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Does a currency union need a capital market union?☆

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ABSTRACT

We compare risk sharing in response to demand and supply shocks in four types of currency unions: segmented markets; a money market union; a capital market union; and complete financial markets. We show that a money market union is efficient at sharing domestic demand shocks (deleveraging, fiscal consolidation), while a capital market union is necessary to share supply shocks (productivity and quality shocks). In a numerical exercise, we find that the welfare gain of moving from segmented markets to a money market union is of roughly similar magnitude to that of moving from a money market to a capital market union.

Failures of risk sharing lie at the heart of many economic crises, notably the one that threatened the survival of the Eurozone in the early 2010s. Private deleveraging led to a recession in the Eurozone as it did in the US. The Eurozone crisis, however, was greatly amplified by the fragmentation of the Eurozone money and deposit markets. Banks' funding costs diverged between countries as the perceived risk of runs increased in vulnerable countries. Finally, banks passed on their funding costs to private agents, firms and households.

The critical failure was the *increase* in funding costs in countries hit by deleveraging shocks (Martin and Philippon, 2017). This is the exact opposite of what an efficient response should look like. In Eggertsson and Krugman (2012), the optimal response to a deleveraging shock is to lower the interest rate to induce savers to increase their consumption (in a model with only households), or firms to increase their investment (in a model with capital accumulation) to pick up the slack left by deleveraging agents. In

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Table 1
Summary of results.

	Definition	Demand shocks	Supply shocks
SM	$R_{j,t} \neq R_t$ for some (j, t)	worse than MMU	worse than MMU
MMU	$R_{j,t} = \bar{R}_t$ for all (j, t)	= COMP	worse than CMU
CMU	Optimal equity portfolio	= COMP	= COMP
COMP	Backus-Smith-Kollmann condition	= COMP	= COMP

This table summarizes the main theoretical results of Sections 2 and 3. $R_{j,t}$ is the risk-free interest rate for a small country in a currency union j at time t and $\bar{R}_t$ is the union-wide risk-free interest rate. SM = Segmented Markets, MMU = Money Market Union, CMU = Capital Market Union, COMP = Complete Markets.

Europe, the negative feedback loop between economic activity and banks and sovereign spreads led to a collapse in demand from non-deleveraging agents.

To forestall this damaging feedback loop Eurozone policy makers have undertaken a suite of policy initiatives known collectively as a *banking union*. A key feature of the banking union is the creation of what we call a *money market union*: a currency union in which country-specific borrowing rates are invariant to country-specific shocks such as private and public deleveraging shocks.

A large literature explains the specific policy steps that are required to achieve a money market union (Véron, 2012). In this paper, we take these policy steps as given and we ask: how much macro risk-sharing does an ideal money market union provide? To answer this question we develop a model with the minimal elements needed to meaningfully study deleveraging shocks in a currency union: a small open economy model of a currency union, with domestic borrowers and savers, and some degree of nominal rigidity. In this environment, we find that conditional on an economy facing demand, including deleveraging, shocks, the money market union provides as much risk sharing as the complete markets outcome.

What are the limits to risk sharing provided by a money market union (MMU)? We frame this question in the context of a second key policy initiative in the Eurozone, the construction of *a capital market union*. Again, our focus is on studying the key macro implications of these policies. We model a capital market union (CMU) as a market structure that allows frictionless sharing of risk to the market value of private capital. In our model, claims to the value of capital most closely resemble traded corporate equity, and our ideal capital market union is one in which domestic savers optimize portfolio allocations frictionlessly.

We find that, unlike the money market union, the CMU attains the complete markets allocation also conditional on supply shocks. Evaluating numerically the welfare changes of moving from a currency union with the risk of money market segmentation (SMU) to an ideal MMU, and from the MMU to a CMU, we find consumption-equivalent gains of a similar magnitude from each.

Table 1 summarizes our findings. Our theory shows that, even though deleveraging creates an aggregate drag on the economy, borrowing and lending across regions allows an efficient sharing of the burden of adjustment. This result is based on a surprising symmetry in demand effects. Deleveraging causes a recession and therefore initially lowers the labor income of savers; however, the lower debt burden of borrowers leads to higher demand in the future, which increases the future income of savers. In the benchmark small country model with Cole-Obstfeld preferences these two effects exactly offset each other so that neither the net present value of savers' nominal income nor their nominal consumption expenditure changes. The result holds approximately in more general models, including relaxing the assumption of small countries such that idiosyncratic shocks have a spillover effect on other countries. However, it crucially requires that funding costs are equalized across regions.

Our theory therefore lends support to the view that preventing the fragmentation of currency union-wide money markets is the key benefit of a banking union from the point of view of macro stabilization. Of course, risky rates can differ across regions with different economic conditions, but this divergence corresponds to the efficient pricing of credit risk, not to divergence in money market funding costs. Specifically in the context of the Eurozone crisis, Martin and Philippon (2017) and Gourinchas et al. (2016) find that spread divergence per se was one of the key amplifiers of deleveraging-induced recessions.

In the paper, we abstract away from the specific policy steps required to achieve MMU, a subject that is by now well understood.[1] Véron (2012) provides an overview of the main institutional features. Faria-e Castro et al. (2016) explain in detail why a fiscal backstop is required to avoid runs and implement credible stress tests that are the cornerstones of modern banking regulation.

We find that a CMU is necessary for the efficient sharing of other shocks (supply shocks). These shocks have a first order effect on the market values of assets and therefore can only be shared with cross-border claims on private capital. To the extent that these types of shocks are important drivers of the business cycle in a currency union, the welfare benefits of a CMU are likely to be significant. Estimating a three-country model of Spain, the Eurozone and the rest of the world, in't Veld et al. (2014) find that technology and trade shocks account for more than a third of the variance in Spanish GDP and a quarter of the variance in consumption. In a similar exercise focused on Germany, Kollmann et al. (2014) find that both supply and demand shocks contributed significantly to the evolution of the German current account following the introduction of the Euro. Taken together, this body of evidence suggests that there might be significant additional benefits to the creation of CMU over and above the creation of a MMU; consistent with this interpretation, in the numerical exercise of Section 4 we find welfare gains of similar magnitudes for MMU and CMU.

[1] In Appendix B we present a simple banking model that is consistent with our definition of a money market union.

The structure of this paper is the following. Section 1 introduces the basic model structure. Section 2 studies the risk sharing properties of a money market union and Section 3 those of capital market union. Section 4 calculates welfare gains from moving from SMU to MMU and MMU to CMU.

Related literature

Our paper is related to various lines of research in international macroeconomics as well as studies of the causes and consequences of the Eurozone crisis. The optimal currency area pioneered by Mundell (1961) recognized the importance of a risk sharing mechanism. Kenen (1969) argued that such risk sharing should be organized through inter-regional fiscal transfers. However, Mundell (1973) notes that sophisticated financial markets might provide full insurance.

Backus et al. (1992) study a two-country, one-good real business cycle model with complete markets. They find that the model is unable to match key properties of international business cycles. Baxter (1995), Baxter and Crucini (1995) and Kollmann (1996) consider similar models with trading in a non-contingent bond. They find that relaxing complete markets can improve the numerical performance of the model though the difference to the complete markets allocation may depend on the persistence of shocks.

Cole and Obstfeld (1991) analyze a two-country, two-good endowment economy with flexible prices and show that adjustments to the terms of trade provide insurance against country specific shocks. Heathcote and Perri (2002) analyze production economies and find that models with asset market segmentation match cross-country correlations better than the complete markets model. Kehoe and Perri (2002) endogenize the incompleteness of markets by introducing enforcement constraints that require each country to prefer the allocation it receives by honoring its liabilities rather than living in autarky from any given time onward.

Obstfeld and Rogoff (1995) introduce nominal rigidities in the style of New Keynesian business cycle models into the open economy framework. Ghironi (2006) provides a discussion of this literature and emphasizes the difficulties in modeling market incompleteness. Gali and Monacelli (2008) circumvent the issue by assuming complete asset markets. This is also the approach followed by Blanchard et al., 2017 who model the Eurozone as a two-country (core and periphery) model.

There is a large literature on risk sharing in currency unions. Bayoumi and Masson (1995) discuss the issue of risk sharing and fiscal transfer before the creation of the Euro, and Asdrubali et al. (1996) provide evidence for the US. The Eurozone crisis spurred interest in this topic. Lane (2012) provides a detailed account of the principal drivers of the Eurozone crisis; the specific role of the boom/bust cycle in capital flows is analyzed by Lane (2013) and Gourinchas and Obstfeld (2012). Martin and Philippon (2017) provide a framework and an identification strategy to study the Eurozone crisis. They decompose each country's dynamics into three components: private leverage cycles, sovereign risks, and sudden stops/banking crises. They find that credit spreads play an important role in exacerbating the Eurozone crisis. We extend their analysis to study analytically what type of market integration is necessary for the efficient sharing of different types of shocks. We also enhance their analysis by modeling aggregate demand spillovers and monetary policy. in't Veld et al. (2014) study the joint dynamics of real activity and capital flows for the Spanish economy in a three-country model; they find a prominent role for a tightening of collateral constraints in driving the 2010s crisis in Spain. Bolton and Jeanne (2011) analyze the transmission of sovereign debt crises through the banking systems of financially integrated economies. Hepp and von Hagen (2013) provide evidence from Germany and Afonso and Furceri (2008) from the EMU. Schmitt-Grohe and Uribe (2016) emphasize the role of downward wage rigidity. Farhi and Werning (2017) analyze risk sharing in a currency union in a model with nominal rigidities. They show that fixed exchange rates increase the value of risk sharing and that complete markets do not lead to constrained efficient risk sharing. Using a similar model, Auray and Eyquem (2014) argue that complete markets can lead to lower welfare than financial autarky. Hoffmann et al. (2018) find that the introduction of the euro led to a more integrated interbank market, yet had little effect on cross-border bank-to-firm lending.

A common thread in both IRBC and NOE research is that the composition of financing flows is not discussed in detail beyond distinguishing between complete markets and non-contingent bond economies, as explained in Devereux and Sutherland (2011b) and Coeurdacier and Rey (2012). The authors provide a simple approximation method for portfolio choice problems in general equilibrium models that are solved using first-order approximations around a non-stochastic steady state. A few papers address specifically one of the enduring puzzles in open economy macroeconomics, the home equity bias puzzle. Coeurdacier and Gourinchas (2016) solve jointly for the optimal equity and bond portfolio in an environment with multiple shocks. In Heathcote and Perri (2013), home bias arises because endogenous international relative price fluctuations make domestic assets a good hedge against labor income risk. Sihvonen (2018) studies the aggregate effects of equity home bias in a model that features nominal rigidities and fixed exchange rates. Fornaro (2018) and Benigno and Romei (2014) study the effect of deleveraging shocks in open economies with nominal rigidities. Fornaro (2018) compares the consequences of a tightening of the exogenous borrowing limit in Bewley economies with and without nominal rigidities and fixed exchange rates. Benigno and Romei (2014) consider a two-country model in which one country is a net debtor and the other is a creditor. They analyze the effect of a tightening in the borrowing limit. The literature on sudden stops in emerging markets (Mendoza and Smith, 2006; Mendoza, 2010; Chari et al., 2005) focuses on the imposition of an external credit constraint. These models are couched in representative agent frameworks and do not account for domestic credit flows. On the other hand, borrower-saver models, (see e.g. Eggertsson and Krugman, 2012), and more generally two-agent New Keynesian models (Bilbiie, 2008; Debortoli and Gali, 2017) lack the international dimension. Our paper instead presents a model that can account for both domestic and external capital flows, which is important for our results.

Finally, some papers have studied the insurance properties of a riskless bond in partial equilibrium settings or endowment economies. Yaari (1976) shows that a patient consumer can self-insure against transitory income shocks through borrowing and lending. This self-insurance property is generally important in heterogeneous agent models with incomplete markets (see e.g. Aiyagari, 1994). Levine and Zame (2004) consider a single good endowment economy. They show that when agents are perfectly patient and endowment shocks are transitory and idiosyncratic, the equilibrium with trading in a single bond attains the complete market outcome. However, we do not assume that shocks are transitory. Rather, we endogenize transitory income effects and we show that, due to general equilibrium effects, demand shocks do not affect savers' nominal wealth or nominal consumption even when they are permanent. Unlike Levine and Zame (2004), we do not assume that agents are perfectly patient and allow for a discount factor below one.

1. Model

We consider a currency union consisting of a continuum of small countries, each of which is populated by a measure of infinitely lived households (as in Gali and Monacelli, 2008); as we explain below, many of our results extend to the case of a finite number of countries. Each country produces a tradable domestic good and households consume both domestic and foreign goods. Following Mankiw (2000) and Eggertsson and Krugman (2012) we assume that households are heterogeneous in their degree of time preference: within each country there is a fraction χ of impatient households, and a fraction $1-\chi$ of patient ones. Patient households (indexed by s for savers) have a higher discount factor than borrowers (indexed by b for borrowers): $\beta \equiv \beta_s > \beta_b$. To economize on notation we treat parameters as constant when we present the model, although later we will be studying shocks to some of the model parameters.

We consider three different versions of this currency union, that differ in the menu of traded assets available to savers. In the segmented market union (SMU), savers can save in nominal bonds with a return that may differ from that in other countries in the union. In the money market union (MMU), the return on nominal bonds is equalized across the union. The capital market union (CMU) adds to MMU the possibility of saving in stocks, which are claims on profit streams from all other countries in the union.

1.1. Preferences and technology

We introduce equilibrium conditions for the home country, but they are defined analogously for the other countries. Households of each type (borrower or saver) derive utility from consumption and labor through Cole-Obstfeld preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta_i^t\left[\log \mathbf{C}_{i,t}-\nu(N_{i,t})\right],\ \text{for}\ i=b,s,$$

where $\mathbf{C}_{i,t}$ is a composite good that aggregates goods produced by the home (C_h) and foreign (C_f) countries

$$\log \mathbf{C}_{i,t}=(1-\alpha)\log\left(C_{h,i,t}\right)+\alpha\log\left(C_{f,i,t}\right),$$

and $\alpha<\frac{1}{2}$ is a measure of the openness to trade of the economy; equivalently, $1-\alpha$ measures home bias in consumption.[2] The home good is a composite bundle of intermediate goods produced and aggregated into the final consumption home good using the following constant elasticity (ε) of substitution technologies:

$$C_{h,i}=\left[\int_0^1 c_i(j)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The foreign good is a composite bundle of goods produced in the different countries and aggregated into a final good via the technology

$$\log C_{f,i}=\int_0^1\log\left(C_{k,i}\right)\,\mathrm{d}k.$$

[2] With discount rate shocks the borrowers problem is

$$\mathbb{E}_t\sum_{t=0}^{\infty}\prod_{k=0}^{t}\beta_{b,k}\left[\log C_{b,t}-\nu(N_{b,t})\right]$$

.

