long or short.

There are certain differences in terms of the effects of conditioned and primary reinforcement on choice. Specifically, regardless of whether terminal links signal terminal-link entry, the effect of primary reinforcement on RS remains relatively unchanged while the effects of conditioned reinforcement are more variable. Specifically, the information conveyed by terminal-link stimuli in Experiment 5 is redundant. This is because the average amount of time which passes between consecutive reinforcers, primary and conditioned remains unchanged. In terms of whether each terminal link is judged to be long or short, entry into either terminal link always results in a equal delay before each presentation of food. From the perspective of terminal-link duration, the signalling of terminal-link stimuli conveys no new information.

This result is consistent with previous research which examined the effects of cued and uncued terminal-link stimuli on choice. Spetch et al. (1990) found that the effects of terminal links signalling the richer alternative occurred when terminal link durations were varied across conditions. Since the effects of signalling terminal-link stimuli are correlated with terminal-link duration, this effect of signalling not occurring in Experiment 5 is consistent with their study. Since the terminal-link and initial-link durations also remain the same across cued and uncued conditions. This means that there is also no possible effect of initial-link duration on choice.

Similar levels of preference can be explained in terms of the type of information conveyed by signalled terminal-link stimuli. When terminal-link stimuli signal information that is associated with changes in terminal-link duration, preference is strengthened. This explanation has been supported by results from previous research (Colton & Moore, 1997). When the signalling of terminal-links is associated only with terminal-link entry ratio, the information conveyed by signalling terminal-link stimuli does not appear to strengthen preference. Coupled with the results of Experiment 4, this suggests that the signalling effect of terminal-link stimuli is contingent on initial-link and terminal-link duration but not necessarily on terminal-link entry ratio.

Our results from Experiment 5 were contrary to those of Alsop et al. (1994). In their Experiment 1, left and right initial links were concurrent VI 25s VI 100s or concurrent VI 100s VI 25s. They had cued and uncued conditions for terminal-link stimuli similar to Experiment 5, but used a steady-state design. They found that sensitivity to the shorter initial link was higher in the uncued condition. Although reasons for the discrepancy in results are unclear, the extended CDM predicted little difference between the cued and uncued conditions (Figure 7.01). Also, we observed order effects: Sensitivity was greater when pigeons were first exposed to the uncued condition. If our pigeons had been repeatedly exposed to the cued and uncued conditions (as in Alsop et al., 1994), it is possible that increased sensitivity with uncued terminal links might have emerged.

7.5 Concluding Points for Interpretation of Results

7.6 What Results in Matching

The equation for the GML presented in Equation (1.2) was translated from the equivalent of its power function as it provides a better description of experimental data than Equation (1.1). Heavy support for this has been conducted by Myers and Myers (1977), Staddon (1972), and Baum (1979). The parameter for sensitivity, *s*, denotes sensitivity to the exponent of reinforcer ratio. The sensitivity value of 0.5 (severe undermatching) means that subjects “matched” response allocation to the square root of reinforcer ratio. This result is particularly important in Experiment 1 as this is a result rarely observed in concurrent schedules of choice.

The current study provides a detailed account of what causes matching of initial-link response allocation to conditioned reinforcement in the form of terminal-link entry ratio in terms of initial-link and terminal-link duration. Preference for the more probabilistically occurring terminal link approaches matching with shorter terminal-link durations but remains unaffected by initial-link duration. This result is supported by a study by Ito and Fantino (1986) who found that as equal terminal-link durations increased, preference approaches matching.

Preference for the terminal link assigned a higher probability of occurring was not affected by signalling effects of terminal-link stimuli when initial-link and terminal-link durations did not change, reaching only severe undermatching. When left and right terminal links occur with equal probability and discrimination for the richer terminal-link is based on differing left and right terminal-link durations, preference for the shorter terminal-link approximates matching for the non-signalled terminal links and reaches severe overmatching in the signalled condition. When discrimination for the richer alternative is based on the shorter terminal-link, preference approximates matching as supported by previous research (Omino, 1993).

One unexpected result which has been demonstrated across our experiments is effect of bias on choice. This is shown by strong position preferences which varied between our experiments which used concurrent chains. This indicates that the amount of extraneous behaviour was different even when the same subjects were used in each experiment. This implies that reinforcement for extraneous behaviour was uneven depending on manipulated experimental contingencies of terminal-link entry ratio, terminal-link duration, initial-link duration as well as whether terminal links signalled the richer alternative. Previous analyses support this view by questioning whether the rate of background reinforcement can remain constant because the reinforcement rate of the desired behaviour is changing (Baum, 1981; Dallery & Soto, 2004; Davison, 1993; Pear, 1975; Soto, McDowell & Dallery, 2005). This is an important issue as it potentially affects response allocation for the target behaviour and therefore, whether matching occurs.

7.7 Limitations and Future Research

Nevin et al. (1984) stated that a major goal of model fitting research was that it hopefully resulted in applications that extend beyond the scope of a model-fitting exercise. In answer to this, the GML has been used to fit the obtained data from numerous real-world behaviours. This is especially important as there is ample evidence of the occurrence of this phenomenon in studies which use the GML to predict behaviour in sports such as shot selection in basketball and play calling in American football (e.g., Vollmer & Bourret, 2000; Reed, Critchfield & Martens, 2006). The GML provides an accurate prediction of the measures of shot selection and play calling in those studies. The model which we have developed achieves this goal of predicting asymptotic preference resulting in matching and severe undermatching.

However, there are certain phenomena which our decision model cannot account for. For example, Alsop and Davison (1992) conducted a key-switching procedure where probability of terminal-link entries on each alternative was 0.50. They found that discriminability towards the richer alternative based on relative obtained reinforcer ratio was approximately equal to other conditions where probability of reinforcement from each alternative was unequal (1:8, 1:4, 4:1 and 8:1). Our decision model would predict indifference when the reinforcer from consecutive trials is rapidly switching during the schedule when left versus right reinforcer ratio is 1:1. Moreover, pigeons respond almost exclusively on the richer alternative but occasionally respond on the other alternative. This is explained by the “fix and sample” theory proposed by Aparicio and Baum (2006) which states that responding is consistently made on the richer alternative and therefore “fixed” while occasional responding occurs on the other alternative and is therefore “sampled” in terms of overall response allocation.

The extended CDM’s predictions of asymptotic preference provide a good account of exhibited preference. This applies to concurrent schedules and concurrent chains when run under rapid acquisition conditions. In terms of asymptotic responding, plots of the model’s simulated log response ratios plotted against obtained log ratios are shown in appendix E. This indicates that the extended CDM serves as a viable framework for developing a predictive model of choice for steady-state and rapid acquisition research. Although this is true for concurrent chains, further research is required to extend it to concurrent schedules.

In terms of research using chained schedules, there have been few studies on preference to terminal-link entry as a measure of choice. The MinVar and MaxVar reinforcer schedules in Experiment 1 should be replicated using concurrent chains to determine the effects of probabilistic reinforcement in concurrent chains. This would allow further investigation of whether probabilistic primary reinforcement and probabilistic conditioned reinforcement have similar effects on choice. Moreover, the signalling effects should be added to a replication of Experiment 2 and Experiment 3 to determine if signalling terminal-link stimuli correlated with terminal-link entry affect preference is contingent on initial-link and terminal-link durations when probabilistic conditioned reinforcement varies rapidly.

Since position preferences vary, extraneous reinforcement needs to be taken into account. The quantitative law of effect denotes reinforcement of extraneous behaviour by parameter r_e in Equation (1.03). Because r_e is interpreted as the obtained amount of background reinforcement, direct tests of this parameter through experimentation are not possible because r_e cannot be measured unambiguously. Therefore, r_e has been assumed to be constant. Equation (1.03) has normally been used in single-alternative studies and has been modified to include multi-alternative environments but that presents a problem. This is because r_e is likely to covary with r (McDowell, 2013).

Log c of the GML describes extraneous behaviour and is therefore directly affected by variations in r_e. As previous research questions whether the amount of extraneous reinforcement remains constant, this implies that extraneous behaviour also varies as evidenced by differing strengths of position preference in our study. It is important that future models of choice account for this phenomenon. The extended CDM could be modified to incorporate precursors to RS by adding another parameter but applied to Log c. Since reinforcement contingencies affect the strength of position preference, best-fit values for this parameter should be obtained for different experimental conditions. Doing this would also allow for a direct investigation of reinforcement contingencies which affect extraneous behaviour, something which has not been examined in detail.

References

Alsop, A., & Davison, M. (1988). Concurrent-chain performance: Effects of absolute and relative terminal-link entry frequency. *Journal of the Experimental Analysis of Behavior, 49*(3), 351-365. doi: https://doi.org/10.1901/jeab.1988.49-351

21-36.

Alsop, A., & Davison, M. (1992). Discriminability between alternatives in a switching-key concurrent schedule. *Journal of the Experimental Analysis of Behavior,*57(1), 51-65. doi: https://doi.org/10.1901/jeab.1992.57-51

Alsop, B., & Elliffe, D. (1988). Concurrent-schedule performance effects of relative and overall reinforcer rate. *Journal of the Experimental Analysis of Behavior, 49*(1), 21-36, doi: https://doi.org/10.1901/jeab.1988.49-21

Alsop, A., Stewart, K. E., & Honig, W. K. (1994). Cued and uncued terminal links in concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior*, *62*(3), 385-397. doi: https://doi.org/10.1901/jeab.1994.62-385

Aparacio, C. F., & Baum, W. M. (2006). Fix and sample with rats in the dynamics of choice. *Journal of the Experimental Analysis of Behavior,*86(1), 43-63. doi:10.1901/jeab.2006.57-05

Autor, S.M. (1960). The strength of conditioned reinforcers as a function of frequency and probability of reinforcement. [Unpublished doctoral dissertation], Harvard University.

Bailey, J. T., & Mazur, J. E. (1990). Choice behaviour in transition: Development of preference in a free-operant procedure. *Journal of the Experimental Analysis of Behavior*, *53*(3), 409–422.

Baum, W.M. (1974). On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior, 22(*1), 231-242. doi:10.1901/jeab.1974.22-231

Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. *Journal of the Experimental Analysis of Behavior, 32*, 269–281. doi:10.1901/jeab.1979.32-269

Baum, W. M. (1981). Optimization and the matching law as accounts of instrumental behavior. *Journal of the Experimental Analysis of Behavior, 36*(3), 387–403. doi:10.1901/jeab.1981.36-387

Baum, W. M. (2003). The molar view of behaviour and its usefulness in behaviour analysis. *Behaviour Analyst Today*, *4*(1), 78-81.

Baum, W. M., (2010). Dynamics of choice: A tutorial. *Journal of the Experimental Analysis of Behavior, 94*(2), 161-174.

Baum, W. M., & Rachlin, H. C. (1969). Choice as time allocation. *Journal of the Experimental Analysis of Behavior, 12*(6), 861-874. doi: 10.1901/jeab.1969.12-861

Berg, M. E., & Grace, R. C. (2006). Initial-link duration and acquisition of preference in concurrent chains. *Learning and Behavior, 34*(1), 50-60. doi:https://doi.org/10.3758/BF03192871

Caraco, T., Martindle, S., & Whitham, T. S. (1980). An empirical demonstration risk-sensitive- foraging preferences. *Animal Behavior*, *28*(3), 820-830. doi:10.1016/S00033472(80)80142-4

Christensen, D. R., & Grace, R. C. (2008). Rapid acquisition in concurrent chains: Effects of initial-link duration. *Behavioural Processes*, *78*(2), 217–223. doi:10.1016/j.beproc.2008.01.006

Christensen, D. R., & Grace, R. C. (2009a). Response allocation in a rapid-acquisition concurrent-chains procedure: Effects of overall terminal-link duration. *Behavioural Processes*, *81*(2), 233-237. doi:10.1016/j.beproc.2009.01.006

Christensen, D. R., & Grace, R. C. (2009b). Response allocation in concurrent chains when terminal-link delays follow an ascending and descending series. *Journal of the Experimental Analysis of Behavior*, *91*(1), 1-20. doi:10.1901/jeab.2009.91-1

Christensen, D. R., & Grace, R. C. (2010). A decision model for steady-state choice in concurrent chains. *Journal of the Experimental Analysis of Behavior*, *94*(2), 227-240. doi:10.1901/jeab.2010.94-227

Chung, S., & Herrnstein, R. J. (1967). Choice and delay of reinforcement. *Journal of the Experimental Analysis of Behavior*, *10*(1), 67-74.

