Dynamic Corporate Liquidity[*]

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May 19, 2017

Abstract

We examine the determinants of corporate liquidity management through the lens of an estimated dynamic model of corporate investment and financing. When external finance is costly, firms can absorb shocks and cover liquidity needs by holding cash and by drawing down credit lines. In contrast to cash, we model credit lines as providing liquidity contingent on economic news, but limited by collateral constraints and covenants. The option to draw down credit lines creates value as it allows firms to take advantage of investment opportunities in an effective way, facilitating firm growth. We find that our estimated model matches well the levels and joint dynamics of cash, credit lines, leverage, equity financing and investment when firms can collateralize roughly one third of their assets. In the cross-section, the model provides novel empirical predictions and rationalizes a wide range of stylized facts regarding credit line usage, covenant violations, and cash holdings.

Keywords: corporate liquidity, cash, credit lines, collateral, covenants, leverage, corporate investment, structural estimation, linear programming, efficient computation.

JEL Classification Numbers: G31, G32.

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The ongoing buildup of cash reserves by US corporations has sparked renewed interest in understanding how firms manage their liquidity and risk. Indeed, average corporate cash-to-asset ratios in the US more than doubled since 1980[1]. A proposed rationale[2] relates this steady growth in corporate cash holdings to more pronounced cash flow risks that firms face. Apart from holding cash, however, firms can draw on credit lines to cover liquidity needs and absorb cash flow shocks. As documented for example in Sufi (2009), a significant fraction of firms now has access to credit lines. While a literature has emerged examining either cash holdings or credit line usage, there is little work connecting the two and identifying the mechanisms that shape their distinct role in firms' liquidity management. In this paper, we take a step in this direction by developing and estimating a dynamic model of corporate liquidity management by means of cash and credit lines.

In our model, a rationale for corporate liquidity management arises from costs of external finance. Indeed, when external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks by allowing them to avoid raising funds in capital markets. Making effective use of liquid funds entails transferring them to times and states where they are most valuable. Liquid funds may be valuable because they aid financing of a profitable investment opportunity, or because they help cover cash shortfalls. Anticipations of such future states thus provide a rationale for corporate liquidity management and renders it inherently dynamic.

We model cash and credit lines as serving different economic roles in the manner that they allow to transfer resources across time and states. Cash provides *uncontingent liquidity* in that it transfers liquid funds across all future states symmetrically. On the other hand, our model highlights the role of credit lines as providing *contingent liquidity* in that firms can draw on them so as to transfer funds to specific states only. While this suggests that credit lines provide a more effective liquidity management instrument, they typically are not only secured and require sufficient collateral, but also subject to a variety of covenants. The main economic mechanism that emerges in our model is therefore a trade-off between effective contingent liquidity by means of credit lines subject to collateral constraints and covenants and uncontingent liquidity provision through cash holdings.

We embed this tradeoff into a flexible dynamic model of corporate investment and financing in which firms' debt capacity is limited by the fraction of assets they can collateralize. More

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[1] See, for example, Bates, Kahle, and Stulz (2009).

[2] See, for example, Boileau and Moyen (2016), Bates, Kahle, and Stulz (2009) and Comin and Philippon (2006).

1

specifically, firms exploit time-varying and stochastic investment opportunities in the presence of costs of external finance, that we model as underwriting costs associated with equity offerings. Apart from equity issuances, investment expenditures can be funded by taking out loans, by drawing on revolving lines of credit, or by tapping cash reserves accumulated internally. Different from loans, the availability of the credit line gives the firm the option to obtain funding contingent on the realization of investment opportunities. In the presence of loan and credit line limits, which arise endogenously in the model through collateral constraints, corporations can accumulate cash in anticipation of future funding needs. For added realism, we also consider agency costs that impede cash holdings, capital adjustment costs, and a variety of covenants prevalent in credit line agreements. The result is a dynamic model in which a firm's economic environment is described by the endogenous state variables firm net worth, capital stock, and profitability.

We test the model using data on credit line usage from CapitalIQ and empirically assess its ability to rationalize key patterns in corporate liquidity policies by means of a structural estimation via simulated method of moments (SMM). Our estimation results show that the model matches quantitatively well the levels and joint dynamics of cash, credit limits, undrawn credit, leverage and equity issuance, along with investment and profits, when firms can collateralize up to roughly one third percent of their assets, and cash holdings are subject to minimal agency costs only.

The model highlights the importance of collateral as a determinant of corporate liquidity management. Similarly, it corroborates the notion that cash is not negative debt, in that it rationalizes the empirical evidence that firms simultaneously hold cash and debt. Within the context of our model, the intuition is simple. While debt and credit lines jointly allow for state-contingency within the limits of debt capacity, holding cash allows to transfer liquidity beyond collateral constraints in case of high financing needs. Such high financing needs most prominently arise when firms have ample profitable investment opportunities. Our model is thus consistent with the time series evidence in the data that firms tend to draw down their credit lines and tap their cash reserves to fund investment projects. Similarly, consistent with the empirical evidence in McLean (2011), firms resort to issuing equity to replenish their cash reserves once their debt capacity is exhausted.

Our model also rationalizes a wide range of empirical patterns regarding cross-sectional determinants of credit line usage, as documented in Sufi (2009). In particular, the dependence of credit line

usage on high profitability, emphasized in Sufi (2009), emerges naturally in our model. Intuitively, this is because current and lagged profitability are correlated with ample investment opportunities, exercising which builds up the collateral that helps securing credit lines. Similarly, as in the data, more profitable firms therefore rely relatively more on credit lines than cash in their liquidity management strategies.

Apart from profitability, the model also makes a number of cross-sectional predictions regarding the link between liquidity management policies and model state variables, that we find strong support in the data for. As cash holdings in the model are more valuable when collateral is scarce, we naturally find that smaller firms accumulate larger cash reserves, as in the data. Similarly, large firms with few growth options tend to rely less on liquidity management, and when they do so they draw on credit lines in a contingent manner. Interestingly, in sharp contrast to small firms, corporations with low net worth tend to rely more on contingent liquidity management through credit lines. Intuitively, this is because low net worth firms need not be small and thus may have ample collateral. However, rather than to fund investment opportunities they draw on credit lines to cover cash shortfalls.

The distinction between capital and net worth as determinants of corporate liquidity management arising in the model has direct counterparts in empirical work. We can think of small firms with little net worth as financially constrained firms with profitable investment opportunities, whose growth has only been impeded by the lack of sufficient funds and collateral. On the other hand, large firms with low net worth are more naturally described as financially distressed, as their funds have been erased by a sequence of bad shocks after a period of rapid growth. In the model, as in the data, such companies manage their liquidity quite differently. This is relevant, as in the literature the term 'financially constrained' is often used in conjunction with both types of firms. We find strong empirical support for our notions of distress and constraints, respectively, by relating commonly used indicators to our model state variables.

Interpreted in terms of these notions of financial constraints and financial distress, our model thus predicts that financially constrained and distressed firms manage their liquidity quite differently, in line with Sufi (2009). In particular, while financially constrained firms tend to rely mostly on cash and save out of cash flows, distressed firms' usage of credit lines is mostly restricted through

3

impending covenant violations. Indeed, when we extend our model to account for some of the most widely used covenants, such as net worth or debt-to-cash flow covenants, we find that more financially distressed and less profitable firms are more likely to violate covenants, and thus to incur restrictions on credit line access, as in the data. In this sense, while credit lines provide firms with contingent liquidity, that access is conditional on not violating covenants.

Our results suggest that access to revolving lines of credit creates significant value for firms. Indeed, in counterfactual experiments, we show that reductions in credit line access as a tool for liquidity management come with substantially reduced firm values. Intuitively, the contingent nature of credit lines gives firms an instrument to take advantage of investment opportunities in an effective manner, and thus gives them valuable flexibility to grow. Similarly, we show that a reduction in collateral comes with significant value losses, higher cash holdings, and less slack on credit lines and thus less flexibility to absorb shocks.

From a computational viewpoint, we introduce solution methods based on linear programming into dynamic corporate finance. Accounting for conditional liquidity management by means of state-contingent policies introduces a large number of control variables into our setup which would render our model subject to the curse of dimensionality for standard computational methods. We exploit and extend linear programming methods to circumvent this problem and efficiently solve for the value and policy functions in this class of problems. Linear programming methods, while common in operations research, have been introduced into economics and finance in Trick and Zin (1993, 1997). We extend their methods to setups common in corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin's constraint generation algorithm.

Related Literature. Our paper is at the intersection of several converging lines of literature. First, we build on the growing literature using dynamic models to explain quantitatively corporate investment and financing policies in the presence of financial frictions. Starting with Gomes (2001), a number of recent contributions include Cooley and Quadrini (2001), Hennessy and Whited (2005, 2007), Moyen (2004, 2007), Hackbarth and Mauer (2012), Sundaresan, Wang, and Yan (2015), or Hackbarth and Sun (2016). We add to this literature by explicitly considering firms' liquidity management.

4

In that regard, our paper is closely related to the growing literature modeling corporate cash policies. A non-exhaustive list of recent papers in this context includes Nikolov and Whited (2014), Gamba and Triantis (2008), Anderson and Carverhill (2012), Hugonnier, Morellec, and Malamud (2015a,b), Bolton, Chen, and Wang (2011, 2013, 2015), Falato, Kadyrzhanova, and Sim (2013), and Eisfeldt and Muir (2015). We differ from this line of work by not focusing exclusively on cash holdings as an instrument to manage liquidity, but by explicitly considering the dynamic choice between different liquidity management tools, namely cash and credit lines. In that respect, our work is most closely related to Bolton, Chen, and Wang (2011) and Boileau and Moyen (2016). While in the theoretical work of Bolton, Chen, and Wang (2011) firms do not hold cash and credit lines simultaneously, in Boileau and Moyen (2016) credit lines are modeled as defaultable short term debt.

Our model emphasizes the special nature of credit lines as a state-contingent instrument, in that firms can draw on them in particular states, subject to constraints. Our approach thus builds on the literature on optimal state-contingent dynamic contracts under limited commitment, such as Albuquerque and Hopenhayn (2004), and especially Rampini and Viswanathan (2010, 2013), Li, Whited, and Wu (2016), Ai and Li (2016), Ai, Kiku, and Li (2016), Zhang (2016), and Sun and Zhang (2016), for example. While this line of work operates in a dynamic optimal contracting framework, we take the form of the contracts as exogenously given and interpret them in the wider context of commonly used frictions in the dynamic financing literature, such as equity issuance costs and investment frictions. Most importantly, we represent the implied state-contingent payments by means of important real world securities, namely credit lines.

Another closely related representation of state-contingent payments, pioneered by Rampini and Viswanathan (2010, 2013), is by means of risk management practices. In this context, we could equally view credit lines as hedging instruments. Our work is thus also related to the recent models of dynamic risk management, such as Vuillemey (2016) and Bretscher, Schmid, and Vedolin (2016). More broadly, our paper is also related to recent work on structural estimation in corporate finance such as Taylor (2010), Taylor (2013), Korteweg (2010), Glover (2015), Morellec, Nikolov, and Schurhoff (2012), Morellec, Nikolov, and Schurhoff (2016), and Hackbarth and Sun (2016).

This paper is structured as follows. In section 1, we begin by illustrating the basic concepts through a simple snapshot into the lifecyle of a firm. A simple numerical example introduces

the main concepts and the trade-off between uncontingent and contingent liquidity management subject to collateral constraints. We embed this trade-off into a fully blown dynamic model of firm investment, financing and liquidity management in section 2. Section 3 introduces our empirical approach using a simulated method of moments (SMM) estimator and describes the data. We report and discuss our results in section 4, along with some additional empirical tests. Section 5 offers a few concluding remarks.
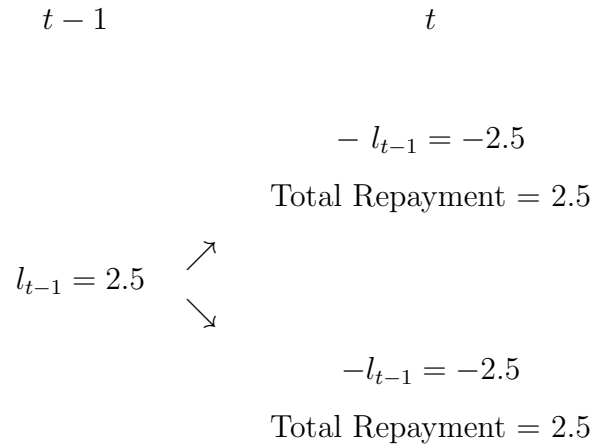
1   An Example

Before laying out our full dynamic model that we take to the data, in this section, we illustrate the main concepts and the key economic tradeoffs by means of simple examples. These examples are meant as simple snapshots in the lifecycle of a firm. For simplicity, we abstract from intertemporal discounting and interest payments in this section. Similarly, we take cash flows as exogenous for the sake of illustration, and show how they emerge endogenously from firms' dynamic optimization in the next section.

Contingent and Uncontingent Liquidity.   Consider a firm wishing to raise funds at some time $t-1$ in a stochastic environment. More precisely, the firm anticipates a good (G) and a bad (B) cash flow state to occur at time t with equal probabilities. We think of the good cash flow state at time t as triggering additional funding needs, for example through the arrival of valuable investment opportunities.

Clearly, one way to raise funds at time $t-1$ is by arranging a loan in the amount of, say, $l_{t-1} = 2.5$. Absent intertemporal discounting and assuming that the loan is riskfree[3], this arrangement triggers uncontingent liabilities of 2.5, to be repaid in both states at time t, as follows:

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[3]We will introduce conditions so that this is guaranteed later on.

$$t-1 \qquad\qquad\qquad\qquad t$$

$$-\ l_{t-1} = -2.5$$
$$\text{Total Repayment} = 2.5$$

$$l_{t-1} = 2.5 \qquad \nearrow$$
$$\searrow$$

$$-l_{t-1} = -2.5$$
$$\text{Total Repayment} = 2.5$$

Observe that, as of time $t-1$, the expected repayment at time t, $E_{t-1}[R_t]$, is equal to $l_{t-1} = 2.5$. The notion of a credit line that we entertain, and later on empirically evaluate in this paper, is just a slight generalization of this straightforward repayment schedule. More specifically, the next figure depicts an alternative repayment schedule, which, importantly, involves the same *ex-ante* expected repayment in that $E_{t-1}[R_t] = 2.5$. It differs from the standard loan repayment schedule in that the *ex-post* repayments are *contingent* on the realized state.

$$t-1 \qquad\qquad\qquad\qquad t$$

$$\text{Total Repayment} = 0$$

$$l_{t-1} = 2.5 \qquad \nearrow$$
$$\searrow$$
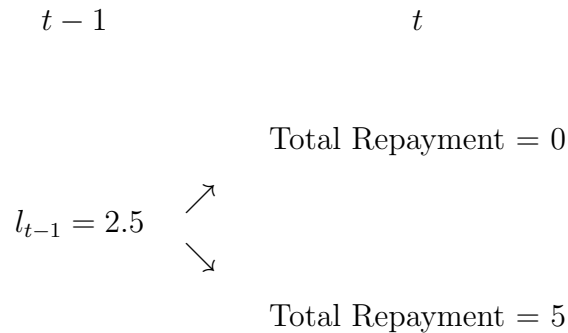
$$\text{Total Repayment} = 5$$

The concept of a credit line that we propose in this paper is based on an interpretation of the cash flows arising in this example. In particular, we can think of the zero net repayment in the good state at time t as the contractual repayment on the loan, in the amount of $l_{t-1} = -2.5$ which is offset by a draw on a previously established revolving credit facility with a bank, in the same amount of 2.5. Under the assumption that the good state triggers additional funding needs, we can interpret this draw on the credit line as providing liquidity *contingent* on the realization of the

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good state. This is because the reduced net repayment in this state effectively frees up resources for alternative use, such as financing investment.

