EVALUATING GRADE 10 LEARNERS' CHANGE IN UNDERSTANDING OF SIMILAR TRIANGLES FOLLOWING A CLASSROOM INTERVENTION

By

Amokelo Given Maweya

Submitted in fulfilment of the requirements for the degree of

MASTER OF EDUCATION IN MATHEMATICS

At the

University of South Africa

2022

DECLARATION OF ORIGINALITY

I, **Amokelo Given Maweya,** declare that this is my own work, submitted in partial fulfilment of the degree of Master of Education at the University of South Africa. I further declare that this dissertation has never been submitted to any other university or institution for any purpose, academic or otherwise. All the sources that I have used or quoted have been acknowledged be means of references.

Amokelo Given Maweya

DEDICATION

This study is dedicated to:

- My father, Nelson Khaorisa Maweya, and my mother, Dimakatso Christina Maweya, for their unequivocal support throughout. More especially for calling me “Professor”.
- My wife, Molebogeng Maweya, for her support, encouragement, and limitless love in my journey.
- My son, Hlompho Sabelo Success Radebe, and daughter Molebogeng Vuhlari Lentsela, who though I was devoted to my study, should always know that “I LOVE YOU”.
- Lastly, my siblings, Khanyisa, Ntiyiso and Junior Maweya, for the trust and belief they have in me.

ACKNOWLEDGEMENTS

My sincere gratitude and appreciation are extended to:

- My supervisor, for her invaluable motivation, guidance, professional advice, and support throughout the study. Her remarks and comments in this research study were constructive, beneficial, and fruitful. Your professionalism is highly appreciated.
- My friend Ramashia V.J., “academic giant”, for his support in my academic journey.
- My colleague Molebogeng Esther Thage, for inspiring me by receiving her master’s degree towards her retirement age and still going for her PhD.

ABSTRACT

Geometry, in particular Euclidean geometry, has been highlighted as a subject in mathematics that presents a variety of challenges to many secondary school learners. Many students struggle to gain appropriate knowledge of geometrical ideas as well as to display solid reasoning and problem-solving abilities. Mathematics educators, parents, and the government, represented by the Department of Basic Education (DBE), have all expressed worry over students' poor performance in Euclidean Geometry. The purpose of this study was to evaluate Grade 10 learners' changes in the understanding of similar triangles following a classroom intervention. This study explored the following main research question: How can a classroom intervention be designed to improve Grade 10 learners' understanding of similar triangles?

The study followed the social constructivism paradigm since learners should learn under a cooperative learning method to construct conceptual knowledge. The van Hieles' model assisted the researcher to determine and identify Grade 10 learners' geometric level. Furthermore, participatory action research was employed because the researcher was part of the study, responsible for the teaching design. The study adopted a mixed method design. A qualitative approach used the following instruments to collect data: an intervention, observation, and semi-structured interview. Eight (8) participants contributed to the qualitative data and were chosen via purposeful sampling to determine their understanding. These data were subjected to analysis.

The quantitative approach used baseline test and the post-test as data collection instruments. The population consisted of (43) FET band learners who chose mathematics as an area of study at a Secondary School. The baseline test was used to determine the learners' present knowledge of geometry considering the van Hieles' levels (VHL). After the designed intervention, the post-test, in comparison with the baseline test, was used to determine the effectiveness of the intervention and the change in understanding of the learners' concepts of similarity within the topic, geometry. Statistical and descriptive data analysis was deployed to describe the effect of the change. The study shows that the designed intervention was effective, and the results indicate that half of the learners in this group improved in their understanding of similar triangles.

Keywords: Mathematics, Geometry, Similar triangles, Learner, Intervention, Triangle, Knowledge and Similarity.

ACRONYMS

ATP	Annual Teaching Plan
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
FAL	First Additional Language
FET	Further Education and Training
GET	General Education and Training
GDE	Gauteng Department of Education
LLI	Learner-to-Learner Interaction
LOLT	Language of Learning and Teaching
MaWiGa	Mabopane-Winterveldt-Ga-rankuwa
MEO	Multiple Examination Opportunities
NSC	National Senior Certificate
PAR	Participatory Action Research
SIP	School Improvement Plan
SSIP	Secondary School Improvement Programme
TLI	Teacher-to-Learner Interaction
VHL	Van Hieles' Levels
ZPD	Zone of Proximal Development

TABLE OF CONTENTS

CHAPTER 4

FIGURES IN THIS STUDY

LIST OF TABLES

CHAPTER 1

INTRODUCTION AND BACKGROUND TO THE STUDY

1.1 INTRODUCTION

An overview of the study's background, the explanation of the problem statement, the research questions, and the research methodology are presented in this chapter. The significance and limitations of the research are also described.

The trend in learners' performance in mathematics examinations shows a decline in each successive grade, and this regression culminates in the high failure rate in the final school examination (Adolphus, 2011, p.144).

In response to the dismal results in mathematics in Grade 12, the Secondary School Improvement Programme (SSIP), a programme of the Gauteng Department of Education (GDE), was introduced to support Grade 12 learners; it is intended to improve the results of priority schools. During 2015, the SSIP programme supported more than 70 000 Grade 12 learners in almost 450 schools. The SSIP was implemented at 164 schools in Gauteng Province. Tutoring takes place on weekends and during holidays, and mathematics is one of the subjects that is offered. The intensive examination preparation camps are a programme that takes place at the beginning of October. The SSIP programme is the major schooling intervention that plays a significant role in priority schools that are credited with having a positive impact on the provincial pass rate (GDE Sci-Bono Annual Report 2016, p.34). Priority schools are secondary schools that have less than an 80% pass rate in their matric class.

Table 1.1: Four-year mathematics performance (Gauteng Province and Tshwane West District) against SSIP learners

Years	Gauteng percentage	No. enrolled in Tshwane West D15	Tshwane West D15 district %	Geometry (Similarity question) average performance	Number attended SSIP
2016	68,7%	2870	59,4%	< 36%	58 568
2017	67,7%	2552	61,6%	< 41%	55 686
2018	74,7%	2331	72,7%	< 31%	66 068
2019	67,8%	2414	62,2%	< 30%	68 001

Despite this programme running from prior to 2010, the performance of mathematics continues to be low, and it is decreasing yearly. The SSIP programme offers camps for progressed learners and learners at risk, and these camps are offered in two formats: walk-in learners and residential learners. Looking at the declining trend in Table 1.1, the SSIP programme appears not to be a remedy for the poor performance of mathematics learners. This observation is especially true in geometry, as it carries a proportionately greater weight in Paper 2, than the other topics, and because learners perform poorly in this topic, it adversely affects the pass mark. The 2018 performance increased due to multiple examination opportunities (MEO), which has been the system used for progressed learners and learners at risk of not writing certain major subjects like mathematics and physical sciences. From 2017 to 2019, a greater percentage of learners who are doing mathematics, physical sciences, and mathematical literacy in township schools did not write the November National Senior Certificate (NSC) to increase the pass percentage of schools. This practice has been discontinued in 2020 after realising the effect it caused on learners.

Amongst others, Ngirishi and Mamali (2015), Sadiki (2016), and Mabotja (2017) have conducted numerous studies in geometry over the past years. Idris and Tay (2004) reported that geometry is a content area that presents many problems in mathematics for secondary school learners. Mathematics teachers, parents, and the government have been reported to be concerned about learners' poor performance in Euclidean geometry (Adolphus, 2011, p.145). The distinguishing feature of Euclidean geometry is that it requires deeper reasoning at a higher cognitive level than is expected from other sub-topics in mathematics (Atebe, 2008). Factual knowledge and routine procedures, even complex procedures, are sufficient for many other topics in geometry, but Euclidean proofs show that problem-solving skills are needed too. This distinction implies that learners need a deep conceptual understanding of the ideas, logical reasoning, and connections that need to be made when dealing with Euclidean proofs.

The researcher decided to research ways of improving one of the troublesome areas that he observed in his mathematics classrooms, namely the similarity of triangles as a sub-topic in Euclidean geometry. Similarity, including congruency (as a special case of similarity), is a topic within which the foundations of logical Euclidean proof reasoning can be established. If this fundamental skill is learned intensively based on the true understanding of underpinning ideas, a successful transfer can be expected to be made to further, more complex Euclidean proofs, such as the compound geometry problems in circle geometry that regularly appear at the end of Paper 2.

The researcher intended to improve his learners' Euclidean reasoning by evaluating the effect of a unique strategy that he developed for Grade 10 learners to understand the similarity of triangles. He planned to eventually consolidate the strategy as a remedial tool in Grades 10-12. In this regard, the researchers' personal experiences are outlined in the next section.

1.1.1 Learning Geometry at secondary school level

The researcher attended a township school in the North-West Province from 2006–2008 (Grade 10–12). While still in Grade 10, a keen interest led me to become acquainted with Grade 12 geometry through self-study. In the year 2006, my love and understanding of geometry led me to start assisting Grade 12 learners in mathematics. He also recalled that during his final school year, Grade 12 learners received the minimum guidance and teaching from our ever busy and often absent mathematics teacher, so he started taking up the role of peer teacher for my classmates. Because of obtaining a distinction in mathematics in matric, it followed naturally that he furthered his tertiary studies in mathematics education. Analysing the factors that contributed to his love for and success in mathematics, particularly geometry, one could speculate the following:

- Given an intense desire to master geometry, self-exploration, self-discovery, and self-motivation, in other words, self-study, seems to be key.
- Through peer-teaching, the insight and understanding of geometry concepts came to full fruition, developing within and through the process of guiding others.

1.1.2 Teaching Geometry at secondary school level

After completing his BEd degree, he returned to his educational roots in the area known as MaWiGa (Mabopane-Winterveldt-Ga-rankuwa) to assist in improving the quality of mathematics teaching there. The researcher’s passion for geometry held strong. The poor socio-economic state of learners that he taught included child-run households, extreme poverty and various social ills, behavioural problems, and unsuccessful school careers. Currently, he is teaching Grades 9 – 12 mathematics, and the average performance of these learners is below 50%, with geometry marks falling below 20%. After analysing the factors that contributed to his learners’ low performance, particularly in geometry, he speculates as follows:

- Learners have a negative attitude towards geometry.
- Grade 10 learners struggle with proving the similarity and congruence of triangles with logical reasoning, both inductive and deductive. Hence, building based on axioms and theorems and delivering proofs seem to be too high a mental leap for learners to take.
- Understanding proportionality in triangles without having been given an angle size seems to pose a serious challenge.

The researcher took steps towards improving the situation. These steps included conducting extra classes for Grades 10 to 12 to break the fear and resistance to geometry; introducing the topic by beginning with the basics that should have been covered in Grade 8; and implementing a strategy that I dubbed “doing grocery” – writing down all the information they have in the diagram and answering selfconstructed questions with that information without looking at the questions. In addition, he requested the learners to categorise as well as form relationships among the theorems as a *centre group*, a *tangent group,* and *no centre group* in Grades 11 and 12. This approach seemed to assist in improving learners' understanding of geometry. The Curriculum Assessment Policy Statement (CAPS) documents stipulate that revision from earlier Grade 9 work must be addressed carefully, especially the conditions for polygons to be similar.

1.2 PROBLEM STATEMENT

In defining the research problem, a brief discussion of van Hieles' theory was followed by approaches from various perspectives: an objective view; a subjective view; a teaching view; and a learning view.

1.2.1 The van Hieles' model

The van Hieles' theory is the most widely used model in mathematics education worldwide, and it focuses on how learners can best learn geometry. In this study, the theory puts more focus on similar triangles as a sub-topic of geometry. Dina van Hiele-Geldof and Pierre Marie van Hiele (wife and husband) were the developers of the van Hieles' theory, which was used to guide instruction and measure learners' abilities (Crowley, 1987). The model proposed five consecutive and developmental levels of thinking in geometry, which are dependent on learner experiences (Stols, Long, & Dunne, 2015, p.1). The level describes the thinking process and the type of geometric idea that was processed, rather than the knowledge (van Hiele, 1986). According to the theory, learners need to conceptualise the basic level (Level 1) before they can move to the next level. Figure 1.1 below shows knowledge that explains how learners can best move from one level to another.

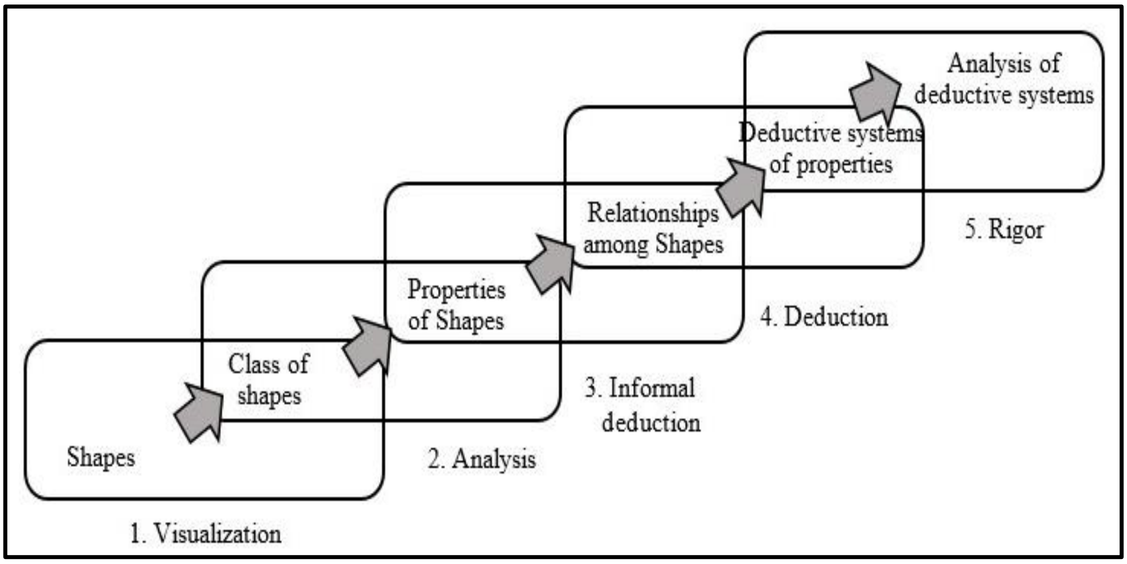


Figure 1.1: The van Hieles' theory of geometric thought (Van de Walle 2006, p.306)

1.2.1.1 Level 1: Recognition (Visualisation)

van Hiele Level 1 can also be called the foundational level of Euclidean geometry. At this level, learners make judgements about figures based on their appearance only. Figures have visual meaning at this level. The arguments of figures are based on a statement of common knowledge or belief, not on logical deductions. Learners recognise angles, parallel lines, triangles, parallelograms, and so forth by their shapes, "how they look".

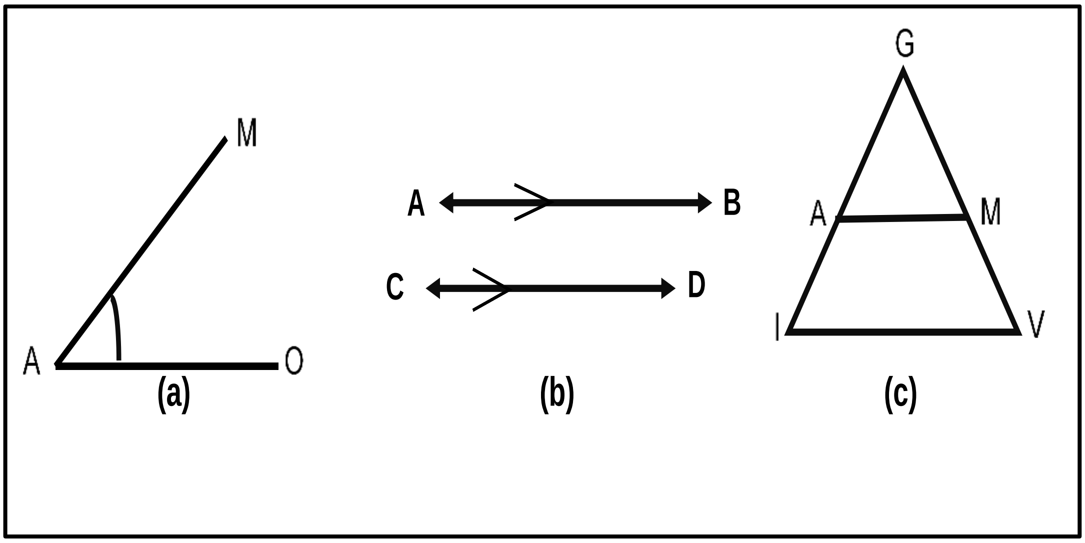


Figure 1.2: Different figures or shapes

Figure 1.2 represents figures that are based on common knowledge. Learners are supposed to know these figures by means of a visual approach, which results in an inability to distinguish between shapes. If learners can give names of these shapes or figures, then we can conclude to say that they are able to work within van Hiele Level 1 of geometric thinking. From Figure 1.2*,* learners must be able to know that in Figure 1.2 (c) we have two triangles, in Figure 1.2 (b) we have parallel lines, and in Figure 1.2 (a) is an acute angle and/or angle $\hat{A}$.

1.2.1.2 Level 2: Analysis

van Hiele Level 2, Analysis, is the case where figures are recognised based on their properties (1984). Learners analysed figures in terms of their special parts and property. For example, in Figure 1.2 (c), a triangle, the word "tri" means three, which is why it has three sides and three angles. Figures and their properties are interdependent on each other. For example, a learner may consider a rectangle to be a figure with four sides and name all shapes with four sides as rectangles, yet he or she may refuse to recognise a square as a rectangle just because it is square. At this level, learners no longer depend on common knowledge or beliefs to name or know figures but support their common knowledge by giving properties of figures or substantiating their answers. In this research study, learners needed to recognise that the similar triangles have conditions.

1.2.1.3 Level 3: Informal deduction (Ordering)

At this level, van Hieles' explains the properties of figures that are orderly. Learners understand the properties of figures and can determine the relationship between properties. Learners grasp the relationships between forms, such as how opposing sides of a parallelogram are parallel and equal; or how a square has all the characteristics of a rectangle, and therefore it must also be a rectangle. For the current study, learners can recognise that congruency is a special case of similarity since it has the properties of similarity, but similar triangles are not congruent. In Grades 11 and 12, learners can now make a relationship of grouping theorems based on the centre group, the tangent group, and the no centre group.

1.2.1.4 Level 4: Formal deduction

van Hiele Level 4 enables learners to construct a geometric proof of a theorem and understand the role of axioms and definitions. This level is where the thinking of learners constructs the meaning of deduction through procedural knowledge. From Figure 1.2 (c), that is where learners need to prove that those two triangles are similar. Learners at this level must memorise proof in its entirety and replicate it in its entirety (it is procedural). Furthermore, this is the stage at which learners construct and develop sequences of statements that logically explain conclusions.

1.2.1.5 Level 5: Rigour

At this level, learners can master more formal deductions, where they establish and analyse the axioms and theorems of different mathematical systems. Since this level is beyond the secondary school level, it did not concern learners in this study. As such, that discussion was not sufficiently provided.

1.2.2 The problems as seen from an objective perspective

Learners' performance in Euclidean geometry is a cause for concern in South Africa and for all stakeholders. The Department of Education implemented an intervention strategy called the Secondary School Improvement Plan (SSIP) to try to remedy the situation. Schools were also asked to implement their own School Improvement Plan (SIP). Despite these efforts, the performance in mathematics remains poor, and the lowest attainment among the major topics is in Euclidean geometry. This perceivably causes a lack of a proper foundation in the concepts building up to logical mathematical reasoning in the lower grades, which becomes especially evident in Grades 9 and 10.

1.2.3 The problems as seen from a subject perspective

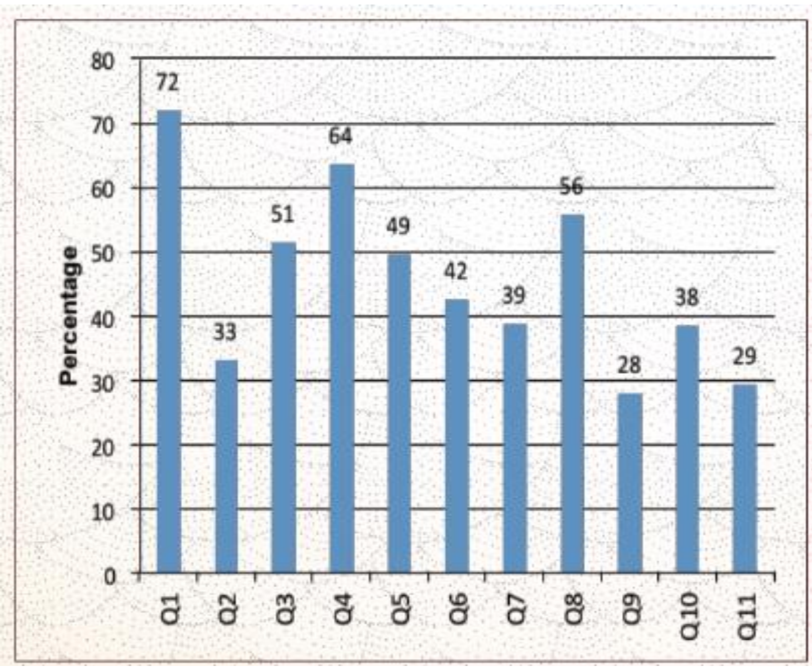
According to the National Assessment Circular 3 of 2015, the problems with mathematics arise from early Grades 7 to 9 (the senior phase), when the marks or the percentage of the promotion requirement for mathematics were increased to moderate achievement 40% (level 3) and above. The following year, in 2015, three subjects were prioritised, and mathematics was one of them. Table 1.2 below indicates how marks were adjusted from 2015 to 2018. Learners and teachers are less likely to take other subjects seriously because of these mark adjustments, particularly math learners who are aware that their exam results will be adjusted.

Table 1.2: Yearly mark adjustment

Years	2015	2016	2017	2018
Mark adjustment in percentage	3 Subjects with 7%	3 Subjects with 6%	3 Subjects with 5%	Mathematics with 2 % or any other subject

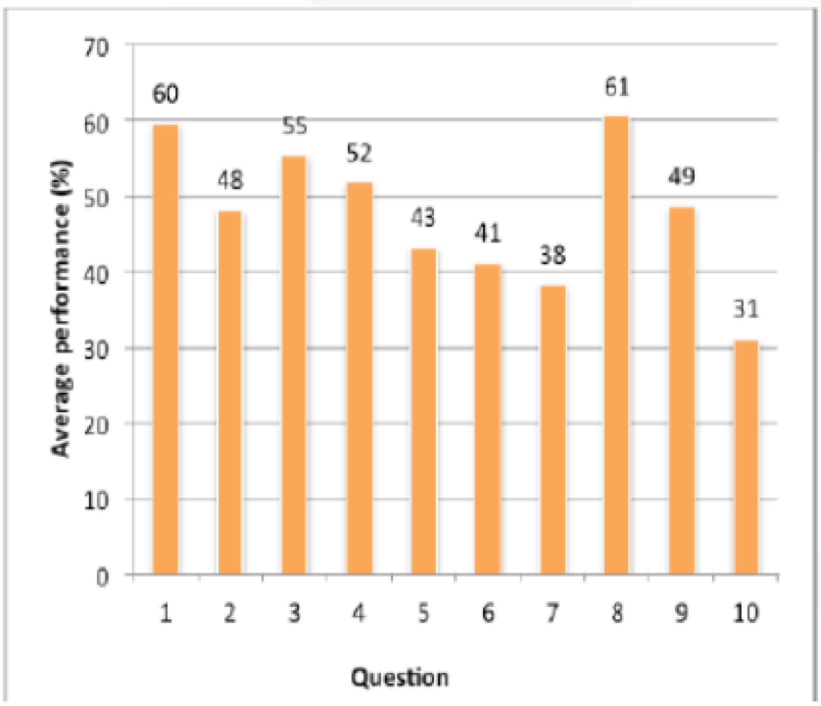
A special dispensation for learners in the senior phase must be applied according to the National Assessment Circulars 3 of 2016 and 1 of 2017. It further elaborates that a learner can be condoned to the next grade if he or she failed mathematics but obtained a minimum grade of 20% in mathematics and meets the promotion requirement. which states that a learner must not continue with mathematics in Grade 10. According to the researcher's observation, irrespective of the promotion requirement for special condonation in public schools, learners are still enrolled to continue with mathematics in Grade 10, and the performance of mathematics continues to be poor. Furthermore, the National Assessment Circular 2 of 2019 supports his observation as it stipulates that those learners have an option to continue with mathematics in Grade 10 even though they obtain a percentage below 30%, even if it is 2%. All the national assessment circulars that were introduced to remedy the situation in the senior phase created a problem in the Further Education and Training (FET) phase.

Geometry is the field of mathematics that examines various shapes or figures and their properties (Paulina, 2007). Euclidean geometry is a major topic. One of the comments in the Diagnostic Report of the NCS (2018, p.151), referring to Grade 12's answers, stated: "In Q10.1.2, candidates proved the two triangles congruent instead of proving them similar. Some candidates did not name the angles correctly, e.g., $\hat{A} = \hat{C}$ instead of $\hat{A}_2 = \hat{C}_2$". These concepts form part of the Grades 8 to 10 syllabus and should have been established there. According to van Hiele (1986), a learner needs to be at the ordering level and tend towards a formal deduction level to cope well in the axiomatic system. The National Diagnostic Report of DBE (2015) indicated that some learners do not understand and know theorems and their applications, and others do not even know or recognise a diameter. According to the van Hieles' geometric levels, we can conclude that some learners in Grade 12 are below Level 3 informal deduction, and others are below Levels 2 and 1 since they are unable to recognise such concepts.



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry
Q11	Euclidean Geometry

Figure 1.3: Extract from Diagnostic Report (2015, p.163)



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

Figure 1.4: Extract from Diagnostic Report (2018, p.143)

Question 10 and 11 from Grade 12 Paper 2 is a section containing two questions that usually involve congruency and similarity of triangles. As a result, it becomes clear from the figure above that the performance of these two questions is the lowest. Seroto (2006) confirms that the performance in this section is much lower than in other sections across Grades 10 to 12. We do not exclude the possibility that, because this question is the last on the paper, time constraints could also play a vital part in the scores. The Diagnostic Report of the NCS (2015, p. 151) reported that “this question [Question 11] was very poorly answered”.

1.2.4 The problems as seen from a teaching perspective

Traditionally, teachers start a lesson by explaining concepts, followed by doing some examples for learners, and then giving them classwork or homework exercises from textbooks. Most textbooks have answers at the back and prompt learners to look for the "correct" answer long before they have engaged with or mastered the concept. This kind of teaching method is teacher-centred and discourages learner involvement and disrupts their own construction of meaning. Gunhan (2014) views this teaching method as being characterised by teachers giving definitions while making no use of concrete materials and not investigating practical ways of explaining mathematical concepts. De Villiers (1997) blames this traditional approach as the main cause of a poor understanding of geometry. The use of this traditional method for conceptualising congruency and similarity of triangles in certainly counterproductive.

To teach geometry effectively in General Education and Training (GET) and Further Education and Training (FET), mathematics teachers need to develop a solid understanding of its content themselves. Jones (2002, p.122) supports the idea that teachers are required to know theorems and recognise geometrical problems, geometric context, and understand the practices of geometry in our daily lives.

The researcher’s observation in the neighbouring schools and the school where he is teaching is that learners prove similarity and congruency by copying processes from previous examples. When asked “why”? – they would refer to examples (which they did not understand) and say – “It is not the same as the previous example” and therefore they do not understand how to do it.

1.2.5 From a learner's perspective

From a learner's perspective, Fabiyi (2017) found that senior secondary learners perceived geometry concepts in mathematics as difficult to learn. Euclidean geometry is observed as the most problematic topic for teachers to teach and learners to learn (Adolphus, 201, p. 144). Furthermore, Sears and Chavez (2015) carried out a descriptive study to examine 1 936 learners' performances on a proof task related to congruent triangles. Their findings indicate that learners generally experience difficulty with the construction of proofs.

Culture and the environment also contribute to the learner's perspective on mathematics, most especially in geometry. Many mathematics teachers have attempted to identify important mathematical issues related to Euclidean geometry. Despite all these attempts, the problem of low performance in mathematics has persisted. It has generally been observed that most learners see geometry as a boring and difficult topic in mathematics. Given all these facts, the researcher intends to evaluate the Grade 10 learners' change in understanding of similar triangles following a classroom intervention. In trying to get an in-depth analysis of the method and make recommendations on how teachers can help learners learn Euclidean geometry more effectively, the research questions outlined below will guide the study:

1.3 RESEARCH QUESTION

Considering the research problem and the purpose above, the following research question led to the study:

1.3.1 Main research question

How can a classroom intervention be designed to improve Grade 10 learners' understanding of similar triangles?

1.3.2 Sub-Research question

The following sub-research questions have been formulated to answer the main research question:

- Which foundational knowledge and skills are needed as a basis for a thorough conceptual understanding of the similarity of triangles?

- What is Grade 10 learners' present levels of foundational knowledge and skills?
- What elements, both conceptual and pedagogical, does the researcher include to mediate the understanding of the concept of similarity of triangles in his specific teaching environment?
- How does the implemented designed intervention impact the Grade 10 learners' understanding of the similarity and congruence of triangles?
- What changes in understanding from the baseline (prior to the intervention) to the post-test provide evidence for improved understanding?

1.4 THE AIMS AND OBJECTIVES OF THE STUDY

The aim and objectives inform the steps to be followed when gathering information to seek answers to the main question and sub-questions.

1.4.1 Aim

The study aims to evaluate Grade 10 learners' change in understanding of similar triangles following a classroom intervention.

1.4.2 Objectives

The study's objectives are as to:

- Identify the foundational knowledge and skills needed for the conceptual understanding of similarity, proportionality, and proof.
- Identify the present level of the mentioned knowledge and skills of the learners in this research experiment.
- Describe the elements, both conceptual and pedagogical, that the researcher includes to mediate the understanding of the similarity of triangles in his specific teaching environment.
- Evaluate how the implemented designed intervention impacts the Grade 10 learners' understanding of the concept of similarity of triangles.
- Identify the evidence of changes for improved understanding from the baseline (prior to the intervention) to the post-test

1.5 RESEARCH METHODOLOGY

The methodology of research relates to the strategies, methods, and processes used to apply the concept of research (Creswell, 2009, p. 18). According to Luneta (2013), the research design is a road map of how the research will be conducted. Furthermore, it is defined as a collection of rules and tools for tackling the issue of research (Creswell, 2009, p. 107).

Research paradigm: Social Constructivism is the paradigm of choice since the intervention planned for the present research swivels around self- and group-exploration of similarity with the aim of constructing meaning.

Research design: Action research was used as the design approach for the current study, which Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p. 196) relate to Kurt Lewin's work, which regards this approach as a "cyclical, dynamic, and collaborative method to solve questions impacting participants." Cohen, Manion, and Morrison (2000, p.226) describe action research as a method to enhance education via change and the repercussions of change. Moreover, action research is participative research i.e., a study that enables individuals to better their own practices and, secondly, those of others.

The type of research: The study employed a mixed-method approach in terms of the research data since both qualitative and quantitative data were generated to address the research problem. Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p.199) explain that action research often uses a mixed-methods approach, where qualitative and quantitative methodologies are combined to execute actual research.

1.6 TRUSTWORTHINESS, RELIABILITY, AND THE VALIDITY OF RESEARCH

In this study, learners were assessed in pre-and post-tests to generate quantitative data, and qualitative data was generated through observation and interviews.

Trustworthiness is when the research establishes credibility. The researcher explained to participants how important this study is when they are answering questions during the interview and test. In doing so, it assisted the researcher in gaining trust and a detailed understanding of the answers of the participants.

Credibility is how the participants believe and have trust in the collected and analysed data. Du Plooy-Cilliers, Davis, and Bezuidenhout define credibility as the correctness of the researcher's interpretation of data that was supplied by the participants (2014, p.258).

Transferability is the researcher's findings that are applicable outside the current study context and are transferable or generalisable. Transferability is explained as the applicability of outcomes that can be used in certain situations and yield similar results (Du Plooy-Cilliers, Davis, and Bezuidenhout, 2014, p.258).

Confirmability is the degree to which outcomes can be confirmed and supported by other researchers (Kumar, 2011, p.172). It means that confirmability measures the standard of trustworthiness. In this research study, the sample learners' interviews were recorded and transcribed verbatim. Open-ended questions were presented to allow learners to give their views or opinions, and for the researcher to be flexible during the interview. Field notes were taken during the designed intervention lesson.

Dependability corresponds to the reliability of getting the same consistent outcomes under identical conditions. According to Anney (2014, p,278), dependability is "the stability of findings over time". Therefore, it emphasises the researcher's accountability to be thorough and meticulous in reporting the research method and analysis.

Reliability focuses on how reliable and consistent the measurement of quantitative research is. Whether the instruments, which are the tests (baseline and post-test) in this study, are consistent and were, when repeated, produce the same results using a similar group of unknown participants.

Validity determines how the instrument measures what should be measured (du Plooy-Cilliers, Davis, and Bezuidenhout, 2014, p.256). From various forms of validity, the researcher focused on the following aspects relating to their instruments:

- Face validity refers to participants' perceptions of the test. Face validity is concerned with the way the instrument appears to the participants (Bless, Higson-Smith, and Sithole, 2013, p.234).
- Content validity refers to whether the test or instrument is representative and specific to the content. In this study, the validity of the baseline and post-test for similarity will be determined by using face validity and content validity.

1.7 THE SIGNIFICANCE OF THE STUDY

The relevance of the study is a written statement which refers to the importance and necessity of the study for the larger field of research, that specifies the subject of investigation and the target group under examination. The significance of this study described the researcher's intervention methods that were of assistance to other teachers and learners. As a result, the research provided learners with a deeper understanding of the similarity of the triangles in geometry. This study endeavoured to improve the quality of answers learners wrote in the tasks and examinations. It has provided me with strategies and knowledge on how to approach the topic. These results had significance for all educational stakeholders in improving the performance of mathematics, especially in the geometry subsection.

1.8 STUDY LIMITATIONS

The present study was carried out at Tshwane West Educational District (D15) in a Secondary (Ga-Rankuwa Cluster) in the province of Gauteng (South Africa). It is therefore a local experiment in a confined sample space. However, it is done with the view of extending the sample space in a later study and to generalising findings following more and wider research.

1.9 DISSERTATION OUTLINE

The following five chapters are part of this study:

Chapter 1

The introduction and background of the study are included in Chapter 1, as well as the description of the problems. The research questions, approaches and procedures, the relevance, and the limits of the study, as well as the dissertation outline, are also highlighted.

Chapter 2

This chapter comprises a review of the relevant literature covering the subject of geometry in mathematics education, focusing mainly on the topic of similarity.

Chapter 3

This chapter addressed the study's research paradigm, research design, and methodologies utilised for completing the investigation.

Chapter 4

It presented the data and the analyses.

Chapter 5

In this chapter, there is a discussion of the major findings from the study. These include both findings from a qualitative and a quantitative perspective. Follow-up limitations, recommendations, and conclusions respectively are presented.

1.10 ETHICAL CONSIDERATIONS

Tracy (2010) defines ethics as rudimentary processes that are important to consider for the study to be principled and of quality. Ethical issues highlight conditions that clarify the procedures that must be in place to follow the moral guidelines and principles of the research study. The researcher explained the aim and proposed outcomes of the research to all the participants. The information that was described included the roles of the participants and the researcher, as well as the possible benefits that participants gained from participating in the study. Research ethics deals with respecting the rights of the participants in research.

In line with best ethical research practice, the researcher promoted confidentiality, anonymity, the rights of participants, and avoided bias. Consent forms were issued to all relevant stakeholders, including the parents or guardians of the participants, informing participants about their confidentiality and anonymity, their right to withdraw at any stage without explanation, and the certainty that procedures were duly followed. For ethical reasons, neither the school nor the participant’s name were divulged.

1.11 SUMMARY

In summary, the teacher-researcher identified the problem of poor performance in Euclidean geometry, in spite of large-scale intervention. In his classroom, he observed difficulties identified as fundamental knowledge and skills within the subject of Euclidean geometry, particularly in proving congruent and similar triangles. The research primary aim was examined, the intervention was conducted, and the report was on teaching geometry techniques that contributed to the existing knowledge of similarity in Euclidean geometry. Consequently, learners will be the ultimate beneficiaries. The highlighted objectives are met to fulfil the study aims.

An overview of the study was presented in this chapter, including the background, the results of the study, and mathematics performance. The research topic was presented and addressed quickly from Van Hieles' theory and a variety (perspectives) of points of view, questions, aims and objectives, research technique, credibility, reliability, and validity. To conclude, the relevance, limitations, the dissertation, and ethical problems have been underlined. In the next chapter, the researcher offers the study's literature review.

LITERATURE REVIEW

2.1 INTRODUCTION

The literature review examines the topic that the current study focuses on (Bless, Higson-Smith, and Sithole 2013, p.49). It involves searching for relevant literature and reading and summarising the available literature that relates both directly and indirectly to the topic. The researcher was primarily concerned with the approach taken to learning and teaching the similarity of triangles at a Grade 10 level. At the start of the literature review, the educational and pedagogical approach to knowledge acquisition was selected.

2.2 CONSTRUCTIVISM

It was only in the process of formalising the classroom study into a master's proposal that the researcher realised he was following a constructivist approach. Sharma and Bansal (2017, p.209) adhere to the belief that people learn best through observation and scientific study; that is, a constructivist perspective. Bada and Olusegun (2015, p.66) emphasise that constructivism is a psychological learning theory that explains the enquiry of knowledge and how people might learn. The views about constructivism originate from the work of cognitive constructivist Piaget and social constructivist Vygotsky.

Constructivism is a philosophy that promotes and increases cognitive thinking and knowledge, resulting in "mental construction" (Ramsaroop, 2017, p.183). The Merriam-Webster online dictionary (https://www.merriam-webster.com/dictionary/constructivism) defines "constructing" as meaning "setting in logical order". For learners, this implies the need to incorporate their prior knowledge and new knowledge to construct their own meaning and understanding. In geometry, learners need to use a foundation of geometry and keywords to construct meaning and understanding.

According to Kepceoglu (2018, p.1), the constructivist approach means that learners create and gain knowledge through active participation themselves, rather than from the teacher or the environment. The learning environment must be conducive to learning in such a manner that learners are directly exposed to interactive learning where the interests of every learner are valued. Goodwin and Webb (2014) support the idea of constructivism, which explicates the learner's role in how to receive knowledge in the learning process; this role is described as an "active participant". Learners must apply the combination of knowledge and experience when deciding and a conclusion based on their own findings. Learners are motivated when they understand a concept and can explain it to other learners. In Grades 10 to 12, learners who understand geometry concepts can explain them to other learners and to their teacher.

Bada and Olusegun (2015, p.67) define constructivism as a philosophy that enhances the common-sense and conceptual growth of learners. This possibility will help the researcher get an answer to his primary research question, which centres around "conceptualising the similarity of triangles". The teacher has the role of assisting learners to enhance their logic and concept formation by using the teaching approach of collaboration within a classroom, managed by applying constructivist principles. Thus, teachers could use cooperative learning or the jigsaw method of learning to facilitate learning when learners are trying to conceptualise the similarity of triangles. Sari and Haji (2021, p.3) explain the jigsaw method as a kind of cooperative learning that consists of 4 to 5 learners with heterogeneous learning teams. They also highlighted that the learner provides information in text form and each learner is accountable for the mastery and training of their peers. Furthermore, Weegar and Pacis (2012, p.11) support the researcher's opinion by explaining constructivism as a philosophy where learners can create their own knowledge through interactions with their environment and with other people.

In Piaget and Vygotsky's work, two types of knowledge can be discerned: the logico-mathematical knowledge and social knowledge. Lutz and Huitt (2004) define logico-mathematical knowledge as cognitive thinking where reason goes beyond physical interaction. The researcher argues that learners gain knowledge through logical reasoning, when they interrogate the root causes of phenomena, asking the question, "Why?". This would result in learners gaining cognitive thinking. Social or conventional knowledge includes conventions created, and generally accepted by people over a period (Kamii and Russell 2012). Supporting the notion of social knowledge, the researcher is of the opinion that learners as collective gain knowledge through common agreement and acceptance of the existing convention about, in this case, similarity and congruency. This happens during a group discussion.

2.2.1 The Zone of proximal development

Vygotsky describes the Zone of Proximal Development (ZPD) as a development that is related to social interaction and this interaction increases the learner's cognitive development. The interaction between the learner and more knowledgeable peers or facilitators increases the ZPD of the learner as a symbolic space. In geometry, the learner will be able to easily learn geometric symbols, e.g., "///", which means similar, through interaction. Vygotsky (1979, p.16) explains the ZPD as the gap between the actual level of development measured by autonomous problem solving and the prospective level of growth indicated by adult guiding problems or in cooperation with more able peers. Woolfolk (2007) supports Vygotsky's learning as a social collaborative construction that is centred around knowledge and values and occurs through social interaction.