Colton, L., & Moore, J. (2017). The effects of terminal-link stimulus arrangements on preference in concurrent chains. *The Psychological Record*, *47*(1), 145-166. doi:

Dallery, J., & Soto, P. L. (2004). Herrnstein’s hyperbolic matching equation and behavioral pharmacology: Review and critique. *Behavioural Pharmacology*, *15*(7), 443–459. doi: 10.1097/00008877-200411000-0000110.1007/BF03395217

Davis, D. G., Staddon, J. E., Machado, A., & Palmer, R. G. (1993). The process of recurrent choice. *Psychological Review*, *100*(2), 320–341. https://doi.org/10.1037/0033295X.100.2.320

Davison, M. (1983). Bias and sensitivity to reinforcement in a concurrent-chain schedule. *Journal of the Experimental Analysis of Behavior*, *40*(1), 15-34. doi:10.1901/jeab.1983.40-15

Davison, M. (1993). On the dynamics of behavior allocation between simultaneously and successively available reinforcer sources. *Behavioural Processes*, *29*(1-2), 49–63. doi:10.1016/0376-6357(93)90027-O

Davison, M., & Baum, W. M. (2000). Choice in a variable environment: Every reinforcer counts. *Journal of the Experimental Analysis of Behavior*, *74*(1), 1-24. doi: 10.1901/jeab.2000.74-1

Davison, M., & Hunter, I. (1976). Performance on variable-interval schedules arranged singly and concurrently. *Journal of the Experimental Analysis of Behavior, 25*(3), 335-345. doi:https://doi.org/10.1901/jeab.1976.25-335

Davison, M., & Jenkins, P. E. (1985). Stimulus discriminability, contingency discriminability and schedule performance. *Animal Learning and Behavior*, 13(1), 77-84. doi:https://doi.org/10.3758/BF03213368

Davison, M., & McCarthy, D. (1988). *The matching law: A research review*. Hillsdale, NJ: Erlbaum.

Dunn, R., & Spetch, M. L. (1990). Choice with uncertain outcomes. *Journal of the Experimental Analysis of Behavior, 53*(2), 201-218. doi:10.1901/jeab.1990.53-201

Dunn, R., Williams, B., & Royalty, P. (1987). Devaluation of stimuli contingent on choice: Evidence for conditioned reinforcement. *Journal of the Experimental Analysis of Behavior, 48*(1), 117-131. doi: https://doi.org/10.1901/jeab.1987.48-117

Fantino, E. (1969). Choice and rate of reinforcement. *Journal of the Experimental Analysis of Behavior, 12*(5), 723-730.

Findley, J. D. (1958). Preference and Switching under Concurrent Scheduling. *Journal of the Experimental Analysis of Behavior, 1*(2), 123-144. doi:10.1901/jeab.1958.1-123

Fleshler, M., & Hoffman, H. S. (1962). A progression for generating variable-interval schedules. *Journal of the Experimental Analysis of Behavior, 5*(4), 529-530. doi: 10.1901/jeab.1962.5-529

Grace, R. C. (1994). A contextual model of concurrent-chains choice. *Journal of the Experimental Analysis of Behavior, 61*(1), 113-129.

Grace, R. C. (2002a). Acquisition of preference in concurrent chains: Comparing linear-operator and memory-representational models. *Journal of Experimental Psychology*: *Animal Behavior Processes, 28*(3), 257-276.

Grace, R. C. (2004). Temporal context in concurrent chains: I. Terminal-link duration. *Journal of the Experimental Analysis of Behavior*, *81*(3), 215-237.

Grace, R. C. (2016). Acquisition of choice in concurrent chains: Assessing the cumulative decision model. *Behavioural Processes*, *127*, 74-85. doi:https://doi.org/10/1016/j.beproc. 2016.03.011

Grace, R. C., Bragason, O., & McLean (2003). Rapid acquisition of preference in concurrent chains. *Journal of the Experimental Analysis of Behavior, 80*(2), 235-252. doi:10.1901/.jeab.2003.80-235

Grace, R. C., & Hucks, A. D. (2012). The allocation of operant behaviour. In G. J. Madden (Ed.) *APA Handbook of Behavior Analysis: Methods and Principles*, (pp. 307-337) Washington, DC: American Psychological Association.

Grace, R. C., & McLean, A. P. (2006). Rapid acquisition in concurrent chains: Evidence for a decision model. *Journal of the Experimental Analysis of Behavior, 85*(2), 181-202.

Grace, R. C., & McLean, A. P. (2015). Evidence for a magnitude effect in probability discounting with pigeons. *Journal of Experimental Psychology: Animal Learning and Cognition, 41*(4), 406.

Grace, R. C., Sargisson, R. J., & White, K. G. (2012). Evidence for a magnitude effect in temporal discounting with pigeons. *Journal of Experimental Psychology*: *Animal Behaviour Processes, 45*(1), 115-127.

Grace, R. C., & Savastano, H. I. (2000). Temporal context and conditioned reinforcement value. *Journal of Experimental Psychology: General, 129*(4), 427-443.

Green, L. (1980). Preference as a function of the correlation between stimuli and Reinforcement outcomes. *Learning and Motivation, 11*(2), 238-255. doi:https://doi.or g /10.1016/0023-9690(80)90015-6

Herrnstein, R. J. (1964). Aperiodicity as a factor in choice. *Journal of the Experimental Analysis of Behavior, 7*(2), 179-182. doi:https://doi.org/10.1901/jeab. 1964.7-179

Herrnstein, R. J. (1964). Secondary reinforcement and rate of primary reinforcement. *Journal of the Experimental Analysis of Behavior, 7*(1), 27-36. doi:https://doi.org/10.1901/jeab. 1964.7-27

Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior, 13*(2), 243-266. doi:10.1901/jeab.1970.13-243

Heyman, G. M., & Monaghan, M. M. (1987). Effects of changes in response requirement and deprivation on the parameters of the matching law equation: New data and review. *Journal of Experimental Psychology, 13*(4), 384-394.

Heyman, G. M., & Monaghan, M. M. (1994). Reinforcer magnitude (sucrose concentration) and the matching law theory of response strength. *Journal of the Experimental Analysis of Behavior, 61*(3), 505-516.

Horney, J., & Fantino, E. (1984). Choice for conditioned reinforcers in the signalled absence of primary reinforcement. *Journal of the Experimental Analysis of Behavior, 41*(2), 193-201. doi:10.1901/jeab.1984.41-193

Hunter, I., & Davison, M. (1985). Determination of a behavioural transfer function: White-noise analysis of session-to-session response-ratio dynamics on concurrent VI VI schedules. *Journal of the Experimental Analysis of Behavior, 43*(1), 43-59. doi:10. 1901/jeab.1985.43-43

Ito, M., & Fantino, E. (1986). Choice, foraging, and reinforcer duration. *Journal of the Experimental Analysis of Behavior, 46*(1), 93-103. doi: 10.1901/jeab.1986.46-93

Kendall, S. B. (1974). Preference for intermittent reinforcement. *Journal of the Experimental Analysis of Behavior, 21*(3), 463-473. doi: 10.1901/jeab.1974.21-463

Killeen, P. (1981). Averaging theory. In C. Bradshaw, E. Szabadi, & C. Lowe (Eds.), *Quantification of steady state operant behaviour* (pp. 21–34). North Holland, Amsterdam: Elsevier.

Kyonka, E. G. E., & Grace, R.C. (2007). Rapid acquisition of choice and timing in pigeons. *Journal of Experimental Psychology: Animal Behaviour Processes*, *33*(4), 392-408. doi: https://doi.org/10.1037/0097-7403.33.4.392

Kyonka, E. G. E., & Grace, R.C. (2008). Rapid acquisition of preference in concurrent chains when alternatives differ on multiple dimensions of reinforcement. *Journal of Experimental Analysis of Behaviour*, 89(1), 49-69.

Kyonka, E. G. E., & Grace, R. C. (2009). Effects of unpredictable changes in initial-link duration on choice and timing. *Behavioural Processes*, *81*(2), 227-232. doi:10.1016/j. beproc.2008.12.024

Kyonka, E. G. E., & Grace, R. C. (2010). Rapid acquisition of choice and timing and the provenance of the terminal-link effect. *Journal of the Experimental Analysis of Behavior*, *94*(2), 209-225. doi:10.1901/jeab.2010.94-209

Landon, J., Davison, M., & Elliffe, D. (2002). Concurrent schedules: Short- and long-term effects of reinforcers. *Journal of the Experimental Analysis of Behavior*, *77*(3), 257-271. doi: https://doi.org/10.1901/jeab.2002.77-257

Luco, J. E. (1990). Matching, delay-reduction, and maximizing models for choice in concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior*, *54*(1), 53-67. doi:10.1901/jeab.1990.54-53

MacEwen, D. (1972). The effects of terminal-link fixed-interval and variable-interval schedules on responding under concurrent chained schedules. *Journal of the Experimental Analysis of Behavior, 18*(2), 253-261.

Maguire, D. R., Hughes, C. E., & Pitts, R. C. (2007). Rapid acquisition of preference in concurrent schedules: Effects of reinforcement amount. *Behavioural Processes, 75*(2), 213-219. doi:10.1016/j.beproc.2007.02.019

Mazur, J. E. (1984). Tests of an equivalence rule for fixed and variable reinforcer delays. *Journal of Experimental Psychology: Animal Behaviour Processes, 10*(4), 426-436. doi: 10.1037/0097-7403.10.4.426

Mazur, J. E. (1992). Choice behaviour in transition: Development of preference with ratio and interval schedules. *Journal of Experimental Psychology: Animal Behaviour Processes, 18*(4), 364-378.

Mazur, J. E. (1997). Effects of rate of reinforcement and rate of change on choice behaviour in transition. *Quarterly Journal of Experimental Psychology, 50*(2), 111–128. doi:10. 1080/713932646

Mazur, J. E. (2001). Hyperbolic value addition and general models of animal choice. *Psychological Review*, *108*(1), 96-112. doi: 10.1037/0033-295X.108.1.96

Mazur, J. E. (2002a). Concurrent-chain performance in transition: Effects of terminal-link duration and individual reinforcers. *Animal Learning & Behaviour, 30*(3), 249-260.

Mazur, J. E., Blake, N., & McManus, C. (2001). Transitional choice behaviour in concurrent-chain schedules. *Behavioural Processes, 53*(3), 171-180.

Mazur, J. E., & Ratti, T. A. (1991). Choice behaviour in transition: Development of preference in a free-operant procedure. *Animal Learning & Behaviour*, *19*(3), 241–248.

McDevitt, M., Spetch, M., & Dunn, R. (1997). Contiguity and conditioned reinforcement in probabilistic choice. *Journal of the Experimental Analysis of Behavior, 68*(3), 317-327. doi:10.1901./jeab.1997.68-317

McDowell, J. J. (2013). On the theoretical and empirical status of the matching law and matching theory. *Psychological Bulletin*, *139*(5), 1000–1028. doi:https://doi.org/10.1037/a0029924

Moore, J. (1984). Choice and transformed interreinforcement intervals. *Journal of the Experimental Analysis of Behavior*, *42*(2), 321-335. doi: https://doi.org/10.1901/jeab.1984.42-321

Myers, D. L., & Myers, L. E. (1977). Undermatching: A reappraisal of performance on concurrent variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 27(1), 203–214. doi:10.1901/jeab.1977.27-203

Navarick, D. J., & Fantino, E. (1972). Transitivity as a property of choice. *Journal of the Experimental Analysis of Behavior*, *18*(3), 389-401. doi:https://doi,org/10.1901/jeab.1972.18-389

Nevin, J. A. (1984). Quantitative analysis. *Journal of the Experimental Analysis of Behavior*, *42*(3), 421-434.

