Machine Learning with Applications 9 (2022) 100383

Contents lists available at ScienceDirect

Machine Learning with Applications

journal homepage: www.elsevier.com/locate/mlwa

Hourly electricity price forecasting with NARMAX

Catherine McHugh *, Sonya Coleman, Dermot Kerr

Intelligent Systems Research Centre (ISRC), Ulster University, Northern Ireland, UK

ARTICLE INFO

Keywords:
Energy market
NARMAX
Machine learning
Energy-related factors

ABSTRACT

Electricity price prediction through statistical and machine learning techniques captures market trends and would be a useful tool for energy traders to observe price fluctuations and increase their profits over time. A Nonlinear AutoRegressive Moving Average model with eXogenous inputs (NARMAX) identifies key energy-related factors that influence hourly electricity price through prediction modelling. We propose to use a transparent NARMAX model and analyse Irish Integrated Single Electricity Market (ISEM) data from May 2019 until April 2020 to determine which external factors have a significant impact on the electricity pricing. The experimental results indicate that historical electricity price, demand, and system generation are the most significant factors with historical electricity price being the most weighted factor and the largest Error Reduction Ratio (ERR). A NARMAX model generated using correlated lags was also considered to identify key energy-related lag factors that influence the electricity price. For justification, the significant lag factors are included as inputs in a Seasonal AutoRegressive Integrated Moving Average model with eXogenous input (SARIMAX) to determine if model performance improves with refinement. To conclude, using the NARMAX methodology with energy-related input factors helps to determine the significant factors and results in accurate predictions of electricity price.

1. Introduction

Electricity price data exhibit complex behaviours which result in price fluctuations thus making price forecasting difficult (Mosbah & El-Hawary, 2016). The Integrated Single Electricity Market (ISEM) is a new electricity market in Ireland providing energy traders with greater control. Awareness of future electricity prices and appropriate selection of input factors would help ISEM market traders know when to buy or sell price units. Considering the ISEM market, the system marginal price, which is the cheapest bid placed by generators to meet customer demand (Li, Arci, Reilly, Curran, & Belatreche, 2016), is influenced by many energy-related factors. These factors including electricity demand, weather temperature or wholesale gas prices can contribute strongly to electricity generation costs and prices (Pandey & Upadhyay, 2016) and therefore necessary to include as input variables in electricity price prediction models to determine an optimal forecasting tool.

Machine learning algorithms use historical data to develop optimal models used to predict future prices (Gao, Lo, & Fan, 2017). Recent literature examined various regression models to predict solar radiation and evaluated the best model by calculating the statistical errors [Mean Absolute Error (MAE), Mean Squared Error (MSE) and Root Mean Squared Error (RMSE)] and selecting the best model with the smallest error (Karasu, Altan, Sarac, & Hacioglu, 2017). Price data display nonlinear traits and for this reason a hybrid model is also worth considering to improve model performance accuracy (Altan, Karasu, & Bekiros, 2019).

A Nonlinear AutoRegressive Moving Average model with eXogenous inputs (NARMAX) observes the relationship between inputs and outputs while considering past error to improve future predictions (Acuna, Ramirez, & Curilem, 2012). A NARMAX model helps to identify the statistically significant input factors influencing electricity price by removing redundant factors. The size of the forecasting window is also important, with short-term (days/weeks) being more desirable for energy forecasting to manage supply and demand fluctuations (Amjady & Hemmati, 2006).

In this paper, the main contribution is a transparent NARMAX model implemented with multiple hourly inputs (corresponding to various energy-related factors and their correlated peak lags) to determine contributing factors for accurately forecasting electricity price. The aim of the correlated lags model is to enhance prediction accuracy and improve market performance. This was justified by using the significant NARMAX lag factors as a hybrid approach to refine a Seasonal AutoRegressive Integrated Moving Average with eXogenous inputs (SARIMAX) lag model to determine if day-ahead prediction accuracy can be improved. Although the study presented here only uses hourly electricity data between May 2019 and April 2020 and has minimal parameter optimisation, we demonstrate excellent results thereby illustrating the robustness of our modelling approach.

* Corresponding author.
E-mail addresses: mchugh-c24@ulster.ac.uk (C. McHugh), sa.coleman@ulster.ac.uk (S. Coleman), d.kerr@ulster.ac.uk (D. Kerr).

https://doi.org/10.1016/j.mlwa.2022.100383
Received 30 August 2021; Received in revised form 16 March 2022; Accepted 6 July 2022
Available online 11 July 2022

The remainder of this paper is organised into the following sections: Section 2 discusses related work using the NARMAX methodology and the methodology is described in detail in Section 3. The procedure used to identify correlated lags is outlined in Section 4. Results are presented and summarised in Sections 5 and 6 concludes with key findings.

2. Related work

Accurate day-ahead forecasting is required for optimal energy trading. Nogales, Contreras, Conejo, and Espínola (2002) found that short-term forecasting methods produced accurate predictions, when proposing models to forecast the Spanish and Californian energy markets, with results displaying small error. It is also beneficial to have a prediction model that can deal with price volatility (Voronin & Partanen, 2013) to remain competitive in the market. Huurman, Ravazzolo, and Zhou (2012) concentrated on weather variables to predict day-ahead energy prices and noted that information from weather forecasts can be useful in improving accuracy. Wind is often considered an important energy-related factor as it is chaotic and difficult to predict accurately thus resulting in the largest volatility in one study (Cerjan, Matijaš, & Delimar, 2014). Li et al. (2016) remarked that other factors such as demand, interconnectors, and power generation were dominant energy-related factors. Kavanagh (2017) utilised historical load data to perform day-ahead forecasting and observed daily and weekly patterns rising in peaks and troughs.

Several computational intelligence forecasting techniques have been applied in the energy sector. The general price forecasting steps include: (i) analysing historical data, (ii) data preparation, (iii) model selection, and (iv) model refinement (Pandey & Upadhyay, 2016). Vijayalakshmi and Girish (2015) researched short-term electricity price forecasting and examined the accuracy between time-series models and an Artificial Neural Network [ANN] model. Gao et al. (2017) compared an ANN with an AutoRegressive Integrated Moving Average [ARIMA] model and discovered that both models were less precise as the prediction window size increased and hence demonstrated that short-term forecasts were highly accurate.

Nonlinear regression models apply a nonlinear combination of independent variables and lagged terms to calculate the dependent variable (Ghalehkhondabi, Ardjmand, Weckman, & Young, 2017). As energy market data display nonlinear relationships, nonlinear regression models such as NARMAX could be useful to identify key external factors (independent variables) and lags. A NARMAX model can also be considered for seasonal data as it is able to identify periodic series (Acuna et al., 2012). A polynomial NARMAX model, which is considered transparent, is desirable since it has a simple model structure with a small number of parameters (Zito & Landau, 2005) and thus can be easily analysed. A NARMAX model is fast to compute and incorporates the relationship between input–output variables (Nehmzow, 2006). NARMAX models have been used in a variety of industry studies: to identify key features influencing China house prices (Zhang, Hua, & Zhao, 2012); predicting cash demand of ATMs with seasonal input variables (Acuna et al., 2012); modelling air pressure and turbine relationships in diesel engines (Zito & Landau, 2005); modelling solar wind in magnetosphere evolution (Boynton, Balikhin, Billings, Wei, & Ganushkina, 2011) and modelling monthly West Africa rainfall (Amisigo, van de Giesen, Rogers, Andah, & Friesen, 2008).