Similarly to the home good, each such foreign good is in turn a composite bundle of intermediate goods:

$$C_{k,i} = \left[\int_0^1 c_{k,i}(j)^{\frac{\varepsilon - 1}{\varepsilon}} \, \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon - 1}}.$$

With these preferences, the home consumption-based price index (CPI) is

$$\mathbf{P} = (P_h)^{1 - \alpha}\left(P_f\right)^{\alpha}.$$

Here the domestic producer price index is

$$P_h = \left[\int_0^1 p(j)^{1 - \varepsilon} \, \mathrm{d}j\right]^{\frac{1}{1 - \varepsilon}},$$

where $p(j)$ are prices of intermediate goods and

$$P_f = \exp \int_0^1 \log (P_k) \, \mathrm{d}k.$$

Similarly for each foreign country the producer price index is

$$P_k = \left[\int_0^1 p_k(j)^{1 - \varepsilon} \, \mathrm{d}j\right]^{\frac{1}{1 - \varepsilon}}.$$

The production of intermediate goods in all countries is linear in labor, $y_{k,i} = A_k N_i$, where A_k is total factor productivity in country k and N_i is firm i's labor demand. In Section 4 we introduce capital into the production function in order to conduct more plausible welfare analysis.

1.2. Wages and prices

We assume a general form for the wage setting function $W_t = g(z^t)$, where z^t denotes the history of state variables up to time t. Following Martin and Philippon (2017), we assume that labor demand is rationed uniformly across households, which makes the analysis more tractable (as we do not need to keep track separately of the labor income of patient and impatient households within a country), and is empirically more plausible than assuming that savers and borrowers have separate labor supply curves.[3] Anticipating the results in Sections 2 and 3, the specific form of wage setting is immaterial to our theoretical results, which characterize the behaviour of *nominal* variables (output and consumption of savers). To be sure, the precise form of wage setting determines the dynamics of *real* variables, and hence the welfare properties of the economies we study. In Section 4 we compare welfare across different types of types of currency unions and to do so we assume sticky wages and introduce a wage Phillips curve of the form:

$$\mathbb{E}_t\left[\varepsilon_L \frac{N_t^{\sigma+1}}{W_t} + (1-\varepsilon_L)\frac{1}{\mathrm{P}_t}\mathrm{C}_{s,t}N_t - \kappa(W_t - W_{t-1}) + \kappa\beta_s(W_{t+1} - W_t)\right] = 0.$$

This curve is employed also when generating the impulse response figures in the theoretical section.

Monopolistically competitive intermediate goods producers set prices as a fixed markup over marginal cost. In a symmetric equilibrium:

$$p_t(j) = P_{h,t} = \mu \frac{W_t}{A}, \ \ \forall j, t,$$

where $\mu \equiv \varepsilon/(\varepsilon - 1)$ is a markup over the marginal cost $\frac{W_t}{A}$. Since intermediate goods producers charge a markup over marginal cost, they earn profits

$$\Pi_t = (\mathrm{A}P_{h,t} - W_t)N_t = (\mu - 1)W_t N_t.$$

[3] In response to a negative shock, impatient households' consumption would fall by more than patient households' and when all households earn the same wage rate, impatient households would increase labor supply by more than patient households. The implication that earnings increase more for credit constrained households (which have higher MPC) is counter-factual (see Patterson, 2022). See Galí et al. (2007) for a microfoundation of a uniformly rationed labor market in a two-agent model.

We assume that intermediate goods producers set prices flexibly (although note that if wages are sticky, prices inherit this stickiness conditional on shocks other than productivity (A) or markup (μ) shocks).

1.3. Borrowers' budget constraint

The nominal budget constraint of impatient households (borrowers) in each country is given by

$$\frac{B_{t+1}}{R_t}+W_tN_t-T_t^b=\mathbf{P}_t\mathbf{C}_{b,t}+B_t.$$

Where B_t is the face value of debt issued in period $t-1$ by borrowers, R_t is the nominal interest rate between t and $t+1$, and T_t are lump sum taxes. Borrowing is denominated in units of the currency of the monetary union and is subject to an exogenous limit $\bar{B}$:

$$B_{t+1}\leq\bar{B}.$$

1.4. Monetary and fiscal policy

The monetary authority sets the policy interest rate $\bar{R}_t$ for the currency union, and does not react to idiosyncratic (country-level) shocks. By implication, the central bank cannot offset the effects of idiosyncratic (country-specific) shocks, including deleveraging shocks, unlike in closed economy models such as Eggertsson and Krugman (2012). This highlights the relevance of studying risk sharing specifically in a currency union: we assume that countries lack the policy tools to stabilize the domestic economy in response to idiosyncratic shocks, and ask how much risk sharing is provided by different asset market configurations.

The government budget constraint is:

$$\frac{B_{t+1}^g}{R_t}=P_{h,t}G_t-T_t+B_t^g. \tag{1}$$

The rate on government debt is R_t and tax receipts are $T_t=\chi T_t^b+(1-\chi)T_t^s$. Here we assume the government consumes only domestic goods, which is important for our results concerning government spending shocks. Since the focus of our paper is not on fiscal policy, we assume away from state-contingent fiscal transfers between governments; on fiscal unions see for example Farhi and Werning (2017).[4] Our key result, Proposition 1, requires some weak technical conditions on the form of taxes (see Appendix A.2).

1.5. Savers' budget constraint in each of the economies

1.5.1. Segmented markets (SMU) and money market union (MMU)

Savers save at the rate R_t. The savers' budget constraint is

$$S_t+W_tN_t-T_t^s+\frac{\Pi_t}{1-\chi}=\mathbf{P}_t\mathbf{C}_{s,t}+\frac{S_{t+1}}{R_t},$$

where Π_t are per-capita profits from intermediate good producers. Only savers in each country have claims to these profits, so $\frac{\Pi_t}{1-\chi}$ are profits per saver. Under MMU, the interest rate at home is always equal to the interest rate in the union: $R_t=\bar{R}_t$ for all t.

Under SMU we assume instead that $R_t=g(\bar{R}_t,Z_{i,t})$, that is, the interest rate at which domestic agents can save and borrow is a function of country-specific financial conditions $Z_{i,t}$; currency union money markets are therefore segmented in the sense that residents of different countries face different interest rates. Martin and Philippon (2017) find that this money market segmentation is the leading explanation for output losses during the Eurozone crisis of the 2010s. In Section 4 we use a calibrated model to estimate the welfare gains from moving from SMU to MMU.

1.5.2. Capital market union (CMU)

In a capital market union savers can additionally trade a continuum of stocks. Each such stock k represents a claim to the aggregate profit stream in country k. The savers' budget constraint in the home country is

$$S_t+W_tN_t-T_t^s+\int_k\varphi_{t,k}(V_{t,k}+\Pi_{t,k})=\int_k\varphi_{t+1,k}V_{t,k}+\mathbf{P}_t\mathbf{C}_{s,t}+\frac{S_{t+1}}{R_t},$$

[4] Note that in our complete markets case the savers, but not governments, can write state-contingent contracts.

where $\varphi_{t,k}$ are the home savers' aggregate holdings of the country k stocks and $V_{t,k}$ is the price of country k stock. In an (ideal) CMU this stock trading is frictionless and savers optimize portfolio allocations (Proposition 2 characterizes the optimal portfolio in CMU).

1.5.3. Complete markets

In the complete markets economy, savers have access to a full set of state contingent securities. We denote purchases at time t of securities paying off one unit of currency at time $t+1$ contingent on the realization of state z_{t+1} following history z^t by $D_{t+1}(z_{t+1}, z^t)$; this security has a time t price $Q_t(z_{t+1}, z^t)$:

$$S_t + W_t N_t - T_t^s + \frac{\Pi_t}{1-\chi} + \int_{z_{t+1}} Q_t\left(z_{t+1}, z^t\right) D_{t+1}\left(z_{t+1}, z^t\right) dz_{t+1} = D_t\left(z^t\right) + \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t}.$$

1.6. Equilibrium conditions

Demand functions for the home and foreign consumption bundles by savers and borrowers are given by

$$P_{h,t} C_{i,t} = (1-\alpha) \mathbf{P}_t \mathbf{C}_{i,t}, \quad \text{for } i = b, s. \tag{2}$$

Savers are unconstrained and their consumption is determined by their Euler equation and budget constraint (which differs depending on which assets are available, as discussed in Section 1.5):

$$\frac{1}{\mathbf{P}_t \mathbf{C}_{t,s}} = \beta_s R_t \mathbb{E}_t \left[\frac{1}{\mathbf{P}_{t+1} \mathbf{C}_{t+1,s}} \right]. \tag{3}$$

When borrowers are unconstrained their consumption is characterized by a similar Euler equation. Market clearing in goods is given by

$$\mathrm{A} N_t = \int_k \left(\chi_k c_{k,h,b,t} + (1-\chi_k) c_{k,h,s,t} \right) + G_t \ , \tag{4}$$

where $c_{k,h,b,t}$ and $c_{i,h,s,t}$ are consumption of home goods by borrowers and savers from country k. Finally, market clearing for borrowing requires

$$\int_k (1-\chi_k) S_{t+1,k} = \int_k \chi_k B_{t+1,k} + \int_k B_{t+1,k}^g \ , \tag{5}$$

and (if available) that for stocks $\int_k (1-\chi_k)\psi_{t+1,k} = 1$ and for Arrow-Debreu securities $\int_k (1-\chi_k) D_{t,k}(z_{t+1}, z^t) = 0$ for all z_{t+1}.

2. Money market union

In this section we study risk sharing of demand shocks in an MMU, specifically, shocks that come from private borrowing or fiscal policy. Our key theoretical result shows analytically that an ideal MMU provides perfect risk sharing with respect to these shocks. We also show that this result, as concerns deleveraging shocks, is robust to departures from Cole-Obstfeld preferences and the small open economy assumption.

We first give an outline for the coming proofs. Lemma 1 writes down the inter-temporal current account condition for the country and establishes that in an MMU the present value of a country's nominal income depends only on net savings and the present value of exports. Lemma 2 establishes that, since the share of aggregate nominal income that accrues to savers does not react to deleveraging and fiscal shocks, it is also the case that the present value of savers' nominal income does not react to these shocks. Given log preferences, it follows that savers' nominal consumption does not react to these types of shocks (Lemma 3 shows that, to a first-order approximation, this result holds for non-unitary intertemporal elasticity of substitution). The main result in this section, Proposition 1, establishes that, conditional on deleveraging and fiscal shocks, the MMU equilibrium obtains the complete market outcome for savers. Intuitively, absent fragmentation in currency union money markets, which is the key feature of MMU, savers are able to smooth the effects of these shocks by borrowing and saving from the rest of the union.

Under MMU, the funding cost is the same in all countries. Let us define the k-period discount rate from the savers' perspective as $R_{t,k} \equiv R_t \times \ldots \times R_{t+k-1}$, with the convention $R_{t,0} = 1$. We also define $\tilde{Y}_t \equiv P_{h,t} N_t - T_t$ as nominal private disposable income and F_t as nominal exports.

The first step is to write the current account equilibrium in market values. We prove the following Lemma:

Lemma 1. *The inter-temporal current account condition (for each country) is*

$$\alpha\left((1-\chi)S_t-\chi B_t+\sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}}{R_{t,k}}(z^{t+k})\right)=(1-\chi)S_t-\chi B_t-B_t^g+\sum_{k=0}^{\infty}\frac{F_{t+k}}{R_{t,k}}(z^{t+k}) \tag{6}$$

for each path of history z^{∞}.

Proof. See Appendix. □

On the left we have the net present value of all future imports, which is a share α of private wealth, which itself equals financial wealth plus the present value of disposable income. On the right we have net foreign assets plus the present value of nominal exports (F_t). Solving Eq. (6) gives

$$\sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}}{R_{t,k}}(z^{t+k})=\frac{1}{\alpha}\left(\sum_{k=0}^{\infty}\frac{F_{t+k}}{R_{t,k}}(z^{t+k})-(1-\alpha)(\chi B_t-(1-\chi)S_t)-B_t^g\right),$$

so the inter-temporal current account condition pins down the NPV of disposable nominal income as a function of current assets and foreign demand.

The result requires an intratemporal unit demand elasticity over home and foreign goods, such that nominal exports are exogenous to a small country, but not log intertemporal preferences (which will be necessary for the following results). The result does not depend on the form of the production function, the labor supply condition, fiscal policy or whether prices are sticky or flexible. In an open economy model with unit demand elasticities and a fixed α (in the next section we consider treat α as a random variable; such "quality" shocks of course do affect the NPV of nominal income), the NPV of exports and country's net wealth fully determine the NPV of disposable income.

How do savers respond to different types of shocks? We prove the following lemma:

Lemma 2. *Nominal spending by savers* ($\mathbf{P}_t\mathbf{C}_{s,t}$) *does not react to private credit shocks* ($\bar{B}_{t+1}$)*, to borrowers' discount rate shocks* ($\beta_{b,t}$) *or to fiscal policy (neither* G_t *nor* T_t*).*

Proof. See Appendix. □

Lemma 2 clarifies the behavior of savers. The *nominal* spending of savers does not react to deleveraging shocks or fiscal shocks. Deleveraging affects the savers in two ways. First, it results in repayments of debt, but savers can substitute these repayments by lending more to foreign countries. The fact that this direct effect does not affect the *net present value* of savers income and therefore their spending is perhaps not surprising. Second, deleveraging also lowers the demand of borrowers. Since currency union-wide monetary policy does not react to the idiosyncratic deleveraging shock, this causes a fall in employment, lowering labor income and profits received by savers. Intuitively, savers' consumption should therefore fall. But borrowers' demand in future periods increases by virtue of their reduced debt burden, which increases savers' future income. For any distribution of deleveraging shocks this future increase in income exactly offsets the initial fall so that the NPV of savers income does not change. As a result, patient agents keep their nominal spending constant.

To gain further intuition for this result, consider the following partial equilibrium reasoning about the effects of deleveraging. Assume a fixed interest rate R and that during the first period borrowers reduce debt (and therefore consumption) by 1 euro. This reduces GDP by $(1-\alpha)\chi$ euros in the first period but increases it by $(1-\alpha)(R-1)\chi$ euros in all the following periods. The total effect on the NPV of the country's GDP and income is

$$-(1-\alpha)\chi+\frac{(1-\alpha)(R-1)\chi}{R}+\frac{(1-\alpha)(R-1)\chi}{R^2}+\frac{(1-\alpha)(R-1)\chi}{R^3}\ldots=0.$$

A saver can fully smooth this shock, that does not affect her permanent income, by borrowing in the first period. Lemma 2 shows that this reasoning is *exactly* valid in *general equilibrium* assuming a continuum of small countries, Cole-Obstfeld preferences and that the NPV of savers' income is a function of the NPV of the country's income.

As regards fiscal policy, Lemma 2 implies that changes in government spending do not affect the nominal consumption of savers; by extension, if there are no borrowers, the result implies that changes in government spending have no effect on nominal household consumption[5]. This result is different from Ricardian equivalence and obtains because Cole-Obstfeld preferences and the SOE assumption imply a nominal fiscal consumption multiplier of zero. This implication can be seen as a version of the Cole and Obstfeld (1991) result and is discussed further in Lemma 5 in Appendix C. In simple economic terms this is because: i) the interest rate does not react due to the small country assumption, ii) nominal exports do not react due to Cole-Obstfeld preferences, i.e. there is (no "leakage"). We discuss below the robustness of this result to deviations from Cole-Obstfeld preferences.

