

Increasing mathematics students' involvement in research activities at teacher education universities

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Abstract

The paper explores the problem of creating interest in mathematics students' research endeavors at teacher preparation institutions. The purpose of the study is to determine whether it is possible to organize students' scientific investigation using the 5E Instructional Model during a workshop on approximation theory and the Fourier series. The study takes into account the survey responses from the students who assisted in assessing the workshop participants' emotional states. The Differential Emotion Scale, a tool for expressing evaluations of positive and negative emotionality, was used to gather the data. The benefits of the atmosphere created by the 5E Instructional Model on students' emotional states and how it shapes their motivation in scientific inquiry when setting up workshop classes are covered in the article. We have reason to believe that running workshops based on the 5E Instructional Model is effective. The students' involvement throughout the session and enthusiasm in research activities increased as a result of the index reduction of their negative feelings.

Keywords: research activities; 5E Instructional Model; Approximation Theory and Fourier Series.

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INTRODUCTION

The determining factor of training a competitive teacher is the organization of his/her scientific activities that allow the teacher to create independently new means of pedagogic activities, create new ideas and approaches that correspond to the changeable requirements of the present time. Yarullin, Bushmeleva, and Tsyrukun ([2015](#)) called the organization of the student's scientific activities one of the mechanisms to form their research competence. The scientists: Jahnke et al. ([1983](#)), Turner ([2010](#)), Vintere and Zeidmane ([2014](#)), Proulx ([2015](#)), Koichu and Pinto ([2018](#)) determine the research competence as one of the components of mathematics teacher's professional competence that guarantees further research activities of the graduate in mathematics. Regarding this, the issue of organizing research activities of pre-service mathematics teacher stays constantly open in pedagogical studies.

Many authors have emphasized the necessity to support active students' research activities. Lithner ([2000](#)) pointed out that international tendency

in mathematics education is acquiring mathematical knowledge not only in terms of context but in terms of getting skills connected with carrying out mathematical research. Bonwell and Eison (1991), quoted in Fallon, Walsh, and Prendergast (2013), stated that students have to do more than just listen. They have to read, discuss and do research on the problems. Scientists emphasized the importance of organizing intensive students' activities, forming their positive attitude to research projects. This idea is agreed with the conclusions made by Hernandez-Martinez and Vos (2018), that described the critical state of the question to form students' interest in research activities. In their work Jones, Black, and Coles (2019) confirm that at every level of university students' training to mathematics teacher's activities, it is necessary to form their creative thinking and their investigative skills. Scientists emphasize that the organization of students' research activities during their training encourages the development of research competence, necessary both for solving practical problems and for being able to adapt fast to changeable conditions of the modern time and master their skills constantly.

We agree with Yore (2001), who states that forming an interest in research activities is the first stage in the development of research competence during mathematics learning. We also took into account the ideas of Dreyfus et al. (2018), who considers research activities during Mathematics learning as a natural part of the educational process, which is directed at forming research competence among students. In order to organize such activities, scientists offer to use special courses dedicated to special scientific researches in the priority areas of modern mathematics. This fact is evidenced by the opinion of the mathematical community (Yarullin, Bushmeleva, & Tsyrukun, 2015; Biza, Giraldo, Hochmuth, Khakbaz, & Rasmussen, 2016; Telegina, Drovosekov, Vasbieva, & Zakharova, 2019) about the significant potential in the researches on forming a positive attitude to students' research activities using the materials of different mathematical branches. In scientists' opinion, the usage of interesting mathematical theories encourages students to get more meaningful education of theoretical materials, facts, and methods of solving mathematical problems and it allows getting particular experience. We can also meet the confirmation of this opinion in the works by Matejko and Ansari (2018), Sevinc and Lesh (2018), who investigated the organization of research activities related to particular branches of mathematics. The idea caught on, that is why guided by the conclusions made by the above-mentioned scientific researches we decided to research the creation of interest among students of mathematics departments in research activities through organizing a workshop on Approximation Theory. The choice of this branch results from its extensive use in practice. This is explained by the fact that the modern stage of science and technology development is characterized by the use of a considerable amount of information. As experience shows this tendency will only enhance in the future – the development of computer science, telecommunication, and registration equipment lead to steady growth of the data amount. Therefore, tools and methods of their processing and

analysis are growing. The creation of a single methodical approach based on general mathematical principles is actual for several tasks such as to get, model, register and process data. The series finds a mass use as a tool to represent a considerable class of functions, carrying out analytical transformations, approximate calculations in many applied tasks. Algorithmic and computer software that is created on their basis is characterized by high universality and is included in computer and hardware – computer complexes of different purposes, which is confirmed by the numerous researches by Malvar (1992), Dedus, Makhortykh, and Ustinin (2002), Pankratov et al. (2009), Novikov and Rovenska (2017).