When determining external factors to use for a predictive model, it is often useful to identify the peak lags as they can be highly correlated with the dependent variable. Peak lags are determined using autocorrelation testing with exogenous variables and output checked to see if any lagged term shows robust correlation or a relationship with the dependent variable (Ghalehkhondabi et al., 2017; Li et al., 2016). A common approach is to determine the top influencing lags for each input factor (Ghalehkhondabi et al., 2017). The work in Li et al. (2016) suggested using a 24-hour lag as an input as their experimental energy study found that same hour data from the previous day provided strong correlation. A limitation was noted in another energy market study with Swedish data as they considered no lagged terms (Xie, Sandels, Zhu, & Nordström, 2013). Therefore, to improve model accuracy peak lagged energy-related factors should be included as model inputs.

3. NARMAX methodology

In a NARMAX model, unknown parameters are estimated through simple regression algorithms (Pagano, Filho, & Plucenio, 2006) considering the input and output variables (Zito & Landau, 2005). The NARMAX model identifies the structure by finding the relationship between previous inputs and current output using a nonlinear equation (Billings & Coca, 2001). The NARMAX methodology uses an error measure to refine model structure and improve prediction accuracy (Acuna et al., 2012). Leontaritis and Billings described a polynomial NARMAX model (Korenberg, Billings, & Liu, 1987) as:

$$y(t) = F^l[y(t-1), \dots, y(t-N_y), u(t), \dots, u(t-N_u), \mathcal{E}(t-1), \dots \mathcal{E}(t-N_{\mathcal{E}})] + \mathcal{E}(t) \tag{1}$$

where $y(t)$ is the output time-series, F^l is an unknown non-linear function either linear, quadratic, or cubic, N_y is the output lag regression, $u(t)$ is the input time-series, N_u is the input lag regression, $N_{\mathcal{E}}$ is the prediction error lag regression, and $\mathcal{E}(t)$ is the prediction error.

Firstly the model estimates F^l, removes unnecessary terms leaving one large polynomial function (Warnes, Glasseyfl, Montague, & Kara, 1996). It can be difficult to decide which degree of polynomial and interaction terms to use for the initial model structure thus trial and error combinations are required to select the inputs, degree, and interaction terms (Warnes et al., 1996). By analysing all possible combinations, unbiased estimates can be obtained and the key model terms identified (Boynton et al., 2011). Next, the parameter terms can be estimated. These two steps are important to remove insignificant coefficients during the iterative learning process (Billings & Fadzil, 1985) and hence ensure a compact model is obtained.

In total there are five stages to the NARMAX methodology in order to estimate and identify the suitable model terms: (1) structure selection, (2) parameter estimation, (3) model validation, (4) prediction and (5) analysis. NARMAX splits the data into model estimation and model validation subsets (Nehmzow, 2006). Structure selection applies orthogonal estimation algorithms (Korenberg et al., 1987) to identify model terms and reduce coefficients. The algorithm first estimates linear parameters excluding $\mathcal{E}(t)$ allowing extra terms to be added without re-estimating the model. Next an initial $\mathcal{E}(t)$ is estimated, and then refined in each iteration for all coefficients (Korenberg et al., 1987). Each coefficient is independent as each term is orthogonal, allowing the model coefficients to be estimated independently (Korenberg et al., 1987). This results in a compact model and avoids over- or under-fitting by adding each term one at a time and checking the coefficient's significance against the output's variance (Billings & Coca, 2001).

When considering models with a high number of inputs there is increased difficulty in reaching the desired accuracy. The NARMAX methodology uses an error estimation algorithm to calculate the Error Reduction Ratio (ERR) which is the percentage reduction from the total Mean Squared Error (MSE) signifying the model term contribution (Taib, 1993). The ERR is calculated as follows (Zito & Landau, 2005):

$$ERR_i = \frac{g_i^2 \sum_{k=1}^{N} w_i^2(t)}{\sum_{k=1}^{N} y_i^2(t)} \tag{2}$$

where g_i is the parameter (in this research, the energy-related factors), w_i is the regressor (in this research, the electricity price) and y_i is the output regressor (in this research, the day-ahead electricity price). ERR develops a parsimonious model through ranking regressors from high to low MSE reduction (Amisigo et al., 2008). Validation tests are necessary to confirm an accurate model fit (Korenberg et al., 1987). Model validation is verified with unseen data to predict and check

Table 1
Energy-related factors.

Energy-related Factors	Unit	Model Input
Historical Electricity Price	GBP per Megawatt Hour	U1
System Generation	Megawatt	U2
Demand	Megawatt	U3
Wind	Megawatt	U4
East–West Interconnector	Megawatt	U5
Moyle Interconnector	Megawatt	U6
CO2 intensity	Kilowatt Hour	U7
CO2 emissions	CO2 intensity per Hour	U8
Load	Megawatt	U9
Temperature	Celsius	U10

the accuracy (Billing & Voon, 1985). The final model displays the statistically significant model terms ranked in order (Zhang et al., 2012). Throughout all stages of the methodology the outcome variable is dependent on the chosen inputs therefore it is crucial to choose correctly to reach maximal model accuracy (Billings & Fadzil, 1985).

4. ISEM dataset

A new unique cross-border energy market, the Integrated Single Electricity Market (ISEM), went live in October 2018 in Northern Ireland and Republic of Ireland, increasing transparency and competition in the market. The ISEM consists of multiple markets allowing traders to purchase electricity units beforehand in the Day-Ahead or Intra-Day markets. If market traders do not purchase electricity units in either of these two markets, they have to pay the balancing market price as well as any financial costs through the imbalance settlement price if the electricity price has increased. The ISEM brings complexity in purchasing and selling electricity units with a need to forecast as it adapts to the European Target Model process.

The ISEM 2019–2020 data and exogenous variables were downloaded from multiple sources: hourly electricity price data were retrieved from the day-ahead trading market Single Electricity Market Operator (SEMOpx, 2020), half-hourly forecast load generation were collected from SEMO (SEMO, 2020), energy-related data recorded in 15-minute intervals were retrieved from EirGrid smart dashboard (EirGrid, 2020): actual demand (predicted electricity production), system generation (total electricity production), forecast wind (total all island wind farms), East–West interconnector (connected from Ireland to Wales), Moyle interconnector (connected from Scotland to Northern Ireland), CO2 intensity (average of CO2 emissions), CO2 emissions (estimated total of all power stations), and hourly temperatures were collected and averaged from five weather Met Office stations across Northern Ireland (MET Office UK, 2020) and the Republic of Ireland (Met Office IE, 2020). Table 1 presents a summary of all of the energy-related factors used in this work. During data pre-processing, the non-hourly data (e.g. half-hourly load) was averaged per hour per day. Data preparation involved merging each of the individual datasets together keeping date, hour, unit and unit value for each energy-related factor.