[5] We discuss this result further and show impulse response function for this case in Appendix D

Note that irrespective of preferences the *real* fiscal consumption multiplier, a statistic studied for example by Farhi and Werning (2013), is generally not zero.

Having shown that savers' consumption does not react to deleveraging and fiscal shocks, we establish the main result on risk sharing in MMU:

Proposition 1. *The Money Market Union achieves the Complete Markets allocation subject to country-specific private and public demand shocks* ($\bar{B}_{t+1}$, $\beta_{b,t}$, G_t, T_t) *using dynamic cross-country borrowing.*

Proof. Under MMU, the interest rate is the same in all countries and is independent of idiosyncratic shocks to the SOE. The complete markets outcome is characterized by the risk sharing (Backus-Smith-Kollmann) condition, which, with log preferences, takes the form

$$\frac{\mathbf{C}_{s,t,j}}{\mathbf{C}_{s,t}} \sim \frac{\mathbf{P}_t}{\mathbf{P}_t^j},$$

for arbitrary foreign country *j*. Since shocks to an SOE do not affect foreign prices or quantities, it follows that the complete markets condition is also that $\mathbf{P}_t\mathbf{C}_{s,t}$ remains constant. Given Lemma 2 in response to deleveraging shocks coming either from a change in the borrowers' credit constraints or the discount rate (or both simultaneously), the MMU replicates the complete markets economy. □

Proposition 1 shows that a money market union is sufficient to deal with any cross-sectional distribution of debt deleveraging and fiscal shocks in a currency union. Martin and Philippon (2017) show that segmented markets, in contrast, can be very inefficient. They find that spreads go up during episodes of private deleveraging, mostly because of stress in the banking sector. This leads savers (or firms under Q-theory) to cut spending precisely when the economy is in recession, exacerbating the downturn. We quantify the welfare gains from moving from SMU to MMU in Section 4.3.

The Proposition is different from previous hedging results in the international macroeconomics literature, such as those in Coeurdacier and Gourinchas (2016) and Coeurdacier et al. (2010). Those authors consider two country models with trading in two real bonds as well as equity claims. They find that countries can share risks using static positions in the real bonds. In contrast, we consider a setting with trading in one nominal bond with a common interest rate and show that countries can share risks through dynamic cross-country borrowing. Our result also differs from the results in Engel and Matsumoto (2009), who show that agents can hedge risks through a static forward position in foreign exchange.

Fig. 1 plots the impulse responses to a domestic deleveraging shock. Deleveraging causes borrowers to reduce consumption and, since the nominal interest rate does not react, results in a recession. Savers smooth this fall in income by borrowing more from foreign countries. After the first period, this deleveraging has a small positive effect on output, wages and profits as borrowers' lower interest expenses boost demand. This additional income offsets the lower interest rate income received by savers who now hold a smaller stock of savings. As implied by Proposition 1, savers' nominal expenditure does not react to these changes. This is because the negative and positive income effects of deleveraging exactly offset each other so that the NPV of savers' income does not change.

2.1. Proposition 1: Beyond Cole-Obstfeld

Similarly to for example Gali and Monacelli (2008), Heathcote and Perri (2013) and Martin and Philippon (2017) our framework assumes Cole-Obstfeld preferences. That is, we assume log-preferences and a unit elasticity of substitution between all goods. However, we next show that Proposition 1 holds approximately for deleveraging shocks with more general preferences.

2.2. Different demand elasticities

To relax the unit elasticity of demand assumption we now consider the aggregators:

$$\mathbf{C}_{i,t} = \left((1-\alpha)^{\xi_1} \left(C_{h,i,t}\right)^{(\xi_1 - 1)/\xi_1} + \alpha^{\xi_1} \left(C_{f,i,t}\right)^{(\xi_1 - 1)/\xi_1} \right)^{\xi_1/(\xi_1 - 1)}, \text{ for } i = b, s,$$

$$C_{f,i} = \left(\int_0^1 C_{k,i}^{(\xi_2 - 1)/\xi_2} \, \mathrm{d}k \right)^{\xi_2/(\xi_2 - 1)}.$$

Here ξ_1 is the demand elasticity between the home good and the aggregate foreign good and ξ_2 is the demand elasticity between different varieties of foreign goods.

We now consider private deleveraging shocks. Fig. A.4 in the appendix shows the response in savers' nominal consumption for three different values of elasticity of substitution between home and foreign goods $\xi_1 = \in \{1, 10, 1000\}$. The results are virtually

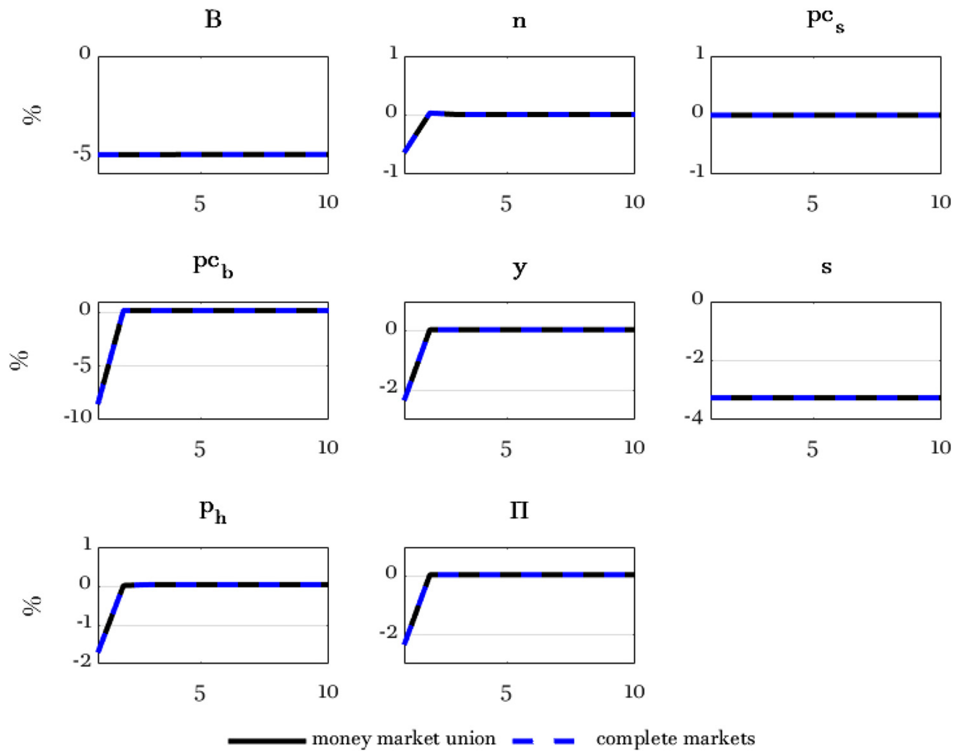

Fig. 1. Private deleveraging shock in an SOE. This figure plots impulse responses to a permanent-5% shock to $\bar{B}$ for the small open economy of Section 1. Parameter values: $\beta_s = 0.98$, $\varepsilon = 4$, $\alpha = 0.25$, $\sigma = 3$, $\chi = 0.5$, $\varepsilon_L = 4$, $\kappa = 0.1$. The calibration of β_b is such that borrowers are impatient enough that they always borrow up to the constraint ($B_{t+1} = \bar{B}$). The two lines in each panel show impulse responses under money market union and complete markets, which per Proposition 1 are identical.

identical for these different values. When demand elasticity is high, nominal consumption stays roughly constant because prices and real consumption do not react. When this elasticity is low, the response in nominal consumption is small because increases in real consumption are offset by a lower price of the home good.

We repeat this exercise but now with different values of elasticity of substitution between different varieties of foreign goods $\xi_2 \in \{1, 10, 1000\}$.[6] The results are given in Fig. A.4b in the appendix and look similar to those before.

In some cases we can actually show that the key result of 1 holds up to first order for any values of the demand elasticity parameters $\xi_1$and ξ_2. In particular we have the following lemma:

Lemma 3. *Assume the labor supply condition is of the form $W_t = h(N_t, \mathbf{P}_t^i, \chi \mathbf{P}^i \mathbf{C}_{b,t} + (1-\chi)\mathbf{P}_t^i \mathbf{C}_{s,t})$. Now Proposition 1 holds in a first order approximation for private deleveraging shocks ($\bar{B}_{t+1}$, $\beta_{b,t}$) for any demand elasticity parameters ξ_1 and ξ_2*

Proof. See Appendix □

This generalizes the results in Lemmas 1 and 2. Now the result of Proposition 1 follows immediately. Changing the demand elasticity parameters alters the response of GDP to a deleveraging shock due to price adjustments. However, in a first order approximation the NPV of these price adjustment effects is still zero. Therefore Proposition 1 still holds up to first order. Alternatively, Cole-Obstfeld preferences imply a type of linearity in the demand effects induced by deleveraging. This linearity is why the effects of deleveraging net out so that the NPV of savers' income does not change. Such linearity still holds in a first order approximation for any values of demand elasticities. In the appendix we examine the robustness of Proposition 1 with respect to fiscal shocks.

2.3. CRRA

We now consider CRRA preferences over the final good

$$\frac{\mathbf{C}_{i,t}^{1-\gamma}}{1-\gamma} \quad \text{for} \quad i = b, s,$$

[6] Here the demand elasticity changes in all countries simultaneously

Lemma 1 still holds with CRRA preferences as the proof makes no assumption concerning the preference over the final good. But what about Proposition 1? The following lemma shows that it can also be generalized:

Lemma 4. *Assume the labor supply condition is of the form* $W_t = h(N_t, \mathbf{P}_t^i, \chi \mathbf{P}^i \mathbf{C}_{b,t} + (1-\chi)\mathbf{P}_t^i \mathbf{C}_{s,t})$. *Now Proposition 1 holds in a first order approximation for private demand shocks* $(\bar{B}_{t+1}, \beta_{b,t})$ *for any CRRA parameter* γ

Proof. See Appendix □

The logic of this lemma is that an CRRA agent prefers to smooth consumption by holding marginal utility constant. In a first order approximation the price effects of demand shocks add up to zero in NPV terms so that keeping marginal utility constant is affordable. These results hold for arbitrary combinations of demand elasticities and CRRA parameters. However, when all demand elasticities equal one, Lemma 4 holds also for fiscal shocks.

We conclude that the key results of the section hold up to first order with general CRRA preferences and arbitrary demand elasticities. That is while they hold exactly with commonly used log-preferences, they also hold approximately in more general models.

2.4. Proposition 1: Beyond small countries

Consider now the case of deleveraging shocks hitting a large economy. Proposition 1 is exactly correct in a small open economy or with a continuum of countries; with two economies, foreign demand depends (partly) on domestic demand and, therefore, on domestic deleveraging. In addition, the central bank reacts by changing the risk free rate.

Fig. 2 shows the impulse responses to a domestic deleveraging shock (credit shock) in a two-country version of our model in which monetary policy follows a Taylor rule (Eq. (7); Taylor rule parameters are as in Table 2). The responses of all variables are virtually the same under MMU and under complete markets (the results are similar if we assume the shock is large enough to make the ZLB bind). In the two-country case, the home deleveraging shock causes a recession at home, which causes the central bank to react by cutting the nominal interest rate (R, bottom right panel of Fig. 2). As a result, both home and foreign savers increase consumption by the same amount. In spite of the endogeneity of foreign demand and the nominal interest rate, we find Proposition 1 holds.

The intuition is as follows: first, we know that savers do not react in a SOE. With two countries, foreign demand is endogenous, but this effect is quantitatively small because it depends on two consecutive cross-border spillovers: the pass-through of domestic demand onto foreign income and then from foreign income back to foreign demand for home goods. Proposition 1 is also approximately correct for reasonable values of the elasticity of substitution other than one.

The second important difference is the Taylor rule. Of course, the reaction of the monetary authority has a direct impact on the dynamics of the currency union. But the key point is that this impact is the same under MMU and under complete markets. Why? Because savers face the same interest rate in both countries.

We conclude that an ideal money market union – a union that guarantees that risk-free rates are equalized across regions – provides as much risk sharing as complete markets conditional on various demand shocks, and this result is robust to deviations from the small country and Cole-Obstfeld preference assumptions.

3. Capital market union

In this section we focus on the benefits of an ideal capital market union over and above an money market union. We pay special attention to technology shocks in the form of "quality" shocks to the goods sold by firms. Formally, we model these shocks as changes to quality parameters α_i (possibly correlated across countries). These shocks imply fluctuations in imports and exports and alter the relative profitability of firms in different countries. The money market union will not be able to share this kind of risk, but the capital market union could, at least in principle. This is because the value of equity claims are sensitive to firm profits. The following proposition characterizes the types of shocks that can be shared efficiently in a CMU.

Proposition 2. *Assume borrowers are impatient enough to borrow up to the borrowing constraint. Equilibrium in the capital market union replicates the complete markets allocation subject to (an arbitrary cross-sectional distribution) of quality* (α_t), *TFP* (A_t), *monetary policy, and various preference shocks (that can be correlated across countries). This equilibrium features static equity positions and no cross-country borrowing.*

Proof. See Appendix. □

The proof shows that the savers' optimal portfolio allocation is such that a saver holds $-\frac{1-\chi}{\mu-1}$ home stocks $1+\frac{1-\chi}{\mu-1}$ foreign stocks, split equally across foreign countries. To efficiently share quality shocks, savers hence underweight home stocks. In practice, various frictions often lead savers to do the opposite and overweight home stocks; we conceive of the ideal CMU as resulting from the removal of all such equity market frictions. The equilibrium in this CMU would feature complete sharing of supply shocks. However, if these frictions cannot be removed perfectly, a full CMU might be unattainable. Here a capital market union with partially segmented equity markets is able to share some but not all of the risks associated with the shocks. We do not explicitly