The learner learns easily within this zone when they interact and participate effectively with others, or their peers, and they can get help or guidance from the facilitator. The first thing that learners require is self-motivation to learn and acquire mathematical knowledge. In this case, the learning within their zone (ZPD) becomes positive. Then, after this initial encounter, learners need to achieve the goals of the first activity, or exercise, where their knowledge is assessed, and once this is achieved, their zone (the area of comfortable learning) is extended, and they are motivated to do more. This kind of learning is supported by social constructivist theorists, where learners act on their own or with peers through interaction before the assistance of the facilitator or educator is invited. The researcher concurs with Vygotsky's notion of the ZPD, which suggests that learners are autonomous if they can construct knowledge independently, and whenever they cannot, guidance is needed from the facilitator.

2.2.2 Constructivist classroom

Geometry should be taught in a classroom environment where the theory of constructivism supports learning, all learners participate fully, and cooperative learning takes place. Foldnes (2016, p.39) defines cooperative learning as learners working in groups through discussions and peer feedback aiming towards the same learning goals. Constructivist beliefs support the practice that tasks that are beyond a learner's level of mastery need to be given to them. Brownstein (2001) supports constructivism by emphasising that those tasks above the current level of mastery increase motivation and improve on previous successes. This engagement, thus, increases opportunities for active involvement and participation in the learning process. Trust and openness between educators and learners play a vital role in a constructivist classroom, and it also increases engagement and participation. Teachers as facilitators need to develop strategies on how to approach and teach geometry that align with a constructivist classroom, for example, engaging in dialogue, asking a question, discussing, and inquiring. The more prepared and comfortable the facilitator is when using these strategies, the easier it is for learners to adapt to them and apply them as well.

Learners learn the easiest and best through examples that they can relate to. Albert Einstein said, “Example isn’t another way to teach, it is the only way to teach”. Educators should engage with the learners and help them construct their own ideas through examples. Constructivist classrooms create an environment where learners share ideas, methods, and results, compare them, and exchange ideas while reasoning with each other to reach an agreement. This might help learners to have a conceptual understanding of geometry, and through this deeper engagement; improve the results.

2.2.3 Constructivist educator

The researcher’s approach is that of a facilitator who mediates and assists learners to improve and understand the concepts rather than that of the teachers being the owners of knowledge and transmitting it to passively absorbing learners. The researcher recognises the need to consider learners’ personal and unique experiences and problems for effective teaching to take place. From a constructivist standpoint, the teacher’s responsibility is to cultivate an environment that promotes collaborative problem-solving, where learners construct knowledge and make meaning, and where teachers facilitate or guide (Sharma and Bansal, 2017, p.211). When teaching geometry, the teacher will facilitate and guide learners while they are grappling with similarity in geometry by requesting reasons when solving problems – "Why do you say so?" or "Well done, please explain to us more".

Furthermore, teacher intervention and teaching methods are tools that may be used to activate learners' prior knowledge and aid in the advancement of geometric understanding. Subjective experiences are vital sources for creating the knowledge of learners. Geometric comprehension and ZDP improve when learners build their own knowledge. Teachers should emphasise the characteristics of thinking, comprehension, reasoning, and applying geometry knowledge. Teachers are supposed to promote reflective thinking among learners since geometry involves learners' knowledge and abilities to form shapes around them. The role of a constructivist educator as a facilitator is to ask rather than to teach or tell (a teacher should not always be speaking), provide support, provide guidelines on how to reach a conclusion, and enable continuous dialogue.

2.2.4 Constructivist learner

A learner can be described as a "constructivist" learner when regarded by the teacher as unique with special needs, skills, and backgrounds. Social constructivist beliefs include the notion that teaching, and learning should be learner centred. Learner-centred learning focuses on the needs and interests of the learner. Learners will gain knowledge in alignment with the theory of social constructivism when they are grouped together. Collaborative learning takes place in a classroom conducive to learning, where the theory of constructivism and its consequent practices are foregrounded.

Through the cognitive construction of knowledge, the learner’s potential to develop is actualised and grows to maturity. The cognitive maturity of the learners can be extended through the support of the facilitator or/and peers. This is supported by Vygotsky’s theory, which includes ZPD. When learners are given a geometry activity as a group, they use their cognitive ability to reason and convince one another about the soundness of their reasoning. Therefore, for example, they call this line a tangent or why two triangles can be described as similar.

2.2.5 Conceptual knowledge

Schneider and Stern see conceptual knowledge as a broad and abstract understanding of the key principles and their links in one field (2010, p.179). The term "concept" is defined by Merriam Webster as a wide or abstract idea extrapolated from specific cases (https://www.merriam-webster.com/dictionary/concept). According to Dane, Çetin, Bas, and Sagirli, (2016, p.82), conceptual knowledge is defined as knowledge that creates meaning. Luneta (2013) argues that this knowledge refers to the interconnection and relationships of ideas, which gives explanation and meaning to the procedures in mathematics. Conceptual knowledge is considered as the knowledge that is rich in relationships. In this study, a learner used this relationship to make meaning using a given statement and a diagram. Morris and Mather (2008) further explain that learners must understand concepts underlying fundamental skills and, in other words, they need to gain conceptual knowledge in mathematics. Furthermore, research concurs that in geometry, the main concept is the "keyword(s)".

Finally, when learners develop conceptual thinking skills, they can tackle new and unexpected situations, and new information or ideas are formed as a result. In the process of teaching, conceptual comprehension necessitates information or keywords that are applicable to the real world. Teachers must draw links between their geometric knowledge, their learners, and their teaching. In my opinion, geometric knowledge and learner knowledge must be linked to classroom practice if teachers are to assist learners in developing geometry mastery.

2.3 SIMILARITY IN EUCLIDEAN GEOMETRY

The word 'geometry' comes from two Greek words, *geo*, meaning "earth" and *metry,* "to measure". Semple and Kneebone (1959, p.1) provide an explanation of geometry, as follows:

> *"Geometry is the study of spatial relations, and in its most elementary form, it is conceived as a systematic investigation into the properties of figures subsisting in the space familiar to common sense … even the most abstract geometrical thinking must retain some link, however, attenuated, with spatial intuition; and... throughout the long development of mathematics, geometers have... given a fresh impulse to formal mathematics by going back once more for inspiration to the primitive geometrical sense".*

Geometry is an essential branch of mathematics, and as such, its education (teaching and learning) should be prioritised. The Curriculum Assessment Policy (CAPS) describes geometry as a study of shapes and space, which aims to enhance knowledge and enjoyment of the natural and cultural forms of patterns, accuracy, achievements, and beauty. Geometry is also important in many other domains, such as architecture, civil engineering, informatics, robotics, art, and culture. In geometry, similarity is one of the important concepts linking many mathematical areas, such as fractions, ratios, proportions, and the congruency of triangles. Similarity reappears as the relationship between triangles with specific properties in Grades 10-12 mathematics. According to the Annual Teaching Plan (ATP) that is derived from CAPS, learners in Grade 10 should understand the basic knowledge required in earlier grades: lines, angles, congruence, and similarity.

Several studies have been conducted, and the findings show that many learners find studying Euclidean geometry tough (Ngirishi & Bansilal, 2019; Van Putten et al., 2010). The 2018 National Senior Certificate mathematics examinations provide support to the findings since it was reported that learners performed the lowest in Euclidean geometry questions of similar triangles in the second paper of mathematics (Department of Basic Education, 2019, p. 143). Researchers discovered that several factors, including teachers' teaching methods, learners' learning methods, geometric terminology, visualising skills, gender differences, poor reasoning skills, insufficient teaching time, a limited school curriculum, and learners' lack of proof skills, all impede and have bad effects on both teachers and learners when learning geometry (Uduosoro, 2011; Aysen, 2012; Mashingaidze, 2012). For to the reasons, most learners are unable to acquire and comprehend geometric vocabulary and terminology.

Learners' geometric knowledge is mostly at the entry level or below, with only a few learners reaching the required and advanced knowledge levels. According to research, many learners in most South African schools are functioning at a lower level based on van Hieles' theory of geometric thinking than predicted (Atebe, 2008; Alex & Mammen, 2014; Luneta, 2015). Many learners enter high school with little or no geometric experience. It is an area that must be addressed in order to assist learners develop abstract thinking, intuition, visualisations, and tackle everyday practical difficulties, just to mention a few. (Sunsuma, Masocha & Zezekwa, 2012). Gunhan (2014) suggests that learners' geometrical understanding and visual perceptions are sometimes lacking or poor, resulting in their inability to make mathematical arguments that are constructive. Learners who had no prior expertise in geometry will find it challenging to comprehend geometric principles as well as they should. Previous experiences have a substantial impact on learners' acquisition of geometry because they assist learners establish mental structures to create networks with new and diverse mathematical circumstances (Mabotja, Chuene, Maoto, and Kibirige (2018). Tall (2008) argues that learners develop geometric concepts based on experiences that they had in their previous grade.

Triangles are the most basic two-dimensional forms in geometry, and they may be found everywhere, especially in architectural buildings. The notion of the triangle, which may be considered the cornerstone of teaching geometry, is frequently employed in the teaching of more complex topics. To completely appreciate the notion of the triangle, it is necessary to master all of its aspects and attributes. Angles, areas, surface area, volume, and sides of a triangle, as well as their attributes, are included as the main aspects of the triangle in the school curriculum. The issue of similarity is one of the ideas that learners have difficulties comprehending and learning, even though it contributes to geometric reasoning. Since the beginning of geometry, one of the most significant aspects of geometry training has been the equality and resemblance of triangles, and we frequently see instances of these in everyday life (Baykul, 2009).

Identifying and comprehending learners' mistakes and misconceptions throughout the construction of their knowledge and learning has absorbed teachers' attention. Teachers, on the other hand, will benefit from such an endeavour since they will be able to use knowledge of these mistakes and misconceptions as a teaching strategy in their classrooms. Teachers want learning opportunities that will allow them to hone their abilities in eliciting learners' thoughts about their mistakes, and researchers discovered that when teachers concentrated on comprehending learners' mistakes, their own mathematical knowledge increased (Chauraya & Brodie, 2018). Other research on geometry education has claimed that most educators' teaching approaches are based on abstract ideas, such as deductive inquiry, which is not practical and does not allow for the investigation of geometric reasoning (Steel, 2013). However, Bankov (2013) indicates that learners' mathematical thinking, communication abilities, and creativity improve when they engage in hands-on geometry instruction. Furthermore, Chiphambo (2011) believes that theoretical mathematical notions are quickly forgotten but learning through doing assists in the retention of the principles taught. Thus, it might be claimed that learners should be capable of applying figure qualities to solve specific geometric issues, which goes beyond simply intellectual comprehension of geometry (Mabotja et al., 2018).

Conceptualising geometric knowledge does not happen spontaneously; it needs an educational process that aligns elementary figures and concepts through intervention tactics and well-incorporated teaching and learning materials (Bussi & Frank, 2015). The most primary and secondary schools require interventions to enhance learners' arithmetic proficiency and performance. Alex and Mammen (2018) argue that to improve geometrical knowledge, proposed interventions should include both visual and verbal representations. However, Smith and Hughes (2015) indicate that learners must have a concrete vocabulary foundation that are flexible and fluent, and experience that includes numbers, symbols, and diagrams, as well as understanding ability, to communicate mathematically. According to research, the way the topic is taught at all levels, from elementary to high school, causes learners' difficulties in learning geometry (Fujita & Keith, 2003). However, to enhance and conceptualise the similarity of triangles and learners' geometric knowledge, the researcher designed an intervention that addresses early childhood years and promotes learners' interaction.

The concept of "tri" means three, which defines three sides and three angles of a triangle. Triangles that are similar are those that have the same shape but different sizes. What matters in similarity are two characteristics or conditions: (a) that all three corresponding angles of the triangles are the same size and (b) that all three corresponding sides of the triangles are in the same proportion. There are three conditions for triangles to be considered as similar, namely:

- **AAA** (angle, angle, angle) – three pairs of angles are the same size.
- **SSS** (side, side, side) – three pairs of sides have the same proportion.
- **SAS** (side, angle, side) - two pairs of sides have the same proportion, and a pair of angles is the same size.

A conceptualisation of similarity is based on concepts of number sense, and it involves a comparison of the relationship of proportionality between corresponding lengths in similar shapes (Seago, Jacobs, Heck, Nelson, and Malzahn 2014). According to Euclid's notion of congruency of line segments, the shapes that can be overlaid on top of each other are congruent, and for similar shapes, the proportions must be the same (Cochrane and McGettigan 2015, p.18). Therefore, if the triangles are congruent in all respects, this means that congruence is a unique case of similarity. Sanwidi and Swastika (2018) state that even if similar shapes have the same shape, it does not mean that the length must be the same size or equal. In conclusion, all congruent triangles are similar, but similar shapes are not congruent. By implication, therefore, the researcher reasons that a crucial element of conceptualising similarity is a clear understanding of ratio and equality, the understanding of which can be transcended to geometry applications, as becomes evident in the next paragraph.

2.4 RATIO AND PROPORTIONAL THINKING

Ekawati, Lin, and Yang explain ratios as the relationship between two or more quantities, which can be of the same quantity, or of different quantities, as long as there is a relationship between them (2014, p.4). The most interconnected notion of ratio is proportion. The ratio is a multiplicative comparison of two quantities. This idea is central to topics in mathematics such as linear functions, similarity, trigonometry, and probability. According to Misnasanti, Utami and Suwanto (2017, p.2), the idea of ratio and proportion are key in many fields of knowledge and are significant in mathematics, most especially in geometry, as a major section in Paper 2. The researcher further indicates that proportionality plays a significant role when dealing with concepts of similarity in Euclidean geometry. In addition, Cunningham and Rappa (2016, p.2) also explain the idea that similar figures have corresponding sides that are in proportion and corresponding angles are equal. With regard to the present study, incorporating the concepts of ratio and equality ideas into the topic of similarity is probably central, yet most complex, mathematical understanding that teachers need to explain.

In this study, the researchers sought to evaluate Grade 10 learners' changes in the understanding of similar triangles following a classroom intervention. He therefore concentrated on the similarity of triangles in relation to ratio and proportion. Similar triangles of the same shape with corresponding different sizes can be classified as triangles that are proportional. The ratios of linear measures in one triangle are equal to the corresponding ratios in the other triangle. The corresponding pairs of sides all have the same ratio; that is, they are proportional to each other. For example,

- 2:3 (one triangle's sides are, $\frac{2}{3}$ the length of the other).

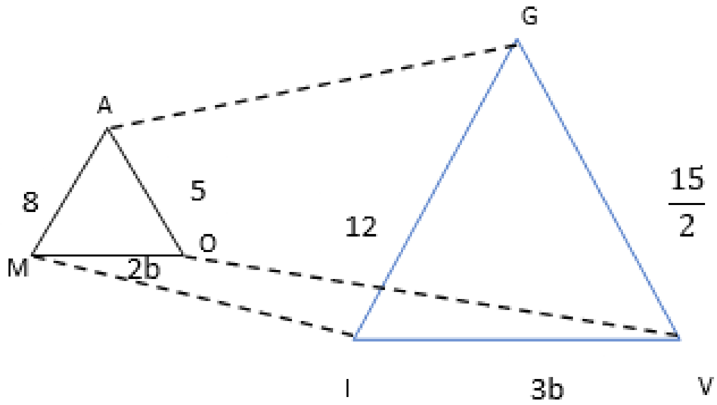


Figure 2.1: Similar triangles with three pairs of angles of the same size

In Figure 2.1, $\Delta AMO\ and\ \Delta GIV$ are similar since they have the same shape, their corresponding sides are in proportion. According to transformation geometry, ΔAMO has been enlarged by a certain factor (k) to form ΔGIV. The transformation can also be expressed in terms of proportion or ratio, ΔAMO:(k)ΔGIV. In this case, the scale factor is k >1. According to Figure 2.1, the ratio of the two triangles can be expressed in the form:

$$\frac{\boldsymbol{AM}}{\boldsymbol{GI}} = \frac{\boldsymbol{MO}}{\boldsymbol{IV}} = \frac{\boldsymbol{AO}}{\boldsymbol{GV}}$$

$$\frac{\mathbf{8}}{\mathbf{12}} = \frac{\mathbf{2}\boldsymbol{b}}{\mathbf{3}\boldsymbol{b}} = \frac{\mathbf{5}}{\frac{\mathbf{15}}{\mathbf{2}}}$$

$$\therefore \frac{\mathbf{2}}{\mathbf{3}} = \frac{\mathbf{2}}{\mathbf{3}} = \frac{\mathbf{2}}{\mathbf{3}}$$

This can be expressed in the form ΔAMO: (k) $\Delta GIV = 2:3$ with a scale factor of k =1,5 units.

- 1:1 (one triangle's sides are the same length as the other's).

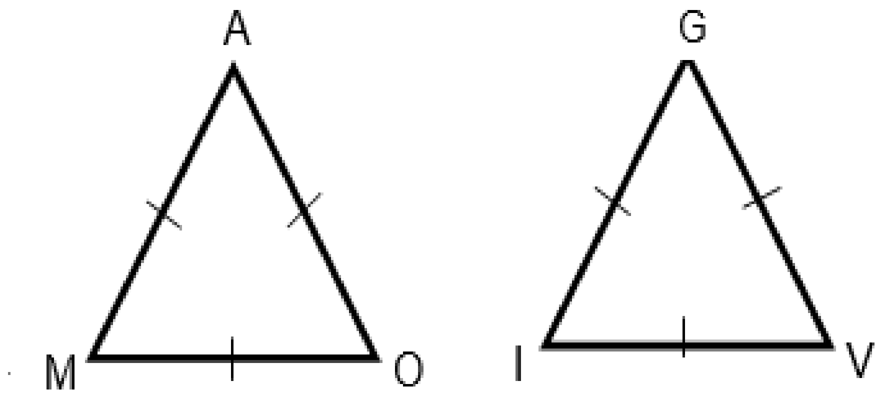


Figure 2.2: Similar triangles with scale factor 1

In Figure 2.2, $\Delta AMO\ and\ \Delta GIV$ are similar since they have the same shape and have equal sides. All corresponding pairs of sides are in the same proportions, namely, $\frac{AM}{GI} = \frac{MO}{IV} = \frac{AO}{GV}$ which results in a ratio of 1:1:1. The ratio of these two triangles can also be expressed differently, $\frac{MO}{GI} = \frac{AO}{IV} = \frac{AM}{GV}$ because they are equiangular, equilateral, and similar. It can be further concluded that all triangles that have equal sides will have a ratio that is equals to 1. According to Tourniaire and Pulos (1985, p.186), a proportion is when two ratios are equal, that is, $\frac{a}{b} = \frac{c}{d}$ ". Euclid emphasises proportionality in (Elements, book V, definition 5 and 6) as a natural static that deals with 'magnitudes' pairings of figures instead of numbers.

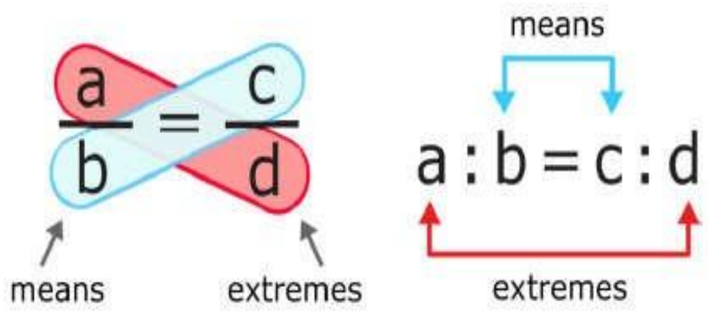


Image is retrieved from https://www.ck12.org/book/ck-12-middle-school-math-concepts-grade-7/r3/section/5.11/

Figure 2.3(a): Means and Extremes on Multiplication Cross (Kershaw, 2014)

When this notion is extended to the similarity of triangles, one will have, for example, $a \; and \; c$ as two sides of triangle M in *Figure 2.3 (b)*, and $b \; and \; d$ as the two corresponding sides of triangle N. Then, if $a \div b = c \div d$, then the two lengths are in similar proportion. Suppose $a = 10cm, b = 2cm, c = 15cm \; and \; d = 3cm$, then $\frac{a}{b} = \frac{c}{d} = \frac{10}{2} = \frac{15}{3} = 5$ and at the same time $a.d = b.c = 30$.

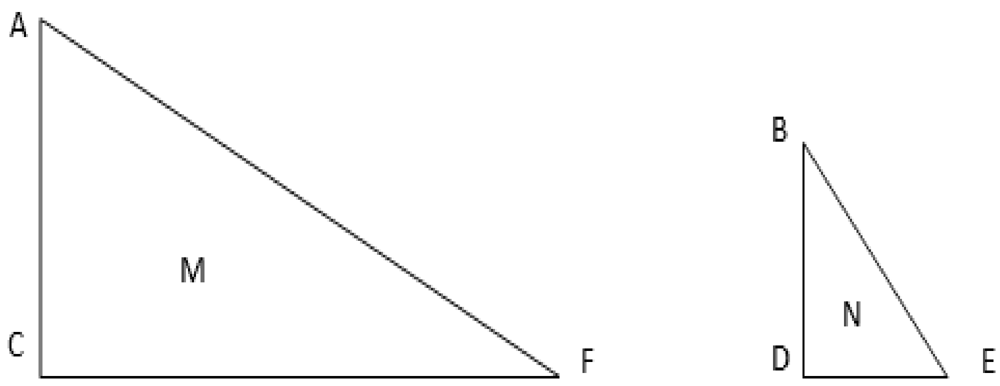


Figure 2.3 (b): Means and extreme are illustrated in similar triangles.

Proportional concepts are traditionally problematic in school mathematics. Learners find it hard to distinguish between ratios and fractions, where a ratio is expressed either as part in relation to the whole, or as one part in relation to the other part. An example is a group of 8 girls and 6 boys. The part-of-a-whole can be represented as fractions, $\frac{8}{14}$ (the girl part of the group) and $\frac{6}{14}$ (the boy part of the group) respectively. The part-to-part ratio between 8 girls and 6 boys, expressed as 8:6 (eight is to six), means the girls are $\frac{8}{6}$ (eight) to the number of boys and the boys are $\frac{6}{8}$ (six) the number of girls.

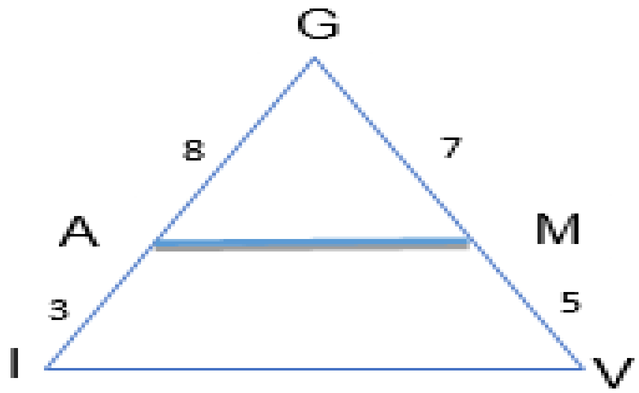


Figure 2.4: Midpoint proportional triangles

In Figure 2.4, $GI = 11units$ which is a whole, and the part-of-a-whole can be represented as: $\frac{GA}{GI} = \frac{8}{11}$ and $\frac{AI}{GI} = \frac{3}{11}$. The part-to-part ratio between $GA\ and\ GM$, can be expressed as 8:7, which means GA is $\frac{8}{7}$ unit of GM and GM is $\frac{7}{8}$ units of GA. These ratios can be expressed in percent too. For example, a proportion of $\frac{3}{11} = 27{,}3\%$. Furthermore, a ratio can be expressed as a rate, as a special type of ratio where two measurements with different units are compared, like R15 per litre of petrol. The denominator in the rate is 1, in this case, which means one litre of petrol costs R15.

2.5 SUMMARY

In brief, this chapter reviewed literature related to the problem investigated in this study. It highlighted the learning and teaching approach with reference to constructivist theory. Under the constructivist theory, the researcher focused on the following aspects: the zone of proximal development; the constructivist classroom; the educator; the learner; and conceptual knowledge. Finally, the researcher focused on the similarity between Euclidean geometry, including its literature, a ratio, and proportional thinking. The research methodology for the study is presented in the following chapter.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter explains the methods for collecting data to address the research in Chapter 1. It begins by reiterating the research questions, aims, and objectives of this study. The chapter then describes the paradigm of research and design, before emphasising the study context. It then discusses the processing of data and the analysis procedure. Finally, the ethical issues surrounding the study are presented in this chapter.

3.2 AIMS OF THE RESEARCH

The aim of the study is to evaluate Grade 10 learners' changes in understanding of similar triangles following a classroom intervention. This study therefore sought to:

- Identify the foundational knowledge and skills that are needed for a thorough conceptual understanding of similarity, proportionality, and proof.
- Identify the present level of the mentioned knowledge and skills of the learners in this research experiment.
- Determine the strategies used to mediate the understanding of the similarity of triangles in his specific teaching environment.
- Determine how the implemented intervention impacts the Grade 10 learners' understanding of the similarity of triangles.
- Identify the evidence of changes for improved understanding from the baseline (prior to the intervention) to the post-test**.**

3.2.1 A Critical Research Question

The main research question for the current study was: How can a classroom intervention be followed to improve Grade 10 learners' understanding of similar triangles?

The following sub-questions are answered in this chapter:

- Which foundational knowledge and skills are needed as a basis for a thorough conceptual understanding of the similarity of triangles?
- What is the Grade 10 learners, in this class, present level of foundational knowledge and skills?
- What strategies does the researcher use to mediate the understanding of the similarity of triangles in his specific teaching environment?
- How does the implemented intervention impact the Grade 10 learners' understanding of the similarity of triangles?
- What changes in understanding from the baseline (prior to the intervention) to the post-test provide evidence for improved understanding?

3.3 METHODOLOGICAL APPROACH

The methodologies of research employed in the implementation of research designs are techniques, methods, and procedures (Creswell, 2009, p.18). This study followed a mixed-method approach in terms of the research data since both qualitative and quantitative data were generated to address the research problem. Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p.199) explain that action research often uses a mixed-methods approach, where qualitative and quantitative methodologies are combined to execute the actual research.

3.3.1 Quantitative Approach

Bless, Higson-Smith, and Sithole (2013, p.16) state that a quantitative research technique is based on what natural scientists are doing: collecting data (either by measuring or by counting frequencies) in accordance with a certain number of steps and trying to stay as impartial and unbiased as possible. For the present study, the researchers designed a baseline test to assess the participants' state of foundational prior knowledge and skills. The baseline test questions were based on the previous grade's knowledge of angles, parallel lines, similarity, and proportion. Following a series of interventions, the growth curve of learner understanding was established. Quantitative data was collected using the following three phases, namely:

- Phase 1: The diagnostic pre-knowledge and skills assessment.
- Phase 2: A series of interventions, including formative assessments.
- Phase 3: A final summative assessment.

The series of assessments allowed the researcher to collect data on changes in learner performance. At the end of the designed intervention, the researcher assessed the learners with a post-test to measure how effective the lesson was compared to their baseline test. This study aims to measure the differential effects of an intervention on learner understanding of the concepts of the similarity of triangles.

3.3.2 Qualitative approach

Cohen, Manion, and Morrison (2007) noted that qualitative research methods present descriptive data, which involves collecting verbal or textual data about observable behaviour. Firstly, the researcher obtained qualitative data from classroom observations when he presented his own strategy to make learners conceptualise the similarity of triangles. The purpose of classroom observation was to establish the nature of teaching and learning geometric problems. According to Creswell (2009, p.16), qualitative research is intended to collect genuine information to create an understanding of the participants' answers and the researchers' observations.

Secondly, an interview was conducted with participants following the intervention. Du Plooy-Cilliers, Davis, and Bezuidenhout state that a qualitative data collection technique allows you to ask participants about their thoughts, attitudes, and beliefs about a certain geometrical phenomenon for present research in an interview. An interview is an instrument of data collection (2014, p.188).

3.4 RESEARCH DESIGN

Creswell describes research design as a collection of rules and tools for tackling the research challenge (2009, p.107). Luneta (2013) states that the design of the research is a road map for the conducting of research. The design approach of the current study was action research. This technique is linked to Kurt Lewin's work by Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p.197), who saw it as a cycle, a dynamic and collaborative process in which participants' problems are addressed. An action research strategy is one that aims to improve education by altering it and learning from the consequences of change (Cohen, Manion & Morrison 2000, p. 226). Participatory research is also a hallmark of action research; researchers are encouraged to improve their own practices, as well as the practices of others, through this type of study.

According to Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p.197), action research should involve participation. Because he would be a variable in the outcomes, the researcher chose participatory action research (PAR) for this study. De Vos, Strydom, Fouche and Delport (2011, p.492) define participatory action research (PAR) as a research paradigm in which the researcher serves as a resource to those being researched – generally a disadvantaged group – so that they can act in their own best interest.

Similarly, Du Plooy-Cilliers, Davis and Bezuidenhout (2014, p.198) claim that this sort of research is appropriate if societal concerns restrict individual lives, as is the situation in the present study. Collaboration with participants is imperative because the planned action may lead to changes in the lives of participants. McCraig and Dahlberg (2010, p.97) argue that the researcher's assessment of the problem and their ability to reflect on their own practice determine their ability to take active action and to effect change.

3.5 PILOT STUDY

The aim of the pilot study was to test the reliability and validity of the instrument as well as to determine whether it needed to be modified. Participants in the pilot research are identical in many respects to those in the main trial, including their age and gender. A pilot study allows the researcher to discover issues with the equipment and to fine-tune them to guarantee a smooth procedure in the main study (De Vos, Strydom, Fouche & Deport, 2011, p. 237). The purpose of the pilot study is to establish whether the instrument is relevant, the time allotted is sufficient, the instructions and questions are clear to the participants, and whether it will provide the data necessary to answer the important research questions.

The pilot study was conducted in a different school. The school was selected on the basis that the participants are doing Grade 10 and must have previous grade knowledge of geometry. After the pilot study, the pilot test was marked and analysed to check whether the instructions and questions were clear to the participants. Table 3.1 below represents the results of the pilot test in sequence order. This table is the one that alerted the researcher about certain questions. Some of the questions and diagrams were modified after the pilot study because the researcher realised that they were not clear.

For example, Question 5 of the multiple-choice section was answered correctly by only three learners. Also, Questions 2.1 and 4.2 required attention. Note that the top learner, Number 14, did not answer Question 2.1 correctly; neither did he answer Question 5 of the multiple-choice questions correctly. The second top learner, Number 4, did not answer the three questions noted here. By ranking items, we see Questions 3.1, 3.2, and 4.1 as the easiest questions, whereas Questions 2,1 and 4.2 appear to be the hardest. However, it may be that the language of the questions was not clear.

Table 3.1: Results of the pilot test in sequence order

	CONCEPTUAL		SECTION C: MULTIPLE CHOICE					SECTION D: STRUCTURED QUESTION										
Question (Marks)	1 (2)	2 (2)	1 (1)	2 (1)	3 (1)	4 (1)	5 (1)	1.1.1 (2)	1.1.2 (2)	1.2.1 (2)	1.2.2 (2)	2.1 (4)	3.1 (1)	3.2 (4)	4.1 (1)	4.2 (3)	Total	%
Learners	2	2	1	1	1	1	1	2	2	2	2	4	1	4	1	3	30	
1	0	1	0	0	1	0	0	1	0	0	0	0	0	2	1	0	6	20
2	2	0	1	0	1	0	1	1	0	1	0	0	1	4	0	0	12	40
3	1	0	1	1		1	0	1		1		0	1	4	0		11	37
4	2	0	1	0	1	0	0	1	1	1	1	0	1	4	1	0	14	47
5	2	1	0	0	1	0	1	0	1	0	0	0	1	4	1	0	12	40
6	0	0	0		0	0	0	2	2	1	1	0	1	4	1	0	12	40
7	1	0	0	0	0	0	0	1	1	1	0		0	2	0		6	20
8	1	0	0	1	1	1	0	1	1	1	0	0	1	4	1	0	13	43
9	1	1	0	0	0	0	1	1	1	1	0	0	1	2	0	0	9	30
10	0	1	1	1		1	0	1				0	1	4	1	0	11	37
11	1	0	1	1	1	0	0	2	0	2	0	1	1	4	1	0	15	50
12	1	0	0	0	0	1	0	0	0	0	1	0	1	4	1	0	9	30
13	1	1	0	0	0	0	0	0	0	0	0	0	0	4	1	0	7	23
14	2	1	0	1	1	1	0	2	2	2	2	0	1	4	1	1	21	70
15	1	0	1	0	1	0	0	1	1	1	1	0	1	3	0	0	11	37
16	2	0	0	1	0	0	0	1	1	1	2	0	0	2	1	0	11	37
	18	6	6	6	8	5	3	16	11	13	8	1	12	55	11	1		
Total	32	32	16	16	16	16	16	32	32	32	32	64	16	64	16	48		
%	**56**	**19**	**38**	**38**	**50**	**31**	**19**	**50**	**34**	**41**	**25**	**2**	**75**	**86**	**69**	**2**		

3.6 CONTEXT OF STUDY

Du Plooy-Cilliers, Davis, and Bezuidenhout define population as the total group of all members from which information could potentially be gleaned (2014, p.132). The population of this study targeted Grade 10 learners from the Secondary School in Gauteng, Tshwane West district. All the participants consider Setswana as their home language and English as their first additional language (FAL). The school is a Quintile 2, which means that the learners in this school do not pay school fees and the school depends on the government for funding to meet all its financial needs, including the learners' needs. For the past years, the school has been struggling to achieve a high pass rate in mathematics in matric. In this study, the researcher was dealing with two Grade 10 classes from the Secondary School, namely Grade 10 A1 and 10 A2. Learners in Grade 10 A1 are majoring in MATHEMATICS, PHYSICAL SCIENCES, LIFE SCIENCES, and GEOGRAPHY, while learners in Grade 10 A2 study the following majors: MATHEMATICS, BUSINESS STUDIES, and ECONOMICS.

3.7 DATA PROCESSING

The research study involved secondary school, where the research questions assisted the researcher in being able to determine the strategies for the collection and analysis of data. The data collection strategies employed in this research study took the form of a baseline test, classroom observation, a post-test, and a semi-structured interview, which helped to accomplish the research aim. Luneta (2013) explains data collection as the research plan that must be executed. The data collection in this present study was executed in three phases.

Phase 1 of data collection was the baseline test. The purpose of this test was to identify foundational geometry knowledge and skills in 10th grade learners. The next phase of data collection involved observation during a designed intervention lesson. During this phase, data were collected through a non-participation observation. The purpose of this method was to check whether learners understood a designed intervention through their group participation and their written records. Phase three of data collection was collected through an assessment namely, a post-test and immediately after the test, a semi-structured interview with eight (8) selected learners. The purpose of post-test was to determine whether the designed intervention produced what was intended, and the purpose of the interview was to determine how learners understood the designed intervention and to gauge their confidence in the topic of similar triangles and geometry. The table below indicates the summary of data collection, instruments, grade, and Van Hieles' levels (VHL).

Table 3.2: A summary of data collection, instruments, grade, and Van Hieles' levels

Phase	Sub-research questions	Type of instrument	Grade and VHL
1	Which foundational knowledge and skills are needed as a basis for a thorough conceptual understanding of the similarity of triangles?	Baseline test	Grade 9 VHL 1 – 3
	What is the Grade 10 learners' present level of foundational knowledge and skills?	Baseline test	Grade 9 VHL 1 – 3
2	How does the researcher mediate the understanding of the similarity of triangles in his specific teaching environment?	Observation	
3	How does the followed intervention impact the Grade 10 learners' understanding of similarity of triangles?	Post-test Interview	Grades 9 and 10 VHL 1 – 3

Since this research study employed action research that is participatory, and action research can be cyclic and dynamic as stated by Kurt Lewin, the researcher decided to use the action research cycle of Coghlan and Brannick (2005, p.22).

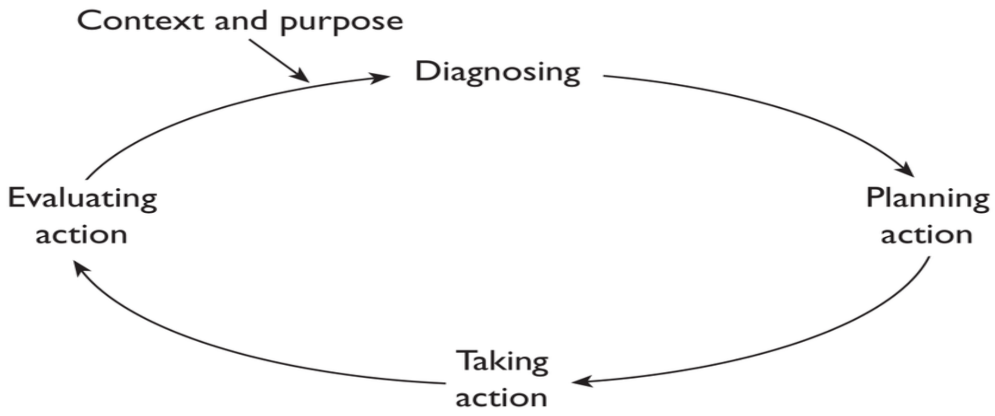


Figure 3.1: Action- research cycle by Coghlan and Brannick (2005, p.22)

Diagnosing involves identifying the problem or issue as the main working theme or objective. Planning action follows analysing the context or the problem, acting whereby the plans are implemented, and intervention is taking place. Lastly, evaluating an action involves looking at the outcome of an action, where everything is examined. The next Figure 3.2 is adopted from the previous research cycle of Coghlan and Brannick (2005).

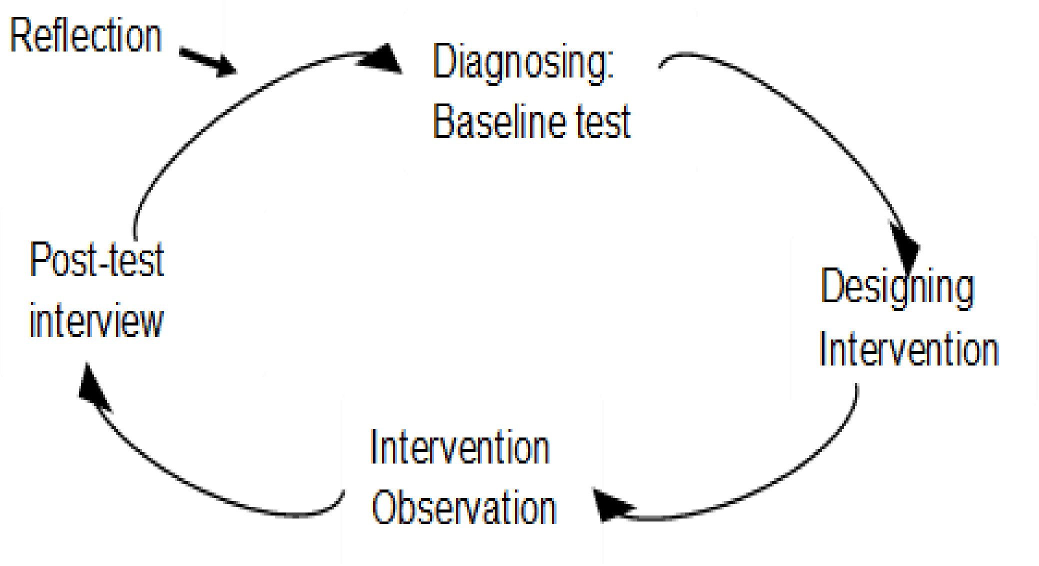


Figure 3.2: Research cycle adopted from Coghlan and Brannick (2005)

Figure 3.2 above was adopted from Coghlan and Brannick (2005), its purpose is to help and guide the researcher in the process of executing the research study based on its paradigm. The researcher diagnosed the root problem from the background of geometry in South Africa and internationally. He further used the baseline test to substantiate the background. Planning action took place by designing an intervention that would impact learners' understanding of geometry (similarity). Acting involved implementing the designed intervention and observing its effectiveness and impacts. Post-test and semi-structured interviews were used to evaluate the effectiveness of the implemented action. Finally, reflection was conducted to analyse the context and purpose of the research study.

3.8 BASELINE TEST AND POST-TEST

As mentioned earlier, the baseline test and post-test are the primary instruments for collecting data in this research study. These tests are pen and paper, and questions are asked in English to cater to all learners and to serve university policy and the language of learning and teaching (LOLT). Some of the items from the baseline test are also used in the post-test because they helped the researcher measure and compare the participant's performance after intervention.

The tests are structured to identify the nature of the geometric understanding expected in Grade 10 and, according to van Hieles', the geometric knowledge is Level 3 and a bit of Entry-level 4. This means that all learners in Grade 10 doing mathematics are expected to have the following geometrical conceptual knowledge:

- Recognise shapes based on their appearance and properties.
- Form abstract definitions.
- Know the conditions for a concept.
- They relate shapes to each other and demonstrate knowledge of their relationships.

According to Van Hieles', geometric knowledge Levels 4 and 5 should not be assessed in Grade 10 because these types of questions are high-order questions. Many studies have been carried out to show that those levels are difficult for learners to conceptualise. Level 5 (rigor) It is generally targeted at the tertiary level, not in further education training (FET) or high school geometry.