5. Correlated lags

Autocorrelation shows the relationship between data points in terms of the time lag function. Autocorrelation testing can find the correlations when applied to lagged time-series input. An Autocorrelation Function (ACF) plot displays data trends and is a key diagnostic tool to identify correlation among the data. Splitting a time-series into 24-hour time periods instead of using the complete time-series in a model has been shown to help to improve model accuracy (García-Martos, Rodríguez, & Sánchez, 2007).

The correlated lag plots for each energy-related factor were examined to identify peak lags for the ISEM market. Autocorrelation testing was performed first selecting initial lags as any positive lag

Table 2
ISEM peak lags.

Energy-related Factors	Unit	Peak Lags
Historical Electricity Price	GBP per Megawatt Hour	Lags 1, 2, 23, 24
System Generation	Megawatt	Lags 1, 2, 3, 22, 23, 24
Demand	Megawatt	Lags 1, 2, 3, 22, 23, 24
Wind	Megawatt	Lag 1
East–West Interconnector	Megawatt	Lags 1, 2, 3, 24
Moyle Interconnector	Megawatt	Lags 1, 2, 3, 24
CO2 intensity	Kilowatt Hour	Lag 1
CO2 emissions	CO2 intensity per Hour	Lags 1, 2, 3, 22, 23, 24
Load	Megawatt	Lags 1, 2, 3, 22, 23, 24
Temperature	Celsius	Lags 1, 2, 3, 22, 23, 24

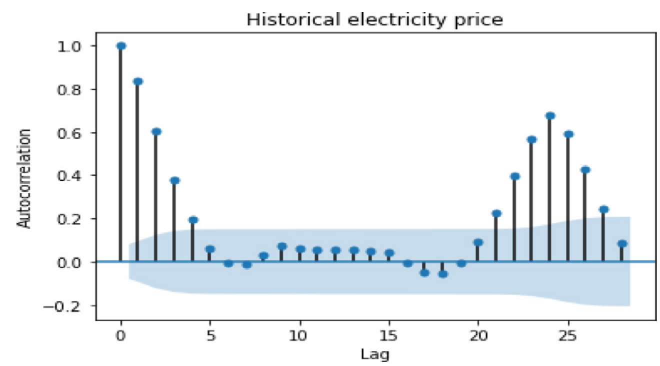

Fig. 1. Autocorrelation plot of historical electricity price. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

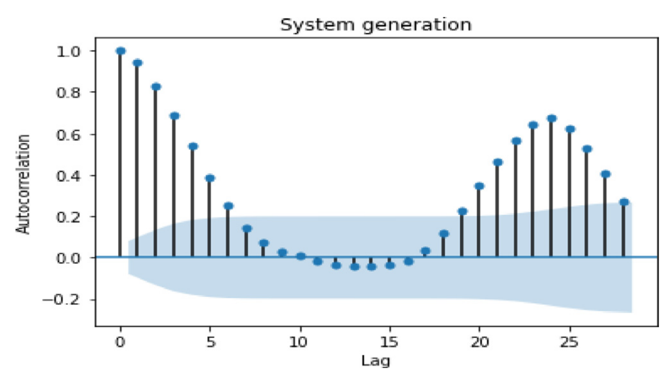

Fig. 2. Autocorrelation plot of system generation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

falling outside the 95% confidence interval (interval is displayed as a blue boundary in the figures). The ACF plots for each individual energy-related factor are shown in Figs. 1–10.

For the majority of the factors, the lagged term at hour 24 demonstrated a strong correlation, which compares to Li et al. (2016) where the same hour data displays strong correlation, therefore in Figs. 1–10 the first 28 h are displayed for each factor. All figures, except Figs. 4 and 7, displayed a multimodal distribution with a peak and trough pattern indicating correlation. Figs. 4 and 7 display weak to no correlation and hence in these situations we only use the first six lagged values.

Using all the lags for each of the exogenous variables, we identify and extract the peak lags using autocorrelation testing. The peak lags for each of the exogenous variables are used as individual inputs in the multiple input single output NARMAX model to determine if correlated lags do improve model accuracy. Table 2 displays the 44 peak lags for the ISEM market, from each individual ACF plot, to be used as inputs in the NARMAX model. For example the peak lags from Fig. 1 were identified as Lags 1, 2, 23, and 24.

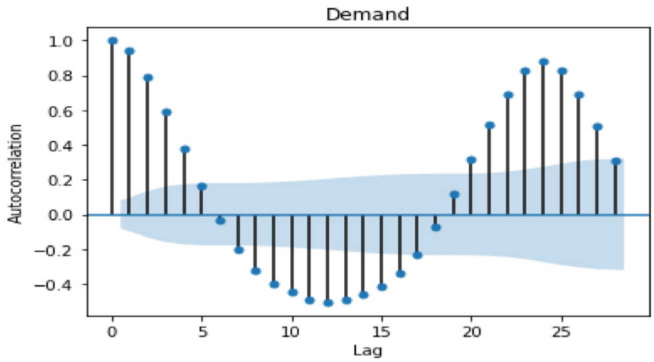

Fig. 3. Autocorrelation plot of demand. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

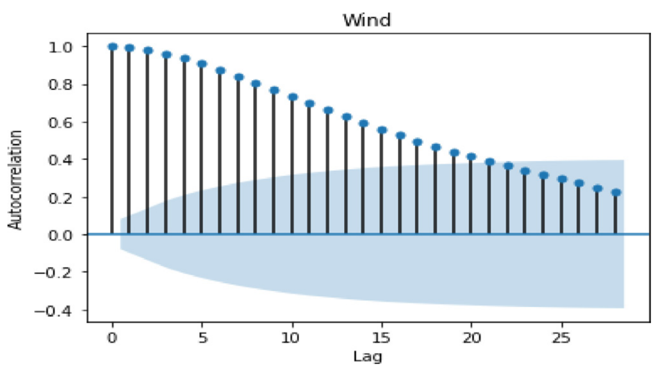

Fig. 4. Autocorrelation plot of wind. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

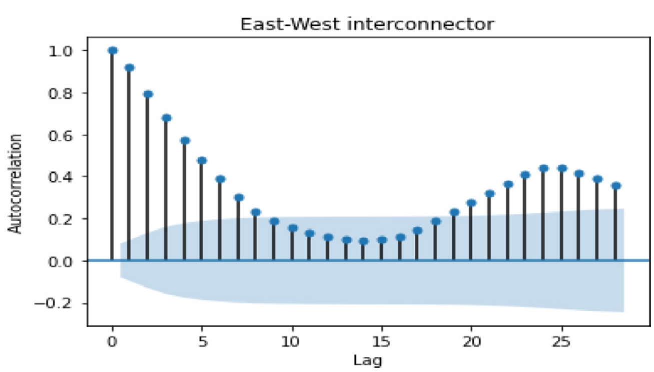

Fig. 5. Autocorrelation plot of East–West interconnector. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