The article is aimed at describing an introduction of a seminar in Approximation Theory and Fourier Series searching for a method to implement a workshop on that was developed to enhance interest in research activities in mathematics students in teacher training universities.

METHOD

At the first stage of the research, we used the survey method to evaluate students' interest in research activities. We used the Differential Emotions Scale by Izard (1977) to survey students.

The relevance of involving this methodology to evaluate students' interest in research activities is proven by the researches where the direct dependency between the subject's interest in cognitive activities and their emotional state during its implementation is emphasized. Since the feeling is a dynamic component of the emotion (Panksepp, 2003) and two psychobiological processes connected with it – fascination and individuation (Langer, 1967), motivating, managing and informative functions of feelings allow them to capture or simplify and organize the thing that can become (especially in difficult situations) a great number of impulses in concentrated cognitive processes. During 2015 – 2019 we surveyed master's degree students of physics – mathematics departments of Kryvyi Rih State Pedagogical University and Berdyansk State Pedagogical University. 49 master's students took part in the survey (17 male students and 32 female students aged from 20 to 28). The use of the online survey, first through Google form, posted on the Internet, and then, moved to the forum of the platform «Higher School Mathematics Teacher» (n/d) had an advantage in comparison to the survey on paper as it encouraged the respondents' frankness and prevented missed questions.

According to the chosen methodology, we selected Likert scale to evaluate each of the basic emotions 1 – 'feeling is completely absent'; 2 – 'feeling is slightly expressed'; 3 – 'feeling is moderately expressed'; 4 – 'feeling is strongly expressed'; 5 – 'feeling is fully expressed'. At the beginning of the research, the most significant (> 9 points) positive emotion related to the experience of research activities was 'interest', negative – 'shame' and 'fear'. Students usually face the last two emotions while learning mathematics.

Students believe that the key problem of learning mathematical theory is the absence of the connection between the theory and practice and abstract character of the subject.

During the second stage of the research, we developed the subjects of the workshop on Approximation Theory.

The workshop program consists of six classes.

1. The history of the development of approximation theory and Fourier series.
2. The ways of periodic function classification.
3. Approximation methods that are based on matrix series summing.
4. Main tasks of approximation theory: approximation of individual function, class approximation, precise and asymptotically precise ratio.
5. Examples of researches by subject.
6. Examples of using approximate aggregates in computer complexes of broad purpose.

The workshop was aimed at the formation of students' interest in research activities through their implementation into the real process of using series in applied tasks.

The workshop was held for a group of 7 – 8 students twice a month for three months. Every class included two hours of classwork and three hours of extracurricular work. The classes were held by the prominent teachers of mathematics departments who took part in the development of the workshop and looked for the method, the implementation of which would encourage the formation of students' interest in research activities during the workshop.

With the help of the Deductive Approach to Content Analysis, we determined the possibility to involve the 5E Instructional Model. During the selection of resources for the analysis of the model usability, we were oriented at those representing the usability of the 5E Instructional Model while learning. Among them we can name TeachThought, Lesley, The National Academies Board on Science Education, Alberta Education ([Table 1](#)).

We also found out what the purpose of using the 5E Instructional Model by other scientists was. Cheng, Yang, Chang, and Kuo ([2016](#)) noted the efficiency of using the model stages to increase the motivation of students' learning. L. Duran and E. Duran ([2004](#)) describe the use of this model in programs of professional development in education. Supasorn and Promarak ([2015](#)) see the use of the 5E Instructional Model as an efficient method of improving students' understanding of natural processes. In conclusions of scientific researches done by Bybee et al. ([2006](#)), Abdi ([2014](#)), Ong et al. ([2018](#)) we also find the confirmation of the 5E Instructional Model efficiency in improving students' achievements in science. This model consists of the stages: Engagement, Exploration, Explanation, Elaboration, and Evaluation. Every stage provides a specific pedagogical function and encourages teaching processes from the teacher's side and forming a better understanding of scientific knowledge, abilities, and skills from the student's side.