Symmetrically, absent additional funding needs for valuable investment projects, in the bad state the firm will not only honor its outstanding loan of 2.5, but also restore the credit line by an additional payment to the bank in the amount of 2.5, leading to a total repayment of 5.

More formally, let us denote the previous balance of the credit line as cl_{t-1}, and the balances in the good and the bad state respectively as $cl_t(G)$ and $cl_t(B)$. The following figure illustrates the ensuing repayment schedule and the payments on the corresponding securities.

$$t - 1 \qquad\qquad\qquad\qquad t$$

$$-\ l_{t-1} = -2.5$$
$$cl_t(G) - cl_{t-1} = +2.5$$
$$\text{Total Repayment} = 0$$

$$l_{t-1} = 2.5 \qquad \nearrow$$
$$\searrow$$

$$-l_{t-1} = -2.5$$
$$cl_t(B) - cl_{t-1} = -2.5$$
$$\text{Total Repayment} = 5$$
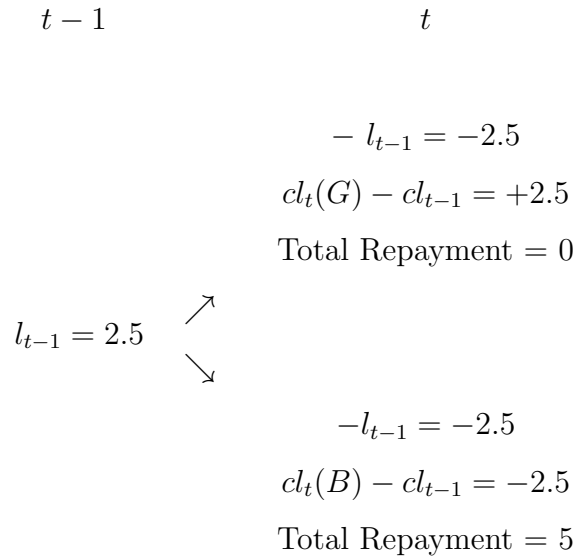
At time $t - 1$, the firm arranges a riskfree loan $l_{t-1} = 2.5$, to be repaid in both states at time t. At time t, the firms draws 2.5 from the credit line in the good state, increasing the previously drawn part cl_{t-1} to $cl_t(G)$, such that the firm raises liquid funds with a cash inflow $cl_t(G) - cl_{t-1} = 2.5$. By drawing this amount, the total debt repayment in the good state is

$$l_{t-1} - (cl_t(G) - cl_{t-1}) = 0.$$

Similarly, in the bad state, the firm decreases the previously drawn part cl_{t-1} to $cl_t(B)$, with a cash outflow $cl_{t-1} - cl_t(B) = 2.5$, and an effective repayment

$$l_{t-1} - (cl_t(B) - cl_{t-1}) = 5.$$

8

Under this interpretation, compared to the case with a riskfree loan only, the firm effectively transfers liquidity to the good state in the amount of 2.5 at time t, contingent on that state being realized. In this sense, credit lines provide *contingent* liquidity. Notice that all securities are fairly priced in this setting in that the present value of the actual repayments at time t is exactly the amount the firm raises at time $t-1$.

Observe that the concept of contingent liquidity we entertain here differs from setups in which the availability to borrow from a credit line is also conditional on not violating certain covenants, as Sufi (2009) and Acharya, Almeida, Ippolito, and Perez (2014) document. We explore the role of covenants as restrictions to access credit lines in section 4.3.

Cash, Loans, Credit Lines and Negative Debt. In this example, firms have access to three different securities to raise debt financing and implement liquidity management, namely cash, loans, and credit lines. Uncontingent loans and credit lines are distinct debt instruments because of the state-contingent nature of credit lines.

Clearly, in the absence of a revolving credit facility providing contingent liquidity, a firm anticipating funding needs in the good state at time t might consider saving at time $t-1$ by setting aside cash for use in the good state. These savings will be available to the firm even upon realization of the bad state, absent funding needs. Accordingly, we think of cash holdings as providing *uncontingent* liquidity. In our example, setting aside cash at time $t-1$ to offset the loan payment of 2.5 in the good state requires a cash balance of 2.5 in that state. This cash balance will equally offset the loan payment in the bad state. As a consequence, in this simple case, cash is negative debt in the absence of revolving lines of credit.

When a credit line is introduced, the firm can engage in liquidity management more effectively, by drawing liquid funds only in those specific states where those resources are needed. In this case, contingent liquidity allows for example to fund investment in the good state at time t, where a profitable opportunity is available. More generally, contingent liquidity can also be used to have a larger available buffer to hedge income shortfalls in bad states and avoid engaging in costly asset fire sales. As a consequence, implementing contingent liquidity management using credit lines helps firms transfer resources to states where funds are most valuable.

9

If contingent liquidity management is more efficient, why do firms use cash to transfer uncontingent liquid funds at all? The answer is that the amount of resources the firm can transfer to a specific state is limited by the presence of collateral constraints, whose presence ultimately results in a limit on the line of credit. For example, in the good state at time t it might be valuable to have more liquidity available in the good state for future investment than what the firm can transfer only in a contingent way. Thus, the firm could find it beneficial to combine contingent and uncontingent liquidity management in order to take advantage of investment opportunities. In sum, a tradeoff between contingent liquidity (effective but constrained), and uncontingent liquidity (less effective but unconstrained) emerges. Unlike in the case with loans and cash only, this tradeoff endogenously generates the co-existence of cash and debt in firms' balance sheets. To see this, notice that if the good state realizes the firm simultaneously holds cash and debt in the form of drawn credit to be eventually repaid to the lenders.

2   Dynamic Model

We now embed the notions of contingent and uncontingent liquidity by means of credit lines and cash, respectively, into a dynamic model that we can take to the data. A continuum of firms i make corporate investment, financing and liquidity management decisions. For added realism, we also consider capital adjustment costs, taxes, issuance and agency costs, and a credit line fee. Due to costs of external financing, corporations will benefit from the availability of liquid funds, either to fund investment opportunities or to cover cash shortfalls. Firms can provide liquid funds by saving through cash holdings, or by drawing on credit lines in a contingent manner. We assume that the availability of credit lines is restricted by collateral constraints that limit firms' debt capacity.

2.1   Model Setup

Technology and Investment.   We consider the problem of a value-maximizing firm in a perfectly competitive environment. Time is discrete. After-tax operating profits for firm i in period t depend upon the capital stock k_{it} and a shock z_{it} and are given by

$$\pi(k_{it}, z_{it}) = (1 - \tau)(z_{it} k_{it}^\alpha - f), \tag{1}$$

10

where $0 < \tau < 1$ denotes the corporate tax rate, $0 < \alpha < 1$ is the capital share in production, and $f > 0$ is a fixed cost incurred in the production process. Note that a capital share less than unity captures decreasing returns to scale. The variable z_{it} reflects shocks to demand, input prices, or productivity and follows a stochastic process with bounded support $Z = [\underline{z}, \overline{z}]$, with $-\infty < \underline{z} < \overline{z} < \infty$, and described by a transition function $Q_z(z_{it}, z_{it+1})$[4].

At the beginning of each period the firm is allowed to scale its operations by choosing its next period capital stock k_{it+1}. This is accomplished through investment i_{it}, which satisfies the standard capital accumulation rule

$$k_{it+1} = k_{it}(1 - \delta) + i_{it}, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of capital. Investment is subject to capital adjustment costs. As in Bolton, Chen, and Wang (2011), for example, we follow the neoclassical literature (Hayashi, 1982) and consider convex adjustment costs for simplicity. We parameterize capital adjustment costs with the functional form

$$\Psi(k_{it+1}, k_{it}) \equiv \frac{1}{2}\psi \left(\frac{i_{it}}{k_{it}} \right)^2 k_{it}, \tag{3}$$

where the parameter ψ governs the severity of the adjustment cost.

Financing and Liquidity Management. To fund investment opportunities and to cover cash shortfalls, firms have access to a realistic set of securities to transfer funds across time and states. Apart from using internally generated cash flows, they can contract riskfree loans l_{it} in debt markets, draw on or restore a credit line with balance cl_{it} at a bank, hoard cash c_{it}, or issue equity. We slightly generalize the simple example of the previous section to add realism in that loans benefit from a tax advantage and credit lines provide contingent liquidity, but both are limited by collateral constraints. Additionally, holding a balance on a credit line comes with an additional fee ξ. Cash provides unlimited but uncontingent liquidity and is subject to agency costs. Finally, issuing equity comes with flotation costs.

More formally, we assume that issuing a loan l_{it} at time $t - 1$ requires a repayment of the principal plus interest net of a tax deduction in the amount of $(1 + r(1 - \tau))l_{it}$ at time t. The

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[4]In our empirical work, we parameterize z_{it} so as to provide a discrete approximation to a continuous AR(1) process with persistence ρ_z and conditional volatility σ_z.

tax deduction recognizes the preferential tax treatment of interest rate payments. On the other hand, setting aside at time $t - 1$ an amount of cash c_{it} will leave the firm with additional resources $(1 + r(1 - \tau) - \gamma)c_{it}$ at time t. This specification captures that cash holdings earn interest, but are disadvantaged through an agency cost γ, similar to Nikolov and Whited (2014).

We emphasize that both loans and cash holdings are decided at time $t - 1$ and are therefore uncontingent on the realization of the profitability shock z_{it} at time t. In contrast, credit lines provide liquidity contingent on the realization of z_{it}. At time $t - 1$, given the credit line, firms and banks agree on next period's balance on the credit line cl_{it}, contingent on next period's realization of z_{it}. Formally, therefore, we have $cl_{it} \equiv cl_{it}(z_{it})$. That balance reflects the amounts the firm has drawn on the credit line over time, and therefore, has to be restored in the future. These draws accumulate interest, and banks need to monitor them, resulting in extra costs ξ for the firm over the risk-free interest rate. Such fees are common in credit line contracts.

At the beginning of period $t - 1$, the balance outstanding on the credit line is thus $(1 + r)cl_{it-1}(z_{it-1})$. That amount needs to be restored over time. On one hand, some of that restoration can take place in period $t-1$, lowering future balances, while the firm might as well find it beneficial to draw additional funds from the credit line, raising expected balances. Fair pricing requires that when firms and bank agree on next period's balances $cl_{it}(z_{it})$, expected discounted future balances amount to $E_{t-1}\left[cl_{it}(z_{it})\right]$. Accordingly, the current restoration is given by

$$cl_{it-1}^{R}(z_{it-1}) = (1 + r)cl_{it-1}(z_{it-1}) - E_{t-1}\left[cl_{it}(z_{it})\right]. \tag{4}$$

A negative restoration corresponds to an additional draw on the credit line. Choosing next period's balances $cl_{it}(z_{it})$ in a state-contingent manner thus effectively allows to redistribute the need for future restorations and the possibility for additional draws across states. In this sense, the credit line allows to transfer resources across states and provides an instrument for contingent liquidity management.

We assume that the balance on the credit lines and the loans need to be fully collateralized by the amount of capital that lenders could recover in case of a liquidation, so that both credit lines and loans are effectively riskfree. We denote by $0 < \theta < 1$ the fraction of capital that can be

12

pledged as collateral. We interpret $\theta < 1$ as an indication that some capital may be intangible and cannot be pledged as collateral. An alternative interpretation is that some capital may be sold off in fire sales at discounted prices. Formally, this requires that

$$(1 + r(1 - \tau))l_{it+1} + (1 + r(1 - \tau) + \xi)cl_{it+1}(z_{it+1}) \leq \theta(1 - \delta)k_{it+1}, \qquad \forall z_{it+1}. \tag{5}$$

In other words, $cl_{it+1}^L \equiv \frac{\theta(1-\delta)k_{it+1} - (1+r(1-\tau))l_{it+1}}{1+r(1-\tau)+\xi}$ provides a time-varying limit on credit line draws, which is decreasing in loan commitments.

It is convenient to define w_{it}, the net worth of firm i at time t, as the resources that are available at the beginning of period t, after the realization of the shock. We have

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\begin{aligned}
w_{it} & \equiv & \pi(k_{it}, z_{it}) + (1 - \delta(1 - \tau))k_{it} + (1 + r(1 - \tau) - \gamma)c_{it} \\
& & -(1 + r(1 - \tau))l_{it} - (1 + r(1 - \tau) + \xi)cl_{it}(z_{it}).
\end{aligned} \tag{6}
$$

In words, current resources available to the firm are realized profits, the depreciated capital (net of a tax allowance for depreciation), cash set aside last period net of loan repayments committed last period, net of the current balance on the credit line cl_{it}. Since net worth measures available resources after the realization of the shock, and the current balance can be high, requiring higher future restorations, or low, enabling higher draws, depending on the shock, the credit line indeed provides the firm with contingent liquidity.

Firms can use such new draws net of restorations, net worth and new loans to finance equity payouts, investment expenditures or to set aside cash for the future. Firms' budget constraint therefore requires that resources to fund equity payouts e_{it} to shareholders are given by

$$e_{it} = w_{it} + l_{it+1} + E_t\left[cl_{it}(z_{it+1})\right] - c_{it+1} - k_{it+1} - \Psi(k_{it}, k_{it+1}). \tag{7}$$

When $e_{it} \geq 0$, the firm is making distributions to shareholders, while we interpret $e_{it} < 0$ as a seasoned equity offering, which entails flotation costs. Equity issuance costs are given by

$$(\lambda|e_{it}|)\mathbf{1}_{\{e_{it}<0\}}. \tag{8}$$

13

The indicator function denotes that the firm faces these costs only in the region where the net payout is negative. Accordingly, distributions to shareholders d_{it} are the equity payout net of issuance costs:

$$d_{it} = e_{it} - (\lambda|e_{it}|)\mathbf{1}_{\{e_{it}<0\}}. \tag{9}$$

Credit Lines, Loans and Cash. The budget constraints (6) and (7) show that in our setup the credit line can be equivalently described as a contingent security which generates a payoff $(1+r)cl_{it}(z_{it})$ in state z_{it} at time t given an upfront payment $E_t[cl_{it}(z_{it})]$ at time $t-1$. As in the example of section 1, credit lines, unlike straight loans and cash, are state-contingent. The firm debt capacity is limited by collateral constraints, to which both loans and credit lines are subject and that determine the credit line limit cl_{it+1}^L. In addition, all three securities are affected by taxes, which generate a tax shield for the two debt instruments and reduces the effective return the firm earns on its cash savings. Finally, the three securities differ because cash holdings are affected by agency costs and credit lines entail monitoring costs, as the parameters γ and ξ, respectively, capture.