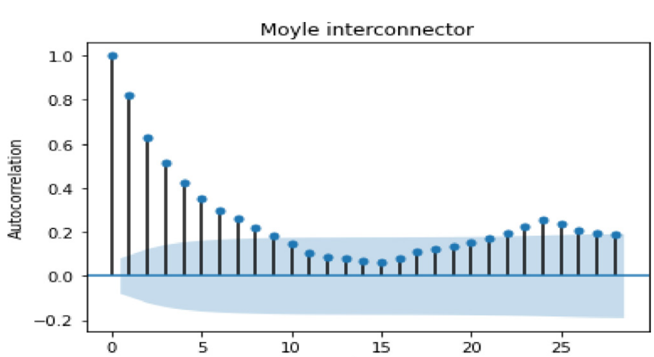

Fig. 6. Autocorrelation plot of Moyle interconnector. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

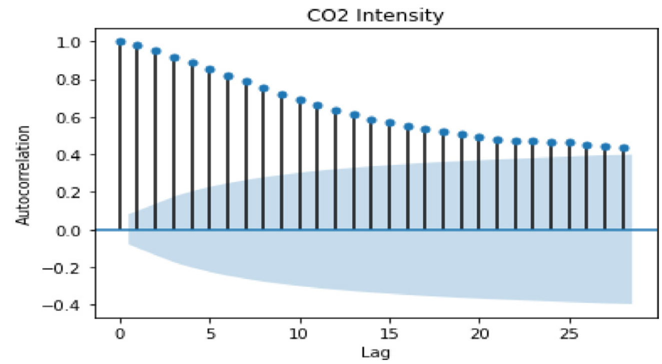

Fig. 7. Autocorrelation plot of CO_2 intensity. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

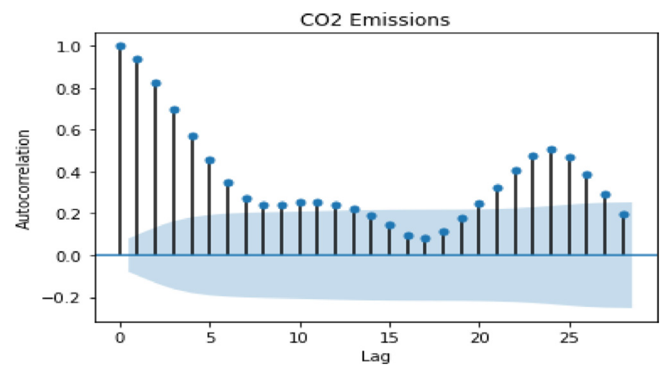

Fig. 8. Autocorrelation plot of CO_2 emissions. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

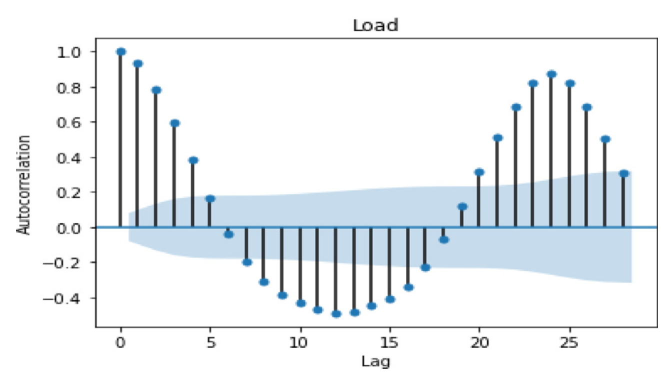

Fig. 9. Autocorrelation plot of load. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

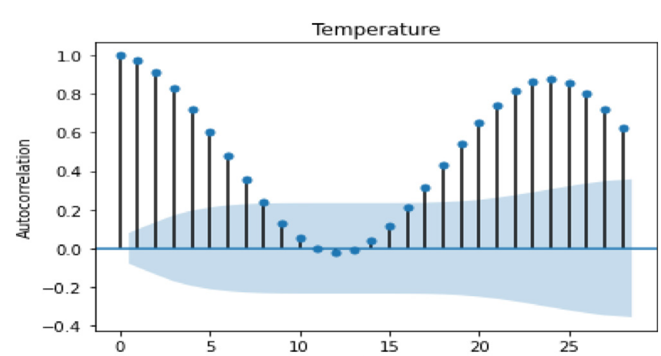

Fig. 10. Autocorrelation plot of temperature. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

6. Results and discussion

Using the NARMAX methodology, we generate a linear polynomial NARMAX model for prediction which utilises only those significant factors that have been deemed to be highly correlated. Including influential energy-related factors as model inputs will help to determine an accurate forecasting tool for the ISEM market. The experiments used ISEM data from May 2019 until April 2020 with a total of 8760 records. The input data ranged from 01st May 2019 until 29th April 2020 and the output data, which was the target day-ahead electricity price, ranged from 02nd May 2019 until 30th April 2020. Each experiment resulted in a linear polynomial model, where the ERR was set to 0.05.

Initially all energy-related factors were included as inputs and after each iteration NARMAX removed redundant factors until the model converged and contained only the significant factors. The resulting NARMAX model is represented as:

$$Y_t = 0.38U_1 - 0.00053U_2 + 0.00086U_3 + 0.00082U_7 - 0.00080U_8 + 0.0021U_9 + 0.26U_{10} + 31.72 \tag{3}$$

where each of the parameters are as defined in Table 1. Here we use 50% of the data for model estimation and 50% for model validation. The Root Mean Squared Error (RMSE) value for the model estimation was 12.58 and the RMSE value for the model validation was 15.15. The final model retained seven significant factors which were historical electricity price, system generation, demand, CO2 intensity, CO2 emissions, load, and temperature.

The percentage variance of each significant factor is presented in Table 3, ranking from largest to smallest ERR value. The three largest ERR values approximated to 35.01 proportion of the variance and this proportion was made up of historical electricity price (30.65), demand (2.85), and system generation (1.51). Historical electricity price was the most weighted factor (0.38) and the largest ERR value (30.65). From the results, both historical electricity price and demand have a large influence on predicting electricity prices. These findings are consistent with Li et al. (2016) who found energy production and demand to be key forecasting factors.

The results from the final NARMAX model validation are displayed in Fig. 11. From Fig. 11, it is clear that the predicted electricity price is a reasonable fit with the actual electricity price however the predicted values struggled to reach the peaks and troughs found in the actual price data. This finding is similar to Kavanagh (2017) noting the appearance of daily load patterns moving in peaks and troughs.