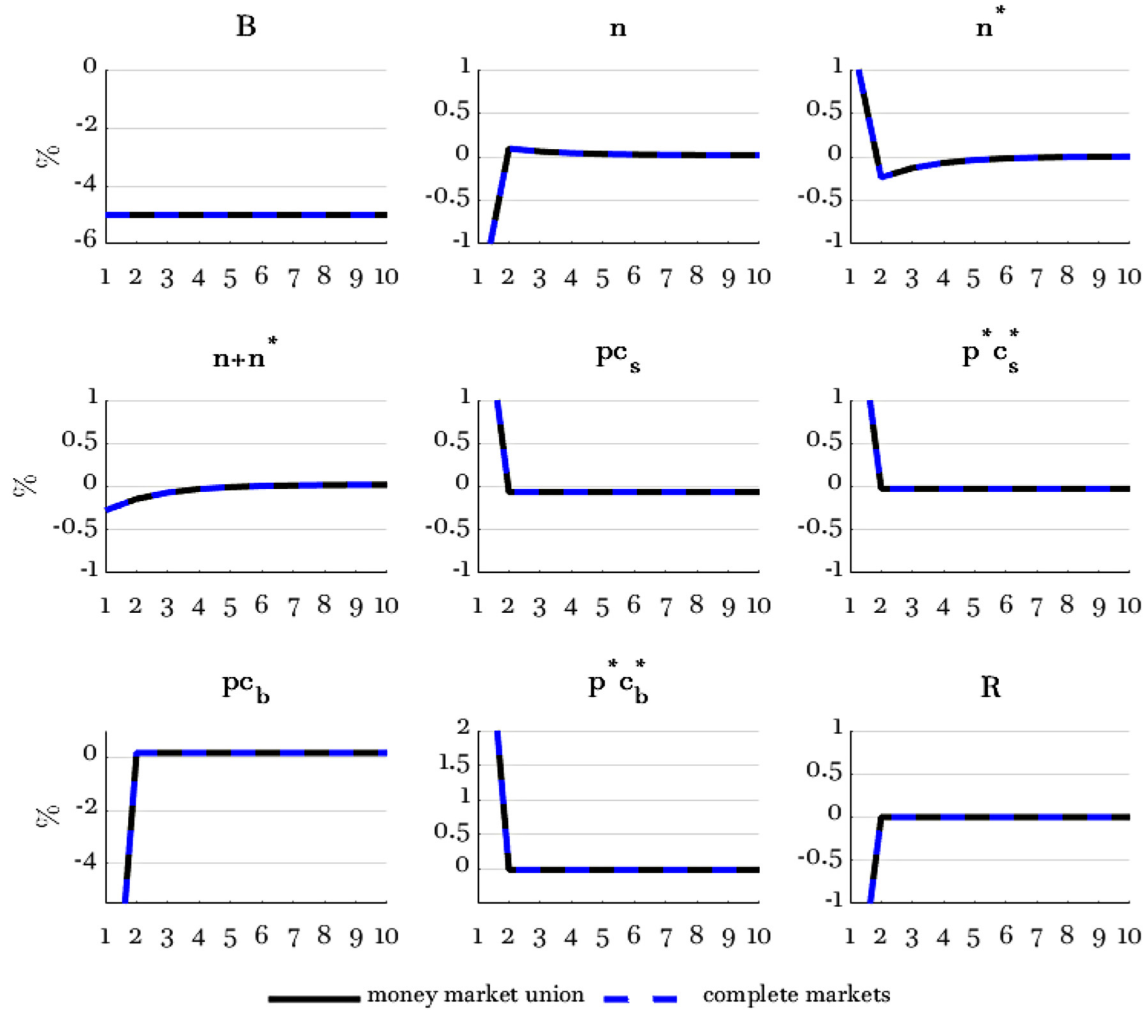

Fig. 2. Private deleveraging in a 2-Country model. This figure plots impulse responses to a permanent-5% shock to $\bar{B}$ (the home borrowing limit) for a two-country version of the model of Section 1. Parameter values (symmetric across countries): $\beta_s = 0.98$, $\varepsilon = 4$, $\alpha = 0.25$, $\sigma = 3$, $\chi = 0.5$, $\varepsilon_L = 4$, $\kappa = 0.1$. Taylor rule parameters as in Table 2. The calibration of β_b is such that borrowers are impatient enough that they always borrow up to the constraint ($B_{t+1} = \bar{B}$). The black solid line and blue dashed line show impulse responses for MMU and complete markets, respectively. Variables with stars indicate the foreign country. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

model such frictions in this paper; for more elaborate micro-foundations of equity home bias and related discussions see, for example, Coeurdacier and Rey (2012) and Sihvonen (2018).

Note that the proposition holds for various different types of shocks, including quality shocks, TFP shocks and monetary policy shocks. It also holds for all types of preference shocks that do not alter the complete markets condition. This includes shocks to the disutility of labor that typically affects the relationship between labor supply and wages. Moreover, the number of shocks can be higher than the number of assets; this is in contrast to the usual finding that obtaining the complete markets outcome requires at least as many assets as shocks (see e.g. Coeurdacier and Gourinchas, 2016).

The exact theoretical result hinges on log-preferences as well as the assumed form of the production function.[7]However, it does not require a unit elasticity of substitution or a continuum of countries. What would happen with more general preferences over the aggregate good such as CRRA? Up to first order, one could still hedge a single type of shock, e.g. quality shocks, using static equity positions. However, replicating the complete market outcome for all of the shocks discussed above would require additional assets. Here the equilibrium stock positions would also be affected by price hedging effects.

The assumption that the borrowers borrow up to the constraint rules out cases in which a supply shock would indirectly induce leveraging or deleveraging. We relax this assumption in Proposition 3. Note that as explained by Lemma 5, due to Cole-Obstfeld preferences, TFP shocks do not affect nominal consumption assuming fixed quality parameters α_i. However with

[7] The production function implies a perfect correlation between dividends and labor income. The result would also hold in a model with a fixed capital stock but not in a model with investment. However, it holds approximately in a model with investment with realistic investment adjustment costs.

Table 2
Calibration of baseline parameters.

Parameter	Description	Value
χ	Fraction of impatient	0.5
β_s	Discount factor of savers	0.99
α	Openness to trade	0.25
ε	Elasticity domestic intermediates	4
ε_L	Elasticity wage setting	4
σ	Labor supply elasticity parameter	3
θ	Capital share	0.36
δ	Depreciation rate	0.015
φ_Y	Taylor rule - output gap	0.5
φ_π	Taylor rule - inflation	1.5

stochastic quality parameters, sharing TFP shocks can require diversification in stock positions. In any case quality shocks seem to imply more interesting GDP dynamics than TFP shocks alone.

Fig. 3 shows the outcomes of a home quality shock in a money market union, a partial capital market union (with equal weights on home and foreign stocks) and a CMU with optimal portfolio allocations (equivalently, complete markets). In the ideal CMU, savers' spending reacts neither in the home country nor in the foreign countries. Proposition 2 shows that if stock positions are chosen correctly, the capital market outcome coincides with the complete markets case. With equal weights on home and foreign stocks, savers' spending in the home country increases. This increase, however, is smaller than in a money market union without cross-border equity claims.

In an ideal CMU cross-border equity holdings provide full insurance against supply shocks and savers have no incentives for cross-country borrowing. However, in a partial CMU savers also borrow more from foreign countries to gain additional smoothing.

3.1. Simultaneous supply and demand shocks

Proposition 1 shows that by using dynamic borrowing a MMU is able to share demand shocks. Proposition 2 argues that by using static equity positions a CMU can share quality shocks. In a first-order approximation these results add up in a fairly straightforward way. In our framework we also obtain the following exact result:

Proposition 3. *Equilibrium in the capital market union replicates the complete markets allocation subject to country-specific private deleveraging as well as foreign quality, productivity, monetary policy, and various preference shocks. This equilibrium features static equity positions and dynamic cross-country borrowing.*

Proof. See Appendix. □

3.2. Shocks that can be shared neither in MMU or CMU

We have provided results for the types of shocks that can be shared perfectly either in MMU or CMU. We have covered a broad array of shocks including credit, discount rate, taxation, government spending, quality, productivity, monetary policy and disutility of labor shocks. Are there shocks, then, for which the CMU does not attain replicate the complete markets outcome? Yes: a salient example is a redistributive shock such as a mark-up shock that alters the relative shares of labor and dividend income. In case of such shocks neither a MMU nor a CMU exactly obtains the complete markets outcome.

What could be done to attain the complete markets outcome in the case of mark-up shocks? The issue with such shocks is that they tend to redistribute income between borrowers and savers in way the savers cannot hedge using bond or equity positions. However, this effect could be offset using redistributive fiscal transfers[8]. However, a detailed analysis of such fiscal policies is beyond the scope of this paper.

3.3. On empirical tests of model predictions

Testing the empirical validity of our theoretical results about the types of shocks that can be shared efficiently either in MMU or CMU is challenging because our results describe counterfactuals. For example, according to Proposition 1 an idealized MMU, in which risk-free rates are fully equalized, could efficiently share deleveraging shocks. However, actual deleveraging episodes such as those observed during the Eurozone crisis tend to be associated with segmentation in risk-free rates. Perhaps the best way to test this proposition would be to consider a region such as US that is closer to a money market and banking union type arrangement with smaller regional differences in state level funding costs. If the Eurozone is also able to implement a well-functioning banking union, future deleveraging periods could also be used for such tests. However, note that this would require carefully

[8] Introducing additional financial instruments can of course help in attaining the complete market case with respect to such shocks.

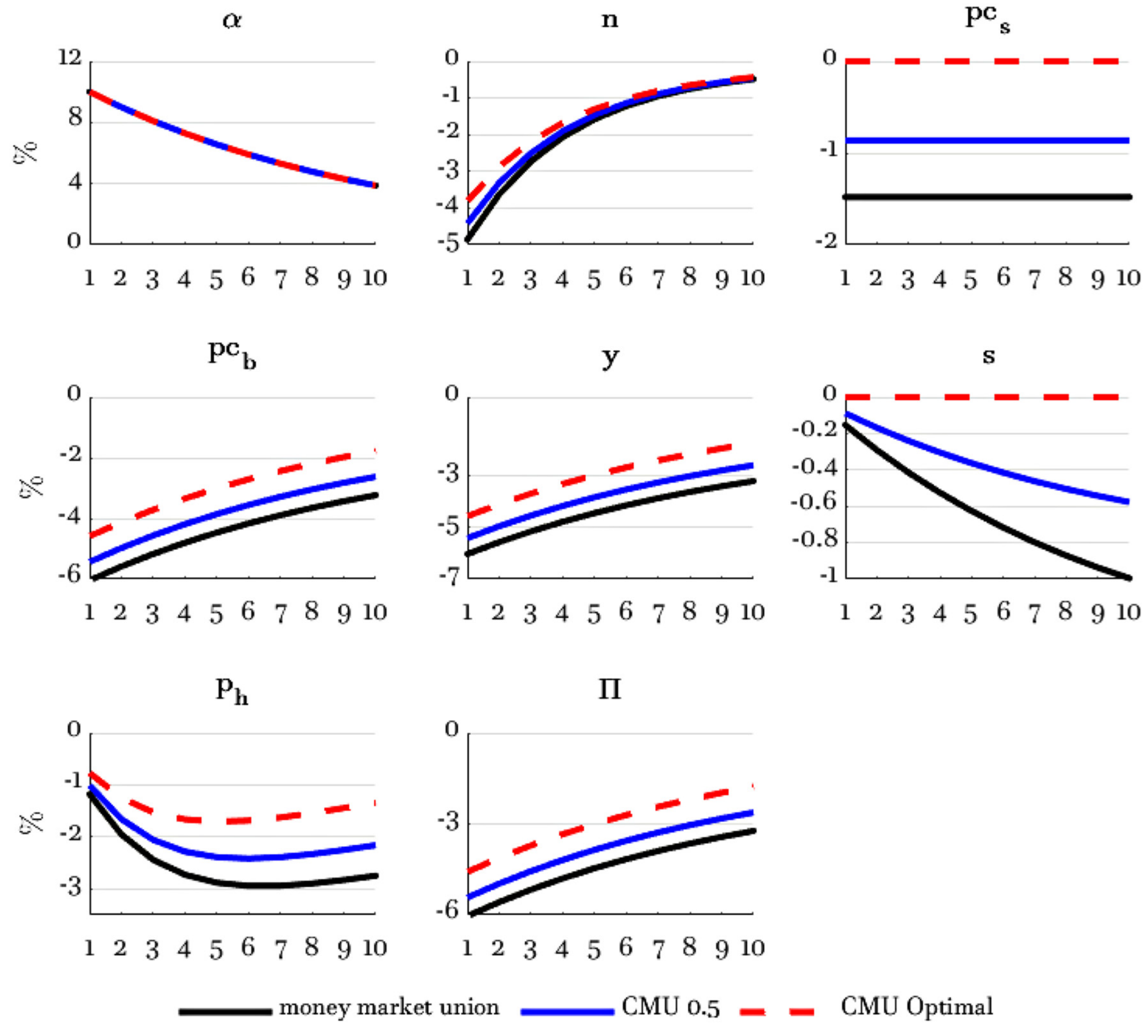

Fig. 3. Quality shocks in MMU and CMU. This figure plots impulse responses to a transitory 10% shock to α for the small open economy of Section 1. Parameter values: $\beta_s = 0.98$, $\varepsilon = 4$, $\bar{\alpha} = 0.25$, $\sigma = 3$, $\chi = 0.5$, $\varepsilon_L = 4$, $\kappa = 0.1$. CMU 0.5 (solid blue line) has a weight of 0.5 on each of the home stock foreign stocks. Complete markets is equivalent to a CMU with optimal weights (red dashed line) (Proposition 2). MMU is CMU with zero weight on foreign stocks (solid black line).

identifying a deleveraging shock. Similarly, Proposition 2 could be tested using a region with a high level of capital market integration such as the US. Again, this would require identifying supply shocks.

Giroud and Mueller (2016) show that the pattern of investment and employment across US locations during the great recession is consistent with what we call a money market union. Following the terminology of Holmström and Tirole (1997), they show that there is no local credit crunch but there is some collateral squeeze. Using census data, Giroud and Mueller (2016) find that the employment of manufacturing establishment does not respond to local house price shocks. This is what our model predicts for traded goods and assuming that costs of funds are not affected by local shocks. Aggregating at the firm level they find results consistent with money market union (no local credit crunch) together with balance sheet/cash flow channels. When firms with low leverage are hit by local demand shocks they do not decrease investment. Instead, they increase short and long term debt to smooth the shocks. This shows that funding costs are equalized in the cross section, or, in the the terminology of Holmström and Tirole (1997), there are no local credit crunches. This does not mean, however, that there are no credit constraints: in fact, Giroud and Mueller (2016) find that firms with high leverage do not smooth these shocks. This is exactly what we assume in our model, except that we focus on household credit constraints (the model works in the same way with credit constrained small firms, as explained in Gourinchas et al., 2016).

4. Numerical welfare gains

In this section, we evaluate quantitatively the welfare benefits of a money market and capital market union. To do so, we extend the model to include physical capital, an important feature in assessing the benefits of capital market integration because investment lowers the correlation between dividends and labor income, which reduces the hedging benefits of foreign equity.[9] We also specify a monetary policy rule and the relationship between wages and labor supply.

4.1. Model structure

4.1.1. Final goods producers

As before, competitive final goods producers produce the consumption good using a CES technology that aggregates intermediate goods:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

4.1.2. Intermediate goods producers

Intermediate goods are produced by monopolistically competitive firms using a Cobb-Douglas technology with labor and capital as inputs:

$$Y_{j,t} = A_t N_{j,t}^{1-\theta} K_{j,t}^{\theta}.$$

Where A_t is an aggregate, country-specific productivity shock. Intermediate goods producers are owned by shareholders in the home and foreign countries and maximize dividend payoffs to shareholders $(d_{j,t})$, discounted using the average discount factor $(\bar{m}_{0,t})$ of savers in the countries

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \bar{m}_{t,t+s} d_{j,t+s}$$

The weights for the discount factors are given by the stock positions. For example if home savers hold most of the equity of home firms, home firms put more weight on the discount factor of home savers. The firms can transfer the aggregate consumption good into capital through investment. Dividends are:

$$d_{j,t} = P_{j,t} Y_{j,t} - W_t N_{j,t} - \mathbf{P}_t I_{j,t} - \mathbf{P}_t f(I_{j,t}).$$

Where $I_{j,t}$, $P_{j,t}$, $N_{j,t}$ and $Y_{j,t}$ are intermediate producer j's investment, price, employment and output at time t and W_t is the wage rate in the country. Moreover, $f(I_{j,t})$ is the investment adjustment cost. Here we set

$$f(I_{j,t}) = \frac{\zeta}{2}\left(\frac{I_{t,j}}{I_{t-1,j}} - 1\right)^2.$$

Firm j's capital evolves according to:

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}.$$

And it faces a downward sloping demand curve from producers of the final good:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_{h,t}}\right)^{-\varepsilon} Y_t.$$

Intermediate goods producers set prices flexibly. It follows that they all set the same price, labor demand and investment level.

$$N_t = N_{j,t},\ \ I_t = I_{j,t},\ \ P_{h,t} = P_{j,t},\ \ K_t = K_{j,t}.$$

Optimal investment is then determined by the following recursive equation:

$$\mathbf{P}_t + \mathbf{P}_t \zeta\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)\frac{1}{I_{j,t-1}} = \mathbb{E}_t \bar{m}_{t,t+1}\left[P_{j,t+1}\eta\frac{Y_{j,t+1}}{K_{j,t+1}} + \mathbf{P}_{t+1}\zeta\left(\frac{I_{j,t+1}}{I_{j,t}} - 1\right)\frac{I_{j,t+1}}{I_{j,t}^2} + A_{t+1}\right].$$

[9] This is because firms invest in good times, which therefore lowers dividends in booms. This logic is explained e.g. in Heathcote and Perri (2013).