Table 1. Analysis of the resources that represent the 5E Instructional Model usage

Resources	Used while learning a subject	Model features	What are the efficiency grounds
Teach Thought (Lesley University Online & TeachThought, n/d)	Biochemistry and Molecular Biology Education, Mathematics	Joint activities	The solid knowledge foundation through an active part
Lesley University (Lesley University, n/d)	Mathematics, Life sciences	Constructing knowledge based on experience	Possibility for the full cycle of education
The National Academies Board on Science Education (Bybee, 2009)	Biological sciences	Structure and sequence of education are directed at creating a challenging situation	Integration of learning activity with laboratory experience
Alberta Education (Branch & Oberg, 2004)	Librarianship, Work with information	Student's involvement in metacognition; encouragement of critical and creative thinking	Focus on achieving defined learning outcomes in different subjects

We developed the recommendations that were implemented during workshop classes in order to use the stages of the 5E Instructional Model during the organization of workshop classes on Approximation Theory.

Engagement. At this stage, the teacher has to determine what students already know regarding the concept that is considered and what kind of knowledge they still need. In order to master new educational material, it is necessary to help students to revise mathematics sections such as Algebra, Mathematical Analysis, Functional Analysis, Function Theory. Moreover, at this stage, the teacher is only a consultant who helps students to prepare short reports encouraging students' interest and motivation. For this purpose, the teacher presents the actuality of the researches dedicated to learning approximate features of approximation methods that are generated by certain transformations of partial sums of Fourier series and allow building the sequence of trigonometric polynomials that would equally coincide for any function ([Table 2](#)).

Table 2. Recommendation for the teacher on organizing the state of Engagement

Appropriate	Inappropriate
- encourage students to raise their questions; - offer to compare their ideas with others.	- read the lecture; - give definitions to terms; - explain or give tasks.

Exploration. The stage is directed at strengthening students' activities regarding knowledge and skills. At this stage, students can revise the tasks that use the methods of approximation theory on special subjects that they learn. As a rule, students cite examples of tasks on periodic signal approximation in the theory of control engineering, pattern recognition, nondestructive

testing, etc. Students can discuss and write down approximation methods in every particular case. The teacher is only a consultant who offers students such research methods as observation, hypothesis generation, forecasting. Students' communication and work in groups without the direct teacher's involvement are encouraged, equally coincide for any function ([Table 3](#)).

Table 3. Recommendation for the teacher on the organization of Exploration

Appropriate	Inappropriate
<ul style="list-style-type: none"> - encouragement of search for several ways to solve the problems; - comparison of ideas; - self and mutual survey. 	<ul style="list-style-type: none"> - use of traditional explanation; - implementation and involvement of a great amount of terminology.

Explanation. At this stage, students can describe their point of view regarding the search for solving extreme problems of approximation theory. After this, the teacher has to introduce common terminology and acquaint the students with the general scheme of researching integral images of trigonometric polynomials variations that are generated by linear methods of summing Fourier series, from periodic functions. Generating students' new ideas on methods of approximation improvement, their comparison with the ideas of the previous stage is possible. At this stage, the teacher also has to prevent possible mistakes while explaining misconceptions that could arise at the stage of engagement and exploration. During the classes of this stage, the teacher involves interactive methods and presentations for mathematical modeling of periodic processes ([Table 4](#)).

Table 4. Recommendation for the teacher on the organization of Explanation

Appropriate	Inappropriate
<ul style="list-style-type: none"> - teacher's explanation; - expression of the ideas using generally accepted terms; - idea review and forming new ones. 	<ul style="list-style-type: none"> - forming a great amount of terminology; - focus on independent work.

Elaboration. After getting an explanation about the research main scheme regarding integrated images of trigonometric polynomials variations on the classes of periodic functions it is important to involve students in further research activities. Further work includes significant analytical calculations connected with exact and approximate methods. Starting from the integral image students can learn asymptotic behavior of exact upper bounds of trigonometric polynomials variations from periodic functions to infinity. The stage is aimed at helping students to develop a deeper understanding of general methods of mathematical analysis and the use of approximation processes in practical tasks. Students can carry out additional researches, develop new approximation methods, exchange ideas and use acquired research experience to integrate approximation theory in practice. It is possible to create algorithmic and computer algorithmic products based on built methods ([Table 5](#)).