Valuation. Managers choose investment, financing, and liquidity management to maximize the wealth of shareholders. Hence, in period t, they decide over real capital k_{it+1}, loans l_{it+1}, cash c_{it+1}, and credit line balances $cl_{it+1}(z_{it+1})$, for each state z_{it+1}. These investment, financing, and liquidity management decisions are intimately related as they need to satisfy firms' budget identities between sources and uses of funds both at time t, and for each state at time $t+1$:

$$w_{it} + E_t[cl_{it}(z_{it+1})] + l_{it+1} = e_{it} + k_{it+1} + \Psi(k_{it}, k_{it+1}) + c_{it+1}, \tag{10a}$$

$$w_{it+1}(z_{it+1}) = \pi(k_{it+1}, z_{it+1}) + k_{it+1}(1-\delta) - (1+r(1-\tau))l_{it+1}$$
$$+ (1+r(1-\tau)-\gamma)c_{it+1} + \tau\delta k_{it} - (1+r(1-\tau)+\xi)cl_{it-1}(z_{it-1}), \quad \forall z_{it+1}. \tag{10b}$$

This formulation suggests that we can use w_{it} as a convenient state variable to describe the relevant economic environment of the firm, together with k_{it} and z_{it}. Indeed, firms' equity value $V(w_{it}, k_{it}, z_{it})$

satisfies the following dynamic programming problem:

$$V(w_{it}, k_{it}, z_{it}) \equiv \max_{k_{it+1}, l_{it+1}, cl_{it+1}(z_{it+1}), c_{it+1}} \left\{ d_{it} + \frac{1}{1+r} E_t \left[V(w_{it+1}, k_{it+1}, z_{it+1}) \right] \right\}. \tag{11}$$

Managers' optimization problem is subject to the budget constraints (10a) and (10b), the collateral constraints (5), and the capital accumulation constraint (2). In addition, we require that loan amounts, cash, and importantly, credit line balances be non-negative.

Covenants. In practice, as Sufi (2009) documents, credit lines are often not only secured, but subject to additional restrictions such as covenants. Most covenants come in the form of bounds on certain financial ratios. Any violation of such bounds comes with a freeze of the facility in that further draws are infeasible. We now provide an extension of the baseline model, in which we account for some of the covenants most widely found in credit line agreements.

We focus on two covenants that are both among the most common ones, and also readily representable in our framework, namely net worth and debt-to-cash flow covenants. The net worth covenant requires that scaled net worth exceed a threshold t_w, in that

$$\frac{w_{it}}{k_{it}} \geq t_w \tag{12}$$

at all times. Similarly, we model a debt-to-cash flow covenant by assuming that the ratio of debt to EBITDA be bounded by a threshold t_{CF}, in that

$$\frac{k_{it} - w_{it}}{(1-\tau)z_{it} \left(\frac{k_{it}}{1-\delta}\right)^{\alpha} - f} \leq t_{CF} \tag{13}$$

at all times. This way of modeling covenants amounts to imposing additional restrictions in firms' dynamic optimization programs, captured in (11). We assume that a covenant violation leads to a freeze of the credit line in that the drawn part can no longer increase. More formally, we require that in such scenarios, we have that

$$(1+r)cl(z_{it}) \geq cl(z_{it+1}), \forall z_{it+1}.$$

15

This formulation makes it clear that in the presence of covenants, credit lines do provide contingent liquidity, but only *conditional* on not violating the covenants.

2.2   Determinants of Liquidity Management

In the presence of costs of external finance, firms can benefit from liquidity management. To provide some qualitative guidance on the determinants of liquidity management through the lens of the model, we first examine its optimality conditions and then inspect the policy functions obtained from a numerical solution.

Obtaining a numerical solution of the model is significantly complicated by the large number of control variables associated with state-contingent draws on the credit line, and the presence of occasionally binding constraints, rendering standard discrete state-space value and policy function iteration methods infeasible. Rather, we develop a novel and efficient approach to solving dynamic models in corporate finance and overcome this difficulty by exploiting the linear programming (LP) representation of dynamic programming problems with infinite horizon (Ross (1983)), building on Trick and Zin (1993), and Trick and Zin (1997). This solution technique is more widely applicable to high-dimensional dynamic models. Appendices A and B provide details on the implementation of the solution method.

To illustrate the value of liquidity management qualitatively, we start by attaching multipliers λ_w, $\lambda_w(z_{it+1})$, and $\lambda_c(z_{it+1})$, respectively, to the budget constraints (10a) and (10b), and to the collateral constraints (5), and examine the first order conditions of the firm's optimization problem (11)[5]. In the following, we discuss the optimality conditions pertaining to firms' liquidity, investment and payout policies.

Formally, the envelope condition $V_w(w_{it}, k_{it}, z_{it}) = \lambda_w$ shows that the multiplier λ_w is simply the marginal value of net worth. In a frictionless world, clearly, the marginal value of net worth is just one. On the other hand, from the first order condition for dividend payments, in our model we have that

$$1 - \lambda_w = \Lambda(d_{it}), \tag{14}$$

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[5]The multipliers on (10b) and (5) are scaled by the corresponding transition probabilities $Q_z(z_{it}, z_{it+1})$ divided by $1 + r(1 - \tau)$ for analytical convenience. In addition, for the sake of illustration, we assume that the optimization problem is sufficiently differentiable.

so that the marginal value of net worth is different from one in the presence of equity issuance costs, and time-varying. Close to the issuance boundary, an additional dollar is more valuable as it allows firms to avoid the flotation costs that come with issuance. Firms thus have an incentive to engage in liquidity management so as to avoid paying these costs. Intuitively, firms find themselves at the issuance boundary in times and states of elevated funding needs, due to investment expenditures or to cover cash shortfalls.

Optimal liquidity management thus entails transferring funds to times and states where these are most valuable, that is, to high $\lambda_w(z_{it+1})$ states. Clearly, saving cash in anticipation of future funding needs is one way to transfer funds across time. However, as the first order condition for cash savings indicates[6],

$$\overbrace{E_t\left[\frac{1+r(1-\tau)-\gamma}{1+r(1-\tau)}\lambda_w(z_{it+1})\right]+\lambda_{c+}}^{\text{marginal benefit of }c_{it+1}} = \overbrace{\lambda_w}^{\text{marginal cost of }c_{it+1}},$$

the marginal benefits of cash holdings are reduced through holding costs γ and, importantly, spread out over future states, including such in which net worth provides little value. In this sense, holding cash provides uncontingent liquidity.

In contrast, drawing on credit lines allows to transfer funds in a contingent manner to specific states according to the relevant marginal value of net worth. Specifically, the first order condition shows that optimal use of credit lines entails allocating the costs of additional net worth today across states next period. These costs, however, are not restricted to lower net worth in the future, but also involve tightening of the collateral constraints, as reflected by the appearance of the multiplier $\lambda_c(z_{it+1})$ in the first order conditions:

$$\underbrace{\lambda_w}_{\text{marginal benefit of }cl_{it+1}(z_{it+1})} = \overbrace{\frac{(1+r(1-\tau)+\xi)}{(1+r(1-\tau))}\left(\lambda_w(z_{it+1})+\lambda_c(z_{it+1})\right)}^{\text{marginal cost of }cl_{it+1}(z_{it+1})}.$$

The state-contingent nature of credit lines requires additional monitoring from banks, reflected in the fee ξ. Straight loans provide funds today more cheaply, but the costs are spread out over future

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[6]λ_{c+} denotes the multiplier on the non-negativity constraint that defines the lower bound of the state space for cash.

states, as the first order condition illustrates

$$\overbrace{\lambda_w}^{\text{marginal benefit of } l_{it+1}} = \overbrace{E\left[\lambda_w(z_{it+1}) + \lambda_c(z_{it+1})\right]}^{\text{marginal cost of } l_{it+1}}.$$

Formally, therefore, as $\frac{(1+r(1-\tau)+\xi)}{(1+r(1-\tau))} > 1$, loans are preferred for the state-uncontingent inframarginal units.

The first-order conditions with respect to cash, loans, and credit balances unfold the intertemporal tradeoff that drives dynamic corporate liquidity choices in our model. Contingent liquidity is preferable to uncontingent liquidity, but limited through collateral constraints. Firms can preserve limited contingent liquidity in anticipation of future states in the form of undrawn credit or exhaust their debt capacity to fund current investment expenditures and equity distributions.

In our model with financial frictions, liquidity management is intimately intertwined with firms' investment decisions. Indeed, investing today means giving up funds in exchange for state contingent proceeds in the future:

$$\overbrace{E_t\left[M_{t,t+1}^w\left(\frac{\frac{\partial\Psi(k_{it+1},k_{it+2})}{\partial k_{it+1}}}{1+r} + \frac{\pi_k(k_{it+1},z_{it+1}) + (1-(1-\tau)\delta)}{(1+r(1-\tau))} + \frac{\lambda_c(z_{it+1})\theta(1-\delta)}{\lambda_w(z_{it+1})(1+r(1-\tau))}\right)\right]}^{\text{marginal benefit of investment}} = \overbrace{1 + \frac{\partial\Psi(k_{it},k_{it+1})}{\partial k_{it+1}}}^{\text{marginal cost of investment}},$$

where $M_{t,t+1}^w \equiv \frac{\lambda_w(z_{it+1})}{\lambda_w}$ is the effective stochastic discount factor for the firm. In other words, in the presence of financial frictions, when the marginal value of net worth is state dependent, the firm behaves as if effectively risk averse since the firm's value function is (weakly) concave in net worth[7]. In addition, investing today helps not only reducing adjustments costs in the future (first term) and generating anticipations of future profits (second term), but also builds up collateral (last term).

Figure 1 illustrates the optimal investment and liquidity management policies in a numerical solution of the model. To begin with, in panel A, the equity value as a function of net worth w_{it} is slightly concave when net worth is low. This reflects, in line with first-order condition (14), that in the presence of financial frictions the marginal value of net worth depends on the state of the firm, and makes the firm effectively endogenously risk averse. As panel B illustrates, firms with low

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[7]Standard dynamic programming arguments ensure that the Bellman operator is a contraction mapping and has a unique fixed point. In addition, the value function is continuous, increasing, and weakly concave in net worth.

net worth do not pay dividends and possibly issue costly external equity. In the rightmost region of panel B, firms are unconstrained in financing investment (as panel E shows) and they allocate a part of their net worth to dividend payments. In this region, the marginal value of net worth is one and the value function in panel A becomes linear, as the envelope condition formally shows.

The policy functions for cash and draws on the credit line show how firms transfer funds to states in which the marginal value of net worth is high. Such a transfer is most effectively done by state-contingent draws on the credit line. Indeed, panels D and I show that firms exhaust their credit limits against high profitability states. With persistence, high profitability signals valuable investment opportunities in the future that generate funding needs. In this illustration, firms do not draw against low profitability states, implying that funding needs for investment override those stemming from liquidity shortfalls.

[Insert Figure 1 Here]

This result is sensitive to parameterizations as absent persistent investment opportunities, firms equally draw against high and low states, and thus use credit lines to cover unexpected cash shortfalls in bad states, as well as investment opportunities, as panel A of figure 2 shows[8]. Increasing persistence makes current shock realizations more informative about future realizations. Panels B and C of figure 2 illustrate that with increasing persistence, firms tilt their draw towards recently realized state. High current realizations increase firms' motives to use credit lines to cover capital expenditures triggered by future high cash flow states, while they start drawing against low states after a sequence of adverse shocks. These effect harmonize with the survey evidence in Lins, Servaes, and Tufano (2010), who show that corporate CFOs draw from credit lines not only against negative profitability shocks, but also to fund future investment opportunities.

[Insert Figure 2 Here]

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[8]Observe that the amounts drawn in panel A are almost flat because when investment opportunities are i.i.d. as current profitability is uninformative about both future investment opportunities or cash shortfalls.

Panel I of figure 1 shows that larger firms draw more credit and are given higher credit limits, as they possess more collateral to pledge. On the other hand, we see in panel J that smaller firms possess more investment opportunities that require funding. Panel H shows that these firms manage these liquidity needs by substituting drawn credit with cash holdings. While liquidity management by means of cash is less effective, it does not require the collateral that small firms first need to build up.

Notably, firms with high net worth manage their liquidity quite differently than large firms, as panels D and E indicate. This is because in our model with capital adjustment costs, large firms can possess excess capital which consumes net worth. On the other hand, small firms also may have low net worth as they have not yet grown sufficiently to accumulate net worth. This illustrates how, in our model, size (as measured by capital) and net worth are economically different, albeit likely correlated, firm characteristics.

Firms with high net worth invest more in the model, as net worth is also a result of high past profitability. This investment is financed both using cash and by drawing on credit lines. Notably, while firms with higher net worth exhaust their credit limits against high profitability states, just as larger firms do, they draw less and less against low profitability states. This is intuitive, as for firms with low net worth, covering cash shortfalls with credit line draws becomes increasingly important because, as discussed above, they behave as effectively risk averse. Small firms, in turn, need not have low net worth so that they are not necessarily subject to funding constraints.

The distinction between capital and net worth as determinants of corporate liquidity management, emphasized in our model and illustrated with the policy functions, has direct counterparts in empirical work. We can think of small firms with little net worth, as financially constrained firms with profitable investment opportunities whose growth has only been impeded by the lack of sufficient funds and collateral. On the other hand, large firms with low net worth are more naturally described as financially distressed, as their funds have been erased by a sequence of bad shocks after a period of rapid growth. In our model, such companies manage their liquidity quite differently. This is relevant, as in the literature the term 'financially constrained' is often used in conjunction with both types of firms. We provide empirical content supporting this distinction in section 4.3.

20

3 Structural Estimation

We formally estimate the parameters of our model by means of a simulation-based estimator, namely the simulated method of moments (SMM). In this section, we describe the data and our estimation method and provide a discussion on identification. We present results, counterfactuals, and further empirical tests in the next section.

3.1 Data Sources

Estimating the dynamic corporate liquidity model requires merging data from different sources. In particular, we obtain financial statements data from the Compustat annual files and credit line data from Capital IQ. We remove all regulated (SIC 4900-4999) and financial firms (SIC 6000-6999). Observations with missing total assets, market value, gross capital stock, cash, long-term debt, debt in current liabilities, credit line limit, drawn portion of the credit line, and SIC code are excluded from the final sample. We obtain a panel dataset with 19,796 observations for 3,424 firms for the period of 2002 to 2011 at the annual frequency.

3.2 Measurement and Implementation

Appendix D provides definitions of variables used. Most variable definitions are standard in the literature. We measure leverage as short and long-term debt (DLC + DLTT) minus the drawn part of the credit line (RC) over total assets. We subtract the drawn part of the credit line because in the model as well as in the structural estimation we distinguish between straight debt and credit lines.

As indicators of collateral, we construct two proxies of tangibility and two proxies of intangibility. We measure tangibility as the ratio of fixed assets (PPENT) to total assets (AT), and as in Berger, Ofek, and Swary (1996) and Almeida and Campello (2007), that is 0.715 RECT + 0.547 INVT + 0.535 PPEGT divided by AT, in which RECT is total receivables, INVT is total inventory, and PPEGT is gross property, plant and equipment. We measure intangibility using a standard perpetual inventory method as in Falato, Kadyrzhanova, and Sim (2013). We respectively capitalize

21

real R&D expenditures (XRD) at a rate of 0.15, and real SG&A expenditures (XSGA) at a rate of 0.2. Both capitalized quantities are scaled by the sum of their own value plus real net book assets (AT-CHE). All real variables are deflated by the consumer price index and expressed in 2000 dollars.

3.3 Estimation

We estimate the key structural parameters of interest using the simulated method of moments (SMM). However, we estimate some of the model parameters separately. For example, we set the risk-free interest rate, r, equal to the average one-year Treasury rate over the sample period. We set the depreciation of capital, δ, equal to 12%, which is the average depreciation rate in the Compustat dataset. Finally, we set the corporate tax rate equal to 20%. This rate is an approximation of the corporate tax rate relative to personal taxes.