3.8.1 Baseline Test

In Creswell's definition, a baseline test is an instrument that measures the characteristics of the participants before receiving the intervention (2012, p.297). The purpose of this baseline test was to diagnose the participants' knowledge and skills. As Cohen, Manion, and Morrison (2007, p.418) point out, there are weaknesses and strengths to the study, which may be used to determine its preparedness. They refer to this baseline assessment as a diagnostic assessment since it helps to identify the strengths and flaws of the characteristics that the researcher is investigating. There are four sections and sixteen questions in the baseline assessment test. As shown in Table 3.3 below, the types of questions and Van Hieles' levels were specified for the baseline test.

Table 3.3: A specification of the baseline test.

SECTIONS AND QUESTIONS	TYPE OF QUESTIONS	GRADE AND VAN HIELE LEVELS
SECTION B	**CONCEPTUAL KNOWLEDGE**	
1	Angle	Grades 7 and 9 VHL 1
2	Proportion	Grade 9 VHL 2
SECTION C	**MULTIPLE CHOICE**	
1	Angle expression	Grades 8 and 9 VHL 1
2	Property of Similarity	Grades 9 and 10 VHL 2
3	Conditions of Similarity	Grades 9 and 10 VHL 2
4	Relating Conditions (Similar triangles)	Grade 10 VHL 3

5	Rate	Grade 9 VHL 2
SECTION D	**STRUCTURED QUESTIONS**	
1.1.1.	Identification of equal angle with reason (Parallel lines)	Grade 9 VHL 2
1.1.2.	Identification of equal angle with reason (Parallel lines)	Grade 9 VHL 2
1.2.1.	Identification of equal angle with reason (Parallel lines)	Grade 9 VHL 2
1.2.2.	Identification of equal angle with reason (Parallel lines)	Grade 9 VHL 2
2.1.	Proving similar triangles	Grades 9 and 10 VHL 3
3.1.	Proportional concepts	Grade 9 VHL 2
3.2.	Drawing proportional diagrams	Grades 8 and 9 VHL 2
4.1.	Showing similar triangles	Grade 9 VHL 1
4.2.	Midpoint theorem	Grade 10 VHL 2

The example below is Question 1.1.1 of Section D. This question was also included in the post-test for the purpose of comparing performance and understanding.

Given the diagram below, AB // CD. Name two angles which are equal to x and give reason.

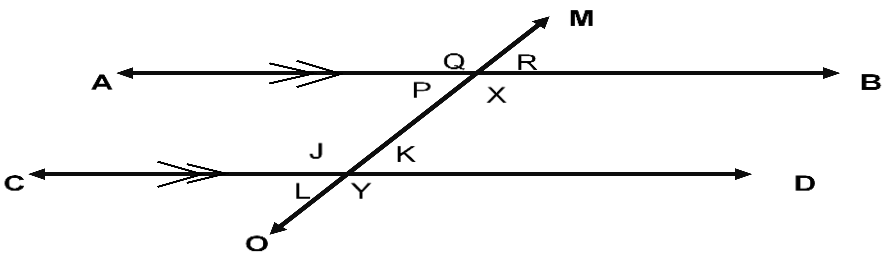


Figure 3.3: Diagram for Question 1.1.1

3.8.2 Post-test

The post-test is a tool that assesses the influence of a change in behaviour or pattern alterations following an intervention that has been carefully planned and implemented. According to Creswell (2012, p.298), this test measures some of the qualities that are examined in the experiment after an intervention has been administered. The objective of a post-test is to assess if the intervention was successful (Cohen, Manion, and Morrison, 2007, p.418). Because it offers feedback on learning, it can also be termed a summative assessment test. In addition, the post-test consists of a total of twelve questions. A post-test is specified by the type of question and Van Hiele levels in Table 3.4 below.

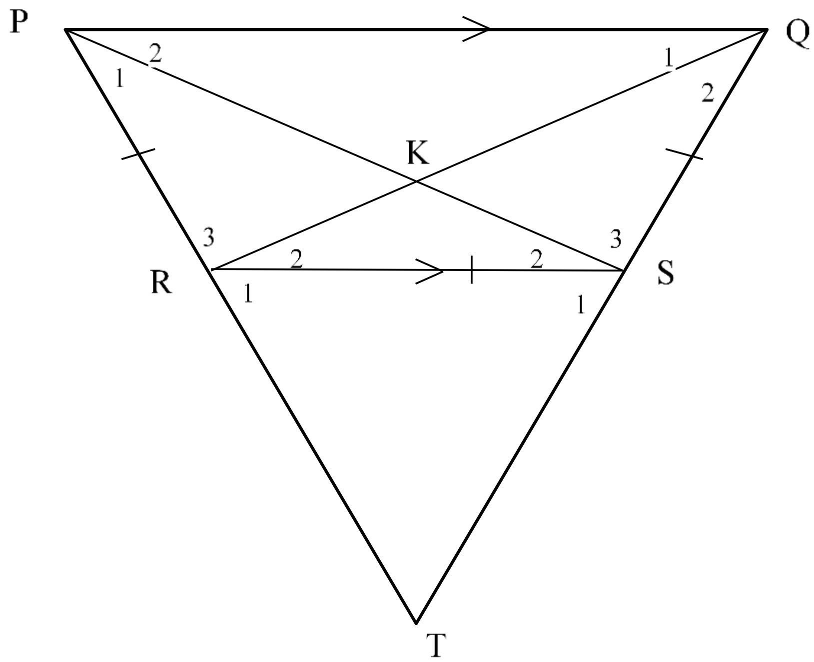
Table 3.4: A specification of post-test

QUESTIONS	TYPE OF QUESTIONS	GRADE AND VAN HIELE LEVELS
QUESTION 1		
1.1.	Proving similar triangle	Grade 10 VHL 3
1.2.	Proving similar triangle	Grade 10 VHL 3
1.3.	Proving similar triangle	Grade 10 VHL 3
1.4.	Identification of equal angle with reason (Parallel lines)	Grade 9 VHL 2
1.5.	Proving similar triangle	Grade 10 VHL 3
QUESTION 2		
2.1.1.	Proving similar triangles	Grades 9 and 10 VHL 3
2.1.2.	Proportional calculation	Grade 10 VHL 3
2.2.1.	Bisecting angle	Grade 10 VHL 3
2.2.2.	Proving similar triangles	Grades 9 and 10 VHL 3
QUESTION 3		

3.1.1.	Showing similar triangles	Grade 9 VHL 1
3.1.2.	Midpoint theorem	Grade 10 VHL 2
3.2.	Midpoint theorem	Grade 10 VHL 3

The example of Question 2.2

2.1. In the diagram above PR=RS=SQ, PQ||RS and $R\hat{P}Q = P\hat{Q}S$.



2.2.1 Show that RQ bisects $\hat{Q}$.

2.2.2 Prove that ΔPKQ|||ΔSKR.

Figure 3.4: Diagram for Question 2.2

3.9 DESIGN INTERVENTION LESSON

An intervention is a program or practice that focuses on people’s behaviour or performance, and its intention is to effect some changes or enhance the situation in that environment. Halse and Boffi (2014, p.2) characterise design intervention as a new method to enable experiences and generate awareness of emerging challenges. Intervention is a daily word that describes a goal-driven orientation to promote a particular state in the environment. The objective of design intervention is to prompt reflective action in the context of interest (Halse & Boffi, 2014, p.2). In conclusion, they emphasised that it is an innovative method that is oriented towards exploring opportunities. However, Udousoro recommended that teachers of mathematics should be aware of their learners' perceptions and try to implement instructional strategies that ensure that whatever they perceive as easy is easy, and that whatever they perceive as difficult is properly addressed in order to improve learners' achievement (2011, p.363). As a results, the researcher is of the opinion that the intervention promotes understanding of the conditions you are working with and their solution.

The researcher designed the interventions to promote a conceptual understanding of the similarity of triangles. In this intervention, the researcher promoted interaction amongst the learners to support a constructivist approach to learning. According to Charreire Petit and Huault (2008, p.4), a constructivist approach suggests that interaction is between subject and object, and that there is a method that develops knowledge. Since social constructivism promotes interaction, it was adopted in this research study. Fabiyi emphasises that learners identified challenging geometry principles in mathematics should be taught employing appropriate teachers' teaching methods (2017, p.17). Therefore, the interactions that took part in the study were teacher-to-learner interaction (TLI) and learner-to-learner interaction (LLI).

3.9.1 Teacher-learner interaction

The researcher who designed the intervention prefers teacher-to-learner interaction (TLI) because it enables learners to participate actively in the process of learning to gain mathematical knowledge. Learners are given an open-ended question to debate among themselves or with the researcher over the course of the intervention. What is this? What do you have to say about this situation? Such questions encourage learners to be themselves and express their opinions about the diagram or what they perceive. In this way, learners learn by doing, and they also help others to discover answers to difficulties. If this interaction is not effective, it indicates that the learners are confused or lost. The teacher acts as a facilitator to facilitate learning by guiding learners towards solutions to problems. This approach motivates learners to participate more effectively and gain confidence and knowledge of mathematics by reasoning accordingly.

3.9.2 Learner-learner interaction

The researcher's designed intervention also promotes learner-to-learner interaction (LLI) because it enables and enhances social interaction as learners share knowledge about the concepts and how to arrive at the answers. Intuitive introverts are helped by social contact in the learning process, whereas smart learners can aid others who are having difficulty understanding topics. In this way, the researcher obtains a better knowledge of the viewpoints of the participants, builds a relationship with them, and hears, sees, and learns to experience mathematical ideas and skills just as the other participants do. They provide a complete or holistic view of the subject under inquiry. From his personal experience and reflections, the researcher also gains valuable insight into the subject matter.

3.9.3 Observation

The observation was conducted during the designed intervention lesson. The focus of classroom observation was on learners' interaction when solving similarity tasks. Learner-to-learner interaction is supported by constructivist principles where learners generate and receive knowledge via active involvement and communication among themselves. Marshall and Rossman (1989, p.89) define observation as the methodical practice of capturing the events, conduct, and artifacts of participants in a social setting. According to Human and Karen (2016, p.15) observation generates data about behaviour, roles, and actions and facilitates understanding of what people do in a specific context. A participant-observer and a non-participant observer are the two forms of observation that are used in a research investigation.

- A participant observer is an observational role where the researcher actively takes part in the activities of the people being observed because the observer is an "insider". In this case, it is difficult for the researcher to take notes on the research site as it may disrupt the normal flow of the event.
- In this case, a non-participant observer does not interact with the group because of his or her status as an "outsider." This person attends the location or site to take notes only but does not participate in the activities of the group. As a non-participant observer, your role is passive, as your main aim is to stay impartial.

The researcher was a participant in this study since it used action research and participatory action research (PAR) as the research strategy. The type of observation employed was participant observation since the researcher was monitoring the participants' interactions throughout the course presentation in the classroom. The researcher engages with the participants as they interact with each other in the scenario under observation and may intervene in the activity and even try to modify it.

3.10 INTERVIEWS (SEE APPENDIX Q)

Alshenqeeti (2014, p.39) pointed out that the interview is not only about the information you retrieve, but it enables the participants to raise their own voices and express their views and emotions. A semi-structured interview for 8 (eight) selected Grade 10 learners was scheduled to collect data for this study. Open-ended questions were used for this type of interview to allow the participants to express their views and create options about geometry and the designed intervention. The researcher chose semi-structured interviews because these interviews are more flexible and new questions can be asked during the process of the interview to seek clarity or to allow the interviewee to express his or her own views.

Alshenqeeti (2014, p.39) supports interviews as opposed to other qualitative techniques of data collection since interviews are more powerful in the natural setting and the interviewee's perspective may be investigated deeply. The research study used personal interviews, which required face-to-face contact, whereby the interview was scheduled after the post-test and privately in the researcher’s office. The advantage of using an interview is that it provides useful and detailed personal information (Creswell, 2010, p.218) and face-to-face, the researcher can also pick up some non-verbal signs.

A conducive atmosphere was created to allow the interviewees to be free and express themselves fully without any fear. A total of five questions were asked in the interview. The following are some of the examples of questions:

- What is your understanding of similar triangles? Please explain.
- How best can you explain to other peers your understanding of similarity? Please explain.

- How do the researchers’ designed methods change your understanding of similar triangles and geometry as a whole?

3.11 TRUSTWORTHINESS

Trustworthiness is the process through which research is judged to be credible and reliable. The researcher explained to participants how important this study is when they are answering questions during the tests and interviews. In doing so, it helped the researcher gain trust and an in-depth understanding of the participant’s answers. Credibility, transferability, confirmability, and dependability were cited as reasons for the trustworthiness of this research study.

3.11.1 Credibility

Credibility is the extent to which participants believe and trust data and data analysis. A researcher's ability to understand the data supplied by participants accurately is characterised by Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p. 258). The degree to which analogies and research findings match is referred to as credibility. According to Lincoln and Guba (1985), credibility is how accurate and appropriate the qualitative data is. In this research study, the interview was used as a qualitative instrument to collect data. To ensure honesty and accuracy in the participants’ responses, the researcher explained to the participants how important their answers were and that there were no incorrect or correct answers to the questions. The research required that they expressed themselves. Issues of transferability are discussed in the next section.

3.11.2 Transferability

There is a notion of transferability, which means that a researcher's results may be transferred or generalised to other situations outside of the current study setting. Du Plooy-Cilliers, Davis and Bezuidenhout define transferability as the capacity to apply findings to similar conditions and produce similar outcomes (2014, p.258). Similarly, Merriam (1991) defines transferability as the facility to use the researcher’s results in other similar contexts. The researcher reported the demographic information of the participants, a description of the research site, research methods, descriptive findings, and direct quotes from the work of the participants. Since the researcher provided this necessary information, it means that it is possible for the findings to be transferable to other similar contexts. Be that as it may, it is important to note that the research can be useful and valuable to other learners and teachers. Issues of confirmability are discussed in the next section.

3.11.3 Confirmability

According to Kumar (2011, p.172), the degree to which the results might be validated or corroborated by others is considered confirmability. It means that confirmability assesses the quality of trustworthiness. Therefore, the research is considered not biased to make sure that the findings are a true reflection of the ideas of the participant rather than what he wants to achieve. Holloway and Wheeler (1996, p.196) consider the following auditing measures that could be used to analyse information for confirmability:

- Tape recordings and notes,
- The findings of the study through analysed data,
- The research process, design and procedures used and followed,
- The first intention of the study, and
- Data collection instruments should be developed, e.g., semi-structured interview.

In this research study, the researcher recorded the sampled learners' interviews and transcribed them verbatim. The field notes were taken during the designed intervention lesson. Open-ended questions for the semi-structured interview were used to allow the learners to give their opinion and for the researcher to be flexible during the interview. Issues of dependability conclude trustworthiness in the next section.

3.11.4 Dependability

Dependency equates to reliability, which means that the same outcome is detected in identical settings. Anney references Bitsch's work and says that reliability means finding steadiness through time (2014, p.278). Dependability describes the consistency of research findings and strengthens confirmability when the researcher produces data as it is without omitting information. Therefore, it emphasises the researcher’s accountability.

3.12 RELIABILITY AND VALIDITY

The term "reliability" refers and focuses on how consistent and reliable quantitative research measurement is. That is, whether the instruments, which are the tests in this study, are consistent and will, when repeated, produce the same results using a similar group of participants. The validity of the measurement was determined using a research instrument (du Plooy-Cilliers, Davis, and Bezuidenhout, 2014, p.256). The present researcher focused on the following instruments linked to the different forms of validity: validity of content, face validity, validity of construction, and criterion-related validity:

- Face validity refers to a participant’s perceptions of the test. Bless, Higson-Smith, and Sithole (2013, p.234) define face validity as being concerned with the way the instrument appears to the participant.
- Content validity refers to whether the test or instrument is representative and specific to the content. In this study, the validity of the baseline and post-test for similar triangles was determined by using face validity and content validity.

3.13 ETHICAL ISSUES

Research ethics deals with respecting the rights of participants in research. Tracy (2010) defines ethics as rudimental to consider the study to be of quality. According to Maziri and Madinga (2016, p.5), ethical issues are conditions that are clarified by following the moral guidelines and principles of the conducted study. All stakeholders, including the University of South Africa (UNISA) Ethics Committee, the Gauteng Department of Education or Tshwane West District, the school principal, parents or guardians, and the learners as participants, granted permission to pilot my study.

The researcher explained the aim and proposed outcome of the research to all the participants. The information that the researcher explained includes the role of participants, the role of the researcher, and the possible benefits that the participants may acquire from participating in the study. All participants were issued with consent forms, which they were asked to sign and date to consent to being part of the study.

In line with best ethical research practice, the researcher considered the promotion of confidentiality, anonymity, the rights of participants, and the avoidance of bias. Consent forms were issued to all relevant stakeholders, including the parents or guardians of the participants, informing participants about their confidentiality and anonymity, their right to withdraw at any stage without explanation, and the certainty that procedures would be duly followed. For ethical reasons, no school nor participant’s name has been mentioned. Du Plooy-Cilliers, Davis, and Bezuidenhout (2014, p. 200), reference Stringers' (1996) proposal, which advocated the use of deliberate or purposeful sampling to assure the participation of participants that suffer from the problem in the action research study.

3.14 DATA ANALYSIS

Dataset analysis is the approach used systematically for the purpose of defining, explaining, condensing, recapturing, and analysing data using statistical and/or logical methods. Mamali (2015, p.58) points out that data analysis is a means to minimise gathered data into a manageable subject, generating summaries and interpretations. This study employed two kinds of methodologies for analysing data; analysing quantitative data using descriptive analysis, which is concerned with "what is" and not with "why," and analysing qualitative data using subjective analysis, which carries out systematic and rigorous analysis procedures.

3.15 SUMMARY

This chapter described the research methodology. The researcher focused on the methodological approach, which included quantitative and qualitative approaches. In addition, the study was piloted in a community with similar contextual factors. The research was designed to process data using the following instruments: baseline test, designed interventions, post-test, and interviews. The following factors were also taken into consideration: trustworthiness, dependability, reliability, and validity. Ethical issues were also considered. Moreover, data analysis was discussed in brief. In the following chapter, the researcher will focus on the presentation and analysis of data.

CHAPTER 4

DATA PRESENTATION AND ANALYSIS

4.1 INTRODUCTION

This chapter presents and analyses the data on the Grade 10 learners' changes in understanding. The purpose of this study was to implement a program or intervention strategy that attempts to deepen the learners' understanding of geometry and thereby change their perceptions of geometry. The method that was used in this research was a mixed method (qualitative and quantitative), and the following analyses were conducted: statistical and descriptive analysis; and subjective analysis.

The following questions guided the study:

- Which foundational knowledge and skills are needed as a basis for a thorough conceptual understanding of the similarity of triangles?
- What is a Grade 10 learner's present level of foundational knowledge and skills?
- What strategies does the researcher use to mediate the understanding of the similarity of triangles in his specific teaching environment?
- How does the implemented intervention impact the Grade 10 learner's understanding of similar triangles?

The data were presented, discussed, and analysed according to the research questions mentioned above. The following instruments were used to answer the above-mentioned questions: baseline test, intervention lesson, post-test, and semi-structured interview.

4.2 BASELINE TEST FOR LEARNERS (See Appendix H)

The baseline test for learners contains four sections. Section A contained learner demographic information, Section B conceptual knowledge (pre-knowledge and knowledge foundation), Section C multiple-choice questions, and Section D a structured questionnaire. In section A, which is demographic information, there were five (5) questions. The information needed in those questions was about their gender, age-group, their previous history of Grade 9 and their passion for mathematics. Section B deals with conceptual knowledge. It entailed two questions, scenarios, or statements. The scenario required the interpretation of meaning from geometry. In Section C, the multiple-choice questions were about the following: angles, similarity, conditions of similarity, and proportion. Lastly, section D, which contained all the questions from Sections B and C in a structured form, was administered to 43 learners who participated in this baseline test.

4.2.1 Section A: Demographic information

4.2.1.1 Learner Codes

Table 4.1: Code of learners

Learner code	Grade 10 A1	Percentage	Grade 10 A2	Percentage	Total	Total Percentage
Correct code e.g., NMT 01	15	62.5 %	18	94,7 %	33	76,7 %
Incorrect code (Number only)	4	16,7 %	0	0	4	9,3 %
Incorrect code (Misspell and mistakes)	5	20,8 %	1	5,3 %	6	14,0 %
Total	**24**		**19**		**43**	**100**

Table 4.1 above shows that 76.7% (33) of the learners were able to follow the instructions and write the correct learner code; 9.3% (4) of the learners wrote the numbers only; and 14.1% (6) are those learners who misspelled and made mistakes when writing the learner codes. Both incorrect codes amounted to 23.3% (10) of the learners. Since 23.3% (10) of learners were unable to follow simple instructions, it simply means that even the instructions for the questions were not followed or understood. It may, of course, have been that the learners did not give attention to this aspect of the research.

4.2.1.2 Learner Gender

Table 4.2 below shows that 48.8% (21) of the learners were males, while 37.2% (16) were females, and 14% (6) did not specify their gender. Even though they did not specify their gender, I was able to trace their genders using their codes, for example, NMT 01. The data also indicates that there were more males than females that completed the baseline test, so we can further conclude that there were more males than females doing Mathematics.

Table 4.2: Gender of learners

Gender	Grade 10 A1	Percentage	Grade 10 A2	Percentage	Total	Total Percentage
Male	12	50,0 %	9	47,4 %	21	48,8 %
Female	10	41,7 %	6	31,6 %	16	37,2 %
Unspecified	2	8,3 %	4	21,0 %	6	14,0 %
Total	**24**		**19**		**43**	**100**

4.2.1.3 Learners grouped by Age

From Table 4.3 below, it can be observed that no learners fall within the 14-year age range or below. 74.4% (32) of the learners were from 15 to 17 years of age, while 16.3% (7) were 18 years of age or older. The learners' distribution of age, as reflected in Table 4.3, conveys that most of the learners were at the correct age at the correct grade, which also implies that it is the accepted age in secondary school. A few students in the table reveal that they were repeating a grade or that they began primary school very late. Only 9.3% (4 learners) did not specify their age group. It might mean that these learners were very old, and as such scared to reveal their age.

Table 4.3: Learners' distribution of age

Age group	Grade 10 A1	Percentage	Grade 10 A2	Percentage	Total	Total Percentage
14 and below	0	0	0	0	0	0
15 – 17	20	83,3 %	12	63,2 %	32	74,4 %
18 and above	3	12,5 %	4	21,0 %	7	16,3 %
Unspecified	1	4,2 %	3	15,8 %	4	9,3 %
Total	24		19		43	

4.2.1.4 Data presentation pertaining to Question 3, 4 and 5 of Grade 10A (1) demographic information

Table 4.4: Results showing Questions 3, 4 and 5

			Reasons for continuing			Aspects or topics of mathematics				
	All Grade 10`s	Average level (1 - 7)	Career choice	Enjoying and love	Other reasons	Algebra	Geometry	Function	Others	Percentage of reasons and or topics
No. of learners who passed Grade 9	14	3	5	6	3	6	3	2	3	58,3 %
No. of progressed Grade 9	4	1	3		1	2	1		1	16,7 %
No. of learners repeating Grade 10	3	1	2		1	3				12,5 %
No response	3	1	1	1	1	1	1		1	12,5 %
TOTAL	24		11	7	6	12	5	2	5	

4.2.1.5 Data presentation pertaining to Questions 3, 4 and 5 of Grade 10a (2) demographic information

Table 4.5: Results showing Questions 3, 4 and 5.

			Reasons for continuing			Aspects or topics of mathematics				
	All Grade 10`s	Average level (1 - 7)	Career choice	Enjoying and love	Other reasons	Algebra	Geometry	Function	Others	Percentage of reasons and aspects or topics
No. of learners who passed Grade 9	10	4	5	1	4	5	1		4	52,6 %
No. of progressed Grade 9	6	2	1	1	4	2	2	1	1	31,6 %
No. of learners repeating Grade 10	2	1	1		1	1		1		10,5 %
No response	1	1	1						1	5,3 %
TOTAL	19		8	2	9	8	3	2	5	

Tables 4.4 and 4.5 above show that 58.3% (14) of the learners and 52.6% (10) of the learners passed mathematics with 40% or more in Grade 9 respectively. The

progression is as follows: 16.7% (4) and 31.6% (6) of learners. These are learners whose mathematics marks have been adjusted or who have progressed to the next grade due to special condonation of mathematics according to National Assessment Circulars. Tables 4.4 and 4.5 further indicate that 12.5% (3) and 10.5% (2) of the learners are repeating the grade. The data for Grade 10 A1 is accurate as it can be compared with the age group presented in Table 4.3 above, and Grade 10 A2 data is inaccurate because it does not tally with the age group of 18 and above, which are learners who are more likely repeating or started school very late. The tables show that the number and percentage of learners who did not respond to the question are: 12.5% (3) and 5.3% (1) of the learners. This might be because the learners are not comfortable revealing the demographic information or they are repeating the grade.

The tables also indicate that 44.2 % (19) of learners were driven by career choices, and 20.9 % (9) of learners enjoyed and loved mathematics. Those were the reasons for continuing with mathematics in Grade 10. The remaining 34.9 % (15) of learners had other reasons. Their reasons above indicate that it is not about conceptualising the concepts and engaging with the real-life situations in which mathematics occurs, but about their personal aspirations. Lastly, the tables show that 46.5 % (20) of learners enjoyed algebra, 18.5 % (8) of learners enjoyed geometry, 9.3 % (4) of learners enjoyed functions, and 23.3% (10) of learners enjoyed other topics in mathematics. This shows that less than 20% of the learners who are doing mathematics in these groups enjoy geometry as one of the major topics assessed in Paper 2.

4.2.2 Section B: Conceptual knowledge (pre-knowledge, knowledge foundations)

Firstly, in Table 4.6 below, the performance of learners according to the way they have responded to the two questions, identified as the foundational knowledge and skills needed, is presented. As already stated in section B above, two questions from the baseline are mentioned below. Table 4.6 below shows the learners who did not attempt to answer, incorrectly answered, partially answered correctly, and correctly answered the questions.

1. The hour hand clock moves a quarter away from 12 o'clock. To which number will it now point? Through what angle will it have moved? (2)

2. An artist did a drawing of the Union Buildings in Pretoria. An observer said that the artist got the proportions wrong. What does he mean? (2)

Table 4.6: Summary of learners' marks

CODE	Descriptions	The number of learners in Question 1	Percentage (%)	The number of learners in Question 2	Percentage (%)	Total learners in both Question	Percentage (%)
-1	**No attempt**	3	6,98	3	6,98	6	6,98
0	**Incorrect**	16	37,20	36	83,72	52	60,46
1	**Partial Correct**	14	32,56	0	0	14	16,28
2	**Correct**	10	23,26	4	9,30	14	16,28
TOTAL		**43**	**100**	**43**	**100**		

Secondly, the table indicates that the performances are as follows: 6.98% (3) of the learners did not attempt to answer both Questions 1 and 2; 37.20% (16) and 83.72% (36) of the learners attempted the questions but answered them incorrectly, 32.56% (14) of the learners partially answered Question 1, and for Question 2 none of the learners, 23.26% (10) of the learners answered Question 1, and 9.30% (4) of the learners answered Question 2 correctly. From Code -1, this might mean that learners do not understand or misinterpreted the questions.

Code 0 performance indicates those learners who were unable to answer the questions correctly. It means that those learners did not recognise a quarter in the context of time, nor did they understand proportion in the context of art? Therefore, it became even more difficult for them to identify the angle and explain what it meant. Some of these learners did not even know how the clock moves, "clockwise or anti-clockwise", as Figure 4.1 below proves.

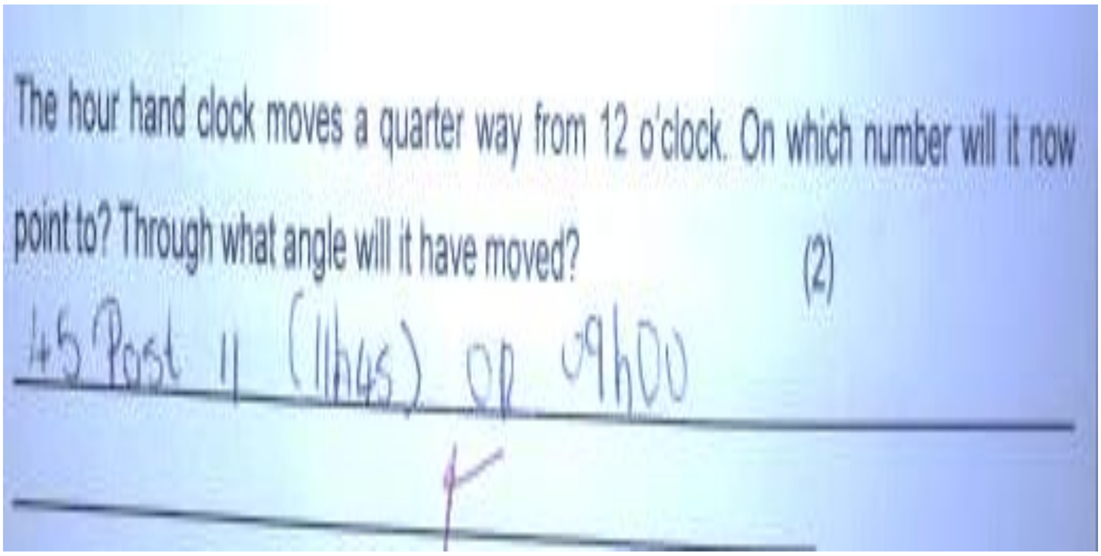


Figure 4.1: Learner NMT18 response

Furthermore, learners who fall within Code 1 are those who managed to correctly answer Question 1 partially correctly. Some of those learners understood what the term “quarter” meant but were unable to identify the angle, while others were only able to identify the angle, and still, others knew how a clock moves, or rotates “clockwise”.

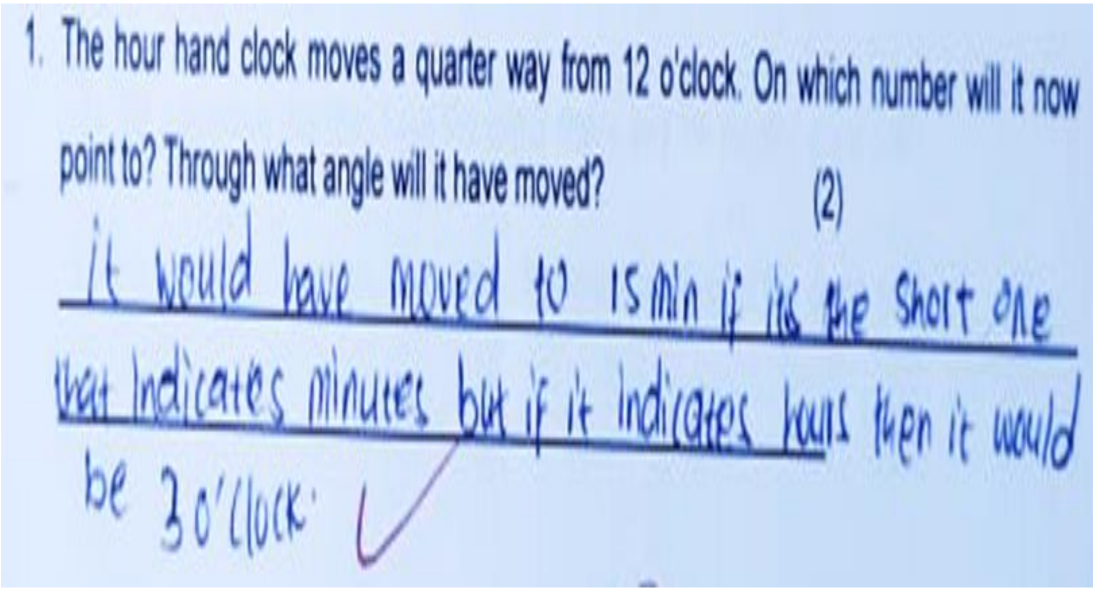


Figure 4.2 (a): Learner NMT22 response

Figure 4.2 (a) above shows learners who were able to understand the term “quarter” or the question partially. Figure 4.2 (b) below indicates those that managed to identify the angle. Through what angle will it have moved? Most of the learners answered this question with “right angle,” with only a few mentioning the angle of “90°."

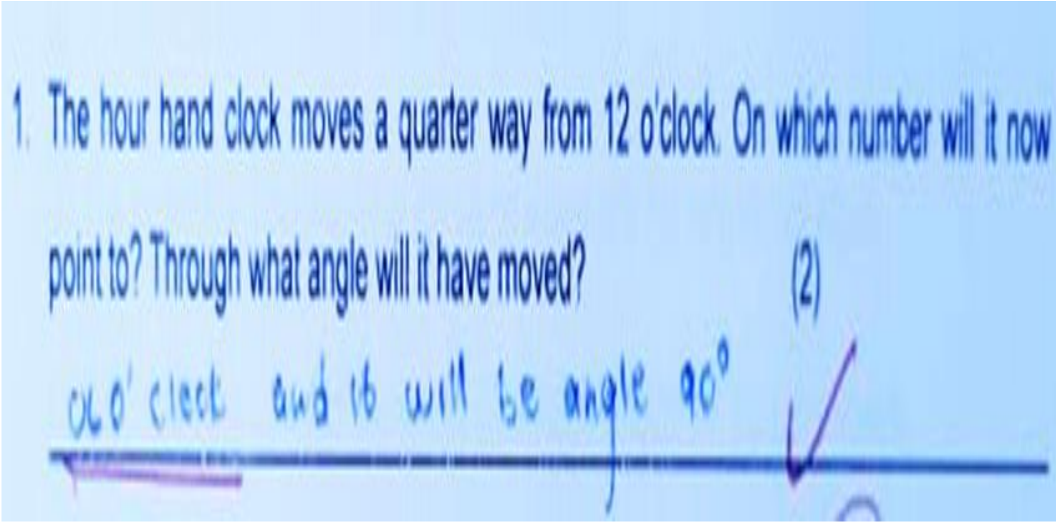


Figure 4.2 (b): Learner NMT15 response

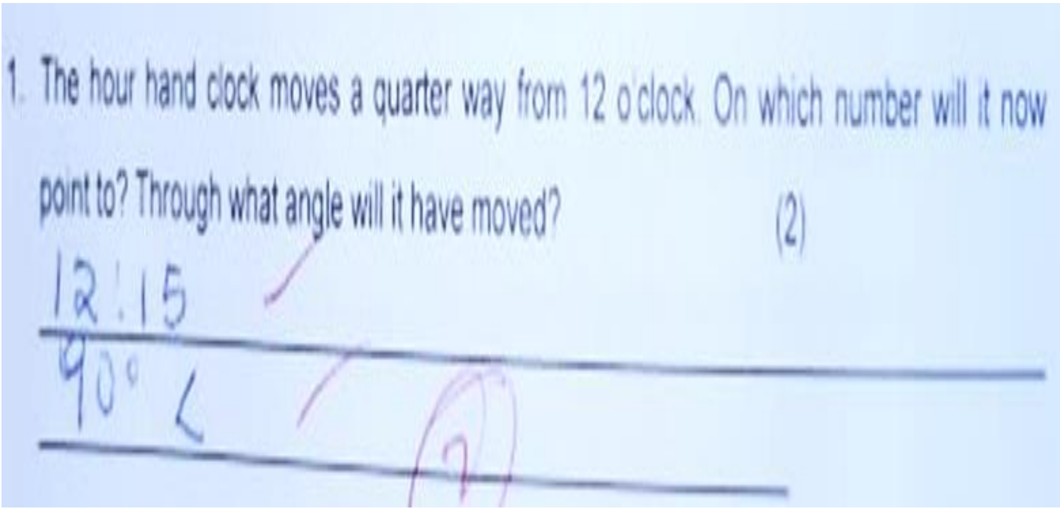


Figure 4.3: Learner NMT28 response

Code 2 refers to learners who managed to correctly answer the question as expected. These learners understood the meaning of words and the questions. Figure 4.3 shows that the learner understood the word quarter and how the hand clock moves and the angle. This explains why learners can create their own knowledge through interactions with the environment and other people (Weegar and Pacis, 2012, p.11). The learner's conceptual knowledge increases. As Luneta (2013) argues, this knowledge is the interconnection and relationships of ideas that give explanation and meaning to the procedures in mathematics.

Finally, the researcher can conclude that those learners whose conceptual knowledge can create partial meaning from the statements, or the scenario as explained in

Chapter 2 have sufficient conceptual knowledge. Furthermore, he can say these learners fall moderately within VHL1 and VHL2 since they are able to visualise the hand clock and know or analyse how it moves according to Van Hieles'levels as explained in Chapter 2. Learners whose conceptual knowledge cannot create meaning from the statements or the scenario, are learners who do not fall within VHL1 and VHL2, since they are unable to visualise the hand clock and know or can analyse how it moves according to Van H. Learners whose conceptual knowledge created full meaning from the statements, or the scenario, are learners who fall within VHL1 and VHL2, since they are able to visualise and know or analyse according to the Van Hieles' model of geometry. Their foundational knowledge and skills for conceptualising geometry are concrete.

4.2.3 Section C: Multiple choice

The table below represents the performance of Section C, which entails the multiple-choice questions. Since it was a multiple-choice question where learners had the choice to choose from all the questions, all the questions were answered by learners. The first question tested their knowledge, naming the angle in different ways. Just over half, 53% (23) of the learners, were able to answer it correctly. The second question entailed two similar triangle conditions that are related to conditions of congruency. About a third, 37% (16) of the learners, got that relationship correct. The third question determined whether the learners knew all the conditions of similarity, and it was found that only 16% (7) of the learners knew the condition of similarity. The fourth question contained two statements of similarity; 33% (16) managed to relate the statements accordingly. The last question was about ratios and proportions. Only 5% (2) learners got it correct.

Table 4.7: Summary of learner's percentage in multiple choice

Section C	Number of Learners	Percentage %
Question 1	23	53
Question 2	16	37
Question 3	7	16
Question 4	14	33
Question 5	2	5

4.2.4 Section D: Structured question

Table 3.3: specification of the baseline test shows the structured questions, and Figure 3.3: diagram is the first question. Learners were able to answer most of the questions in this question. Questions 1.1 and 1.2 were answered by most of the learners, and the performance was average.

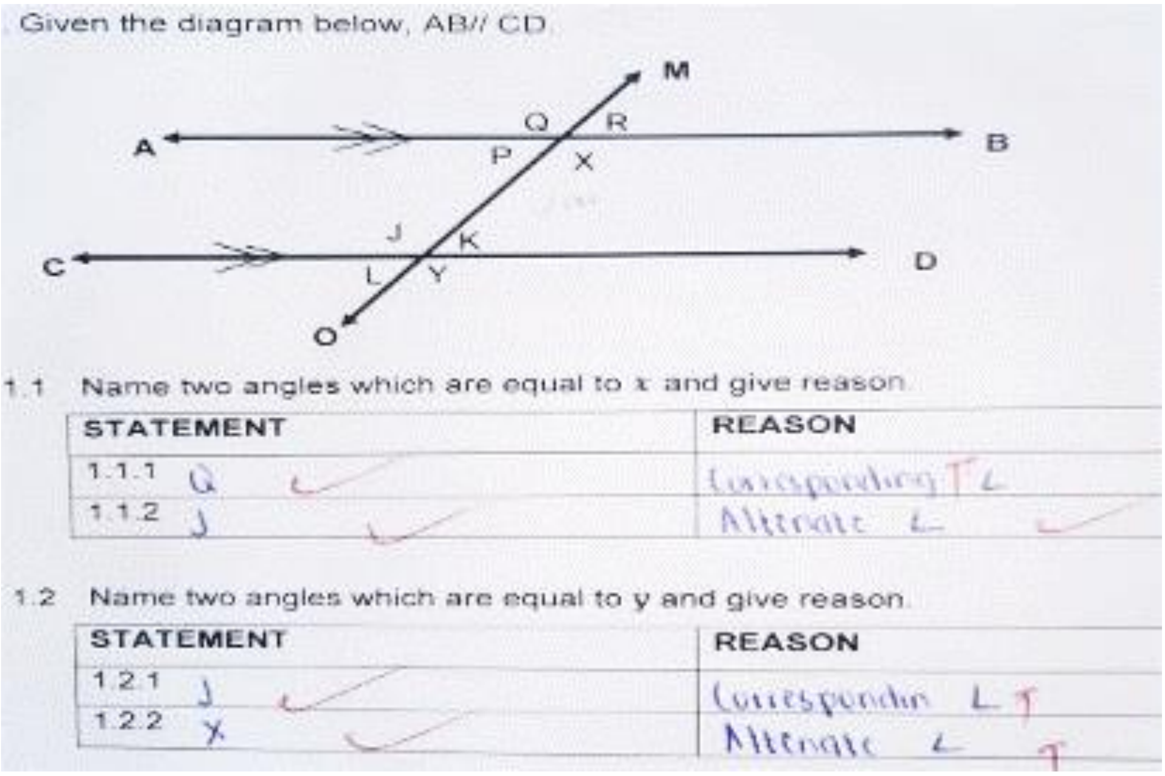


Figure 4.4: Learner NMT01 response in Section D

Figure 4.4 above shows that the learner responded correctly to Question 1.1, but her or his reason for 1.1.1, “corresponding angles,” was incorrect; the correct answer was vertically opposite angles. The learners also failed to give the correct reasons for Question 1.2. In Question 1.2.1, the learner said $y = x$ because angles are alternative instead of saying corresponding angles. This indicated a lack of deeper knowledge of the words or terminology used in Euclidean geometry, even though the learner recognised angles that are equal. The results showed that approximately 30,2% of the learners were able to score 2 marks and 14% of the learners were able to score the whole 4 marks in both the questions. This shows that more than 40 % of the learners from the group were able to conceptualise the concept and work at the visualisation and analysis levels of geometric thinking.