Omino, T. (1993). A quantitative analysis of sensitivity to the conditioned reinforcing value of terminal-link stimuli in a concurrent-chains schedule. *Journal of the Experimental Analysis of Behavior*, *60*(3), 587-594. doi: https://doi.org/10.1901/jeab.1993.60-587

Pear, J. J. (1975). Implications of the matching law for ratio responding. *Journal of the Experimental Analysis of Behavior*, *23*(1), 139–140. doi:10.1901/jeab.1975.23-139

Reed, D. D., Critchfield, T. S., & Martens, B. K. (2006). The Generalized matching law in elite sport competition: Football play calling as operant choice. *Journal of Applied Behaviour Analysis*, *39*(3), 281-297. doi: 10.1901/jaba.2006.146-05

Schofield, G., & Davison, M. (1997). Non-stable concurrent choice in pigeons. *Journal* *of the Experimental Analysis of Behavior, 68*(2), 219-232. doi: 10.1901/ jeab.1997.68-219

Soto, P. L., McDowell, J. J., & Dallery, J. (2005). Effects of adding a second reinforcement alternative: Implications for Herrnstein's interpretation of r_e. Journal of the Experimental Analysis of Behavior, 84(2),185–225. doi:10.1901/jeab.2005.09-05

Spetch, M. L., Belke, T. W., Barnet, R. C., Dunn, R., & Pierce, W. D. (1990). Suboptimal choice in a percentage-reinforcement procedure: Effects of signal condition and terminal-link length. *Journal of the Experimental Analysis of Behavior, 53*(2), 219-234. doi:https: //doi.org/10.1901/jeab.1990.53-219

Staddon, J. E. R. (1970). On Herrnstein's equation and related forms. *Journal of the Experimental Analysis of Behavior*, *28*(2), 163-170.

Staddon, J. E. R. (1972). Temporal control and the theory of reinforcement schedules. In R. M. Gilbert & J. R. Millenson (Eds.), *Reinforcement: Behavioral analyses* (pp. 209–262). New York, NY: Academic Press.

Staddon, J. E. R., & Davis, G. S. (1990). Memory for Reward in Probabilistic Choice: Markovian and Non-Markovian Properties *Behaviour, 114*(1-4), 37-64. doi: https://doi.org/10.1163/156853990X00040

Sutton, N. P., Grace, R. C., McLean, A. P., & Baum, W. M. (2008). Comparing the generalized matching law and contingency discriminability model as accounts of concurrent schedule performance using residual meta-analysis. *Behavioural Processes, 78*(2), 224-230. doi: https://doi.org/10.1016/j.beproc.2008.02.012

Vollmer, T. R., & Bourret, J. (2000). An application of the matching law to evaluate the allocation of two-and three-point shots by college basketball players. *Journal of Applied Behaviour Analysis, 33*(2), 137-150. doi: https://doi.org/10.1901/jaba. 2000.33-137

Williams, B. A., & Fantino, E. (1978). Effects on choice of reinforcement delay and conditioned reinforcement. *Journal of the Experimental Analysis of Behavior, 29* (1), 77-86. doi: 10.1901/jeab.1978.29-77

Williams, B. A., & Dunn, R. (1991). Preference for conditioned reinforcement. *Journal of the Experimental Analysis of Behavior, 55* (1), 37-46. doi:https://doi.org/10.1901/jeab.1991.55-37

Appendices

Appendix A

Obtained Versus Predicted Log Ratios by Response Allocation

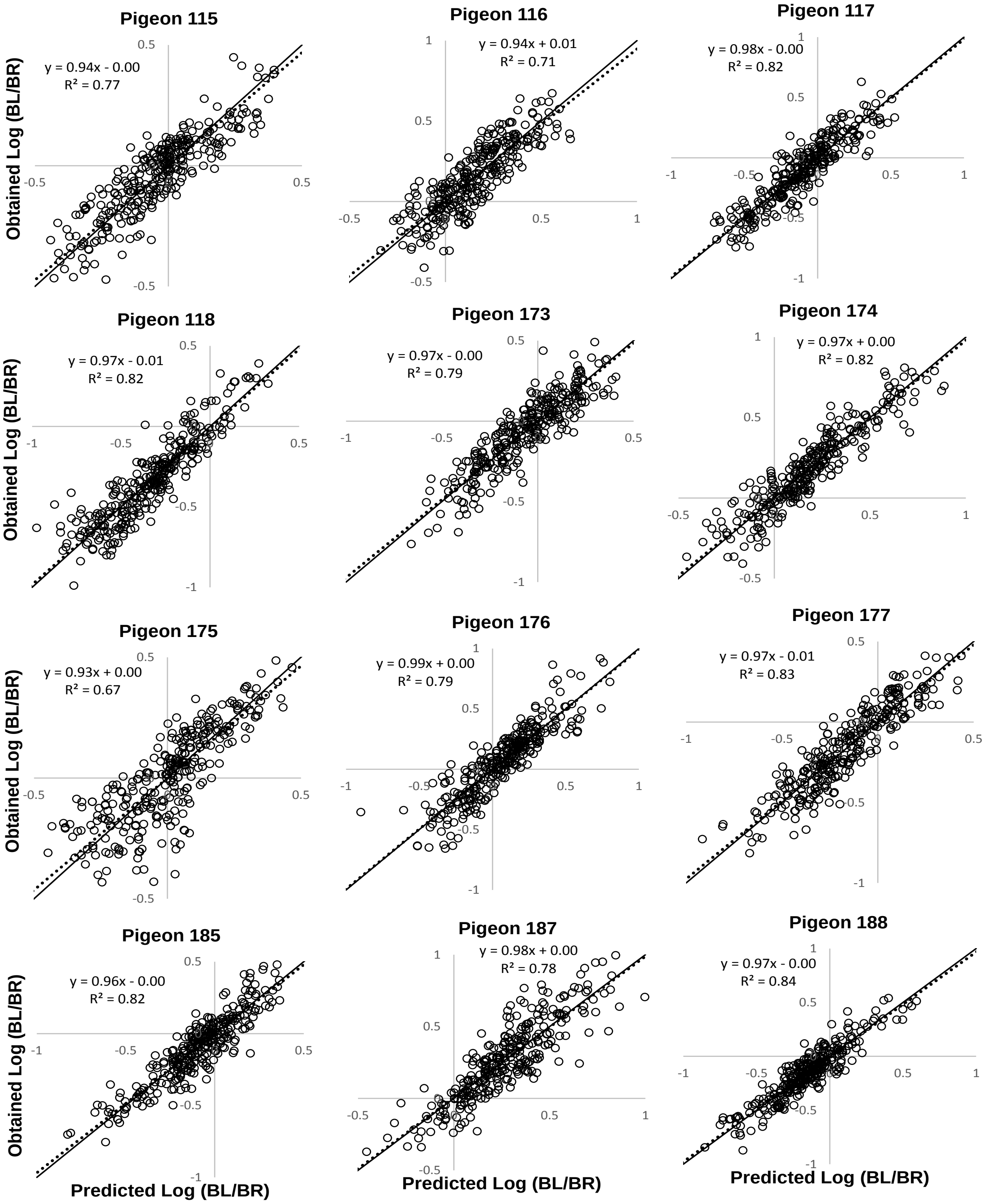


Figure A1. Log ratios of MaxVar pigeons. Data points are based on session sixth responding. For purposes of emphasizing the scatter pattern of data points, not all axes are of equal limits.

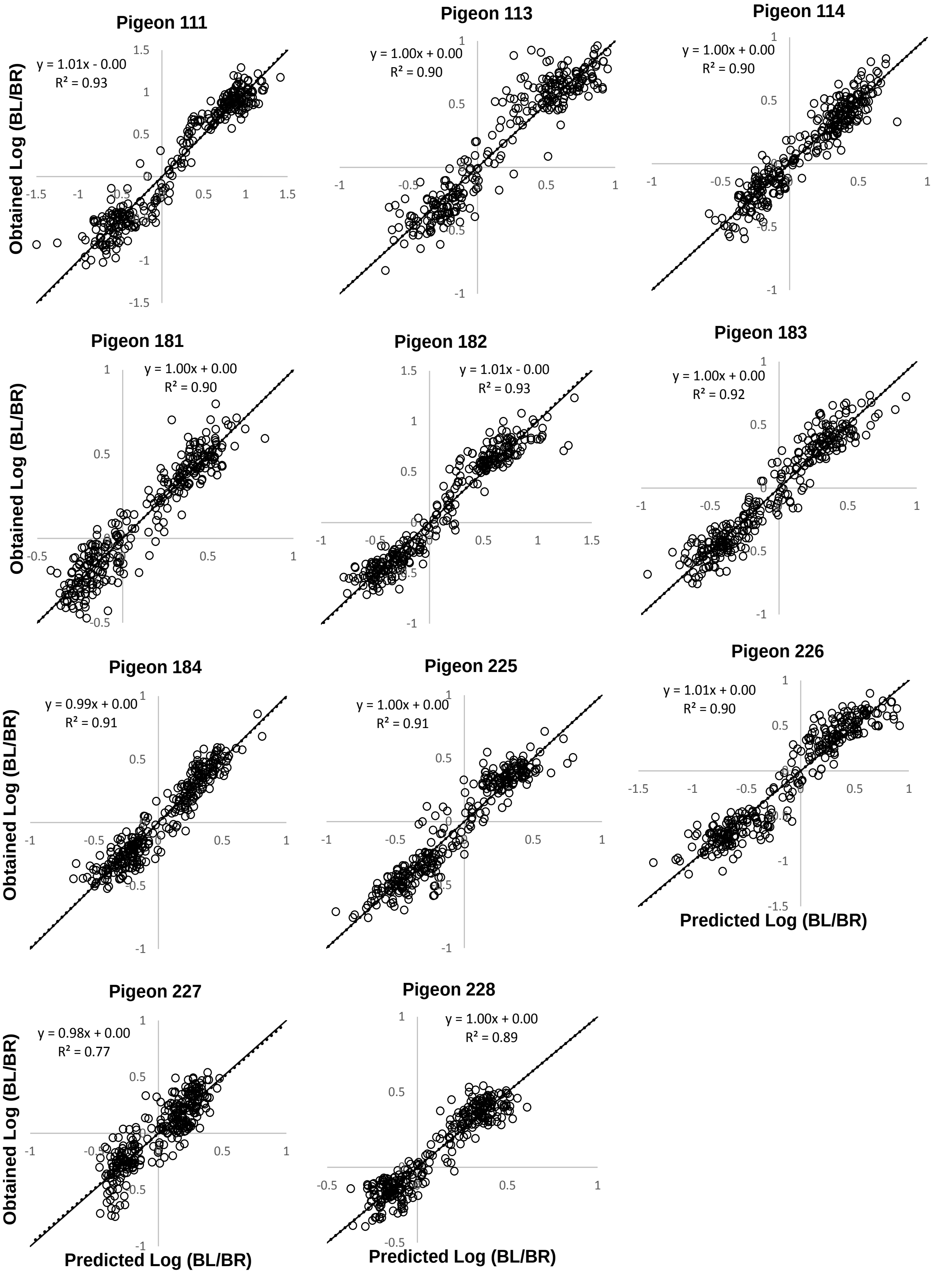


Figure A2. Log ratios of MinVar pigeons. Data points are based on session sixth responding. For purposes of emphasizing the scatter pattern of data points, not all axes are of equal limits.