We then estimate 9 parameters using the simulated method of moments: the curvature of the profit function, α; the fixed production cost, f; the serial correlation of $ln(z)$, ρ_z; the standard deviation of the innovation of $ln(z)$, σ_z; the capital adjustment cost, ψ; the debt capacity, θ; the equity flotation cost, λ, the agency cost parameter, γ, and the credit line fee, ξ.

The simulated method of moments, although computationally intensive, is conceptually simple. We solve the model numerically using the linear programming approach described in appendix B and generate simulated data. Then, we compute relevant moments from both simulated and observed data. The SMM estimator selects the parameters such that a weighted distance between simulated and actual moments is minimized.

One important aspect of the SMM estimation is the choice of the weighting matrix. A natural candidate for this choice is the identity matrix. While intuitive, this choice comes with a significant drawback as it allocates more weight to the moments that are larger in absolute value. Given the lack of a valid economic interpretation, we rather choose to use the optimal weighting matrix obtained as the inverse of the covariance matrix of the moments. Intuitively, this method allocates more weight on the moments that are measured with greater precision. To compute the covariance matrix of the moments, we follow the influence function approach of Erickson and Whited (2000).

Finally, one last aspect of the estimation relates to unobserved heterogeneity. Indeed, our model generates predictions for a representative firm. However, for the estimation, we use data from Compustat and Capital IQ, i.e. a panel data set. To have consistency between the simulated and the observed data, we need to either add heterogeneity to the simulated data or remove heterogeneity from the observed data. We select the second approach. To do so, we use firm and year fixed effects when we estimate variances, covariances, and regression coefficients. The details of the SMM procedure are relegated to appendix C.

3.4   Identification

Before proceeding with the estimation of the model, it is important to understand how we can identify the model parameters in the data. A sufficient condition for identification is a one-to-one mapping between the structural parameters and a set of data moments of the same dimension. Obtaining such a closed-form mapping is challenging in any economic model. As a consequence, to achieve identification, we select a set of moments such that every estimated parameter has a differential impact on this set of moments. Heuristically, a moment h is informative about an unknown parameter β if that moment is sensitive to changes in the parameter and the sensitivity differs across parameters. Formally, local identification requires the Jacobian determinant, $det(\partial h/\partial \beta)$, to be nonzero. To aid in the intuition of the identification of the model parameters, we compute elasticities of the model-implied moments with respect to the parameters, $(\partial h/\partial \beta)/(\beta/h)$. Inspection of these elasticities reveals that the condition $det(\partial h/\partial \beta) \neq 0$ holds, so that we can separately identify the parameters of the model.

More specifically, we select 17 moments that relate to the distributions of cash, credit lines, leverage, operating income, investment, and equity issuance. Average profitability primarily identifies the curvature of the profit function α. Next, the variance and autocorrelation of profits directly identify the parameters σ_z and ρ_z. The fixed operating cost, f, increases the need for liquidity management and is identified by average cash and undrawn credit. The capital adjustment cost ψ directly affects the pace and size of investment changes and is identified by the variance and autocorrelation of investment. Average credit line limit along with average leverage help identify the debt capacity parameter θ. Average equity issuance and cash help identify equity flotation

23

costs. The agency cost parameter, γ, is identified by average cash and undrawn credit. Finally, the credit line fee parameter, ξ, is primarily identified by average undrawn credit.

3.5 Comparative Statics

Figures 3 and 4 provide further insights into the moment sensitivity to the parameters by means of a comparative statics analysis of simulated moments with respect to some key model parameters. The parameter values underlying the figures are the benchmark estimates detailed in section 4.1 and reported in table 1.

Figure 3 reports the sensitivity of simulated moments of the cash-to-asset ratio, undrawn credit and leverage with respect to some key technological parameters. When the curvature of the profit function falls (that is, when α increases), firms tend to invest in a lumpier fashion. This not only triggers larger liquidity needs but also builds up collateral. At the same time, profits are less spread out across states, so that cash holdings become relatively more attractive relative to credit line draws and loans. Higher operating leverage through higher fixed costs (f) requires higher liquidity across all states which is mainly absorbed in higher cash holdings, while firms conserve more debt capacity by choosing lower leverage. Perhaps not surprisingly, contingent instruments are not as effective at absorbing costs accruing equally across all states, underlying the insensitivity of credit line draws to changes in fixed costs. Corporations react to more pronounced cash flow risks (higher σ_z and ρ_z) by accumulating cash and preserving debt capacity reflected in less drawn credit.

[Insert Figure 3 Here]

Figure 4 documents moment sensitivity to key investment, financing, and agency parameters. Investment adjustment costs only accrue conditional on investment, so that the higher funding needs associated with higher adjustment costs are absorbed using contingent instruments, explaining higher credit line draws and leverage. Higher debt capacity leads firms to issue more loans and keep more slack on their credit limits as this gives them the flexibility to respond effectively to shocks. Higher equity flotation costs raise funding needs so that firms accumulate more cash, and preserve slack on the credit limit so as to be able to respond to funding contingencies. Clearly, raising the agency costs of cash makes cash holdings less attractive relative to debt instruments. A higher credit line fee renders contingent liquidity more expensive and results into larger undrawn credit.

24

[Insert Figure 4 Here]

4 Empirical Results

We now present our quantitative results. We start by discussing the structural estimation results along with some counterfactuals. We then present a number of additional empirical implications and tests guided by our framework.

4.1 Estimation Results

Table 1 presents the main results of the structural estimation. Panel A reports simulated and actual moments. Panel B reports structural parameter estimates and their corresponding standard errors.

Overall, panel A shows that our dynamic liquidity model fits the data reasonably well. In particular, the model performs very well in matching average cash, credit line limit, operating income, and investment. The t-statistics that accompany these moments show that the difference between the actual and the simulated moments is insignificant. The model performs reasonably well in matching average undrawn credit and leverage. For these moments, even if the difference between actual and simulated moments is statistically significant, economically this difference is negligible. In addition, the model quantitatively rationalizes leverage and cash holdings simultaneously well, corroborating the fact that cash is not negative debt in our setting. The average moment that is poorly matched is equity issuance.

The model also captures well moments that describe the dynamic nature of corporate liquidity decisions. Indeed, the model matches both variances and autocorrelations of all variables of interest at the exception of the variances of undrawn credit and leverage. Although the difference between the actual and the model moments here is statistically significant, these moments have the same order of magnitude, so that they are economically comparable.

Panel B shows that all model parameters are economically plausible and statistically significant. In addition, the estimates of the standard parameters pertaining to technology and investment are

25

in line with those reported in the previous literature. The debt capacity parameter, θ, is estimated at 34%. This implies that the average firm can use close to a third of its assets as collateral to seek financing. This estimate is consistent with the results obtained in Li, Whited, and Wu (2016). The issuance cost parameter, λ, is in the range of the estimates in Hennessy and Whited (2007). The agency costs parameter, γ, estimated at 1 basis point, is in the range of agency cost estimates in Nikolov and Whited (2014). Finally, the credit line fee is estimated at 12 basis points. Sufi (2009) reports a median value of 150 basis points. Our estimate is lower as our sample is composed of large firms that typically exhibit lower financing costs. We examine the economic impact of these parameters in next section.

[Insert Table 1 Here]

4.1.1 Sample Splits

So far, our estimation results show that the model performs well in explaining dynamic corporate liquidity policies for the average firm in our sample. In this section, we seek further empirical validation for the determinants of corporate liquidity management in the model by testing if the model can explain dynamic liquidity management for sub-samples of firms that vary with key firm characteristics. Specifically, because collateral is an important determinant of the trade-off in the model, we focus on firms that exhibit high and low collateral. To do so, we present sample splits both based on firm size as measured by capital as well as based on tangibility.

Table 2 reports the estimation results for two sub-samples based on firm size, representing the top and bottom terciles, respectively. Overall, the model delivers a good fit to the data across samples, with actual and simulated moments that are largely statistically and economically indistinguishable. Moreover, the results are consistent with the key trade-off in the model. Indeed, panel A shows that small firms hold higher cash balances, have higher credit line limits, lower undrawn credit, lower leverage, are less profitable, and invest more. This is in line with the notion of firms that exhibit higher liquidity needs due to profitable investment opportunities, but limited collateral. Interestingly, small firms exhibit higher credit line limits than large firms, despite the fact that

26

small firms have less collateral. This is because small firms preserve contingent liquidity through credit lines by selecting low leverage, while the opposite holds true for large firms.

Panel B reports the corresponding parameter estimates. All estimated parameters are statistically different from zero and economically plausible. The panel reveals that small firms face higher fixed operating costs, higher profit shock volatility, and higher external financing costs. These results point to a higher need for liquidity management for small firms. The latter effect is exacerbated through the higher curvature of the profit function. In addition, for small firms the fraction of capital that can be pledged as collateral is lower. Large firms also have a lower agency cost of hoarding cash. While the fraction of cash wasted due to agency frictions is lower, the actual dollar amount is larger. This result is in line with the evidence documented Nikolov and Whited (2014). In line with the notion that the fee on the credit line, ξ, captures costs related to monitoring, smaller firms incur substantially higher spreads.

[Insert Table 2 Here]

Table 3 presents estimation results for two sub-samples based on the top and bottom tangibility terciles, respectively. Our measure of tangibility here follows Berger, Ofek, and Swary (1996) and Almeida and Campello (2007), defined as Tangibility 1 in appendix D. The model provides a good fit across those, both economically and statistically. Through the lens of our model, it is intuitive that low tangibility firms share many characteristics with smaller firms, such as lower leverage, higher cash holdings and investment, as they possess less collateral. On the other hand, there are intriguing differences between the two classes of firms, in that lower tangibility firms exhibit lower credit limits and higher undrawn credit, so that they are more reluctant to exhaust their debt capacity. In the sense that low tangibility firms may have substantial intangible assets that are hard to collateralize, they would particularly benefit from further access to credit lines.

The parameter estimates reported in panel B are all statistically significant and economically meaningful. Consistent with the notion that lower tangibility comes with a higher information asymmetry, we find that both the agency costs of cash and the credit line fee are substantially higher for such firms.

[Insert Table 3 Here]

27

4.2 The Value of Contingent Liquidity

The sample splits suggest that in the presence of frictions, cross-sectional differences in size and tangibility not only come with substantial differences in cash holdings, undrawn credit, credit limits, and corporate policies more broadly, but significant variation in estimated debt capacity, agency costs, and credit line fees, for example. Before we provide a detailed cross-sectional analysis of corporate liquidity policies, we provide some quantitative guidance on how these differences are reflected in firm values through counterfactual analysis. We do so by considering the effects of 5% increases and decreases in $\hat{\theta}$, $\hat{\gamma}$, and $\hat{\xi}$ while keeping the remaining parameters at their baseline values. The results are reported in table 4.

Table 4 shows that an increase in debt capacity leads to a lower cash balance and an increase in undrawn liquidity. While the first implication is straightforward, the latter is the result of two distinct forces. Clearly, an increase in debt capacity raises the credit line limit. At the same time, an increase in debt capacity facilitates the use of the credit line. Quantitatively, the first effect dominates the second and we obtain an increase in undrawn liquidity. Overall, with an increase in debt capacity, liquidity management is more effective as it allows firms to rely relatively more on contingent liquidity. As a result, firm value increases.

An increase in the agency cost of hoarding cash lowers firms' cash balances and firms' undrawn credit. The latter effect occurs as agency costs have little effect on the credit line limit, while making cash holdings less attractive leads firms to draw relatively more on the credit line. Overall, there is less liquidity, undrawn liquidity is reduced, and as a consequence firm value falls.

An increase in the credit line fee lowers firms' cash balances and increases firms' undrawn credit. The latter effect occurs as the credit line fee has little effect on the credit line limit, while making the credit line less attractive leads firms to draw relatively less on the credit line. The overall effect on firm value is negative.

[Insert Table 4 Here]

To better understand the mechanism underlying the value creation through credit line access, we propose the following counterfactual experiment. Based on the parameter estimates in the

28

tangibility split in table 3, we confront the dynamics and valuations of a high tangibility firm ($\hat{\theta} = 0.361$, with a credit line limit of 0.208) with an otherwise identical firm, but endowing it artificially with the credit line limit of low tangibility firms ($\hat{\theta} = 0.361$, but with a credit line limit of 0.151). More specifically, we simulate the dynamics of two such firms and compare their evolution when they are subject to the exact same sequence of shocks. To consider the relevance for firm growth, we initialize the capital stock at one-tenth of the steady-state capital stock. This design allows us to trace the implications of reduced financial flexibility through more restrictive credit line access in an empirically grounded setting.

The results are in table 5. We focus on the the initial growth phase, in which firms expand and where we consider the first 25, 50 and 100 simulated periods, respectively. The first two rows report the fraction of time that firms spend being financially constrained, defined as the fraction of simulated periods in which firms either issue equity or pay no dividends, in both specifications, while the remaining rows trace out implications for equity valuations and policies.

The table points to firms' initial growth phase as the source of the most significant value gains. Specifically, it documents that differential access to credit lines affect firm *growth* in a major way. Indeed, firms with restricted access to credit lines are substantially more constrained in the early expansion phase in their life cycle, and thus grow significantly more slowly. Such slowdowns in growth come with substantial value losses initially, and delay maturing considerably. Overall, access to credit lines and the option to implement contingent liquidity management thus allows firms to take advantage of investment opportunities in an effective way, facilitating corporate growth and creating value along the growth path.

[Insert Table 5 Here]

4.3   Corporate Liquidity in the Cross-Section

A number of patterns regarding cross-sectional determinants of corporate liquidity management emerge in our model. These predictions are discussed in section 2.2. We now provide direct empirical tests guided by our model as well as further simulation results.

In the model, cross-sectional variation in liquidity policies is driven by differences in the firm-level state variables, namely the capital stock, k_{it}, net worth, w_{it}, and profitability, z_{it}. We document that these variables also give rise to meaningful cross-sectional variation in corporate liquidity in the data. Notably, we show that our model naturally rationalizes a broad set of key stylized facts about credit line usage, as documented in Sufi (2009).

Determinants of Corporate Liquidity. The panel regressions in table 6 report suggestive evidence on how the state variables in the model, namely the capital stock, k_{it}, net worth w_{it}, and profitability z_{it}, affect observed corporate liquidity choices, both in terms of the total amount of liquidity transferred and in terms of the mix between contingent and uncontingent liquidity. The dependent variable in columns (1) and (2) is the total amount of liquidity firms transfer both by preserving undrawn credit and holding cash (scaled by total assets). The dependent variable in columns (3) and (4) is the fraction of corporate liquidity that firms choose to preserve in a contingent way, measured as undrawn credit divided by total liquidity. Finally, the dependent variable in columns (5) and (6) is the fraction of corporate liquidity firms could choose to preserve in a contingent way, as bounded by the credit line limit (scaled by total liquidity). The specifications in columns (1), (3), and (5) are based on the sample we describe in section 3, while those in columns (2),(4), and (6) are averages across 100 simulated panels of 1,000 firms for 20 years under the baseline model estimation in table 1.

The regressions in columns (1) and (2) show that both in the model and the data, the total amount of corporate liquidity is positively related to net worth and negatively to capital. Intuitively, small firms have more growth options and larger total liquidity needs than large firms, consistent with the policy functions in figure 1. On the other hand, the marginal value of net worth is declining in net worth itself, so that firms with higher net worth can afford to allocate more resources to liquidity management. Finally, more profitable firms need to transfer less liquidity since they internally generate resources for future needs. These predictions find support in the data.