Question 2 was answered very poorly; only one learner got only one (1) mark from the whole group. This shows that learners were unable to prove that the two triangles are like each other by means of calculations. It also means that 97.7% of learners in this group did not comprehend Grade 9 geometry knowledge. According to the CAPS document for Grades 7 to 9, learners must be able to identify and describe the properties of similar shapes. Question 3 was the easiest; the requirements of the questions were about the learner’s own reasoning and the drawing of that. Prior knowledge of the sizes of pictures or photos is needed, and also that the pictures or photos have a rectangular or square shape. The performance of open-mind response was 48.8% (21) of learners were able to comprehend the question (see Appendix H). This means that these learners think outside the box and can step outside their comfort zone. Furthermore, learners are able to make decisions on their own and appreciate the different choices and opinions of other learners. The results of drawing photos based on the instruction and opinion were 44.2% (19) learners (see Appendix H). These are the learners who were able to draw the original photo and/ or all the photos correctly.

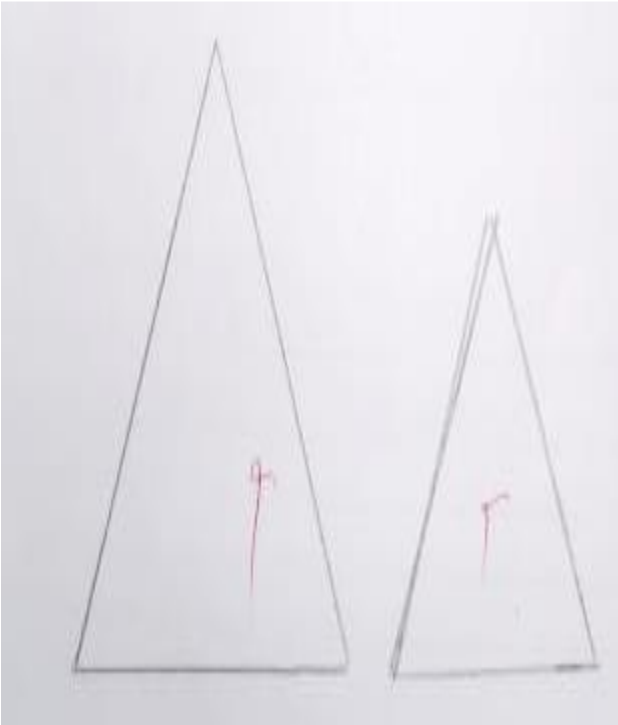


Figure 4.5: Learner NMT15 response in Section D

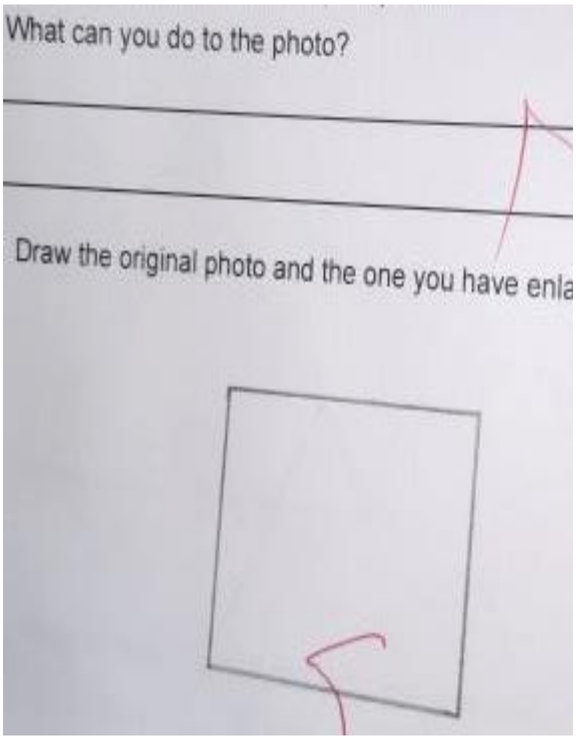


Figure 4.6: Learner NMT25 response in Section D

Figures 4.5 and 4.6 above show that learners **NMT15** and **NMT25** can comprehend the concepts of the questions where they were requested to draw, and indeed, they drew even though they did not conceptualise the whole question. Figure 4.5 indicates that the learner drew the unique original photo and enlarged the photo. However, he misinterpreted the statement because the original photo was a rectangle with a length of 3 cm and a height of 6 cm. He then drew a triangle instead of the rectangle. Figure 4.6 shows that the learner had challenges with literacy or with English, the language of learning and teaching (LOLT), since he or she was unable to answer the first question, which requires a multiple-choice response or an open-mind response. Furthermore, he or she could not conceptualise the second question of drawing two photos. The results show that more than 40% of learners from this group scored 2 or more marks on this question.

The last question was about the mid-point theorem and similarity. This prior knowledge necessitates an understanding of the diagram in Figure 3.3. In the first part, learners were able to state which triangle is like triangle AOZ but struggled to give a reason. The reasons are referred to as the conditions of similarity (**AAA** or **SAS**). The second part of the question was about the application of the mid-point theorem, and learners answered the question very badly, with 20.9% (9) of the learners answering it correctly and 79.1% (34) of the learners failing to respond correctly. The knowledge of the midpoint theorem was lacking.

Table 4.8: Summary of learners' marks in 4.1

CODE	Code Description	Number of Learners	Percentage (%)
-1	Not answered	1	2.3
0	All Incorrect	8	18,6 %
1	One Correct	28	65,1 %
2	Both Correct	6	14,0 %
Total		43	100 %

Table 4.8 indicates that 20.9% (9) of the learners were unable to answer the question (out of the 9 learners, only 2.3% (1) learner did not attempt to answer the question and 18.6% (8) of the learners answered incorrectly). 65.1% (28) of the learners managed to provide an answer by stating the similar triangle, but with the wrong reasons or without reasoning, and 14% (6) of the learners were able to state the triangle and give the correct reason why triangles are similar. Of the 6 learners, 4.65% (2) of the learners answered both 4.1 and 4.2 correctly. The results further show that more than 75% of the learners from the group were able to conceptualise the concept and work under Levels 1 and 2 of van Hieles'.

4.3 OVERALL PERFORMANCE OF BASELINE TEST

Table 4.9 below represents the performance of learners that wrote the baseline test. The results indicate that 58.13% (25) of the learners did not achieve (fail) the baseline test and 41.87% (18) of the learners achieved by meeting the passing requirement for mathematics in FET. The results revealed that 27.91% (12) of the learners achieved elementary achievement, that is, the entry level, 9.30% (4) of the learners on adequate achievement, 2.3% (1) learner on moderate achievement, and 2.3% (1) learner on substantial achievement.

Table 4.9: Baseline performance

RANGE OF LEARNERS' MARKS (%)	NUMBER OF LEARNERS	PERCENTAGE (%)
0	0	0
1 – 29	25	58,13
30 – 40	12	27,91
41 – 50	4	9,30
51 – 60	1	2,33
Above 61	1	2,33
TOTAL	43	100

4.4 OBSERVATION OF INTERVENTION STRATEGY

This lesson was designed to help learners discover the properties of similar triangles and develop a different approach to doing geometry problems. This lesson was intended to solidify the basic knowledge of Euclidean geometry and similar triangles. The lesson was presented for 4 days, and the following were the objectives of the lessons: revision of previous grade knowledge, understanding equal angles, recognising similar triangles, understanding the definition of a similar triangle, and understanding proportionality.

4.4.1 Lessons 1 and 2 observations

In this lesson intervention, learners were given Euclidean geometry exercises that require the following knowledge: angle, type of angle, parallel lines and transversal, properties of triangles and quadrilaterals. Since the lesson was participatory, the exercises (*more information can be found in Figure 4.7, 4.9(a) and 4.9(b)*) were sampled for the purpose of observing what learners were saying and writing or doing. This was important and key to collecting data.

The first exercise given to learners was for them to name the angle and express the name in different ways.

Pose a question to learners: what is the name of the angle below?

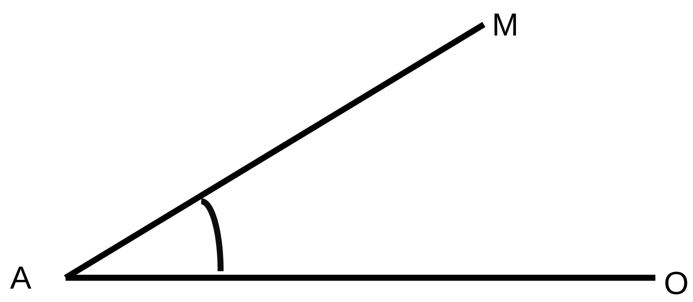


Figure 4.7: Learner question

Discussion with learners on how they can write or express the angle in different ways.

The possible and expected answers or responses are angle $\hat{A}, M\hat{A}O \text{ and } O\hat{A}M$.

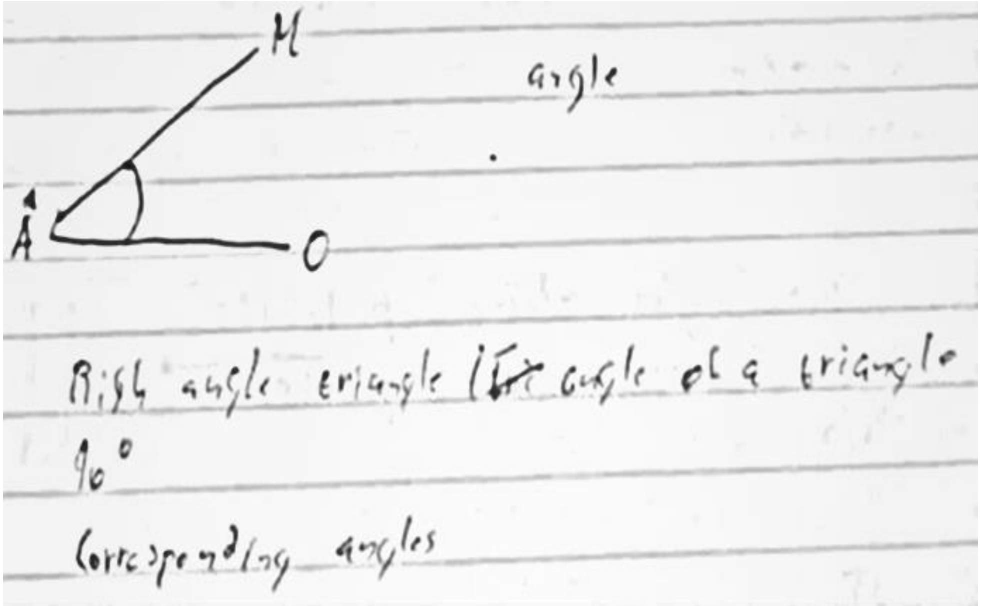


Figure 4.8: Learner NMT24 response

The reply of the learner during the first exercises is shown in Figure 4.8. According to the figure above, it was found that learners could not answer the name of the angle. The most popular names among the learners are "Acute Angle," while some refer to the "Obtuse Angle" and the "Right Angle". Since the learners were not able to name it, all learners could not express or write the angle differently. Rather than describing the angle, they explained why they said that kind of angle.

The second exercise given to learners was for them to discuss and write about the types of angles. It was observed that the discussion was effective because learners knew most of the types of angles that they have learned in the previous grade, even though some of them had challenges in convincing or giving reasons to others about why they were saying, “vertically opposite angles” or “supplementary angles”.

The parallel lines with a transversal were the third exercise given to the learners. This exercise was practiced and focuses on “FUNX”. The analogy of “FUNX” was that every time when you remember your “X” friend, your memories must be “FUN”. Where “X” can also mean “*Mabakabaka”*, which means “vertically opposite angle”.

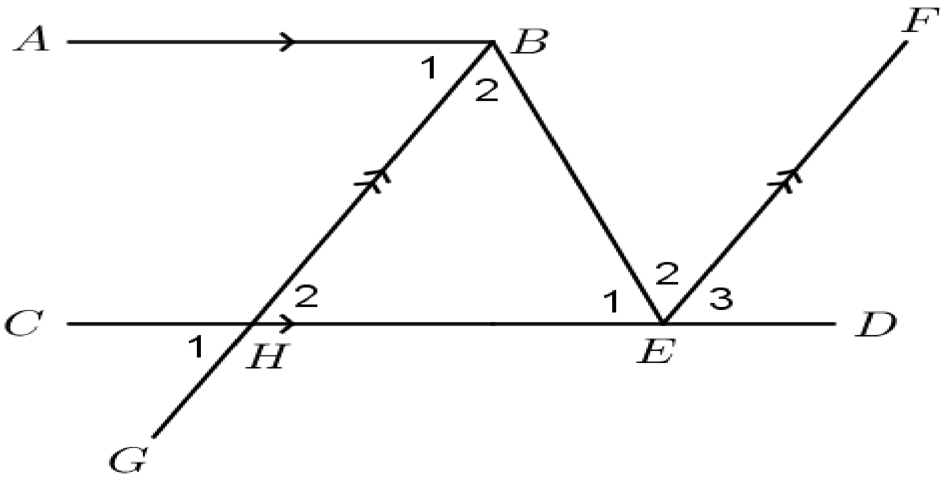


Figure 4.9 (a): Learner exercise 3

1. Name any angle and express it in 2 different ways.

2. Show which angle is equal to which one.

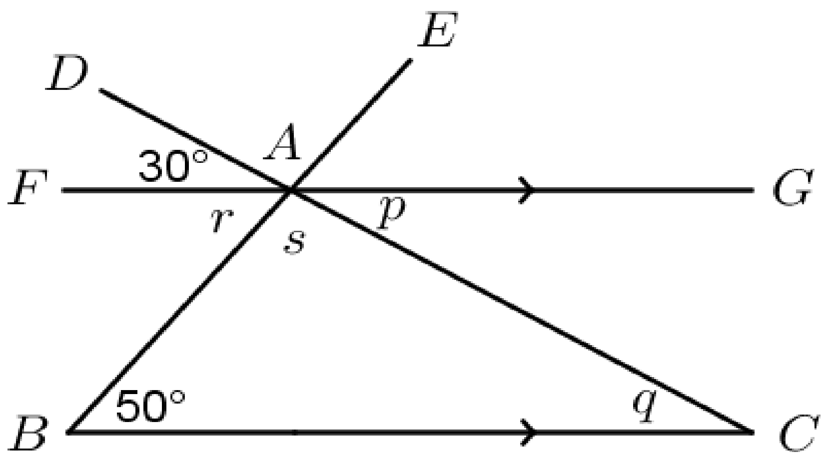


Figure 4.9 (b): Learner exercise 4

1. Fill in the missing angles.

In their attempt to solve these exercises, learners seemed to have the ideas and knowledge of the diagram and question provided to them, even though they were generic. They understood that they had to utilise the diagram and the information provided to solve the questions. Following the learners' discussion carefully, it revealed that learners had ideas of what was expected to answer the questions. However, their present knowledge was not enough.

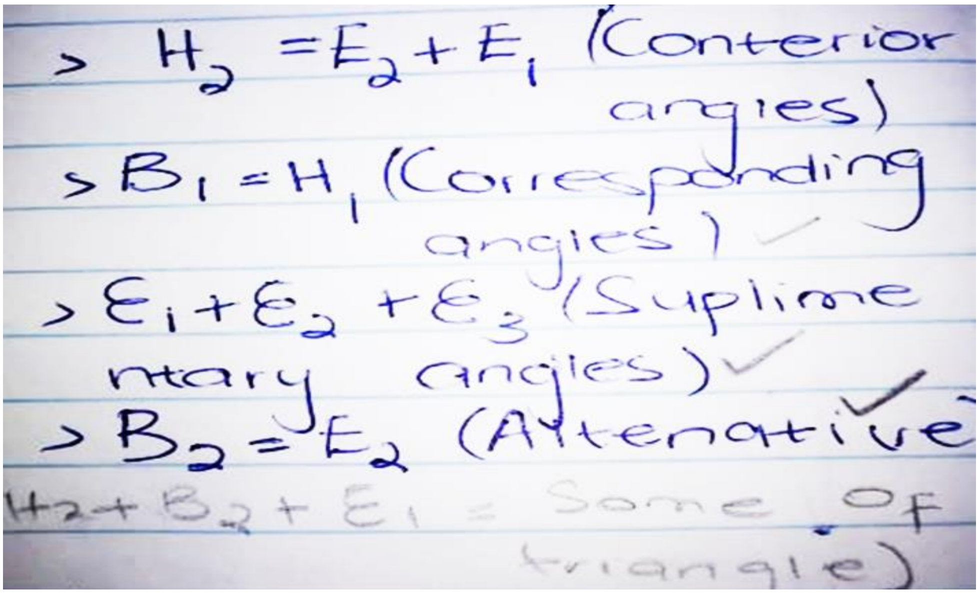


Figure 4.10: Learner NMT16 response

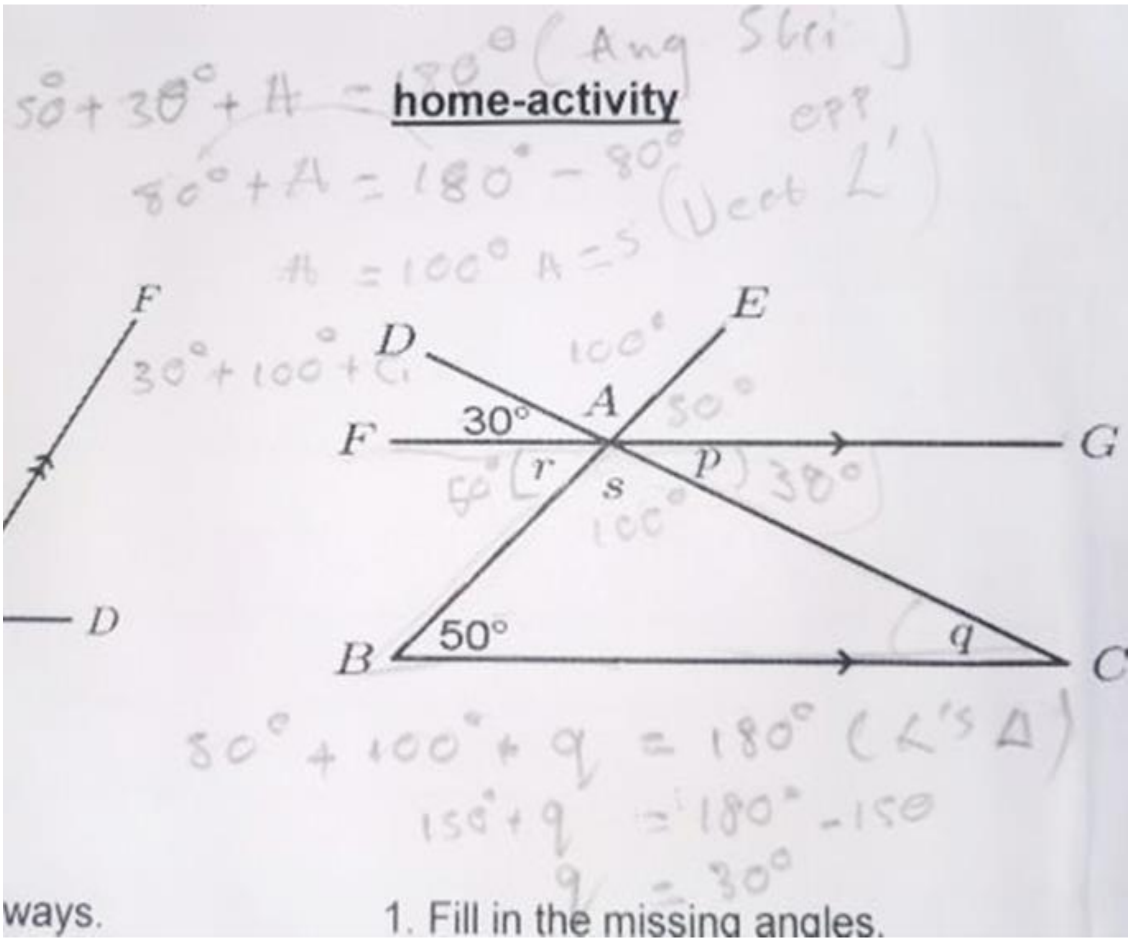


Figure 4.11: Learner NMT20 response

Figures 4.10 and 4.11 show that, while working in groups, learners will comprehend more and will improve their performance and confidence. Working in a group in a class allows learners to develop effective teamwork skills and enhance their communication skills in cooperative learning environments. The group technique is one of the best and most efficient ways for learners in the field of geometry. When the question was answered, group discussions were successful and fruitful.

Finally, naming triangles and quadrilaterals and their properties was an exercise given to the learners. Various triangles were drawn, and the learners were asked to name the attributes. It was debatable because, during their interview, they all knew the name of the triangle, but it was challenging to know which name and how it belonged to the diagrams. As a result, some of their names misrepresented some diagrams, as they could not deduce them accordingly. For quadrilaterals, it was easy since they wrote it at home. The probability is that they used a textbook as a reference.

4.4.2 Lessons 3 and 4 observations

Throughout this lesson intervention, Euclidean geometry exercises that demand knowledge were presented to the learners: introduction, similarity conditions, ratio and proportion, and midpoint theorem. The session was participative as well, the activities were sampled to see what learners had to say and write or do. This was crucial and essential for data collection.

The introduction of the lesson was about the discussion and demonstration, where the researcher posed two questions to the learners. The following are the questions and discussions:

First question: what can you say about you and your shadow?

Learners' responses:

"Wherever I go, it goes with me".

"They are equal".

"Same shape".

"It is my reflection".

Second question: What can you say about you and your mirror image?

Learner’s responses:

“It is your reflection”.

“We are the same”.

“They are opposite”.

The demonstration was as follows: cutting the folded page into a triangle and posing a question. How many triangles do you see? Most of the learners said that they saw one triangle, which was true according to what they saw. How many triangles do I have? Learners gave many different answers.

After the discussion and demonstration, the misconceptions and errors that were made during the discussion were corrected. The researcher then linked the discussion and demonstration to the sub-topic of similarity. The first question was used to explain the ratio and proportion concepts and one of the conditions of similarity, which is of (**SSS**) and demonstration (**AAA**). The researcher explained how to determine the corresponding sides using his method, dubbed “The Russian or Happy Face.”

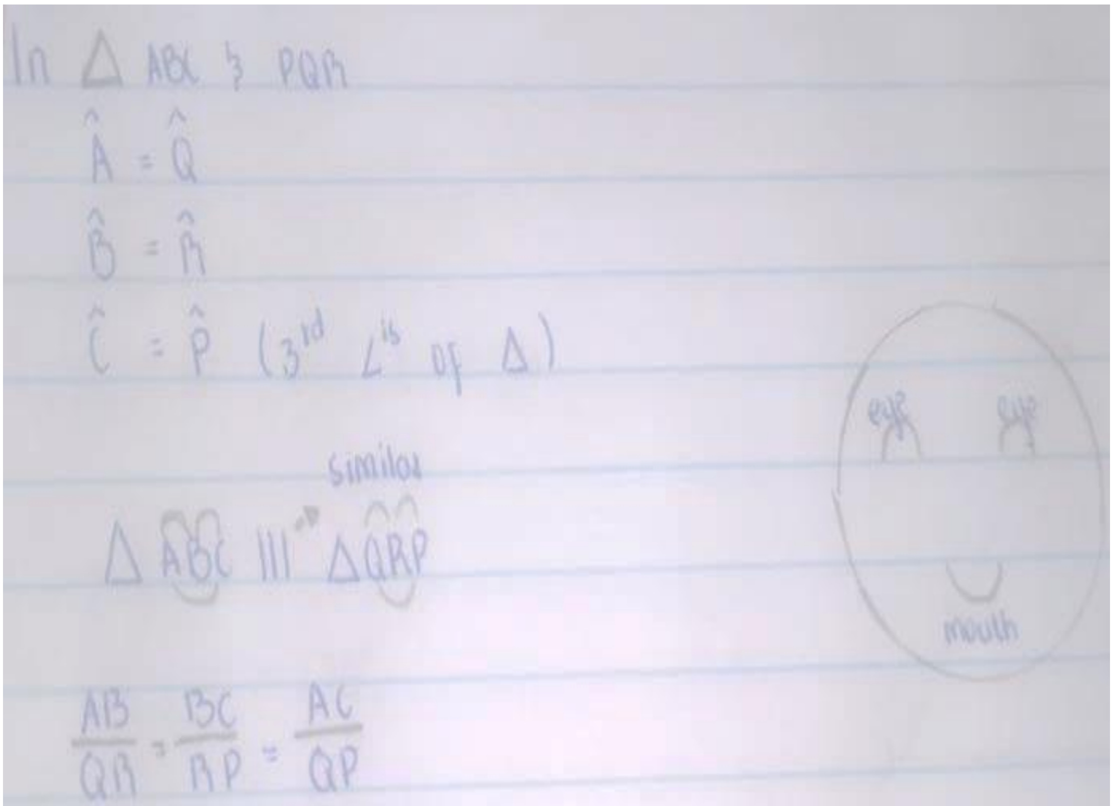


Figure 4.12: NMT01 expression of the Russian or Happy face

Figure 4.12 above shows the Russian strategy, which refers to an eye for an eye times two and a mouth for the mouth. In triangles $\Delta ABC \ and \ \Delta QRP$ that are similar, AB is the eye and QR is another eye, BC is the eye, and RP is another eye and AC is the mouth and QP is another mouth. Therefore, it implies that AB and QP, BC and RP and AC and QP are corresponding sides of each other.

Determine whether the following are similar or not?

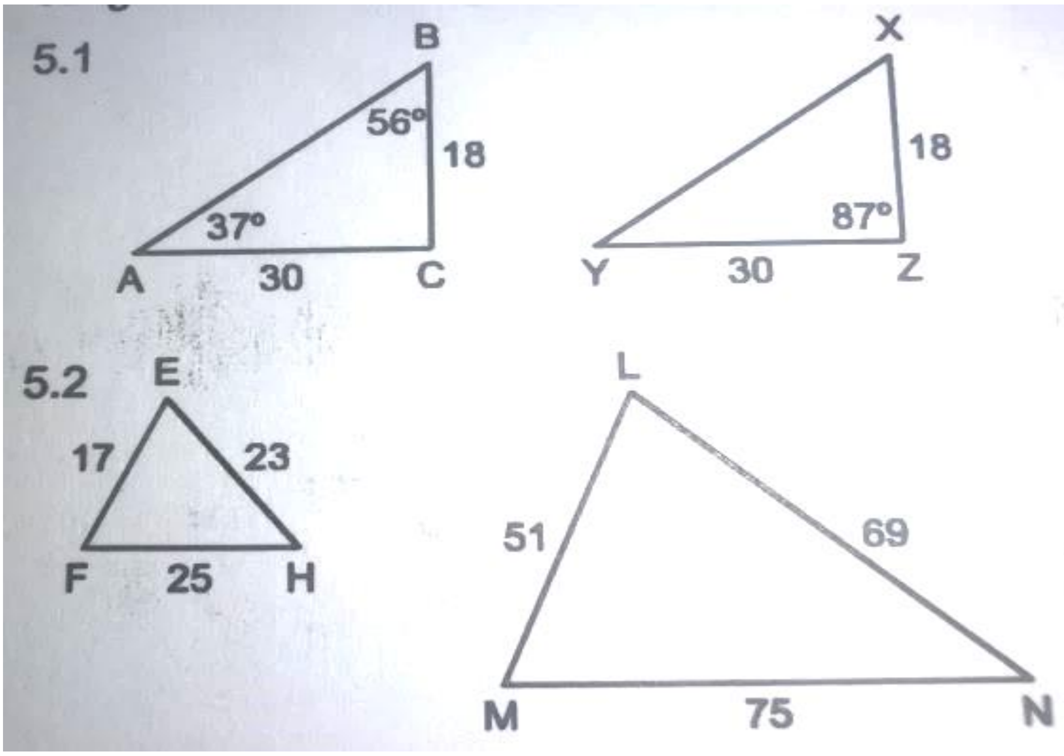


Figure 4.13: Learner exercise 5

Figure 4.13 above is the sample exercise that was given to learners to check whether they understand the concept of similarity. Figure 4.14 below shows the answers of learners to Figure 4.13 questions above. Figure 4.14 also indicates that the learners conceptualise the concepts of similarity even though 5.1 answers show that the two triangles are congruent instead of similar according to the question. Even though congruency is a special case of similarity, they made use of the wrong sign to indicate similarity. Furthermore, they did not conclude which triangle is like which one, but yet they made the correct reason for (**SAS**).

In both 5.1 and 5.2, learners did not indicate which triangles they were going to prove, but they started with the conclusion. In 5.2, they did not indicate that corresponding sides are proportional, but rather that they are equal, despite the fact their conclusion of (**SSS**) is correct “proportional or same ratio”.

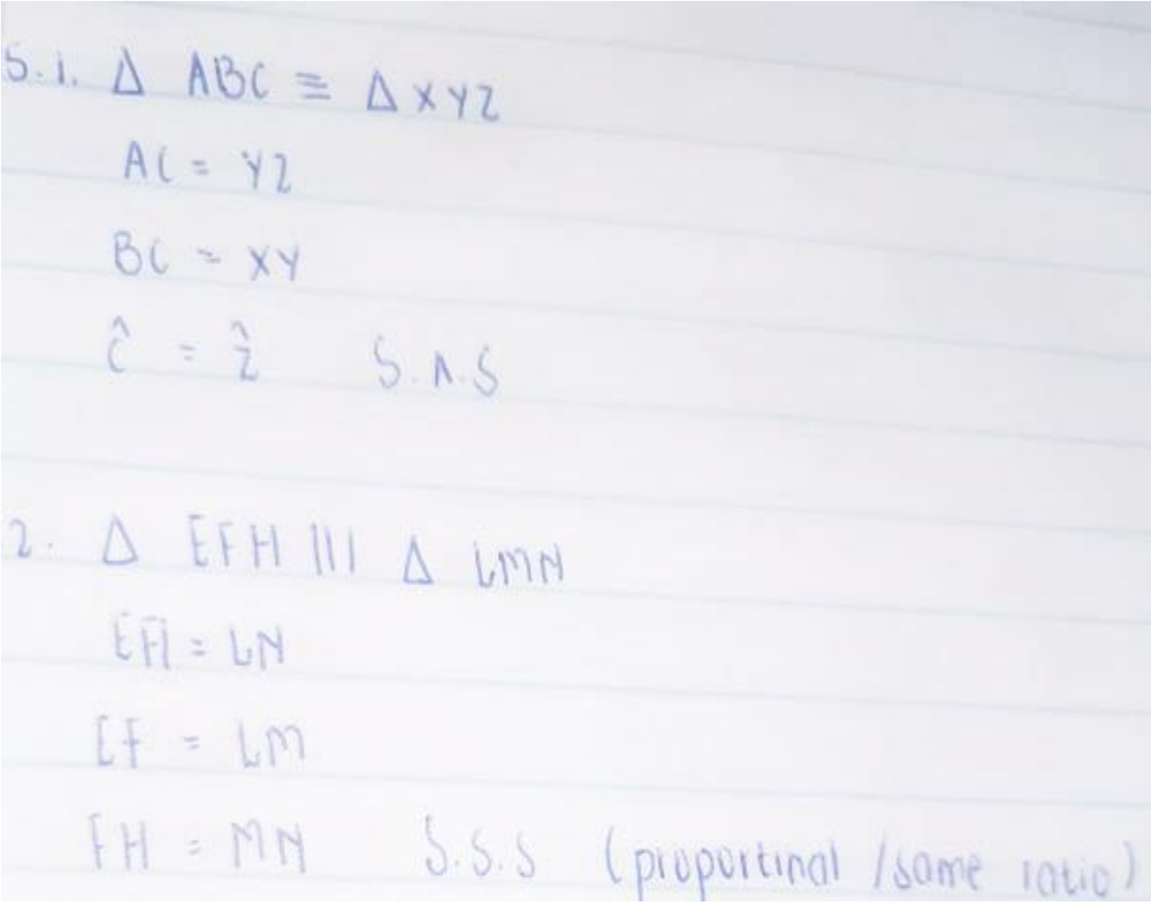


Figure 4.14: NMT01 learner response

4.5 POST-TEST

The post-test for learners is the test that contained three questions, and its standards were a bit more difficult compared to the baseline test. Question 1 expected learners to use conditions of similarity and perform calculations to state whether the given triangles are similar or not. Post-test Question 4 (P4) was assessed in the baseline test. Question 2 entailed diagrams where learners were required to prove similar triangles. P6 was also assessed in the baseline, and in Question 3 is P11 and P12. Table 4.8 below shows the performance of the post-test per question. The code description “0” denotes learners who received 0%, “1” denotes those who received 25%, “2” denotes those who received half of the marks allocated to the question, “3” denotes those who received 75%, and “4” denotes those who received all of the marks for that question.

Table 4.10: Performance per question of post-test

CODE	Marks in Percentage (Code description)	NUMBER OF LEARNERS PER QUESTION											
		QUESTION 1				QUESTION 2					QUESTION 3		
		P1 (4)	P2 (4)	P3 (4)	P4 (4)	P5 (4)	P6 (4)	P7 (4)	P8 (4)	P9 (4)	P10 (2)	P11 (2)	P12 (4)
0	0%	24	28	35	13	40	39	42	41	38	10	22	20
1	25%	7	3	3	5	0	1	0	1	3	-	-	7
2	50%	8	5	3	10	2	1	0	0	1	29	12	1
3	75%	2	1	0	5	0	1	0	0	0	-	-	9
4	100%	1	5	1	9	0	0	0	0	0	3	8	5
TOTAL		42	42	42	42	42	42	42	42	42	42	42	42

Table 4.10 above shows that a greater proportion of learners from Question 1 (P1) to Question 3 (P12) got 0 out of the total mark of the sub-questions. Furthermore, Figure 4.15 below shows the box plot of learners in Code 0. The minimum is 10, which means this is the lowest number under Code 0 and is P10. It also means that this is the item that performed better compared to others. The maximum is 42, which means this is the highest number under Code 0 and is P7. It also means that this is the item that performed the worst out of all the items, since all the learners got 0%. It also reveals that the median is closer to the maximum size of the box, and the whisker is shorter on that side, indicating that the data distribution is negatively skewed or skewed to the left, indicating that most of the learners performed poorly or negatively.

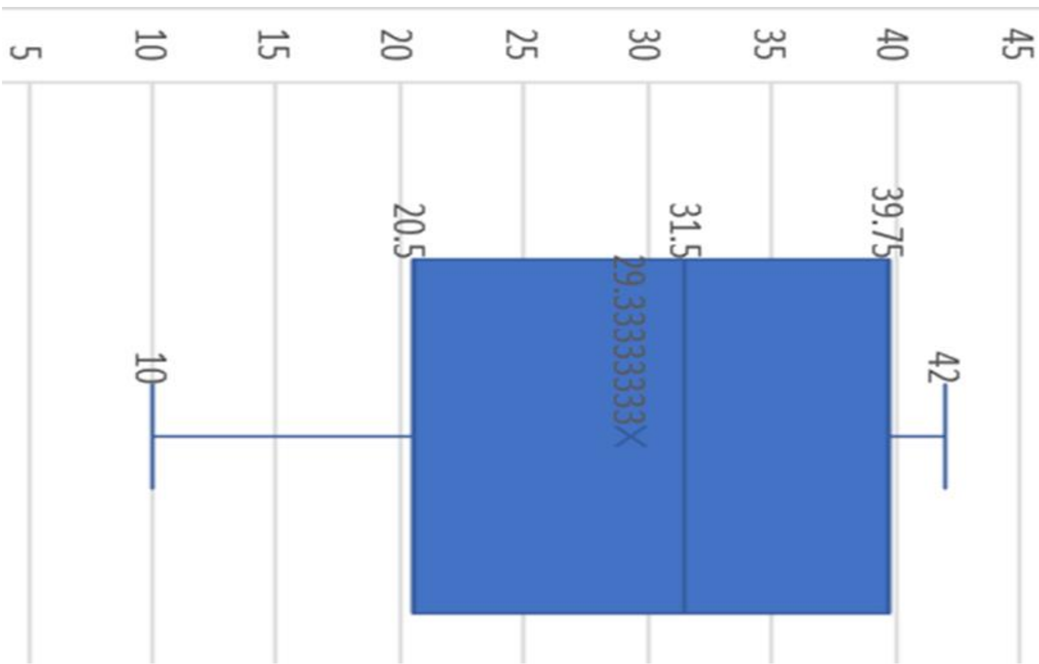


Figure 4.15: Box plot for learners in Code 0

The performance reveals that the learners who fall within Code 1 are as follows from P1 to P12: 7, 3, 3, 5, 0, 1, 0, 1, 3, and 7 respectively; and P10 and P11 do not have that code since the mark allocation is 2. P1 and P12 are items that performed the highest, and P5 and P7 are items that performed the lowest under Code 1. Furthermore, the following are learners who obtained 50 percent on items 1 to 12 of the post-test: 8, 5, 3, 10, 2, 1, 0, 0, 1, 29, 12, and 1 respectively. The item that got greater performance is P10, with 29 learners falling within Code 2, and P7 and P8 are the lowest-performing items. Moreover, 2, 1, 0, 5, 0, 1, 0, 0, 0, and 9 are the learners who managed to attain 75 percent from P1 to P12, respectively, and P10 and P11 are not mentioned since their full mark is 2. Nine (9) of the learners from Code 3 performed better in P12 than in P3, P5, P7, P8 and P9, with no learners obtaining 75% of the mark allocated. Finally, these are the learners who got full marks from P1, P2, P3, P4, P10, P11, and P12: 1, 5, 1, 9, 3, 8, and 5 respectively, and from P5 to P9, there are no learners who got full marks.

4.5.1 Additional items

Additional items are the questions that were not administered on the baseline, and there were 8 items. Those items were more challenging since learners were proving three (3) triangles to be similar. The purpose of adding those items was to stretch and evaluate how much knowledge the learners had gained. The results showed that 20,9% (9) of the learners from this group were able to answer some of those added items. This means that those learners conceptualise the Euclidean Geometry of Grade 10. Eventually, they will be able to be competitive in Grade 12 and tertiary geometry.

4.5.2 Overall performance of post-test (See Appendix N)

Table 4.11 below indicates the overall performance of learners after the post-test was administered. It shows that 6.9% (3) of the learners obtained a zero percentage, 37.2% (16) of the learners, and 25.6% (11) of the learners performed badly, 20.9 % (9) of the learners, 4.7 % (2) of the learners performed averagely, and 4.7 % (2) of the learners performed above average. It further shows that 69.8 % (30) of the learners who wrote failed and 30.2 % (13) of the learners passed the post-test.

Table 4.11: Post-performance

RANGE OF MARKS (%)	No. OF LEARNERS	PERCENTAGE (%)
0	3	6,9
1 – 29	36	83,7
30 – 39	2	4,7
40 – 50	2	4,7
Above 51	0	0
TOTAL	**43**	**100**

4.6 BASELINE AND POST-TEST

The baseline and post-test were compared based on the following: per item comparison between the baseline and post-test and same item comparison between the baseline and post-test. The purpose was to evaluate whether learners had improved understanding or not. The learners who improved positively might mean that they took the intervention very seriously, while the learners who declined might mean that they were comfortable and relaxed during the intervention. Figures 4.13 and 4.14 below indicate those comparisons.

4.6.1 Comparison of same items from baseline and post-test (See Appendix P)

Table 4.13 below indicates the comparison of the same question items from the baseline test and post-test. The following are the items: B8, B9, B12, B15, and B16 from the baseline and P4, P6, P10, and P11 from the post-test. The above items were compared, and Table 4.13 below shows that 48.8 % (21) of the learners' marks improved, 18.6 % (8) of the learners' marks remained the same, and 32.6 % (14) of the learners' marks declined (*see Appendix P*). When looking at learners whose marks remained the same, they scored the marks from different items, which shows that learners were not sure.

Table 4.12: Same items comparison

	Number of learners	Percentage (%)
Positive improvement	21	48,8
No changes	8	18,6
Negative improvement	14	32,6
TOTAL	43	100

4.6.2 Comparison per item from baseline and post-test

Table 4.14 below indicates the comparison per item from the baseline test and post-test looking at the order of improvement based on average. It shows that the average mark percentage from baseline and post-test items is 28.5 % for B8 and B9, and 48.8 % for P4, 0.58 % for B12, and 3.5 % for P6, 46.5 % for B15, and 44.2 % for P10, and 20.9 % for B16, and 36.1 % for P11. It also indicates that B8, B9, and P4, B12, and P6, B16, and P11 are items that increased, and B15 and P10 are the only items that decreased slightly. It further shows that B8, B9, and P4, B12, and P6, B16, and P11, their average mark percentage increased positively, and B15 and P10 decreased slightly.