For the second experiment, all energy-related factors and their correlated peak lags were included and after each iteration NARMAX removed redundant factors until the optimal model was obtained. The resulting NARMAX model that included the correlated lags is represented as:

$$\begin{aligned} Y_t = {} & 0.32U_1 - 0.074U_{1(t-2)} + 0.032U_{1(t-23)} + 0.029U_{1(t-24)} - 0.0020U_2 \\ & + 0.0010U_{2(t-1)} + 0.0002U_{2(t-2)} + 0.0006U_{2(t-3)} - 0.0008U_{2(t-22)} \\ & + 0.0007U_{2(t-23)} + 0.0016U_3 - 0.0015U_{3(t-1)} + 0.0033U_{3(t-2)} \\ & - 0.0036U_{3(t-3)} - 0.0012U_{3(t-22)} + 0.0005U_5 + 0.0038U_7 \\ & + 0.0015U_8 - 0.0005U_{8(t-1)} - 0.0021U_{8(t-2)} + 0.0028U_9 \\ & + 0.0021U_{9(t-1)} - 2.52U_{10} + 2.54U_{10(t-1)} + 0.37U_{10(t-22)} + 12.34 \end{aligned} \tag{4}$$

The RMSE value for the model estimation was 12.10 and the RMSE value for the model validation was 15.02. These results are an improvement compared with the initial NARMAX model which contained no lagged terms. The resulting model retained 8 significant factors and 17 peak lags which consisted of historical electricity price, system generation, demand, East–West interconnector, CO2 intensity, CO2 emissions, load, and temperature. Similar to the initial NARMAX model, both wind and the Moyle interconnector were insignificant and therefore removed from the resulting model. However, when using the identified highly correlated lagged terms, NARMAX identified that the East–West interconnector data was significant.

For the 44 identified peak lags from the ACF plots, only 17 remained in the resulting model. Historical electricity price lag 1 was removed; system generation lag 24 was removed; demand lags 23 and 24 were removed; all the lags for wind, East–West interconnector, Moyle interconnector, and CO2 intensity were completely removed; CO2 emissions lags 3, 22, 23 and 24 were removed; load lags 2, 3, 22, 23 and 24 were removed; and temperature lags 2, 3, 23 and 24 were removed. For the majority of energy-related factors, the importance of Lags 1 and 2 indicates the importance of recent observations.

The percentage variance of each significant factor is presented in Table 4, ranking from largest to smallest ERR value. The three largest ERR values approximated to 34.20 proportion of the variance and this

Table 3
Error Reduction Ratio NARMAX model.

ERR	Energy-related Factors (Model Input)
30.653281	Historical electricity price (U1)
2.847335	Demand (U3)
1.511024	System generation (U2)
0.355038	Load (U9)
0.284880	CO2 intensity (U7)
0.179546	Temperature (U10)
0.059359	CO2 emissions (U8)

Table 4
Error Reduction Ratio correlated lags NARMAX model.

ERR	Energy-related Factors (Model Input)
30.653281	Historical electricity price (U1)
2.673473	Demand (U3)
0.879478	System generation (U2)
0.670583	Historical electricity price lag 23 (U1)
0.668477	Demand lag 3 (U3)
0.591910	Historical electricity price lag 24 (U1)
0.484539	System generation lag 23 (U2)
0.476113	Demand lag 22 (U3)
0.453832	Demand lag 2 (U3)
0.451905	System generation lag 3 (U2)
0.443696	Temperature lag 1 (U10)
0.353654	Demand lag 1 (U3)
0.298665	Load (U9)
0.262550	Historical electricity price lag 2 (U1)
0.236631	CO2 emissions lag 1 (U8)
0.214814	Temperature (U10)
0.199812	CO2 emissions lag 2 (U8)
0.166720	System generation lag 2 (U2)
0.110188	Temperature lag 22 (U10)
0.098272	CO2 intensity (U7)
0.091632	System generation lag 1 (U2)
0.081114	East–West interconnector (U5)
0.068008	System generation lag 22 (U2)
0.065651	Load lag 1 (U9)
0.050782	CO2 emissions (U8)

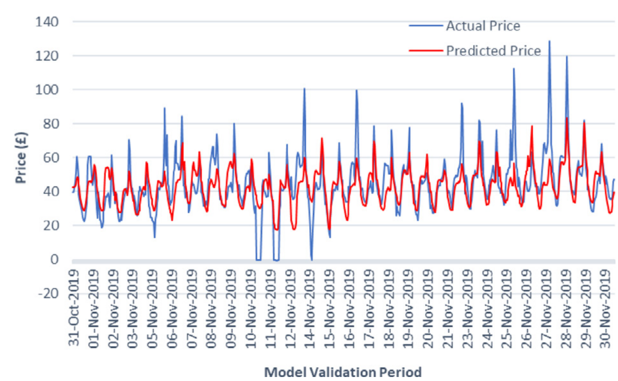

Fig. 11. Model validation for NARMAX model.

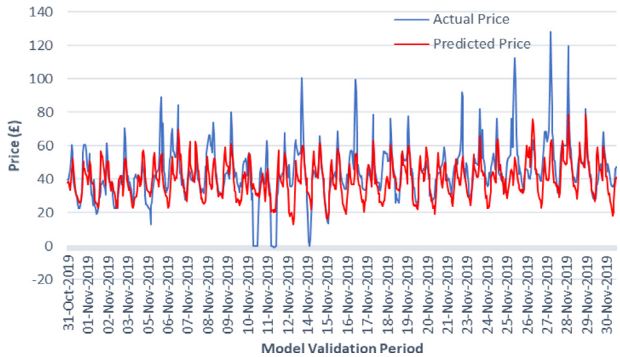

Fig. 12. Model validation for correlated lags NARMAX model.

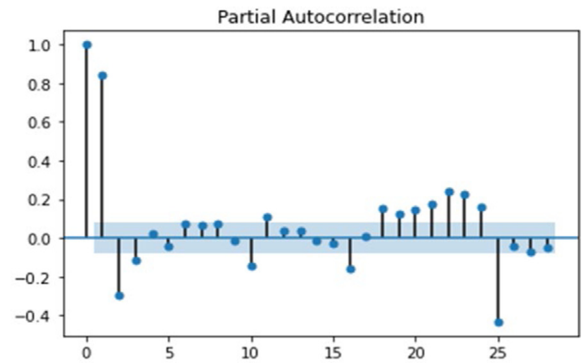

Fig. 13. PACF plot to determine p.

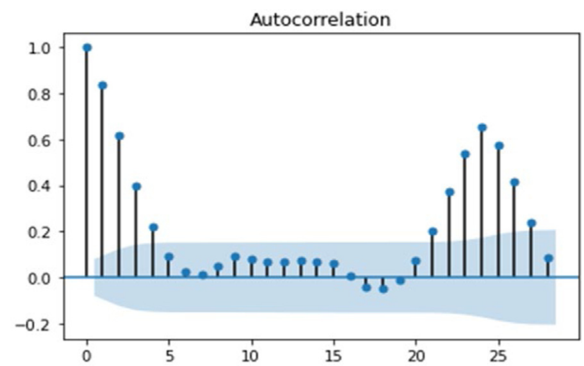

Fig. 14. ACF plot to determine q.

proportion was made up of historical electricity price (30.65), demand (2.67), and system generation (0.88). Temperature Lag 1 was the most weighted factor (2.54) and historical electricity price had the largest ERR value (30.65). From the results, historical electricity price has a large influence on predicting electricity prices as it has the largest ERR value for both the initial model and that with correlated lags. A visual representation of the actual and predicted prices using the resulting correlated lags NARMAX model are displayed in Fig. 12. From Fig. 12, the predicted electricity prices consistently matched the actual electricity prices but the predicted values still struggled to reach the peaks and troughs.