The estimates in columns (3) and (4) reflect the economic tradeoff between contingent and uncontingent liquidity in our model. Firms with a larger capital stock have more collateral to pledge, and prefer to use contingent liquidity to fulfill their needs for future resources. In practice,

30

the slack they keep on their lines of credit is a large part of their total liquidity. Given their capital stock, firms with higher net worth have a lower marginal cost of transferring liquidity, and they do so by hoarding cash. Finally, more profitable firms relay on credit lines relatively more. In the model, this is because more profitable firms have better investment opportunities allowing them to accumulate collateral.

The results in columns (5) and (6) confirm that more profitable firms and firms with a higher capital stock have relatively higher credit limits, and thus better access to contingent liquidity. In the model, that access is facilitated by more collateral. On the other hand, consistent with the estimates in (3) and (4), firms with higher net worth find it easier to transfer liquidity by holding cash, given the capital stock.

Overall, our estimates confirm the empirical relevance of the model state variables as determinants of corporate liquidity policies. Notably, our results are in line with, and effectively, rationalize the key empirical patterns documented in Sufi (2009), table 6, who highlights the role of profitability for credit line access. Our model rationalizes that notion, as profitability facilitates accumulation of collateral through investment and thus relaxes the access to credit lines. More broadly, our model highlights the notion that many of these key empirical patterns can be understood through the lens of collateral availability. We next providence some evidence on this link based on a common empirical proxy for collateral, namely tangibility. Specifically, our model predicts that all else equal, firms with more tangible assets to pledge can fulfill their liquidity needs primarily in a contingent way.

[Insert Table 6 Here]

Table 7 provides suggestive stylized empirical evidence to that effect. Clearly, although these estimates do not *per se* formally extend far beyond suggestive correlations in the absence of a valid instrument, we view them nevertheless as informative. The table reports panel regressions of total liquidity, defined as the ratio of total liquidity (cash plus undrawn credit) to total assets (panel A), the fraction of contingent-to-total liquidity (panel B), and credit line limit scaled by total liquidity (panel C), on two proxies of tangibility (Berger, Ofek, and Swary (1996), and fixed-to-total-asset ratio), and two proxies of intangibility (the fraction of organization capital measured

31

with the perpetual inventory method applied to SG&A expenses, and the fraction of knowledge capital measured with the perpetual inventory method applied to R&D expenses) employed in the empirical literature. The results in panel B emphasize that firms with more pledgeable assets fulfill their liquidity needs mostly in a contingent way. The results in panel A suggest instead that firms with more intangible assets tend to transfer more liquidity in total, but mostly in an uncontingent way using cash. This result is consistent with Falato, Kadyrzhanova, and Sim (2013), who find that high-growth firms with a large fraction of intangible assets hold disproportionately more cash. Finally, panel C corroborates the view that firms with more tangible assets have higher credit limits.

[Insert Table 7 Here]

Financial Constraints versus Distress.   In our model, the availability of collateral in the form of capital as a determinant of liquidity management evolves endogenously, and is jointly determined with the other state variables net worth and profitability. The evidence in table 6 shows nuanced links between firm characteristics and liquidity policies. In particular, the evidence highlights the importance of distinguishing between small and low net worth firms in shaping corporate liquidity choices. We now interpret these differences in the light of financing constraints and financial distress, and provide further empirical support.

The notion of financing constraints is ubiquitous in finance, and captures the notion of young firms whose growth is inhibited by financing frictions. In our model, this notion intuitively corresponds to small firms with net worth sufficiently low that their growth and investment policies depend on external financing. Subtly different from that notion is a firm with ample collateral, but similarly low net worth that has been eroded through a long sequence of adverse shocks. That notion is most readily captured by financial distress. Our results indicate that these two types, as captured by their relative capital and net worth, manage their liquidity needs quite differently. In the following, we provide further empirical content to that observation.

Table 8 reports the results. It presents projections of common empirical distress indicators, as well as a constraint indicator, on the model implied state variables, capital stock, net worth, and profitability, in the data. Regarding distress indicators, we choose a bankruptcy indicator, from the

32

updated bankruptcy data in Chava and Jarrow (2004), and the commonly used O- and Z-Scores, respectively. High bankruptcy indicator, as well as a high O-Score, suggest impending financial distress, while a high Z-Score suggests that a firm is in relatively good financial health. On the other hand, we use the Whited-Wu index as a financial constraints measure. A high Whited-Wu index indicates that a firm is financially likely constrained.

Our main empirical result, which is in line with our theoretical prediction and the notions of financial constraints and distress, respectively, developed in the model, is that both distress and constraint indicators are similarly related to net worth and profitability, but load differently on the capital stock. Indeed, while higher net worth and profitability uniformly reduce the likelihood of distress and constraints, for given net worth, a higher capital stock increases the risk of financial distress (columns 1 to 3), but reduces the risk of financial constraints (column 4). Intuitively, a firm with low net worth and capital stock is financially constrained as it lacks the collateral it still needs to build up, while low net worth and a high capital stock are the results of bad shocks, that have depleted internal funds over time, increasing the likelihood of financial distress. Table 8 thus gives empirical support to the notions of distress and constraints, respectively, inherent in the model, and emphasizes the need of distinguishing these concepts. Not least, that need arises from the observation, predicted by our model, and documented in table 6, that such firms manage their liquidity needs quite differently.

The Role of Covenants. Our model endogenously links characteristics of financially distressed firms to liquidity management strategies. This link is especially relevant, as in practice many credit line agreements between firms and banks contain covenants designed to protect lenders from borrowers' distress. Indeed, as Sufi (2009) reports, 72 percent of all credit line contracts in his sample have information about financial covenants. In particular, bounds on debt-to-cash flow ratios, net worth or coverage ratios are extremely prevalent. Any violation of such bounds leads to a freeze of the credit facility.

We now examine the interplay between various covenants on credit facilities and liquidity policies through the lens of our model. First of all, our model endogenously accounts for a particular covenant, namely the collateral constraint. A binding collateral constraint restricts credit line

access directly. As this sort of restriction on credit facilities is not directly observable in the data, we relate its usage to two more commonly observed covenants that can readily be represented in the model, namely net worth and debt-to-cash flow covenants, as implemented in the model extensions in (12) and (13). In our implementation, we set the thresholds t_w and t_{CF} to be consistent with evidence from Chava, Fang, and Prabhat (2015) and Nini, Smith, and Sufi (2012), respectively.

Table 9 presents a first set of results. Panel A gives an account of the distribution of states in which the baseline collateral constraints, as well as net worth and/or cash flow covenants, bind and restrict access to revolving credit facilities. In particular, these results give a sense of the extent to which we can interpret real world covenants as representing binding collateral constraints. We proceed as follows. For scenarios with i) active collateral constraints only (baseline model), ii) additionally active net worth covenants, iii) additionally active cash flow covenants, and iv) both net worth and cash flow covenants are active, we identify in simulations firm-year observations in which an individual state variables is in the top and bottom 30 percent of values. For all such terciles we check in what fraction of total observations, we find binding constraints or covenants. We find this to be an economically intuitive representation of the distribution of binding constraints over the state space.

For example, the first two entries on the first row in panel A imply that among all firm-year observations in the baseline model with only collateral constraints, in which net worth ended up in the bottom thirty percent of values, the collateral constraint was binding in 9.7 percent of all cases, while in the top thirty percent of the net worth distribution, binding collateral constraints only occurred in 5.9 percent of the observations. Overall, the first line thus suggests that, perhaps unsurprisingly, collateral constraints are most likely to bind when either net worth, capital, or profitability are low.

Comparing with the scenarios with active covenants in rows 2 to 4 in panel A, we see that, qualitatively, collateral constraints and covenants limit access to credit lines in similar states, namely in low net worth, capital, or profitability states. This observation thus suggests that qualitatively, through the lens of the model, covenants can be naturally represented by collateral constraints. In other words, covenants lead to credit line freezes in anticipation of states in which collateral constraints are expected to bind. Quantitatively, there are differences, however. Covenants tend to

be more restrictive in low net worth and low profitability states, while collateral constraints restrict credit line access more frequently for small firms, when collateral is scarce. Finally, the last line in panel A shows that states in which net worth and cash flow covenants do not entirely coincide, implying, consistent with the evidence in Sufi (2009), that cash flow covenants are non redundant. This suggests that these covenants are designed to mitigate agency conflicts beyond the limited commitment problem underlying our collateral constraints.

Panel B of Table 9 reports and compares the first moments of key variables such as profitability, investment, leverage, undrawn credit, as well as cash, computed across entire samples, and conditioning on binding constraints or covenant violations. Comparing these moments brings out some subtle differences between collateral constraints and covenants. While rows 1 and 3 indicate that firms are less profitable and more levered in states when collateral constraints bind or covenants are violated, row 2 suggests that while there are no differences in investment between states in which covenants are violated or not, collateral constraints significantly restrict investment. In this sense, covenants apply mostly to distressed firms, while collateral constraints also restrict constrained firms. In contrast, collateral constraints and covenants similarly affect liquidity policies, as rows 4 and 5 suggest. Total liquidity uniformly falls, indicating that in the presence of binding constraints or covenant violations firms' immediate need for resources override liquidity management concerns.

Table 10 presents further evidence on covenant violations, through the lens of the model. Specifically, the table reports the results of projecting the probability of covenant violations on the model state variables, in panel regressions on simulated data. Columns (1), (3), and (5) corroborate, and effectively rationalize, the evidence in Sufi (2009) that profitability emerges as the quantitatively most important driver of violations. In the light of the model, this has a simple intuition in that persistent shocks not only drive violations in the cash flow covenant, but also erode net worth, so as to lead to violations in both covenants. Similarly, columns (2), (4), and (6) indicate, again in line with the evidence in Sufi (2009), that in our model, covenant violations are declining in net worth, and increasing in the capital stock. Remarkably, this is precisely the pattern we uncover for the likelihood of entering financial distress, as reported in table 8, and distinct from the emergence of financial constraints. Summarizing, our results corroborate the intuitive notion that the likelihood of violating a covenant is closely tied to entering financial distress, rationalizing the evidence in Sufi (2009).

4.4 Corporate Liquidity in the Time-Series

Corporate liquidity policies reflect anticipations of future funding needs and thus are inherently dynamic. In particular, time series variation in funding needs gives rise to time series variation in firms' liquidity positions that arise jointly with their investment and financing policies. We now report some time series predictions about the co-determination of corporate investment, financing and liquidity policies and present some supportive empirical evidence by means of correlations.

Correlations Table 11 reports the average time-series correlations among variables that describe firms' investment, financing, and liquidity policies. The first column considers the sample we describe in section 3, while the second column refers to a simulated sample of 1,000 firms for 100 years under the estimated parameter values in table 1. These correlations are not directly targeted by our estimation procedure. Therefore, they not only capture the dynamic nature of the model by quantifying co-movements among relevant variables, but also serve as an informal out-of-sample test of the model.

We consider correlations among several variables that describe firms' sources and uses of funds, namely credit line draws, changes in cash balances, equity issuances, internally generated operating income, and investment expenses. Table 11 shows that the model is broadly consistent with co-movements observed in our sample. For example, changes in drawn credit, that reflect firms' uses of contingent liquidity, are positively correlated to investment so that firms tend to draw down their credit lines in order to finance investment expenses. Similarly, firms tend to deplete their cash holdings when they invest, suggesting that they save partly in anticipation of high investment expenditures. In the model, corporations' savings reflect anticipations of expenditures that exhaust their debt capacity. In addition, equity issuances are positively correlated with changes in cash suggesting that firms use parts of the proceeds to replenish their cash reserves.

[Insert Table 11 Here]

5 Conclusion

We develop and estimate a dynamic model of corporate liquidity management. Firms can use cash reserves and draw down credit lines to absorb shocks and cover liquidity needs. We model credit lines as providing contingent liquidity in the sense that they can be drawn down contingent on the realizations of economic shocks, but subject to collateral constraints and covenants. The economic mechanism emerging in our model is thus a trade-off between effective contingent liquidity by means of credit lines subject to collateral constraints, and uncontingent liquidity provision through cash holdings. We embed this trade-off into a flexible model of corporate financing and investment that we can take to the data by means of structural estimation.

We find that our estimated model rationalizes well the levels and joint dynamics of cash, credit lines, leverage, equity financing and investment when firms can collateralize roughly one third of their assets. Our model thus highlights the importance of collateral as a determinant of corporate liquidity management. The availability of collateral drives the cross-sectional and time-series predictions of the model, for which we find substantial empirical support. In particular, our model naturally rationalizes key stylized facts on credit line usage, cash holdings, and the likelihood of covenant violations in the cross-section. Access to contingent liquidity through credit lines creates significant value as it allows firms to take advantage of investment opportunities in an effective way, as the data suggest.

One limitation of our setup is that our modeling of collateral constraints implies that all debt is risk free. On one hand, the observation that firms with ample collateral have better access to credit lines and debt because higher recovery rates reduce default spreads[9] suggests that our main trade-off will still be at work in a setup with default. On the other hand, it would nevertheless be important and interesting to take this challenging extension to the data by means of structural estimation. We leave this task for future work.

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[9]See for example Davydenko and Strebulaev (2007) and Jankowitsch, Nagler, and Subrahmanyam (2014) for empirical evidence.

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Appendix

In the appendix, we outline the recursive formulation of firms' optimization problem on which our solution strategy and simulated moments estimator (SMM) is based. Appendix A details the recursive formulation, appendix B gives an account of the solution algorithm based on a linear programming representation of the optimization problem, and appendix C provides an overview of our implementation of the SMM estimator.

A. Full Dynamic Program

Using primes to denote next period variables and dropping the firm index i for readability, we can represent the firm's optimization problem recursively as follows:

$$V(w,k,z) \equiv \max_{k',l,cl(z'),c',d} \left\{ d - \Lambda(d) + \frac{1}{1+r} E\left[V(w',k',z')\right] \right\}$$

s.t.

$$\Lambda(d) = \lambda|d|1_{\{d<0\}} \qquad\qquad \text{[Costly Equity Issuance]}$$

$$w(z') \leq \pi(k',z') + (1 - \delta(1-\tau))k' + (1 + r(1-\tau) - \gamma)c' \qquad \forall z' \qquad \text{[Budget Constraints } t+1]$$
$$\qquad -(1 + r(1-\tau))l + -(1 + r(1-\tau) + \xi)cl(z')$$

$$w + l + E[cl(z')] \geq k' + d + c' + \Psi(k,k') \qquad\qquad \text{[Budget Constraint } t]$$

$$(1 + r(1-\tau))l + (1 + r(1-\tau) + \xi)cl(z') \leq \theta(1-\delta)k' \qquad \forall z' \qquad \text{[Collateral Constraints]}$$

B. Solution by Mixed-Integer Programming

Because of the presence of several occasionally non-binding collateral constraints and because equity issues depend on indicator functions, the model cannot be solved numerically by interior point methods. The model could instead be solved on a discrete grid by standard iterative methods as value function iteration or policy function iteration. The Bellman operator in the functional equation (11) is in fact a contraction mapping because Blackwell's sufficient conditions are satisfied and theorems 9.6 and 9.8 in Stokey and Lucas (1989) apply.