Table 4.13: Item by item comparison

Items (marks)	Baseline average mark performance (%)	Post-test average mark performance (%)
B8 (2), B9 (2) and P4 (4)	28,5	48,8
B12 (4) and P6 (4)	0,58	3,5
B15 (2) and P10 (2)	46,5	44,2
B16 (2) and P11 (2)	20,9	36,1

6.7 SEMI-STRUCTURED INTERVIEW

6.7.1 Interview schedule for learners

The following are questions, follow-up questions, or clarity-seeking questions, and the views expressed by the learner concerning the similarity of triangles and the intervention of the researcher.

Question 1: What is your understanding of similar triangles? Please explain.

NMT 01: "My understanding about similar triangles is that they have the same shape, but they do not have equal angles and proportion. They have the same ratio and fractions."

NMT 09: "I do not understand anything."

NMT 21: "I do not understand them clearly." FQ: Which parts? "There are some parts I do not understand."

NMT 22: "I understand that similarity…similar triangles...any other kind of shape should have same sides and angles and they are proportional...I think that is all."

NMT 25: "I understand through angels that they have different sides and angles, but they are the same."

NMT 30: "My understanding is that they may have the same shape of triangles but not the equal size. They can reflect each other, for a triangle to be similar when you calculate the triangles must be equal to one number." FQ: When you say equal to one number what are you referring too? "Like… they must have the same value."

NMT 33: "My understanding is that they two same triangles, but the angles differ, and they are not equal."

NMT 35: "Similar angles have the same angles but different sizes." FQ: What do you mean by same angles? "Sir… their angles are equal but different in size." FQ: Sizes of what? "Their lines."

From what the learners said above, some of the learners indicated that their understanding of similar triangles is when the triangles have the same shape but a different size. Other learners also indicated that the angles are equal, but the lines have different sizes (proportional), and there are those who do not understand anything about similar triangles.

Question 2: How can you best explain your understanding of similarity to your peers? Please explain.

NMT 01: "I could just tell them that…maybe I could draw a triangle, like similar triangles then I'll show them…the other one will be big and the other one small, but they will have the same angles and sides, but the sides will not be equal, like the ratio of the triangle can be 1:3 and the sides will be equals to 3 and the other side will be a 1 because the ratio is 1:3."

NMT 09: "How they differ, how you calculate but here and there…I just do not understand anything."

NMT 21: "I do not know how to explain to other people."

NMT 22: "Hmm… I think I can say to them that to see that angle or rather triangles are similar it is not about the shape, or the...Hmmm…let me see…triangles can be similar and not have the same size, but they are proportional and have the same angles." FQ. What do you mean by the same angles? "It is because I remember you said that we can see similar angles through the sides which is side...side…side which means all sides will be equal and angles too...are also equal."

NMT 25: “I cannot explain to another person.” FQ: Why? “I master something after a while.”

NMT 30: “I will say that they are triangles that are the same but not equal when you add angles and sides if they are similar the value and sides must be the same.”

NMT 33: “They are triangles, let me rather say they are objects that are the same when looking at them.”

NMT 35: “Like I said similar angles have the same angles but different sizes.” FQ: Can you briefly explain. “I will say similar angles have the same angles but different shapes, but the sizes differ that is all …they will understand me because they are my peers.”

Two learners can clearly explain their understanding of similar triangles, **NMT01,** stating that she or he will use a drawing of two triangles, and **NMT22,** mentioning different sizes, which is proportionality. Other learners also indicated that they are unable to explain their action to other peers because of their lack of understanding or clarity. From what other learners said, it was clear that there were not sure about what they were saying because of being confused.

Question 3: What challenges do you have in similarity? Please explain.

NMT 01: “Sometimes I cannot identify the problem, I only manage to get the side and the angle, and I fail to understand why I fail to get the third one.” FQ: Calculation part? “I am fine with that.”

NMT 09: “Everywhere…” FQ: Why, what could be the problem? “I cannot focus on the classroom.” FQ: When you discuss with your peers is it helpful? “They help a bit, but the problem is that when I get home I do not practice.”

NMT 21: “Solving x and y specifically calculations, but reasoning and proving I am fine.”

NMT 22: “Proving that angles are similar without coordinates (without given numbers).” FQ: One more? “Proving that angles are similar without…proving that angles are similar while they are not in the same shape or size.”

NMT 25: “Adding gives me a challenge.” FQ: Adding what exactly? “Numbers and angles.”

NMT 30: “Shapes are a challenge they can be the same but not equal, but when you calculate its difficult because when you divide sides, they must be proportional.”

NMT 33: “When I look at them, they are the same but when I calculate they have different answers.” FQ: what do you mean different answers? “I am referring to the different ratios.”

NMT 35: “Calculations are challenging me.” FQ: What specifically are you referring to when you say calculations? “You will be given a number outside the triangle so those numbers outside the triangle give me a challenge.” FQ: What do you think those numbers given outside represent? “Sizes of lines.”

Most of the learners had challenges with calculations, especially when dealing with proportionality or ratio, solving for variables and identifying the problem or what is required. Only one of the learners had challenges of everything based on similar triangles.

Question 4: What methods or strategies were employed by your teacher to best explain similarity? Please explain.

NMT 01: “To work in groups, you showed us examples by drawing triangles…By cutting a paper …you made sort of…what is it called…something like… I forgot.” FQ: How was the paper cut? “You folded the paper…I cannot remember how many triangles were there.”

NMT 09: “We are taught well…the problem it is me...my body it is in the classroom, but my mind is somewhere else.” FQ: What causes that? “A lot of things gallivanting and girls…just a whole lot of things.”

NMT 21: “When you enter the classroom the first time, you do is to introduce the chapter and then teach and ask if there is anything we did not understand.” FQ: How do I introduce the chapter? “I do not know how to explain it.”

NMT 22: "The teacher used some strategies like fun. Explaining alternative angles through the method of fun, he also used strategies saying, "an eye for an eye", meaning what you do must be equal…okay I just forgot…."

NMT 25: "I forgot."

NMT 30: "yes…"

NMT 33: "Group work, explaining to each other." FQ: which concepts did you understand better? "Is that triangles can be the same but differ in sizes."

NMT 35: "He explained what similarities are and an explanation was given…I just do not remember."

Leaners indicates that the method used by the researcher was group work, fun, and the Russian or Happy Face (*see Figure 4.8*). One learner could not remember the method that was used during the intervention. Other learners are unable to describe the method or strategy used. It might mean that other learners do not understand; what is the method?

Question 5: Does the researcher`s design methods improve your understanding of similar triangles and geometry as a whole?

NMT 01: "Yes, it did."

NMT 09: "No…"

NMT 21: "No, I am still blank."

NMT 22: "Yes…It is because the strategies he used make it a bit easy to use."

NMT 25:" I forgot."

NMT 30: "yes…but not all"

NMT 33: "Yes"

NMT 35: "A bit."

Most of the learners indicated that the researcher's method improved their understanding of Euclidean geometry and geometry. Two learners mentioned that they did not conceptualise the designed method. The other learner was sceptical, and the other one forgot the strategies.

6.8 SUMMARY

In summary, the researcher presented data using figures that contained learners' responses, tables that contained information on the learners' performance, and comparisons of results from both the baseline and the post-test. Quotations included learners' responses, which were noted verbatim. Furthermore, data were also analysed using statistical and descriptive methods for quantitative data and subjective analysis for qualitative data.

In the next chapter, the major findings, recommendations, and conclusions of the study will be discussed.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this final chapter, the study's focus was on evaluating the change in Grade 10 learners' comprehension of similar triangles after a developed intervention technique. Because of the statement problems, literature review, and methodology, this chapter summarises the study's findings based on the obtained data. It is hoped that the study's findings will contribute towards solving the national math problem regarding Euclidean geometry, as well as improving the teaching method, particularly in terms of similarity. Moreover, a summary of the findings will be provided, followed by legitimate recommendations, as well as limitations to the study, and then, finally, the drawing of conclusions.

In Chapter 1, the introduction and background of the study, the problem statement from many perspectives including the Van Hieles' theory, the research questions, the purpose and objective of the study, the research design and methodology, the significance and limits, and ethical concerns were briefly explored.

Chapter 2 provided a review of important literature related to the issues in geometry as a mathematical discipline, either directly or indirectly. The purpose of the literature review was to find out what other authors had discovered about geometry learners based on conceptualisation.

In Chapter 3, the researcher described the research strategy and methods. To acquire data, the study used both qualitative and quantitative methods. A baseline test, a planned intervention, a post-test, and semi-structured interviews were used as study instruments. A pilot study was conducted to make it easier for the researchers to identify and use the right instruments that would yield the accurate expected results.

In Chapter 4, data were presented and analysed using statistical, descriptive, and subjective methods pertaining to learners' answers for both the baseline and post-test.

In Chapter 5, the researcher presented the study overview, major findings, recommendations, and summed up with conclusion.

The researcher followed constructivism as a guide, which argues that learners obtain information by creating it for themselves. Under constructivism, the learner is responsible for his or her own education.

5.2 RESEARCH MAJOR FINDINGS

Answers to the primary research question and its sub-questions may be found in the findings of this study.

5.2.1 Major Findings from Research Question 1

Which foundational knowledge and skills are needed as a basis for a thorough conceptual understanding of the similarity of triangles?

In Chapter 2 of the literature review, the researcher discussed the constructivist approach, which demonstrates that learners' cognitive thinking and knowledge are created and enhanced from multiple perspectives. Merriam Webster's online dictionary (*Tuesday, January 2, 2022*) emphasises the importance of knowledge being organised logically. As a result, learners' basic knowledge and abilities must be progressive and rational in nature. The following are some of the findings of aspects or bases that have been discovered in conceptualising understanding:

5.2.1.1 Read with understanding

Learners must know the LOLT adequately. Reading with comprehension will help learners make connections between what they are reading and their existing knowledge. Learners can comprehend and successfully interpret the given texts and integrate them with what they already know. As a result, they can contrast their personal meaning. Their comprehension is strengthened if they are right about this. Learners have to anticipate the examiner and forecast future thoughts and responses to the questions.

5.2.1.2 Improve the ZPD

Learning knowledge requires a learner's ZPD, especially in geometry. To narrow the knowledge gap between learners' past knowledge and anticipated knowledge, new knowledge must continually be created. Knowing what learners already know and what they anticipate learning should always be a priority. When it comes to ZPD, Vygotsky says that it is the difference between a learner's current level of development, as assessed by autonomous problem solving, and their future degree of development, as measured by adult guidance and/or through conjunction with more able peer groups (1979, p.16). Learners who are well-versed in a subject are more likely to interact and participate in ways that promote learning and conceptual similarity.

5.2.1.3 Know geometric concepts

When it comes to mathematics and other subjects, each area has its own set of concepts and language. Geometric concepts and terminology should be well-understood and improved by learners. Morris and Mather (2008) concur that learners must develop conceptual knowledge in mathematics to grasp the concepts that underlie foundational abilities. As a basis for comprehending similarity in Euclidean geometry, the following are some essential terms, language, and concepts that are utilised:

i. ***///*** - similarity sign
ii. AB ***//*** CD **–** line AB is parallel to line CD.
iii. Supplementary angle – add up to 180^0.
iv. Proportional
v. Four types of triangles and their properties e.g., Isosceles, equilateral, and right-angled etc.
vi. Types of quadrilaterals and their properties e.g., Parallelogram, square, and rectangle etc.

Lines, angles, congruence, and similarity are among the basic mathematical concepts that should be understood by learners in Grade 10 as outlined in the ATP and CAPS documents of mathematics. To conceptualise similarity, the following authors: Seago, Jacobs, Heck, Nelson, and Malzahn suggested using ideas of number sense and comparing proportional relationships (2014).

5.2.1.4 Interpretation of statement-to-figure connection

When learning Euclidean geometry, learners must be aware that there are several sorts of information that are provided. It is very important that learners interpret this information correctly and make statements-to-figure connections. The following are the useful types of information:

i. Hidden – is the information that is hidden in terms of mathematical symbols and geometric terminologies.
ii. Graphical – is the information that is represented if the figure or diagram.
iii. Given – is the information that is very clear on the statement without any complications.

Since various sorts of information are all related in some way, the statement must speak the same language as the diagram or figure to be effective. If the learner has a good vocabulary, he or she will be able to conceptualise mathematical symbols and geometric language when creating statements-to-figure connections.

5.2.2 Major Findings from Research Question 2

What is Grade 10 learners' present levels of foundational knowledge and skills?

Grade 10 learners' present level of foundational knowledge is measured by both general perspective and analytical perspective, which is data found from the present study.

5.2.2.1 General and Scientific perspective

Numerous studies have shown that learners in Grade 10 lack the conceptual understanding of concepts, logical reasoning, and connections needed for basic knowledge. Atebe (2008) confirms this when he points out that mathematics needs a higher cognitive capacity and level than anticipated thinking. As the mathematics stream is chosen, the kind of negative attitudes that leaners have towards mathematics and that are increased by brothers and sisters in the community have an impact on the knowledge of Grade 10 learners. Furthermore, Fabiyi (2017) points out that secondary school learners have the perception that geometry and geometric concepts are difficult.

Since 2014, up to date, most learners in the senior and intermediate phases (over 80%) have moved from one grade to the next without passing mathematics. In the exceptional permission for progression, the national assessment circulars allude to the fact that learners may be advanced to the next level notwithstanding their failure in mathematics. The consequences are detrimental to the progressive mathematical understanding and knowledge of learners, and further cause learners to have an enormous amount of content that is not understood. It is also evident in Grade 12 mathematics diagnostic reports (2015 and 2018) that the candidates can not accurately name angles and some of the candidates cannot even identify a diameter. If Grade 12 learners do not have the knowledge of such content, what about Grade 10 learners? It is logical to assume that there are gaps in knowledge all the way down to Grade 7 where these terms should be introduced.

5.2.2.2 Analytical perspective

The analytical perspective presents the findings found from the baseline and observation.

a) Baseline

The study reveals that learners in the baseline test performed in accordance with the knowledge of a learner who is in Grade 7 and below. The results show that the present prior knowledge or fundamental knowledge and skills of learners are not solid and not according to their grade level. Thus, learners are not reasoning logically and according to geometric reasoning or accepted geometric abbreviations, but rather to their level of maturity rather than their grade.

This was shown in cases where learners did not conceptualise the question, some reasoned wrongly or gave reasons for the wrong statement, and others wrote the statements without giving reasons. For example, in the conceptual knowledge section B learning activity, **NMT34** and other learners were unable to understand how the hand of the clock moves, such as “it will point to 11:45” and how, as it moves, certain angles are formed with reference to geometry. As the hands of the clock rotate, forming a revolution in geometry, which is equal to $360°$, and it forms other angles; namely, acute, obtuse, right angle etc., as it moves a quarter, it will form an angle which is a quarter of $360°$, and a quarter of $360°$ is equals to $90°$.

Learners in Grade 10 need to know that plans for buildings are drawn on paper, and later they will be transferred or translated from paper to the building structure. Therefore, learners should know that scale and proportion are relative concepts that are needed in construction. According to CAPS, architecture and civil engineering are other fields where geometry is essential. The scale or ratio on the paper must be accurate, correct, and realistic, and learners should know what it means. The results reveal that those learners lack fundamental knowledge and the ability to link geometry (proportion) to architecture or building plans, as most of the learners were unable to respond to the questions, and of those who responded, only (4) four learners managed to comprehend and correctly answer the question. Learner **NMT30,** for example, responded, “the artist did not measure the right proportions of the building”.

Secondly, it was evident in Section C of multiple choice, where all learners in Grade 10 are expected to know how to name angles, which is the knowledge concept that is introduced and taught in Grade 7. The results of the study show that some learners in Grade 10 were unable to name an angle, which means they lack Grade 7 knowledge and have a geometric content gap. Another aspect was about the conditions of similarity and congruency as a special case, which is also part of the concepts of Grades 8 and 9 knowledge. The results of this study reveal the poor performance of learners in these aspects.

The last question was based on a daily life situation that involved ratio and proportional thinking. Ratio and proportion are also related to the following concepts as well: rates, decimal, fraction, and percentage. The above-mentioned concepts are taught in Grade 4 as stipulated in the mathematics

CAPS document for Grades 4 to 6. This implies that learners in Grade 10 should be mastering those concepts because they are progressive from Grade 4, but the study reveals a different picture in which learners were unable to perform simple multiplication calculations. This question was one of the most difficult, and it was where learners performed the worst.

Finally, in the structured question, which entails Grades 8 and 9 content, all those concepts are evident to learners' fundamental and present knowledge. The parallel line with a transversal line is introduced in Grades 8 and 9 and consists of the basics of geometry, for example, straight line, alternative and vertically opposite angle, just to mention a few. This study indicates that learners can identify other shapes of geometric figures, but their reasoning does not justify the correct statement. Thus, learners use alphabet symbols to conceptualise and learn different types of angles. For example, in the previous chapter, it was discovered that learners can recognise the symbolic alphabets such as "Z" on a geometric figure that is associated with an alternative angle, but they struggle to recognise it when the symbol is skewed or not straight. Then learners make a conclusion and reason why the angles associated with such symbols are equal.

In the baseline, proving similar triangles requires background knowledge of isosceles triangles and their properties. Learners were expected to know and understand the following properties: "an equal side equals opposite angles in a triangle" and the condition of similarity that is relevant to the figure, since it is previous content knowledge. The results indicate that learners cannot translate the given information into mathematical writing or symbols. For example, angle $\hat{E}_2$ is equal to angle $\hat{C}_2$ which simply means, $\hat{E}_2 = \hat{C}_2$. Mathematically, and the reason is also clear, because it is the information that you are given in the statement, therefore the reason must be "given".

The study findings indicate that some learners do not understand the enlargement concepts, which deal with ratio and proportional thinking. Learners did not know what they had to the photo if they had to enlarge it or increase it to a different size. Much the same thing can be said about cases where learners were also expected to show, by means of drawing, that they were given the condition of a two-dimensional (**2D**) figure, the height and length of which formed the rectangular figure. The results reveal that learners drew three-dimensional (**3D**) figures while others drew a figure of a person without dimensions. This concept is general and occurs in real life. Sharma and Bansal (2017, p.209) support the constructivist belief that people learn best through observation and scientific study. This implies that learners are supposed to observe it in their daily lives and learn from it.

The last question under the structured questions was about the midpoint theorem. The study reveals that learners performed the worst on this question. The midpoint theorem is a Grade 10 concept, and learners were given this question to diagnose and classify learners accordingly.

b) Observation

During a designed intervention, it was observed that most of the learners were struggling with the following: LOLT, topic vocabulary, and a lack of proper foundation in the subject. The LOLT is English, and it is the first additional language in the school. Since English FAL is not their home language or their mother language, learners found it difficult to comprehend most of the lessons that are taught daily. It caused learners to have a barrier to learning due to language and vocabulary barriers. During discussions, it was observed that other learners were using their home language (Setswana) to explain certain concepts to their peers.

Since LOLT is a barrier and challenging, it results in learners having difficulty conceptualising geometric terminology, which leads to unsatisfactory limited fundamental skills and knowledge. The vocabulary of this topic is very important and makes it easy for learners to understand geometry. The study reveals that learners are struggling with reasoning using the correct abbreviations according to the accepted abbreviations of geometry. When discussing Figure 4.8, for example, learners point out that "$\angle' s\Delta$" and "ang stri" agreed to give such reasons. It is evident that lack of proficiency in the LOLT and in the geometric vocabulary are some of the contributing factors to learners having limited knowledge.

Based on the above perspective and observation points, one can tell that the present level of foundational knowledge of Grade 10 learners in this group is equivalent to the basic knowledge required of Grade 7 learners and below, that is, if one follows the expectations of the CAPS curriculum.

5.2.3 Major Findings from Research Question 3

What elements, both conceptual and pedagogical, does the researcher include to mediate the understanding of the concept of similarity of triangles in his specific teaching environment?

The study reveals that the following elements are conducive when teaching geometry (similarity of triangles): contextual content and interaction techniques.

5.2.3.1 Contextual content

The content of this topic was built from an early grade. The researcher saw a need to do so by introducing his lesson with the basics from Grade 8. Showing learners how important and useful it is to know how to name an angle and express it in different ways without changing the name of the angle. Then follow up with geometric concepts that are associated with symbols and angles. For example, what is a statement, and for every statement that you make, especially in the beginning, you must give a reason, "//" mean this sign symbolises or means parallel and similarity sign "///", etc.

5.2.3.2 Interaction techniques

The teacher-learner and learner-learner interactions were the techniques used during the designed intervention. Where the teacher demonstrated techniques such as "The Russian or Happy Face" to solve similar triangles with reference to the corresponding (see Figure 4.9), the teacher was making relationships and connections between mental conceptions and existing imaginations. Instructional teaching was one of the elements where the researcher used cooperative learning so that he could play the role of facilitator to guide learners through learning. Interaction between learners was the major element used to make sure that learners understood the concept of the similarity of triangles. During the designed intervention, learners' interaction improved the attitudes and interests of other learners towards learning geometry, from very poor to good. As a result, it promotes willingness and confidence to learn geometry. Learners' interest in geometry is a useful aspect as it is related to many careers.

Furthermore, learners' interaction promotes cooperative learning, which is when learners in a group productively work together. The results reveal that learners understand more when they work in groups rather than in isolation. As a result, group work improves performance, and it does not only end at the school level; it also helps and prepares learners to develop teamwork skills, ethics, and improve communication abilities needed outside of the school level, e.g., in the corporate world. One of the most effective methods used in learning and increasing understanding is group work.

5.2.4 Major Findings from Research Question 4

How does the designed intervention promote the Grade 10 learners' understanding of the similarity triangles?

The study provides clear evidence of the gain of knowledge and skills after the intervention was implemented, demonstrating that the knowledge of the learners was very important to other learners since it employed cooperative learning. The study indicates that the following items increased positively: the first items (20.3%), the second items (2.92%), and the fourth items by 15.2%. Although the third items declined slightly by 2.3%, it was further established that 67.4% (29) of the learners showed an improvement when comparing equivalent items from the baseline and post-tests. There were also positive responses from a proportion of the learners from both the tests and the designed intervention lessons. The additional items also provide evidence with more than nine (9) learners responding positively to similarity questions. Although the mitigating factors were that the language of teaching and learning and the associated geometrical concepts were not well understood, the intervention promoted Grade 10's understanding of similar triangles.

5.2.5 Major Findings from Research Question 5

What changes in understanding from the baseline (prior to the intervention) to the post-test provide evidence for improved understanding?

The study shows that the designed intervention produced educational gains in knowledge and skills related to similar triangles and geometry. It also indicates that in the baseline test, most learners were struggling with the following:

- Writing the correct symbol for an angle
- Interpretation of diagrams
- Writing the correct statement
- Giving the correct reason for the correct statement
- Proofing of two similar triangles

After the designed intervention, learners were given a post-test to assess the effectiveness of the intervention and the understanding of learners through similar triangles and geometry. The following changes are visible in the post-test results and performance, as shown in the figures below:

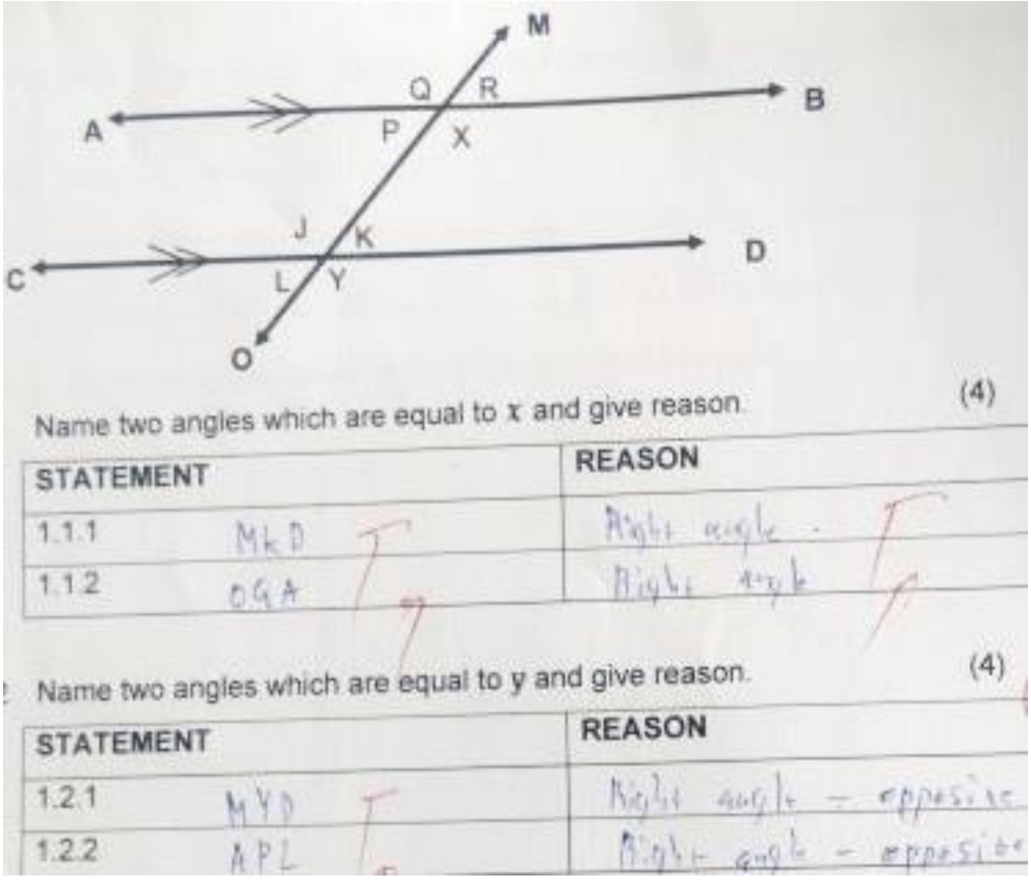


Figure 5.1(a): NMT23 baseline test

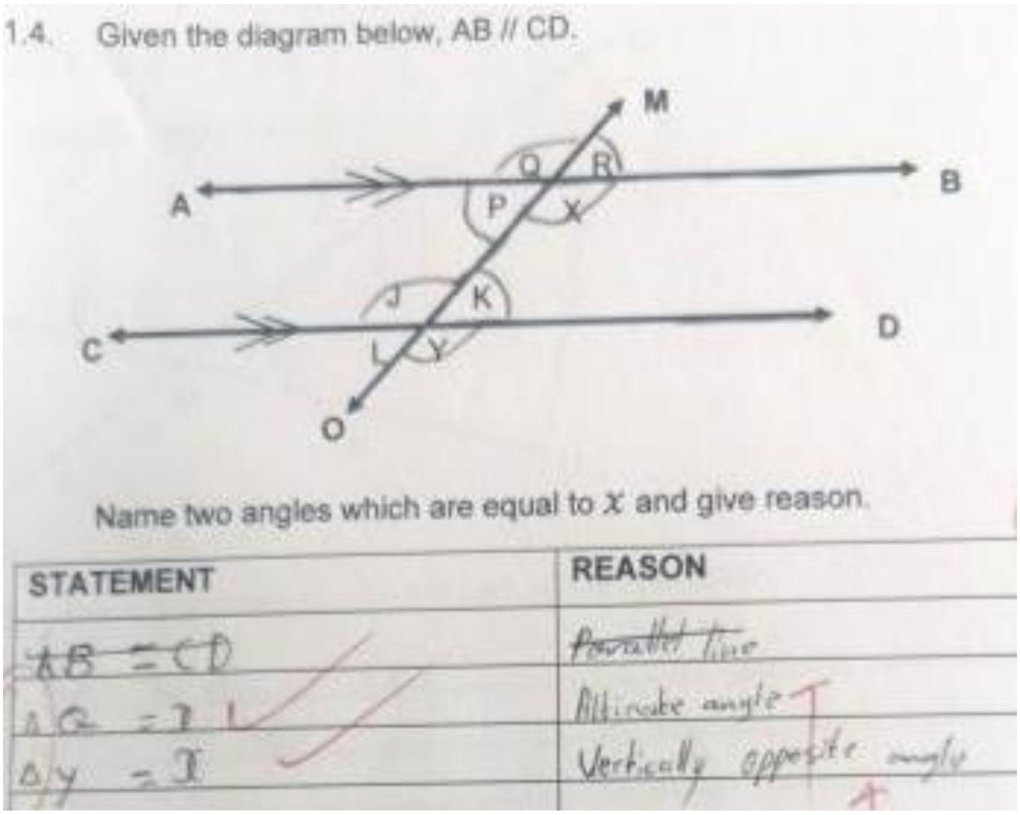


Figure 5.1(b): NMT23 post-test

Figure 5.1. (a) of the baseline shows that a learner was unable to write a correct statement, such as “MKD and DQA”, and instead gave a wrong or incorrect reason that is not even applicable in geometry, such as “right angle opposite”. Figure 5.1 (b) provides evidence for an improved understanding where a learner could now write a correct statement with reasons. Even though his or her reasons are correct, in this case, he or she made a correct statement and substantiated it with an incorrect reason.

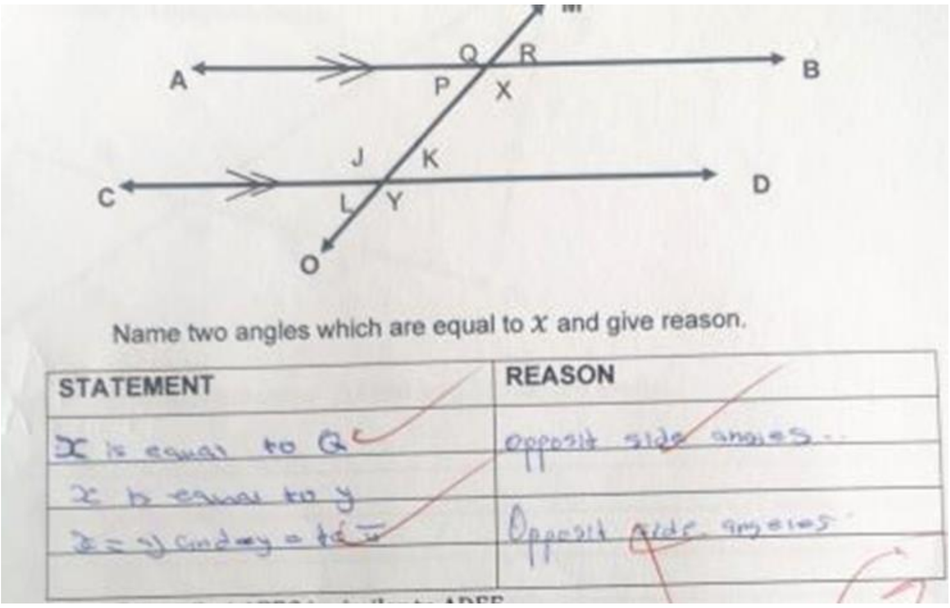


Figure 5.2(a): NMT12 baseline test

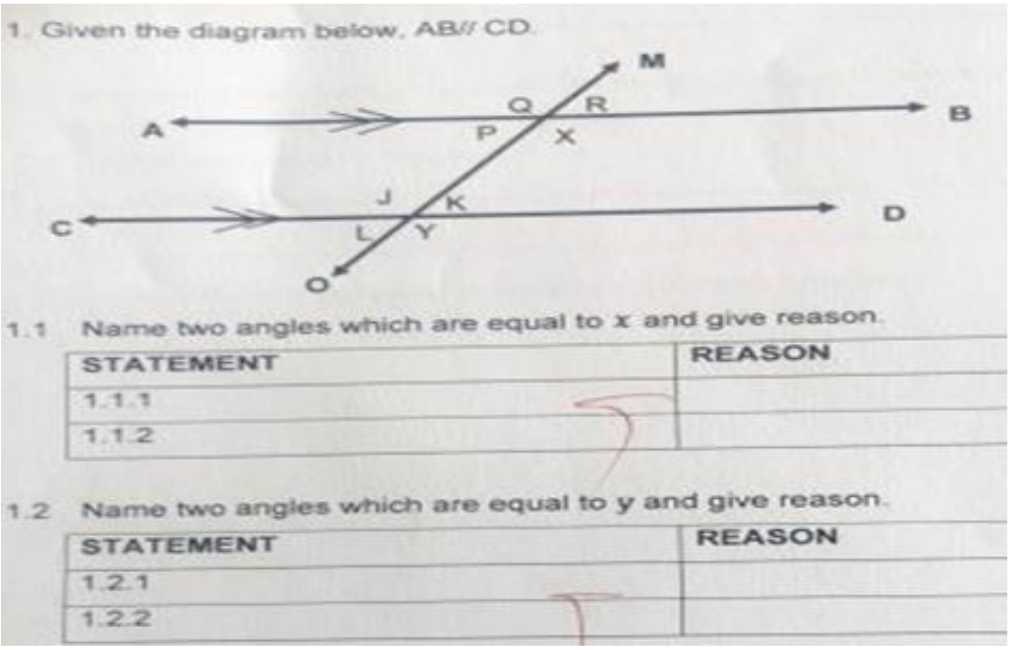


Figure 5.2(b): NMT12 post-test

The figures above provide further evidence where in Figure 5.2 (a), which is a baseline test, a learner could not even interpret the diagram because he or she did not even know what was required by not answering the question. The learners' knowledge and skills improved in Figure 5.2 (b) post-test, where they could write the correct statement and provide the correct reason, but it appears that the learner mastered and understood vertically opposite angles more than other concepts.

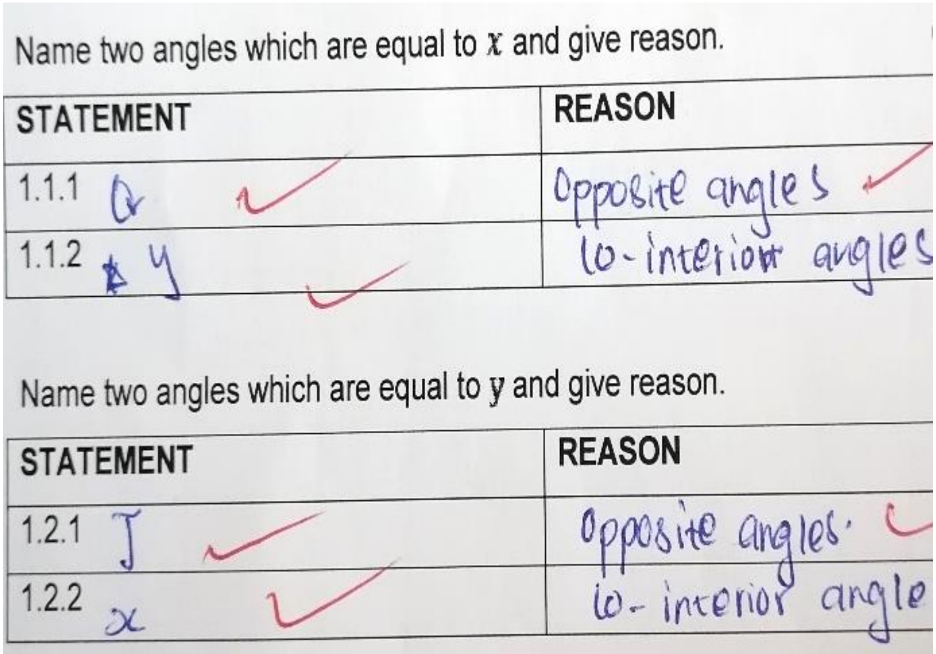


Figure 5.3(a): NMT22 baseline test

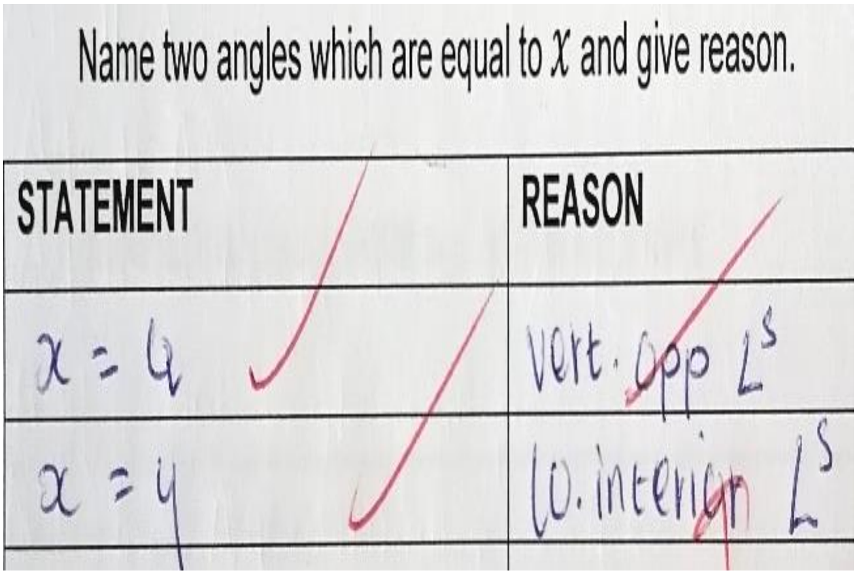


Figure 5.3(b): NMT22 post-test

Furthermore, in Figure 5.3 (a) above, it shows that the learner had fundamental skills and knowledge of geometry. It also indicates that the learner is at the correct grade with the relevant required knowledge of mathematics. Figure 5.3 (b) clearly shows that the learner's reasoning skills and knowledge improved, as shown by the learner's ability to write reasons in an accepted geometry abbreviation, such as angle as "∠" or vertically opposite angles as "$vert.\, opp.\, \angle's$". ***"Yes…,"*** the learners said in response to Question 4 and 5, and gave this explanation "*It is because the strategies he used make it a bit easy to use."* Where he or she elaborated more that ***"****The teacher used some strategies like fun. Explaining alternative angles through the method of fun, he also used strategies saying, "an eye for an eye", meaning what you do must be equal…okay I just forgot…."* These provides additional evidence.

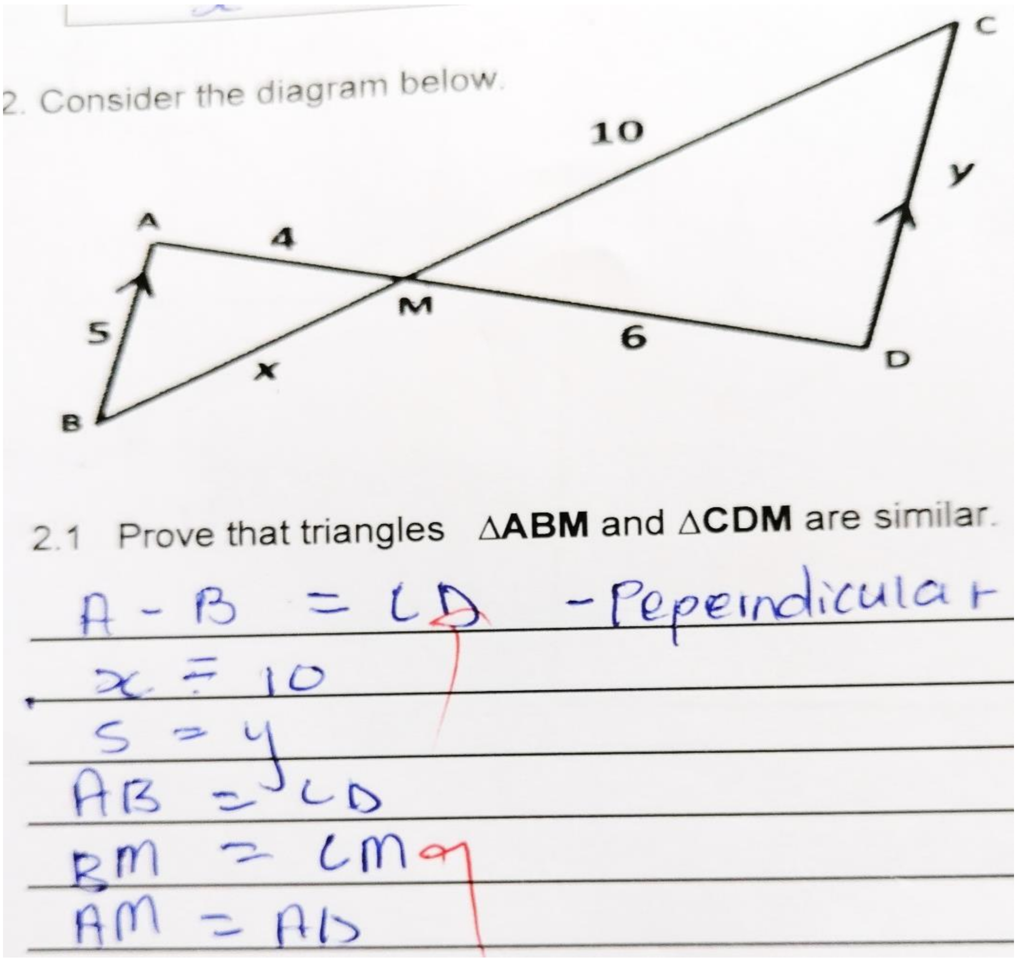


Figure 5.4(a): NMT22 baseline test

In Figure 5.4 (a) above, the learner could not use his or her previous knowledge from Figure 5.3 (a) to solve similar triangles since the diagrams are related but different.