Appendix B

Obtained Versus Predicted Log Ratios by Time Allocation

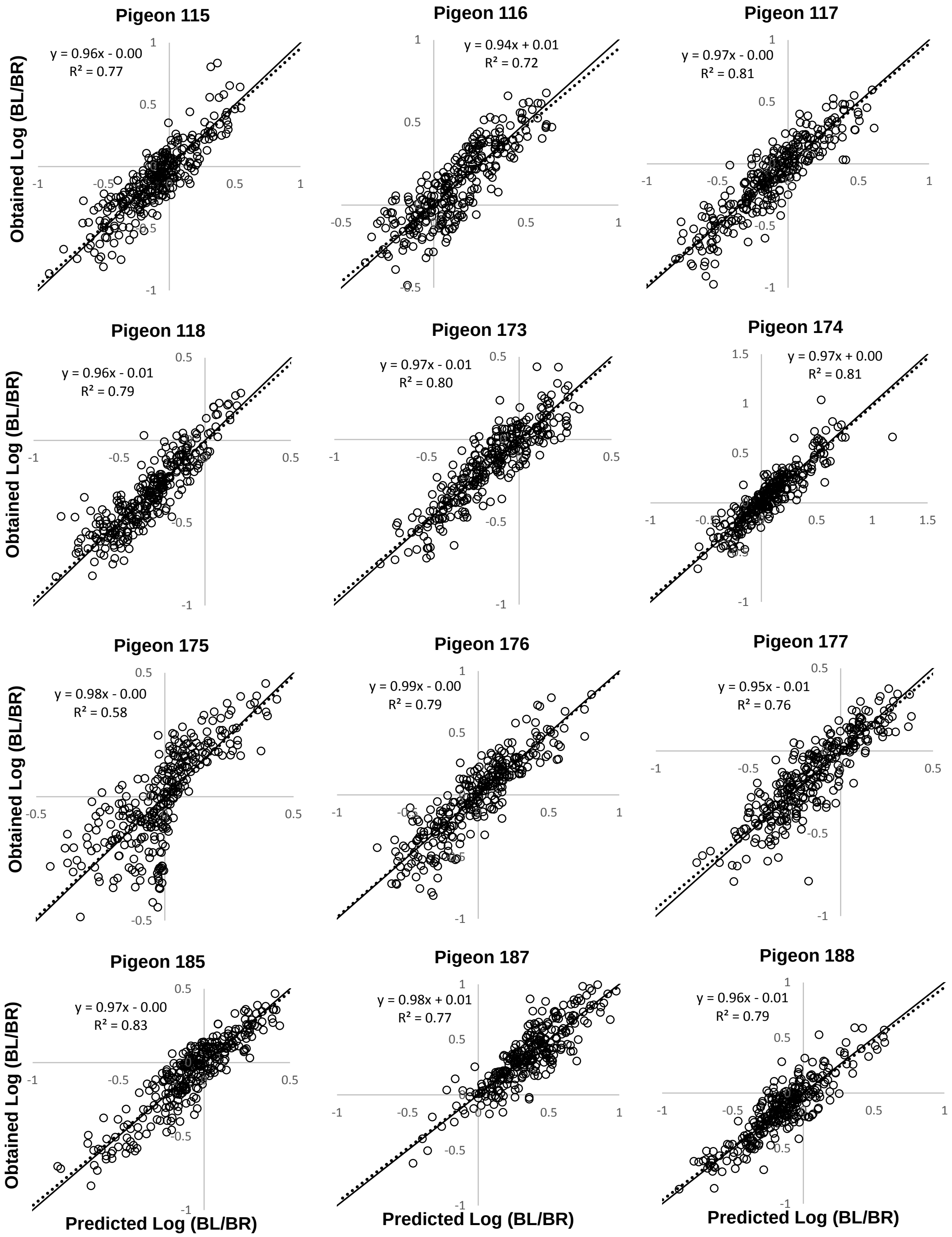


Figure B1. Log ratios of MaxVar pigeons. Data points are based on session sixth responding. For purposes of emphasizing the scatter pattern of data points, not all axes are of equal limits.

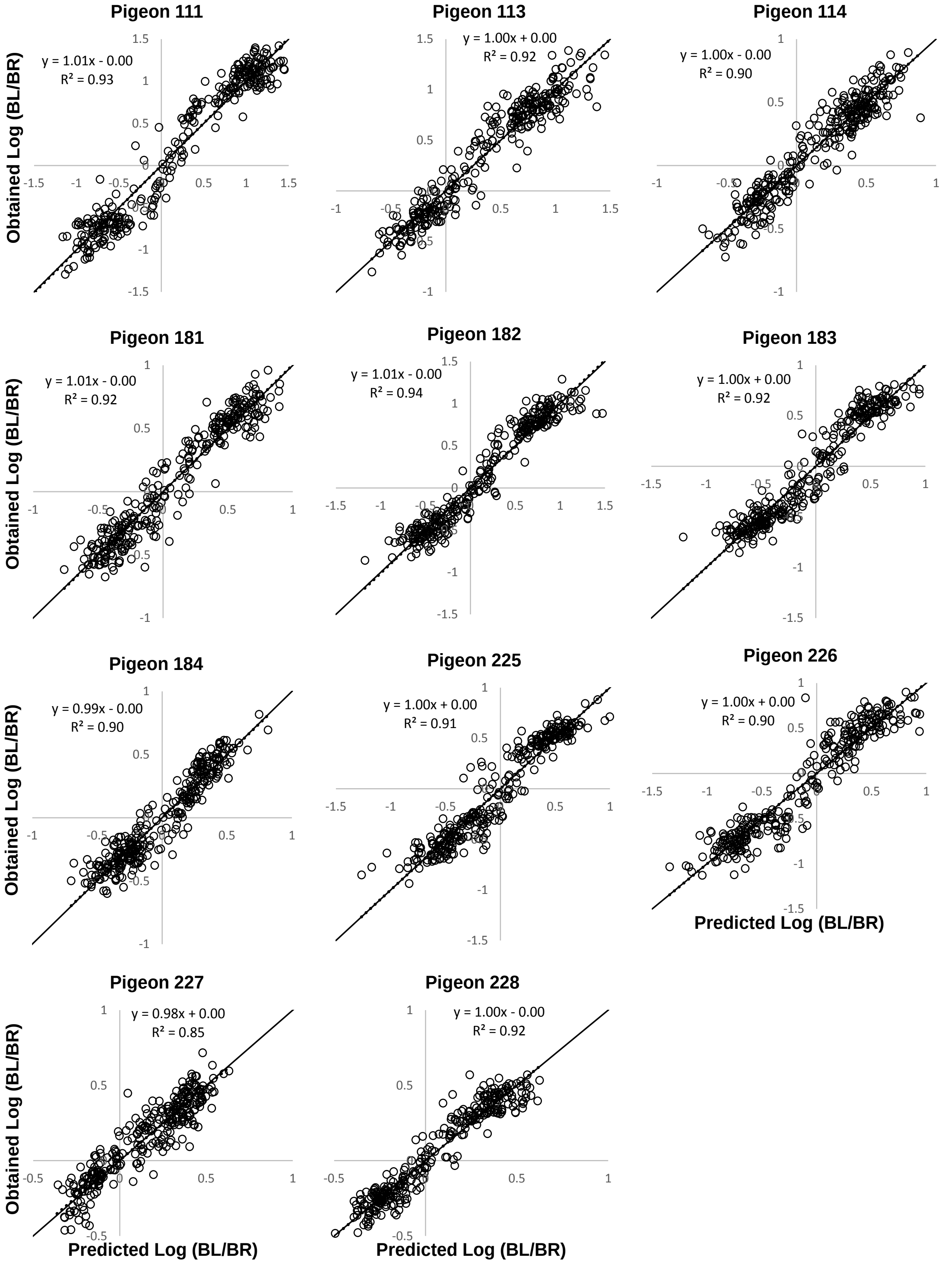
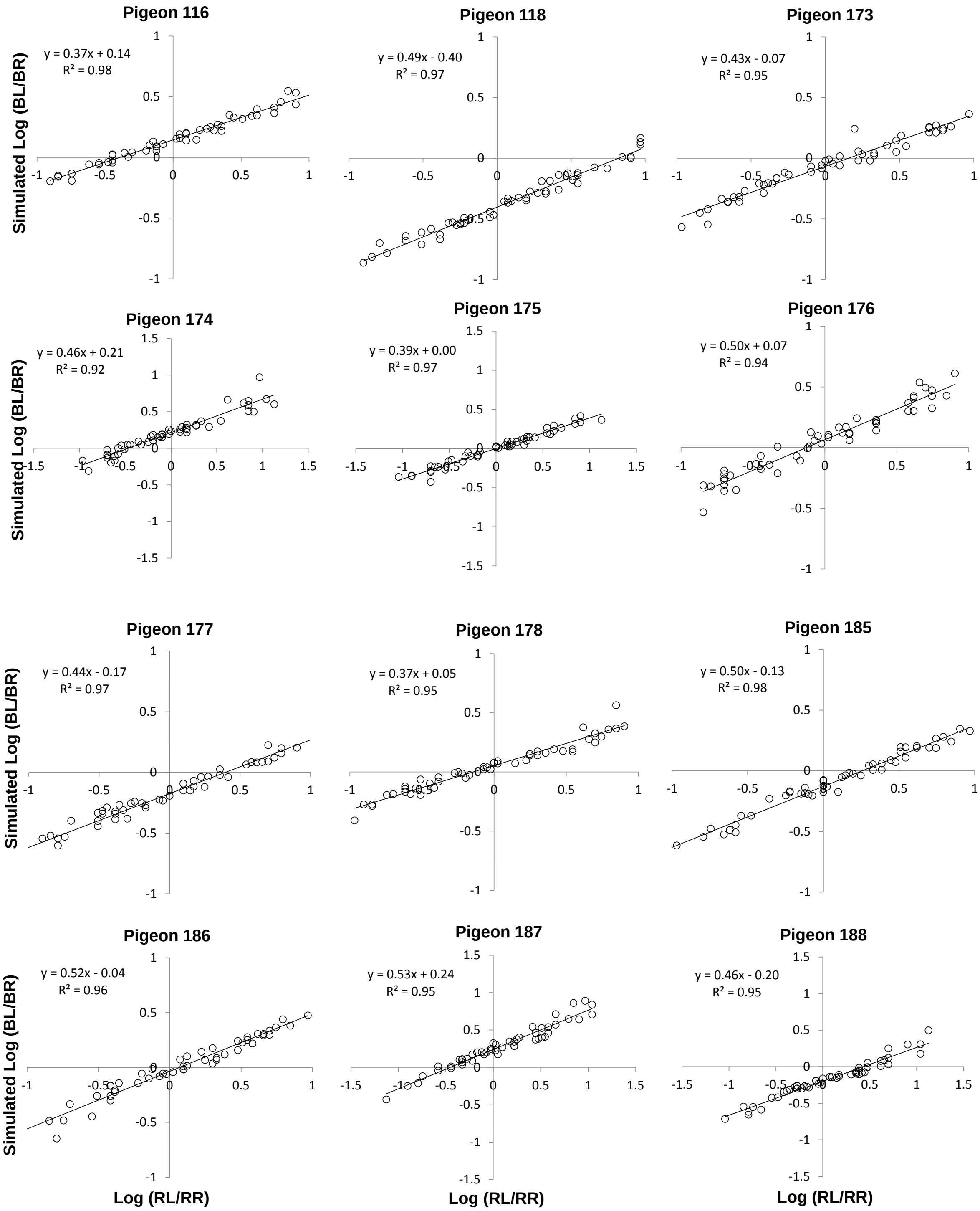


Figure B2. Log ratios of MinVar pigeons. Data points are based on session sixth responding. For purposes of emphasizing the scatter pattern of data points, not all axes are of equal limits.

Appendix C

Simulated Asymptotic Preference by Groups



Pigeon 116 Pigeon 118 Pigeon 173 Pigeon 174 Pigeon 175 Pigeon 176 Pigeon 177 Pigeon 178 Pigeon 185 Pigeon 186 Pigeon 187 Pigeon 188

Figure C1. Simulated log ratios plotted against log reinforcement ratios for MarVar pigeons. Data points are based on responding during the second half of each session over the last 50 sessions.