Where,

$$A_{t+1} = (1-\delta)\left[\mathbf{P}_{t+1} + \mathbf{P}_{t+1}\zeta\left(\frac{I_{j,t+1}}{I_{j,t}} - 1\right)\frac{1}{I_{j,t}} - \mathbb{E}_{t+1}\bar{m}_{t+1,t+2}\mathbf{P}_{t+2}\zeta\left(\frac{I_{j,t+2}}{I_{j,t+1}} - 1\right)\frac{I_{j,t+2}}{I^2_{j,t+1}}\right].$$

The price is a constant markup over marginal cost

$$P_{h,t} = \mu MC_t.$$

Where the markup over marginal cost MC_t is given by $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$ and $MC_t = \frac{W_t}{(1-\theta)Y_t/N_t}$.

4.1.3. Wage setting

As in the theory part labor is rationed uniformly across households. Assume the country is split to small labor unions, each representing a particular saver-borrower pair. Each union chooses its wage subject to a Rotemberg utility cost, $\frac{\kappa}{2}(W_t - W_{t-1})^2$. We assume the union is run by savers who set wages to maximize the objective:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta_s^t\left[\log \mathbf{C}_{s,t} - \nu(N_t) - \frac{\kappa}{2}(W_t - W_{t-1})^2\right]$$

The individual labor choices are aggregated using a CES technology, with substitution parameter ε_L. Hence labor demand is $\left(\frac{W_{t,i}}{W_t}\right)^{-\varepsilon_L} N_t$. We also assume a labor disutility function $\nu(N) = \frac{N_t^{1+\sigma}}{1+\sigma}$. All unions set the same wage. The FOC gives the wage Phillips curve

$$\mathbb{E}_t\left[\varepsilon_L\frac{N_t^{\sigma+1}}{W_t} + (1-\varepsilon_L)\frac{1}{\mathbf{P}_t\mathbf{C}_{s,t}}N_t - \kappa(W_t - W_{t-1}) + \kappa\beta_s(W_{t+1} - W_t)\right] = 0.$$

4.1.4. Monetary policy rule

For the small open economy model we assume a constant policy rate from the perspective of the home country. For the two country version considered later we assume the central bank sets the interest rate according to

$$\bar{R}_t = R_{ss}\left(\left(\frac{Y_t}{Y_{ss}}\right)\left(\frac{Y_t^*}{Y_{ss}^*}\right)\right)^{\varphi_Y}\left(\left(\frac{\pi_t}{\pi_{ss}}\right)\left(\frac{\pi_t^*}{\pi_{ss}^*}\right)\right)^{\varphi_\pi}, \tag{7}$$

where R_{ss}, Y_{ss} and π_{ss} are the steady state interest rate, output and inflation.

4.1.5. Measuring welfare gains

We quantify welfare gains from moving to a money market and capital market union similarly to Kollmann (2002). We solve the model using a 2nd order approximation and express the utility gain as permanent increase ξ in the steady state consumption of both savers and borrowers $\chi U(\mathbf{C}_s^{ss}(1+\xi), N) + (1-\chi)U(\mathbf{C}_b^{ss}(1+\xi), N)$.

4.1.6. Symmetric calibration

Table 2 shows the calibrated parameters.

4.2. Numerical welfare benefits of a money market union

We now use the model with capital to estimate the welfare benefits of a money market union. Under segmented markets, the private costs of funds are not equalized across regions. It is important to understand that we do not start from any segmented market model. We start from a model that actually describes the behavior of the Eurozone. Martin and Philippon (2017) and Gourinchas et al. (2016) quantify the extent of the dispersion in funding costs during the Eurozone crisis. The simplest interpretation is that domestic banks intermediate savings and investment, and, thus, the private cost of funds is pinned down by the banking system. Formally, in log-deviations from steady state, we have

$$r_t = r_t^b$$

where r_t^b is the banks' funding cost. We can then consider a small country subject to a spread shock r_t^b and a private leverage shock $\bar{B}_t$. We estimate these shocks using data from the Eurozone as in Martin and Philippon (2017) but otherwise consider the baseline calibration discussed in the next section. The idea is to model the joint dynamics of spreads and private debt. Debt is well described by an AR(2) process and spreads by an AR(1) process. The processes are correlated because negative shocks cause spread to rise and banks to cut lending. Our calibration uses data from a volatile period, the Eurozone crisis, so our welfare calculations capture the value of a money market union during periods of heightened financial risks. To include some supply side shocks we include quality shocks. The quality shock process, and the investment and wage adjustment cost parameters are estimated in the next subsection.[10] Spread differences between countries increase consumption volatility and lower welfare. The money market union reduces consumption volatility by equalizing interest rates between countries. In the model the welfare gain of money market union is equivalent to a 0.1% permanent increase in the steady state consumption of both savers and borrowers. Both savers and borrowers gain but the former benefit relatively more. In particular the welfare gain for savers is equivalent to a 0.14% permanent increase in consumption but the welfare gain for borrowers is equivalent to a 0.06% increase in consumption.

Finally note that there is considerable uncertainty around our estimated welfare gains. For example these depend on assumptions concerning the relevant importance of different types of shocks. Moreover, moving towards a MMU might have other unmodeled effects, for example it might affect the borrowing constraints or the distribution of related shocks.

4.3. Numerical welfare benefits of a capital market union

In this section we argue that the welfare gains of moving from a money market union to a capital market union can be also be significant. As before we employ the model with capital but now with two countries. We assume two kinds of (home and foreign) shocks: deleveraging and quality. The deleveraging shocks are estimated directly from the data, as in the previous section. The quality shocks and the investment and wage adjustment parameters are estimated from the data to match consumption and export data from France.

We estimate the parameters using a stock position of $\varphi = 0.8$. That is, we start from a reasonable empirical benchmark with low levels of within union cross-border equity holdings. We then solve numerically for the optimal home stock position from an individual saver's perspective using the method described by Devereux and Sutherland (2011b). The zero order optimal home stock position is given by $\varphi = \varphi^* = 0.26$. We do not model the friction that leads agents to choose a larger-than-optimal home stock position. As in Tille and van Wincoop (2010), for example, we can think of this friction as a higher-order term that affects macroeconomic conditions through its impact on stock positions.

The welfare gain of moving from an equilibrium with home bias to the frictionless equilibrium is equivalent to a 0.1% permanent increase in the steady state consumption of both savers and borrowers. Here both savers and borrowers gain roughly equally (0.1%).

4.4. Comparison to existing studies

How do the above welfare gains compare to those reported in previous studies? First, the welfare exercise for MMU lacks an obvious analogy in the previous literature that has focused on comparing financial autarky with free trading in a non-contingent bond. Here for example Kim et al. (2003) apply a general equilibrium model to report a relative gain of 0-0.2%. Devereux and Sutherland (2011a) and Brunnermeier and Sannikov (2015) find that free bond trading can even reduce welfare relative to financial autarky.

However, these papers do not consider the kind of segmentation witnessed during the Eurozone crisis. The key feature is that spreads tend to *increase* during deleveraging episodes when it would be efficient for countries to smooth shocks by borrowing. The comparison here is between a model with a counter-cyclical spread (segmented markets) and a model with no spread in riskless borrowing rates (a MMU) not between free bond trading and financial autarky.[11]

The exercise for CMU is more closely comparable to those in existing studies. Partial equilibrium analyses on the costs of equity home bias often imply fairly high losses: for example French and Poterba (1991) argue that home bias results in losses equivalent to several hundred basis point reductions in equity returns. However, loss estimates from general equilibrium macroeconomic models are typically smaller. For example Kim et al. (2003) find that the consumption equivalent welfare gain of moving from a bond-only economy to complete markets is at most, but typically much less than, 2%. They also argue that when shocks are purely transitory, the gains become negligible. However, overall their numbers are roughly consistent with the welfare gains of CMU calculated above.

Finally, note that our model features elements typically missing from models used to calculate welfare gains such as that in Kim et al. (2003). These include wage rigidities and borrowing constrained non-Ricardian agents. However, welfare gains

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[10] The borrowing limit follows the process

$$\log\bar{B}_{i,t} - \log\bar{B}_{i,t-1} = -0.01 \times (\log\bar{B}_{i,t-1} - \log\bar{B}) + 0.85 \times (\log\bar{B}_{i,t-1} - \log\bar{B}_{i,t-2}) + 0.04\varepsilon^b_{i,t}$$

and the spread the process

$$r^b_{i,t} = 0.9 r^b_{i,t-1} + 0.003\varepsilon^r_{i,t}$$

and the correlation between the two shocks is

$$\text{corr}\left(\varepsilon^b_{i,t}, \varepsilon^r_{i,t}\right) = -0.3.$$

The investment adjustment cost is estimated in the next section.

[11] In any case, financial autarky implying a zero trade balance and net wealth is not a realistic policy option.

would likely be larger if we assumed higher risk aversion and recursive preferences, which are also more consistent with asset price data, as in Lewis and Liu (2015).

Regarding the relative magnitudes of the gains of moving from SMU to MMU and MMU to CMU, our finding that they are similar is consistent with the evidence from estimated models of the drivers of real activity and trade flows in Eurozone countries in Kollmann et al. (2014) (Germany) and in't Veld et al. (2014) (Spain). Those papers find that both deleveraging and other demand shocks, and trade and technology shocks are important drivers of real activity, consumption and trade flows.

4.5. Role of a currency union

What is the role of a currency union for our results? First, as explained before, our theoretical results do not hinge on nominal rigidities. In particular we could assume perfectly flexible wages on top of flexible prices. On the other hand, in such a setting the assumption of a fixed exchange rate is irrelevant and the model is effectively equivalent to an RBC model. In this model the union wide interest rate is also determined through household equilibrium conditions rather than central bank policies.

Because flexible wages may facilitate adjustment to shocks, the benefits of risk sharing tend to be higher compared to a flexible wage model. The welfare benefits of a money market union would be 60% lower if we assumed perfectly flexible wages. Perfectly flexible wages would also wipe out more than 90% of the gains of CMU. Here perfect price and wage adjustment would largely offset the effects of quality shocks.[12]

What about a setting with sticky wages but flexible exchange rates and hence no currency union? Theoretically we would expect flexible exchange rates to offset at least some of the effects of sticky wages and hence reduce the benefits of risk sharing. Flexible exchange rates would also give countries more room to pursue independent monetary policies, which can for example help in countering deleveraging episodes as in Eggertsson and Krugman (2012). However, because modeling empirically realistic exchange rate behavior poses challenges to standard models, this question is ultimately beyond the scope of this paper.

5. Conclusion

Failures of risk sharing lie at the heart of many economic crises. Such crises are particularly acute in the context of a currency union in which constituent countries are hit by large, asymmetric shocks; the Eurozone crisis of 2009–2012 stands as a particularly striking example.

This paper presents two main theoretical findings. The first is that in the case of demand shocks - for example, private or public deleveraging - an idealized money market union in which risk-free rates are equalized across constituent members of the currency union provides the same level of insurance as complete markets. The second finding illustrates the limitations of this ideal money market union: in the case of supply shocks, the money market union does not provide full insurance, but an idealized capital market union, in which savers frictionlessly choose optimal portfolios, does. We find that the welfare benefits of moving from segmented markets to a money market union, and from a money market union to a capital markets union are of similar magnitudes.

Declaration of Competing Interest

None

Appendix A. Proofs

A.1. Proof of lemma 1

Define the k-period ahead discount rate for $k \geq 1$ from the savers' perspective:

$$R_{t,k} \equiv \prod_{i=1}^{k} \left(1 + r_{t+i-1}\right).$$

and the convention $R_{t,0} = 1$. Let us start from market clearing for the home good:

$$Y_t = (1-\alpha)\left(\chi \mathbf{P}_t \mathbf{C}_{b,t} + (1-\chi)\mathbf{P}_t \mathbf{C}_{s,t}\right) + F_t + P_{h,t} G_t,$$

where Y_t is nominal GDP. Using the budget constraints of the agents and of the government we get

$$\alpha \tilde{Y}_t = (1-\alpha)\chi \left(\frac{B^h_{t+1}}{1+r_t} - B^h_t\right) - (1-\alpha)(1-\chi)\left(\frac{S_{t+1}}{1+r_t} - S_t\right) + F_t + \frac{B^g_{t+1}}{1+r_t} - B^g_t,$$

[12] Note that this result would generally not hold if we assumed more shocks making it difficult for prices and wages alone to provide sufficient smoothing.

where $\tilde{Y}_t = W_t N_t + \Pi_t - T_t$ is total disposable income. Summing and rearranging the terms, we get

$$\alpha\left(\tilde{Y}_t + \frac{\tilde{Y}_{t+1}}{R_{t,1}}\right) = (1-\alpha)\chi\left(\frac{1}{R_{t,1}}\frac{B^h_{t+2}}{1+r_{t+1}} - B^h_t\right) - (1-\alpha)(1-\chi)\left(-S_t + \frac{1}{R_{t,1}}\frac{S_{t+2}}{1+r_{t+1}}\right) + F_t + \frac{F_{t+1}}{R_{t,1}} + \frac{1}{R_{t,1}}\frac{B^g_{t+2}}{1+r_{t+1}} - B^g_t$$

to write:

$$\alpha\left(\tilde{Y}_t + \frac{\tilde{Y}_{t+1}}{R_{t,1}} + \frac{\tilde{Y}_{t+2}}{R_{t,b}}\right) = -(1-\alpha)\chi\left(B^h_t - \frac{1}{R_{t,2}}\frac{B^h_{t+3}}{1+r_{t+2}}\right) + (1-\alpha)(1-\chi)\left(S_t - \frac{S_{t+3}}{R_{t,c}}\right) + F_t + \frac{F_{t+1}}{R_{j,t,1}} + \frac{F_{t+2}}{R_{t,2}} - B^g_t + \frac{1}{R_{t,2}}\frac{B^g_{t+3}}{1+r_{t+2}}.$$