Table 5. Recommendation for the teacher on the organization of Elaboration

Appropriate	Inappropriate
<ul style="list-style-type: none"> - understanding enhancement through strengthening the ideas acquired by experience; - use of algorithms that are close to new situations; - grounds for conclusions; - support of forming student's proper ideas. 	<ul style="list-style-type: none"> - development of the ideas that do not have a connection with previous experience; - generating a great number of ideas without deepening in the essence of the theory.

Evaluation. Evaluation is considered to be a permanent process during which the teacher only observes the students and supports during report presentations, idea introduction, and question tasks. The use of peer assessment is relevant. Such a form of evaluation can be complemented by students' self-assessment of their level. During the classes of this stage, the teacher involves interactive methods and presentations for mathematical modeling of periodic processes ([Table 6](#)).

Table 6. Recommendation for the organization of the concluding stage

Appropriate	Inappropriate
<ul style="list-style-type: none"> - evaluate the progress in general in comparison to the initial level; - evaluate the ability to use approximate methods to solve complex problems; - give students feedback regarding the feasibility of their ideas; - encourage questions that enhance a deeper understanding of the influence of individual function features on the approximation order. 	<ul style="list-style-type: none"> - evaluate single facts and separate elements of approximation theory; - offer a survey in a test form.

Using the 5E Instructional Model does not oblige the teacher to strictly follow the implementation of five stages. If necessary, it is possible to repeat the stages of Explanation and Elaboration several times.

RESULTS

During the preparation stage, we selected the target type as a selection strategy, because the selection had to include the students who have a high achievement level in mathematical branches. By high level, we understand the absence of the final mark «satisfactory» and lower following the national 4-level scale «unsatisfactory», «satisfactory», «good», «excellent» for each of the subjects «Algebra», «Mathematical analysis», «Functional analysis» and «Function theory». The target selected analysis provided us with sample size $n=49$ of students that represents 23.0% of the general number of master's degree students of the first year during 2015 – 2019. At the stage of organizing data collection, we used the tool of express-evaluation of positive and negative emotional state the Differential Emotion Scale ([Izard, 1977](#)), which ensures

diagnostics of a wide range of emotional states. Each of the ten basic emotions ($x_i, i=1,2,\dots,10$) is represented by three independent changeable 5-character scales for factors that describe emotional states. The points on every scale correspond to the level of emotional feedback and can be in total from 3 to 15 points. The stage of data analysis of every profile implies the selection of significant (>9 points) emotions, creation of 'emotion profile', determination of the dominant emotional state.

At the beginning of the research, the most significant positive emotions regarding the experience of research activities is 'interest', negative – 'shame' and 'fear' (Table 7).

Table 7. Distribution of significant emotions at the beginning of the survey Font styles for a reference to a journal article

Emotion	Number of students who have this emotion as dominant (>9 points)	Comparison with the general number of students
Interest	32	65.3%
Fear	45	91.8%
Shame	27	55.1%

While processing every profile we defined the indexes of emotional states that characterize the level of subjective students' emotional attitude to the present experience of research activities. The Index of positive emotions and Index of critically negative emotions could range from 9 to 45 points, the Index of anxio-depressive emotions ranged from 12 to 60 points. We defined that the positive emotional state turned out to be dominant among 69.4% of students, a strong level (> 36 points) of expressing positive emotional state was marked only among 6.1% of respondents. Also, a distinct (from 29 to 36 points) level of positive emotional state was fixed among 10.2% of students. Other students (53.1%) showed moderate (from 20 to 28 points) and weak (< 20 points) level. So, most students' attitude to the research process can be mainly characterized as positive. However, this positive attitude is weakly expressed, unstable and cannot ensure the proper motivation in overcoming difficulties that inevitably arise during research activities. This fact plays an important (if not the most important) role in the failure of attempts to involve an unprepared student to research activities in any area, including mathematics.

The dominant critically negative emotional state regarding the present experience of research activities was fixed among 12.2% of respondents, half of whom had a strong (>32 points) or distinct (from 25 to 32 points) level. It is important that among all the students who had the critically negative state as dominant, the factor "Dull" took no less than 4 points, and, accordingly, made the greatest contribution to the calculation. It testifies a stereotype regarding the complexity and absence of interest in research activities among young people. We considered this aspect while searching for methods of workshop implementation.