Moreover, the learner did not use the conditions to prove similarity. Figure 5.4 (b) of the post-test below shows that the learner's knowledge improved enough to be able to understand how to write geometric symbols, name or separate angles differently and use conditions of similarity when proving similar triangles. Where the learner showed that EG // DF could also state the correct statement, although he or she did not provide reasons for his or her statements. The learner managed to prove that triangle EFG and triangle DEF are similar by providing 3 angles that are equal and unnecessary, including the side, based on his or her statements, but could not reach the correct conclusion of (AAA) instead of (SAS) due to the unnecessary side.

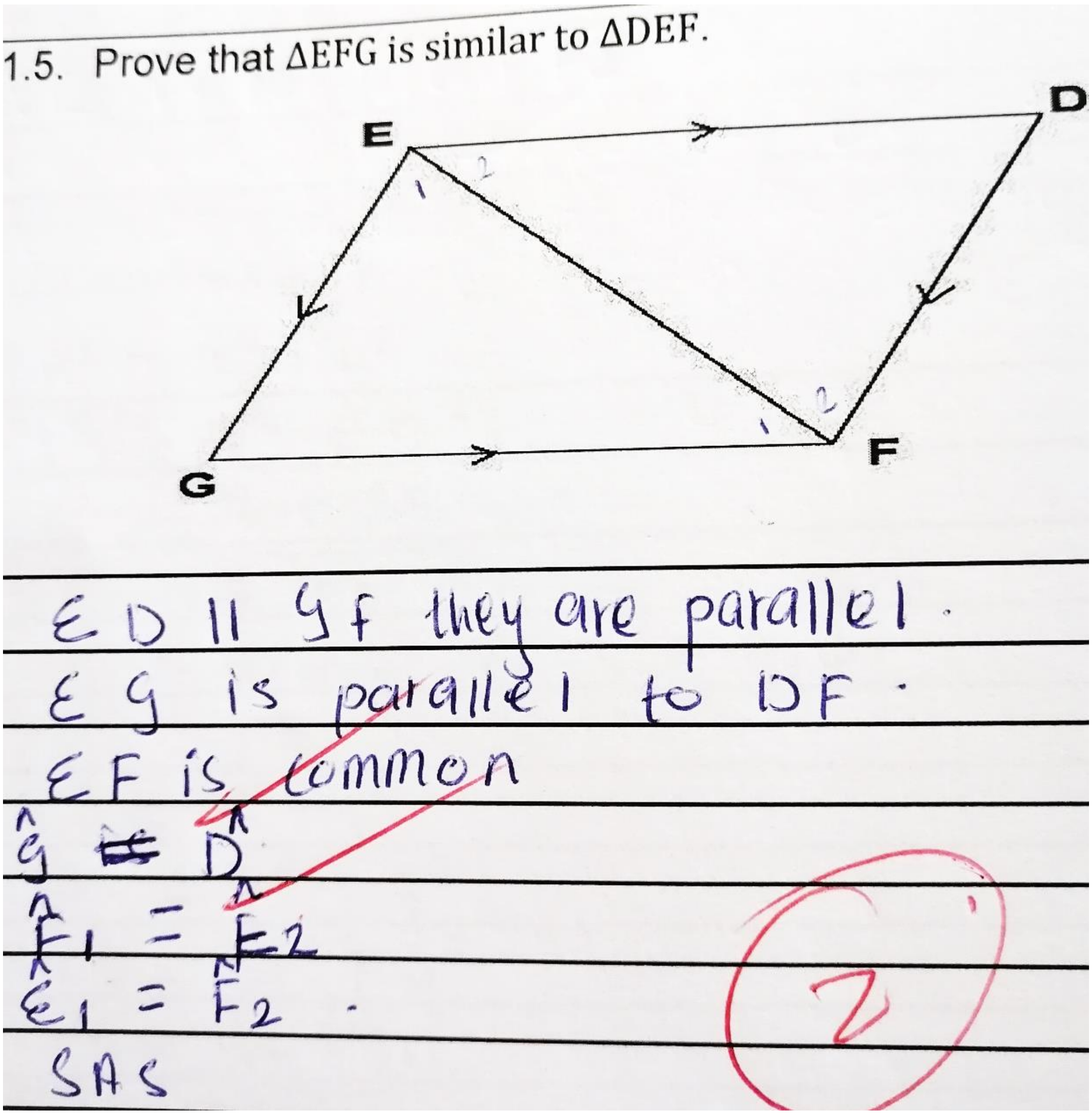


Figure 5.4(b): NMT22 post-test

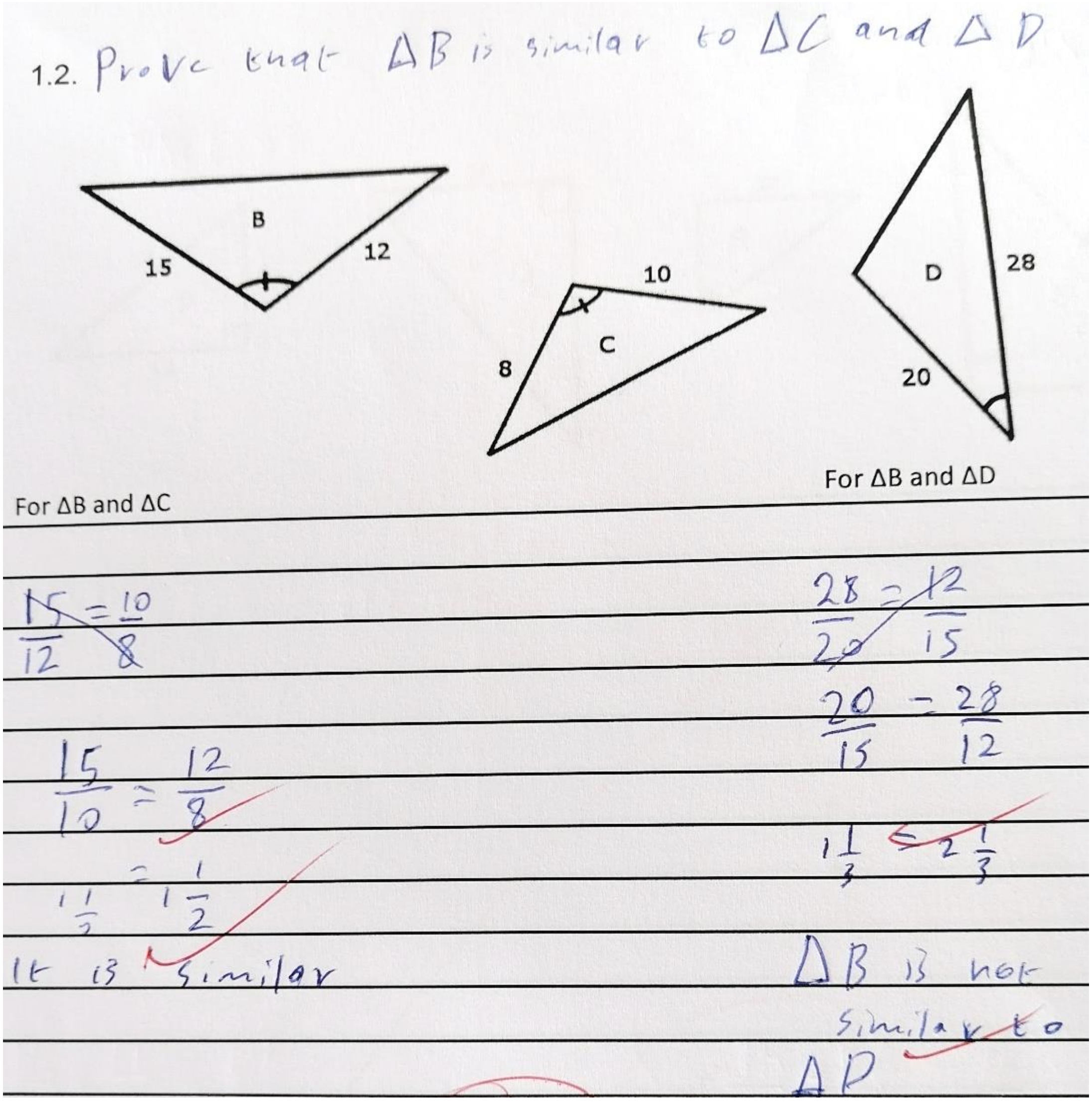


Figure 5.5: NMT36 post-test

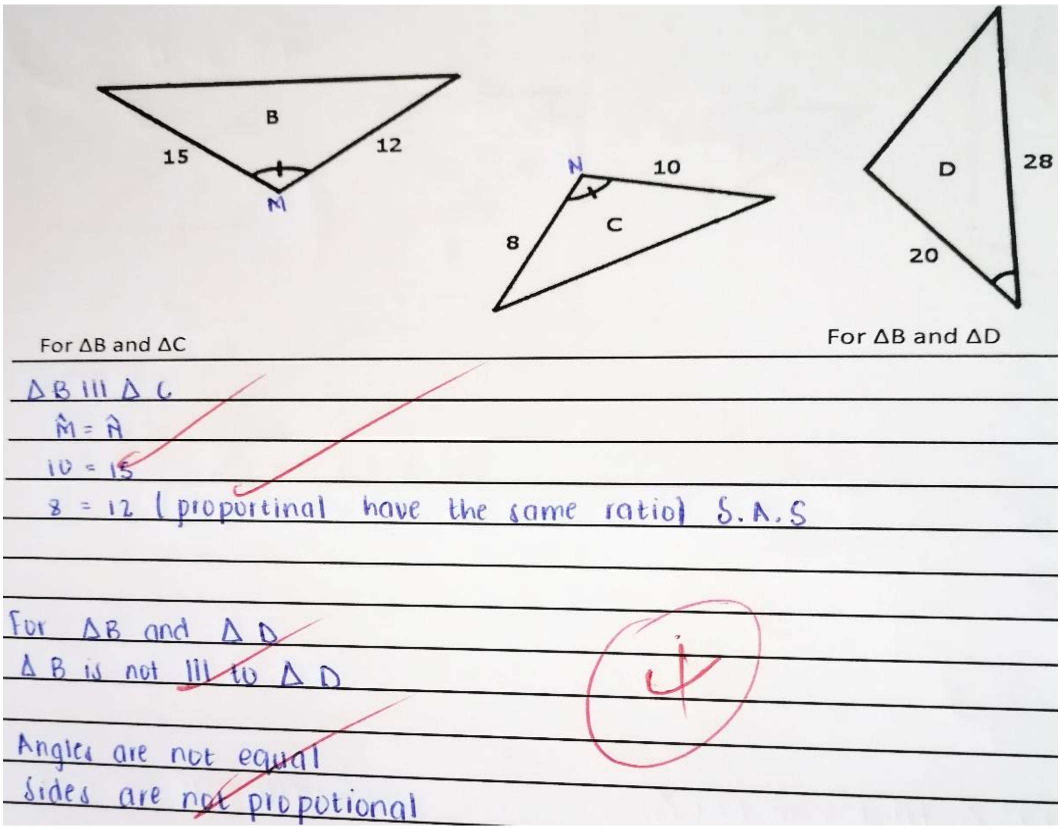


Figure 5.6: NMT01 post-test

In Figures 5.5 and 5.6 above, one could tell that the questions from the post were more challenging than the ones on the baseline (*See Figure 5.4 (a)),* since they were proving similarity for three and four triangles. While using different approaches, **NMT01** and **NMT36** were able to prove that triangles B and C are similar, but B and D are not similar. Both learners interpreted the diagram and identified the corresponding sides. **NMT36** calculated the ratio to verify proportionality, and **NMT01** equated the angles and compared them to the size of their corresponding sides. Both learners made the correct conclusion after using their different methods, but **NMT36** did not use geometric symbols, but **NMT01** used geometric symbols and gave the correct reason, considering similarity conditions “S.A.S”.

Finally, the results of the same items from both the baseline and post-test reveal that the mean mark (average) and percentage of tests are 2.5, with 20.9% and 3.4, with 28.1%, respectively, with 21 (48.8%) of the learners showing improvement in the study after the designed intervention.

5.3 CRITICAL FINDINGS

There are other findings that are critical to this study, which include:

- The researchers’ intervention strategy was effective.
- Learners' geometric knowledge of the similarity of triangles is mostly at the entry level or below.
- Many learners are functioning at a lower level based on van Hiele's theory of geometric thinking.
- Geometric vocabulary and language still remain a challenge.
- The grade 10 mathematics learners have knowledge equivalent to grade 7 learners or below.

5.4 LIMITATIONS OF THE STUDY

There are several limitations to this study, which include:

- Only a few participants were included in the sample.
- The study was only performed at a Secondary School in the Tshwane West District of Gauteng Province. As a result, the findings of this study may not be generalisable to wider populations, as is the case with (designed intervention) teaching. If the study had been expanded to other areas in Gauteng Province, the results may have been quite different. As a result, the study's findings cannot be extrapolated to a larger, regionally based population.
- The study was done in a single school environment, which may have impacted the study's findings. If this had been done in two or more different schools, it would have offered a good image of a designed intervention for improving Grade 10 learners' conceptualisation of similar triangles.
- Because the Ga-rankuwa cluster represents a rural area, the findings are confined to rural perspectives.

5.5 RECOMMENDATIONS

Regardless of its limitations, this study suggests that providing a learning environment in which learners actively interact is favourable to expanding learners' reasoning and conceptualisation in similar triangle geometry. As a result, the following suggestions are made in light of the study's findings:

- To promote effective instructional decisions that will aid learners' geometric knowledge growth, mathematics teachers should use teaching methods that encourage self- and peer interaction.
- Teachers should use the learners' native or home language when teaching mathematics to promote effective self- and peer interaction and understanding of concepts.
- Mathematics teachers should establish a learning environment in which learners may engage with one another while tackling geometry problems.
- The topic of similar triangles should be progressive and across the FET band, to ensure that prior knowledge of similarity is introduced through the band.
- The concepts and skills required for the similarity of triangles, namely proportionality and knowledge of angles and sides of triangles, should be given adequate attention in the prior grades.
- More research should be done on this designed intervention for mathematics teaching and learning.

5.6 CONCLUSION

The enhancement of learner performance is dependent on all stakeholders, since they must all work together to achieve good learner performance in geometry. The result of everyone's commitment will considerably improve student performance in geometry, assuring the development of quality education in South Africa. Improved learner performance will have a significant influence in increasing learners' self–esteem. Geometry learners' performance can be enhanced if teachers educate in an efficient manner.

The study's primary goals have been met. This study was able to evaluate the change or improvement in understanding of similar triangles among Grade 10 learners. This study concludes that learners' basic knowledge should be strengthened and solidified as a potential variable influencing learners' mathematics performance. The intervention of the researcher should be implemented to strengthen and solidify fundamental knowledge and to teach similar triangles in Euclidean geometry. For the past two years, the results have increased by more than 50% in mathematics, 72,7%

in 2018 and 85,7% in 2019 after the intervention of the researcher was implemented in Grade 12.

REFERENCES

Acquah, S. (2011). *Pre-Service teachers' difficulties in learning geometric transformations and perception of factors inhibiting the development of their mathematical knowledge for teaching: A case of two college of education.* University of Education, Winneba. Retrieved from DOI:10.18404/ijemst.78424

Adolphus, T. (2011). *Problems of teaching and learning of geometry in secondary schools in Rivers State,* 1(2), 143 - 152. Nigeria. Retrieved from http://dspace.stir.ac.uk/handle/1893/26189#

Albert Einstein. (n.d.) AZQuotes.com Retrieved July 15, 2021. from AZQuotes.com Website: https://www.azquotes.com/quotes/535129

Alex, J.K., & Mammen, K.J. (2014). An assessment of the readiness of grade 10 students for geometry in the context of curriculum and assessment policy statement (CAPS) expectation*. International Journal of Educational Sciences*, *7*(1), 29–39. https://doi.org/10.1080/09751122.2014.11890167

Alex, J.K., & Mammen, K.J. (2018). Students' understanding of geometry terminology through the lens of Van Hiele theory. *Pythagoras*, *39*(1), a376. https://doi.org/10.4102/pythagoras.v39i1.376

Ally, N., & Christiansen, M. (2013). Opportunities to develop mathematical proficiency in Grade 6 mathematics classroom in Kwazulu-Natal. *Perspectives in Education*, *31*(3), 106-121. Retrieved from: http://perspectives-in-education.com

Alshenqeeti, H. (2014). Interviewing as a data collection method: a critical review. English linguistics research, 3(1), 39-45. Retrieved from http://dx.doi.org/10.5430/elr.v3n1p39.

Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: looking at trustworthiness criteria. *Journal of Emerging Trends in Educational Research and Policy Studies* (JETERAPS*),* *5*(2), 272 - 281. Retrieved from http://www.scholarlinkresearch.com.

Atebe, H.U., 2008. Students van Hiele levels of geometric thought and conception in plane geometry: a collective case study of Nigeria and South Africa.

Unpublished doctoral dissertation. Rhodes University, Grahamstown. South Africa. Retrieved from http://hdl.handle.net/10962/d1003662

Aysen, O. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *International Journal of New Trends in Arts, Sports and Science Education, 1(4), 1-13.* Retrieved from *doi: 10.1016/j.sbspro.2012.09.557*

Bada, S.O & Olusegun, S. (2015). Constructivism learning theory: A paradigm for teaching and learning. *Journal of Research and Method in Education*, *5*(6), 66 - 70. Retrieved from http://dx.doi.org/10.9790/7388-05616670.

Bankov, K. (2013). Teaching of geometry in Bulgaria. *European Journal of Science and Mathematics Education, 1*(3). https://doi.org/10.30935/scimath/

Bless, C., Higson-Smith, C., & Sithole, S. L. (2013). *Fundamental of Social Research Methods an African Perspective* (5th ed.). Cape Town, South Africa: JUTA.

Brownstein, B. (2001). Collaboration: The foundation of learning in the future. *Education*, 122(2).

Bussi, M. G. B., & Frank, A. B. (2015). Geometry in early years: sowing seeds for a mathematical definition of squares and rectangles. *ZDM Mathematics Education, 47*(3), 391-405.

Charreire Petit, S., & Huault, I. (2008). From practice-based knowledge to the practice of research: revisiting constructivist research works on knowledge. *Management Learning*, 39(1), 73 – 91

Chauraya, M., & Brodie, K. (2018). Conversations in a professional learning community: An analysis of teacher learning opportunities in mathematics. *Pythagoras*, *39*(1), 1–9. https://doi.org/10.4102/pythagoras.v39i1.363

Cochrane, R., & McGettigan, A. (2015). *Math centre*. Retrieved 03 31, 2019, Retrieved from http://www.mathcentre.co.uk

Coghlan, D. & Brannick, T. (2005). *Doing action research in your own organisation* (2nd ed.). London: Sage Publication.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research Methods in Education* (5th ed.). London and New York: Routledge Falmer.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research Methods in Education* (6th ed.). London and New York: Routledge.

Creswell, J. W. (2009). *Research Design: Quantitate and Qualitative Approaches.* London: Sage Publishers Ltd.

Creswell, J. W. (2010). *Educational Research: Planning, Conducting and Evaluating Quantitative and Qualitative Research.* New Jersey: Pearson Prentice Hall.

Creswell, J. W. (2012). *Educational Research: Planning, Conducting and Evaluating Quantitative and Qualitative Research (4th ed.).* New Jersey: Pearson Prentice Hall.

Crowley, M. L. (1987). The van Hiele model of the development of geometric thought. *Learning and Teaching Geometry, K-12*, 1-16.

Cunningham, R. F., & Rappa, A. (2016). Survey of mathematics teachers' static and transformational performance and perspectives for teaching similarity. *European Journal of Sciences and Mathematics Education, 4*(4), 440-446. Retrieved from https://eric.ed.gov/?id=EJ1118144.

Dane, A., Cetin, Ö. F., Bas, F., & Sagirli, M. Ö. (2016). A Conceptual and Procedural Research on the Hierarchical Structure of Mathematics Emerging in the Minds of University Students: An Example of Limit-Continuity-Integral-Derivative. *International Journal of Higher Education*, *5*(2), 82-91. Retrieved from https://files.eric.ed.gov/fulltext/EJ1099870.pdf

Data Analysis. (n.d.). Responsible Conduct in Data Management. Retrieved March 2, 2021, from https://ori.hhs.gov/education/products/n_illinois_u/datamanagement/datopic.html#:~:text=Data%20Analysis,and%20recap%2C%20and%20evaluate%20data.&text=An%20essential%20component%20of%20ensuring,appropriate%20analysis%20of%20research%20findings.

De Villiers, M. D. (1997). The role of proof in investigation, computer-based geometry: some personal reflections. (J. King, & D. Schattschneider, Eds.) *Geometry Turned on, Mathematical Association of America*.

De Vos, A. S., Strydom, H., Fouche, C. B., & Delport, R. (2011). *Research at grass roots: for the social sciences and human service professions* (4th ed.). Pretoria: Van Shaik Publishers.

Department: Basic Education South Africa. (2011). *Curriculum and Assessment Policy Statement (CAPS): Further Education and Training. Grades 10-12. Mathematics.* Pretoria, South Africa: Author.

Department: Basic Education South Africa. (2015). *National Assessment Circular 3 of 2015: Mark adjustment for learners in the senior phase (Grades 7-9).* Pretoria, South Africa: Author. Retrieved from https://www.education.gov.za/LinkClick.aspx?fileticket=WAwpa7CJOJY%3D&tabid=587&portalid=0&mid=4485

Department: Basic Education South Africa. (2016). *National Assessment Circular 3 of 2016: Special condonation dispensation for learners in the senior phase (Grades 7-9).* Pretoria, South Africa: Author. Retrieved from https://www.education.gov.za/LinkClick.aspx?fileticket=SVxpisoe170%3D&tabid=587&portalid=0&mid=4484

Department: Basic Education South Africa. (2017). *National Assessment Circular 1 of 2017: Special condonation dispensation for the senior phase (Grades 7-9).* Pretoria, South Africa: Author. Retrieved from https://wcedonline.westerncape.gov.za/circulars/minutes17/CMminutes/DAM/dam14_17.pdf

Department: Basic Education South Africa (2018). *2018 National Diagnostic Report on Learners Performance.* Pretoria: South Africa: Author.

Department of Basic Education. (2019). *National senior certificate 2018. Diagnostic report.* Pretoria: DBE.

Dündar, S., & Gündüz, N. (2017). Justification for the subject of congruence and similarity in the context of daily life and conceptual knowledge. *Journal on Mathematics Education,8(1)*, 35-54. Retrieved from http://dx.doi.org/10.22342/jme.8.1.3256.35-54.

Du Plooy-Cilliers, F., Davis, C., & Bezuidenhout, R.-M. (2014). *Research Matters.* Cape Town, South Africa: Juta.

Ekawati, R., Lin, F.-L., & Yang, K.-L. (2014). Developing an instrument for measuring teachers' mathematics content knowledge ratio and proportion: A case of Indonesian primary teachers. *International journal of sciences and mathematics education*, 1-24. Retrieved from https://link.springer.com.

Fabiyi, T.R. (2017). Geometry concepts in mathematics perceived difficult to learn.

by senior secondary school students in Ekiti State, Nigeria. *Journal of Research & Method in Education, 7*(1), 83 -90. Retrieved from: http://doi.org/10.9790/7388-0701018390

Foldnes, N. (2016). The flipped classroom and cooperative learning: Evidence from a randomised experiment. *Active Learning in Higher Education*, *17*(1), 39-49. Retrieved from: https://journals.sagepub.com/doi/pdf/10.1177/1469787415616726

French, D. (2004). *Teaching and learning geometry.* London: Continuum International Publishing Group.

Fujita, T., & Keith, J. (2003). The place of experimental tasks in geometry teaching: learning from the textbooks design of the early 20th Century. Research in Mathematics Education, *5*(1), 47–62. https://doi.org/10.1080/14794800008520114

Gauteng Department of Education (2016). *2015/2016 Sci-Bono Annual Report*. Johannesburg: South Africa: Author

Goodwind, D., & Webb, M. A. (2014). Comparing teachers' paradigms with the teaching and learning paradigm of their state`s teacher evaluation system. *Research in Higher Education Journal, 25*, 1-11. Retrieved from https://eric.ed.gov/?id=EJ1055341.

Gunhan, B. C. (2014). A case study on the investigation of reasoning skill in geometry. *South Africa Journal of Education, 34*(2), 1-19. Retrieved from http://www.sajournalofeducation.co.za

Halse, J., & Boffi, L. (2014). Design interventions a form of inquiry. Retrieved from http://scholar.google.com.

Human, S., & Karen, P. (2016). An exploration of streets as social spaces as informative for urban planning and design. *Challenges of modern technology*, 7(4), 11-27. http://doi.org/10.5604/01.3001.0010.8785

Idris, N., & Tay, B.L. (2004). Teaching and learning of geometry: problems and prospect. *Masalah Pendidikan Jilid, 27*, 165 - 178. Retrieved from http://scholar.google.com.

Jones, K. (2002). Issues in the teaching and learning of Geometry. In L. Haggarty (Ed). *Aspects of Teaching Secondary Mathematics: perspective on practice* (pp. 121 - 139). London: Routledge Falmer.

Jones, K., & Tzekaki, M. (2016). Research on the teaching and learning of geometry. In A. Gutiérrez, G. Leder & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education: The Journey Continues* (pp. 109-149). Rotterdam: Sense. Retrieved from http://brill.com/view/book/edcoll.

Jupp, V. (2006). *The SAGE Dictionary of Social Research Methods.* London: SAGE Publication.

Kamii, C. (2012). Elapsed time: Why is it so difficult to teach. *Journal for Research in Mathematics Education, 43*, 296-315.

Kepceoglu, I. (2018). *Strategies of constructing shapes in Cabri.* Kastamonu University, Department of Sciences and Mathematics Education, 8(4), 1-8. Turkey: Canadian Centre of Science and Education. Retrieved from https://eric.ed.gov/?id=EJ1189598.

Kumar, R. (2011). *Research methodology a step-by-step guide for beginners.* Los Angeles: London: Sage.

Leedy, P. D., & Ormond, J. E. (2001). *Practical research planning and design* (7th ed.). Merrill Prentice-Hall, Upper Saddle River, New Jersey Columbus: Ohio.

Lincoln, Y. S., & Guba, E. G., (1985). Naturalistic inquiry (pp. 289 – 331). Beverly Hills: Sage.

Luneta, K. (2013). *Teaching Elementary Mathematics. Learning to teach elementary mathematics through mentorship and professional development.* Saarbruken: LAP Lambert Academic Publishing GmbH & Co. KG.

Luneta, K. (2015). Understanding students' misconceptions: An analysis of final Grade 12 examinations in geometry. *Pythagoras*, *36*(1), art261. https://doi.org/10.4102/Pythagoras.v36i1.261

Lutz, S., & Huitt, W. (2004). Connecting cognitive development and constructivism: Implication from theory for instruction and assessment. *Constructivism in the Human Sciences, 9*(1), 67-90. Retrieved from http://scholar.google.com

Mabotja, S., Chuene, K., Maoto, S., & Kibirige, I. (2018). Tracking Grade 10 students' geometric reasoning through folding back. *Pythagoras*, *39*(1), a371. https://doi.org/10.4102/pythagoras.v39i1.371

Mamali, N. R. (2015). *Enhancement of learner's performance in geometry at secondary schools in Vhembe District of the Limpopo Province* (Doctoral dissertation). Retrieved from http://scholar.google.com

Marshall, C. & Rossman, G.B., (1989). *Designing qualitative research*. Newbury Park, CA: Sage.

Mashingaidze, S. (2012). The Teaching of Geometric (Isometric) Transformations at Secondary School Level: What Approach to Use and Why? *Asian Social Science, 8*(15), 197-210. https://doi.org/10.5539/ass.v8n15p197

Maziriri, E. T., & Madinga, N. W. (2016). A qualitative study on the challenges faced by entrepreneurs living with physical disabilities within Sebokeng Township of South Africa. *International Journal of Research*, 1.

McCraig, C., & Dahlberg, L. (2010). *Practical research and evaluation: A start-to-finish guide for practitioners.* London: Sage.

Merriam, S. B., (1991). Case study research in education: A qualitative approach (2nd ed.). San Francisco: Jossey-Bass Publishers.

Merriam-Webster. (n.d.). Logical. In Merriam-Webster.com dictionary. Retrieved January 2, 2022, from https://www.merriam-webster.com/dictionary/logical

Misnasanti, Utami, R. W., & Suwanto, F. R. (2017). Problem based learning to improve proportional reasoning of students in mathematics learning. *The 4th international conference on research, implementation and education of mathematics and sciences* (p. 8). Yogyakarta, Indonesia: AIP Publishing.

Morris, R. J., & Mather, N. (Eds.). (2008). *Evidence-based interventions for students with learning and behavioural challenges*. Routledge.

Muttaqin, H., Putri, R. I., & Somakin. (2017). Design research on ratio and proportion learning by using ratio table and graph with Oku Timur context at the 7th grade. *Journal on mathematics education, 8*(2), 211-222. Retrieved from https://eric.ed.gov/?id=EJ1150229.

Ngirishi, H., & Bansilal, S. (2019). An exploration of high school learners' understanding of geometric concepts. *Problems of Education in the 21st Century, 77*(1), 82–96. https://doi.org/10.33225/pec/19.77.82

Paulina, M. M. (2007). *Perspectives on the teaching of geometry for the 21st century.* Dordrecht: Kluwer.

Sari, W. P., & Haji, S. (2021). Improving conceptual understanding through inquiry learning by using a jigsaw method in abstract algebra subject. In *Journal of Physics: Conference Series* (Vol. 1731, No. 1, p. 012052). IOP Publishing. Retrieved from http://scholar.google.com.

Sanwidi, A., & Swastika, G. T. (2018). Lectora Inspire in Learning Congruence Triangle in Higher Education. *Journal IImiah Pendidikan Matematika (JIPM), 7*(1), 66-73. Retrieved from http://e-journal.unipma.ac.id/

Schneider, M., & Stern, E. (2010). The developmental relations between conceptual and procedural knowledge: A multimethod approach. *Developmental Psychology*, *46*(1), 178.

Seago, N. M., Jacobs, J. K., Heck, D. J., Nelson, C. L., & Malzahn, K. A. (2014). Impacting teachers understanding of geometric similarity: results from field testing of the learning and teaching Geometry Professional development materials. *Professional Development in Education, 40*(4), 627-653. Retrieved from http://www.tandfonline.com/doi/abs/10.1080.

Semple, J. G., & Kneebone, G. T. (1959). *Algebraic curves.* Oxford: Clarendon Press.

Seroto, N. (2006). Exploration of geometrical concepts involved in the traditional circular building and their relationship to classroom learning. *Masters Dissertation*.

Singh, R.I. (2006). *An investigation into learner understanding of the properties of selected quadrilaterals using manipulatives in a grade eight mathematics classes.* Unpublished masters' thesis, University of KwaZulu-Natal, Durban, South Africa.

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. *Journal of Mathematics Teacher Education, 16*, 245–268

Stols, G., Long, C., & Dunne, T. (2015). An application of the Rasch measurement theory to an assessment of geometric thinking levels. *African Journal of Research in Mathematics, Science and Technology Education*, *19*(1), 69-81.

Sunzuma, G., Masocha, M., & Zezekwa, N. (2012). Secondary school students' attitudes towards their learning of geometry: A survey of Bindura urban secondary schools. *Greener Journal of Educational Research, 3*(8), 402- 410. https://doi.org/10.15580/GJER.2013.8.051513614

Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24. https://doi.org/10.1007/BF03217474

Tracy, S. J. (2010). Qualitative quality: Eight "big-tent" criteria for excellent qualitative research. Qualitative inquiry, 16(10), 837-851. Retrieved from http://scholar.google.com.

Uduosoro, U.J. (2011). Perceived and actual learning difficulties of students in secondary school mathematics. *International Multidisciplinary Journal, Ethiopia, 5 (5), 357-366.*

Usiskin, Z. (2002). Teachers need a special type of content knowledge. *ENC Focus, 9*(3), 14–15.

Van Putten, S., Stols, G., & Howie, S.J. (2010). Making Euclidean geometry compulsory: Are we prepared? *Perspectives in Education, 28*(4), 22–31.

Wang, Z., Wang, Z., & An, S. (2018, July). Error analysis of 8th graders` reasoning and proof of congruent triangles in China. *Journal of Mathematics Education, 11(2)*, 85-120. Retrieved from https://www.educationforatoz.com

Weegar, M. A., & Pacis, D. (2012, January). A Comparison of two theories of learning-behaviourism and constructivism as applied to face-to-face and online learning. In *Proceedings e-leader conference, Manila*.

Van de Walle, J. A. (2006). *Elementary and middle school mathematics: Teaching developmentally*. New York: Pearson Education

Van Hiele, P. M. (1986). Structure and insight: A theory of mathematics education. Orlando Fla: Academic Press.

EDITORS CERTIFICATE

EDITORIAL
CERTIFICATE

Author: Mr Amokelo Given Maweya

Document title: EVALUATING GRADE 10 LEARNERS' CHANGE IN UNDERSTANDING OF SIMILAR TRIANGLES FOLLOWING A CLASSROOM INTERVENTION

Date issued: 06/04/2022

This document certifies that the above manuscript was proofread and edited by Prof Gift Mheta (PhD, Linguistics).

The document was edited for proper English language, grammar, punctuation, spelling and overall style. The editor endeavoured to ensure that the author's intended meaning was not altered during the review. All amendments were tracked with the Microsoft Word "Track Changes" feature. Therefore, the authors had the option to reject or accept each change individually.

Kind regards

Prof Gift Mheta (Cell: 073 954 8913)

INFORMED CONSENT LETTER TO THE DEPARTMENT OF EDUCATION

1192 Block FF
Soshanguve
0152

04 April 2020

The Head of Education
Gauteng Department of Education
Private Bag X895
Pretoria
0001

Dear Sir/ Madam

I, **AMOKELO GIVEN MAWEYA (**STUDENT NUMBER**: 55232558),** from the School of Education at the University of South Africa (UNISA) and currently registered for a Master's in Education with a specialisation in Mathematics, hereby request permission to conduct a research study with the learners in one of the schools in the Tshwane West district (D15). The title of the research is: **EVALUATING GRADE 10 LEARNERS' CHANGE IN UNDERSTANDING OF SIMILAR TRIANGLES FOLLOWING A CLASSROOM INTERVENTION.** The university, Unisa, has accepted the proposal.

The purpose of the research is to implement a teaching design that aims to change how learners perceive geometry, that is develop a deeper understanding of the principles, and thereby improve learners' performance in mathematics in this school, and ultimately in the Tshwane West district. The research aims to improve the quality of learning geometry in mathematics, which will eventually benefit the school and the district.

This study will employ both quantitative and qualitative methods to collect data from the learners. The ethics policy of the university requires the Department of Education to grant me permission to conduct research in this school. This ethical procedure is to protect all participants' anonymity and keep their information confidential and safe. A copy of this study will be made available once it is completed.

Thanks in advance for reading this letter. Should you require any further information, feel free to contact my supervisor using the information below.

Yours faithfully

Mr. Maweya Amokelo

072 261 5345

E-mail: amokelogiven@yahoo.com

or 55232558@mylife.unisa.ac.za

INFORMED CONSENT LETTER TO THE PRINCIPAL OF THE SCHOOL

1192 Block FF

Soshanguve

0152

04 April 2020

4221 Motsuminyani Str.
Zone 3
Ga-rankuwa
0208

To the Principal

Dear Sir/ Madam

REQUESTING PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

I, **AMOKELO GIVEN MAWEYA (**STUDENT NUMBER**: 55232558),** from the School of Education at the University of South Africa (UNISA) and currently registered for a master's in education with a specialisation in mathematics, request permission to conduct a research study with the learners at your institution. The title of the research is: **EVALUATING GRADE 10 LEARNERS' CHANGE IN UNDERSTANDING OF SIMILAR TRIANGLES FOLLOWING A CLASSROOM INTERVENTION**. The university, UNISA, has accepted the proposal.

The Grade 10 mathematics learners who are currently struggling with mathematics, especially Euclidean geometry, will be asked to participate in the study that has the following phases: baseline test, lesson presentation (intervention) by the researcher, post-test, and interview for selected learners during May 2020. Since the research will be conducted during school hours, and is aligned with the curriculum, it will not affect and disrupt learners' learning in any way. The purpose of the teaching design is to implement a program which will attempt to deepen learners' understanding of geometry and thereby change their perceptions of geometry. It is anticipated that this intervention will improve learners' performance in mathematics in this school, and ultimately in the Tshwane West district. Insights gained by me, the teacher researcher, will be reported. Also, reflections on the improvement of the intervention will be reported.

Participation of the learners is completely voluntary. Participants are not obliged to participate in this study and may feel free to withdraw at any point of the study without any penalty. There is no compensation for participating in this research. The anonymity and confidentiality of the learners' responses will be respected. The confidential information will be kept in a separate file (Google Drive) and kept safe in the locker.

A brief summary of the findings and recommendations will be shared with the learners and other participants upon completion of the research. The findings might also be presented at academic conferences and published in academic journals.

Thanks in advance for reading this letter. Should you require any further information, feel free to contact my supervisor using the information below.

Yours faithfully

Mr. Maweya Amokelo

072 261 5345

E-mail: amokelogiven@yahoo.com

or 55232558@mylife.unisa.ac.za

INFORMED CONSENT LETTER TO THE PARENTS/ GUARDIANS

Dear Parent/ Guardian

REQUESTING PERMISSION FOR YOUR CHILD TO PARTICIPATE IN THE RESEARCH

I, **AMOKELO GIVEN MAWEYA,** am a teacher at a Secondary School. I am currently doing my master's in education with a specialisation in mathematics at the University of South Africa. I am conducting a research study with the following title: **EVALUATING GRADE 10 LEARNERS' CHANGE IN UNDERSTANDING OF SIMILAR TRIANGLES FOLLOWING A CLASSROOM INTERVENTION.** The university has approved the proposal. Mathematics learners in Grade 10 at NM Tsuene Secondary School are selected to participate in the study, and your child is one of them.

The purpose of the research is to implement change in how learners perceive geometry and improve learners' performance in mathematics in your school and Tshwane West district. The research will improve the quality of learning geometry in mathematics, which will eventually benefit your child. During the study, your child will be asked to participate in the following phases: writing a baseline test, participating in the lessons (intervention) by the researcher, writing a post-test, and taking part in an interview. The interview will be recorded and be saved in a computer or cloud (Google Drive), which is password protected. Your child is not obliged to participate in this study, and should your child participate, he or she is free to withdraw at any point of the study without any penalty.

If you decide that your child will not be part of or participate in the study, the researcher will request that your child to be in a position where he or she is not disadvantaged at all in the lesson and will not write the pre and post-test. He or she will be attending the same lesson as the others, but his or her work will not be considered in the study. The anonymity and confidentiality of your child's response will be protected. I therefore request permission from you as the parent or guardian to work with your child in the research study.

If you have any questions that require clarity, feel free to ask at any time using any mode of communication.

Thank you in advance.

Yours faithfully,

Mr. Maweya Amokelo

072 261 5345

E-mail: amokelogiven@yahoo.com

or 55232558@mylife.unisa.ac.za

INFORMED CONSENT LETTER TO GRADE 10 LEARNERS

Dear Learner

<u>REQUESTING YOUR PERMISSION TO PARTICIPATE IN THE RESEARCH</u>

I am conducting research on Euclidean geometry, in particular the topic of similarity, with Grade 10 learners as part of my master's degree at the University of South Africa (Unisa). The Department of Education, together with the school principal, has given me permission to conduct this study at school, and I would be very glad and humbled if you agreed to participate. The researcher intends to use his designed interventions to teach with the aim of enhancing and improving the concept of geometric knowledge, focusing more on similar triangles. This will eventually shift their mindset about how they perceive geometry and improve their performance in mathematics at school.

The purpose of this letter is to explain to you how the study is planned and what I would like you to do. If there is any part of this letter that needs clarity, you can ask any teacher or any other adult to explain it to you. Your guardians or parents are also being sent a letter to ask their permission for you to participate in this research. It is advisable that you involve your parents or guardian when deciding to take part in this research.

If you agree to participate, you will be asked to write a baseline test, attend the classes (the intervention) that will be happening during school time for about a week, and write a post-test. You may be one of the selected learners that will be interviewed. Your identity will be anonymous as you will not be writing your name. The information that is presented will also be confidential. There is no harm or potential risk during the research, as it follows a normal class routine. There are no rewards or compensation if you participate, and no penalties if you decide not to participate in this research. Participation in this research is completely voluntary; you are not obliged to participate in this study and may feel free to withdraw at any point of the study without any penalty or punishment.

If you have any questions that require clarity, feel free to ask at any time using any mode of communication.