Therefore for a generic horizon K

$$\sum_{k=0}^{K}\frac{\alpha\tilde{Y}_{t+k}}{R_{t,k}} = (1-\alpha)\Big((1-\chi)S_t - \chi B^h_t\Big) - B^g_t + \sum_{k=0}^{K}\frac{F_{t+k}}{R_{t,k}} - (1-\chi)(1-\alpha)\frac{S_{t+K+1}}{R_{t,K+1}} + \frac{1}{R_{t,K}}\left(\frac{(1-\alpha)\chi B^h_{t+K+1}}{1+r_{t+K}} + \frac{B^g_{t+K+1}}{1+r_{t+K}}\right).$$

We take the limit and we impose the No-Ponzi conditions

$$\lim_{K\to\infty}\frac{S_{t+K+1}}{R_{t,K+1}}(z^{t+K}) = 0$$
$$\lim_{K\to\infty}\frac{1}{R_{t,K}}\frac{B^h_{t+K+1}}{1+r_{t+K}}(z^{t+K}) = 0$$
$$\lim_{K\to\infty}\frac{1}{R_{t,K}}\frac{B^g_{t+K+1}}{1+r_{t+K}}(z^{t+K}) = 0.$$

The inter-temporal current account condition is

$$\alpha\sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}}{R_{t,k}}(z^{t+k}) = \sum_{k=0}^{\infty}\frac{F_{t+k}}{R_{t,k}}(z^{t+k}) - (1-\alpha)\Big(\chi B^h_t - (1-\chi)S_t\Big) - B^g_t.$$

A.2. A technical condition on taxes

Lemma 1 provides a mapping between the net present value of the country's income, the net wealth of the country and the net present value of exports. Lemma 2 argues that this implies restrictions on the net present value of savers' income. This argument requires the following technical condition:

$$\sum_{k=0}^{\infty}\frac{\tilde{Y}^s_{t+k}}{R_{t,k}}\left(z^{t+k}\right) \sim \sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}}{R_{t,k}}\left(z^{t+k}\right). \tag{8}$$

That is for each history z^∞, the present value of savers' disposable income is a function of the present value of average/aggregate disposable income and does not depend on any other variable. Savers disposable income consists of income before taxes deducted by tax payments. In our model for MMU $Y^s_t = \frac{1}{\mu}\left(1 + \frac{\mu-1}{1-\chi}\right)Y_t$, that is savers income before taxes is proportional, period by period, to nominal output. This form, which rests on constant mark-ups implies the above condition holds for before tax income. However, the condition should hold for disposable income and therefore imposes restrictions on admissible tax policy. Given our model, the condition holds if we impose the following condition on the form of taxes:

Condition 1. For each history z^∞, the present value of total taxes paid by savers and borrowers are functions of the present value of total (before tax) income and do not depend on any other variable.[13]

$$\sum_{k=0}^{\infty}\frac{T^s_t}{R_{t,k}}\left(z^{t+k}\right) \sim \sum_{k=0}^{\infty}\frac{Y_{t+k}}{R_{t,k}}\left(z^{t+k}\right), \quad \sum_{k=0}^{\infty}\frac{T^b_t}{R_{t,k}}\left(z^{t+k}\right) \sim \sum_{k=0}^{\infty}\frac{Y_{t+k}}{R_{t,k}}\left(z^{t+k}\right).$$

Condition 1 ensures that the net present value of aggregate income is a sufficient statistic for the net present value of savers' income. Condition 1 imposes some restrictions on tax policy, but it holds in many natural settings and in all the applied models

[13] To rule out ill-defined cases we require that this function be differentiable.

that we have studied. The simplest example is uniform flat taxation of all income at rate τ, i.e., $T_t^b = \tau W_t N_t + T_t^{b,LS}$ and $T_t^s = \tau\left(W_t N_t + \frac{\Pi_t}{1-\chi}\right) + T_t^{s,LS}$ with lump-sum taxes such that $\sum_{k=0}^{\infty} \frac{T_t^{s,LS}}{R_{t,k}}(z^t) \sim \sum_{k=0}^{\infty} \frac{Y_{t+k}}{R_{t,k}}(z^{t+k})$ and $\sum_{k=0}^{\infty} \frac{T_t^{b,LS}}{R_{t,k}}(z^t) \sim \sum_{k=0}^{\infty} \frac{Y_{t+k}}{R_{t,k}}(z^{t+k})$. For example when the lump-sum taxes are zero $\tilde{Y}_t^b = (1-\tau)W_t N_t$ and $\tilde{Y}_t^s = (1-\tau)\left(W_t N_t + \frac{\Pi_t}{1-\chi}\right) = (1-\tau)W_t N_t\left(1 + \frac{\mu-1}{1-\chi}\right)$. Therefore, all taxes, income and profits are proportional to $W_t N_t$. In particular, $\tilde{Y}_t = \mu(1-\tau)W_t N_t$, and therefore $\tilde{Y}_t^s = \frac{1}{\mu}\left(1 + \frac{\mu-1}{1-\chi}\right)\tilde{Y}_t$. Here all disposable incomes are directly proportional, period-by-period, which is stronger than Condition 1.

The condition allows for savers to be taxed at a higher rate than borrowers. It is also consistent with some forms of progressive taxes in which a temporary increase in income leads to an increase in tax rate. To see this consider a deleveraging episode in which the country's income first declines but then increases keeping the NPV of the country's income constant. This progressivity initially decreases but then increases savers' taxes. As long as these effects are symmetric the condition remains valid. The condition is also consistent with redistribution following a change in the present value of country's output as long as the present value of this redistribution depends only on the present value of total income.

A.3. Proof of lemma 2

Consider the program of the savers. With log-preferences, we can reformulate the savers' problem as a choice of nominal consumption:

$$\max \mathbb{E}\sum_{t\geq 0}\beta_s^t \log\left(\mathbf{P}_t\mathbf{C}_{s,t}\right)$$
$$s.t.\ \ \mathbf{P}_t\mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t} = S_t + \tilde{Y}_t^s.$$

The inter-temporal budget constraint of savers is

$$\sum_{k=0}^{\infty}\frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}(z^{t+k}) = S_t + \sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}^s}{R_{t,k}}(z^{t+k}), \tag{9}$$

where $\tilde{Y}_t^s = W_t N_t - T_t^s + \frac{\Pi_t}{1-\chi}$ is the disposable income of savers. Lemma 1 shows that the net present value of aggregate disposable income is a function of exactly four variables:

$$\sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}}{R_{t,k}}\left(z^{t+k}\right) \equiv \Omega\left(S_t, B_t, B_t^g, \sum_{k=0}^{\infty}\frac{F_{t+k}}{R_{t,k}}(z^{t+k})\right),$$

where the first three variables (saving, household debt, public debt) are predetermined at time t and the last one (exports in euros) is exogenous given a unit demand elasticity. Assuming further that Condition 1 is satisfied, it follows that:

$$\sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}^s}{R_{t,k}}\left(z^{t+k}\right) \equiv \Omega\left(S_t, B_t, B_t^g, \sum_{k=0}^{\infty}\frac{F_{t+k}}{R_{t,k}}(z^{t+k})\right), \tag{10}$$

For example with a flat tax rate on all income $\tilde{Y}_t^s = \frac{1}{\mu}\left(1 + \frac{\mu-1}{1-\chi}\right)\tilde{Y}_t$, which satisfies Condition 1. But Eq. (10) would also hold in a more general model, as long as it remains the case that the present value of aggregate income is a sufficient statistic for the present value of savers' income. Given Eq. (10), Eq. (9) is, in fact,

$$\sum_{k=0}^{\infty}\frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}\left(z^{t+k}\right) \sim S_t + \Omega_t\left(z^{\infty}\right).$$

Given log-preferences, savers' current nominal expenditure ($\mathbf{P}_t\mathbf{C}_{s,t}$) is a constant fraction of the expected present value of wealth, and depends only on S_t and Ω_t and the path of nominal interest rates. In particular, for given Ω_t and interest rates, it does not depend on contemporaneous or future private credit, borrowers' discount rate, or fiscal policy.

A.4. Proof of lemma 3

Now the market clearing condition is

$$Y_t = (1-\alpha)(\mathbf{P}_t^i/P_{t,i})^{\xi_1 - 1}\left(\chi \mathbf{P}_t^i \mathbf{C}_{b,t} + (1-\chi)\mathbf{P}_t^i \mathbf{C}_{s,t}\right) + (P_{t,i}^{1-\xi_2}/\mu)F.$$

Log-linearizing

$$\hat{y}_t = (1-\alpha)\left((1-\chi)(\hat{\mathbf{P}}_t + \hat{\mathbf{C}}_{s,t}) + \chi(\hat{\mathbf{P}}_t + \hat{\mathbf{C}}_{b,t})\right) + a_1 \widehat{p}_t.$$

Here hats denote log-deviations and

$$a_1 = (1-\alpha)(\xi_1 - 1) - (\xi_1 - 1)(1-\alpha)^2 + \alpha(1-\xi_2).$$

Using the borrowers' and savers' budget constraints and rearranging

$$\alpha \hat{y}_t = (1-\alpha)\left(\chi\left(\frac{\hat{b}_{t+1}^h}{R} - \hat{b}_t^h\right) - (1-\chi)\left(\frac{\hat{s}_{t+1}^h}{R} - \hat{s}_t^h\right)\right) + a_1 \widehat{p}_t.$$

The interest rate is constant. Now we have $W_t = h(N_t, \mathbf{P}_t^i, \chi \mathbf{P}_t^i \mathbf{C}_{b,t} + (1-\chi)\mathbf{P}_t^i \mathbf{C}_{s,t})$. Using the price setting condition, this implies $P_{t,i}/\mu = h(Y_t/P_{t,i}, \mathbf{P}_t^i, \chi \mathbf{P}^i \mathbf{C}_{b,t} + (1-\chi)\mathbf{P}_t^i \mathbf{C}_{s,t})$. Using the savers' and borrowers' budget constraints, $P_{t,i}/\mu = h(Y_t/P_{t,i}, \mathbf{P}_t^i, \chi\left(\frac{B_{t+1}}{1+r_t} - B_t\right) - (1-\chi)\left(\frac{S_{t+1}}{1+r_t} - S_t\right))$. Linearizing we obtain

$$\hat{y}_t = a_2 \widehat{p}_t + a_3\left(\chi\left(\frac{\hat{b}_{t+1}^h}{R} - \hat{b}_t^h\right) - (1-\chi)\left(\frac{\hat{s}_{t+1}^h}{R} - \hat{s}_t^h\right)\right)$$

for some a_2 and a_3. Plugging this back to the market clearing condition implies

$$\widehat{p}_t = \left(\chi\left(\frac{\hat{b}_{t+1}^h}{R} - \hat{b}_t^h\right) - (1-\chi)\left(\frac{\hat{s}_{t+1}^h}{R} - \hat{s}_t^h\right)\right)a_4,$$

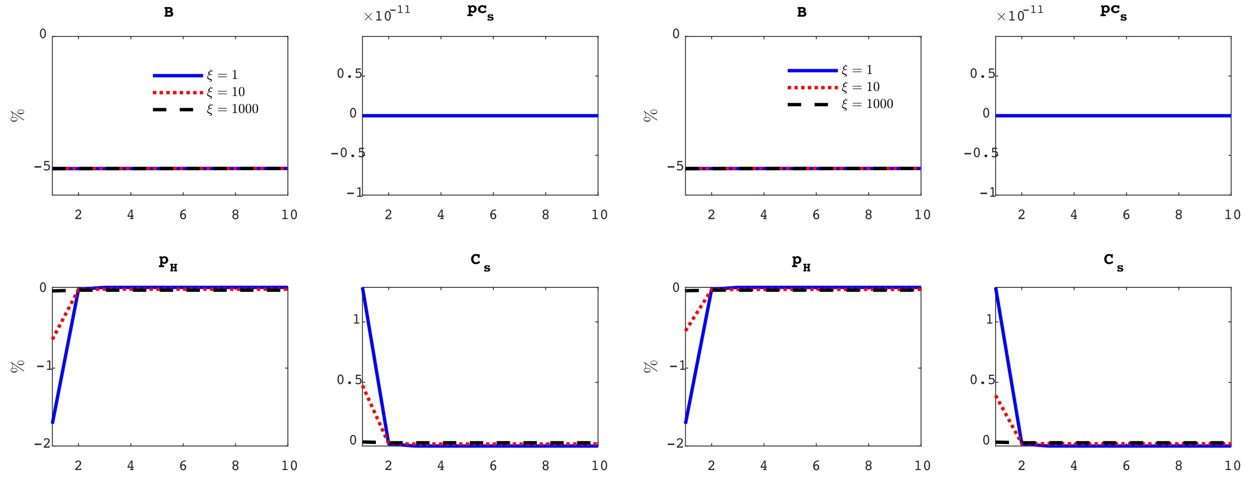
where $a_4 = \frac{1-\alpha-\alpha a_3}{\alpha a_2 - a_1}$. But therefore

$$\alpha \hat{y}_t = \left(\chi\left(\frac{\hat{b}_{t+1}^h}{R} - \hat{b}_t^h\right) - (1-\chi)\left(\frac{\hat{s}_{t+1}^h}{R} - \hat{s}_t^h\right)\right)((1-\alpha) + a_1 a_4).$$

Now iterating similarly to before and imposing transversality conditions

$$\alpha \sum_{k=0}^{\infty} \frac{\hat{y}_{t+k}}{R_k} = \left((1-\chi)\hat{s}_t^h - \chi \hat{b}_t^h\right)((1-\alpha) + a_1 a_4).$$

A.5. Proof of lemma 4



(a)Elasticity between home and foreign goods (ξ1) (b)Elasticity between varieties of foreign (ξ2)

Fig. A.4. Private deleveraging in a money market union for different values of elasticity of substitution between home and foreign goods (a) and different varieties of foreign goods (b). This figure plots impulse responses to a permanent-5% shock to $\bar{B}$ for the small open economy of Section 1, replacing the log aggregator with CES, for applicable parameters using the calibration in Table 2. The calibration is such that borrowers are impatient enough that they always borrow up to the constraint ($B_{t+1} = \bar{B}$). The blue solid line, red dotted line and black dashed line show impulse responses for elasticity of substitution equal to $\{1, 10, 1000\}$, respectively.