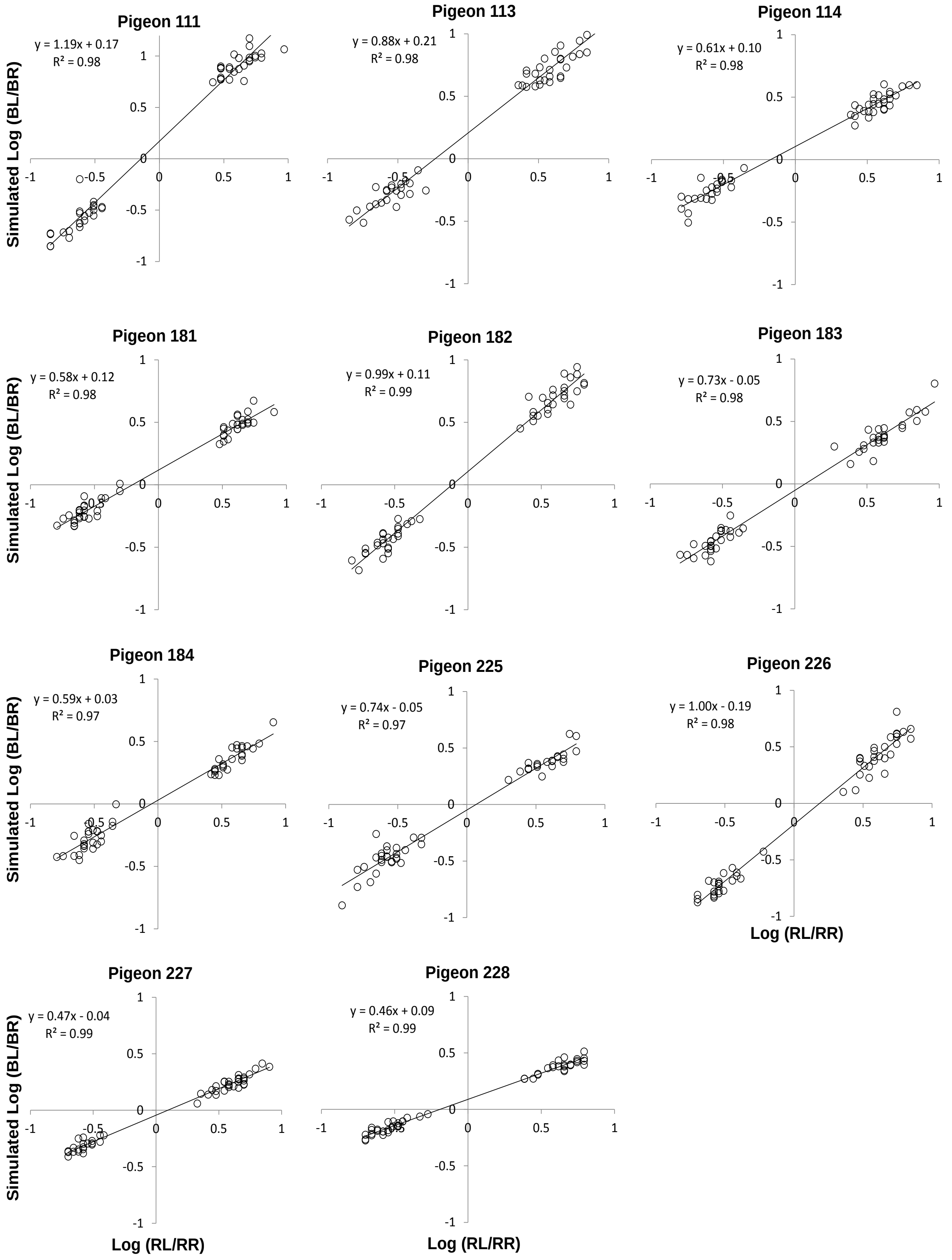


Figure C2. Simulated log ratios plotted against log reinforcement ratios for MinVar pigeons. Data points are based on responding during the second half of each session over the last 50 sessions.

Appendix D

Obtained Asymptotic Preference by Groups

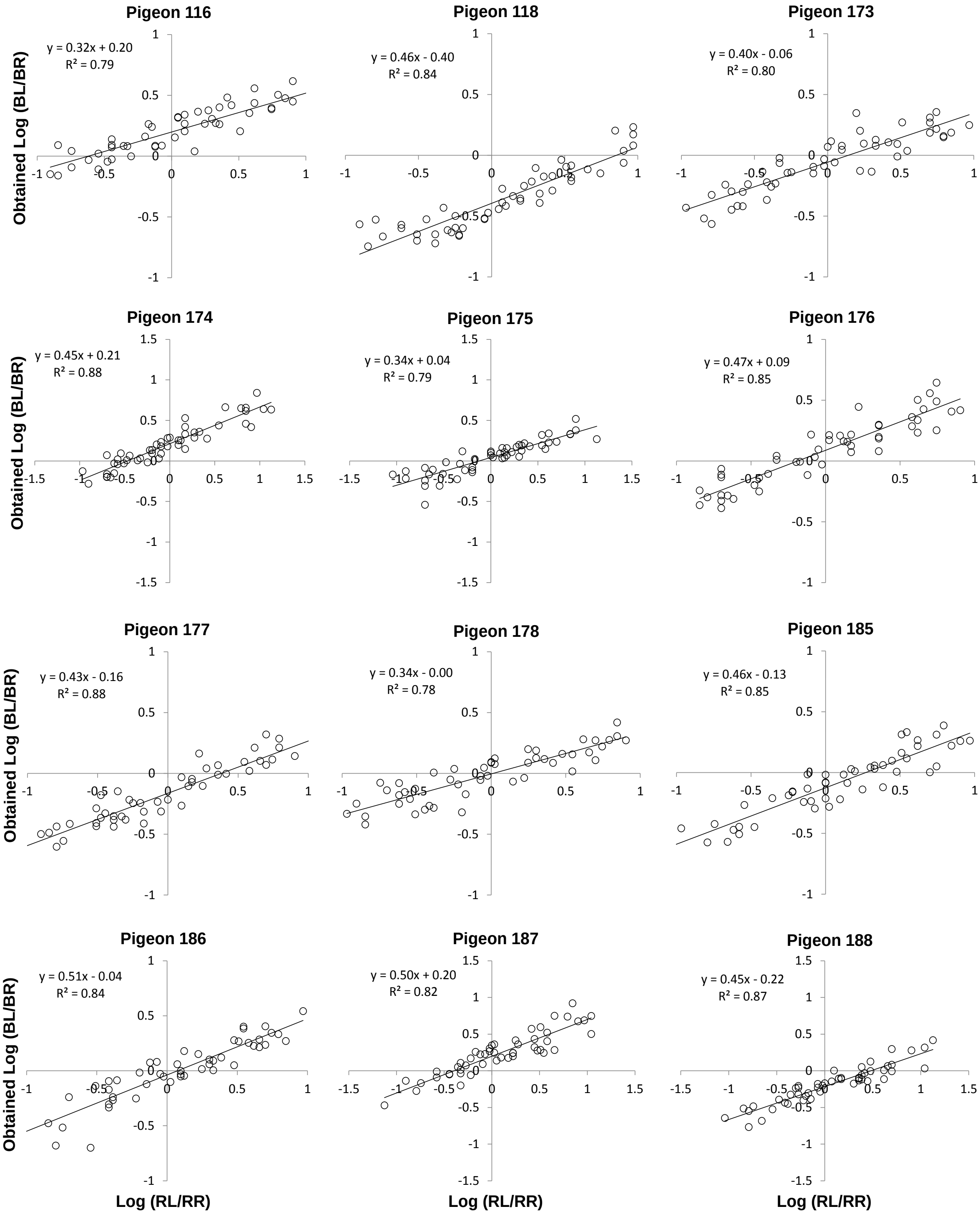


Figure D1. Obtained log ratios plotted against log reinforcement ratios for MarVar pigeons. Data points are based on responding during the second half of each session over the last 50 sessions.

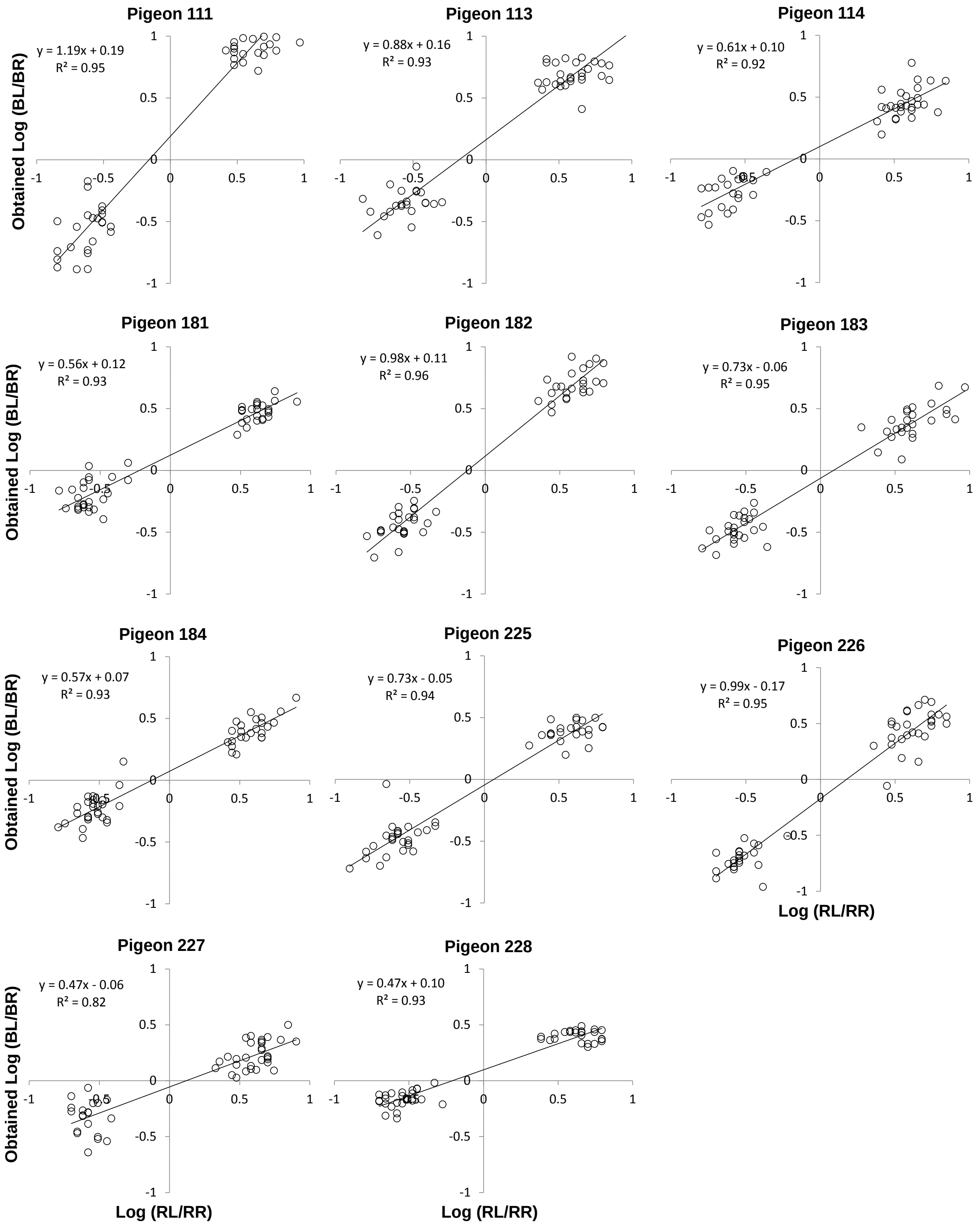


Figure D2. Obtained log ratios plotted against log reinforcement ratios for MinVar pigeons. Data points are based on responding during the second half of each session over the last 50 sessions.

Appendix E

Obtained Versus Predicted Asymptotic Log Response Ratios

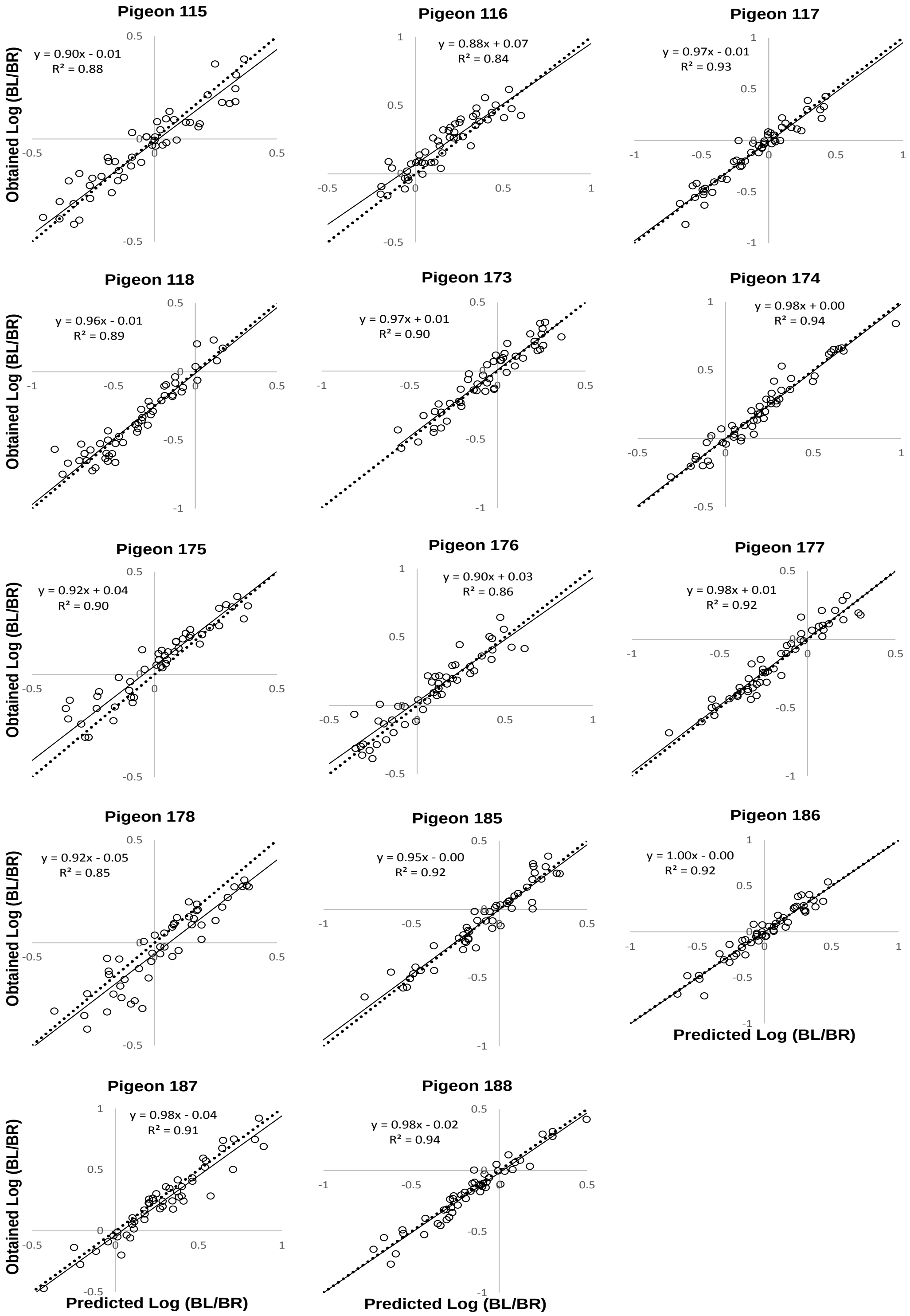


Figure E1. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for MaxVar pigeons in Experiment 1. Data points are from the last 50 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.