Yours faithfully

Mr. Maweya Amokelo

072 261 5345

E-mail: amokelogiven@yahoo.com or 55232558@mylife.unisa.ac.za

BASELINE TEST FOR LEARNERS'

Learner Code: ______________________ **Date: MAY 2020**

Please remember that the answers you will provide on this baseline test will be treated as confidential. Please use the provided space to answer the following questions.

Answer all questions.

SECTION A: Demographic information

1. Gender:

M	**F**

2. Age group:

14 and below	15 - 17	18 and above

3. Did you pass mathematics in Grade 9 (state if you are repeating the grade)? What was the level of your mathematics in Grade 9?

____

____

4. Why are you continuing with mathematics in Grade 10? Give one reason.

____

____

5. What aspects of mathematics do you enjoy?

____

____

SECTION B: Conceptual knowledge (pre-knowledge, knowledge foundations)

1. The hour hand clock moves a quarter way from 12 o'clock. On which number will it now point to? Through what angle will it have moved? (2)

____

____

2. An artist did a drawing of the Union Buildings in Pretoria. An observer said that the artist got the proportions wrong. What does he mean? (2)

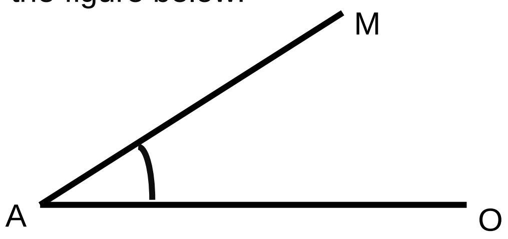
____

____

SECTION C: Multiple choice **<u>5 Marks</u>**

In the following question circle the correct answer.

1. Given the figure below:



Which of the following is correct when we are referring to an angle?

A. $M\hat{A}O$

B. $A\hat{M}O$

C. $M\hat{O}A$

D. All of (A - C) is correct

E. None of (A - C) is correct

2. If two figures ΔAMO and ΔGIV are similar but not congruent then ….

A. The bases and heights of the respective triangles are equal in length.

B. The heights of the respective triangles are equal in length.

C. The corresponding bases are aligned horizontally.

D. The respective triangles have the same shape, and the lengths are equal.

E. The respective triangles have sides of different lengths but the same shape.

3. Which one of the following options is **NOT** a property or condition of similarity?

   A. $\angle, \angle, \angle$

   B. S, S, S

   C. $90°, \mathrm{H}, \mathrm{S}$

   D. $\mathrm{S}, \mathrm{A}, \mathrm{S}$

   E. $\angle, \angle, \mathrm{S}$

4. Given the following triangles $\Delta\mathrm{PQR}$ and $\Delta\mathrm{XYZ}$:

   Statement I: their corresponding angles are equal.

   Statement II: their corresponding sides are in proportion.

   Which option is correct?

   A. Both **I** and **II** are true.

   B. Both **I** and **II** are false.

   C. If **I** is true, then **II** is false.

   D. If **II** is true, then **I** false.

   E. All (A – D) are true.

5. The price of petrol is increasing rapidly and decreasing slowly; the estimated price of petrol per litre at midyear in 2020 could be R28,50 per litre. Your parents fill 15 litres of petrol at a price of R427,50. The increase rate of price can be expressed in many ways.
   Which of the following is correct?

   A. 3.5%

   B. $\frac{2}{57}$

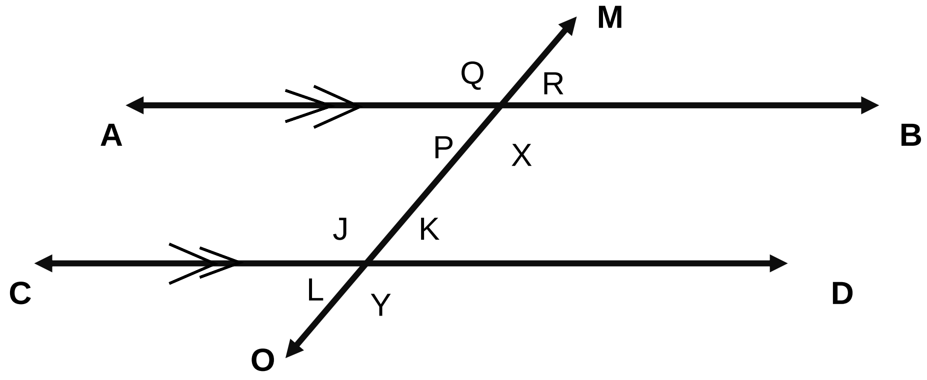
   C. 228: 8

   D. All of (A - C) is correct.

   E. None of (A - C) is correct.

SECTION D: Structured question

1. Given the diagram below, AB// CD.



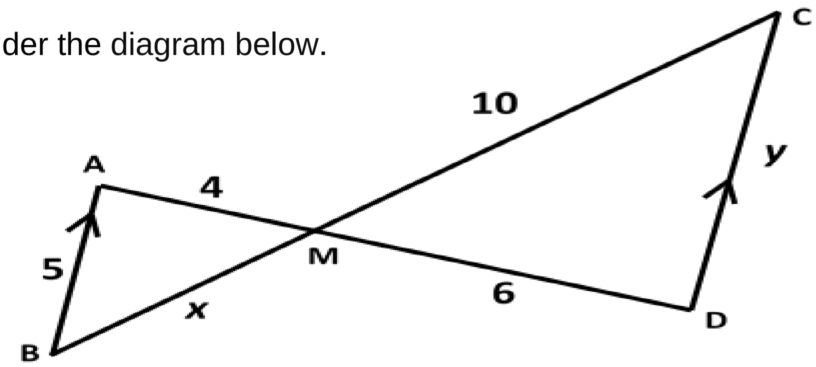
1.1 Name two angles which are equal to x and give a reason. (4)

STATEMENT	**REASON**
1.1.1	
1.1.2	

1.2 Name two angles which are equal to y and give a reason. (4)

STATEMENT	**REASON**
1.2.1	
1.2.2	

2. Consider the diagram below.



2.1 Prove that triangles Δ**ABM** and Δ**CDM** are similar. (4)

___

___

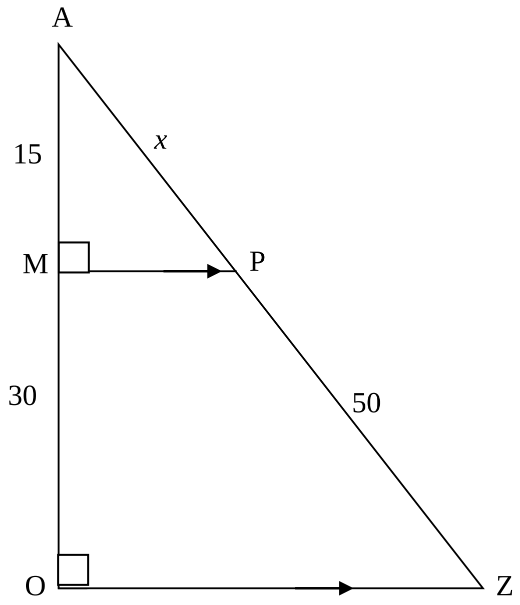
___

3. You have original photo of yourself with the length equal to 3cm and height equal to 6cm. You want to enlarge the size of your photo.

   3.1 What can you do to the photo? (1)

   3.2 Draw the original photo and the one you have enlarged. (4)

4. In the diagram below, MP // OZ.



4.1 Which triangle is similar to Δ AOZ and give reason? (2)

4.2 Determine the value of x. (2)

ADDITIONAL SPACE

TOTAL MARKS [30]

!!!!!!!!!!!!!!!!!!THANKS FOR PARTICIPATING IN THIS STUDY!!!!!!!!!!!!!!!

MEMO BASELINE TEST FOR LEARNERS'

Learner Code: ______________________ **Date: MAY 2020**

Please remember that the answers you will provide on this baseline test will be treated with confidence. Please use the provided space to answer the following questions.

SECTION A: Geographical background

Responses depend on individual background and needs.

SECTION B: Conceptual knowledge (Pre-knowledge, Knowledge foundations)

1. 12H15 or Three (3). √ 90° OR RIGHT ANGLE. √
2. It means the corresponding side of the union building and the drawing are having different proportion and ratio. √√

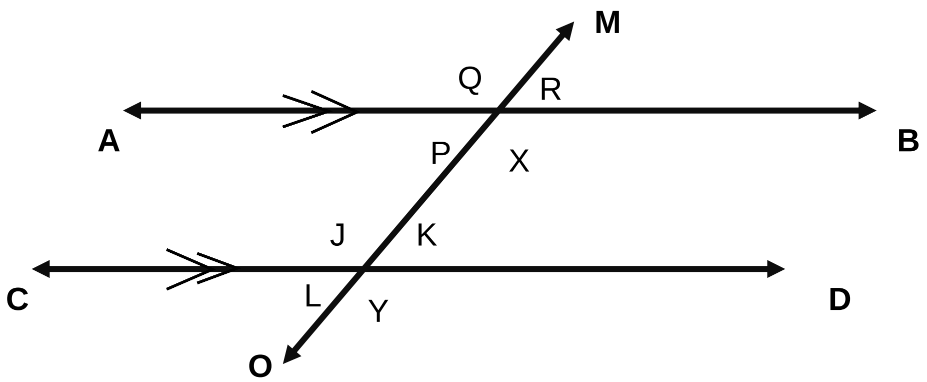
SECTION C: Multiple choice

In the following question circle the correct answer.

1. A √
2. E √
3. E √
4. A √
5. D √

SECTION D: Structured question

1. Given the diagram below, AB// CD.



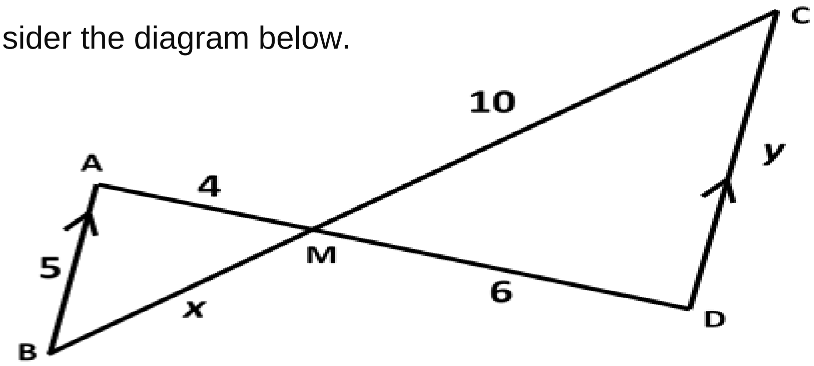
1.1 Name two angles which are equal to x and give reason. (4)

STATEMENT	REASON
1.1.1 $x = \hat{G}_1$ √	vertically opposite angle. √
1.1.2 $x = y$ √	corresponding angle. √

1.2 Name two angles which are equal to y and give reason. (4)

STATEMENT	REASON
1.2.1 $y = \hat{V}_1$ √	vertically opposite angle. √
1.2.2 $x = y$ √	corresponding angle. √

2. Consider the diagram below.



Prove that triangles Δ**ABM** and Δ**CDM** are similar. (4)

$\hat{A} = \hat{D}$ $(alt. \angle`s\ PQ//RS)$ $\sqrt{}\ S/R$

$\hat{B} = \hat{C}$ $(alt. \angle`s\ PQ//RS)$ $\sqrt{}\ S/R$

$\hat{M} = \hat{M}$ $(Vert. opp.\ \angle`s\ )$ $\sqrt{}\ S/R$

$\therefore \Delta ABM ||| \Delta DCM$ (AAA) √

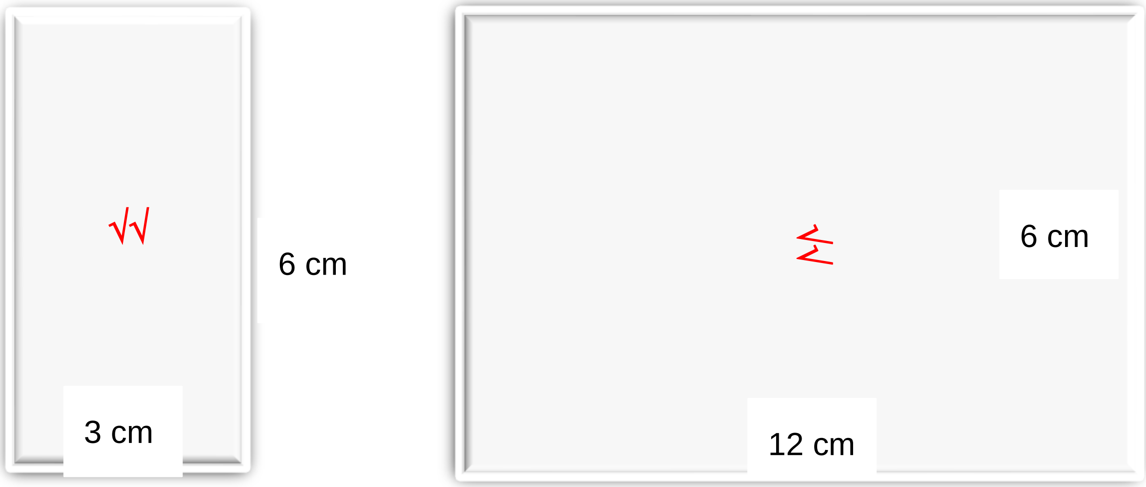
3. You have original photo of yourself with the length equal to 3cm and height equal to 6cm. You want to enlarge the size of your photo.

   3.3 What can you do to the photo? (1)

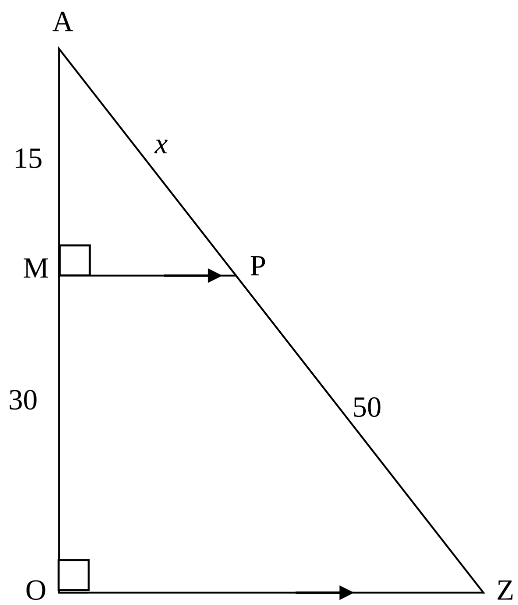
   enlarge both the length and height by 2 cm. √ **(ANY FACTOR)**

   3.4 Draw the original photo and the one you have enlarged. (4)

Original photo **enlarged photo.**



4. In the diagram below, MP // OZ



4.1 Which triangle is similar to Δ AZO and give reason? (2)

$\Delta AZO\ ///\ \Delta AMPG$ √ $(A\ A\ A)$ √

4.2 Determine the value of x. (2)

P is the midpoint of AZ (Converse Midpt. th) √

$x = 25$√

BASELINE TEST RESULTS

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16		
Question (Marks Allocation)	1 (2)	2 (2)	1 (1)	2 (1)	3 (1)	4 (1)	5 (1)	1.1.1 (2)	1.1.2 (2)	1.2.1 (2)	1.2.2 (2)	2.1 (4)	3.1 (1)	3.2 (4)	4.1 (2)	4.2 (2)	Total 30	%
NMT01	0	0	1	0	0	1	0	1	2	1	1	1	1	4	2	0	15	50
NMT02	1	0	0	1	0	0	0	1	0	1	0	0	1	2	1	2	10	33
NMT03	1	0	0	1	0	1	0	2	2	0	0	0	0	0	0	2	9	30
NMT04	0	0	1	0	0	0	0	2	2	0	0	0	0	4	0	0	9	30
NMT05	1	2	0	0	0	0	0	0	1	0	0	0	0	0	1	0	5	17
NMT06	0	0	1	1	0	0	0	0	1	0	1	0	0	0	0	0	4	13
NMT07	1	0	1	0	0	0	0	0	1	0	1	0	1	4	1	0	10	33
NMT08	1	0	1	0	0	0	0	0	0	0	0	0	1	4	1	0	8	27
NMT09	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	2	7
NMT10	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	3	10
NMT11	1	0	0	0	0	1	0	1	1	1	1	0	1	4	1	0	12	40
NMT12	1	0	1	0	0	1	0	0	0	0	0	0	0	0	2	0	5	17
NMT13	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	3	10
NMT14	2	0	0	0	1	0	0	0	0	0	0	0	1	4	1	0	9	30
NMT15	1	0	1	0	0	0	1	2	1	2	2	0	1	0	2	2	15	50
NMT16	1	0	1	0	0	1	0	0	2	2	0	0	0	0	1	0	8	27
NMT17	2	2	1	1	0	0	0	0	2	2	0	0	1	4	1	2	18	60
NMT18	0	0	1	1	1	1	0	1	1	1	1	0	1	0	0	0	9	30
NMT19	0	0	1	0	1	0	0	0	0	0	0	0	1	2	0	0	5	17
NMT20	2	0	1	0	0	0	0	0	0	0	0	0	0	4	1	0	8	27
NMT21	1	0	1	0	0	0	0	1	0	1	0	0	1	4	1	2	12	40
NMT22	1	0	1	0	0	0	0	2	1	2	1	0	1	2	1	0	12	40
NMT23	2	0	1	1	0	0	0	0	0	0	0	0	0	2	0	0	6	20
NMT24	0	0	1	0	0	0	0	1	1	0	1	0	0	0	1	2	7	23
NMT25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	3
NMT26	0	0	1	1	0	1	0	1	0	1	0	0	1	4	1	0	11	37
NMT27	2	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	5	17
NMT28	2	2	0	1	0	1	0	1	0	0	0	0	1	2	1	2	13	43
NMT29	2	0	1	0	1	0	0	0	0	0	0	0	1	3	1	0	9	30
NMT30	2	2	1	1	0	1	0	2	2	2	2	0	0	4	1	0	20	67
NMT31	0	0	0	0	1	0	0	0	0	0	0	0	0	3	1	0	5	17
NMT32	0	0	1	0	0	1	0	0	0	0	1	0	0	2	2	0	7	23
NMT33	2	0	0	1	0	0	0	0	1	1	0	0	1	4	1	0	11	37
NMT34	0	0	0	1	0	1	0	0	0	0	0	0	1	4	1	0	8	27
NMT35	0	0	1	1	0	1	0	2	1	2	0	0	1	2	2	2	15	50
NMT36	1	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	4	13
NMT37	1	0	0	0	0	0	0	1	0	0	0	0	1	0	1	2	6	20
NMT38	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2	7
NMT39	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	3	10
NMT40	2	0	1	1	0	0	0	0	1	0	0	0	1	0	2	0	8	27
NMT41	1	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	5	17
NMT42	0	0	0	0	0	1	0	0	0	1	0	0	0	2	1	0	5	17
NMT43	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	3	10
Total	35	8	23	16	7	14	2	23	26	21	14	1	21	76	40	18		
Total	86	86	43	43	43	43	43	86	86	86	86	172	43	172	86	86		
%	**41**	**9**	**53**	**37**	**16**	**33**	**5**	**27**	**30**	**24**	**16**	**1**	**49**	**44**	**47**	**21**		

INTERVENTION LESSON

This lesson is designed to help learners discover the properties of similar triangles. They will be asked to determine the general conditions required to verify or prove that two triangles are similar and specifically understand the concept of proportionality. This lesson is intended to solidify the basic knowledge of Euclidean geometry and similar triangles.

TOPIC : EUCLIDEAN GEOMETRY (SIMILAR TRIANGLES)

PERIOD : 4 DAYS (1HR PER DAY)

LEARNING OBJECTIVE:

- ✓ Revision of previous grade knowledge
- ✓ Understand equal angles.
- ✓ Recognise similar triangles.
- ✓ Understand the definition of a similar triangle.
- ✓ Understand proportionality.

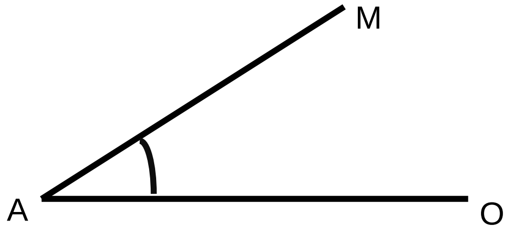
GRADE 9	GRADE 10	GRADE 12
PRE-KNOWLEDGE	**CURRENT/ OBJECTIVE**	**LOOKING FOR FORWARD**
• Revise basic results established in earlier grades lines, angles, congruency, similarity. • Revise theorem of Pythagoras and properties of Quadrilaterals	**PROVE THE FOLLOWING THEOREMS: Examinable** • Midpoint theorem • Proportional theorem • Solve problems and prove riders using the properties of parallel lines, triangles, and quadrilaterals.	• Proportionality theorems • Similar triangles • Theorem of Pythagoras (proof).

GRADE 10: PRE-KNOWLEDGE

DAY 1 AND 2

INTRODUCTION

Pose a question to learners: what is the name of the angle below?



Discussion with learners how can we write or express the angle in a different way.

Vocabulary

Theorem - A statement that has been proved based on what has already been established.

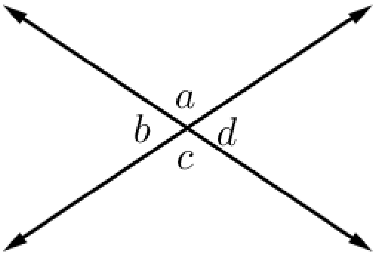
Converse - A statement formed by interchanging what is given in a theorem and what is to be proved.

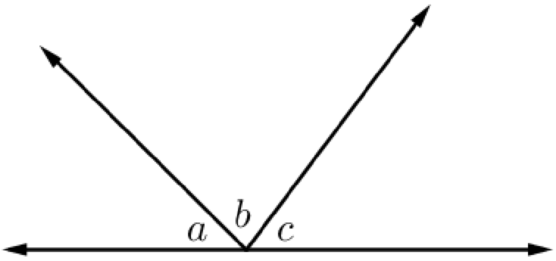
NB!!!!!!!

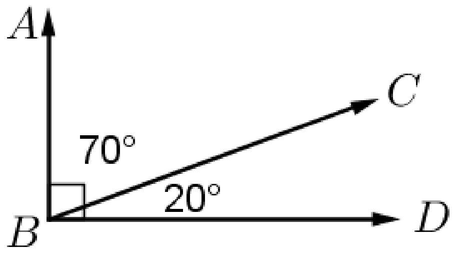
Fact - you must have a reason for each and every statement you make or write.

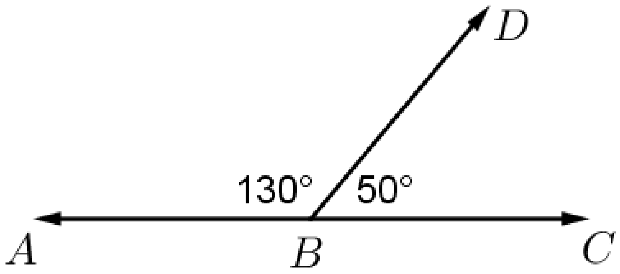
LESSON

Ask learners to discuss the types of angles in their groups. After choosing learners randomly to give answers.



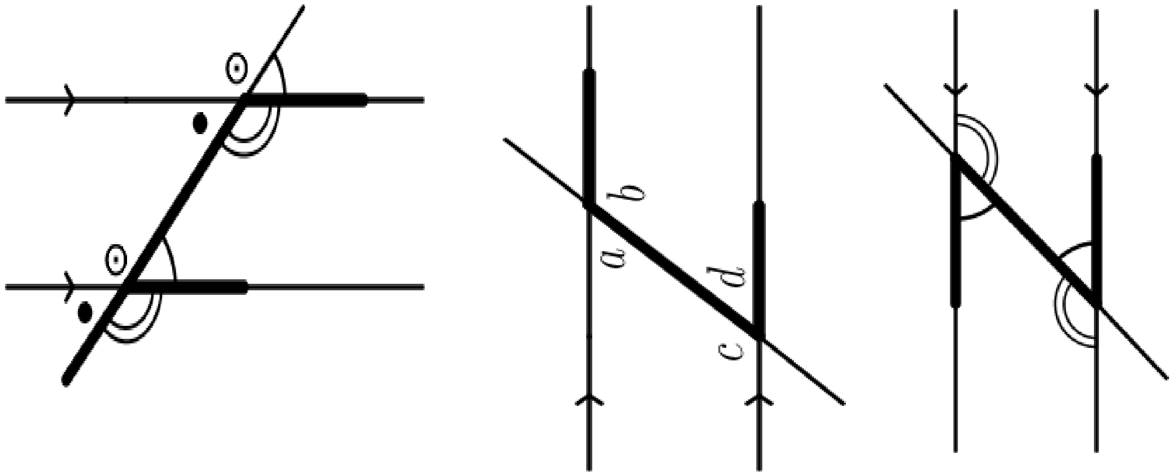




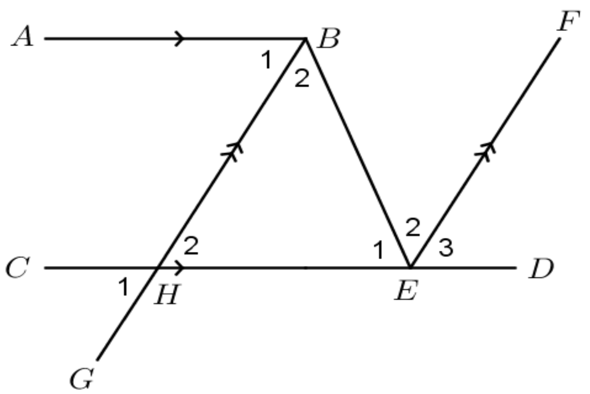


FUN

NB!!!!! Parallel lines

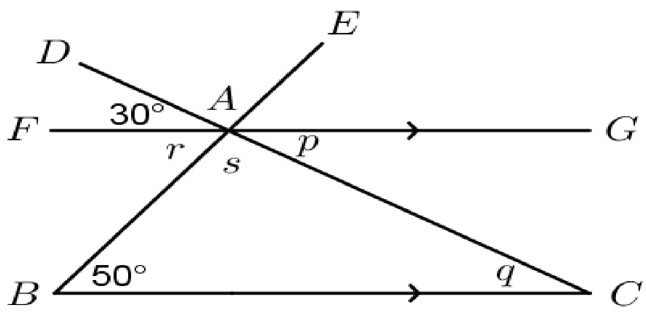


Class-activity



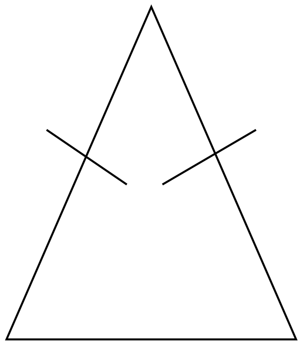
1. Name any angle and express it in 2 different ways.
2. Show which angle is equal to which one.

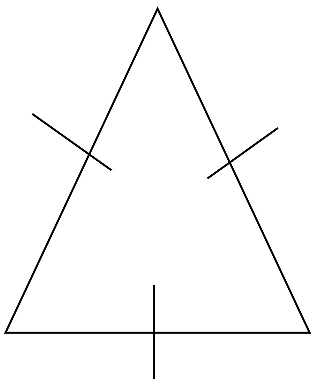
home-activity

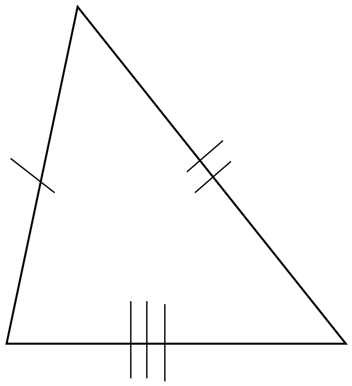


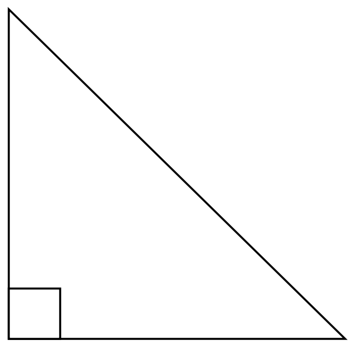
1. Fill in the missing angles.

Draw/ give learners different triangles and ask them to write what type of triangle and why?









SUMMARY

Ask learners to write notes Quadrilaterals and their properties.

- **Quadrilateral -** A 4-sided closed shape (polygon).
- **Parallelogram -** A quadrilateral with two pairs of parallel sides.
- **Square -** A four-sided polygon with all four sides equal in length and all four angles are right angles.
- **Rectangle -** A four-sided polygon with both pairs of opposite sides equal in length and all four angles are right angles.
- **rhombus** A quadrilateral with two pairs of parallel sides and all four sides equal length
- **trapezium** A quadrilateral with one pair of parallel sides.
- **kite** A quadrilateral with two pairs of adjacent sides equal.

GRADE 10: SIMILAR TRIANGLES

DAY 3 AND 4

INTRODUCTION

Discussion

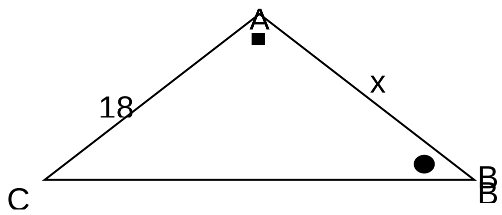
What can you say about you and your shadow?

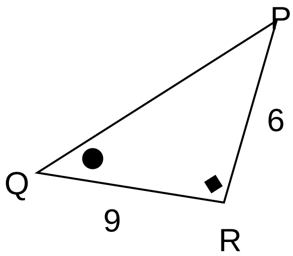
What can you say about yourself and your mirror image?

LESSON

Similar figures have the same shape but are different sizes. In the case of triangles:

- Corresponding angles are equal.
- Corresponding lengths are in the same ratio.



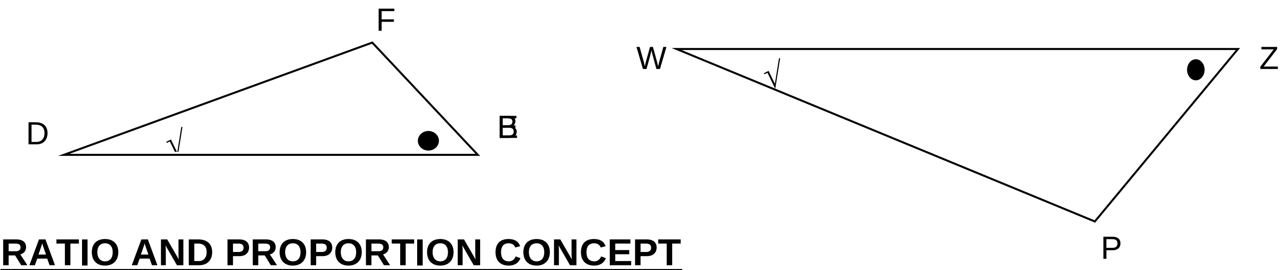


Notes:

- The symbol for similarity is |||
- The order of the points in the names of the triangles is important. Equal angles of the two triangles must coincide.

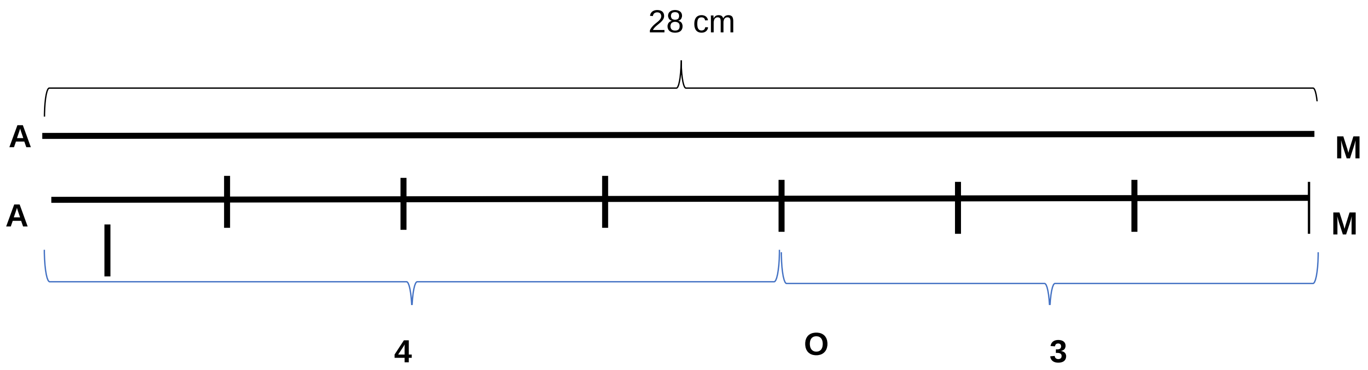
CONDITIONS TO DETERMINE IF TWO TRIANGLES ARE SIMILAR.

1. **AAA similarity**: If in two triangles, the corresponding angles are equal, the triangles are similar. (The 3rd set of angles will be equal, because of the sum of the interior angles of a triangle being 180°.)
2. **SSS similarity**: If the corresponding sides of two triangles are proportional the triangles are similar.
3. **SAS similarity**: If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.



RATIO AND PROPORTION CONCEPT

Consider the line segment AM. If AM = 28 cm and O divides AM in the ratio AO:MO = 4: 3, it is possible to find the actual lengths of AO and OM.



$$AO:OM = \frac{AO}{MO} = \frac{16cm}{12cm} = \frac{4}{3}$$

Means and Extremes on Multiplication Cross

$3:2 = 6:4$

$\frac{\mathbf{3}}{\mathbf{2}} = \frac{\mathbf{6}}{\mathbf{4}}$ play with cross multiplication (by interchanging)

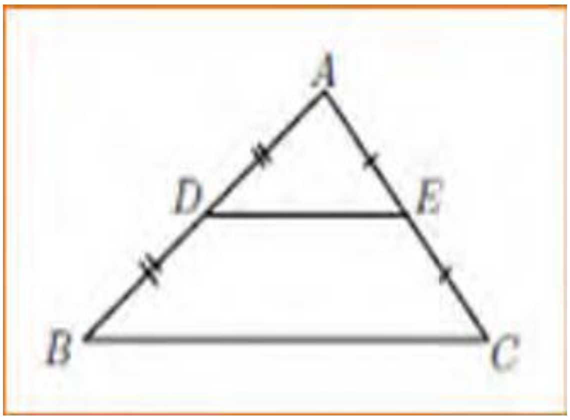
In general, for the ratio $\frac{\boldsymbol{a}}{\boldsymbol{b}} = \frac{\boldsymbol{c}}{\boldsymbol{d}}$ **then:**

Consider the following statement:

If $\frac{\boldsymbol{AB}}{\boldsymbol{CD}} = \frac{\boldsymbol{WX}}{\boldsymbol{YZ}}$ then:

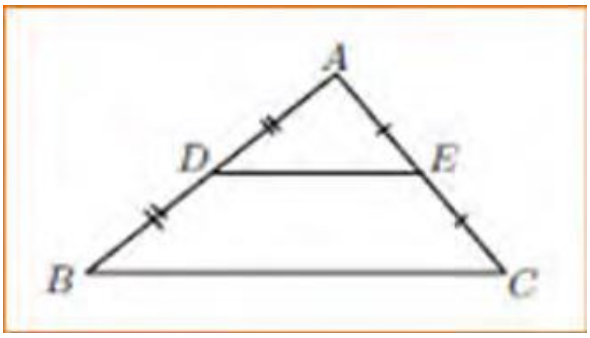
MIDPOINT THEOREM

The line segment joining the midpoints of two sides of a triangle, is parallel to the 3rd side of the triangle and half the length of that side.



If AD = DB and AE = EC, then DE// BC and DE = $\frac{1}{2}$BC.

A line drawn parallel to one side of a triangle that intersects the other two sides, will divide the other two sides proportionally.



If DE// BC AD: DB = AE: EC

SUMMARY

NB!!!!!!! To prove that two triangles are similar, we must show that **one** (not all) of the following statements is true:

- The three sides are in the same proportion.
- Two sides are in the same proportion, and their included angle is equal.
- The three angles of the first triangle are equal to the three angles of the second triangle.

DAY 5 IS FOR EXAMINATION QUESTIONS (EXERCISES)

POST TEST FOR LEARNERS'

Learner Code: ______________________ **Date: MAY 2020**

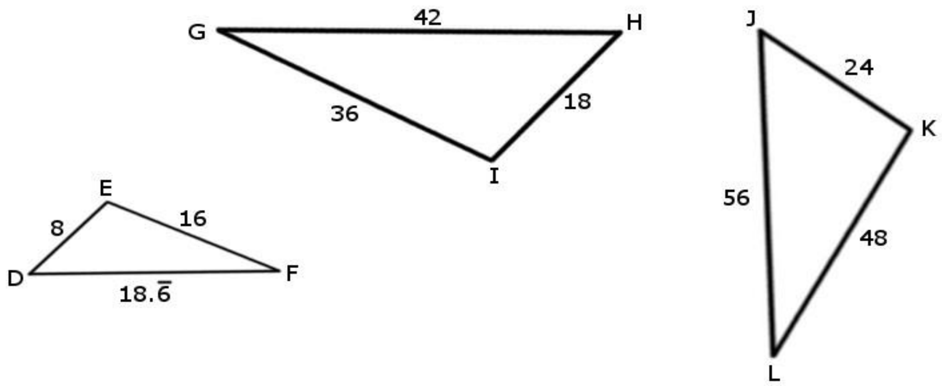
Please remember that the answers you will provide on this post-test will be treated with confidence. Please use the provided space to answer the following questions.

Answer all questions.

QUESTION 1 [20 MARKS]

1. State whether the following triangles are similar or not? do calculations to prove.

   1.1.



For Δ**DEF** and Δ**GHI** For Δ**DEF** and Δ**JKL** (4)

____

____

____

____

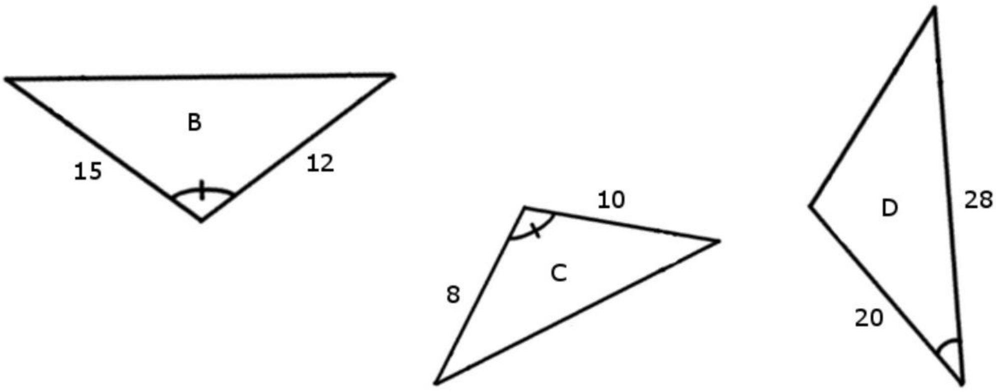
____

____

____

____

1.2.

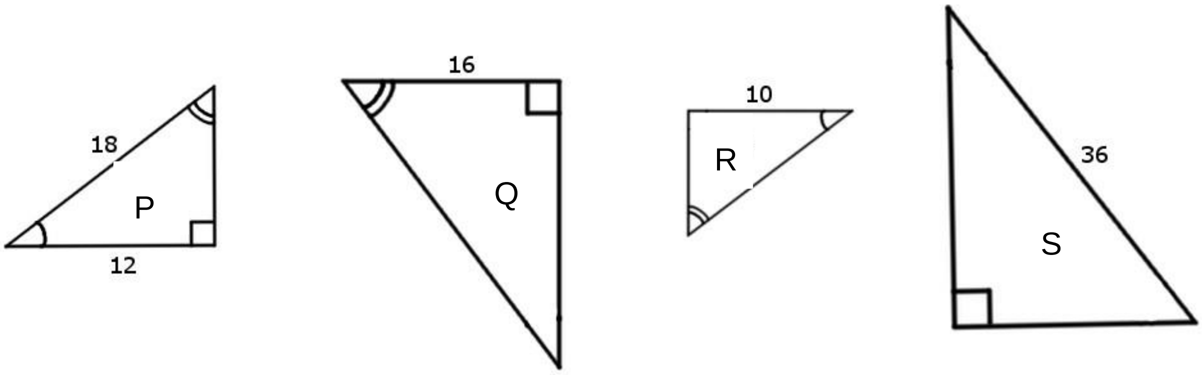


For ΔB and ΔC

For ΔB and ΔD

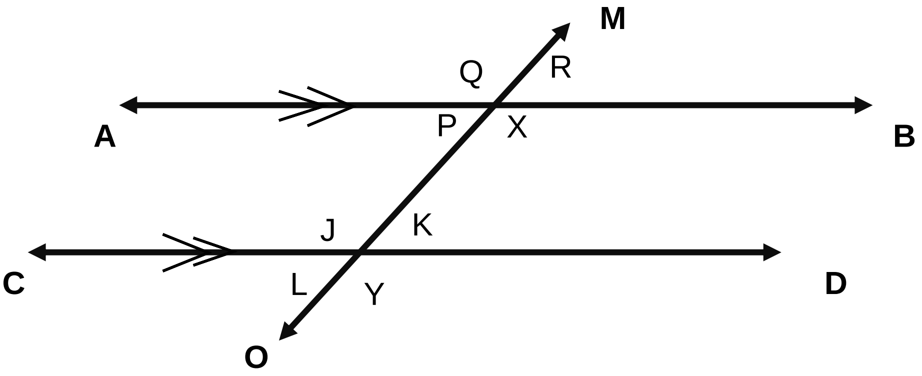
(4)

1.3.