The Euler equation is

$$\beta \mathbb{E}_t \left(\frac{\mathrm{C}_{t+\mathbf{1}}}{\mathrm{C}_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} R = 1.$$

$$\frac{\mathbf{C_t}^{-\gamma}}{P_t} = \beta R \mathbb{E}_t \frac{\mathbf{C_{t+1}}^{-\gamma}}{P_{t+1}}.$$

In a first order approximation this becomes

$$mu_t = \mathbb{E}_t mu_{t+1}$$

Here mu_t is log-marginal utility. We can solve

$$mu_t = \lim_{T \to \infty} \mathbb{E}_t mu_{t+T}.$$

Hence the agent maintains constant marginal utility following the shock. We next argue that she keeps it at the pre-shock level. Up to first order the present value of consumption is

$$\sum_{k=0}^{\infty} \frac{\hat{\mathbf{C}}_{t+k} + \hat{\mathbf{P}}_t^i}{R_k}.$$

Note that as argued in the previous proof

$$\widehat{p_t} = \left(\chi \left(\frac{\hat{b}_{t+1}^h}{R} - \hat{b}_t^h \right) - (1-\chi) \left(\frac{\hat{s}_{t+1}^h}{R} - \hat{s}_t^h \right) \right) a_4.$$

Hence

$$\sum_{k=0}^{\infty} \frac{\widehat{p_t}}{R_k} = -\left(\chi \hat{b}_t^h + (1-\chi) \hat{s}_t^h \right) a_4.$$

is predetermined as is $\sum_{k=0}^{\infty}\frac{\hat{\mathbf{P}}_t^i}{R_k}$. The condition for constant marginal utility can be linearized as

$$-\gamma\hat{\mathbf{C}}_t-\hat{\mathbf{P}}_t^i=0$$

which implies

$$\hat{\mathbf{C}}_t+\hat{\mathbf{P}}_t^i=(1-\frac{1}{\gamma})\hat{\mathbf{P}}_t^i$$

and

$$\sum_{k=0}^{\infty}\frac{\hat{\mathbf{C}}_{t+k}+\hat{\mathbf{P}}_t^i}{R_k}=(1-\frac{1}{\gamma})\sum_{k=0}^{\infty}\frac{\hat{\mathbf{P}}_t^i}{R_k}.$$

which is predetermined and hence does not respond to the shock. Hence choosing the old marginal utility is feasible. Now if some other marginal utility than the previous one were optimal, it would have been so already so before the shock. Therefore following the shock marginal utility remains constant which is also the complete markets condition.

A.6. Proof of proposition 2

To highlight that the result does not depend on the assumption of a continuum of countries we show it in an I country version of the model from which we can see that it holds also when $I\to\infty$.[14] The equilibrium conditions for this version of the model are very similar to those with a continuum of countries. Here the mass of each country is $\frac{1}{I}$. We assume symmetric countries but relax this in the appendix. Given symmetric countries and log preferences the complete markets condition is $\mathbf{P}_t^i\mathbf{C}_{s,t,i}=\mathbf{P}_t^j\mathbf{C}_{s,t,j}$. Imposing symmetric and constant stock positions as well as constant taxes, government spending and borrowing, and borrowing limits, the savers' budget constraints in countries i and j are

$$\mathbf{P}_t^i\mathbf{C}_{s,t,i}=\bar{B}\left(1-\frac{1}{R_t}\right)+W_{t,i}N_{t,i}+\varphi\frac{(\mu-1)W_{t,i}N_{t,i}}{1-\chi}+\sum_{j\neq i}\frac{(1-\varphi)}{I-1}\frac{(\mu-1)W_{t,j}N_{t,j}}{1-\chi}.$$

Proof. where we used the assumption for the production function and the fact that taxes and transfers cancel assuming no new borrowing by government. Moreover, to simplify expressions in this case of symmetric countries, but without loss of generality, we here choose a different normalization of stock supply. Namely, each unit of the home stock entitles a saver to a dividend of $\frac{\Pi_t}{1-\chi}$. Deducting the conditions for two countries i and $j\neq i$ we obtain

$$\begin{aligned}\mathbf{P}_t^i\mathbf{C}_{s,t,i}-\mathbf{P}_t^j\mathbf{C}_{s,t,j}&=W_{t,i}N_{t,i}-W_{t,j}N_{t,j}+\varphi\frac{(\mu-1)W_{t,i}N_{t,i}-(\mu-1)W_{t,j}N_{t,j}}{1-\chi}\\ -(\mu-1)\frac{W_{t,i}N_{t,i}-W_{t,j}N_{t,j}}{1-\chi}\frac{1-\varphi}{I-1}&=\left(W_{t,i}N_{t,i}-W_{t,j}N_{t,j}\right)\left(1+\varphi\frac{\mu-1}{1-\chi}-\frac{1-\varphi}{I-1}\frac{\mu-1}{1-\chi}\right).\end{aligned}$$

Imposing the complete markets condition and ignoring the indeterminacy case (discussed in the appendix), we need

$$1+\varphi\frac{\mu-1}{1-\chi}-\frac{1-\varphi}{I-1}\frac{\mu-1}{1-\chi}=0. \tag{11}$$

From this one can solve

$$\varphi=\frac{1}{I}-\frac{I-1}{I}\frac{1-\chi}{\mu-1}. \tag{12}$$

With these stock positions the complete markets condition holds for arbitrary labor income realizations. The complete markets condition also ensures that the Euler equations for stocks and borrowing hold. Therefore, the above stock positions and no-cross country borrowing constitute an equilibrium that replicates the complete markets outcome. In the small open economy limit $I\to\infty$, a saver should hold $-\frac{1-\chi}{\mu-1}$ home stocks and $1+\frac{1-\chi}{\mu-1}$ foreign stocks split equally. □

[14] In the limit there is a countable infinity of countries instead of a continuum of countries. However, the limiting model is effectively equivalent to a continuum economy, see Sihvonen (2019) for a discussion. Moreover, we could prove all the results by imposing a continuum of countries a priori.

A.7. Proof of proposition 3

We need to first extend the argument in Proposition 1 to include static equity positions. For simplicity we directly impose a continuum of countries as in the proof of Proposition 1. Similarly to the proof of Lemma 1, using market clearing and the agents' budget constraints we can write

$$W_t N_t(\mu - (1-\alpha)(1+\varphi(\mu-1))) = F_t + (1-\chi)(1-\alpha)\left(\frac{B_{t+1}}{R_t} - B_{t,i}\right) - \chi(1-\alpha)\left(\frac{S_{t+1}}{R_t} - S_t\right) + (1-\alpha)(1-\chi)\Gamma_t.$$

Here Γ_t is the savers' income from foreign stocks. We also assumed away from public deleveraging and spending shocks. From this we can solve

$$W_t N_t = a_1 F_t + a_2\left(\frac{B_{t+1}}{R_t} - B_t\right) - a_3\left(\frac{S_{t+1}}{R_t} - S_t\right) + a_4 \Gamma_t,$$

where

$$a_1 = \frac{1}{\mu - (1-\alpha)(1+\varphi(\mu-1))}, a_2 = \frac{(1-\chi)(1-\alpha)}{\mu - (1-\alpha)(1+\varphi(\mu-1))},$$

$$a_3 = \frac{\chi(1-\alpha)}{\mu - (1-\alpha)(1+\varphi(\mu-1))}, a_4 = \frac{(1-\alpha)(1-\chi)}{\mu - (1-\alpha)(1+\varphi(\mu-1))} \quad .$$

The savers' budget constraint is

$$S_t + W_t N_t + \varphi(\mu-1)\mathrm{WN}_t + \Gamma_t = \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t}.$$

Plugging in the previous result and rearranging we obtain:

$$\mathbf{P}_t \mathbf{C}_{s,t} = (1+\varphi(\mu-1))a_1 F_t + (1+\varphi(\mu-1))a_2\left(\frac{B_{t+1}}{R_t} - B_t\right) - (1+\varphi(\mu-1))a_3 + 1)\left(\frac{S_{t+1}}{R_t} - S_t\right) + (1+\varphi(\mu-1))a_4 + 1)\Gamma_t.$$

Similarly to the proof of Lemma 1, using iteration and a no-Ponzi condition, it now follows that $\sum_{k=0}^{\infty} \frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}(z^t)$ is only a function of S_t, B_t, $\sum \frac{F_{t+k}}{R_{t,k}}$ and $\sum_{k=0}^{\infty} \frac{\Gamma_{t+k}}{R_{t,k}}$ that do not react to domestic deleveraging shocks:

$$\sum_{k=0}^{\infty} \frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}(z^{t+k}) = \widehat{\Omega}\left(S_t, B_t, \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t,k}}(z^{t+k}), \sum_{k=0}^{\infty} \frac{\Gamma_{t+k}}{R_{t,k}}(z^{t+k})\right)$$

This generalizes the argument of Proposition 1 to static equity positions.

A.7.1. The main argument

Given symmetric borrowing patterns the optimal stock positions perfectly share shocks affecting labor income such as quality shocks by the argument in Proposition 2. These shocks need not be idiosyncratic. Idiosyncratic deleveraging shocks do not distort symmetry and the savers' consumption expenditure stays constant by the argument in Proposition 1. While the proof assumes that the home quality stays constant it also goes through with unanticipated home quality shocks. Moreover, it works for preference shocks that do not alter the complete markets condition such as shocks to the disutility of labor. Under certain further restrictions on fiscal policy, the proof can be generalized to public deleveraging.

Appendix B. A simple banking model

Generally, a natural definition of an ideal banking union is that borrowing (and lending) rates depend only on the risk characteristics of the borrower. In particular the borrowing rate should not depend on the location of the borrower, after controlling for risk attributes. Effectively there is just a single union wide bank market for borrowing and saving and banks have no incentive to discriminate between customers based on location. Local banking conditions and the local health of banks do not affect the borrowing rates. This implies in particular that the risk-free rates are equalized across countries. As explained before we call this feature of an ideal banking union a money market union.

In practice these money market flows would plausibly be intermediated by banks. However, we have abstracted away from explicitly modeling banks as this would simply complicate the model without bringing new insights. For illustrative purposes, we now sketch a simple banking model consistent with our interpretation. Note that this is not the only model consistent with our interpretation.

B.1. Segmented markets

Household saving and borrowing is intermediated by banks. In each country there is a competitive representative bank. Domestic households can only transact with this local bank but the bank can also borrow from foreign banks. As in the main text, we assume all contracts are one period and there is no default[15]. For simplicity, we abstract away from bank equity[16] so that

$$B^l_{t+1,i} = B^d_{t+1,i} + B^f_{t+1,i},$$

where $B^l_{t+1,i}$ is loans to domestic households, $B^d_{t+1,i}$ is deposits of domestic savers and $B^f_{t+1,i}$ is total borrowing from foreign banks (can be negative). Here we could add lending to the government to the model without altering the key results. The bank's profit is given by

$$\frac{B^l_{t+1,i}}{R^l_{t,i}} - \frac{B^d_{t+1,i}}{R^d_{t,i}} - \frac{B^f_{t+1,i}}{R^f_{t,i}}.$$

Here $R^l_{t,i}$, $R^d_{t,i}$ and $R^f_{t,i}$ are the interest rates for loans, deposits and borrowing from foreign banks. The bank profits are distributed to households according to some rule. The local bank faces a (generally multi-dimensional) constraint of the form

$$\Xi(B^l_{t+1,i}, B^d_{t+1,i}, \mathbf{B}^f_{t+1,i}, Z_{t,i}) \geq 0,$$

for some function Ξ, where $Z_{t,i}$ is a set of state variables. The $\mathbf{B}^f_{t+1,i}$ includes the bank's borrowing from foreign banks but also its portfolio of loans to banks in other countries. This constraint represents country specific lending and borrowing frictions. This constraints plausibly becomes tighter during a crisis period as captured by the state variable $Z_{t,i}$. Moreover, the form of constraint implies that a bank can be particularly constrained to lending banks in a specific country, such as a country with bad economic conditions. This might capture, in a reduced form way, effects similar to bank default risk. The high bank funding costs would then generally be transmitted to the rates faced by households in that country.

The problem of the bank is to choose $B^l_{t,i}$, $B^d_{t,i}$and $\mathbf{B}^f_{t+1,i}$ to maximize profits subject to this constraint. This problem then defines the rates faced by households in each country as well as the rates in the bank funding market. However, fully specifying and solving a model with a continuum of local banks is beyond the scope of this paper. Therefore in the numerical part we follow Martin and Philippon (2017) and take spreads as exogenous, matching them to data from the Eurozone.

B.2. Money market union

In a MMU there is just one competitive representative bank and households in each country transact with this bank. The constraint takes the form

$$\Xi(B^l_{t+1}, B^d_{t+1}, Z_t) \geq 0,$$

where B^l_{t+1} is aggregate loans to domestic households and B^d_{t+1} is aggregate deposits and Z_t is a set of aggregate state variables. Because there is no default and country specific variables affecting the constraint, all households face the same interest rates and there are no country specific spreads. This would still hold if we assume a continuum of non-identical banks. In a MMU the banks are not constrained to lend to households in a particular country and have no incentive to discriminate between households in different countries.

This equalization of interest rates in different countries is the key condition to facilitate risk sharing within the currency union. Because of the constraint, the rates faced by households might still differ from the policy rate.[17] For example borrowers in all countries might face a slightly higher rate than savers. The effect of this would be immaterial to our results. For example consider the exercise in Fig. 1 but now assuming the borrowers face 1% higher rate than the savers. A deleveraging shock of 5% increases

[15] We considered default in a previous version of the paper but removed it because it brought additional complications yet few additional insights..

[16] We could think of this as a limit of a model with no bank equity.

[17] We can justify the effect of monetary policy in the standard way of assuming the households can also hold money but considering the cashless limit of this economy.

the savers' nominal spending by roughly 0.002%. The effect is not generally zero because deleveraging is not any more a zero NPV transaction when calculated using the savers' rate. However, this effect is numerically extremely small. Note that our numerical exercise for segmented markets instead shows that volatile and counter-cyclical country specific spreads can instead be highly detrimental to welfare.

Hence under a money market union, we can consider a bank problem with no constraint. By bank profit maximization the lending and borrowing rates in each country are now equal and the bank makes zero profits. Such a banking model is homeomorphic to our model of a MMU. For the purposes of this paper, in an ideal MMU, banks are largely a veil.

Appendix C. Productivity and government spending shocks only

Due to Cole-Obstfeld preferences, price adjustments give a natural hedge against productivity and government spending shocks. This can be formalized in the following lemma that generalizes the famous Cole and Obstfeld (1991) result to a borrower-saver agent economy with rigidities. Note also the limitations of the lemma: it considers a setting with only productivity shocks and government spending shocks. That it does not hold for example in an environment with both productivity and quality shocks in which case the CMU still attains the complete markets outcome.