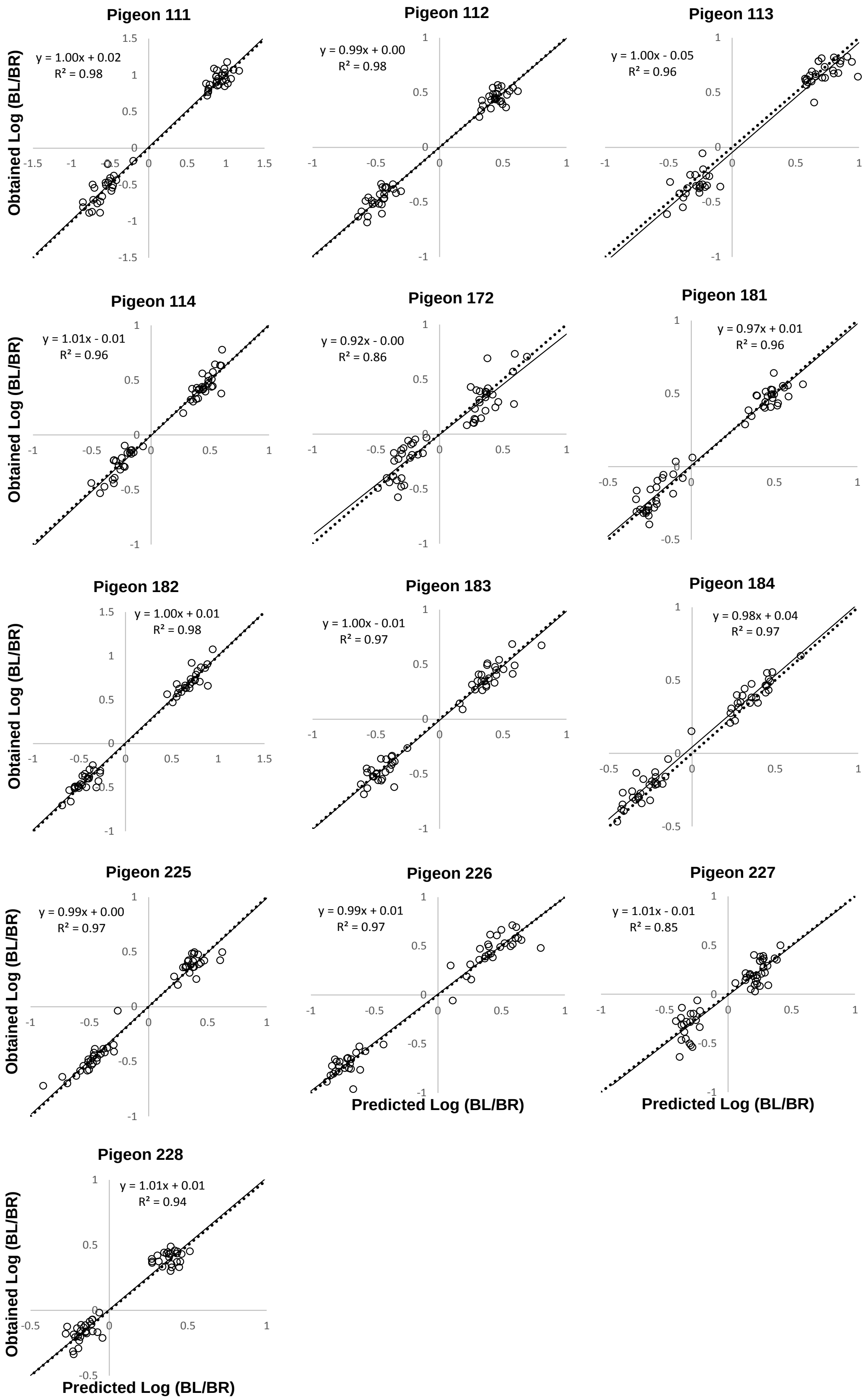
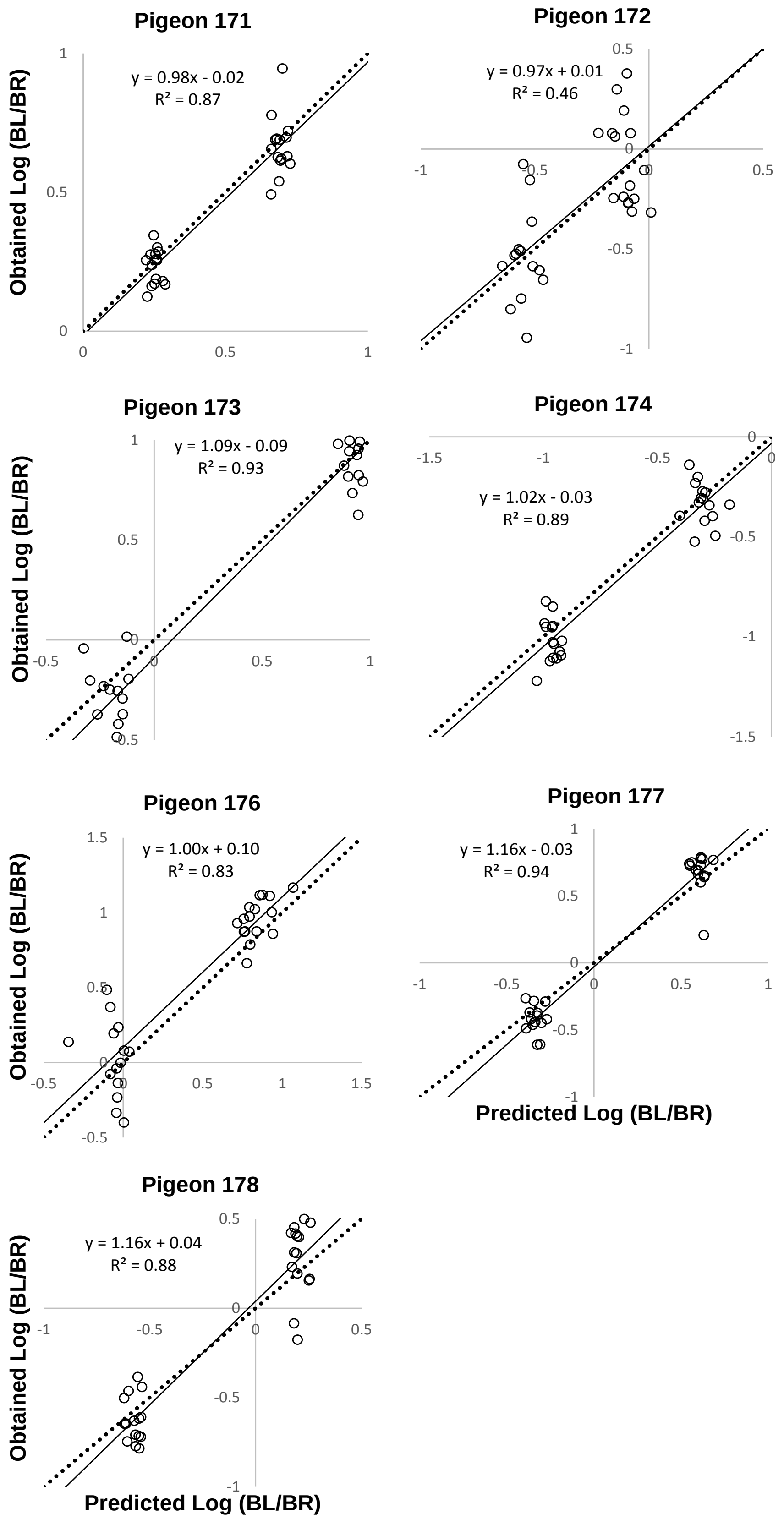


Figure E2. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for MaxVar pigeons in Experiment 1. Data points are from the last 50 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.



Pigeon 171

Pigeon 172

Pigeon 173

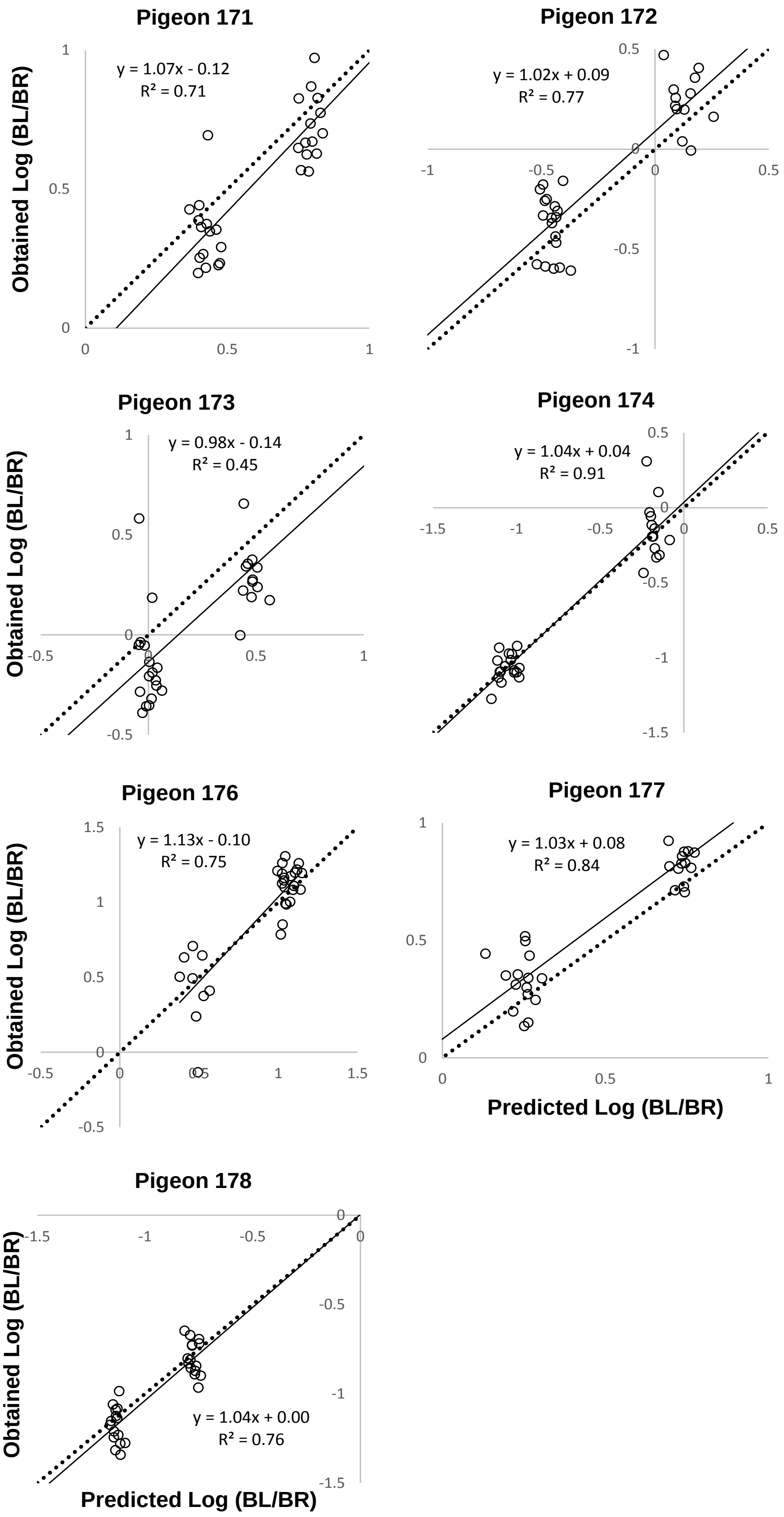
Pigeon 174

Pigeon 176

Pigeon 177

Pigeon 178

Figure E3. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for short terminal links in Experiment 2. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.