Unfortunately, a computational hurdle complicates the solution of the model with standard techniques. The large number of control variables (capital, cash, debt, and one drawn credit variable for each future state) makes the practical implementation of value function and policy iteration problematic. Specifically, the maximization step, that is determining for each state the combination of control variables that maximizes the sum of distributions and the continuation value, is critical. For each grid point for the state variables, it requires to store and maximize over a vector of nk x nc x nb x ncl^{nz} elements, where nk, nc, nb, ncl and nz are respectively the number of grid points for capital, cash, standard debt, drawn credit and the profitability shock. Our optimization problem is therefore plagued by a severe curse of dimensionality (Rust (1997)). The amount of computer memory and CPU time required increases exponentially with the number of control variables. As a consequence, even for modest values for nz, such a vector becomes too large even to be stored.

44

We overcome this difficulty by exploiting the linear programming (LP) representation of dynamic programming problems with infinite horizon (Ross (1983)), as in Trick and Zin (1993), and Trick and Zin (1997). Historically, the LP solution method has not been widely used. Although it often achieves considerable speed gains over iterative methods, the linear programming method requires in turn to store enormous matrices and arrays. This computational requirement makes the standard LP approach not scalable and impractical for complex enough models. We therefore extend the constraint generation algorithm developed by Trick and Zin (1993). More precisely, we introduce a separation oracle, an auxiliary mixed-integer programming problem, which avoids dealing with large vectors. As in Trick and Zin (1993), the constrained generation algorithm converges to the fixed point much faster than traditional iterative methods. Moreover, the separation oracle efficiently implements the maximization step because of a remarkable feature of our model, namely the relatively small number of state variables in spite of the large number of control variables. With this method, we manage to solve the model in a reasonable time (around two minutes on a regular eight-core workstation).

Any finite dynamic programming problem with infinite horizon can be equivalently formulated as a linear programming problem (LP), where for each grid point on the state space, every feasible decision corresponds to a constraint in the LP. Specifically, our model can be formulated as an LP as follows:

$$\min_{v_{k,w,z}} \sum_{k=1}^{nk} \sum_{w=1}^{nw} \sum_{z=1}^{nz} v_{k,w,z} \tag{A1}$$

$$s.t.$$

$$v_{k,w,z} \geq d_{k,w,z,a} + \sum_{z'=1}^{nz} Q(z'|z) \frac{1}{1+r} v_{k'(a),w'(a),z'} \qquad \forall k,w,z,a, \tag{A2}$$

where nk, nw, and nz are the number of grid points on the grids for $k_{i,t}$, $w_{i,t}$, and $z_{i,t}$ respectively, $v_{k,w,z}$ is the value function on the grid point indexed by k, w and z, a is an index for an action on the grid for both future capital, cash, and state-contingent debt repayments, and $d_{k,w,z,a}$ denotes the payout corresponding to the action a starting from the state indexed by k, w and z. $k'(a)$ and $w'(a)$ denote the future values for the state variables given the current firm's decisions. For a formal proof, we refer to Ross (1983).

As Trick and Zin (1993) discuss, solving the LP above would require to store a huge matrix, because the number of constraints in the problem is very large. Computational requirements would therefore be enormous. Thus, we implement constraint generation, a standard method in operation research to solve problems with a large number of constraints. First we solve a relaxed problem with the same objective. Second, we identify the remaining constraints in the problem that are violated by the current solution. Third, we add a subset of the violated constraints to the relaxed problem according to a selection rule. We iterate the procedure is iterated until all constraints are satisfied. The following constraint generation algorithm converges to the unique fixed point of our Bellman problem.

1. solve the problem in (A1) with an initial random subset of constraints for each state (k, w, z);
2. if all constraints $a \in \Gamma^n(k, w, z)$, for all (k, w, z), are satisfied, terminate the algorithm (where $\Gamma^n(k, w, z)$ is the set of feasible actions at iteration n);
3. for each state (k, w, z), add the constraint $a \in \Gamma^n(k, w, z)$ that generates the highest violation in (A2) with respect to the current solution $v^n(k, w, z)$;
4. solve the problem with the current set of constraints;
5. go back to step 2.

To practically implement the above procedure, another issue must be addressed. The selection of the most violated constraint in the third step involves searching over an extremely large vector of grid points for all the decision variables. The computational burden would still be excessive for a model with many controls variables. We therefore use a separation oracle in the third step. A separation oracle is an auxiliary mixed-integer programming problem that identifies the most violated constraint[10]. We specify the separation oracle for this problem below:

Definition 1 (Separation Oracle)

$$\max_{a=\{k',c',l',cl(z')\}} d_{k,w,z,a} - \Lambda(d_{k,w,z,a}) + \sum_{z'=1}^{nz} Q(z'|z)\frac{1}{1+r}v_{k'(a),w'(a),z'} - v_{k,w,z} \tag{A3}$$

$s.t.$

$$cl(z') \geq 0 \quad \forall z' \tag{A4}$$

$$0 \leq c' \leq \overline{C} \qquad \forall z' \tag{A5}$$

$$l' \geq 0 \qquad \forall z' \tag{A6}$$

$$(1+r(1-\tau)+\xi)cl(z') + (1+r(1-\tau))l' \leq \theta k'(1-\delta) \quad \forall z' \tag{A7}$$

$$0 \leq p(i_k) \leq 1 \qquad \forall i_k = 1,...,n_k \tag{A8}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{A9}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k)k^G(i_k) \tag{A10}$$

$$d_{k,w,z,a} = w - k' - \Psi(k,k') - c' + l' + \sum_{z'=1}^{nz} Q(z'|z)cl(z') \tag{A11}$$

$$f(k') = \sum_{i_k=1}^{n_k} p(i_k)(k^G(i_k))^\alpha \tag{A12}$$

$$w(z') = (1-\tau)(z'f(k')-f) + k'(1-\delta) - (1+r(1-\tau))l' - (1+r(1-\tau)+\xi)cl(z') \tag{A13}$$
$$+(1+r(1-\tau)-\gamma)c' + \tau\delta k' \qquad \forall z' = 1...nz.$$

Equations (A4), (A5), and (A6) define bounds for drawn credit, loans and cash, equations (A6) are collateral constraints, equations (A8) and (A9) define the variables $p(i_k)$ that have the role to select a grid point for capital on the grid $k^G(i_k)$ and linearize the term k'^α in the production function, equation (A10) picks the grid point for the chosen capital stock from $k^G(i_k)$, equation (A11) defines dividends, equation (A12) computes the nonlinear term in capital in the production function, and equation (A13) defines future net worth in each state z'. The computation of the law of motion for future net worth is obtained by interpolation with the logarithmic formulation of Vielma and Nemhauser (2011). Capital adjustment costs $\Psi(k,k')$ and equity flotation costs $\Lambda(d_{k,w,z,a})$ are instead incorporated using a Big-M formulation. The solutions of the separation oracle for cash and state-contingent debt are continuous variables and are interpolated to the nearest point on the corresponding grid. When dividends are negative equity issuance costs $-\Lambda(d_{k,w,z,a})$ are added using a big-M formulation. In the extension with covenants of section 4.3,

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[10]Separation oracles are standard tools in operation research. See for example Schrijver (1998) and Vielma and Nemhauser (2011).

in case a covenant is violated in the current state, the law of motions of the drawn part of the credit line and of future net worth, which in turn determine the continuation equity value, are computed using SOS2 formulations.

We implement the codes with Matlab$^®$, and the solver for the mixed-integer programming problems is CPLEX$^®$. The two applications are interfaced through the CPLEX Class API$^®$. Our workstation has a CPU with 8 cores and 64GB of RAM. The model is solved with seven grid points for the idiosyncratic shock, 21 grid points for capital, 17 grid points for current net worth, and 500 points for cash, loans, and each state-contingent drawn credit variable. Following McGrattan et al. (1997), the grids for net worth and capital are not evenly spaced, but more points are collocated in the low net worth region, where the curvature of value function is more relevant.

C. Estimation Procedure

We provide a brief discussion of the estimation procedure[11]. Let x_i be an *i.i.d.* data vector, $i = 1, \ldots, n$, and let $y_{is}(\beta)$ be an *i.i.d.* simulated vector from simulation s, $i = 1, \ldots, n$, and $s = 1, \ldots, S$. Here, n is the length of the simulated sample, and S is the number of times the model is simulated. We pick $n = 20,000$ and $S = 10$, following Michaelides and Ng (2000), who find that good finite-sample performance of a simulation estimator requires a simulated sample that is approximately ten times as large as the actual data sample.

The simulated data vector, $y_{is}(\beta)$, depends on a vector of structural parameters, β. In our application $\beta \equiv (\alpha, f, \rho_z, \sigma_z, \psi, \theta, \lambda, \gamma, \xi)$. The goal is to estimate β by matching a set of *simulated moments*, denoted as $h(y_{ik}(\beta))$, with the corresponding set of actual *data moments*, denoted as $h(x_i)$. The candidates for the moments to be matched include for example simple summary statistics or OLS regression coefficients. Define

$$g_n(\beta) = n^{-1} \sum_{i=1}^{n} \left[h(x_i) - S^{-1} \sum_{s=1}^{S} h(y_{is}(b)) \right].$$

The simulated moments estimator of β is then defined as the solution to the minimization of

$$\hat{\beta} = \arg \min_{\beta} g_n(\beta)' \hat{W}_n g_n(\beta),$$

and in which $\hat{W}_n$ is a positive definite matrix that converges in probability to a deterministic positive definite matrix W. In our application, we use the inverse of the sample covariance matrix of the moments, which we calculate using the influence function approach in Erickson and Whited (2000).

The simulated moments estimator is asymptotically normal for fixed S. The asymptotic distribution of β is given by

$$\sqrt{n}\left(\hat{\beta} - \beta\right) \xrightarrow{d} \mathcal{N}\left(0, \text{avar}(\hat{\beta})\right)$$

in which

$$\text{avar}(\hat{\beta}) \equiv \left(1 + \frac{1}{S}\right) \left[\frac{\partial g_n(\beta)}{\partial \beta} W \frac{\partial g_n(\beta)}{\partial \beta'}\right]^{-1} \left[\frac{\partial g_n(\beta)}{\partial \beta} W \Omega W \frac{\partial g_n(\beta)}{\partial \beta'}\right] \left[\frac{\partial g_n(\beta)}{\partial \beta} W \frac{\partial g_n(\beta)}{\partial \beta'}\right]^{-1} \quad \text{(A14)}$$

in which W is the probability limit of $\hat{W}_n$ as $n \to \infty$, and in which Ω is the probability limit of a consistent estimator of the covariance matrix of $h(x_i)$.

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[11]The exposition closely follows Nikolov and Whited (2014).

D. Variable Definition

The following tables summarize variable definitions with reference to Compustat and Capital IQ items and, when applicable, their model counterparts. The subscript "-1", when applied to data items, denotes lagged variables.

Variable	Construction	
	Data	Model
Cash	$\frac{CHE}{AT}$	$\frac{c_{it}}{k_{it}+c_{it}}$
Credit Line Limit	$\frac{RC+UNDRAWN}{AT}$	$\frac{cl_{it}^L}{k_{it}+c_{it}}$
Undrawn Credit	$\frac{UNDRAWN}{RC+UNDRAWN}$	$\frac{cl_{it}^L-cl_{it}}{cl_{it}^L}$
Leverage	$\frac{DLTT+DLC-RC}{AT}$	$\frac{l_{it}}{k_{it}+c_{it}}$
Operating Income	$\frac{OIBDP}{AT}$	$\frac{(1-\tau)(z_{it}k_{it}^\alpha-f)}{k_{it}+c_{it}}$
Investment	$\frac{CAPX}{PPENT}$	$\frac{k_{it+1}-(1-\delta)k_{it}}{k_{it}}$
Equity Issuance	$\frac{SSTK}{AT}$	$\frac{d_{it}1_{\{d_{it}<0\}}}{k_{it}+c_{it}}$
Tobin's q	$\frac{DLTT+DLC+PRCC\_F\cdot CSHO}{AT}$	$\frac{l_{it}+E_t[cl_{it+1}(z_{it+1})]+v_{it}}{k_{it}+c_{it}}$
Net Worth	CEQ	w_{it}

Variable	Construction (Data Only)
Tangibility 1	$\frac{0.715\ RECT+0.547\ INVT+0.535\ PPEGT}{AT}$
Tangibility 2	$\frac{PPENT}{AT}$
Intangibility 1	Perpetual inventory on real XRD (rate=0.15) scaled by AT-CHE
Intangibility 2	Perpetual inventory on real XSGA (rate=0.2) scaled by AT-CHE
O-Score	$-1.32 - 0.407\ \ln(AT) + 6.03\frac{LT}{AT} - 1.43\frac{WCAP}{AT}$ $+0.076\frac{LCT}{ACT} - 2.37\frac{NI}{AT} - 1.83\frac{FOPT1}{LT} - 0.521\frac{NI-NI_{-1}}{|NI|+|NI_{-1}|}$ -1.72 if $LT > AT + 0.285$ if a net loss realized for the last two years
Z-Score	$1.2\frac{WCAP}{AT} + 1.4\frac{RE}{AT} + 3.3\frac{EBIT}{AT} + 0.6\frac{PRCC\_F \cdot CSHO}{LT} + \frac{REVT}{AT}$
Whited-Wu Index	$-0.091\frac{IB+DP}{AT} + 0.021\frac{DLTT}{AT} - 0.044\ln(AT)$ $-0.035\frac{SALE}{SALE_{-1}} - 0.062$ if $DVC + DVP > 0$ $+0.102$ average industry sale growth (three digit SIC)

Table 1
Simulated Moments Estimation

Calculations are based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets. The sample period is from 2002 to 2011. The estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. Panel A reports the simulated and the actual moments and the t-statistics for the differences between the corresponding moments. Panel B reports the estimated structural parameters. α is the curvature of the profit function. f is the fixed production cost. ρ_z is the serial correlation of $ln(z)$. σ_z is the standard deviation of the innovation of $ln(z)$. ψ is the variable adjustment cost. θ is the debt capacity. λ is the equity flotation cost. γ is the agency cost parameter. ξ is the credit line fee. Standard errors are in parenthesis under the parameter estimates.

Panel A: Moments

	Actual moments	Simulated moments	t-stats
Average cash	0.139	0.129	1.13
Variance of cash	0.004	0.005	-0.30
Autocorrelation of cash	0.688	0.895	-0.78
Average credit line limit	0.172	0.179	-0.88
Average undrawn credit	0.868	0.608	2.19
Variance of undrawn credit	0.011	0.066	-3.08
Autocorrelation of undrawn credit	0.101	0.133	-0.02
Average leverage	0.171	0.153	2.78
Variance of leverage	0.010	0.002	2.27
Autocorrelation of leverage	0.725	0.926	-0.53
Average operating income	0.138	0.128	1.12
Variance of operating income	0.003	0.005	-0.23
Autocorrelation of operating income	0.562	0.585	-0.15
Average investment	0.116	0.120	-0.39
Variance of investment	0.005	0.001	1.69
Autocorrelation of investment	0.339	0.625	-1.35
Average equity issuance	0.020	0.066	-4.40

Panel B. Parameter estimates

α	f	ρ_z	σ_z	ψ	θ	λ	$\gamma \times 10,000$	$\xi \times 10,000$
0.662	5.133	0.635	0.283	5.428	0.335	0.123	1.054	11.911
(0.080)	(0.880)	(0.203)	(0.035)	(0.328)	(0.110)	(0.006)	(0.150)	(5.207)

Table 2
Simulated Moments Estimation: Large Firms versus Small Firms

Calculations are based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets. The sample period is from 2002 to 2011. The estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. Panel A reports the simulated and the actual moments and the *t*-statistics for the differences between the corresponding moments. Panel B reports the estimated structural parameters. α is the curvature of the profit function. f is the fixed production cost. ρ_z is the serial correlation of $ln(z)$. σ_z is the standard deviation of the innovation of $ln(z)$. ψ is the variable adjustment cost. θ is the debt capacity. λ is the equity flotation cost. γ is the agency cost parameter. ξ is the the credit line fee. Standard errors are in parenthesis under the parameter estimates.