Lemma 5. Cole-Obstfeld 91 Result with Borrowers *Consider the baseline model of the paper but subject to productivity shocks only. The optimal stock positions are indeterminate and the equilibrium always attains the complete markets allocation for both borrowers and savers (absent any cross-country borrowing). The result holds also when we add idiosyncratic government spending shocks financed through (potentially distortionary) taxes absent government borrowing. This effectively implies a nominal fiscal consumption multiplier of zero.*

Proof. Similarly to before we perform the proof in an Icountry version of the model. For any country i

$$A_{t,i}N_{t,i}P_{t,i} = \mu W_{t,i}N_{t,i}.$$

Here $P_{t,i}$ is the price of the good produced by country i. Conjecture that the model attains the complete markets outcome for both savers and borrowers. That is for any countries i and j:

$$\mathbf{C}_{s,t,i}\mathbf{P}_t^i = \mathbf{C}_{s,t,j}\mathbf{P}_t^j,$$

where P_t^i is the consumer price index in country i and

$$\mathbf{C}_{b,t,i}\mathbf{P}_t^i = \mathbf{C}_{b,t,j}\mathbf{P}_t^j.$$

Now we have,

$$\frac{A_{t,i}N_{t,i}}{A_{t,j}N_{t,j}} = \frac{(1-\alpha)\chi\mathbf{C}_{s,t,i}\mathbf{P}_t^i/P_{t,i} + (1-\alpha)(1-\chi)\mathbf{C}_{b,t,i}\mathbf{P}_t^i/P_{t,i} + \frac{\alpha}{I-1}\sum_{k\neq i}(\chi\mathbf{C}_{s,t,k}\mathbf{P}_t^k/P_{t,i} + (1-\chi)\mathbf{C}_{b,t,k}\mathbf{P}_t^k/P_{t,i})}{(1-\alpha)\chi\mathbf{C}_{s,t,j}\mathbf{P}_t^j/P_{t,j} + (1-\alpha)(1-\chi)\mathbf{C}_{b,t,j}\mathbf{P}_t^j/P_{t,j} + \frac{\alpha}{I-1}\sum_{k\neq j}(\chi\mathbf{C}_{s,t,k}\mathbf{P}_t^k/P_{t,j} + (1-\chi)\mathbf{C}_{b,t,k}\mathbf{P}_t^k/P_{t,j})}.$$

Then applying the complete markets conditions, we obtain

$$\frac{A_{t,i}N_{t,i}}{A_{t,j}N_{t,j}} = \frac{\chi\mathbf{C}_{s,t,i}\mathbf{P}_t^i + (1-\chi)\mathbf{C}_{b,t,i}\mathbf{P}_t^i}{\chi\mathbf{C}_{s,t,i}\mathbf{P}_t^i + (1-\chi)\mathbf{C}_{b,t,i}\mathbf{P}_t^i}\frac{P_{t,j}}{P_{t,i}} = \frac{P_{t,j}}{P_{t,i}}.$$

Prices and output levels move inverse one-to-one. But this implies

$$W_{t,i}N_{t,i} - W_{t,j}N_{t,j} = 0.$$

Now one can see that the budget constraints support the complete markets conditions for both savers and borrowers for any symmetric stock positions. Note that α can be arbitrary so the result also holds with respect to symmetric quality shocks. However, it does not hold with respect to arbitrary quality shocks such as shocks that only affect some countries.

What is the intuition behind the result? Assume that markets are complete. Now due to Cole-Obstfeld preferences relative output levels and prices must move one-to-one. This means that the value of output in each country must be the same. Higher production implies lower prices. But the assumption for production technology implies that labor income is a constant fraction of the total value of output in each country. This means that total labor income in each country must be the same. Finally, this implies that the budget constraints support the complete markets allocation. The result holds also in the SOE limit $I\rightarrow\infty$.

To see that the result holds when adding idiosyncratic government spending shocks financed through current taxes (in the SOE limit) note that in the proof of lemma 1, we have the line

$$\alpha\tilde{Y}_t = (1-\alpha)\chi\left(\frac{B^h_{t+1}}{1+r_t} - B^h_t\right) - (1-\alpha)(1-\chi)\left(\frac{S_{t+1}}{1+r_t} - S_t\right) + F_t + \frac{B^g_{t+1}}{1+r_t} - B^g_t.$$

Now absent any borrowing this becomes

$$\alpha\tilde{Y}_t = F_t.$$

In SOE government spending shock does not affect F_t and therefore $\tilde{Y}_t$ does not react. Foreign demand solely determines income. By Condition 1 neither the borrowers' nor the savers' income reacts. By the budget constraints the nominal consumption levels do not react either. Because private consumption does not react in any country, the total value of production in the home country must rise by the value of nominal government spending. Therefore the fiscal multiplier is one. A similar simplified argument could be used for productivity shocks, but the former proof highlights that this first result holds also in the finite country case.

Appendix D. Government spending shocks: An example

Fig. D.5 shows the impulse responses subject to a government spending increase. Here we assume the spending increase is financed by a contemporaneous increase in a single tax rate on all income. Nominal spending by households stays constant. Higher government demand for the home good pushes its price up. Before tax wages increase. The reason why nominal consumption does not react is roughly the following. An increase in government spending implies higher taxes, which reduces the disposable income of households. However, at the same time higher government spending increases production improving nominal profits and labor income. With Cole-Obstfeld preferences these two effects exactly offset each other so that the disposable income of savers does not react. This implies that nominal consumption also stays constant. Because disposable income stays constant each period, savings do not react either.

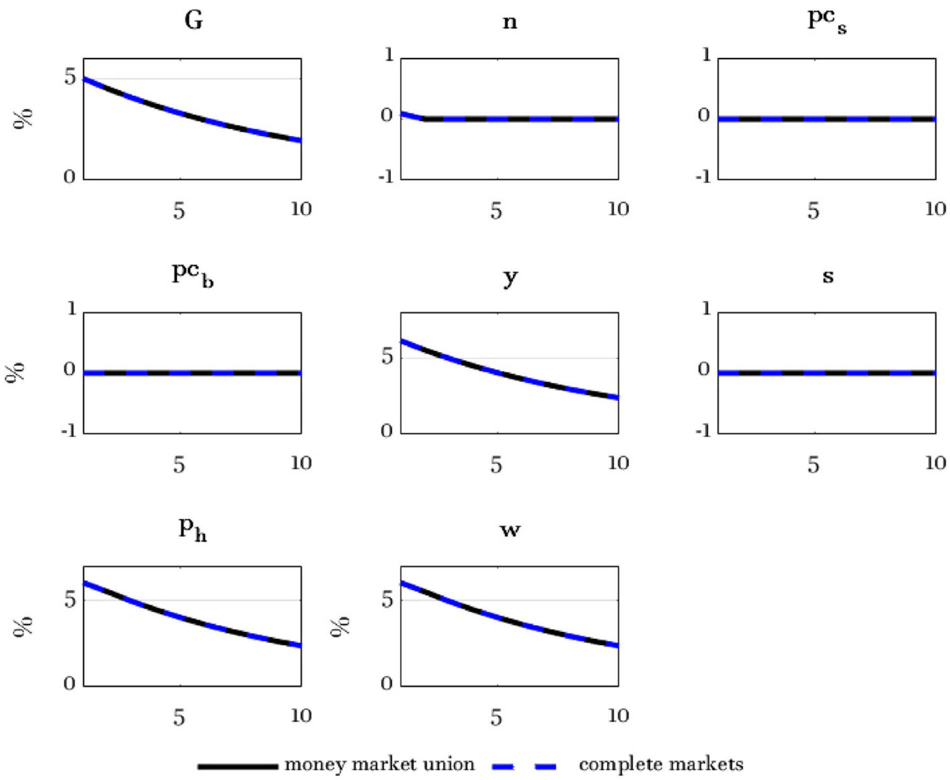


Fig. D.5. Government spending shock. Note: Impulse responses to a 5% shock to G_t. y_t is disposable income.

Appendix E. Asymmetries

We now generalize the results concerning equity to asymmetric initial stock positions, mark-ups and shares of savers. The complete markets condition is $\mathbf{P}_t^i\mathbf{C}_{s,t,i} = \lambda_{i,j}\mathbf{P}_t^j\mathbf{C}_{s,t,j}$, where $\lambda_{i,j}$ is the relative Pareto weight. We first show the result in a two country version of the model and then tackle the I country case. The budget constraints are

$$\bar{B} + N_tW_t - T + \varphi(\mu-1)N_tW_t + (\frac{1}{1-\chi} - \frac{1-\chi^*}{1-\chi}\varphi^*)(\mu^*-1)N_t^*W_t^* = \mathbf{P}_t\mathbf{C}_{s,t} + \frac{\bar{B}}{R_t}$$

$$\bar{B} + N_t^*W_t^* - T + (\frac{1}{1-\chi^*} - \frac{1-\chi}{1-\chi^*}\varphi)(\mu-1)N_tW_t + \varphi^*(\mu^*-1)N_t^*W_t^* = \mathbf{P}_t^*\mathbf{C}_{s,t}^* + \frac{\bar{B}}{R_t},$$

where starred values refer to the foreign country. Deducting the budget constraints and imposing the complete markets condition yield

$$N_tW_t\left(1+(\mu-1)(\varphi - \frac{1}{1-\chi^*} + \frac{1-\chi}{1-\chi^*}\varphi)\right) - N_t^*W_t^*\left(1+(\mu^*-1)(\varphi^* - \frac{1}{1-\chi} + \frac{1-\chi^*}{1-\chi}\varphi^*)\right) = (\lambda-1)\mathbf{P}_t^*\mathbf{C}_{s,t}^*.$$

or

$$N_tW_t\left(1+(\mu-1)(\varphi - \frac{1}{1-\chi^*} + \frac{1-\chi}{1-\chi^*}\varphi) - \frac{\lambda-1}{1+\lambda}\mu\right) - N_t^*W_t^*\left(1+(\mu^*-1)(\varphi^* - \frac{1}{1-\chi} + \frac{1-\chi^*}{1-\chi}\varphi^*) + \frac{\lambda-1}{1+\lambda}\mu^*\right) = 0.$$

From this we solve

$$\varphi = \frac{1}{2-\chi-\chi^*} + \frac{\lambda-1}{1+\lambda}\frac{\mu}{\mu-1}\frac{1-\chi^*}{2-\chi-\chi^*} - \frac{1}{\mu-1}\frac{1-\chi^*}{2-\chi-\chi^*}$$

and

$$\varphi^* = \frac{1}{2-\chi-\chi^*} + \frac{\lambda-1}{1+\lambda}\frac{\mu^*}{\mu^*-1}\frac{1-\chi^*}{2-\chi-\chi^*} - \frac{1}{\mu^*-1}\frac{1-\chi^*}{2-\chi-\chi^*}.$$

The relative Pareto weight λ depends on initial conditions and can be solved numerically. φ is increasing in λ and φ^* decreasing. The result can be generalized to different tax rates. The above derivations generalize Proposition 2. Proposition 3 can be generalized similarly.

With I countries the budget constraints are:

$$\bar{B} + N_{t,i}W_{t,i} + \sum\varphi_{\mathrm{i,k}}(\mu_k-1)N_{t,k}W_{t,k} = \mathbf{P}_t^i\mathbf{C}_{s,t} + \frac{\bar{B}}{R_t}, i = 1,..,I.$$

The complete market condition is$\mathbf{P}_t^i\mathbf{C}_{s,t,i} = \lambda_{i,j}\mathbf{P}_t^j\mathbf{C}_{s,t,j}$. Deducting the budget constraints and using this condition we obtain:

$$N_{t,i}W_{t,i}(1+\varphi_{\mathrm{i,i}}(\mu_i-1)) - N_{t,j}W_{t,j}(1+\varphi_{\mathrm{i,j}}(\mu_j-1)) + \sum_{k\neq i,j}(\varphi_{\mathrm{i,k}} - \varphi_{j,k})(\mu_k-1)N_{t,k}W_{t,k} = (\lambda_{\mathrm{ij}}-1)\mathbf{P}_t^i\mathbf{C}_{s,t,i}, j\neq i.$$

Using the fact that value of total consumption equals value of total output as well as the complete market conditions:

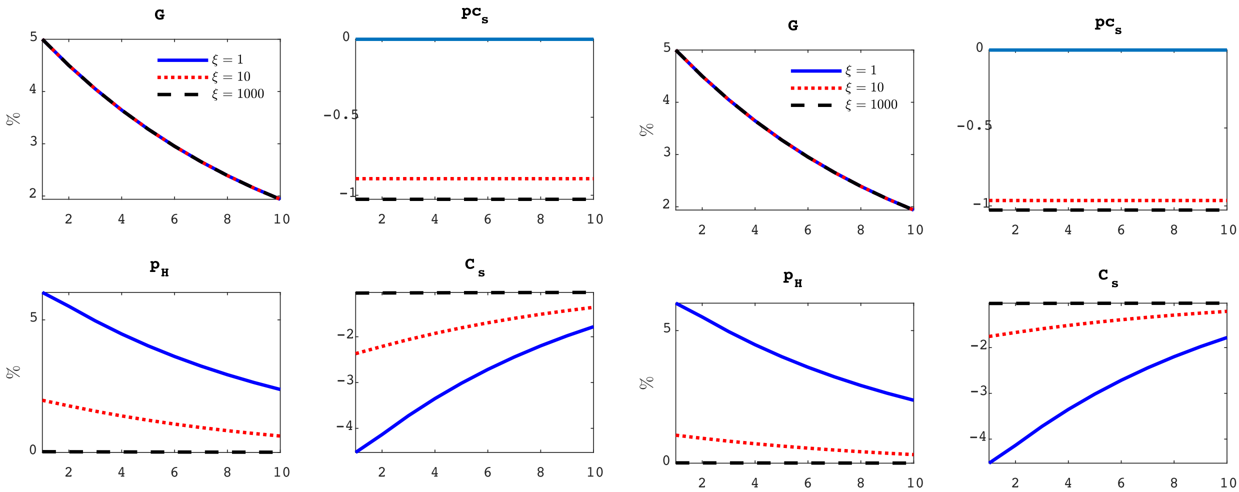
$$\begin{aligned} &N_{t,i}W_{t,i}(1+(\varphi_{\mathrm{i,i}} - \varphi_{j,i})(\mu_i-1)) - N_{t,j}W_{t,j}(1+(\varphi_{\mathrm{i,j}} - \varphi_{\mathrm{j,j}})(\mu_j-1)) \\ &+\sum_{k\neq i,j}(\varphi_{\mathrm{i,k}} - \varphi_{j,k})(\mu_k-1)N_{t,k}W_{t,k} = (\lambda_{\mathrm{ij}}-1)\frac{\sum_k\mu_kW_{t,k}N_{t,k}}{1+\sum_{k\neq j}\lambda_{\mathrm{jk}}}, j\neq i. \end{aligned}$$

We need to set the multiplier on each $N_{t,k}W_{t,k}$ to zero, which gives a well-defined problem. For each j we get I restrictions in total. There are $I-1$ such equations. Together with the stock market clearing conditions we have $I \times I$ equations and unknowns and can now solve for the static equity positions replicating the complete market outcome. The result holds also in the small country limit $I\rightarrow\infty$.

Appendix F. Proposition 1: Beyond cole-obstfeld for fiscal shocks

Pure government *deleveraging*, that does not affect purchases G_t, works similarly to private deleveraging and is not sensitive to the demand elasticity parameters. However, government *spending* shocks can have a large effect on the overall level of taxation not just the timing of taxes. When government spending increases are financed through distortionary taxes, demand elasticities can have a larger effect on how nominal consumption reacts. Fig. F.6 repeats the exercise of Fig. D.5 but now with different values of the elasticity of substitution between home and foreign goods $\xi_1 \in \{1, 10, 1000\}$. Note that a demand elasticity of one (for home and foreign countries) is close to empirically reasonable values (see e.g. Heathcote and Perri, 2013 for a discussion).

Fig. F.6b performs the exercise in Fig. F.6a but now with different values of elasticity of substitution between different varieties of foreign goods $\xi_2 \in \{1, 10, 1000\}$ [18]. The results are similar to those before.



(a)Elasticity between home and foreign goods (ξ1) (b)Elasticity between varieties of foreign (ξ2)

Fig. F.6. Government spending shock in a money market union for different values of elasticity of substitution between home and foreign goods (a) and different varieties of foreign goods (b). Note: Impulse response to a 5% shock to G_t.

The result that government spending shocks do not affect nominal consumption is also sensitive to the assumption that the government purchases only home goods. However, this sensitivity vanishes as the demand elasticity approaches infinity.

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[18] Here the demand elasticity changes in all countries simultaneously

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