(4)

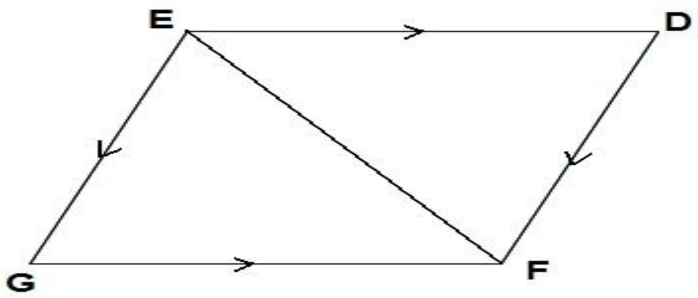
1.4. Given the diagram below, AB // CD.



Name two angles which are equal to x and give reason. (4)

STATEMENT	REASON

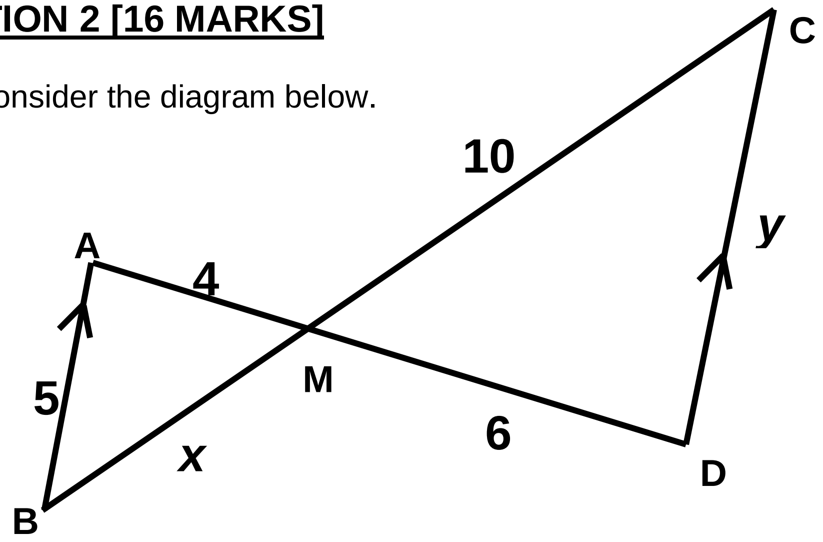
1.5.



(4)

QUESTION 2 [16 MARKS]

2.1. Consider the diagram below.



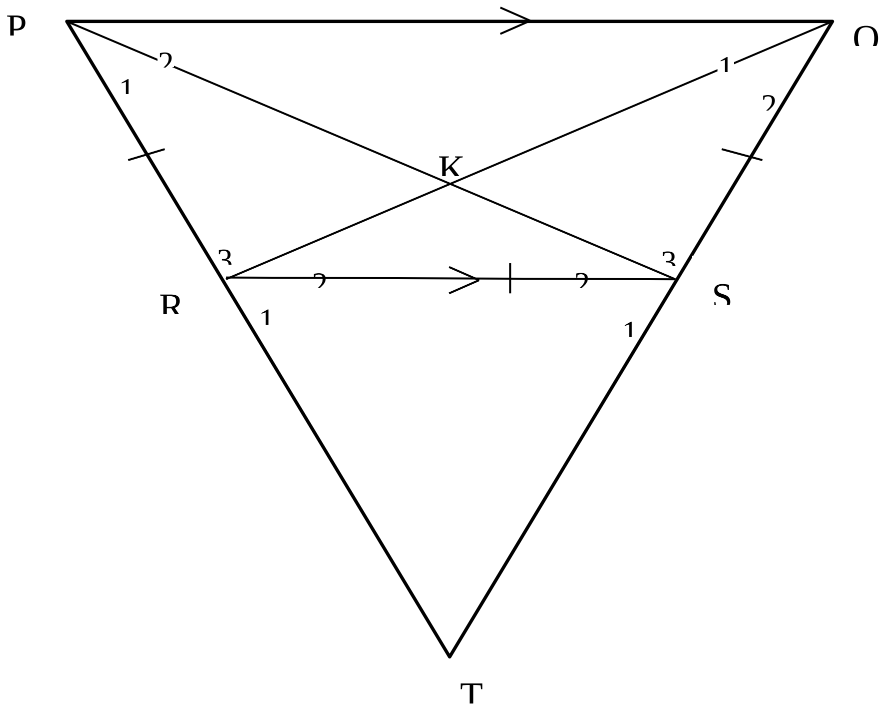
2.1.1. Prove that triangles Δ**ABM** and Δ**CDM** are similar.

(4)

2.1.2. Calculate **x** and **y**.

(4)

2.2. In the diagram above PR=RS=SQ, PQ||RS and $R\hat{P}Q = P\hat{Q}S$.



2.2.1 Show that RQ bisects $\hat{Q}$.

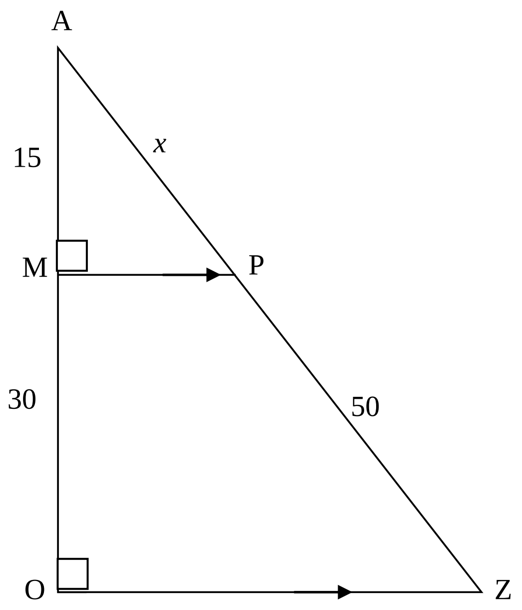
(4)

2.2.2 Prove that ΔPKQ|||ΔSKR.

(4)

QUESTION 3 [9 MARKS]

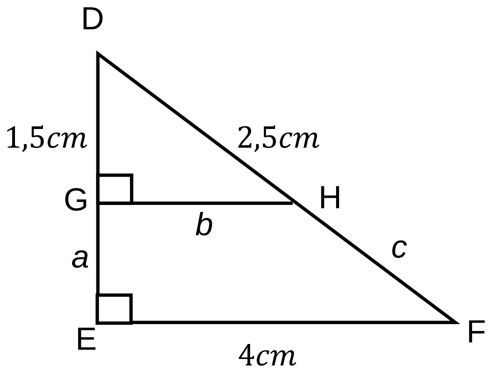
3.1 In the diagram below, MP // OZ.



3.1.1 Which triangle is similar to Δ AZO and give reason? (2)

3.1.2 Determine the value of x. (2)

3.2. In the diagram below, given DF = $5\ cm$ calculate the value of a , b and c, **with reasons.**



(5)

TOTAL MARKS: 45 MARKS

!!!!!!!!!!!!!!!!!!THANKS FOR PARTICIPATING IN THIS STUDY!!!!!!!!!!!!!!!

MEMO FOR POST TEST FOR LEARNERS'

Learner Code: ______________________ **Date: MAY 2020**

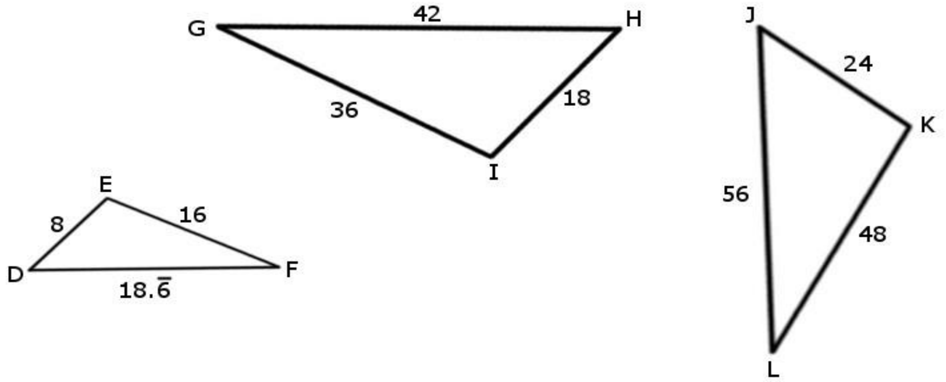
Please remember the answers you will provide in this post-test will be treated with confidence. Please use the provided space to answer the following questions.

Answer all questions.

QUESTION 1 [20 MARKS]

1. State whether the following triangles are similar or not? do calculations to prove.

1.1



For ΔDEF and ΔGHI

$\frac{8}{18} = \frac{4}{9}$

$\frac{16}{36} = \frac{4}{9}$

$\frac{18,6}{42} = \frac{4}{9}$

For ΔDEF and ΔJKL (4)

$\frac{24}{8} = 3$

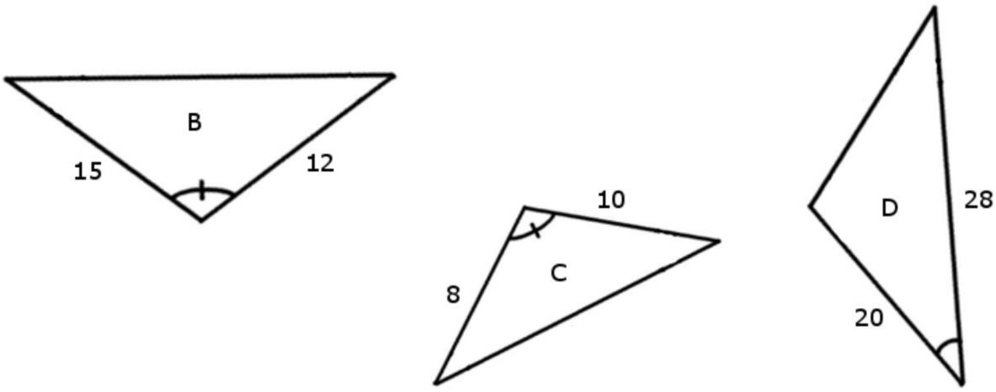
$\frac{48}{16} = 3$

$\frac{56}{18,6} = 3$

$\therefore \Delta DEF\ ///\ \Delta GHI \text{ and } \Delta DEF\ ///\ \Delta JKL$

All triangles are similar to each other.

1.2



For ΔB and ΔC

$\frac{15}{10} = \frac{3}{2}$

$\frac{12}{8} = \frac{3}{2}$

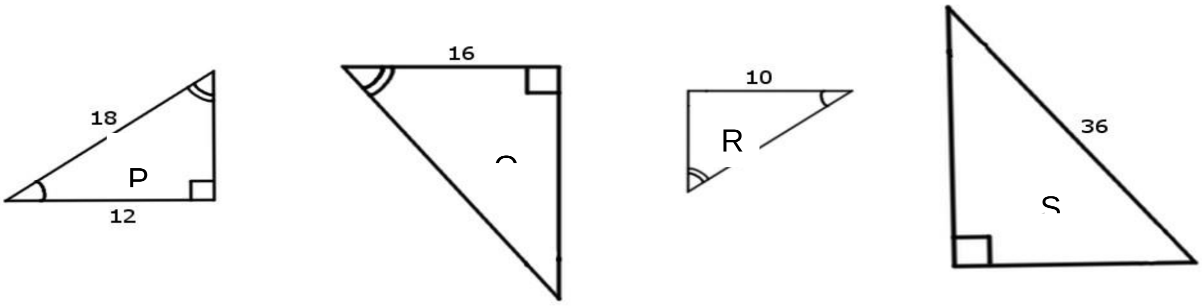
For ΔB and ΔD (4)

$\frac{15}{20} = \frac{3}{4}$

$\frac{12}{28} = \frac{3}{7}$

∴ 1 corr. ∠`s are equal $\therefore \Delta B /// \Delta C$ $\therefore \Delta B$ *is not similar* ΔD and other ΔC.

1.3



In ΔP *and* ΔQ (4)

Two corr. ∠`s are equal $\therefore \Delta P /// \Delta Q$ (AAA)

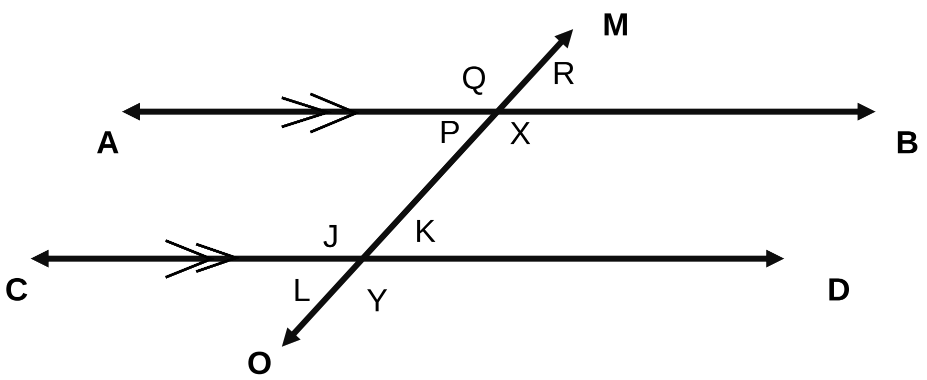
In ΔP *and* ΔR

Two corr. ∠`s are equal $\therefore \Delta P /// \Delta R$ (AAA)

In ΔP *and* ΔS

1 corr. ∠`s are equal and 1 corr. side are proportional $\therefore \Delta P$ *is not similar* ΔR and any other Δ

1.3 Given the diagram below, AB // CD.

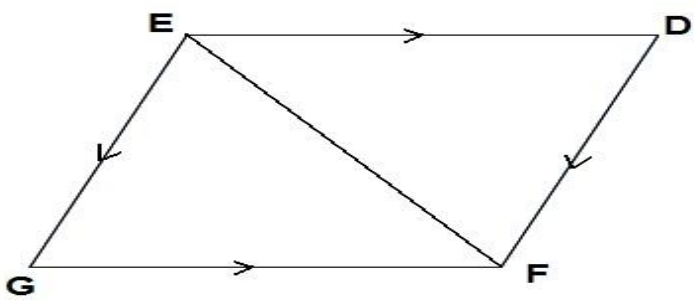


Name two angles which are equal to x and give reason.

(4)

STATEMENT	REASON
1.1.1 $x = \hat{G}_1$ √	vertically opposite angle. √
1.1.2 $x = y$ √	corresponding angle. √

1.5



$G\hat{E}F = D\hat{F}E$ (alt. ∠`s GE//DF) √ S/R

$E\hat{F}G = D\hat{E}F$ (alt. ∠`s ED//GF) √ S/R

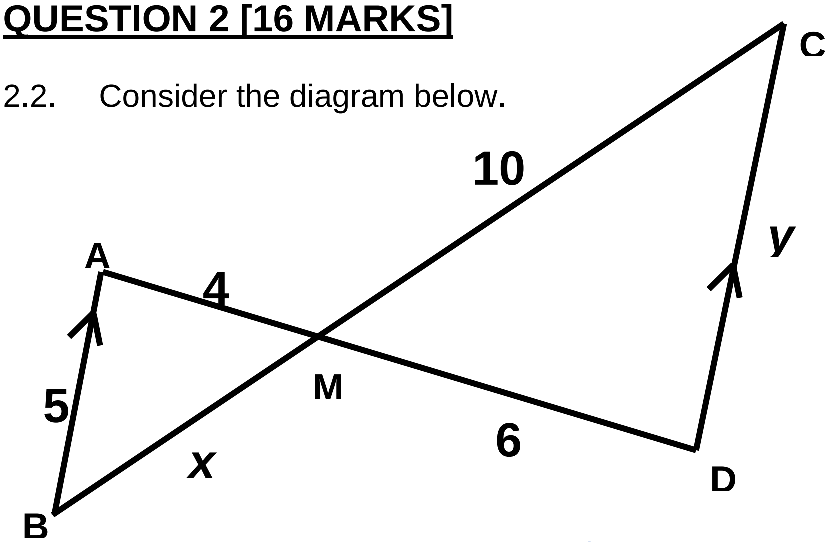
$\hat{G} = \hat{D}$ (3rd. ∠ of Δ) √ S/R

∴ ΔEFG|||ΔDEF (AAA) √

(4)

QUESTION 2 [16 MARKS]

2.2. Consider the diagram below.



2.2.1. Prove that triangles Δ**ABM** and Δ**CDM** are similar.

(4)

$\hat{A} = \widehat{D}$ $(alt. \angle`s\ PQ//RS)$ $\surd\ S/R$

$\hat{B} = \hat{C}$ $(alt. \angle`s\ PQ//RS)$ $\surd\ S/R$

$\widehat{M} = \widehat{M}$ $(Vert. opp.\ \angle`s\ )$ $\surd\ S/R$

$\therefore \Delta ABM|||\Delta DCM\ (AAA)$ $\surd$

2.2.2. Calculate **x** and **y**.

$\frac{AB}{DC} = \frac{BM}{CM} = \frac{AM}{DM}$ $(prop. div. theorem)$ $\surd S/R$ (4)

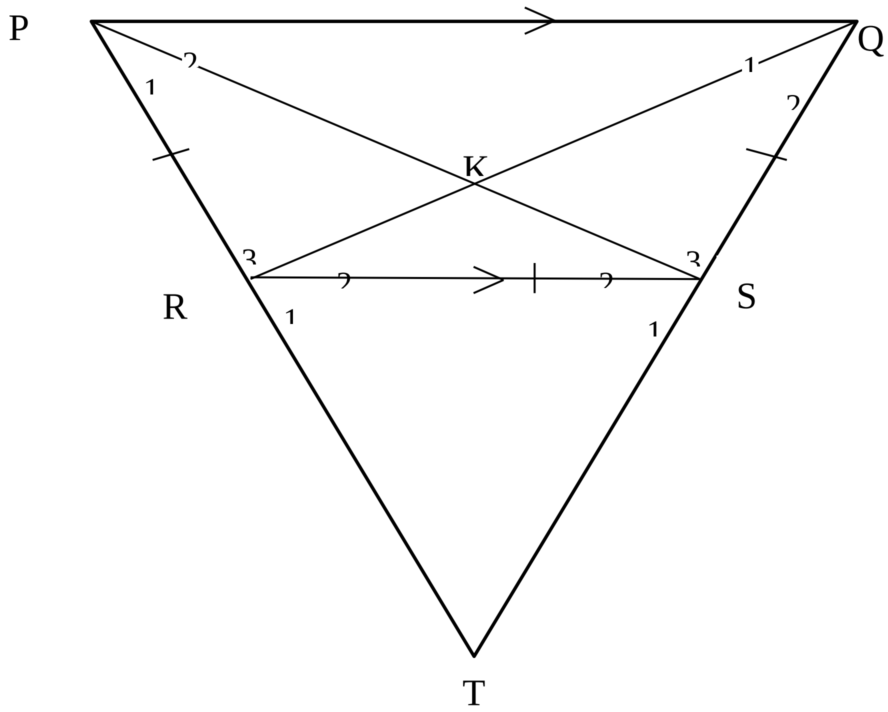
$\frac{BM}{10} = \frac{4}{6}$ $\surd$

$x = \frac{10 \times 4}{6} = 6{,}67$ $\surd$

$\frac{DC}{5} = \frac{6}{4}$

$y = \frac{5 \times 6}{4} = 7{,}5$ $\surd$

2.3. In the diagram above PR=RS=SQ, PQ||RS and $R\hat{P}Q = P\hat{Q}S$.



2.2.3 Show that RQ bisects $\hat{Q}$.

(4)

$\hat{Q}_1 = \hat{R}_2$ $(alt.\angle`s\ PQ//RS)$ $\surd\ S/R$

$\hat{Q}_2 = \hat{R}_2$ $(\angle\ = opp.side\ of\ \Delta)$ $\surd\ S/R$

$\therefore \hat{Q}_1 = \hat{Q}_2$ $\surd$

$RQ\ bisects\ \hat{Q}$ $\surd$

2.2.4 Prove that ΔPKQ|||ΔSKR.

(4)

$\hat{Q}_1 = \hat{R}_2$ $(PROVEN)$ $\surd\ S/R$

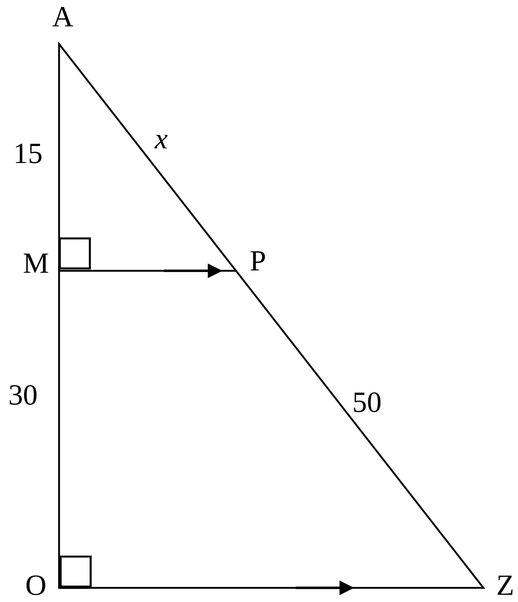
$\hat{S}_2 = \hat{P}_2$ $(alt.\angle`s\ PQ//RS)$ $\surd\ S/R$

$\widehat{K} = \widehat{K}$ $(Vert.opp.\ \angle`s\ )$ $\surd\ S/R$

$\therefore$ ΔPKQ|||ΔSKR (AAA) $\surd$

QUESTION 3 [9 MARKS]

3.2 In the diagram below, MP // OZ



3.2.1 Which triangle is similar to Δ AZO and give reason? (2)

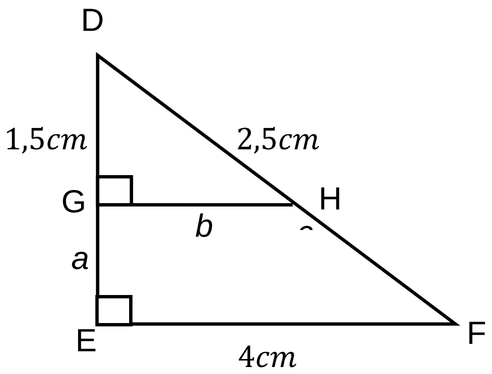
ΔAZO /// ΔAMPG √ (A A A) √

3.2.2 Determine the value of x. (2)

P is the $midpoint$ of AZ (Converse Midpt th) √

x = 25√

3.3. In the diagram below, given DF = $5\ cm$ calculate the value of a , b and c, **with reasons.**



(5)

$G\ is\ the\ midpoint\ \ of\ DE$ (Converse Midpt th) √

$a\ =\ 1{,}5$ cm √

$H\ is\ the\ midpoint\ \ of\ DF$ (Converse Midpt th) √

$c\ =\ 1{,}5$ cm √

$b\ =\ 2$ cm √

TOTAL MARKS: 45 MARKS

!!!!!!!!!!!!!!!!!!THANKS FOR PARTICIPATING IN THIS STUDY!!!!!!!!!!!!!!!!

POST-TEST RESULTS

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12		
Question (Marks Allocation)	1.1 (4)	1.2 (4)	1.3 (4)	1.4 (4)	1.5 (4)	2.1.1 (4)	2.1.2 (4)	2.2.1 (4)	2.2.2 (4)	3.1.1 (2)	3.1.2 (2)	3.2 (5)	Total 45	%
NMT01	0	4	2	4	0	1	0	0	1	2	2	5	21	47
NMT02	0	0	0	0	0	0	0	0	1	1	1	3	6	13
NMT03	0	0	0	1	0	0	0	0	0	1	2	0	4	9
NMT04	0	0	1	4	0	0	0	1	1	0	0	0	7	16
NMT05	0	0	0	0	0	0	0	0	0	1	0	0	1	2
NMT06	0	0	0	3	0	0	0	0	0	0	0	0	3	7
NMT07	0	0	0	2	0	0	0	0	0	1	2	3	8	18
NMT08	1	0	0	4	0	0	0	0	2	2	1	0	10	22
NMT09	0	1	0	0	0	0	0	0	0	1	0	1	3	7
NMT10	0	0	0	0	0	0	0	0	0	1	0	1	2	4
NMT11	1	0	0	2	0	0	0	0	0	0	0	1	4	9
NMT12	2	0	0	3	0	3	0	0	0	0	0	0	8	18
NMT13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NMT14	0	0	0	2	0	2	0	0	0	0	0	0	4	9
NMT15	2	0	0	4	2	0	0	0	0	0	0	0	8	18
NMT16	0	2	2	4	0	0	0	0	0	1	1	1	11	24
NMT17	0	0	0	3	0	0	0	0	0	1	1	0	5	11
NMT18	2	0	0	1	0	0	0	0	0	1	2	3	9	20
NMT19	0	0	0	0	0	0	0	0	0	1	0	3	4	9
NMT20	0	0	0	2	0	0	0	0	0	1	0	0	3	7
NMT21	0	0	0	2	0	0	0	0	0	1	0	0	3	7
NMT22	1	1	4	3	2	0	0	0	0	1	2	5	19	42
NMT23	1	0	0	2	0	0	0	0	0	0	2	5	10	22
NMT24	0	0	0	1	0	0	0	0	0	1	1	0	3	7
NMT25	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NMT26	2	4	1	0	0	0	0	0	0	1	0	3	11	24
NMT27	3	0	0	4	0	0	0	0	0	1	1	0	9	20
NMT28	2	4	0	4	0	0	0	0	0	1	2	5	18	40
NMT29	1	2	0	2	0	0	0	0	0	1	1	1	8	18
NMT30	2	4	0	3	0	0	0	0	0	2	0	2	13	29
NMT31	0	3	0	1	0	0	0	0	0	1	1	4	10	22
NMT32	2	1	1	2	0	0	0	0	0	1	1	3	11	24
NMT33	2	2	0	4	0	0	0	0	0	1	1	3	13	29
NMT34	1	0	0	0	0	0	0	0	0	1	0	0	2	4
NMT35	1	2	2	4	0	0	0	0	0	1	1	1	12	27
NMT36	3	4	0	2	0	0	0	0	0	1	1	3	14	31
NMT37	0	0	0	0	0	0	0	0	0	1	0	0	1	2
NMT38													0	0
NMT39	0	0	0	2	0	0	0	0	0	0	0	0	2	4
NMT40	4	2	0	0	0	0	0	0	0	1	0	0	7	16
NMT41	0	0	0	1	0	0	0	0	0	1	0	1	3	7
NMT42	0	0	0	0	0	0	0	0	0	1	0	0	1	2
NMT43	0	0	0	0	0	0	0	0	0	1	2	3	6	13
Total	33	36	13	76	4	6	0	1	5	35	28	60		
Total	172	172	172	172	172	172	172	172	172	86	86	215		
%	**19**	**21**	**8**	**44**	**2**	**3**	**0**	**1**	**3**	**41**	**33**	**28**		

ORDER OF IMPROVEMENT RESULTS

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16		
Question	1 (2)	2 (2)	1 (1)	2 (1)	3 (1)	4 (1)	5 (1)	1.1.1 (2	1.1.2	1.2.1	1.2.2	2.1 (	3.1 (	3.2 (	4.1 (	4.2 (	Tota	%
NMT36	1	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	4	13
NMT31	0	0	0	0	1	0	0	0	0	0	0	0	0	3	1	0	5	17
NMT27	2	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	5	17
NMT43	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	3	10
NMT22	1	0	1	0	0	0	0	2	1	2	1	0	1	2	1	0	12	40
NMT23	2	0	1	1	0	0	0	0	0	0	0	0	0	2	0	0	6	20
NMT12	1	0	1	0	0	1	0	0	0	0	0	0	0	0	2	0	5	17
NMT32	0	0	1	0	0	1	0	0	0	0	1	0	0	2	2	0	7	23
NMT09	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	2	6.7
NMT16	1	0	1	0	0	1	0	0	2	2	0	0	0	0	1	0	8	27
NMT25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	3.3
NMT01	0	0	1	0	0	1	0	1	2	1	1	1	1	4	2	0	15	50
NMT28	2	2	0	1	0	1	0	1	0	0	0	0	1	2	1	2	13	43
NMT08	1	0	1	0	0	0	0	0	0	0	0	0	1	4	1	0	8	27
NMT10	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	3	10
NMT39	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	3	10
NMT06	0	0	1	1	0	0	0	0	1	0	1	0	0	0	0	0	4	13
NMT38	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2	6.7
NMT19	0	0	1	0	1	0	0	0	0	0	0	0	1	2	0	0	5	17
NMT33	2	0	0	1	0	0	0	0	1	1	0	0	1	4	1	0	11	37
NMT41	1	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	5	17
NMT18	0	0	1	1	1	1	0	1	1	1	1	0	1	0	0	0	9	30
NMT13	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	3	10
NMT40	2	0	1	1	0	0	0	0	1	0	0	0	1	0	2	0	8	27
NMT26	0	0	1	1	0	1	0	1	0	1	0	0	1	4	1	0	11	37
NMT29	2	0	1	0	1	0	0	0	0	0	0	0	1	3	1	0	9	30
NMT05	1	2	0	0	0	0	0	0	1	0	0	0	0	0	1	0	5	17
NMT42	0	0	0	0	0	1	0	0	0	1	0	0	0	2	1	0	5	17
NMT04	0	0	1	0	0	0	0	2	2	0	0	0	0	4	0	0	9	30
NMT07	1	0	1	0	0	0	0	0	1	0	1	0	1	4	1	0	10	33
NMT24	0	0	1	0	0	0	0	1	1	0	1	0	0	0	1	2	7	23
NMT37	1	0	0	0	0	0	0	1	0	0	0	0	1	0	1	2	6	20
NMT02	1	0	0	1	0	0	0	1	0	1	0	0	1	2	1	2	10	33
NMT20	2	0	1	0	0	0	0	0	0	0	0	0	0	4	1	0	8	27
NMT03	1	0	0	1	0	1	0	2	2	0	0	0	0	0	0	2	9	30
NMT14	2	0	0	0	1	0	0	0	0	0	0	0	1	4	1	0	9	30
NMT34	0	0	0	1	0	1	0	0	0	0	0	0	1	4	1	0	8	27
NMT35	0	0	1	1	0	1	0	2	1	2	0	0	1	2	2	2	15	50
NMT11	1	0	0	0	0	1	0	1	1	1	1	0	1	4	1	0	12	40
NMT15	1	0	1	0	0	0	1	2	1	2	2	0	1	0	2	2	15	50
NMT21	1	0	1	0	0	0	0	1	0	1	0	0	1	4	1	2	12	40
NMT30	2	2	1	1	0	1	0	2	2	2	2	0	0	4	1	0	20	67
NMT17	2	2	1	1	0	0	0	0	2	2	0	0	1	4	1	2	18	60
Total	35	8	23	16	7	14	2	23	26	21	14	1	21	76	40	18		
Total	86	86	43	43	43	43	43	86	86	86	86	172	43	172	43	129		
%	41	9.3	53	37	16	33	4.7	26.74	30	24	16	0.6	49	44	93	14		

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12				
Question	1.1 (4	1.2 (4	1.3 (4	1.4 (4	1.5 (4	2.1.1	2.1.2	2.2.1	2.2.2	3.1.1 (2)	3.1.2 (2)	3.2 (5	Total	%		
NMT36	3	4	0	2	0	0	0	0	0	1	1	3	14	31.1		18
NMT31	0	3	0	1	0	0	0	0	0	1	1	4	10	22.2		6
NMT27	3	0	0	4	0	0	0	0	0	1	1	0	9	20		3
NMT43	0	0	0	0	0	0	0	0	0	1	2	3	6	13.3		3
NMT22	1	1	4	3	2	0	0	0	0	1	2	5	19	42.2		2
NMT23	1	0	0	2	0	0	0	0	0	0	2	5	10	22.2		2
NMT12	2	0	0	3	0	3	0	0	0	0	0	0	8	17.8		1
NMT32	2	1	1	2	0	0	0	0	0	1	1	3	11	24.4		1
NMT09	0	1	0	0	0	0	0	0	0	1	0	1	3	6.67		0
NMT16	0	2	2	4	0	0	0	0	0	1	1	1	11	24.4		-2
NMT25	0	0	0	0	0	0	0	0	0	0	0	0	0	0		-3
NMT01	0	4	2	4	0	1	0	0	1	2	2	5	21	46.7		-3
NMT28	2	4	0	4	0	0	0	0	0	1	2	5	18	40		-3
NMT08	1	0	0	4	0	0	0	0	2	2	1	0	10	22.2		-4
NMT10	0	0	0	0	0	0	0	0	0	1	0	1	2	4.44		-6
NMT39	0	0	0	2	0	0	0	0	0	0	0	0	2	4.44		-6
NMT06	0	0	0	3	0	0	0	0	0	0	0	0	3	6.67		-7
NMT38													0	0		-7
NMT19	0	0	0	0	0	0	0	0	0	1	0	3	4	8.89		-8
NMT33	2	2	0	4	0	0	0	0	0	1	1	3	13	28.9		-8
NMT41	0	0	0	1	0	0	0	0	0	1	0	1	3	6.67		-10
NMT18	2	0	0	1	0	0	0	0	0	1	2	3	9	20		-10
NMT13	0	0	0	0	0	0	0	0	0	0	0	0	0	0		-10
NMT40	4	2	0	0	0	0	0	0	0	1	0	0	7	15.6		-11
NMT26	2	4	1	0	0	0	0	0	0	1	0	3	11	24.4		-12
NMT29	1	2	0	2	0	0	0	0	0	1	1	1	8	17.8		-12
NMT05	0	0	0	0	0	0	0	0	0	1	0	0	1	2.22		-14
NMT42	0	0	0	0	0	0	0	0	0	1	0	0	1	2.22		-14
NMT04	0	0	1	4	0	0	0	1	1	0	0	0	7	15.6		-14
NMT07	0	0	0	2	0	0	0	0	0	1	2	3	8	17.8		-16
NMT24	0	0	0	1	0	0	0	0	0	1	1	0	3	6.67		-17
NMT37	0	0	0	0	0	0	0	0	0	1	0	0	1	2.22		-18
NMT02	0	0	0	0	0	0	0	0	1	1	1	3	6	13.3		-20
NMT20	0	0	0	2	0	0	0	0	0	1	0	0	3	6.67		-20
NMT03	0	0	0	1	0	0	0	0	0	1	2	0	4	8.89		-21
NMT14	0	0	0	2	0	2	0	0	0	0	0	0	4	8.89		-21
NMT34	1	0	0	0	0	0	0	0	0	1	0	0	2	4.44		-22
NMT35	1	2	2	4	0	0	0	0	0	1	1	1	12	26.7		-23
NMT11	1	0	0	2	0	0	0	0	0	0	0	1	4	8.89		-31
NMT15	2	0	0	4	2	0	0	0	0	0	0	0	8	17.8		-32
NMT21	0	0	0	2	0	0	0	0	0	1	0	0	3	6.67		-33
NMT30	2	4	0	3	0	0	0	0	0	2	0	2	13	28.9		-38
NMT17	0	0	0	3	0	0	0	0	0	1	1	0	5	11.1		-49
Total	34	36	14	84	4	6	1	1	5	38	31	64				
Total	172	172	172	172	172	172	172	172	172	86	86	215				
%	19.8	20.9	8.14	48.8	2.33	3.49	0.58	0.58	2.91	44.186	36.0465	29.8				

TESTS SAME ITEMS COMPARISON

Question (Marks Allocation)	B8 1.1.1 (2)	B9 1.1.2 (2)	B12 2.1 (4)	B15 4.1 (2)	B16 4.2 (2)	TOTAL	%		P4 1.4 (4)	P6 2.1.1 (4)	P10 3.1.1 (2)	P11 3.1.2 (2)	TOTAL	%		% Change
NMT35	2	1	0	2	2	7	58.3		4	0	1	1	6	50		-8.3
NMT15	2	1	0	2	2	7	58.3		4	0	0	0	4	33.3		-25.0
NMT01	1	2	1	2	0	6	50.0		4	1	2	2	9	75		25.0
NMT03	2	2	0	0	2	6	50.0		1	0	1	2	4	33.3		-16.7
NMT30	2	2	0	1	0	5	41.7		3	0	2	0	5	41.7		0.0
NMT17	0	2	0	1	2	5	41.7		3	0	1	1	5	41.7		0.0
NMT24	1	1	0	1	2	5	41.7		1	0	1	1	3	25		-16.7
NMT28	1	0	0	1	2	4	33.3		4	0	1	2	7	58.3		25.0
NMT22	2	1	0	1	0	4	33.3		3	0	1	2	6	50		16.7
NMT04	2	2	0	0	0	4	33.3		4	0	0	0	4	33.3		0.0
NMT21	1	0	0	1	2	4	33.3		2	0	1	0	3	25		-8.3
NMT02	1	0	0	1	2	4	33.3		0	0	1	1	2	16.7		-16.7
NMT37	1	0	0	1	2	4	33.3		0	0	1	0	1	8.33		-25.0
NMT16	0	2	0	1	0	3	25.0		4	0	1	1	6	50		25.0
NMT41	1	1	0	1	0	3	25.0		1	0	1	0	2	16.7		-8.3
NMT11	1	1	0	1	0	3	25.0		2	0	0	0	2	16.7		-8.3
NMT40	0	1	0	2	0	3	25.0		0	0	1	0	1	8.33		-16.7
NMT27	1	0	0	1	0	2	16.7		4	0	1	1	6	50		33.3
NMT12	0	0	0	2	0	2	16.7		3	3	0	0	6	50		33.3
NMT33	0	1	0	1	0	2	16.7		4	0	1	1	6	50		33.3
NMT07	0	1	0	1	0	2	16.7		2	0	1	2	5	41.7		25.0
NMT36	0	1	0	1	0	2	16.7		2	0	1	1	4	33.3		16.7
NMT32	0	0	0	2	0	2	16.7		2	0	1	1	4	33.3		16.7
NMT18	1	1	0	0	0	2	16.7		1	0	1	2	4	33.3		16.7
NMT26	1	0	0	1	0	2	16.7		0	0	1	0	1	8.33		-8.3
NMT05	0	1	0	1	0	2	16.7		0	0	1	0	1	8.33		-8.3
NMT08	0	0	0	1	0	1	8.3		4	0	2	1	7	58.3		50.0
NMT29	0	0	0	1	0	1	8.3		2	0	1	1	4	33.3		25.0
NMT14	0	0	0	1	0	1	8.3		2	2	0	0	4	33.3		25.0
NMT31	0	0	0	1	0	1	8.3		1	0	1	1	3	25		16.7
NMT06	0	1	0	0	0	1	8.3		3	0	0	0	3	25		16.7
NMT20	0	0	0	1	0	1	8.3		2	0	1	0	3	25		16.7
NMT39	0	0	0	1	0	1	8.3		2	0	0	0	2	16.7		8.3
NMT09	0	1	0	0	0	1	8.3		0	0	1	0	1	8.33		0.0
NMT10	0	0	0	1	0	1	8.3		0	0	1	0	1	8.33		0.0
NMT42	0	0	0	1	0	1	8.3		0	0	1	0	1	8.33		0.0
NMT34	0	0	0	1	0	1	8.3		0	0	1	0	1	8.33		0.0
NMT25	0	0	0	1	0	1	8.3		0	0	0	0	0	0		-8.3
NMT13	0	0	0	1	0	1	8.3		0	0	0	0	0	0		-8.3
NMT23	0	0	0	0	0	0	0.0		2	0	0	2	4	33.3		33.3
NMT43	0	0	0	0	0	0	0.0		0	0	1	2	3	25		25.0
NMT19	0	0	0	0	0	0	0.0		0	0	1	0	1	8.33		8.3
NMT38	0	0	0	0	0	0	0.0						0	0		0.0
Total	23	26	1	40	18		900		84	6	38	31		1208		
Total	86	86	172	86	86		20.9		172	172	86	86		28.1		
%	26.7	30.2	0.6	47	20.93				49	3.49	44.19	36				

LEARNERS' QUESTIONS FOR SEMI-STRUCTURED INTERVIEW

TITLE: EVALUATING GRADE 10 LEARNERS' CHANGE IN UNDERSTANDING OF SIMILAR TRIANGLES FOLLOWING A CLASSROOM INTERVENTION.

After the researcher's intervention

1. What is your understanding of similar triangles? Please explain.

2. How best can you explain to other peers your understanding of similarity? Please explain.

3. What challenges do you have in similarity? Please explain.

4. What methods or strategies were employed by your teacher to best explain the similarity? Please explain.

5. How does the researcher's designed methods improve your understanding of similar triangles and geometry as whole?