Panel A. Moments

	Large firms			Small firms		
	Actual moments	Simulated moments	*t*-stats	Actual moments	Simulated moments	*t*-stats
Average cash	0.088	0.079	1.41	0.181	0.179	0.58
Variance of cash	0.002	0.003	-0.05	0.006	0.009	-0.54
Autocorrelation of cash	0.521	0.889	-0.81	0.621	0.940	-0.82
Average credit line limit	0.149	0.157	-0.82	0.186	0.186	0.02
Average undrawn credit	0.888	0.621	2.24	0.855	0.705	2.02
Variance of undrawn credit	0.010	0.086	-1.79	0.012	0.060	-1.87
Autocorrelation of undrawn credit	0.093	0.353	-3.98	0.102	0.112	-0.01
Average leverage	0.230	0.201	4.72	0.093	0.083	0.97
Variance of leverage	0.010	0.001	2.16	0.010	0.001	2.18
Autocorrelation of leverage	0.523	0.874	-0.54	0.654	0.803	-0.27
Average operating income	0.150	0.142	1.29	0.114	0.120	-1.11
Variance of operating income	0.002	0.007	-1.12	0.004	0.006	-0.12
Autocorrelation of operating income	0.543	0.574	-0.12	0.583	0.549	0.15
Average investment	0.111	0.120	-1.12	0.121	0.119	0.12
Variance of investment	0.002	0.001	0.39	0.005	0.001	1.24
Autocorrelation of investment	0.497	0.649	-0.42	0.325	0.188	0.44
Average equity issuance	0.014	0.057	-2.38	0.026	0.057	-2.01

Panel B. Parameter estimates

	α	f	ρ_z	σ_z	ψ	θ	λ	$\gamma \times 10,000$	$\xi \times 10,000$
Large Firms									
	0.666	5.137	0.630	0.298	5.670	0.332	0.061	0.448	8.184
	(0.077)	(0.482)	(0.030)	(0.045)	(0.443)	(0.054)	(0.011)	(0.062)	(1.065)
Small Firms									
	0.661	5.149	0.620	0.311	5.856	0.299	0.152	1.105	26.680
	(0.185)	(1.786)	(0.042)	(0.070)	(0.656)	(0.016)	(0.009)	(0.166)	(10.107)

51

Table 3
Table 3
Simulated Moments Estimation: High versus Low Tangibility Firms

Calculations are based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets. The sample period is from 2002 to 2011. The estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. Panel A reports the simulated and the actual moments and the t-statistics for the differences between the corresponding moments. Panel B reports the estimated structural parameters. α is the curvature of the profit function. f is the fixed production cost. ρ_z is the serial correlation of $ln(z)$. σ_z is the standard deviation of the innovation of $ln(z)$. ψ is the variable adjustment cost. θ is the debt capacity. λ is the equity flotation cost. γ is the agency cost parameter. ξ is the credit line fee. Standard errors are in parenthesis under the parameter estimates.

Panel A. Moments

	High tangibility firms			Low tangibility firms		
	Actual moments	Simulated moments	t-stats	Actual moments	Simulated moments	t-stats
Average cash	0.076	0.081	-0.91	0.205	0.209	-0.55
Variance of cash	0.003	0.002	0.07	0.005	0.011	-1.01
Autocorrelation of cash	0.635	0.840	-0.50	0.617	0.948	-0.67
Average credit line limit	0.203	0.208	-0.15	0.145	0.151	-0.71
Average undrawn credit	0.842	0.595	2.11	0.894	0.609	2.38
Variance of undrawn credit	0.012	0.077	-1.74	0.010	0.080	-1.81
Autocorrelation of undrawn credit	0.130	0.364	-1.92	0.073	0.176	-0.11
Average leverage	0.182	0.177	0.51	0.129	0.094	3.23
Variance of leverage	0.012	0.001	2.84	0.009	0.002	1.63
Autocorrelation of leverage	0.685	0.827	-0.24	0.760	0.829	-0.21
Average operating income	0.152	0.145	0.45	0.119	0.111	0.54
Variance of operating income	0.003	0.007	-0.21	0.002	0.006	-0.65
Autocorrelation of operating income	0.519	0.631	-0.46	0.611	0.552	0.52
Average investment	0.112	0.119	-1.23	0.131	0.120	1.78
Variance of investment	0.004	0.001	0.80	0.005	0.001	0.31
Autocorrelation of investment	0.295	0.576	-0.86	0.315	0.261	1.46
Average equity issuance	0.014	0.071	-3.50	0.026	0.056	-2.36

Panel B. Parameter estimates

	α	f	ρ_z	σ_z	ψ	θ	λ	$\gamma \times 10,000$	$\xi \times 10,000$
High tangibility firms									
	0.673	5.134	0.679	0.289	5.321	0.361	0.080	0.502	7.665
	(0.010)	(1.387)	(0.149)	(0.017)	(0.882)	(0.010)	(0.021)	(0.038)	(1.873)
Low tangibility firms									
	0.652	5.152	0.629	0.340	5.119	0.263	0.119	1.009	12.908
	(0.030)	(0.917)	(0.168)	(0.029)	(0.529)	(0.005)	(0.015)	(0.029)	(0.337)

Table 4
Counterfactuals

The table reports the value loss and differences in cash balances and undrawn credit due to the debt capacity and agency cost. The value loss is expressed in terms of Tobin's q. $\hat{\theta}$, $\hat{\gamma}$, and $\hat{\xi}$ are respectively the estimated debt capacity, the agency cost, and the credit line fee parameters as reported in table 1.

	Value loss (in %)	Cash difference (in %)	Undrawn credit difference (in %)
5% decrease in $\hat{\theta}$	-1.54%	11.28%	-4.61%
5% increase in $\hat{\theta}$	1.93%	-14.17%	6.94%
5% decrease in $\hat{\gamma}$	0.98%	9.34%	5.83%
5% increase in $\hat{\gamma}$	-0.74%	-10.21%	-6.57%
5% decrease in $\hat{\xi}$	0.43%	4.65%	-7.91%
5% increase in $\hat{\xi}$	-0.58%	-6.72%	8.18%

Table 5

The Value of Contingent Liquidity

The table is based on a comparison of two panels of simulated data based on the SMM estimation corresponding to the parameter values in table 3. The first panel ("Regular CL Availability") refers to a simulated economy with the parameter values for high tangibility firm. The second panel ("Reduced CL Availability") refers to a benchmark economy with the same parameter values but in which firms have a lower credit line limit, corresponding to the estimated value for the low tangibility economy in table 3. Both simulated panels include 1000 firms and 1000 time periods. Firms in the two economies are exposed to the same sequence of exogenous profitability shocks. Firms enter both economies with a small value of capital, namely one-tenth of the steady state level. Columns one to three refer to the first 25, 50, and 100 simulated periods respectively. Rows one to seven respectively report: the fraction of periods firms are financially constrained in the baseline economy with credit lines and in the benchmark economy without credit lines, the average fractional increase in equity value, net worth, capital, debt, and equity issuances between the baseline and the benchmark economy. For all variables, the fractional increase for a given firm-year is defined as the difference between the variable value in the baseline and the one in the benchmark economy, scaled by the latter.

	Growth ($T = 25$)	Growth ($T = 50$)	Growth ($T = 100$)
Constrained Periods (Regular CL Availability)	23.6%	22.3%	20.7%
Constrained Periods (Reduced CL Availability)	53.3%	35.0%	22.6%
Change in Equity Value	13.6%	6.4%	1.8%
Change in Net Worth	24.8%	10.8%	1.9%
Change in Capital	17.4%	9.8%	5.2%
Change in Debt	6.2%	5.8%	7.4%
Change in Equity Issuances	-64.3%	-46.8%	-18.4%

Table 6
Net Worth, Size, Profitability and Corporate Liquidity

The table reports estimates from linear panel regressions of total liquidity, the proportion of contingent to total liquidity, and the credit line limit as a proportion of total liquidity on the determinants of liquidity identified by the model as its state variables. The column denoted as 'Data' is based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets. The sample period is from 2002 to 2011. The column denoted as 'Model' is based on averages across 100 simulated panels of 1000 firms for 20 years. Contingent liquidity is measured as the undrawn amount from firms' lines of credit, and uncontingent liquidity is measured as firms' cash holdings. Total liquidity is the sum of contingent and uncontingent liquidity. All liquidity variables are scaled by total assets. In panel A, the dependent variable is total liquidity. In panel B, the dependent variable is the ratio of contingent to total liquidity. In panel C, the dependent variable is the ratio of the credit line limit to total liquidity. The determinants of liquidity are (log) net worth, (log) capital, and profitability. Following Rampini, Sufi, and Viswanathan (2014), net worth is defined as the book value of equity. Capital is the book value of property, plant and equipment. Profitability is measured as operating profits before depreciation scaled by capital. All specifications include firm fixed effects, and standard errors are clustered at the firm level. T-statistics are in parentheses.

	A) Total Liquidity		B) Contingent-to-Total Liq.		C) CL Limit-to-Total Liq.	
	(1) Data	(2) Model	(3) Data	(4) Model	(5) Data	(6) Model
Net Worth	0.046	0.660	-0.010	-1.361	-0.118	-3.573
	(17.063)	(58.911)	(-2.806)	(-40.702)	(-10.350)	(-4.479)
Capital	-0.071	-0.579	0.054	2.340	0.165	4.288
	(-21.235)	(-11.497)	(13.312)	(15.573)	(12.734)	(1.196)
Profitability	-0.002	-0.243	0.001	0.596	0.003	2.411
	(-2.941)	(-12.974)	(1.240)	(10.648)	(1.433)	(1.804)
Adj. R^2	27.157	78.975	15.363	64.379	8.770	1.772

Table 7
Tangibility and Corporate Liquidity

The table reports estimates from linear panel regressions of total and contingent corporate liquidity on different proxies of asset tangibility and intangibility. Calculations are based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets. The sample period is from 2002 to 2011. Contingent liquidity is measured as the undrawn amount from firms' lines of credit, and uncontingent liquidity is measured as firms' cash holdings. Total liquidity is the sum of contingent and uncontingent liquidity. All variables are scaled by total assets. In panel A, the dependent variable is total liquidity. In panel B, the dependent variable is the ratio of contingent to total liquidity. In panel C, the dependent variable is the ratio of the credit line limit to total liquidity. The proxies for tangibility and in tangibility are the following: 'Tangibility1' is measured as in Berger, Ofek, and Swary (1996) and Almeida and Campello (2007); 'Tangibility 2' is the ratio of fixed assets over total assets; following Falato, Kadyrzhanova, and Sim (2013), 'Intangibility 1' is the fraction of knowledge capital measured with the perpetual inventory method applied to selling, general, and administrative expenses, and 'Intangibility 2' is the fraction of organization capital measured with the perpetual inventory method applied to R&D expenses. All specifications include firm fixed effects, and standard errors are clustered at the firm level. T-statistics are in parentheses.

	A) Total Liq.				B) Contingent-to-Total Liq.				C) CL Limit-to-Total Liq.			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Tangibility 1	-0.309 (-15.959)				0.338 (14.033)				0.544 (7.510)			
Tangibility 2		-0.516 (-22.969)				0.375 (11.261)				0.882 (8.586)		
Intangibility 1			0.728 (30.303)				-0.167 (-6.183)				-0.405 (-6.231)	
Intangibility 2				0.503 (21.547)				-0.201 (-8.581)				-0.562 (-8.736)
Adj. R^2	17.758	16.036	38.519	11.478	18.267	9.909	23.286	6.187	11.005	8.755	12.982	4.699

56

Table 8
Financial Constraints and Distress

The table reports estimates of three measures of financial distress and one measure of financial constraints on the determinants of liquidity identified by the model as its state variables. The first column estimates a logit model in which the dependent variable is the bankruptcy indicator as in Chava and Jarrow (2004). The second, third and fourth columns estimate fixed effects panel regression in which the dependent variables are Ohlson's O-score, Altman's Z-score, and the Whited-Wu index respectively. Results are based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets, merged with the updated bankruptcy data in Chava and Jarrow (2004). The sample period is from 2002 to 2011. The column denoted as 'Model' is based on averages across 100 simulated panels of 1000 firms for 20 years. The determinants of liquidity are (log) net worth, (log) capital, and profitability. Following Rampini, Sufi, and Viswanathan (2014), net worth is defined as the book value of equity. Capital is the book value of property, plant and equipment. Profitability is measured as operating profits before depreciation scaled by capital. Standard errors are clustered at the firm level. T-statistics are in parentheses.

	Bankrupt $\{0,1\}$	O-Score	Z-Score	Whited-Wu
Net Worth	-0.344	-1.606	2.655	-0.052
	(-3.718)	(-42.789)	(23.289)	(-35.371)
Capital	0.398	1.167	-2.756	-0.020
	(4.416)	(28.478)	(-19.740)	(-12.272)
Profitability	-0.042	-0.264	0.235	-0.004
	(-1.119)	(-24.493)	(7.632)	(-10.477)
N	35950	30204	33674	30746
R^2	2.09	31.80	10.05	62.60

Table 9
Covenants and Collateral Constraints

The table reports results from simulations for the model with collateral constraints and covenants. The results refers to panels of 1000 simulated firms for 100 years from four specifications, for which the parameters are set to the estimated values of table 1. "Coll. Cons. Only" refers to the baseline model with no covenants, "NW Cov. Only" to the baseline model augmented with a net worth covenant, "Debt/EBITDA Cov. Only" to the baseline model augmented with a Debt/EBITDA covenant, and "Both Cov." to the baseline model augmented with the two aforementioned covenants. The thresholds for the net worth and to the Debt/EBITDA covenants are set to the values of 0.7 and 4. Covenant violations result in the impossibility to draw additional liquidity from the credit line. Panel A reports the fraction of firm-year observations for which the collateral constraint is binding ("Coll. Cons. Only"), for which the net worth covenant is violated ("Debt/EBITDA Cov. Only"), for which the Debt/EBITDA covenant is violated ("Debt/EBITDA Cov. Only"), and for which the two covenants are simultaneously violated ("Both Cov."). Results are reported for the top and bottom thirty percent of firm-year observations sorted on net worth, capital, and profitability. Panel B reports moments for the four specifications for the simulated sample and for the subset of firm-year observations in which collateral constraints are binding and covenants are violated.