Pigeon 171

Pigeon 172

Pigeon 173

Pigeon 174

Pigeon 176

Pigeon 177

Pigeon 178

Figure E4. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for long terminal links in Experiment 2. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.

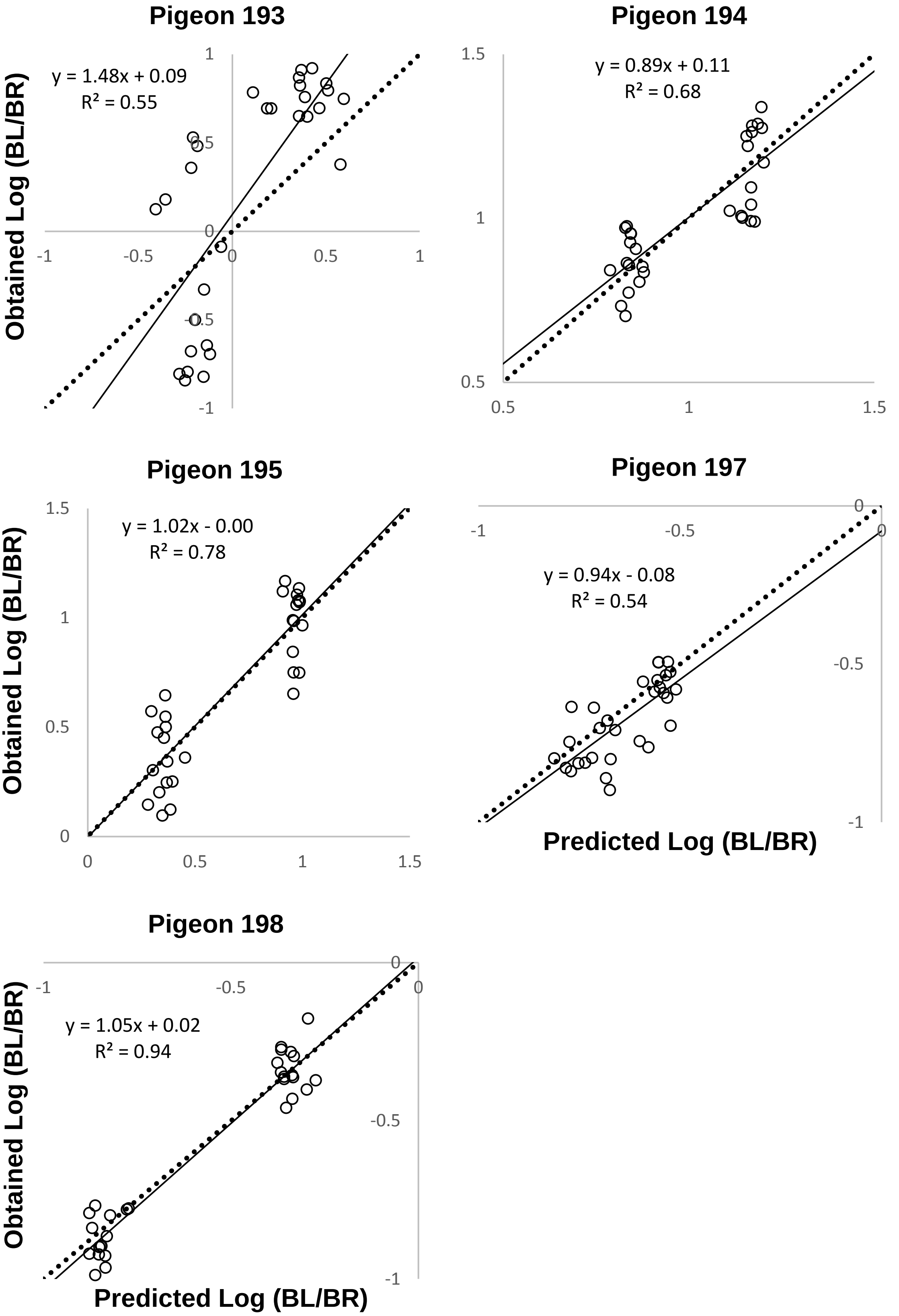


Figure E5. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for short initial links in Experiment 3. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.

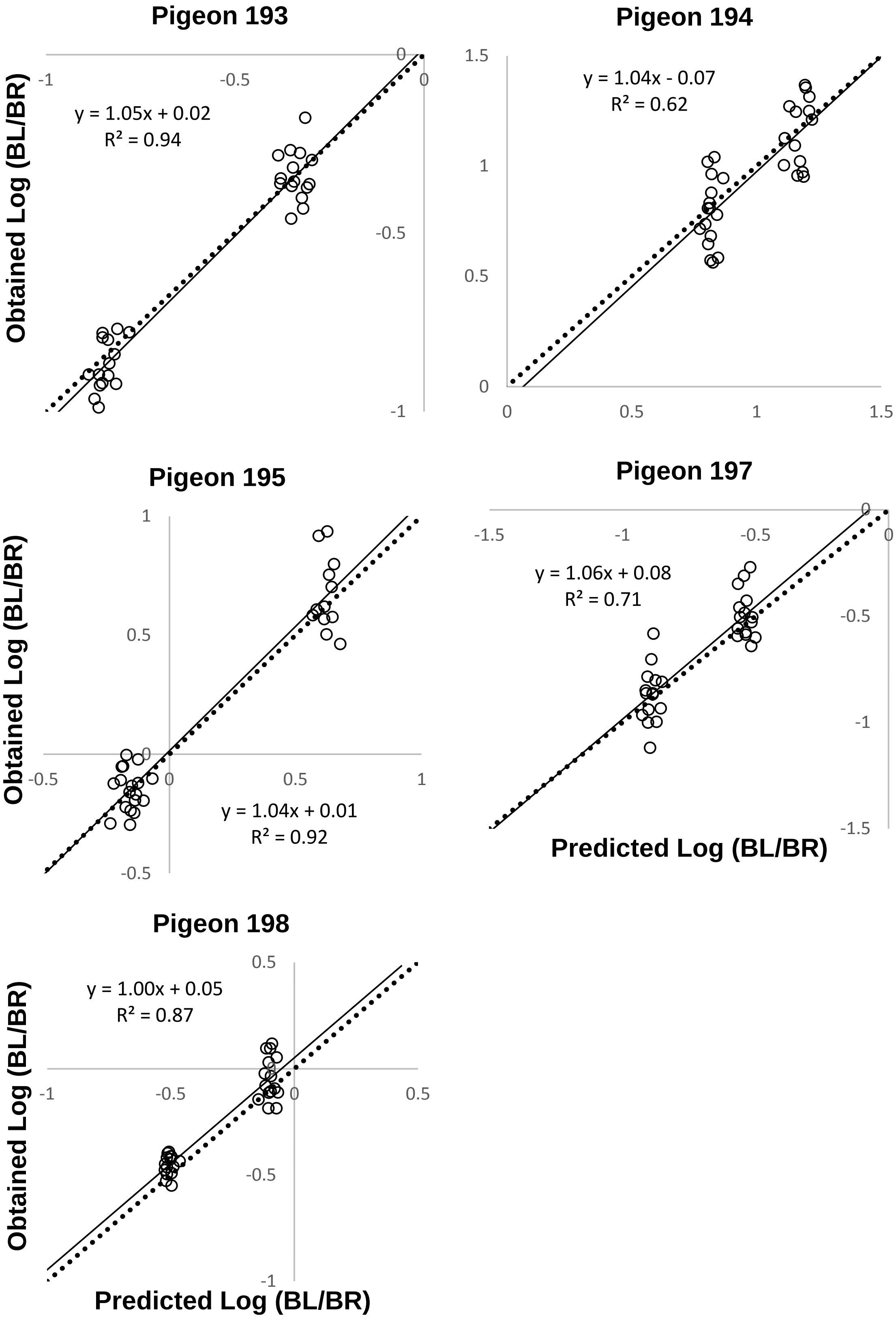


Figure E6. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for long initial links in Experiment 3. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.