To justify the NARMAX results, a SARIMAX model was considered using correlated lag data:

$$\varphi_p(B)\Phi_P(B^S)(1-B)^d\left(1-B^S\right)^D Y_t = \beta_k x'_{k,t} + \theta_q(B)\Theta_Q(B^S)\varepsilon_t \tag{5}$$

where $\varphi_p(B)$ is the non-seasonal autoregressive term, $(1-B)^d$ is the non-seasonal differencing term, and $\theta_q(B)$ is the non-seasonal moving average term. $\Phi_P(B^S)$ is the seasonal autoregressive term, $\left(1-B^S\right)^D$ is the seasonal differencing term, $\Theta_Q(B^S)$ is the seasonal moving average term, Y_t is the prediction output, $\beta_k x'_{k,t}$ is the exogenous variable of the kth input at time t and ε_t is the error term (Vagropoulos, Chouliaras, Kardakos, Simoglou, & Bakirtzis, 2016).

There are four stages to the SARIMAX methodology in order to estimate and identify the suitable model terms: (1) model identification, (2) parameter estimation, (3) diagnostic checking, and (4) prediction. First, the order *terms p, d, q, P, D, Q* and *S* must be identified to determine an appropriate SARIMAX model. To remove non-stationarity, differencing was applied between current and previous electricity prices to make the series trend stationary; d is set to 1. Due to heterogeneity within the data, it was decided not to include seasonal differencing and thus D is set to 0. For seasonal data, the seasonal pattern must repeat itself over a time span S (Xie et al., 2013); S is set to 24 to capture the daily 24-hour recurring cycle.

The ranges for the parameter order terms (p, q, P, Q) are selected from the Partial AutoCorrelation Function (PACF) and AutoCorrelation Function (ACF) plots. From Fig. 13, the last significant autoregressive lag for p ranges between 1 and 4 and the significant lag for seasonal order P ranges between 1 and 2. From Fig. 14, the last significant moving average lag for q ranges between 1 and 4 and the significant lag for seasonal order Q ranges between 1 and 2.

The order terms are selected by applying the AIC technique, which uses a brute force search of all the combinations within the set range, to measure the quality of the model fit and verify the order terms chosen. The optimal order terms ($p = 3$, $q = 3$, $P = 2$, and $Q = 1$) are selected as they outputted the lowest AIC value (27478.86). The SARIMAX(3, 1, 3) (2, 0, 1, 24) correlated lags model function is:

$$\begin{aligned}
Y_t = {} & 0.023\nabla Y_{t-1} + 0.28\nabla Y_{t-2} + 0.26\nabla Y_{t-3} - 0.0019\nabla\varepsilon_{t-1} - 0.36\nabla\varepsilon_{t-2} \\
& - 0.37\nabla\varepsilon_{t-3} + 0.19S^{24}Y_{t-24} - 0.053S^{48}Y_{t-48} - 0.22S^{24}\varepsilon_{t-24} \\
& + 0.28U_1 - 0.023U_{1(t-1)} + 0.0008U_{1(t-2)} + 0.012U_{1(t-23)} \\
& + 0.10U_{1(t-24)} - 0.0004U_2 - 6.71^{e-06}U_{2(t-1)} + 0.0001U_{2(t-2)} \\
& + 0.0003U_{2(t-3)} - 6.61^{e-05}U_{2(t-22)} + 0.0006U_{2(t-23)} - 0.0002U_{2(t-24)} \\
& + 0.0014U_3 + 5.54^{e-05}U_{3(t-1)} - 2.03^{e-05}U_{3(t-2)} - 0.0009U_{3(t-3)} \\
& + 9.79^{e-05}U_{3(t-22)} + 0.0004U_{3(t-23)} - 0.0004U_{3(t-24)} + 0.0002U_4 \\
& + 2.17^{e-05}U_{4(t-1)} + 0.0004U_5 + 0.0002U_{5(t-1)} + 0.0003U_{5(t-2)} \\
& + 0.0004U_{5(t-3)} + 2.38^{e-05}U_{5(t-24)} - 0.0002U_6 + 5.92^{e-05}U_{6(t-1)} \\
& - 0.0007U_{6(t-2)} - 0.0004U_{6(t-3)} + 0.0001U_{6(t-24)} - 0.0013U_7 \\
& + 0.0006U_{7(t-1)} + 0.0006U_8 + 0.0001U_{8(t-1)} - 0.0003U_{8(t-2)} \\
& - 0.0004U_{8(t-3)} - 0.0002U_{8(t-22)} + 0.0001U_{8(t-23)} - 5.54^{e-05}U_{8(t-24)} \\
& + 0.0002U_9 + 0.0016U_{9(t-1)} + 0.0011U_{9(t-2)} + 0.0001U_{9(t-3)} \\
& - 0.0005U_{9(t-22)} - 0.0003U_{9(t-23)} + 0.0002U_{9(t-24)} - 0.38U_{10} \\
& + 0.48U_{10(t-1)} + 0.45U_{10(t-2)} - 0.58U_{10(t-3)} - 0.16U_{10(t-22)} \\
& - 0.44U_{10(t-23)} - 0.38U_{10(t-24)} + 30.77
\end{aligned} \tag{6}$$

To confirm if SARIMAX(3, 1, 3) (2, 0, 1, 24) is an appropriate model fit, diagnostic checking was performed on the standardised residuals. Fig. 15(A) plots the standardised residuals which fluctuate around 0, however there are many peaks and troughs from outliers; Fig. 15(B) is a histogram with a density plot (orange line) that is normally distributed with a narrow bell-shaped pattern symmetrical around 0; Fig. 15(C) is a normal quantile–quantile plot with the quantiles mainly on, or close to, the red line suggesting a normal distribution but the sharp curves at the ends highlight extreme data values that the model is unable to fit; Fig. 15(D) is a correlogram plot with slight autocorrelation present as a few lag errors fall outside the blue boundary.

Therefore, SARIMAX correlated lag model is an accurate prediction with a RMSE value of 14.36. Fig. 16 displays the actual and predicted electricity prices and the model fit is reasonably accurate with no extreme declines.

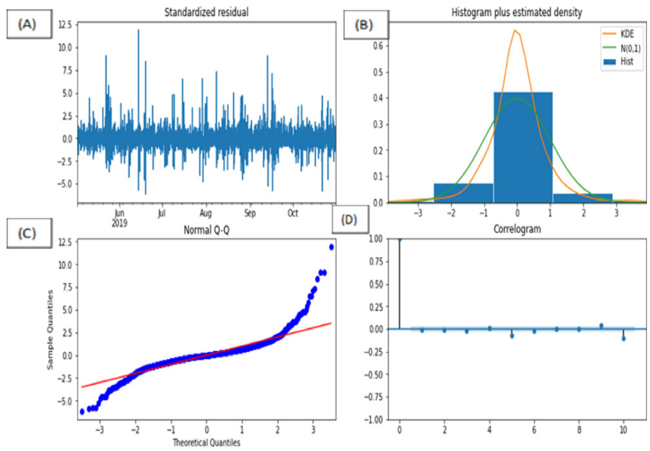

Fig. 15. Residual diagnostic checks for correlated lags SARIMAX model. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

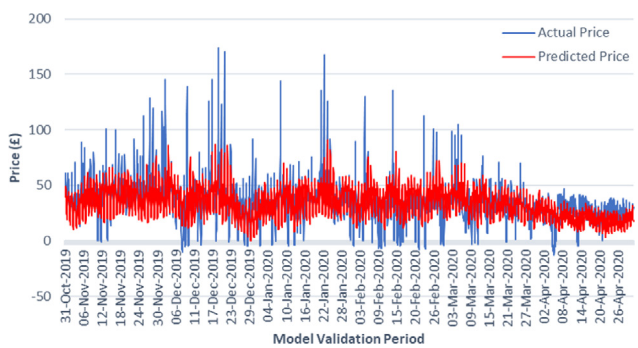

Fig. 16. Model validation for correlated lags SARIMAX model.