As mentioned above, the emotions ‘fear’ and ‘shame’ were detected as significant among 91.8% and 55.1% of respondents. These emotions are included in the third group of emotions that determine the anxio-negative emotional state of the subject regarding the experience of research activities. Despite this fact, the given state is dominant only among 18.4% of students. It demonstrates that these two emotions influence the formation. 4.1% of respondents have strong (> 30 points) level of emotional state, distinct (from 21 to 30 points) – 10.2%, moderate (from 12 to 20 points) and 4.1% of respondents – weak (< 12 points). Such a noticeable selection of two emotions in the general image of the emotional state confirms the idea that fear and shame prevent students from implementing their interest in the research process and take an active position while conducting research.

The repetitive survey was carried out after finishing the workshop. The distribution of significant emotions after taking a workshop is represented (Table 8).

Table 8. **Distribution of significant emotions after taking a workshop**

Emotion	Number of students who have this emotion as dominant (>9 points)	Comparison with the general number of students
Interest	44	89.7%
Surprise	18	36.7%
Fear	39	79.5%
Shame	5	10.2%

Interest turned out to be a significant positive emotion among 44 students. We can note that the number decrease in students who had shame as a significant negative emotion is well seen – 17 respondents. At the same time, the number decrease of students who had fear as a significant emotion is minor – 6 students (Fig. 1).

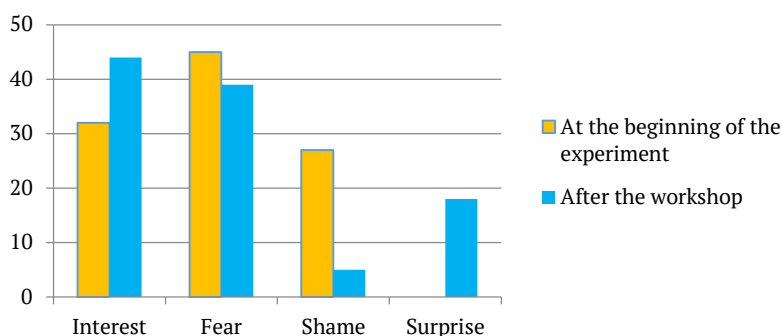


Figure 1. **Distribution of significant emotion**

Despite this fact it is impossible to claim that this emotion in the context of the given research is badly adapted. The profile analysis of respondents’ emotions shows the decrease of fear expression to varying degrees among 77.5% of students. The presence of surprise among the significant emotions, as well as interest, which is included in the positive group, is predictable.

More detailed analysis of the feasibility of implementing a workshop that was carried out using the index calculations of students' emotional states. We detected the increase of students with the dominant positive emotional state up to 81.7%, where 63.2% of respondents had a strong and distinct level. At the beginning of the workshop, the same indicator was 16.3%. Thus, we managed to form a stable positive attitude to research activities among more than half of the workshop participants.

The number of students who have a critically negative emotional state as dominant remained at the level of 12.2%, though the qualitative structure of this subgroup changed. In our opinion, it is connected with a greater amount of working practice in small groups during workshop classes in comparison to individual work. As teachers pointed out certain students perceived such format negatively.

The dominant anxio-negative subject's attitude to experience of research activities after taking a workshop was fixed among 6.1% of students. Among them 4.0% of respondents have moderate and 2.1% - weakly expressed level of emotional state. The comparative analysis of the students' number regarding dominant emotional states is displayed ([Fig. 2](#)).

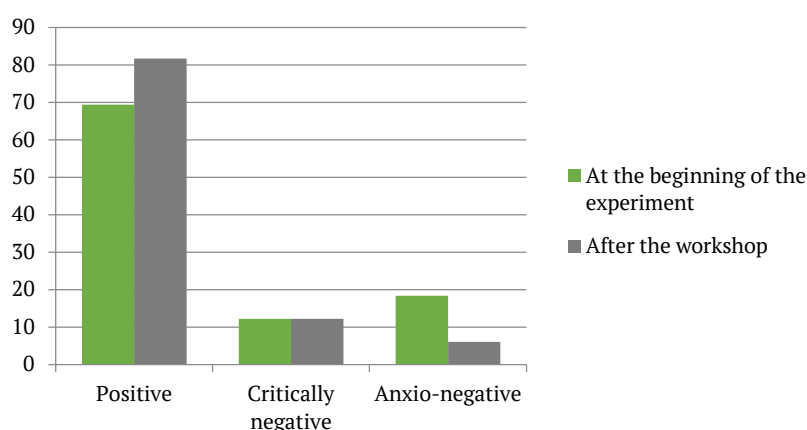


Figure 2. **Distribution of dominant states**

The analysis of the results proved that creating the environment based on the 5E Instructional Model during the scientific workshop where students did not feel negative emotions to research activities encouraged the increase of their interest in research activities.