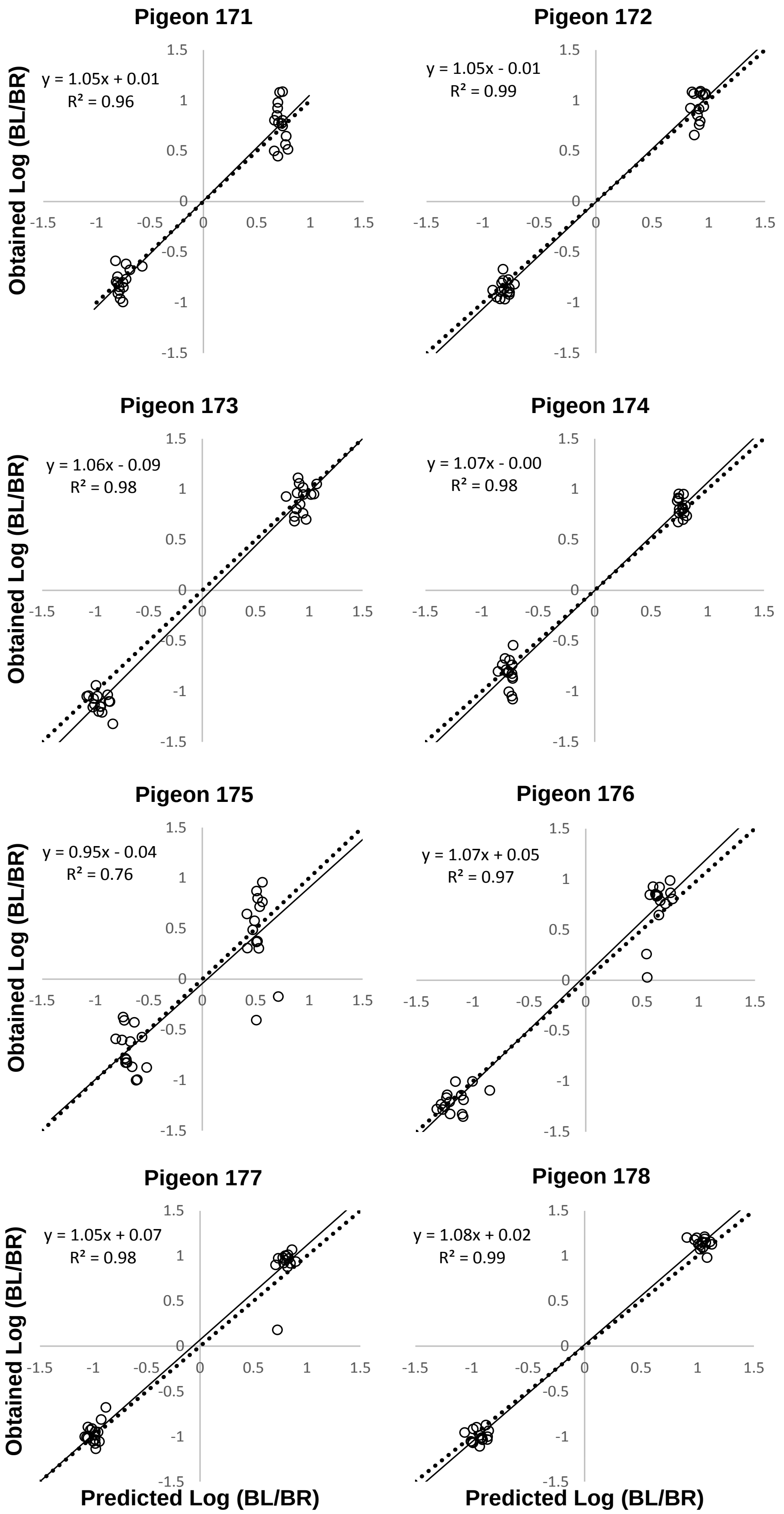
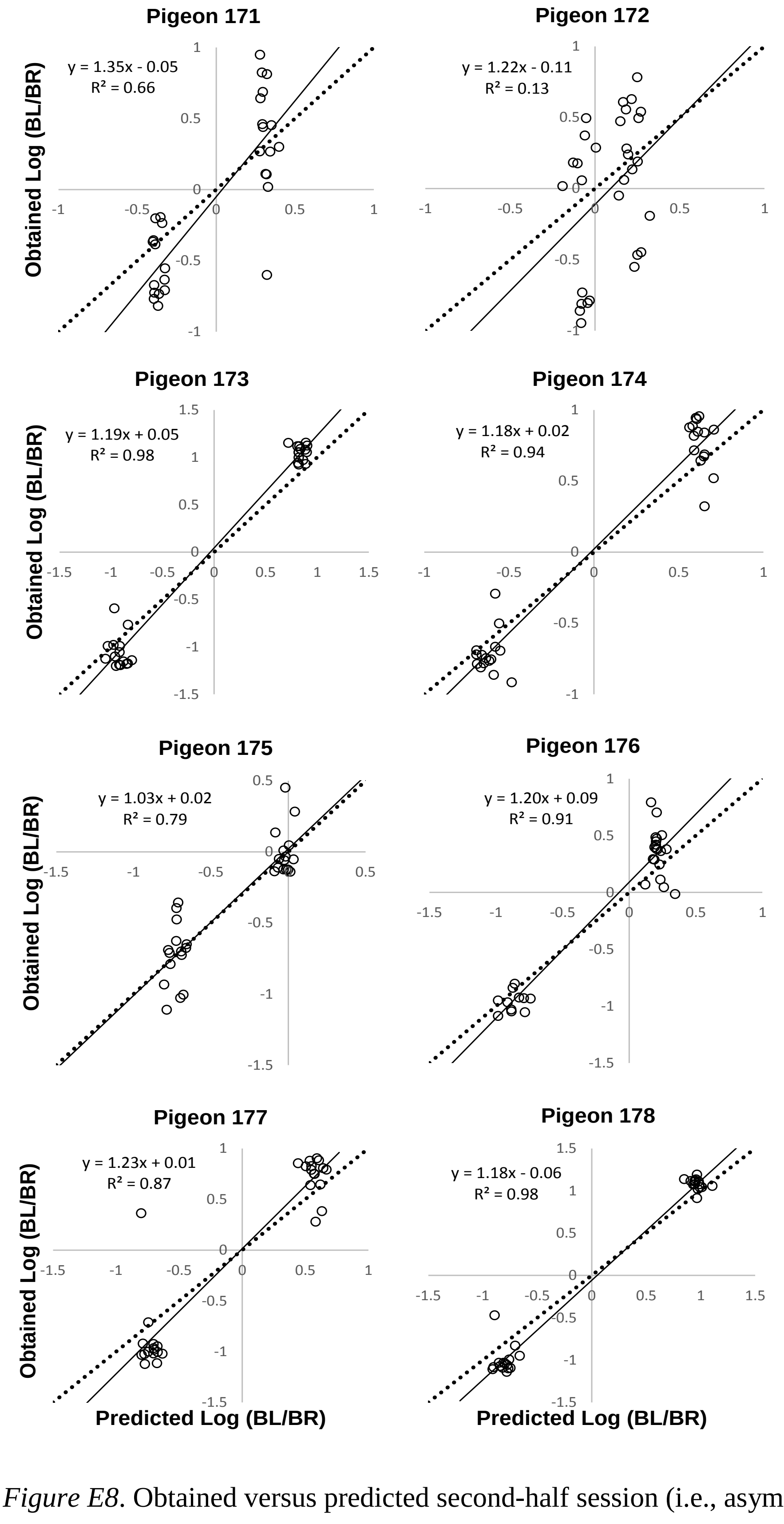


Figure E7. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for cued terminal links in Experiment 4. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.



Pigeon 171

Pigeon 172

Pigeon 173

Pigeon 174

Pigeon 175

Pigeon 176

Pigeon 177

Pigeon 178

Figure E8. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for uncued terminal links in Experiment 4. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.

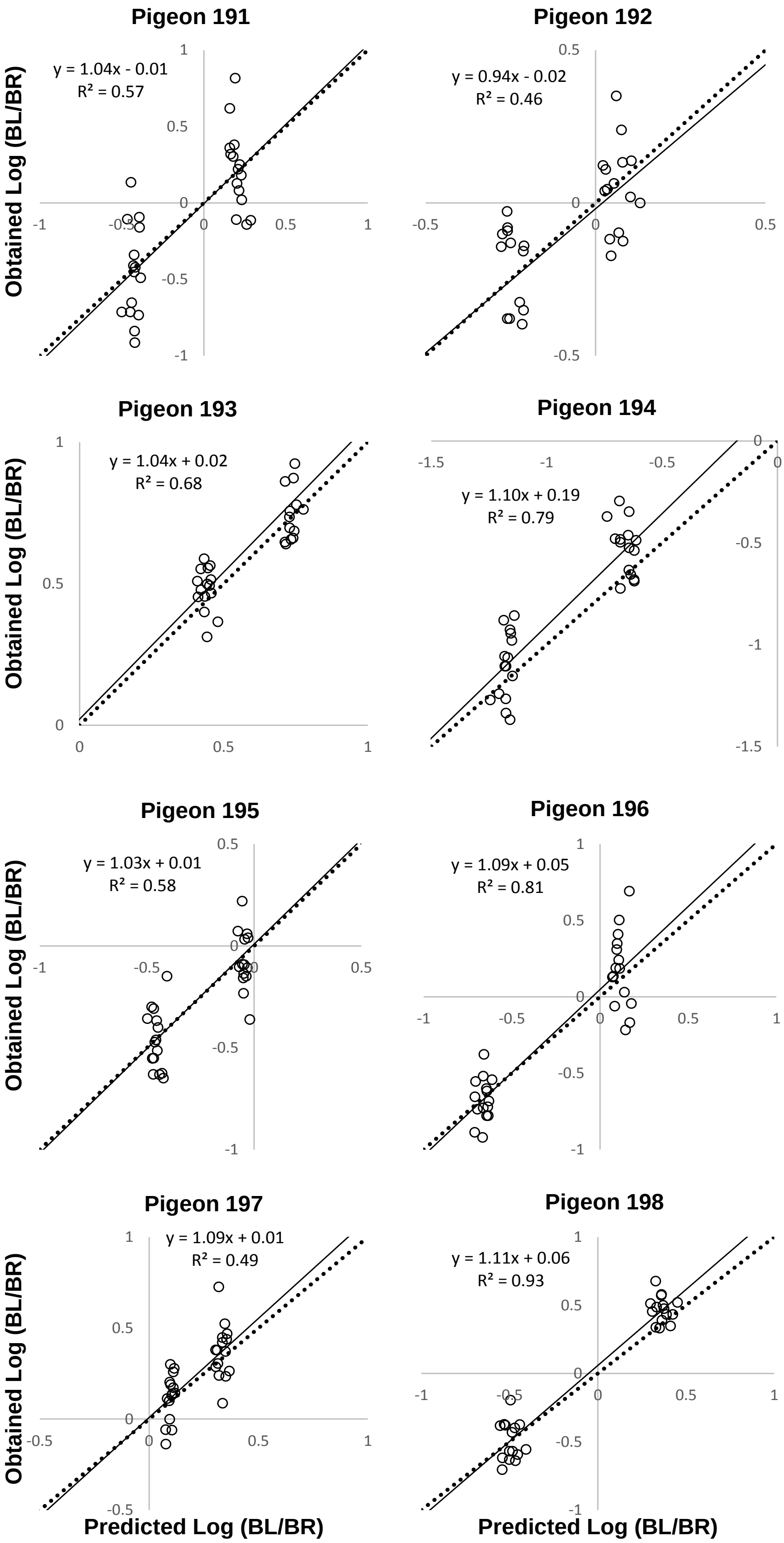


Figure E9. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for cued terminal links in Experiment 5. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.

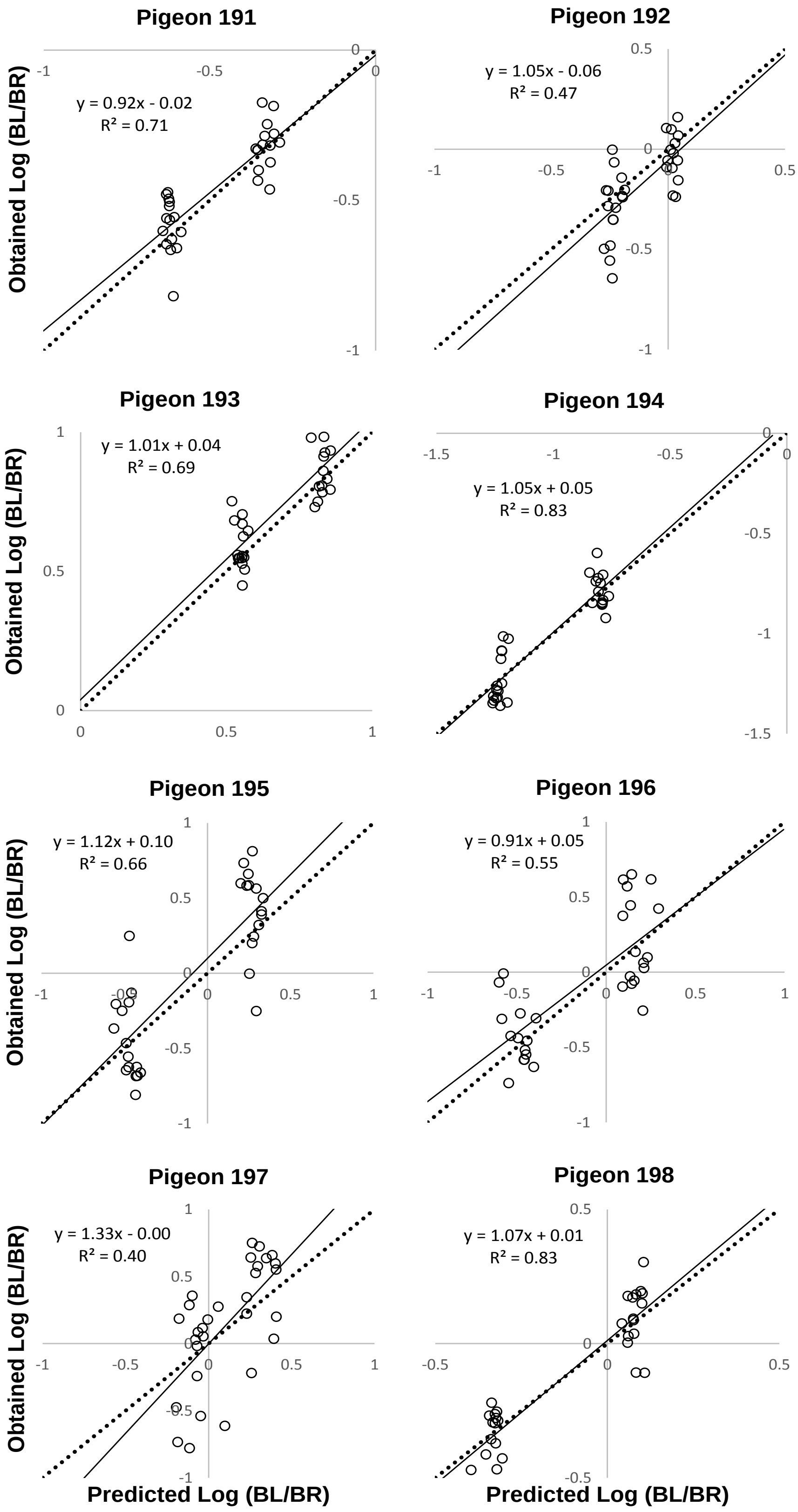


Figure E10. Obtained versus predicted second-half session (i.e., asymptotic) log response ratios for uncued terminal links in Experiment 5. Data points are from the last 30 sessions. For purposes of emphasizing the scatter pattern of data points, not all axes have the same scale.