Table 5
RMSE values for each model.

Model	RMSE
NARMAX	15.15
Correlated NARMAX	15.02
Correlated SARIMAX(3, 1, 3)(2, 0, 1, 24)	14.36
Refined correlated SARIMAX(3, 1, 3)(2, 0, 1, 24)	13.99

An approach was applied to determine if the significant NARMAX factors could refined the statistical SARIMAX model and improve accuracy. The SARIMAX(3, 1, 3)(2, 0, 1, 24) correlated lags model was refined and the model function is:

$$\begin{aligned}
Y_t = {}& 0.065\nabla Y_{t-1} + 0.27\nabla Y_{t-2} + 0.20\nabla Y_{t-3} - 0.031\nabla\varepsilon_{t-1} - 0.36\nabla\varepsilon_{t-2} \\
& - 0.33\nabla\varepsilon_{t-3} + 0.17S^{24}Y_{t-24} - 0.053S^{48}Y_{t-48} - 0.19S^{24}\varepsilon_{t-24} \\
& + 0.30U_1 + 0.035U_{1(t-2)} + 0.013U_{1(t-23)} + 0.11U_{1(t-24)} \\
& - 0.0004U_2 - 5.36^{e-05}U_{2(t-1)} + 0.0003U_{2(t-2)} + 0.0004U_{2(t-3)} \\
& - 0.0002U_{2(t-22)} + 0.0006U_{2(t-23)} + 0.0013U_3 + 0.0003U_{3(t-1)} \\
& - 3.04^{e-05}U_{3(t-2)} - 0.0010U_{3(t-3)} - 0.0001U_{3(t-22)} + 0.0004U_5 \\
& - 0.0042U_7 + 0.0005U_8 + 0.0003U_{8(t-1)} - 0.0003U_{8(t-2)} \\
& + 0.0005U_9 + 0.0008U_{9(t-1)} - 0.48U_{10} + 0.30U_{10(t-1)} \\
& - 0.37U_{10(t-22)} + 31.09 \qquad (7)
\end{aligned}$$

The refined model's RMSE value of 13.99 was lower than the previous model's RMSE value of 14.36 highlighting that keeping significant factors helps improve model performance. Fig. 17 displays an accurate model fit between the actual and predicted electricity price values.

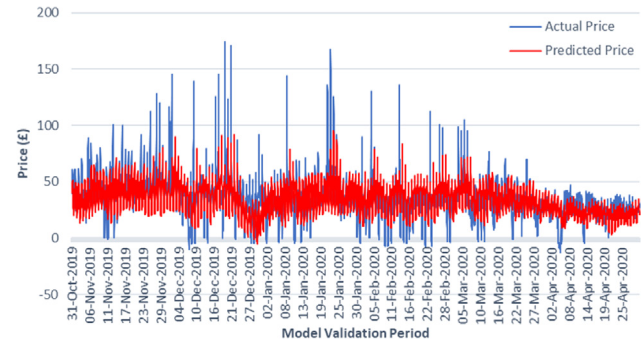

Fig. 17. Model validation for refined correlated lags SARIMAX model.

For each of the four experiments, RMSE values were compared to determine if any of the correlated lags or refined models could further improve model performance and day-ahead electricity price forecasting. From Table 5, it clear that both correlated lags and significant factors together improve model performance for the ISEM market.

7. Conclusion

This paper examined the performance of a polynomial NARMAX model with energy-related factors and evaluated the model's suitability to accurately predict electricity price in the ISEM market. Correlated peak lags were identified through autocorrelation testing of energy-related factors. For both the original and correlated lags models, the RMSE values were compared to determine if model accuracy can be further improved with the inclusion of correlated peak lags. The findings in this paper emphasise that correlated lags for significant energy-related factors identified from a regression model do help to refine a statistical model and improve model accuracy. The significant energy-related factors from the ISEM market were found to be historical electricity price, demand, and system generation. One limitation of this paper is that the NARMAX models only considered one threshold error (ERR = 0.05) and another limitation is that the input data included all historical records with no split for weekends, holidays, etc. Future work will explore applying the identified significant energy-related factors from NARMAX as inputs to other machine learning models to further improve prediction accuracy.

8. Remarks

This paper applied autocorrelation testing to energy-related factors from the ISEM market to determine peak correlated lags. A NARMAX regression model identified the significant energy-related factors and refined the SARIMAX statistical model to determine if performance accuracy improved. This was determined by the RMSE values which highlighted that the inclusion of significant factors and their respective correlated lags do improve model accuracy for day-ahead forecasting in the ISEM market.

CRediT authorship contribution statement

Catherine McHugh: Conceptualization, Methodology, Validation, Writing – original draft. **Sonya Coleman:** Conceptualization, Supervision, Writing – review & editing. **Dermot Kerr:** Conceptualization, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This research was funded by DfE CAST scholarship, UK in collaboration with Click Energy.

References

Acuna, G., Ramirez, C., & Curilem, M. (2012). Comparing NARX and NARMAX models using ANN and SVM for cash demand forecasting for ATM. In *Proceedings of the international joint conference on neural networks, Vol. 1* (pp. 0–15). http://dx.doi.org/10.1109/IJCNN.2012.6252476.

Altan, A., Karasu, S., & Bekiros, S. (2019). Digital currency forecasting with chaotic meta-heuristic bio-inspired signal processing techniques. *Chaos, Solitons & Fractals*, *126*, 325–336. http://dx.doi.org/10.1016/j.chaos.2019.07.011.

Amisigo, B. A., van de Giesen, N., Rogers, C., Andah, W. E. I., & Friesen, J. (2008). Monthly streamflow prediction in the volta basin of west Africa: A SISO NARMAX polynomial modelling. *Physics and Chemistry of the Earth*, *33*(1–2), 141–150. http://dx.doi.org/10.1016/j.pce.2007.04.019.

Amjady, N., & Hemmati, M. (2006). Energy price forecasting: Problems and proposals for such predictions. *IEEE Power and Energy Magazine*, *4*, 20–29. http://dx.doi.org/10.1109/MPAE.2006.1597990.

Billing, S. A., & Voon, W. S. F. (1985). *Correlation based model validity tests for nonlinear models*: *Acse report 285*, Retrieved from http://eprints.whiterose.ac.uk/76926.