DISCUSSION

Searching for the ways of forming interest of master's students of pedagogical universities in research activities we faced the researches done by Sandoval and Reiser ([2004](#)), Rocard et al. ([2007](#)). The scientists point out that in order to form students' impression of the real world it is necessary to show them how to organize their activities as real scientists do, experiencing the process of learning and knowledge grounding. Fallon et al offered to seek

the possibilities to organize students' research activities through method selection and forms of a learning organization that influences active students' involvement.

Traditional educational methods, which are oriented at the teacher, don't provide an active involvement of students to research activities ([Yore, 2001](#); [Lin et al., 2014](#); [Vlasenko, Chumak, Sitak, Lovianova, & Kondratyeva, 2019](#)). The scientists emphasize the importance of searching for educational models that encourage the strengthening of students' learning activities. The Deductive Content Analysis Method helped us to choose the 5E Instructional Model as the foundation of developing a scientific environment of students' education.

The efficiency of the 5E Instructional Model to encourage students' research activities is proved in the works by L. Duran and E. Duran ([2004](#)), Bybee and Landes ([1990](#)), Supasorn and Promarak ([2015](#)), Cheng, Yang, Chang, & Kuo ([2016](#)). Also, we support the opinion by Vlasenko, Chumak, Sitak, Chashechnikova, and Lovianova ([2019](#)), who believe that learning has to be built so that students can research, explain, extend and estimate their progress, and the introduction of ideas assumes students' awareness of the reason or necessity of their use. Such an approach is completely agreed with the structure, functioning, and consistency of the 5E Instructional Model.

Alshehri ([2016](#)) believes that while organizing research activities it is necessary to direct students to the main models of subject matters. For the pre-service mathematics teacher, one of the key subject matters is Approximation Theory, its broad influence on the modern state of innovation and technology development is widely known. The research is aimed at searching for ways of implementing a workshop on Approximation Theory to form students' interest in research activities. The main research result testifies that the use of the 5E Instructional Model influenced efficiently the formation of students' positive attitude towards research activities. Within this model, the involvement of the workshop on Approximation Theory encouraged the increase of the level of expressing students' positive emotional state (particularly interest, surprise increase) and decrease of anxiety level. These results are agreed with the conclusions by Coulson ([2002](#)), Chin and Lin ([2013](#)), Abdi ([2014](#)), Jung, Wranke, Hamburger, and Knauff ([2014](#)), Ong et al. ([2018](#)), who studied the connection between interest growth and persons' emotional state. This justifies the use of methodology Differential Emotions Scale by Izard ([1977](#)) during the experiment.

CONCLUSION

The actuality of involving students to research activities in education arises from the fact that research competence is considered as one of the components of the professional competence of the mathematics teacher. Enhancing students' interest in research content and research activities during the studies

also requires the use of a model that implies complete students' awareness of the importance of the research problem. The Deductive Approach to Content Analysis helped us determine the possibility to involve the 5E Instructional Model to the organization of workshop classes on Approximation Theory, point out the characteristics of the model and parameters of its efficiency. The model stages provide the formation of a better understanding of scientific knowledge, abilities, and skills from the students' side. We developed the content of workshop classes on approximation theory according to the model stages. Relying on the analysis of the present recommendations on the model use in learning different subjects, we offered the recommendations on the organization of workshop classes, It should be noted that the course should be provided following the stages Engagement, Exploration, Explanation, Elaboration, and Evaluation following certain recommendations that encourage students' interest in research activities.

Forming a positive attitude to research activities is the first step to the development of the research competence of pre-service mathematics teachers. The analysis of works on the connection between the person's interest and emotional state allowed formulating the most important positive and negative emotions that are connected with the experience of the research activities. The results of calculating the indexes of students' emotional states proved that the creation of the environment based on the 5E Instructional Model where students do not feel negative emotions to research activities encourages emotional state and interest in research activities.

The perspectives of future research involve the creation of the courses that use Inquiry-based approaches with the purpose of further research on forming research competence among students of pedagogical universities.

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