Panel A: Frequency of Active Constraints and Covenants

	Net Worth		Capital		Profitability	
	Low (Lo 30%)	High (Hi 30%)	Low (Lo 30%)	High (Hi 30%)	Low (Lo 30%)	High (Hi 30%)
Coll. Cons. Only	0.097	0.059	0.096	0.059	0.114	0.051
NW Cov. Only	0.176	0.000	0.063	0.053	0.168	0.000
Debt/EBITDA Cov. Only	0.167	0.002	0.067	0.057	0.186	0.000
Both Cov.	0.097	0.000	0.035	0.029	0.094	0.000

Panel B: Moments

	Coll. Cons.		Net Worth		Debt/EBITDA		Both Cov.	
	All	Binding	All	Violation	All	Violation	All	Violation
Profitability	0.154	0.127	0.147	0.051	0.147	0.033	0.147	0.036
Investment	0.120	0.103	0.120	0.120	0.120	0.120	0.120	0.120
Leverage	0.153	0.189	0.153	0.240	0.151	0.208	0.151	0.242
Undrawn Credit	0.197	0.021	0.105	0.064	0.106	0.101	0.106	0.066
Cash	0.129	0.014	0.129	0.001	0.130	0.050	0.130	0.002

58

Table 10
Covenant Violations

The table reports results from panel regressions on simulated data relating the probability of a covenant violation to the state variables of the model, namely profitability, net worth, and capital. The reported coefficients are averages from 20 simulations of 50 firms for 100 time periods. The parameters are set to the estimated values of Table 1. "Net Worth" refers to the baseline model augmented with a net worth covenant, "Debt/EBITDA Cov. Only" to the baseline model augmented with a Debt/EBITDA covenant, and "Both Cov." to the baseline model augmented with the two aforementioned covenants. The thresholds for the net worth and to the Debt/EBITDA covenants are set to the values of 0.7 and 4. Covenant violations result in the impossibility to draw additional liquidity from the credit line. Columns 1 and 2 refer to the probability that the net worth covenant is violated, columns 3 and 4 to the probability that the Debt/EBITDA covenant is violated, and columns 5 and 6 to the probability that the two covenants are simultaneously violated ("Both Cov.").

| | Probability of Covenant Violation {0,1} | | | | | |
| | Net Worth Cov. | | Debt/EBITDA Cov. | | Both Cov. | |
	(1)	(2)	(3)	(4)	(5)	(6)
Profitability	-0.806	-0.236	-0.934	-0.579	-0.523	-0.195
	(-21.297)	(-6.298)	(-25.265)	(-14.810)	(-17.880)	(-6.386)
Net Worth		-0.015		-0.009		-0.008
		(-35.961)		(-21.433)		(-25.550)
Capital		0.016		0.007		0.009
		(3.315)		(1.498)		(2.416)

59

Table 11
Liquidity Dynamics

The table reports pairwise time-series correlations among changes in drawn credit, changes in cash, debt issues, equity issues, investment, and operating income. The column denoted as 'Data' is based on a sample of nonfinancial, unregulated firms from the annual COMPUSTAT and Capital IQ datasets. The column denoted as 'Model' is based on a simulated panel of 1000 firms for 100 years. All correlations are equally-weighted averages of correlations for individual firms. All variables are scaled by total assets and are winsorized at the 1 percent level. The sample period is from 2002 to 2011.

Correlation	Data	Model
Change in Drawn Credit and Investment	0.189	0.062
Change in Drawn Credit and Debt Issuance	-0.156	-0.119
Change in Drawn Credit and Equity Issuance	-0.120	-0.085
Change in Drawn Credit and Change in Cash	-0.017	-0.607
Change in Cash and Investment	-0.165	-0.281
Change in Cash and Debt Issuance	-0.022	-0.035
Change in Cash and Equity Issuance	0.106	0.165
Debt Issuance and Investment	0.686	0.151
Debt Issuance and Equity Issuance	0.106	0.314
Operating Income and Investment	0.109	0.110
Operating Income and Drawn Credit	0.017	0.158
Operating Income and Equity Issuance	-0.170	-0.467

Figure 1
Optimal Policies: Investment, Payouts, Liquidity, and Equity Value

The figure illustrates firms' optimal policies as a function of current net worth w_{it} (Panels A-E) and current capital stock k_{it} (Panels F-J). In panels D and I, the solid blue line shows the credit line limit, the solid black line shows drawn credit for the one state up the current state, the dashed black line shows drawn credit for the current state, and the dashed-dotted black line shows drawn credit for one state down the current state. Parameters are set to the values in the baseline estimation of table 1.
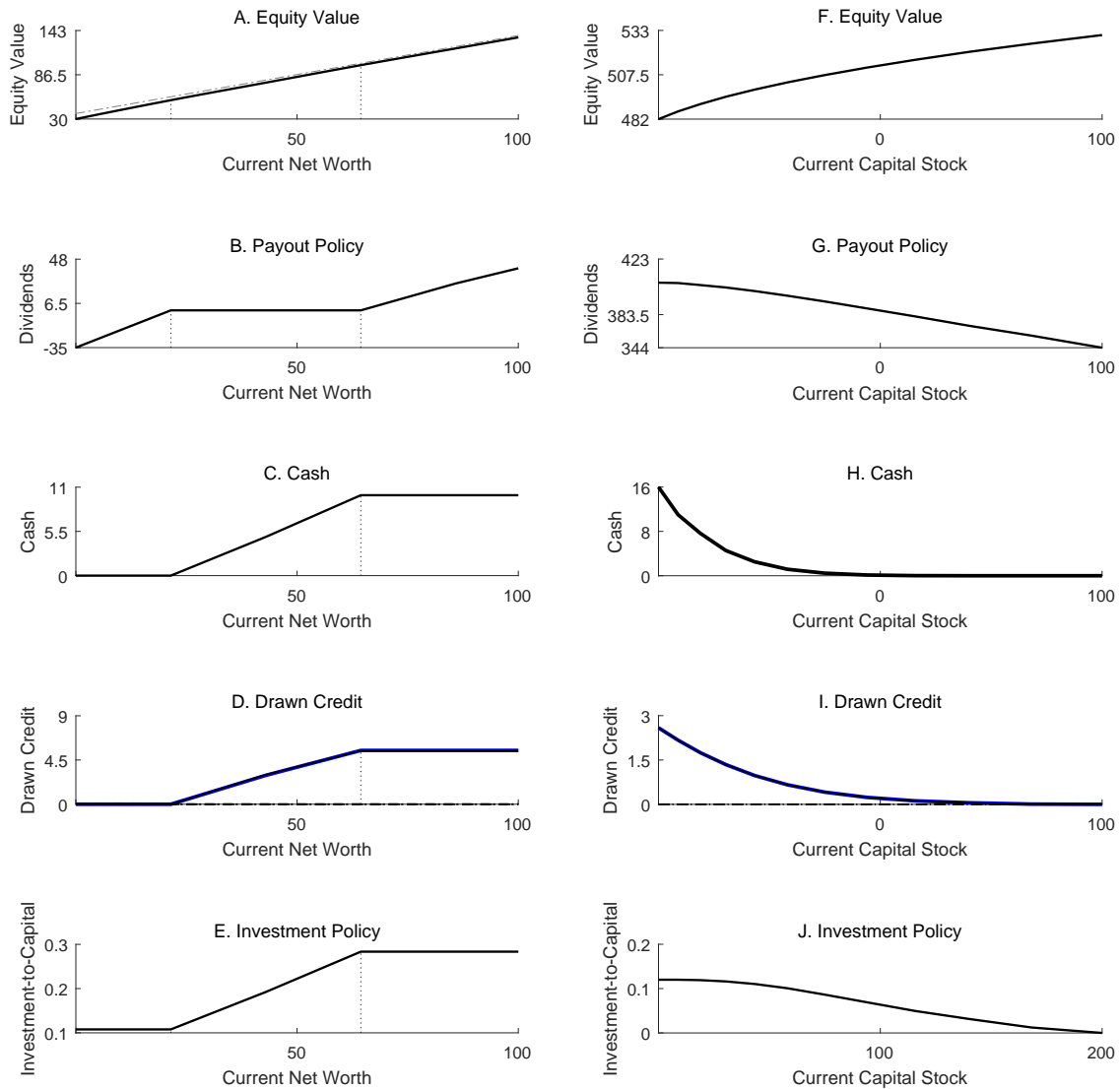
Figure 2
Credit Line Usage with Persistent Investment Opportunities

The figure depicts drawn credit in high, medium and low cash flow states as a fraction of the total drawn credit in all states as a function of current profitability z_{it}. Panel A refers to the case of no persistence in investment opportunities ($\rho_z = 0$), panel B to the case of moderate persistence in investment opportunities $\rho_z = 0.3$ and panel C to the case of high persistence in investment opportunities ($\rho_z = 0.6$). The solid line refers to the highest future cash flow states (top tercile), the dashed line refers to the middle future cash flow states (middle tercile), and the dashed-dotted black line refers to the lowest future cash flow state (bottom tercile). Parameters are set to the values in the baseline estimation of table 1.
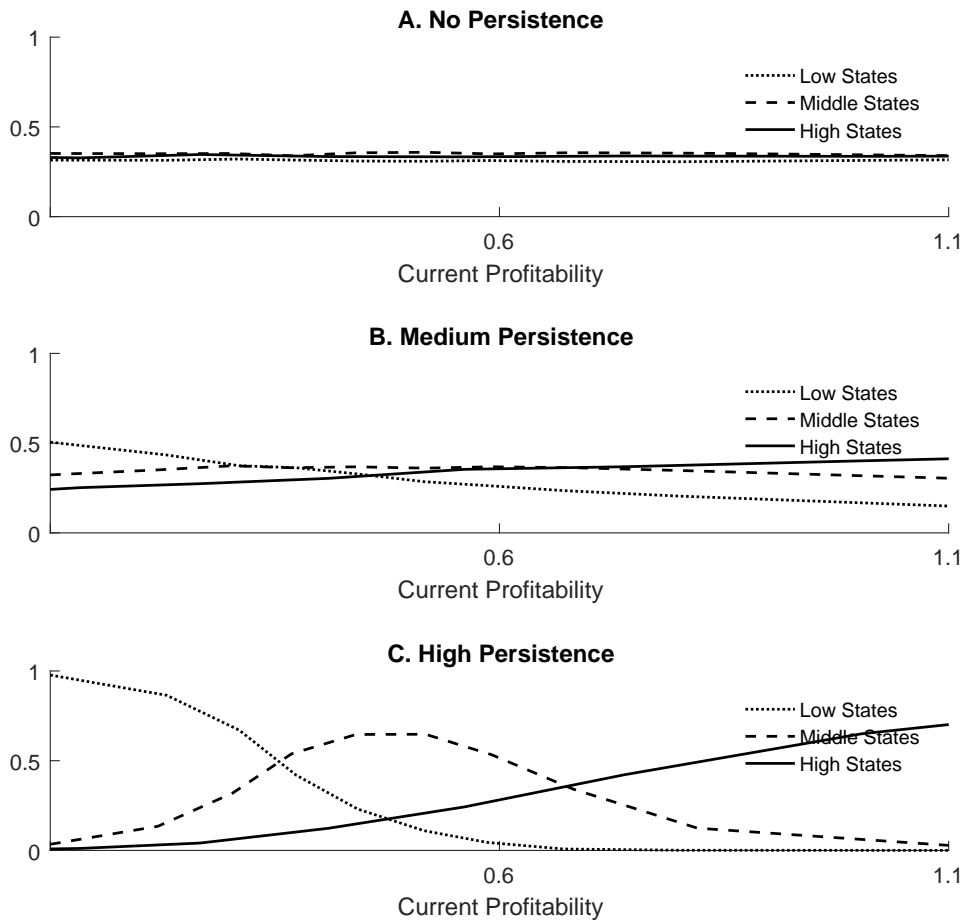
Figure 3
Comparative Statics - Technology

Figure 3 depicts the relation between the curvature of the profit function, α, the fixed production cost, f, the serial correlation of $ln(z)$, ρ_z, and the standard deviation of the innovation of $ln(z)$, σ_z, and i) the cash-to-asset ratio, ii) the fraction of undrawn credit over the credit line limit, and iii) the leverage ratio.
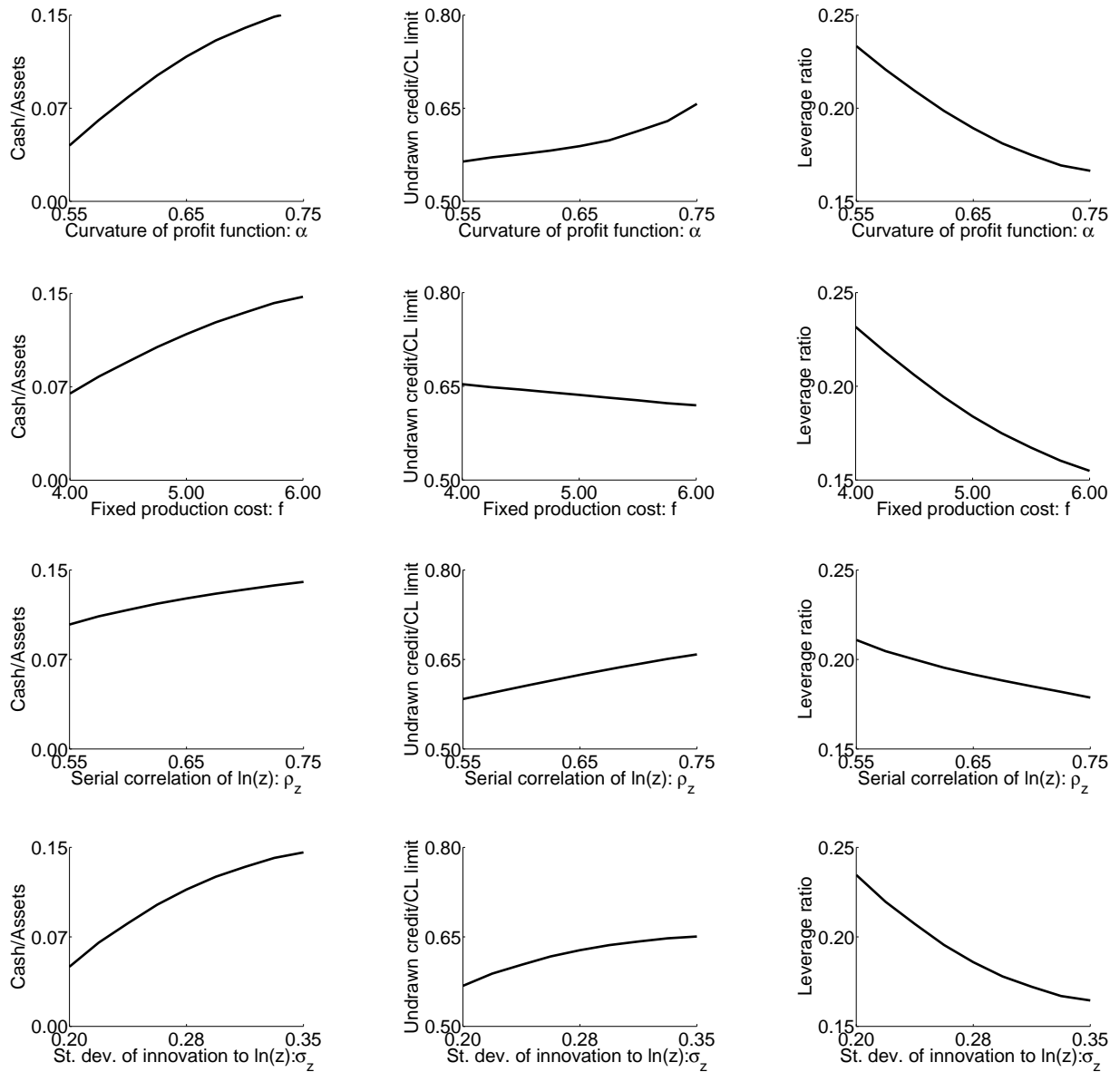
Figure 4
Comparative Statics - Agency, Investment and Financing

Figure 4 depicts the relation between the capital adjustment cost, ψ, the equity flotation cost, λ, the agency cost parameter, γ, the credit line fee parameter, ξ, and debt capacity θ, and i) the cash-to-asset ratio, ii) the fraction of undrawn credit over the credit line limit, and iii) the leverage ratio.