Billings, S., & Coca, D. (2001). Identification of NARMAX and related models. In *Control systems, robotics, and automation, Vol. VI*. Retrieved from http://www.eolss.net/sample-chapters/c18/E6-43-10-03.pdf%5Cnhttp://www.eolss.net/sample-chapters/c18/e6-43.pdf.

Billings, S. A., & Fadzil, M. B. (1985). The practical identification of systems with nonlinearities. *IFAC Proceedings Volumes*, *18*(5), 155–160. http://dx.doi.org/10.1016/S1474-6670(17)60551-2.

Boynton, R. J., Balikhin, M. A., Billings, S. A., Wei, H. L., & Ganushkina, N. (2011). Using the NARMAX OLS-ERR algorithm to obtain the most influential coupling functions that affect the evolution of the magnetosphere. *Journal of Geophysical Research, Space Physics*, *116*(5), 1–8. http://dx.doi.org/10.1029/2010JA015505.

Cerjan, M., Matijaš, M., & Delimar, M. (2014). Dynamic hybrid model for short-term electricity price forecasting. *Energies*, *7*(5), 3304–3318. http://dx.doi.org/10.3390/en7053304.

EirGrid (2020). Energy-related generation. Retrieved November 2, 2020, from https://www.smartgriddashboard.com/#all.

Gao, G., Lo, K., & Fan, F. (2017). Comparison of ARIMA and ANN models used in electricity price forecasting for power market. *Energy and Power Engineering*, *09*(04), 120–126. http://dx.doi.org/10.4236/epe.2017.94B015.

García-Martos, C., Rodríguez, J., & Sánchez, M. J. (2007). Mixed models for short-run forecasting of electricity prices: Application for the Spanish market. *IEEE Transactions on Power Systems*, *22*(2), 544–552. http://dx.doi.org/10.1109/TPWRS.2007.894857.

Ghalehkhondabi, I., Ardjmand, E., Weckman, G. R., & Young, W. A. (2017). An overview of energy demand forecasting methods published in 2005–2015. *Energy Systems*, *8*(2), 411–447. http://dx.doi.org/10.1007/s12667-016-0203-y.

Huurman, C., Ravazzolo, F., & Zhou, C. (2012). The power of weather. *Computational Statistics & Data Analysis*, *56*(11), 3793–3807. http://dx.doi.org/10.1016/j.csda.2010.06.021.

Karasu, S., Altan, A., Sarac, Z., & Hacioglu, R. (2017). Prediction of solar radiation based on machine learning methods. *The Journal of Cognitive Systems*, *2*(1), 16–20, Retrieved from https://www.dergipark.gov.tr/jcs.

Kavanagh, K. (2017). Short term demand forecasting for the integrated electricity short term demand forecasting for the integrated electricity market market short term demand forecasting for the integrated single electricity market. *Student Journal of Energy Research*, *2*(1), http://dx.doi.org/10.21427/D75G90.

Korenberg, M., Billings, S. A., & Liu, Y. P. (1987). *An orthogonal parameter estimation algorithm for nonlinear stochastic systems*: *Acse report 307*.

Li, P., Arci, F., Reilly, J., Curran, K., & Belatreche, A. (2016). Using artificial neural networks to predict short-term wholesale prices on the Irish single electricity market. In *2016 27th irish signals and systems conference* (pp. 1–10).

Met Office IE (2020). Temperature. Retrieved November 2, 2020, from https://www.metoffice.gov.uk/research/climate/maps-and-data/historic-station-data.

MET Office UK (2020). Temperature. Retrieved November 2, 2020, from https://www.metoffice.gov.uk/services/data/datapoint.

Mosbah, H., & El-Hawary, M. (2016). Hourly electricity price forecasting for the next month using multilayer neural network. *Canadian Journal of Electrical and Computer Engineering*, *39*, 283–291. http://dx.doi.org/10.1109/CJECE.2016.2586939.

Nehmzow, U. (2006). *Scientific methods in mobile robotics*. Springer-Verlag LondonLimited2006.

Nogales, F. J., Contreras, J., Conejo, A. J., & Espínola, R. (2002). Forecasting next-day electricity prices by time series models. *IEEE Transactions on Power Systems*, *17*(2), 342–348. http://dx.doi.org/10.1109/TPWRS.2002.1007902.

Pagano, D. J., Filho, V. D., & Plucenio, A. (2006). Identification of polinomial Narmax models for an oil well operating by continuous gas-lift. *IFAC Proceedings Volumes*, *39*(2), 1113–1118. http://dx.doi.org/10.3182/20060402-4-BR-2902.01113.

Pandey, N., & Upadhyay, K. G. (2016). Different price forecasting techniques and their application in deregulated electricity market : A comprehensive study. In *International conference on emerging trends in electrical, electronics and sustainable energy systems* (pp. 1–4). http://dx.doi.org/10.1109/ICETEESES.2016.7581342.

SEMO (2020). Daily load forecast. Retrieved November 2, 2020, from https://www.sem-o.com/market-data/dynamic-reports/index.xml#BM-010.

SEMOpx (2020). Day-ahead electricity price. Retrieved November 2, 2020, from https://www.semopx.com/market-data/market-results/.

Taib, M. N. (1993). *Time series modelling and prediction using neural networks*. Thesis, University of Sheffield.

Vagropoulos, S. I., Chouliaras, G. I., Kardakos, E. G., Simoglou, C. K., & Bakirtzis, A. G. (2016). Comparison of SARIMAX, SARIMA, modified SARIMA and ANN-based models for short-term PV generation forecasting. In *Energycon 2016*, *2016 IEEE international energy conference* (pp. 1–6). http://dx.doi.org/10.1109/ENERGYCON.2016.7514029.

Vijayalakshmi, S., & Girish, G. P. (2015). Artificial neural networks for spot electricity price forecasting: A review. *International Journal of Energy Economics and Policy*, *5*(4), 1092–1097, Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84944450808&partnerID=tZOtx3y1.

Voronin, S., & Partanen, J. (2013). Price forecasting in the day-ahead energy market by an iterative method with separate normal price and price spike frameworks. *Energies*, *6*(11), 5897–5920. http://dx.doi.org/10.3390/en6115897.

Warnes, M. R., Glasseyfl, J., Montague, G. A., & Kara, B. (1996). On data-based modelling techniques for fermentation processes, vol. 31. (2), (pp. 147–155).

Xie, M., Sandels, C., Zhu, K., & Nordström, L. (2013). A seasonal ARIMA model with exogenous variables for elspot electricity prices in Sweden. In *2013 10th international conference on the european energy market* (pp. 1–4). http://dx.doi.org/10.1109/EEM.2013.6607293.

Zhang, Y., Hua, X., & Zhao, L. (2012). Exploring determinants of housing prices: A case study of Chinese experience in 1999–2010. *Economic Modelling*, *29*(6), 2349–2361. http://dx.doi.org/10.1016/j.econmod.2012.06.025.

Zito, G., & Landau, I. D. (2005). A methodology for identification of NARMAX models applied to diesel engines. In *IFAC world congress, Vol. 16* (pp. 374–379). Elsevier, http://dx.doi.org/10.3182/20050703-6-CZ-1902.00063.